

THE ROLE OF FINITE-DIFFERENCE METHODS IN DESIGN AND ANALYSIS FOR SUPERSONIC CRUISE

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SUMMARY

Finite-difference methods for analysis of steady, inviscid supersonic flows are described, and their present state of development is assessed with particular attention to their applicability to vehicles designed for efficient cruise flight. As an illustration, calculations of the supersonic flows over delta wings are compared with experimental pressure distributions. The overall agreement with experiment is very good even well beyond the angles of attack where linearized theory methods are applicable. Current work is described which will allow greater geometric latitude, improve treatment of embedded shock waves, and relax the requirement that the axial velocity must be supersonic. The evolved finite-difference methods are expected to complement the design capability of linearized theory methods by identification of circumstances (the presence of shocks and critical pressures) which will impose constraints on linearized theory solutions. Thus they will allow refinement of designs before models are constructed, eliminating unnecessary wind tunnel tests of unsuitable designs. Further, they will accurately predict loadings required for structural design.

INTRODUCTION

Linearized theory methods are very familiar to those involved in supersonic cruise aircraft design and analysis. Over a period of 20 years or so these methods (such as refs. 1 and 2) have been developed into extremely useful tools for the aerodynamicist; and, as other papers in this conference show, they still have great potential for further development. However, because these methods are linearized, they have inherent limitations: They cannot account for the nonlinear effects of shock waves or of large flow angles, effects which can be important in aircraft design. Thus, there is a need to supplement the linearized theory methods with methods which do not have these limitations.

As part of the effort to meet this need, the study reported herein was initiated to assess the current status of finite-difference methods for computation of steady, inviscid, supersonic flows, to identify their present limitations, and to explore their potential role in aircraft design. This paper reports some of the study results obtained up to this time. Specifically, it will address four topics: (1) Distinguishing features of finite-difference

methods, (2) some past applications of these methods, (3) current developments aimed at overcoming limitations and (4) some applications in the design process to which these methods are ideally suited.

SYMBOLS

b	span
c	local chord
\bar{c}	mean geometric chord
C_n	section normal force coefficient, $\frac{1}{S} \int_0^1 \frac{c}{\bar{c}} \Delta C_p d\left(\frac{x}{\bar{c}}\right)$
C_p	pressure coefficient, $(p - p_\infty)/q_\infty$
ΔC_p	lifting pressure coefficient, $C_{p, \text{lower}} - C_{p, \text{upper}}$
M	Mach number
M_A	Mach number of velocity component along x axis
p	local static pressure
p_∞	free-stream static pressure
q_∞	free-stream dynamic pressure
S	wing reference area
x	longitudinal distance from model apex
y	spanwise distance from model centerline
α	angle of attack, deg
Λ	leading-edge sweep angle, deg

GENERAL DESCRIPTION OF FINITE-DIFFERENCE METHODS

The finite-difference approach to solving the flow equations is very different from that of linearized theory. Linearized theory methods solve a simplified set of flow equations in which all nonlinear terms are neglected. The overall solution to a given problem is built up by superposing the independent solutions for the complete flow field about each of the elementary panels composing the aircraft. Finite-difference methods, on the other hand, solve the complete equations for steady, inviscid flow. Since these equations are nonlinear, superposition of elementary solutions does not apply; instead, the overall solution is found by numerical integration over an extensive grid to obtain the complete solution at one point at a time. In supersonic flow, the points influencing the solution at a given point all lie upstream of that point, and this fact is the basis of the marching technique used in supersonic finite-difference methods to achieve greater computational efficiency.

The familiar method of characteristics can be used to illustrate the idea of point-by-point solution of the flow equations, for although it is usually classified separately, it employs basic finite-difference concepts (ref. 3). A two-dimensional characteristics network is shown schematically on the left in figure 1. The network is constructed a point at a time by following the Mach lines (characteristics) from each pair of neighboring points having known flow conditions (on the left in the figure) to their intersections. The flow conditions at the intersection point are determined by applying a local solution to the flow equations, which shows that certain quantities are invariant along the characteristics. This construction process is repeated, working back along the model using the new points for initial conditions. The intersection of two characteristics of the same family indicates that a shock wave is beginning to form; it must be inserted into the mesh with its strength determined by the Rankine-Hugoniot relation.

Because the characteristics network is determined as part of the solution, the user has no direct control over the size or direction of the multitude of individual steps that make up a complete flow field solution. For complex three-dimensional flows this lack of control can lead to a chaotic situation which makes the method of characteristics ill-suited to computer implementation. Thus, although there is some continuing interest in methods using characteristics (ref. 4), most research has turned to the development of the methods conventionally classified as finite-difference methods.

