TECHNIQUES FOR COMPUTING REGIONAL RADIANT EMITTTANCES OF THE EARTH-ATMOSPHERE SYSTEM FROM OBSERVATIONS BY WIDE-ANGLE SATELLITE RADIOMETERS

By José F. Pina and Frederick B. House

EARTH ENERGY EXPERIMENT (E³) PROJECT

NASA CONTRACT NAS 1-11871

drexel university

DEPARTMENT OF PHYSICS AND ATMOSPHERIC SCIENCE
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Final Report
Phase III

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The present report contains results of an investigation of critical problems related to satellite systems for long-term earth energy budget (EEB) observations, performed under Langley Research Center Contract No. NAS1-11871 for the National Aeronautics and Space Administration.

This research study, performed by Drexel University, is one part of a much larger effort by several institutions, including Colorado State University, The University of Wisconsin, Virginia Polytechnic Institute and State University, Link Temco Vought as well as cognizant personnel at NASA Langley Research Center. This team is studying satellite systems for performing long-term EEB measurements over geographical regions, hemispheres, and the entire earth for periods of 10 to 30 years. A major portion of the total effort is responsive to the AAFE proposal, and the proposed LZEEBE system (reference 1) which employs three balloon radiometers.

The decision was made to expand the scope of the total effort beyond that envisioned in the AAFE proposal. This broadened scope includes Phase A type of efforts concerning other concepts of performing EEB observations besides the balloon system. For example, systems employing spinning plate radiometers and/or scanning radiometers could be developed for long-term space application. Regardless of the geometric characteristic of the observational system, the problems of data analysis and interpretation are similar for all wide-angle systems with only an adjustment required for viewing geometry.

The current investigation was performed during the twelve month period 1 January 1974 through 31 December 1974. This period is denoted as Phase III in subject contract. The express purpose of the investigation is outlined
in the following objectives, contained in the statement of work.

1. Accuracy assessment of Sampling Bias by Candidate Satellite Systems for EEB Observations

2. Development of Procedures for Analysis and Interpretation of EEB Observations by Wide-field, Broad-band Detectors

3. Accuracy Assessment of Procedures for Analysis and Interpretation of EEB Observations

4. Specification of Spectral Absorbing Characteristics of Broad-band Detectors and Calibration Requirements for Accurate Observation of the EEB

5. Variance Spectrum Analysis of EEB Observations from ESSA VII Satellite

6. Professional Support and Advisory Effort as Co-Principal Investigator of LZEEBE


Gratitude is extended to several NASA/LaRC personnel for their encouragement, interest, stimulating discussions and suggestions provided during the present investigation. Among these scientific personnel are included: Messrs. George Sweet (technical monitor), Charles Woerner, Jack Cooper, Dr. Louis Smith and other members of the LaRC team.

Frederick B. House, Project Director
Associate Professor of Physics and
Atmospheric Science
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* The quantities and names used throughout this report are based on those selected by Craig (reference 2).
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</tr>
<tr>
<td>$W^A_{ei}$</td>
<td>Weighted average of $W_e$ (weight is the area $A$)</td>
<td>4-59</td>
</tr>
<tr>
<td>Symbol</td>
<td>Definition</td>
<td>First used in Equation</td>
</tr>
<tr>
<td>--------</td>
<td>------------</td>
<td>------------------------</td>
</tr>
<tr>
<td>$W_{ei}^F$</td>
<td>Weighted average of $W_e$ (weight is the configuration factor $F$)</td>
<td>4-58</td>
</tr>
<tr>
<td>$W_{eij}$</td>
<td>The $j$th determination of the value of $W_e$ for the $i$th region</td>
<td>4-58</td>
</tr>
<tr>
<td>$A_{ij}$</td>
<td>Total area seen of the $i$th region during the $j$th observation set</td>
<td>4-59</td>
</tr>
<tr>
<td>$F_{ij}$</td>
<td>Sum of all the configuration factors of the $i$th region which entered in the $j$th observation set.</td>
<td>4-60</td>
</tr>
<tr>
<td>$r_{ijk}$</td>
<td>Directional reflectance by $\Delta A_{ijk}$</td>
<td>4-63</td>
</tr>
<tr>
<td>$r_k(o)$</td>
<td>Directional reflectance of the $k$th region for zero solar zenith angle</td>
<td>4-63</td>
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<tr>
<td>$R_{ijk}$</td>
<td>Reading from figure 5, reference 3</td>
<td>4-63</td>
</tr>
<tr>
<td>$R'_{ijk}$</td>
<td>Reading from figures B-3, B-4, or B-5 of reference 3</td>
<td>4-69</td>
</tr>
<tr>
<td>$R''_{ijk}$</td>
<td>Defined as $R_{ijk}(\zeta) \cos(\zeta)$</td>
<td>4-86</td>
</tr>
</tbody>
</table>

**Acronyms**

- **EEB**: Earth Energy Budget
- **AAFE**: Advanced Application Flight Experiment
- **LZEEBE**: Long-term Zonal Earth Energy Budget Experiment
- **GARP**: Global Atmosphere Research Project
The study of the total energy budget of the planet earth and its atmosphere is based on analysis of the exchange of radiant energy between the earth-atmosphere (E-A) system and space. Radiometers on earth-orbiting satellites can effectively measure this exchange of radiant energy at observation points in space external to the E-A system. The current effort is concerned with the analysis and interpretation of observations by wideangle, spherical and flat radiometers with regard to the general problem of the earth energy Budget (EEB); but in particular, it is concerned with the problem of determining the energy budget of regions smaller than the field of view (FOV) of these radiometers. Before considering specific reasons or motivations for conducting the present research effort, the background to the overall problem will be discussed.

Background

The energy budget at a specific time \( t \) for a given region of the E-A system's surface can be described by the following expression which shows the relationship of the net flux \( Q \) to the three fluxes \( H_s, W_r, \) and \( W_e \).

\[
Q = H_s - (W_r + W_e)
\]  

(1-1)

The meaning of the three fluxes is shown schematically in figure 1-1. The symbols \( Q, H_s, W_r, \) and \( W_e \) are defined in the List of Symbols in the front matter of this report. Introducing the concept of albedo \( A = W_r / H_s \), which is also depicted in Figure 1-1, one can rewrite (1-1) as

\[
Q = H_s (1 - A) - W_e
\]  

(1-2)
Figure 1-1. Radiant power balance on an earth surface element.
A time average of the net flux $\bar{Q}$ over any convenient time scale can be readily computed from the data available for any of the E-A system regions that has been under study.

The net flux $Q$ defined by equation (1-1) can represent the time average conditions for a broad range of spatial dimensions. These can range from the dimensions of a GARP grid, about $5^\circ \times 5^\circ$ great circle arc, to the dimensions of the entire globe. Different observational satellite systems must be employed to perform the required measurements over this broad range of spatial dimensions. For example, a scanning radiometer would be needed to observe the components of net flux for a GARP grid. On the other hand, a wide-angle radiometer can best fulfill the sampling requirements for the entire globe.

In terms of conventional meteorological dimensions a GARP grid maybe classed as a mesoscale feature. Wide-angle radiometers become more effective observational tools than scanning (narrow angle) radiometers for some meteorological features between the mesoscale and global dimensions. It is suggested here that the synoptic scale meteorological feature may be the cross over point between the usefulness of scanning and wide-angle radiometers.

The current effort focuses on the problem of interpreting wide-angle observations in terms of the components of the net flux for regions whose dimensions are synoptic scale and larger, i.e., spatial dimensions whose areas are $1-5 \times 10^6 \text{ km}^2$ to the entire area of the earth. It should be emphasized that this problem of data interpretation is common to both spherical and plate radiometers.

The important point to be noted here is that $H_s$, $W_r$, and $W_e$, can be determined from observational data of satellites orbiting the earth. It follows then, that $Q$ can be computed for any region for which satellite data are available. Consequently, determination of the fluxes $W_r$ and $W_e$ mentioned above for regions smaller than the FOV of the satellite becomes important, and
techniques to accomplish this task have been sought for some time.

At Drexel University, two techniques involving matrix inversions were developed for computing the two fluxes under discussion for regions smaller than the FOV. The motivation behind the efforts to determine $W_r$ and $W_e$ for surface areas smaller than the FOV will now be discussed.

Motivation

One can think of two reasons for developing techniques to interpret wide-angle observations for synoptic scale regions. These reasons are:

(a) Knowledge of the radiant energy budget over several areas (such as eastern continental U.S.A., the polar caps, the Northwestern part of Africa, and the Caribbean - Gulf of Mexico region) possessing meteorological significance in the dynamical analysis of the atmosphere are of practical and scientific value. These areas are smaller than the FOV of wide-angle radiometers on satellites at orbital altitudes commonly used.

(b) Several of the regions of interest in studies of the earth's energy budget may have dimensions equal to, or larger than, the FOV of the radiometer (e.g. subtropical oceans). However, often only a portion of the FOV covers part of the area of interest during an observation; and it is, therefore, impossible to determine from this measurement alone what fraction of the power measured originated at the area of concern.

The above arguments provide sufficient reason for endeavoring to develop the types of techniques sought. The essentials of the two techniques developed at Drexel follow.
In one technique, the number of observations \( m \) equals the number \( n \) of regions under study, and a unique solution is obtained for each region. The resulting system of \( n \) simultaneous equations is solved by operating on the observed power column matrix with the inverse of the configuration factor matrix. If all the observations are made at one time during a single pass of the satellite, the solutions are termed INSTANTANEOUS. For this reason, the technique is called Instantaneous/Inversion Technique.

In the other technique, the number of observations \( m \) is larger than the number \( n \) of regions being observed. The \( n \) simultaneous equations required to seek a solution is obtained by using an extension of the method of least squares to determine the surface which best fits the data. For this reason, this technique is termed the Best Fit/Inversion technique. Again here, the system of \( n \) simultaneous equations is solved by the use of a matrix inversion subroutine.

The values of \( W_e \) obtained by both techniques from error-free observations were totally acceptable. However, when the measurements included uncertainties, not all the instantaneous values of \( W_e \) computed by the first technique were acceptable. Nevertheless, a prediction scheme was developed which forecasts the quality of the instantaneous \( W_e \) values to be determined by the first technique. In this scheme, the square matrix of the coefficients of the \( n \) simultaneous equations found by the first technique was used to predict the acceptability of the instantaneous values of \( W_e \) obtained for each of the regions. The computer program calculates the elements of the matrix and then
proceeds to analyze the matrix in order to predict the quality of the data to be obtained. The actual results computed were always compared with the pertinent tolerances listed in TABLE 4-7, Accuracy Requirements for Radiation Budget Components, in order to determine the acceptability of the results. In all cases, the predictions were in agreement with the results of these comparisons. The results obtained by both techniques are now presented.

Accuracy of Results

Two types of errors were selected to perturb the power measurements made by the radiometers. These were, systematic errors (0.3, 0.6, and 0.9 watts/m²) and gaussian random errors (the gaussian distribution had a sigma value σ = 0.5 watts/m²). From TABLE 4-7, the pertinent "desired" and "minimum useful" accuracy requirements for \( W_e \) are ±3 watts/m² and ±15 watts/m², respectively. To evaluate the Instantaneous/Inversion Technique, ten sets of six observations each were included in the analysis. The predictions and the uncertainties in the values of \( W_e \) obtained by using a combination of the gaussian errors mentioned and 0.9 watts/m² systematic errors were as follows (refer to TABLE 4-8)

<table>
<thead>
<tr>
<th>Predictions, sphere</th>
<th>( W_{e1} )</th>
<th>( W_{e2} )</th>
<th>( W_{e3} )</th>
<th>( W_{e4} )</th>
<th>( W_{e5} )</th>
<th>( W_{e6} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prediction, plate</td>
<td>Poor</td>
<td>Poor</td>
<td>Accept</td>
<td>Accept</td>
<td>Poor</td>
<td>Accept</td>
</tr>
<tr>
<td>Sphere</td>
<td>15.2</td>
<td>20.1</td>
<td>6.9</td>
<td>3.1</td>
<td>17.4</td>
<td>5.8</td>
</tr>
<tr>
<td>Plate</td>
<td>15.4</td>
<td>21.9</td>
<td>9.1</td>
<td>4.2</td>
<td>33.8</td>
<td>9.3</td>
</tr>
</tbody>
</table>

As it can be seen from these results, only the values of \( W_{e2} \) and \( W_{e5} \) exceed significantly the minimum useful accuracy requirements (±15 watts/m²) given for both satellites in TABLE 4-7. However, it is noticed that consistently the plate exhibits larger uncertainties than the sphere. It should
be remarked here that different satellite orbits that include larger segments within their FOV's of regions one and five would serve to provide $W_e$ values with lesser errors for these regions.

In order to test the Best Fit/Inversion Technique, thrity-six observations of six regions were included. The values of $W_e$ obtained when the power measurements were assumed to exhibit the same combination of errors used above for the Instantaneous/Inversion Technique were as shown below. The six values used as a standard for comparing the values of $W_e$ retrieved were the averages of the given $W_e$ values computed for each of the six-regions.

<table>
<thead>
<tr>
<th>$W_{e1}$</th>
<th>$W_{e2}$</th>
<th>$W_{e3}$</th>
<th>$W_{e4}$</th>
<th>$W_{e5}$</th>
<th>$W_{e6}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average (Standard)</td>
<td>179.4</td>
<td>200.4</td>
<td>220.4</td>
<td>239.8</td>
<td>259.6</td>
</tr>
<tr>
<td>Results</td>
<td>179.3</td>
<td>200.6</td>
<td>222.2</td>
<td>243.0</td>
<td>257.4</td>
</tr>
</tbody>
</table>

As it can be seen from these data, the highest uncertainty in the value of $W_e$ is exhibited by $W_{e4}$, which is 3.2 watts/m$^2$. However, even this error is below the minimum useful accuracy requirement shown in TABLE 4-7.

**BASIC IDEAS**

The basic ideas and concepts required for deriving the expression for the total radiant power $P$ (watts) intercepted by a radiometer orbiting the earth are discussed in the following subsections.

**Shape or Configuration Factor**

This factor $F$ which is dimensionless, appears often in radiative
transfer literature and is defined in the following expression (reference 3)

\[ P = F A W \]  \hspace{1cm} (2-1)

where

- \( P \) = radiant power (watts) intercepted by the radiometer.
- \( A \) = characteristic area of the radiometer. This area is \( A = \pi a^2 \) for both a sphere of radius \( a \) and a flat circular plate of radius \( a \).
- \( W \) = radiant flux (watts/m\(^2\)) per unit time, or radiant power per unit area, which is either emitted (radiant emittance \( W_e \)) or reflected (radiant reflectance \( W_r \)) by the earth-atmosphere (E-A) system.
- \( F \) = shape, or configuration factor (dimensionless). This factor represents that fraction of an observed area flux \( W \) that is intercepted by the radiometer per unit of characteristic area.

The power **absorbed** by the radiometer is given by,

\[ P = \alpha P = \alpha F A W \]  \hspace{1cm} (2-2)

where

- \( \alpha \) = absorptance of the radiometer (dimensionless). It is the ratio of the power absorbed to the power intercepted.

**Radiometer Characteristics**

Two types of radiometer will be treated in this report: (a) spherical, and (b) horizontal, flat, circular.

The characteristic area \( A = \pi a^2 \) of both of the above radiometers will be assumed to be \( A = 1m^2 \). This assumption serves to simplify the expressions without affecting the physical significance of the results.

Both satellites will be assumed to be blackbodies i.e., the spectral emissivity \( \varepsilon_\lambda \) and absorptivity \( \alpha_\lambda \) are assumed to be unity
\[ \varepsilon_{\lambda} = \alpha_{\lambda} = 1 \] (2-3)

Therefore, all the radiant power \( P \) intercepted by either radiometer (sphere, or plate) is totally absorbed, and one has

\[ P = P_{\alpha} \] (2-4)

Hence, the power absorbed by the radiometer can be written as

\[ P = F \varepsilon \] (2-5)

Radiance and Radiant Emittance

These two quantities are of great importance in the discussion of longwave radiation (LWR) emitted by the E-A system. One is interested in the expression that relates the radiance \( N(\theta, \psi) \) (watts/m^2 - sr) and the radiant emittance \( W_e \) (watts/m^2) of an area element \( dA(m^2) \) whose centroid is located at longitude \( \lambda \) and latitude \( \phi \). The radiance is the radiant power per unit area emitted by \( dA \) within an element of solid angle \( d\Omega \). The radiant emittance is the total radiant power per unit area (or radiant flux per unit area) emitted by \( dA \) into a \( 2\pi \) steradians solid angle. The expression relating the instantaneous values of \( W_e \) and \( N \) for a given \( dA(\lambda, \phi) \) is

\[ W_e(\lambda, \phi, t) = \int_0^{2\pi} d\psi \int_0^{\pi/2} N(\theta, \psi; \lambda, \phi, t) \sin \theta \cos \theta \, d\theta \] (2-6)

Where the radiance is being considered as a function of the zenith angle \( \theta \) and the azimuthal angle \( \psi \) which define the direction of \( N(\theta, \psi; \lambda, \phi, t) \). This quantity and \( W_e \) are functions of the position of the area observed as well as of the specific time at which the observation is made. In order to simplify the notation, the dependence of \( N \) on \( \lambda, \phi, \) and \( t \) will not be shown.
A reasonable approach in the interpretation of LWR is to assume that the radiance is a function of the zenith angle \( \theta \) only (reference 4). Then, one can write,

\[
N(\theta, \psi) = N(\theta) = N^Z f(\theta)
\]  \hspace{1cm} (2-7)

where \( N^Z \) = Zenith radiance, i.e., the radiance in the zenith direction, \( \theta = 0^\circ \).

\( f(\theta) \) = limb darkening function (LDF) which gives a measure of the anisotropy of the radiation field.

Hence, by substituting equation (2-7) into (2-6) one obtains

\[
W = \frac{W_e}{2\pi} N^Z \int_0^{\pi/2} f(\theta) \sin \theta \cos \theta \, d\theta
\]

(2-8)

or

\[
N^Z = \frac{W_e}{2\pi \int_0^{\pi/2} f(\theta) \sin \theta \cos \theta \, d\theta}
\]

(2-9)

The integral appearing in (2-8) and (2-9) is used often and hence, it is convenient to define it as,

\[
I(f) = \int_0^{\pi/2} f(\theta) \sin \theta \cos \theta \, d\theta
\]

(2-10)

Then, (2-8) and (2-9) can be rewritten as .
\[ W_e = 2 \pi N^2 I(f) \]

and
\[ N^2 = \frac{1}{2\pi I(f)} W_e \]  \hspace{1cm} (2-11)

For an ISOTROPIC radiation field, \( f(\theta) = 1 \), \( I(f) = \frac{1}{2} \), and the isotropic radiance \( N^{iso} \) is then
\[ N^{iso} = \frac{W_e}{\pi} \]  \hspace{1cm} (2-12)

Figure 2-1 shows a satellite \( S \) at a height \( H \) above a spherical E-A system of radius \( R \). The geometric nomenclature to be used in this report is presented in this figure for future reference. A flat earth is depicted tangential to the spherical system at the sub-satellite point (SSP). The concept of a flat earth will be used to make certain calculation simplifications and it will be thoroughly discussed in the subsection entitled "Flat Earth Application". From this figure one can readily write.

\[ \theta = \gamma + \alpha \]

\[ R \sin \theta = (R + H) \sin \alpha \]  \hspace{1cm} (2-13)

\[ R \sin \gamma = r \sin \alpha \]  \hspace{1cm} (2-14)

\[ r^2 = R^2 + (R + H)^2 - 2R(R+H)\cos \gamma \]  \hspace{1cm} (2-15)
Figure 2-1. Pictorial definition of the symbols used.
Irradiance, Radiance, Radiant Reflectance, and Albedo

These four quantities are of great importance when the solar shortwave radiation (SWR) reflected by the earth is treated. One of the expressions of interest is the relation between the radiance $N_r$ (watts/(m$^2$ - sr)) and the radiant reflectance $W_r$ (watts/m$^2$) of an area element $dA$ (m$^2$) whose centroid is located at longitude $\lambda$ and latitude $\phi$. The radiance in this case is the radiant power per unit area reflected by $dA$ within an element of solid angle $d\Omega$. The radiant reflectance is the total radiant power per unit area (or radiant flux per unit area) reflected by $dA$ into a $2\pi$ steradians solid angle.

