PILOT-MODEL MEASUREMENTS OF PILOT RESPONSES IN A LATERAL-DIRECTIONAL CONTROL TASK

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Pilot response during an aircraft bank-angle compensatory control task has been measured by using an adaptive modeling technique. The results show that in the main control loop, which is the bank angle to aileron command loop, the pilot response is the same as that measured previously in single-input, single-output systems. The results also show that the pilot uses a rudder to aileron control coordination that cancels up to 80 percent of the vehicle yawing moment due to aileron deflection.

### Key Words
- Lateral-directional stability and control
- Pilot models
- Handling qualities
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SUMMARY

Pilot response during an aircraft bank-angle compensatory control task has been measured by using an adaptive modeling technique. Two different aircraft configurations were tested with a fixed-based cockpit. One configuration had no yawing moment due to aileron deflection and was easy to control; the other configuration had adverse yaw and was difficult to control. Measurements were made to determine the manner in which the pilot combined the aileron and rudder control.

The results of the study showed that in the main bank angle to aileron control loop the pilots responded in a manner similar to that measured in previous tests of single-loop control tasks. The pilots used the rudder control to cancel up to 80 percent of the yawing moment due to aileron deflection.

INTRODUCTION

This report presents measurements of pilot response in coupled control tasks. In previous reports, references 1 and 2, pilot responses in single-loop control tasks (that is, single-input, single-output tasks) were reported. These reports covered tasks which correspond to partial aircraft control tasks such as pitch tracking only or bank-angle control only. Other previous reports, references 3 and 4, have covered multiaxis tasks (multiple outputs with a separate control for each output), which correspond to aircraft control tasks such as combined pitch and roll control. References 3, 5, and 6 have described pilot response in multiloop control tasks (that is, tasks with multiple inputs but only one control output). These tasks correspond to aircraft control tasks such as glide-slope control or helicopter-hover control. In contrast to these efforts, the present investigation presents pilot response on multiple-input, multiple-output control tasks. In particular, lateral aircraft control in which there is both coupling in the aircraft response and adverse yaw due to roll control is studied. In reference 4, similar control tasks were studied in which, however, there was only control cross coupling, and very strong cross coupling at that. The task was, therefore, not directly applicable to aircraft lateral control. Aircraft lateral-control tasks were also studied in reference 7; however, no
usable pilot model resulted from reference 7. The present study attempts to provide clear and descriptive definitions of pilot response so as to provide a usable pilot model for use in aircraft lateral-control studies.

SYMBOLS

g gravity, 9.81 m/sec$^2$

$I_x, I_y, I_z$ rolling, pitching, and yawing moments of inertia, kg-m$^2$

$I_{xz}$ product of inertia, kg-m$^2$

$K$ gain

$K$ gain

$K_\phi$ pilot-model static gain

$L$ rolling moment, N-m

$L_i' = \left(1 - \frac{I_{xz}^2}{I_x I_z}\right)^{-1} \left(L_i + \frac{I_{xz}}{I_x} N_i\right)$ where $i = \beta, p, r, \delta_a, \delta_r$

$L_p = \frac{1}{I_x} \frac{\partial L}{\partial p}$, per sec-rad

$L_r = \frac{1}{I_x} \frac{\partial L}{\partial r}$, per sec-rad

$L_\beta = \frac{1}{I_x} \frac{\partial L}{\partial \beta}$, per sec$^2$-rad

$L_\delta_a = \frac{1}{I_x} \frac{\partial L}{\partial \delta_a}$, per sec$^2$-rad

$m$ mass, kg

$N$ yawing moment, N-m

$N_i' = \left(1 - \frac{I_{xz}^2}{I_x I_z}\right)^{-1} \left(N_i + \frac{I_{xz}}{I_z} L_i\right)$ where $i = \beta, p, r, \delta_r, \delta_a$

$N_p = \frac{1}{I_z} \frac{\partial N}{\partial p}$, per sec-rad
\( N_r = \frac{1}{I_Z} \frac{\partial N}{\partial r}, \text{ per sec-rad} \)