The two main classes of finite-difference methods are shown at the center and right in figure 1. These methods, like the method of characteristics, use a step-by-step computation of flow conditions starting from conditions known in an initial data plane. However, for the finite-difference methods each step is of uniform size so that each new set of points lies in a plane parallel to the initial plane. In addition, the computation is made by direct integration of the flow equations in such a way that each new point corresponds to a single initial point. The following paragraphs discuss the distinguishing features of the finite-difference methods without going into the mathematical detail. For a general treatment of the mathematics involved, see reference 5.

The shock-capturing technique, illustrated in the center of figure 1, uses the flow equations in their conservation form (refs. 6 to 8). The equations in this form apply across shock waves, so the integration can proceed right on through all shock waves which occur in the flow. These captured shocks have large gradients, which can introduce numerical oscillations into the solution, and it is customary to add an artificial viscosity term in the equations in order to dampen these oscillations. (See ref. 9, for example.) Since the equations are written in terms of the conserved quantities (e.g. momenta), an additional calculation is needed in order to recover the physical flow variables (e.g. pressure, velocity).

In the earliest applications of this method, a complete rectangular grid was entirely preset before the computation was begun. In more recent applications, the grid has usually been fitted between the body and the bow shock, with the latter computed as a discrete shock rather than being captured in the mesh. This technique effects a savings by avoiding the repetitive calculations of free-stream conditions at mesh points outside the bow shock. Since captured shocks are distributed over a number of mesh points, a fine grid is required for satisfactory resolution. Grid enrichment, in which extra grid points are introduced in the vicinity of shock waves, is a way of providing the fine mesh only where required and of retaining a coarser mesh where lower resolution is sufficient. This adaptive mesh is illustrated in the figure.

The shock-fitting finite-difference method forms the other major class (ref. 10). It is illustrated schematically on the right side of figure 1. Its principal difference from the shock-capturing class is in the treatment of shock waves. In the shock-fitting method each shock is treated discretely, with the flow through it computed to satisfy the Rankine-Hugoniot relation explicitly. In the implementation shown in the figure, the mesh is adjusted so that the shock lies along mesh lines. Program logic is required to readjust the mesh when appropriate. Since the flow equations are not integrated through the shock waves, the equations can be in terms of the physical variables and no artificial viscosity is needed. Also, a coarser mesh can be used with good results.

To achieve good resolution with a relatively coarse mesh, the mesh points need to be concentrated in high gradient regions but can be more spread out in regions of nearly uniform flow. One way to achieve this favorable distribution of mesh points is through the use of conformal mapping techniques. If, for example, a body-wing cross section is mapped to a near circle and the circle divided evenly into mesh points, the corresponding points in the physical plane tend to cluster toward the wing tip, a high gradient region. Thus conformal mapping can provide a measure of automatic mesh control.

PAST AND PRESENT APPLICATIONS

Figure 2 shows a few representative configurations of the many for which finite-difference calculations of flow fields have been made. The first is a real tour-de-force application which attempted to calculate the entire flow

field around a complete B-1 bomber under an Air Force contract (reference 11). The calculation, which used shock-capturing even for the bow shock, was made in an entirely preset square grid, graduated in spacing from 0.0007 of the (full scale) length at the nose to six times that in the far field. While the method used for this long-term demonstration project did not incorporate recently developed features which improve computational efficiency and provide ease of use, its results demonstrate the potential of more developed finite-difference methods.

The second part of the figure shows an early U.S. shuttle orbiter configuration for which the flow field was computed by a Russian shock-capturing method (ref. 12). In the reference, good agreement is shown with the results from the method of characteristics and from a shock-capturing finite-difference method given by Rakich and Kutler (ref. 13).

The last part of figure 2 shows results on a fighter-type configuration computed by a shock-fitting finite-difference method (ref. 10). Like some of the shock-capturing methods, this method was originally developed with emphasis on flow-field calculations about shuttle orbiter configurations. Consequently, while it has some advanced capabilities, such as real gas effect calculations, it has only limited capability to perform calculations about the complex geometries typical of supersonic cruise aircraft. Its application to a fighter forward fuselage section (as shown in the figure) is part of the subsequent effort to extend the method to more general configurations.

As part of the study of the possibilities of finite-difference methods for aircraft design, this same shock-fitting finite-difference method has been applied to one of a series of delta wings for which an extensive set of experimental data and results computed by other theories are available (refs. 14 and 15). The following three figures describe this delta wing and a few of the results obtained.