Before writing the expressions connecting $N_r$ and $W_r$ it will prove helpful to define the solar irradiance $H$ (watts/m$^2$). This is the amount of radiant power per unit area (or radiant flux per unit area) impinging upon $dA(\lambda, \phi)$ from all directions contained within a $2\pi$ steradians solid angle. Hence, the definitions of $W_e$, $H$, and $W_r$ are very similar, except that they refer to emitted, incident, and reflected radiations, respectively. Then, one can write, using the nomenclature of reference 3.

$$H(L, \zeta, \lambda, \phi, t) = SL \cos \zeta$$

(2-16)

where

- $S$ = Solar constant ($S = 1353 \pm 21$ watts/m$^2 = 1.940 \pm 0.03$ cal/(cm$^2$-min), see reference 5).
- $L = \frac{d^2}{d^2 \overline{d}}$ and $d$ are the mean and true sun-to-earth distances, respectively.
- $\zeta$ = Zenith angle of the sun at $dA(\lambda, \phi)$.
- $\lambda$ = Longitude of the observed area element $dA$.
- $\phi$ = Latitude of the observed area element $dA$. 

2-7
t = specific time at which observation was made.

The instantaneous albedo $A(\zeta, \lambda, \phi, t)$ for the area element $dA(\lambda, \phi)$ is defined by

$$A(\zeta, \lambda, \phi, t) = \frac{W_r(L, \zeta, \lambda, \phi, t)}{H(L, \zeta, \lambda, \phi, t)}$$

(2-17)

In order to simplify this expression, one can assume that $L$ does not change significantly during the time interval of the observations. Then, for a given $L$, the dependence of $W_r$ and $H$ on $L$ does not have to be shown. In a similar way, $\lambda$, $\phi$, and $t$ can be dropped from the above expressions, and

$$A(\zeta) = \frac{W_r(\zeta)}{H(\zeta)}$$

(2-18)

From the definitions given for $N_r$ and $W_r$ at the beginning of this subsection, one can write the following expression relating these two quantities, which is similar to equation (2-8).

$$W_r(\zeta) = \int_0^{2\pi} d\psi \int_0^{\pi/2} N_r(\theta, \psi, \zeta) \sin \theta \cos \theta d\theta$$

(2-19)

Dividing this expression through by $H(\zeta)$, one has

$$A(\zeta) = r(\zeta) = \int_0^{2\pi} d\psi \int_0^{\pi/2} \rho(\theta, \psi, \zeta) \sin \theta \cos \theta d\theta$$

(2-20)

Where,

$$A(\zeta) = r(\zeta) = \frac{W_r(\zeta)}{H(\zeta)}$$

(2-21)

$r(\zeta)$ is the directional reflectance which in this case is the same as the instantaneous albedo.
\[\rho(\theta, \psi, \zeta) = \frac{N_r(\theta, \psi, \zeta)}{H(\zeta)} \quad (2-22)\]

\(\rho(\theta, \psi, \zeta)\) is the bidirectional reflectance.

Thus, the directional reflectance is the ratio of the total radiant power per unit area \(W_r(\zeta)\) which is reflected by \(dA\) onto a \(2\pi\) steradians solid angle to the total solar radiant power per unit area \(H(\zeta)\) incident on \(dA\) at a zenith angle \(\zeta\). The bidirectional reflectance involves two directions, namely, the direction of the incident radiation which is given by the zenith angle \(\zeta\), and the direction of the observer. The latter direction is defined by the zenith angle \(\theta\) and the azimuthal angle \(\psi\). The bidirectional reflectance is the ratio of that amount of radiant power per unit area per unit solid angle \(N_r\) which is reflected by \(dA\) onto an element of solid angle \(d\Omega(\theta, \psi)\) to the total solar radiant power per unit area \(H(\zeta)\) incident on \(dA\) at the zenith angle \(\zeta\).

The integral over the angle \(\theta\) appearing in equation (2-20) is similar to the one appearing in equations (2-8) and can be treated in a similar manner. That is, one can define \(I(\rho)\) as

\[I(\rho) = \int_0^{\pi/2} \rho(\theta, \psi, \zeta) \sin \theta \cos \theta d\theta \quad (2-23)\]

Then, equation (2-20) can be rewritten as

\[A(\zeta) = r(\zeta) = \int_0^{2\pi} I(\rho) \, d\psi \quad (2-24)\]

A method proposed for utilizing these equations will be discussed at the end of the section entitled Instantaneous/Inversion Technique.
Instantaneous Net Flux

Once the representative values of $H$, $W_r$, and $W_e$ are determined for a particular region from a given set of $n$ observations $i$, taken during a time interval centered at $t_j$, one can determine the net radiant power, or net flux, $Q_i(t_j)$ for this region. Hence, $Q_i(t_j)$ represents the instantaneous value of the net radiant flux for the specific time of day $t_j$.

Average Net Flux

It follows from the above discussion of the instantaneous energy budget that a weighted average (using the configuration factor as a weight) for a given region can be obtained from the $m$ values of $Q_i(t_j)$ computed for $m$ sets of observations.

If, for instance, one is interested in obtaining the average of $Q(t_j)$ during a month for a unique value of $t_j$ (say 3:00 PM) from $m$ values, one has

$$Q(t_j) = \frac{\sum_{i=1}^{m} F_i Q_i(t_j)}{\sum_{i=1}^{m} F_i}$$  

(2-25)

Other types of averages which are more suitable for the user's needs can be computed in a similar manner.

NOTE: In this report only the LWR component will be treated. The SWR component will be discussed in a subsequent report.
Radiative Equilibrium

When the temperature of the sensor reaches a steady state, it is said that the sensor is in radiative equilibrium. In this state, the power input $P_{in}$ is equal to the power output $P_{out}$. On the basis of the definition of $N(\theta)$ for the LWR presented in previous sections, one can write the expressions for $P_{in}$ for both satellites, considered as blackbodies, as

SPHERE

$$P_{in} = A_s \int_{0}^{2\pi} \int_{0}^{\alpha_m} N[\theta(\alpha)] \sin \alpha \, d\alpha$$

(2-26)

PLATE

$$P_{in} = A_s \int_{0}^{2\pi} \int_{0}^{\alpha_m} N[\theta(\alpha)] \sin \alpha \cos \alpha \, d\alpha$$

(2-27)

where $\alpha_m$ is the maximum value of the nadir angle.

The expression for $P_{out}$ for both satellites follows from Stefan-Boltzmann law,

SPHERE

$$P_{out} = S \sigma T^4 = 4 A_s \sigma T^4$$

(2-28)

PLATE

$$P_{out} = A_s \sigma T^4$$

(2-29)

Where, $\sigma = 5.6697 \times 10^{-8}$ watts/(m$^2$·K$^4$); Stefan-Boltzmann constant.

It will be assumed only for convenience that the characteristic areas of both radiometers is $A_s = 1 \text{m}^2$. Also, $N[\theta(\alpha)]$ will be written in terms of $N^2$ and $f[\theta(\alpha)]$ per equation (2-7).

Then, the above four equations can be rewritten as,
SPHERE  \( P_{\text{in}} = 2\pi N_z z^m f[\theta(a)] \sin \alpha \, d\alpha \) \quad (2-30)

PLATE  \( P_{\text{in}} = 2\pi N_z z^m f[\theta(a)] \sin \alpha \cos \alpha \, d\alpha \) \quad (2-31)

SPHERE  \( P_{\text{out}} = 4 \sigma T^4 \) \quad (2-32)

PLATE  \( P_{\text{out}} = \sigma T^4 \) \quad (2-33)

Substituting in (2-30) and (2-31) the value of \( N_z \) as given by (2-9), one obtains,

SPHERE  \[ P_{\text{in}} = \frac{\int_0^\alpha f[\theta(a)] \sin \alpha \, d\alpha}{\int_0^{\pi/2} f(\theta) \sin \theta \cos \theta \, d\theta} \quad W_e \] \quad (2-34)

PLATE  \[ P_{\text{in}} = \frac{\int_0^\alpha f[\theta(a)] \sin \alpha \cos \alpha \, d\alpha}{\int_0^{\pi/2} f(\theta) \sin \theta \cos \theta \, d\theta} \quad W_e \] \quad (2-35)

But comparing equation (2-5), that is \( P = FW \), with equations (2-34) and (2-35) one sees that the expressions for the shape factors for both radiometers are
or, using the definition of $I(f)$ given by (2-10), one can write

$$F_{sp} = \frac{1}{I(f)} \int_0^\alpha \int_0^{\pi/2} f[\theta(\alpha)] \sin \alpha \cos \alpha \, d\theta \, d\alpha$$  \hspace{2cm} (2-38)$$

For the special case of isotropic radiation

$f[\theta(\alpha)] = 1$ and $I(f) = \frac{4}{3}$. Then,

$$F_{iso}^{sp} = 2(1 - \cos \alpha)$$  \hspace{2cm} (2-40)$$

$$F_{iso}^{pl} = \sin^2 \alpha$$  \hspace{2cm} (2-41)$$
The only piece of information that a single satellite observation can yield is the total radiant power intercepted by the radiometer, as given by either equation (2-30) for a sphere, or (2-31) for a plate. This power represents the sum of those radiances originating at each of the area elements within the FOV and directed toward the radiometer. Therefore, it is impossible to obtain from a single, wide-angle measurement, and without additional information of any kind, an exact or rigorous solution for either $W_e$ or $W_r$ for any area within the FOV of the radiometer.

A similar situation exists for measurements by narrow-angle radiometers. The only information that is contained in a single observation of an area element is the radiance in the direction of the radiometer on the satellite. Scanning by this radiometer only adds information about adjacent area elements and nothing additional about the original area element under consideration. Hence, the fundamental difficulty one faces when trying to determine the $W_e$ field from a measurement of the radiance $N$ (or $W_r$ from $N_r$) is that the total angular distribution of $N$ (or $N_r$) can not be deduced from the measurement, and hence, $W_e$ can not be deduced from the measurement either.

It follows from the above discussion that it is impossible to determine the $W_e$ (or $W_r$) field of an area of any size - i.e., of an area smaller than the FOV, equal to the FOV, or larger than the FOV of the radiometer - unless the angular distribution of $N$ is also made available. On the other hand, it is possible that by utilizing representative values of the angular distributions of $N$ and $N_r$ obtained from previous observations the problem can be rendered solvable in a satisfactory manner. Furthermore, these angular distribution values can be refined in the future by increasing the number of measurements of these distributions for different regions of the E-A system, as well as by
improving the accuracy and precision of these measurements.

Therefore, one can utilize an empirical model which portrays the angular emitting and reflecting characteristics of the E-A system, based on previous observations (satellite, aircraft, etc.) in order to solve the problem. At Drexel University, this approach was followed by making certain assumptions which are justifiable on the basis of physical processes and observational data accumulated by other investigators (reference 4). These assumptions are discussed in the following subsection.

Basic Assumptions

The following basic assumptions were sufficient to develop a technique for computing $W_e$ and $W_r$ from radiant power measurements taken by spherical and horizontal flat circular radiometers.

**Division into regions.** The surface area of the earth–atmosphere system is assumed divisible into regions which have emitting and reflecting characteristics significantly distinct from those of adjacent regions. The criterion for implementing the division must ultimately be based on results of previous satellite observations. The values of $W_e$ and $W_r$ computed for a given region are considered to represent the mean of the meteorological variations taking place in the time interval $\Delta t$ (e.g., a month) during which the observations were made.

**Position and extent of each region.** The position and extent of each region is assumed to be available or can be approximated from data gathered by previous investigators.

**Angular distribution of $N$ and $N_r$.** The angular dependence of $N$ for LWR, and of $N_r$ for the reflected solar SWR are available, or approximations can be made, from previous observations.
Although two different techniques for computing the radiant emittance and radiant reflectance fields have been developed at Drexel, the initial computational steps are common to both techniques. These common steps are discussed in detail in the following subsection.

**Mathematical Bases**

In this subsection, the mathematical bases which are common to both techniques are treated in detail. At the end of the subsection, the principal differences in the two techniques for accomplishing the final goal are listed.

The problem to be solved is illustrated in figure 3-1, which depicts a satellite (which for simplification is assumed to be crossing the equator) with three different regions \((R_1, R_2, \text{ and } R_3)\) falling, partially or totally, within the FOV of its radiometer. The following development is for a spherical radiometer; however, the treatment for horizontal flat sensor is similar.

A spherical radiometer of unit cross-sectional area is considered. Using \(P = FW\), as given by equation (2-5), one can write for the total power contributed by those portions of the three regions within the FOV of a radiometer on board the satellite at position S in figure 3-1 the following,

\[
P_{in} = P_1 + P_2 + P_3 \quad (3-1)
\]

or

\[
P_{in} = F_1 W_1 e_1 + F_2 W_2 e_2 + F_3 W_3 e_3 \quad (3-2)
\]
Figure 3-1. Regions within the FOV of a satellite crossing the equator.
That is, by using (2-30) to represent the power originating at region $R_1$ and which is intercepted by the satellite, one can write

$$P_1 = F_1 W_{el} = N_{1}^{z} \int_{\psi_{1n}}^{\psi_{1u}} \int_{\alpha_{1l}}^{\alpha_{1u}} \psi_1 \ln \frac{\theta_0}{\alpha_1} \sin \alpha \, d\alpha \, d\psi$$

(3.3)

Where,

- $P_1$ = radiant power from region $R_1$ intercepted by the radiometer.
- $N_{1}^{z}$ = zenith radiance of region $R_1$.
- $\psi_{1n}$ = lower boundary for $\psi$ in region $R_1$.
- $\psi_{1u}$ = upper boundary for $\psi$ in region $R_1$.
- $\alpha_{1l}$ = lower boundary for $\alpha$ in region $R_1$.
- $\alpha_{1u}$ = upper boundary for $\alpha$ in region $R_1$.

To facilitate the discussion, an isotropic radiation field will be assumed. Hence, $f[\theta(\alpha)] = 1$, and one obtains

$$P_1 = F_1 W_{el} = N_{1}^{iso} \int_{\psi_{1n}}^{\psi_{1u}} \int_{\alpha_{1l}}^{\alpha_{1u}} \psi_1 \ln \sin \alpha \, d\alpha \, d\psi$$

(3.4)

Where

- $N_{1}^{iso}$ = Isotropic radiance of region $R_1$.

Using expressions similar to (3.4) for $P_2$ and $P_3$ we can rewrite (3.1) for the power input to the radiometer, as

$$P_{1} = \int_{R_1} d\psi \int_{R_1} \sin \alpha \, d\alpha + \int_{R_2} d\psi \int_{R_2} \sin \alpha \, d\alpha + \int_{R_3} d\psi \int_{R_3} \sin \alpha \, d\alpha$$

(3.5)
Where $R_1$, $R_2$, and $R_3$ imply that the integration limits are those corresponding to the regions 1, 2, and 3, respectively.

Equation (3-5) can be rewritten as

$$P = \sum_{i=1}^{3} N_{iso} \int \int \sin \alpha \, d\alpha \, d\psi$$

(3-6)

By using equation (2-12), $P$ can be expressed in terms of $W_{ei}$ rather than $N_{iso}$, that is,

$$P = \frac{1}{\pi} \sum_{i=1}^{3} W_{ei} \int \int \sin \alpha \, d\alpha \, d\psi$$

(3-7)

The main problem at this point is to decide which is the most advantageous and efficient manner to perform the integration indicated in equations (3-6) or (3-7).

Two main approaches were considered; these are discussed in the following paragraphs together with the advantages that each presents.

The following different methods were considered.

(a) The integrations indicated in equation (3-6) could be performed numerically (e.g., by using the trapezoidal rule) between the boundaries given for each region in terms of longitude and latitude.

(b) The integrations could be performed geometrically. That is, by dividing the surface area of the E-A system into a large number $L$ of area elements, one could simply sum up the radiances that each of the area elements emits in the direction of the satellite in order to accomplish the integration indicated by (3-6).

It was decided that the second approach was more advantageous for the following reasons:
(a) In the first approach, since the boundaries of the regions do not necessarily follow longitudinal and latitudinal lines of constant value, the number of separate integrations to be performed could often be impractically high. Furthermore, the limits of integration for the angles \( \alpha \) and \( \psi \) could involve difficult trigonometric expressions which could make the implementation of this procedure highly impractical.

(b) In the second method, once the centroids and boundaries of each of the area elements have been accurately defined, the regions can be easily defined in terms of a group of adjacent area elements.

(c) In the second method, a procedure for testing whether a particular area element \( \Delta A \) should be considered to be within the FOV or not can be easily implemented if the longitude and latitude of the centroid of \( \Delta A \) is known. If the centroid is on, or inside of the perimeter of the FOV, \( \Delta A \) is considered to be within the FOV; otherwise, it is considered to be outside the FOV. On the average, it is expected that as many area elements will be accepted as will be rejected in each integration. Furthermore, since it is at the limb that the area elements are tested to be accepted or rejected, and since these area elements have much smaller shape factors than those situated close to the SSP, it is clear that the error would be negligible even if the number of area elements accepted do not match the number rejected.

Therefore the second method will be followed in which the surface of the E-A system is divided into a large number \( L \) of area elements \( \Delta A \), and each area element is identified by the longitude and latitude of its centroid.
In order to illustrate the computation of the radiant power intercepted by a satellite radiometer whose FOV comprises more than one region, a FLAT E-A system will be assumed. This type of system greatly simplifies the results. Only three regions and three FOV's will be considered, which are depicted in figure 3-2. The portion of the flat E-A system shown in this figure is comprised between 0° and 70° longitude, and between -50° and +50° latitude, which is adequate for the purposes of this discussion. An area element is considered within the FOV if its centroid lies on, or within, the perimeter of the FOV, as per the criterion stated earlier. In figure 3-2, the area elements have been numbered in order to identify them more easily. In TABLE 3-1, the elements that are considered to be contained in each of the three FOV's are tabulated. It is seen that in this instance, each FOV contains exactly twenty-six area elements.

The equations used to calculate the radiant power that an area element ΔA contributes to one observation are presented below. The symbols entering in these expressions are shown in figure 3-3 which is a schematic of a radiometer at a position S over the flat E-A system.

Let \( \Delta P_{ijk} \) be the power that the ith area element in the kth region contributes to the jth observation. Then, one writes for a spherical radiometer

\[
\Delta P^S_{ijk} = N_{ijk} \Delta A_{ijk} \cos \theta_{ijk} \frac{A_s}{d_{ijk}^2}
\]  

(3-8)

For a horizontal flat circular radiometer, one writes,

\[
\Delta P^P_{ijk} = N_{ijk} \Delta A_{ijk} \cos \theta_{ijk} \frac{A_s \cos \alpha_{ijk}}{d_{ijk}^2}
\]  

(3-9)

\( N_{ijk} \) = Radiance of \( \Delta A_i \) in the direction of the radiometer

\( A_s \) = Characteristic area of the satellite radiometer, which for the sphere and plate \( \pi a^2 \) (\( a \) = radius of sphere and plate).

