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CONCEPTS FOR THE DEVELOPMENT OF LIGHT-WEIGHT COMPOSITE STRUCTURES FOR ROTOR BURST CONTAINMENT

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TECHNICAL PAPER to be presented at the NASA Workshop on an Assessment of Technology for Turbojet Engine Rotor Failures conducted by the Massachusetts Institute of Technology
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CONCEPTS FOR THE DEVELOPMENT OF LIGHT-WEIGHT COMPOSITE STRUCTURES FOR ROTOR BURST CONTAINMENT

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ABSTRACT

Based on published results on rotor burst containment with single materials, and on body armor using composite materials, a set of hypotheses is established as to what variables might control the design of a weight - efficient protective device. Based on modern concepts for the design and analysis of small optimum seeking experiments, a particular experiment for evaluating the hypotheses and materials was designed. The design and methods for the analysis of results are described.

SUMMARY

The purpose of the research reported herein was to plan an experimental program, the results of which could provide a basis for the design of weight efficient full circumferential containment devices to protect passengers and critical aircraft systems from the devastating effects of turbine engine disk bursts. The conclusions about the needed experiment were synthesized from three areas of information, namely, (1) prior disk burst protection experiments, (2) personnel body armor research, and (3) modern concepts in the design and analysis of small optimum seeking experiments.

Based on both the prior disk burst experiments and the body armor research, a list of hypotheses was established as to what factors might be controlling in the design of a weight efficient protective device. The consequence of such hypotheses is that the device should consist of as many as four concentric rings, each to consist of a material uniquely chosen for its position in the penetration sequence. Four unique classes of materials are proposed for the four rings and

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particularly attractive examples of each are identified. Experimenting is proposed to evaluate the hypotheses and material choices.

Because the materials are expensive, because their processing is difficult to control, and because the results of disk burst containment experiments are difficult to evaluate, some modern concepts for the design and analysis of small optimum seeking experiments were examined and are discussed. Based on such concepts, a particular experiment for evaluating the hypotheses and materials was designed, and the design and the method for the analysis of results is described.

INTRODUCTION

Recent statistics on turbine engine rotor failures in commercial aviation show that failures of several types occur (Mangano and De Lucia (1975)). The probability of successful containment of such failures depends on whether the failures to contain the fragments are due to: (1) full wheel bursts, (2) failed rim segments, or (3) failed blades. Engine containment of full wheel bursts (Table 1) has never occurred. Containment of rim fragments occurs in only a minority of failures. Containment of failed blades usually occurs, but this is not surprising because the FAA requires (Federal Regulations, Title 14) that failed blades be contained. Another FAA requirement is that failed disks be contained if the turbine is internal to the fuselage, as in the case of auxiliary power units.

The results of a long series of rotor burst protection experiments have been described by Mangano (1972). These results seem to imply that the weight penalties associated with full circumferential disk burst containment are prohibitive. The problem must be regarded as a research problem for which a major breakthrough is needed.

The possibility of using something less than full circumferential containment is currently being explored. Devices are under investigation to protect just a sector of a full circumference. The technique is called shadow shielding and the devices that have been proposed are called deflectors. Future research will undoubtedly separate those design situations (small angle of protection) where deflectors have the best weight efficiency from those situations (large angle of protection) where full circumferential containment has the best efficiency. Such a delineation cannot properly be made until optimization studies have been completed for both types.

The purpose of the present research was to plan some rotor burst containment experimenting that could result in procedures of general applicability for the design of weight efficient full circumferential rotor burst containment devices. To that end three areas of information were examined. The first was that provided by the bursting of turbine rotors into containment rings in a spin pit (Mangano (1972)). That investigation presented the results of a large amount of testing of mostly similar (steel) containment materials. The second area of information is that provided by the ballistic materials research of the Department of Defense to develop weight efficient personnel body armor. Although the response of targets to projectiles is basically different from the response of containment rings to disk bursts, the research does compare the ballistic properties of very dissimilar materials.

The joint examination of these two areas of research provides a list of physical hypotheses on how materials of widely different ballistic properties might be used in combination (composite rings) to product a more weight efficient containment than could be achieved with monolithic rings.

The main hypothesis from the rotor burst tests (Mangano (1972)) is that the containment device should absorb large amounts of energy in tensile straining. The main hypotheses from the body armor research (Rolston(1968)) is that the material properties should vary through the thickness of the device. In military armor, such variations are exemplified by dual hardness steel and by ceramics backed by fiber reinforced plastics.

The physical hypotheses should be subjected to critical experimentation so that they can be evaluated. Because the materials are expensive, because their processing is difficult to control, and because the results of disk burst containment experiments are difficult to evaluate, some modern concepts for the design and analysis of small optimum seeking experiments were reviewed. A specific design of an experiment is proposed. Because the materials and their processing are expensive, the experiment was designed so that preliminary conclusions can be drawn on completing just one half of the total design. On completion of the first half, the results can be examined to see whether the composite rings are superior to, or inferior to, the simpler monolithic rings (which have been extensively investigated). If the composite rings are not clearly superior to monolithic rings, the investigation can be terminated and further costs avoided. If the composite rings are superior, then the second half should be performed. Because the experiment is a telescoping design (Holms (1967)) or Addelman (1969) the data from both halves can be combined to produce valid estimates of the direct effects of the variables and their synergistic combinations.

In addition to providing containment design methods, a second purpose of the proposed experiment is to determine the weight penalty associated with a weight efficient containment system.

The results of the experiment might also identify concepts and materials applicable to the lesser problems of fan, compressor, and turbine blade containment.

