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TIDAL ANALYSIS OF MET ROCKET WIND DATA

Final Report

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J. Bedinger
E. Constantinides

Prepared by
GCA CORPORATION
GCA/TECHNOLOGY DIVISION
Bedford, Massachusetts

for
NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
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ABSTRACT

A new method of analyzing Met Rocket wind data is described. Modern tidal theory and specialized analytical techniques are used to resolve specific tidal modes and prevailing components in observed wind data. The objective is to formulate a representation of the wind which is continuous in both space and time. Such a representation allows direct comparison with theory, allows the derivation and other quantities such as temperature and pressure which in turn may be compared with observed values, and allows the formation of a wind model which extends over a broader range of space and time.

The current results for a single series of measurements indicates that significant diurnal tidal modes with wavelengths of 10 and 7 km are present in the data and are resolved by the analytical technique. The application to a wider data sample is required before the full potential of method can be determined.
SECTION I

INTRODUCTION

The primary purpose of this contract is the development of a new method for the analysis of upper atmospheric wind and temperature data, and the application of these methods to data obtained by the standard Meteorological Rocket systems. The method embraces the premise that the most useful form of the data is one which allows a continuous representation in both space and time. In order to achieve this representation, the wind data must be resolved into distinct components, each component having a distinct set of temporal and spatial characteristics. Such a resolution has predictive value, in the sense that it can predict the winds at a different place and time, and also in the sense that it can infer values of other observables, such as temperature and pressure. Furthermore, such a resolution allows direct comparison with theoretical predictions.

The analysis utilizes a data sample consisting of a series of wind measurements taken over a time interval of 24 (or more) hours, and covering a substantial portion of the altitude range from 20 to 80 kilometers. The tools of the analysis are (1) the theory of atmospheric motions in the stratosphere and mesosphere, and (2) an analytical procedure specifically suited to the Met Rocket data from Wallops Island. The method requires the resolution of the observed winds into distinct tidal modes as well as prevailing components. Each tidal mode has a characteristic period as well as spatial propagation characteristics. The number of measurements is always limited. Accordingly, harmonic analysis can resolve the observed winds according to their temporal characteristics only imperfectly. Further, there are several modes present with the same temporal period. Thus, a method to include the vertical propagation characteristics of each mode was incorporated into the analysis. The method removes modes with short wavelengths, and the remaining long-wavelength modes are then defined by use of harmonic analysis. The method relies heavily on the general results of tidal theory, and the analytical procedure is based on temporal and vertical "filtering" operations in order to resolve the observed measurements according to their temporal and spatial characteristics.

A brief survey of past observations and analysis of similar data are contained in Section II of this report. The analytical method is described in Section III and the current results are presented in Section IV.
SECTION II

PREVIOUSLY REPORTED TIDAL ANALYSIS

The presence of tidal forces in the atmosphere has long been recognized from the observations of surface pressure changes from which it was determined also that the driving force is thermal since the solar tide is much larger than the lunar. The semidiurnal component of the observed solar tide has a larger amplitude than the diurnal. For many years, this was the principal support for the "resonance theory" which assumed that the atmosphere had a natural resonance period close to 12 hours. During the past few years, however, it has been determined that the motions are actually forced oscillations in which the amplitudes are affected by varying efficiencies of the coupling to the energy source. It is now generally accepted that the thermally forced tides result from absorption of ultraviolet solar insolation by ozone in the stratosphere and mesosphere and from infrared absorption by water vapor in the troposphere. Contemporary tidal theory and limited observational data indicate that the amplitudes of tidal components increase from a few centimeters per second at the surface to several meters per second around 50 km and become the dominant component of the wind field above 100 km with amplitudes of over 100 meters per second. The development of the theory was hindered by the fact of the equations are nonlinear which prohibit a complete global solution and also by the lack of observational data to furnish guidance for determination of useful approximations.

The data are difficult to analyze because the atmospheric wind system has several components that vary greatly in character and result from different sources. These differences appear in both size and time scales. Some have actually been observed and identified; others have been proposed as an explanation of observational results or on purely theoretical grounds. The shortest period, smallest size variations are in the latter group. The smallest sizes are usually categorized as turbulence and are generally attributed to nonlinearity of flow in high shear regions. The next larger scale size is often attributed to gravity waves that may result from a variety of sources including many kinds of disturbances in the atmosphere. Although these two smallest size components have not been unambiguously identified, small variations have been reported by many observers and most treatises on atmospheric dynamics include them in some form.

The next larger scale of variations is thermally driven tides with periods of 24 hours and its harmonics. Some of these motions have been identified and related through the appropriate theory to the energy source that produces them; i.e., absorption of solar insolation by ozone and water vapor. Diurnal and semi-diurnal components have been reported in the Met Rocket data and several other harmonics have been reported from other sources.

Planetary scale waves with periods of several days have been observed also. These are usually attributed to large disturbances in the atmosphere such as magnetic storms.
The largest period variations are seasonal, semiannual, annual, and quasi-biennial oscillation. The source of the 2-year oscillation is not yet known but the major seasonal changes are well described by the thermal wind equation in which the zonal wind component is defined by the meridional temperature gradient.

The observational data have been subjected to harmonic analyses in order to detect and evaluate components with periods of 24 and 12 hours. The data have been analyzed from hourly averages over varying periods of several days to months and from series of sequential measurements over a day or two. For short periods of observation, the results will be affected by the short period irregular components such as interval gravity waves or short period planetary waves. For larger observing periods, planetary waves and seasonal variations produce uncertainties. In all cases, the best fit parameters were determined by a purely mathematical process and then compared to theory. Early reports by Smith (1960) who found large diurnal and semi-diurnal components from 23 observations in Nevada and Johnston Island, and by Lenhard (1963) who reported similar components with amplitudes of 10 m sec\(^{-1}\) at 60 km at Eglin, Florida were compared to the resonance theory of atmospheric tides. The observed phases were not compatible with the predictions of that theory.

Since the introduction of the forced oscillation theory and the acquisition of more data, a large number of tidal analyses of Met Rocket data have been reported.

Miers (1965) reported results from three series of rockets launched every 2 hours throughout the day from White Sands, New Mexico and from Eglin, Florida. Diurnal components with amplitudes from 10 to 20 m sec\(^{-1}\) and semi-diurnal component with amplitudes of 3 to 5 m sec\(^{-1}\) were determined. Reed et al. (1966) reported diurnal amplitudes of 5 to 10 m sec\(^{-1}\) near the stratosphere from hourly values averaged over several months.

Diurnal oscillations with amplitude of 12 m sec\(^{-1}\) around 56 km were determined from 16 observations within 51 hours at White Sands by Beyers and Miers (1966). An associated temperature variation of 80°C was also reported but attempts to relate the wind and temperature variations through the generalized thermal wind equation were unsuccessful.

The presence of a semidiurnal tide with amplitude of 6 m sec\(^{-1}\) at 60 km was confirmed by Reed (1969), and Beyers and Miers (1968) confirm tidal components in the equatorial stratosphere from a series of 24 measurements within a 2-day period over Ascension Island.

In summary, it appears that tidal components with periods of 12 and 24 hours, are consistently present in the middle atmosphere but the relation of these components to the variation of temperature and pressure and to the appropriate thermal driving forces remains to be determined. The new analytical procedure which is described in Section III is specifically designed to determine these relations.
SECTION III

ANALYTICAL METHOD

A. INTRODUCTION

The primary purpose of the analysis of wind data is to obtain a representation of the winds that is continuous in space and time. In order to achieve a useful representation, the wind data must be resolved into distinct components, each component having a distinct set of temporal and spatial characteristics. Such a resolution has predictive value, in the sense that it can predict the winds at a different place and time, and also in the sense that it can infer values of other observables, such as temperature and pressure. Furthermore, such a resolution allows direct comparison with theoretical predictions.

The analysis suggested in this proposal assumes a data sample consisting of a series of wind measurements taken over a time interval of 24 (or more) hours, and covering a substantial portion of the altitude range from 20 to 80 kilometers. The tools of the analysis are (1) the theory of atmospheric motions in the stratosphere and mesosphere, and (2) an analytical procedure specifically suited to the Met Rocket data from Wallops Island. A brief account of pertinent theoretical results and of the analytical procedure is given below.

B. THEORETICAL RESULTS

In the altitude region under consideration nonlinear effects can be neglected. Accordingly the total wind is represented by a linear superposition of distinct contributions. These contributions are broken down into three general categories: (1) large-scale, long-period motions, (2) tidal oscillations, and (3) small-scale, short-period motions. The characteristics of these three types of motions are as follows:

1. Large-Scale, Long-Period Motions

Large-scale, long-period motions vary slowly or not at all during the period of observation. In other words, they are nonoscillatory with a characteristic time in excess of 24 hours. Their variation with altitude has a characteristic length in excess of 10 kilometers. The thermal wind and planetary waves are included in this category. Although these have been studied theoretically, it is not considered feasible at this time to determine their properties in detail from data confined to a single observing site.

