ANALYSIS OF GaAs AND Si SOLAR ENERGY HYBRID SYSTEMS

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Prepared by
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Various silicon hybrid systems are modeled and compared with a Gallium Arsenide hybrid system. The hybrid systems modeled produce electric power and also thermal power which can be used for heating or air conditioning. Various performance indices are defined and used to compare the system performance: capital cost per electric power out; capital cost per total power out; capital cost per electric power plus mechanical power; annual cost per annual electric energy; and annual cost per annual electric energy plus annual mechanical work. These performance indices indicate that concentrator hybrid systems can be cost effective when compared with present day energy costs. Realistic cost and efficiencies of GaAs and Si are respectively $35,000/m² for 15% efficient solar cells and $1000/m² for 10% efficient solar cells. Limiting values for annual costs are 10.3 ¢/kwh and 6.8 ¢/kwh for Si and GaAs respectively. Results demonstrate that for a given flow rate there is an optimal operating condition for maximum photovoltaic output associated with concentrator systems. Also concentrator hybrid systems produce a distinct cost advantage over flat plate hybrid systems.
PREFACE

This work was performed during the summer and fall of 1975. Basic data and conclusions are thought to be valid during this period. The system analysis contained in this report can be updated with improved performance and cost information.

J.H. Heinbockel
A.S. Roberts, Jr.

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ANALYSIS OF GaAs AND Si SOLAR ENERGY HYBRID SYSTEMS

By

John H. Heinbockel\textsuperscript{1} and A.S. Roberts, Jr.\textsuperscript{2}

SUMMARY

Various silicon hybrid systems are modeled and compared with a Gallium Arsenide hybrid system. The hybrid systems modeled produce electric power and also thermal power which can be used for heating or air conditioning. Various performance indices are defined and are used to compare the system performance. The performance indices are: capital cost per electric power out; capital cost per total power out; capital cost per electric power plus mechanical power; annual cost per annual electric energy; and annual cost per annual electric energy plus annual mechanical work. These performance indices indicate that concentrator hybrid systems can be cost effective when compared with present day energy costs.

Realistic costs and efficiencies of GaAs and Si are respectively $35,000/m\textsuperscript{2} for 15\% efficient solar cells and $1000/m\textsuperscript{2} for 10\% efficient solar cells. The performance indices show that limiting values for annual costs are 10.3 \textcent/kwh and 6.8 \textcent/kwh for Si and GaAs respectively. Results demonstrate that for a given flow rate there is an optimal operating condition for maximum photovoltaic output associated with concentrator systems. Also concentrator hybrid systems produce a distinct cost advantage over flat plate hybrid systems.

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LIST OF SYMBOLS

\( \text{Si} \) = silicon

\( \text{GaAs} \) = gallium arsenide

\( P_{\text{max}} \) = maximum power

\( \eta_3 \) = efficiency of solar cell

\( \eta_r \) = reference efficiency

\( T_r \) = reference temperature

\( \beta \) = slope coefficient

\( T \) = temperature °C

\( T_K \) = temperature °K

\( E \) = solar intensity [w/m²]

\( I_{\text{sc}} \) = short circuit current density [amp/cm²]

\( V_{\text{oc}} \) = open circuit voltage [Volts]

\( \text{FF} \) = fill factor

\( \text{VOCO, ISCO, FIFO} \) = scale factors

\( A_i \) = constants \( (i = 1, 2, 3, 4, 5) \)

\( Q_{\text{ELEC}} \) = electric output per square meter of absorber area [w/m²]

\( Q_{\text{IN}} \) = energy absorbed by system per square meter of absorber [w/m²]

\( Q_{\text{LOSSES}} \) = loss terms per unit area due to convection and radiation losses [w/m²]

\( I_N \) = incident solar flux density [w/m²]

\( Q_{\text{THERMAL}} \) = energy removed by coolant [w/m²]

\( T_{\text{CK}} \) = temperature (°K) of plate or cell

\( T_{\text{FK}} \) = temperature (°K) of cooling fluid

\( h_1 \) = heat loss coefficient [w/m² °K]

\( T_{\text{AIRK}} \) = ambient temperature (°K)
\[\epsilon = \text{emissivity}\]
\[\sigma = \text{Stephan-Boltzmann constant}\]
\[.m = \text{mass flow rate [kg/hr]}\]
\[C_p = \text{specific heat of fluid}\]
\[A_{\text{abs}} = \text{area of absorbing surface}\]
\[\eta_2 = \text{efficiency of flat plate collector}\]
\[P_i = i = 1, 2, 3, 4, 5 \text{ performance indices}\]
\[A_{\text{ap}} = \text{aperture area}\]
\[C_{\text{TH}} = \frac{A_{\text{ap}}}{A_{\text{abs}}} = \text{theoretical concentration ratio}\]
\[Q_{\text{WORK}} = .5 \left(1 - \frac{T_{FK}}{T_{CK}}\right)Q_{\text{THERMAL}} = \text{high quality thermal energy}\]
\[Q_{\text{LT}} = \text{low quality thermal energy}\]
\[K = \frac{.mC_p}{A_{\text{abs}}} = \text{effective thermal conductance per unit area of absorber}\]
\[\eta_1 = \text{efficiency of absorption}\]
\[n = \text{month of year (1 to 12)}\]
\[Q_{\text{ELEC}}(n) = \text{electric output for nth month per unit area of absorber}\]
\[Q_{\text{THERMAL}}(n) = \text{thermal output for nth month per unit area of absorber}\]
\[Q_{\text{WORK}}(n) = \text{work output for nth month per unit area of absorber}\]
\[I_N(n) = \text{average solar flux density for nth month}\]
\[i = \text{interest rate}\]
\[m = \text{maintenance cost}\]
\[n_1 = \text{number of years}\]
\[\text{minP}_4 = \text{minimum value of performance index 4}\]
1. INTRODUCTION

A typical solar energy hybrid system is illustrated in figure 1. The system is a hybrid system in that solar energy can be converted to (a) electrical energy by utilizing photovoltaic devices and (b) thermal energy which in turn can be used for heating or air conditioning.

In this report, various types of solar collectors will be analyzed so that electric and thermal energy outputs received from the collectors can be compared. The photovoltaic devices considered for the conversion of solar energy to electric energy will be limited to silicon (Si) and gallium arsenide (GaAs) solar cells. The system comparisons will incorporate both energy output comparisons and cost comparisons. The cost comparisons are based upon current costs and projected costs for Si and GaAs solar cells. Comparisons are obtained by defining various system performance indices such as capital costs per unit power out and annual costs per annual energy out.

In section 2, mathematical models are constructed to simulate the conversion efficiencies of both Si and GaAs photo cells under a variety of temperature and light flux conditions. These models are in turn utilized in sections 3 and 4 where various collector systems are modeled and compared under nominal and annual solar insolation simulations. The results of the system comparisons are presented graphically and can be found in these later sections. Section 3 is limited to system comparisons under nominal solar insolations and section 4 deals only with annual solar insolation system performance.

The results of the system performance on an annual basis are in turn utilized in section 5 to illustrate various design considerations necessary to meet specific power requirements.

Section 6 discusses the various assumptions that have been made throughout the study and section 7 presents a rationale for future cost projections of Si and GaAs solar cells. Section 8 presents the conclusions of the study.
The various graphs presented in this report were obtained from the computer programs presented in Appendices A and B, by utilizing the appropriate conditions in the programs.

2. MODELS FOR GaAs AND Si EFFICIENCIES

Linear Model

Figures 2, 3, and 5 are taken from references 1, 2, and 3, and depict the behavior of GaAs and Si solar cells under a wide variety of temperatures and light flux intensities. Figure 4 comes from H. Hoval and J.M. Woodall, "Optimization of Solar Cells for Air Mass Zero Operation and a Study of Solar Cells at High Temperatures," which is a quarterly progress report for the period June 1974 to October 1974, NASA contract NAS1-12812. This report is not a readily available reference. If one assumes that the maximum power out is directly proportional to the solar intensity $E$, then one can write

$$P_{\text{max}} = a(T) \cdot E \, [\text{mw}]$$

where $a(T)$ is temperature dependent proportionality constant, which in general decreases with temperature. From the definition of solar cell efficiency

$$\eta_3 = \text{efficiency} = \frac{\text{power out}}{\text{power in}} = \frac{P_{\text{max}}}{E \cdot (\text{cell area})} = \frac{a(T)}{\text{cell area}}$$

one can conclude that the solar cell efficiency depends upon temperature only. Thus, it is assumed that

$$\eta_3 = \eta_3(T) = \eta_r \left[1 - \beta (T - T_r)\right] \quad (1)$$

where $\eta_r$ is reference efficiency at reference temperature $T_r$ and $\beta$ is proportionality constant.

For Si, $\beta$ is chosen as .0041, this value of $\beta$ gives a zero efficiency when $T - T_r = 243.9^\circ \text{C}$. For GaAs, $\beta$ is chosen as .0024, which gives a zero efficiency when $T - T_r = 416.66^\circ \text{C}$ (reference for
the above coefficients is "On Heat Rejection from Terrestrial Solar Cell Arrays with Sunlight Concentration" by L.W. Florschuetz of the Mech. Engn. Dept., Arizona State University, Tempe, AZ. This paper was received in a private correspondence with the author and is not yet readily available).

Nonlinear Model

The assumed solar cell efficiency $\eta_3$ was modeled to conform with the experimental data from reference 2 (fig. 3b). The modeled efficiency decreases with increasing light intensity. It should be noted that cell design will determine the actual behavior of the efficiency. In several GaAs cells, efficiency has been observed to increase with increasing light intensity (refs. 4, 5, and 6). For the purposes of this study, the more conservative assumption of decreasing efficiency with intensity was utilized in the effort to simulate worst characteristics of mass produced cells.

The following formulas were used to model the efficiency $\eta_3$ of Si and GaAs solar cells and were derived on the basis of all parametric data taken from various sources (refs. 2, 3, 4, 5, 6, 7, 8, 9, 10).

$$\eta_3 = \eta_3(E, T) = \frac{I_{SC}(E, T) \cdot V_{OC}(E, T) \cdot FF(E, T) \cdot 10^4}{E}$$

where

- $I_{SC}$ = short circuit current density [amp/cm²],
- $V_{OC}$ = open circuit voltage [Volts],
- $FF$ = fill factor,
- $E$ = solar flux density [w/m²], and
- $T$ = temperature [°C].