3-8
Figure 3-2. Three satellite positions over three regions.
TABLE 3-1. Data from three satellite observations.

<table>
<thead>
<tr>
<th>Observation No.</th>
<th>Region No.</th>
<th>Area Element No.</th>
<th>No. of Elements</th>
<th>Elements Per Obs.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>201-203</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>186-190</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>172-176</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>3</td>
<td>158</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>159-162</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>3</td>
<td>144</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>145-148</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>131-133</td>
<td>3</td>
<td>26</td>
</tr>
<tr>
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<td>2</td>
<td>159-161</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>144</td>
<td>1</td>
<td></td>
</tr>
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<td>145-148</td>
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<td></td>
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<td>4</td>
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<td>89-91</td>
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<td>3</td>
<td>102</td>
<td>1</td>
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<td>103-106</td>
<td>4</td>
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<td>88</td>
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<td>60-64</td>
<td>5</td>
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<td>3</td>
<td>47-49</td>
<td>3</td>
<td>26</td>
</tr>
</tbody>
</table>
Figure 3-3. Radiance $N_{ijk}$ emitted by $\Delta A_{ijk}$ of a flat E-A system and intercepted by the radiometer S.
\( \Delta A_{ijk} \) = ith area element in the kth region within the FOV of the jth observation.

\( \theta_{ijk} \) = Zenith angle of the radiometer at \( \Delta A_{ijk} \)

\( \alpha_{ijk} \) = Nadir angle of \( \Delta A_{ijk} \) at the radiometer

\( d_{ijk} \) = Distance from \( \Delta A_{ijk} \) to the radiometer

Let it be assumed that \( N_{ijk} \) is a function of the zenith angle \( \theta_{ijk} \) only (i.e., that it is independent of the azimuthal angle \( \psi_{ijk} \)). Then, one can write

\[
N_{ijk} = N^z_{ijk} f(\theta_{ijk})
\]  

(3-10)

Where,

\( N^z_{ijk} \) = radiance in the zenith direction at \( \Delta A_{ijk} \)

\( f(\theta_{ijk}) \) = limb darkening function (LDF) or anisotropic factor at \( \Delta A_{ijk} \)

It will be further assumed that a region \( K \) is characterized by the following:

(a) The value of the LWR radiant emittance in the kth region has a representative average value \( W_{ek} \).

(b) The LWR radiance in the zenith direction has also a representative average value \( N^z_k \) in the kth region.

(c) The LDF has a representative functional form \( f_k(\theta) \) throughout the kth region.

Hence, for the kth region one rewrites (3-10) as

\[
N_{ijk} = N^z_k f_k(\theta_{ijk})
\]  

(3-10')
To avoid carrying $A_s$ in the following expressions, it is assumed that $A_s = 1 \text{ m}^2$. This assumption does not affect the validity of the results.

By substituting (3-10) into (3-8) and (3-9), and making $A_s = 1 \text{ m}^2$, one obtains,

$$\Delta P^s_{ijk} = N^z_k f_k (\theta_{ijk}) A_{ijk} \cos \theta_{ijk} \frac{1}{d^2_{ijk}}$$  \hspace{1cm} (3-11)

$$\Delta P^p_{ijk} = N^z_k f_k (\theta_{ijk}) A_{ijk} \cos \theta_{ijk} \frac{\cos \alpha_{ijk}}{d^2_{ijk}}$$  \hspace{1cm} (3-12)

In the section BASIC IDEAS, the following expressions were introduced to show the relationship between $N^z_k$ and $W_{ek}$. From equations (2-10) and (2-11) one then writes, respectively,

$$I\left(f_k\right) = \int_0^{\pi/2} f_k(\theta) \sin \theta \cos \theta \, d\theta$$ \hspace{1cm} (3-13)

$$N^z_k = \frac{1}{2\pi I\left(f_k\right)} W_{ek}$$ \hspace{1cm} (3-14)

Substituting (3-14) into (3-11) and (3-12) one obtains,

$$\Delta P^s_{ijk} = \left[\frac{f_k(\theta_{ijk})}{2I\left(f_k\right)}\right] \frac{\Delta A}{\pi} \frac{\cos \theta_{ijk} W_{ek}}{d^2_{ijk}}$$  \hspace{1cm} (3-15)

$$\Delta P^p_{ijk} = \left[\frac{f_k(\theta_{ijk})}{2I\left(f_k\right)}\right] \frac{\Delta A}{\pi} \frac{\cos \theta_{ijk} \cos \alpha_{ijk} W_{ek}}{d^2_{ijk}}$$  \hspace{1cm} (3-16)
Although the subscripts in $\Delta A_{ijk}$ identify the position, the observation, and the region, respectively, of the area element under consideration, these subscripts can be dropped here since they appear in other variables in the same expression. Also, the magnitude of $\Delta A$ has been assumed to be the same for all area elements; and hence, it is unnecessary to identify it with the subscripts.

It was shown in expression (2-5) that if $A_s = 1m^2$, the power $P$ is related to the radiant emittance $W_e$ by means of the shape factor $F$, that is

$$P = F W_e \quad (3-17)$$

Hence, using the definition of the shape factor, one obtains from (3-15) and (3-16),

$$P^s_{ijk} = \left[ \frac{f_k(\theta_{ijk})}{2I(f_k)} \right] \frac{\Delta A}{\pi} \cos \theta_{ijk} \frac{\cos \alpha_{ijk}}{d_{ijk}^2} \quad (3-18)$$

$$P^p_{ijk} = \left[ \frac{f_k(\theta_{ijk})}{2I(f_k)} \right] \frac{\Delta A}{\pi} \cos \theta_{ijk} \frac{\cos \alpha_{ijk}}{d_{ijk}^2} \quad (3-19)$$

From figure 3-3, it is seen that for a flat E-A system (which is the type of system being used in this development) $d_{ijk}^2$ is given by

$$d_{ijk}^2 = H^2 + r_{ijk}^2 \quad (3-20)$$

which is different from the equivalent expression for a spherical E-A system, which from figure 2-1 can be seen to be

$$r_{ijk}^2 = R^2 + (R+H)^2 - 2R(R+H)\cos \gamma \quad (3-21)$$
where,

\[ r_{ijk} = \text{distance from } \Delta A_{ijk} \text{ to the radiometer in the spherical E-A system.} \]

\[ R = \text{radius of the E-A system.} \]

\[ \gamma = \text{angle between } A_{ijk} \text{ and the SSP, measured at the center of the earth.} \]

In order to obtain the power \( P_{jk} \) contributed by the \( k \)-th region to the \( j \)-th observation of both, the sphere and the plate, equations (3-15) and (3-16) are summed up over \( i \).

\[
P^S_{jk} = \sum_{i=1}^{I} \Delta P^S_{ijk} = \sum_{i=1}^{I} F^S_{ijk} W_{ek} \tag{3-22}
\]

\[
P^P_{jk} = \sum_{i=1}^{I} \Delta P^P_{ijk} = \sum_{i=1}^{I} F^P_{ijk} W_{ek} \tag{3-23}
\]

where,

\[ I = \text{total number of area elements } \Delta A_{ijk} \text{ of the } k \text{-th region that were under the FOV of the } j \text{-th observation.} \]

Similarly, the configuration factor \( F_{jk} \) represents the contribution by the \( k \)-th region to the \( j \)-th observation and is obtained as follows,

\[
F^S_{jk} = \sum_{i=1}^{I} F^S_{ijk} \tag{3-24}
\]

\[
F^P_{jk} = \sum_{i=1}^{I} F^P_{ijk} \tag{3-25}
\]

And for the three contributions from the regions \( R_k (k = 1, 2, 3) \) to the \( j \)-th observation one obtains by combining (3-22) with (3-24) and (3-23) with (3-25),
\[ \mathbf{p}_s = \sum_{j=1}^{3} \sum_{k=1}^{3} \mathbf{p}_{jk} \mathbf{w}_{ek} \]  
(3-26)  

\[ \mathbf{p}_p = \sum_{j=1}^{3} \sum_{k=1}^{3} \mathbf{p}_{jk} \mathbf{w}_{ek} \]  
(3-27)  

Or, writing in detail these expressions, one obtains for the sphere,

\[ \mathbf{p}_s^1 = \mathbf{F}_{11} \mathbf{w}_{11} + \mathbf{F}_{21} \mathbf{w}_{12} + \mathbf{F}_{31} \mathbf{w}_{13} \]  

\[ \mathbf{p}_s^2 = \mathbf{F}_{21} \mathbf{w}_{11} + \mathbf{F}_{22} \mathbf{w}_{12} + \mathbf{F}_{23} \mathbf{w}_{13} \]  
(3-28)  

\[ \mathbf{p}_s^3 = \mathbf{F}_{31} \mathbf{w}_{11} + \mathbf{F}_{32} \mathbf{w}_{12} + \mathbf{F}_{33} \mathbf{w}_{13} \]  

And for the plate one has,

\[ \mathbf{p}_p^1 = \mathbf{F}_{11} \mathbf{w}_{11} + \mathbf{F}_{12} \mathbf{w}_{12} + \mathbf{F}_{13} \mathbf{w}_{13} \]  

\[ \mathbf{p}_p^2 = \mathbf{F}_{21} \mathbf{w}_{11} + \mathbf{F}_{22} \mathbf{w}_{12} + \mathbf{F}_{23} \mathbf{w}_{13} \]  
(3-29)  

\[ \mathbf{p}_p^3 = \mathbf{F}_{31} \mathbf{w}_{11} + \mathbf{F}_{32} \mathbf{w}_{12} + \mathbf{F}_{33} \mathbf{w}_{13} \]  

By denoting the matrix of the coefficients for the sphere and plate, respectively, as \( \mathbf{F}_s \) and \( \mathbf{F}_p \), one can express (3-28) and (3-29) in matrix form as follows,

\[ \mathbf{F}_s \{ \mathbf{w} \} = \{ \mathbf{p}_s \} \]  
(3-30)  

\[ \mathbf{F}_p \{ \mathbf{w} \} = \{ \mathbf{p}_p \} \]  
(3-31)
Where, \( \{W_e\} \) is the column matrix of the radiant emittance whose elements are the \( W_{ek} \) \( (k = 1,2,3) \) values of the three regions. \( \{P^s\} \) and \( \{P^p\} \) are column matrices whose elements are the individual power measurements by the spherical and flat radiometers, respectively.

The mathematical expressions presented up to this point are common to both of the techniques developed at Drexel. From this point on the two techniques proceed by different paths. The fundamentals of these two techniques will be broadly discussed in the next paragraphs before proceeding to the next sections in which each of the techniques and the results obtained by applying them are discussed in detail.

**Instantaneous/inversion technique.** In this technique, the number \( m \) of observations matches the number of regions \( n \) (i.e., the \( n \) unknown values of \( W_e \)), and hence, the solutions are unique. It is termed instantaneous since the procedure is implemented by taking all the \( m=n \) observations during one single pass or orbit of the satellite. Several sets of instantaneous results obtained at different time intervals can be grouped together and weight-averaged over the complete time period comprising the total number of observations. The weights to be used are the configuration factors of those region segments appearing within each of the FOV's, or the areas of the region segments.

Therefore, in this procedure, the solution to the \( n \) simultaneous equations of the type shown in the set of equations (3-22) for \( n=3 \) can be accomplished by inverting the configuration factor matrix. The inverted matrix then, when multiplied by the column matrix of the \( n \) power measurements \( P_j \) yields the \( n W_{ek} \) values one seeks.
Best-fit/inversion technique. In this technique, the number of observations $m$ is larger than the number of regions $n$ (i.e., the $n$ unknown values of $W_e$). The $n$ simultaneous equations required to solve the problem are obtained by using an extension of the method of least squares. These $n$ simultaneous equations are solved by using a matrix inversion subroutine. The $n W_{e_k}$ values determined in this manner represent the partial coefficients of a surface which best fits the data resulting from the observations. These $W_e$ represent a mean atmospheric situation portraying the overall condition for a significantly large time period (e.g., a month), and no instantaneous results are ever obtained from application of this technique. Here again, each of the $m$ observations will be represented by an expression similar to those shown in the set of equations (3-26) or (3-27).

The details of this procedure are presented by means of an illustration in the section entitled "Best Fit/Inversion Techniques."
INSTANTANEOUS/INVERSION TECHNIQUE

A mathematical technique for computing the value of $W_e$ will first be presented. Afterwards, the technique will be tested by applying it to a simplified case of a flat E-A system.

Mathematical Development

The set of simultaneous equations (3-28) for the case of $n$ unknowns can be written in matrix form as follows:

$$\{P\} = F \{W_e\} \quad (4-1)$$

where

$$\{P\} = \begin{bmatrix} p_1 \\ p_2 \\ \vdots \\ p_n \end{bmatrix}$$

is the column matrix whose elements are the $n$ power measurements,

$$\{W_e\} = \begin{bmatrix} W_{e1} \\ W_{e2} \\ \vdots \\ W_{en} \end{bmatrix}$$

is the column matrix whose elements are the hypothetical values of $W_e$ for each of the $n$ regions,

$$F = \begin{bmatrix} F_{11} & F_{12} & \cdots & F_{1n} \\ F_{21} & F_{22} & \cdots & F_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ F_{n1} & F_{n2} & \cdots & F_{nn} \end{bmatrix}$$

is the $n$ by $n$ configuration factor matrix whose elements $F_{jk}$ are given by (3-18), and which can be written as

$$F = \begin{bmatrix} F_{11} & F_{12} & \cdots & F_{1n} \\ F_{21} & F_{22} & \cdots & F_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ F_{n1} & F_{n2} & \cdots & F_{nn} \end{bmatrix} \quad (4-2)$$

$F^{-1}$, the inverse of $F$, is then computed in order to solve equations (4-1) for $W_e$ by operating with $F^{-1}$ on $\{P\}$, that is,

$$F^{-1} \{P\} = \{W_e\} \quad (4-3)$$
So far, it has been tacitly assumed that \( F \) and \( \{P\} \) are exact (i.e., do not contain errors). Now, however, it will be assumed that the observations \( \{P\} \) include uncertainties \( \{\delta P\} \).

Then, the actual equation to be solved is not the exact equation (4-1) but the perturbed equation (reference 6)

\[
F \{W'_e\} = \{P + \delta P\} \tag{4-4}
\]

where \( \{W'_e\} \) has been introduced to represent the exact solution of the perturbed equation (4-4) as opposed to \( \{W_e\} \) which is the exact solution of the exact equation (4-1). One can then write

\[
\{W'_e\} = \{W_e + \delta W_e\} \tag{4-5}
\]

Again, using the inverse of \( F \) one obtains from (4-4) and (4-5),

\[
F^{-1}\{P + \delta P\} = \{W_e + \delta W_e\} \tag{4-6}
\]

or,

\[
F^{-1}\{P\} + F^{-1}\{\delta P\} = \{W_e\} + \{\delta W_e\} \tag{4-7}
\]

Subtracting (4-3) from (4-7) one obtains

\[
F^{-1}\{\delta P\} = \{\delta W_e\} \tag{4-8}
\]

If the elements of the error matrix \( \{\delta W_e\} \) are large for small error elements \( \{\delta P\} \), the perturbed equation (4-4) is known as ILL-CONDITIONED, the INVERSE MATRIX \( F^{-1} \) is termed UNSTABLE, and the matrix \( F \) is called ILL-CONDITIONED (references 6 and 7).

If the elements of \( \{\delta W_e\} \) are small, or acceptable according to some
prescribed accuracy requirements, then expression (4-8) has solved the problem of retrieving the values of \( W_e \), and nothing else needs to be done. On the other hand, if the resulting elements of \( \{ \delta W \} \) do not meet the accuracy requirements for a given set of small uncertainties \( \{ \delta P \} \), then the problem has just begun. This is the situation that will be treated in the following paragraphs.

Let it be assumed that the elements of \( \{ \delta W \} \) are unacceptable, but that the unstable matrix \( F^{-1} \) can somehow be modified to become a stable matrix \( \~{F}^{-1} \), which, when operating on \( \{ \delta P \} \), yields a set of new errors in \( W_e \) which are now acceptable. These errors \( \~{\delta W} \) are the elements of the new column matrix \( \{ \~{\delta W} \} \) resulting from the following equations.

\[
\~{F}^{-1}(P + \delta P) = \{ W_e + \~{\delta W} \}
\]

(4-9)

or,

\[
\~{F}^{-1}(P) + \~{F}^{-1}(\delta P) = \{ W_e \} + \{ \~{\delta W} \}
\]

(4-10)

But it is known that \( \{ W_e \} \) is related to the exact power column matrix \( \{ P \} \) by (4-3). That is

\[
\{ W_e \} = F^{-1} \{ P \}
\]

(4-11)

Subtracting (4-11) from (4-10) one obtains,

\[
\~{F}^{-1}(P) - F^{-1}(P) + \~{F}^{-1}(\delta P) + \{ \~{\delta W} \}
\]

(4-12)

or

\[
[\~{F}^{-1} - F^{-1}] \{ P \} + \~{F}^{-1} \{ \delta P \} = \{ \~{\delta W} \}
\]

(4-13)
Equation (4-13) shows that the new inverse matrix can contribute in the following two ways to the value of \( \delta W_e \).

(a) The \( n \) by \( n \) matrix resulting from subtracting the old inverse matrix from the new one appears multiplying the power matrix \( \{ P \} \).

(b) The new \( n \) by \( n \) inverse matrix \( F^{-1} \) multiplies the power uncertainty matrix \( \{ \delta P \} \).

In the subsection entitled "Matrix Stabilization," a detailed account of a scheme found to stabilize the inverse matrix \( F^{-1} \) is given. In that subsection is also explained the physical significance of modifying the unstable inverse matrix.

In order to implement this technique, a computer program would be required for performing the numerical integrations, matrix inversions, matrix stabilizations, and tests that would be deemed necessary. Furthermore, another computer program would be required for dividing the surface of the spherical earth-atmosphere system into a finite number of area elements. The output data of the latter program would be used in the former for accomplishing the numerical integrations.

Nevertheless, before engaging in writing these large and sophisticated computer programs, it was decided to subject the technique to a test. By considering a hypothetical flat earth-atmosphere system, some of the equations, as well as the overall computer program could be greatly simplified. On the other hand, the procedure developed to stabilize the inverses of the matrices would be rigorously tested. In the next subsection, the flat E-A system is discussed.
Flat Earth Application

By considering the E-A system to be flat rather than spherical, the task was greatly simplified and less time consuming, while the results obtained can be interpreted as a representation of actual physical situations in a spherical system. This flat E-A system was assumed to be a rectangle of 360° of longitude by 180° of latitude. The longitude is measured westward (to the left) from 0°, at a hypothetical Greenwich meridian, to 360°. The latitude is measured northward (upward) from 0°, at a hypothetical equator, to 90°; and southward (downward) from 0° to -90°. It was found convenient, for interpreting some of the results, to consider the flat E-A system to be tangential to the spherical E-A system at the SSP, as shown in figure 4-1. At any point in this flat E-A system, one degree of longitude is the same as one degree of latitude and each is equal to 100 km. The surface area of this system was divided into a total of 2592 equal area elements ΔA of 5° by 5°. The area of an area element is given by ΔA = 500 km x 500 km = 250,000 km² = 2.5x10¹¹ m². The surface area of the system was also divided into 162 regions of 20° by 20°, or 2000 km by 2000 km. It is to be noted that the area elements are used to perform the numerical or geometrical summations or integrations. Each area element ΔA is identified by the longitude and latitude of its centroid, and hypothetical values of $W_e$ and albedo can be assigned to ΔA. However, in the present report only $W_e$ is included. Direct and reflected SWR will be discussed in a subsequent report. A region is characterized by a representative value of $W_e$ (and/or $W_r(0)$), and variations of $W_e$ (or $W_r(0)$) within it are negligibly small or undetectable by a radiometer.

