Statistical Distribution of Time to Crack Initiation and Initial Crack Size Using Service Data

by

R. A. Heller and J. N. Yang
Professor and Associate Professor

Final Report
NASA NSG. 1099
Supplement 1.
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ACKNOWLEDGEMENTS

The work reported here has been performed in the Department of Engineering Science and Mechanics of Virginia Polytechnic Institute and State University, under Research Grant No. NSG-1099, Supplement 1, sponsored by NASA Langley Research Center, over a two year period between September 15, 1974 and September 14, 1976.

The continued interest of the NASA Technical Officer, Dr. John R. Davidson, Materials Division, is gratefully acknowledged.

The results of this investigation were generated through the joint effort of Mr. William S. Johnson, Mr. Jagdish Shah, Mr. George H. Stevens, Dr. Jann N. Yang and Dr. Robert A. Heller.

ABSTRACT

Crack growth inspection data gathered during the service life of the C-130 "Hercules" airplane is used in conjunction with a crack propagation rule to estimate the distribution of crack initiation times and of initial crack sizes.

Because at early inspections many small cracks are missed a Bayesian statistical approach is used to calculate the fraction of undetected initiation times as a function of the inspection time and the reliability of the inspection procedure used.
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<td>crack length, crack initiation size = .03 in, initial crack size, ultimate crack size</td>
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<td>A, Aᵏ</td>
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<td>f (l/r)</td>
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<td>fₓ(x), f'(x), f''(x)</td>
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<td>r</td>
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<td>fatigue strength</td>
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$S(t)$ laboratory flight hrs

t, $t_i$ inspection time, crack initiation time

$V(\cdot)$ coefficient of variation

$x, y$ variables

$x_0$ Weibull location parameter

$Z_\alpha$ $\alpha$th percentile for normal distribution

$\alpha, \bar{\alpha}$ Weibull shape parameters

$\beta, \bar{\beta}$ Weibull scale parameters

$\beta_1, \beta_2$ moment parameters

$\gamma$ Johnson $S_B$ distribution location parameter

$\Gamma(\cdot)$ Gamma function

$\delta$ Dirac function

$\Delta k$ stress intensity range

$\Delta \sigma$ stress range

$\epsilon$ Johnson $S_B$ distribution shape parameter

$\eta$ Johnson $S_B$ distribution shape parameter

$\lambda$ Johnson $S_B$ distribution shape parameter, Poisson failure rate

$\xi$ severity index
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Section I

INTRODUCTION

The Lockheed C-130 aircraft has been in use by the U.S. Air Force for more than two decades in various theaters of operation under several commands.

The center wing box of the airplane has been periodically inspected and the size of cracks growing around rivet holes in the skin has been recorded. Small cracks were permitted to grow through several inspections. Eventually cracks were repaired or in some cases the whole center wing box was replaced.

The aim of this paper is to reduce the C-130 service-flight inspection data by means of fracture mechanics and statistical analysis into a form from which the probability of time to crack initiation and the distribution of initial flaw sizes for various locations on the aircraft can be determined.

The importance of the time to crack initiation for the purposes of reliability analysis and maintenance schedules for aircraft has been indicated by numerous authors [1-8]. Though such data has been available for laboratory test specimens [9-10] they have not been computed from actual service inspection records for a large number of full scale structures. For the purposes of crack propagation analysis the time to crack initiation, defined here as the time required for a crack to grow to an inspectable size, is not needed if the initial crack size is known [11-15].

Because initial crack sizes are noninspectable they are computed from crack propagation relations by backward extrapolation [11,12].
Previous attempts to define the distribution of time to crack initiation for the C-130 and C-141 aircraft [16-18] have not yielded satisfactory results. Impellizzeri [19] discussed methods to evaluate time to crack initiation for the F-4 aircraft but the lack of sufficient data made the problem difficult. In these reports the data was not normalized, that is, the time at first crack sighting was used as the time of crack initiation regardless of crack size. This type of approach is reasonable only if inspection intervals are short so that all the cracks are small and of approximately the same size at first sighting. It is, however, impractical and uneconomical to perform inspections at such short service intervals [5,6].

The inspection records do not contain cracks smaller than .03 inches (.0762 cm) in length. Therefore, this size has been arbitrarily chosen as the minimum inspection size (crack initiation size) \( a_i \).

Since the objectives of this investigation are to determine the distributions of the times of growth to minimum inspection size (crack initiation) and of initial flaw sizes, crack sizes are normalized with the use of fracture mechanics. A growing crack is extrapolated backwards from its first recorded crack size to .03 inches (minimum inspection size) yielding the time to crack initiation \( t_i \) [6]. In a similar fashion, the crack is grown backwards to a time equal to zero to yield an initial flaw size \( a_0 \).

All of the C-130 crack inspection data examined refer to rivet holes in the center wing box section of the aircraft (Fig. 1). The center wing box was divided into nearly one hundred inspection locations which were symmetric about the center line of the aircraft.
The original inspection data was received from Warner Robbins Air Logistic Center (ALC/ACDCJ), on a nine track magnetic tape.

The following information is available for each aircraft:

- aircraft series
- aircraft serial number
- total flying hours (at time of inspection)
- date of inspection
- military command
- military base
- facility where inspected
- number of inspection
- crack locations
- crack sizes
- crack numbers (there may be more than one crack at a location)

In order to perform the discussed computations, crack growth constants were computed for each individual location and were analyzed statistically.

Two approaches were used to describe the rate of crack growth at various locations. One is the more common practice of fixing the crack growth parameters. The second approach is to let the crack growth parameters vary from location to location even though the material, geometry and type of loading are similar between locations.

Only cracks that grew were utilized in the present analysis. These comprise approximately fifteen percent of the total number of rivet holes and consequently conclusions reached are valid for this reduced population.

Statistical distributions were fitted to time to crack initiation and to initial flaw sizes in order to describe their expected behavior at each location. Since only the cracked population was considered, probabilities are conditional on the presence of a crack [20].
The influence of the modes of operation within the various commands is reflected in the rates of crack propagation indicating more or less severe usage. The data has, as a consequence, been also used to estimate the parameters of the load distributions applied to the airplanes.

Because the method of inspection is not a hundred percent reliable some cracks present in the structure were presumably not discovered. Utilizing an assumed inspection reliability curve combined with a Bayes' approach, the distribution of crack initiation times undetected during an inspection has been estimated [21,22].
Section II
METHODS OF ANALYSIS

1. Time to Crack Initiation and Initial Flaw Size

Inspection of the aircraft took place at irregular intervals. As a result raw inspection data points could not be directly compared because the length of time during which cracks have propagated differed for each crack.