The following empirical relations were used in the nonlinear model for efficiency.
GaAs

\[ V_{oc}(E,T) = VOC_G + 7.361(10^{-3})E^{0.57} - 2.45(10^{-3})T \]

\[ I_{sc}(E,T) = ISC_G \cdot (0.148E^{0.36} + 0.388)T \]

\[ FF_G(E,T) = FIFO_G + 1.922E^{-0.162} - 1.11(10^{-3})T \]

Si

\[ V_{oc}(E,T) = VOC_S + \left[ 2.9(10^{-3})E^{0.172} - 2.23(10^{-3})T \right] \]

\[ I_{sc}(E,T) = ISC_S \left( A_0 + A_1T + A_2T^2 + A_3T^3 + A_4T^4 + A_5T^5 \right) E/10 \]

\[ FF_S(E,T) = FF_G(E,T) \]

The above equations were derived to fit the data from figures 2 through 5 and the \( I_{sc}(E,T) \) for Si was taken from reference 3. The quantities \( VOC, ISC, FIFO \) are scaling parameters in order that the above equations can satisfy the following reference conditions at \( T = 25^\circ C, E = 1000 \text{ w/m}^2 \):

**GaAs**

\( V_{oc} = 1.0; \quad FF_G = 0.85; \quad I_{sc} = \frac{\eta_r E}{10^6V_{oc}FF_G} \)

**Si**

\( V_{oc} = 0.60; \quad FF_S = 0.85; \quad I_{sc} = \frac{\eta_r E}{10^6V_{oc}FF_G} \)

where \( \eta_r \) = efficiency at reference conditions. The constants \( A_i \) for Si are:

\( A_0 = 0.914727 \)
\( A_1 = 0.108713(10^{-2}) \)
\( A_2 = -0.695706(10^{-5}) \)
\( A_3 = 0.226603(10^{-7}) \)
\( A_4 = 0.17109(10^{-9}) \)
\( A_5 = -0.144039(10^{-11}) \)
Comparison of the linear and nonlinear solar cell efficiency models are illustrated in figure 6 for a variety of solar flux intensities using a reference efficiency of 15%.

The models depicted by (1) and (2) will be utilized to represent the solar cell efficiency in the following sections.

3. HYBRID SYSTEM MODELS AND ANALYSIS

A hybrid system can be visualized in figure 1. A hybrid system can produce electrical power and heating or air conditioning. Actually a hybrid system produces electric power by photovoltaic conversion of the incoming solar flux density and also thermal power by absorption of that light flux density which is not converted to electrical power. The thermal power can then be utilized in heating or air conditioning. Figure 1 illustrates a system that could be used for an ammonia-water absorption cooling system.

Various systems for the absorption of the solar flux are considered in this report. The various systems are illustrated in figure 7 and can be summarized as follows:

**System I:** Two flat collectors, one for the collection of solar flux to be converted into thermal power and the other having silicon (maintained at 25°C) for the production of electrical power.

**System II:** A single flat plate collector upon which is placed silicon solar cells. It is assumed that the temperature of the silicon cells is the same as that of the flat plate and the silicon efficiency is a function of this temperature.

**System III:** A concentrator system having GaAs solar cells with efficiency as a function of temperature. The temperature can be controlled by a fluid passing through the absorber.

**System IV:** A concentrator system having Si solar cells with efficiency as a function of temperature. The temperature can be controlled by a fluid passing through the absorber.
System V: A concentrator system with no solar cells and a flat plate collector with silicon solar cells. The concentrator system is for thermal power production and the flat plate collector is assumed maintained at 25°C for the production of electrical power.

Analysis and Assumptions for Systems I and II

In system I, illustrated in figure 7, it is assumed that 84% of the incident solar flux is absorbed by the flat plate collectors. The electric power output per unit area of absorber is

\[ Q_{\text{ELEC}} = \eta_3 Q_{\text{IN}} \ [w/m^2] \]

where \( Q_{\text{IN}} = .84I_N \ [w/m^2] \) and \( \eta_3 \) is cell efficiency.

The energy loss from the collector is

\[ Q_{\text{LOSS}} = h_1(T_{\text{CK}} - T_{\text{AIRK}}) - \varepsilon\sigma(T_{\text{CK}}^4 - T_{\text{AIRK}}^4) \] (5)

It is assumed that the thermal energy obtained from flat plate 1 is the energy remaining after losses are accounted for. An energy balance produces

\[ Q_{\text{IN}} - Q_{\text{LOSS}} - Q_{\text{THERMAL}} = 0 \]

or

\[ Q_{\text{THERMAL}} = .84I_N - h_1(T_{\text{CK}} - T_{\text{AIRK}}) - \varepsilon\sigma(T_{\text{CK}}^4 - T_{\text{AIRK}}^4) \]

\[ = \frac{mC_p}{A_{\text{abs}}} (T_{\text{CK}} - T_{\text{FK}}) \] (6)

where

\[ I_N \] = incident solar flux (assumed to be 500 w/m²)
\[ T_{\text{CK}} \] = temperature of flat plate [°K]
\[ T_{\text{AIRK}} \] = ambient temperature [°K]
\[ \varepsilon \] = emissivity = .04
\[ \sigma = \text{Stephan-Boltzmann constant} = 5.6697 \times 10^{-8} \text{ w/m}^2 \cdot \text{k}^4 \]
\[ \dot{m} = \text{mass flow rate [kg/hr]} \]
\[ C_p = \text{specific heat of fluid [kJ/kg °C]} \]
\[ T_{FK} = \text{temperature of fluid entering collector [°K]} \]
\[ A_{abs} = \text{area of absorbing surface [m}^2\text{]} \]

The cost of the flat plate collectors is assumed to be a linear function of the thermal efficiency \( \eta_2 \) where

\[ \eta_2 = \text{efficiency of collector} = \frac{Q_{\text{THERMAL}}}{Q_{IN}} \]

and the cost is given by

\[ \text{cost of collector} = 400 \eta_2 + 10 \ \text{[$/m}^2\text{]} \]

This corresponds to current day costs which range between $53.82/m^2 ($5/ft^2) and $645.90/m^2 ($60/ft^2) for flat plate collectors.

The silicon solar cell costs are analyzed in section 7 and are taken as $1000/m^2 for 10% efficient cells and the cost of maintaining cell temperatures at 25° C is assumed to have the minimal value of $56/m^2. This gives a total cost for the flat plate 2 of $1056/m^2.

System II is illustrated in figure 7. Assuming that 84% of the incident flux energy is absorbed by the solar cells an energy balance on system II gives

\[ Q_{IN} - Q_{ELEC} - Q_{LOSSES} - Q_{\text{THERMAL}} = 0 \]

or

\[ (1 - \eta_3)(.84)I_N - h_1(T_{CK} - T_{AIRK}) - \varepsilon \sigma (T_{CK}^4 - T_{AIRK}^4) \]

\[ = \frac{\dot{m}C_p}{A_{abs}} (T_{CK} - T_{FK}) \]
where

\[ I_N = \text{incident flux density [w/m}^2\text{]} \ (500 \ \text{w/m}^2) \]

\[ h_1 = \text{heat loss coefficient [w/m}^2\ \text{OK]} \ (500 \ \text{w/m}^2 \ \text{OK}) \]

\[ T_{CK} = \text{temperature of cells = temperature of flat plate [}^\circ\text{K]} \]

\[ \varepsilon = \text{emissivity (.04)} \]

\[ \sigma = \text{Stephan-Boltzmann constant} = 5.6697(10^{-8}) \ \text{w/m}^2\text{k}^4 \]

\[ T_{AIRK} = \text{ambient temperature [}^\circ\text{K]} \]

\[ \dot{m} = \text{mass flow rate of fluid} \]

\[ C_p = \text{specific heat of fluid} \]

\[ A_{abs} = \text{area of absorber} \]

It is known that the second law of thermodynamics gives an expression for the maximum thermal efficiency of a heat engine, which is the Carnot efficiency, \( \eta_C = 1 - \frac{T_{FK}}{T_{CK}} \)

where \( T_{FK} \) is fluid temperature and \( T_{CK} \) is cell temperature. Here it is assumed that the fluid will enter collector at a temperature \( T_{FK} \) and exit from collector at a temperature \( T_{CK} \). This assumption is consistent with the fact that typical large power plants have overall efficiencies of 50 to 60% of the Carnot efficiency (ref. 11).

Actually most solar heat engines go through a Rankine cycle which is approximately 50% the efficiency of a Carnot cycle. Define

\[ Q_{\text{WORK}} = .5 \left(1 - \frac{T_{FK}}{T_{CK}}\right) Q_{\text{THERMAL}} \]  \hspace{1cm} (8)

as the high quality thermal energy that can be extracted via turbine from the thermal energy received by the solar collector.

In order to compare the various systems in figure 7, various performance indices are defined which will characterize the various forms of energy obtainable from a hybrid system.
For both systems I and II the following performance indices are defined:

\[ P_1 = \frac{\text{CAPITAL COST}}{\text{PEAK(ELEC + MECH)POWER}} \]

\[ = \frac{(\text{COST FP}_1)_\text{abs} + (\text{COST FP}_2 + \text{COST CELL})_\text{abs}}{Q_{\text{ELEC}} \cdot A_{\text{abs}} + 0.5 \left(1 - \frac{T_{FK}}{T_{CK}}\right) Q_{\text{THERMAL}} \cdot A_{\text{abs}}} \]

\[ P_2 = \frac{\text{CAPITAL COST}}{\text{TOTAL POWER OUT}} \]

\[ = \frac{(\text{COST FP}_1)_\text{abs} + (\text{COST FP}_2 + \text{COST CELL})_\text{abs}}{(Q_{\text{ELEC}} + Q_{\text{THERMAL}})_\text{abs}} \]

\[ P_3 = \frac{\text{CAPITAL COST}}{\text{ELEC POWER}} \]

\[ = \frac{(\text{COST FP}_1)_\text{abs} + (\text{COST FP}_2 + \text{COST CELL})_\text{abs}}{Q_{\text{ELEC}} \cdot A_{\text{abs}}} \]

where

\[ A_{\text{abs}} = \text{area of absorber [m}^2\text{]} \]

\[ Q_{\text{ELEC}} = \eta_3 (.84) I_N \]

\[ Q_{\text{THERMAL}} = K(T_{CK} - T_{FK}), \quad K = \frac{m_c P}{A_{\text{abs}}} = \text{flow rate parameter [kw/m}^2\text{ °C]} \]

\[ Q_{\text{WORK}} = 0.5 \left(1 - \frac{T_{FK}}{T_{CK}}\right) Q_{\text{THERMAL}} \]

is the fraction of thermal energy which is converted to mechanical work assuming a Rankine cycle which is modeled as 50% of a Carnot cycle efficiency.
The same cost figures are used in system II as those presented for system I. Systems I and II are summarized in figures 8 and 9.

Analysis and Assumptions for Systems III and IV

Systems III and IV are concentrator systems having GaAs and Si solar cells respectively and are illustrated in figure 7.

The following assumptions and notations are used to describe and model these systems.

Let \( A_{ap} \) = aperture area, \( A_{abs} \) = absorber area of receiver, \( C_{TH} = \frac{A_{ap}}{A_{abs}} \) = theoretical concentration ratio, \( A_s \) = surface area of receiver. It is assumed that \( \frac{A_s}{A_{abs}} = 2 \). Then for both systems III and IV the various energy terms per unit area of absorber are:

\[
Q_{IN} = \eta_1 I_N C_{TH}
\]

where \( \eta_1 \) = optical efficiency of concentrator (assumed to be .7)

\[
Q_{ELEC} = \eta_3 Q_{IN}
\]

where \( \eta_3 \) = solar cell efficiency

\[
Q_{LOSSES} = 2h_1 (T_{CK} - T_{AIRK}) - 2\epsilon \sigma (T^h_{CK} - T^h_{AIRK})
\]

\[
Q_{THERMAL} = \frac{mC_P}{A_{abs}} (T_{CK} - T_{FK}) = K(T_{CK} - T_{FK})
\]

\[
Q_{WORK} = \frac{1}{2} \left( 1 - \frac{T_{FK}}{T_{CK}} \right) Q_{THERMAL}
\]

where the notation is the same as that employed to model systems I and II.