2. Tidal Oscillations

The dominant tidal motions in this region of the atmosphere are the diurnal oscillations (with a period of 24 hours), and the semidiurnal oscillations (with a period of 12 hours). No evidence has been presented to date of the existence, to any significant extent, of tidal oscillations with
shorter periods. Pertinent features of five diurnal modes and three semidiurnal 
modes are listed in Table 1, which is based on the results of Lindzen (1967, 
1968). All diurnal modes have the same temporal period (24 hours), but dis-
trict variations with respect to altitude and latitude. Two types of diurnal 
modes are listed in Table 1: trapped and propagating. Mathematically, trapped 
and propagating modes are associated with negative and positive, respectively, 
"equivalent depths." ("Equivalent depths" are the eigenvalues of the tidal 
equation.) Physically, for trapped modes, the response of the atmosphere is 
confined to the immediate vicinity of the regions of excitation. Propagating 
modes are so-named because they propagate energy away from the region excita-
tion. The term "phase" in Table 1 denotes the hour of maximum. The trapped 
diurnal modes, whose phase does not vary with altitude, represent motions which 
attain their maximum value simultaneously at all altitudes. There is evidence 
(Lindzen and Blake, 1970) that the trapped diurnal modes are not significant 
below the mesopause.

The following characteristics are true of each mode listed in 
Table 1. The amplitude contains a factor which grows with altitude as \((\rho_0/\rho)^{1/2}\) 
where \(\rho\) is the local density and \(\rho_0\) is the ground density. This is essentially 
an exponential growth. The relative amplitudes of the eastward and northward 
wind components are a function of latitude only, and are independent of alti-
tude. Finally, the northward component leads the eastward component by a 
quarter of a period (i.e., the northward component achieves its maximum a 
quarter of a period before the eastward component). All of the above charac-
teristics are independent of the nature of the exciting source. For the known 
sources of excitation (due to absorption by ozone and water vapor), the relative 
magnitudes of the different modes are known. Mathematical formulations of 
the stated characteristics are presented in the following subsection.

3. Small-Scale, Short-Period Motions

Small-scale, short-period oscillations may be representative of 
internal gravity waves or random motions. Since the continuous spectrum of 
allowed gravity waves is broad in terms of both periods and wavelengths, there 
are no general characteristics of such motions that are pertinent in the present 
context.

C. ANALYTICAL PROCEDURE

1. Previous Analyses

Previous analyses of Met Rocket data have treated data separately 
at each altitude; further, each wind component was analyzed separately. Basi-
cally, the method consisted of removing, in some fashion, a "prevailing" wind 
and a "trend", and then dealing with the residual wind as representing the 
diurnal oscillations (or the diurnal and semidiurnal oscillations together). 
Mathematically this procedure expresses the eastward component of the wind as

\[ u(z,t) = A_1(z) \cos \omega_1 t + B_1(z) \sin \omega_1 t \]
Table 1. Characteristics of some tidal modes

<table>
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<th>Period (hours)</th>
<th>Name and Characteristics</th>
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<td>24</td>
<td>First Trapped Diurnal. Phase does not vary with altitude. Amplitude varies smoothly with altitude. Probably not significant.</td>
</tr>
<tr>
<td>24</td>
<td>Second Trapped Diurnal. Phase does not vary with altitude. Amplitude varies smoothly with altitude. Probably not significant.</td>
</tr>
<tr>
<td>24</td>
<td>First Propagating Diurnal. Approximate wavelength: 28 km. Dominant diurnal mode.</td>
</tr>
<tr>
<td>24</td>
<td>Third Propagating Diurnal. Approximate wavelength: 7 km.</td>
</tr>
<tr>
<td>12</td>
<td>First Semidiurnal. Approximate wavelength: 150 km.</td>
</tr>
<tr>
<td>12</td>
<td>Second Semidiurnal. Approximate wavelength: 54 km.</td>
</tr>
<tr>
<td>12</td>
<td>Third Semidiurnal. Approximate wavelength: 34 km.</td>
</tr>
</tbody>
</table>
\[ + A_2(z) \cos \omega_2 t + B_2(z) \cos \omega_2 t \]
\[ + P_0(z) + P_1(z) t, \]

where

\[ u = \text{eastward component of the wind} \]
\[ \omega_1 = \text{frequency of diurnal oscillation} \]
\[ \omega_2 = \text{frequency of semidiurnal oscillation} \]
\[ z = \text{altitude} \]
\[ t = \text{time} \]
\[ P_0 = \text{prevailing wind} \]
\[ P_1 = \text{"Trend"} \]

and the amplitudes (C) and phases (\( \delta \)) are determined from the equations

\[ C_i^2 = A_i^2 + B_i^2, \quad \delta_i = \frac{1}{\omega_i} \arctan \left( \frac{B_i}{A_i} \right) \quad i = 1, 2. \]

A similar expression pertains to the northward component of the wind. The \( A \)'s, \( B \)'s and \( P \)'s are treated as parameters to be determined by the method of least squares from \( N \) observations

\[ u(t_1), \ldots, u(t_N). \]

Despite the simplicity of this approach, results fairly consistent with theory were obtained by Reed (1966, 1967), Miers (1965), and Bevers and Miers (1966, 1968). These results are shown in Figures 1 and 2 (after Lindzen, 1967).

2. Data Sample and Errors

For an adequate data sample (say nine or more observations over a 24-hour period), the analysis can be improved by taking into account general theoretical results, as well as temporal and spatial "filtering" techniques. Some of the details of those techniques depend on the temporal resolution of the data, and on the ability to ascertain the magnitude of the errors in the data. In general, it is best to acquire the data at evenly spaced intervals.
Figure 1. Altitude distribution of the amplitude of the solar diurnal component of $u$ at 30° latitude for winter, equinoctial and summer conditions. Also shown are some distributions based on observations.
Figure 2. Altitude distribution of the phase of the solar diurnal component of $u$ at 30° latitude for winter, equinoctial and summer conditions. Also shown are some distributions based on observations.
(e.g., at 3-hour intervals for nine observations). A theoretical estimate of the errors in the data is usually obtained from the characteristics of the measuring apparatus as well as from the procedure utilized to translate direct observations (e.g., radar signals) to wind values. Another estimate can be obtained by averaging the wind values over a specified height interval (e.g., half a kilometer) and treating the root-mean-square deviation from the average as the error figure. Owing to the difficulty of obtaining reliable theoretical estimates of errors, the second approach appears preferable. The error figure at each altitude determines the weight to be attached to the wind values at that altitude. For simplicity, consideration of error figures and weights are omitted from the following discussion. The methods of error assessment and data smoothing with the computer codes are contained in Appendix B of this report.

3. Analytic Representation of the Measurements

To formulate the technique of temporal and vertical filtering, it is assumed that the measured winds can be represented as a linear combination of tidal oscillations, a slowly-varying contribution, gravity waves, and noise. Analytically,

\[
\begin{align*}
\mathbf{u}(x, \theta, t) &= \exp(x/2) \sum_{n=1}^{2} M(n) \sum_{m=1}^{M(n)} U_{nm}(\theta) \left[ A_{nm}(x) \cos \omega_n t + B_{nm}(x) \sin \omega_n t \right] \\
&\quad + P_{u0}(x) + P_{u1}(x)t + P_{u2}(x)t^2 + G_u(x,t) + N_u(x,t) \\
\mathbf{v}(x, \theta, t) &= \exp(x/2) \sum_{n=1}^{2} M(n) \sum_{m=1}^{M(n)} V_{nm}(\theta) \left[ A_{nm}(x) \cos \omega_n t - B_{nm}(x) \sin \omega_n t \right] \\
&\quad + P_{v0}(x) + P_{v1}(x)t + P_{v2}(x)t^2 + G_v(x,t) + N_v(x,t)
\end{align*}
\]

where

\[u = \text{eastward component of the wind}\]
\[v = \text{northward component of the wind}\]
\[\theta = \text{colatitude} \quad (\theta = 0^\circ \text{ at the north pole})\]
\[t = \text{local time}\]
\[ x = \int_0^z \frac{dz}{H(z)} = \text{"reduced" altitude} \]

\[ z = \text{altitude} \]

\[ H(z) = \text{scale height of altitude } z \]

\[ \omega_n = \text{frequency of oscillation for } n\text{th tidal harmonic, } n = 1, 2 \]

\[ U_{nm}(\theta) = \text{latitudinal variation of } m\text{th mode of the } n\text{th harmonic for the eastward wind (known)} \]

\[ V_{nm}(\theta) = \text{latitudinal variation of } m\text{th mode of the } n\text{th harmonic for the northward wind (known)} \]

\[ P_{u0} + P_{u1}t + P_{u2}t^2 = \text{representation of long-period eastward wind} \]

\[ P_{v0} + P_{v1}t + P_{v2}t^2 = \text{representation of long-period northward wind} \]

\[ G_u = \text{collective label for short-period contributions to eastward wind} \]

\[ G_v = \text{collective label for short-period contributions to northward wind} \]

\[ N_u = \text{noise in the eastward wind component} \]

\[ N_v = \text{noise in the northward wind component} \]

\[ N(n) = \text{number of modes for } n\text{th tidal harmonic} \]

Noise, in the present context, denotes a short-lived transient that affects only a few of the measurements. It is distinguished from the short-period contributions \( C \), that may be present in all measurements. The coefficients \( A_{nm} \) and \( B_{nm} \) determine the amplitude \( C_{nm} \) and phase \( \delta_{nm} \) of each mode as follows

\[ C_{nm}^2 = A_{nm}^2 + B_{nm}^2, \quad \delta_{nm} = \frac{1}{\omega_n} \arctan \left( \frac{B_{nm}}{A_{nm}} \right) \]  

