METHODS FOR THE EVALUATION
OF ALTERNATIVE DISASTER
WARNING SYSTEMS

by C. E. Agnew, R. J. Anderson, Jr., and W. N. Lanen

MATHTECH, Inc.
P. O. Box 2392
Princeton, N. J. 08540

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## Methods for the Evaluation of Alternative Disaster Warning Systems

This study identifies several methods that can be used in the evaluation of alternative disaster warning systems. For each of the methods identified, a theoretical basis is provided and an illustrative example is described. The example includes sufficient realism and detail to enable an analyst to conduct an evaluation of other systems. The methods discussed in the study include equal capability cost analysis, consumers' surplus, and statistical decision theory. Also included is a brief history of disaster warning programs and program evaluations.

### Key Words (Suggested by Author(s))
- Disaster Warning
- Benefit-Cost Analysis
- Evaluation Methods
- Communication Satellites
- Ground Stations
FOREWARD

This study is the result of a one year effort by MATHTECH to identify and illustrate methods for the evaluation of alternative disaster warning systems.

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SUMMARY

Unlike many government programs, a disaster warning system requires the participation of two distinct decision-making parties: first, the government must decide on the type of transmission-reception system to employ; second, the individual must decide whether to purchase a receiver and, given a warning, whether to take action. This two-party nature of disaster warning decisions suggests that traditional "single-decision-maker" approaches to the evaluation of alternative systems may not be fruitful. For example, different warning systems may provide services that individual citizens desire to a greater or lesser degree. A complete analysis must take account of these differences in valuations on the part of individual citizens as well as differences in value from the perspective of the government decision-maker.

This report summarizes the results of a study of methods for estimating the economic costs and benefits of the transmission-reception and reception-action segments of a disaster warning system (DWS). Specifically, the objectives of this study were to: (i) identify methods for the evaluation of alternative disaster warning systems; (ii) perform example analyses using the methods identified.

The methods considered in this study included those from the economics of information, benefit-cost analysis, general economic theory, and statistical decision theory. In order to insure that the methods proposed were applicable to disaster warning systems, a brief review of system technologies that have been previously developed was conducted.
The examples in the report indicate that the methods identified in the study provide the user with a set of proper, and practical, tools with which to evaluate alternative disaster warning systems. However, it is important to emphasize that, as with all analytic tools, the methods proposed are only an aid (albeit an important one) to the decision maker.
NOTE TO THE READER

The purpose of this study is to identify appropriate methods for the evaluation of alternative disaster warning systems and to illustrate their application by examples. As such, it is designed so that individual chapters are, for the most part, sufficiently complete to be read separately. There are, however, three important consequences of this design that should be explicitly noted:

First, appendices and references are located at the end of the chapter in which they are used;

second, examples in different chapters cannot be compared since the assumptions used in each (discount rates, for example) are not necessarily comparable. The reason is that each example is designed to illustrate specific points about the individual methods;

third, because the examples are designed to illustrate specific points, they cannot be considered to be complete analyses of any actual system.
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CHAPTER I:  
INTRODUCTION AND SUMMARY

A. Overview

This report presents the results of a study of methods for estimating the economic costs and benefits of the transmission-reception segment of a forecasting-warning system. Specifically, the objectives of this study are to:

- identify methods for the evaluation of the transmission and reception portions of alternative forecasting-warning systems;
- perform example analyses using the methods identified.

The nature of these tasks and our findings become clearer if we focus on the individual components of a forecasting-warning system rather than the overall system itself. Such a system can be thought of as being made up of the following, functionally distinct, components:

- **Sensing** - detection of a potential disaster before it occurs
- **Forecasting** - the use of sensor data to predict the nature, time and location of a disaster
- **Transmission** - sending the forecast to the public
- **Reception** - receipt of the forecast by individuals
- **Action** - doing something to mitigate the losses that result from disasters.
While additional functions could be added (most notably post-disaster efforts), these five functions usefully delineate the bounds of a forecasting-warning system without cutting across the jurisdictional responsibilities of several agencies.

In this report, we do not consider the sensing and forecasting components of the system. There are two related reasons for this. First, the design of transmission and reception components does not depend significantly on the design of the sensing and forecasting components. Second, since the sensing and forecasting components serve other purposes (namely, routine weather forecasting) they are generally taken as given.

Figure I-1 depicts the relationship among the five components and the dotted line encloses those functions that we refer to as a disaster warning system (DWS) and that were considered in this study. 1/

Looking at the components within the dotted lines in Figure I-1, we see that there are decisions that must be made by the government (i.e., what transmission system to invest in) and by the individual (e.g., whether or not to invest in a home warning receiver). The fact that a disaster warning system provides a service directly to the public makes it something of a novelty as far as system evaluations since value of service to individuals as well as cost considerations may arise in analyses of alternative systems. We need, therefore, to analyze both governmental decisions about transmission systems and individual decisions about receivers.

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1/ These components include the transmission of routine forecasts as well but in this study attention is directed primarily toward the disaster warning role.
Figure I-1

The Five Components of a Forecasting-Warning System
Unless the decisions of the individuals are taken into account in an evaluation of a particular disaster warning system, the results of the analysis may not be relevant. For example, if the government were to select a transmission system that required very expensive receivers, it is likely that not many people would buy the receivers. Therefore, very few people receive any warning that is issued over the system and the system would be worthless as a means for providing warnings to the vast majority of the public.

The methods we have considered for dealing with these and other problems include those from the economics of information, benefit-cost analysis, general economic theory, and statistical decision theory. In order to insure that the methods we propose are applicable to disaster warning systems, we have also conducted a brief review of system technologies that have been previously developed. Some currently available data sources have also been reviewed for use in the example analyses.

In addition to the identification of appropriate evaluation methods, two additional topics are included in this report. The first is the theoretical foundation of the method. This discussion is intended to explain when each method is appropriate and in what sense each provides a useful indicator of the value of a system.

The second topic we discuss is the practical implementation of each method by way of an example analysis using the method. In this study, the examples that have been developed are a compromise between realism and simplicity. Clearly, an example that entirely abstracts from reality would not serve as a useful guide for an actual application of
the method in the real world. Just as clearly, however, an example that incorporates all the complexities associated with the evaluation of disaster warning system components would not provide a clear illustration. For this reason, as well as missing data and the incomplete specification of alternative systems, these examples are illustrations of the methods only and are not to be interpreted as analyses of actual systems.

B. A Framework for Economic Evaluation of Disaster Warning Systems

Given the diverse and diffuse nature of the benefits generated by a DWS, how can we compare government costs, private costs, and the benefits that accrue as a result of a DWS being implemented? In this section, we describe the methodological framework that we will use throughout this report. Naturally, no methodological tool can make the determination of what system the government "should" invest in. However, economic evaluations, by providing information to the decision maker about relative costs and benefits of alternative systems, can aid in the decision-making process.

1. Principles

The general principles underlying the economic evaluation of alternative systems are well-known; they are those of benefit-cost analysis. Benefit-cost analysis is simply the application of decision making techniques used by private decision makers to governmental decision problems. The application of benefit-cost analysis is depicted graphically in Figure I-2. There, we let E be a measure of system effectiveness. By "effectiveness"
we mean a measure of how "well" the system performs with respect to one or more characteristics (in Figure I-2, effectiveness is in terms of one characteristic). The measures used will depend upon the characteristics determined to be relevant in a comparative analysis of two or more systems. In an analysis of disaster warning systems, for example, an important characteristic is coverage and a measure of effectiveness might be the percentage of the population that could receive a warning.
The cost function, \( C(E) \), gives the \textbf{minimum} cost for a given level of effectiveness.\(^1\) This corresponds directly to the private firm's cost function which depends on the amount produced. Similarly, \( B(E) \) gives some measure of society's "well-being" (benefits)\(^2\) that are obtained from a given level of benefits.\(^3\) This corresponds to the private firm's revenue function which, again, depends on the amount sold. Conceptually, at least, it is also an easy matter to estimate the costs of system alternatives for public projects. One simply follows known engineering-economic principles. Of course, it is important to ensure that the true opportunity costs of resources are used in the calculation. For example, the opportunity costs of facilities used must be included—even if no new facilities are required—as long as facilities used have an alternative public or private use.

It is somewhat more difficult, both in concept and practice, to estimate the benefits of alternative transmission-reception segments of a DWS. This portion of the system derives its value from the information that it provides to decision-makers—in this case, households, businesses, governments, institutions, etc.—that are the target audience for natural disaster warnings. How to go about placing a value on this information—and the system which conveys it—is, however, a difficult problem.

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\(^1\) This curve (or surface in cases where there is more than one effectiveness dimension) can be determined via equal capability cost analysis, to be described below.

\(^2\) The controversy surrounding the use of what essentially is a social welfare function is too involved for satisfactory discussion here. The interested reader is referred to Quirk & Saposnik. All we require here is that some method is used to relate effectiveness to well-being.

\(^3\) We shall have more to say in Chapters IV and V about how this curve may be determined.
2. Practical Methods

The fact that the disaster warning system can be divided into transmission and reception segments suggests that some of these problems can be simplified by using different methods for each segment. We have, therefore, identified three methods for the evaluation of alternative disaster warning systems, each suitable for a specific segment. For the transmission-reception link, equal capability cost analysis (discussed in Chapter III) provides a suitable method. For the reception-action link, and the benefits to be derived by the individual from a disaster warning system, consumers' surplus (discussed in Chapter IV) is an appropriate measure and can be derived directly from the demand curve, if it is known, or can be calculated from a demand curve derived through the use of decision theory (as we illustrate in Chapter V). The relationship among these methods and the benefit-cost framework is described briefly in the remainder of this section.

The solution to the government's decision, again like the solution to the private decision, is to maximize the (positive) difference between benefits and costs (net benefits) which corresponds to the maximization of profits. This occurs in Figure 1-2 at effectiveness \( E^* \) where the slopes of the benefits and cost curves are equal. To provide a higher level of effectiveness, say \( \widetilde{E} \), means that the increase in well-being \( \Delta B(=B(\widetilde{E}) - B(E^*)) \) is less than the increase in cost \( \Delta C(=C(\widetilde{E}) - C(E^*)) \). Therefore, society would prefer to save the amount of resources \( \Delta C \) to receiving \( \Delta B \) (expressed in the same units as costs) in benefits.

\[ \text{1/ This is another application of the widely used marginal analysis so common in economics. In this case, the optimal point is D, where } B'(E) = C'(E) \text{ -- marginal benefits equal marginal costs (primes denote derivatives). If } B'(E) > C'(E), C'(E) > B'(E) \text{ for all } E, \text{ no system is optimal.} \]
While it is a simple matter to talk about curves labelled "benefits" and "costs", it is not so simple to estimate these curves in practice. However, by considering different situations in which the evaluation is to take place, appropriate techniques for analysis can be identified.

Suppose, first, we have a situation where two, or more, systems can be made to have varying degrees of effectiveness at some cost and within a limited range. This is illustrated in Figure I-3. The two systems $S_1$ and $S_2$ while similar in effectiveness and cost do show some differences, namely, in the cost associated with increased effectiveness (measured by the slope of the lines in Figure I-3).

![Graphical Depiction of Equal Capability Cost Analysis](image)

**Figure I-3**

Graphical Depiction of Equal Capability Cost Analysis
How can the decision-maker make a choice in this situation? If we assume certain requirements, say $R_0$ (see Figure I-3), then one appropriate way would be to compare cost at that level of effectiveness and $S_1$ would be selected. We can term this the "equal capability cost comparison approach."

The danger with this method, of course, is that at a higher level, $R_1$ (see Figure I-3), the decision would change and depending on how "close" $R_0$ and $R_1$ are, the sensitivity of the result to the presumed requirements level may be extremely high. Therefore, a careful sensitivity analysis is called for when this approach is used. In Chapter III, we will illustrate the use of this method in the disaster warning framework.

Generally, the decision-maker is not confronted with a choice among systems of similar effectiveness but one where different systems provide quite different levels of effectiveness. In this situation, one solution is not to redesign the systems but to relate costs to effectiveness levels and determine the additional resources that must be foregone to get a system with additional benefits.

This process is displayed in Figure I-4. There, effectiveness ($E$) and cost ($C$) are placed on the two axes and alternative systems, $S_i$, with different degrees of effectiveness and cost, are shown in relation to one another. If a particular system is lower in effectiveness and higher in cost than some other system, it is obviously dominated by that other system. Therefore, for system $S_3$, in Figure 1-4, we have drawn dashed lines delineating the northwest quadrant (the shaded area) in
relation to the system. If any system falls into this quadrant, we know it is dominated by system $S_3$. Thus, we know $S_5$ is dominated and we do not consider it further.

\[ S_i = \text{SYSTEM } i. \]

**Figure I-3**
Graphical Depiction of Cost-Effectiveness Analysis

Often, a minimum level of effectiveness shall be taken to be "required." If this means that the decision-maker cannot even consider systems whose effectiveness is below that level, then such systems can also be ignored. In Figure I-4 we have set a minimum level of effectiveness at $R$ thus removing $S_1$ from consideration.

Finally, note the solid lines connecting the remaining systems $S_2$, $S_3$ and $S_4$. These represent the "trade-offs" between effectiveness and cost that only a decision-maker can make. Represented as the slope
of the line \( \Delta E/\Delta C \), it is the increase in effectiveness that can be obtained for the increase in cost \( \Delta C \). The goal of cost-effectiveness analysis is to provide the decision-maker with these trade-offs.

While the method of cost-effectiveness analysis avoids the problem associated with dissimilar systems, it leaves the decision-maker confronted with a set of costs and effectiveness levels (and there is generally more than one measure of effectiveness) from which to make his choice. Thus, we do not consider cost-effectiveness analysis in this report.

A solution to the problem associated with systems providing different levels of effectiveness is to translate effectiveness into a measure commensurate with costs, in other words, to estimate the benefits function, \( B(E) \), described above. Therefore, a large part of the report is concerned with the development of methods and measures for finding the benefits associated with a particular level of effectiveness. In Chapter IV, we describe and illustrate the estimation of the generally accepted measure of benefits (consumers' surplus) in the context of disaster warning. As we will see there, knowledge of the demand curve is necessary for the calculation of consumers' surplus. Therefore, in Chapter V, we describe and illustrate a method called statistical decision theory that is used to estimate the demand curve if one is not known.