\( N_\beta = \frac{1}{I_Z} \frac{\partial N}{\partial \beta}, \text{ per sec}^2\text{-rad} \)

\( N_{\delta a} = \frac{1}{I_Z} \frac{\partial N}{\partial \delta a}, \text{ per sec}^2\text{-rad} \)

\( N_{\delta r} = \frac{1}{I_Z} \frac{\partial N}{\partial \delta r}, \text{ per sec}^2\text{-rad} \)

\( p, q, r \) roll, pitch, and yaw rates, rad/sec

\( s \) Laplace variable, per sec

\( T_R \) aircraft roll time constant, sec

\( T_S \) aircraft spiral-mode time constant, sec

\( T_1 \) pilot-model lag time constant, sec

\( T_2 \) pilot-model lead time constant, sec

\( V \) resultant inertial velocity, m/sec

\( Y \) side force, N

\( Y_\beta = \frac{1}{mV} \frac{\partial Y}{\partial \beta}, \text{ per sec-rad} \)

\( \alpha \) incremental angle of attack, rad

\( \alpha_o \) trim angle of attack, rad

\( \beta \) sideslip angle, rad

\( \delta \) control deflection, rad

\( \delta_a, \delta_r \) aileron and rudder control deflection, rad

\( \zeta \) system-response mode damping ratio
\( \zeta_d \) aircraft Dutch roll damping ratio

\( \zeta_\phi \) damping ratio appearing in numerator quadratic of \( \phi/\delta_a \) transfer function

\( \theta, \phi, \psi \) pitch, roll, and heading angle, rad

\( \omega \) system-response mode frequency, rad/sec

\( \omega_d \) aircraft Dutch roll frequency, rad/sec

\( \omega_\phi \) undamped natural frequency appearing in numerator quadratic of \( \phi/\delta_a \) transfer function, rad/sec

Subscripts:

\( c \) command

\( o \) initial or trim value

Dots over symbols indicate derivatives with respect to time. A prime designates a transformation from body axis to principal axis.

**EXPERIMENTAL PROCEDURES**

The pilot responses analyzed in the investigation were obtained in a fixed-base simulator. The signals to this simulator were generated by a digital computer which solved the equations of motion presented in the appendix. In these equations, note that body-axis longitudinal acceleration is set equal to zero. The pilot's controls consisted of an aircraft center control stick and rudder pedals. Preloaded centering springs were incorporated in these controls to minimize hysteresis and provide a force gradient. The control sensitivity (the gearing between the cockpit and the aircraft control surfaces) was adjusted until it was satisfactory to all subjects. The display presented to the pilot consisted of a three-axis attitude indicator set in a typical instrument panel. Although other instruments were present in the panel, they were not used in this investigation.

The subjects used were three experienced NASA test pilots. Each has had several years (up to 20 years for pilot B and 7 years each for pilots M and K) of flight test experience. Also, each of these subjects had participated in several studies on pilot response that were similar in nature to the present study.
ANALYSIS

The method used to obtain the transfer function which describes the pilot's response was the model-matching method presented in reference 1; in this method a random signal is used as a forcing function. In the present study the random signal was obtained from a white noise source, which was filtered with a single first-order filter with a break frequency at 1 radian per second, and then added to the rolling-moment equation for the aircraft. The amplitude of the random signal was adjusted so that the maximum roll angle of the aircraft, with no pilot control, was approximately 60° peak to peak. The pilot's task was to keep bank angle as near zero as possible.

The forcing function was confined to the roll axis because of the intention to measure pilot response for roll control only; therefore, no forcing function was applied to any other axis. Tests in which a forcing function was applied to all axes are described in reference 3. It should also be noted that, even though the simulation involved five degrees of freedom, the pilot was asked to control only the roll angle.

The method assumes a fixed form for the output-input relationship of the pilot model and also assumes that time histories of the human pilot input and output are available. The pilot-model form is

\[
\frac{\text{Output}}{\text{Input}} = \frac{K\phi(1 + T_2s)}{(1 + T_1s)^2}
\]

The input is a displayed quantity such as the angle between the horizontal line on the attitude indicator and a body-axis fixed reference line. The output is the control deflection produced by the pilot. The method minimizes the root-mean-square difference between the real-pilot control deflection and the model-pilot control deflection using an on-line, steepest-descent procedure.

