A.R.A.P. WORKING PAPER NUMBER 76-2

GYC: A PROGRAM TO COMPUTE
THE TURBULENT BOUNDARY LAYER
ON A ROTATING CONE

Roger D. Sullivan

August 1976

Prepared under Contract No. NAS2-8014
by
Aeronautical Research Associates of Princeton, Inc.
50 Washington Road, Princeton, New Jersey 08540

for
AMES RESEARCH CENTER
NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
# TABLE OF CONTENTS

1. **INTRODUCTION**  
2. **ANALYSIS**  
3. **NUMERICAL METHOD**  
4. **AUXILIARY COMPUTATIONS**  
5. **CONTROL OF THE MESH**  
6. **PROGRAM STRUCTURE**  
7. **INPUT AND OUTPUT**  
8. **FINAL REMARKS**  
9. **REFERENCES**  

**APPENDIX**

A. **THE EQUATIONS BEFORE MODELING**  
B. **THE MODELS**  
C. **THE EQUATIONS AFTER MODELING**  
D. **THE FINAL EQUATIONS**  
E. **TENSOR NOTATION EXAMPLES**  
F. **GLOSSARY**  

**TABLE**

1. **INPUT CARDS**  
2. **EDGE VALUES AND WALL VALUES**  
3. **INITIAL CONDITIONS, FIRST SET**  
4. **INITIAL CONDITIONS, SECOND SET**  
5. **DEPENDENT VARIABLES**
1. INTRODUCTION

The Ames Research Center of NASA is engaged in an effort to enhance its understanding of the behavior of turbulent boundary layers and to improve its ability to predict their effect on bodies in supersonic flow. As a part of this effort, it has entered into a contract with A.R.A.P. Under this contract, A.R.A.P. has delivered to ARC a computer program, GYC, which is capable of computing the properties of a compressible turbulent boundary layer on a rotating axisymmetric cone-cylinder body, according to the principles of invariant modeling. The program, which was delivered at the beginning of 1976, is to be extended during the current year to include the calculation of the turbulence scale by a differential equation.

In the meantime, GYC has been in operation on the ARC CDC-7600 computer and has undergone several corrections and improvements as a result of the experience gained with it there and at A.R.A.P. However, so far there has been lacking a comprehensive document giving the theoretical basis for the program and the method of implementation, as well as information on its operation. This paper is intended to fulfill this need, but in a tentative way. When an extended version of the program is delivered in a few months, it is hoped that a report describing it — an improved version of this — will be included. The anticipated improvements will be generated largely by the reactions to this paper.

Section 2 is a concise review of the means used to derive the equations. The models used are listed but not justified. Some background on the models can be found in Reference 1 for the models that apply to incompressible flows, and in Reference 2 for others.
In Section 3, the numerical method used to solve the equations is set forth. Some extra computations are described in Section 4, while the means for controlling the finite-difference mesh are reported in Section 5.

Sections 2 through 5 are essentially free of program considerations, which are covered in Sections 6 and 7. Section 6 describes GYC as a whole and then considers in more or less detail the main sections of the program. The part concerned with the numerical integration of the equations is given the most attention. External aspects of the program operation are described in Section 7. The paper is brought to a close with a few remarks in Section 8.

The full equations under consideration are set forth in appendices. An extensive glossary of Fortran names and Fortran-like names is included as Appendix F.
2. ANALYSIS

The basic equations used in this study are the following:

**Continuity**

\[ \rho_t + (\rho u^j)_{,j} = 0 \]

**Momentum**

\[ \rho u_{i,j} + \rho u^j u_{i,j} = -p_{,i} + \tau_{i,j} \]

where

\[ \tau_{i,j} = \mu \left( u_{i,j}^2 + u_{j,i}^2 \right) + \mu^* \delta_{i,j} u^l_{,l} \]

**Enthalpy Version of Energy**

\[ \rho h_t + \rho u^l h_{,l} = p_t + u^p p_{,l} + \tau_{m,l} u_{,m} + \left( kT_{,l} \right)_{,l} \]

The thermodynamic relations of a "calorically perfect" gas are used

\[ p = \rho RT \]

\[ h = c_p T \]

where \( R \) and \( c_p \) are constants.

Writing the dependent variables in these equations as the sum of mean and fluctuating parts (\( \rho = \bar{\rho} + \rho' \), for example), we can deduce equations for the mean quantities and for the second-order correlations of the fluctuations by protracted manipulations. Two assumptions are made immediately:

1) Fourth-order correlations are neglected.

2) Third-order correlations involving \( \mu \), \( \mu^* \), or \( k \) are neglected.
The resulting equations for a steady mean flow are displayed in Appendix A.

Fluctuations in the transport parameters are handled by assuming

\[ \mu' = \mu_{TT} \]
\[ \mu_{i} = \mu_{TT,i} \]
\[ \mu'' = \mu_{TT}'' \]
\[ \mu_{i}'' = \mu_{TT,i}'' \]
\[ k' = k_{TT} \]
\[ k_{i} = k_{TT,i} \]

where the subscript \( T \) denotes a derivative with respect to temperature evaluated at \( T \).

The models used to close the equations are given in Appendix B. As in previous A.R.A.P. models, these are invariant under coordinate and Galilean transformations and, of course, are dimensionally consistent. The goal of simplicity has been sacrificed to some extent in favor of generality. An obvious feature of these models is that many of them are expressed in terms of quantities that are themselves modeled. This results in some of the parameters (coefficients) appearing only in products with other parameters in the applications. (This is exemplified by the models for \( \overline{p'h_{i}} \) and \( \overline{p'\rho_{i}} \) where the substitution has already been carried out.) Thus some of the parameters are redundant, but they are left that way for the sake of flexibility in case it is found necessary to modify them in the future.

Three- or four-character names, suitable for direct use in Fortran, have been chosen for the modeling parameters except for \( a \), \( b \), and \( \beta \), in the velocity-dissipation model, which are left in that form out of nostalgia.

It will be seen later that all of the terms containing models for correlations involving the divergence of the velocity.
fluctuations, \( u_{i}^{2} \), drop out. For this reason, the following parameters do not appear in the final equations: BDV, VBS, WMD, WWU2, WWD1, WWD2, WWH2, and WWR2. In addition, terms involving PMH, PMR, PTHM, and PTUM do not occur in the final set. Of the remaining 38 independent parameters, about a quarter are set to zero until better information on them is available, and the majority of the others are evaluated by analogy. Thus 0.1 has been found a good value for VUU in constant density tests; VRU, VUH, VRR, VRH, and VHH don't occur in constant-density modeling but, by analogy, 0.1 is used for all of them, until better information is available.

So far the equations have been written using mechanical units for heat quantities. In order to introduce additional flexibility, a conversion factor is introduced, allowing arbitrary units for heat. This factor is

\[
\text{GMOMS} = \frac{1}{J}
\]

where \( J \) is the mechanical equivalent of heat. For example, if English units are used with heat measured in Btu, \( \text{GMOMS} = 0.001285 \text{ Btu/ft-lb} (= 1/(778 \text{ ft-lb/Btu})) \). If mechanical units are used for heat, \( \text{GMOMS} = 1 \). It is often desirable to make the equations nondimensional. If this is done using \( u_r \) and \( h_r \) as reference values for velocity and enthalpy per unit mass, the nondimensional version of GMOMS is

\[
\text{GMOMS}_{ND} = \text{GMOMS} \cdot \frac{u_r^2}{h_r} = \frac{u_r^2}{Jh_r} = (\gamma - 1)M_r^2
\]

(Hence the name GMOMS - Gamma Minus One \( M_r \) Squared.)

The equations with the models substituted are exhibited in Appendix C. Here the notation is that devised for input to TENS, A.R.A.P.'s software for the expansion of tensor equations. This notation is like Fortran in several respects. Thus,
variable names may consist of more than one character; multiplication is always made explicit with a *; multiplication and division proceed from left to right so that $A/B/C*D$ is to be evaluated as $((A/B)/C)*D = (A*D)/(B*C)$. On the other hand, superscripts and subscripts are represented in a manner completely different from the Fortran representation of subscripts. Superscripts are preceded by "; subscripts by '. Superscripts and subscripts consist of a single letter or digit only; two or more together represent separate indices. The metric tensor is denoted by $\otimes$; the Kronecker delta by $\delta$. Covariant differentiation is denoted by a , preceding an index; and "contravariant differentiation" (that is, covariant differentiation with the index raised) by $!$ preceding an index. All these forms are illustrated in Appendix E.

The Glossary (Appendix F) gives the meaning of the variable names used. Thus, for RS, US, and RU in the first equation of Appendix C, we find $\rho$, $u$, and $\rho^T u^T$ respectively. With the help of Appendix E, we see that the first equation of Appendix C is a transcription of the first equation in Appendix A, as it should be since there are no terms needing modeling. Proceeding thus step by step, the formidable expressions of Appendix C can gradually be seen to make sense.

A cross-section of the axisymmetric coordinate system is shown in the sketch on the following page.
The coordinates of the point P are \( x^1 = x = OQ \), \( x^2 = y = PQ \), \( x^3 = \phi \) = the azimuthal angle. The half angle of the cone, \( \theta_c \), and the "origin radius," \( r_0 \), are parameters of the system. The coordinates \( x \) and \( y \) are related to the cylindrical coordinates \( \xi = SR \) and \( r = PR \) by

\[
\xi = \cos \theta_c x - \sin \theta_c y \\
r = r_0 + \sin \theta_c x + \cos \theta_c y
\]

It is understood that \( y = 0 \) always represents the cone surface. The location of the origin in the \( x \) direction is quite arbitrary and hence, in general, the value of \( r_0 \) is arbitrary. The exception is the limiting case of a cylindrical surface (\( \theta_c = 0 \) or \( \theta_c = 180^\circ \)) for which \( r_0 \) represents the radius of the cylinder.

Note that for \( \theta_c \) in the second and third quadrants, the flowfield (for which \( y > 0 \)) is on the inside of a conical surface.
It is convenient to have the equations apply also for a Cartesian coordinate system \((x,y,z)\). To accomplish this, another parameter is defined by

\[
\text{DIMFL} = \begin{cases} 
1. & \text{axisymmetric} \\
0. & \text{Cartesian}
\end{cases}
\]

Now let

\[
S = \text{DIMFL} \times \sin \theta_c
\]

\[
C = \text{DIMFL} \times \cos \theta_c
\]

\[
R = \text{DIMFL} \times r_0 + Sx + Cy + 1. - \text{DIMFL}
\]

Then the metric tensor can be written

\[
\mathbf{g}_{ij} = \begin{pmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & R^2
\end{pmatrix}
\]

The nonvanishing Christoffel symbols of the second kind are

\[
\Gamma^1_{33} = -SR
\]

\[
\Gamma^2_{33} = -CR
\]

\[
\Gamma^3_{31} = \Gamma^3_{13} = S/R
\]

\[
\Gamma^3_{32} = \Gamma^3_{23} = C/R
\]

These apply for both Cartesian coordinates and the axisymmetric coordinates of the sketch.
Note that

\[ R = \begin{cases} 
  r & \text{axisymmetric} \\
  l & \text{Cartesian}
\end{cases} \]

so that it not only has different meaning but even different dimensions in the two cases.

The next stage of the analysis was carried out by TENSr, A.R.A.P.'s software for the expansion of tensor expressions. The following information was supplied to TENSr:

1) The values of the metric tensor and the Christoffel symbols of the second kind in terms of \( R, S, \) and \( C \), as given above.

2) The information that terms involving derivatives with respect to \( x^3 (\phi) \) are zero, since the flow is assumed to be axisymmetric.

3) The substitutions required so the output involving the mean velocity could be expressed in terms of the physical components, \( \bar{u}, \bar{v}, \) and \( \bar{w} \) (or \( U_s, V_s, \) and \( W_s \)), instead of the covariant and contravariant components, and similarly for correlations involving \( u' \).

4) The equations of Appendix C.

The output for the whole set of 18 equations consisted of 9843 terms. This number is somewhat deceptive, since TENSr is naive algebraically and does not combine like terms.

These equations were submitted to more A.R.A.P. software known as TATTR. The first stage of TATTR does combine terms and reduced the 18 equations to a total of only (1) 4348 terms.

The main function of TATTR is to drop terms according to order-of-magnitude assumptions. Its operation in this case can be described as follows.
The equations were interpreted to be written in nondimensional form. The (nondimensional) boundary-layer thickness was assumed to have a small value, $\delta$. The mean quantities and the radius, $R$, were assumed to be of order one except for the normal component of the velocity, $\bar{v}$ (VS), which was taken to be of order $\delta$. Derivatives with respect to $x$ and $y$ were assumed to be of order one and $\delta^{-1}$, respectively. The transport coefficients $\mu$, $\mu^*$, and $k$ were assumed to be of order $\delta^2$. All of the above corresponds to standard boundary layer assumptions. In addition, correlations involving a velocity fluctuation were taken to be of order $\delta^{1/2}$ and those involving $\rho'$ or $h'$ of order $\delta^{1/3}$ for each such appearance. (For example, $\bar{u}'u'^r$, $\bar{u}'v'^r$, $\bar{\rho}'w'^r$, and $h'h'^r$ (UU, UV, RW, and HH) were assigned orders $\delta$, $\delta$, $\delta^{5/6}$, and $\delta^{2/3}$, respectively.) The scale length $\Lambda$ was assumed to be of order $\delta^{4/3}$. Finally, GMOMS, which, as has been seen, is $(\gamma - 1)N_T^2$ when the equations are interpreted nondimensionally, was taken to be of order $\delta^{-1/3}$.

All quantities not mentioned, including the modeling parameters, were assumed to be of order one. These ratios were arrived at after some trial and error, using Cartesian coordinates. They were chosen so as to insure the retention of terms known to be important, while allowing the elimination of terms known to be unimportant.

TATTR computed the exponent, $T$, representing the order of magnitude $\delta^T$, for each term of each equation, given the information in the paragraph above concerning the factors that can appear. For example, the continuity equation was analyzed as follows (where :1 and :2 stand for partial differentiation with respect to $x^1$ and to $x^2$).
The final stage of TATTR dropped terms for which the value of $T$ is greater than a maximum appropriate for the particular equation. In the case of the continuity equation, the cutoff value is zero, so the final form is


or

$$(\bar{\rho}u)_x + (\bar{\rho}v)_y + \frac{S}{R}\bar{\rho}u + (\bar{\rho}Tv)_y = 0$$

The whole set of equations was reduced from 4348 terms to 529 terms by the final stage of TATTR. The second component (normal to the surface) of the momentum equation reduced to

$$T$$

$${}-C*RS/R*WS*WS$$

$$+ (RS*VV):2$$

$$= -(PS):2$$

The values of $T$ indicate that, according to TATTR, the remaining terms are seriously out of balance. But it is well known in boundary-layer theory, though not to TATTR, that the derivative of pressure in the normal direction is not large like other normal derivatives; $(PS):2$ is really of order one $(\delta^0)$, not $\delta^{-1}$. Therefore, the terms containing $(PS):2$ in
the other equations were dropped manually if the $T$ value for that term was within one of the cutoff value for the equation. In this way, 63 terms were dropped leaving 466 in the 18 equations. Two of the final equations have been presented above. The others, which have been slightly edited manually, are presented in Appendix D. (The reason for the organization of the equations into "passes" is given in the next section.)

As has been indicated, the criteria used to drop terms were arrived at pragmatically. This can be defended on the basis that (1) the process is more systematic than that heretofore used for modeled equations for compressible turbulent flow and (2) the results are not sensitive to the details of the assumptions used—e.g., it makes little difference whether $\rho'$ is assumed to be of order $\delta^{1/3}$ or order $\delta^{1/2}$. Nevertheless, it would be valuable to find a rational basis for completely specifying the criteria. Such a basis should take into account the fact that the balance of terms differs markedly in different parts of the boundary layer.

Some features of the final set of equations should be noted. (1) All terms involving models for $u'_{,x}^3$ correlations, as noted above, have dropped out. This is a result, of course, not only of the criteria used, but also of the particular models used for such correlations. (2) All terms involving the second coefficient of viscosity, $\mu^*$, have dropped out. (For the most part, these are terms that also involve $u'_{,x}^3$. ) (3) If $S$ and $C$ are both zero (as they are if $\text{DIMFL} = 0$), the equations consisting of the remaining terms are each homogeneous in $R$. This fact (which is true of the equations before terms are dropped) shows that the change in dimensionality of $R$ as $\text{DIMFL}$ changes from one to zero does not violate any principles.

A new equation

$$\text{BLAM} = \text{GYCSZ}$$
has been added to the set and included in Appendix D. Its presence here is merely symbolic. BLAM (A) is evaluated as a known function of \( y \) and the gross features of the flow, quantified by input parameters (the scale parameters). It is included here and in the program as a pseudo-unknown, satisfying a "differential" equation of zero order which becomes a "difference" equation of zero order. The reason for this treatment is that it is intended to introduce a true differential equation for \( A \) and incorporate its solution in the program in the near future. The program modification will be much simplified by having \( A \) already treated as an unknown.

The sixteen differential equations of Appendix D are parabolic in the sense that they have first-order derivatives with respect to \( x \) and second-order derivatives (as well as first-order) with respect to \( y \). Thus \( x \) is the time-like variable and \( y \) is the space-like variable. The dependent variables determined by the equations are referred to as parabolic variables.

The boundary conditions needed for this set consist of the initial conditions, the wall conditions, and the free-stream conditions. The initial conditions consist of profiles of the parabolic variables, that is, their values as functions of \( y \) at the \( x \) station where computation is to start. The wall conditions on all the turbulence correlations and on \( \bar{u} \) are that they are zero there. The wall condition on \( \bar{w} \) is that it is equal to the lateral velocity of the cone surface, that is, the product of the angular velocity and the local cone radius. The wall condition on \( \bar{h} \) is either its value at the wall or the value of its gradient (a measure of the heat flux) at the wall. Either of these may be functions of \( x \).

The free-stream conditions, or "edge conditions," are that the parabolic variables approach given values asymptotically. For the turbulence correlations and for \( \bar{w} \), the given values are constant in \( x \), but for \( \bar{u} \) and \( \bar{h} \) they may be functions
of $x$. In practice the concept "approach asymptotically" is handled as follows. At the largest value of $y$ currently used in the computation, the boundary condition $\frac{\partial f}{\partial y} = 0$, where $f$ stands for any of the unknowns, is used in solving the equations. The resulting value of $f$ at that $y$ is compared with the given value and if they agree within a specified tolerance, the given value is substituted for the calculated value (for the sake of uniformity and neatness) and the solution continues. If the tolerance is not satisfied, for any of the parabolic variables, the maximum value of $y$ is increased by adding another point, and the process is repeated.

These boundary conditions are not the most general that can be posed for the system of equations; rather they are the conditions provided for in the GYC program.
3. NUMERICAL METHOD

A finite-difference mesh is introduced in the $x, y$ plane as shown in the sketch.

The time-like variable, $x$, is indexed by $n$ and the space-like variable, $y$, by $j$. The spacing is not uniform in either direction; furthermore, the spacing in the $y$ direction is adjusted as the solution proceeds in the $x$ direction, as described in Section 5.