Figure 3 shows the $\Lambda = 76^\circ$ uncambered, clipped-delta wing studied. It had a 4-percent-thick circular-arc airfoil section with sharp leading and trailing edges. A faired body of circular cross section was added to provide a sting attachment for the experimental model. As shown in the figure, two modifications were made in the numerical model in order to meet geometric limitations imposed by the method. First, the forward 1 1/2-percent of the total length was replaced by a 24° half-angle cone faired to the original body and wing. This modification was necessary in order to obtain cone-flow starting solutions at angles of attack near 20° . The second, and more significant, modification was the replacement of the outer half of the wing by a thicker section providing an elliptical cross section. This was necessary since at the Mach numbers of interest, the flow normal to a leading edge swept 76° is subsonic; in this case, the computational method used requires the leading edge to be blunt.

It should be noted that several nose shape variations were tried unsuccessfully before this one was found for which complete runs could be made routinely. Each complete run at a single Mach number and angle of attack used less than 30 minutes of control processor time on a CDC6600 computer.

Figure 4 shows comparisons of computed centerline pressure coefficients with wind-tunnel measurements at Mach 3.5 and 4.6 and angles of attack typical of cruise conditions. The shock-fitting finite-difference method results (solid line) agree well with the experimental pressure distributions and span loadings. Also shown in the figure are curves (dashed lines) from a linearized-theory calculation (Woodward method, refs. 15 and 2). These curves also agree fairly well with the experimental results; this fair agreement at Mach numbers above the usual range of validity for linearized theory can be attributed to the extreme slenderness of the configuration.

Figure 5 shows a more complete comparison at a Mach number of 3.5 at an angle of attack near 20° . At this high angle, the agreement between the shock-fitting finite-difference and experimental results is still good. (The poor agreement at the wing tip is in a region dominated by the added leading-edge bluntness.) However, the linearized theory does not give useful estimates at this condition. From these two figures, then, it is seen that, although both the finite-difference method and the linearized theory can give good results at cruise conditions, only the finite-difference method can give usable loading estimates at high angles of attack, which are likely to produce critical design conditions.

CURRENT DEVELOPMENTAL AREAS

Since finite-difference methods show promise of being able to compute loadings at critical design conditions, work is progressing on improving their efficiency, range of applicability, and ease of use. The next three figures show three areas of current work on the shock-fitting finite-difference method used to obtain the preceding results.

Figure 6 indicates the proposed generalized conformal mapping currently being developed by Moretti (ref. 16). As mentioned previously, mapping is used to transform each cross section of the aircraft into a near circle about which a regular rectangular grid can be generated to form the computational mesh. The present mapping was set up to work well for shuttle orbiter cross sections but will not work for some typical cruise-aircraft cross sections. In particular the mapping relations require that the cross section be single-valued in polar coordinates, a requirement which makes the highly cambered midsection and the detached aft section unmappable for configurations like that shown in the figure. The new mapping, by careful placement of singularities, will open up a cross section such as that shown at the bottom of the figure into the desired near circle. The open section between the wing and the body will form parts of the boundary (BC and EF), for which a special flow-through boundary condition is required in order to provide for the correct flow in that region.

Figure 7 illustrates a new method for handling embedded shock waves in the shock-fitting finite-difference method. In the present method (shown on the left) the mesh is adjusted so that the shock lies along mesh lines. The mesh points on the shock are actually double points, with the quantities on

each side of the shock calculated to satisfy the Rankine-Hugoniot relation explicitly. As the shock moves through the flow, the mesh must be readjusted as shown in the figure. Unfortunately, the mesh distortions and the partitioning of the flow field can sometimes lead to difficulties, particularly in three-dimensional flows. In the new method, shown on the right and called floating shock fitting, the shock is not required to be a boundary of the flow. It is still treated as a discontinuity satisfying the Rankine-Hugoniot relation, but it can move freely, or float, through the undisturbed mesh. In order to provide this capability, the relations required for evaluating the derivatives are modified for mesh points near the shock.

The floating shock-fitting finite-difference method has been developed for the two-dimensional case (ref. 17) and has shown good results for complex flows involving multiple shock interactions. Its extension to three-dimensional flows is currently underway.