IMPLICATIONS OF BODY ARMOR RESEARCH FOR ROTOR BURST CONTAINMENT

A basic concept that has proven widely useful in the design of weight efficient armor is the concept that the material properties should vary through the thickness of the armor. An elementary example is provided by the use of dual-hardness steel. The projectile first encounters a hard material that contributes to the deformation of the projectile, but because the hard material cannot be ideal in energy absorption, it is backed up by tougher material that sacrifices hardness in favor of better energy absorption. Such a concept was further investigated by Wong and Prifti (1972) at the Army Mechanics and Materials Research Center, Watertown, MA, who showed the existence of synergistic combinations of metals.

More complex systems were described by Rolston, Bodine, and Dunleavy (1968). They described some body armor in which a very hard material (a ceramic) is used in combination with a very strong material (a fiber reinforced plastic).

Materials that have proven weight efficient in protecting against slower moving projectiles have included nylon cloths (MIL-C-12369F(GL) (1974)) nylon felts (MIL-C-43635 (1969)) and aramid cloths (LP/P DES 32-75 (1975)). The use of aramid cloth for rotor burst protection was discussed by Gerstle (1975), in which he suggested that multi-material devices might be superior to monolithic devices.

PHYSICAL HYPOTHESES

The process by which a projectile is defeated by body armor is assumed to have some characteristics in common with, and some characteristics which

differ from, the process of a full circumferential disk burst containment. The common characteristics are assumed to occur in the initial stages where resistance to shear and resistance to spalling are important. The stage of disk burst containment that is assumed to be different from the operation of body armor is the final stage where the protective ring undergoes very large circumferential tensile and bending strains (Mangano (1972)).

The literature of body armor and the literature of rotor burst protection thus suggest a large number of physical hypotheses that might describe the rotor burst protection process. If all of these hypotheses were operative, the most efficient devices would be quite complex. The appropriate research would seem to consist of investigating the indicated complex device with a view to determining which features contribute to weight efficiency and which features do not.

Thus the long list of hypotheses to be considered should not be viewed as listing factors to be included in a design manual, but instead should be regarded as listing factors to be included in a research program. Many of the factors might prove to be insignificant and could be so identified in a design manual.

The hypotheses are as follows:

1. The protective device should consist of a nested set of four concentric cylinders, each having unique ballistic properties.

2. The innermost cylinder should be very strong in shear because:
   a. It should provide some blunting of the sharp edges of the projectile.
   b. It should dissipate some energy through projectile deformation.
   c. It should resist penetration by achieving a wider distribution of the load.

3. The first and second layers function in the immediate vicinity of the impact points as beams in bending. The first layer acts as the compressively stressed part of the beam and the second layer acts as that part of a beam that sustains high tensile stresses. The bond between them must sustain the "neutral axis shear stresses" and should also delay the spalling failure of the hard layer. The first layer should be very strong in compression and the second layer should be very strong in tension, and the combination should be of very low mass so as to minimize the distortions from circularity that result from inertia effects. The preservation of circularity would improve the uniformity of the load that is transferred to the outer layers. The particular desirability of low inertia for these layers suggests that hardness in the first layer is to be sought from a ceramic or a glass instead of a metal, and that strength in the second layer should be sought from a fiber reinforced plastic.

4. The third layer should be the result of a "hedge" strategy, that is, it should be a material proven weight efficient in tests of monolithic rings, namely, a high-toughness metal. As such, it would have some of the attributes of the other three layers.

5. The fourth layer should be chosen solely for its ability to absorb large amounts of energy in tensile straining. It should be a ballistic fabric or felt.

The experiment should serve two types of objectives.

1. It should test the truth or falsity of each of the preceding hypotheses.

2. It should show whether an optimum device (on a weight basis) would consist of more than one of the previously defined layers, and on a rough quantitative basis, it should give the optimum proportions of each.

So that the experiment will be representative of the weight efficiencies that are appropriate to aircraft usage, the four layers should each consist of materials that have maximum probability of performing the hypothesized-function on a weight efficient basis. Classes of materials that are though to be appropriate are as follows:

Layer	Class of material
First	Ceramic or glass
Second	Fiber reinforced plastic
Third	Metal
Fourth	Ballistic fabric or felt

Some materials that are regarded as being illustrative of the preceding four classes of materials are listed in Table 2. The listing does not differentiate between materials as to their practicality for the cold section or the hot section of a turbine engine. The assumption is that the experiment will evaluate basic interactions among the disk burst and containment material variables. When this has been done, the containment designer must then select materials that will retain the appropriate dynamic properties at the engine temperature conditions. For example, if an aramid fiber reinforced epoxy were found to be weight efficient in the second layer, then a containment device in the turbine hot section might use a tungsten fiber reinforced nickel in the second layer.

A high strength adhesive is proposed to be used between the first and the second layers. Detailed information on high strength adhesives was given by Shields (1970). High strength adhesives are specified by MMM-A-132. Some examples of high strength adhesives are provided by the cyanoacrylates (MIL-A-46050) and the epoxy-nylons.

DESIGN AND ANALYSIS OF SMALL OPTIMUM SEEKING EXPERIMENTS

Many strategies for the experimental attainment of optimum conditions have been investigated and described in the literature. Of them, the particular set of concepts known as "Box-Wilson methods" (Box and Wilson (1951)) or "Response Surface Methodology" (Box and Hunter (1957)) is now well established as the most rational and efficient approach. These methods have a flow sequence as depicted by Fig. 1 and as described as follows.

Step 1. - Using all prior knowledge, select a set of independent variables that are to be investigated for their effect on the dependent variable that is to be optimized. (In the present instance the dependent variable could be chosen as the ratio of rotor burst energy divided by the containment weight for just marginal containment, or it could be chosen as the ratio of rotor burst momentum divided by the containment weight for just marginal containment, or it could be chosen as some other function of the rotor variables and the containment weight).