At the beginning of each $x$ step, the values of the dependent variables are known at $x = x_{n-1}$ for all $y_j$. Derivatives with respect to $x$ are approximated by

$$
    f_x = \frac{f^n_j - f^{n-1}_j}{\Delta x}
$$

where

$$
    f^n_j = f(x_n, y_j)
$$
\[ \Delta x = x_n - x_{n-1} \]

For derivatives with respect to \( y \), some additional notation is convenient:

\[ h_+ = y_{j+1} - y_j \]

\[ h_- = y_j - y_{j-1} \]

\[ h_t = y_{j+1} - y_{j-1} \]

\[ H_+ = \frac{h_-}{h_t} \]

\[ H_- = \frac{h_+}{h_t} \]

\[ G_+(g) = \frac{g_{j+1} - g_j}{h_t} \]

\[ G_-(g) = \frac{g_j - g_{j-1}}{h_t} \]

Then we have

\[ f_y = H_+ (f_{j+1} - f_j) + H_- (f_j - f_{j-1}) \]  

(2)

\[ (gf_y)_y = g_+(g) (f_{j+1} - f_j) - g_-(g) (f_j - f_{j-1}) \]  

(3)

If \( g \) is constant, equation (3) can be written

\[ f_{yy} = \frac{2}{h_+h_t} (f_{j+1} - f_j) - \frac{2}{h_-h_t} (f_j - f_{j-1}) \]  

(4)
One way of arriving at equations (2) and (4) is by passing a parabola through the three points \( (y_{j-1}, r^j_{j-1}) \), \((y_j, r^j_j)\), \((y_{j+1}, f^n_{j+1})\) and evaluating its derivatives at \( y_j \). Equation (3) is an obvious generalization of equation (4) except that \( g \) is evaluated at \( x_{n-1} \) instead of at \( x_n \). This is done to linearize the numerical procedure since \( g \), in general, depends on the unknowns. The other nonlinear terms in Appendix D are handled similarly; that is, in any product, at most one factor is evaluated at \( x_n \) while the rest of the term is evaluated at \( x_{n-1} \). This artifice has been successfully used in similar equations for several years at A.R.A.P. Of course, it is necessary to keep \( \Delta x \) small enough so that the changes in the dependent variables are small in each step. The linearized system is in the form referred to in the literature as the (complete, or fully) implicit method for the numerical solution of parabolic differential equations.

For terms of the form \( \bar{\rho} v f_y \), equation (2) is optionally replaced by an upwind differencing formulation, described in Note 4 of the Glossary (Appendix F).

The equations are coupled, but the finite-difference versions can be decoupled to any extent desired by the same device that is used to make them linear. That is, terms or factors which are unknowns if evaluated at \( x_n \) become known if evaluated at \( x_{n-1} \). Further, if some unknowns are solved for before others, they become known at both \( x_{n-1} \) and \( x_n \) for the later solutions. If \( m \) equations are solved at once, at each point of the flowfield a matrix of size \( m \times m \) must be inverted and two matrix multiplications of \( m \times m \) matrices must be performed. Since these procedures require of the order of \( m^3 \) operations each, there is strong motivation for keeping \( m \) down. On the other hand, experience at A.R.A.P. has shown that some equations are better solved together in order to have stability and well-behaved solutions with reasonable step sizes.
For GYC, these considerations have led to the division of the solution process, for each step in $x$, into six passes. In the first pass, $\vec{w}$ and $\vec{v}'\vec{w}'$ are determined so $m = 2$. In the rest of the passes $m = 3$: $\vec{u}$, $\vec{u}'\vec{v}'$, and $\vec{u}'\vec{w}'$ are determined in the second; $\vec{u}'\vec{u}'$, $\vec{v}'\vec{v}'$, and $\vec{w}'\vec{w}'$ in the third; $\vec{h}$, $\vec{h}'\vec{v}'$, and $\vec{h}'\vec{h}'$ in the fourth; $\vec{p}'\vec{u}'$, $\vec{p}'\vec{v}'$, and $\vec{p}'\vec{w}'$ in the fifth; and $\vec{h}'\vec{u}'$, $\vec{h}'\vec{w}'$, and $\vec{A}$ in the last.

In each pass, the $m$ unknowns are thought of as forming a vector, $\phi_j$, of $m$ elements at each $y_j$ (for $x = x_n$). Thus in the first pass

$$\phi_j = \begin{pmatrix} -n_w \\ w_j \\ \vec{v}'\vec{w}'_j \end{pmatrix}$$

in the second

$$\phi_j = \begin{pmatrix} -n_u \\ u_j \\ \vec{u}'\vec{v}'_j \\ \vec{u}'\vec{w}'_j \end{pmatrix}$$

etc.

Then the finite-difference equations can be written

$$A_j \phi_{j-1} + B_j \phi_j + C_j \phi_{j+1} = d_j$$

(5)

where $A_j$, $B_j$, and $C_j$ are square matrices of order $m$ and $d_j$ is a vector of dimension $m$. The elements are known functions of the dependent variables at $x_{n-1}$ (and in a few instances at $x_n$ for variables determined on previous passes), the independent variables, and, through equations (1)-(4), the mesh spacing.
Equation (5) applies for $2 \leq j \leq J - 1$ where $J = J_{\text{TOP}}$ is the index for the largest $y$ in use and $j = 1$ corresponds to the wall ($y_1 = 0$). The finite-difference version of the edge boundary condition, that the derivative with respect to $y$ be zero for each variable, can be expressed as

$$\phi_{j-1} - \phi_J = 0 \quad (6)$$

The wall boundary conditions can be written

$$\phi_1 = d_1 \quad (7)$$

except when $\bar{h}$ is involved and its gradient at the wall is specified.

When the gradient of $\bar{h}$ at the wall is specified, the boundary condition is handled this way. It is required that there be no blowing so both components of the mean velocity, as well as all correlations, are zero at the wall. Then, on the wall the differential equation for $\bar{h}$ (the first equation of the fourth pass) reduces to

$$0 = \bar{u} u_y + (R \bar{w}) (\bar{w}_y) + \left( \frac{K}{c_p} \bar{h}_y \right)_y$$

assuming mechanical units so $\text{GMOMS} = 1$. The condition on the gradient is written

$$\bar{h}_y = g$$

where $g$ is the specified value. These two equations are written in finite difference form using three points, the wall, $y_1 = 0$, the first point off the wall, $y_2$, and a "phantom" point, $y_0$, inside the wall. (Extrapolation is used to evaluate "known" quantities at the phantom point.) Thus there are two equations involving the unknowns $h^n_0$, $h^n_1$, $h^n_2$. 

19
Eliminating $h_0^n$ between these equations leaves one equation which can be written

$$\alpha h_1^n + \beta h_2^n = \gamma$$  \hspace{1cm} (8)

where $\alpha$, $\beta$, and $\gamma$ depend on the known variables and the mesh spacing at the wall.

In any case, we conclude from equations (7) and (8) that the wall boundary conditions for a pass can be written

$$B_1 \phi_1 + C_1 \phi_2 = d_1$$  \hspace{1cm} (9)

where $B_1$ is diagonal and $C_1$ is zero except in the fourth pass when the gradient of $h$ is specified, in which case the upper left element of $C_1$ is the $\beta$ of equation (8) and the rest of $C_1$ is zero.

Similarly, equation (6) can be written

$$A_J \phi_{J-1} + B_J \phi_J = d_J$$  \hspace{1cm} (10)

where $A_J$ is the unit matrix, $B_J$ is the negative of the unit matrix, and $d_J$ is zero.

Equations (5), (9), and (10) can be written

$$
\begin{pmatrix}
B_1 & C_1 \\
A_2 & B_2 & C_2 \\
A_3 & B_3 & C_3 \\
\cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot \\
A_{J-1} & B_{J-1} & C_{J-1} \\
A_J & B_J
\end{pmatrix}
\begin{pmatrix}
\phi_1 \\
\phi_2 \\
\phi_3 \\
\cdot \\
\cdot \\
\cdot \\
\phi_{J-1} \\
\phi_J
\end{pmatrix}
= 
\begin{pmatrix}
d_1 \\
d_2 \\
d_3 \\
\cdot \\
\cdot \\
\cdot \\
d_{J-1} \\
d_J
\end{pmatrix}
$$
\[ M\Phi = D \]

where \( M \) is a tridiagonal matrix of order \( J \), each element of which is an \( m \times m \) matrix, and \( \Phi \) and \( D \) are vectors of dimension \( J \), each element of which is a vector of dimension \( m \).

This set is solved by a standard algorithm for tridiagonal systems. The process consists of two parts called the upsweep and the downsweep. In the upsweep the values of auxiliary matrices, \( F_j \), and auxiliary vectors, \( \bar{\phi}_j \), are computed by

\[
\Gamma_1 = B_1^{-1}C_1 \\
\bar{\phi}_1 = B_1^{-1}d_1 \\
\Gamma_j = \left(B_j - A_j\Gamma_{j-1}\right)^{-1}C_j \quad j = 2, 3, \ldots, J-1 \quad (11) \\
\bar{\phi}_j = \left(B_j - A_j\Gamma_{j-1}\right)^{-1}\left(d_j - A_j\bar{\phi}_{j-1}\right) \quad j = 2, 3, \ldots, J \quad (12)
\]

In the downsweep the solution is obtained by

\[
\phi_j = \bar{\phi}_j \\
\phi_j = \bar{\phi}_j - \Gamma_j\phi_{j+1} \quad j = J-1, J-2, \ldots, 1 \quad (13)
\]

( \( A_1 \) and \( C_J \) do not appear in the system. If we define \( A_1 = C_J = 0 \), equations (11) and (12) can be used for \( j = 1, 2, \ldots, J \); equation (13) can be used for \( j = J, J-1, \ldots, 1 \); and the special equations for \( j = 1 \) and \( j = J \) are not needed.)
As mentioned in the previous section, the edge boundary condition is really asymptotic. The imposition of equation (6) or (10) at \( y_j \) is only tentative. At the end of the upsweep, the values of \( \phi_j = \bar{\phi}_j \) are compared with the known free-stream values. If they are not equal within specified tolerances, \( J \) is increased by one; the already known solutions are suitably extrapolated; \( A_j, B_j, C_j, \) and \( d_j \) for \( j = J_0 \), the old value of \( J \), are recomputed using expressions valid for \( j < J \), the upsweep is redone for \( j = J_0 \) and done for \( j = J \); and the test is repeated. When the tolerances are satisfied, the given free-stream values are substituted for the computed values at \( J \) (from which they differ by less than the tolerances) and the downsweep is performed.

Consider a regime where \( \bar{w} = 0 \), the turbulence is negligible, and \( y \) derivatives are negligible. The equations for the means reduce to

\[
\bar{\rho}\bar{u}\bar{u}_x = -\bar{p}_x \tag{14}
\]

\[
\bar{\rho}\bar{u}\bar{h}_x = \bar{u}\bar{p}_x \tag{15}
\]

(taking \( \text{GMOMS} = 1 \)). If the edge conditions on \( \bar{u}, \bar{h}, \) and \( \bar{p} \) satisfy these relations (\( \bar{p} \) being determined by the equation of state), this handling of the free-stream boundary conditions works well. Otherwise, an unreasonable number of points may be added, satisfying finite-difference versions of equations (14) and (15) at the added points while trying to match conflicting given edge values. In this case, it is probably not right to assume \( y \) derivatives are negligible; but if they are not, it isn't clear how the edge of the calculation field should be determined.

If there is free-stream turbulence, another situation arises. Consider the simplest case with \( \bar{w} = 0 \), the other mean quantities constant, \( y \) derivatives negligible, and
\[ u^i u^j = \delta^i_1 q^2 / 3 \]

Then it is found that \( q^2 \) is governed by

\[ \overline{\rho} u (q^2)_x = - \frac{2 \overline{\rho}}{\Lambda^2} \left( a + b \frac{\overline{\rho} q A}{\overline{u}} \right) q^2 \]

That is, dissipation will make free-stream turbulence decrease downstream unless \( \Lambda \to \infty \), according to the equations being used. There is no provision in the program as it stands for the turbulence-level edge condition to vary with \( x \) so excessive points will be added if free-stream turbulence is specified unless \( \Lambda \) is made to increase for large \( y \). The scale parameters which control the calculation of \( \Lambda \) allow for such increase.

Excessive adding of points at the edge can always be avoided by increasing the tolerance used to check the matching of edge conditions for the variables giving trouble. This may give results of questionable validity.

There are three dependent variables that are not determined from parabolic equations. These are \( \overline{p}, \overline{\rho}, \) and \( \overline{v} \). The mean pressure is calculated at \( x = x_n \) during the downsweep of the first pass from the normal momentum equation displayed in Section 2. The integration starts with a given edge value of \( \overline{p} \) (which may be a function of \( x \)) and proceeds inward to the wall. The mean density is calculated at \( x = x_n \) during the downsweep of the fourth pass from the mean equation of state, which can be written

\[ \overline{\rho} = \frac{c_p \overline{p}}{R (\overline{h} - \overline{h}' h'/\overline{h})} \]

The normal component of the mean velocity is calculated at \( x = x_n \) after the completion of all the passes, using the
continuity equation which is also displayed in Section 2. The integration starts with a given wall value of $\bar{v}$ (which may be a function of $x$) and proceeds outward to the edge. Trapezoidal integration is used for both $\bar{p}$ and $\bar{v}$.

Another quantity, namely $\bar{u}_x$, is grouped with $\bar{p}$, $\bar{\rho}$, and $\bar{v}$. Since its value for the current step is not available in the first pass, its value saved from the previous step is used there. The contribution of the terms in which it appears is expected to be small, so the error involved in lagging it this way should also be small. The value from the current step is set aside for use in the next step as part of the downsweep of the second pass.
4. AUXILIARY COMPUTATIONS

Some functionals of the dependent variables are calculated at the completion of each x step (as well as during the initialization stage). These calculations serve three main purposes: (1) they provide output which helps the user interpret the results of the program, (2) they provide data used in determining the finite-difference mesh for the next step, and (3) they provide data on which the calculation of the local value of A is based. In the last case, the process is not really auxiliary since A is basic to the modeling and strongly influences the course of the solution.

Many of the calculations are repeated for each of the parabolic variables. To avoid long descriptions, such procedures are set forth here as if there were only one independent variable, named F.

These quantities include: (1) the "max" which is the maximum over \( y_j \) of \(|F|\) at \( x_n \), (2) the value of y at which the max occurs, (3) the "global max" which is the maximum over \( y_j \) and \( x_n \) (up to the current \( x_n \)) of \(|F|\), (4) the "max change" which is the maximum over \( y_j \) of

\[
|F(x_n, y_j) - F(x_{n-1}, y_j)|
\]

(5) the integral (according to the trapezoidal rule)

\[
I(F) = \int_0^{y_J} |F|\,dy
\]

(6) the "integral spread" which is \( I(F) \) divided by the max of \( F \), and (7) the "percent spread" which is the value of \( y \) for which \( F \) is a designated fraction of its max.
In addition, \( \delta_{99} \), which is the value of \( y \) for which \( \overline{u} \) is 99 percent of its free-stream value, is found. A typical length, \( L_t \), is defined as a linear combination of \( \delta_{99} \) and the percent spread of one of the parabolic variables. The coefficients are inputs as is the choice of which percent spread to use.

The calculation of \( A \), the turbulence scale, is given by

\[
A = \min(c L_t, A_0 + d_i y)
\]

\[
A = \begin{cases} 
A_i & y \leq \delta_{99} \\
A_i + d_o (y - \delta_{99}) & y > \delta_{99}
\end{cases}
\]

The quantities \( c \), \( d_i \), \( d_o \), and \( A_0 \) are input parameters. For normal values of these constants, \( A \) equals \( A_0 \) at the wall and increases with slope \( d_i \) until it reaches the value \( c L_t \). It stays at that value the rest of the way out if \( d_o \) is zero. Otherwise, it starts increasing again at \( y = \delta_{99} \) with slope \( d_o \). The parameter \( A_0 \) is made greater than zero to simulate wall roughness. The parameter \( d_o \) is made greater than zero to allow for free-stream turbulence without dissipation there, as mentioned in Section 3. Otherwise, both \( A_0 \) and \( d_o \) are normally zero.

The rest of the auxiliary calculations are done to provide informative output only. They have no effect on the course of the program. The output labels and definitions are:

momentum thickness

\[
\Theta = \int_0^y \frac{\rho \overline{u}}{\rho_{e} u_{e}} \left( 1 - \frac{\overline{u}}{u_{e}} \right) dy
\]
displacement thickness

\[ \Delta^* = \int_0^y \left(1 - \frac{\overline{\rho u}}{\rho_e u_e}\right) dy \]

shape factor

\[ \text{SHAPE} = \frac{\Delta^*}{\theta} \]

kinematic displacement thickness

\[ DTI^* = \int_0^y \left(1 - \frac{\overline{u}}{u_e}\right) dy \]

wall shear

\[ \tau_w = \left[ \frac{1}{\mu} \frac{\partial \overline{u}}{\partial y} \right]_{y=0} \]

local skin friction coefficient

\[ \text{CF} = \frac{\tau_w}{\frac{1}{2} \rho_e u_e^2} \]

wall heat flux

\[ Q_{\text{WALL}} = - \left[ \frac{k}{c_p} \frac{\partial}{\partial y} \left( \overline{h} + \frac{\overline{\mu c_p}}{2k} u^2 \right) \right]_{y=0} \]

total Stanton number

\[ \text{STANT} = \frac{Q_{\text{WALL}}}{\left[ \rho_e u_e \left( h_w - h_e - \frac{1}{2} u_e^2 \right) \right]} \]
Reynolds numbers

\[ \text{RETHETA} = \text{THETA} \frac{\rho_e u_e}{\mu_e} \]

\[ \text{REXW} = \text{XW} \frac{\rho_e u_e}{\mu_e} \]

laminar stress

\[ \text{TAULAM} = \bar{\mu} \frac{\partial \bar{u}}{\partial y} \]

total stress

\[ \text{TAUTOT} = \bar{\mu} \frac{\partial \bar{u}}{\partial y} - \bar{\rho} u' v' \]

total heat flux

\[ \text{QTOT} = -\frac{K}{c_p} \frac{\partial \bar{h}}{\partial y} + \bar{\rho} h' v' \]

The integrals are done by the trapezoidal rule. The derivatives are evaluated by equation (2) of Section 3 except at the wall where a two-point formulation is used. In this regard it should be pointed out that the \( \bar{u}^2 \) term in the formula for QWALL is superfluous as a derivative (since \( \bar{u} = 0 \) at the wall, the derivative of its square is zero there), but in finite-difference terms the form given can be shown to be more accurate, at least if \( \bar{w} = 0 \) and \( \bar{v} = 0 \) at the wall.

None of the calculations in this group take into account the possible effects of nonzero \( \bar{w} \). Modifications or additional calculations to show such effects could easily be introduced into the program.
5. CONTROL OF THE MESH

In order to provide reasonable accuracy without excessive computing; the mesh is modified by the program as the run progresses. Parameters supplied by the user allow him to control the balance between accuracy and computing time.

The value of $\Delta x$ for the first step of a run is supplied as an input. Thereafter, a new $\Delta x$ is computed before each $x$ step. To do this, tentative values of the step size are computed for each of the 17 parabolic variables (that is, the variables determined by parabolic equations), and the smallest of these is selected. Each tentative value is determined by multiplying the previous $\Delta x$ by the ratio of a criterion established for the particular variable to the max change of that variable for the previous step. The criterion is a nominal change that can be specified either as an absolute quantity or as some factor times the global max of the variable with an optional lower limit. The new $\Delta x$, the minimum of the tentative values, is then modified if necessary so that it doesn't exceed either a specified factor times the old $\Delta x$ or a specified $\Delta x_{\text{max}}$. On the other hand, if the new $\Delta x$ is less than a specified fraction of the old, "backup" occurs: That is, some change generated by the previous step is considered unacceptably large so the results of the solution for conditions at $x_n$ just computed are discarded, and a smaller step from the conditions at $x_{n-1}$ is tried, using the new smaller $\Delta x$. A minimum $\Delta x$ may also be specified. If $\Delta x$ becomes less than the minimum, the run is halted.

A further complication results from the existence of so-called time breaks. These are values of $x$ at which the user requires output. In order to avoid an abnormally small $\Delta x$ as the last step to such a point, the program looks several steps ahead and modifies the $\Delta x$ found by the process described above, if need be.
The initial distribution of points in the \( y \) (normal) direction is part of the input of the initial conditions. Before the first and before each succeeding \( x \) step, a complex process is used to monitor and, if indicated, to adjust the spacing of points in the \( y \) direction. The basic idea is to examine each set of three adjacent points, \( Y_{j-1}, Y_j, Y_{j+1} \). If, for all the dependent variables, the values for those points fall almost on a straight line, the mid-point can be dropped. On the other hand, if, for at least one of the dependent variables, there is an excessive departure from a straight line, a point should be inserted in one of the subintervals \( Y_{j-1}, Y_j \) or \( Y_j, Y_{j+1} \), or in each. (These notions are made quantitative by comparing a nondimensional measure of the departure from a straight line, for each dependent variable, with input parameters. The measure is the absolute value of the second derivative, as given by equation (4), divided by the global max and multiplied by the square of the length of the appropriate interval.) However, it is also required that the ratio of the lengths of adjacent intervals be less than a specified value. This requirement can prevent a point from being dropped, that otherwise would be, but does not prevent an insertion. Instead, insertions are made in other intervals until the ratio requirement is satisfied everywhere.