(Note that the growth factor \( \exp(x/2) \) has been explicitly introduced as a coefficient of the tidal components in Equations (3) and (4). Note also that the general theoretical relations between the eastward and northward components pertaining to each mode have been incorporated into Equations (3) and (4) as well.)
4. Temporal "Filtering"

"Filtering" usually refers to an operation on a continuous signal. In the present case the term is used to denote a linear combination of \( K \) distinct measurements conducted at the times \( t_k, k = 1, \ldots, K \). Let \( w(k) \) be a set of coefficients and consider the combination

\[
S_u(x) = \sum_{k=1}^{K} w(k) U(x, t_k)
\]

where \( u(x, t) \) is the eastward wind measured at time \( t \). The sum \( S \) will be free of diurnal and semidiurnal contributions provided the coefficients \( w \) are chosen so as to satisfy the equations

\[
\sum_{k=1}^{K} w(k) \cos \omega_n t_k = 0, \quad n = 1, 2 \quad (7)
\]

\[
\sum_{k=1}^{K} w(k) \sin \omega_n t_k = 0, \quad n = 1, 2 \quad (8)
\]

Then

\[
S_u = P_{u0} \sum_{k=1}^{K} w(k) + P_{u1} \sum_{k=1}^{K} w(k) t_k + P_{u2} \sum_{k=1}^{K} w(k) t_k^2
+ \sum_{k=1}^{K} w(k) G_u(t_k) + \sum_{k=1}^{K} w(k) N_u(t_k)
\]

Equations (7) and (8) together impose four conditions on the \( w(k) \) coefficients, leaving \( K-4 \) degrees of freedom. (It is assumed that \( K \geq 9 \)). Accordingly \((K-4)\) linearly independent combinations of the form of equation (9) are formed. From these combinations, the three coefficients \( P_{u0}, P_{u1}, P_{u2} \) are determined by a method of least squares, assuming the contributions from \( N \) and \( G \) cancel out. This assumption is justified provided the following conditions are met:

1. The coefficients \( w(k) \) are of comparable magnitude, so that noise present in a few measurements does not lead to a spurious result.

2. The combination does not enhance excessively any short-period contribution contained in \( G_u \). Let \( \omega_G \) be the frequency of such a contribution. Forming a combination of the form of equation (9) enhances (or attenuates, as the case may be) \( G_u \) by a factor \( F_G \), where
\[ F_G^2 = \left( \sum_{k=1}^{K} w(k) \cos \omega_G t_k \right)^2 + \left( \sum_{k=1}^{K} w(k) \sin \omega_G t_k \right)^2. \] (10)

These conditions are met provided there is sufficient redundancy in the data. For \( K=9 \) (measurements) there are \( K-4=5 \) equations of the form (9) from which the three \( P \) coefficients can be determined.

The first step of the temporal filtering procedure thus yields an initial estimate of the long-period wind component separately at each altitude:

\[ P_{u0}(x), P_{u1}(x), P_{u2}(x) \] determined at each altitude \( x \).

The second step is the imposition of the requirement that the long-period wind component vary slowly (and smoothly) with altitude. This is equivalent to vertical filtering that establishes the smooth representations

\[ P_u(x, t_k) \quad k = 1, \ldots, K \] (11)

These are approximations to the long-period contributions to the wind at the times of the observations.

The next step of the temporal filtering procedure is to form the quantities

\[ Q_u(x, t_k) = u(x, t_k) - P_u(x, t_k) \] (12)

and the combination

\[ T_u(x) = \sum_{k=1}^{K} \alpha(k) Q_u(x, t_k). \] (13)

\( Q_u \) is thus the eastward wind with the long-period contribution removed. If the requirement

\[ \sum_{k=1}^{K} \alpha(k) \cos \omega_2 t_k = \sum_{k=1}^{K} \alpha(k) \sin \omega_2 t_k = 0 \] (14)
is imposed on the coefficients $a(k)$, then the combination (13) is free of semidiurnal contributions. Since there are $K$ coefficients $a(k)$ and two conditions imposed by equation (14), $(K-2)$ linearly independent combinations of the form of equation (13) can be obtained. If conditions (1) and (2) mentioned above are observed, then the $T_u$ can be considered essentially free of noise and short-period contributions and thus contain the diurnal modes only.

From equation (3)

$$T_u(x, \ell) = \exp(x/2) \sum_{m=1}^{M(1)} U_{1m}(\theta) \left[ A_{nm}(x) \sum_{k=1}^{K} \alpha(k, \ell) \cos \omega_1 t_k \right. \left. + \beta_{nm}(x) \sum_{k=1}^{K} \beta(k, \ell) \sin \omega_1 t_k \right] \quad \ell = 1, \ldots, K-2$$  \hspace{1cm} (15)

Using equation (5) this can be written as

$$\exp(-x/2) T_u(x, \ell) =$$

$$-F_1(\ell) \sum_{m=1}^{M(1)} U_{1m}(\theta) C_{1m}(x) \cos \omega_1 \left[ t_1(\ell) - \delta_{1m}(x) \right] \quad \ell = 1, \ldots, K-2$$  \hspace{1cm} (16)

The same steps applied to the northward wind component result in the equation

$$\exp(-x/2) T_v(x, \ell) =$$

$$-F_1(\ell) \sum_{m=1}^{M(1)} U_{1m}(\theta) C_{1m}(x) \sin \omega_1 \left[ t_1(\ell) - \delta_{1m}(x) \right] \quad \ell = 1, 11, \ldots, K-2$$  \hspace{1cm} (17)

where

$$F_1(\ell)^2 = \left( \sum_{k=1}^{K} \alpha(k, \ell) \cos \omega_1 t_k \right)^2 + \left( \sum_{k=1}^{K} \beta(k, \ell) \cos \omega_1 t_k \right)^2$$  \hspace{1cm} (18)

$$t_1(\ell) = \frac{1}{\omega_1} \arctan \left[ \left( \sum_{k=1}^{K} \alpha(k, \ell) \sin \omega_1 t_k \right) / \left( \sum_{k=1}^{K} \beta(k, \ell) \cos \omega_1 t_k \right) \right]$$  \hspace{1cm} (19)
Since the notation is cumbersome, it seems appropriate to review the significance of equations (16) and (17). Given \( k \) arbitrary coefficients \( a \) there are \((k-2)\) linearly independent ways of choosing these coefficients while satisfying the two conditions of equation (14). These \((k-2)\) sets of coefficients are labeled

\[ \alpha(k, \ell), \quad K = 1, \ldots, k, \quad \ell = 1, \ldots, k-2. \]

There are, therefore \((k-2)\) linearly independent combinations \( T_u \) and \( T_v \) that can be formed:

\[ T_u(x, \ell), \quad T_v(x, \ell), \quad \ell = 1, (k-2). \]

For each \( \ell \), \( T_u(x, \ell) \) and \( T_v(x, \ell) \) represent, respectively, the total eastward and northward diurnal wind at the time \( T \), enhanced by the factor \( F(x, \ell) \). The unknowns in equations (16) and (17) are the amplitudes \( C_{1m}(x) \) and phases \( \delta_{1m}(x) \), for \( m = 1, \ldots, M(1) \), where \( M(1) \) is the number of diurnal modes.

In an entirely analogous manner the semidiurnal winds are isolated as well.

5. Vertical "Filtering"

It has been mentioned previously that trapped diurnal modes are insignificant below the mesopause. Thus only the three propagating modes need be considered. These have wavelengths of approximately 28, 11, and 7 kilometers. Owing to the very slow variation with altitude of the amplitude factors \( C_{1m}(x) \), the shorter-wavelength modes can be removed by vertical filtering. To illustrate the filtering procedure, let us consider the function

\[ f(x) = \cos (\omega t - \frac{2\pi}{\lambda} x), \]

and the integral

\[ I(a, x) = \int_{x-a/2}^{x+a/2} dy f(y) = \frac{\lambda}{\pi} \sin \frac{2\pi a}{\lambda} \cos (\omega t - \frac{2\pi}{\lambda} x). \]
If \( a \approx \lambda/2 \), then \( \tilde{I}(a,x) \approx 0 \). Thus averaging the combinations \( T_u \) and \( T_v \) of equation (16) and (17) over 7 km and 11 km successively will remove essentially all but the first propagating diurnal mode multiplied by known factors. The result can be written in the compact form

\[
\begin{align*}
C_{11}(x) \cos \omega_1 t_1(\lambda) - \delta_{11}(x) &= Y_u(x,\lambda) \quad \lambda = 1, \ldots, k-2 \\
C_{11}(x) \sin \omega_1 \bar{t}_1(\lambda) - \delta_{11}(x) &= Y_v(x,\lambda) \quad \lambda = 1, \ldots, k-2
\end{align*}
\]

where \( Y_u \) and \( Y_v \) are known. Consider now the graphical representations of \( Y_u \) and \( Y_v \) versus \( x \). It follows that

1. where \( Y_u \) has a maximum:

\[
\delta_{11}(x) = \bar{t}_1(\lambda), \quad C_{11}(x) = Y_u(x,\lambda)
\]

2. where \( Y_u \) has a minimum:

\[
\delta_{11}(x) = \bar{t}_1(\lambda) + \frac{\pi}{\omega_1}, \quad C_{11}(x) = Y_u(x,\lambda)
\]

3. where \( Y_v \) has a maximum:

\[
\delta_{11}(x) = \bar{t}_1(\lambda) - \frac{\pi}{2\omega_1}, \quad C_{11}(x) = Y_u(x,\lambda)
\]

4. where \( Y_v(x,\lambda) \) has a minimum:

\[
\delta_{11}(x) = \bar{t}_1(\lambda) + \frac{\pi}{2\omega_1}, \quad C_{11}(x) = -Y_v(x,\lambda)
\]

From the \( 2(k-2) \) such graphical representations a sufficient number of points can be established in the vertical profiles of \( C_{11}(x) \) and \( \delta_{11}(x) \) to allow a smooth analytic representation of these functions.