The independent variables would be chosen to represent the environment of the impact process together with the design and material variables of the containment device. The test levels chosen for the independent variables would be based on prior knowledge of the physical process. A statistically optimal design of experiment is then selected to be maximally efficient for the model fitting. The data is to be fitted with a simple mathematical model (which is usually a polynlmial equation of first degree augmented by a few higher degree terms as may be permitted by the small experiment).

The experiment is performed and a statistically optimal procedure is used to select a mathematical model of maximum predictive accuracy in terms of the actual data. The next step depends upon the nature of the selected model, as displayed by the relative magnitudes of the first degree and higher order terms. If the first degree terms are clearly predominant the response function is essentially planar and the "method of steepest ascents" is appropriate. The next step is therefore Step 2. If the second degree terms cannot be ignored, the response surface is warped or curved and the "method of local exploration" is appropriate, and the next step is therefore Step 3.

Step 2. - The situation is that a planar surface represents the response as a first degree equation in the independent variables and the equation is used to determine the direction of steepest ascent in terms of the independent (coordinate) variables. The situation is analogous to a mountain climber at a river's edge who decides to walk in a straight line over the meadow in its direction of steepest ascent (for example, 30 degrees east of north, which is to say, some fixed ratio of the independent variables "miles east" and "miles north").

Having established such a direction, a sequence of experimental points is laid out in that direction. With the completion of the indicated experimenting, the location in the experiment space is identified for the maximum of the dependent variable. If the achieved maximum is adequate or if experimenting must be stopped for other reasons, the next step is Step 4. Otherwise the next step is to go back to Step 1 (but with newly acquired empirical and other information).

Step 3. - The experiment plan of "Step 1" was minimally adequate for a first degree equation. It must be augmented by sufficient "hypercube blocks" (Box and Hunter (1957)) or (Holms (1967)) to evaluate two-factor-interaction terms. It must also be augmented by a "star block" (Box and Wilson (1951)) or (Box and Hunter (1957)). Performance of the experiment allows the fitting and selection of a model that is a statistically optimal representation of the data. The practical interpretation of the equation can be performed as described by Box and Wilson (1951), by Davies (1960), or by Myers (1971).

The predictive model and its geometrical interpretation (often by the "method of canonical reduction") can be used to decide that a true maximum has been located, or that it has not. If a true maximum has been located, or if experimenting is to be discontinued for other reasons, the next step is Step 4. If not, then the canonical reduction would be used to identify a line of steepest ascents along a "rising ridge", and the procedure would otherwise be that of Step 2.

Step 4. - Stop the experimenting and write the report, or build the prototype, or both.

ILLUSTRATIVE EXAMPLE - A PARTICULAR EXPERIMENT FOR PRELIMINARY OPTIMIZATION OF A ROTOR BURST CONTAINMENT DEVICE USING COMPOSITE MATERIALS

To test the stated hypotheses, and to evaluate the listed materials, the experimenting would consist of spin-pit burst containment testing using a representative turbine wheel. The wheel to be burst is surrounded by the containment ring assembly to be tested. The number of equally sized wheel fragments and the burst speed are controlled by saw cuts radially oriented in the rim of the test wheel. The result of each burst test would be measured by the weight of the containment assembly, the wheel speed at burst, and whether the ring assembly contained or did not contain the wheel fragments.

In the design and analysis of a sequence of optimum seeking experiments, one object function, such as the protective efficiency, would be selected as the dependent variable. In any case, in the fitting of models to the data from a single experiment, more than one dependent variable can be tried. One dependent variable that might be tried is the ratio of kinetic energy stored in the rotor just prior to burst divided by that weight of containment that provides marginal or threshold containment. Another dependent variable that might be tried is the ratio of angular momentum stored in the rotor just prior to burst divided by that weight of containment that provides marginal or threshold containment. If two or more such dependent variables are compared for their correlation with a set of independent variables, the comparison might show that one of them is superior as a containment design variable.

Two classes of independent variables can be defined.

1. Variables that involve the <u>attacking fragments</u> such as (a) the number of them, (b) their mass, (c) their speed, and (d) the initial clearance between the rotor and the protective device.

2. Variables that involve the <u>containment design</u> such as the mechanical properties of the containment materials and the weight of each material used.

The experiment should provide some information on what might approximate an optimum condition among the second class of variables. It should also provide some information on how the conditions within the first class of variables might affect the optimum among the second class. The experiment

should be designed so that it can be fitted by a model equation containing first degree terms in all the variables and containing cross product terms involving independent variables both within and between these two classes of variables.

Fragment Variables

The fragment variables selected for the experiment are (a) the number of equally sized sectors and (b) the initial radial clearance between the rotor and the inside surface of the containment device. The test wheels will be modified so that on a controlled basis, the nature of the bursts will include two, three, and six piece bursts. Thus the sector sizes will be, respectively, 180°, 120°, and 60°. These pieces will differ widely in their masses, so that their speeds for threshold containment will probably be different.

Differing speeds are likely to require differing relative weights of the different layers for maximum overall weight efficiency. Such a result is equivalent to saying that there are interactions between the sector size variable and the variables expressing the relative weights of the layers.