It has been shown, for example in reference 8, that the model can account for between 50 to 80 percent of the variance of the pilot's output. It is assumed that the remaining 20 to 50 percent is noise generated in the pilot, which is not in any way related to the input. Attempts to account for more of this noise by adding additional factors to the pilot model have not been very effective, as shown in reference 9.

Previous investigations to determine pilot response have either involved single-axis tests where there was only one input and one output or multiloop control tasks where there was more than one input but only one output. There was no question in these tests concerning which control output was to be matched. In the present investigation, however, there are two control outputs, aileron and rudder, and at least two possible inputs, roll angle and yawing velocity. Were the pilot moving either of the two controls as a function
of either of the two inputs, four relationships could exist. There is a question, therefore, concerning which output is being prompted by which input; and this is the difference between the present investigation and the previous single-axis, multiaxes, and multiloop control-task investigations. In the present study the model-matching method was used to determine if a relation did exist between aileron deflection and bank angle, rudder deflection and bank angle, and rudder and yawing velocity. Because the possibility of a relation between aileron deflection and yawing velocity is considered to be very remote, this combination was not tested.

In flight tasks, sideslip angle $\beta$ or lateral acceleration would have to be considered as possible inputs. Since, however, the present investigation was conducted with a fixed-base simulator, there was no lateral acceleration and $\beta$ was not displayed on an instrument; therefore, these two quantities were not considered as possible inputs. In some flight tasks the pilot would control heading, but heading control is a complex, multiloop control task. The present study was not intended to be a multiloop analysis. Consequently, the subjects were instructed to ignore any heading change that might occur on the attitude indicator so as to insure that heading would not be an input to the pilots' control output. However, it was not implied that they should not use yaw rate if they found it would improve the roll response.

The analysis proceeded in the following manner. Tasks of 3 minutes duration were conducted. While the model-matching method can identify the parameters in the pilot model in 1 minute, or less, 3-minute tests were conducted to insure that steady-state values were obtained. First, the bank-angle deviation from zero was used as the input to the pilot model, and the steepest descent method was used to determine the values of $K_\phi$, $T_1$, and $T_2$ that provided the best possible match to the aileron-deflection time history. In this way the relation used by the pilot between bank angle and aileron deflection was determined. Next, the model-matching method, using bank angle as the input again, was used to match rudder deflection, thereby determining the relation between bank angle and rudder control. Last, yawing velocity was used as the input, and rudder deflection output was matched.

This same procedure, that is, testing all possible input-output pairs, was used in reference 4. In reference 7 a more elegant method was used which required no prior assumption on the relations that might exist. However, from a practical viewpoint, there are only a few input-output pairs likely to be used by a pilot; consequently, the requirement of making a prior assumption as to which relation should be tested is not restrictive.

Two different aircraft configurations were included in the investigation. One configuration, designated aircraft D, had adverse yawing moment due to aileron deflection; in contrast, the other configuration, designated aircraft E, had none. There were other small differences in the stability derivatives; the stability derivatives are presented in
table I, along with the response characteristics of the two configurations. A variable stability aircraft was used to simulate these two configurations; the results of the flight tests are presented in reference 10. The Cooper-Harper pilot ratings (see ref. 10) for these two configurations are 7 (unacceptable) for aircraft D, the configuration with the adverse yaw, and 2 (satisfactory) for aircraft E, the other configuration. Characteristics of pilot models derived from measured pilot response when controlling these two configurations in a fixed-base simulator in a roll regulation task are presented in the present paper.