After the longest-wavelength mode has been isolated in this fashion, it can be subtracted from the total diurnal contributions of equations (16) and (17) and the other modes can be investigated as well.
6. Temperature and Pressure Investigation

It is a general result of tidal theory that, for each tidal mode separately, the temperature and pressure variation can be determined from the wind components. Explicitly, the pressure variation is

\[
\Delta P_{nm}(x, \theta, t) = \frac{4a \omega_e^2}{R g \omega_n} \mathcal{H}_{nm}(\theta) \frac{1}{H} \exp(x/2)
\]

\[
x - A_{nm}(x) \cos \omega_n t + B_{nm}(x) \sin \omega_n t
\]

and the temperature variation is

\[
\Delta T_{nm}(x, \theta, t) = \frac{4a \omega_e^2}{R g \omega_n} \mathcal{H}_{nm}(\theta) \frac{1}{H} \exp(x/2)
\]

\[
v \left\{ \left[ H(x) A_{nm}(x) - \mathcal{H} A_{nm}(x) \right] \cos \omega_n t \\
- \left[ H(x) B_{nm} - H B_{nm}(x) \right] \sin \omega_n t \right\}
\]

where

\( \Delta P_{nm} \) = pressure variation due to mth mode of nth harmonic

\( \Delta T_{nm} \) = temperature variation due to mth mode of nth harmonic

\( a \) = radius of the earth

\( \omega_e \) = frequency of rotation of the earth

\( R \) = gas constant

\( g \) = gravitational constant

\( H_{nm}(\theta) \) = latitudinal variation (known).
The altitude-dependent functions $A_{nm}$ and $B_{nm}$ are the same ones involved in (and determined by) the wind components. Primes in equation (23) indicate derivatives with respect to the reduced height. In taking derivatives, it is always the case that small errors get greatly magnified. Thus, in practice one would integrate equation (23) over a small altitude interval (1/2 to 2 kilometers, depending on the accuracy of the data) and obtain a mean value for the interval.

The total tidal contribution to temperature and pressure variations is obtained by summing over all modes

$$\Delta P \text{ (tidal)} = \sum \sum P_{nm}$$

$$\Delta T \text{ (tidal)} = \sum \sum T_{nm}.$$ 

It should be noted that the same filtering procedures developed for the winds can be applied to temperature and pressure measurements of comparable accuracy.
SECTION IV
CURRENT RESULTS

The methods described in Section III were applied to a series of measurements from Wallops Flight Center in June 1974. After application of the error assessment and smoothing procedures contained in Appendix B, the vertical profiles of the northward component of the winds for that series is shown in Figure 3. It was found that the smoothing procedure had to be applied twice in succession in order to reduce the random variations to levels which allowed the extraction of small tidal oscillations. It was also found that a broader data base was required to define the vertical profiles of prevailing zonal wind and ambient temperature which are required in the analysis. The values of these quantities which were obtained from the CIRA 1965 model are given in Appendix A of this report.

Plots such as Figure 3 are important to the analytical method especially during the initial phases. The location of maxima and minima on the vertical profiles are used to determine the relative phases of the predominant tidal modes. The values of phases which are actually utilized in the analysis are computed from tidal theory for the identified modes. Some examples of this procedure for flight nos. 5, 6, 7, and 8 of the June series are shown in Figure 4 through 7. The phase of the 7 km (diurnal) component expressed as multiples of \( \pi \) is shown over the observed altitude range. The solid curve was obtained from tidal theory utilizing the prevailing temperature profile. The points are from visual observation of maxima and minima on the vertical wind profiles. The points are generally alternatively the eastward maxima (e), the northward maxima (o), the eastward maxima (g) and the northward minima (z).

Verification of the amplitudes of the various superimposed oscillations in the June data was obtained by applying a standard harmonic analysis to that series of data. The results are discussed in Appendix D of this report.

The computer codes for determining the tidal modes with the filtering method are contained in Appendix C. Some examples of the results obtained from the June 1974 data are shown in Figures 8, 9, and 10. The phase of a 10 km diurnal component which could not be determined visually as was the 7 km component was, however, determined in the filtering procedure and is shown in Figure 8 for the first eight measurements in the June series.

The short wavelength residual after the removal of the 7 km and 10 km wavelength diurnal modes is shown in Figure 9. This remainder strongly resembles a 5 km wavelength semidiurnal mode.

Finally, the very long wavelength contribution to the northward wind is shown in Figure 10.

Application of the program to data from other observational series at different time has been initiated but could not be completed in time for this report.
Figure 3. Smoothed northward components of the winds from June 1974.
Figure 4. Phase of the 7 km diurnal mode from flight 5 in Figure 3 (see text for explanation and meaning of symbols).
Figure 5. Phase of the 7 km diurnal mode from flight 6 in Figure 3 (see text for explanation and meaning of symbols).
Figure 6. Phase of the 7 km diurnal mode from Flight 7 in Figure 3 (see text for explanation and meaning of symbols).
Figure 7. Phase of the 7 km diurnal mode from flight 8 in Figure 3 (see text for explanation and meaning of symbols).
Figure 8. Derived phase of the Northward 10 km diurnal component.
Figure 9. Short wavelength components after removal of 7 km and 10 km components.
Figure 10. Long wavelength contribution to the northward wind.
SECTION V

CONCLUSIONS

Difficulties associated with the organization of the original data and the restructuring of the smoothing process were much greater and more time consuming than were initially anticipated. Thus the amount of actual processed data was less than initially expected. However, the primary objective of the contract was accomplished in that the development of the new analytical method was completed. It was demonstrated that the computer programs produce results with the vertical filtering procedure which are verified by the totally separate harmonic analysis process. Assessment of the full potential of the method must await the processing of a much longer amount of data from different times and locations.
SECTION VI

REFERENCES

Lindzen, R.S., Quart, J.R. (1967), Met. Soc. 93, 18-42.
Miers, B.T., (1965), J. Atmos. Sciences, 22, 382-387.
APPENDIX A

PREVAILING WINDS AND AMBIENT TEMPERATURE PROFILES

Detailed profiles of the ambient temperature and of the prevailing zonal wind have been established by interpolation from tables found in the CIRA 1965 handbook. Values appropriate to the dates and latitudes of the measurements of interest were obtained from the tables at 5 kilometer intervals. Using interpolation techniques, we calculated the required profiles at 0.25 km intervals and smoothed the profiles using the procedures described in Appendix B. Graphical representations of the ambient temperature and prevailing zonal winds are presented in Figures A-1 through A-8.
Figure A-2
Figure A-4
Figure A-6
Figure A-7
Figure A-8
APPENDIX B

REMOVAL AND ASSESSMENT OF RANDOM ERRORS IN THE WIND AND TEMPERATURE DATA

A. INTRODUCTION AND SUMMARY

A smoothing technique has been applied to the wind and temperature data supplied by the Wallops Flight Center in order to remove random errors. The technique is based on the assumption that, over an altitude interval of 2 kilometers, physically meaningful variations with respect to altitude can be represented by a three-term (quadratic) polynomial. Wavelike variations with wavelengths as short as 5 kilometers are totally unaffected by this technique. At each altitude, the technique yields an improved ("smoothed") value, the probable error associated with the smoothed value, and the random error, defined as the difference between the smoothed and original values. The random errors have been analyzed statistically. In general, the random errors associated with the wind measurements are less than 0.5 meter/second up to an altitude of about 55 kilometers. Above that altitude, the magnitude of the error tends to increase to a value between 3 and 5 meters/second at 80 kilometers. The random errors associated with temperature measurements are generally less than 0.3°C below 60 kilometers and increase to about 0.8°C at 70 kilometers.

The smoothing technique, the analysis of random errors, and estimates of errors obtained by other observers are presented in the following subsections. The computer codes utilized in the smoothing procedure and the determination of random errors is reproduced in subsection 5, along with the necessary documentation.

B. METHOD OF SMOOTHING

The smoothing procedure is first outlined in general form, and the specific form appropriate to the current data is then presented.

Let \( Y_k \) = the observed value of a physical quantity \( Y \) (e.g. temperature) corresponding to the value \( z_k \) of the independent variable \( z \) (e.g. altitude)

\[ S_k = \text{the probable error in } Y_k \quad \text{(B-1)} \]

\[ X_k = \sum_{m=1}^{M} C_m f_{m,k} \]

= a suitable analytic representation of \( Y \),

where \( f_{m,k} = f_m(z_k) \), \( m = 1, M \)

= the value of the expansion function \( f_m \) at \( z_k \)
\[ C_m = \text{expansion coefficient} \]
\[ M = \text{number of terms in the analytic representation.} \]

Define \( D_k = X_k - Y_k \) \hspace{1cm} (B-2)

\[ \bar{s}^2 = \left( \sum_{k=1}^{K} S_k^2 \right) / K \] \hspace{1cm} (B-3)

\[ W_k = \left( \frac{\bar{s}^2}{S_k^2} \right) \] \hspace{1cm} (B-4)

\[ Q = \left( \sum_{k=1}^{K} W_k D_k^2 \right) \] \hspace{1cm} (B-5)

where \( K = \text{number of observations of the quantity Y} \)

The optimum value of the coefficients \( C_m \) is obtained by minimizing \( Q \) with respect to the variations of the \( C_m \) (the least squares procedure):

\[ \frac{\partial Q}{\partial C_m} = 0, \ m = 1, M. \]

The resulting equations can be formulated in terms of the matrix \( H \) and the vector \( U \), where:

\[ H_{km} = \sum_{k=1}^{K} f_{ik} f_{mk} W_k, \quad U_m = \sum_{k=1}^{K} f_{mk} Y_k W_k, \quad m, f, = 1, M \]

Then \( U = C \cdot H \) and \( C = U H^{-1} \).