The radial clearance is defined as the radial distance between the outer surface of the disk and the inner surface of the container. This definition ignores the presence of the blades. Blades were concluded to be relatively unimportant by Mangano (1972) who wrote as follows:

"Therefore, the blades on a rotor fragment do not significantly influence the distribution of the impact loads that are induced in a ring (provided the ring thickness approaches that required to effect containment and the fragment hub to blades mass ratio is large), nor do the blades absorb significant amounts of energy through their deformation during the containment process. The blades serve only to influence the fragment trajectory during the initial stages of impact. This also means that in cases where the rotor tip-to-ring clearance is small (test or operational clearances) the blade radial length becomes in effect the radial clearance that influences the orientation of the hub or disk portion of the fragment."

As defined, the radial clearance would be relatively small for the last stage of a compressor and for the first stage of a turbine, and would be relatively large for the first stage of a compressor or for the last stage of a turbine.

The radial clearance determines the amount that a disk sector rotates before contacting the container. Thus making the radial clearance an independent variable will vary the orientation of the attacking fragment to the inner surface. This variation might affect the optimum fraction of total weight that is assigned to the inner layer. Thus there might be an interaction between clearance and first layer weight.

Container Variables

The container consists of four layers. The fractions of the total weight assigned to three of the layers are independent variables. The fraction of total weight assigned to the fourth layer is correlated with the other three and is therefore not an independent variable. Such a variable is sometimes called a slack variable.

Two variations of a basic experiment plan will be described. In one variation of the plan, the fraction of total weight assigned to the third (metallic) layer will be the balance of weight variable, while in the other variation, the fraction of total weight assigned to the fourth (cloth) layer will be the balance of weight variable. In any case, the materials for each layer would be selected from Table 2.

Plan of Experiment

The plan of the experiment is indicated in Table 3. The treatment symbols represent the combinations of independent variable conditions in Yates notation and they are listed in the first column. They are the same as those in Table 7 of Holms (1967) which also describes the notation and further characterizes the plan.

The independent variables x_A, x_B, x_C, x_D, and x_E are to be assigned relative levels that are consistent with the levels implied by the treatment symbols in the first column. In Table 3 the plan variables have the meanings listed in Appendix A.

As listed in Table 7 of Holms (1967) all the treatments are intended to be performed in a single time span, or stage, or block. As such, the experiment is highly efficient in producing orthogonal estimates of all direct effect coefficients and all two-factor interaction coefficients. As such the experiment would

not ordinarily be subdivided. For the purposes of multi-layer rotor burst containment experimenting, each specimen will be terribly expensive. Furthermore, as described in Appendix B, each treatment (each combination of independent variables) will require about four specimens to produce a single value of the associated dependent variable.

Because the evaluation of the treatments will be so terribly expensive, the experiment plan as listed in Table 3 has been divided into two blocks, so that depending on the results from the first block, a decision can be made to either continue or not continue with the second block. This division means that on completion of the experiment, one two-factor interaction effect will not be capable of being estimated. To improve the precision of each block and to improve the precision of the combined experiment, some center point treatments not in Table 7 of Holms (1967) have been added to each block of Table 3.

One basis for deciding whether or not to continue from the first to the second block of Table 3 could consist of a comparison of the performance of the multi-material containers with the performance of monolithic containers. The standard of comparison might be the performance of a metal container, or it might be the performance of a cloth container. In either case, the standard of comparison need not be established by data external to the experiment. It could be established from results obtained from the first block. If a metal were desired as the standard of comparison, then the variable x_C would be assigned to the weight fraction of ballistic cloth, and the variable z would be assigned to the weight fraction of metal, namely z would be the weight fraction of metal in the third layer which would be specified by the z-column of Table 3 (and the metal would be chosen from Table 2(c)).

If the standard of comparison were to be a ballistic cloth, then the weight fraction of metal in the third layer would be specified by x_C of Table 3 and the weight fraction of cloth would be as specified by the z column of Table 3. (The cloth would be chosen from Table 2(d).)

The criteria used in assigning the treatments of Table 3 to the two blocks are given in Appendix C. Also given in Appendix C is an illustration of how the results from the first block, and from the combined blocks, would be interpreted if the standard of comparison were a metal.

Model Selection and Interpretation

If the experiment were that given by both blocks of Table 3 then the model initially fitted to the data would be that given by equation (5) of Appendix C. Such an equation might contain a few coefficients consisting mostly of experimental error, and the equation could be improved by deleting such terms as described by Holms (1974). Terms could also be deleted using a more conventional deletion procedure such as that given by Sidik (1972).

Suppose equation (5) has been fitted to the data and the insignificant terms deleted. The coefficients of x_A, x_B, and x_C would be examined for negative signs. Any such term having a negative sign would thereby suggest that the associated material was less weight efficient than the "others". (The "others" would always include the "balance of weight" material that is not explicitly represented in the model.) The larger positive coefficients of x_A, x_B, and x_C (if any are found) identify associated materials as being particularly weight efficient.

Numerically large coefficients of the two factor interactions would show important interaction (synergistic) effects. Their interpretation would follow from the definitions given to the independent variables.

CONCLUDING REMARKS

Preliminary to some proposed empirical development of design methods for weight efficient full circumferential rotor burst containment devices, three areas of information were reviewed, namely: (1) rotor burst protection experiments, (2) personnel body armor materials, and (3) modern methods for the design and analysis of small optimum seeking experiments.

Review of the information on rotor burst protection and body armor suggested that the following hypotheses should be evaluated:

1. The device should consist of four concentric cylinders, each having unique ballistic properties.

2. The innermost cylinder should be strong in shear to: (a) provide blunting of the sharp edges of the rotor fragments (b) dissipate some energy through fragment deformation, and (c) resist penetration by achieving a wider distribution of the load.

3. In the vicinity of each impact point the first and second layers should act as a beam in bending with: (a) the first layer having high compressive strength, (b) the second layer having high tensile strength, (c) the bond between them (the neutral axis shear area) having high shear strength, and (d) the combination should be of low mass to minimize distortions from the original shape due to inertia effects. The bond and the second layer should also be strong to inhibit spalling in the first layer.