There are also means of specifying maximum and minimum interval lengths. The maximum prevents the dropping of a point which would cause it to be exceeded. If the insertion of a point creates an interval length smaller than the minimum, a warning message is printed.

Furthermore, if the insertion of points would make \( J = J_{TOP} \) bigger than its upper limit, \( J_{MAX} \) (an input), the user has the option of allowing the relaxation of the criteria governing the dropping and insertion of points. If the relaxation is not allowed or is unsuccessful in keeping \( J \leq J_{MAX} \), the run halts.
When a point is inserted, it is not necessarily put at the mid-point of the interval. Rather it is weighted toward the point with the largest (in absolute value) normalized second derivative considering all the dependent variables. The restriction on ratio between the lengths of adjacent intervals is observed, of course.

The values of the dependent variables that are assumed for an inserted point are obtained by linear interpolation. This seems like an anticlimax, but it is the only simple scheme that has been found here that does not lead to undesirable results in some cases.
6. PROGRAM STRUCTURE

The principles of invariant modeling have been applied at A.R.A.P. to several different types of turbulent flows that can be described by parabolic equations. The technique developed for solving those problems has been used for the GYC program; and, to a large extent, the actual subprograms have been carried over, with varying amounts of modification.

What has been referred to as the GYC program is actually a main program, GYCON, and a number of subprograms. GYCON itself is quite simple. Basically, it consists of a call to GYCIN, which handles input and initialization, followed by a loop consisting of calls to: (1) GYCAL, which handles the auxiliary computations, the determination of the new \( \Delta x \), and output; (2) GYCAA, which handles the distribution of the points \( y_j \); (3) GYCSS, which does the actual solution for one \( x \) step; and (4) GYCRV, which determines \( \bar{v} \). The call to GYCRV is followed by a return to the call to GYCAA. This basic flow of the program is modified by certain special conditions; for example, if backup is required (see Section 5), the call to GYCAA is skipped. When the run is halted, GYCIN is called again. Depending on the input cards supplied, this can result in a restart of the same run, the start of a new run, or job termination.

Before describing the operation of the principal subprograms mentioned above, an aspect of data handling should be mentioned. Some of the other programs from which this one evolved needed more data storage during the solution process than was available in core (especially when the A.R.A.P. computer had only 8,192 words of 16 bits each in core). Such programs used external (disk) storage; and the GYC program, although contained entirely in core, mimics that arrangement by keeping a good part of the working data in a two-dimensional
array called STOR. The elements of STOR are never addressed directly; instead, parts of STOR are copied to and from other arrays in core, much as if STOR were an external device.

STOR consists of 52 elements for each \( j \), that is, for each normal mesh point. The first element is the value of \( y_j \) itself. The next 17 elements are the old values of the parabolic variables, that is, the values evaluated at \( (x_{n-1}, y_j) \) in the order they are listed in Table 5. Then come the old values of \( \frac{\partial}{\partial t}, \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \) and \( \frac{\partial}{\partial x} \). The following 17 elements are used for the new values of the parabolic variables and also for the values of \( \frac{\partial}{\partial j} \), in the notation of equation (12), while generating the new values. The next four elements are the new values of \( \frac{\partial}{\partial t}, \frac{\partial}{\partial y}, \frac{\partial}{\partial x}, \) and \( \frac{\partial}{\partial x} \). The rest of the 52 elements are used to hold the matrix \( \Gamma_j \).

The copying or moving in and out of STOR is done by a subprogram, SFVMV. With parameters \( A, B, \) and \( N \), SFVMV is equivalent to the simple DO loop:

\[
\text{DO 8 I = 1,N}
\]

\[
8 \ B(I) = A(I)
\]

It exists as a subroutine because it is used so often; at A.R.A.P. it is implemented in firmware to speed its operation. A companion subprogram, SFVFL, fills \( N \) elements of the vector \( B \) with the value of the variable \( A \), when the parameters are \( A, B, \) and \( N \).

Of the principal subprograms, GYCIN is the most straightforward. It initializes certain quantities, it reads the first data card, it calls GYCRI to read the rest of the data cards if the first card indicates that there are more, it calls GYCI to do more initialization if a new run is being started, and finally it calls GYCP to print the title pages of the output.
GYCAL first calls GYCQD and GYCAQ which do the auxiliary computations described in Section 4 and which also compute the new $\Delta x$ as described in the first part of Section 5, except for the modification due to time breaks which is done by a call to GYCTB. The bulk of GYCAL is concerned with calls to GYCOI and GYCOT if the various criteria so indicate. GYCOI produces a minor printout, whereas GYCOT produces a major printout including tables of the dependent variables as functions of $y$.

GYCAA is the most complex of the subprograms. It is also the least changed from versions in use for other problems. Its function is to monitor and, if indicated, to adjust the spacing of points in the $y$ direction, as described in Section 5. Suffice it to say that it works as advertised.

GYCSS, together with its subprograms, forms the heart of the system. It performs the upsweep and the downsweep, repeating the process for each of the passes described in Section 3. Since three-point differencing is used in the $y$ direction, a section of COMMON is used for values of variables at $j-1$, that is, variable names ending with the letters M and I (compare the sketches at the beginning of Section 3 and the beginning of the Glossary, Appendix F); another section of COMMON is used for values of variables at $j$, with names ending with the letters Z and J; and a third section is used for values of variables at $j+1$, with names ending with the letters P and K.

As $j$ is incremented at the beginning of a stage of the upsweep of a typical pass, the values in the Z-J section are shifted to the M-I section, and those of the P-K section to the Z-J section. Then GYCFF moves to the P-K section from STOR the values that are available there and calculates the values of others. Obviously, special provisions are made for $j = 1$. The values of the elements of $A_j$, $B_j$, $C_j$, and $d_j$ are calculated by the subroutines GYCB (for $j = 1$), GYCMC (for $2 \leq j \leq J - 1$), and GYCO (for $j = J$). After making some
calculations common to all passes, GYCMC in turn calls separate subroutines for each pass. (Due to some confusion in the early stages of the development, the subprogram for the first pass is named GYCP2 and that for the second pass is named GYCP1.) It is in these pass subprograms that the actual equations being solved (Appendix D) make themselves felt. They were automatically generated, for the most part, by DIFFR, another A.R.A.P. software package.

DIFFR took the equations of Appendix D, one pass at a time, along with some information about which symbols to treat as unknowns, as known variables, or as constants, and generated Fortran statements evaluating the elements of \( A_j \), \( B_j \), \( C_j \), and \( d_j \). These statements, after some minor manual editing, constitute the bulk of the pass subprograms.

Returning to the description of a stage of the upsweep, \( \Gamma_j \) and \( \Phi_j \) (equations (11) and (12) of Section 3) are evaluated using the special matrix subroutines GYCMP and GYCMI. Finally \( \Gamma_j \) and \( \Phi_j \) are moved to STOR, and \( j \) is incremented again.

The handling of the free-stream boundary conditions has already been described. When they are satisfied, the downsweep according to equation (13) of Section 3 is trivial. It is slightly complicated in the first, second, and fourth passes by the incorporation of the calculation of \( \bar{p} \), \( \bar{U}_x \), and \( \bar{p} \), respectively.

There are several other subprograms which have not been mentioned here. All are included in the Glossary (Appendix F) with brief descriptions. At the beginning of the listing of each subprogram are comments giving another view of its role.

Three fictitious subprograms are included in the Glossary, and places for calls to them are suggested by comments in the program. These provide for the possibility that it may be
desirable to create files in external storage for later plotting. Two types of files are anticipated. One, the "running file," would be of quantities that depend on \( x \) only, such as skin friction and momentum thickness. The other, the "profile file," would be of quantities that are functions of both \( x \) and \( y \), such as any of the dependent variables. The three subprograms are GYCPs which would initiate both files, GYCRF which would write to the running file, and GYCPF which would write to the profile file.
7. INPUT AND OUTPUT

Table 1 lists the variables read on each input card along with the format for each. Definitions of the variables are included in the Glossary (Appendix F).

There must be at least two and no more than two hundred Cards 16.n (see Table 2). They produce an internal table specifying the velocity, enthalpy, and pressure at the edge of the boundary layer and the wall blowing velocity and either the enthalpy or enthalpy gradient (heat transfer) at the wall, all as functions of downstream distance \( x \). The program uses linear interpolation in this table. For values of \( x \) outside the range of the table, it extrapolates using the first two or last two entries. The input values of \( x \) must be in ascending order. The end of the table is flagged by a nonincreasing value for \( x \) or by \( n \) reaching 200. In the former case, the values on the terminating card are not used. If all three quantities are constant with \( x \), only two cards, including the terminating card, need be used; the program will automatically generate another point to store in its table for interpolation. As noted at the foot of Table 2, \( h_e \) and \( -p_e \) need not be input. (An exception occurs for \( p_e \) when \( GMOMS = 0 \). In this case, the values supplied for \( p_e \) will be used whether positive, zero, or negative.) When the program needs them at a particular \( x \), the interpolated values from the internal table are first determined. If these are zero (or negative), values computed from the homentropic relations for a perfect gas are substituted. (If the inputs \( HSTAG = h_s \) and \( PSTAG = p_s \) are not needed for these calculations, they are not used at all.)

There must be at least five and no more than \( JMAX \) Cards 31.j (see Table 3). The values of \( y \) must be in ascending order. The end of the set is flagged by a nonincreasing value of \( y \) or by \( j \) reaching \( JMAX \). In the former case, the values on the terminating card are not used.
Provision is made for specifying the initial profile of \( A \) on Cards 31.j. This is intended chiefly for use when \( A \) is solved for by a differential equation in the future. If zero is supplied for \( A \), the values will be determined in the usual way from the scale parameters (see Note 1 of the Glossary, Appendix F).

Cards 32 and 33.j are not read at all if the flag terminating the set of Cards 31.j is negative. The corresponding dependent variables (see Table 4) are initialized to zero for all values of \( y \) in that case. If the flag terminating Cards 31.j is zero or positive, then Cards 32 and 33.j are read. The end of the set is flagged by the first value of \( Y \) scaled by \( GV(1) \) which fails to equal the corresponding value of \( Y \) scaled by \( FV(1) \) in Cards 31.j, taken in order, or by \( j \) reaching the total number of cards in that set. In the former case, the corresponding dependent variables are initialized to zero for the remaining values of \( y \).

As indicated in Tables 2, 3, and 4, the card sets designated 16.n, 31.j, and 33.j are multiplied, or scaled, by elements of the vectors \( EV \), \( FV \), and \( GV \), respectively. It is recommended that the beginner set all values of these vectors equal to one and forget them until a need is felt for them. Two typical situations where they would be useful are: (1) The initial conditions are supplied as a function of \( y \) in, say, inches, and it is desired to use feet in the program. Then \( FV(1) \) (and \( GV(1) \) if needed) could be set at 0.0833333 and \( y \) values in inches punched in Cards 31.j (and 33.j if needed). (2) After one run starting with a certain level of turbulence, it is decided to make another under the same conditions except that the initial turbulence level is different, or zero. Then it is only necessary to change one card to prepare the input to the new run.
Input may be in any consistent system of units; outputs will be the same units. Nondimensional inputs may be used, producing nondimensional output, but they must be consistent. For example, if BLAMO is made nondimensional with respect to a reference length, $L_r$, then all other input lengths (DELTI, DTMAX, etc.) must be supplied as ratios to $L_r$ and all output lengths ($X$, $Y$, $R$, etc.) will be referred to $L_r$. The introduction to the Glossary contains additional information on dimensions and nondimensionalization.

A deck for one job might be set up as follows:

Card 1 with INFLG = 1
Cards 2-31.j, input for first run
Card 1 with INFLG = 2
Cards 2-29, restart input for first run
Card 1 with INFLG = 1
Cards 2-31.j, input for second run
Card 1 with INFLG = 0

This assumes a negative flag terminating the profile input is included in Cards 31.j. The above job, then, consists of two completely separate runs, the first being restarted after its first halt. The restart could be used to introduce a different cone angle, for example. If a run which halts on an error condition is set up for a restart, the restart probably won't be what's intended but the error condition is likely to recur soon. On the other hand, it's possible to design the changes introduced on restart precisely to overcome an anticipated error condition. See Note 7 of the Glossary (Appendix F) for some specific information on restarts.

Notes 5 and 6 of the Glossary contain information on inputs for special modes of operation.
Output consists of title pages, minor printouts, and major printouts. The title pages display the information input on Cards 1-29, for the most part in the same order. The labels are included in the Glossary, but one item near the top of the second title page is not labeled. Normally this is F but if the option of zeroing out correlations involving $\rho'$ and $h'$ has been selected, a T appears there.

A minor printout gives basic information on the progress of the run and includes most of the auxiliary quantities described in Section 4. Two unlabeled items appear in the top line. On the left, there appears the name of the parabolic variable for which the smallest tentative $\Delta x$ for the next step was computed, as described in Section 5. The letter Y appears there if such a calculation has not just been made, e.g., at the start of a run. On the right, there appears the name of a variable which did not satisfy its edge tolerance until a point was added, as described in Section 3. It is not necessarily the only such variable. If no points have been added at the outer edge since the last minor printout, the letter Y appears there.

A major printout consists of a minor printout plus profiles of the dependent variables and of the auxiliary quantities which are functions of $y$.

A minor printout occurs alone only if NSTEP is a multiple of NIOLP. A major printout occurs: (1) if NSTEP is a multiple of NFOLP, (2) at time breaks (TBRKV), (3) if the maximum or the percent spread has changed by more than PCFMX or PCFSP times the value at the last major printout, (4) if NSTEP reaches NSTMX, or (5) if an error condition occurs. If NFOLP is one (a major printout at every step is specified), two extras are included: (1) profiles are printed after points have been inserted in or deleted from the $y$ mesh by GYCAA and (2) a major printout shows the discarded results generated before each backup.

40
A run halts when the absolute value of a negative time break is reached, when NSTEP reaches NSTMX, or when an error condition occurs.

Error conditions produce messages indicating the nature of the problem. A.R.A.P. should be consulted if a cure doesn't suggest itself.
8. FINAL REMARKS

Along with the GYC program, an "SSF" listing of it was delivered to ARC. The SSF listing (named after the A.R.A.P. program that produced it) is an alphabetical index of variable names and subprogram names, locating every reference to each by subprogram and position within it, and printing the lines so located. Since GYC makes much use of COMMON, the SSF listing is a valuable aid in tracing the modification and uses of variables.

Anyone who wants to understand the details of the operation of GYC must turn to the program listings. Considerable effort has been expended (with varying degrees of success) to reduce clutter and to make the listings readable. With rare exceptions, the program flow is from top to bottom. Statement numbers appear in order according to an obvious scheme. DO loops are indented and occasionally indentation is used to set off other statements. Natural breaks in the program are signalled by blank lines. (Rows of asterisks or other symbols are not used. It has been realized for several hundred years that decorated pages are not easy to read, attractive though they may be.)

With the descriptions of Section 6 as a start and with the aid of the SSF listing, those familiar with Fortran should be able to determine what is going on throughout GYC.
9. REFERENCES


APPENDIX A. - THE EQUATIONS BEFORE MODELING

\[
\left( \frac{\bar{\rho} u}{\bar{u}} \right)_z + \left( \frac{\rho' u'}{u'} \right)_z = 0
\]

\[
-\bar{\rho} u \frac{\bar{u}}{u} + \rho' u' \frac{u'}{u} + \left( \bar{\rho} u \frac{\bar{u}}{u} + \bar{\rho} u \frac{\bar{u}}{u} + \rho' u' \frac{u'}{u} \right)_z =
\]

\[
= -p_z + \left( \frac{\mu u}{u} + \frac{\mu u}{u} + \mu' u' \frac{u'}{u} + \mu' u' \frac{u'}{u} \right)_z +
\]

\[
+ \left( \frac{\mu k}{k} + \frac{\mu k}{k} \right)_z
\]

\[
-\bar{\rho} h \frac{\bar{h}}{h} + \rho' h' \frac{h'}{h} + \left( \bar{\rho} h \frac{\bar{h}}{h} + \bar{\rho} h \frac{\bar{h}}{h} + \rho' h' \frac{h'}{h} \right)_z =
\]

\[
= -\frac{\bar{u}}{\bar{p}}, \bar{u} + \frac{u'}{p'}, \frac{u'}{u'} + \left( \frac{\mu u}{u} + \frac{\mu u}{u} + \mu' u' \frac{u'}{u} + \mu' u' \frac{u'}{u} \right)_z \frac{\bar{u}}{m} +
\]

\[
+ \left( \frac{\mu k}{k} + \frac{\mu k}{k} \right)_z \frac{\bar{u}}{u} + \frac{\bar{u}}{m} \frac{\bar{u}}{m} +
\]

\[
+ \bar{\mu} \left[ \left( \frac{\bar{u}}{m} \frac{\bar{u}}{m} \right), \frac{\bar{m}}{m} \frac{\bar{m}}{m} - 2 \left( \frac{\bar{u}}{m} \frac{\bar{u}}{m} \right)_m + \frac{\bar{u}}{m} \frac{\bar{u}}{m} \right] +
\]

\[
+ \left( \frac{\bar{u}}{m} + \frac{\bar{u}}{m} \right)_m \mu' u' \frac{u'}{u} + \bar{\mu} \left( \frac{\mu u}{u} + \frac{\mu u}{u} + \mu' u' \frac{u'}{u} + \mu' u' \frac{u'}{u} \right) +
\]

\[
+ \left( \frac{\bar{k}}{T}, \frac{\bar{k}}{T} \right)_z + \left( \frac{\bar{k}}{T}, \frac{\bar{k}}{T} \right)_z
\]