The radius $r$ of that area of the flat E-A system within the FOV of the radiometer can be quickly calculated with the aid of figure 4-1. From this figure, it can be seen that
Figure 4-1. Flat E-A system parameters.
\[ \alpha_m = \sin^{-1}\left(\frac{R}{R + H}\right) \]  

(4-14)

and,

\[ \rho_m = H \tan \alpha_m \]  

(4-15)

Hence,

\[ \rho_m = H \tan \left[\sin^{-1}\left(\frac{R}{R + H}\right)\right] \]  

(4-16)

For a height \( H = 800 \) km, \( \alpha_m = 62.74^\circ \), and

\[ \rho_m (H=800 \text{ km}) = 1550 \text{ km} \]  

(4-17)

Since in the flat E-A system \( 1^\circ = 100 \) km, one has

\[ \rho_m (H=800 \text{ km}) = 15.5^\circ \]  

(4-18)

As shown in figure 4-1, \( \rho \) is the distance between the SSP and the centroid of the area element \( \Delta A \) under consideration. Therefore, according to the criterion previously discussed for deciding if a given \( \Delta A_i \) should be considered within the FOV or not, one should compare the corresponding value \( \rho_i \) with \( \rho_m \). If \( \rho_i < \rho_m \), \( \Delta A_i \) is considered to be within the FOV. Therefore, one can write into the computer program a simple scanning scheme for testing all area elements within the area of interest. Of course, the total of 2592 \( \Delta A \)'s can be scanned each time a particular \( \Delta A \) is considered for inclusion in the computations, but computer time is saved if one can program the boundaries of an area somewhat larger than the area of interest. This point is illustrated in detail in the following exercise which was actually used to evaluate the validity of the technique being discussed.
In this subsection, a spherical radiometer is simulated in a straight line trajectory 800 km above the E-A system. A horizontal flat circular radiometer will be assumed to be coincident with the spherical radiometer at all times and hence the surface area of the E-A system intercepted by both FOV's will be identical. Arbitrarily, six regions and six satellite positions have been selected which are portrayed in figure 4-2. Although a total of fifteen regions are shown in this figure, only six of them are observed at one time or another by the satellites; and hence, only the $W_e$ values for these six regions needed to be shown. Nevertheless, the $W_e$ values of eight regions were actually included. The six satellite positions are identified by numbers one through six as the satellites travel from south to north in their common trajectory. The longitude and latitude of the SSP for each observation is shown in parentheses next to the number of the observation. The longitudes and latitudes of the centroids of all area elements can be easily found by referring to the longitudes and latitudes indicated at the margins. For example, the longitudes and latitudes of the centroids of the two area elements positioned north and south of SSP No. 4 are as follows.

North: long. 22.5°, lat. +12.5°. South: long. 22.5°, lat. +7.5°. All the data pertaining to those area elements contained within the eight regions bounded by longitudes 0° and 40°, and by latitudes −20° and 60° were fed into the computer program generated to test the technique.

In order to illustrate how the results for the situation portrayed in figure 4-2 were obtained, one may consider the area element ΔA(12.5, 32.5), that is the area element whose centroid has longitude 12.5° and latitude 32.5°. The perimeter of this area element is shown marked with a broken line in figure 4-2. For purposes of this illustration, satellite position No. 6 which has a SSP at 24° longitude and 20° latitude is selected. The first
Figure 4-2. Six Satellite FOV's over six regions of the flat E-A system.
step is to compute \( \rho_6 (12.5, 32.5) \) for this situation and compare it with \( \rho_{6m} \) to determine if \( \Delta A (12.5, 32.5) \) is or is not within the FOV of the satellites at position No.6. In this notation the \( \rho_6 (12.5, 32.5) \) means the distance (in m) between SSP No.6 and the centroid of \( \Delta A (12.5, 32.5) \); while \( \rho_{6m} \) means the radius of the FOV for SSP No.6, which is the maximum value \( \rho \) can attain for observation No.6.

As shown in the previous subsection for \( H = 800 \text{ km} \), \( \rho_{6m} = 1.55 \times 10^6 \text{ m} \).

Using primes to indicate the distances in degrees, one calculates \( \rho'_6 (12.5, 32.5) \) as follows,

\[
\rho' = \left( \Delta_{\text{long}}^2 + \Delta_{\text{lat}}^2 \right)^{\frac{1}{2}}
\]

where,

\[
\Delta_{\text{long}}^2 = (12.5 - 24.)^2 = (-11.5)^2 = 132.25
\]

\[
\Delta_{\text{lat}}^2 = (32.5 - 20)^2 = (12.5)^2 = 156.25
\]

\[
\rho' = \text{distance from SSP to } \Delta A \text{ in degrees}
\]

Then,

\[
\rho'_6 = (288.5)^{\frac{1}{2}} = 17.0^0; \rho'_{6m} = 15.5^0
\]

Hence, \( \rho'_6 > \rho'_{6m} \) which means that the area element \( \Delta A (12.5, 32.5) \) is NOT within the FOV of the radiometer at position No.6.

In order to calculate the shape factor and the power increment contributed by an area element (which must lie within the FOV of the satellites) an area element lying inside the FOV of observation No.1 is selected. This element is \( \Delta A (7.5, -12.5) \) and its boundaries are also marked with a broken line in figure 4-2. Since the whole trajectory is at a height of 800 km, the value of \( \rho_m \) for position No.1 is the same as No.6, i.e., \( \rho'_{1m} = 15.5^0 \). The calculation...
of \( \rho_1' (7.5, -12.5) \) is then,

\[
\begin{align*}
\Delta_{\text{long}}^2 &= (7.5 - 19)^2 = 132.25 \\
\Delta_{\text{lat}}^2 &= (-12.5 - (-5))^2 = 56.25 \\
\rho_1' &= (188.5)^{1/2} = 13.7^\circ
\end{align*}
\]  

(4-21)

Thus, \( \rho_1' < \rho_{1m} \) and \( \Delta A (7.5, -12.5) \) contributes radiant power to the radiometers of the satellites at position No.1.

In order to simplify the computations of this run, the radiation field was assumed isotropic. This means that in equations (3-18) and (3-19),

\[
f(\theta_1) = 1 \quad \text{and} \quad I(f) = \frac{1}{2}
\]

Hence, for a spherical satellite the shape factor for the \( i \)-th area element in the \( k \)-th region and within the FOV of the \( j \)-th observation is

\[
F_{ijk}^s = \frac{\Delta A}{\pi} \frac{\cos \theta_{ijk}}{d_{ijk}^2}
\]

(4-22)

From figure 4-1, one obtains for the flat E-A system,

\[
\cos \theta_{ijk} = \frac{H}{d_{ijk}}
\]

(4-23)

\[
d_{ijk}^3 = (H^2 + \rho_{ijk}^2)^{3/2}
\]

Hence,

\[
F_{ijk}^s = \frac{\Delta A}{\pi} \frac{H}{(H^2 + \rho_{ijk}^2)^{3/2}}
\]

For the case under consideration \( \rho_{ijk} = 13.7 \times 10^5 \text{m} \), and

\[
\Delta A = 2.5 \times 10^{11} \text{m}^2
\]

\[
H = 8 \times 10^5 \text{m}
\]

Then,

(4-24)
By (3-24) all the shape factors of the area elements within region No. 1, and within the FOV of SSP No. 1 are added to yield the first element $F_{11}^S$ of the configuration factor matrix. The value of this element was calculated by the computer program TARA 7-15-1, and its value, shown in figure 4-3, is

$$F_{11}^S = 0.484847428 \quad (4-25)$$

Then, according to (3-17) for a spherical radiometer with a characteristic area $A_s = \pi a^2 = 1 \text{ m}^2$, one can write for the power $P_{11}^S$ which region No. 1 contributes to observation No. 1

$$P_{11}^S = F_{11}^S W_{el} \quad (4-26)$$

From figure 4-2, $W_{el} = 236.0 \text{ W/m}^2$. Hence,

$$P_{11}^S = 114.4239930 \text{ W} \quad (4-27)$$

In a similar manner, the radiant power contributed by regions 2, 3, and 4 are computed and added to get the total radiant power intercepted by the sphere at position No. 1. It is noted that since regions 5 and 6 do not appear within the FOV, their contributions are zero. Then, one writes,

$$P_1^S = \Delta P_{11}^S + \Delta P_{12}^S + \Delta P_{13}^S + \Delta P_{14}^S \quad (4-28)$$

$$P_1^S = F_{11}^S W_{el} + F_{12}^S W_{e2} + F_{13}^S W_{e3} + F_{14}^S W_{e4} \quad (4-29)$$

This power was calculated by the computer program and its value, shown in figure 4-3, is

$$P_1^S = 262.892068914 \text{ W} \quad (4-30)$$
An identical procedure would be followed in order to obtain $P^p_1$ for the horizontal flat circular satellite. There is only one difference between the two calculations which can be seen from comparing the expressions for the shape factor of the sphere (3-18) and for the plate (3-19). The latter has an additional factor, namely, $\cos \alpha_{ijk}$. The angle $\alpha_{ijk}$ is the nadir angle of $\Delta A_{ijk}$ measured at the satellite, and $\cos \alpha_{ijk}$ is used in order to account for the projection of the plate area onto a plane perpendicular to the direction of the radiance $N_{ijk}$ (refer to figure 3-3). In the flat E-A system, $\alpha_{ijk} = \theta_{ijk}$, and hence, $\cos \alpha_{ijk} = \cos \theta_{ijk}$. Thus, from (3-19), the expression for the shape factor for the plate, assuming an isotropic radiation field, is

$$P^p_{ijk} = \frac{\Delta A}{\pi} \frac{\cos^2 \theta_{ijk}}{d_{ijk}^2}$$

(4-31)

The values of the variables in this expression are the same as for the sphere. Using the value of $\cos \theta_{ijk}$ given by (4-23) then, one has,

$$P^p_{ijk} = \frac{\Delta A}{\pi} \frac{H^2}{d_{ijk}^4}$$

(4-32)

$$P^p_{ijk} = 0.008$$

(4-33)

In this case this value is one half that of the sphere; however, this ratio varies for different area elements as would be expected. The area element here considered is at the limb where $\cos \alpha_{ijk}$ attains its smallest value. For an area element at the nadir position, $\cos \alpha_{ijk} = 1$, and the shape factors for both sphere and plate have identical values.

Again here, adding all the contributions from region No.1 to the plate in position No.1, one obtains the value of $P^p_{11}$ shown in figure 4-4, that is,
Then, again, according to (3-17) for horizontal flat circular radiometer with a characteristic area \( A_s = \pi a^2 = \text{lm}^2 \) one can write for the power \( P_{11}^P \) which region No.1 \( (W_{e1} = 236.0 \text{ w/m}^2) \) contributes to observation No.1

\[
P_{11}^P = 0.378000655 \quad (4-34)
\]

Again here, the powers contributed by regions 2, 3, and 4 are calculated in a similar manner and added in order to obtain the total radiant power intercepted by the plate at position No.1. Since the FOV's of the sphere and plate coincide, regions 5 and 6 must not appear in the FOV of the plate at position No.1 either. The expression for \( P_{1}^P \) equivalent to (4-29) is

\[
P_{1}^P = \sum_{k=1}^{4} F_{lk} W_{ek} \quad (4-36)
\]

The value of \( P_{1}^P \) calculated by the computer program TARA 7-15-1 is shown in figure 4-4; this value is

\[
P_{1}^P = 190.073940996 \text{ W} \]

In the previous paragraphs, it has been shown in detail how to calculate the elements of the configuration factor matrices \( F \) and power column matrices \( \{P\} \) for spherical and horizontal flat circular radiometers. The elements of the column matrix \( \{W_e\} \) were assumed known in order to simulate and compute the radiant power observed by both satellites. In this manner, the elements of the column matrices \( \{P\} \) for both radiometers were calculated by the computer program. Figures 4-3 and 4-4 show all the elements of the square \( F \) matrix and of the two column matrices \( \{W_e\} \) and \( \{P\} \). These figures show the matrix products for the sphere and plate, which can be written symbolically
Figure 4-3. Matrix product computed for the sphere.
<table>
<thead>
<tr>
<th>$\text{COFHOR}(K,1)$</th>
<th>$\text{COFHOR}(K,2)$</th>
<th>$\text{COFHOR}(K,3)$</th>
<th>$\text{COFHOR}(K,4)$</th>
<th>$\text{COFHOR}(K,5)$</th>
<th>$\text{COFHOR}(K,6)$</th>
<th>$\text{We}$</th>
<th>$\text{PHORIZ}(k)$</th>
</tr>
</thead>
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<tr>
<td>0.378000855</td>
<td>0.279486438</td>
<td>0.078892673</td>
<td>0.063693275</td>
<td>0.000000000</td>
<td>0.000000000</td>
<td>236.0</td>
<td>190.073940996</td>
</tr>
<tr>
<td>0.200205230</td>
<td>0.200205230</td>
<td>0.200205230</td>
<td>0.200205230</td>
<td>0.000000000</td>
<td>0.000000000</td>
<td>238.0</td>
<td>191.396199412</td>
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<td>0.000000000</td>
<td>0.000000000</td>
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<td>193.360972805</td>
</tr>
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<td>0.251441001</td>
<td>0.471341713</td>
<td>0.008805215</td>
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<td>242.0</td>
<td>187.635182979</td>
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<td>0.000000000</td>
<td>0.172299495</td>
<td>0.473440034</td>
<td>0.040187252</td>
<td>0.091648780</td>
<td>244.0</td>
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<td>0.000000000</td>
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<td>0.312709653</td>
<td>0.087326867</td>
<td>0.312709653</td>
<td>246.0</td>
<td>194.868514638</td>
</tr>
</tbody>
</table>

Figure 4-4. Matrix product computed for the plate.
as

\[ \{P\} = F \{W_e\} \quad (4-37) \]

The first test to which the technique was subjected is the following. Consider the error-free power measurements contained in the column matrices entitled PSPHER(k) and PHORIZ(k) in figures 4-3 and 4-4 respectively. Compute the six values of \( W_e \) by operating with the inverses of the configuration factor matrices on the power column matrices as indicated by equation (4-3), that is, by using

\[ F^{-1}\{P\} = \{W_e\} \quad (4-38) \]

For the case in which the observations are free of uncertainties, the six values of \( W_e \) retrieved should be the same as the six hypothetical values of \( W_e \) originally given.

The inverses \( F^{-1} \) of the original configuration factor matrices for the sphere and plate are presented in figures 4-5 and 4-6, respectively. It should be noted that the inverses appear in the transposed form which is the order in which the computer, ordinarily, prints out matrices.

The six \( W_e \) values obtained when performing the matrix multiplications (for the sphere and the plate) given symbolically by (4-38) were identical to the six original hypothetical \( W_e \) values to at least eight decimal places, for both satellites.

The technique was then subjected to the following test. The observations were assumed to include gaussian random uncertainties. The procedure followed and the results obtained are described in detail in the next subsection.
| .126406637E 02 | -.136852019E 02 | .128539834E 01 | -.240860110E 00 | -.093927101E 01 | .210357918E 01 |
| -.299509655E 02 | .403290804E 02 | -.137579890E 02 | .699220887E 01 | -.191839507E 02 | .386080888E 01 |
| .322632341E 02 | -.463025861E 02 | .339027825E 02 | -.198634305E 02 | .103020142E 03 | -.252470553E 02 |
| -.195404147E 02 | .286083638E 02 | -.286083638E 02 | .195404147E 02 | -.145767511E 03 | .379092813E 02 |
| .491022939E 01 | -.718887652E 01 | .718887652E 01 | -.491022930E 01 | .833969093E 02 | -.249340703E 02 |
| -.287768318E-01 | .421310440E-01 | -.421310441E-01 | .287768319E-01 | -.163760570E 02 | .778493063E 01 |

Figure 4-5. Inverse of original configuration factor matrix, in transposed form, for the sphere.
\begin{table}
\begin{tabular}{cccccc}
.131424995E 02 & -.146981203E 02 & .269545375E 01 & -.113983297E 01 & .272309581E 01 & -.373343722E 00 \\
-.292298882E 02 & .422074149E 02 & -.157270628E 02 & .774441065E 01 & -.445058032E 02 & .907613661E 01 \\
.323672225E 02 & -.498062519E 02 & .378035854E 02 & -.203645559E 02 & .152739147E 03 & -.328461452E 02 \\
-.201552571E 02 & .315809718E 02 & -.315809718E 02 & .201552571E 02 & -.209818167E 03 & .472575341E 02 \\
.499057827E 01 & -.781966270E 01 & .781966270E 01 & -.499057827E 01 & .120475873E 03 & -.308370566E 02 \\
-.264366054E 00 & .414231230E 00 & -.414231229E 00 & .264386054E 00 & -.228348995E 02 & .942600877E 01 \\
\end{tabular}
\end{table}

Figure 4-6. Inverse of the original configuration factor matrix, in transposed form, for the plate.
Perturbed Observations

In order to generate the gaussian random errors needed toperturb the six radiant power measurements, a computer program using two subroutines (RANDU and GAUSS) was generated. The sigma value of the gaussian distribution is \( \sigma = 0.5 \text{ W/m}^2 \). The gaussian errors \( \varepsilon \) were added to the power values originally computed (i.e., the unperturbed powers) in order to obtain the gaussian-perturbed power matrix \( \{ P_g \} \), that is,

\[
\{ P \} + \{ \varepsilon \} = \{ P + \varepsilon \} = \{ P_g \} \tag{4-39}
\]

where

\[
P_g = \text{Gaussian perturbed power elements of the new matrix.}
\]

The test to which the technique will now be subjected consists in carrying out the multiplication of the configuration factor matrix by the new column power matrix \( \{ P_g \} \). This multiplication is similar to that in (4-38), except that in the latter, the original power column matrix \( \{ P \} \) is used instead of \( \{ P_g \} \). Hence, the old elements of \( \{ W_e \} \) in (4-38) will be modified by uncertainties \( \varepsilon_{wg} \) resulting from the uncertainties \( \varepsilon \) in the elements of \( \{ P_g \} \). That is, in matrix form,

\[
\{ W_e \} + \{ \varepsilon_{wg} \} = \{ W_e + \varepsilon_{wg} \} = \{ W_{eg} \} \tag{4-40}
\]

and the matrix product to be calculated can be written as

\[
F^{-1} \{ P_g \} = \{ W_{eg} \} \tag{4-41}
\]

Where \( F^{-1} \) are the inverse matrices given in transposed form for the sphere and the plate in figures (4-5) and (4-6), respectively. The first three results obtained by adding Gaussian random perturbations to the exact power
measurements are listed in TABLE 4-1. There are three sets of data in this table. For each set, the gaussian error for each of the regions appears on the first row of the set. The second row contains the uncertainties $e_{wg}$ in the values of $W_e$ for the six regions, resulting from the perturbations $e$ of the power measurements of the spherical radiometer. Similarly, the third row contains the corresponding uncertainties $e_{wg}$ in the values of $W_e$ for the six regions, for the flat radiometer. Each of the quantities in the last column represents the root-mean-square (rms) values of the six errors shown in the corresponding row. Two important facts are to be noticed from the last column of TABLE 4-1; these are

(a) The plate consistently exhibits a larger error than the sphere.