3. A Caveat

The use of any of these related methods requires care. Like other formal methods, they may provide a false sense of precision about the costs and/or benefits associated with a particular alternative. It is
unfortunate, perhaps, that uncertainty is a pervasive part of our lives. (Otherwise, of course, there would not be any requirement for a disaster warning system.) If the results of a benefit-cost analysis are presented as, say, X dollars in net present value, this may mask uncertainties about costs (e.g., perhaps communication cost will rise, or fall, in the future) or uncertainty about benefits (what will the demand actually be?). If these uncertainties are not made explicit, the results can be misleading.

In addition, these analyses cannot, practically, capture all of the benefits and costs associated with a project. It is unlikely any method can. Reliance on the results of such analyses alone is apt to lead to trouble.

C. Report Organization

The history of disaster warning concepts and analyses appears in Chapter II which is designed to provide a brief review of disaster warning and a basis for the methods described in the following chapters. In Chapter III, a specific method of evaluating systems with the same effectiveness (benefits) is described and illustrated by use of an example.

In Chapter IV, the measurement of consumers' surplus, generally accepted as the proper way to calculate the benefits associated with a specific system given a demand function, is discussed and an example calculation provided. Chapter V addresses the problem of assessing consumer demand in the absence of market survey or other demand data. This method does not rely on any survey and, in addition, can be used to estimate a demand curve from which benefits can be calculated. The relationship between the chapters and the components identified in Section A above is shown in Figure I-5.
Figure I-5
Relation Between Functional Components and Chapters in this Report

Component:

Sensing

Forecasting and Warning

Transmission

Reception

Action

Applicable Methods Discussed In:

Chapter III

Chapter IV

Chapter V

Components dealt with in this report.
D. Using the Report

While the report provides a framework in which to evaluate alternative disaster warning systems, a specific analysis need not require the use of all of the methods described. In order to assist the reader who has a specific analysis to perform, we have prepared the flowchart appearing in Figure I-6. By working through the flowchart, the relationships among the methods can be seen and the appropriate tool for the specific analysis can be selected.

The first question that is asked is: Do the systems that the analyst is considering appear to have roughly equal benefits? (The concept and measure of economic benefits are discussed in Chapter IV). If the answer to that question is "yes", an equal capability cost analysis (described in Chapter III) can be performed. If the benefits of the systems are sufficiently different, or if there is uncertainty about the relative benefits, then some estimate of the benefits of each system should be made. If the demand for the home receiver (or, equivalently, for the information provided by the receiver) has been previously estimated, the associated consumers' surplus (discussed in Chapter IV) can be calculated directly. If this demand information is not available (or as a check on the information) statistical decision theory (Chapter V) can be used to assess the potential demand and, from this, the associated consumers' surplus. When the net benefits have been estimated, they can be compared with the costs of portions of the system not covered by user chargers to provide information on the relative merit of the alternative systems.
Figure I-6
Selecting the Proper Method
In addition to providing a valuable decision-making aid, the methods we have identified can also be used to aid in the planning process. For example, reviewing results of various decision theory analyses can lead to the identification of services that would be highly valued by the individual and, therefore, these services become promising candidates for inclusion in a disaster warning or other system.

Finally, the usefulness of the methods described in this report is not limited to disaster warning systems. Any proposed project providing information to the public for which there exists a demand could be evaluated using these methods.

E. Report Summary

As noted above, the provision of services directly to the home by a disaster warning system means that analyses based only on transmission system costs may not always be correct for overall system evaluation. We have found, however, that by considering components of a system separately we have been able to identify and illustrate three methods which are suitable as evaluation techniques.

More specifically, we have observed as a result of this study that:

• unlike many government investment alternatives, a disaster warning system often requires an investment on the part of the individual. Therefore, the cost of the transmission portion of the system alone is not a sufficient criterion on which to base a government investment decision;
because the methods we propose incorporate the "private" decision and the individual's benefit-cost calculus, the economic benefits and costs of disaster warning systems can be analyzed with the methodological base we present;

given the receiver demand curve (from whatever source), consumer surplus is one measure of the economic benefits to be derived from a disaster warning system that, while not value-free, does have intuitive appeal and is a generally accepted measure of benefits;

statistical decision theory, because it provides a way of incorporating the inherent uncertainty associated with natural disasters is one method of assessing potential consumer demand that does not require an extensive and expensive market survey;

by making explicit the benefits and costs associated with the individual's receiver acquisition decision, statistical decision theory can often be useful in generating new alternatives that may provide greater economic benefits;

by removing much of the problem of unequal effectiveness inherently associated with different transmission systems, the equal capability cost analysis method of comparing alternative transmission systems can be usefully applied when the requirements for the system are clearly defined;
All methods have certain characteristics that make them more or less appropriate in specific situations. Therefore, the strengths and weaknesses of each of the methods described in this report should be considered before implementation of specific methods is attempted.

The methods described and illustrated in the following chapters provide the user with a set of proper, and practical, tools with which to evaluate alternative disaster warning systems. However, it is important to emphasize that, as with all analytic tools, the methods we propose in this report are only an aid (albeit an important one) to the decision-maker.
BIBLIOGRAPHY

CHAPTER II:
A REVIEW OF DISASTER WARNING PROGRAMS

A. Introduction

Before we describe the methods we propose, it will prove useful to review briefly the various disaster warning program evaluations that have been undertaken in the past. In particular, we review and critique four analyses of disaster warning systems to provide the motivation for the methods we propose in the following chapters. As a part of this review, a brief history of disaster warning is presented. This history provides a perspective from which to understand how current concepts and policies have evolved. In the Appendix, we describe several alternative technologies for disaster warning. This technology review is concerned only with the warning function and not with the functions of sensing and forecasting.

The next section traces the government's involvement in disaster warning from the early 1950's to the present. In Section C, four analyses that have addressed the evaluation of alternative systems are reviewed. Summary remarks are provided in Section D.

B. History of Disaster Warning Systems

The Federal Government's involvement with disaster warning, which has occurred primarily in the post-World War II period, originated with the concern about nuclear attack. In any emergency situation, including nuclear attack or natural disaster, early warning of those affected can

1/ This brief history is based on the discussion in [3].
be important in reducing losses. To this end, the government originally supported the installation of community sirens. Soon thereafter, in an effort to apply radio technology to the warning problem, CONELRAD was developed. This system provided the public with disaster information over two frequencies (both in the AM band) while other broadcasting was suspended. The purpose of using the two particular frequencies was to make it difficult for enemy aircraft to use the radio signals to navigate.

When more sophisticated navigation technologies became widely used (about 1963), CONELRAD was replaced by the Emergency Broadcast System (EBS). With EBS, selected stations broadcast warnings on their normal frequencies during emergencies.

A widely recognized problem with the CONELRAD and the EBS concepts was that people received warnings only if they happened to be listening to radio or watching television when the warning was broadcast. To overcome this problem, a method was sought to provide warnings even when the radio and television were turned off. The first concept considered was the National Emergency Alarm Repeater (NEAR), which was based on powerlines transporting a signal to turn-on radios and television sets. Development was halted when interference from other electrical appliances became a problem.
Following the Palm Sunday tornadoes in 1965, an interagency group was formed to study procedures that would reduce the impact of natural disasters. The study resulted in recommendations to combine many of the existing systems into a coordinated entity. The primary system was to consist of the weather teletype system feeding warnings to the public via the media and of outdoor sirens. A backup system, consisting of amateur radio, the National Warning System, and the NOAA Weather Radio System (see below) would support the primary system. Termed the Nationwide Natural Disaster Warning System (NADWARN), it was never actually implemented.

Two systems that are operational are the National Warning System (NAWAS) and NOAA's Weather Radio. The NAWAS of the Defense Civil Preparedness Agency (DCPA) is a telephone system which was designed to issue warnings of enemy attack to government agencies. Although designed specifically for attack warnings, it has also been used to provide warnings of natural disasters since 1958. The NAWAS is currently the primary system for issuing attack warnings.

The Weather Radio System of the National Oceanic and Atmospheric Administration (NOAA) is a network of VHF radio transmitters that provide local weather forecasts and warnings on a 24 hour basis. This is unique in that it has the capability to signal a warning by demuting a specially designed receiver. Until recently almost all such demutable receivers were in the hands of agencies that require quick notification in an
emergency which was apparently due to their relatively high cost. Relatively inexpensive demutable receivers are now becoming available and are being marketed in those areas where NOAA Weather Radio transmitters are operational. Less expensive receivers that do not have the demute capability but that are set to the proper frequency are also available.

In 1971, an interagency group chaired by the Office of Telecommunications Policy (OTP) conducted a study of the various warning systems then operating or planned for the future. Specifically, the group considered the EBS, the VHF-FM weather radio (NOAA Weather Radio), the commercial telephone system, a warning satellite, the Decision Information Distribution System (DIDS), and a system based on nighttime use of television stations (CHAT-TV). As a result of this study, (which was also concerned with the Administration's policy with respect to home receivers) it was established that purchase of a home receiver would be voluntary, and that the "DIDS system appears capable of providing the greatest coverage and geographical selectivity...and the fastest response time".  

Following this study, OTP studied the probable market penetration of home receivers. This study was based on several factors including cost, the time required to bring alternative systems to

1/ These systems are described in the Appendix.
2/ See Section C.2 below for a review of OTP study.
completion, and additional services that could be provided by the home receiver. As a result, it concluded in 1975 that NOAA's Weather Radio "is the best choice for a Federal radio warning system" [14]. Although DIDS as a warning system per se was not evaluated, it became official policy that DIDS would no longer be used for direct, to the home, warning. [14]

The General Accounting Office recently (April 9, 1976) issued a report criticizing the lack of coordination in the development of disaster warning systems. [9] It singled out three systems for particular criticism on operational and cost grounds: DIDS, NOAA's Weather Wire, and the Disaster Warning Satellite System (DWSS). As will be seen, however, the systems all vary in capability and functions performed, and, therefore, these cost comparisons may be misleading. [1]

This is, of course, an abbreviated history of disaster warning in the post-WWII era. It does indicate, however, the types of systems that have been conceived for this purpose. Several specialized systems also exist. These are designed for forest fires, river flooding, and tsunamis (to name a few). These are not all considered separately in the appendix since the procedures, the technology, and, in some cases, actual parts of other warning systems are used. However, in any evaluation of disaster warning systems, care must be exercised to include as relevant costs, changes in these specialized systems induced by changes in the disaster warning system.

Table II-1 presents a summary of the values for several descriptive characteristics discussed in the Appendix for some alternative disaster warning systems. The values in this table do not have the precision to make comparisons among systems unless the differences are very great. For example, the values under lead time that involve seconds or minutes cannot be differentiated without further information. Some of the systems we discuss in the Appendix were not included since they were similar to others. From the table it is clear that mass telephone ringing is dominated (i.e., it's more costly and less effective) by other systems. CHAT-TV, NOAA Weather Wire, and EBS also appear to be inferior solutions for disaster warning through home receivers. Of the remaining four, it is clear that there must be a trade-off between performance and cost in deciding which system is best.

Of course, these values are not all precise and subject to exact comparison. Generally, unless reliability is an important part of the system (as it is for DIDS), there may not be exact measurements. In those cases, we have attempted to provide at least a subjective value that can be used for comparisons. The current objectives for several of the programs are given in Table II-2.

One possibility, not considered in the comparison summarized in Table II-1 is that of combining two or more of the systems to provide both attack and natural disaster warning capability. Such a system might be a combination of NAWAS and NOAA Weather Radio (as suggested by GAO) or of DWSS and NOAA Weather Radio as developed in the CSC report.
Table II-1

Values of Descriptive Characteristics for Selected Alternative Disaster Warning Systems

<table>
<thead>
<tr>
<th>System</th>
<th>Coverage</th>
<th>Reliability 2/</th>
<th>Cost 3/</th>
<th>Selectivity</th>
<th>Hours</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Mech.</td>
<td>False</td>
<td>Surv.</td>
<td>(Not Necessarily Comparable)</td>
</tr>
<tr>
<td>DWSS</td>
<td>100  100</td>
<td>Good</td>
<td>Good</td>
<td>Excellent</td>
<td>$72 million</td>
</tr>
<tr>
<td>DIDS</td>
<td>96%  91%</td>
<td>Less than 1 min.</td>
<td>.99</td>
<td>.00001</td>
<td>Good (Good)</td>
</tr>
<tr>
<td>Weather Radio</td>
<td>90%  (55-85)</td>
<td>All</td>
<td>30 sec.</td>
<td>.99</td>
<td>.01</td>
</tr>
<tr>
<td>NAWAS</td>
<td>90%  (75)</td>
<td>All</td>
<td>25 sec.</td>
<td>Good</td>
<td>Good</td>
</tr>
<tr>
<td>Weather Wire</td>
<td>15%  (75)</td>
<td>Nat. Indeter-</td>
<td>Short (if during Broadcast)</td>
<td>Fair</td>
<td>Good</td>
</tr>
<tr>
<td>EBS</td>
<td>Variable All</td>
<td>Short (if during broadcast)</td>
<td>Good</td>
<td>Good</td>
<td>Poor</td>
</tr>
<tr>
<td>CHAT-TV</td>
<td>Variable All</td>
<td>12-24 (for preparation) 1-2 minutes (act. warning)</td>
<td>Good</td>
<td>Good</td>
<td>Poor</td>
</tr>
<tr>
<td>Mass Telephone</td>
<td>80%  (All)</td>
<td>No</td>
<td>Estimate</td>
<td>Available</td>
<td>Fair</td>
</tr>
</tbody>
</table>