44
\[
\begin{align*}
\bar{\rho}\bar{u}\left(\bar{u}_1^\ell u_j^\ell \right)_\ell + & \left(\frac{\bar{\rho} u^\ell u_1^\ell u_j^\ell + \bar{u} \rho u_1^\ell u_j^\ell}{\bar{\rho}^\ell u_1^\ell u_j^\ell}\right)_\ell - \left(\rho u^\ell u_1^\ell\right)_\ell u_1^\ell u_j^\ell + \\
& \bar{\rho} u_1^\ell u_j^\ell, \ell + \rho u_1^\ell u_1^\ell, \ell + \bar{\rho} u_1^\ell u_j^\ell, \ell + \\
& \bar{\rho} u_1^\ell u_j^\ell, \ell + \rho u_1^\ell u_1^\ell, \ell + \rho u_1^\ell u_j^\ell, \ell = \\
& = -u_1^\ell p_1^\ell, j - u_1^\ell p_1^\ell, i + \\
& + \bar{\mu} \left(\frac{u_1^\ell u_j^\ell}{u_1^\ell, j}, \ell - 2u_1^\ell u_1^\ell, j, \ell + u_1^\ell u_1^\ell, \ell + u_1^\ell u_j^\ell, \ell\right) + \\
& + \bar{\mu}, \ell \left(u_1^\ell u_j^\ell, \ell + u_1^\ell u_1^\ell, \ell + u_1^\ell u_j^\ell, \ell\right) + \left(u_1^\ell, \ell + u_1^\ell, i\right) u_j^\ell \mu_j^\ell, \ell + \\
& + \left(u_j^\ell, \ell + u_1^\ell, j\right) u_1^\ell, \ell + \left(u_1^\ell, \ell + u_1^\ell, i\right) \mu_j^\ell, \ell + \\
& + \left(u_j^\ell, \ell + u_1^\ell, j\right) u_1^\ell, \ell + \bar{u}_1^m \left(u_1^\ell, j^*, \ell + u_j^\ell, j^*, \ell\right) + \\
& + \bar{u}_1^m, m u_1^\ell, j^*, \ell + \bar{u}_1^m, j^*, \ell u_1^\ell, m + \bar{u}^* \left(u_1^\ell, m, j^* + u_j^\ell, m, j^*\right) + \\
& + \bar{u}_j^m, m u_1^\ell, j^*, \ell + \bar{u}_j^m, j^*, \ell u_1^\ell, m + \\
& + \bar{u}^* u_1^\ell, m + \bar{u}_j^* u_1^\ell, m
\end{align*}
\]
\[
\begin{align*}
\bar{\rho} u^\ell(h^i h^i)_l &+ (\bar{\rho} u^\ell h^i h^i + \bar{u}^\ell \rho^i h^i h^i)_l - (\rho^i u^\ell)_l h^i h^i + \\
&+ 2 \rho^i h^i \bar{u}^\ell h^i_{,l} + 2 \rho_\ell h^i u^\ell h^i_{,l} + 2 \rho_\ell \rho^i u^\ell h^i_{,l} = \\
&= 2 h^i p^i_{,l} + 2 u^\ell h^i p^i_{,l} + 2 \rho_\ell h^i u^\ell + 2 h^i u^\ell p^i_{,l} + \\
&+ 2 (\bar{\mu} u^\ell_{,m} + \bar{u} u^\ell_{,m} + \mu^i u^\ell_{,m} + \mu^i u^\ell_{,m}) h^i u^m_{,l} + \\
&+ 2 (\bar{\mu} u^k_{,k} + \mu^i u^k_{,k}) h^i u^\ell_{,l} + 2 \bar{u} u^\ell_{,m} (h^i u^m_{,l} + h^i u^m_{,l}) + \\
&+ 2 \bar{u} u^\ell_{,m} (\bar{u} u^m_{,l} + \bar{u} u^m_{,l}) u^i h^i_{,l} + 2 \bar{u} u^\ell_{,m} \mu^i h^i_{,l} + 2 \bar{\mu} u^\ell_{,m} h^i u^m_{,l} + \\
&+ 2 k h^i T^i_{,l} + 2 k h^i k^i_{,l} + 2 T^i k^i_{,l} = 2 T^i h^i k^i_{,l} + 2 T^i h^i k^i_{,l} \\
\end{align*}
\]
APPENDIX B. - THE MODELS

\[ u_{1}^{i} u_{j}^{i} u_{k}^{i} = -VUuqA \left( (u_{1}^{i} u_{j}^{i})_{k} + (u_{j}^{i} u_{k}^{i})_{j} + (u_{k}^{i} u_{i}^{i})_{j} \right) \]

\[ u_{1}^{i} u_{j}^{i} \rho ^{i} = -VRuqA \left( (\rho ^{i} u_{1}^{i})_{j} + (\rho ^{i} u_{j}^{i})_{i} \right) \]

\[ u_{1}^{i} u_{j}^{i} h ^{i} = -VUhqA \left( (h ^{i} u_{1}^{i})_{j} + (h ^{i} u_{j}^{i})_{i} \right) \]

\[ u_{1}^{i} \rho ^{i} \rho ^{i} = -VRqA \left( \rho ^{i} \rho ^{i} \right)_{i} \]

\[ u_{1}^{i} \rho ^{i} h ^{i} = -VRHqA \left( \rho ^{i} h ^{i} \right)_{i} \]

\[ u_{1}^{i} h ^{i} h ^{i} = -VHqA \left( h ^{i} h ^{i} \right)_{i} \]

\[ u_{i}^{t} u_{j}^{t} \rho _{m}^{t} = \frac{a}{\Lambda ^{2}} u_{1}^{i} u_{j}^{i} + b \frac{\rho q}{\Lambda \mu } \left( u_{1}^{i} u_{j}^{i} + (1 - \beta) g_{i} u_{j} \frac{u_{1}^{i} u_{j}^{i}}{3} \right) \]

\[ u_{i}^{t} h ^{i} \rho _{m}^{t} = \left( \frac{AHU}{\Lambda ^{2}} + BHU \frac{\rho q}{\Lambda \mu } \right) h ^{i} u_{1}^{i} \]

\[ u_{i}^{t} h ^{i} \rho _{m}^{t} = \left( ARU \frac{\rho q}{\Lambda ^{2}} \right) \rho ^{i} u_{1}^{i} \]

\[ h ^{i} h ^{i} \rho _{m}^{t} = \left( AHH \frac{\rho q}{\Lambda ^{2}} \right) h ^{i} h ^{i} \]

\[ u_{1}^{i} u_{j}^{i} = \pi _{g} u_{1}^{i} u_{j}^{i} \]

\[ h ^{i} u_{1}^{i} = \pi _{g} h ^{i} u_{1}^{i} \]

where \( \pi _{g} = -BDV \frac{\bar{p} \rho \gamma _{p}}{\gamma ^{p}} \)
\[
\begin{align*}
\overline{\rho' u'_1} &= \pi_{\rho} \rho' u'_1, \\
\overline{\rho' u'_1 u'_1} &= \overline{VBS}_{\rho} \rho' u'_1 u'_1, \\
\overline{u'_1 k' u'_1} &= \pi_{\nu} \nu u'_1 k'_1, \\
\overline{h' u'_1} &= \frac{1}{2} \pi_{h} \overline{h' u'_1}, \\
\overline{\rho' u'_1} &= \frac{1}{2} \pi_{\rho} \overline{\rho' u'_1}, \\
\overline{\rho' h'} &= \frac{1}{2} \pi_{\rho} \overline{\rho' h'}, \\
\overline{u'_1 u'_1}_{j,k} &= \frac{1}{2} \left( \overline{u'_1 u'_1}_{j,k} \right) - \frac{1}{4} \left[ \overline{g_{jkl}} \overline{u'_1 u'_1}_{j,k} \overline{u'_1 u'_1}_{j,k} + \right. \\
&\quad \left. + \frac{1}{2} \left( \overline{g_{jkl}} \overline{u'_1 u'_1}_{j,k} - \overline{g_{ijkl}} \overline{u'_1 u'_1}_{j,k} \right) \right] \\
\overline{h'_1}_{j} &= \overline{W} \left[ \overline{h'_1}_{j} \right] - \frac{1}{3} \overline{g_{ij}} \overline{h'_1}_{j} + \frac{1}{3} \overline{g_{ij}} \overline{h'_1}_{j} \\
\overline{\rho'_1}_{j} &= \overline{W} \left[ \overline{\rho'_1}_{j} \right] - \frac{1}{3} \overline{g_{ij}} \overline{\rho'_1}_{j} + \frac{1}{3} \overline{g_{ij}} \overline{\rho'_1}_{j} \\
\overline{u'_1}_{j} &= \overline{W} \left[ \overline{u'_1}_{j} \right] - \frac{1}{3} \overline{g_{ij}} \overline{u'_1}_{j} + \frac{1}{3} \overline{g_{ij}} \overline{u'_1}_{j} \\
\overline{u'_1 u'_1}_{j} &= (WGU/A^2) \left[ \overline{g_{mj}} \overline{u'_1}_{j} \overline{u'_1}_{j} + \overline{g_{nm}} \overline{u'_1}_{j} \overline{u'_1}_{j} + \frac{1}{9} \overline{g_{ij}} \overline{g_{nm}} \overline{u'_1}_{j} \overline{u'_1}_{j} \right].
\end{align*}
\]
\[ u_{i, j}^{\text{m}} u_{n, m}^{\text{n}} = \left(\frac{\text{WWGUq}}{\Lambda^3}\right) \left(\frac{u_{i, j}^{\text{m}}}{u_{i, j}^{\text{n}}} - \frac{1}{3} \varepsilon_{i, j, k} q^2 \right) \]

\[ u_{i, n}^{\text{m}} u_{n, m}^{\text{n}} = \left(\frac{q}{\Lambda}\right) \left[ \text{WWU} (u_{i, n}^{\text{m}} u_{n, m}^{\text{n}}) + \text{WWU}2 u_{i, n}^{\text{m}} \right] \]

\[ u_{n, m}^{\text{m}} u_{n, n}^{\text{n}} = \left(\frac{q}{\Lambda}\right) \left[ \text{WWD} (u_{n, m}^{\text{m}} u_{n, n}^{\text{n}}), k + \text{WWD}2 u_{n, m}^{\text{m}} \right] \]

\[ h_{i, n}^{\text{m}} u_{n, m}^{\text{n}} = \left(\frac{q}{\Lambda}\right) \left[ \text{WWh} (h_{i, n}^{\text{m}} u_{n, m}^{\text{n}}) + \text{WWH}2 h_{i, n}^{\text{m}} \right] \]

\[ \rho_{i, n}^{\text{m}} u_{n, m}^{\text{n}} = \left(\frac{q}{\Lambda}\right) \left[ \text{WWR} (\rho_{i, n}^{\text{m}} u_{n, m}^{\text{n}}) + \text{WWR}2 \rho_{i, n}^{\text{m}} \right] \]

\[ p_{i, n}^{\text{m}} = \bar{p} \Lambda^2 \left[ \text{PMU} (u_{i, n}^{\text{m}} u_{n, m}^{\text{n}}) + \text{PMU}2 u_{i, n}^{\text{m}} \right] \]

\[ p_{i, n}^{\text{m}} = \bar{p} \Lambda^2 \left[ \text{PMH} (h_{i, n}^{\text{m}} u_{n, m}^{\text{n}}) + \text{PMH}2 h_{i, n}^{\text{m}} \right] \]

\[ p_{i, n}^{\text{m}} = \bar{p} \Lambda^2 \left[ \text{PMR} (\rho_{i, n}^{\text{m}} u_{n, m}^{\text{n}}) + \text{PMR}2 \rho_{i, n}^{\text{m}} \right] \]

\[ p_{i, j, k}^{\text{m}} u_{j, i}^{\text{n}} u_{i, m}^{\text{n}} = \left(\frac{q}{\Lambda}\right) \left[ \text{PGU} (u_{j, i}^{\text{m}} u_{i, m}^{\text{n}}) + \text{PGU}2 u_{j, i}^{\text{m}} \right] \]

\[ p_{i, j, k}^{\text{m}} u_{j, i}^{\text{n}} u_{i, m}^{\text{n}} = \left(\frac{q}{\Lambda}\right) \left[ \text{PGH} (u_{j, i}^{\text{m}} u_{i, m}^{\text{n}}) + \text{PGH}2 u_{j, i}^{\text{m}} \right] + \]

\[ + \frac{\rho q}{\Lambda} h_{i, j}^{\text{m}} \]

\[ p_{i, j, k}^{\text{m}} \rho_{i, j, k}^{\text{n}} = \left(\frac{q}{\Lambda}\right) \left[ \text{PGR} (\rho_{i, j, k}^{\text{m}} u_{j, i}^{\text{n}}) + \text{PGR}2 u_{j, i}^{\text{n}} \right] + \]

\[ + \frac{\rho q}{\Lambda} \rho_{i, j, k}^{\text{m}} \]
\[
\begin{align*}
\overline{u_i^p}'_t + \overline{\mu_i^u p'_t} = \text{PTM} \left( (\overline{u_i^p}'_t + \overline{\mu_i^u p'_t}) \right) \\
\overline{h_i^p}'_t + \overline{\mu_i^h p'_t} = \text{PTHM} \left( (\overline{h_i^p}'_t + \overline{\mu_i^h p'_t}) \right) \\
\rho_i' h_i' = -h_i' \rho_i' \overline{\rho / h} \\
\rho_i' \rho_i' = -\rho_i' h_i' \overline{\rho / h} \\
\rho_i' h_i' h_i' = 0 \\
\overline{u_i^k u_i^p}'_i, \overline{\gamma} = 0 \\
\overline{h_i^u p_i^p}'_i, \overline{\gamma} = 0
\end{align*}
\]

The following constraints apply:

\[
3^\# \text{WGH1} + \text{WGH2} + \text{WGH3} = 0 \\
3^\# \text{WGR1} + \text{WGR2} + \text{WGR3} = 0
\]
APPENDIX C.- THE EQUATIONS AFTER MODELING

# MEAN CONTINUITY

\[(RS*US"L)\*L + RU"L\*L = 0\]

# MEAN MOMENTUM

\[RS*US"L*US'I,L + RU"L*US'I,L + (RS*UU"L*I + US"L*RU*I)L\]
\[- VRU*(QL*(RU*I)L + RU"L*I))L\]
\[- 2/3*EMUT*(WMT*TU"M*M - BDGP*TU"M*PS,M),I\]
\[+ (EMS*US"M,M + EMST*BDGP*TU"M*PS,M),I\]

# MEAN ENTHALPY

\[RS*US"L*HS*L + RU"L*HS*L + (RS*HU"L + US"L*RH),L\]
\[- VRH*(QL*RH*L)L\]
\[= GMOMS*(US*L*PS,L + (RS*BLAM*BLAM*(PMU*US"M,N*(1/2*UU"LN,M\]
\[1/4*(&"N*M*UU"LK,K = &"L*NUUU"NK,K)\]
\[+ 1/2*(&"N*M*BDGP*UU"LK*PS,K - &"L*BUU"NK*PS,K)\]
\[+ PMU2*Q/BLAM*(WWU1*UU"KL,K + WWU2*BDGP*UU"KL*PS,K)))\]
\[- RS*BLAM*BLAM*(PGU*US"M,M*(WMD*((BDGP*UU"NL*PS,L)*M\]
\[1/3*(&"N*M*(BDGP*UU"KL*PS,L)*K)\]
\[+ 1/3*(&"N*M*BDGP*BUU"KL*PS,K)\]
\[+ PGU2*Q/BLAM*(WMD1*(BDGP*UU"KL*PS,K),L\]
\[+ WMD2*BDGP*BDGP*UU"KL*PS,K*PS,L)\]
\[+ (EMU*(US"M,L + US"L,M\]
\[+ EMUT*(WMT*(TU"M,L + TU"L,M - 2/3*UU"L,M*TU"K,K\]
\[+ 2/3*UU"L,M*TU"KPS,K)))*US"M,L\]
\[(EMU*US"K,K + EMST*BDGP*TU"K*PS,K)*US"L,L\]
\[+ EMU*ALAM*3LAM*UU"L,L + BLAM*RS*Q*UU"L,L\]
\[+ EMU*(UU"LM,L - 2*(BDGP*UU"ML*PS,L)*M\]
\[+ BDGP*BUU"ML*PS,M*PS,L)\]
\[+ EMUT*US"L*M + US"M,L)*(WMT*(TU"L,M - 1/3*UU"L,M*TU"N,N\]
\[+ 1/3*UU"L,M*BDGP*TU"N*PS,N)\]
\[+ FMS*BDGP*BDGP*UU"ML*PS,M*PS,L\]
\[+ EMST*US"M,M*BDGP*TU"M*PS,L\]
\[+ (CAP*HS*L)L + 1/2*CAYT*TT*L*L\]
RS*US"L*HH* L - VHH*(RQL*HH* L) !L - RU"L* L*HH
  + 2*RH*US"L*HS* L + 2*RS*HS* L*HU"L - 2*VRH*QL*HS* L*RH* L
  = 2*GMO*MS* (PMH*US"M* N* (RS*BLAM*BLAM* (PMH*US"M* N* (WMH*
    (HU"N* M - 1/3* M* HU"K* K)
    + 1/3* M* BDGP* HU"K* PS* K)
    + PMH2*Q/BLAM* (WH1*HU"K* K + WH2*BDGP* HU"K* PS* K))) ) ) L
+ PS* L* HU"L
+ (EMU* (US"M* L + US"L* M )
+ EMUT* (WMT* (RU* M* L + TU"M* L - 2/3* M* TU"K* K)
  + 2/3* M* BDGP* TU"K* PS* K)) * (WHM* (HU"N* L
  - 1/3* M* HU"N* N) + 1/3* M* BDGP* HU"N* PS* N)
+ (EMS*US"K* K + EMST*BDGP*TU"K* PS* K) * BDGP* HU"L* PS* L
+ (CAP*HH* L) IL - 2* CAP* HHD* MF* HH
+ CAVT/CP/CP* (2* HS* L* L* HH + HS* L* HH* L)

RS*US"L*RU"I* L - (VRU* ROL* (RU"I* L + RU"L* I)  
+ VRR*QL* US"L* RR* I * L - RU"L* L* RU" I
+ RR* US"L* US"L* I + US"L* RS* L* RU" I
+ RS* US"I* L* RU" L + RS* RS* L* US"I* L
- VRR* QL* RR* I* US" I* L - VRU* QL* RS* L* (RU" I* L + RU"L* I)
= -(RS*BLAM*BLAM* (PMR* (WMR* (US"L* M* RU"N* M - 1/3* US"M* M* RU"L* L)
  + 1/3* US"M* M* BDGP* RU"L* PS* L)
  + PMR2*Q/BLAM* (WWR1*RU"L* L + WWR2*3DG* RU"L* PS* L)) * I
+ PGR*RS* (WGR1*US"N* M* RU" I + WGR2*US"M* I* RU" I
+ WGR3*US"I* M* RU" M)  + PGR2*WGR* RS* Q/BLAM* RU" I
- RS*BDGP* UU"L* I* PS* L - 2*RS* US"L* L* RU" I
+ 2*VBS* VRU* ROL* 3DG* PS* L* (RU" I* L + RU"L* I)
+ VRR* QL* US"L* L* RR* I
+ EMU* (WMR* (RU" I* L - 1/3* I* L* RU" K* K)
  + 1/3* I* L* BDGP* RU" K* PS* K) * I - EMU* RU" MF* RU" I
+ 1/2* (EMU + EMS) * (BDGP* RU" L* PS* L) * I
+ EMU* L* WMR* (RU" I* L + RU"L* I - 2/3* I* L* RU" K* K)
+ 2/3* I* L* BDGP* RU" K* PS* K)
+ EMS* I* BDGP* RU" M* PS* M
APPENDIX D.– THE FINAL EQUATIONS

FIRST PASS

\[ \begin{align*}
\text{RSUS} & \times (R \times WS) : 1 \\
+ \text{RSVS} & \times (R \times WS) : 2 \\
+ \text{RV} & \times (R \times WS) : 2 \\
+ (\text{RS} \times R \times VW) & : 2 \\
= (\text{FMU} \times (R \times WS) : 2) : 2
\end{align*} \]

\[ \begin{align*}
\text{RSUS} & \times (R \times VW) : 1 \\
+ \text{RSVS} & \times (R \times VW) : 2 \\
- 2 \times r \times RW \times WS \\
- 2 \times (VUU \times RQL \times (R \times VW) : 2) : 2 \\
- RV2 \times R \times VW \\
+ US \times RV \times (R \times WS) : 1 \\
+ VS \times RV \times (R \times WS) : 2 \\
- C \times WS \times RW \times WS \\
+ RS \times UV \times (R \times WS) : 1 \\
+ RS \times VW \times (R \times WS) : 2 \\
+ RS \times R \times VW \times VS2
\end{align*} \]

\[ \begin{align*}
= -1/4 \times PMU \times R \times R \times VW \times (RBLS \times (1/R \times WS) : 2) : 2 \\
- 1/4 \times PMU \times RBL \times WS \times R2 \times (R \times VW) : 2 : 2 \\
+ PGU \times WGU \times RS \times R \times UV \times (1/R \times WS) : 1 \\
+ PGU \times WGU \times RS \times R \times VW \times (1/R \times WS) : 2 \\
+ 1/3 \times S \times PGU \times WGU \times RS \times VW \\
+ S \times PGU \times WGU \times RS \times UV \times WS \\
+ C \times PGU \times WGU \times RS \times VW \times WS \\
+ 1/3 \times PGU \times WGU \times RS \times VS2 \times R \times VW \\
- C \times PGU \times WGU \times RS \times WS \times VW \\
- 2/3 \times PGU \times WGU \times RS \times US1 \times R \times VW \\
- ROOL \times R \times VW \\
+ (EMU \times (R \times VW) : 2) : 2 \\
- 2 \times A \times EMU \times BLAM / BLAM \times R \times VW \\
- 2 \times A \times BETA \times ROOL \times R \times VW \\
+ 1/4 \times EMU2 \times (R \times VW) : 2 \\
+ (1 - WMT/3) \times EMUT \times TV2 \times (R \times WS) : 2 \\
+ EMUT \times TV \times ((R \times WS) : 2) : 2
\end{align*} \]
SECOND PASS