Figure 8 shows a third area of improvement, development of a method for continuing the calculation through regions in which the Mach number of the flow component in the marching direction is subsonic even though the total velocity remains supersonic (ref. 18). This condition arises fairly frequently in the vicinity of canopies and blunt-leading-edge wings, which turn the flow strongly away from the marching direction. In the general flow calculation method illustrated at the left of the figure, flow conditions at point B' are calculated using the conditions at points A, B, and C, known for the previous step. For numerical stability, the step must be small enough so that the characteristics through B' (shown as dashed lines) pass between A and C. This condition, which is essentially the Courant-Friedricks-Levy (CFL) condition for an explicit marching scheme, can always be met if the axial Mach number M_A is greater than 1. However, if the flow is at a high angle, the axial Mach number may become subsonic. As shown in the middle of the figure, a characteristic is then swept forward relative to the marching direction, and it is impossible to meet the CFL criterion, so the marching stops. The method proposed for continuing the computation in this case makes use of the fact that the flow deflection to a high angle is caused by a boundary, as shown on the right in the figure. Thus, although the conditions at B' cannot be computed directly (because of the forward-inclined characteristic), the condition at A' can be computed from the known conditions at A and B plus the boundary conditions at A'. However, even with the conditions at A' known, the CFL criterion cannot be met for B' since the characteristic falls outside of A'ABC. The conditions at A'' are also needed, but getting them directly would require knowing the conditions at B' first. Fortunately, the conditions at A'' and B' can both be obtained through a simultaneous solution of the relations for both points and the boundary conditions at A''. Thus, the CFL criterion can be met for both B' and A'' and the computation can continue. This procedure can be extended to include in the simultaneous solution as many points along the oblique front as fall within the region of subsonic axial Mach number. In reference 18, Marconi and Moretti have successfully applied this method in conjunction with the shock-fitting finite-difference method to the three-dimensional flow over an aircraft with a region of subsonic axial Mach number embedded at the front of the canopy.

When these three improvements have been incorporated into the shock-fitting finite-difference method, it will be able to compute the steady inviscid supersonic flow about a very general class of aircraft configurations. Shock-capturing finite-difference methods are undergoing similar improvements. For example, this method also has been extended to the case of supersonic flow with subsonic axial Mach number (ref. 19). Thus, finite-difference methods are new tools which are now becoming available to aid aircraft designers in their work.

APPLICATION TO DESIGN

In order to use this new tool effectively and efficiently, it is necessary to understand its capabilities relative to those of linearized theory, the basis of so many of the present methods for aerodynamic analysis and design. Figure 9 indicates how the capabilities of the two kinds of methods complement one another. The most important asset of the linearized theory is its capability for direct design; for example, linearized theory methods are able to determine directly the camber surface required to produce a given aerodynamic pressure distribution on a wing. This capability is the result of reducing the problem of finding the flow over an aircraft to one of solving a large number of simultaneous linear algebraic equations. Since digital computers are able to solve such systems of equations quickly and efficiently, linearized theory methods are quick enough and inexpensive enough to allow the evaluation of a large number of design variations. With further increases in computational speed, linearized methods will make true interactive man-in-the-loop aerodynamic design a practical reality.

However, linearized theory has the inherent limitation of not being able to predict or analyze nonlinear effects. Thus finite-difference methods also have a role in the design process. Their most important asset is their accurate representation of the flow, through solution of the complete equations for inviscid, steady flow. The practical limit to this accuracy for inviscid flows comes only through the limitation of resources committed to it. That is, the limit is related to the number of mesh points used, and more points require more computer storage and more time.

The accurate analysis afforded by finite-difference methods makes them ideal for use in design critique. After a good design candidate has been found through use of the linearized theory methods, analysis of a finite-difference method will allow the detection of such potential problem areas as shock waves occurring in unfavorable locations or pressures reaching critical values. With this kind of information in hand the aerodynamicist can refine the design to alleviate the problems. For example, the configuration may have to be changed to avoid shock impingement on an inlet. As suggested by the figure, the avoidance of a critical condition revealed by the finite-difference calculation may impose new design restraints. Then the design procedure by linearized theory can be reinstated using the new restraints to provide a refined candidate design. Use of this linearized-theory design and finite-difference critique iteration procedure will provide an aerodynamically efficient and practical design while avoiding the costly and time-consuming building and testing of

wind-tunnel models in the early design stages. Thus many more candidate configurations may be considered in choosing the best overall design for confirmation by wind-tunnel tests. Refinement through the detail design of fillets, inlets, etc. can then take place with each change checked using the accurate analysis afforded by the finite-difference method (fig. 10). The result of designing by this process will be a better aerodynamic design in less time and at lower cost.

But this may not be the most important role of finite-difference methods in the design process. Because of their accuracy, they are ideally suited for rapidly obtaining the detailed loadings which now can be acquired only by an extremely lengthy experimental process. This is particularly the case at off-cruise-point conditions such as high angles of attack, which cannot be properly treated by linearized theory, but which often form the critical loading conditions for the structural design cycle.

CONCLUDING REMARKS

It has been shown that finite-difference methods for computing steady, inviscid, supersonic flows are becoming developed to the point where they form useful additional tools for the aircraft designer. For example, with the incorporation of new features now under development, the shock-fitting finite-difference method will be able to accurately analyze complex flows over general aircraft configurations, including critical off-cruise-point conditions. The detailed, accurate analysis afforded by finite-difference methods suits them particularly well for a role complementary to the rapid design capabilities of linearized theory methods. Finite-difference methods are also well suited for determining critical loads for structural design purposes.