An energy balance on the concentrator systems produces

\[
Q_{IN} - Q_{ELEC} - Q_{LOSSES} = Q_{THERMAL}
\]
and the performance indices $P_i \; i = 1, 2, 3$ become

\[
P_1 = \frac{\text{CAPITAL COST}}{\text{PEAK (ELEC + MECH) POWER}} = \frac{(\text{COST CON})A_{ap} + (\text{COST CELL})A_{abs}}{Q_{ELEC} \cdot A_{abs} + Q_{\text{WORK}} \cdot A_{abs}}
\]

\[
= \frac{(\text{COST CON}) + \frac{\text(COST CELL)}{C_{TH}}}{Q_{ELEC} + Q_{\text{WORK}}} \frac{C_{TH}}{C_{TH}}
\]

\[
P_2 = \frac{\text{CAPITAL COST}}{\text{TOTAL POWER}} = \frac{(\text{COST CON}) + \frac{\text(COST CELL)}{C_{TH}}}{Q_{ELEC} + Q_{\text{THERMAL}}} \frac{C_{TH}}{C_{TH}}
\]

\[
P_3 = \frac{\text{CAPITAL COST}}{\text{ELEC POWER}} = \frac{(\text{COST CON}) + \frac{\text(COST CELL)}{C_{TH}}}{Q_{ELEC}} \frac{C_{TH}}{C_{TH}}
\]

The cell costs are taken as $35,000/m^2$ for 15% efficient GaAs solar cells and $1000/m^2$ for 10% efficient Si cells. Concentrator costs are assumed to have a constant value of $156/m^2$. This high cost of the concentrator includes two-dimensional tracking and structural supports for wind loads and high concentrations. It is a very conservative value when one compares average concentrator costs of $34/m^2$ from references 12, 13, and 14. Systems III and IV are summarized in figure 10.

Analysis and Assumptions for System V

System V is illustrated in figure 7 and consists of a concentrator system for thermal energy and a separate flat plate system for electrical energy (modeled the same as in system I).

Using the same assumptions and notations defined previously, the various energy terms associated with system V are:
\[ Q_{IN} = \eta_1 I_N C_{TH} \]  
(15)

\[ Q_{ELEC} = \eta_3 (0.84) I_N \quad \eta_3 = \text{solar cell efficiency} \]  
(16)

\[ Q_{LOSSES} = 2h_1 (T_{CK} - T_{AIRK}) + 2\varepsilon \sigma (T_{CK}^4 - T_{AIRK}^4) \]  
(17)

\[ Q_{WORK} = 0.5 \left(1 - \frac{T_{FK}}{T_{CK}}\right) Q_{THERMAL} \]  
(18)

\[ Q_{THERMAL} = \frac{mC_P}{A_{abs}} (T_{CK} - T_{FK}) = K(T_{CK} - T_{FK}) \]  
(19)

It is assumed that the silicon solar cells will operate at maximum efficiency and that the energy balance on the concentrator system is:

\[ Q_{THERMAL} = Q_{IN} - Q_{LOSSES} \]  
(20)

Again, the following performance indices are defined:

\[ P_1 = \frac{\text{CAPITAL COST}}{\text{PEAK (ELEC & MECH) POWER}} \]

\[ = \frac{(\text{COST CON}) A_{ap} + (\text{COST CELL} + \text{COST FP}) A_{ap}}{Q_{ELEC} \cdot A_{ap} + Q_{WORK} \cdot A_{abs}} \]

\[ = \frac{(\text{COST CON}) + (\text{COST CELL} + \text{COST FP})}{Q_{ELEC} + \frac{Q_{WORK}}{C_{TH}}} \]
\[
P_2 = \frac{\text{CAPITAL COST}}{\text{TOTAL POWER}} = \frac{(\text{COST CON}) + (\text{COST CELL} + \text{COST FP})}{Q_{\text{ELEC}}} + \frac{Q_{\text{THERMAL}}}{C_{\text{TH}}}
\]

(21) concl'd

\[
P_3 = \frac{\text{CAPITAL COST}}{\text{ELEC POWER}}
\]

\[
= \frac{(\text{COST CON}) + (\text{COST CELL} + \text{COST FP})}{Q_{\text{ELEC}}} = \text{CONSTANT}
\]

A summary of the equations describing system V are illustrated in figure 11.

Systems I, II, III, IV, and V were compared for a solar flux density of \( I_N = 500 \text{ w/m}^2 \) at various concentrations and flow rates. The computer program used is presented in Appendix A.

The results of the computer program in Appendix A are illustrated in figures 12, 13, and 14. These results utilize the nonlinear model for solar cell efficiencies as a function of intensity and temperature. Results are for a nominal intensity of 500 w/m².

In figures 12, 13, and 14 systems I and II have the highest capital cost per energy output. All three performance indices show capital cost in the neighborhood of $10,000/kw. System V has the next highest capital costs--ranging between $3000 to $9000 per kw of energy produced. The systems with the lowest costs are the concentrator systems with GaAs and Si solar cells (systems III and IV). Capital costs for these systems depend upon concentration and operating temperatures and range between $900 and $2,000 per kw of energy produced. This is slightly above capital costs (installation costs) of nuclear energy which is currently running in the neighborhood of $700/kw and energy from fossil fuels which costs $550 to $600 per kw installed. Gas turbine energy installed costs are around $135/kw. However, fuel cost for this type of energy is $2.46 per million KJ ($2.60 per million BTU) of energy produced which is expensive.
Nuclear fuel costs are only 14¢ per million KJ (15¢ per million BTU) of energy produced. The above figures are data obtained from a personal communication with C.F. Miller of the Federal Power Commission, Washington, DC 20426. In contrast solar energy has no fuel costs and depends only upon weather conditions indigenous to the area where it is to be utilized.

Figures 15 and 16 illustrate solar cell electric output as a function of concentration for various values of the flow rate parameter \( K = \frac{\dot{m}C}{A_{\text{abs}}} \). These curves illustrate that at a fixed flow rate the electric power output increases with concentration up to a point where the temperature of the cell can no longer be maintained at a low value by the cooling fluid. For the higher temperatures at the higher concentrations the efficiency of the solar cell will begin to rapidly drop off and the electric output will go to zero. The maximum power points for the GaAs and Si solar cells occur approximately at temperatures of 245° C and 160° C respectively for an incident flux density of 500 w/m² assuming a 70% optical efficiency of the concentrator.

The triangular graphs of figures 17 and 18 illustrate the percent energy distribution for the concentrator systems III and IV. In these figures

\[ Q_{\text{LT}} = Q_{\text{THERMAL}} - Q_{\text{WORK}} \]

where \( Q_{\text{LT}} \) represents the low quality thermal energy remaining after fluid has undergone a Rankine cycle to extract useful work from the high temperature fluid.

At each point of the triangular graphs the sum of the ordinates will add to 100. The ordinates increase from a to A for \( Q_{\text{ELEC}} \), b to B for \( Q_{\text{WORK}} \), and from c to C for \( Q_{\text{LT}} \).

At low concentrations there is mostly electric energy and low quality thermal energy produced by the concentrator systems. As the concentration increases, the temperature rises and the useful work that can be obtained from the fluid by a turbine
will increase; however, as the temperature increases, the electric output is driven to zero.

Figures 19 and 20 illustrate temperature for various concentrations of systems III and IV.

4. ANNUAL PERFORMANCE MODELS

The analysis of systems I, II, III, IV, and V has indicated that the concentrator systems III and IV have the lowest capital cost. For this reason the following sections will investigate the annual energy production of systems III and IV.

For this purpose, average yearly insolation data from the southwest United States has been selected (ref. 15). These average values are illustrated in figure 21 and represent data for the simulation of two-dimensional tracking concentrators.

Let $I_n(n)$ denote the average intensity for the nth month. Then the energy balance (13) becomes

$$ F = (1 - \eta_3) \eta_1 I_n(n) C_{TH} - 2h_1 (T_{CK} - T_{AIRK}) $$

$$ - 2\varepsilon \sigma (T_{CK}^4 - T_{AIRK}^4) - K(T_{CK} - T_{FK}) = 0 $$

(23)

If one assumes that the solar efficiency $\eta_3$ is a linear function of temperature then (23) can be written as

$$ F = F(T_{CK}, I_n(n), C_{TH}, T_{FK}, \eta_1, h_1, \varepsilon) = 0 $$

(24)

For a fixed concentration $C_{TH}$ one can solve (24) by iteration to determine the temperature $T_{CK}$.

If instead the nonlinear model for efficiency (2) is used, then (23) can be written as
\[ F = F(T_{CK}, I_n(n), \eta_3(E, T_{CK}), C_{TH}, T_{FK}, \eta_1, h_1, \varepsilon) = 0 \]  

where \( E = \eta_1 I_n(n)C_{TH} \). Again, for a fixed concentration \( C_{TH} \) one can solve (25) by iterative techniques and determine the temperature \( T_{CK} \).

For either the linear or nonlinear model for efficiency, \( \eta_3 \), corresponding to each month \( n \), we have

\[ Q_{ELEC}(n) = \eta_1 I_n(n)C_{TH}\eta_3(24)\text{DAYM}(n) \]  

\[ Q_{THERMAL}(n) = K(T_{CK} - T_{FK}) \]  

\[ Q_{WORK}(n) = \left[ 0.5 \left( 1 - \frac{T_{FK}}{T_{CK}} \right) Q_{THERMAL}(n) \right]^{(24)\text{DAYM}(n)} + Q_{ELEC}(n) \]  

where \( \text{DAYM}(n) \) represents the number of days in the \( n \)th month. Then the annual output from the concentrator system is

\[ \text{ANNUAL ELEC} = \left( \sum_{n=1}^{12} Q_{ELEC}(n) \right) \cdot A_{abs} \]

\[ \text{ANNUAL WORK} = \left( \sum_{n=1}^{12} Q_{WORK}(n) \right) \cdot A_{abs} \]

the capital cost of the concentrator system is given by

\[ \text{CAPITAL COST} = (\text{COST CON})A_{ap} + (\text{COST CELL})A_{abs} \]
where \((\text{COST CON})\) represents the concentrator costs with two-dimensional tracking. The concentrator cost is modeled in two different ways. The first representation is

\[
(\text{COST CON}) = \text{CONSTANT} = \$156/\text{m}^2
\]  

(31)

and the second model is

\[
(\text{COST CON}) = \frac{122}{1000} \ C_{\text{TH}} + 34 \ \$/\text{m}^2
\]  

(32)

which is a linear function of concentration.

The capital costs are amortized over a twenty-year period at an 8\% interest with an assumed maintenance cost of 2\% per year. The annual cost can thus be represented as:

\[
\text{ANNUAL COST} = \left[ \frac{i}{1 - [1 + i]^{-n_1}} + m \right] (\text{CAPITAL COST})
\]  

(33)

where \(n_1 = 20\), \(i = .08\), and \(m = .02\).

Two additional annual performance indices are defined. These are

\[
\begin{align*}
P_4 &= \frac{\text{ANNUAL COST}}{\text{ANNUAL ELEC}} \\
P_5 &= \frac{\text{ANNUAL COST}}{\text{ANNUAL WORK}}
\end{align*}
\]  

(34)

where \(\text{ANNUAL ELEC}\) is obtained from (29), \(\text{ANNUAL WORK}\) obtained from (29) and \(\text{ANNUAL COST}\) is obtained from (33). The equations for annual comparison of systems III and IV are summarized in figure 21.