4. The third layer should be the result of a "hedge" strategy, that is, it should be a material proven weight efficient in tests of monolithic rings. Thus it would have some of the attributes of the other three layers.

5. The concentrated loads of the attacking fragments should be assumed to be well distributed by the first three layers, and the fourth layer should be chosen solely for its weight efficiency in absorbing large amounts of energy in tensile straining.

Based on the preceding hypotheses and based on the ballistic properties of different types of armor materials, the four concentric cylinders should consist of materials from inner to outer as follows:

1. A light hard layer, such as a ceramic or a glass.
2. A light high tensile strength layer, such as a fiber reinforced plastic.
3. A tough layer, such as a metal.
4. A stretchable layer, such as a ballistic nylon cloth.

To test the stated hypotheses, and to evaluate the listed materials, the experimenting would consist of spin-pit burst containment testing using a representative turbine wheel. The wheel to be burst is surrounded by the containment ring assembly to be tested. The number of equally sized wheel fragments and the burst speed are controlled by saw cuts radially oriented in the rim of the test wheel. The result of each burst test would be measured by the weight of containment assembly, the wheel speed at burst, and whether the ringassembly contained or did not contain the wheel fragments. In addition to the containment system variables, other variables (representing engine design) were included in the experiment. Thus interactions can be observed between engine variables and containment material variables. The engine variables consist of the radial distance between the disk and the inner containment ring, and the combined effects of the mass and speed of the attacking fragments.

The attributes of the proposed experiment plan are as follows:

1. The experiment can be performed in two stages. Completion of the first stage results in a direct comparison of the weight efficiency of the composite ring concept with that of a monolithic ring.

2. If the comparison is unfavorable to the composite ring, the investigation can be terminated.

3. If the investigation is continued to the completion of the second stage, then the major hypotheses will be quantitatively evaluated. That is, the fitted model equation will contain 14 empirical coefficients and their values will provide 14 conclusions about the direct influences of the variables together with the ways that they combine (interact) to produce synergistic effects.

4. The orthogonal design of the experiment results in the observed effects of the variables being free of error correlations with each other and free from variations entering the experiment between the performances of the two stages.

APPENDIX A

SYMBOLS

x_A	weight fraction of first (innermost) layer
x_B	weight fraction of second layer
x_C	weight fraction of third (or fourth) layer
x_D	number of equally sized sector fragments of test rotor
x_E	radial clearance (-1 means small clearance, +1 means large, and 0 means mean of other two)
z	balance of weight (weight fraction not included in x_A, x_B, and x_C)
Y	dependent variable. SFE_{50} is a possible dependent variable
SFE_{50}	ratio of kinetic energy stored in the rotor at burst divided by the weight of containment providing marginal, or threshold, or 50 percent probability of containment

APPENDIX B

TEST STRATEGY

Threshold containment is to be evaluated for each of the treatment conditions of Table 3. The dependent variable could be the stored kinetic energy prior to burst divided by the container weight, or it might be the angular momentum prior to burst divided by the container weight, or it might be otherwise defined. In any case, the threshold condition is defined here as that condition which results in a 50-percent probability of containment. The object of the testing is therefore to determine a rotor speed representing a 50-percent probability of containment. Each test usually has an identifiable result that can be called contained and labeled "C" or not contained labeled "NC". The NC results will usually occur at higher speeds than the C results (although material property variations can sometimes result in a C at a higher speed (RPM) than one or more speeds that resulted in NC). From the data, a quantity called rpm_{50} must be determined which will be an estimated speed for a 50-percent probability of containment. For the purposes of the experiment defined by Table 3, a good enough estimate of rpm_{50} is believed to be attainable if the experimenting includes four burst tests for every treatment. The test wheels would be modified with radial cuts to induce the 2, 3, and 6 sector bursts as listed in Table 3. The depths of the cuts would be such as to result in approximations to the desired burst speed. The first test at any given treatment condition should be at a speed which (based on all prior information) is equally likely to result in a C or an NC. Subsequent speeds are to be computed using a stepping factor, f_s. If the experimenter had good prior knowledge of the performance of the containment system, he might choose f_s such that $1 < f_s < 2$. With little prior information on the containment system, he might choose $f_s \geq 2$. If the first result is a C then each new speed rpm_{i+1} at point $i + 1$ in the sequence following a C at rpm_i should be

$$rpm_{i+1} = \sqrt{f_s} * rpm_i$$

If the first test in a sequence results in an NC, then each new test that follows an NC shall be at speed rpm_{i+1} determined from the previous speed rpm_i as follows:

$$rpm_{i+1} = rpm_i \Big/ \sqrt{f_s}$$

After a test result has been followed by a test result of opposite type (C followed by NC or NC followed by C) the next test shall be at rpm_{i+1} determined from the smallest speed for NC, $rpm_{min,\,NC}$ and from the largest speed for C, $rpm_{max,\,C}$ as follows:

$$rpm_{i+1} = \left[\left(rpm^2_{min,\,NC} + rpm^2_{max,\,C}\right)\Big/ 2\right]^{1/2}$$

Illustrations of how such a test strategy might proceed are given by Fig. 2.

The final estimate of rpm_{50} would be obtained from the preceding equation with $i = 4$, except that if all four results were only C or only NC, then rpm_{50} would not be estimable.

APPENDIX C

DESIGN OF TWO-STAGE EXPERIMENT

This section presents some background information on the design of the experiment for multi-material containment rings. The terminology and use of symbols is that of Davies (1960) or of Holms (1967).