Once the crack propagation parameters have been calculated, cracks could be mathematically grown to either a common size or to a common inspection time. Such a "normalization" procedure permits the determination of statistical distributions for "crack initiation times" defined here as the time at which the crack size was .03 in (minimum inspectable crack length), and for "initial crack sizes", the size of cracks at time equal to zero (Fig. 2).

In order to determine the time to crack initiation (t_i) and the initial flaw size (a_0) the "Power Law" crack growth relation [23]

\[
\frac{da}{dN} = C(\Delta k)^n
\]

was used.

Here \( \Delta k = \Delta \sigma \sqrt{a} f(1/r) \) = Stress Intensity Range

\[
\frac{da}{dN} = \text{crack growth per cycle}
\]

\( C \) & \( n \) = material constants

\( \Delta \sigma \) = stress range

\( a \) = crack length

\( f(1/r) \) = geometric correction factor
Eq. 1 may be rewritten utilizing Eq. 2 and a rate-of-cycling term, dN/dt as

\[
\frac{da}{dt} = C \left( \frac{\Delta \sigma}{\pi a} f(l/r) \right)^n \frac{dN}{dt}
\]

(3)

\[
= C \left( \frac{\Delta \sigma}{\pi} f(1/r) \right)^n \frac{dN}{dt} \cdot a^{n/2}
\]

Letting \( C* = C \left( \frac{\Delta \sigma}{\pi} f(1/r) \right)^n \frac{dN}{dt} \)

(4)

and

\[ n* = n/2 \]

(5)

where \( C* \) is in units of \([\text{in./in.}^{n/2}/\text{flight hours}]\), Eq. 3 becomes

\[
\frac{da}{dt} = C* \cdot a^{n*}
\]

(6)

Eq. 6 is the crack growth relation used in this analysis. Taking the logarithm of Eq. 6 a straight line relation, whose slope is \( n* \) and intercept is \( \log C* \), is obtained.

\[
\log (\frac{da}{dt}) = \log C* + n* \log a
\]

(7)

Several methods were tried in attempts to evaluate the parameters of Eq. 6. To determine the crack growth constants \( C* \) and \( n* \) for each growing crack individually the average crack growth rate between two inspections is calculated as the difference in crack sizes divided by the inspection interval evaluated at the average of the two crack sizes and at the midpoint of the interval. Because two constants are to be determined a minimum of three successive inspections during which a crack grew are required to produce two points for Eq. 7 (Figs. 2 & 3). Not many cracks were permitted to grow through three inspections.
Due to this limitation only a relatively small number of the voluminous data could be utilized to obtain crack growth parameters for individual cracks.

2. **Linear Least Squares Regression**

In order to increase the number of available data points an attempt was made to evaluate crack growth rate versus crack length data for each pair of successive inspections and then to use all of these values at a specific location on the aircraft for individual commands in a linear regression analysis as shown in Fig. 4.

In a first attempt all the \( \frac{da}{dt} \) vs. \( a_{\text{mean}} \) data were utilized in order to find characteristic \( n^* \) and \( C^* \) values for each location. A computer program (Statistical Analysis System was used to perform a linear least squares fit to Eq. 7. The slope and intercept of the line are the \( n^* \) and \( C^* \) values, respectively.

It was found that the exponent, \( n^* \), was lower than what had been anticipated from the examination of aircraft aluminum crack growth curves [11] and varied considerably. An \( n^* \) of approximately 1.5 (\( n=3 \)) had been expected. It is recognized that \( n^* \) should be nearly constant in value since similar material and geometries are used in each aircraft and location. Referring to Eq. 4, \( C^* \) varies with material constant \( C \), material constant \( n \), geometric correction factor \( f(1/r) \) and stress range \( \Delta \sigma \).

The variations in \( n^* \) may be explained as follows: Fig. 5 represents two ideally growing cracks in the same location but on two different aircraft within the same command. The only difference between the two is
that aircraft B has experienced higher stress ranges than aircraft A. Note that both lines have approximately the same slope, n*, but differ in C* (Table 1) reflecting the difference in stress range. On the other hand a linear least squares fit of all data points (six) from both aircraft results in an n* = .92 instead of an average value of 1.01. If each crack is treated individually and average values of n* and C* are calculated less error is introduced.

3. Non-linear Regression

In another attempt to use all available data directly the rate of growth equation (Eq. 6) was integrated

$$\frac{da}{dt} = C*a^{n*}$$

therefore $$a^{-n*} da = C* dt$$

Integrating both sides

$$\int_{a_i}^{a} a^{-n*} da = C* \int_{t_i}^{t} dt$$

$$\frac{1}{1-n*} \left( a (1-n*) - a_i (1-n*) \right) = C* (t - t_i)$$

$$a_i (1-n*) = C* (1-n*) (t - t_i) + a_i (1-n)$$

$$a = \left[ C* (1-n*) (t - t_i) + a_i (1-n*) \right]^{\frac{1}{1-n*}}$$

(8)

where $t_i$ = time to crack initiation

$a_i$ = crack length at crack initiation = .03 in. is obtained (Fig. 6).

By assuming certain values of n* Eq. 8 may be simplified:
for \( n^* = 2 \)

\[
\frac{1}{a} = -C^*(t-t_1) + \frac{1}{a_i}
\]

\[
\frac{1}{a} = C_1 t + C_2
\]  \hspace{1cm} (9)

where \( C_1 = -C^* \)

\[
C_2 = C^* t_i + \frac{1}{a_i}
\]

For \( n^* = 1.5 \)

\[
\frac{1}{a} = \left[ -\frac{C}{2} (t-t_0) + \frac{1}{\sqrt{a_0}} \right]^2
\]

carrying out the operations and factoring yields

\[
\frac{1}{a} = C_1 t^2 + C_2 t + C_3
\]  \hspace{1cm} (10)

where

\[
C_1 = \frac{C^*^2}{4} ; C_2 = \frac{C^*}{\sqrt{a_i}} - \frac{C^*^2}{2} t_i
\]

\[
C_3 = \frac{C^*^2}{4} t_i^2 - \frac{C^* t_i}{\sqrt{a_i}} + \frac{1}{a_i}
\]

Equations 9 and 10 were evaluated again using regression analysis for all \( a \) versus \( t \) data for a given location in each command (Fig. 6). Results were again unsatisfactory because \( n^* \) values were afflicted with a large scatter.
4. **Conjugate Gradients Method**

The next approach was an attempt to minimize the square difference between observed and predicted crack length by the method of conjugate gradients.