The computer program for the comparison of systems III and IV, for annual performance, is given in Appendix B. Graphical results
are illustrated in figures 22 through 27. For these figures silicon costs were $1000/m² at 10% efficiency and gallium arsenide costs were $35,000/m² at 15% efficiency.

Figure 22 illustrates performance index $P_4$ vs. concentration for various values of the flow rate parameter $K$. Observe that there is a minimum value of ANNUAL COST/ANNUAL ELEC for each value of the flow rate parameter. Figure 23 illustrates performance index $P_4$ vs. performance index $P_5$ for various values of the flow rate parameter $K$. In this figure, note that there is a distinct minimum value of ANNUAL COST/ANNUAL ELEC for various $K$ values. As the concentration is further increased the value of $P_5$ decreases but under the penalty of increasing electric costs. Stated differently, the increased concentration raises average cell temperature and reduces electric output.

The performance index $P_5$ is not a representative performance index of true costs as the additional cost of converting a high temperature fluid to usable work has not been added to the capital costs. Approximate additional cost for converting this energy is $333/kw for a high temperature fluid. [One possible model for this additional cost would be $333/(T_{CK} - T_{FK})$ where $T_{CK}$ is temperature of cells and $T_{FK}$ is fluid temperature leaving heat engine.]

Figures 24 and 25 depict annual energy outputs from the concentrator systems III and IV. Figures 26 and 27 illustrate solar cell cost vs. minimum value of $P_4$ for various flow rate parameter values of $K$. Listed alongside these curves are approximate concentrations where minimum values are obtained.

5. DESIGN CONSIDERATIONS

A hybrid system used in conjunction with conventional heating and air conditioning equipment must be designed to carry a certain percentage of the daily load requirement. The following are calculations to give a first estimate for cost and sizing of a hybrid system to supply energy to a residential house.
The following assumptions will be made:

1. Heating requirements: \( = 69.9 \times 10^3 \) kwh/yr

\[ = (25 \text{ BTU/hr ft}^2)(265 \text{ days/year})(1500 \text{ ft}^2) \]

2. Air conditioning requirements: \( = 26.4 \times 10^3 \) kwh/yr

\[ = (25 \text{ BTU/hr ft}^2)(100 \text{ days/year})(1500 \text{ ft}^2) \]

3. Electrical power demand: \( = 8 \times 10^3 \) kwh/yr

4. Area of house: \( = 139 \text{ m}^2 = (1500 \text{ ft}^2) \)

5. Cost of GaAs: \( = $35000/\text{m}^2 \)

At a concentration ratio of 400, one can assume the following energy values per square meter of aperture area (values taken from fig. 25)

\[ \text{ANNUAL } Q_{\text{ELEC}} = 300 \text{ kwh/m}^2 \text{ yr} \]

\[ \text{ANNUAL } Q_{\text{THERMAL}} = 1700 \text{ kwh/m}^2 \text{ yr} \]

Assume total thermal load is \( 69.9 \times 10^3 \) kwh/yr and total electric load is \( (26.4 + 8) \times 10^3 = 34.4 \times 10^3 \) kwh/yr.

Let \( A_{\text{ap}} \) denote aperture area and \( A_{\text{ab}} \) denote absorber area (photocell area). Then to meet the thermal demand one would require

\[ 1700 \text{ kwh/m}^2 \text{ yr} \times A_{\text{ap}} = 69.9 \times 10^3 \text{ kwh/yr} \]

or

\[ A_{\text{ap}} = 41.12 \text{ m}^2 = (442 \text{ ft}^2) \]
is the aperture area required to meet this demand. This aperture area would supply the following electrical power

\[ Q_{ELEC} = 300 \text{ kwh/m}^2 \text{ yr} \cdot 41.12 \text{ m}^2 = 12.34 \text{ kwh/yr} \]

which is 35.8% of electrical energy requirements. The required solar cell (absorber) area necessary is

\[ A_{ab} = \frac{A_{ap}}{400} = 0.1028 \text{ m}^2 = 1.106 \text{ ft}^2 \]

The approximate cost of such a system can be divided as in table 1.

The cost of such a system depends upon concentrator costs which could be major costs of the system. There is currently no reliable data on concentrator costs and if one takes the cost figures as representing lower and upper bounds for system cost, an average cost would be $8738.47.

Of course this is only a rough estimate of system cost. There will be economic variations with respect to geographic location and weather conditions. It is envisioned that such systems will be in widespread use in commercial and industrial applications within the next 25 years.

6. DISCUSSION OF MODELS, ASSUMPTIONS, AND RESULTS

The models constructed in this report are for steady-state operating conditions of the systems under investigation. The models represent average operating conditions of the various systems and do not consider transient responses.

The concentrating devices are assumed to give uniform solar cell illumination and \( K = \frac{mC_P}{A_{abs}} \) represents an effective uniform
thermal conductance per unit area of absorber. Values of $K$ between .01 and .1 represent passive cooling such as wind cooling. In comparison, values of $K$ between .1 and 10 represent active cooling with high mass flow rates. In the report $K$ is the heat removed from the absorber area per unit of absorber (cell) area per unit temperature difference between coolant inlet and outlet temperatures. Via a secondary loop this heat is available to drive a vapor engine at an assumed 50% Carnot efficiency operating between some temperature limits. Heat exchanger losses are ignored.

There is a wide spectrum of costs in the solar field for flat plate and concentrator collectors. There is also a wide spectrum of costs for solar cell devices utilizing silicon and gallium arsenide. The cost values used in this study were chosen as conservative values under the assumption that the costs of $1000/m^2$ for 10% efficient Si and $35000/m^2$ for 15% efficient GaAs solar cells together with concentrator costs are greater than "other" subsystem costs. This assumption is valid for concentrator costs between $30/m^2$ and $150/m^2$ and concentrations ranging between 230 and 1200 for GaAs systems and concentrations between 6 and 30 for Si systems.

There is not a great deal of information in the literature on concentrator costs for full 2-D sun tracking concentrator devices and so throughout most of this study the concentrator costs were assumed constant ($156/m^2$) (refs. 12, 13, 14, 16, 17, 18, 22, 23, 24, 25). In figure 26, the concentrator cost was assumed to vary linearly with concentration $(34.0 + 0.122 C_{TH})$/m$^2$.

The annual energy produced by the concentrator systems was divided into electrical energy, high quality thermal energy, and low quality heat. The high quality thermal energy was that energy that can be extracted from a high temperature fluid via a vapor engine with an assumed 50% Carnot efficiency. The annual values of electrical energy and high quality thermal energy ($Q_{\text{work}}$) are illustrated in figure 25. The cost models do not consider the additional cost of converting the high quality energy to useful work. A more detailed cost analysis would be necessary for such a comparison.
There are some differences in the literature as to how solar cells behave under high concentrations of solar flux energy. In this report it was assumed that solar cell efficiencies decreased with increased illumination. Some investigators report that the opposite is true—that is, solar cell efficiencies increase with increased illumination (refs. 4, 5, 6). The solar cell design will determine the actual behavior of efficiency. If cell efficiencies increase with increased illumination, then the results of this study can be taken to represent very conservative estimates as to how the various systems perform and the cost estimates given in the figures of results must be lowered. Another area where no data is available is lifetime performance of solar cells under high flux densities.

Concentrators with high concentration ratios 100 to 10,000 have been developed for the space program (ref. 19). The technology exists for constructing high concentration systems but little data is available on costs for such concentrators.

**7. COST PROJECTIONS FOR GaAs SOLAR CELLS FOR TERRESTRIAL APPLICATIONS**

The hybrid system performance and economic analysis has been undertaken with uncertain component cost information, but no apologies are necessary. The intent was to find the limits of performance and to establish comparisons between GaAs and Si solar cell hybrid systems. It was realized from the outset that solar cell costs—especially for \( \text{Ga}_{1-x}\text{Al}_x\text{As-GaAs} \) cells—would be pivotal, and system performance results are reported over parameter domains where this assumption remains valid. The cells of interest, grown by the liquid-phase epitaxy process, are currently "hand-made" for experimental purposes; though requiring little material per wafer, their cost is understandably high because of the labor (skilled labor) intensiveness of the processing. Cost reductions must ultimately be predicated on market development for photovoltaic devices and mass production techniques.

Even while costs remain high for the GaAs heterojunction cells, there are compelling advantages relative to Si cells which motivate
continued basic research and stimulate system feasibility studies. Among other researchers, Stevenson in the proposal report, D.A. Stevenson, "Thin Film Gallium Arsenide for Low Cost Photovoltaic Solar Energy Conversion," Report No. CMR-P-73-17, Center for Materials Research, Stanford University, Stanford, CA, 1973 (not readily available), has pointed out these advantages:

1. The bandgap is a better match to the solar spectrum, therefore better efficiencies can be obtained.

2. The bandgap is direct, therefore considerably thinner cells can be used to absorb the solar energy.

3. The bandgap is greater, therefore (a) the power output with increasing temperature is greater, and (b) the output voltage is greater.

4. The minority carrier lifetimes and diffusion lengths are less....

Stevenson also argues convincingly concerning the natural abundances of the prime materials, gallium and arsenic. Materials are available for large quantities of thin (5 to 100 μm) GaAs cells; there is needed only the stimulus to reduce the labor-intensive current manufacturing processes.

It will be the intent of the remaining portion of this section to describe the current cost basis used in this report for Si cells and for GaAs heterojunction cells. Also a scenario is outlined depicting a plausible cost reduction projection for GaAs cells based on an expanding market for photovoltaic electric power generation.

In 1975, a few materials laboratories would quote a price for small quantities of experimental heterojunction GaAs cells, and the number was in the neighborhood of $800,000/m², a staggering figure to face for the photovoltaic/thermal power system designer. Prominent researchers in the field were pragmatically uninterested in predicting where current costs might go should a market develop. A number of people in the solid-state electronic device industry were asked if they would be willing to extrapolate their experience with LED and solid-state laser developments over to GaAs
solar cell production, and estimate a de-escalating cost curve with growing demand--most were not willing to do this, including H. Kressel of RCA Laboratories and M.B. Panish of Bell Laboratories (ref. 30). The reason given was basic dissimilarity of the devices. However, it was learned that two different producers of LED devices were employing epitaxial structures. They were currently growing an epitaxial layer on a GaAs substrate, dicing-up the wafers, and providing electrical contacts at a cost of about $3.50/cm² ($23/in.²). This yields a "current" cost estimate for GaAs cells equivalent to $35,000/m² assuming the technology extrapolates to solar cell production in large numbers. Although it has no impact on "current" cell costs, the continuous cell growth processes which are being studied (ref. 26), must inevitably contribute to diminishing GaAs solar cell costs.

Solar cell array manufacturing costs (exclusive of substrate and encapsulation costs) for "current" Si cells for terrestrial application are variously quoted in recent literature: $2000/m² (refs. 27 and 15); $1250/m² (ref. 28). For the sake of this study, however, a value of $1000/m² is used (personal communication with Gilbert H. Walker of NASA Langley Research Center, Hampton, VA 23665), based on the knowledge that Solarex Corporation of Rockville, MD is producing commercially small Si cell arrays at a cost of $10/peak watt. For a 10% efficient cell this is equivalent to the $1000/m² cost value which is used in the analysis.