1/ See Appendix for sources.
2/ Although some numerical values exist for the characteristics, we have included subjective values that we would have applied in the absence of this information. Thus, we are implying, for example, that .99 Mechanical reliability is "Good". We do this solely to provide a comparison with the other systems.
3/ For DIDS and DWSS, we show two cost estimates. This does not represent a possible range, but rather two different point estimates whose variance could well be caused by inflation.
<table>
<thead>
<tr>
<th>OBJECTIVE</th>
<th>FY 1975 PROGRAM CHANGES</th>
<th>FY 1976 PROGRAM CHANGES</th>
<th>FY 77-80 PROGRAM ESTIMATES</th>
<th>REMARKS</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>WARNING DISSEMINATION</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>NWWS</strong></td>
<td>Complete coverage of NOAA Weather Wire Service to all conterminous states in U.S.</td>
<td>Complete coverage in states of: NV, UT, NM, MS, MA, CT, RI, NH, ND, SD, ID(N), WA, CA (42/1425K)</td>
<td></td>
<td>Flow 1976 increases will complete the program with service available in the conterminous 48 states.</td>
</tr>
<tr>
<td><strong>VHF-FM Weather Transmission</strong></td>
<td>NOAA operational in 35 states.</td>
<td>Install equipment at 46 locations. Continue negotiations of purchase/lease arrangements for remainder of locations. (0/3560K)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Weather-by-Phone</strong></td>
<td>NOAA Install NOAA Weather Radio at 10 locations, bringing total stations operating to 77.</td>
<td>NOAA Install equipment at 23 sites. FY 77 Install equipment at 87 sites. FY 78 Install equipment at 50 sites. FY 79 Install equipment at final 48 locations.</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Funding by TELCO.</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Funding by TELCO.</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Source: [7]
<table>
<thead>
<tr>
<th>OBJECTIVE</th>
<th>FY 1975 PROGRAM CHANGES</th>
<th>FY 1976 PROGRAM CHANGES</th>
<th>FY 77-80 PROGRAM ESTIMATES</th>
<th>REMARKS</th>
</tr>
</thead>
<tbody>
<tr>
<td>DIDS</td>
<td>Provide a low cost radio warning system known as the Decision Information Distribution System (DIDS) to disseminate attack warning to selected Federal agencies, local governments and institutions, home of selected officials and emergency services key personnel, the broadcast stations, and by interface, over the NOAA Weather Radio System to private homes. Use DIDS whenever practicable to disseminate natural disaster warnings. Test interface of DIDS with the National Weather Service disaster warning operations.</td>
<td>DCPA Continue to operate the first DIDS transmitter located at Edgewood, Md. Conduct special tests. Continue the general weather announcements and time announcements now being made over DIDS. Continue to operate the 375 voice receivers now deployed and now operational in a 10-state area. FY 75 increased $500K over FY 74. DCPA and NOAA are developing procedures for dissemination of attack warnings over NOAA Weather Radio.</td>
<td>DCPA Continue operation of the first transmitter and the 375 voice receivers. Add 1,000 voice receivers. Conduct operational test of the 1,000 voice receivers deployed in specific geographical areas prone to natural disasters and other emergencies. FY 76 decreased $179K from FY 75. DCPA and NOAA will finalize and implement procedures for disseminating attack warnings by NOAA Weather Radio.</td>
<td>[NOTE: DIDS testing is currently being held in abeyance]</td>
</tr>
<tr>
<td>NAVAS</td>
<td>Provide additional DCPA National Warning System Circuits, in Weather Service Offices and communities. Provide interstate connections to Weather Service Offices for speeding the warning process when tornadoes cross state boundaries.</td>
<td>DCPA Only 38 out of a planned 200 cities and counties were added to the National Warning System due to a freeze on U.S. Army Communications Command leasing funds. The program is funded through the Dept. of the Army. This was a decrease of $40K from the FY 74 effort.</td>
<td>DCPA Add 77 National Weather Service Stations to the DCPA National Warning System giving them a total of 356 stations on the system. Add a total of 123 cities and counties to the National Warning System. The program is funded through Dept. of Army. This increase FY 75 funds by $80K.</td>
<td>DCPA Continue to add a total of 200 cities and counties to the National Warning System each fiscal year.</td>
</tr>
<tr>
<td>Future Dissemination System</td>
<td>Conduct investigations and studies needed to develop for mid 1980s a low cost national warning system that will make warnings available in all homes.</td>
<td>NOAA/NASA Continued work following initial jointly-funded feasibility study which showed a Disaster Warning Satellite System (DWSS) was technologically feasible.</td>
<td>NOAA/NASA Undertake system definition studies for a DWSS.</td>
<td>NOAA/NASA Further developmental work contingent on outcome of FY 1976 studies. Favorable outcome could lead to prototype launch in 1981.</td>
</tr>
</tbody>
</table>
C. Previous Analyses

1. Introduction

This section reviews and critiques previous analyses of the cost-effectiveness of alternative disaster warning systems. We consider four different analyses done over the period 1971-1976. The goals of this review are two-fold: first, to describe some alternative methods by which disaster warning systems have been evaluated; and, second, to motivate the methods we propose in the following chapters.

The four analyses that we review are: the Office of Telecommunications Policy (OTP) report of 1971 [11]; the General Accounting Office's report of 1976 [8]; the Computer Sciences Corporation (CSC) study of 1974 [1]; and the 1974 study by Rosen and Haines [4]. The first two of these represent evaluations of several systems while the last two represent more detailed studies of particular systems.

2. The Office of Telecommunication Policy Study

Because the definition of warning requirements overlaps the responsibility of several government agencies, a Warning Steering Group
was established under the auspices of the Office of Telecommunications Policy (OTP). This group was composed of representatives from the Office of Civil Defense (OCD), the National Oceanic and Atmospheric Administration (NOAA), the Office of Emergency Preparedness (OEP), the Federal Communications Commission (FCC), the Department of Transportation (DOT), and OTP.

To support the Steering Group, a Warning Working Group was established to:

... survey and summarize the current state of knowledge about the capabilities and costs of present and potential systems for alerting and informing the public. [12, p. 1]

There were three system requirements underlying the Group's work: provision of warning to the home; provision of both natural disaster and attack warning; and provision of a selective addressing capability.

In much the same way as we have done in the Appendix, various criteria were established by which to evaluate the different systems. These criteria were in terms of:

- coverage
- time constraints (time availability and the lead time required)
survivability
security (against unauthorized signals)
assessability and control (includes message priority capability, demuting capability and selectivity)
input and output (modes of messages and strength of signals).

These criteria served both to determine the systems chosen for evaluation and the comparison among systems.

Five systems were eventually selected by OTP for detailed evaluation: telephone warning systems, DIDS, CHAT-TV, FM-Broadcast, and VHF-FM (Weather Radio). In particular, a satellite system was not considered by OTP because: [12]

... the Working Group feels that satellite warning systems cannot meet the Minimum Acceptable Performance Standards, and are likely to be more costly than other systems.

However, the Group saw satellites as becoming technically feasible in the late 1970's.

The method employed in the study was cost-effectiveness analysis, i.e., the values for different performance criteria (effectiveness) were ascertained and costs were estimated. Based upon these

1/ The minimum values for the various criteria are discussed in [12], Annex B.
results, a determination could be made of the "cost-effective" system. As an example, we report the summary results for two systems (DIDS and VHF-FM) in Table II-3.

A major area of weakness in the study is the inconsistency of the cost estimates. This is understandable given the varying degrees with which each of the proposed systems is defined. Thus, the costs for DIDS are relatively detailed compared to the others. This makes it extremely difficult to compare the costs properly.

In addition, proper cost analysis concepts were not always followed. A prime example is the fact that total discounted costs were not presented. Costs for DIDS, for example, are presented as one-time and annual costs. Because the phasing of the different systems may be quite different, even the same information for all systems would not provide a proper basis for comparison.

Cost-effectiveness analyses, in general, also have two other characteristics that should be discussed. First, the benefits of the system are not explicitly calculated. This is because the multidimensional trade-off, relating several measures of effectiveness to benefits, is assumed to be specific to the decision-maker. This is often required for services for which data on the individuals valuations do not exist. We show in both Chapters IV and V that this data, however imperfect, does exist in the case of disaster warning. Without an estimate of the benefits associated with a system, it is difficult to relate the cost of a project to the benefits of the project.
Table II-3
Example of Results From OTP Study

**DIDS**

- **Response Time:** Excellent
- **Coverage:** Excellent; 97%
- **Survivability:** Very good
- **System Reliability:** Very good; has built-in safeguards against false alerts
- **Cost:** Nominal
- **General:** Meets all minimum requirements for attack and natural disaster home warning.

**VHF-FM**

- **Response Time:** Excellent
- **Coverage:** Good, 85%
- **Survivability:** Poor - subject to weapon affects damage.
- **System Reliability:** Good, however, there are hundreds of points where partial false alerts can be initiated.
- **Cost:** Moderate
- **General:** Does not meet all requirements; has insufficient address selectivity.

Source:[12].
The second area of weakness is the dependence of the results on the requirements. By a suitable specification of the requirements, any system can be made to appear "cost-effective" by virtue of the fact that it is the only one which meets the "requirements."

An example of this is the requirement for response time. The requirement, as stated in the OTP report is 30 seconds. The values for each of the systems evaluated are:

- Mass telephone ringing -- 33-48 seconds
- DIDS -- 20-29 seconds
- CHAT-TV -- "minutes"
- FM Broadcast -- 30-40 seconds
- VHF-FM -- 30-40 seconds

On the basis of this one requirement alone, only DIDS meets the "Minimum Acceptable Standard." 1/

Lest we appear overly critical, the type of approach does have valuable uses. In the early phases of any program, there will be many systems in alternative stages of development. A report such as the OTP study can be extremely useful in concisely describing and comparing alternative systems.

1/ In the following chapter, we discuss one method that is designed to deal with this problem.
Additionally, a study such as this can often be useful in giving a good indication of relative total costs even though refined cost estimates are not available. For example, the cost estimates provided for mass telephone ringing indicate (probably correctly) that this method would be significantly higher in cost than others.

3. **The General Accounting Office's Report**

At the request of Congressman Clarence J. Brown, the GAO conducted a study into the disaster warning system effort and the coordination problems encountered. The study was, therefore, not a study designed to determine the most cost-effective system *per se* but a study designed to improve the managerial and coordination efforts of the many Federal agencies involved.

Although this was the underlying theme of the study, the GAO also used this report to derive conclusions about which systems were cost-effective. It is, therefore, proper to inquire into the methods used to make this determination. Before we discuss the methods, it will be useful to describe the study.

The GAO identified twelve different warning systems currently planned and/or operated. Of these, seven are specialized systems (i.e., designed in response to specific disasters) while five are general purpose. The report is concerned primarily with these five, which are:
These systems are first described briefly and then the report discusses the reasons it believes "NAWAS and Weather Radio should provide an adequate means of warning the general public of natural disasters and enemy attacks." [9, p. 7]

The basic reason for this conclusion is that these two systems provide "sufficient" capability and at lower "cost." Thus, the analysis is again of the cost-effectiveness type and many of the comments in the previous section apply here also. In addition, in this study, the requirements are not clearly specified. This is recognized by the GAO and in fact their findings include the fact that "requirements of a consolidated warning system [are] not defined." However, the result of the GAO report is just the opposite of the OTP study. In the OTP report, only one system met all requirements while in the GAO study all systems met them, at least implicitly.

Second, benefits are never explicitly discussed. Although there is a recognition that the potential receiver market may be small and that the effectiveness of a system depends on the ability of the system to provide information to those who value it, no attempt is made to evaluate differences in this type of effectiveness.
Finally, the costs reported by GAO are mainly funding requests rather than estimated actual costs. Also, the costs are not reported as total discounted costs. This may make cost comparisons among systems misleading.

In summary, then, the GAO report suffers from the same defects and has the same strengths as the OTP report. Although these reports are useful in summarizing alternative system performance parameters, there generally is not sufficient information on which to base a cost-effectiveness decision. Additionally, these reports concentrate solely on the costs of the transmission system and neglect benefits and the costs of the receivers. As we will see below, without careful study to insure that alternative systems that have benefits which exceed total cost have been considered the "best" system may be overlooked. We are not suggesting that another system or set of systems is to be preferred to those recommended by GAO (or OTP). Rather, we would argue, neither study provides complete enough information on which to make a specific recommendation.

4. The Computer Sciences Corporation Study

In a complete and thorough study that meets many of the criticisms leveled above, CSC [1] has provided a detailed cost comparison between two alternative disaster warning technologies -- a satellite system
and a terrestrial system. The study was a cost-effectiveness study only and was not intended to be anything more. The report proceeds in the following manner:

- definition of requirements;
- design of a terrestrial system that satisfies the requirements;
- design of a satellite system that satisfies the requirements;
- cost comparison; and
- sensitivity analyses.

Because the assumed requirements were explicitly stated, the effect of small changes in any one of them can be (and is) analyzed in sensitivity analyses. Also, the cost estimates for both alternatives were prepared to the same level of detail using similar factors and methods. Therefore, the cost comparison provides useful information about the relative cost of each system at the functional level (i.e., warning, coordination, data acquisition, and spotter reporting).

The design of the terrestrial system is, basically, the NOAA Weather Radio System for broadcasting with dedicated landlines for certain coordination functions. The concept is illustrated in Figure II-1, which has been reproduced from [1, p. 6-18]. The satellite system was the DWSS described in the Appendix and illustrated in Figure II-2.

The cost estimates for the two systems were $1 billion for the terrestrial system and $1.6 billion for the satellite system. These
Figure II-1
Terrestrial System Concept

Source: [1]
Figure II-2

Satellite System Concept

Source: [1]
do not include receiver costs. The baseline systems were then modified to lower the costs. These resulted in costs of $.87 billion and $1.32 billion for the terrestrial and satellite system respectively.

As we stated above, benefits were not estimated in this study. Thus, it cannot be shown that either system is or is not cost-effective. However, it is an excellent example of a cost comparison that provides information on relative costs for two types of systems.

5. The Rosen and Haimes Study

In an imaginative and interesting study, Rosen and Haimes (hereafter, "RH") [4] investigated the costs and benefits of a home warning system for delivering messages about impending national (e.g., nuclear attack) and/or natural disaster to the general public at home. RH concentrated on costs and benefits of alternative home warning systems for providing warnings of natural disasters.

Three alternative transmission systems were investigated: DIDS, DWSS, and VHF-FM (NOAA Weather Radio). All transmission systems are designed to broadcast to demutable home receivers which demute upon receiving a suitable signal and then to broadcast a warning tone followed by a voice message concerning an impending disaster.

In the RH analysis, the costs of the three alternative systems included the investment and operating costs of both the transmission system and the home receivers. Benefits estimates included the estimated incremental reduction in property loss (estimated in dollars) and loss of life (estimated in lives) that would be achieved if one of the alternative
home warning systems were adopted and the existing system of radio and television warnings were maintained. An important step in the evaluation of benefits was predicting the rate and extent of adoption of home receivers by the public. RH concluded that in no case do monetary (property) benefits outweigh transmission system costs, and that about 300-540 lives would be saved by a home warning system (over a twenty-year period). ([4], p. vii)

The RH analysis of benefits is based on the following basic premises:

1. households that do not have home receivers receive no benefits from a home warning system,

2. benefits accrue to a home warning system only when warnings are received from it and from no other source (e.g., radio and/or television), and

3. benefits may be evaluated as reductions in property damage and lives saved as a result of defensive action taken, initiated by receipt of a warning.