The function to be minimized may be written as

\[
F = \sum_{k=1}^{N} (A_k - a_k)^2 = \sum_{k=1}^{N} \left( A_k - \left[ a_i (1-n^*) + C^* (1-n^*) (t_k - t_i) \right] \right)^2
\]

where
- \( A_k \) = observed crack length
- \( a_k \) = predicted crack length
- \( t_k \) = observed time

To minimize the function, its partial derivatives

\[
\frac{\partial F}{\partial C^*} = \sum_{k=1}^{N} -2 (t_k - t_i) \left[ A_k - B_k \left( \frac{1}{1-n^*} \right) \right] B_k \left( \frac{n^*}{1-n^*} \right)
\]

\[
\frac{\partial F}{\partial t_i} = 2C^* \left[ A_k - B_k \left( \frac{1}{1-n^*} \right) \right] B_k \left( \frac{n^*}{1-n^*} \right)
\]

\[
\frac{\partial F}{\partial n^*} = \sum_{k=1}^{N} \frac{2n^*}{(1-n^*)^2} \left[ A_k - B_k \left( \frac{1}{1-n^*} \right) \right] B_k \left( \frac{1}{1-n^*} \right) \ln B_k \left[ a_i (1-n^*) \ln a_i + C^*(t_k - t_i) \right]
\]

are set equal to zero and the nonlinear equations, Eqs. 12-14 are solved simultaneously.
In the above relations

\[ B_k = a_i (1-n^*) + C^*(1-n^*) (t_k-t_i) \]

The gradients, \( \frac{\partial F}{\partial C^*} \), \( \frac{\partial F}{\partial t_i} \), and \( \frac{\partial F}{\partial n^*} \), were used in a computer program (SSP subroutine FMCG). It was not possible to set bounding values in the subroutine. Therefore, certain physical limitations, such as positive crack length, were violated. The term, \( B_k (1-n^*) \), represents crack length and was in some cases forced into the negative region by the FMCG program in order to minimize the function. The method was consequently abandoned.

5. Variable \( n^* \) Method

Crack growth parameters for individual cracks were also calculated and were then averaged to find representative \( C^* \) and \( n^* \) values.

For this method only those \( da/dt \) versus \( a_{\text{mean}} \) data were used in which a crack grew through at least two inspections (three points on an \( a \) versus \( t \) curve). Since stress ranges could vary greatly from one inspection to another, not all \( n^* \) and \( C^* \) values calculated were acceptable. Therefore, limitations were set on \( n^* \) (1.0 < \( n^* \) < 2.5), and on \( C^* \) (0.05 > \( C^* \)). Many of the cracks that grew sufficiently to be investigated yielded \( n^* \) and \( C^* \) outside the acceptable range.

The acceptable parameters for cracks within a given location and command were averaged to yield mean values of \( \bar{n^*} \) and \( \bar{C^*} \). These average parameters were then used to extrapolate all cracks that grew, from their first sightings backwards to the time of crack initiation (\( t_i \)) and to initial flaw size (\( a_0 \)).

Table 2 lists the raw data of crack length and inspection times for the relevant locations.
This method yielded mean values of n* from approximately 1.2 to 1.9 (Table 3). When these n* values were used to calculate $a_0$, the initial flaw size varied with n*; a lower n* produced a smaller $a_0$ (see Figs. 7 and 8). Since initial flaw size in an aircraft should be independent of the life of the aircraft, the values of $a_0$ at various locations should be of the same order of magnitude. These results indicated that n* should indeed be a fixed material property.

6. Constant n* Method

A value for the material property n was selected after evaluating crack growth data on 7075 Aluminum from the Rockwell International Fracture Mechanics Data Bank [9] and comparing this to a NASA Langley study on laboratory crack growth of C-130 components [10]. Based on these studies, n=3, i.e., n*=1.5, was chosen.

With n* established, only one additional parameter needs to be determined, hence individual crack growth data, da/dt versus $a_{\text{mean}}$ could be used (Fig. 9). It has been mentioned previously that when n* and C* were both allowed to vary, the values of n* and C* were frequently outside an acceptable range. By fixing n* and letting only C* vary, the results yielded a reasonable C* value every time. Due to the increase in the number of reasonable C*'s for all crack locations, more crack locations could be evaluated.

The values of C* for the various growing cracks within a location were averaged to yield a $\overline{C*}$. This $\overline{C*}$ value and n*=1.5 were used to extrapolate the $a$ versus $t$ data of the first crack that grew at a particular location.
on an aircraft. The resulting values of $t_i$ and $a_0$ are shown in Table 3. It might be noted that all the $a_0$ values for the various locations were, as expected, of the same order of magnitude (see Section III.2).
Section III
STATISTICAL DESCRIPTION OF THE DATA

Statistical distribution functions have been fitted to the times to crack initiation and to initial flaw sizes. The probability of time to crack initiation will be discussed first.

1. Time to Crack Initiation Distributions
   a. Two Parameter Weibull Distribution

   Time to crack initiation (or failure) has in the past been described in terms of the Weibull distribution [24]. The two parameter Weibull probability of failure is given as
   \[ F_{t_i}(x) = 1 - e^{-\left(\frac{x}{\beta}\right)^\alpha} \]  
   (15)
   where  \( \alpha \approx 1.2 \) \( (\sigma/\bar{x}) \) is the shape parameter
   \( \beta = \bar{x} \Gamma(1 + 1/\alpha) \) is the scale parameter
   \( \bar{x} \) = mean
   \( \sigma \) = standard deviation

   This distribution fits most of the data but fails to fit the end points (see Figs. 10 through 17 and Table 4). The lower portion of the data (earliest crack initiation) is considered extremely important, therefore it is necessary to try other statistical models that might fit these points better.

   b. Three Parameter Weibull Distribution

   The three parameter Weibull distribution
\[
F_{t_1}(x) = 1 - e^{-\left(\frac{x-x_0}{\beta-x_0}\right)}
\]  \hspace{1cm} (16)

has also been used. Because of the possible existence of flaws at time \(t=0\) this distribution function will have a negative lower limit, \(x_0\) and Eq. 16 will have to be restricted to values for \(t_i > 0\). Consequently the distribution of crack initiation times becomes:

\[
F_{t_i}(x) = 0 \quad \text{for} \quad x < 0 \quad (17a)
\]
\[
F_{t_i}(0) = 1 - e^{-\left(\frac{-x_0}{\beta-x_0}\right)} \quad \text{for} \quad x = 0 \quad (17b)
\]

and

\[
F_{t_i}(x) = 1 - e^{-\left(\frac{x-x_0}{\beta-x_0}\right)} \quad \text{for} \quad x > 0 \quad (17c)
\]

that is, the probability that the crack initiation time is zero has a fixed, nonzero value. The corresponding density function consists of a Dirac function at \(t_i = 0\) with an area given by Eq. 17b and a truncated density function [21]. These functions are schematically shown in Figs. 18 and 19.