The system closed-loop response $\phi/\phi_c$, given in table II, was obtained from the combination of the three aircraft lateral-directional equations (eqs. (1) to (3)) and the single pilot-model equation (eq. (4)) as follows:

$$-\dot{\beta} + Y_\beta \beta - r + \alpha_o p + \frac{\delta}{V} \phi = 0$$

(1)

$$N'_\beta \beta - \dot{r} + N'_r r + N'_p p + N'_{\delta r} \delta_r + N'_{\delta a} \delta_a = 0$$

(2)

$$L'_\beta \beta + L'_r r - \dot{p} + L'_p p + L'_\delta \delta_a = 0$$

(3)

$$\ddot{\delta}_a = -\frac{K_\phi}{T_1} (\phi - \phi_c) - \frac{K\phi T_2 \dot{\phi}}{T_1} - \frac{2\dot{\delta}_a}{T_1} - \frac{\delta_a}{T_1}$$

(4)

When necessary, an additional equation for $\delta_r$ was included. The combined pilot-aircraft equations, when no rudder control is used, result in transfer functions of the form

$$\frac{\phi}{\phi_c} = \frac{K_\phi (1 + T_2 s) L_{\delta a} (s^2 + 2\zeta \omega s + \omega^2)}{(s^2 + 2\zeta_1 \omega_1 s + \omega_1^2)(s^2 + 2\zeta_2 \omega_2 s + \omega_2^2)(s^2 + 2\zeta_3 \omega_3 s + \omega_3^2)}$$

(5)

where the numerical subscripts 1, 2, and 3 used with $\zeta$ and $\omega$ represent first, second, and third modes.

RESULTS

The measured pilot responses are given in table II for both aircraft configurations. By comparing these results it is possible to see the compensation that is added by the pilots to stabilize aircraft D.
In order to interpret the pilot-model coefficients given in table II, it should be noted that if the pilot does not make any control deflection proportional to the time rate of change of the displayed error, then the lead time constant $T_2$ will be zero. Previous tests designed to make the pilot employ the maximum lead possible in single-loop tasks have resulted in measured lead time constants of 1 second. Values of between 0 and 0.2 second therefore represent a low level of effort on the part of the pilot. The numerical values of lag time constant $T_1$ must be interpreted in an entirely different manner. A lag time constant of zero would be physically impossible to achieve since it would require the pilot to completely overcome the inertia of his arm and control stick. The minimum lag time constant that has been measured is 0.03 second. At the other end of the scale a lag time constant of infinity, which would represent a pure integration of the displayed error, is also difficult, but not impossible, to achieve. Tests in which pilots have controlled plants that appear as a pure gain or can be approximated as a gain (such as $K$ or $\frac{K^2}{s^2 + 1.4Ks + K^2}$), which require an integration for good control, have shown that the pilots do provide this integration. However, any deviation from a value of around 0.2 second for the lag time constant is always accompanied by a poor pilot rating.

Table II shows that all of the subjects (pilots B, M, and K) responded to the task of controlling aircraft E with low lead time constants ($T_2 \approx 0.2$) and with an optimal lag time constant, $T_1$, of 0.2 second. Also, all of the control response was confined to the aileron; the rudder was not used. The pilots felt rudder was not needed with aircraft E.

With aircraft D, various amounts of control compensation were applied by the subjects. The compensation added by pilots B and M consisted of increased lead in the $\delta_a/\phi$ response and the injection of some control coordination of rudder to aileron crossfeed. Pilot K used no control coordination; he also eliminated the lead which he had used with aircraft E. These items represent an apparent incorrect adjustment on the part of pilot K for aircraft D because computation has shown that adding control coordination and increasing lead will provide better (that is, tighter) system response. Pilot K did increase his static gain with aircraft D, in comparison with aircraft E.

The measurements of $\delta_a/\phi$ and $\delta_r/\phi$ for aircraft D show that nearly the same lead and lag are used for each of these two transfer functions by each of the two subjects who used the rudder. This is particularly true for pilot M, and less so for pilot B. These results indicate that the rudder and aileron are being used in a coordinated manner by these subjects. Time histories of these cases are shown in figure 1, where the forcing function is the random signal described previously. These time histories show that for pilot M the rudder and aileron are very much in phase and, again, show that this is less true for subject B. The time histories also illustrate that there are instances in which there is no rudder movement corresponding to an aileron movement, but on the average
there is some control coordination being generated and the pilot model measurements reflect this best average response.