A scenario is outlined here demonstrating how GaAs cell costs might ultimately reach a competitive level; the scenario is based on the work of Wolf (ref. 28) and Baum (ref. 29) combining an electric power market penetration projection with the past cost history of Si solar cells. Wolf sees the rising costs of conventional sources of power and growing public awareness as setting the stage for rapid power generation market penetration by photovoltaic systems. Significant prototype application is envisioned to commence around 1980 providing electric (and thermal) power for buildings, residential and commercial. From figure 3 in the paper by Wolf (ref. 28), table 2 is constructed showing anticipated annual electric energy from photovoltaic units on buildings. These
values are converted to average kilowatts for a year and then to "peak" kilowatts or installed capacity by assuming a plant capacity factor of 50%.

Wolf has argued the plausibility of this level of penetration over the next 50 to 60 years. Although the numbers appear large they represent a small percentage of projected United States total energy consumption in the given years. For later reference attention is called to the value for installed or "peak" kilowatts for the year 1990, a point in time representing the starting date for rapid commercial addition of photovoltaic equipment.

Although GaAs solar cell fabricators have been reluctant to predict future cell costs, VARIAN of California has released cell fabrication "experience" factors which gauge costs of epitaxially grown cells as a function of peak photovoltaic kilowatts installed; this is based on their work with GaAs cells under sunlight concentration (unpublished data from R.L. Bell VARIAN Co., Palo Alto, CA). Based on the VARIAN projections a market growth to $10^6$ kw peak installed will be required to achieve a 100-fold decrease in current GaAs cell costs. This kilowatt level coincides with the year 1990 according to the projections of Wolf.

Historically the unit cost of production falls off exponentially with rapid market penetration (refs. 27 and 29). If the Si solar cell costs of Baum (ref. 29) are plotted along with the "current" cost figure of $10/watt (the Solarex Corporation cost derived assuming a 10% cell and a one-sun value of 1 kw/m²), a semi-logarithmic slope of -.23 yr⁻¹ is found. This curve is shown in figure 28. For GaAs heterojunction cells the "current" cost is $35,000/m². If the number is modified assuming a 15% efficient cell and a one-sun value of 1 kw/m², a dimensional conversion yields a current cost of $233/watt, compatible with the data for Si solar cells. Extrapolating from this cost using the slope of -.23 yr⁻¹, a value is derived in figure 28 for the point in time where the GaAs cell cost will have dropped by a factor of 100 as suggested by VARIAN. Fortuitously, this date is 1995, coinciding approximately with the date of significant market penetration (1990) suggested by Wolf and being consistent with the level of installed
kilowatts necessary to stimulate cell production as indicated by VARIAN.

In terminating this section of the report the following conclusions are offered:

1. With significant prototype testing commencing by 1980, installed (peak) photovoltaic capacity could reach $10^6$ kw by 1990 (Wolf, ref. 28).

2. A 100-fold cost reduction for GaAs cells is feasible as the market achieves $10^6$ kw installed (VARIAN).

3. If silicon cell cost reductions observed from 1958 to 1975 are used as a gauge and the same semi-log slope (-.23 yr$^{-1}$) is applied to "current" GaAs cell costs, a 100-fold reduction is predicted to occur by 1995 concurrent with Wolf's market projections to $10^6$ kw installed.

4. Convergence of Si and GaAs cell costs are indeterminate at this time looking ahead 20 to 40 years. Convergence may be possible, however, because GaAs cells (a) require less material per unit cell area and (b) can display higher efficiencies relative to Si cells, especially under the desirable condition of sunlight concentration.

8. CONCLUSIONS

Current costs and efficiencies for GaAs and Si solar cells are: $35,000/m^2$ for 15% efficient GaAs cells and $1000/m^2$ for 10% efficient Si cells, both efficiencies at AM1.

Limiting values for annual energy costs from GaAs and Si concentrator systems have the following range of values: a GaAs concentrator system ranges between 2¢ and 6.8¢ per kwh and a Si concentrator system ranges between 2.5¢ and 11¢ per kwh. The ranges in annual energy costs reflect the different assumptions on concentrator costs which include full two-dimensional tracking.

For a given flow rate, there is an optimal operating condition for maximum photovoltaic output of both GaAs and Si hybrid systems. This can be seen by examining figures 15 and 16.
The high concentration hybrid systems offer a distinct cost advantage over flat plate hybrid systems because the concentration increases solar flux density and decreases the solar cell area.

Solar hybrid systems for the heating and cooling of buildings seem to be economically competitive with existing energy sources. Additional cost studies should be undertaken to calculate "total system costs" as this study did not figure in costs of cooling equipment, hot water storage for heating, or turbine energy conversion costs.

As the cost of solar cells decreases, optimum system performance from Si and GaAs hybrid systems can be achieved at lower concentrations. For Si cell costs of $50/m² (NSF goal) and GaAs a factor of 20 more expensive, optimal Si performance can be achieved at concentrations around 10 while optimal GaAs performance can be achieved at concentrations of around 100.

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Table 1. Approximate cost for solar photovoltaic system.

<table>
<thead>
<tr>
<th>Component</th>
<th>Cost Formula</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. GaAs solar cell cost:</td>
<td>$(35000)(.1028)$</td>
<td>$3598.00</td>
</tr>
<tr>
<td>2. Concentrator cost with tracking:</td>
<td>$(34/m^2)(41.12 m^2)$ lower bound or $(156/m^2)(41.12 m^2)$ upper bound</td>
<td>$1398.08 $(6414.72)</td>
</tr>
<tr>
<td>3. Battery storage cost:</td>
<td>$(40/kwh)(1/.65)(3 HRS)(.1028 m^2)(8 kw/m^2)$</td>
<td>$151.83</td>
</tr>
<tr>
<td>4. Conversion cost:</td>
<td>$(100/kw)(8 kw/m^2)(.1028 m^2)$</td>
<td>$82.24</td>
</tr>
<tr>
<td>5. Hot water storage costs:</td>
<td>$(.2642/litre)(3785 litres) = $(1/gal)(1000 gal)$</td>
<td>$1000.00</td>
</tr>
</tbody>
</table>

**TOTAL COST**: $6230.15 $(11,246.79)
Table 2. Solar/photovoltaic market penetration.

<table>
<thead>
<tr>
<th>Year</th>
<th>Electric Kilowatt-hours per year</th>
<th>Average Kilowatts</th>
<th>Installed Kilowatts</th>
</tr>
</thead>
<tbody>
<tr>
<td>1990</td>
<td>$5.0 \times 10^9$</td>
<td>$5.7 \times 10^5$</td>
<td>$1.1 \times 10^6$</td>
</tr>
<tr>
<td>2000</td>
<td>$2.6 \times 10^{11}$</td>
<td>$3.0 \times 10^7$</td>
<td>$6.0 \times 10^7$</td>
</tr>
<tr>
<td>2010</td>
<td>$1.0 \times 10^{12}$</td>
<td>$1.1 \times 10^8$</td>
<td>$2.2 \times 10^8$</td>
</tr>
<tr>
<td>2020</td>
<td>$1.8 \times 10^{12}$</td>
<td>$2.0 \times 10^8$</td>
<td>$4.0 \times 10^8$</td>
</tr>
<tr>
<td>2030</td>
<td>$2.8 \times 10^{12}$</td>
<td>$3.2 \times 10^8$</td>
<td>$6.4 \times 10^8$</td>
</tr>
<tr>
<td>2040</td>
<td>$3.5 \times 10^{12}$</td>
<td>$4.0 \times 10^8$</td>
<td>$8.0 \times 10^8$</td>
</tr>
</tbody>
</table>
Figure 1. A solar energy hybrid system.
Fig. 2. Optimum power $P$ as function of light flux. Cell $A-I_0 = 10^{-9} \text{A/cm}^2$, cell $B-I_0 = 10^{-8} \text{A/cm}^2$; the dashed curves are calculated.

Fig. 3. Fill factor $\varepsilon(a)$ and voltage characteristic (b) of the cells for different high light flux levels.

Figure 2. GaAs solar cell performance curves from reference 1.
Figure 2. Relationship between $U_{n,L}$ and optimal power (with index $a$) for silicon PB and temperature, $E = \text{var}$: 1) 940; 2) 5600; 3) 12000; 4) 18500; 5) 24000 W m$^{-2}$.

Figure 3. Relationship between $I_{s,c}$ for silicon PB and temperature for $E = \text{var}$ (same symbols as in Fig. 2).

Figure 5. Efficiency of silicon PB as function of temperature for $E = \text{var}$ (designations the same as in Fig. 2).

Figure 3. Si solar cell characteristics from reference 2.
Figure 3. Ga$_{1-x}$Al$_x$As-GaAs solar cell parameters as a function of temperature using a Zenon light source as a solar simulator. Device efficiencies were calibrated with the solar simulator at JPL.

Figure 4. GaAs characteristics from Progress Report cited on page 5.
Fig. 3. Short-circuit current output as a function of intensity and cell temperature

Fig. 4. Open-circuit voltage output as a function of intensity and cell temperature

Figure 5a. Si performance from reference 3.
Fig. 5. Maximum power output as a function of intensity and cell temperature

Figure 5b. Si performance from reference 3.
Figure 6. Comparison of models for Si and GaAs efficiency.
Figure 7. Solar collectors for hybrid systems.
SYSTEM I

FLAT PLATE COLLECTOR FOR THERMAL ENERGY
FLAT PLATE COLLECTOR (SILICON SOLAR CELLS) FOR ELECTRICAL ENERGY

\[ Q_{I_N} = 0.84 I_N \text{ W/m}^2 \]

\[ Q_{LOSS} = h_1(T_{CK} - T_{AIR}) + \varepsilon(T_{CK} - T_{AIR}) \text{ W/m}^2 \]

\[ Q_{ELEC} = n_3 Q_{I_N} \]

\[ Q_{THERMAL} = \frac{\dot{m}C_p}{A_{abs}}(T_C - T_F) \]

ENERGY BALANCE (SILICON PLATE)

\[ Q_{I_N} - Q_{ELEC} - Q_{LOSSES} = 0 \]

\[ (1 - n_3)(0.84)I_N - h_1(T_{CK} - T_{AIR}) - \varepsilon(T_{CK} - T_{AIR}) = 0 \]

ENERGY BALANCE FOR THERMAL ENERGY

\[ Q_{I_N} - Q_{LOSSES} - Q_{THERMAL} = 0 \]

\[ Q_{THERMAL} = 0.84 I_N - h_1(T_{CK} - T_{AIR}) - \varepsilon(T_{CK} - T_{AIR}) = \frac{\dot{m}C_p}{A_{abs}}(T_{CK} - T_{FK}) \]

COST CON = 400\(\eta_2\) + 10 $/m² \quad \eta_2 = \text{EFFICIENCY OF COLLECTOR} = \frac{Q_{THERMAL}}{Q_{I_N}}

COST FLAT PLATE PLUS SILICON - $1000. + 56.