Based upon these assumptions, RH constructed a probability simulation model to simulate the distribution of benefits that may be expected from adoption of each of the three home warning systems they studied. The details of their procedure are far too numerous and complex to discuss here. Instead, we have tried to develop a simpler version of their model which hopefully does it justice. To develop this stylized version of their model, we shall need a modest amount of notation:
Pr(E) = probability that a warning is received by the household via the existing warning system.

Pr(\overline{E}) = probability that a warning is not received by the household via the existing system.

Pr(H) = probability that a warning is received by the household via a home warning system.

L = loss per disaster per household in absence of warning.

M = fraction of household loss remaining if warning is received ("mitigated" loss fraction).

F = number of households.

In terms of this notation, the RH estimate of benefits (in terms of expected reduction in loss) per disaster for the existing system and for the home warning system may be explained as follows.

First, consider the expected reduction in losses per disaster per household using the existing system. Expected losses per household under the existing system are:

\[ L \Pr(\overline{E}) + M L \Pr(E) \]

That is, expected loss is loss in the absence of warning (L) times the probability that no warning is received (Pr(\overline{E})), plus the "mitigated" loss fraction (M) times unmitigated loss (L) times the probability that a warning is received (Pr(E)). The expected benefit per disaster per household of having the existing system over the no-system alternative, therefore, is

\[ L - L \Pr(\overline{E}) - M L \Pr(E) = L [1 - (1 - \Pr(E)) - M \Pr(E)] \]

\[ = L \Pr(E) [1 - M] \]

since

\[ \Pr(\overline{E}) = 1 - \Pr(E) \]
If a home warning system is added to the existing system, then the probability that a warning will be received by the household is:

\[ \Pr(E) + \Pr(H) - \Pr(E) \cdot \Pr(H) \]

and the expected reduction in losses per disaster over household over the no-warning case \((L)\) is:

\[
L \cdot \left[1 - \Pr(E) - \Pr(H) + \Pr(E)\Pr(H)\right] - \\
ML(\Pr(E) + \Pr(H) - \Pr(E) \cdot \Pr(H))
\]

\[
= L(\Pr(E) + \Pr(H) - \Pr(E)\Pr(H))[1 - M]
\]

The incremental reduction in expected loss per household obtained by adding a home warning system to the existing system is given by equation (2) minus equation (1), or:

\[
L(1 - M)[\Pr(E) + \Pr(H) - \Pr(H) \cdot \Pr(E) - \Pr(E)]
\]

\[
= L(1 - M)[\Pr(H) - \Pr(H) \cdot \Pr(E)]
\]

and the total loss over all households is then just,

\[
L(1 - M) \left[ \Pr(H) - \Pr(H) \cdot \Pr(E) \right] \cdot F
\]

This is the basic logic behind the RH estimates of benefits accruing as reductions in property loss (where \(L\) measures baseline property loss in dollars) and reductions in loss of life (where \(L\) is in lives). Based upon this logic, RH compute the estimates of benefits and lives saved for the alternative home warning systems shown in Table II-4. (The estimated costs are also shown there.)
Table II-4
Summary of Results of Rosen and Haines Study

<table>
<thead>
<tr>
<th></th>
<th>DIDS</th>
<th>DWS</th>
<th>VHF - FM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Present Value of Property</td>
<td>43.3 - 50.9</td>
<td>32.8 - 39.0</td>
<td>50.4 - 59.4</td>
</tr>
<tr>
<td>Savings (10^6 Dollars)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of Lives Saved</td>
<td>304 - 362</td>
<td>324 - 384</td>
<td>461 - 539</td>
</tr>
<tr>
<td>Over 20-Year Horizon</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Present Value of Trans-</td>
<td>60.9</td>
<td>180.2</td>
<td>25.8</td>
</tr>
<tr>
<td>mitters' Costs</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Present Value of</td>
<td>209.4 - 426.7</td>
<td>208.8-268.5</td>
<td>190.9-231.3</td>
</tr>
<tr>
<td>Receivers' Costs</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>


Obviously, several assumptions are required to compute the probabilities used by RH to estimate benefits and lives saved. For example, RH use assumptions about frequency of occurrence of disasters, the time of day at which they occur, the probability of hearing a warning on radio or television and so forth. RH have done a commendable job in documenting their assumptions and we do not intend to appraise their assumptions here.

The RH measure of monetary benefits is based on assumed market values of property damage reduced. This measure understates to some extent the benefits to be gained from receiving a warning for three reasons. First, it is doubtful that all costs that result from a disaster can be enumerated. For example, psychological costs borne by victims are not included in the RH measure. Second, there will be damage to some objects considered by victims to be "priceless." Third, there are inevitable transactions costs to be incurred in the replacement of damaged property.
A more usual method of assessing benefits is to inquire into how much the individual would pay to have the additional warning. As we show in Chapter IV using the RH data, benefits estimated in this way are substantially larger than those estimated by RH.

In summary, the RH analysis is useful in aiding our understanding of the stochastic nature of the problem and the way in which disaster losses are generated. In addition, their approach to benefits measurement provides at least a lower bound on which to conduct an analysis. Lacking any information on demand for receivers, this would be the best that could be hoped for. In Chapter IV we describe more completely the willingness-to-pay approach to benefits estimation and, as an illustration of the method, use the same data as RH to derive revised benefits estimates.

D. Summary

In this brief review we have identified several applications of cost-effectiveness analysis and one application of benefit-cost analysis. Two of the cost-effectiveness analyses, the OTP report of 1971 and the GAO report of 1976 illustrate the dependence of the conclusions of the analysis on the assumed requirements. The CSC report illustrated one approach to mitigating the effects of requirement definition on the relative costs of the systems to be compared.
The Rosen and Haimes study was the only one of the four analyses reviewed that attempted to measure benefits. The method they used was to estimate the property savings and lives saved associated with alternative systems. Although this measure generally underestimates true benefits (see Chapter IV) this study represents a valuable first step in the proper evaluation of alternative disaster warning systems.
APPENDIX

REVIEW OF ALTERNATIVE DISASTER WARNING SYSTEM TECHNOLOGIES

A. Introduction

1. Candidate Technologies

The alternative technologies to be described in this appendix include systems that are currently in operation and two that have not been implemented but for which considerable documentation exists. Current technologies that will be described are the Emergency Broadcast System (EBS), National Warning System (NAWAS), NOAA Weather Wire (NWWS), and NOAA Weather Radio (NWRS). The other two systems are the Disaster Warning Satellite System (DWSS) and Defense Information Distribution System (DIDS). 1/ For each alternative, the hardware will be described, system operation discussed, and the values for the descriptive characteristics described below will be provided.

Section B describes the current systems while Section C is concerned with those systems that have not yet been implemented but that still represent technically feasible approaches. Section B also includes a brief discussion of systems that have been reviewed in the past but have not been implemented, nor are they currently being studied.

---

1/ As a result of a recent OTP Policy Statement [14], DIDS is not currently being considered for home warning. However, because the technology represents a technically feasible approach, we have included DIDS in our review.
2. Disaster Warning System Characteristics

The alternative technologies to be described below possess very
different characteristics. Thus, it is necessary to develop some
characteristics that may be used to compare them. The characteristics
listed below will be used throughout to describe and compare
the various disaster warning systems. Due to the nature of the different
systems and the differences in their level of development, not all
characteristics can be assigned a numerical value for all systems. 1/
For example, the values for reliability (described below) are not available
for most systems and therefore have been assigned rather subjective
values. The cost estimates are not necessarily
comparable because of differences in system capabilities. In addition,
many of the "costs" reported here are appropriations and may understate
or overstate the true economic costs of a particular system.

a. Coverage

The coverage achieved by a system can be classified as being one of
three types. First, there is demographic coverage which describes the
characteristics of the population that will be affected by the new technology.
The primary measure of demographic coverage is the percentage of the total
U.S. population that is able technically to receive a warning from a particular
system. The demographic coverage characteristic is important because

1/ The values assigned to the characteristics of each system are
taken from different sources. This is in itself a major cause of
noncomparability.
it indicates the potential for accomplishing the main goal of any warning system: to get the warning to as many people who may be affected by a national disaster as possible.

The second type of coverage characteristic is geographic coverage. This refers to the land area reached by the system's signals (e.g. the forty-eight contiguous states). Geographic coverage is important since there may be areas that are prone to natural disasters of a particular type that will not be covered.

Finally, we will be interested in the types of natural disasters that a particular system is designed to warn against. In general, the warning system tends to be independent of the underlying disasters. There may be technological or design reasons, however, for a limitation on the types of disasters reported. If a system is designed primarily for natural disaster warning, for example, it is unlikely that it will be hardened against nuclear attack. Although technically capable of providing such a warning, it would be vulnerable in an actual attack.

b. Lead Time

The second important characteristic that can be used to describe a system is the lead time. This is the time from the issuance of the warning to the receipt of the message. The shorter the lead time, the greater is the time available to take precautionary actions.
c. Reliability

In any communications system, an important aspect of its effectiveness is its reliability. Reliability can be further divided into equipment availability, possibility of generating a false warning, 1/ and (for want of a better term) survivability.

A measure of equipment availability is the probability that, at any point in time, the system will not function. Availability is particularly important with respect to disaster warning systems because of their relatively infrequent use, coupled with the great potential loss should the system fail to operate as expected.

Another aspect of reliability is the probability of issuing a false warning. Since false warnings are costly, both because of the costs incurred from taking unnecessary action and from the decreased confidence in the system, false alarms should be infrequent.

Survivability, on the other hand, refers to the capability of the system to survive the disaster and to withstand the environment even after the impact of the disaster. This may include, for example, radiation after a nuclear attack or heavy winds during and after a hurricane.

1/ In this case, we mean false warnings due to equipment failure and not to the issuance of a forecast later shown to be false.
d. **Cost**

Clearly, an important characteristic is the cost of the system. This means the total cost, which includes investment costs, operating and maintenance costs, receiver costs, etc. A complication with this measure is that the system may also be used for another purpose and that some sort of cost allocation to the disaster warning function will be necessary.

e. **Selectivity**

Especially in the case of natural disasters, selectivity (the capability to address a specific subset of the population) is important if the system is to be accepted. Numerous warnings for disasters occurring elsewhere will, sooner or later, cause citizens to ignore warnings that actually concern them. The degree of selectivity can be measured in several ways including the number of addresses, \( \frac{1}{n} \) or the smallest geographical location that can be addressed. Selectivity is also important because different disasters may make it desirable to select the target audience in different ways. For example, flooding will tend to affect those in low-lying areas and along rivers. Tornadoes, however, follow a less predictable path.

f. **Readiness**

The final characteristic considered here is the readiness of the system, as measured by its hours of operation. The closer the system

\[ \frac{1}{n} \] The number of subsets of the population.
is to a 24-hours, seven days a week operation, the more effective it will be. Several systems, particularly those that depend on sources outside the Federal Government for transmission, may operate only on a part-time basis.

B. Present Warning Systems

There are currently several systems that are, or could be, used for natural disaster warning. Some of these systems are already in operation while others are in various stages of development. Below, we briefly describe the hardware used by these systems and the effectiveness, measured in terms of the characteristics described above, for the task of natural disaster warning.

1. Emergency Broadcast System

As described in the introduction of this chapter, the Emergency Broadcast System (EBS) was designed to replace CONELRAD and to provide the President with a direct means of communication with the public in times of attack or other national emergency. Because it uses the broadcasting facilities of private stations, it is dependent on their cooperation. EBS circuits lead to the nationwide dissemination path for the broadcast industry so that more than 8,500 stations are accessed [9]. This makes for broad geographic coverage but it also depends on the station transmitting being on-the-air and individual's listening
or watching. Although this system is not used for local warnings, the FCC could approve procedures for "linking" other warning systems with local broadcast stations [9].

The basic problem with this system, for the purposes of warning, is the lack of capability of receiving a warning when the receiver is off. This deficiency lowers the probability of an individual home receiving any one warning. Thus, while the potential coverage may be high, the prospects for receipt of a message are not. Of course, this system also does not require the purchase of an additional receiving device and may, therefore, be as effective as a system that requires purchase of a home receiver whose price is sufficiently high to discourage purchase.

The nature of the EBS makes the assignment of values to the various characteristics difficult since the voluntary nature and the lack of control that can be exercised by a central agency often results in considerable variability. However, some descriptive values may be given.

a. **Coverage**

As was stated above, the percentage of the population that is potentially "alertable" is equal to that number of households that have a radio or television and that are in the vicinity of EBS stations. In terms of the alertable population, coverage is close to 100 percent [9].

b. **Lead Time**

The lead time for the EBS is less than one minute, assuming that the alert occurs during normal broadcasting hours.
c. **Reliability**

The mechanical reliability of the EBS is good. However, only 600 of the stations have backup power. [11] Thus, EBS might not be usable during and after disasters.

d. **Cost**

The only cost of the EBS is the annual circuit rental for the leased line to the network distribution point, which is approximately $127,000 annually. [9] All other facilities are provided by the broadcaster and no estimate of these costs (if any) is available. There is no additional home receiver cost.

e. **Selectivity**

For those in areas that are able to receive a broadcast the selectivity is good because, while all in the viewing and listening areas are being warned, it is possible to describe the affected areas.

f. **Readiness**

The hours that the system is operational varies according to the broadcast schedule of the stations in the system. Although many of the EBS stations will operate on a full-time basis, the period usually taken is 6:00 a.m. - 1:00 a.m., the period of when most stations are generally transmitting and the audience is of sufficient size for the warning to have any effect. Since there is no positive alert feature (demuting, sirens, etc.) associated with EBS, we take 6:00 a.m. - 1:00 a.m. as representative of the readiness of the system.
2. National Warning System

The National Warning System (NAWAS), which is under control of the Defense Civil Preparedness Agency (DCPA), is operated and funded by the U. S. Army Communications Command. The system is designed to provide warning of an attack upon the United States. Under the Disaster Relief Act of 1970, the NAWAS can also be used by the National Weather Service (NWS) to provide warnings to local officials of impending natural disasters. Figure II-A-1 shows the location of the NWS offices that have "drops" off of the NAWAS. By using the NAWAS, information can be passed expeditiously between NWS offices. NAWAS is controlled by three National Warning Centers located in protected facilities in various parts of the country. The system can be divided into eight separate areas controlled by a DCPA regional office. Any one National Warning Center can control the entire system, and any state can operate the system as a warning system while continuing to monitor the regional circuit.