Such distributions have also been fitted to the data. There is significant improvement over the fit of the two parameter function, but for very short initiation times this distribution does not exhibit a good fit either. The distributions are presented in Figs. 20-27 and Table 5.

c. Johnson Distributions

Johnson distributions were examined next, due to the fact that the Johnson \(S_B\) distribution has four adjustable parameters [24]. In
order to determine which one of three types of Johnson distributions would be applicable, values of $\beta_1$ and $\beta_2$ were calculated for each data set.

$$\beta_1 = \left( \frac{m_3}{(m_2)^{3/2}} \right)^2$$

$$\beta_2 = \frac{m_4}{(m_2)^2}$$

where $m_2$, $m_3$, and $m_4$ are second, third and fourth moments of the data, respectively:

$$m_2 = \frac{1}{n} \sum_{i=1}^{n} (x_i - \bar{x})^2$$

$$m_3 = \frac{1}{n} \sum_{i=1}^{n} (x_i - \bar{x})^3$$

$$m_4 = \frac{1}{n} \sum_{i=1}^{n} (x_i - \bar{x})^4$$

From an examination of Table 4 and Fig. 28 [24] it is noted that the Johnson $S_L$ or $S_B$ distributions apply.

The $S_B$ distribution has two shape parameters, $\gamma$ and $\eta$, one location parameter (lower bound), $\epsilon$, and one scale parameter, $\lambda$. The distribution is bounded by $\epsilon$ and $\epsilon+\lambda$. In the case of time to crack initiation, the range, $\lambda$, is unknown but can be estimated by the following equation:

$$\lambda = (x_{0.5-\epsilon}) \frac{(x_{0.5-\epsilon}) (x_{1-\alpha-\epsilon}) + (x_{0.5-\epsilon}) (x_{1-\alpha-\epsilon})-2 (x_{\alpha-\epsilon}) (x_{1-\alpha-\epsilon})}{(x_{0.5-\epsilon})^2 - (x_{\alpha-\epsilon}) (x_{1-\alpha-\epsilon})}$$
where $x_{0.5}$ = median of the data

$x_{\alpha}$ = $\alpha$ percentile of data

$x_{1-\alpha}$ = (1-$\alpha$) percentile of data

Since primary interest lies in fitting the extreme data points, $\alpha = 1\%$ was used in most cases. More extreme values were often necessary to assure that $\lambda$ is positive. In some cases the term $(x_{0.5} - \varepsilon)^2 - (x_{\alpha} - \varepsilon)(x_{1-\alpha} - \varepsilon)$ becomes negative if extreme values are not chosen. In the case of location 74-90 TAC, an $x_{\alpha}$ value had to be extrapolated below the lowest data point to obtain a positive $\lambda$.

Once $\varepsilon$ and $\lambda$ are established, the two shape parameters may be calculated as follows:

\begin{equation}
\eta = \frac{Z_{1-\alpha} - Z_{\alpha}}{\ln \left[ \frac{(x_{1-\alpha} - \varepsilon)(\varepsilon + \lambda - x_{\alpha})}{(x_{\alpha} - \varepsilon)(\varepsilon + \lambda - x_{1-\alpha})} \right]} \tag{24}
\end{equation}

\begin{equation}
\gamma = Z_{1-\alpha} - \eta \ln \left( \frac{x_{1-\alpha} - \varepsilon}{\varepsilon + \lambda - x_{1-\alpha}} \right) \tag{25}
\end{equation}

where $Z_{\alpha}$ = the standard normal variate for $\alpha$ 100th percentile

$Z_{1-\alpha}$ = the standard normal variate for (1-$\alpha$) 100th percentile

Once $\varepsilon$, $\lambda$, $\eta$, and $\gamma$ have been calculated for each set of data, the following density function can be numerically integrated to find the distribution function

\begin{equation}
f_{t_1}(x) = \frac{\eta}{\sqrt{2\pi}} \frac{\lambda}{(x - \varepsilon)(\lambda - x + \varepsilon)} \exp \left\{ -\frac{1}{2} \left[ \gamma + \eta \ln \left( \frac{x - \varepsilon}{\lambda - x + \varepsilon} \right) \right]^2 \right\} \tag{26}
\end{equation}
The results of the Johnson $S_B$ distribution for time to crack initiation are shown in Figs. 10 to 17 and the parameters for each location are presented in Table 4.

Because $x_\alpha$ and $x_{1-\alpha}$ may be arbitrarily chosen this family of distribution functions can be made to fit the short initiation time data points, but at the expense of missing the large number of data points in the central region.

d. Statistical Distribution Based on Service Load and Fatigue Performance

Because none of the afore discussed distribution functions fit the data equally well in the central region and the extremes, a technique based essentially on the concept of stress-strength interference will also be examined [6].

It is assumed that the fatigue strength, $R$, of the structure in a laboratory test can be described by a three parameter Weibull distribution. The probability that the fatigue life is shorter than a given value, $x$ is written as

$$F_R(x) = 1 - e^{-\left(\frac{x-x_0}{\beta-x_0}\right)^\alpha}$$

where $R$ and hence $x$ is measured in laboratory hours.

The statistical density function of the number of gusts that produce significant fatigue damage in the aircraft in $t$ flying hours has been described by Yang [6] as a continuous version of the Poisson process

$$f_N(t)(x_1) = (\lambda t)^{x_1} e^{-\lambda t \Gamma(x_1+1)} \quad 0 < x_1 < \infty$$

where the random variable, $N(t)$, is the number of gusts in $t$ flight hrs,
\(\lambda\) is the average number of significant gusts per flight hr, \(\lambda t\) is the average number in \(t\) flight hrs and \(\Gamma(\cdot)\) is the gamma function.

Because the variance of the Poisson distribution is also \(\lambda t\), the coefficient of variation which is a measure of dispersion becomes

\[
V_{N(t)} = 1/\sqrt{\lambda t}
\]  

(29)

The number of gusts \(N(t)\) experienced by an aircraft in \(t\) flight hrs can be related to the number of equivalent laboratory hrs, \(S(t)\) as

\[
S(t) = N(t)/\lambda \xi
\]  

(30)

where \(\xi\) is a so-called severity index that in the present analysis is assumed to be unity. As a consequence the laboratory spectrum is equivalent to the average flight spectrum.