Figure 8. Summary of equations describing system I.
ENERGY BALANCE

\[ Q_{IN} - Q_{ELEC} - Q_{LOSSES} - Q_{THERMAL} = 0 \]

\[(1 - \eta_3)(.84)I_N - h_1(T_{CK} - T_{AIRK}) - \varepsilon\sigma(T_{CK}^4 - T_{AIRK}^4) = \frac{mC_p}{A_{abs}}(T_{CK} - T_{FK})\]

PERFORMANCE INDEX \( P \)

\[
P_1 = \frac{\text{CAPITAL COST}}{\text{PEAK(ELEC + MECH)POWER}} = \frac{(\text{COST FP}_1)A_{abs} + (\text{COST FP}_2 + \text{COST CELL})A_{abs}}{Q_{ELEC}A_{abs} + .5\left(1 - \frac{T_{FK}}{T_{CK}}\right)Q_{THERMAL} \cdot A_{abs}}\]

\[
P_2 = \frac{\text{CAPITAL COST}}{\text{TOTAL POWER OUT}} = \frac{(\text{COST FP}_1)A_{abs} + (\text{COST FP}_2 + \text{COST CELL})A_{abs}}{(Q_{ELEC} + Q_{THERMAL})A_{abs}}\]

\[
P_3 = \frac{\text{CAPITAL COST}}{\text{ELEC POWER}} = \frac{(\text{COST FP}_1)A_{abs} + (\text{COST FP}_2 + \text{COST CELL})A_{abs}}{Q_{ELEC}A_{abs}}\]

Figure 9. Summary of equations describing system II.
### III - GaAs Solar Cells

### IV - Silicon Solar Cells

**Q_{LOSSES}**

**Q_{ELEC}**

**Q_{THERMAL}**

- \( A_{ap} \) = APERTURE AREA
- \( A_{abs} \) = ABSORBER AREA OF RECEIVER
- \( A_s \) = SURFACE AREA OF RECEIVER

\[
C_{TH} = \frac{A_{ap}}{A_{abs}} = \text{THEORETICAL CONCENTRATION RATIO}
\]

\[
\frac{A_s}{A_{abs}} = 2
\]

\[
Q_{IN} = \eta_1 I_n C_{TH}
\]

\[
Q_{ELEC} = \eta_3 Q_{IN}
\]

\[
Q_{LOSSES} = 2h_1(T_{CK} - T_{AIRK}) + 2\varepsilon_0(T^4_{CK} - T^4_{AIRK})
\]

\[
Q_{THERMAL} = \frac{mc_p}{A_{abs}}(T_{CK} - T_{FK})
\]

**COST CON = $100 + \eta_1 80. \$/m^2**

[INCLUDES TRACKING STRUCTURAL (WIND) SUPPORTS]

**ENERGY BALANCE**

\[
Q_{IN} - Q_{ELEC} - Q_{LOSSES} = Q_{THERMAL}
\]

**PERFORMANCE INDEX \( P \)**

\[
P_1 = \frac{\text{CAPITAL COST}}{\text{PEAK(ELEC + MECH)POWER}} = \frac{(\text{COST CON})A_{ap} + (\text{COST CELL})(A_{abs})}{Q_{ELEC} \cdot A_{abs} + .5 \left( 1 - \frac{T_{FK}}{T_{CK}} \right) Q_{THERMAL} \cdot A_{abs}}
\]

\[
= \frac{\text{COST CON} + \frac{\text{COST CELL}}{C_{TH}}}{Q_{ELEC} + .5 \left( 1 - \frac{T_{FK}}{T_{CK}} \right) Q_{THERMAL}}
\]

\[
P_2 = \frac{\text{CAPITAL COST}}{\text{TOTAL POWER}} = \frac{\text{COST CON} + \frac{\text{COST CELL}}{C_{TH}}}{Q_{ELEC} + Q_{THERMAL}}
\]

\[
P_3 = \frac{\text{CAPITAL COST}}{\text{ELEC POWER}} = \frac{\text{COST CON} + \frac{\text{COST CELL}}{C_{TH}}}{Q_{ELEC}}
\]

**Figure 10. Summary of equations describing systems III and IV.**
SYSTEM V

CONCENTRATOR FOR THERMAL ENERGY AND SEPARATE FLAT PLATE COLLECTOR FOR ELECTRIC ENERGY WITH SILICON SOLAR CELLS

\[ Q_{IN} = \eta_1 I_N C_{TH} \]
\[ Q_{ELEC} = \eta_3 (0.84) I_N \]
\[ Q_{LOSSES} = 2h_1 (T_{CK} - T_{AIRK}) + 2\varepsilon\sigma (T_{CK}^4 - T_{AIRK}^4) \]
\[ Q_{THERMAL} = \frac{\dot{m} C_p}{A_{abs}} (T_{CK} - T_{FK}) \]

ENERGY BALANCE

\[ Q_{THERMAL} = Q_{IN} - Q_{LOSSES} \]

COST CON = $100 + \eta_1 80.
COST FLAT PLATE + SILICON = $1056/m^2

PERFORMANCE INDEX \( P \)

\[ P_1 = \frac{\text{CAPITAL COST}}{\text{PEAK(ELEC + MECH)POWER}} = \frac{(\text{COST CON})_{Ap} + (\text{COST CELL} + \text{COST FP})_{Ap}}{Q_{ELEC} \cdot A_{ap} + 0.5 \left( 1 - \frac{T_{FK}}{T_{CK}} \right) Q_{THERMAL} \cdot A_{abs}} \]

\[ = \frac{\text{COST CON} + \text{COST CELL} + \text{COST FP}}{Q_{ELEC} + 0.5 \left( 1 - \frac{T_{FK}}{T_{CK}} \right) \frac{Q_{THERMAL}}{C_{TH}} \cdot C_{TH}} \]

\[ P_2 = \frac{\text{CAPITAL COST}}{\text{TOTAL POWER}} = \frac{\text{COST CON} + \text{COST CELL} + \text{COST FP}}{Q_{ELEC} + \frac{Q_{THERMAL}}{C_{TH}}} \]

\[ P_3 = \frac{\text{CAPITAL COST}}{\text{ELEC POWER}} = \frac{\text{COST CON} + \text{COST CELL} + \text{COST FP}}{Q_{ELEC}} = \text{CONSTANT} \]

Figure 11. Summary of equations describing system V.
$I_N = 500 \text{ W/m}^2$, $\eta_1 = .7$, $h_1 = 15 \text{ W/m}^2 \degree \text{C}$

Figure 12. $P_1$ vs. concentration.
Figure 13. $P_2$ vs. concentration.
Figure 14. $P_3$ vs. concentration.
Si 10% EFF, \( h_1 = 0.015 \frac{\text{KW}}{\text{m}^2 \, \text{°C}} \), \( I_N = 500 \text{ W/m}^2 \)

\( \eta_1 = 0.7, \ TF = 60. \)

Figure 15. Electric output vs. concentration for Si.
GaAs, 15% EFF, \( h_1 = 0.015 \frac{KW}{m^2 \degree C} \), \( I_N = 500 \ W/m^2 \),
\( \eta_1 = 0.7 \), \( TF = 60 \).

**Figure 16.** Electric output vs. concentration for GaAs.
Figure 17. Energy distribution for system IV.
Figure 18. Energy distribution for system III.
Figure 19. Concentration vs. temperature for system IV.
15% GaAs \quad I_N = 500 \text{ W/m}^2

Figure 20. Concentration vs. temperature for system III.
ANNUAL ENERGY PRODUCTION

\[ I_N(n) = \text{AVG. INTENSITY FOR } n\text{th MONTH} \]

\[ (1 - \eta_3)\eta_1 I_N(n)C_{TH} - 2h_1(T_{CK} - T_{AIRK}) - 2E\sigma(T^h_{CK} - T^h_{AIRK}) - K(T_{CK} - T_{FK}) = 0 \]

Calculate

\[ Q_{ELEC}(n) = \eta_1 I_N(n) C_{TH} \eta_3 (24) \text{DAYM}(n) \]

\[ Q_{COOL}(n) = K(T_{CK} - T_{FK}) \]

\[ Q_{WORK}(n) = 0.5 \left(1 - \frac{T_{FK}}{T_{CK}}\right) Q_{COOL}(n) + Q_{ELEC}(n) (24) \text{DAYM}(n) \]

\[ \text{ANNUAL ELEC} = \left(\sum_{n=1}^{12} Q_{ELEC}(n)\right) A_{abs} ; \quad \text{ANNUAL WORK} = \left(\sum_{n=1}^{12} Q_{WORK}(n)\right) A_{abs} \]

\[ \text{(CAPITAL COST)} = (\text{COST CON})(A_{ap}) + (\text{COST CELL})(A_{abs}) \]

\[ \text{ANNUAL COST} = \left[ \frac{(1 + i)^n - i}{(1 + i)^n - 1} + m \right] \text{(CAPITAL COST)} \]

Performance Index 4 \( P_4 = \frac{\text{ANNUAL COST}}{\text{ANNUAL ELEC}} \)

Performance Index 5 \( P_5 = \frac{\text{ANNUAL COST}}{\text{ANNUAL WORK}} \)

\( i = 0.08; \quad n = 20 \text{ years}; \quad m = \text{maintenance cost} = 0.02 \)

Figure 21. Summary of equations for annual output from systems III, IV.
Figure 22. Performance index $P_4$ vs. concentration for various flow rate parameters $K$. 
Figure 23. $P_4$ vs. $P_5$ for various flow rate parameters $K$. 

The graph shows the relationship between the annual cost and the flow rate parameter $K$. The costs are represented as $\frac{\text{ANNUAL COST}}{\text{ANNUAL (ELEC + WOUT)}} \text{[\$KWH]}$.
Figure 24. Annual energy output per square meter of aperture.
Figure 25. Annual energy output per square meter of aperture vs. concentration.
Figure 26. Solar cell cost vs. min $P_4$, for concentrator costs a linear function of concentration.
Figure 27. Solar cell cost vs. min $P_4$ for concentrator costs as a constant.
Figure 28. Extrapolated solar cell cost.
APPENDIX A

COMPUTER PROGRAM FOR COMPARISON OF SYSTEMS I, II, III, IV, V
SUBROUTINE EFFIC(VOCOG, ISCOS, FIFOC, VOSOS, ISCOS, FIFOS)

DIMENSION A(6)

REAL ISCOS, ISCOS, ISCOS, ISCOS

WRITE(3,101)

DO 1 I=1,6

E=A(I)

WRITE(3,101) E

IF(TEMP .GT. 200.) GO TO 3

CALL CHAR(E, TEMP, ISCOS, VOCOG, FILLS, EFFG, I, VOCOG, ISCOS, FIFOS)

CONTINUE

IF(TEMP .GT. 200.) GO TO 4

WRITE(3,12) TEMP, EFFG, ISCOS, FILLS, EFFS, VOCOS, ISCOS, FILLS

FORMAT(9(IX,F9.5))

GO TO 2

WRITE(3,12) TEMP, EFFG, ISCOS, FILLS

CONTINUE

RETURN

END

SUBPROGRAMS CALLED

CHAR

SCALARS AND ARRAYS [ ** NO EXPLICIT DEFINITION - "%" NOT REFERENCED ]

*VOCOS 1  ISCOS 2  *VOCG 3  *EFFG 4  *E 5  *VOCOG 6
*EFFS 7  *FIFOS 10  *FILLS 11  *J 12  A 13  S0000 21
. S0000 22  ISCOS 23  *FIFOC 24  ISCOS 25  *FILLC 26  *TEMP 27

TEMPERARIES

S0000 22

EFFIC [ NO ERRORS DETECTED ]
SUBROUTINE CHAR(E,T,ISC,VOC,FILL,EFF,ICODE,VOCO,ISCO,FIFO)

REAL I,L1,I1,IMP,IMP1,ISC,MI,M2,ISCO

FITG(X,Y)=(2.278*(X+45.6R.)**.112)-(-8.666E-4)*T

FV(X,Y)=.456765*X**.1480-(-2.456-3)*Y

FIS(E,T)=AO+T*(A1+T*(A2+T*(A3+T*(A4+T*A5+T))))*E/10.