Explicitly \[ C_m = \sum_{k=1}^{K} G_{mk} Y_k W_k \] \hspace{1cm} (B-6)

where \[ G_{mk} = \sum_{f=1}^{M} f_{ik} (H^{-1})_{fm} \] \( m = 1, M, k = 1, K \).
Corresponding to the errors $S_k$ in the measured quantities $Y_k$, there are probable errors in the coefficients $C_m$. It can be shown that these errors are represented by the matrix:

$$\Delta C_m \Delta C_k = (H^{-1})_{mk} \cdot \bar{S}^2$$  \hspace{1cm} (B-7)

where the quantity on the left represents the correlation of the probable errors $\Delta C_m$ and $\Delta C_k$. Further, the most probable value of $Q$ is given by:

$$Q^* = (K - M) \cdot \bar{S}^2.$$  \hspace{1cm} (B-8)

Finally the most probable error in $X_k$ is given by:

$$\Delta X_k^2 = \sum_{m=1}^{M} \sum_{k=1}^{M} \Delta C_m \Delta C_k f_{mk} f_{lk}$$  \hspace{1cm} (B-9)

$$= \sum_{m=1}^{M} \sum_{k=1}^{M} (H^{-1})_{mk} f_{mk} f_{lk} \cdot \bar{S}^2$$

If, as in the present instance, the uncertainties $S_k$ in the measurements $Y_k$ are unknown, a somewhat different procedure must be adopted. For the data under consideration, we assume that the uncertainties in the measurements are equal over the interval involved. Thus:

$$S_k^2 = S^2, \quad W_k = 1, \quad k = 1, K$$  \hspace{1cm} (B-10)

and using (B-5) and (B-8):

$$\bar{S}^2 = \sum_{k=1}^{K} D_k^2 / (K - M).$$  \hspace{1cm} (B-11)

For the present data, the analytic representation consists of a three-term polynomial with respect to the altitude. The smoothing procedure involves nine data points (or equivalently, 2 kilometers) centered at the data point in
question. This choice of smoothing interval allows most of the noise to be removed without affecting wavelike components with wavelengths as short as 5 kilometers. When applied to simulated, error-free data resulting from the superimposition of a 5-kilometer wave and an 11-kilometer wave, the method introduced less than 1 percent error. Figure B-1 illustrates the effect of smoothing on a data sample with particularly severe random errors. Note the negative correlation between the eastward and northward components.

C. ANALYSIS OF RANDOM ERRORS

The random errors established during the smoothing procedure have been analyzed statistically. They are presented graphically in Figures B-2 through B-5. Figure B-2 pertains to the random errors in the temperature data, plotted separately for each series as well as cumulatively for all the data from Wallops Island. In each instance the plotted quantity is a 2-kilometer average of the root-mean-square value of the random error \( D \) as defined by equation (B-2). Owing to the nature of the temperature measurement, the random errors are small, as expected. Below 60 kilometers the errors are generally less than 0.3°C, although the Kourou data gives errors closer to 0.5°C. Above 60 kilometers the error increases to a value of about 1°C at 70 kilometers.

Figure B-3 displays the random errors for the cumulative wind data from Wallops Island. Below 55 kilometers the errors vary between 0.5 and 1.0 meter/second for both the eastward and northward wind components. Above 55 kilometers, errors increase to a value of about 5 meters/second at 80 kilometers. The dashed curves represent the uncertainty in the smoothed wind values. These are generally about half as big as the random error at the same altitude. The apparently oscillatory nature of the random error in the eastward component above 55 kilometers is not due to any physical process. It is, rather, a fortuitous result of the superposition of all results from Wallops Island. This can be seen more clearly in Figure B-4, which displays the correlated error, as defined as:

\[
D(E) \cdot D(N)
\]

where \( D(E) \) = Random error in eastward wind,

\( D(N) \) = Random error in northward wind.

Figure B-4 reveals that above 55 km the eastward and northward errors are strongly correlated, possibly owing to the experimental difficulty of accurately establishing a direction of observation at high altitudes.

Figure B-5 is analogous to Figure B-3 but pertains to the data from Kourou.

In summary, it can be said that random temperature errors are insignificant below about 65 kilometers, whereas random wind errors are small below about 55 kilometers. It should be noted that the technique utilized in the current study does not detect systematic errors.
Figure B-1. Altitude Profiles of the Original and Smoothed Wind Components Measured at Wallops Island (23 20 UT, June 29, 1974). Solid Curves are Original Data. Dashed Curves are Smoothed Data.
Figure B-2. Random Error in the Temperature Data as a Function of Altitude. (a) For the June 1974 Series at Wallops Island. (b) For the March 1974 Series at Wallops Island. (c) For the April 1974 Series at Wallops Island. (d) Cumulatively for all the Series at Wallops Island. (e) For the March 1974 Series at Kourou.
Figure B-3. Random Error in the Wind Data From Wallops Island as a Function of Altitude. (a) For the Eastward Component. (b) For the Northward Component. (c) For the Correlated Error. Solid Curves Depict Error in Original Data. Dashed Curves Depict Uncertainty Associated with Smoothed Data.
Figure B-4. The Square of the Correlated Wind Error as a Function of Altitude, Plotted Separately for Each Series (see text for definition). Curves Have Been Drawn Only Where the Error Differs Appreciably from Zero.
Figure B-5. Random Error in the Wind Data from Kourou as a Function of Altitude.
D. OTHER ERROR ESTIMATES

Some of the estimates of error associated with standard Met rocket measurements of wind and temperature are listed in Table B-1. The general estimates were determined by the authors from the various factors involved in the measurements. The observed differences were obtained from comparison of two or more measurements. Susko and Vaughan, tracked Jimsphere wind sensors with two different FPS-16 radars and compared the results. Miller, Wolf, and Finger, compared the results of simultaneous temperature measurements made with a U.S. and a Japanese sensor. Miller and Shmidlin compared the data obtained from pairs of identical rocket systems launched 5 minutes apart, along the same trajectory. Finally, the uncertainty produced by the angular error of the tracking radar is estimated for data taken at altitude increments of 250 meters.

E. THE COMPUTER CODES USED FOR SMOOTHING THE DATA

This subsection contains a listing of the computer codes utilized to obtain smoothed values of the wind and temperature data, as well as to calculate the random errors in the original data. The smoothing procedure utilizes two subroutines, SMOOTH and MISCELZ, together with a calling sequence. MISCELZ is called once to establish parameters common to the smoothing of all of the quantities of interest. Subroutine SMOOTH is called successively to accomplish various tasks, as indicated by the calling sequence. Each subroutine is listed below, together with explanations identifying program variables with the mathematical quantities defined in subsection 2 this appendix.

F. SUBROUTINE MISCELZ

Subroutine MISCELZ is listed in Table B-2. This subroutine defines the four sets of parameters required to carry out the smoothing procedure. The program parameters and their relation to symbols previously defined in subsection 2 are as follows:

- $H^{-1}$ is the inverse of the matrix $H$ as defined in the equation preceding (B-6)
- $F(M,N)$ are the analytic (polynomial) expansion functions defined in (B-1)
- $G(M,N)$ are the auxiliary functions defined by (B-6) and the equation that follows it
- $ESQ(K)$ is the quantity on the right side of (B-9) without the $S^2$.

G. SUBROUTINE SMOOTH

Subroutine SMOOTH is listed in Table B-3. This subroutine is designed to produce smoothed values of the two wind components and the temperature. It was found desirable to do a double smoothing of the original data. In
Table B-1. REPORTED RANDOM ERRORS

<table>
<thead>
<tr>
<th>Windspeed m/s</th>
<th>Temperature °C</th>
<th>References</th>
</tr>
</thead>
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<tr>
<td>General Estimates</td>
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<td></td>
</tr>
<tr>
<td>3 - 5</td>
<td></td>
<td>Smith, J. of Meteor., 17, 296, 1960</td>
</tr>
<tr>
<td>± 2.5</td>
<td></td>
<td>Lenhand, J.G.R. 68, 227, 1962</td>
</tr>
<tr>
<td>± 3</td>
<td>±2</td>
<td>Beyers &amp; Miers, JAS, 25, 155, 1968</td>
</tr>
</tbody>
</table>

| Observed Differences | | |
| ± 0.4 | | Susko & Vaughan, NASA TNX53752, 1968 |
| ± 3 | ±1 | Miller & Schmidlin, J. Appl. Meteor., 10, 320, 1971 |

<p>| Radar Precision FPS-16 | | |</p>
<table>
<thead>
<tr>
<th>Alt. km</th>
<th>Angular Error mils</th>
<th>Fallrate m/s</th>
<th>Wind Speed Error m/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>60</td>
<td>0.1</td>
<td>100</td>
<td>3.4</td>
</tr>
<tr>
<td>20</td>
<td>0.1</td>
<td>10</td>
<td>0.1</td>
</tr>
</tbody>
</table>
Table B-2.