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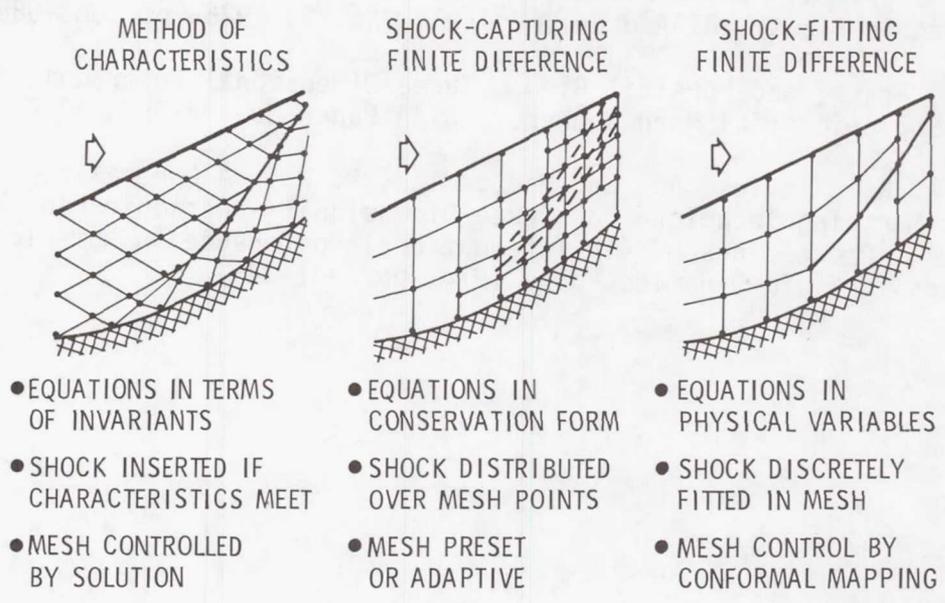


Figure 1.- Comparison of features of method of characteristics and steady, inviscid, finite-difference methods.

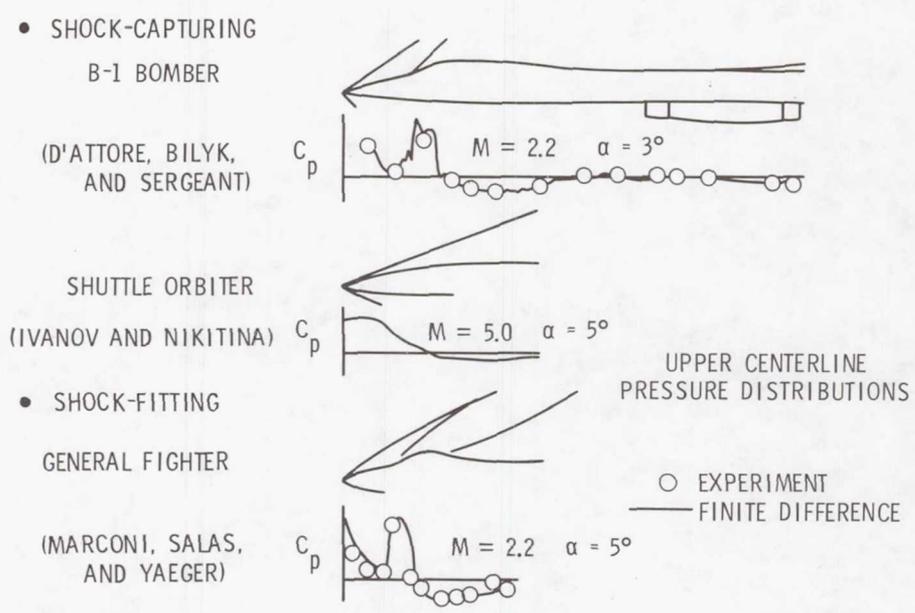


Figure 2.- Examples of past applications of finite-difference methods.