RSUS*(US): 1
  + RSUS*(US): 2
  - S*RS/R*WS*WS
  + RV*(US): 2
  + (RS*UV): 2
  = -NPDX
  + (EMU*(US): 2): 2

RSUS*(UV): 1
  + RSUS*(UV): 2
  - 2*S*RS/R*WS*UVW
  - 2*r*RS/R*WS*UW
  - 2*(VUU*RQL*(UV): 2): 2
  - RV2*UV
  = C/R*WS*WS*RU
  + US*RV*(US): 1
  + VS*RV*(US): 2
  - S/R*WS*WS*RV
  + RS*VS2*UV
  + ∂S*US1*UV
  + RS*VV*(US): 2
  = 2*VRI*QSL*RV2*(US): 2
  - 1/4*PMU*R3LS*US2*(VV): 2: 2
  - (PMU2*WWU1*RQL*(UV): 2): 2
  + 1/3*PGU*WGU*RS*VS2*UV
  - C*PGU*WGU*RS/R*WS*UW
  + 1/3*PGU*WGU*RS*US1*UV
  + PGU*WGU*RS*VV*(US): 2
  - S*PGU*WGU*RS/R*WS*VV
  - 1/3*PGU*WGU*RS/R*US*UV
  - RQOL*UV
  + (EMU*(UV): 2): 2
  - 2*λ*EMU/BLAM/BLAM*UV
  - 2*R*BETA/RQOL*UV
  + 1/4*EMU2*(UV): 2
  + (1 - WMT/3)*EMUT*TV2*(US): 2
  + FMT*TV*((US): 2): 2

RSUS*(R*UW): 1
  + RSUS*(R*UW): 2
  - 2*S*RS/R*WS*W
  - (VUU*RQL*(R*UW): 2): 2
  - RV2*R*UW
THIRD PASS

RSUS*(UU): 1
  + RSUS*(UU): 2
  - 4*C*RS/R*WS*UWJ
  + (VUU*RQL*(UU): 2): 2
  - RV2*UU
  + 2*US*RU*US1
  + 2*VS*RU*US2
  - 2*S/R*WS*WS*RU
  + 2*RS*US1*UU
  + 2*RS*US2*VWJ
  - 2*VR*UQ*US2*(RU): 2
  = 8/9*PGU*WGU*RS*US1*UU
    + 4/3*PGU*WGU*RS*US2*UVJ
    - 2*S*PGU*WGU*RS/R*WS*UWJ
    - 4/9*PGU*WGU*RS*US2*UU
    - 4/9*PGU*WGU*RS*WS*UWJ
    - 2/3*PGU*WGU*RS*WS*UWJ
    - 2*3*PGU*WGU*RS*UWJ
    - 2/3*PGU*WGU*RS/R*US*UU
    - 2/3*PGU*WGU*RS/R*US*VV
    - 2/3*PGU*WGU*RS*US1*VV
    + 2/3*PGU*WGU*RS*US1*VV
    + 2/3*PGU*WGU*RS*US2*VV
    - 2/3*PGU*WGU*RS/R*US*VV
    + 2/3*RQOL*UU
    + 1/3*RQOL*VV
    + 1/3*RQOL*WW
 + (EMU*(UU): 2): 2
  = 2/9*EMU2*WLU1*ROL*(VV)U2: 2
  + P*(1 - WMT)*EMU*US2*(TU): 2
  + P*EMU*TU*US2.

RSUS*(VV): 1
  + RSUS*(VV): 2
  - 4*C*RS/R*WS*VWJ
  + 3*(VUU*RQL*(VV): 2): 2
  - RV2*VV
  - 2*C/R*WS*WS*RV
  + 2*S/R*VS*VV
  = -2*(PMU2*WWU1*RQL*(VV): 2): 2
+ \frac{1}{3}RQOL*R*U
+ \frac{1}{3}RQOL*R*V
+ (EMU*(R*W) \mp 2) ; 2
- 2*A*EMU/BLAM/BLAM*R*W
- 2*B*BETA*RQOL*R*W
- 2/3*B*OMBET*RQOL*R*U
- 2/3*B*OMBET*RQOL*R*V
- 2/3*B*OMBET*RQOL*R*W
+ 1/2*EMU2*R*(V) ; 2
+ 2*(1 - WMT)*EMUT*RWS2*(R*TW) ; 2
+ 2*EMUT*R*TW*RWS22
FOURTH PASS

RSUS*(HV):1
+ RSVS*(HV):2
- 2*C*RS/R*WS*HW
- 2*(VUH*RQL*(HV):2):2
- RV2*HV
+ US*RV*(HV):1
+ VS*R*(HV):2
- C/R*WS*WS*RH
+ RS*VV*(HV):2
+ RS*VS2*HV
- 2*VU*RQL*RV2*(HV):2
= GMOMS*UV*DPDX
+ GMOMS*VV*DPDY
- (PMH2*WGH1*RQL*(HV):2):2
+ PGH*WGH1*RS*US1*HV
+ PGH*WGH1*RS*VS2*HV
+ S*PGH*WGH1*RS/R*US*HV
+ PGH*WGH2*RS*US2*HV
+ PGH*WGH2*RS*VS2*HV
+ PGH*WGH2*RS/WSR2*RH
+ C*PGH*WGH2*RS/WS*HW
+ PGH*WGH3*RS*WS2*HV
- C*PGH*WGH3*RS/R*WS*HW
+ PGH2*WGH2*WQL*HV
+ 3/2*GMOMS*EMU*US2*UVJ2
+ 3/4*GMOMS*EMU*WSR2*VWRJ2
+ 3/4*GMOMS*EMU*WS2*RWJ2
+ GMOMS*EMUT/CP*US2*USJ2*HV
+ GMOMS*EMUT/CP*WS2*WRJ2*RWSJ2*HV
- (1 - 2*WMT/3) *(CAP*(HV):2):2
- CAP*HUDMF*HV
+ CAYT/CP/CP*(HV*(HV):2):2
- 2/3*CAYT/CP/CP*WMT*(HV):2
= EU*HU*HV
+ 4/3*EU*WMT*(HV):2

RSUS*(HH):1
+ RSVS*(HH):2
- VHH*(RGL*(HH):2):2
- RV2*HH

#
\[ + 2*US*RH*(HS):1 \\
+ 2*VS*RH*(HS):2 \\
+ 2*RS*(HS):2*HV \\
- 2*VRH*QL*HS2*(RH):2 \\
= 2*GMOMS*DPDX*HU \\
+ 2*GMOMS*UPU*Y*HV \\
+ 4*GMOMS*EMU*US2*WMH*(HU):2 \\
+ 2*GMOMS*EMU*RWS2*WMH*(1/R*HW):2 \\
+ 2*GMOMS*EMU*WSR2*WMH*(R*HW):2 \\
+ 2*GMOMS*EMUT/CP*US2*USJ2*HH \\
+ 2*GMOMS*EMUT/CP*WSR2*RWSJ2*HH \\
+ (CAP*(HH);2):2 \\
- 2*CAP*HHDMF*HH \\
+ 2*CAYT/CP/CP*HH*(HS):2;2 \\
+ CAYT/CP/CP*HS2*(HH):2 \]
RSUS*(RU): 1
+ RSUS*(RU): 2
- 2*s*RS/R*WS*RW
- (VRU*RQL*(RU): 2): 2
- RV2*RU
+ US*RR*US1
+ VS*RR*US2
- S/R*WS*WS*RR
+ US*RS1*RU
+ VS*RS2*RU
+ 3*RS*US1*RU
+ RS*US2*RV
+ RS*US2*UVJ
- VRR*QL*RR2*US2'
- VRU*QL*RS2*(RU): 2
= PGR*WGR1*RS*US1*RU
+ PGR*WGR1*RS*US2*RU
+ s*PGR*WGR1*RS/R*US*RU
+ PGR*WGR2*RS*US1*RU
+ PGR*WGR2*RS*WSR1*RS*RW
+ s*PGR*WGR2*RS/R*RW
+ PGR*WGR3*RS*US1*RU
+ PGR*WGR3*RS*US2*RV
+ s*PGR*WGR3*RS/R*WS*RW
+ PGR2*WGR*RGR*OL*RU
- P*RS*VS2*RU
- 2*s*RS/R*US*RU
+ EMU*(WGR*(RU): 2): 2
- FMU*RUDMF*RU
+ FMU2*WGR*(RU): 2
+ 1/2*EMU*US2*(RT): 2
+ EMUT*US22*RT

RSUS*(RV): 1
+ RSUS*(RV): 2
- 2*r*RS/R*WS*RW
- 2*(VRU*RQL*(RV): 2): 2
- RV2*RV
- C/R*WS*WS*RR
+ US*RS1*RV
+ VS*RS2*RV
+ 3*RS*VS2*RV
+ RS*RS2*VV
- 2*VRU*QL*RS2*(RV): 2
= -(PMR2*WGR1*RQL*(RV): 2): 2
SIXTH PASS

RSUS*(HU):1
+ RSVS*(HU):2
+ 2*S*RS/R*WS*HW
- (VUH*RQL*(HU):2):2
- RV2*HU
+ US*PU*(HS):1
+ VS*RU*(HS):2
+ US*RH*US1
+ VS*RH*US2
- S/R*WS*WS*IRH
- RS*UV*(HS):2
+ US*RH*US1
+ VS*RH*US2
- VRU*QL*RU2*HSJ2
- VRH*QL*RH2*USJ2

= GMOMS*UU*DPDX
+ 6GOMS*UV*DPDY
+ PGH*WG1*RS*US1*HU
+ PGH*WG1*RS*VS2*HU
+ S*PGH*WG1*RS/R*US1*HU
+ PGH*WG2*RS*US1*HU
+ PGH*WG2*RS*WSR1*R*HW
+ S*PGH*WG2*RS/R*WS*HW
+ PGH*WG3*RS*US1*HU
+ PGH*WG3*RS*US2*HVJ
- S*PGH*WG3*RS*R*WS*/HW
+ PGH2*WG*RO*US1*HU
+ GMOMS*EMU*US2*USJ2
+ 1/2*GMOMS*EMU*RWS*UWRJ2
+ 1/2*GMOMS*EMU*US2*USJ2
+ 1/2*GMOMS*EMU*WSR2*RUWJ2
+ GMOMS*EMUT/CP*US2*USJ2*HU
+ GMOMS*EMUT/CP*US2*UWRJ2
+ (CAP*(HU):2):2
- CAP*HUOMF*HU
- (WMT*CAP*(HU):2):2
+ CAYT/CP/CP*(HU*HS):2):2
- CAYT/CP/CP*HS2*WMT*(HU):2
- EMU*(WMH*(HU):2):2
- EMU*HUOMF*HU
- EMU*WMS*(HU):2
+ 1/2*EMUT/CP*US2*(HH):2
+ EMUT/CP*US2*HH

RSUS*(R*HW):1
+ RSVS*(R*HW):2
- (VUH*RQL*(R*HW):2):2
- RV2*R*HW
+ *US*RW*(HS):1
+ R*VS*RW*(HS):2
+ US*RH*RWS1
+ VS*RH*RWS2
+ RS*R*VW*(HS):2
\[
+ RS*RW*1*HU
+ vRU*QL*RR*2*HS*J2
- V*RH*QL*RH2*RSW*2
= GMOMS*R*UW*DPDX
+ GMOMS*R*VW*DPD*Y
+ PGH*WG*H1*RS*US*1*RS*UHW
+ PGH*WG*H1*RS*US*2*RS*UHW
+ S*PGH*WG*H1*RS*US*1*RS*UHW
+ S*PGH*WG*H2*RS*WS*SU*HW
- C*PGH*WG*H2*RS*WS*HV*J
+ S*PGH*WG*H2*RSUS*1*RS*US*HW
+ PGH*WG*H3*RSUS*1*RS*US*HW
+ PGH*WG*H3*RSUS*2*RSW*2*HV*J
+ S*PGH*WG*H4*RSUS*1*RS*US*HW
- S*PGH*WG*H3*RSUS*1*RS*US*HW
- C*PGH*WG*H3*RSUS*1*RS*US*HW
+ PGH2*WG*H3*RSUS*1*RS*US*HW
+ GMOMS*EMU*US*2*RUW*J2
+ 1/2*GMOMS*EMU*RW*2*WW*J2
+ 1/2*GMOMS*EMU*RSUS*2*RUW*V*J2
+ 1/2*GMOMS*EMU*RSUS*2*RUW*V*J2
+ GMOMS*EMUUT*CP*US*2*WS*J2*RS*HW
+ GMOMS*EMUUT*CP*US*2*WSW*2*RS*HW
+ (CAP(R*HW):2:2
- CAP*HUDMF*R*HW
- (WMT*CAP*(R*W*:2:2)
+ CAYT/CP/CP*(R*HW*+(HS):2:2
+ CAYT/CP/CP*HS*WMT*(R*HW):2
+ EMUT/WMH*(R*HW):2:2
- EMUT/HUDMF*R*HW
+ EMUT/WMH*(R*HW):2
+ 1/2*EMUT/CP*RW*2*(HH):2
+ EMUT/CP*RW*22*(HH)

BLAM = GYCSZ
APPENDIX E.- TENSOR NOTATION EXAMPLES

; is used as a separator in these examples.

<table>
<thead>
<tr>
<th>Common Notation</th>
<th>TENSOR Notation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scalars</td>
<td>A ; PHI ; TIME2 ; NU</td>
</tr>
<tr>
<td>Covariant Vectors</td>
<td>A'I ; PHI'R ; V'A</td>
</tr>
<tr>
<td>Contravariant Vectors</td>
<td>A''I ; PHI''R ; V''A</td>
</tr>
<tr>
<td>Various Tensors</td>
<td>A''IJ or A''I''J</td>
</tr>
<tr>
<td></td>
<td>A'IJ or A'I''J</td>
</tr>
<tr>
<td></td>
<td>A''I'J ; A''I''J</td>
</tr>
<tr>
<td></td>
<td>CAPSIG''IJM''KM</td>
</tr>
<tr>
<td>Covariant Derivative</td>
<td>PHI,I</td>
</tr>
<tr>
<td>Partial Derivative</td>
<td>PHI:I</td>
</tr>
</tbody>
</table>

68
Common Notation

Metric (or Fundamental) Tensor

\[ g_{ij} ; g^{ij} \]

Kronecker Delta

\[ \delta^i_j = g^i_j \]

Covariant Derivative with Index Raised (or "Contravariant Derivative")

\[ g^i_m \phi^m = \phi^i = \phi_i \]
APPENDIX F.- GLOSSARY

This glossary includes:

1) The names of the variables appearing in the tensor equations of Appendix C and in the final equations of Appendix D.

2) The names of all the subprograms of the GYC system.

3) The Fortran names of all the input variables.

4) The Fortran names of some of the important variables of the programs.

5) The labels used on the program output.

Many of the entries fit in several of the above categories.

In the list, several of the names end with __. This is used to indicate that the name may appear with any of the following appearing in place of _: M, Z, P, I, J, K, 1, 2, J2, 22, or none of them. The letters refer to the six-point finite-difference mesh used in the main solution process as indicated in the sketch.

![Diagram](attachment:image.png)

\[ \Delta x = \text{DELT} \quad x = \text{T|ME} \]
The values denoted by names ending in M, Z, and P, at the old x, are known. Those ending in I, J, and K may be known or unknown depending on the stage of the computation. The suffix 1 indicates the first derivative with respect to x between Z and J, 2 the first derivative with respect to y at Z, J2 the first derivative with respect to y at J, and 22 the second derivative with respect to y at Z. A name that has a different meaning without any such suffix is listed both with and without __, e.g., S. On the other hand, a variable that carries the same basic meaning without a suffix is listed once. For example, HH appears as such in Appendices C and D, and as an input variable, as well as in the form HHM, HHZ, etc. In any case it signifies h'h', so it appears in the list only as HH__.

The column labeled Dimensions gives the dimensions of the quantity in terms of a density, D, a velocity, V, a length, L, a temperature, θ, and an enthalpy per unit mass, H. (If mechanical units are used for heat, then H = V^2. The substitutions D = ML^-3 and V = LT^-1 will transform this set into a more familiar one.) Absence of any indication means the quantity is nondimensional. The symbol W means the elements of the vector have the same dimensions as the corresponding elements of the vector XV as given in Table 5. The combination W,N used with EPSBV and EPSTV means W if the element is input as a positive number and nondimensional if it is input as a negative number. Finally, X indicates some other mixture of dimensions for different elements.

At the end of several of the definitions there are expressions in square brackets. These are evaluations in terms of Mach number, Reynolds number, etc. of the variables when they are interpreted as nondimensional quantities under the following conditions:
1) The nondimensionalization is done with respect to a length, \( L_r \), and with respect to a reference density, \( \rho_r \), velocity, \( u_r \), temperature, \( T_r \), and enthalpy per unit mass, \( h_r \), that all refer to the same condition or point in the flow.

2) The reference state for the transport coefficients is that same condition, i.e., \( T_{r_f} = T_r \).

An example of how they are derived is given for the case of GMOMS in Section 2.

Notes to the Glossary appear at the end.