All circuit terminations of the system are in government offices. Since it is not designed for direct (to the general public) warning, the warnings issued over the system must be relayed by sirens or some other method. However, because the system operates as a party-line, post-disaster feedback is provided.

The values for the characteristics for the NAWAS are more readily available.

a. Coverage

The population coverage of the NAWAS will be 90% when completed [9]. The geographic coverage while still unknown appears to be similar to that of other systems with similar population coverage or around 75%. Although designed specifically for warning against attack, NAWAS can also be used for natural disaster warning or as a statewide warning system.
Figure II-A-1

National Weather Service NAWAS "Drops"

Source: [6].
b. **Lead Time**

The lead time required to transmit a message to all points on the network has been estimated at 25 seconds [9, p. 9]. This, however, does not include the time required to warn the general public.

c. **Reliability**

The equipment reliability is good and the three National Warning Centers are hardened. System reliability suffers because, in getting the message from the Center to the public, human action is required at points other than the initiating Center. A backup circuit sends the warning to the UPI and AP news services. From there it is transmitted to "about 6,800 radio and television stations [9]."

d. **Cost**

Estimated costs, through 1980, are $14.4 million [9]. These include both operating costs and investment (for expansion). Annual operating costs only are estimated to be $2.5 million [9, p. 9]. There are no home receiver costs.

e. **Selectivity**

Selectivity varies from state to state because of differences in the state systems.

f. **Readiness**

NAWAS is designed to operate 24 hours per day, seven days per week.

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1/ Although not explicitly stated, it appears that all costs in the GAO report [9] are constant 1976 dollars. The basis for this is that for EBS, the five year total is equal to five times the annual cost.
3. **NOAA Weather Radio**

The NOAA Weather Radio System is designed to provide continuous weather forecasts and warnings over three FM frequencies (162.55MHz, 162.40MHz, and 162.45MHz). These broadcasts can be received on specially designed radios available to the public, some of which have the capability to demute on signal. There are at least two advantages of a system such as this over a system such as NAWAS. First, there is no possibility of congested circuits since the system depends on radio broadcast. Second, the warning is issued directly to the household via a receiver that also provides continuous weather information during normal periods. Thus, the individual is provided with additional capability for his investment.

331 transmitters would be required to cover 90% of the contiguous U. S. (see Figure II-A-2). Each of these transmitters will be equipped with a 330-1200 watt transmitter. In addition to the transmitting equipment, appropriate tape equipment, antennas, circuits to the transmitters, a tone alert transmitter and spare parts are required.

The receivers, in the hands of the general public and organizations, are of two types: a standard VHF receiver with either fixed or tunable frequency, (two or three channel) and a tone-alert receiver with fixed frequency.

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1/ Not all receivers have the capability of receiving all three frequencies.

2/ A different problem is frequency congestion. This affects the completion but not the operation of the system.
NOAA WEATHER RADIO NETWORK

IN OPERATION
EQUIPMENT BEING INSTALLED
EQUIPMENT ON ORDER
PLANNED

1/ As of 1975.
Source: [7].

Figure II-A-2
Procedures for issuing the warning are as follows: The operator takes the equipment out of the normal weather reporting tape mode. The tone alert button is then pushed to activate the demutable receivers and the warning is issued.

a. Coverage

The nature of the Weather Radio System constrains the coverage characteristics to be dependent upon the number of transmitters. Currently, there are 115 active broadcasting transmitters. Their locations are shown on the map in Figure II-A-2. When completed, the system will contain 331 transmitters in all. At that time, the population coverage is estimated to be 90 percent [2, p. 1]. The geographic coverage is unknown but an estimate of 55-85 percent appears reasonable. ¹ While primarily designed for natural disaster warning, the NOAA Weather Radio System can also be used for the dissemination of attack warnings under an agreement between NOAA and DCPA.

¹ The radius of coverage is to be about 64 km (40 mi.) [6]. If 64 km is used, the area covered is 4.25 million square kilometers, or 55 percent of the gross (land and water) area of the 48 contiguous states. If a 80 km (50 mi.) radius is used the coverage increases to 86 percent.
b. **Lead Time**

It has been estimated that the time required to activate the system, once a warning message is received, is less than two minutes [12].

c. **Reliability**

During a 1968 test, the alert signal activated the receiver 99% of the time and a false signaling occurred approximately one percent of the time [12]. The receivers were located up to 80 kilometers from the transmitter.

As described above, there are two ways of transmitting the warning message. When the landlines are broken, however, the alert signal will not be broadcast. Most of the transmitter sites have backup power but no sites are hardened.

d. **Cost**

The cost of the NOAA Weather Radio service can only be estimated at this time since the method of providing service to the remaining 254 sites is not known. On May 13, 1976, in testimony before the House Subcommittee on Communications, estimates of per site costs varied from $6,200 to $20,000 annually depending on the bidder and whether or not the communications equipment was included [2]. Using $15,000 per site per year for the still uninstalled sites and $8,000 annual operating costs for those sites already installed, an estimate of the annual operating costs of the Weather Radio system is $4.4 million. This estimate, however, is subject to error as the actual nature of each site installation is still an unknown.

1/ The cost is calculated using 254 sites @ $15,000/year/site plus 77 sites @ $8,000/year/site.
Costs of the home receivers are projected to be in the range of $15-25 per unit [9]. This range includes both radios with and without automatic demuting, assuming sufficient quantities of each are produced.

e. **Selectivity**

The system can address 331 different areas separately.

f. **Readiness**

The system is designed to be ready 24 hours per day, seven days per week.

4. **NOAA Weather Wire Service**

The NOAA Weather Wire Service (NWWS) was designed as a system that would provide consumer-oriented weather information (forecasts and warnings) directly to the news media. The system consists of a network of leased lines and teletypewriters linking local NWS offices with broadcasters and newsrooms. The service is primarily intrastate with overlay circuits providing the interstate connections. Input to the system is limited to certain NWS offices.

The system is envisioned by NOAA as a backup to the NOAA Weather Radio that will provide broadcasters with a printed copy of the message. This system has essentially the same drawbacks as the EBS discussed earlier: the efficiency of the system is dependent on the broadcaster's cooperation and whether or not the public is "listening". However the NWWS suffers from the added complication that the service is a subscription service. If the individual news organization is not a sub-
scriber, neither it nor its audience receives the warning. The fact that much of the same information is available from Weather Radio, and that the cost of that system is the one time purchase cost for the receiver, lessens the incentive to subscribe to the NWWS.

The characteristics for the NWWS are described below.

a. **Coverage**

The NWWS is currently available in 36 states plus the District of Columbia (see Figure II-A-3) [7]. Originally expected to be operational in all states by FY 1976, progress is currently dependent upon available funding. The population covered is highly variable since it depends on whether or not the local broadcaster is a subscriber. Of the 9,000 broadcasting stations, only about 1,300-1,400 currently subscribe to the service. No estimate of the percentage of the population covered is available. The NWWS is solely used for weather forecasts and warnings but there is no technical reason why it could not be used for other warnings.

b. **Lead Time**

As in the case of EBS, the lead time for the NWWS is dependent on whether or not the stations are broadcasting. If they are on the air, a warning should be broadcast within a few minutes of being typed into the network.

c. **Reliability**

Since the NWWS is a subscription weather service, no extraordinary precautions have been taken to insure the system is protected from natural disaster. Like the EBS, the use of NWWS during and after disasters is doubtful. The NWWS is not hardened.
Figure II-A-3

d. **Cost**

The estimation of the costs of the NOAA Weather Wire service involves two complicating factors: the existence of common personnel costs and the revenues produced by subscribers. Using figures from [9], the incremental costs (costs that could be avoided by discontinuing NWWS) can be estimated as $2.68 million annually (49 personnel @ $20,000 annually plus $1.7 million in annual circuit cost). Annual subscriber revenues, assuming $75/month1/ and 2,000 subscribers would be $1.8 million. The net cost of NWWS is thus about $880,000 annually.

e. **Selectivity**

Only those areas that are covered by the subscribing news organizations can be addressed. Within those areas, the selectivity is the same as for the EBS.

f. **Readiness**

The hours that this system would be effective are the hours that the station is broadcasting, generally 6:00 a.m. to 1:00 a.m. although, again, many stations will operate 24 hours per day.

5. **Other Systems & Concepts**

There are several other possible systems either currently used for specialized warnings or that have been proposed as candidates for a disaster warning system. We will briefly mention some of those systems and indicate the advantages and disadvantages for use as a natural disaster warning system.

1/ The GAO [9] gives estimates of $50 - $100/month.
a. Washington Area Warning System

The Washington Area Warning System (WAWAS) is used to provide warnings of attack or natural disaster to Federal, State, and local governments in the Washington, D.C. area. It consists of two-way voice channels with a two-way record copy backup. In terms of technology and concept, WAWAS does not appear to be much different from NAWAS and therefore the comments of the latter system apply here.

b. Tsunami Warning System

This system, which operates between Alaska, Hawaii, and the West Coast, is designed to warn against a specific type of danger, the tsunami. It does this over various circuits including the teletype circuits of DoD and the NAWAS. It too is similar to the NAWAS and has many of the same characteristics.

c. U.S. Coast Guard Marine Weather Broadcast

This system is designed to provide weather information to the marine community on the coasts and the Great Lakes. This is done over approximately 300 HF, VHF, and VHF-FM Coast Guard stations. Since this system is also a radio broadcast system, it will have much in common with the NOAA Weather Radio.

d. Multiple-Access Recorded Telephone Systems

The telephone system that provides weather forecasts and warnings to many areas of the country cannot be considered a viable alternative for disaster warning since it requires that the public take the initiative and call for the information. The location of these telephone services is shown in Figure II-A-4. This illustrates that large areas of the country are without service.
Figure II-A-4

Multiple-Access Recorded Telephone Announcement Systems

Source: [6]
e. Mass Telephone Ringing

One system that has been proposed in the past is the ringing of telephones in the individual households. The system would operate as follows: The local central office would receive notification signal. It would automatically disconnect all calls in progress, ring telephones (with a distinctive ring), deliver a voice warning, and then resume normal telephone service. The coverage for this service was estimated to be approximately 80% [12]. The cost was estimated to be on the order of $300-500 million investment cost plus an annual cost on the order of 40% of the investment cost [12]. From these figures, it is reasonable to say that this system is not as good an option as some of the others, since its costs are considerably higher and its effectiveness no better.

f. Crisis Home Alert - T. V. (CHAT-TV)

This system is another warning system that takes advantage of the household television set as a warning receiver. The procedure under CHAT would be as follows. When there is a crisis or the probability of a natural disaster, the public would be advised to tune their television to a predetermined "silent" channel for the night and leave it there. When an attack or the disaster was imminent, the broadcaster would broadcast this warning over the channel at higher than normal volume. If no disaster or attack occurred, the television could be turned off in the morning. Two things about this system are obvious. First, the attack or the disaster must be predicted well in advance. Second, it assumes that people are already watching television (to obtain notification of possible warning). In fact, in the case of
a natural disaster, if people are tuned to a distant station (e.g., through cable television) they may not receive the warning. In addition, this system again depends on the cooperation of the individual broadcaster.

C. Other General Systems

1. Decision Information Distribution System (DIDS)

In 1972, the Defense Civil Preparedness Agency (DCPA) of the Department of Defense started work on the Decision Information Distribution System (DIDS). The system was intended to provide a warning capability for both (nuclear) attack and major natural disasters. As warning system in addition to the current NAWAS system, DCPA has stated that DIDS would provide faster warning, wider coverage, greater reliability, more complete information, direct warning with home receivers, and the capability to warn of possible weapons impacts locations. Although no longer being considered for home warning, the technical details of DIDS warrant its inclusion in this review.

The distribution system would be terrestrial, low-frequency radio system that provided coverage to the 48 contiguous states. The transmitter sites would be constructed to withstand natural disasters and hardened for protection in a nuclear attack. The first DIDS transmitter was completed in 1974 and has undergone some tests although it is not active currently.

The distribution system would consist of the following components:
• Three National Warning Centers located throughout the country and already in operation as part of NAWAS. These centers are located near Colorado Springs, Denton, Texas, and Washington, D. C. Their function was discussed above under NAWAS.

• Two control stations with high power (200 KW), low-frequency (61.15 KHz) transmitters designed to be automatic and hardened for protection.

• Ten distribution stations with medium power (50 KW), low-frequency (167-191 KHz) transmitters, having the same protective features of the control stations.

Figure II-A-5 shows the proposed locations of these stations and their proposed coverage. A backup system using leased landlines and dial-up AUTOVON circuits (the telephone network of DoD) would interconnect the three Centers, two control stations and ten distribution stations.

The receiving portion of DIDS would consist of 40,000 receivers located at various Federal, state, and local offices, as well as the homes of selected government officials [8]. These receivers would vary in their capability to receive messages in different formats. The eight DCPA regional offices would have receivers with voice and radioteletypewriter capability. These capabilities would also be available to state and local governments. The offices of the state governors, the state adjutants general, and the state civil defense offices would have receivers with
DIDS TRANSMISSION COVERAGE
High Reliability Service

Source: [7].

Figure II-A-5
voice capability only. The emergency operating centers of the U. S. Army Areas would have both types of receiving capability. The approximately 5,000 control rooms where siren systems are controlled would be equipped with receivers that have the capability to control the sirens automatically. The concept of the system is illustrated in Figure II-A-6.

The DIDS would have the capability to operate in two modes: fully automatic nationwide and "ad-lib". In the automatic mode, the warning officer (see the description of NAWAS above), upon command from the Director of Civil Preparedness, would activate the control and distribution systems. The Warning Center from which the alert is issued would transmit the alert to the two control stations and simultaneously, through the landlines, send it to the two control stations and the ten distribution stations. The control stations would then transmit the alert to the distribution stations. Upon receiving the broadcast, the distribution station would select a prerecorded tape and transmit a signal that would turn on the receivers and that would automatically activate the community sirens. When these actions were completed, the system would then transmit a signal to turn the receivers off (remote) unless another message was to be sent. In the ad-lib mode, the warning could be limited to those areas affected.