(b) The results of the matrix inversion are completely unacceptable when the observations include gaussian uncertainties.

At this point, based on the results stated in item (b) above, the applicability of this technique was questioned and therefore, it was decided to investigate the possibility of improving the situation. However, before going into a discussion of how the problem was solved, it will prove helpful to introduce now the results obtained when the observations included systematic uncertainties only, as well as when combinations of systematic and gaussian uncertainties were included.

TABLE 4-2 shows the errors $e_{ws}$ in $W_e$ when systematic errors $e_s$ were added to the power elements of $\{P\}$. In this table, two sets of data are presented. Each set has in the first row, the systematic errors $e_s$ for the six regions. The second and third rows, as before, contain the errors $e_{ws}$ in $W_e$ for the sphere and plate, respectively. Systematic errors $e_s$ of equal magnitude and opposite sign produce errors $e_{ws}$ of equal magnitude but opposite sign.

Two conclusions can be made from TABLE 4-2: (a) Systematic errors in
TABLE 4-1. Results obtained from observations having gaussian uncertainties.

<table>
<thead>
<tr>
<th>Set No.</th>
<th>Type of Error</th>
<th>Region 1</th>
<th>Region 2</th>
<th>Region 3</th>
<th>Region 4</th>
<th>Region 5</th>
<th>Region 6</th>
<th>rms of All Columns</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Gaussian, $\varepsilon_g^*$</td>
<td>- 1.1430</td>
<td>- 0.3780</td>
<td>0.0730</td>
<td>0.7630</td>
<td>- 0.3030</td>
<td>0.7480</td>
<td>0.6693</td>
</tr>
<tr>
<td></td>
<td>Sphere, $\varepsilon_{wg}$*</td>
<td>- 17.1903</td>
<td>21.0556</td>
<td>- 17.8317</td>
<td>12.6009</td>
<td>- 126.0356</td>
<td>36.5961</td>
<td>55.4374</td>
</tr>
<tr>
<td></td>
<td>Plate, $\varepsilon_{wg}$*</td>
<td>- 18.6985</td>
<td>23.9852</td>
<td>- 21.1519</td>
<td>13.9772</td>
<td>- 188.8153</td>
<td>47.0500</td>
<td>81.0685</td>
</tr>
<tr>
<td>2</td>
<td>Gaussian, $\varepsilon_g$</td>
<td>- 0.6290</td>
<td>- 0.0900</td>
<td>0.0220</td>
<td>1.1540</td>
<td>0.0010</td>
<td>0.5750</td>
<td>0.5869</td>
</tr>
<tr>
<td></td>
<td>Sphere, $\varepsilon_{wg}$</td>
<td>- 27.1069</td>
<td>36.9908</td>
<td>- 31.8555</td>
<td>21.6465</td>
<td>- 169.1907</td>
<td>45.9727</td>
<td>75.6367</td>
</tr>
<tr>
<td></td>
<td>Plate, $\varepsilon_{wg}$</td>
<td>- 28.3301</td>
<td>41.0255</td>
<td>- 36.1231</td>
<td>22.9781</td>
<td>- 249.4868</td>
<td>58.6197</td>
<td>108.0112</td>
</tr>
<tr>
<td>3</td>
<td>Gaussian, $\varepsilon_g$</td>
<td>- 0.6060</td>
<td>- 0.0060</td>
<td>- 0.5100</td>
<td>- 0.1780</td>
<td>- 0.0350</td>
<td>0.3750</td>
<td>0.3654</td>
</tr>
</tbody>
</table>

* $\varepsilon_g$ is in watts
** $\varepsilon_{wg}$ is in w/m².
TABLE 4-2. Results obtained from observations having systematic errors.

<table>
<thead>
<tr>
<th>Set No.</th>
<th>Type of Error</th>
<th>Region 1</th>
<th>Region 2</th>
<th>Region 3</th>
<th>Region 4</th>
<th>Region 5</th>
<th>Region 6</th>
<th>rms of All Columns</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Systematic, $\epsilon_g^*$</td>
<td>-0.3</td>
<td>-0.3</td>
<td>-0.3</td>
<td>-0.3</td>
<td>-0.3</td>
<td>-0.3</td>
<td>0.3</td>
</tr>
<tr>
<td></td>
<td>Sphere, $\epsilon_{ws}^{**}$</td>
<td>0.0882</td>
<td>0.5409</td>
<td>0.0094</td>
<td>0.4641</td>
<td>0.5549</td>
<td>0.4432</td>
<td>0.4123</td>
</tr>
<tr>
<td></td>
<td>Plate, $\epsilon_{ws}$</td>
<td>0.2552</td>
<td>0.5636</td>
<td>0.1789</td>
<td>0.5007</td>
<td>0.3662</td>
<td>0.5109</td>
<td>0.4205</td>
</tr>
<tr>
<td>2</td>
<td>Systematic, $\epsilon_g$</td>
<td>0.9</td>
<td>0.9</td>
<td>0.9</td>
<td>0.9</td>
<td>0.9</td>
<td>0.9</td>
<td>0.9</td>
</tr>
<tr>
<td></td>
<td>Sphere, $\epsilon_{ws}$</td>
<td>0.2646</td>
<td>1.6226</td>
<td>-0.0283</td>
<td>1.3922</td>
<td>-1.6648</td>
<td>1.3297</td>
<td>1.2370</td>
</tr>
<tr>
<td></td>
<td>Plate, $\epsilon_{ws}$</td>
<td>0.7657</td>
<td>1.6907</td>
<td>0.5368</td>
<td>1.5022</td>
<td>-1.0987</td>
<td>1.5328</td>
<td>1.2614</td>
</tr>
</tbody>
</table>

* $\epsilon_g$ is in watts  
** $\epsilon_{ws}$ is in w/m²
the observations do NOT produce large errors in $W_e$; and (b) the plate shows larger errors $\varepsilon_{ws}$ than the sphere.

Combinations of systematic and gaussian errors $\varepsilon_{gs}$ were included in the power measurements in order to compute the errors $\varepsilon_{wgs}$ resulting in the $W_e$ values. As expected, these $\varepsilon_{wgs}$ errors exhibited more sensitivity to the gaussian than to the systematic uncertainties. The results obtained are shown in TABLE 4-3. The results in this table again show that the sphere yields better results than the plate when gaussian random uncertainties are included in the power measurements. So far, however, the two radiometers yield acceptable results only when the errors are systematic.

Thus, as can be seen from the data in TABLES 4-1 and 4-3, the gaussian errors are highly magnified through the matrix multiplication of the inverse of the configuration factor matrix and the perturbed power column matrix. This instability of the inverses of both configuration factor matrices (sphere and plate) was corrected as explained in the following section.

Matrix Stabilization

Prior to describing the scheme developed for stabilizing the two matrices, it is necessary to return to the topic of ill-conditioned perturbed equations in order to introduce the concept of the CONDITION NUMBER of a matrix.

Consider the set of $n$ simultaneous equations represented in (4-1) in matrix form as

$$F(W_e) = P$$

where

$$F(W_e) = \{P\} \quad (4-42)$$

4-24
TABLE 4-3. Results obtained from observations having combinations of gaussian and systematic errors.

<table>
<thead>
<tr>
<th>Set No.</th>
<th>Type of Error</th>
<th>Region 1</th>
<th>Region 2</th>
<th>Region 3</th>
<th>Region 4</th>
<th>Region 5</th>
<th>Region 6</th>
<th>rms of All Columns</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Gauss, $\varepsilon^*$</td>
<td>-1.1430</td>
<td>0.3780</td>
<td>0.0730</td>
<td>0.7630</td>
<td>0.3030</td>
<td>0.7480</td>
<td>0.6693</td>
</tr>
<tr>
<td></td>
<td>Systematic, $\varepsilon_s^*$</td>
<td>0.3000</td>
<td>0.3000</td>
<td>0.3000</td>
<td>0.3000</td>
<td>0.3000</td>
<td>0.3000</td>
<td>0.3000</td>
</tr>
<tr>
<td></td>
<td>Total $\varepsilon_{gs}$</td>
<td>0.8430</td>
<td>0.0780</td>
<td>0.3730</td>
<td>1.0630</td>
<td>0.0030</td>
<td>1.0480</td>
<td>0.7170</td>
</tr>
<tr>
<td></td>
<td>Sphere $\varepsilon_{wgs}$ **</td>
<td>17.1021</td>
<td>21.5965</td>
<td>-17.8411</td>
<td>13.0649</td>
<td>126.5905</td>
<td>37.0394</td>
<td>55.7449</td>
</tr>
<tr>
<td></td>
<td>Plate $\varepsilon_{wgs}$</td>
<td>18.4433</td>
<td>24.5488</td>
<td>-20.9730</td>
<td>14.4779</td>
<td>189.1815</td>
<td>47.5609</td>
<td>81.2855</td>
</tr>
<tr>
<td>2</td>
<td>Gauss, $\varepsilon$</td>
<td>-1.1430</td>
<td>0.3780</td>
<td>0.0730</td>
<td>0.7630</td>
<td>0.3030</td>
<td>0.7480</td>
<td>0.6693</td>
</tr>
<tr>
<td></td>
<td>Systematic, $\varepsilon_s$</td>
<td>0.6000</td>
<td>0.6000</td>
<td>0.6000</td>
<td>0.6000</td>
<td>0.6000</td>
<td>0.6000</td>
<td>0.6000</td>
</tr>
<tr>
<td></td>
<td>Total $\varepsilon_{gs}$</td>
<td>0.5430</td>
<td>0.2220</td>
<td>0.6730</td>
<td>1.3630</td>
<td>0.2970</td>
<td>1.3480</td>
<td>0.8718</td>
</tr>
<tr>
<td></td>
<td>Sphere $\varepsilon_{wgs}$</td>
<td>17.0139</td>
<td>22.1374</td>
<td>-17.8506</td>
<td>13.5290</td>
<td>127.1454</td>
<td>37.4826</td>
<td>56.0537</td>
</tr>
<tr>
<td></td>
<td>Plate $\varepsilon_{wgs}$</td>
<td>18.1881</td>
<td>25.1123</td>
<td>-20.7940</td>
<td>14.9786</td>
<td>189.5478</td>
<td>48.0718</td>
<td>81.5040</td>
</tr>
<tr>
<td>3</td>
<td>Gauss, $\varepsilon$</td>
<td>-1.1430</td>
<td>0.3780</td>
<td>0.0730</td>
<td>0.7630</td>
<td>0.3030</td>
<td>0.7480</td>
<td>0.6693</td>
</tr>
<tr>
<td></td>
<td>Systematic, $\varepsilon_s$</td>
<td>0.9000</td>
<td>0.9000</td>
<td>0.9000</td>
<td>0.9000</td>
<td>0.9000</td>
<td>0.9000</td>
<td>0.9000</td>
</tr>
<tr>
<td></td>
<td>Total $\varepsilon_{gs}$</td>
<td>0.2430</td>
<td>0.5220</td>
<td>0.9730</td>
<td>1.6630</td>
<td>0.5970</td>
<td>1.6480</td>
<td>1.0890</td>
</tr>
<tr>
<td></td>
<td>Sphere $\varepsilon_{wgs}$</td>
<td>16.9257</td>
<td>22.6782</td>
<td>-17.8600</td>
<td>13.9931</td>
<td>127.7003</td>
<td>37.9258</td>
<td>56.3639</td>
</tr>
<tr>
<td></td>
<td>Total $\varepsilon_{wgs}$</td>
<td>17.9328</td>
<td>25.6759</td>
<td>-20.6151</td>
<td>15.4793</td>
<td>189.9140</td>
<td>48.5828</td>
<td>81.7242</td>
</tr>
</tbody>
</table>

* $\varepsilon$, $\varepsilon_s$, and $\varepsilon_{gs}$ are in watts  
** $\varepsilon_{wgs}$ is in W/m²
\[ F = n \text{ by } n \text{ configuration factor matrix} \]
\[ \{W_e\} = \text{column matrix of } n W_e \text{ elements} \]
\[ \{P\} = \text{column matrix of } n P \text{ elements} \]

It is possible that these quantities may include errors resulting from either one, or both, of the following (reference 6).

(a) The data (i.e., the observations \(\{P\}\)) are inexact.
(b) Rounding errors are generated during computations.

Samples were run with error-free observations and all the \(W_e\) values retrieved were accurate to at least eight decimal places. Consequently, one is justified in assuming that computational errors have not caused difficulties, and that only errors of type (a) should be treated in this discussion.

The perturbed equation to be solved is of the form given by (4-4), that is,

\[ F\{W'\} = \{P + \delta P\} \]  

(4-43)

where, as indicated in (4-5),

\[ \{W'\} = \{W_e + \delta W_e\} \]  

(4-44)

In this expression, the elements of \(\delta P\) are considered to be either gaussian errors \(\varepsilon_g\), or systematic errors \(\varepsilon_s\), or a combination of both \(\varepsilon_g \varepsilon_s\).

It is assumed for purposes of this section that the perturbed equation (4-43) is ill-conditioned, that is, that the inverse matrix \(F^{-1}\) is unstable. It can be shown that for ill-conditioning of this type the following relationship exists (reference 6).

\[ \text{rel} \{W_e\} = C_2 \cdot \text{rel} \{P\} \]  

(4-45)
where
\[ \text{rel}(W_e) = \frac{||\delta W_e||}{||W_e||} \]  \hspace{1cm} (relative error) \hspace{1cm} (4-46)
\[ \text{rel}(\mathcal{P}) = \frac{||\delta \mathcal{P}||}{||\mathcal{P}||} \]  \hspace{1cm} (relative error) \hspace{1cm} (4-47)
\[ C_2 = ||F|| \cdot ||F^{-1}|| \]  \hspace{1cm} (condition number of \( F \)) \hspace{1cm} (4-48)

The symbol \( || \cdot || \) denotes the column or row norm of the matrix inside it, as will be explained below.

**Condition number.** Even though there are several definitions of the condition number of a matrix (references 6 and 7) only two of them will be introduced in this report.

\[ C_1 = \frac{\text{max} |\lambda_i|}{\text{min} |\lambda_i|} \]  \hspace{1cm} (4-49)
\[ C_2 = ||F|| \cdot ||F^{-1}|| \]  \hspace{1cm} (4-50)

where
\[ \text{max} |\lambda_i| = \text{largest modulus eigenvalue of } F \]
\[ \text{min} |\lambda_i| = \text{smallest modulus eigenvalue of } F \]
\[ ||F|| = \text{Column or row norm of } F \text{ defined, respectively, as (reference 5)} \]
\[ ||F||_1 = \text{max} \sum_j |F_{ij}| \]  \hspace{1cm} (4-51)
\[ ||F||_\infty = \text{max} \sum_i |F_{ij}| \]  \hspace{1cm} (4-52)

For the ill-conditioned situation considered here and represented by the perturbed equation (4-44), it can be seen from (4-46) that ill-conditioning
depends on the size of the condition number $C_2$ and on the relative error $\text{rel } \{F\}$. Thus, for a given set of power errors $\{\delta_\xi\}$, one would expect that if the original ill-conditioned matrices could somehow be transformed into two well-conditioned matrices, the condition numbers of the latter would be smaller than those of the former.

A computer program was used to obtain the eigenvalues of the two configuration factor matrices and to compute the values of $C_1^2$ for both matrices. This program also verified that the eigenvalues computed are correct by making use of the following relations (reference 8)

$$\text{Tr}_F = \sum \lambda_i$$

$$|F| = \prod \lambda_i$$

where,

$$\text{Tr}_F = \text{Trace of the matrix } F$$

$$\lambda_i = \text{ith eigenvalue of } F$$

$$|F| = \text{The determinant of } F$$

The values of the condition numbers $C_2$ for both of the original ill-conditioned matrices were calculated. These values are shown in TABLE 4-4 below.

**TABLE 4-4. Condition numbers of the two original matrices.**

<table>
<thead>
<tr>
<th>Type of Sensor</th>
<th>$C_1$</th>
<th>$C_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spherical</td>
<td>131.6</td>
<td>693.9</td>
</tr>
<tr>
<td>Horizontal flat circular</td>
<td>126.4</td>
<td>684.7</td>
</tr>
</tbody>
</table>
After considerable effort, a scheme was found which rendered both configuration factor matrices well-conditioned and their corresponding values of $C_1$ and $C_2$ became much smaller than those shown in TABLE 4-4.

Stabilization procedure.- Essentially, this procedure consists in translating the smallest elements in the six by six configuration factor matrix. The translation of each element is performed along the row of the element to the position of the diagonal element in that row and it is added to the diagonal element. In this manner, the sum of the elements in the row in question is preserved. This means that the sum of the row elements always adds up to the configuration factor of the total FOV, which is a desirable feature, as will be explained below.

Referring to the two original matrices in figures 4-3 and 4-4, for the sphere and the plate, respectively, one sees that the elements $F_{41}$ and $F_{45}$ are the lowest value elements in both matrices. For the sphere, these two elements have the common value 0.017068977; while for the plate the common value is 0.008805215. In each of the matrices, these two elements were translated and added to the diagonal element $F_{44}$. For the sphere, the original value of this element was 0.595573384; while for the plate, the value was 0.471341713. The new values of $F_{44}$ for the sphere and plate become, respectively, 0.629711338 and 0.488952143. The physical meaning of this element translation can be best seen when evaluating the error it introduces into the power calculations. The translation of an element means that the element will appear multiplying the $W_e$ value of the column corresponding to the diagonal element rather than the $W_e$ value corresponding to the column where the element originally appeared. As an example of the magnitude of the error introduced, the case of element $F_{45}$ is calculated. $W_{e5} = 244.0 \text{ W/m}^2$, and $W_{e4} = 242.0 \text{ W/m}^2$. Hence, $\Delta W_e = 242.0 - 244.0 = -2.0 \text{ W/m}^2$, and the power error $\Delta P$ introduced is approximately $\Delta P = (-2.0)(0.017)$, or, $\Delta P = -0.034 \text{ W}$, which is certainly a negligible error. Physically, this
error is equivalent to saying that the radiometers at position No. 4 looked a little more at the limb of region No. 4 and did not see region No. 5 at all. Figures 4-7 and 4-8 show the new stabilized matrices for the sphere and plate, respectively. The captions of these figures include the title of the computer program (TARA 7-15-1) which stabilized the matrices and then used them to compute the values of $W_e$. The stabilization scheme used in this program establishes lower limits for the magnitudes of the matrix elements for each of the two matrices; any element whose value is below the lower limit assigned to its matrix is to be translated as previously described. The limits determined after several trials were 0.032 for the sphere and 0.016 for the plate. A different computer program (TARA 7-15-2) used different lower bounds as the criteria to carry out the translations. These limits were 0.04 for the sphere and 0.02 for the plate. The results obtained with the latter limits were not as good as those obtained with the former.

The errors in $W_e$ obtained with computer program TARA 7-15-1 by first using the two original ill-conditioned matrices and later using the new well-conditioned matrices are presented in TABLE 4-5. Three different data groups are tabulated there. These groups refer to the first set of data appearing in each of TABLES 4-1, 4-2, and 4-3, as is indicated in the first column of TABLE 4-5. These three groups of data correspond to the three types of uncertainties previously introduced, Gaussian $\epsilon_g$, systematic $\epsilon_s$, and combinations of gaussian and systematic $\epsilon_{gs}$. The type of inverse matrix $F^{-1}$ used (original or stable), as well as the class of radiometer (sphere or plate), is specified for each row of $W_e$ errors.