SUBROUTINE DISCEL2
contains miscellaneous data
C
CONT:<H1S2/H1(3,3),F(2,3),G(3,9),FSG(9)
LATA H1/2.554E-01, 0, -3.4632E-01,
1 0, 2.6667E-01, 0,
2 -3.4632E-01, 0, 8.3117E-01/

C+++
DO 140 I=1,9
F(1,I)=1.0
L0 110 M=2,3
110 F(K,M)=(6.25*(I,-5))**K*(M-1)
DO 130 N=1,3
CC=0.0
DO 120 L=1,3
CC=CC+F(L,M)*F(L,N)
130 G(K,L)=CC
140 CONTINUE

C----------
DO 165 J=1,9
CC=0.0
DG 164 I=1,3
DG 164 I=1,3
164 CC=CC+HI(K,I)*F(H,J)*F(H,J)
165 ESC(J)=CC
RETURN.
END
Table B-3.

SUBROUTINE SMOOTH (I, ITLX, MIN, MAX, FA, FB, ES, DS, F1)

Given FA(X), I = MIN, MAX at 0.25 km intervals
Smooth FA using three-term polynomial fit over two kilometers
(9 points). MT = smoothed values of FA
For ITER=2 calculates probable errors ES, absolute errors DS
For I=3 and ITER=2, 3 calculates F1=derivative of FA

COMMON /DATA5/ VVT (3, 361), S (3, 361), ESR (3, 361), DSM (3, 361), DIM (361)

C
COMMON /ISC/ I (3, 3), T (3, 9), G (3, 9), ES (9)
DIMENSION FA (361), FB (361), ES (361), DS (361), F1 (361), C (3)

C
initialize output quantities

LO 190 N = MIN, MAX
FB (N) = 999.0
ES (N) = 999.0
DS (N) = 999.0
F1 (N) = 999.0

190 CONTINUE

C

JMAX = 9
J2MAX = 5

DIM = MAX/3
JG0 = 1
JH = 1
JLOW = MAX/2 + 1
JHIG = MAX - 1

C

ORIGINAL PAGE IS OF POOR QUALITY.
Table B-3 (continued).

C 5=COL:1, IL:1, INTERVALS AT A TIME
200 DO=ILC+1
300 IF (IL < L< J, AX-1)
1F (L>1, L 'A) 10 210
JH=J-AX/2+1
JMAX=J-AX
JGC=3
C FIRST EXPANSION COEFFICIENTS
210 DC 236 M=1,3
CC=0.0
DC 228 JN=1, JMAX
L=NLG+JN-1
220 CC=CC+G(JN-1, JN) # FA(N)
230 C(J)=CC
IF (ITER,JF,2) GO TO 251
C CALCULATE ERRORS
SUR=0.
DO 250 JN=1, JMAX
L=NLG+JN-1
CC=0.0
DC 240 M=1,3
240 CC=CC+C(JN) # (L, JN)
CC=CC- VVT(I,JN)
250 SUR=SUR+CC##2
251 CONTINUE
GO TO (770, 760, 270), JGC
C JGC=2 (MIDDLE INTERVALS)
260 L=NLG+JMAX/2
FE(:)=C (1)
F1(N)=C (2)
IF (ITFR, NE, 2) GO TO 200
ES(JN)=E(JN)-VVT(I,JN)
ES(JN)=SUB(T(SUR#ES(JN)/JN))
GO TO 200
C JGC=1,3 (FIRST, LAST INTERVALS)
270 DO 285 JN=JH; II, JMAX
L=NLG+JN-1
CC=0.0
DC 280 M=1,3
280 CC=CC+C(JN) # (L, JN)
FB(N)=CC
F1(N)=C (2)+2.0 # C (3) # F(2, JN)
IF (ITFR, NE, 2) GO TO 295
DS(JN)=CC- VVT(I, JN)
ES(JN)=SUB(T(SUB(ES(JN)/JN))
285 CONTINUE
IF (JGC=50, 3) GO TO 290
JGO=2
GO TO 200
290 CONTINUE
RETURN:
END
other words, the smoothed values obtained on the first iteration were used as the input data to obtain a new set of smoothed values. Errors are defined as the difference between the original data and the doubly smoothed data. For temperature data, the first and second derivatives with respect to altitude were evaluated as well. Initial values for the first derivative were obtained during the second smoothing of the temperature. These initial values were then smoothed to obtain the final set of first derivatives and the initial values of the second derivative. Finally the second derivative of the temperature was smoothed.

The subroutine arguments have the following significance:

I = index specifying quantity being smoothed
   = 1 for eastward wind
   = 2 for northward wind
   = 3 for temperature

ITER = Index specifying iteration number
   = 1 for first smoothing
   = 2,3 for subsequent smoothings and derivative calculations

A = Limits on height indices. These are determined by the availability of data subject to the restriction that a 0.25 kilometer grid is assumed.

FA = Vector containing values to be smoothed

FB = Vector containing smoothed values

DS = Vector containing differences between smoothed values and original data (as defined by (B-2))

ES = Vector containing probable error in smoothed data (as defined by equation (B-9))

F1 = Vector containing derivative of smoothed quantity.

Within the main body of the subroutine the index JGO serves to select the procedure to be used at the beginning of the altitude range (JGO = 1), the end (JGO = 3) and the middle (JGO = 2). This distinction is necessary because smoothed values at each altitude are obtained by considering the four altitudes that precede it and the four altitudes that follow it, except at the beginning and end of the altitude range.
1. **Calling Sequence**

The calling sequence used to accomplish the tasks described previously is evident from the listing provided in Table B-4. It is assumed that it is preceded by CALL MISCELZ which establishes the necessary parameters, as well as by the definition of the index I which serves to define the quantity to be smoothed. The variables that appear in the listing have the following significance.

- **VVT (I,N)** is a matrix that contains the eastward and northward winds and the temperature.
- **SM (I,N)** = Matrix containing final set of smoothed values for quantity I.
- **ESM (I,N)** = Matrix containing probable error in smoothed value
- **DSM (I,N)** = Matrix containing random error in original data
- **DIM (N)** = Vector containing first temperature derivative
- **D2M (N)** = Vector containing second temperature derivative
Table B-4. Calling Sequence

```
C DEFINE QUANTITY TO BE SMOOTHED
DO 110 I=MIN,MAX
  110 FS(I) = W(I)(I,J)
C FIRST SMOOTHING OF QUANTITY I
  IITER=1
  CALL SMOOTH (I,ITER,MIN,MAX,FS,FR,ES,DS,F1)
C SECOND SMOOTHING OF QUANTITY I (WITH PROBABLE ERROR ES, ABSOLUTE
  DIFFERENCE DS, FIRST DERIVATIVE F1 FOR I=3)
  IITER=2
  CALL SMOOTH (I,ITER,MIN,MAX,FS,FR,ES,DS,F1)
C SAVE CALCULATED QUANTITIES
D0 120 J=MIN,MAX
  ES(I,J) = ES(I,J)
  DS(I,J) = DS(I,J)
120 IF (J.LT.3) GO TO 140
C FOR TEMPERATURE (I=3) ONLY
C SMOOTH FIRST TEMPERATURE DERIVATIVE, FIND SECOND DERIVATIVE
  IITER=3
  CALL SMOOTH (I,ITER,MIN,MAX,F1,L1,ES,DS,FR)
C SMOOTH SECOND TEMPERATURE DERIVATIVE
  IITER=4
  CALL SMOOTH (I,ITER,MIN,MAX,FR,D2,ES,DS,F1)
140 CONTINUE
```
APPENDIX C

COMPUTER CODES FOR DETERMINATIONS OF TIDAL MODES

This program resolves short-wavelength modes from vertical wind profiles. The following subroutines are required:

1. RPARV - Reads parameters for vertical analysis.
2. RFILEV - Reads data from smoothed vertical profile.
3. RPREV - Reads prevailing wind, temperature and temperature derivatives.
4. ZFUNS - Defines height as a function of height index.
5. VFUNS - Calculates expansion functions for vertical analysis.
6. VFilter - Extracts short-wavelength modes.
PROGRAM TMoDES(1:INPUT,OUTPUT,TAPE1=INPUT,TAPE2=OUTPUT,TAPE11=TAPE12)
C RESOLVES SHORT-WAVELENGTH MODES FROM VERTICAL PROFILES
C REQUIRES VERTICAL FILE, VFILE, VFUN, VFIL, ZFILE, ARRAY, MIN, PHASE
C
C 000003 COMMON, KODE/INIT,NIT, KTAPE, LTAPE, PI
C 000003 COMMON/W60/LS,L+1,LMAX, TCE(7), FLAT, TLOC, FHNG(14), NHR(14),
C 000003 KIP(14), FPREV(14), TFILE, VFILE,вер (14), VEHICLE(14)
C 000003 COMMON, DATA3/VT(3,260), SH(3,260), ESMT(3,260), DMH(3,260), D1N(260)
C 000003 COMMON, PREV/PREV(3,260), LMN, LMAX
C 000003 COMMON, PARV/KPD, Z3, SCALE, KHR, NH(3), NH(3), HNN(3), NOPT, HINT
C 000003 COMMON/CODES/1FFPRINT, IFTAPE
C 000003 COMMON/KFNS/FH0(7,260)+AI(2,260)+X1(260)
C
C 000003 NIT=1
C 000004 NIT=2
C 000005 KTAPE=11
C 000006 LTAPE=12
C 000007 PI=3.14159265
C
C READ PARAMETERS FOR VERTICAL ANALYSIS
C
C 000011 CALL PPAHV
C 000012 WRITE (NOT,100)
C 000013 100 FORMAT (1H1/# PARAMETERS FOR VERTICAL ANALYSIS#/
C 000014 WRITE (NOT,110) KPD, Z3, SCALE
C 000015 110 FORMAT (13=F5.2,# SCALE=F5.2)
C 000016 WRITE (NOT,120) NH
C 000017 120 FORMAT (13=K10.0)
C 000018 DO 130 K=1,NH
C 000019 130 WRITE (NOT,140) K, NH(K), HN(K), HN(K)
C 000020 140 FORMAT (313,F5.4)
C 000021 WRITE (NOT,150) NOPT, HINT
C 000022 150 FORMAT (13=F5.2)
C
C READ PRINT/TAPE CODES
C
C 000065 READ (INIT,140) IFFPRINT, IFTAPE
C
C READ FILE NUMBER LIMITS
C
C 000075 READ (INIT,140) LMN, LMAX, LMAX
C
C READ KODE/IN\T=NIT, NIT=140
C
C CALL RPNEV
C
C DO 160 N=1,260
C 000112 160 SM(3,N)=PREV(3,N)-273.0
C
C L=1
C
C NN(K,L)=1
C
C NN(K,L)=260
C
C TLOC(L)=0.0
C
C DEFINE EXPANSION FUNCTIONS
C
C 000130 CALL VFUN
C 000131 DO 600 LL=LMN,LMAX
C
C CALL RFLEV
C
C CALL VFILTER
C
C 800 CONTINUE
C
C 000140 END
SUBROUTINE RPARV
C READ PARAMETERS FOR VERTICAL ANALYSIS
C 24 MAY 75
C COMMON/IODEV/HIT,NOT,PTAPE,LTAPIE,PI
C COMMON/PARV/KPUL,ZS,SCAF,KHR,NH(3),NH(3),HMN(3),NOPT,HINT
C READ NUMBER OF POLYNOMIAL COEFFICIENTS
C READ OF POLYNOMIAL COEFFICIENTS
C READ HARMONIC FUNCTION DATA
C READ OPTION (NOPT=-1 FOR FIXED FITTING INTERVAL HINT)
C READ (NIT,100) NOPT,HINT
C RETURN
C END.

ORIGINAL PAGE IS OF POOR QUALITY
SUBROUTINE HFILEV
C
READS DATA FROM SMOOTHED VERTICAL FILE
C
53 FAY 75
C
COMMON/I,DEV/INIT,NOT,MTAPE,LTape,P1
C
COMMON/ICANS/ISER,L,LSMAX,TCE(L),FLAT,FLONG,TLOC(L),NHR(L),
1   NMI(L),DAY(L),NMO(L),NVE(L),NFI(L),NFP(L),VEHICLE(L)
C
COMMON/DATA'S/VVT(3,260),SM(3,260),ESH(3,260),DSH(3,260),D2M(260)
1   ,DNM(260),NM(3,14),LMX(3,14)
C
READ (MTAPE,210) ISER,L,LSMAX,TCE,FLAT,FLONG,TLOC,L,NHR(L),
1   NMI(L),DAY(L),NMO(L),NVE(L),NFI(L),VEHICLE(L)
C
DO 240 I=1,3
C
READ (MTAPE,220) NMIN,NMAX
C
220 FORMAT (2I3)
C
IMIN(I,L)=NMIN(I)
LMX(I,L)=NMAX(I)
C
READ (MTAPE,230) ( SM(I,N),N=NMIN,NMAX)
C
READ (MTAPE,230) ( ESH(I,N),N=NMIN,NMAX)
C
READ (MTAPE,230) ( VVT(I,N),N=NMIN,NMAX)
C
IF (I,N.E,3) GO TO 240
C
READ (MTAPE,230) ( D1(IN,N),N=NMIN,NMAX)
C
READ (MTAPE,230) ( D2M(N),N=NMIN,NMAX)
C
READ (MTAPE,230) ( D2M(N),N=NMIN,NMAX)
C
230 FORMAT (10F8.2)
C
240 CONTINUE
C
READ (MTAPE,130) A
C
130 FORMAT(A1)
C
IF (EOF,MTAPE) 160,140
C
WRITE (NOT,150) L
C
150 FORMAT (/* EOF NOT THERE AFTER FILE*/13/)
C
160 RETURN
C
END
SUBLRINTNE NAME

READS PREVAILING KIND, TEMPERATURE AND TEMPERATURE DERIVATIVES

COMMON/IOREV/HT, MTTAPE, LTAPF, P1

COMMON/IHDOS/ISERT, LS MAX, TCB (7), FLAT, FLONG, TLOC (14), NHR (14),

1

M1 (1 4), NDAY (14), NM (1 4), N Y (1 4), NF1 (1 4), NTP (1 4), VEHICLE (1 4)

COMMON/DATA/VVT (3, 260), SM (3, 260), ES (3, 260), DSM (3, 260), D1M (260)

1

D2M (260), NM1 (3, 14), NM (3, 14)

COMMON/PREV5/PREV (3, 260), NM1 (3, 260), NMAX

COMMON/DU N (80)

DIMENSION U, MY (80)

NMAX = 760

READ (110), ISERT

DO 150 I = 1, 3, 2

READ (110), NMAX

IF (1, E0, 3) GO TO 120

READ (110), (PREV (1, N), N = NM1, NMAX), BUNK

READ (110), (DUMMY, PREV (1, N), N = NM1, NMAX), BUNK

READ (110), (DUMMY, D1M (N), N = NM1, NMAX), BUNK

READ (110), (DUMMY, D2M (N), N = NM1, NMAX), BUNK

CONTINUE

RETURN

END
FUNCTION ZFUN(N)
C DEFINES HEIGHT AS A FUNCTION OF HEIGHT-INDEX
C ZFUN(1)=20.00 KM, ZFUN(2)=20.25 KM, ..., ZFUN(281)=90.00 KM
ZFUN=19.75+0.25*N
RETURN
END
SUBROUTINE VFUN5

C CALCULATE EXPANSION FUNCTIONS FOR VERTICAL ANALYSIS

000002 COMMON/HEADS/ISER,L,LSMAX,TCD(7),FLAT,FLOD,TOC(14),NHR(14),
000002 1 H1(14),H2(14),H3(14),NHR(14),NRE(14),NRE(14),NHR(14),NHR(14),NHR(14),NHR(14),
000002 1 VEHICLE(14)

000002 COMMON/DATAS/VVT(3,260),SM(3,260),ESN(3,260),DSM(3,260),D1M(260)

000002 COMMON/PARV/KPOL(2),SCALE,KHR,NH(3),KH(3),HMIN(3),NMIN,NMAX

000002 COMMON/IFPP/ALP(3),OXN(3),QXH(3),QXR(3),WAVE(3)

000002 COMMON/FKNS/FLM(7,260),AL(2,260),XI(260)

000002 COMMON/NOPT/IFPP,ALP(3),OXN(3),QXH(3),QXR(3),WAVE(3)

DIMENSION ALP(3),OXN(3),QXH(3),QXR(3),WAVE(3)

C

000002 HMIN=HHR(3,L)
000002 NMAX=HHR(3,L)

000011 IF (HMIN,EQL,NMAX) GO TO 500

000012 DEFF POLYNOMIAL FUNCTIONS

000014 FKNS(K,N) = 1.0

000017 Z=SCALE*Z(1+HUN(K,N)-ZB)

000024 DO 100 K=2,KPOL

000025 100 FKNS(K,N) = ZAK*K(K-1)

C DEFF HARMONIC FUNCTIONS

000045 CHT=2.932E-02
000046 CGAM=2.0/7.0
000050 DZ=2.025
000051 NHID=101

C INITIALIZE

000052 DO 110 K=1,KHR

000054 110 AL(K,NMIN)=0.0

000062 XI(HMIN)=0.0

000064 LINES=60

000065 ITER=1

000066 120 N=NMIN-1

000070 130 N=N+1

000072 IF (N,GTE,NMAX) GO TO 180

000075 IF (N,EQL,NMIN) GO TO 150

C FOR (N,GTE,NMIN) REDEFINE QUANTITIES

000076 HH=HN

000077 DO 140 K=1,KHR

000080 140 OXH(K)=OXN(K)

000081 150 CONTINUE

C CALCULATE NEW SCALE HEIGHT AND ITS DERIVATIVES WRT HEIGHT Z

000086 HN=CHT*SM(3,N)+273.0

000093 H1=CHT*DH(N)

000095 H2=CHT*DHM(N)

000117 PH=HNK*(H1+CGAM)

000118 C CALCULATE NEW WAVELENGTHS AND WAVELENGTHS

000122 DO 160 K=1,KHR

000124 CC=-1.25*PH/HHN(K)

000130 160 OXN(K)=SORT(CC)