The closed-loop pilot-aircraft system response characteristics for $\phi/\phi_c$, obtained by using the measured pilot models in combination with the aircraft equations of motion, are also shown in table II. These results show that pilots B and M, by changing their response when the aircraft response is changed, maintain approximately the same system response. In each configuration there is a fast control mode of motion, which is sometimes oscillatory at about 5 radians per second and sometimes overdamped, so that the roots appear as two nearly equal real roots of about 5. There is also a midfrequency oscillatory mode of motion near the Dutch roll frequency of about 2.5 radians per second. This mode is driven to near zero damping by the pilot loop closures. There is also a low frequency mode of motion — the bank-angle mode of response — which is sometimes well damped, sometimes overdamped, and appears as two nearly equal real roots. For pilot K, the system response is degraded with aircraft D in that the smallest real root is greatly reduced in magnitude to a value of -0.31 radian per second. When compared with values of other systems, this -0.31 value indicates a slow system response. An approximation was used in determining the closed-loop system characteristics mentioned previously. It was noted that the lead and lag of the $\delta_a/\phi$ and the $\delta_r/\phi$ pilot models were nearly the same for both pilot B and pilot M. Therefore, the same equation (or transfer function) was used to implement the dynamics for each of these control functions. The gains on the two outputs of this single dynamic function were adjusted to give the desired static gain.

In the representation of the aircraft, both the roll effectiveness of the aileron, $L_{\delta a}$, and the yaw effectiveness of the rudder, $N_{\delta r}$, were purposely set at -1.0. Therefore, the given static gains of the pilot models for $\delta_a/\phi$ and $\delta_r/\phi$ represent the total forward loop control effectiveness of the respective control channels. With pilot B, the results therefore indicate that 50 percent of the yawing moment due to aileron was canceled with use of the rudder, and with pilot M, 80 percent was canceled. This fact is deduced from the results which show that the ratios of the static gains on the $\delta_a/\phi$ and $\delta_r/\phi$ pilot models were 0.05 and 0.08 for the two subjects, whereas the ratio of $N_{\delta a}/L_{\delta a}$ for aircraft D was -0.10.

In order to examine the importance of the control coordination supplied by pilots B and M, the system characteristics were determined by using the same $\delta_a/\phi$ transfer function in each respective case and leaving out the $\delta_r/\phi$ control coordination. The configuration used in this system was aircraft D, of course. The results are shown in table III. By comparing the results with those in which control coordination was included, it is possible to obtain an idea of the system response that the pilot is trying to achieve. For pilot M, leaving out the control cross coupling results in a real root of very small magnitude, -0.089 radian per second. For pilot B, leaving out the control coordination
results in a slower response than is obtained when the control coordination was included. As noted before, pilot K did not use control coordination but did adjust his static gain and, therefore, achieved a system response with a real root of -0.31 radian per second. It appears from these results that the pilots will make adjustments in their response, whether it is by adding lead or by control coordination or by increasing the static gain, so as to insure a real root more negative than -0.3 radian per second.

Magnitude of pilot-aircraft system real roots has been a matter of some concern in past studies that attempted to predict aircraft handling qualities. For example, the prediction of longitudinal-handling-qualities ratings depended in part on specifying a real root more negative than -0.38 radian per second (a time constant less than 2.6 seconds). This specification insured that only a short time would elapse before the pilot would have knowledge of the system response. The handling-qualities rating was determined as a function of the pilot compensation required to achieve this minimum system time characteristic. In a study of lateral handling qualities, the indication was that a real root of -0.3 radian per second should be part of the system-response specification, although the requirement was not needed in the cases studied. It is for these reasons that the result of having system real roots more negative than -0.31 radian per second obtained in the present study is given emphasis. This result is further evidence that pilots desire a certain minimum system time characteristic.