The striking differences exhibited by the results compared in TABLE 4-5 is indicative of the effect that stabilization of the configuration factor
Figure 4-7. Stabilized matrix for the sphere. Computer program TARA 7-15-1.
<table>
<thead>
<tr>
<th>( \text{COFHOR}(K,1) )</th>
<th>( \text{COFHOR}(K,2) )</th>
<th>( \text{COFHOR}(K,3) )</th>
<th>( \text{COFHOR}(K,4) )</th>
<th>( \text{COFHOR}(K,5) )</th>
<th>( \text{COFHOR}(K,6) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.378000655</td>
<td>0.279486438</td>
<td>0.078892673</td>
<td>0.063693275</td>
<td>0.000000000</td>
<td>0.000000000</td>
</tr>
<tr>
<td>0.200205230</td>
<td>0.200205230</td>
<td>0.200205230</td>
<td>0.200205230</td>
<td>0.000000000</td>
<td>0.000000000</td>
</tr>
<tr>
<td>0.063693275</td>
<td>0.078892673</td>
<td>0.279486438</td>
<td>0.378000653</td>
<td>0.000000000</td>
<td>0.000000000</td>
</tr>
<tr>
<td>0.000000000</td>
<td>0.018591209</td>
<td>0.251441001</td>
<td>0.488952143</td>
<td>0.000000000</td>
<td>0.018591209</td>
</tr>
<tr>
<td>0.000000000</td>
<td>0.000000000</td>
<td>0.172299495</td>
<td>0.473440034</td>
<td>0.040187252</td>
<td>0.091648780</td>
</tr>
<tr>
<td>0.000000000</td>
<td>0.000000000</td>
<td>0.087326867</td>
<td>0.312709653</td>
<td>0.087326867</td>
<td>0.312709653</td>
</tr>
</tbody>
</table>

Figure 4-8. Stabilized matrix for the plate. Computer program TARA 7-15-1.
TABLE 4-5. Comparisons of results of original and stabilized matrices for the first set of data shown in TABLES 3-2, 3-3, and 3-4.

<table>
<thead>
<tr>
<th>Table No.</th>
<th>Type of Error</th>
<th>Type of Matrix</th>
<th>Region 1</th>
<th>Region 2</th>
<th>Region 3</th>
<th>Region 4</th>
<th>Region 5</th>
<th>Region 6</th>
<th>rms of All Columns</th>
</tr>
</thead>
<tbody>
<tr>
<td>3-2</td>
<td>Gauss</td>
<td></td>
<td>1.1430</td>
<td>0.3780</td>
<td>0.0730</td>
<td>0.7630</td>
<td>0.3030</td>
<td>0.7480</td>
<td>0.6693</td>
</tr>
<tr>
<td></td>
<td>Sphere, Original;</td>
<td>$e_g$</td>
<td>17.1903</td>
<td>21.0556</td>
<td>17.8317</td>
<td>12.6099</td>
<td>126.0356</td>
<td>36.5961</td>
<td>55.4374</td>
</tr>
<tr>
<td></td>
<td>Sphere, Stable;</td>
<td>$e_wg$</td>
<td>4.3780</td>
<td>2.2977</td>
<td>0.9263</td>
<td>0.2114</td>
<td>30.4585</td>
<td>11.7397</td>
<td>13.4839</td>
</tr>
<tr>
<td></td>
<td>Plate, Original;</td>
<td>$e_wg$</td>
<td>18.6985</td>
<td>23.9852</td>
<td>21.1519</td>
<td>13.9772</td>
<td>188.8153</td>
<td>47.0500</td>
<td>81.0685</td>
</tr>
<tr>
<td></td>
<td>Plate, Stable;</td>
<td>$e_wg$</td>
<td>6.1246</td>
<td>4.2833</td>
<td>1.4500</td>
<td>1.4032</td>
<td>57.9194</td>
<td>17.5682</td>
<td>24.9106</td>
</tr>
<tr>
<td>3-3</td>
<td>Systematic</td>
<td></td>
<td>0.3</td>
<td>0.3</td>
<td>0.3</td>
<td>0.3</td>
<td>0.3</td>
<td>0.3</td>
<td>0.3</td>
</tr>
<tr>
<td></td>
<td>Sphere, Original;</td>
<td>$e_g$</td>
<td>0.0882</td>
<td>0.5409</td>
<td>0.0094</td>
<td>0.4641</td>
<td>0.5549</td>
<td>0.4432</td>
<td>0.4123</td>
</tr>
<tr>
<td></td>
<td>Sphere, Stable;</td>
<td>$e_wg$</td>
<td>0.1034</td>
<td>0.8243</td>
<td>0.2928</td>
<td>0.6576</td>
<td>1.9990</td>
<td>0.8188</td>
<td>0.9895</td>
</tr>
<tr>
<td></td>
<td>Plate, Original;</td>
<td>$e_wg$</td>
<td>0.2552</td>
<td>0.5636</td>
<td>0.1789</td>
<td>0.5007</td>
<td>0.3662</td>
<td>0.5109</td>
<td>0.4205</td>
</tr>
<tr>
<td></td>
<td>Plate, Stable;</td>
<td>$e_wg$</td>
<td>0.1036</td>
<td>0.8011</td>
<td>0.0586</td>
<td>0.6523</td>
<td>1.9446</td>
<td>0.8664</td>
<td>0.9673</td>
</tr>
<tr>
<td>3-4</td>
<td>Gauss</td>
<td></td>
<td>1.1430</td>
<td>0.3780</td>
<td>0.0730</td>
<td>0.7630</td>
<td>0.3030</td>
<td>0.7480</td>
<td>0.6693</td>
</tr>
<tr>
<td></td>
<td>Systematic</td>
<td>$e_g$</td>
<td>0.3</td>
<td>0.3</td>
<td>0.3</td>
<td>0.3</td>
<td>0.3</td>
<td>0.3</td>
<td>0.3</td>
</tr>
<tr>
<td></td>
<td>Total</td>
<td>$e_g$</td>
<td>0.8430</td>
<td>0.0780</td>
<td>0.3730</td>
<td>1.0630</td>
<td>0.0030</td>
<td>1.0480</td>
<td>0.7170</td>
</tr>
<tr>
<td></td>
<td>Sphere, Original;</td>
<td>$e_wg$s</td>
<td>17.1021</td>
<td>21.5965</td>
<td>17.8411</td>
<td>13.0649</td>
<td>126.5905</td>
<td>37.0394</td>
<td>55.7449</td>
</tr>
<tr>
<td></td>
<td>Sphere, Stable;</td>
<td>$e_wg$s</td>
<td>4.1861</td>
<td>2.6868</td>
<td>1.0686</td>
<td>0.1490</td>
<td>30.2402</td>
<td>11.9819</td>
<td>13.4409</td>
</tr>
<tr>
<td></td>
<td>Plate, Original;</td>
<td>$e_wg$s</td>
<td>18.4433</td>
<td>24.5488</td>
<td>20.9730</td>
<td>14.4779</td>
<td>189.1815</td>
<td>47.5609</td>
<td>81.2855</td>
</tr>
<tr>
<td></td>
<td>Plate, Stable;</td>
<td>$e_wg$s</td>
<td>5.8109</td>
<td>4.7553</td>
<td>1.1795</td>
<td>1.8456</td>
<td>57.6776</td>
<td>17.9422</td>
<td>24.8656</td>
</tr>
</tbody>
</table>
matrices has upon the magnitudes of the uncertainties in $W_e$. For instance the rms values for $\varepsilon_{wg}$ (last column, second and fourth rows) were 55.4374 W/m$^2$ and 81.0685 W/m$^2$ when $W_e$ was computed with the original matrices for the sphere and plate, respectively. However, when the corresponding stable matrices were used, the values of $\varepsilon_{wg}$ were 13.4839 w/m$^2$ and 24.9106 W/m$^2$, respectively, as seen from the results in the third and fifth rows of the last column.

The next question that arises is: how do the condition numbers of the stable matrices compare with those of the two original matrices? In TABLE 4-6, the two condition numbers for each of the four matrices are listed.

TABLE 4-6. Condition numbers of the original and stabilized matrices for the sphere and the plate.

<table>
<thead>
<tr>
<th>TYPE OF SATELLITE</th>
<th>TYPE OF MATRIX</th>
<th>COMPUTER PROGRAM</th>
<th>$C_1$</th>
<th>$C_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sphere</td>
<td>Original (Unstable)</td>
<td>TARA 7-15-1</td>
<td>131.6</td>
<td>693.9</td>
</tr>
<tr>
<td>Sphere</td>
<td>Stabilized</td>
<td>TARA 7-15-1</td>
<td>59.9</td>
<td>223.4</td>
</tr>
<tr>
<td>Sphere</td>
<td>Partially Stable</td>
<td>TARA 7-15-2</td>
<td>71.4</td>
<td>293.6</td>
</tr>
<tr>
<td>Plate</td>
<td>Original (Unstable)</td>
<td>TARA 7-15-1</td>
<td>126.4</td>
<td>684.7</td>
</tr>
<tr>
<td>Plate</td>
<td>Stabilized</td>
<td>TARA 7-15-1</td>
<td>39.0</td>
<td>218.2</td>
</tr>
<tr>
<td>Plate</td>
<td>Partially Stable</td>
<td>TARA 7-15-2</td>
<td>58.6</td>
<td>317.1</td>
</tr>
</tbody>
</table>

In the first and second rows of TABLE 4-6 are presented the condition numbers for the original and stable matrix of the spherical radiometer. The fourth and fifth rows show the corresponding condition numbers for the horizontal flat plate radiometer. These are the four matrices whose results have been discussed up to now and which were used in computer program TARA 7-15-1. However, the third and sixth rows of TABLE 4-6 present the
condition numbers of the two matrices obtained by using slightly different lower bounds (0.04 for the sphere and 0.02 for the plate) as was discussed previously. The results obtained by using these matrices did not meet the accuracy requirements and are termed "partially stable" in TABLE 4-6, and were of no further use. These matrices were generated by, and used in, computer program TARA 7-15-2.

Data Quality Prediction

TABLE 4-5 indicates that the errors in \( W_e \) for some of the regions are much more significant than those for other regions. For instance, region 5 shows consistently larger errors than the remaining regions for both types of satellites and for all matrices. Hence, even though the results obtained with the stable matrix (third and fifth rows of each data group) are much more acceptable than those obtained with the original matrices (second and fourth rows of each data group), not all of those results obtained with the stable matrices appear equally acceptable. At this point, two important pertinent questions need to be answered.

(a) What are the accuracy requirements for \( W_e \)?

(b) Does the form of the original matrix and the magnitude of its elements bear any relation to the errors in \( W_e \) obtained for each of the regions? If this relationship exists, can it be linked to the requirements in (a)?

These two questions are thoroughly dealt with in the following two subsections.

Accuracy requirements. - TABLE 4-7 lists the desired and minimum useful tolerances for each of the quantities to be measured for radiation budget determinations. It is seen from this table that the minimum useful
TABLE 4-7. Accuracy Requirements for Radiation Budget Components.

(Recommendations of investigators conference, 1975)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Accuracy</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solar Intensity</td>
<td>± 1 to 2 W/m²</td>
<td>Monthly</td>
</tr>
<tr>
<td>Solar Spectrum In Ozone Bands Δλ = 50Å</td>
<td>± 10%</td>
<td></td>
</tr>
<tr>
<td>Components For Global Net:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Albedo</td>
<td>± 0.004</td>
<td>Long Term With</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Monthly Resolution</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Desired - Seasonal Is</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Minimum Useful Period</td>
</tr>
<tr>
<td>Longwave Exitance</td>
<td>± 1 W/m²</td>
<td></td>
</tr>
<tr>
<td>Components For Regional Net:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Albedo</td>
<td>± 0.02</td>
<td>Monthly For 10° Of Great Circle Latitude And</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Longitude</td>
</tr>
<tr>
<td>Longwave Exitance</td>
<td>± 3 W/m²</td>
<td></td>
</tr>
<tr>
<td>Medium Resolution Scanning:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Albedo</td>
<td>± 0.04</td>
<td>Monthly Averaged Determined From Scanning Data</td>
</tr>
<tr>
<td></td>
<td></td>
<td>- 10⁴ to 10⁵ km² Spatial Resolution</td>
</tr>
<tr>
<td>Longwave Exitance</td>
<td>± 6 W/m²</td>
<td></td>
</tr>
</tbody>
</table>
accuracy for monthly averages of the longwave exitance \( (W_e) \) of small regions is \( \pm 15 \text{ w/m}^2 \). In the cases presented in TABLE 4-5, however, the values of \( W_e \) are instantaneous for all practical purposes since the time interval during which the measurements were taken is of the order of minutes. Hence, the tolerances for these results should be less stringent than those for monthly averages stated above.

It should be pointed out that the accuracy requirements listed in TABLE 4-7 are the results of recommendations made at the Chicago investigators conference of 1975. On the basis of the above tolerance of \( \pm 15 \text{ w/m}^2 \), one can see that from all the \( W_e \) values obtained with the two stabilized matrices the only ones which are not acceptable are

(a) Those of region 5 resulting from gaussian, or combinations of gaussian/systematic power errors for the spherical satellite.

(b) Those of regions 5 and 6 resulting also from gaussian or gaussian/systematic power errors for the plate.

The possibility of a connection existing between the structure of the original configuration factor matrix and the stability of its inverse matrix was thoroughly investigated and excellent results were obtained as discussed in the following paragraphs.

Matrix Parameters.— The original configuration factor matrices shown in figures 4-3 and 4-4 for the sphere and plate, respectively, are again introduced here in figures 4-9 and 4-10. However, these latter figures display additional information which will be needed in this discussion. Each of the first six quantities in the last column consist of the sums of the elements in their corresponding rows, and should be approximately equal to the shape factor of the total FOV of the observation corresponding to that row. The seventh quantity in the same column is the sum of the first six quantities. When this sum is divided by six, it yields the average shape factor for the total FOV, which is shown in the last row of the last
<table>
<thead>
<tr>
<th>$\odot FSPH(K, 1)$</th>
<th>$\odot FSPH(K, 2)$</th>
<th>$\odot FSPH(K, 3)$</th>
<th>$\odot FSPH(K, 4)$</th>
<th>$\odot FSPH(K, 5)$</th>
<th>$\odot FSPH(K, 6)$</th>
<th>$SPHROW(K)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.484847428</td>
<td>0.384856899</td>
<td>0.127704158</td>
<td>0.108360067</td>
<td>0.000000000</td>
<td>0.000000000</td>
<td>1.105768551</td>
</tr>
<tr>
<td>0.276829292</td>
<td>0.276829292</td>
<td>0.276829292</td>
<td>0.276829292</td>
<td>0.000000000</td>
<td>0.000000000</td>
<td>1.105768551</td>
</tr>
<tr>
<td>0.108360067</td>
<td>0.127704158</td>
<td>0.384856899</td>
<td>0.484847428</td>
<td>0.000000000</td>
<td>0.000000000</td>
<td>1.105768551</td>
</tr>
<tr>
<td>0.017068977</td>
<td>0.035500329</td>
<td>0.355753027</td>
<td>0.595573384</td>
<td>0.017068977</td>
<td>0.035500329</td>
<td>1.056465024</td>
</tr>
<tr>
<td>0.000000000</td>
<td>0.000000000</td>
<td>0.247608706</td>
<td>0.596414856</td>
<td>0.068729515</td>
<td>0.143711947</td>
<td>1.056465024</td>
</tr>
<tr>
<td>0.000000000</td>
<td>0.000000000</td>
<td>0.137012641</td>
<td>0.415871635</td>
<td>0.137012641</td>
<td>0.415871635</td>
<td>1.105768551</td>
</tr>
</tbody>
</table>

**COLUMN SUMS** 0.887105765 0.824890678 1.529764721 2.477896661 0.222811133 0.595083912 6.537552870

**PERCENTAGES, %** 81.416314223 75.706371568 140.397921212 227.415062849 20.449039970 54.615290183 1.089592145

Figure 4-9. Original matrix for the spherical radiometer.
<table>
<thead>
<tr>
<th>COFHOR(K,1)</th>
<th>COFHOR(K,2)</th>
<th>COFHOR(K,3)</th>
<th>COFHOR(K,4)</th>
<th>COFHOR(K,5)</th>
<th>COFHOR(K,6)</th>
<th>HORROW(K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.37800655</td>
<td>0.279486438</td>
<td>0.07892673</td>
<td>0.063693275</td>
<td>0.000000000</td>
<td>0.000000000</td>
<td>0.800073041</td>
</tr>
<tr>
<td>0.200205230</td>
<td>0.200205230</td>
<td>0.200205230</td>
<td>0.200205230</td>
<td>0.000000000</td>
<td>0.000000000</td>
<td>0.800820918</td>
</tr>
<tr>
<td>0.063693275</td>
<td>0.07892673</td>
<td>0.279486438</td>
<td>0.37800655</td>
<td>0.000000000</td>
<td>0.000000000</td>
<td>0.800073041</td>
</tr>
<tr>
<td>0.008805215</td>
<td>0.018591209</td>
<td>0.251441001</td>
<td>0.471341713</td>
<td>0.008805215</td>
<td>0.018591209</td>
<td>0.777575561</td>
</tr>
<tr>
<td>0.000000000</td>
<td>0.000000000</td>
<td>0.172299495</td>
<td>0.473440034</td>
<td>0.040187252</td>
<td>0.091648780</td>
<td>0.777575561</td>
</tr>
<tr>
<td>0.000000000</td>
<td>0.000000000</td>
<td>0.087326867</td>
<td>0.312709653</td>
<td>0.087326867</td>
<td>0.312709653</td>
<td>0.800073041</td>
</tr>
</tbody>
</table>

**COLUMN SUMS**

| 0.650704375  | 0.577175549 | 1.069651704 | 1.899390561 | 0.136319334 | 0.422949643 | 4.756191165 |

**PERCENTAGES**

| 82.087244037 | 72.8114824591 | 34.9380207892 | 39.610708856 | 17.196869788 | 53.355674072 | 0.792698527 |

Figure 4-10. Original matrix for the horizontal plate radiometer.
column. This average value is denoted by $x$ for the sphere, and by $y$ for the plate. Each of the quantities in the row entitled "column sums" is the sum of the six quantities above it. The first six elements of the row termed "percentages, $x,y$" are obtained by dividing each column sum by the average shape factor for the total FOV and multiplying the result by 100.

NOTE: Since the systematic errors do not produce unacceptable results, the following discussion applies only to those cases in which the errors are totally or partially gaussian.

Prediction scheme.—A qualitative prediction classification based solely on the structure of the original matrices was found. It yielded excellent results when compared with the rms's of several $W_e$ errors computed for each of the regions. This rms is based on ten sets of error data; each set has six $\varepsilon$ values required to perturb the six power measurements of each observation group.

In order to predict the quality of the data to be retrieved, three distinct data classes were arbitrarily selected. The quality of the $W_e$ values to be retrieved were predicted to be either ACCEPTABLE; POOR, or REJECTABLE, according to the criteria detailed below.