The values for the characteristics for DIDS, taken from available documents, are less refined since the system has not been implemented. Where there are disagreements among the sources, we have generally taken an "average" or "representative" value.
Figure II-A-6

Source: [6].
a. **Coverage**

An estimated 96-98% of the population would be covered by DIDS [3, 8]. These values are not precisely comparable with other coverage estimates, because there are some areas where voice warnings will not be received but sirens will be activated. We use 96% as the population coverage since that represents a reasonable possibility for the reception of voice warning. Voice warning is the appropriate figure for DIDS since it is the direct home warning aspect of DIDS that makes it substantially different from NAWAS for our review.

Geographic coverage of DIDS is estimated at 91% for voice and 99-100% for city and community sirens [12]. DIDS is capable of providing warnings against both attack and natural disasters.

b. **Lead Time**

The lead time for DIDS is estimated to be 30 seconds [3].

c. **Reliability**

The overall reliability for DIDS is estimated to be .99. The probability of a false alarm (a mechanical failure resulting in a warning being issued) is estimated at 0.00001. In terms of failure rates, the MTBF for all but the transmitters is 5 years. The MTBF for the transmitters is 3000 hours [12]. The DIDS is hardened.

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1/ This value refers to cities and communities and not to 99-100% of the total land area.
d. **Cost**

Investment cost estimates for DIDS range from $59 million [8] to $73 million [9].\(^1\) This does not include the cost of household receivers. (It is unclear whether these figures include receivers in government offices.)

e. **Selectivity**

Up to 5,000 separate codes for states, counties, and up to 300 cities can be selected in DIDS [3].

f. **Readiness**

DIDS would be designed to operate 24 hours per day, seven days per week.

2. **Disaster Warning Satellite System (DWSS)**

Because of advances in satellite technology, and particularly in the technology of communications satellites, the use of a satellite for disaster warning is technically feasible. In 1969, NOAA and NASA entered into an agreement to study the possibility of using a satellite for this purpose. Although still in the study phase, and hence not completely defined, a warning system based on the use of satellites can certainly be described operationally, and the characteristics as far as the design goals are concerned can be spelled out.

Although the first study was completed in 1970, a more complete review was undertaken in 1972 to determine the feasibility of a satellite system in comparison to a terrestrial system [1]. (The terrestrial system

\(^1\) This difference may well be the result of inflation because of the different times at which these estimates were published.
was very similar, in its warning function, to the NOAA Weather Radio
and hence is not discussed further in this chapter. 1/

There are three basic hardware components in the warning
system portion of the DWSS. 2/ The first is the satellite(s) used for the
communications. Depending on the capabilities of the satellite, more
than one may be required (for example, if insufficient battery power is
available during eclipse). The second is the antenna at (or remotely
located from) the local Weather Service Offices (WSO's) used to trans-
mit warnings. Third, is the home receiver which must be capable of
receiving the satellite signal.

The procedure for issuing a warning is much the same as in
the NOAA Weather Radio System. However, if there are no continuous
weather forecast broadcasts, 3/ there would be no tape to halt. The
warning is issued by the WSO via the uplink channel. The satellite
issues the demuting signal to the selected addresses followed by the
warning. The concept of the DWSS is illustrated in Figure II-A-7.

Since the DWSS is still being designed, the following characteristic
values are based on design goals.

1/ This study is discussed in Section C.4 above.

2/ The DWSS as conceived in the CSC study [1] can also be used for data
collection and distribution, spotter reports and spotter coordination.
These are addressed in the next chapter. Here we are concerned
only with describing the warning function of each of the systems.

3/ The GAO report states (apparently incorrectly) that "...this system...would
transmit weather forecasts and warnings directly to special receivers
purchased by the public." [9, p. 4]
SATELLITE SYSTEM CONCEPT

Source: [1]  Figure II-A-7
a. **Coverage**

Both demographic and geographic coverages are 100 percent\[6,7\]. The geographic coverage includes coverage of portions of the Atlantic and Pacific Oceans. Both types of disaster warnings could be issued with this system.

b. **Lead Time**

The lead time is estimated to be less than one minute\[10\].

c. **Reliability**

The design life for the satellite is five years\[1\]. An on-the-ground backup satellite could be provided.

d. **Cost**

The system cost for the DWSS naturally depends on the number of satellites used. In addition, the number of communication channels available is an important cost driver. An estimate of system cost is $81 million (in constant, 1976 dollars).\[9\] A second estimate is $72 million for the R&D program (including a first satellite) plus $25 million for each additional satellite\[10\]. These costs are in constant, 1974 dollars. Because of the design uncertainties, costs are less certain for the DWSS than the costs provided for the past or present systems. An important factor in the cost associated with the satellite (and all other systems) is the extent to which other services are provided.
e. **Selectivity**

One of the stated requirements for the DWSS is the capability to address a part of one county implying great selectivity (up to 20,000 separate areas) [1].

f. **Readiness**

The DWSS is designed for full-time availability.
REFERENCES


CHAPTER III
EVALUATION OF SYSTEMS WHEN BENEFITS
HAVE NOT BEEN ESTIMATED

A. Introduction

In the previous chapters we emphasized the importance of benefits estimation in a benefit-cost analysis. There may be times, however, when benefits estimates are not available or they are suspect. Also, as is sometimes the case for disaster warning systems, transmission systems with the same coverage, reliability, readiness, etc., (and hence with roughly equal benefits) may have to be compared. In such a situation the best system is the one with the lowest cost. 1/

The purpose of this chapter is to illustrate that a method that we have called 2/ the "equal capability cost comparison" approach can be used to derive implications about the relative cost-effectiveness of two such systems without having to consider a multi-dimensional effectiveness measure. Thus, we avoid the need to trade off, say, population coverage with geographic coverage, etc.

As we have illustrated graphically in Chapter I, equal capability cost analysis consists of the following steps:

- specify the requirement(s) each DWS alternative is to meet;
- design each alternative to meet each requirement in the least-cost manner;
- Calculate the costs associated with each alternative
- perform sensitivity analyses;

1/ Note that this assumes that a disaster warning system has positive net benefits. If not, no system, regardless of relative cost, is "best."

2/ See Chapter I.B above.
Of course, simplifications such as the avoidance of specific tradeoffs do not come without their price. Often in attempts to design two or more systems to "equal capability" we must mold the designs to meet the assumed (required) capability, which often results in a hybrid system design for which a minimum cost design is more difficult to identify. Thus, we may run into many of the same problems that were criticized in Chapter II, namely, by specifying the capability, we may be implicitly selecting the system. However, as we stated in Chapter II, this approach can often be useful in indicating dominated alternatives and allowing the decision-maker to focus on fewer, more desirable alternatives.

In the next section, we use this approach with an example comparison of two possible disaster warning transmissions systems. In Section C, we remark on the use and efficacy of this approach.

B. Illustration of the Equal Capability Cost Comparison Approach

1. Problem Definition

This section presents a cost comparison between two alternative systems designed for disaster warning using the equal capability cost analysis method described above. The two systems considered are the Disaster Warning Satellite System (DWSS) and the VHF-FM NOAA Weather Radio System (NWRS). (Although the DWSS provides many other functions such as coordination and data transfer, the impetus behind the satellite has been disaster warning.) The configuration of the satellite used for the cost analysis for the DWSS is described in NASA Technical
Memorandum TM X-730407 [4], while the NWRS system design is based on the CSC report of September, 1974 [2]. The period covered by the analysis is 1976-2010. This example is laid out in the way outlined in Section A above and in a way that an analysis of this sort should proceed. Therefore, any specific equal capability cost analysis problem could follow the steps just as they are given here.

Because of the inherent differences in the effectiveness, functions, capabilities, and benefits of the two systems, a complete benefit-cost analysis would be desirable. However, problems associated with the measurement of the benefits (for example, the assignment of dollar values to the number of lives lost) may tend to cloud some important comparisons being addressed in the cost analysis itself. To avoid these problems, we have chosen to redesign the two systems so that their capabilities and performance parameters are as equal as possible. (Clearly, the two systems can never be perfectly equal. For example, the DWSS may provide more "survivability" -- i.e., the capability to function even in the disaster environment. On the other hand, the NWRS, given the multitude of transmitters, does not rely on a single transmission source for the entire country. These qualitative differences cannot be overcome without making the two systems identical, rather than merely equal in capability.)

1/ In performing this example analysis, we have used data from published studies. Therefore, the designs we have arrived at do not necessarily represent the current design of either system. Again, this example is designed to illustrate the method only.
The purpose of any cost analysis is to provide information -- specifically, information on cost. This analysis is no different. However, where most cost analyses evaluate one or more systems to provide information for a choice on systems, our analysis attempts to determine the relative costs for various system functions as well as total costs. Furthermore, it is intended to provide information on fruitful avenues of future investigation. Because the systems have qualitative differences that will make their relative cost-effectiveness dependent upon the assumed requirements, we will identify the effect of the assumptions on the relative costs of the two systems.

The next section describes the functions the two systems are assumed to be capable of providing. Following that, the analysis for the DWSS is provided in Section 3. Section 4 contains the analysis for the NWRS. Finally, Section 5 provides the results of a sensitivity analysis of some of the major assumptions.

2. System Requirements

The following system specifications are assumed to be required and met by both systems.

a. Disaster Warning

The system must be capable of transmitting a message to 99 percent of the population in the 50 states. Ocean coverage is not assumed
to be a requirement since it is not feasible for the NWRS system at reasonable cost. (This, of course, causes the NWRS to appear less costly.) Other performance capabilities (time required to transmit a warning message, reliability, etc.) are assumed approximately equal for the two systems (see Chapter II). Only the coverage parameter appears to be a cost driver for our purposes.

b. Continuous Weather Forecasts

Local continuous weather forecasts must be available to 90 percent of the population with either system, resulting in greater costs for the DWSS. The 90 percent figure is based on current plans of the National Weather Service (NWS) for 331 transmitters.

c. Home Receivers

Demutable receivers that individuals can purchase must be available for either system. Additionally, the receiver must be capable of the reception of both local continuous weather forecasts and warning transmissions. (This requirement implicitly assumes that all purchasers of one capability desire the other.)

d. Coordination

Both systems must provide for coordination among the 300 Weather Service Offices (/WSO's/) and spotter coordination. In addition, both systems must have the capability of interrogating 20,000 [4] data collection platforms and receiving data transmissions in a manner consistent with the GOES program. Because of technical advantages of the DWSS, this requirement leads to higher costs for the NWRS.

[1/ Including Centers and Weather Service Forecasting Offices.]

III-5
Both systems must provide the capability for the reporting of 2,000 spotters. The original requirement of 100,000 spotters [4] has been reduced after discussions with NASA-LeRC personnel.

3. DWSS Cost Analysis

The assumptions used in the DWSS cost analysis are shown in Figure III-4. We discuss each function separately below.

For the warning function a satellite as described in the Disaster Warning Satellite Study Update [4] will be used. The first unit cost (including R&D) is $82 million [4]. \(^1\) This cost is distributed over the period 1976-1984 as shown in Figure III-4. Additional units are estimated to cost $29 million [4] and to be launched at five year intervals (due to the five year life [4] of each satellite). No spares will be launched or maintained in orbit. Launch costs, using the Shuttle, are estimated to be $15 million [3]. \(^2\)

The local continuous weather broadcast requirement will be met by 331 transmitters like those currently in use and planned for the NOAA Weather Radio. The lease costs of the transmitters include maintenance and are based on testimony given by the National Weather Service in congressional hearings on NOAA Weather Radio [5].

\(^1\) These costs are adjusted to 1976 dollars from those in [4] by using an assumed 7 percent annual rate of inflation.

\(^2\) This figure is based on a 1246 kg payload (including apogee motor) with a Spin-Stabilized Upper Stage (SSUS-A) being 9.14 meters (30 feet) in length. Such a payload would take up one half of the Shuttle bay and given some inefficiency in loading additional payloads, we assume to represent 75% of the total Launch payload. The cost of an SSUS-A is estimated to be $2-3 million [3] in 1975 dollars while the total cost of a launch is estimated at $16-18 million again in 1975 dollars. Taking the midpoint and using 7% to adjust to 1976 dollars gives approximately $15 million.
### DWSS Costs

<table>
<thead>
<tr>
<th>Requirement</th>
<th>Cost Basis 1/</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Hardware (1st unit) $72 \times 10^6 with R&amp;D</td>
<td>NASA letter [4]</td>
</tr>
<tr>
<td></td>
<td>Phased as follows</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1976 $ 2 \times 10^6</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1977 $ 4 \times 10^6</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1978 $ 6 \times 10^6</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1979 $10 \times 10^6</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1980 $12 \times 10^6</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1981 $12 \times 10^6</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1982 $12 \times 10^6</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1983 $12 \times 10^6</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1984 $12 \times 10^6</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Operational in 1985</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Additional units $29 \times 10^6</td>
<td>NASA letter [4]</td>
</tr>
<tr>
<td></td>
<td>Launch costs $15 \times 10^6</td>
<td>NASA [3]</td>
</tr>
<tr>
<td></td>
<td>Launch dates 1984, 89, 94, 99, 2004 (5 year life)</td>
<td>(5 year life)</td>
</tr>
<tr>
<td>2. Continuous Weather Forecasts</td>
<td>331 transmitters @$15K/yr. (includes equipment lease and maintenance)</td>
<td>Testimony [5]</td>
</tr>
<tr>
<td></td>
<td>Phased in as follows</td>
<td></td>
</tr>
<tr>
<td></td>
<td>77 in place</td>
<td></td>
</tr>
<tr>
<td></td>
<td>85 in 1977</td>
<td></td>
</tr>
<tr>
<td></td>
<td>85 in 1978</td>
<td></td>
</tr>
<tr>
<td></td>
<td>84 in 1979</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$15/unit through 1984</td>
<td>GAO [1]</td>
</tr>
<tr>
<td>4. Coordination</td>
<td>300 Earth Stations @$121K in 1985 plus $3M R&amp;D in 1985</td>
<td>CSC [2]</td>
</tr>
<tr>
<td></td>
<td>1 Central Control Station (CCS) @$6M plus annual operations cost $2M 1986-2010</td>
<td>CSC [2]</td>
</tr>
<tr>
<td>5. Spotter Reporting</td>
<td>2,000 tranceivers w/spares and parts @$6K 1985</td>
<td>CSC [2]</td>
</tr>
</tbody>
</table>

1/ All costs adjusted to 1976 dollars using a 7 percent rate of inflation.
Home receivers for the DWSS will probably be more complicated than those required for the NWRS system. This is due to the technical requirement that the receiver antenna have high gain and be capable of receiving two frequencies (the warning and the weather broadcasts). Although Rosen and Haimes [6] use a figure of $20 for the receivers, we have used $25 (1976 dollars) to allow for the additional capability of receiving the local continuous weather broadcasts. The market penetration is assumed to be that given in Rosen and Haimes (about 22% of households) and is reproduced in Figure III-5 (column 2). As an allowance for maintenance and replacement, we have assumed that 10 percent of the stock existing in year $t-1$ is replaced in year $t$ so that the total number of units purchased in year $t$ is:

$$P_t = S_t + 0.10 U_{t-1}$$

where

$U_{t-1}$ = stock of receivers in year $t-1$

$S_t$ = sales in period $t$

This figure is shown in column 4 of Figure III-5. Until the satellite is launched in 1984 all receivers are assumed to be the $15 type. In 1985 and thereafter the receivers are assumed to be the $25 variety.