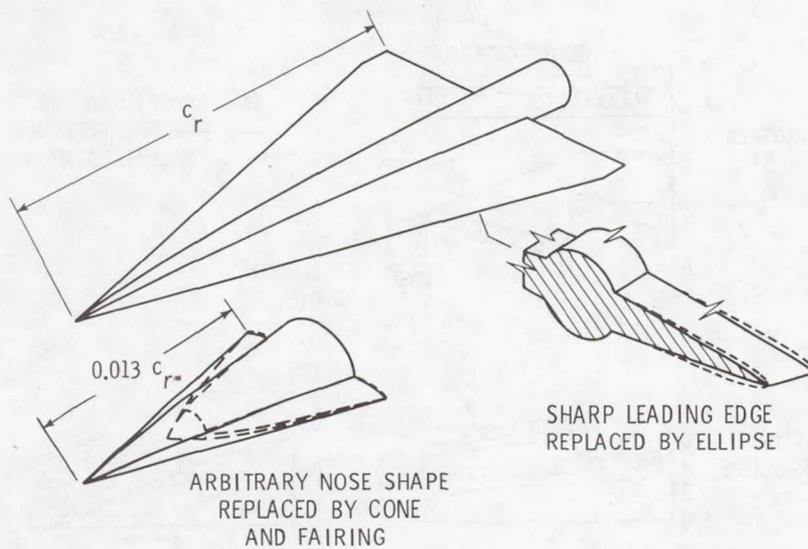


Figure 3.- Delta wing used for experimental comparisons and modifications made for numerical model. Leading-edge sweep angle, $\Lambda = 76^\circ$.

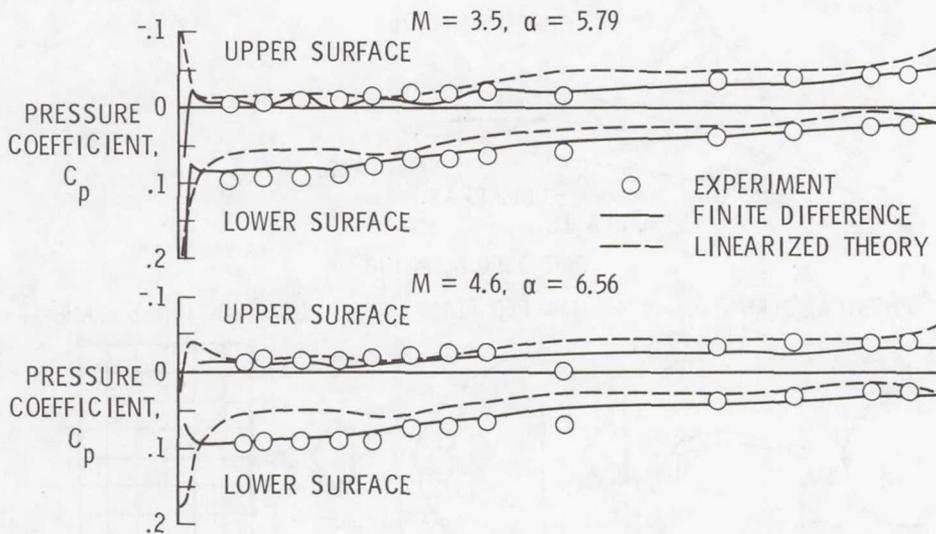


Figure 4.- Centerline pressure distributions for 76° delta wing at low angles of attack. Experiment from reference 14; linearized theory by Woodward method (ref. 2).

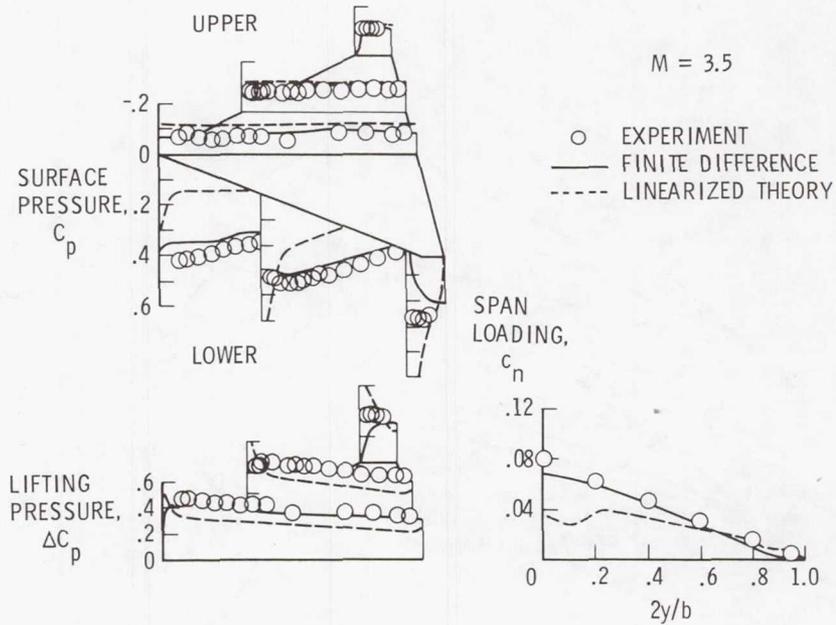


Figure 5.- Pressure distribution and span loading for 76° delta wing at 19.7° angle of attack (ref. 15).