Let $SS_j$ denote the sum of the elements in the $j$th column of the configuration factor matrix for the sphere, and let $SP_j$ denote the equivalent sum for the plate. It is recalled that $x$ was used to denote the average of the six shape factors for the total FOV of the sphere. Similarly, $y$ was used to denote the equivalent quantity for the plate. Then, using these definitions, the first criterion can be stated as follows:

NOTE: In the following, REJECT means reject the $W_e$ value determined for the $j$th region from observations by the sphere, in the case of $SS_j$, or by the plate, in the case of $SP_j$. Similar meanings should be attached to ACCEPT and POOR.
IF $SS_j < 0.2X$ REJECT \hfill (4-53)

IF $SP_j < 0.2Y$ REJECT \hfill (4-53')

Otherwise, $SS_j$ and $SP_j$ are subjected to the following tests.

IF $SS_j > 1.25X$ ACCEPT \hfill (4-54)

IF $SP_j > 1.25Y$ ACCEPT \hfill (4-54')

If this test is not passed, the diagonal element $F_{jj}$ of the $j$-column of both matrices ($F^S_{jj}$ for the sphere and $F^P_{jj}$ for the plate) are subjected to the following test.

IF $F^S_{jj} \leq 0.25 SS_j$ REJECT \hfill (4-55)

IF $F^P_{jj} \leq 0.25 SP_j$ REJECT \hfill (4-55')

Otherwise, the two sets of quantities are subjected to a final test.

IF $F^S_{jj} > 0.6 SS_j$ ACCEPT \hfill (4-56)

IF $F^P_{jj} > 0.6 SP_j$ ACCEPT \hfill (4-56')

For those cases when $F^S_{jj}$ and $F^P_{jj}$ are between the above two limits, the classification is,

IF $0.25 SS_j < F^S_{jj} \leq SS_j$ POOR \hfill (4-57)
The predictions made on the basis of the above criteria were sufficiently satisfactory to indicate that there is a definite relationship between the quality of the data to be retrieved and the configuration factor matrix of the measurements. This is of great value since a simple analysis of the matrix tells in advance which are the regions whose results should be considered for further processing, as will be discussed in the following subsection.

TABLE 4-8 compares the data quality predictions with the rms's of the errors in $W_e$ for each of the regions. As seen from this table, in every instance the predictions agree with the computed errors in $W_e$. From TABLE 4-8 the following can also be seen,

(a) In all cases the uncertainties in $W_e$ for the plate are larger than those for the sphere

(b) The only instance in which a REJECTION was predicted was for the plate in region No.5. The $W_e$ errors computed for the plate in this case were, 33.9284 w/m$^2$ for the gaussian perturbation and 33.8025 w/m$^2$ for the gaussian/systematic error combination.

It is apparent then that before retrieving the values of $W_e$ from a given set of power measurements, the above scheme will tell which $W_e$ values will be good enough for further processing, such as averaging of data discussed in the next subsection.

Weighted Averages

The n observations required to solve the n simultaneous equations for $W_e$ can be taken within a time interval of any arbitrary length. However, the physical situations represented by the results would be different for
TABLE 4-8. Comparisons of data quality predictions with rms's of computed $W_\epsilon$ errors.

<table>
<thead>
<tr>
<th>Type of Satellite</th>
<th>Type of Power Error ($W/m^2$)</th>
<th>Type of Results</th>
<th>Region 1 ($\epsilon_g$)</th>
<th>Region 2 ($\epsilon_g$)</th>
<th>Region 3 ($\epsilon_g$)</th>
<th>Region 4 ($\epsilon_g$)</th>
<th>Region 5 ($\epsilon_g$)</th>
<th>Region 6 ($\epsilon_g$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sphere</td>
<td>Gaussian, $\epsilon_g$</td>
<td>Predicted</td>
<td>Poor</td>
<td>Accept</td>
<td>Accept</td>
<td>Poor</td>
<td>Accept</td>
<td>Accept</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>15.2497</td>
<td>19.9081</td>
<td>6.8638</td>
<td>2.9748</td>
<td>17.4476</td>
<td>5.7264</td>
</tr>
<tr>
<td>Sphere</td>
<td>Gaussian Plus (0.9 $W/m^2$)</td>
<td>Computed</td>
<td>15.1991</td>
<td>20.0824</td>
<td>6.8882</td>
<td>3.1020</td>
<td>17.4142</td>
<td>5.8314</td>
</tr>
<tr>
<td></td>
<td>Systematic, $\epsilon_{gs}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Plate</td>
<td>Gaussian, $\epsilon_g$</td>
<td>Predicted</td>
<td>Poor</td>
<td>Accept</td>
<td>Accept</td>
<td>Reject</td>
<td>Accept</td>
<td>Accept</td>
</tr>
<tr>
<td>Plate</td>
<td>Gaussian Plus (0.9 $W/m^2$)</td>
<td>Computed</td>
<td>15.4480</td>
<td>21.9173</td>
<td>9.0688</td>
<td>4.2407</td>
<td>33.8025</td>
<td>9.2882</td>
</tr>
<tr>
<td></td>
<td>Systematic, $\epsilon_{gs}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
different time interval lengths. At any rate, regardless of the magnitude of the time intervals selected for accomplishing each set of observations, the resulting data can always be averaged out over much longer time periods. For purposes of the present discussion, a set of results will be considered instantaneous if the corresponding set of n observations is taken during a single pass or orbit of the satellite. A total of T sets of data will be considered for averaging in the following discussion.

The following three types of averages were considered,

\[
W_{ei} = \frac{T}{T} \sum_{j=1}^{T} W_{ej}
\]  
(4-58)

\[
W_{ei}^{A} = \frac{T}{T} \sum_{j=1}^{T} \frac{A_{ij} W_{ej}}{A_{ij}}
\]  
(4-59)

\[
W_{ei}^{F} = \frac{T}{T} \sum_{j=1}^{T} \frac{F_{ij} W_{ej}}{F_{ij}}
\]  
(4-60)

Where,

- \( W_{ej} \) = the jth determination of the value of \( W_{e} \) for the ith region
- \( A_{ij} \) = total area seen of the ith region during the jth observation set.
- \( F_{ij} \) = sum of all the configuration factors of the ith region which entered in the jth set of observations.
- \( T \) = total number of results, or observations sets, used in the averaging.
- \( W_{ei} \) = plain average of the value of \( W_{e} \) for the ith region.
\[ W^A_{ei} = \text{Weighted average of } W_e \text{ for the } i\text{th region, using the area } A_{ij} \text{ as weight.} \]

\[ W^F_{ei} = \text{Weighted average of } W_e \text{ for the } i\text{th region, using the configuration factor } F_{ij} \text{ as weight.} \]

The average \( W^A_{ei} \) given by (4-58), assigns equal weights to all the \( W_e \) results entering into the averaging process, regardless of the sizes of the segments of the \( i\text{th region} \) that were within the FOV's of the radiometer, and regardless of the positions that these segments occupied within the radiometer's FOV. Hence, this type of averaging was considered inadequate. The type of average defined by (4-59) takes into account the size of the region segments that entered into the observations; however, two segments of equal area but making different contributions due to their different positions within the FOV are given equal weights, which might be undesirable in some cases. The third type, defined by (4-60) was considered adequate since it does not suffer from either of the shortcomings mentioned above for (4-58) and (4-59).

Thus, weekly or monthly weighted averages (for any time of the day) can be easily obtained from instantaneous results (for the time of day selected) by using the expression (4-60). Although the SWR component is not discussed in detail in this report, a proposed method that is currently under investigation for tackling the problem of reflected SWR will now be discussed.

**Proposed Procedure for Computing \( W_r \)**

When the \( W_e \) regions of the E-A system previously defined were considered, it was assumed that the variations of \( W_e \) within the region were negligible or barely detectable by the satellite radiation sensing system. Now, one can assume another common characteristic to all
area elements within a given region; this is the ALBEDO $A(\zeta_0)$ for a specific solar zenith angle $\zeta_0$. For reasons which will be apparent later, the zenith angle chosen for this common characteristic is $\zeta_0 = 0^\circ$. Before proceeding further, it is advisable to recall the following relationships that were introduced in the section entitled "BASIC IDEAS".

$$W_r(\zeta) = \int_0^{2\pi} d\psi \int_0^{\pi/2} N_r(\theta, \psi, \zeta) \sin \theta \cos \theta \, d\theta \tag{2-19}$$
$$A(\zeta) = r(\zeta) = \int_0^{2\pi} d\psi \int_0^{\pi/2} \rho(\theta, \psi, \zeta) \sin \theta \cos \theta \, d\theta \tag{2-20}$$
$$A(\zeta) = r(\zeta) = \frac{W_r(\zeta)}{H(\zeta)} \tag{2-21}$$
$$\rho(\theta, \psi, \zeta) = \frac{N_r(\theta, \psi, \zeta)}{H(\zeta)} \tag{2-22}$$

Then, one can write for those characteristics that have been assumed common to all area elements within a given region the following,

$$W_r(0) = \int_0^{2\pi} d\psi \int_0^{\pi/2} N_r(\theta, \psi, 0) \sin \theta \cos \theta \, d\theta = \text{constant} \tag{4-61}$$
$$A(0) = r(0) = \int_0^{2\pi} d\psi \int_0^{\pi/2} \rho(\theta, \psi, 0) \sin \theta \cos \theta \, d\theta = \text{constant} \tag{4-62}$$

However, since at any instant of time the solar zenith angle exhibits spatial variations within any given region; then, $r(\zeta)$, $A(\zeta)$, and $W(\zeta)$ are not necessarily constant within the given region. Then, the problem is, how is one to determine the $W_r(\zeta)$ of a region if the value of $W_r(0)$ for that region is known?

The following is a simple procedure which uses data available from previous satellite observations to provide a quick solution to the above problem.
Figure 5 on page 11 of reference 4 shows several curves which depict relationships between $r(\zeta)$ and $r(\phi)$ for different types of surface regions of the E-A system. One or several of these curves are selected to represent the reflecting characteristics of the hypothetical regions one has assumed. Consider now the $i$th area element of the $k$th region which is within the FOV of the $j$th observation of a satellite radiometer. This area element is denoted by $\Delta A_{ijk}$, and the radiant power it contributes to the $j$th measurement is $\Delta P_{ijk}$. The reading from the curve in figure 5 that corresponds to $\Delta A_{ijk}$ is denoted by

$$R_{ijk}(\zeta) = r_{ijk}(\zeta)/r_k(0) \quad (4-63)$$

The albedo $A$ and directional reflectance $r$ for $\Delta A_{ijk}$ is given by

$$A_{ijk}(\zeta) = r_{ijk}(\zeta) = R_{ijk}(\zeta) r_k(0) \quad (4-64)$$

It should be noted that $r_k(0)$ does not have the subscripts $i$ and $j$ since the value $r_k(0) = A_k(0)$ changes only if the region changes.

From (2-21) shown above, one writes

$$W_{ijk}(\zeta) = r_{ijk}(\zeta) H_{ijk}(\zeta) = A_{ijk}(\zeta) H_{ijk}(\zeta) \quad (4-65)$$

Then, substituting (4-64) into (4-65) one obtains

$$W_{ijk}(\zeta) = R_{ijk}(\zeta) r_k(0) H_{ijk}(\zeta) \quad (4-66)$$

The expression for the power increment $\Delta P_{ijk}$ which the area element $\Delta A_{ijk}$ emits in the direction of the satellite is

$$\Delta P_{ijk} = N_{ijk}(\delta, \psi, \zeta) \frac{A_k}{d_{ij}^2} \Delta A_{ijk} \cos \theta_{ijk} \quad (4-67)$$
where,

\[ N_{rijk}(\theta, \psi, \zeta) = \text{Reflection radiance of } \Delta A_{ijk} \text{ in the direction } (\theta, \psi) \text{ due to solar radiation incident from a direction given by the solar zenith angle } \zeta. \]

\[ A_s = \pi a^2 \quad = \text{Characteristic area of the radiometer, where } a \text{ is the radius of the radiometer.} \]

\[ d_{ijk} = \text{Distance from } \Delta A_{ijk} \text{ to the radiometer.} \]

\[ \theta_{ijk} = \text{Zenith angle of the radiometer as seen from } \Delta A_{ijk}. \]

From (2-22) shown above, one writes

\[ N_{rijk}(\theta, \psi, \zeta) = H_{ijk}(\zeta) \rho_{ijk}(\theta, \psi, \zeta) \quad (4-68) \]

Substituting (4-68) into (4-67) and assuming that \( A_s = 1m^2 \), one obtains

\[ \Delta P_{ijk} = H_{ijk}(\zeta) \frac{\Delta A_{ijk} \cos \theta_{ijk}}{d_{ijk}^2} \quad (4-69) \]

Now, from the corresponding curve in one of the figures B-3, B-4, or B-5 of reference 3, one can read directly (or by using linear extrapolation) the value of the ratio \( r(\zeta)/\pi \rho(\theta, \psi, \zeta) \) which will be denoted by \( R' \), that is,

\[ R'_{ijk}(\theta, \psi, \zeta) = \frac{r_{ijk}(\zeta)}{\pi \rho_{ijk}(\theta, \psi, \zeta)} \quad (4-70) \]

or,

\[ \rho_{ijk}(\theta, \psi, \zeta) = \frac{r_{ijk}(\zeta)}{\pi R'_{ijk}(\theta, \psi, \zeta)} \quad (4-71) \]

Substituting (4-71) into (4-69), one obtains

\[ \Delta P_{ijk} = H_{ijk}(\zeta) \frac{r_{ijk}(\zeta)}{\pi R'_{ijk}(\theta, \psi, \zeta)} \frac{\Delta A_{ijk} \cos \theta_{ijk}}{d_{ijk}^2} \quad (4-72) \]
But from (2-21) above, one writes

\[ H_{ijk}(\zeta) \cdot r_{ijk}(\zeta) = W_{rijk}(\zeta) \]  \hspace{1cm} (4-73)

Substituting (4-73) into (4-72) one obtains

\[ \Delta P_{ijk} = \left( \frac{\Delta A_{ijk}}{\pi} \right) \left[ \frac{\cos \theta_{ijk}}{R_{rijk}(\theta,\psi,\zeta) d_{ijk}^2} \right] W_{rijk}(\zeta) \]  \hspace{1cm} (4-74)

By the definition of the shape, or configuration factor given by (2-2), with \( A_s = \text{Im}^2 \), one writes

\[ \Delta P_{ijk} = F_{ijk} W_{rijk}(\zeta) \]  \hspace{1cm} (4-75)

Comparison of (4-74) and (4-75) indicates that

\[ F_{ijk} = \left( \frac{\Delta A_{ijk}}{\pi} \right) \left[ \frac{\cos \theta_{ijk}}{R_{rijk}(\theta,\psi,\zeta) d_{ijk}^2} \right] \]  \hspace{1cm} (4-76)

In the following discussion, a new type of shape factor will be introduced in order to develop a simple expression for computing the solutions for the reflected SWR component of the energy budget. This new type of shape factor will be termed PSEUDOSHAPE FACTOR.

One can represent \( H_{ijk}(\zeta) \) as

\[ H_{ijk}(\zeta) = H_k(0) \cos \zeta_{ijk} \]  \hspace{1cm} (4-77)

Note that since the value of \( H_k(0) \) changes only with changes of region, only the subscript \( k \) of the region need be used, just as in the case of \( r_k(0) \) and \( A_k(0) \).
Substituting (4-77) into (4-65) one obtains

\[ W_{rijk}(\zeta) = r_{ijk}(\zeta) H_r(0) \cos \zeta_{ijk} \]  \hspace{1cm} (4-78)

For the case \( \zeta = 0 \), one obtains from this expression,

\[ W_{rk}(0) = r_k(0) H_r(0) \] \hspace{1cm} (4-79)

Again here, only the subscript \( k \) is necessary since these three quantities change value only with region changes.

From (4-63) one writes

\[ r_{ijk}(\zeta) + R_{ijk}(\zeta) r_k(0) \] \hspace{1cm} (4-80)

Substituting (4-80) into (4-78) one obtains

\[ W_{rijk}(\zeta) + R_{ijk}(\zeta) r_k(0) H_k(0) \cos \zeta_{ijk} \] \hspace{1cm} (4-81)

And substituting (4-79) into (4-81),

\[ W_{rijk}(\zeta) + W_{rk}(0) R_{ijk}(\zeta) \cos \zeta_{ijk} \] \hspace{1cm} (4-82)

Substitution of (4-82) into (4-74) yields

\[ \Delta P_{ijk} = \left( \frac{\Delta A^*_{ijk}}{\pi} \right) \left[ \frac{R_{ijk}(\zeta) \cos \theta_{ijk} \cos \zeta_{ijk}}{R'_{ijk}(\theta, \psi, \zeta) d^2_{ijk}} \right] W_{rk}(0) \] \hspace{1cm} (4-83)

From this expression one defines the PSEUDOSHAPE FACTOR \( F'_{ijk} \) as

\[ F'_{ijk} = \left( \frac{\Delta A^*_{ijk}}{\pi} \right) \left[ \frac{R_{ijk}(\zeta) \cos \theta_{ijk} \cos \zeta_{ijk}}{R'_{ijk}(\theta, \psi, \zeta) d^2_{ijk}} \right] \] \hspace{1cm} (4-84)
By comparing the expressions (4-76) for \( F_{ijk} \) and (4-84) for \( F'_{ijk} \) one sees that these two qualities are related by the following expression.

\[
F'_{ijk} = F_{ijk} R_{ijk}(\zeta) \cos \zeta_{ijk}
\]  

(4-85)

Let the following quantity be defined,

\[
R''_{ijk}(\zeta) = R_{ijk}(\zeta) \cos \zeta_{ijk}
\]  

(4-86)

Then, (4-85) can be rewritten as

\[
F'_{ijk} = F_{ijk} R''_{ijk}(\zeta)
\]  

(4-87)

For future use, the weighted average value of \( R''_{ijk}(\zeta) \) over all those area elements \( \Delta A_{ijk} \) of the kth region which appeared at least once within one of the FOV's of the radiometer will now be calculated. Thus, one writes

\[
\frac{1}{I} \sum_{i=1}^{I} \frac{1}{J} \sum_{j=1}^{J} F'_{ijk} = \sum_{i,j} F_{ijk} R''_{ijk}(\zeta) = R''_{k} \sum_{i,j} F_{ijk}
\]  

(4-88)

where,

\[
I = \text{total number of area elements of the kth region within the FOV of the jth observation.}
\]

\[
J = \text{the number of observations in which the kth region appears.}
\]

\[
R''_{k} = \text{weighted average of } R''_{ijk}(\zeta) \text{ for the kth region.}
\]

Thus,

\[
R''_{k} = \frac{\sum_{i,j} F_{ijk}}{\sum_{i,j} F_{ijk}}
\]  

(4-90)
Returning to equation (4-83), one obtains the power $P_{jk}$ contributed by the kth region to the jth observation, by adding up over all the area elements in the kth region. Hence,

$$P_{jk} = \sum_{i=1}^{I} \Delta P_{ijk}$$

(4-91)

Then, one obtains the total power detected in the jth observation by adding up all the powers $P_{jk}$ contributed by the K regions, that is

$$P_j = \sum_{k=1}^{K} P_{jk}$$

(4-92)

These $P_j$'s are the elements of the power column matrix $\{P\}$. In exactly the same manner, one obtains for the configuration factor $F_{jk}$ and the pseudo-configuration factor $F'_{jk}$, that the kth region contributed to the jth observation

$$F_{jk} = \sum_{i=1}^{I} F_{ijk}$$

(4-93)

$$F'_{jk} = \sum_{i=1}^{I} F'_{ijk}$$

(4-94)

These $F'$ 's constitute the elements of the pseudoconfiguration factor matrix $F'$. Thus, one can write in matrix form,

$$F'\{W_r(o)\} = \{P\}$$

(4-95)

where the elements of the column matrix $\{W_r(o)\}$ are the hypothetical values of $W_r(o)$ which were assigned originally to the different regions. Then, by operating on $\{P\}$ with $F'^{-1}$, the inverse of $F'$, one obtains back $\{W_r(o)\}$, that is
\[ F^{-1}(P) = \{ W_r(o) \} \quad (4-96) \]

If this matrix operation retrieves the correct values of \( W_r(o) \) that were originally assigned to each of the regions, then one proceeds, as in the one of \( W_e \), to perturb \( (P) \) with gaussian and systematic errors to test the stability of \( F^{-1} \).