The density function, of gust loading, Eq. 28 can be transformed with the aid of Eq. 30 into a density function in terms of equivalent laboratory flight hours

\[
f_{S(t)}(x) = \lambda t^x e^{-\lambda t} \frac{\lambda^x}{\Gamma(\lambda x+1)}
\]  

(31)

The probability that a crack will initiate in \(t\) flight hours is equivalent to the probability that the fatigue strength, \(R\), is exceeded by the service loading converted to laboratory flight hrs, \(S(t)\)
Eq. 32 is a form of the classical stress-strength interference relation [25] and can be expressed as

\[ F_T(t) = \int_0^\infty F_R(x) f_S(t)(x) \, dx \]  

(33)

Where \( F_R(x) \) is given by Eq. 27 and \( f_S(t)(x) \) by Eq. 20. Therefore

\[
F_T(t) = \int_0^\infty \left[ 1 - e^{-\left(\frac{x-x_0}{\beta-x_0}\right)^\alpha} \right] \frac{(\lambda t)^{\frac{x-x_0}{\beta-x_0}} e^{-\lambda t}}{\Gamma(\lambda t+1)} \, dx
\]

(34)

For convenience of integration a change of variables is introduced:

\[ y = \lambda x \quad , \quad dy = \lambda dx \]

(35)

Substituting into Eq. 34 the probability of failure becomes

\[
F_T(t) = \int_0^\infty \left[ 1 - e^{-\left(\frac{y-x_0^\lambda}{\beta-x_0^\lambda}\right)} \right] \frac{(\lambda t)^y e^{-\lambda t}}{\Gamma(y+1)} \, dy
\]

(36)

Eq. 36 is a convolution integral that requires numerical integration. The values of \( \alpha, \beta \) and \( x_0 \) are estimated by fitting a three parameter Weibull distribution to the central part of the crack initiation time data (Table 5).

Because \( 1/\sqrt{\lambda \beta} \) is a measure of the dispersion in \( \beta \) hrs, a small value of \( \lambda \) produces a large dispersion. Choosing \( \lambda \) values by trial and error (\( \lambda \beta \) is usually less than 100) Eq. 36 is evaluated and is
fitted to the crack initiation time data so that the curve approaches the data in the lower tail (short initiation times) of the distribution as shown in Figs. 29 to 36.

Smaller values of $\lambda$ tend to bend the lower tail upward and the upper tail downward.

Because short crack initiation times are of interest, the poorer fit at the upper tail is not considered significant.

2. Initial Flaw Size Distributions

It is not unexpected that material imperfections, manufacturing methods and the limitations of inspection techniques produce a certain number of undetected minute cracks even before the aircraft is put into service. Once the average parameters of the crack growth relation are evaluated and $t_i$ is calculated for each crack, Eq. 8 can be further extrapolated backwards to time $t=0$ to yield an initial crack size as shown in Fig. 2. Only data with extrapolated initial crack sizes greater than zero can be used.

Initial flaw sizes were plotted on normal probability paper. The plots indicated that the distribution functions were not normal; the data did not fall on straight lines. As an alternative the Johnson $S_B$ distribution was again employed. Since .03 inches (.0762 cm) had been chosen as the crack initiation size, the nondefect population (those locations with initial flaws less than .03 inches) ranges from 0.0 to 0.03 inches. Thus, $\varepsilon = 0.0$ and $\lambda = 0.03$. The shape parameters $\eta$ and $\lambda$, were found as previously shown from Eqs. 24 and 25 with $\alpha = 1-2\%$. 
The parameters are listed in Table 6. Distribution functions, are 
presented in Figs. 37 to 44. It is seen that the Johnson \( S_B \) distri-
bution fits these extrapolated points very well. Because initial 
flaw sizes should be independent of loading their distributions 
should be similar. It is seen from Table 6 that the means and 
standard deviations are of similar magnitude. As a consequence all 
initial crack sizes could be considered to belong to the same 
population. An overall distribution function has been derived for 
the combined data and is presented in Fig. 45. The mean initial 
crack size is .00432 in. and has a coefficient of variation of .2%.

The two Commands considered here, TAC (Tactical Air Command and 
PACAF (Pacific Air Command), had 263 and 223 C-130 Aircraft inspected 
respectively.

Since two symmetric locations and two holes at each location 
were all considered to be identical the total number of potential crack 
sites at one location is 1052 and 892 for TAC and PACAF respectively. 
Of these 16% in TAC and 30% in PACAF have actually developed growing 
cracks at the most vulnerable location (75-91, see Table 3). Aircraft 
operated by PACAF have a greater proportion of growing cracks at all 
locations while aircraft in the Tactical Air Command developed fewer 
inspectable and hence extrapolatable cracks. This fact would indicate 
that PACAF aircraft have been subjected to more severe loading condi-
tions. Such a conclusion is not always born out by the data shown in 
Table 3. The value of \( C^* \) for locations 75-91 is greater for PACAF 
than for TAC but at the same time other locations indicate an opposite 
trend.
Section IV

THE RELIABILITY OF INSPECTION

1. Distributions of Detected and Undetected Cracks

The probability that an existing crack is detected is a function of a number of circumstances. These include: a.) The method of detection such as radiography, thermography, dye penetrant, ultrasonic or visual inspection, etc., and b.) The sensitivity and accuracy of the instrument or the alertness of the inspector. These conditions are influenced primarily by the size of the crack at the time of inspection.

As a consequence not all cracks are found and the probability, \( P[D|a] \), that a crack is detected, given that its length is \( a \), is a function of the crack size.

The methods of inspection or the reliability of the procedure used for crack detection in C-130 airplanes are not available. Data can however be found in the literature on the probability of crack detection in aluminum using various ultrasonic techniques [26]. A set of data from Ref. 26 is shown in Fig. 46 with a Weibull type probability function

\[
P[D|a] = 1 - e^{-\left(\frac{a}{\bar{a}}\right)^\alpha},
\]

where \( \bar{\alpha} = 2.0 \) and \( \bar{\beta} = .075 \) in. (1.19 cm), fitted to it. Similar probability functions have been incorporated in wing failure analyses and have been reviewed by Eggwerts [27].

The C-130 inspection data indicates that cracks shorter than .03 in have not been found. The assumed probability function used here predicts
that a small fraction, 15%, of such cracks are located, while the probability of detecting a crack .2 in. in length is almost unity.