000136 IF (ITER,EQL,2) GO TO 200

000140 IF (N,EQL,NMIN) GO TO 130

000142 CC=0.5*DZ

000144 XI(N)=XI(N-1)+CC*((1.0/HH+1.0/HN)

000153 DO 170 K=1,KHR

62.
C
SHIFT XI, PHASES AT END OF FIRST ITERATION

DO 165 K=1,KHR
ALP(K)=AL(K,NMIN)

DO 190 N=NMIN,NMAX
XI(N)=XI(N)-XI

DO 190 K=1,KHR
AL(K,N)=ALP(K)-AL(K,N)

ITER=ITER+1
GO TO 120

C
SECOND ITERATION ONLY

Z=ZFUK (N)

EX=EXP (0.5*XI(N))

PHI=0.5*(PHI+H2*HN**2)

DO 210 K=1,KHR

QXR(K)=PHI/(HN(K)**2)

CC=EX/SQRT(QXR(K))

CC=1.0

M=KPOL+2*K-1

FKN(K,H)=CC*COS(ALP(K,N))

WRITE ((50,K+1),210)

ALP(K)=ALP(K,N)/PI

IF (IFPR;T,EQ,1) GO TO 300

IF (LKES.LT,60)-GO TO 2300

WRITE ((50,K+1),210)

LINE5=LINE5+1

CONTINUE

GO TO 130

END
SUBROUTINE VFILTER

C EXTRACTS SHORT WAVELENGTH NODES
COMMON/REV/INIT,FIV,TAPE,TAPE,PI
COMMON/HEADS/ISER,L,LSMAX,TCB(7),FIV,TLOC,TLOC(14),NRH(14),
  LML(14),HOAY(14),RHO(14),NYE(14),FF1(14),NTP(14),VEHICLE(14) 1
COMMON/DATA1/VVT(3,260),SH(3,260),ESH(3,260),DSM(3,260),D1M(260)
  D2M(260),NHR(3,14),NHX(3,14)
COMMON/PAR/POL,SCALE,KHR,NH(3),NH(3),NH(3),NOPT,HINT
COMMON/ CODE/IF,PRINT,IF,TAPE
COMMON/FLNS/FLN(7,260),AL(2,260),X1(260)
COMMON/PREVS/PREV(3,260),NMIN,NMAX
DIMENSION H(10,10),H1(10,10),U(10),C(10),SV(100),AMP(3),PH(3)
DIMENSION YC(81),ER(81)
DIMENSION Y(260),W(260)
DO 400 I=1,3
  HMIN=HMIN(I,L)
  NMAX=MAX(I,L)
  IF (I:NMIN,E1,NMAX) GO TO 400
  IF (1,EQ,2) GO TO 110
  CC=0.0
  IF (1,EQ,3) CC=273.0
  DO 100 N=NMIN,NMAX
    Y(N)=Y(N)
    IF (ESM(1,N),LT,1.0) ESM(1,N)=1.0
    W(N)=1.0/(ESM(1,N)**2)
  CONTINUE
  WRITE (MOT,130) ISER,L
  130 FORMAT (1H1/314,*) = SERIES, FLIGHT, QUANTITY, 3/X1,1HN,7X,TX,1HZ,6X,
    2&XYC,6X,1HY,6X,2HER,5X,3HSUM,6X,2HYP,2(5X,3HAMP,6X,2HPH) /
  JMAX=4*(HINT+0.01)+1
  KMAX=KPOL+2*KHR
  DMN=JMAX-(KMAX)
  JGO=1
  JMIN=1
  JSMAX=JMAX/2+1
  NLO=NMIN-1
  JLO=NLO+1
  NH1=NLO+JMAX-1
  IF (NH1.LT,JMAX) GO TO 210
  JMIN=JMAX/2+1
  JMAX=JMAX
  JGO=3
  CONTINUE
  210 CONTINUE
C DEFINE/REDEFINE MATRIX H, VECTOR U
  IF (NLO.GT,JMIN) GO TO 250
C FIRST DEFINITION OF H,U
  DO 240 K=1,KMAX
    CC=0.0
    DO 220 JC=1,KMAX
      N=NLO+JN-1
      CC=CC+W(N)*FNN(K,N)*Y(N)
    CONTINUE
    U(K)=CC
    DO 240 N=1,KMAX
  CONTINUE
  WRITE (MOT,130) ISER,L
CC=0.0
DO 230 J=1,JMAX
N=N0+J-1
230 CC=CC+CM(N)*FKN(K,N)*FKN(M,N)
240 H(K,N)=CC
GO TO 270

250 CONTINUE
C REDEF H, U
NF=NL-1
NL=NL+1
DO 260 K=1,KMAX
U(K)=U(K)+W(NL)*FKN(K,NL)*U(NL)+W(NF)*FKN(K,NF)*U(NF)
260 H(K,N)=H(K,N)+W(NL)*FKN(K,NL)*FKN(M,NL)*U(NL)-W(NF)*FKN(K,NF)*FKN(M,NF)
DO 270 K=1,KMAX
270 CONTINUE
C INVERT MATRIX H
MODE=2
CALL ARRAY(MODE,KMAX,KMAX,KK,10,10,SV,H)
CALL MINV(SV,KMAX,KK,DI)
MODE=1
CALL ARRAY(MODE,KMAX,KMAX,KK,10,10,SV,H)
C FIND EXPANSION COEFFICIENTS
DO 290 K=1,KMAX
CC=0.0
DO 280 T=1,KMAX
CC=CC+U(T)*FKN(K,T)
280 C(K)=CC
DO 300 K=1,KMAX
CC=CC+U(K)*FKN(K,N)
300 YC(JN)=CC
ER(JN)=CC-Y(N)
310 SUM=SUM+W(N)*Y(JN)**2
320 SUM=SUM+W(N)*Y(N)**2
330 GO TO 320
340 N=N0+JN-1
350 Z=ZFU(N)
360 YP=YC(JN)+YP
370 N=NL+JN-1
380 Y=YP+YP+C(K)*FKN(K,N)
390 YP=YM(C(K))+YP
400 IF (JGO.EQ.3) GO TO 400
410 JGO=0
1 GO TO 1
N=NL+JN-1
200 NN=NL+JN-1
210 YP=YC(JN)+YP
220 N=NL+JN-1
230 Y=YP+YP+C(K)*FKN(K,N)
240 YP=YM(C(K))+YP
250 IF (JGO.EQ.3) GO TO 400
260 JGO=2
400 CONTINUE
C INVERT MATRIX H
MODE=2
CALL ARRAY(MODE,KMAX,KMAX,KK,10,10,SV,H)
CALL MINV(SV,KMAX,KK,D)
MODE=1
CALL ARRAY(MODE,KMAX,KMAX,KK,10,10,SV,H)
C INVERT MATRIX H
MODE=2
CALL ARRAY(MODE,KMAX,KMAX,KK,10,10,SV,H)
CALL MINV(SV,KMAX,KK,D)
MODE=1
CALL ARRAY(MODE,KMAX,KMAX,KK,10,10,SV,H)
00535  GO TO 200
00535  400 CONTINUE
00537  RETURN
00540  END
APPENDIX D

HARMONIC ANALYSIS OF WIND DATA

Standard harmonic analysis techniques have been applied to the data from the June 1974 series at Wallops Island. The results are of a preliminary nature, in that the vertical resolution techniques have not been applied. Nevertheless, the preliminary results are sufficient to demonstrate the validity of the elements of tidal theory that have been previously mentioned. Of particular interest in the present context is the fact that the eastward (X) wind component lags the northward (Y) wind component by a quarter of a period. Let the temporal behavior of the wind components be expressed analytically as:

\[ X = C_x \cos \omega t + S_x \sin \omega t \]  \hspace{1cm} (D-1)

\[ Y = C_y \cos \omega t + S_y \sin \omega t \]

Then it follows from tidal theory that:

\[ C_y = K \cdot S_x \quad S_y = -K \cdot C_x \]  \hspace{1cm} (D-2)

where \( K \) is a constant determined by the latitude. This result holds for each tidal mode individually.

The correlation implied by Equation (D-2) is evident in Figures D-1 and D-2. Figure 1 displays the coefficients \( C_x, S_x, C_y, \) and \( S_y \) for the diurnal wind component, as determined by harmonic analysis. It can be seen that the correlation between \( C_x \) and \( -S_y \), and between \( C_y \) and \( S_x \), is good, even though the diurnal component is made up of several modes. In Figure D-2(a), the correlation in the semidiurnal component is even more remarkable. Some degree of correlation is also evident in Figure D-2(b) for the terdiurnal component.

Figure D-3 displays the prevailing winds for the eastward (curve (a)) and northward (curve (c)) wind components. Also plotted at 5 kilometer intervals are values of the eastward prevailing wind for July 1 at the latitude of Wallops Island, as listed in the 1965 Cospar Reference Atmosphere. The general agreement between these values and the prevailing wind deduced by harmonic analysis is good. The significance of the eastward trend (curve (b)), is unclear at this time. The same remark applies to the northward prevailing wind and trend (curves (c) and (d), respectively).

In summary it can be stated that the preliminary results of the harmonic analysis are in general agreement with the results of tidal theory.
Figure D-1. Harmonic Coefficients for the Diurnal Wind Component.
Figure D-2. Harmonic Coefficients for the Semidiurnal and Terdiurnal Wind Components.
Figure D-3. Coefficients for the Prevailing and Trend Functions for the Eastward and Northward Wind Components.