The next step is to obtain the values of \( W_r(\xi) \) for each of the regions from the corresponding values \( W_r(o) \) found through \( (4-95) \). This can be done as follows.

Assuming that the \( k \)th region appeared in the FOV's of \( J \) observations, one can find the total power that this region contributed to the \( J \) observations by adding over \( j \) the \( P_{jk} \) powers given by \( (4-91) \), that is,

\[ P_k = \sum_{j=1}^{J} P_{jk} \quad (4-97) \]

Similarly, the contribution of all the configuration and pseudoconfiguration factors by the \( k \)th region to all the \( J \) observations are, from \( (4-93) \) and \( (4-94) \),

\[ F_k = \sum_{j=1}^{J} F_{jk} \quad (4-98) \]

\[ F_k' = \sum_{j=1}^{J} F_{jk}' \quad (4-99) \]

And from \( (4-91) \) and \( (4-97) \) is obtained

\[ P_k = \sum_{j,i} \Delta P_{ijk} \quad (4-100) \]

By comparing this expression with \( (4-75) \), one can write,

\[ P_k = \sum_{j,i} F_{ijk} W_{ijk}(\xi) \quad (4-101) \]
If one defines \( W_{rk} \) to be the weighted average of \( W_r(\zeta) \) over the \( k \)th region for all the \( J \) observations, one writes

\[
W_{rk} = \frac{\sum_i F_{ijk} W_{ijk}(\zeta)}{\sum_j F_{ijk}} \quad (4-102)
\]

Then, (4-101) becomes

\[
P_k = W_{rk} \sum_i F_{ijk} \quad (4-103)
\]

If now one compares expression (4-100) with equation (4-83) and uses (4-84), one can write,

\[
P_k = W_{rk}(0) \sum_i F'_{ijk} \quad (4-104)
\]

Equating (4-103) and (4-104), one obtains

\[
W_{rk} = \frac{\sum_j F'_{ijk}}{\sum_i F_{ijk}} W_{rk}(0) \quad (4-105)
\]

But the ratio on the right is the same as that in (4-90). Hence, one can say that

\[
W_{rk} = W_{rk}(0) R'\kappa' \quad (4-106)
\]

Where \( R'\kappa' \) is the weighted average of \( r(\zeta) \cos \zeta / r(0) \) for the \( k \)th region for all observations.

Therefore, the essence of the procedure when applied to an actual set
of power measurements is as follows. Compute the elements of the configuration and pseudoconfiguration factor matrices. Obtain the inverse matrix $F^{-1}$ of the pseudoconfiguration factor matrix and operate with it on the column matrix of the power measurements in order to obtain the column matrix $\{W_r(0)\}$. From these elements, the corresponding $W_{rk}$ elements are obtained by application of (4-106).
BEST FIT/INVERSION TECHNIQUE

As was mentioned in the last paragraph of the section entitled "FUNDAMEN-TALS OF THE TECHNIQUES," the main feature of the Best Fit/Inversion Technique is that the number of observations m is larger than the number of unknowns n. An extension of the method of least squares is used to determine the approximating surface in the n+1 dimensional space. The procedure followed to find the values of $W_e$ is illustrated by a simplified three dimensional case as follows.

Let it be assumed that a satellite has made eighteen observations over two regions. The FOV of each observation is either totally filled by one of the regions, or totally filled by segments of both of the regions. Furthermore, let it be assumed that the $W_e$ values of the two regions, identified as regions 1 and 2, are in the ranges $W_{e1} = 240.0 \pm 5 \text{ w/m}^2$ and $W_{e2} = 280.0 \pm 5 \text{ w/m}^2$, as shown in TABLE 5-1. This table shows the configuration factors $F_1$ and $F_2$ that regions 1 and 2 contribute to each of the measurements. The hypothetical values of $W_{e1}$ and $W_{e2}$ at the time each observation is taken are also presented in this table, as well as the two partial powers for each of the measurements. These partial powers, for the $i$th observation, are given by

$$P_i = P_{i,1} + P_{i,2} = W_{e1,1} F_{1,1} A_s + W_{e1,2} F_{1,2} A_s$$  \hspace{1cm} (5-1)

Where

$$A_s = 1 \text{ m}^2 = \text{characteristic area of the radiometer.}$$

Therefore, the eighteen observations can be written as

$$W_{e1,1} F_{1,1} + W_{e1,2} F_{1,2} = P_1$$
$$W_{e2,1} F_{2,1} + W_{e2,2} F_{2,2} = P_2$$
$$W_{e3,1} F_{3,1} + W_{e3,2} F_{3,2} = P_3$$
$$W_{e18,1} F_{18,1} + W_{e18,2} F_{18,2} = P_{18}$$  \hspace{1cm} (5-2)
### Table 5.1: Hypothetical data about satellite observations of two regions.

<table>
<thead>
<tr>
<th>Measurement No.</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Parameter</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$F_1$</td>
<td>0.4</td>
<td>0.5</td>
<td>0.6</td>
<td>0.3</td>
<td>0.7</td>
<td>0.2</td>
<td>0.8</td>
<td>0.1</td>
<td>0.9</td>
</tr>
<tr>
<td>$F_2$</td>
<td>0.6</td>
<td>0.5</td>
<td>0.4</td>
<td>0.7</td>
<td>0.3</td>
<td>0.8</td>
<td>0.2</td>
<td>0.9</td>
<td>0.1</td>
</tr>
<tr>
<td>$F = F_1 + F_2$</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>$W_{e1} (\text{W/m}^2)$</td>
<td>240</td>
<td>241</td>
<td>242</td>
<td>243</td>
<td>239</td>
<td>238</td>
<td>237</td>
<td>244</td>
<td>245</td>
</tr>
<tr>
<td>$W_{e2} (\text{W/m}^2)$</td>
<td>280</td>
<td>279</td>
<td>278</td>
<td>277</td>
<td>281</td>
<td>282</td>
<td>283</td>
<td>276</td>
<td>275</td>
</tr>
<tr>
<td>$P_1 (\text{W})$</td>
<td>96.0</td>
<td>120.5</td>
<td>145.2</td>
<td>72.9</td>
<td>167.3</td>
<td>47.6</td>
<td>189.6</td>
<td>24.4</td>
<td>220.5</td>
</tr>
<tr>
<td>$P_2 (\text{W})$</td>
<td>168.0</td>
<td>139.5</td>
<td>115.2</td>
<td>193.9</td>
<td>84.3</td>
<td>225.6</td>
<td>.56.6</td>
<td>248.4</td>
<td>27.5</td>
</tr>
<tr>
<td>$P = P_1 + P_2$</td>
<td>264.0</td>
<td>260.0</td>
<td>260.4</td>
<td>266.8</td>
<td>251.6</td>
<td>273.2</td>
<td>246.2</td>
<td>272.8</td>
<td>248.0</td>
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</table>
TABLE 5-1. Hypothetical data about satellite observations of two regions. (Continuation)

Measurement No.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
<th>16</th>
<th>17</th>
<th>18</th>
</tr>
</thead>
<tbody>
<tr>
<td>( F_1 )</td>
<td>0.0</td>
<td>1.0</td>
<td>0.45</td>
<td>0.55</td>
<td>0.65</td>
<td>0.25</td>
<td>0.35</td>
<td>0.75</td>
<td>0.15</td>
</tr>
<tr>
<td>( F_2 )</td>
<td>1.0</td>
<td>0.0</td>
<td>0.55</td>
<td>0.45</td>
<td>0.35</td>
<td>0.75</td>
<td>0.65</td>
<td>0.25</td>
<td>0.85</td>
</tr>
<tr>
<td>( F = F_1 + F_2 )</td>
<td>1.0</td>
<td>1.0</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
</tbody>
</table>

\[ \text{W}_{e_1} \ (w/m^2) \]

| \( \xi_3 \) | 240 | 236 | 235 | 240 | 241 | 242 | 243 | 239 | 238 |

\[ \text{W}_{e_2} \ (w/m^2) \]

| \( \xi_3 \) | 280 | 284 | 285 | 280 | 281 | 282 | 283 | 279 | 278 |

\[ \text{P}_1 \ (w) \]

| \( \xi_3 \) | 0.0 | 236.0 | 105.75 | 132.0 | 156.65 | 60.5 | 85.05 | 179.25 | 35.7 |

\[ \text{P}_2 \ (w) \]

| \( \xi_3 \) | 280.0 | 0.0 | 156.75 | 126.0 | 98.65 | 211.5 | 183.95 | 69.75 | 236.3 |

\[ \text{P} = \text{P}_1 + \text{P}_2 \]

| \( \xi_3 \) | 280.0 | 236.0 | 262.50 | 258.0 | 255.30 | 272.0 | 269.0 | 249.0 | 272.0 |
The $i$th equation of this set represents the $i$th plane in a three-dimensional space whose rectangular coordinates are $F_{i1}$, $F_{i2}$, and $P_i$. The form of the equations indicates that all planes cross the origin of the coordinate system. One assumes further that the data obtained from the observations (i.e., the values of $F_{i1}$, $F_{i2}$ and $P_i$) when plotted in this three dimensional coordinate system become a scatter diagram which can be represented by an APPROXIMATING PLANE (see reference 9) that best fits the data. The equations of this plane will be of the form

$$P = w_1 F_{i1} + w_2 F_{i2}$$

(5-3)

and the partial regression coefficients $w_1$ and $w_2$ will be determined by an extension of the method of least squares as follows.

The general form for the equations (5-2) is,

$$P_i = w_{i1} F_{i1} + w_{i2} F_{i2}$$

(5-4)

One multiplies this equation through by $F_{i1}$ to form 18 equations ($i=1, \ldots, 18$). These 18 equations are added up to generate the first equation of the set of 2 equations needed to find the 2 unknowns. The second equation required is obtained by multiplying (5-4) by $F_{i2}$ to form, again, 18 equations which are then added together as before. Thus

$$\sum_{i=1}^{18} P_i F_{i1} = w_{i1} \sum_{i=1}^{18} F_{i1}^2 + w_{i2} \sum_{i=1}^{18} F_{i1} F_{i2}$$

(5-5)

$$\sum_{i=1}^{18} P_i F_{i2} = w_{i1} \sum_{i=1}^{18} F_{i1} F_{i2} + w_{i2} \sum_{i=1}^{18} F_{i2}^2$$

(5-6)

In order to simplify the notation, the following quantities are defined.
Using (5-7), equations (5-5) and (5-6) can be written as

\[
\overline{PF}_1 = \frac{18}{\sum_{i=1}^{18} P_i F_{i,1}} \\
\overline{PF}_2 = \frac{18}{\sum_{i=1}^{18} P_i F_{i,2}} \\
\overline{F}_1 = \frac{18}{\sum_{i=1}^{18} F_{i,1}} \\
\overline{F}_2 = \frac{18}{\sum_{i=1}^{18} F_{i,2}} \\
\overline{F}_{12} = \frac{18}{\sum_{i=1}^{18} F_{i,1} F_{i,2}}
\]  

(5-7)

As was mentioned before, (5-8) is a set of two equations in the two unknowns \( W_{el} \) and \( W_{e2} \) which is required to solve the problem.

A short program was written to solve the set of equations (5-8) by using a matrix inversion subroutine. This program also computes the average values of \( W_{el} \) and \( W_{e2} \) in order to compare these results with the coefficient values found by the least squares method. As in previous programs, the original matrix and its inverse were multiplied in order to verify that the result was the identity matrix. The values of \( W_{el} \) and \( W_{e2} \) obtained from the eighteen observations are compared in TABLE 5-2 with the average values of the hypothetical distributions of \( W_{el} \) and \( W_{e2} \) given in the problem.

TABLE 5-2. Comparison of \( W_e \) values computed from error-free data with the averages of the given \( W_e \) values.

<table>
<thead>
<tr>
<th>TYPE OF VALUES</th>
<th>( W_{el} ) (w/m²)</th>
<th>( W_{e2} ) (w/m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>By least square method from 18 observations</td>
<td>239.83</td>
<td>279.99</td>
</tr>
<tr>
<td>Averages of the given ( W_e ) values</td>
<td>240.17</td>
<td>280.17</td>
</tr>
</tbody>
</table>
Two important modifications were introduced in the next application.

(a) Six regions, rather than two, were considered and a total of thirty-six observations were made.  
(b) systematic and gaussian uncertainties were incorporated into the power measurements. It should be pointed out that since the space now considered is 7th dimensional, one can no longer speak of the equation of an approximating plane, but rather of the equation of an approximating surface.

A gaussian random error distribution with a sigma value of 0.5 \( \text{w/m}^2 \) was used, as well as three values of systematic errors, 0.3 \( \text{w/m}^2 \), 0.6 \( \text{w/m}^2 \) and 0.9 \( \text{w/m}^2 \).

TABLE 5-3 lists five groups of results based on the following types of uncertainties in the observations.

1. No error in the observations.
2. Gaussian errors only included.
3. Systematic (0.3 \( \text{w/m}^2 \)) combined with gaussian errors.
4. Systematic (0.6 \( \text{w/m}^2 \)) combined with gaussian errors.
5. Systematic (0.9 \( \text{w/m}^2 \)) combined with gaussian errors.

The five sets of data in TABLE 5-3 clearly show that the results are not highly sensitive to the gaussian and systematic uncertainties assumed for the power measurement.

It is recalled that the essential difference between this technique and the Instantaneous/Inverse Technique is that in the latter the number of observations \( m \) equals the number of unknowns \( n \); while in the former, \( m \) is greater than \( n \). However, the basic equations to be used are the same in both cases, namely, equations (3-22) and (3-23) introduced in the section entitled "Mathematical Bases." The configuration factors in these two expressions include the LDF as can be seen from equations (3-18) and (3-19). Furthermore,
TABLE 5-3. Comparison of $W_e$ values computed from data having uncertainties with the averages of the given $W_e$ values.

<table>
<thead>
<tr>
<th>TYPE OF DATA</th>
<th>Region 1</th>
<th>Region 2</th>
<th>Region 3</th>
<th>Region 4</th>
<th>Region 5</th>
<th>Region 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>AVERAGE of Given $W_e$ Values FROM Error-Free Data</td>
<td>179.36</td>
<td>200.42</td>
<td>220.44</td>
<td>239.83</td>
<td>259.56</td>
<td>280.44</td>
</tr>
<tr>
<td>FROM Observations with Gaussian Errors ($\sigma=0.5 \text{ w/m}^2$)</td>
<td>178.09</td>
<td>199.63</td>
<td>221.22</td>
<td>242.15</td>
<td>256.66</td>
<td>280.69</td>
</tr>
<tr>
<td>FROM Observations with (0.3 w/m$^2$) Systematic &amp; Gaussian Errors</td>
<td>178.38</td>
<td>199.65</td>
<td>221.30</td>
<td>242.06</td>
<td>256.45</td>
<td>280.64</td>
</tr>
<tr>
<td>FROM Observations with (0.6 w/m$^2$) Systematic &amp; Gaussian Errors</td>
<td>178.68</td>
<td>199.95</td>
<td>221.60</td>
<td>242.36</td>
<td>257.05</td>
<td>281.24</td>
</tr>
<tr>
<td>FROM Observations with (0.9 w/m$^2$) Systematic &amp; Gaussian Errors</td>
<td>179.28</td>
<td>200.55</td>
<td>222.20</td>
<td>242.96</td>
<td>257.35</td>
<td>281.54</td>
</tr>
</tbody>
</table>
the corresponding expressions for the case of reflected SWR are developed in a manner similar to that employed in the Instantaneous/Inverse Technique.

The results obtained so far by this technique are satisfactory and further work in its application should be pursued.
Once it had been shown that the techniques for computing $W_e$ yield acceptable results, it was decided to return to the original problem in which a spherical earth-atmosphere system was being considered.

As was mentioned previously, the power integrations are to be accomplished by adding up the power increments reaching the satellite simultaneously from the different area elements within the FOV. Hence, the first step was to generate a computer program which would divide the surface area of the earth-atmosphere system into a finite number of area elements of equal area. The data output of this program gives the longitudes and latitudes of the centroid and four boundaries of each of the 2060 area elements. An area element of 250,000 km$^2$ was found to be adequate. This size corresponds to an area of 500 by 500 km, or approximately $5^\circ$ by $5^\circ$ of great circle arc. This yielded a total of 2060 area elements, 2058 of which have equal areas ($2.5 \times 10^{11}$ m$^2$ each). The other two elements are those centered at each of the poles. These have an area of $2.339 \times 10^{11}$ m$^2$ each. This yields a total surface area for the spherical E-A system of $5.149678 \times 10^{14}$ m$^2$. The radius of this system is considered to be $6.40155 \times 10^6$ m, which gives a surface area of $5.149679 \times 10^{14}$ m$^2$. Since the radius of a sphere having an area equal to the area of the earth has a radius of $6.37123 \times 10^6$ m, it follows that an atmospheric spherical shell of 30.32 km thickness is being included. This is approximately the thickness commonly used (30km) since it was first suggested by Dr. Frederick B. House (reference 10). More than 95% of the total atmosphere is contained within this shell.

The output data of this program is used in the main computer program to calculate the $W_e$ values of a spherical E-A system.
The total shape factors for the sphere and plate were calculated also in this main program by adding up the shape factors of each of the area elements within the FOV of the radiometers. The results are compared below with those obtained analytically.

<table>
<thead>
<tr>
<th>Radiometer</th>
<th>Numerically</th>
<th>Analytically</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sphere</td>
<td>1.089592145</td>
<td>1.0838471</td>
</tr>
<tr>
<td>Plate</td>
<td>0.792698527</td>
<td>0.7901660</td>
</tr>
</tbody>
</table>

These results indicate that it is unnecessary to use area elements of smaller size to perform the numerical integrations.

The results of the investigations presently being performed using a spherical E-A system will be discussed in a subsequent report.
CONCLUSIONS

On the basis of the investigation results reported in this document, the following has been concluded:

1. The Instantaneous/Inversion Technique (including the data quality prediction and matrix stabilization schemes) yields excellent results when applied to radiometer data containing gaussian and systematic errors. The $W_e$ values obtained are acceptable according to the pertinent accuracy requirements displayed in TABLE 4-7.

2. The problem of determining the solar radiation reflected by the E-A system has been cast in a form which requires minimum computer time for calculating $W_r$ and the albedo. It is concluded that this simple formulation represents the optimum method to solve this problem and will soon be implemented.

3. The results obtained from preliminary applications of the Best Fit/Inversion Technique clearly indicate that the errors in $W_e$ calculated by the use of this technique are not highly sensitive to gaussian and/or systematic power uncertainties. In all cases investigated the $W_e$ values retrieved met the pertinent accuracy requirements in TABLE 4-7.

4. It has been concluded that division of the surface of the E-A system into 2060 area elements of about $2.5 \times 10^5$ km$^2$ each yields accurate results when used in numerical integrations of configuration factors.

7-1
REFERENCES