If the distribution of crack sizes prior to inspection is known, the distributions of detected and undetected cracks may be calculated utilizing the probability of detection function, Eq. 37 in conjunction with Bayes theorem [28,29].

The probability that a flaw of size, \(a\), will be between \(x\) and \(x+dx\) given that the flaw is detected may be written as

\[
P(x<a<(x+dx)|D) = \frac{P[D|x<a<(x+dx)]P[x<a<(x+dx)]}{P[D]} \tag{38}
\]

where \(D\) stands for detection, \(P[D|x<a<(x+dx)]\) is the probability of detecting a flaw of size \(a\) (from Eq. 37), \(P[x<a<(x+dx)]\) is the probability that the crack size is \(a\) and \(P[D]\) is the total probability of detection independently of size. Eq. 38 may be expressed in terms of density functions as

\[
f_a(x|D) = k_1 P[D|a] f_a(x) \tag{39}
\]

in which \(f_a(x)\) is the density function of flaw sizes prior to detection, \(f_a(x|D)\) is the density function of detected flaw sizes (posterior density), \(P[D|a]\) defined earlier, is called the likelihood function in Bayes' theorem terminology and \(k_1 = 1/P[D]\) is a normalizing constant that will make the area under the \(f_a(x|D)\) curve equal to unity.

In a similar manner the density function of undetected cracks may also be derived:

\[
f_a(x|\overline{D}) = k_2 P[\overline{D}|x] f_a(x) \tag{40}
\]
where $\overline{D}$ stands for no detection, $P[D|x] = 1 - P[D|x]$, and $k_2$ is a second normalizing constant. The above technique has been utilized by Davidson [21] in a slightly different form with exponential detectability and crack size distribution functions assumed.

Adding Eqs. 39 and 40

$$\frac{f_a(x|D)}{k_1} + \frac{f_a(x|\overline{D})}{k_2} = f_a(x) \quad (41)$$

indicates that the reciprocal normalizing constants, $1/k_1$ and $1/k_2$ represent the fractions of detected and undetected cracks.

Because the density function of cracks before inspection is usually unknown, the posterior function, $f_a[x|D]$, of detected cracks may be used to evaluate the prior, $f_a(x)$, and the density of undetected cracks, $f_a[x|\overline{D}]$, from Eqs. 39 and 40 [29].

As indicated by Eq. 8 crack size is a function of the parameters $a_i$ and $t_i$ and the time of inspection $t$. As a result Eqs. 39-40 are also dependent on these quantities. Since inspections were carried out at irregular intervals the above relations could only be utilized if detected cracks were mathematically grown to a common inspection time. Alternatively Eqs. 38-41 can be transformed to yield density functions of crack initiation times, $t_i$, at a common crack size, $a_i = .03$ in (.76 cm).

2. Distributions of Detected and Undetected Crack Initiation Times

Eqs. 39 and 40 may be rewritten in terms of crack initiation times, $t_i$ as

$$f_{t_i}(x|D) = k_3 P[D|t_i] f_{t_i}(x) \quad (42)$$
and

\[
f_{t_i}(x) = k_4 \left(1 - P[D|t_i]\right)f_{t_i}(x)
\]

(43)

where \(f_{t_i}(x|D)\) and \(f_{t_i}(x|\bar{D})\) are the density functions of detected and undetected crack initiation times, \(P[D|t_i]\) is the likelihood function for detection and \(f_{t_i}(x)\) is the prior density function of all crack initiation times.

\(P[D|t_i]\) is derived from Eq. 37 by substitution of Eq. 8. Hence

\[
P[D|t_i] = 1 - \exp\left\{-\frac{1}{\beta^*} \left[\alpha^*(1-n^*)(t-t_i) + a_i(1-n^*)\right]^{\frac{\alpha}{1-n^*}}\right\}
\]

(44)

It should be noted that Eq. 44 becomes equal to unity when

\[
(t - t_i) = -\frac{a_i(1-n^*)}{C^*(1-n^*)}
\]

(45)

because within this time interval a crack will grow to infinite length. Hence \(P[D|t_i]\) will be equal to Eq. 44 for \((t-t_i) < -[a_i(1-n^*)/C^*(1-n^*)]\) and becomes equal to unity for \((t-t_i) \geq -[a_i(1-n^*)/C^*(1-n^*)]\), which indicates that the probability of detecting an initiation time, \(t_i\), depends on the time of inspection, \(t\). As a consequence the density functions of detected and undetected initiation times will also become functions of the inspection time while the density function of all initiation times should be independent of when the inspection is carried out.

The distributions of \(t_i\) described in Section II were derived from long-term records, in some instances covering 7000 hrs of flying time.
It is therefore not unreasonable to assume that the great majority of cracks have grown to inspectable size and that distributions of crack initiation times derived from these by backward extrapolation are good approximations of the total initiation time distributions. Such an assumption is conservative because long initiation times (small cracks) make up the majority of still undetected initiations.

With this assumption it is possible to determine the density functions of discovered and undiscovered initiation times for an inspection carried out earlier in the life of the structure.

The three parameter Weibull distributions, Eqs. 17, fitted to the derived data of Section II (the convolution integral representation is mathematically more complicated)

\[
f_{t_1}(x) = \left[1 - e^{-\left(\frac{x-x_0}{\beta-x_0}\right)^\alpha}\right] \delta(x) + \frac{\alpha}{\beta-x_0} \left(\frac{x-x_0}{\beta-x_0}\right)^{\alpha-1} e^{-\left(\frac{x-x_0}{\beta-x_0}\right)^\alpha}\right]
\]

will be used where \(\delta(x)\) is a Dirac function.

For specified inspection time, \(t\), crack growth parameters \(C^*\) and \(n^*\) and \(a_i = .03\) in., Eqs. 44 and 46 substituted into Eqs. 42 and 43 yield the density functions of detected and undetected initiation times. As in the case of detected and undiscovered cracks the reciprocal normalizing coefficients add to unity

\[
\frac{1}{k_3} + \frac{1}{k_4} = 1
\]

and indicate the fractions of discovered and undiscovered initiation times.
The results of such calculations are shown at location 76-92, TAC Command in Figs. 47-49 for inspection times \( t = 2500, 4000, \) and 5000 hrs. The parameters of the distribution functions are presented in Table 7.

It should be noted that the density functions for detected and undetected initiation times have been divided by their respective normalizing constants so that the areas under the two curves are proportional to the fractions of discovered and undiscovered initiation times and the two areas sum up to the area of the density function for all initiation times.