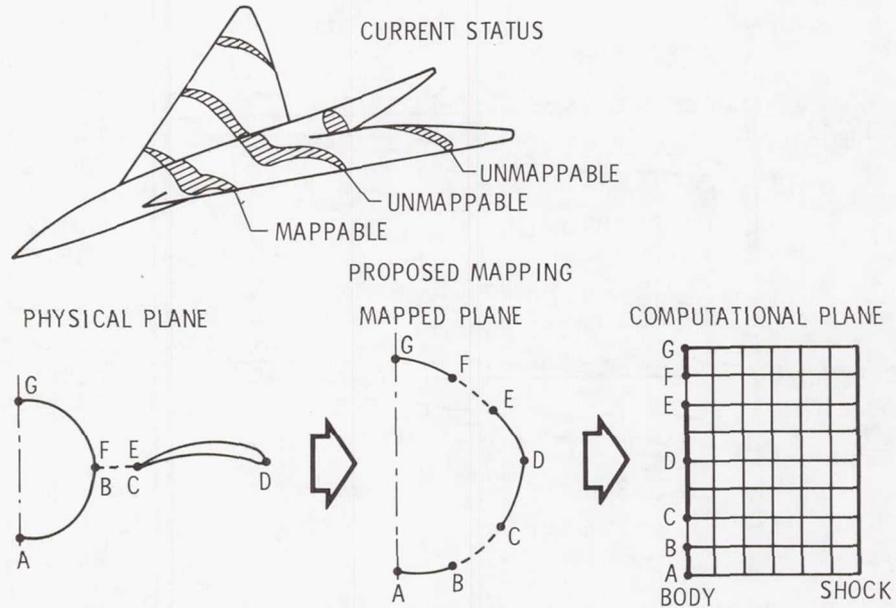


Figure 6.- Current work on generalizing conformal mapping technique used with shock-fitting finite-difference method by Moretti (ref. 16).

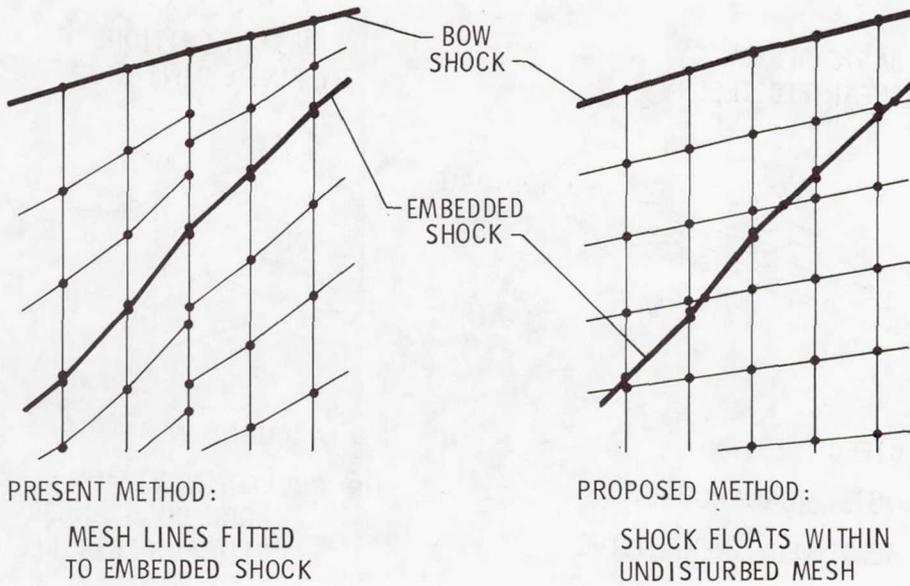


Figure 7.- Current work on improving treatment of embedded shock waves in shock-fitting finite-difference method by Salas (ref. 17).