The first block (eight treatments) might be planned as a resolution 3 design to provide estimates of first-order coefficients for a model equation that would include the five variables. The defining contrasts must then include two three-letter words and one four-letter word. The four-letter word would be the defining contrast for the experiment with two blocks and 16 treatments. The experiment with two blocks would be a resolution 4 design, and therefore, it would be almost worthless with respect to the estimateion of the coefficients of the two-factor interactions. Such a design would be of little value because the physical basis for the research is the hypothesis that certain materials, when used in combination, might interact beneficially, and that furthermore, the beneficial effects of certain materials might be critically dependent on such ballistic variables as fragment orientation and speed. Correspondingly, the estimation of most of the two-factor interaction coefficients is essential to the answering of the main questions of the research.

In line with the preceding criteria, the objective of obtaining a resolution 3 design at the end of the first block will be sacrificed, and as a benefit of that sacrifice a nearly resolution 5 design can be achieved at the conclusion of the two blocks. For the two blocks, the defining contrast will be a five-letter word (which would ordinarily provide a resolution 5 design) but for the priviledge of having the option to stop or to continue the investigation beyond the first block we must pay the price of confounding one two-factor interaction with the block effect.

The defining contrasts for the first block can be

$$I = -ABD = CE = -ABCDE \tag{1}$$

and the defining contrasts for the first two blocks can be

$$I = -ABCDE \tag{2}$$

Let the variables be chosen and labeled as in Appendix A and in particular let x_C be the weight fraction of the fourth layer (cloth) and let z_M be the weight fraction of the third layer (metal). In such a labeling of variables, the variable z_M is obviously a first-degree function of x_A, x_B, and x_C and is therefore not an independent variable. It is called a slack variable or a "balance of weight" variable and would be omitted from any model fitting that included the variables x_A, x_B, and x_C. The association of particular plan variables (letters) with the physical variables might have been arbitrary, but it should not be, because the interaction $x_C x_E$ is confounded with the block effect. Because the coefficient of $x_C x_E$ is in error by the amount of the block effect, the letters C and E should be assigned to the variables thought least likely to interact.

The assignment of physical variables to the letters C and E is based on the following considerations. The impact process begins with the wheel fragments traveling through the clearance distance and the process ends with the transfer of some minor or major strain to the outer layer. This sequence suggests that the physical variables consisting of the initial clearance and the weight fraction of outer layer are the two physical variables least likely to interact. Correspondingly, these variables should be given the symbols C and E, and the order is arbitrary. (As suggested by the body armor data, the speed of impact is a variable that can change the mode of fracture. Thus the speed of impact has a high probability of interacting with the other variables. It was for this reason that the number of fragments is introduced as a controlled variable into the experiment, thus forcing the experiment into differing ranges of speed. Correspondingly, the variables x_C and x_E should not be used to represent the number of fragments.) Some additional concepts for the matching of physical variables to plan variables were given by Sidik (1971).

With the defining contrasts given by equation (1), the treatments and the aliased first- and second-degree model parameters are as shown in Table 4. Performance of the experiment with such treatments and acquisition of the associated observations would permit the numerical evaluation of eight model coefficients. Let these coefficients be labeled b_0, b_1, b_2, b_3, b_4, b_5, b_6, and b_7. Referring to the alias combinations of Table 4, the predictive equation could be written

$$Y = b_0 + b_1 x_A + b_2 x_B - b_3 x_D + b_4 x_C + b_5 x_A x_C + b_6 x_B x_C - b_7 x_C x_D \tag{3}$$

Reference to the aliased pairs in Table 4 shows that any one or more of the terms in the preceding equation can be arbitrarily replaced by its alias as listed in Table 4. (The choices of algebraic signs are based on the assumption that the b's are computed by Yates' method.)

Note that the first degree terms in x_C and x_E are indistinguishable. Furthermore, a basic assumption of the multi-material concept is that the right combination of several materials will provide containment that is more weight efficient than the best material used singly. Consistent with this assumption is the assumption that the two-factor interactions will be large and that the ambiguities among the terms of Table 4 will not permit any conclusions to be drawn with respect to the effects of the variables. What will be achieved is the performance of eight or nine milti-material combinations to be compared with the performance of single material containment rings.

The performance of single material containment rings could be obtained from direct tests with single material rings, however, a crude indirect comparison of the performance of single material rings with multiple material rings is obtainable from just the first block data of Table 3. The crude comparison is obtained by fitting the model

$$Y = \alpha + \beta_M z_M \tag{4}$$

to the data, where α and β_M are the only constants fitted to the nine observations of Y. If the coefficient of correlation were low, or if the coefficient β_M were concluded to be insignificant, then no useful comparison could be drawn between the weight efficiency of metal rings and the weight efficiency of multi-material rings. The experimenter might proceed with block 2 or he might look to other sources of information. On the other hand, if the coefficient of correlation were high, of if the coefficient β_M were tested as significant, the immediate conclusions would be that the variation of the weight fraction of the metallic content was important and that the variations of the weight fractions of the nonmetallic materials were unimportant. (As listed in Table 3, the weight fraction of the metal would have been 0/12, 2/12, 3/12, 4/12, and 6/12.) If in the model fitting, β_M were concluded to be significant, then a negative value would show that the nonmetallic materials were weight efficient and that the investigation should be continued through the second block (at which point the effects of the nonmetallic materials would probably become clear - significant interactions would be displayed). A significant and positive value for β_M from the first block would show that the performance of the metal was

superior to that of the other materials. The implication of such a result would be that all the concepts leading to the design of the experiment should be re-examined, and that the next step should not include a performance of the second block.