Reference 4 examined cases which were similar in some respects to those studied here. In reference 4, the airplane model contained a yaw due to roll control cross coupling equal to the roll control effectiveness, \( \frac{N_{c_a}}{L_{c_a}} = -1.0 \). In the present study the ratio \( \frac{N_{c_a}}{L_{c_a}} \) was only -0.1. In reference 4, only control coupling was present; whereas, in the present study, all of the pertinent aerodynamics were included, which results in output variable couplings as well as control coupling. The results of reference 4 showed that the pilot completely canceled the control coupling. With the more subtle coupling that exists in the present cases, the pilots canceled from 80 percent of the aircraft control coupling for pilot M to 0 percent for pilot K.

The relationship between yawing velocity and rudder deflection, \( \frac{\delta_r}{r} \), was also measured. It was felt that the pilot might be responding to yawing velocity with rudder control to improve the damping of the system. However, the measurements showed that there was no such response. The static gains of the \( \frac{\delta_r}{r} \) response were zero.

In addition to the identification of the pilot models using the random forcing functions, time histories of the response of the pilots to step bank-angle inputs were also obtained. While it cannot be assumed that pilots will respond to a step input with exactly the same kind of control command (same transfer function) they would use in responding to a continuous random signal, the step response of each pilot should have some characteristics in
common with his response to the random input. The step responses of each pilot while controlling aircraft E and then aircraft D are shown in figure 2. It can be seen from this figure that pilot K generates a very slow response with aircraft D. This result is in agreement with the closed-loop system response characteristics, which show that pilot K has the smallest (in magnitude) real root, -0.31 radian per second. It can also be seen from the time histories that for all of the subjects the midfrequency mode has very low damping in most cases and that the low frequency mode is either well damped or over-damped, which, again, is in agreement with the computed closed-loop response.

CONCLUDING REMARKS

The results of tests to measure pilot response for roll control only in aircraft control-coupling systems show that (1) pilots respond in the main control loop, bank angle to aileron deflection, in a manner similar to that used in single-loop control tasks and (2) pilots use the rudder control to cancel up to 80 percent of the yawing moment due to aileron control. No other use of the rudder was measured.

These results can be used to add an additional refinement to the pilot-model data reported in the past. Pilot models have been used to provide preliminary design information on pilot-aircraft system response and to aid in the understanding of simulator and flight test results. By including the bank angle to rudder cross coupling, a better prediction of lateral response can be obtained.

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APPENDIX

EQUATIONS OF MOTION

The equations of motion used in the study are given in this appendix as follows:

\[ a_x = 0 \]
\[ a_y = Y_{\beta \beta} V_{xo} \]
\[ a_z = (L_{\alpha \alpha} + L_o)V_{xo} \]
\[ \dot{p} = L^{'p} p + L^{'\beta \beta} + L^{'r} r + L^{'6a} \delta_a \]
\[ \dot{q} = M_{\alpha \alpha} + M_{qq} + M_{ee} \delta_e \]
\[ \dot{r} = N^{'r} r + N^{'\beta \beta} + N^{'p} p + N^{'r} \delta_r + N^{'6a} \delta_a \]
\[ \phi = p + q \sin \phi \tan \theta + r \cos \phi \tan \theta \]
\[ \theta = q \cos \phi - r \sin \phi \]
\[ \psi = r \cos \phi + q \sin \phi \]

\[ l_1 = \cos \psi \cos \theta \]
\[ l_2 = \sin \psi \cos \theta \]
\[ l_3 = -\sin \theta \]
\[ m_1 = \cos \psi \sin \theta \sin \phi - \sin \psi \cos \phi \]
\[ m_2 = \sin \psi \sin \theta \sin \phi + \cos \psi \cos \phi \]
\[ m_3 = \cos \theta \sin \phi \]
\[ n_1 = \cos \psi \sin \theta \cos \phi + \sin \psi \sin \phi \]
\[ n_2 = \sin \psi \sin \theta \cos \phi - \cos \psi \sin \phi \]
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\[ n_3 = \cos \theta \cos \phi \]

\[ \dot{V}_x = l_1 a_x + m_1 a_y + n_1 a_z \]

\[ \dot{V}_y = l_2 a_x + m_2 a_y + n_2 a_z \]