For the coordination function, we take the requirements listed in the CSC study [2]. We also adopted the CSC report’s [2] $2 million annual cost for the operation of the Central Control System. This was the only place personnel costs were included explicitly for the DWS since the Central Control System is unique to the DWSS.
Figure III-5  
Receiver Sales and Costs  
(Millions)  

<table>
<thead>
<tr>
<th>Year</th>
<th>Net Sales / $S_t$</th>
<th>Previous Year Stock St-1</th>
<th>Replacements</th>
<th>Total Purchases $1 + 2$</th>
<th>Receiver NWRS $3$</th>
<th>Receiver DWSS $4$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.74</td>
<td>.62</td>
<td>.06</td>
<td>1.80</td>
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<td>3.28</td>
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<td>8.48</td>
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<td>3.93</td>
<td>58.95</td>
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<tr>
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<td>2.34</td>
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<td>1.08</td>
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<td>1.35</td>
<td>2.43</td>
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<td>14.21</td>
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<td>15.14</td>
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<td>1.55</td>
<td>23.25</td>
<td>38.75</td>
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<td>1.55</td>
<td>23.25</td>
<td>38.75</td>
</tr>
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<td>1.55</td>
<td>23.25</td>
<td>38.75</td>
</tr>
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<td>15.44</td>
<td>1.54</td>
<td>1.54</td>
<td>23.10</td>
<td>38.50</td>
</tr>
<tr>
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<td>0</td>
<td>15.44</td>
<td>1.54</td>
<td>1.54</td>
<td>23.10</td>
<td>38.50</td>
</tr>
<tr>
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<td>1.54</td>
<td>1.54</td>
<td>23.10</td>
<td>38.50</td>
</tr>
<tr>
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<td>1.54</td>
<td>1.54</td>
<td>23.10</td>
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<td>1.54</td>
<td>1.54</td>
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<tr>
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<td>38.50</td>
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<tr>
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<td>1.54</td>
<td>23.10</td>
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<td>1.54</td>
<td>23.10</td>
<td>38.50</td>
</tr>
<tr>
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<td>15.44</td>
<td>1.54</td>
<td>1.54</td>
<td>23.10</td>
<td>38.50</td>
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<td>1.54</td>
<td>23.10</td>
<td>38.50</td>
</tr>
<tr>
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<td>15.44</td>
<td>1.54</td>
<td>1.54</td>
<td>23.10</td>
<td>38.50</td>
</tr>
<tr>
<td></td>
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<td>15.44</td>
<td>1.54</td>
<td>1.54</td>
<td>23.10</td>
<td>38.50</td>
</tr>
<tr>
<td></td>
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<td>15.44</td>
<td>1.54</td>
<td>1.54</td>
<td>23.10</td>
<td>38.50</td>
</tr>
<tr>
<td></td>
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<td>15.44</td>
<td>1.54</td>
<td>1.54</td>
<td>23.10</td>
<td>38.50</td>
</tr>
<tr>
<td>2010</td>
<td>0</td>
<td>15.44</td>
<td>1.54</td>
<td>1.54</td>
<td>23.10</td>
<td>38.50</td>
</tr>
<tr>
<td></td>
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<td>15.44</td>
<td>1.54</td>
<td>1.54</td>
<td>23.10</td>
<td>38.50</td>
</tr>
</tbody>
</table>

1/ Based on [6], Table 5-10, where new sales in year $t$ ($S_t$) are  

$$S_t = \frac{\Delta f_t + \Delta u_t}{2} \times .01 \times H_t$$  

where $\Delta f_t$, $\Delta u_t$ are lower and upper penetration percentages in year $t$  
and $H_t$ are U.S. households (in millions) in year $t$.  

III-9
Spotters are assumed to be one-fiftieth the number given in [4], or 2,000. Each satellite transceiver (with spares and parts) is assumed to cost $6,000 [2] in 1976 dollars.

The cost estimates for the DWSS are shown in Figure III-6. (Details of the calculations are presented in the Appendix.) A 10 percent rate of discount was assumed. Any additional inflation was assumed to be offset by cost-reducing technical improvements. The total present value of the cost resulting from the operation of the DWSS over the period 1976-2010 is $59.1 million. The major part of the cost is clearly the costs associated with receivers and is estimated to be $424 million or 72 percent.

### Figure III-6
**Present Value of DWSS Cost, 1976-2010**
(In millions of 1976 dollars)

<table>
<thead>
<tr>
<th>Function</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Warning</td>
<td>$ 89</td>
</tr>
<tr>
<td>2. Weather Broadcast</td>
<td>46</td>
</tr>
<tr>
<td>3. Home Receivers</td>
<td>424</td>
</tr>
<tr>
<td>4. Coordination</td>
<td>27</td>
</tr>
<tr>
<td>5. Spotters</td>
<td>5</td>
</tr>
<tr>
<td>6. Total Cost (1+2+3+4+5)</td>
<td>$591</td>
</tr>
<tr>
<td>7. Cost to the Government</td>
<td>$167</td>
</tr>
</tbody>
</table>
4. **NWRS Cost Analysis**

Figure III-7 lists the assumptions used in the development of the NWRS cost estimate. In order to provide the 99 percent population coverage, 750 transmitters (instead of the currently planned 331) are required [2]. The phasing is assumed to follow that of the current plan for the first 331 with the remaining 419 added in 1985 to provide comparable capability with the satellite system. Since lease costs were used, and these include maintenance, no allowance for maintenance or replacement was included. The continuous weather broadcast function will be provided by the same equipment used for the warning function.

The assumptions used for home receiver penetration are the same as those for the DWSS system except that home receivers are assumed to cost $15 per unit. Again, 10 percent of the existing stock is replaced each year as an allowance for maintenance and replacement.

The coordination requirement is assumed to be provided by the landlines, as was the case in the GSC study [2, Table 8-7]. As shown in Figure III-7, the largest single cost is $10 million per year for "Local

---

1/ It is unlikely that such a large number of transmitters would ever be implemented in one year. However, in order to provide "equal capability" we have estimated the cost on this basis. Note that this phasing leads to lower costs for the NWRS.
### NWRS Costs

<table>
<thead>
<tr>
<th>Function</th>
<th>Cost Basis 1/</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Warning</td>
<td>750 transmitters (65km (40 mi.) radius)</td>
<td>CSC [2]</td>
</tr>
<tr>
<td></td>
<td>$15K/yr. (including maintenance)</td>
<td>NWS [5]</td>
</tr>
<tr>
<td></td>
<td>Phased in as follows:</td>
<td></td>
</tr>
<tr>
<td></td>
<td>77 in place</td>
<td></td>
</tr>
<tr>
<td></td>
<td>85 in 1977</td>
<td></td>
</tr>
<tr>
<td></td>
<td>85 in 1978</td>
<td></td>
</tr>
<tr>
<td></td>
<td>84 in 1979</td>
<td></td>
</tr>
<tr>
<td></td>
<td>419 in 1985</td>
<td></td>
</tr>
<tr>
<td>2. Weather Forecast</td>
<td>Included in warning function</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>GAO [1]</td>
</tr>
<tr>
<td>4. Coordination</td>
<td>Landlines: WSO-WSO $4M/Yr.</td>
<td>CSC [2]</td>
</tr>
<tr>
<td></td>
<td>Spotter Control 1M/Yr.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>News Media 2/ 0M/Yr.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>National Org. 1M/Yr.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Local Comm. Officials 10M/Yr.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Total $16M/Yr.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>DCP Required Personnel 1M/Yr.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$17M/Yr.</td>
<td></td>
</tr>
<tr>
<td>5. Spotters</td>
<td>2,000 mobile FM radios @$2,000</td>
<td>CSC [2]</td>
</tr>
<tr>
<td></td>
<td>300 spotter control trans./rec. @$6,000</td>
<td></td>
</tr>
</tbody>
</table>


2/ Because of rounding.
Community Officials. " Although it is unclear whether or not this function will also be provided by the DWSS, this cost was included for the NWRS. The $1 million for personnel is an additional cost for this system required for the handling of the data from the data collection platforms [2]. All other costs for the platforms are equal for the two systems [2] and are, therefore, not included.

The spotter requirement for the NWRS system has been reduced to 2,000 in the same way as for the DWSS. The FM radios are $2,000 as given in [2]. The headquarter transceivers are assumed to cost $6,000 in line with the CSC report [2].

The cost calculations are shown in Figure III-8. Again, any inflation was assumed to be offset by improvements in technology. The total present value of cost is $502 million. Again the major cost is home receivers at $354 million or 71 percent of the total.

Figure III-8
Present Value of NWRS Cost, 1976-2010
(In millions of 1976 dollars)

<table>
<thead>
<tr>
<th>Function</th>
<th>Cost (millions)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Warning</td>
<td>$ 73</td>
</tr>
<tr>
<td>2. Weather Broadcast</td>
<td>0</td>
</tr>
<tr>
<td>3. Home Receivers</td>
<td>354</td>
</tr>
<tr>
<td>4. Coordination</td>
<td>73</td>
</tr>
<tr>
<td>5. Spotters</td>
<td>2</td>
</tr>
<tr>
<td>6. Total Cost (1+2+3+4+5)</td>
<td>$502</td>
</tr>
<tr>
<td>7. Cost to the Government</td>
<td>$148</td>
</tr>
</tbody>
</table>

5. Sensitivity Analysis

This analysis was conducted in an attempt to evaluate relative costs for two potential disaster warning systems providing (approximately)

1/ Again, these costs have been adjusted to 1976 dollars.
equal service. Where assumptions beyond those in the NASA LeRC or CSC documents were required, they tended to reduce the costs (with some exceptions), of the satellite system. As an example, the local community officials cost ($10 million annually) applied to NWRS may be associated with a feature not available on the satellite system. If this function were deleted, the cost of the NWRS would be reduced by $43 million to $459 million.

Since the figures for system costs are dependent on the functional requirements, Table III-1 presents the results of a sensitivity analysis on some of the more important requirements. The first row of figures is based on the baseline assumptions presented above (i.e., Figures III-4 and III-7). The second row shows the results if receiver costs are equal for both systems ($15/unit). The next comparison is based on the assumption that the "Local Community Official" requirement, explicitly accounted for in the NWRS costs is not a requirement. This results in a reduction of $10 million annually (1985-2010) or a total present value of $43 million. The next comparison assumes that only 90 percent population coverage is required. This is equivalent to assuming that the planned 331 transmitters are sufficient for the warning tasks. As shown there, the satellite costs are insensitive to coverage. Finally, weather broadcasts are assumed not to be a requirement and the results for the two systems are shown. In all cases but one, the "Equal Receiver Cost" case (see line 2 of Table III.1), DWSS costs are noticeably higher (greater than 10%) than NWRS costs. The one exception is, for all practical purposes, a tie. As these analyses show, receiver costs are really the key to the relative costs of the two systems.
Table III-1

Sensitivity Analysis - Total Discounted Costs (Millions of 1976 dollars) for Selected Changes in Assumptions (1976 - 2010)

<table>
<thead>
<tr>
<th>Assumptions</th>
<th>DWSS</th>
<th>NWRS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Baseline (Figs. 2 and 4)</td>
<td>$591</td>
<td>$502</td>
</tr>
<tr>
<td>2. Equal ($15/unit) Receiver Costs</td>
<td>$521</td>
<td>$502</td>
</tr>
<tr>
<td>3. No Requirement for &quot;Local Community Official&quot; Communication</td>
<td>$591</td>
<td>$459</td>
</tr>
<tr>
<td>4. 90% Population Coverage Requirement</td>
<td>$591</td>
<td>$475</td>
</tr>
<tr>
<td>5. No Requirement for Weather Broadcasts (receivers for DWSS remain at $25/unit)</td>
<td>$566</td>
<td>$502</td>
</tr>
</tbody>
</table>
The analysis to this point has proceeded on the assumption that the two systems are substitutes. However, it is clear that each system has certain advantages. For example, because the weather broadcasts are assumed to be required, use of the NWRS for disaster warning costs very little. Similarly, once a satellite has been launched for warning purposes, the costs of using it for coordination are low. This may suggest that a more fruitful approach to the analysis of disaster warning systems is the analysis of communication systems (i.e., considering the NWRS and the DWSS as complements).

An example will illustrate this point. Figure III-9 repeats the results of the analysis for the two systems in columns 1 and 2. Column 3 is the cost of a hybrid system where the NWRS is used primarily for the weather broadcasts and warning (to 90% of the population) while the satellite is used for coordination, data transfer, and providing warning services to the remaining 10% and ocean areas. Therefore, the cost of warning is equal to the weather forecasting cost of the DWSS. The continuous weather broadcast costs are included in the warning cost. The receiver cost represents the costs of reaching 90% of the population with NWRS receivers and the remaining 10% with the higher-cost receivers capable of receiving a warning from the satellite.\(^1\) The coordination costs represent the full cost of the satellite (\$89 million) plus the previous satellite coordination costs. Finally, the spotter costs are the same as under the NWRS.

---

\(^1\) Note that because the quantity of the higher cost radios sold is lower, the unit price may increase. However, since the quantities assumed remain relatively large, we assume there is no change in unit price.
Although the total costs are greater than the NWRS alone, this type of analysis suggests that by considering the communications requirements together, a less costly hybrid system may be possible. For example, the satellite design might be different if it were to be used only for NWS communications. Alternatively, by a different design, it might be able to provide redundant warning capability.

One implication of this is that an investigation of NWS communications requirements may be desirable in order to determine other services that could be provided by a disaster warning system. The provision of additional services by the satellite results in economies to warning.