<table>
<thead>
<tr>
<th>Name</th>
<th>Dimensions</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td></td>
<td>Modeling parameter, ( a ). See Note 2.</td>
</tr>
<tr>
<td>AHH</td>
<td></td>
<td>Modeling parameter. See Note 2.</td>
</tr>
<tr>
<td>AHU</td>
<td></td>
<td>Modeling parameter. See Note 2.</td>
</tr>
<tr>
<td>AMAT</td>
<td>X</td>
<td>Matrix ( A ) (Section 3).</td>
</tr>
<tr>
<td>ARU</td>
<td></td>
<td>Modeling parameter. See Note 2.</td>
</tr>
<tr>
<td>AV</td>
<td>W</td>
<td>Vectors of the unknowns at the new ( x ). AVM, AVZ, and AVP would more logically be called XVI, XVJ, and XVK.</td>
</tr>
<tr>
<td>B</td>
<td></td>
<td>Modeling parameter, ( b ). See Note 2.</td>
</tr>
<tr>
<td>BDGP</td>
<td>( 1/(DV^2) )</td>
<td>(-BDV/(\gamma P))</td>
</tr>
<tr>
<td>BDV</td>
<td></td>
<td>Modeling parameter. See Note 2.</td>
</tr>
<tr>
<td>BETA</td>
<td></td>
<td>Modeling parameter, ( \beta ). See Note 2.</td>
</tr>
<tr>
<td>BHH</td>
<td></td>
<td>Modeling parameter. See Note 2.</td>
</tr>
<tr>
<td>BHU</td>
<td></td>
<td>Modeling parameter. See Note 2.</td>
</tr>
<tr>
<td>Name</td>
<td>Dimensions</td>
<td>Definition</td>
</tr>
<tr>
<td>---------</td>
<td>------------</td>
<td>-------------------------------------------------------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>BLAM_</td>
<td>L</td>
<td>Turbulence scale length. See Tables 3 and 5 and Note 1.</td>
</tr>
<tr>
<td>BLAMO</td>
<td>L</td>
<td>Scale parameter. See Note 1.</td>
</tr>
<tr>
<td>BMAT</td>
<td>X</td>
<td>Matrix B (Section 3).</td>
</tr>
<tr>
<td>BREAK POINTS</td>
<td></td>
<td>TBRKV</td>
</tr>
<tr>
<td>BRU</td>
<td></td>
<td>Modeling parameter. See Note 2.</td>
</tr>
<tr>
<td>BUFAC</td>
<td></td>
<td>Maximum ratio by which EPSTV criteria may be exceeded without causing backup.</td>
</tr>
<tr>
<td>C</td>
<td></td>
<td>DIMFL* cos θ_c</td>
</tr>
<tr>
<td>CAP_</td>
<td>DVL</td>
<td>k/cp</td>
</tr>
<tr>
<td>CAY_</td>
<td>DHVL/θ</td>
<td>Heat conductivity, k.</td>
</tr>
<tr>
<td>CAYRF</td>
<td>DHVL/θ</td>
<td>Reference value of k: its value at TMRF. See Note 3. [1/(RePr)]</td>
</tr>
<tr>
<td>CAYT_</td>
<td>DHVL/θ^2</td>
<td>Derivative of k with respect to temperature.</td>
</tr>
<tr>
<td>CF</td>
<td></td>
<td>Skin friction coefficient.</td>
</tr>
<tr>
<td>CHANGE PER STEP</td>
<td>W,N</td>
<td>EPSTV</td>
</tr>
<tr>
<td>CLAMB</td>
<td></td>
<td>Scale parameter. See Note 1.</td>
</tr>
<tr>
<td>CMAT</td>
<td>X</td>
<td>Matrix C (Section 3).</td>
</tr>
<tr>
<td>CMNTS</td>
<td></td>
<td>Up to 76 characters of comments to be printed on the title pages.</td>
</tr>
<tr>
<td>CONGL</td>
<td></td>
<td>Cone half-angle, θ_c, in degrees. See Note 5.</td>
</tr>
<tr>
<td>CP</td>
<td>H/θ</td>
<td>Specific heat at constant pressure, cp, assumed constant. [1.]</td>
</tr>
<tr>
<td>Name</td>
<td>Dimensions</td>
<td>Definition</td>
</tr>
<tr>
<td>---------</td>
<td>------------</td>
<td>------------</td>
</tr>
<tr>
<td>CSUTH</td>
<td>0</td>
<td>Constant in the viscosity law. See Note 3.</td>
</tr>
<tr>
<td>DELT</td>
<td>L</td>
<td>Step size in the x direction, Δx.</td>
</tr>
<tr>
<td>DELTA*</td>
<td>L</td>
<td>Displacement thickness.</td>
</tr>
<tr>
<td>DELTI</td>
<td>L</td>
<td>Input value of DELT. See Note 7.</td>
</tr>
<tr>
<td>DIMFL</td>
<td></td>
<td>Important: see Note 5. Dimension flag. A value of zero signifies flat-plate mode; a value of one signifies axisymmetric mode. Any other input value is converted to one.</td>
</tr>
<tr>
<td>DIN</td>
<td></td>
<td>Scale parameter. See Note 1.</td>
</tr>
<tr>
<td>DH/DY</td>
<td>H/L</td>
<td>The wall value of ( \frac{\partial h}{\partial y} ).</td>
</tr>
<tr>
<td>DLT99</td>
<td>L</td>
<td>Value of y for which ( \bar{u} ) is 99 percent of its free stream value.</td>
</tr>
<tr>
<td>DOUT</td>
<td></td>
<td>Scale parameter. See Note 1.</td>
</tr>
<tr>
<td>DFX</td>
<td>DV^2/L</td>
<td>Partial derivative of ( \bar{p} ) with respect to x.</td>
</tr>
<tr>
<td>DPHY</td>
<td>DV^2/L</td>
<td>Partial derivative of ( \bar{p} ) with respect to y.</td>
</tr>
<tr>
<td>DSM</td>
<td>L</td>
<td>( \text{SZ} - \text{SM} = h_\text{m} ) (Section 3).</td>
</tr>
<tr>
<td>DSP</td>
<td>L</td>
<td>( \text{SP} - \text{SZ} = h_\text{p} ) (Section 3).</td>
</tr>
<tr>
<td>DST</td>
<td>L</td>
<td>( \text{SP} - \text{SM} = h_\text{t} ) (Section 3).</td>
</tr>
<tr>
<td>DTMX</td>
<td></td>
<td>Maximum ratio by which Δx may increase from one step to the next.</td>
</tr>
<tr>
<td>DTI#</td>
<td>L</td>
<td>Velocity thickness.</td>
</tr>
<tr>
<td>DTMAX</td>
<td>L</td>
<td>Maximum Δx.</td>
</tr>
<tr>
<td>DTMIN</td>
<td>L</td>
<td>Minimum Δx.</td>
</tr>
</tbody>
</table>

74
<table>
<thead>
<tr>
<th>Name</th>
<th>Dimensions</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>DU/DX</td>
<td>V/L</td>
<td>$\bar{u}_x$</td>
</tr>
<tr>
<td>DVEC</td>
<td>X</td>
<td>Vector D (Section 3).</td>
</tr>
<tr>
<td>DZFMN</td>
<td></td>
<td>DZFMN*TYPL is the minimum $\Delta y$.</td>
</tr>
<tr>
<td>DZFMX</td>
<td></td>
<td>DZFMX*TYPL is the maximum $\Delta y$.</td>
</tr>
<tr>
<td>DZRMX</td>
<td></td>
<td>Maximum ratio of two adjacent values of $\Delta y$.</td>
</tr>
<tr>
<td>ECMN</td>
<td></td>
<td>Governs dropping of points from the profile.</td>
</tr>
<tr>
<td>ECMNI</td>
<td></td>
<td>Initial value of ECMN at each step.</td>
</tr>
<tr>
<td>ECMX</td>
<td></td>
<td>Governs adding points in the profile.</td>
</tr>
<tr>
<td>ECMXI</td>
<td></td>
<td>Initial value of ECMXI at each step.</td>
</tr>
<tr>
<td>EDGE</td>
<td>W,N</td>
<td>EPSBV</td>
</tr>
<tr>
<td>TOLERANCE</td>
<td></td>
<td></td>
</tr>
<tr>
<td>EDGEV</td>
<td>W</td>
<td>Free stream values of dependent variables.</td>
</tr>
<tr>
<td>EMS</td>
<td>DVL</td>
<td>Second coefficient of viscosity, $\mu^*$.</td>
</tr>
<tr>
<td>EMST</td>
<td>DVL/θ</td>
<td>Derivative of $\mu^*$ with respect to temperature.</td>
</tr>
<tr>
<td>EMU</td>
<td>DVL</td>
<td>First coefficient of viscosity, $\mu$.</td>
</tr>
<tr>
<td>EMURF</td>
<td>DVL</td>
<td>Reference value of $\mu$: its value at TMRF. See Note 3. $[1/Re_r]$</td>
</tr>
<tr>
<td>EMUT</td>
<td>DVL/θ</td>
<td>Derivative of $\mu$ with respect to temperature.</td>
</tr>
<tr>
<td>EPSBV</td>
<td>W,N</td>
<td>The solution is extended in the radial direction until the difference between the calculated value of each parabolic variable and its asymptotic value is less than EPSBV(I) or $-\text{EPSBV}(I)\times \text{GMAXV}(I)$, whichever is positive.</td>
</tr>
<tr>
<td>Name</td>
<td>Dimensions</td>
<td>Definition</td>
</tr>
<tr>
<td>--------</td>
<td>------------</td>
<td>-------------------------------------------------------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>EPSNV</td>
<td>W</td>
<td>When (</td>
</tr>
<tr>
<td>EPSTV</td>
<td>W, N</td>
<td>(\Delta x) is controlled by attempting to keep the change, TCMXV(I), in each of the parabolic variables (in the axial direction) below EPSTV(I) or (-EPSTV*GMAXV(I)), whichever is positive.</td>
</tr>
<tr>
<td>EV</td>
<td></td>
<td>See Table 2.</td>
</tr>
<tr>
<td>EXPMU</td>
<td></td>
<td>Exponent in the viscosity law. See Note 3.</td>
</tr>
<tr>
<td>FAC99</td>
<td></td>
<td>See Note 1.</td>
</tr>
<tr>
<td>FCMX</td>
<td></td>
<td>Maximum factor by which ECMNI and ECMXI may be increased through repeated applications of FCURI.</td>
</tr>
<tr>
<td>FCURI</td>
<td></td>
<td>Factor by which ECMN and ECMX are increased when JMAX impedes adding points in the profile.</td>
</tr>
<tr>
<td>FHM</td>
<td>1/L</td>
<td>(-H_-(Section 3)).</td>
</tr>
<tr>
<td>PHP</td>
<td>1/L</td>
<td>(H_+(Section 3)).</td>
</tr>
<tr>
<td>PHZ</td>
<td>1/L</td>
<td>(H_--H_+(Section 3)).</td>
</tr>
<tr>
<td>FINTV</td>
<td>L</td>
<td>Spreads of the dependent variables based on their integrals (output label: INT SPREAD).</td>
</tr>
</tbody>
</table>

76
<table>
<thead>
<tr>
<th>Name</th>
<th>Dimensions</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>FMAXV</td>
<td>W</td>
<td>Maxima over y of the dependent variables (output label: MAXIMUM).</td>
</tr>
<tr>
<td>FUP</td>
<td></td>
<td>Upwind differencing parameter. See Note 4.</td>
</tr>
<tr>
<td>FUPV</td>
<td></td>
<td>Upwind differencing parameter. See Note 4.</td>
</tr>
<tr>
<td>FV</td>
<td></td>
<td>See Table 3.</td>
</tr>
<tr>
<td>FZMXV</td>
<td>L</td>
<td>Locations of the maxima (output label: Y AT MAX).</td>
</tr>
<tr>
<td>GLOBAL MAX</td>
<td>W</td>
<td>GMAXV</td>
</tr>
<tr>
<td>GMAT</td>
<td>X</td>
<td>Matrix Γ (Section 3).</td>
</tr>
<tr>
<td>GMAXV</td>
<td>W</td>
<td>Maxima over y and x of the dependent variables (output label: GLOBAL MAX).</td>
</tr>
</tbody>
</table>
| GMOMS        | H/V²       | Important: see Note 6.  
The reciprocal of the mechanical equivalent of heat.  
\[
\frac{1}{(\gamma-1)M_r^2}
\] |
<p>| GV           |            | See Table 4.                                                             |
| GYCAAA       |            | Subroutine which adjusts the spacing of points in the y direction.       |
| GYCAL        |            | Subroutine which controls output and auxiliary computations.             |
| GYCAQ        |            | Subroutine which computes spreads and the next step size and which initializes for the next step. |
| GYCBBI       |            | Subroutine which implements the wall boundary conditions.                |
| GYCBBO       |            | Subroutine which implements the outer boundary conditions.              |</p>
<table>
<thead>
<tr>
<th>Name</th>
<th>Dimensions</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>GYCCM</td>
<td></td>
<td>COMMON file.</td>
</tr>
<tr>
<td>GYCCP</td>
<td></td>
<td>Subroutine which calculates the pressure distribution and the density distribution when they need to be determined separately.</td>
</tr>
<tr>
<td>GYCDZ</td>
<td></td>
<td>Subroutine which determines the position in y of a new point which is to be inserted.</td>
</tr>
<tr>
<td>GYCEW</td>
<td></td>
<td>Subroutine which interpolates to determine values of the specified free-stream variables at the current x.</td>
</tr>
<tr>
<td>GYCFP</td>
<td></td>
<td>Subroutine which transfers from STOR, or calculates, the values in the section of COMMON from QP to USK1.</td>
</tr>
<tr>
<td>GYCIC</td>
<td></td>
<td>Subroutine which handles some of the initialization for a new run.</td>
</tr>
<tr>
<td>GYCIN</td>
<td></td>
<td>Subroutine which controls start or restart procedures.</td>
</tr>
<tr>
<td>GYCIP</td>
<td></td>
<td>Subroutine which determines values of the dependent variables at a point inserted in the profile.</td>
</tr>
<tr>
<td>GYCLI</td>
<td></td>
<td>Subroutine for linear interpolation.</td>
</tr>
<tr>
<td>GYCMC</td>
<td></td>
<td>Subroutine which controls the computation of AMAT, BMAT, CMAT, and DVEC.</td>
</tr>
<tr>
<td>GYCMII</td>
<td></td>
<td>Subroutine which inverts 2 \times 2 and 3 \times 3 matrices.</td>
</tr>
<tr>
<td>GYCMPI</td>
<td></td>
<td>Subroutine which computes a matrix product and optionally subtracts the product from a third matrix.</td>
</tr>
<tr>
<td>GYCNS</td>
<td></td>
<td>Subroutine which flags certain unlikely errors.</td>
</tr>
<tr>
<td>Name</td>
<td>Dimensions</td>
<td>Definition</td>
</tr>
<tr>
<td>-------</td>
<td>------------</td>
<td>------------</td>
</tr>
<tr>
<td>GYCOI</td>
<td>Subroutine which produces a minor printout and the first part of a major printout.</td>
<td></td>
</tr>
<tr>
<td>GYCON</td>
<td>Main program of the GYC system.</td>
<td></td>
</tr>
<tr>
<td>GYCOT</td>
<td>Subroutine which produces the three profile tables of a major printout and also prints any error messages.</td>
<td></td>
</tr>
<tr>
<td>GYCPF</td>
<td>Fictitious subroutine which would write to a file for later plotting of profiles.</td>
<td></td>
</tr>
<tr>
<td>GYCP1</td>
<td>Subroutine which prints the first two pages of a new or restarted run.</td>
<td></td>
</tr>
<tr>
<td>GYCP2</td>
<td>Subroutine which prints the first two pages of a new or restarted run.</td>
<td></td>
</tr>
<tr>
<td>GYCP3</td>
<td>Subroutine to GYCMC for the second pass.</td>
<td></td>
</tr>
<tr>
<td>GYCP4</td>
<td>Subroutine to GYCMC for the first pass.</td>
<td></td>
</tr>
<tr>
<td>GYCP5</td>
<td>Subroutine to GYCMC for the third pass.</td>
<td></td>
</tr>
<tr>
<td>GYCP6</td>
<td>Subroutine to GYCMC for the fourth pass.</td>
<td></td>
</tr>
<tr>
<td>GYCP7</td>
<td>Subroutine to GYCMC for the fifth pass.</td>
<td></td>
</tr>
<tr>
<td>GYCP8</td>
<td>Subroutine to GYCMC for the sixth pass.</td>
<td></td>
</tr>
<tr>
<td>GYCP9</td>
<td>Subroutine which calculates various integrals and determines maxima.</td>
<td></td>
</tr>
<tr>
<td>GYCRF</td>
<td>Fictitious subroutine which would write to a file for later plotting of &quot;running variables&quot; as a function of x.</td>
<td></td>
</tr>
<tr>
<td>GYCRV</td>
<td>Subroutine which determines the normal component of the mean velocity.</td>
<td></td>
</tr>
<tr>
<td>Name</td>
<td>Dimensions</td>
<td>Definition</td>
</tr>
<tr>
<td>--------</td>
<td>------------</td>
<td>------------------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>GYCSC</td>
<td>L</td>
<td>Function which determines the value of the turbulent scale. See Note 1.</td>
</tr>
<tr>
<td>GYCSS</td>
<td></td>
<td>Subroutine which controls the main solution process.</td>
</tr>
<tr>
<td>GYCSZ</td>
<td>L</td>
<td>GYCSC(SZ)</td>
</tr>
<tr>
<td>GYCTB</td>
<td></td>
<td>Subroutine which makes a final adjustment to the step size, $\Delta x$, if needed to hit a break point.</td>
</tr>
<tr>
<td>GYCTC</td>
<td></td>
<td>Subroutine which evaluates $k$ and $\mu$ and their derivatives with respect to temperature. See Note 3.</td>
</tr>
<tr>
<td>H</td>
<td>H</td>
<td>$\bar{h}$. See Tables 3 and 5.</td>
</tr>
<tr>
<td>HE</td>
<td>H</td>
<td>Free-stream value of $\bar{h}$. See Table 2.</td>
</tr>
<tr>
<td>HG</td>
<td>H/L</td>
<td>$\bar{h}_y$ at the wall.</td>
</tr>
<tr>
<td>HH</td>
<td>H$^2$</td>
<td>$\bar{h}^2$. See Tables 4 and 5.</td>
</tr>
<tr>
<td>HHDMF</td>
<td>1/L$^2$</td>
<td>$\frac{AHH + BHH*REL}{BLAMZ/BLAMZ}$</td>
</tr>
<tr>
<td>HS</td>
<td>H</td>
<td>$\bar{h}$. See Table 5.</td>
</tr>
<tr>
<td>HSTAG</td>
<td>H</td>
<td>The stagnation enthalpy is $\text{ABS}(HSTAG)$. A positive value indicates that the wall value of the mean enthalpy is specified on Cards 16.n. A negative value indicates that the wall value of the gradient of the mean enthalpy is specified on Cards 16.n. $\left[1 + \frac{\gamma-1}{2} M_p^2\right]$</td>
</tr>
<tr>
<td>HSW</td>
<td>H,H/L</td>
<td>The wall value of $\bar{h}$ or of $\frac{\partial \bar{h}}{\partial y}$. See Table 2.</td>
</tr>
<tr>
<td>HU</td>
<td>HV</td>
<td>$\bar{h}^\prime u^\prime$. See Tables 4 and 5.</td>
</tr>
<tr>
<td>HUDMF</td>
<td>1/L$^2$</td>
<td>$\frac{AHU + BHU*REL}{BLAMZ/BLAMZ}$</td>
</tr>
<tr>
<td>HV</td>
<td>HV</td>
<td>$\bar{h}^\prime v^\prime$. See Tables 4 and 5.</td>
</tr>
<tr>
<td>Name</td>
<td>Dimensions</td>
<td>Definition</td>
</tr>
<tr>
<td>-------------</td>
<td>------------</td>
<td>-------------------------------------------------------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>HW_</td>
<td>HV</td>
<td>$\bar{H}^\text{W}$ I. See Tables 4 and 5.</td>
</tr>
<tr>
<td>I</td>
<td></td>
<td>Index almost always used to range over the dependent variables.</td>
</tr>
<tr>
<td>INFLG</td>
<td></td>
<td>1 flags a new run (all cards are read); 2 flags a restart with changes (Cards 1 through 29 are read); 0 flags end of job. See Note 7.</td>
</tr>
<tr>
<td>INT SPREAD</td>
<td>L</td>
<td>FINTV</td>
</tr>
<tr>
<td>IOFAC</td>
<td></td>
<td>Scale parameter. See Note 1.</td>
</tr>
<tr>
<td>ITEST</td>
<td></td>
<td>Index of variable to which PCFMX and PCFSP apply.</td>
</tr>
<tr>
<td>J</td>
<td></td>
<td>Index used to range across the profile. J=1 corresponds to the wall; J=JTOP corresponds to the edge of the boundary layer (free stream).</td>
</tr>
<tr>
<td>JMAX</td>
<td></td>
<td>Maximum number of points to be allowed across the profile, i.e., the upper limit on JTOP.</td>
</tr>
<tr>
<td>JMXMX</td>
<td></td>
<td>Maximum permissible value of JMAX.</td>
</tr>
<tr>
<td>JTOP</td>
<td></td>
<td>Current number of points across the profile.</td>
</tr>
<tr>
<td>LAST DX</td>
<td>L</td>
<td>$\Delta x$ for the step just completed.</td>
</tr>
<tr>
<td>LBURF</td>
<td></td>
<td>True indicates that the step is to be tried again with a smaller $\Delta x$.</td>
</tr>
<tr>
<td>LDELT</td>
<td></td>
<td>True indicates that $\Delta x$ is less than its minimum.</td>
</tr>
<tr>
<td>LDETZ</td>
<td></td>
<td>True indicates that GYCMII has tried to invert a singular, or nearly singular, matrix.</td>
</tr>
<tr>
<td>LFOLF</td>
<td></td>
<td>True indicates that full output is to occur.</td>
</tr>
<tr>
<td>Name</td>
<td>Dimensions</td>
<td>Definition</td>
</tr>
<tr>
<td>---------</td>
<td>------------</td>
<td>---------------------------------------------------------------------------</td>
</tr>
<tr>
<td>LFOFF</td>
<td></td>
<td>True indicates that output to a profile plot file would occur.</td>
</tr>
<tr>
<td>LHMWK</td>
<td></td>
<td>True indicates that the wall value of enthalpy is specified; false that its gradient is specified.</td>
</tr>
<tr>
<td>LIOLF</td>
<td></td>
<td>True indicates that a minor printout is to occur.</td>
</tr>
<tr>
<td>LIOFF</td>
<td></td>
<td>True indicates that output to a running plot file would occur.</td>
</tr>
<tr>
<td>LJMXA</td>
<td></td>
<td>True indicates that more than JMAX points are needed to satisfy the curvature requirements.</td>
</tr>
<tr>
<td>LNWPG</td>
<td></td>
<td>True indicates that the next printout should start on a new page.</td>
</tr>
<tr>
<td>LPKRF</td>
<td></td>
<td>True indicates that more than JMAX points are needed to satisfy the outer boundary conditions.</td>
</tr>
<tr>
<td>LPOSV</td>
<td></td>
<td>True indicates a non-negative dependent variable.</td>
</tr>
<tr>
<td>LPPFFF</td>
<td></td>
<td>True would indicate that a profile file was full.</td>
</tr>
<tr>
<td>LRHOF</td>
<td></td>
<td>True indicates a problem in the calculation of $\bar{h}$: either $h_1^2 \geq h_2$ or too many iterations in GYCCP.</td>
</tr>
<tr>
<td>LRORG</td>
<td></td>
<td>True indicates that points have been added or deleted in the current step.</td>
</tr>
<tr>
<td>LRPFFF</td>
<td></td>
<td>True would indicate that a running file was full.</td>
</tr>
<tr>
<td>LRSTF</td>
<td></td>
<td>True indicates that a restart is in progress.</td>
</tr>
<tr>
<td>Name</td>
<td>Dimensions</td>
<td>Definition</td>
</tr>
<tr>
<td>------------</td>
<td>------------</td>
<td>---------------------------------------------------------------------------</td>
</tr>
<tr>
<td>LSTFL</td>
<td></td>
<td>True indicates that a new run is starting.</td>
</tr>
<tr>
<td>LTRNF</td>
<td></td>
<td>True indicates that the run is terminating.</td>
</tr>
<tr>
<td>LZHFF</td>
<td></td>
<td>True indicates that $y$ has reached $Y_{\text{MAX}}$.</td>
</tr>
<tr>
<td>LZHRP</td>
<td></td>
<td>True indicates that correlations involving $\rho'$ and $h'$ are to be set to zero.</td>
</tr>
<tr>
<td>MAX CHANGE</td>
<td>W</td>
<td>$T_{\text{CMXV}}$</td>
</tr>
<tr>
<td>MAXIMUM</td>
<td>W</td>
<td>$F_{\text{MAXV}}$</td>
</tr>
<tr>
<td>NEXT DX</td>
<td>L</td>
<td>$\Delta x$ for the next step.</td>
</tr>
<tr>
<td>NFOLP</td>
<td></td>
<td>Steps between major printouts.</td>
</tr>
<tr>
<td>NFOPP</td>
<td></td>
<td>Steps between fictitious writes to a profile plot file.</td>
</tr>
<tr>
<td>NIOLP</td>
<td></td>
<td>Steps between minor printouts.</td>
</tr>
<tr>
<td>NIOPP</td>
<td></td>
<td>Steps between fictitious writes to a running plot file.</td>
</tr>
<tr>
<td>NOISE</td>
<td>W</td>
<td>$E_{\text{PSNV}}$</td>
</tr>
<tr>
<td>THRESHOLD</td>
<td></td>
<td></td>
</tr>
<tr>
<td>NPASS</td>
<td></td>
<td>Number of the pass through the procedure of GYCSS.</td>
</tr>
<tr>
<td>NRUN</td>
<td></td>
<td>Run number.</td>
</tr>
<tr>
<td>NSKIP</td>
<td></td>
<td>BLYAA is skipped for the first NSKIP steps whether starting or restarting.</td>
</tr>
<tr>
<td>NSTEP</td>
<td></td>
<td>Number of the current step in the $x$ direction.</td>
</tr>
<tr>
<td>NSTMX</td>
<td></td>
<td>The run is halted if the number of steps reaches $N_{\text{STMX}}$. See Note 7.</td>
</tr>
</tbody>
</table>