\[ \dot{V}_z = l_3 a_x + m_3 a_y + n_3 a_z + g \]

\[ u = l_1 V_x + l_2 V_y + l_3 V_z \]

\[ v = m_1 V_x + m_2 V_y + m_3 V_z \]

\[ w = n_1 V_x + n_2 V_y + n_3 V_z \]

\[ V = \left( V_x^2 + V_y^2 + V_z^2 \right)^{1/2} \]

\[ \alpha = \tan^{-1} \frac{w}{u} \]

\[ \beta = \sin^{-1} \frac{v}{V} \]

where

\[ a_x, a_y, a_z \quad \text{body-axis acceleration, m/sec}^2 \]

\[ \phi, \theta, \psi \quad \text{Euler angles, rad} \]

\[ l, m, n \quad \text{direction cosines} \]

\[ V_x, V_y, V_z \quad \text{Earth-axis velocity, m/sec} \]

\[ u, v, w \quad \text{body-axis velocity, m/sec} \]

\[ V_{xo} = 188 \text{ m/sec} \]

\[ L_\alpha = \frac{\rho V S}{2m} C_{L\alpha} \quad (1.3 \text{ per sec}) \]

\[ M_\alpha = \frac{\rho V^2 S c}{2I_Y} C_{m\alpha} \quad (-7.79 \text{ per sec}^2) \]

\[ M_q = \frac{\rho V S c^2}{4I_Y} C_{m_q} \quad (-1.70 \text{ per sec}) \]
APPENDIX

\[ M_{\infty} = \frac{\rho V^2}{2I_Y} C_{m_{\infty}} \]  

\[ L_0 = \frac{g}{V} \]  

and where

\[ \delta_e \]  
elaborator control deflection, rad

\[ \rho \]  
air density, kg/m³

\[ S \]  
wing area, m²

\[ C_{L_{\alpha}} \]  
nondimensional lift-curve slope

\[ c \]  
mean aerodynamic chord

\[ C_{m_{\alpha}} \]  
nondimensional pitching-moment slope

\[ C_{m_q} \]  
nondimensional pitching-moment slope due to pitching velocity

\[ C_{m_{\infty}} \]  
nondimensional pitching-moment slope due to elevator deflection

It should be noted that linear aerodynamics and inertial reactions are used and that altitude density \( \rho \) and dynamic pressure \( \frac{1}{2} \rho V_x^2 \) are kept constant in the problem. Those factors provide a constant vehicle response for the approximately 1 minute of test time required to obtain the pilot coefficients in the model-matching method. It is assumed that even the normal type of nonlinear relations would not affect the results because of the small magnitude of maneuvers involved in the task. The main factors of importance were the linear coupling between the system variables and the nonlinear effects that involved gravity.
REFERENCES


TABLE I.- STABILITY DERIVATIVES AND RESPONSE CHARACTERISTICS
OF TWO AIRCRAFT CONFIGURATIONS

<table>
<thead>
<tr>
<th>Stability derivative</th>
<th>Aircraft E</th>
<th>Aircraft D</th>
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<tbody>
<tr>
<td>( L_{\beta} )</td>
<td>-42.14</td>
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<tr>
<td>( L_p )</td>
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<td>( L_T )</td>
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<td>( N_{\beta} )</td>
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<td>( Y_{\beta} )</td>
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<td>-1.0</td>
<td>-1.0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Response characteristic</th>
<th>Aircraft E</th>
<th>Aircraft D</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \omega_d )</td>
<td>2.49</td>
<td>2.49</td>
</tr>
<tr>
<td>( \zeta_d )</td>
<td>0.10</td>
<td>0.10</td>
</tr>
<tr>
<td>( \phi/\beta )</td>
<td>4.8</td>
<td>5.2</td>
</tr>
<tr>
<td>( T_R )</td>
<td>0.37</td>
<td>0.40</td>
</tr>
<tr>
<td>( T_S )</td>
<td>987</td>
<td>997</td>
</tr>
<tr>
<td>( \omega_{\phi} )</td>
<td>2.34</td>
<td>1.198</td>
</tr>
<tr>
<td>( \zeta_{\phi} )</td>
<td>0.093</td>
<td>0.066</td>
</tr>
<tr>
<td>( \omega_{\phi}/\omega_d )</td>
<td>0.94</td>
<td>0.48</td>
</tr>
<tr>
<td>( \zeta_{\phi}/\zeta_d )</td>
<td>0.93</td>
<td>0.66</td>
</tr>
</tbody>
</table>
TABLE II. - MEASURED PILOT TRANSFER-FUNCTION COEFFICIENTS AND
CLOSED-LOOP SYSTEM CHARACTERISTICS