AO=1.416727; A1=.108711E-2; A2=-.695706E-5

A3=.226609E-7; A4=.17109E-9; A5=.144039E-11

IF(ICODE.GT.1) GO TO 22

FILL=FIFO*FITG(E,T)

VOC=VOCO+FV(E,T)

ISC=ISCO*FI(E,T)

GO TO 21

VOC=VOCO+VFS(E,T)

ISC=ISCO*FIS(E,T)

FILL=(FITG+FIS(E,T))

CONTINUE

EFF=FILL*VOC*(ISC**(1.0E6)/E)

RETURN

END

SUBPROGRAMS CALLED

FITG

FV

FVS

FI

FIS

SCALARS AND ARRAYS [ ** NO EXPLICIT DEFINITION - "&" NOT REFERENCED ]

** A4 1       ** FILL 7       ** FIFO 2       ** EFF 3       ** T 4       ** FIFO 11 5       ** IMP 12

. F0001 6

. F0007 15

. F0008 17

** A2 20      ISCO 21      ** M1 22      ISC 22      ** A1 23      ** IMP1

. F0002 27

. F0003 30      ** FILL 32

** VOCO 24      . F0006 25

. F0004 26

** T 46

** A5 31

. F0004 34

. F0001 35

CHAR [ NO ERRORS DETECTED ]
SUBPROGRAMS CALLED

CHAR

SCALARS AND ARRAYS ( "*" NO EXPLICIT DEFINITION - "#" NOT REFERENCED )

<table>
<thead>
<tr>
<th>AREA</th>
<th>#FIFO</th>
<th>#EFF</th>
<th>#T</th>
<th>#FILL</th>
</tr>
</thead>
<tbody>
<tr>
<td>#E</td>
<td>5</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>#VOCO</td>
<td>12</td>
<td>13</td>
<td>14</td>
<td></td>
</tr>
</tbody>
</table>

TEMPORARIES

..MAX1 15

MAX1 I NO ERRORS DETECTED
ONTO

UNIT \texttt{9G8B.FOR} \hspace{.5cm} FORTRAN V.4(210) /XI \hspace{.5cm} 15-SEP-75 \hspace{.5cm} 9:05 \hspace{.5cm} PAGE 1

\begin{verbatim}
00001  SUBROUTINE ONTO(GMAX,CFPS1,IK,IN,FPS,VOCO,ISCO,FIFO,M1,TF,ETA3,
00002     2TEMP,AREA)
00003  C IF IK=1 THIS SUBROUTINE WORKS ON SYSTEM 1
00004  C IF IK=2 THIS SUBROUTINE WORKS ON SYSTEM 2
00005  REAL IF,ISCO,IMP,ISC,IL
00006  IF((I+EFF)*EN-H1**8*(T-TAIRK)=EPS*SIGMA**(1**4)-(TAIRK**4))
00007  DELTA=(IFMAX-FF)/10. ; SIGMA5,6697*E-9 \hspace{.5cm} TAIROC25.
00008  REAL
00009  EN=IN+.84
00100  TAIRO=TAIRO+273. ; TF=TF+273.
00211  IF(IK \textless GT. 1) GO TO 2
00312  CALL \texttt{RIGT}
00413  101 FORMAT(///,I30,'SYSTEM I',/,'II',,'III',,'IV',,'FLAT PLATE COLLECTOR FOR THERMAL F
00514  2ENERGY ANO.',/,'I',,'II',,'FLAT PLATE COLLECTOR (SILICON) FOR ELECTRICAL EN
00615  3ERGY',/)
00716  GO TO 1
00817  2 \texttt{RIGT(3,101)}
00918  102 FORMAT(///,I30,'SYSTEM II',/,'III',,'FLAT PLATE COLLECTOR FOR THERMAL
01019  2ENERGY I',,'II',,'HAVING SILICON (EFFIC. FNC. OF TEMPERATURE)',/)
01120  3 CONTINUE
01221  \texttt{RIGT(3,105)}
01322  105 FORMAT(T3,'T','TEMP,T9','RATE/AL,T20','OTHERK,T31','TOTAL,T42','FRAC,T48,
01423  2','FRAC,T57','P','T64','QU/AL,T76','OTOT,T86','FRAC,T93','FRAC,T101','F1',/)
01524  3T3,'C(T9,T20,'W/2,K,T20,,'W/2,T31,,'W/2,T57,,'W/2,T64,,'W/2,T76,,'W/2,
01625  42,'T86,,'ELEC,T93,,'THEM,T93,,'THEM,T101,,'S/T107,'EFF',/)
01726  5T111,'F3(G/W)',/)
01827  DO 50 I=1,10
01928  TF=TF+DELT+273.
02029  TC=TC-273.
02130  IF(IK \textless GT. 1) GO TO 4
02231  IF(ETA3+NE. 0.) GO TO 77
02332  CALL \texttt{MAXI}\{IN,EN,MAX,TMP,AREA,VOCO,ISCO,FIFO\}
02433  ETA3=ENMAX ; EFF=EFMAX
02534  77 EFF=EFF+2*(TAIRK**2)
02635  AAA=ANH4(TK**2)-(TAIRK**2)
02736  QJ=EN-H1*(TK-TAIRK)-EPS*SIGMA***AAA
02837  QELE=ETAM+10.
02938  GO TO 50
03039  CALL CHANG\{EN,TC,ISC,VOC,FF,E2,VOCO,ISCO,FIFO\}
03140  EFF=EFF/100.
03241  QJ=QJ**K
03342  QELE=EFF**EN
03443  EFF=EFF**100.
03544  5 IF(QJ \textless GT. 0.) GO TO 50
03645  QTOT=QJ+QJ
03746  PE=QJ+QJ + PT=QJ+QJ
03847  RATE=QJ/(TK-TFK)
03948  ETA2=QJ+QJ/EN
04049  EFF=TFK,*,ETA2 \hspace{.5cm} +10.
04150  2\{HERE\_**5,9(1-\(TFK/TK**)*QJ\}
04251  QJ=QJ+QJ**2+QJ**2
04352  PQF=QJ+QJ**2 + PT=PT**2+PT]
04453  F2=(CPK+CFPS1)/QJOT
04554  F2=(CPK+CFPS1)/QJOT
04655  PE=PE+100. ; PT=PT+100.
04756  PE=PE+100. ; PT=PT+100.
\end{verbatim}
SUBPROGRAMS CALLED
CHARGE
 MAXSI  

SCALARS AND ARRAYS: ( "#" NO EXPLICIT DEFINITION - "%" NOT REFERENCED )

*CFPSI  1 *AREA  2  $IL  *FIFO  3  *EN  4  *EFF  5  *PE  6  *FIFOT  7  *F  11  *H  12  *QUAL  1  *TK  14  *PT  15  *ILMP  *TFK  16  *DELT  17  *TF  20  *EPMAX  21  *PTH  22  *FOTOT  23  *TAIPC  24  *ISCO  25  *ETA2  26  *TK  27  *OTHERM  38  *F3  31  *CFF  32  *DELEC  3  *ISC  14  *MORR  35  *F2  36  *ARR  37  *VOCO  40  *IN  41  *ETA3N  42  *TEMP  43  *RATE  44  *TMAX  45  *ASS  46  *N  47  *TF  50  *TC  51  *PE  52  *TAIRK  53  *EPS  54  *AAA  55  *SIGMA  56

TEMPORARIES

ONTWO  325

ONTWO  [ NO ERRORS DETECTED ]
SUBPROGRAMS CALLED

COMMON G

SCALARS 24) ARRAYS ("** NO EXPLICIT DEFINITION - "+ NOT REFERENCED

*CPSP 1  BANCA  345 .......  H90  2 ...  E9F  3  P9E  4
*QOTF 5  *YMON  6  *FF  7  +H1  10  *PL  11  E  12
*QUAL 13  *X  14  *RT  15  B1MP  16  *FRK  17  *DELT  18
*F 23  *P9H8  21  *UT9AL  22  *T9RCE  23  LSC9  24  *TK  25
*Q9ER 2  *F3  27  *J  30  *Q9E  31  LSC  32  .S999  33
*9T 11  .BAA  45  *F2  36  *VOCO  37  IN  40  *C2  41
*RATE 42  *CC04  43  *TRAX  44  *VOC  45  *CTH  46  *I  47
*F  50  *FC  51  *PE  52  *T9R  53  *OD  54  *EPS  55
*Q2 56  *SIGA  57  *CG  60

TEMPORARIES

.THR 16  42

THRPD (4) EXElONS DETECTED}
SUBROUTINE FIVE(TMAX,TF,IX,CPSI,1N,FP,S,VOCO,ISO,FIFO,ETA,TAI)

THIS SUBROUTINE WORKS ON SYSTEM 5

REAL 1N,ISO,TF,1P,1L

G(T)=E-2(H1*(T-TAI))-2*EPS*SIGMA*(T*T4-(T-TAI)**4)

AITE(1,1)

FORMAT(7),(3,1H30,SYSTEM V/,1H14,'CONCENTRATION FOR THERMAL ENERGY')


TAI*K=TAIR*K+273.

IF(TF>273).

DEL=1.*(MAX-TF).15.

0=0.3. : CTH=2.4.

Q=5.8.J=1.8.

CTH=CTH+U0.

FJ=(J-0.4). CTH=CTH+28.

CCO=G*ISO+ETA*180.

CF=CFPSI

IF(ETA>1.G.9.) GO TO 3

CALL MAXSI(1N,EMAX,TEMP,AREA,VOCO,ISO,FIFO)

ETA*EMAX,J=1

IF(3*ETA**2>1.800).