83
<table>
<thead>
<tr>
<th>Name</th>
<th>Dimensions</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>NSVRT</td>
<td>NVART plus one for the dependent variable y (S_); 22.</td>
<td></td>
</tr>
<tr>
<td>NVARP</td>
<td>Number of parabolic variables; 17.</td>
<td></td>
</tr>
<tr>
<td>NVART</td>
<td>Total number of dependent variables saved for the old and the new value of x ; 21.</td>
<td></td>
</tr>
<tr>
<td>NWQRD</td>
<td>Number of words in COMMON representing the M-I, the Z-J, or the P-K conditions; 59.</td>
<td></td>
</tr>
<tr>
<td>NWSAV</td>
<td>Total number of variables saved; 43.</td>
<td></td>
</tr>
<tr>
<td>NWSGM</td>
<td>NWSAV plus the size of GMAT; 52.</td>
<td></td>
</tr>
<tr>
<td>NWSRD</td>
<td>NWSAV; 43.</td>
<td></td>
</tr>
<tr>
<td>NWVEC</td>
<td>Number of unknowns in a pass; 2 in the first pass, 3 in the others.</td>
<td></td>
</tr>
<tr>
<td>OFAC</td>
<td>Scale parameter. See Note 1.</td>
<td></td>
</tr>
<tr>
<td>OMBET</td>
<td>1 - BETA.</td>
<td></td>
</tr>
<tr>
<td>OUTPUT</td>
<td>Parameters input on Card 17.</td>
<td></td>
</tr>
<tr>
<td>CONTROLS</td>
<td></td>
<td></td>
</tr>
<tr>
<td>P</td>
<td>( Dv^2 ) ( p ). See Note 6.</td>
<td></td>
</tr>
<tr>
<td>PCFMX</td>
<td>If FMAXV(ITEST) has changed more than PCFMX times the value it had the last time both BLYOT and BLYPF were called, a call to each is forced.</td>
<td></td>
</tr>
<tr>
<td>PCFSP</td>
<td>If ZPSM(ITEST) has changed more than PCFSP times the value it had the last time both BLYOT and BLYPF were called, a call to each is forced.</td>
<td></td>
</tr>
<tr>
<td>PCTMV</td>
<td>ZPSM(I) is the largest value of y for which ( XV(I) = PCTMV(I) \times FMAXV(I) ).</td>
<td></td>
</tr>
<tr>
<td>Name</td>
<td>Dimensions</td>
<td>Definition</td>
</tr>
<tr>
<td>----------</td>
<td>------------</td>
<td>---------------------------------------------------------------------------</td>
</tr>
<tr>
<td>PCT SPREAD</td>
<td>$L$</td>
<td>ZFSM</td>
</tr>
<tr>
<td>PE</td>
<td>$DV^2$</td>
<td>Free-stream value of $\bar{p}$. See Table 2 and Note 7.</td>
</tr>
<tr>
<td>PGR</td>
<td></td>
<td>Modeling parameter. See Note 2.</td>
</tr>
<tr>
<td>PGR2</td>
<td></td>
<td>Modeling parameter. See Note 2.</td>
</tr>
<tr>
<td>PGR</td>
<td></td>
<td>Modeling parameter. See Note 2.</td>
</tr>
<tr>
<td>PGR2</td>
<td></td>
<td>Modeling parameter. See Note 2.</td>
</tr>
<tr>
<td>PGR</td>
<td></td>
<td>Modeling parameter. See Note 2.</td>
</tr>
<tr>
<td>PGR2</td>
<td></td>
<td>Modeling parameter. See Note 2.</td>
</tr>
<tr>
<td>PGR</td>
<td></td>
<td>Modeling parameter. See Note 2.</td>
</tr>
<tr>
<td>PGR2</td>
<td></td>
<td>Modeling parameter. See Note 2.</td>
</tr>
<tr>
<td>PGR</td>
<td></td>
<td>Modeling parameter. See Note 2.</td>
</tr>
<tr>
<td>PGR2</td>
<td></td>
<td>Modeling parameter. See Note 2.</td>
</tr>
<tr>
<td>PMH</td>
<td></td>
<td>Modeling parameter. See Note 2.</td>
</tr>
<tr>
<td>PMH2</td>
<td></td>
<td>Modeling parameter. See Note 2.</td>
</tr>
<tr>
<td>PMR</td>
<td></td>
<td>Modeling parameter. See Note 2.</td>
</tr>
<tr>
<td>PMR2</td>
<td></td>
<td>Modeling parameter. See Note 2.</td>
</tr>
<tr>
<td>PMU</td>
<td></td>
<td>Modeling parameter. See Note 2.</td>
</tr>
<tr>
<td>PMU2</td>
<td></td>
<td>Modeling parameter. See Note 2.</td>
</tr>
<tr>
<td>PS</td>
<td>$DV^2$</td>
<td>$\bar{p}$. See Note 6.</td>
</tr>
<tr>
<td>PSTAG</td>
<td>$DV^2$</td>
<td>Important: see Note 6. The stagnation pressure is $ABS(PSTAG)$. A positive value indicates a normal run; a negative value indicates that all correlations involving $p'$ and $h'$ are zeroed out at each step. (This is intended as a debugging device.) $\left[ \left( 1 + \frac{\gamma-1}{2} M_{h}^2 \right)^\gamma \right] / \left( \gamma M_{h}^2 \right)$</td>
</tr>
<tr>
<td>PTHM</td>
<td></td>
<td>Modeling parameter. See Note 2.</td>
</tr>
<tr>
<td>PTS</td>
<td>JTOP</td>
<td></td>
</tr>
</tbody>
</table>

85
<table>
<thead>
<tr>
<th>Name</th>
<th>Dimensions</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>PTUM</td>
<td></td>
<td>Modeling parameter. See Note 2.</td>
</tr>
<tr>
<td>Q_</td>
<td>V</td>
<td>$q = \sqrt{u'u'^2 + v'v'^2 + w'w'^2}$</td>
</tr>
<tr>
<td>QL_</td>
<td>VV</td>
<td>$qA$</td>
</tr>
<tr>
<td>QQ</td>
<td>V²</td>
<td>$UUZ + VVZ + WWZ$</td>
</tr>
<tr>
<td>QQJ</td>
<td>V²</td>
<td>$UUJ + VVJ + WWJ$</td>
</tr>
<tr>
<td>QTOT</td>
<td>DHV</td>
<td>Total heat transfer rate in the $y$ direction.</td>
</tr>
<tr>
<td>QWALL</td>
<td>DHV</td>
<td>Heat transfer rate out of the wall.</td>
</tr>
<tr>
<td>R_</td>
<td>L</td>
<td>Distance to the axis of the cone. See Note 5.</td>
</tr>
<tr>
<td>RBLSD</td>
<td>DL²</td>
<td>$\overline{\rho}A^2$</td>
</tr>
<tr>
<td>RDT</td>
<td>1/L</td>
<td>$1/\Delta x$</td>
</tr>
<tr>
<td>REL</td>
<td></td>
<td>$\overline{\rho}qA/\overline{\mu}$</td>
</tr>
<tr>
<td>RETHETA</td>
<td></td>
<td>Reynolds number based on momentum thickness.</td>
</tr>
<tr>
<td>REXW</td>
<td></td>
<td>Reynolds number based on wetted length.</td>
</tr>
<tr>
<td>RGAS</td>
<td>V²/θ</td>
<td>Important: see Note 6.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Gas constant.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\left[\frac{1}{\left(\gamma M^2 \rho \right)}\right]$</td>
</tr>
<tr>
<td>RH_</td>
<td>DH</td>
<td>$\overline{\rho}^1h^1$</td>
</tr>
<tr>
<td>RHO</td>
<td>D</td>
<td>$\overline{\rho}$</td>
</tr>
<tr>
<td>RNOSE</td>
<td>L</td>
<td>Origin radius, $r_0$. See Note 5.</td>
</tr>
<tr>
<td>ROTAT</td>
<td>V/L</td>
<td>Rate of rotation of the cone in radians per unit time.</td>
</tr>
<tr>
<td>Name</td>
<td>Dimensions</td>
<td>Definition</td>
</tr>
<tr>
<td>-------</td>
<td>------------</td>
<td>------------</td>
</tr>
<tr>
<td>RQL_</td>
<td>DVL</td>
<td>$\bar{\rho}qA$</td>
</tr>
<tr>
<td>RQOL</td>
<td>DV/L</td>
<td>$\bar{\rho}q/A$</td>
</tr>
<tr>
<td>RR_</td>
<td>D^2</td>
<td>$\bar{\rho}^T\bar{\rho}^T$</td>
</tr>
<tr>
<td>RRW2</td>
<td>D^2V/L</td>
<td>$(\bar{\rho}\bar{\rho}^T)_y$</td>
</tr>
<tr>
<td>RS_</td>
<td>D</td>
<td>$\bar{\rho}$</td>
</tr>
<tr>
<td>RSUS</td>
<td>DV</td>
<td>RS*US</td>
</tr>
<tr>
<td>RSVS</td>
<td>DV</td>
<td>RS*VS</td>
</tr>
<tr>
<td>RT_</td>
<td>D0</td>
<td>$\bar{\rho}^T\bar{\rho}^T$</td>
</tr>
<tr>
<td>RTW2</td>
<td>DV\theta/L</td>
<td>$(\bar{\rho}\bar{\rho}^T)_y$</td>
</tr>
<tr>
<td>RU_</td>
<td>DV</td>
<td>$\bar{\rho}^T\bar{\rho}^T$. See Tables 4 and 5.</td>
</tr>
<tr>
<td>RUDMF</td>
<td>L/L^2</td>
<td>$(ARU + BRU*REL)/BLAMZ/BLAMZ$</td>
</tr>
<tr>
<td>RUN</td>
<td></td>
<td>NRUN</td>
</tr>
<tr>
<td>RUWJ2</td>
<td>V^2</td>
<td>$(R_u\bar{w}^T)_y$ at the new value of $x$.</td>
</tr>
<tr>
<td>RV_</td>
<td>DV</td>
<td>$\bar{\rho}^T\bar{\rho}^T$. See Tables 4 and 5.</td>
</tr>
<tr>
<td>RVWJ2</td>
<td>V^2</td>
<td>$(R\bar{v}^T\bar{w}^T)_y$ at the new value of $x$.</td>
</tr>
<tr>
<td>RW_</td>
<td>DV</td>
<td>$\bar{\rho}^T\bar{w}^T$. See Tables 4 and 5.</td>
</tr>
<tr>
<td>RWRJ2</td>
<td>V^2L</td>
<td>$(R^2\bar{w}^T\bar{w}^T)_y$ at the new value of $x$.</td>
</tr>
<tr>
<td>RWS_</td>
<td>DV</td>
<td>$\bar{R_w}$</td>
</tr>
<tr>
<td>S</td>
<td></td>
<td>DIMFL* sin $\theta_c$</td>
</tr>
<tr>
<td>S_</td>
<td>L</td>
<td>Normal independent variable, $y$.</td>
</tr>
<tr>
<td>SFVFL</td>
<td></td>
<td>Subroutine which fills a vector.</td>
</tr>
<tr>
<td>SFVMV</td>
<td></td>
<td>Subroutine which moves the contents of one vector to another.</td>
</tr>
<tr>
<td>Name</td>
<td>Dimensions</td>
<td>Definition</td>
</tr>
<tr>
<td>----------</td>
<td>------------</td>
<td>-------------------------------------------------------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>SHAPE</td>
<td></td>
<td>Displacement thickness divided by momentum thickness.</td>
</tr>
<tr>
<td>SHM</td>
<td>$1/L^2$</td>
<td>$1/(h_h_t)$ (Section 3).</td>
</tr>
<tr>
<td>SHP</td>
<td>$1/L^2$</td>
<td>$1/(h_h_t)$ (Section 3).</td>
</tr>
<tr>
<td>SPREAD</td>
<td></td>
<td>PCTMV</td>
</tr>
<tr>
<td>PARAMETER</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SPRO</td>
<td>L</td>
<td>ZPSM(IOFAC). See Note 1.</td>
</tr>
<tr>
<td>STANT</td>
<td></td>
<td>Stanton number.</td>
</tr>
<tr>
<td>STEP</td>
<td></td>
<td>NSTEP</td>
</tr>
<tr>
<td>STOR</td>
<td>X</td>
<td>Storage array mimicking an external device.</td>
</tr>
<tr>
<td>TAULAM</td>
<td>$DV^2$</td>
<td>Laminar stress.</td>
</tr>
<tr>
<td>TAUTOT</td>
<td>$DV^2$</td>
<td>Total stress.</td>
</tr>
<tr>
<td>TAUW</td>
<td>$DV^2$</td>
<td>Wall stress.</td>
</tr>
<tr>
<td>TBRKV</td>
<td>L</td>
<td>Time break vector. When TIME reaches the absolute value of one of the elements of TBRKV, calls to GYCOT and GYCPF are forced. If the element is negative, the run is halted. See Note 7.</td>
</tr>
<tr>
<td>TCMXV</td>
<td>W</td>
<td>Maxima over y of the changes in the dependent variables during the last step (output label: MAX CHANGE).</td>
</tr>
<tr>
<td>THETA</td>
<td>L</td>
<td>Momentum thickness.</td>
</tr>
<tr>
<td>TIME</td>
<td>L</td>
<td>Streamwise independent variable, x.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>See Note 7.</td>
</tr>
<tr>
<td>TMRF</td>
<td>$\theta$</td>
<td>Reference temperature for $\mu$ and $k$.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>See Note 3. $[1.]$</td>
</tr>
<tr>
<td>Name</td>
<td>Dimensions</td>
<td>Definition</td>
</tr>
<tr>
<td>-------</td>
<td>------------</td>
<td>------------</td>
</tr>
<tr>
<td>TT</td>
<td>$\theta^2$</td>
<td>$T_T^T$</td>
</tr>
<tr>
<td>TU_</td>
<td>$V\theta$</td>
<td>$T_{T'}^u$</td>
</tr>
<tr>
<td>TV_</td>
<td>$V\theta$</td>
<td>$T_{T'}^v$</td>
</tr>
<tr>
<td>TW_</td>
<td>$V\theta$</td>
<td>$T_{T'}^w$</td>
</tr>
<tr>
<td>TYPL</td>
<td>L</td>
<td>Typical length. See Note 1.</td>
</tr>
<tr>
<td>U</td>
<td>V</td>
<td>$\bar{u}$. See Tables 3 and 5.</td>
</tr>
<tr>
<td>UE</td>
<td>V</td>
<td>Free-stream value of $\bar{u}$. See Table 2.</td>
</tr>
<tr>
<td>US_</td>
<td>V</td>
<td>$\bar{u}$. See Table 5.</td>
</tr>
<tr>
<td>UU_</td>
<td>$V^2$</td>
<td>$\bar{u}'\bar{u}'$. See Tables 3 and 5.</td>
</tr>
<tr>
<td>UV_</td>
<td>$V^2$</td>
<td>$\bar{u}'\bar{v}'$. See Tables 3 and 5.</td>
</tr>
<tr>
<td>UW_</td>
<td>$V^2$</td>
<td>$\bar{u}'\bar{w}'$. See Tables 4 and 5.</td>
</tr>
<tr>
<td>UWRJ2</td>
<td>$V^2/L^2$</td>
<td>$(\bar{u}'\bar{w}'/R)_y$ at the new value of $x$.</td>
</tr>
<tr>
<td>V</td>
<td>V</td>
<td>$\bar{v}$. See Table 3.</td>
</tr>
<tr>
<td>VARNM</td>
<td></td>
<td>Vector of variable names.</td>
</tr>
<tr>
<td>VBS</td>
<td></td>
<td>Modeling parameter. See Note 2.</td>
</tr>
<tr>
<td>VHH</td>
<td></td>
<td>Modeling parameter. See Note 2.</td>
</tr>
<tr>
<td>VRH</td>
<td></td>
<td>Modeling parameter. See Note 2.</td>
</tr>
<tr>
<td>VRR</td>
<td></td>
<td>Modeling parameter. See Note 2.</td>
</tr>
<tr>
<td>VRU</td>
<td></td>
<td>Modeling parameter. See Note 2.</td>
</tr>
<tr>
<td>VS_</td>
<td>V</td>
<td>$\bar{v}$</td>
</tr>
<tr>
<td>VSW</td>
<td>V</td>
<td>The wall value of $\bar{v}$. See Table 2.</td>
</tr>
<tr>
<td>VUH</td>
<td></td>
<td>Modeling parameter. See Note 2.</td>
</tr>
<tr>
<td>Name</td>
<td>Dimensions</td>
<td>Definition</td>
</tr>
<tr>
<td>--------</td>
<td>------------</td>
<td>------------</td>
</tr>
<tr>
<td>VUU</td>
<td>V</td>
<td>Modeling parameter. See Note 2.</td>
</tr>
<tr>
<td>VV_</td>
<td>V^2</td>
<td>$\bar{v}^T \bar{v}'$. See Tables 3 and 5.</td>
</tr>
<tr>
<td>VW_</td>
<td>V^2</td>
<td>$\bar{v}^T \bar{w}'$. See Tables 4 and 5.</td>
</tr>
<tr>
<td>VWRJ2</td>
<td>V^2/L^2</td>
<td>$(\bar{v}' \bar{w}')_y$ at the new value of $x$.</td>
</tr>
<tr>
<td>W</td>
<td>V</td>
<td>$\bar{w}$. See Tables 3 and 5.</td>
</tr>
<tr>
<td>WGH1</td>
<td></td>
<td>Modeling parameter computed from constraint. See Note 2.</td>
</tr>
<tr>
<td>WGH2</td>
<td></td>
<td>Modeling parameter. See Note 2.</td>
</tr>
<tr>
<td>WGH3</td>
<td></td>
<td>Modeling parameter. See Note 2.</td>
</tr>
<tr>
<td>WGR1</td>
<td></td>
<td>Modeling parameter computed from constraint. See Note 2.</td>
</tr>
<tr>
<td>WGR2</td>
<td></td>
<td>Modeling parameter. See Note 2.</td>
</tr>
<tr>
<td>WGR3</td>
<td></td>
<td>Modeling parameter. See Note 2.</td>
</tr>
<tr>
<td>WGU</td>
<td></td>
<td>Modeling parameter. See Note 2.</td>
</tr>
<tr>
<td>WMD</td>
<td></td>
<td>Modeling parameter. See Note 2.</td>
</tr>
<tr>
<td>WMH</td>
<td></td>
<td>Modeling parameter. See Note 2.</td>
</tr>
<tr>
<td>WMR</td>
<td></td>
<td>Modeling parameter. See Note 2.</td>
</tr>
<tr>
<td>WMT</td>
<td></td>
<td>Modeling parameter. See Note 2.</td>
</tr>
<tr>
<td>WS_</td>
<td>V</td>
<td>$\bar{w}$. See Table 5.</td>
</tr>
<tr>
<td>WSR_</td>
<td>V/L</td>
<td>$\bar{w}/R$</td>
</tr>
<tr>
<td>WW_</td>
<td>V^2</td>
<td>$\bar{w}' \bar{w}'$. See Tables 3 and 5.</td>
</tr>
<tr>
<td>WWD1</td>
<td></td>
<td>Modeling parameter. See Note 2.</td>
</tr>
<tr>
<td>WWD2</td>
<td></td>
<td>Modeling parameter. See Note 2.</td>
</tr>
<tr>
<td>Name</td>
<td>Dimensions</td>
<td>Definition</td>
</tr>
<tr>
<td>---------</td>
<td>------------</td>
<td>---------------------------------------------------------------------------</td>
</tr>
<tr>
<td>WWGH</td>
<td></td>
<td>Modeling parameter. See Note 2.</td>
</tr>
<tr>
<td>WWGR</td>
<td></td>
<td>Modeling parameter. See Note 2.</td>
</tr>
<tr>
<td>WWGU</td>
<td></td>
<td>Modeling parameter. See Note 2.</td>
</tr>
<tr>
<td>WWH1</td>
<td></td>
<td>Modeling parameter. See Note 2.</td>
</tr>
<tr>
<td>WWH2</td>
<td></td>
<td>Modeling parameter. See Note 2.</td>
</tr>
<tr>
<td>WWR1</td>
<td></td>
<td>Modeling parameter. See Note 2.</td>
</tr>
<tr>
<td>WWR2</td>
<td></td>
<td>Modeling parameter. See Note 2.</td>
</tr>
<tr>
<td>WWU1</td>
<td></td>
<td>Modeling parameter. See Note 2.</td>
</tr>
<tr>
<td>WWU2</td>
<td></td>
<td>Modeling parameter. See Note 2.</td>
</tr>
<tr>
<td>X</td>
<td>L</td>
<td>x. See Table 2.</td>
</tr>
<tr>
<td>XSTRT</td>
<td>L</td>
<td>Initial value of XWET. See Note 7.</td>
</tr>
<tr>
<td>XV</td>
<td>W</td>
<td>Vectors of the unknowns at the old x. Compare AV. See Table 5.</td>
</tr>
<tr>
<td>XW</td>
<td>L</td>
<td>Wetted length.</td>
</tr>
<tr>
<td>XWET</td>
<td>L</td>
<td>Wetted length (output label: XW).</td>
</tr>
<tr>
<td>Y</td>
<td>L</td>
<td>y. See Tables 3 and 4.</td>
</tr>
<tr>
<td>Y AT MAX</td>
<td>L</td>
<td>FZMXV</td>
</tr>
<tr>
<td>YMAX</td>
<td>L</td>
<td>The maximum value of y to which the solution may spread.</td>
</tr>
<tr>
<td>ZPSM</td>
<td>L</td>
<td>Spreads of the dependent variables based on PCTMV (output label: PCT SPREAD).</td>
</tr>
</tbody>
</table>