An examination of Figs. 47-49 indicates an expected trend. If inspection is performed too early, the majority of initiation times will remain undetected while more and more crack initiation times are found at later inspections.

The aforegoing discussion has utilized longterm inspection data to derive the prior distribution of crack initiation times. Such records are however usually unavailable for aircraft in service. On the contrary at a particular inspection time the distribution of observed crack initiation times is actually the posterior distribution, \( f_{t_i}(x|D) \), while the prior density, \( f_{t_i}(x) \), of all initiation times and the density function of undetected initiation times, \( f_{t_i}(x|D) \), are unknown.

Because the latter two functions are of primary interest Eqs. 42 and 43 may be rewritten using Eq. 47 as

\[
f_{t_i}(x) = \frac{f_{t_i}(x|D)}{K_3 P[D|t_i]} \tag{48}
\]

and

\[
f_{t_i}(x|D) = \frac{(1-P[D|t_i])f_{t_i}(x|D)}{(K_3-1) P[D|t_i]} \tag{49}
\]
These equations would predict the density functions of all crack initiation times as well as that of the undetected times provided that inspections are carried out at approximately the same number of flight hours on all aircraft in a fleet.
Section V

CONCLUSIONS

The methods presented indicate the feasibility of determining the statistical distributions of initial crack sizes and times to crack initiation using backward extrapolation of inspection data. It appears that of the two crack propagation parameters the value of $n^*$ is a material constant with a value of $n^* = 1.5$ while the second parameter $C^*$, in addition to being dependent on material and geometry, is also a function of the stress range.

Evaluation of $C^*$ for each individual growing crack and averaging such values for specific crack locations and military commands yields satisfactory distributions of crack initiation times and initial crack sizes.

Crack initiation times are dependent on both the initial crack size and on $C^*$, the latter being dependent on location and load, while the distribution of initial crack sizes is independent of these factors.

The three parameter Weibull distribution fits the crack initiation times reasonably well. The convolution integral, that involves the three parameter Weibull distribution of fatigue strength and the Poisson distribution of gust occurrences does not improve the fit appreciably because this method has been developed for the purpose of evaluating laboratory fatigue tests under simulated load spectra rather than for the analysis of service data. Two parameter Weibull or Johnson $S_B$ distributions do not fit the data very well.
The distributions of initial crack sizes on the other hand are well represented by the Johnson $S_B$ distribution. The methods presented permit evaluation of a single initial crack size distribution for all growing cracks independently of the different loads in the two commands or the location of cracks on the wing. Though only four locations were analyzed it is believed that the distribution of initial crack sizes is representative of other locations as well. The information can consequently be used for the prediction of crack growth for the C-130 aircraft as well as for structures made of the same material with similar geometry and having undergone similar manufacturing processes. It is recognized that the distributions of time to crack initiation and initial crack size are not independent of each other but are interrelated through Eq. 8. Consequently a probability transformation can be performed on one of the two distributions to derive the density function of the other. It is obvious that such a transformation will result in distribution functions different from those presented here though it is expected that the derived function would be well approximated by the fitted curves. A transformation will be carried out at a future date.

The Bayesian approach presented here illustrates a method for estimating the proportion of undiscovered crack initiation times at various inspections and indicates the influence of the reliability of crack detection methodology particularly during early inspections when cracks are small.

Because the information concerning the reliability of the particular method of nondestructive examination used for the C-130 aircraft was not available an assumed likelihood function was used. For future systems
the documentation of NDE reliability should become an integral part of service inspection.

A similar approach may be used to determine the fraction of undetected crack sizes at a particular inspection time. For this purpose cracks would have to be analytically grown to common inspection times and their distributions analyzed. Such work may also be carried out in the future.

The present analyses have utilized average values of the crack propagation parameter C*. The statistical distributions of the parameter have not been evaluated. Again calculations, with C* as a random variable, may be performed at a later date.
References


TABLE 1
Crack Growth Constants for Two Hypothetical Aircraft

<table>
<thead>
<tr>
<th>Aircraft A</th>
<th>Aircraft B</th>
<th>Least Squares Fit</th>
<th>Average of A and B</th>
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<tr>
<td>( n^* = 1.02 )</td>
<td>( n^* = 1.01 )</td>
<td>( n^* = 0.92 )</td>
<td>( n^* = 1.015 )</td>
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<td>( C^* = 0.008 )</td>
<td>( C^* = 0.016 )</td>
<td>( C^* = 0.01 )</td>
<td>( C^* = 0.012 )</td>
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TABLE 2
C-130 Inspection Data for Selected Locations
(Continued p. 38-60)

Explanation of columns 1-8

<p>| Column 1. | Aircraft Identification Number |
| Column 2. | Location of Rivet Hole (see Fig. 1) |
| Column 3. | Two Holes at Each Location: (I) Inboard and (O) Outboard |
| Column 4. | Direction of Crack Growth: (F) Forward and (A) Aft |
| Column 5. | Crack No. at Rivet Hole. Only First Growing Crack Used |
| Column 6. | User Command (P) Pacific Air Command, PACAF; (T) Tactical Air Command, TAC |
| Column 7. | Length of Crack in Hundredths of Inches |
| Column 8. | Inspection Time in Flight Hours |</p>
<table>
<thead>
<tr>
<th>Location</th>
<th>n*</th>
<th>C*</th>
<th>t_i hrs.</th>
<th>$\bar{a}_0$ in.</th>
<th>n*</th>
<th>C*</th>
<th>t_i</th>
<th>$a_0$</th>
<th>No. of Data Pts.</th>
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<td>0.00537</td>
<td>3385.</td>
<td>0.00527</td>
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<td>0.750</td>
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<td>0.352</td>
<td>0.934</td>
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<tr>
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<td>0.00525</td>
<td>3750</td>
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<tr>
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<td>Location</td>
<td>Weibull Distribution</td>
<td>Johnson $S_B$ Distribution</td>
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<td></td>
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<td>Location</td>
<td>$\alpha$</td>
<td>$\beta$</td>
<td>$\beta_1$</td>
<td>$\beta_2$</td>
<td>$\epsilon$</td>
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TABLE 5

Statistical Parameters for Fitting Time to Crack Initiation
Three Parameter Weibull Distribution and Convolution

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<th>Location</th>
<th>Location</th>
<th>Three Parameter Weibull Distribution</th>
<th>Convolution</th>
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<td>β</td>
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<td>TAC</td>
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<td>3400</td>
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<td>3750</td>
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<td>3500</td>
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<td>PACAF</td>
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<td>PACAF</td>
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<td>3950</td>
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<td>Location</td>
<td>Mean</td>
<td>Standard Deviation</td>
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<td>76 92 TAC</td>
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<td>Over all</td>
<td>0.00431</td>
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TABLE 7