If a second block is performed, the basic treatments and the first- and second-degree parameter estimates for the two blocks would be as shown in Table 5. Such an experiment would be described as a two-level, half-replicate, fractional-factorial experiment on five variables in two blocks.

Based on the structure exhibited by Table 5, a prediction equation obtained from the parameter estimates from the data observed from the two blocks would be written:

$$Y = b_0 + b_1x_A + b_2x_B + b_3x_Ax_B + b_4x_C + b_5x_Ax_C + b_6x_Bx_C - b_7x_Dx_E + b_8x_D$$

$$+ b_9x_Ax_D + b_{10}x_Bx_D - b_{11}^{*}x_Cx_E + b_{12}x_Cx_D - b_{13}x_Bx_E - b_{14}x_Ax_E - b_{15}x_E \qquad (5)$$

The estimate b_{11}^{*} is not necessarily the correct value for the coefficient of x_Cx_E. The estimate will be in error by the average performance shift in Y caused by any changes that may have occurred between the two blocks. The term in x_Cx_E would be deleted if equation (5) (or any simplification of it) were used as a containment design equation.

The experiment with the two blocks, as just described, can be doubled to a full factorial experiment with parameters estimated for all interactions up to the five variable interaction. If this were done, the coefficients of x_Cx_E, $x_Ax_Bx_D$, and $x_Ax_Bx_Cx_Dx_E$ would still contain any errors caused by block effects. *Confounded with block effect.

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TABLE 1. - DISK BURSTS

	1971	1972	1973	1974
Fan	1	1	0	0
Compressor	7	2	2	1
Turbine	5	2	1	4
Total	13	5	3	5

TABLE 2. - MATERIAL AND PROCESS OPTIONS

(a) Layer 1

Option	Requirements	Description
a	MIL-A-46103 (class 4)	Boron carbide ceramic, monolithic ring.
b	MIL-A-46103 (class 4)	Boron carbide ceramic, adhesively bonded tiles.
c	MIL-A-46103 (class 3)	Boron carbide/silicon carbide/silicon ceramic, monolithic ring.
d	MIL-A-46103 (class 3)	Boron carbide/silicon carbide/silicon ceramic, adhesively bonded tiles.
e	MIL-A-46103 (class 2)	Silicon carbide ceramic, monolithic ring.
f	MIL-A-46103 (class 2)	Silicon carbide ceramic, adhesively bonded tiles.
g	MIL-A-46103 (class 1) and/or MIL-T-46098	Aluminum oxide ceramic, monolithic ring.
h	MIL-A-46103 (class 1) and/or MIL-T-46098	Aluminum oxide ceramic adhesively bonded tiles.
i		Borosilicate glass (Pyrex 7740 or equal) monolithic ring
	MIL-A-46050	Adhesive, cyanoacrylate.
	MMM-A-132	Adhesive, epoxy-nylon.

TABLE 2. - Continued.

(b) Layer 2

Option	Requirements	Description
a	SAE-AMS 3832	Glass roving, filament wound, S-glass, epoxy resin.
b	MIL-A-46103B or MIL-I-17368*	Glass cloth reinforced, polyester resin.
c		Aramid fiber filament wound, phenolic-polyvinyl butyral resin.
d		Aramid cloth reinforced, phenolic-polyvinyl butyral resin.
e		Aramid fiber filament wound, epoxy resin.
f		Aramid cloth reinforced epoxy resin.

*Doron: Glass MIL-C-9084, resin MIL-R-7575.

TABLE 2. - Continued.

(c) Layer 3

Option	Requirements	Description
a	MIL-S-17758	Hadfield steel rings. Billets pierced and roll formed. Fully austentized.
b	MIL-S-13259	Hadfield steel, rolled strip. Fully austentized. Spirally wrapped and tack welded.
c	MIL-S-17249 (ASTM 128, B-3)	Hadfield steel rings, centrifugally cast and finish machined.
d	SAE-AMS 5639 Fed QQ-S-763 (AISI 304)	Stainless steel rings, billets pierced and roll formed, solution treated.
e	SAE-AMS 5515 (AISI 301 or 302)	Stainless steel, rolled strip. Hot rolled and solution treated. Spirally wrapped and tack welded.
f	SAE-AMS 5370 (ACI-CF-8)	Stainless steel rings, centrifugally cast and finish machined.
g		TRIPP steel. Billets pierced and roll formed.
h		TRIPP steel, rolled strip. Spirally wrapped and tack welded.

TABLE 2. - Concluded.

(d) Layer 4

Option	Requirements	Description
a	MIL-C-43635	Felt, ballistic, nylon.
b		Felt, aramid (Kevlar 29).
c	MIL-C-12369	Fabric, ballistic, nylon.
d	LP/P DES 32-75*	Fabric, ballistic, aramid.
e		Polypropelene plastic film, Phillips XP or equal.

* Natick limited use specification.