91
Note 1.

The turbulence scale $A$ that enters in many of the models is computed as follows. First, a typical length is computed from

$$\text{TYPL} = \text{FAC99} \times \text{DLT99} + \text{OFAC} \times ZPSM(\text{IOFAC})$$

where DLT99 is the value of $y$ where $\bar{u}$ is 99 percent of the free-stream value; ZPSM is the vector of spreads of the dependent variables based on PCTMV; and FAC99, OFAC, and IOFAC are inputs. (ZPSM(OFAC) is included in the output with the label SPRO.)

The value of $A$ is made equal to BLAM0 at the wall. This parameter is ordinarily zero but may be made positive to simulate roughness. Off the wall, $A$ increases with slope DIN until it reaches a maximum of CLAMB*TYPL. For $y > \text{DLT99}$, $A$ increases again with slope DOUT. This parameter is ordinarily zero but should be positive if there is free-stream turbulence.

All the parameters determining $A$ are nondimensional except BLAM0.

Note 2.

The modeling parameters are defined by their appearance in the models, as given in Appendix B. They are all nondimensional. Two of them, WGH1 and WGR1, are not input; instead, they are computed from the constraints given at the end of Appendix B.
Note 3.

The viscosity is determined from

$$\bar{\mu} = \mu_{rf} \left( \frac{\frac{r}{T_{rf}}}{\frac{1 + C/T_{rf}}{1 + C/T}} \right)^\omega$$

where $\mu_{rf} = EMURF$, $T_{rf} = TMRF$, $\omega = EXPMU$, and $C = CSUTH$. With $\omega = 1/2$, this is the Sutherland relation. With $C = 0$, it is the power-law relation. With both $\omega = 0$ and $C = 0$, it gives constant $\bar{\mu}$.

The heat conductivity is determined from

$$\bar{k} = k_{rf} \frac{\bar{\mu}}{\mu_{rf}}$$

Since $c_p$ is constant, this is equivalent to assuming that the Prandtl number is constant.

Note 4.

Upwind differencing is suppressed when $FUP = 0$, used when $FUP = 1$. In this case, the coefficient, $\bar{\rho}V$, is evaluated as $(1 - FUPV)$ times the central-point value and $FUPV$ times the upwind-point value. If $FUP = 0$, the value of $FUPV$ is immaterial. Values of $FUP$ other than 0 and 1 produce a mixture of standard and upwind differencing.

Note 5.

If the dimension flag, DIMFL, is input as zero, the program operates in flat-plate mode. In this case, RNOSE and CONGL are not used and the following acquire different meanings:

- $R$ (Dimensionless) is the number 1.
- ROTAT (Dimension: V) is the sideslip velocity of the surface.

93
Thus, the value of $\bar{w}$ at the surface is

$$\text{ROTAT} \cdot R$$

(where $R$ is evaluated at the surface) for either value of $\text{DIMFL}$.

Note 6.

If $\text{GMOMS}$ is input as zero, the program operates in Mach number zero mode. In this case, the following acquire different meanings:

$\text{GMOMS}$ (Dimensionless) is the number 0. The units for expressing heat quantities may still be chosen at will. A conversion factor between mechanical and heat units is not needed in this case.

$P$, $PE$, and $PS_\parallel$ (Dimensions: $DV^2$) all represent pressure differences instead of absolute pressures. The datum for zero values for the outputs is whatever is used for the input ($PE$). If the free-stream value is constant, it would ordinarily be input as zero, and the output pressures would represent departures therefrom. If $PE$ is input as the absolute free-stream pressure, the output pressures are absolute too.

$PSTAG$ (Dimensionless) is one of the numbers $\pm 1$. This is used only to flag runs for which correlations involving $\rho'$ and $h'$ are to be zeroed out.

$\text{RGAS}$ (Dimension: D) is the density corresponding to $\text{HSTAG}$. 

94
Note 7.

On restart (INFLG = 2) two inputs, DELTI and XSTRT, are ignored if input as zero; the current values of DELT and XWET in the halted run are used instead. On the other hand, the input values will be used if they are not zero. The input value of TIME (x) is always used, so if it is desired to have it run continuously the final value from the halted run must be supplied as input. The time break values, TBRKV, are interpreted in terms of TIME, so they may need to be altered on restart. If the run was halted by the use of a negative time break, that should probably be changed for restart. If the run was halted by NSTMX, that should be changed for restart.
<table>
<thead>
<tr>
<th>Card</th>
<th>Description</th>
<th>Format</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>INFLG, NSKIP</td>
<td>(11,3X,I4)</td>
</tr>
<tr>
<td>2</td>
<td>CMNTS, NRUN</td>
<td>(19A4,I4)</td>
</tr>
<tr>
<td>3</td>
<td>DIMFL, RNOS, CONGL, ROTAT, TIME</td>
<td>(5F8.0)</td>
</tr>
<tr>
<td>4</td>
<td>RGAS, CP, HSTAG, PSTAG, GMOMS, XSTRX</td>
<td>(6F8.0)</td>
</tr>
<tr>
<td>5</td>
<td>TMRF, EMURF, CAYRF, CSUSH, EXPMU</td>
<td>(5F8.0)</td>
</tr>
<tr>
<td>6</td>
<td>CLAMB, DIN, DOUT, BLAM0, FAC99, OFAC, IOFAC</td>
<td>(6F8.0,I4)</td>
</tr>
<tr>
<td>7</td>
<td>A, B, BETA</td>
<td>(3F8.0)</td>
</tr>
<tr>
<td>8</td>
<td>AHH, AHU, ARU, BHH, BUA, BRU</td>
<td>(6F8.0)</td>
</tr>
<tr>
<td>9</td>
<td>VUU, VUH, VH, VR, VRH, VRR</td>
<td>(6F8.0)</td>
</tr>
<tr>
<td>10</td>
<td>PMU, PMU2, PMH2, PMR2, PTHM, PTUM</td>
<td>(6F8.0)</td>
</tr>
<tr>
<td>11</td>
<td>PGH, PGH2, PQR, PQR2, PGU, PGU2</td>
<td>(6F8.0)</td>
</tr>
<tr>
<td>12</td>
<td>WGU, WMH, WMR, WMT</td>
<td>(4F8.0)</td>
</tr>
<tr>
<td>13</td>
<td>WGH2, WGH3, WGR2, WGR3</td>
<td>(4F8.0)</td>
</tr>
<tr>
<td>14</td>
<td>WWU1, WWL, WWH1, WWU, WWGR, WWGH</td>
<td>(6F8.0)</td>
</tr>
<tr>
<td>15</td>
<td>EV</td>
<td>(6F8.0)</td>
</tr>
<tr>
<td>16,n</td>
<td>X, UE, HE, PE, VSW, HSW</td>
<td>(6F8.0)</td>
</tr>
<tr>
<td>17</td>
<td>NIOLP, NI OPP, NFOLP, NFOPP, PCFMX, PCFS, ITEST</td>
<td>(4I4,2F8.0,I4)</td>
</tr>
<tr>
<td>18</td>
<td>TBRKX</td>
<td>(10F8.0)</td>
</tr>
<tr>
<td>19</td>
<td>NSTMX, JMAX, YMAX, FUP, FUFV</td>
<td>(2I4,3F8.0)</td>
</tr>
<tr>
<td>20</td>
<td>ECMNI, ECMXI, PCURI, PCMX, DZFMN, DZFMM, DZFMX, DZRMX</td>
<td>(7F8.0)</td>
</tr>
<tr>
<td>21</td>
<td>DELTI, DTMIN, DTMAX, DTFMX, BUFAC</td>
<td>(5F8.0)</td>
</tr>
</tbody>
</table>
### Table 1. Input Cards (Continued)

| Cards 22/23 | PCTMV  | (8F8.0/9F8.0) |
| Cards 24/25 | EPSTV  | (8F8.0/9F8.0) |
| Cards 26/27 | EPSBV  | (8F8.0/9F8.0) |
| Cards 28/29 | EPSNV  | (8F8.0/9F8.0) |
| Card 30     | FV     | (10F8.0)     |
| Cards 31.j  | Y, U, V, W, H, UU, VV, WW, UV, BLAM | (10F8.0) |
| Card 32     | GV     | (10F8.0)     |
| Cards 33.j  | Y, UW, VW, HU, HV, HW, RU, RV, RW, HH | (10F8.0) |
### Table 2. Edge Values and Wall Values

<table>
<thead>
<tr>
<th>Name</th>
<th>Multiplied By</th>
<th>Gives</th>
<th>Dimensions</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>EV(1)</td>
<td>x</td>
<td>(L)</td>
</tr>
<tr>
<td>UE</td>
<td>EV(2)</td>
<td>$u_e$</td>
<td>(V)</td>
</tr>
<tr>
<td>HE</td>
<td>EV(3)</td>
<td>$h_e$</td>
<td>(H)</td>
</tr>
<tr>
<td>PE</td>
<td>EV(4)</td>
<td>$p_e$</td>
<td>(DV$^2$)</td>
</tr>
<tr>
<td>VSW</td>
<td>EV(5)</td>
<td>$v_w$</td>
<td>(V)</td>
</tr>
<tr>
<td>HSW</td>
<td>EV(6)</td>
<td>$h_w$</td>
<td>(H)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(\left(\frac{\partial h}{\partial y}\right)_w)</td>
<td>(H/L)</td>
</tr>
</tbody>
</table>

If $h_e$ is input as zero, it is calculated from $h_e = h_s - \frac{1}{2}u_e^2$ where $h_s$ is the stagnation enthalpy.

If GMOMS > 0 and $p_e$ is input as zero, it is calculated from $p_e = p_s\left(\frac{h_e}{h_s}\right)^{\gamma/(\gamma-1)}$ where $p_s$ is the stagnation pressure.
Table 3. Initial Conditions, First Set

<table>
<thead>
<tr>
<th>Name</th>
<th>Multiplied By</th>
<th>Gives</th>
<th>Dimensions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y</td>
<td>FV(1)</td>
<td>y</td>
<td>(L)</td>
</tr>
<tr>
<td>U</td>
<td>FV(2)</td>
<td>\bar{u}</td>
<td>(V)</td>
</tr>
<tr>
<td>V</td>
<td>FV(3)</td>
<td>\bar{v}</td>
<td>(V)</td>
</tr>
<tr>
<td>W</td>
<td>FV(4)</td>
<td>\bar{w}</td>
<td>(V)</td>
</tr>
<tr>
<td>H</td>
<td>FV(5)</td>
<td>\bar{h}</td>
<td>(H)</td>
</tr>
<tr>
<td>UU</td>
<td>FV(6)</td>
<td>\bar{u}'\bar{u}'</td>
<td>(V^2)</td>
</tr>
<tr>
<td>VV</td>
<td>FV(7)</td>
<td>\bar{v}'\bar{v}'</td>
<td>(V^2)</td>
</tr>
<tr>
<td>WW</td>
<td>FV(8)</td>
<td>\bar{w}'\bar{w}'</td>
<td>(V^2)</td>
</tr>
<tr>
<td>UV</td>
<td>FV(9)</td>
<td>\bar{u}'\bar{v}'</td>
<td>(V^2)</td>
</tr>
<tr>
<td>BLAM</td>
<td>FV(10)</td>
<td>A</td>
<td>(L)</td>
</tr>
</tbody>
</table>

Table 4. Initial Conditions, Second Set

<table>
<thead>
<tr>
<th>Name</th>
<th>Multiplied By</th>
<th>Gives</th>
<th>Dimensions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y</td>
<td>GV(1)</td>
<td>y</td>
<td>(L)</td>
</tr>
<tr>
<td>UW</td>
<td>GV(2)</td>
<td>\bar{u}'w'</td>
<td>(V^2)</td>
</tr>
<tr>
<td>VW</td>
<td>GV(3)</td>
<td>\bar{v}'w'</td>
<td>(V^2)</td>
</tr>
<tr>
<td>HU</td>
<td>GV(4)</td>
<td>\bar{h}'u'</td>
<td>(HV)</td>
</tr>
<tr>
<td>HV</td>
<td>GV(5)</td>
<td>\bar{h}'v'</td>
<td>(HV)</td>
</tr>
<tr>
<td>HW</td>
<td>GV(6)</td>
<td>\bar{h}'w'</td>
<td>(HV)</td>
</tr>
<tr>
<td>RU</td>
<td>GV(7)</td>
<td>\bar{\rho}'u'</td>
<td>(DV)</td>
</tr>
<tr>
<td>RV</td>
<td>GV(8)</td>
<td>\bar{\rho}'v'</td>
<td>(DV)</td>
</tr>
<tr>
<td>RW</td>
<td>GV(9)</td>
<td>\bar{\rho}'w'</td>
<td>(DV)</td>
</tr>
<tr>
<td>HH</td>
<td>GV(10)</td>
<td>\bar{h}'\bar{h}'</td>
<td>(H^2)</td>
</tr>
<tr>
<td>Name</td>
<td>Dimensions</td>
<td>Output Label</td>
<td>Meaning</td>
</tr>
<tr>
<td>--------</td>
<td>------------</td>
<td>--------------</td>
<td>---------</td>
</tr>
<tr>
<td>WS = XV(1)</td>
<td>(V)</td>
<td>W</td>
<td>$\bar{w}$</td>
</tr>
<tr>
<td>VW = XV(2)</td>
<td>(V²)</td>
<td>VW</td>
<td>$\mathbf{v'}w'$</td>
</tr>
<tr>
<td>US = XV(3)</td>
<td>(V)</td>
<td>U</td>
<td>$\bar{u}$</td>
</tr>
<tr>
<td>UV = XV(4)</td>
<td>(V²)</td>
<td>UV</td>
<td>$u'^{T}v'$</td>
</tr>
<tr>
<td>UW = XV(5)</td>
<td>(V²)</td>
<td>UW</td>
<td>$u'^{T}w'$</td>
</tr>
<tr>
<td>UU = XV(6)</td>
<td>(V²)</td>
<td>UU</td>
<td>$u'^{T}u'$</td>
</tr>
<tr>
<td>VV = XV(7)</td>
<td>(V²)</td>
<td>VV</td>
<td>$v'^{T}v'$</td>
</tr>
<tr>
<td>WW = XV(8)</td>
<td>(V²)</td>
<td>WW</td>
<td>$w'^{T}w'$</td>
</tr>
<tr>
<td>HS = XV(9)</td>
<td>(H)</td>
<td>H</td>
<td>$\bar{h}$</td>
</tr>
<tr>
<td>HV = XV(10)</td>
<td>(HV)</td>
<td>HV</td>
<td>$h'^{T}v'$</td>
</tr>
<tr>
<td>HH = XV(11)</td>
<td>(H²)</td>
<td>HH</td>
<td>$h'^{T}h'$</td>
</tr>
<tr>
<td>RU = XV(12)</td>
<td>(DV)</td>
<td>RU</td>
<td>$\rho'^{T}u'$</td>
</tr>
<tr>
<td>RV = XV(13)</td>
<td>(DV)</td>
<td>RV</td>
<td>$\rho'^{T}v'$</td>
</tr>
<tr>
<td>RW = XV(14)</td>
<td>(DV)</td>
<td>RW</td>
<td>$\rho'^{T}w'$</td>
</tr>
<tr>
<td>HU = XV(15)</td>
<td>(HV)</td>
<td>HU</td>
<td>$h'^{T}u'$</td>
</tr>
<tr>
<td>HW = XV(16)</td>
<td>(HV)</td>
<td>HW</td>
<td>$h'^{T}w'$</td>
</tr>
<tr>
<td>BLAM = XV(17)</td>
<td>(L)</td>
<td>BLAM</td>
<td>$\Lambda$</td>
</tr>
</tbody>
</table>