Normalizing Constants and Percentage of Detected and Undetected Crack Initiation Times

<table>
<thead>
<tr>
<th>Inspection Time t hours</th>
<th>( k_3 )</th>
<th>% Detected</th>
<th>( k_4 )</th>
<th>% Undetected</th>
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<tr>
<td>2500</td>
<td>16.60</td>
<td>6.02</td>
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<td>4000</td>
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<td>5000</td>
<td>1.01</td>
<td>98.56</td>
<td>69.34</td>
<td>1.44</td>
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</table>
Fig. 1. Location of Investigated Cracks on C-130 Aircraft.
Fig. 2. Extrapolation of Inspection Data.
Fig. 3. Determination of Crack Growth Parameters. Variable $n^*$ Method.
Fig. 4. Determination of Crack Growth Parameters. Linear Regression Method.
Fig. 5. Two Ideally Growing Cracks.
Fig. 6. Determination of Crack Growth Parameters. Nonlinear Regression Method.
INITIAL FLAW SIZE - $a_0$

Fig. 7. Histogram of Initial Flaw Size Using a Larger $n^*$. 

76-92 TAC

$n^* = 1.86$

--- Johnson $S_B$ Distribution

137 Data Points
Fig. 8. Histogram of Initial Flaw Size Using a Smaller $n^*$. 

74-90 PACAF

$n^* = 1.3$

Johnson $S_B$ Distribution

148 Data Points

INITIAL FLAW SIZE - $a_0$
Fig. 9. Determination of Crack Growth Parameters. Constant n* Method.
Fig. 10. Distribution of Crack Initiation Times. Two Parameter Weibull and Johnson $S_B$ Distributions.
Fig. 11. Distribution of Crack Initiation Times. Two Parameter Weibull and Johnson SB Distributions.
Fig. 12. Distribution of Crack Initiation Times. Two Parameter Weibull and Johnson SB Distributions.
Fig. 13. Distribution of Crack Initiation Times. Two Parameter Weibull and Johnson $S_B$ Distributions.
Fig. 14. Distribution of Crack Initiation Times. Two Parameter Weibull and Johnson $S_B$ Distributions.
Fig. 15. Distribution of Crack Initiation Times. Two Parameter Weibull and Johnson $S_B$ Distributions.
Fig. 16. Distribution of Crack Initiation Times. Two Parameter Weibull and Johnson $S_B$ Distributions.
Fig. 17. Distribution of Time to Crack Initiation. Two Parameter Weibull and Johnson $S_B$ Distributions.
Fig. 18. Three Parameter Weibull Density Function of Crack Initiation Times.

\[ f_{t_1}(x) = \frac{\alpha}{\beta-x_0} \left( \frac{x-x_0}{\beta-x_0} \right)^{\alpha-1} \left( \frac{x-x_0}{\beta-x_0} \right)^\alpha e \]

Area = 1 - $e^{-\left(\frac{-x_0}{\beta-x_0}\right)^\alpha}$
Fig. 19. Three Parameter Weibull Distribution Function of Crack Initiation Times.

\[ F_{t_i}(x) = 1 - e^{-\left(\frac{x}{B-X_0}\right)^\alpha} \]
Fig. 20. Distribution of Crack Initiation Times.
Fig. 21. Distribution of Crack Initiation Time.
TIME TO CRACK INITIATION, $t_i$, ($10^3$ flight hours)

Fig. 22. Distribution of Crack Initiation Times.
Fig. 23. Distribution of Crack Initiation Times.

75-91 PACAF

Three Parameter Weibull
- Data Points

TIME TO CRACK INITIATION, $t_i$, ($10^3$ flight hours)
74-90 TAC

Three Parameter Weibull

Data Points

Fig. 24. Distribution of Crack Initiation Times.
Fig. 25. Distribution of Crack Initiation Times.
Fig. 26. Distribution of Crack Initiation Times.
Fig. 27. Distribution of Crack initiation Times.

TIME TO CRACK INITIATION, $t_i$, ($10^3$ flight hours)

PROBABILITY, $F_{T_i}(t_i)$

73-89 PACAF

- Three Parameter Weibull

- Data Points

Fig. 27. Distribution of Crack initiation Times.
Fig. 28. Johnson Distribution Applicability Chart.
Fig. 29. Lower Tail of Crack Initiation Time Distribution
**76-92 PACAF**

--- Convolution

• Data Points

*Fig. 30. Lower Tail of Crack Initiation Time Distribution.*
Fig. 31. Lower Tail of Crack Initiation Time Distribution.
Fig. 32. Lower Tail of Crack Initiation Time Distribution.
Fig. 33. Lower Tail of Crack Initiation Time Distribution.
Fig. 34. Lower Tail of Crack Initiation Time Distribution.
Fig. 35. Lower Tail of Crack Initiation Time Distribution.
Fig. 36. Lower Tail of Crack Initiation Time Distribution.
Fig. 37. Distribution of Initial Flaw Sizes.
Fig. 38. Distribution of Initial Flaw Sizes.
Fig. 39. Distribution of Initial Flaw Sizes.
Fig. 40. Distribution of Initial Flaw Sizes.
Fig. 41. Distribution of Initial Flaw Sizes.

74-90 TAC
- Johnson $S_B$ Distribution
- Data Points

INITIAL FLAW SIZE, $a_0$

PROBABILITY, $F_{a_0}(a_0)$

0.01 0.02 0.03 0.04 0.05 0.06 cm

0 0.005 0.010 0.015 0.020 0.025 0.030 in
Fig. 42. Distribution of Initial Flaw Sizes.
Fig. 43. Distribution of Initial Flaw Sizes.
Fig. 44. Distribution of Initial Flaw Sizes.
Fig. 45. Distribution of Initial Flaw Sizes for all 1256 Growing Cracks.
Fig. 46. Probability of Detection, "Likelihood Function".

Data From Ref. 26.
I. 

Fig. 47. Distributions of Crack Initiation Times for a Typical Wing Location for an Early Inspection.

Inspection time $t = 2500$ hours
Fig. 48. Distributions of Crack Initiation Times for a Typical Wing Location for an Intermediate Inspection.
CRACK INITIATION TIME - $t_i$

Fig. 49. Distributions of Crack Initiation Times for a Typical Wing Location for a Late Inspection.

Inspection time $t = 5000$ hours

PROBABILITY DENSITY - $f(t_i)$