<table>
<thead>
<tr>
<th>Configuration</th>
<th>Function</th>
<th>Pilot-model coefficient</th>
<th>System characteristics</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$K_{\phi}$</td>
<td>$T_1$, sec</td>
</tr>
<tr>
<td>Aircraft E</td>
<td>$\delta_a/\phi$</td>
<td>3.33</td>
<td>0.22</td>
</tr>
<tr>
<td>Aircraft D</td>
<td>$\delta_a/\phi$</td>
<td>5.0</td>
<td>0.25</td>
</tr>
<tr>
<td></td>
<td>$\delta_r/\phi$</td>
<td>0.25</td>
<td>0.20</td>
</tr>
<tr>
<td></td>
<td>$\delta_r/r$</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Aircraft E</td>
<td>$\delta_a/\phi$</td>
<td>2.56</td>
<td>0.21</td>
</tr>
<tr>
<td>Aircraft D</td>
<td>$\delta_a/\phi$</td>
<td>1.78</td>
<td>0.18</td>
</tr>
<tr>
<td></td>
<td>$\delta_r/\phi$</td>
<td>0.14</td>
<td>0.20</td>
</tr>
<tr>
<td></td>
<td>$\delta_r/r$</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Aircraft E</td>
<td>$\delta_a/\phi$</td>
<td>1.2</td>
<td>0.20</td>
</tr>
<tr>
<td>Aircraft D</td>
<td>$\delta_a/\phi$</td>
<td>3.5</td>
<td>0.22</td>
</tr>
<tr>
<td></td>
<td>$\delta_r/\phi$</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\delta_r/r$</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>
### TABLE III - COMPARISON OF SYSTEM CHARACTERISTICS WITH AND WITHOUT PILOT CONTROL CROSS COUPLING FOR PILOTS B AND M WITH AIRCRAFT D

<table>
<thead>
<tr>
<th>Coupling</th>
<th>Control mode of motion</th>
<th>Dutch roll mode of motion</th>
<th>Bank-angle mode of motion</th>
</tr>
</thead>
<tbody>
<tr>
<td>With $\delta_r/\phi$</td>
<td>Real roots: -6.17, -2.65</td>
<td>$\omega = 2.76; \ \zeta = 0.065$</td>
<td>$\omega = 1.45; \ \zeta = 0.515$</td>
</tr>
<tr>
<td>Without $\delta_r/\phi$</td>
<td>Real roots: -6.1, -2.6</td>
<td>$\omega = 2.98; \ \zeta = 0.064$</td>
<td>Real roots: -0.87, -0.64</td>
</tr>
<tr>
<td>Pilot M</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>With $\delta_r/\phi$</td>
<td>Real roots: -6.40, -5.25</td>
<td>$\omega = 2.45; \ \zeta = 0.058$</td>
<td>Real roots: -1.26, -0.963</td>
</tr>
<tr>
<td>Without $\delta_r/\phi$</td>
<td>Real roots: -6.3, -5.25</td>
<td>$\omega = 2.58; \ \zeta = 0.030$</td>
<td>Real roots: -2.03, -0.089</td>
</tr>
</tbody>
</table>
Figure 1.- Time histories of pilot-aircraft system response with a random rolling-moment forcing function.
(a) Response with aircraft E.

Figure 2. - Response of pilot aircraft system to a step roll command.
(b) Response with aircraft D.

Figure 2. - Concluded.
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—National Aeronautics and Space Act of 1958

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