TABLE 3. - PLAN OF EXPERIMENT AND LEVELS OF VARIABLES

Treatment symbol	Block	Fractions of total weight				Number of sectors, x_D	Disk to ring clearance, x_E
		x_A	x_B	x_C	z		
Center	1	1/4	1/4	1/4	1/4	3	0
(1)	1	1/6	1/6	1/6	3/6	2	-1
ae	2	2/6	1/6	1/6	2/6	2	1
be	2	1/6	2/6	1/6	2/6	2	1
ab	1	2/6	2/6	1/6	1/6	2	-1
ce	1	1/6	1/6	2/6	2/6	2	1
ac	2	2/6	1/6	2/6	1/6	2	-1
bc	2	1/6	2/6	2/6	1/6	2	-1
abce	1	2/6	2/6	2/6	0	2	1
de	2	1/6	1/6	1/6	3/6	6	1
ad	1	2/6	1/6	1/6	2/6	6	-1
bd	1	1/6	2/6	1/6	2/6	6	-1
abde	2	2/6	2/6	1/6	1/6	6	1
cd	2	1/6	1/6	2/6	2/6	6	-1
acde	1	2/6	1/6	2/6	1/6	6	1
bcde	1	1/6	2/6	2/6	1/6	6	1
abcd	2	2/6	2/6	2/6	0	6	-1
Center	2	1/4	1/4	1/4	1/4	3	0

TABLE 4. - FIRST BLOCK ALIASES

Treatments	Parameter aliases		Term aliases	
(1)	β_I	$+\beta_{CE}$	b_0	$b_0x_Cx_E$
ad	β_A	$-\beta_{BD}$	b_1x_A	$-b_1x_Bx_D$
bd	β_B	$-\beta_{AD}$	b_2x_B	$-b_2x_Ax_D$
ab	β_{AB}	$-\beta_D$	$-b_3x_D$	$b_3x_Ax_B$
ce	β_C	$+\beta_E$	b_4x_C	b_4x_E
acde	β_{AC}	$+\beta_{AE}$	$b_5x_Ax_C$	$b_5x_Ax_E$
bcde	β_{BC}	$+\beta_{BE}$	$b_6x_Bx_C$	$b_6x_Bx_E$
abce	$-\beta_{CD}$	$-\beta_{DE}$	$-b_7x_Cx_D$	$-b_7x_Dx_E$

TABLE 5. - TREATMENTS AND ESTIMATES

FOR TWO BLOCKS

Block	Treatment	Parameter estimated
1	(1)	β_I
2	ae	β_A
2	be	β_B
1	ab	β_{AB}
1	ce	β_C
2	ac	β_{AC}
2	bc	β_{BC}
1	abce	$-\beta_{DE}$
2	de	β_D
1	ad	β_{AD}
1	bd	β_{BD}
2	abde	$-\beta^*_{CE}$
2	cd	β_{CD}
1	acde	$-\beta_{BE}$
1	bcde	$-\beta_{AE}$
2	abcd	$-\beta_E$

*Confounded with block effect.

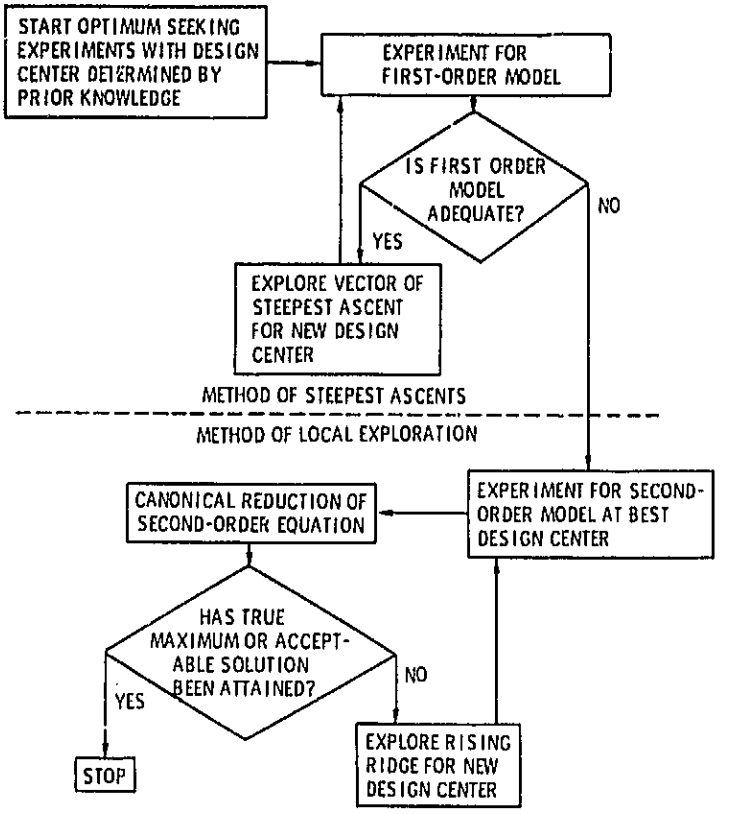

Figure 1. - Box-Wilson methods.

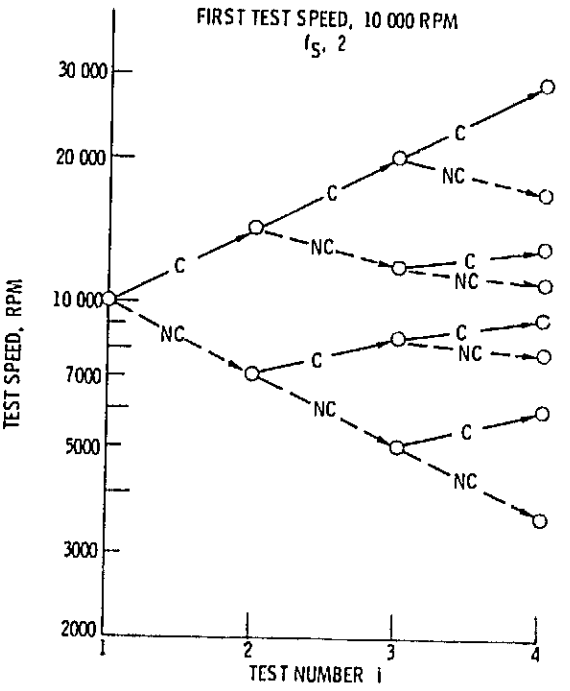

Figure 2. - Illustration of test strategy.