J=ETA**3**3.*1.44**1.1

FORMAT(7),102,'CONCENTRATION = ',F10.2,1X,'FURNACE FACTOR = ',F10.2,1X,'COST OF SILICON = ',F10.2,1X,'COST OF CONCENTRATION = ',F10.2,1X,'COST OF CONDUCTIVITY = ',F10.2,1X,'MAX ECP OF SILICON = ',F10.2,1X,'SILIC = ',F10.2,1X

AITE(3,144)

AITE(3,102) CTH,ETA,CON,CFF,ETA,W,OLFC

AITE(104)

RETURN

END
SUBPROGRAMS CALLED

G

MAXSI

SCALARS AND ARRAYS ( ** NO EXPLICIT DEFINITION - "*" NOT REFERENCED )

*EPIS 1  *AREA 2  *AI 3  *EF 4  *POE 5
*QDOT 6  *P00007  *HI 10  *PT 11  *E 17  *JUAL 13
*QNM 5  *PP 14  *NMP 15  *TFK 15  *DELT 16  *TP 17
*EEMAX 29  *PTH 21  *DOTAL 22  *PAR 23  *ISCO 24  *TK 25
*QIMAX 26  *F3 27  *J 30  *CFP 31  *PSC 32  *ISC 35
*Q0001 33  *ETA 34  *S0000 35  *F2 36  *VOCO 37  *TN 40
*ETAEM 31  *TEMP 42  *RATE 43  *CCON 44  *TMAX 45  *CTH 46
*SIG 47  *F 50  *FC 51  *PF 52  *TIA 53  *B 54
*EPS 55  *Q2 56  *SIGMA 57

TEMPORARIES

*FIV16_302

FIVE 1 NO ERRORS DETECTED 1
SUBR VOS 2/8/75

FIFI

EFFIC 1177, 1178

SCALARS 440 2/8/75 ("*" = EXPLICIT DEFINITION - "*" NOT REFERENCED)

#VOCOS 1 #AREA 7 ISCOG 3 #FIFO 4 #LIL 5 #JJ 6
#EFF 9 #T 7 #FF 12 #H 11 #TIME 12 #E 13
#TK 14 #TP 11 #L 14 #TP 22 #FIFO 23 #V 24 #LSC 25
#KK 26 #LSC 27 #TCP 45 #BAG 31 #MM 42 #ISCO
#FIFO 31 #V 14 #AM 16 #L 14 #TP 36 #LSC 37 #ETA 40
#COS 31 #T 12 #LSC 43 #VOC 44 #CURR 44
#LSC 31 #14 15
APPENDIX B

COMPUTER PROGRAM FOR ANNUAL ENERGY PRODUCTION
OF SYSTEMS III, IV
SUBROUTINE BLOCK1

DIMENSION XK(8)

COMMON EFF, CON,CELL1,CELL2,N,XI,XM,ETA1,XX,TF

WRITE(3,233)

233 FORMAT(*'GA=AS SOLAR CELLS'//)

WRITE(3,200) EFF,CON,CELL1,CELL2,ETA1,TF

200 FORMAT(*'EFFICIENCY = ',F8.3,'X',CONC COST = ',F10.2,'//

21X,'COST GA=AS = ',F10.3,'X',COST SILICON = ',F12.3,'//

31X,'ETA1 = ',F10.2,'X',FLUID TEMP = ',F10.2,'//

CALL PERFOR(1,EFF,CON,CELL1,CELL2,N,XI,XM,ETA1,XX,TF)

RETURN

END

COMMON BLOCKS

/,COMM/(+21)

EFF +0 CON +1 CELL1 +2 CELL2 +3 N +4
XI +5 XM +6 ETA1 +7 XX +8 TF +10

SUBPROGRAMS CALLED

PERFOR

SCALARS AND ARRAYS (*# NO EXPLICIT DEFINITION = "#" NOT REFERENCED )

TEMPORARIES

.BLOC 46

BLOCK1 [ NO ERRORS DETECTED ]
SUBROUTINE BLOCK2
DIMENSION XR(2)
COMMON EFF,CON,CELL1,CELL2,N,XI,XM,ETA1,XX,TF
WRITE(3,234)
234 FORMAT('1*1X,'SILICON SOLAR CELL8.',/)
WRITE(2,236) EFF,CON,CELL1,CELL2,ETA1,TF
236 FORMAT('1X,'EFFICIENCY = ',F6.2,2X,'CONC. COST = ',F10.2,/
           1X,'COST GALASH = ',F12.2,2X,'COST SILICON = ',F12.2,/
           1X,'ETA1 = ',F10.2,2X,'FLUID TEMP = ',F10.2,/
CALL PERFOR(2,EFF,CON,CELL1,CELL2,N,XI,XM,ETA1,XX,TF)
RETURN
END

COMMON BLOCK8

/COMMON,/(+21)

EFF    +8   CON   +1   CELL1  +2   CELL2  +3   N    +4
XI     +5   XM    +6   ETA1   +7   XX    +10  TF   +20

SUBPROGRAMS CALLED

PERFOR

SCALARS AND ARRAYS ("*" NO EXPLICIT DEFINITION = "#" NOT REFERENCED)

TEMPERATURES

.BLOCK 46

BLOCK2 (NO ERRORS DETECTED.)
SUBROUTINE PERFORM (ICODE, EFF, CON, CELL1, CELL2, X, XI, XM, ETA, XK, TF)
DIMENSION X(X), DAYM(12), XIN(12), TAIR(12), GE(12), GUSE(12), QWORR(12)
2
F(TD, X, Z) = ETA*1 + X*ETA + X*CTH + ETA + X*CTH + 2.*X
2 M1 = (C(T) + X)*2.*EPS*SIGMA + (XX(T) + X)*EPS*SIGMA + 2.*X
FP(TC, X, Y, Z) = ETA*BETA*ETA + X*EPS*SIGMA + 2.*EPS*SIGMA + 2.*X
2 X + 2.*X + 2.*X
DAY(1) = 31.
DAY(2) = 30.
DAY(3) = 31.
DAY(4) = 30.
DAY(5) = 31.
DAY(6) = 30.
DAY(7) = 31.
DAY(8) = 31.
DAY(9) = 30.
DAY(10) = 31.
DAY(11) = 30.
DAY(12) = 31.
TAIR(1) = 10.
TAIR(2) = 19.
TAIR(3) = 20.
TAIR(4) = 20.
TAIR(5) = 20.
TAIR(6) = 32.
TAIR(7) = 32.
TAIR(8) = 32.
TAIR(9) = 20.
TAIR(10) = 19.
TAIR(11) = 20.
TAIR(12) = 10.
XIN(1) = 7.6.
XIN(2) = 8.0.
XIN(3) = 8.3.
XIN(4) = 9.8.
XIN(5) = 9.8.
XIN(6) = 9.7.
XIN(7) = 9.5.
XIN(8) = 9.0.
XIN(9) = 8.0.
XIN(10) = 7.3.
XIN(11) = 7.2.
DO 1 J = 1, 12
1 XIN(J) = XIN(J)/24.
WRITE(1, 999)
999 FORMAT(3(J1, 99))
DO 2 K = 1, 8
2 WRITE(3, 100)
100 FORMAT(10X, "(DOT/CP/AREA ABSORBER) =", T10, 48)
WRITE(3, 100)
DO 3 K = 1, 8
3 TCK = TC + 273.
4 TFK = TF + 273.
5 TRK = TR + 273.
6 TKN = TK + 273.
7 TNN = TN + 273.
8 XNN = XN + 273.
9 XKK = XK + 273.
10 XNN = XN + 273.
11 XKK = XK + 273.
12 XNN = XN + 273.
13 XKK = XK + 273.
14 XNN = XN + 273.
15 XKK = XK + 273.
16 XNN = XN + 273.
17 XKK = XK + 273.
SUBPROGRAMS CALLED.

FP
ABS, F

SCALARS AND ARRAYS [ "=" NO EXPLICIT DEFINITION = "#" NOT REFERENCED ]

T AIR 3  EFF 17  DEN2 20  IDEL 21
ETA 24  CCOST 25  K  26  PFI 27
CCON 26  r  33  IDCODE 34  PFI 3 5

CON 72  TP 73  X1  75  XNUM 76
CCOOL 100  J  101  PFI 1 102  Q  103  XNUM 117

DO 1 J=1,12
DEN1=0.; DEN2=0.; DEN3=0.; DO 1 J=1,12
DEN1=DEN1+Q(J)
DEN2=DEN2+QUSE(J)
DEN3=DEN3+QWORK(J)
DEN1=DEN1/CPTH
DEN2=DEN2/CPTH
DEN3=DEN3/CPTH
A=1.*XI**N
B=(AXI)/(A+1.)*XM
XNUM=XNUM+COST
PFI1=XNUM/DEN1
PFI2=XNUM/DEN2
PFI3=XNUM/DEN3
WRITE(3,200) CTH,TT,PFI1,PFI2,PFI3,DEN1,DEN2,DEN3
FORMAT(1X,F7.2,2X,F9.3,2X,G14.5)
CONTINUE
IF( RR .GE. 0.) GO TO 10
RETURN
END
SUBROUTINE SDATA (BETA, TCMAX, H1, ETA1, TF, EFF, TAIR, XK)

REAL H1, EFF

DIMENSION XK(9)

DATA SIGMA = 0.0, 6097E-11 / EPS = 0.0, 1 ETAR=EFF/I00, / T R=25.

WRITE (3, 993) XK(K)

995 FORMAT (1X, 'NON-CP/AREA ABSORBED=', I10, 4X)

WRITE (3, 100)

100 FORMAT (1X, 'NON-CP/AREA ABSORBED=', I10, 4X)

CONTINUE

GO TO 5

A = 2.0*H1/(TC-TAIR)+2.0*EPS*SIGMA*4*(TC-TAIR)+4*XK(K)*(TC-TF)

CTH = H1/(TC-TAIR)+ ETA1+(TF-EFF)*2/XK(K)+4*(TC-TF)

GE = ETA1*K*KH(E2(TAIR))

OUT = 5.0*(TC-TAIR)+4/XK(K)+4*(TC-TF)

WORK = OUT + G

GLT = COOL + MOUT + G

FRA1 = G + G + G + G + G + G + G + G + G

FRA2 = FRA1 + MOUT + G

C = FRA2 + MOUT + G

400 FORMAT (1X, 'NON-CP/AREA ABSORBED=', I10, 4X)

CONTINUE

GO TO 4

RETURN

END

SUBPROGRAMS CALLED

SCALARS AND ARRAYS . ( # NO EXPLICIT DEFINITION = # NOT REFERENCED )

*TAIR 1  *EFF 2  XK 3  *BETA 4  *K 5  *H1 6
*TR 7  *R 8  *GUE 10  *TF 12  *FRA2 13  *COOL 14
*GUE 15  *A 16  *MOUT 17  *ETAR 20  *ETA1 21  *FRA1 22
*X0 20  *N 24  IN 25  *TCM 26  *GLT 27  *CTH 30
*TC 31  *TC 32  *TAIR 33  *EPS 34  *SIGMA 35

TEMPERATURES

*DATA 104

*DATA [ NO ERRORS DETECTED ]
C MAIN PROGRAM FOR SOLAR CONCENTRATOR CHARACTERISTICS

DIMENSION XK(8)

COMMON EFF, CON, CELL1, CELL2, N, XI, XM, ETA1, XK, TF

REAL IN

X(1)=.01; X(2)=.03; X(3)=.06; X(4)=.1; X(5)=.25

X(6)=.5; X(7)=.75; X(8)=.9

ETA=0.024; TCMAX=441.; H1=.015; IN=5.0E-1; ETA1=.7

TF=.69; EFF=15.; TAIK=25.

WRITE(3,233)

233 FORMAT(1*,IX,'GA=AS SOLAR CELLS',/)

CALL SDATA(BETA, TCMAX, H1, IN, ETA1, TF, EFF, TAIK, XK)

WRITE(3,234)

234 FORMAT(1*,IX,'SILICON SOLAR CELLS',/)

CALL SDATA(BETA, TCMAX, H1, IN, ETA1, TF, EFF, TAIK, XK)

WRITE(3,235)

235 FORMAT(1*,IX,'NORMAL END',/)

END

COMMON BLOCKS

END

COMMON BLOCKS

END

SUBPROGRAMS CALLED

SADATA BLOCK 1 BLOCK 2

SCALARS AND ARRAYS ( ** = NO EXPPLICIT DEFINITION = * = NOT REFERENCED )

*TAI K** 1  *BETA  ** 2  *H1** 3  *IN** 4  *TCMAX** 5
REFERENCES


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