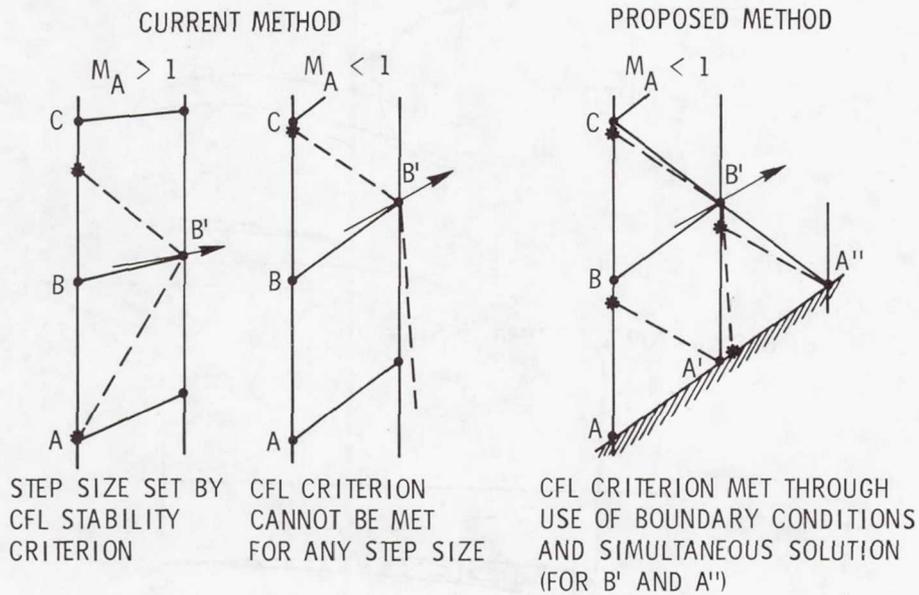


Figure 8.- Current work on extending shock-fitting finite-difference method to include regions of supersonic flow with subsonic axial Mach number by Marconi and Moretti (ref. 18).

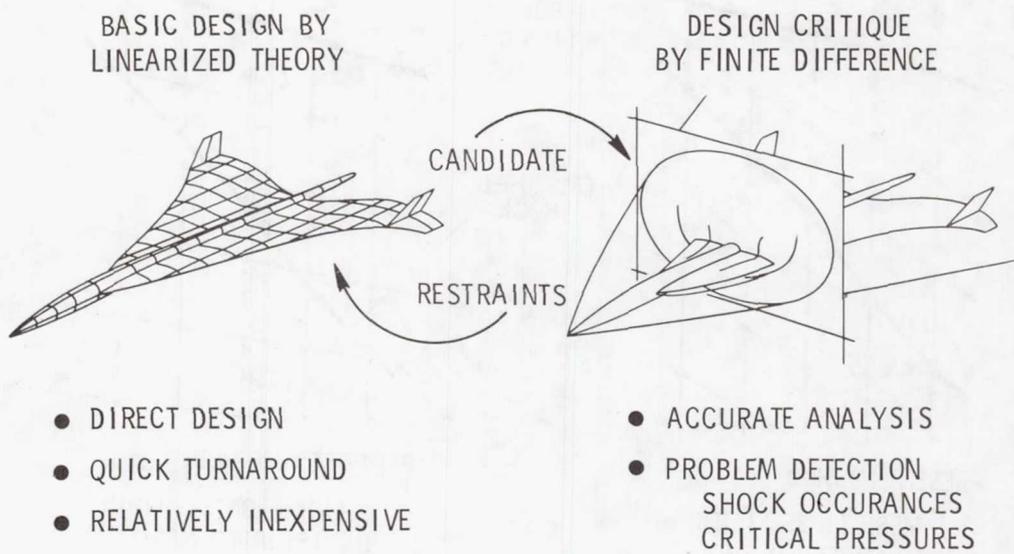


Figure 9.- Application of finite-difference methods in design process.

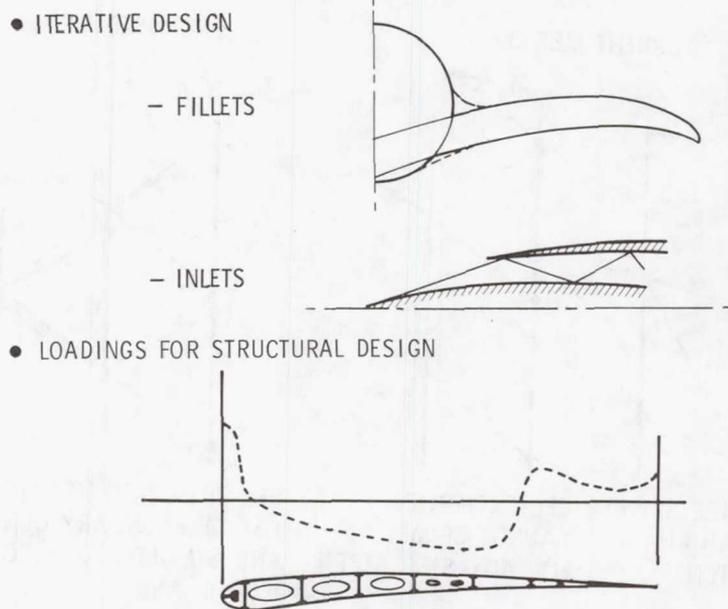


Figure 10.- Additional design application of finite-difference methods made practical by their detailed accuracy.