Figure III-9
Cost of a Hybrid System, 1976-2010
(Present Value in millions of 1976 dollars)

<table>
<thead>
<tr>
<th></th>
<th>DWSS</th>
<th>NWRS</th>
<th>HYBRID</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Warning</td>
<td>$89</td>
<td>$73</td>
<td>$46</td>
</tr>
<tr>
<td>2. Weather</td>
<td>46</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3. Receivers</td>
<td>424</td>
<td>354</td>
<td>378</td>
</tr>
<tr>
<td>4. Coordination</td>
<td>27</td>
<td>73</td>
<td>116</td>
</tr>
<tr>
<td>5. Spotter</td>
<td>5</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>6. Total Cost</td>
<td>591</td>
<td>502</td>
<td>542</td>
</tr>
<tr>
<td>7. Cost to Government</td>
<td>165</td>
<td>148</td>
<td>164</td>
</tr>
</tbody>
</table>
C. Remarks

In this chapter, we have presented a method for evaluating alternative systems when an estimate of benefits is not available. This method relies on defining the competing systems in such a way that they are, for all practical purposes, equally effective. The illustration we have provided in the previous section provides an example of its application.

From that example, we see that the method can be useful as well in identifying, by functional area, where alternative systems might be more desirable. This, in itself, can provide valuable information to the decision-maker as far as alternative functions that might be performed.

We also saw from the example that the sensitivity analysis is an important adjunct in this approach like it is in any benefit-cost analysis. Because the results of the analysis depend so heavily on the definition of the capability to be met by both systems, special care must be exercised to insure that the definition does not rule out all but one system.

Therefore, the actual application of this method should proceed as follows. First, the characteristics that define effectiveness (e.g., coverage) are selected and minimum, "required", values assigned. Second, each of the competing systems is modified to satisfy each of these requirements in the lowest-cost manner. This modification must be reviewed to insure that the modified system is technically feasible and represents the minimum cost way for the system to meet each assumed requirement. Third, the cost of each system is calculated
and the systems compared. Finally, a sensitivity analysis is performed to test the robustness of the results.
APPENDIX

This appendix provides the details of the cost calculations used in the analysis. Figure A-1 illustrates the calculations for the DWSS (in support of Figure III-6 in the text) while Figure A-2 shows the calculations for the NWRS (supporting Figure III-8 in the text). While most of the calculations are straightforward, the formula derived below will be helpful in working through the calculations.

The present value of $1 spent (or received) \ t \ years \ from \ the \ present \ (e.g., \ 1976) \ is \ equal \ to

$$PV = \$1 \ (\frac{1}{1+r})^{t-1976}$$

where \ r \ is \ the \ appropriate \ discount \ rate. \ If \ a \ series \ of \ $1 \ payments \ is \ to \ be \ made \ (received), \ the \ present \ value \ of \ the \ series \ is

$$PV = \$1 \sum_{i=0}^{t} (\frac{1}{1+r})^i = \frac{$1}{r} \left[1 - \frac{1}{(1+r)^t}\right]$$

since it is a finite geometric sum.

Because many of the calculations in this appendix involve uniform annual payments beginning in some year other than the first, the above expression must be modified. Suppose the payments begin in year a and end in year b. Then, the present value (we drop the $1 for convenience) is

$$PV = \sum_{i=a}^{x} (\frac{1}{1+r})^i - \sum_{i=b+1}^{x} (\frac{1}{1+r})^i$$

III-20
These are both infinite sums of terms less than 1. Let \( s = \frac{1}{1+r} \).

Then, since both terms are geometric series, we have

\[
PV = \frac{s^a}{1-s} - \frac{s^{b+1}}{1-s} = \frac{s^a - s^{b+1}}{1-s}
\]

In the following calculations,

\[
s = \frac{1}{1+r} = \frac{1}{1+.10} = .90909...
\]

and \( a \) and \( b \) represent years from 1976. For example, \( a \) in many calculations is

\[
a = 1985 - 1976 = 9
\]

while,

\[
b = 2010 - 1976 = 34
\]

and the multiplier is,

\[
PV = \frac{s^a - s^{b+1}}{1-s} = \frac{(.91)^9 - (.91)^{34}}{1 - .91} = 4.23
\]
Figure A-1
DWSS Cost Analysis

Warning:

(a) First Unit

\[
C = \sum_{t=1976}^{1984} \frac{S_t}{(1 + .10)^{t-1976}} = 52M
\]

where \( S_t \) = cost in year \( t \) (see Figure III-5).

(b) Additional Units

\[
C = \sum_{t=1}^{4} \frac{14}{(1 + .10)^{1983 - 1976 + 5t}} + \sum_{t=1}^{4} \frac{15}{(1 + .10)^{1984 - 1976 + 5t}} = 20M
\]

(c) Launch Costs

\[
C = 15M \cdot \sum_{t=0}^{4} \frac{1}{(1 + .10)^{1984 - 1976 + 5t}} = 17M
\]

Total Warning Cost = 89M

Weather Forecasts:

\[
C = 15K \left[ 77 \times \sum_{t=1976}^{2010} \frac{1}{(1 + .10)^{t-1976}} + 85 \sum_{t=1977}^{2010} \frac{1}{(1 + .10)^{t-1976}} + 85 \sum_{t=1978}^{2010} \frac{1}{(1 + .10)^{t-1976}} + 84 \sum_{t=1979}^{2010} \frac{1}{(1 + .10)^{t-1976}} \right] = 46M
\]
Figure A-1
DWSS Cost Analysis (Continued)

Home Receivers:

\[ C = \sum_{t=1976}^{2010} \frac{R_t}{(1 + .10)^{t-1976}} = $424M \]

where \( R_t \) is undiscounted receiver cost for the DWSS (column 6 of Figure III-5).

Coordination:

(a) Earth Stations

\[ C = \frac{300 \times \$121K}{(1 + .10)^{1985-1976}} + \frac{\$3M}{(1 + .10)^{1985-1976}} = $16.7M \]

(b) CCS

\[ C = \frac{2010}{(1 + .10)^{1985-1976}} \cdot \frac{1}{(1 + .10)^{t-1976}} \]

Total Coordination Costs =

Spotter Reporting:

\[ C = \frac{2000 \times \$6K}{(1 + .10)^{1985-1976}} = $5M \]
Figure A-2

NWRS Cost Analysis

Warning:

\[
C = $46M + 419 \times $15K \sum_{t=1985}^{2010} \frac{1}{(1 + .10^{t-1976})} = $73M
\]

where $46M$ is obtained from Figure A-1 (Weather Forecast).

Weather Forecast: No additional Cost

Home Receivers:

\[
C = \sum_{t=1976}^{2010} \frac{R_t}{(1 + .10)^{t-1976}} = $354M
\]

Coordination:

\[
C = $17M \sum_{t=1985}^{2010} \frac{1}{(1 + .10)^{t-1976}} = $73M
\]

Spotter:

\[
C = \frac{2000 \times $2K}{(1 + .10)^{1985-1976}} + \frac{300 \times $6K}{(1 + .10)^{1985-1976}} = $2M
\]
BIBLIOGRAPHY


IV. MEASURING BENEFITS

A. Introduction

In Chapter III, we saw how to apply the benefit-cost framework when we did not have estimates of benefits. In this chapter we describe a generally accepted measure of benefits from a government project and illustrate how this measure can be estimated for a particular project. This chapter may be less familiar and intuitive in the theoretical foundations than the previous chapter for two reasons. First, cost analysis methods are often much simpler to apply than benefits estimation methods, and, therefore, much more widely applied and understood. Second, there still exists a considerable amount of misunderstanding about the empirical measure of benefits that we propose, even within the economics profession.

Therefore, in Section B, we provide a simple discussion of the benefits measure we propose. This discussion is designed to give the reader a general picture of the concept that is sufficient for an understanding of the example in Section C. In an appendix to this chapter, we provide the theoretical foundations for this measure of benefits. Because of the theoretical nature of this appendix, it is necessarily more complex than the sections in the main body of this chapter. The example presented in Section C is an illustration of estimating the benefits of a home receiver for disaster warning using data provided in Rosen and Haimes [10]. Thus, the estimates we calculate are comparable to those obtained by Rosen and Haimes in their analysis (see Chapter II).
B. The Estimation of Benefits

A generally accepted principle of benefit-cost analysis is that the value of anything is measured simply by what people are willing to pay for it. If, for example, a household buys a home receiver capable of receiving disaster warnings for $25, then we may infer that the value which the household attaches to the services provided by the receiver is at least $25.

In particular, households that purchase a home receiver will, in general, attach a higher value to the receiver than what is actually paid for it. For example, many households might still be willing to purchase the receiver if its price were $35. The difference between what households actually pay and the maximum amount they would be willing to pay rather than go without it is called net willingness-to-pay. It measures what consumers would be willing to pay over and above what they actually pay.

Net willingness-to-pay is thus a measure of economic benefits to project beneficiaries, over and above any user-charges that may be levied upon them. This measure is accepted generally in the economics profession as the appropriate way to value benefits. Benefit-cost analysis consists of comparing net willingness-to-pay with any costs not covered by user charges.

1/ See the discussion in Chapter I on this point.

2/ A rigorous definition of net willingness-to-pay is given in the appendix. Roughly, it is as follows: consider a good whose price is p. Then the amount of money, m, that a consumer would be willing to give up that would make him just indifferent between (i) buying the good at price p and giving up m, and (ii) not buying the good at all, is the consumer's net willingness-to-pay.

3/ See, e.g., Harberger [3] and Mishan [7].
As a practical matter, it is necessary to work with approximate estimates of net willingness-to-pay. This is because it is almost always impossible to perform the kind of experiment or to observe the kind of situation that one needs to make an exact estimate.

The approximation most frequently adopted is to measure consumers' surplus. The basic idea behind consumers' surplus is to use points along the ordinary market demand curve as indications of maximum willingness-to-pay for successive units of product. This is illustrated in Figure IV-1, which presents the market demand curve for a product. In this diagram, $p$ is the market price and $10$ is the quantity purchased. By examining the market demand curve, $DD$, we find that consumers would be willing to pay $p_1$ for a total of one unit. To be induced to buy a second unit, the price would have to be lowered to $p_2$. This amount ($p_2$) is (approximately) the (gross) willingness-to-pay for the second unit, and so forth.

---

1/ The market demand curve represents the demands of all individuals in the market and is constructed by summing all the individual demands at each price $p_1$.

2/ The only requirement on the demand curve is that it is downward sloping. We use linear demand curves to facilitate the exposition.
If $N$ units are sold, the consumer's surplus approximation to net willingness to pay is:

$$CS = \sum_{i=1}^{N} (p_i - \bar{p})$$

If we treat units of goods as infinitely divisible, this approximation becomes:

$$CS = \int_{0}^{N} p(n)dn - p(N) \cdot N$$

where $p(n)$ is the demand function.

An intuitive appreciation for this measure can be gained by considering the benefits of a government project that lowers the cost of an existing product or service. This situation is pictured in Figure IV-2. There demand for the good is given by $D$ and the price before the government program is $p_0$. At price $p_0$, $q_0$ units will be consumed. Let the government project result in a reduction in price to $p_1$. At this price, $q_1$ units will be sold.

A measure of benefits that would often be used in this situation is "cost-savings". Each of the units sold would be sold for $(p_0 - p_1)$ dollars less. Total cost-savings would, therefore, be the difference in unit costs $(p_0 - p_1)$ multiplied by the number of units sold $q_0$ or $q_0(p_0 - p_1)$ (the cross-hatched area in Figure IV-2.).
The consumers' surplus measure includes the same area but, in addition, takes into account the benefits accruing to those who purchase units of the good purchased at the lower price but not at the higher price. These additional benefits are shown as the triangular-shaped region ABC in Figure IV-2. In the case of a price-reducing investment, consumers' surplus includes cost-savings in its measure of benefits.
Knowledge of the market demand curve, therefore, can be used to measure approximately\(^1\) the net willingness-to-pay on the part of individual's for the services of the home receiver. This figure can then be compared directly to the cost of the transmission system to determine the economic value of the particular disaster warning system. In the next section, we illustrate how this concept can be used to estimate the benefits associated with a home receiver. Since the receiver is not associated with a particular sensing, forecasting, or transmission system, the results cannot be attributed to any one system.

C. An Application of the Consumer's Surplus Measure of Benefits

In this section we apply the concept of consumer's surplus to estimate the benefits associated with a disaster warning system. The demand data we use is provided by Rosen-Haimes [10]. Thus, the results we obtain here should be comparable to the benefits they estimate. We caution the reader again, however, that this is merely an application of the method and is not to be taken as a necessarily accurate estimate of the benefits associated with any disaster warning system.

\[^1\] The reasons that this only provide an approximate measure and the error induced by using it are discussed in the Appendix to this chapter.
1. **Demand Curve Estimate**

If we adopt the procedure discussed in the preceding section, all we require to estimate the benefits of a home warning system for natural disasters is a demand function for home warning receivers.\textsuperscript{1/} Fortunately, this is provided by Rosen and Haimes (RH), whose method also requires demand information. In Table IV-1 below, we report the results of a survey conducted by the Opinion Research Corporation \textsuperscript{9} on which RH based their demand analysis.

We have plotted the points for prices of $10, $25, $50 and $100 in Figure IV-3 and have drawn straight-line segments between these points. The graph thus obtained closely resembles a demand curve.\textsuperscript{2/} Indeed, if we specify how many households there are in the nation and assume that each purchasing household buys only one home warning receiver, we may find the quantity of home warning receivers that would be purchased at each and every price by simply multiplying the percentage of households buying at that price (as read off the

\textsuperscript{1/} This is the derived demand function \([x(p)]\) discussed in the appendix.

\textsuperscript{2/} Again, for those readers who have gone through the appendix, this is the ordinary (uncompensated) demand curve.
Table IV-1

Household Demand for Home Receivers

<table>
<thead>
<tr>
<th>Price</th>
<th>Percentage of Households Indicating That They Would Purchase Immediately a Natural Disaster Warning Receiver</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt;$10</td>
<td>25.8%</td>
</tr>
<tr>
<td>$10</td>
<td>25.0%</td>
</tr>
<tr>
<td>$11-19</td>
<td>23.4%</td>
</tr>
<tr>
<td>$20-24</td>
<td>22.6%</td>
</tr>
<tr>
<td>$25</td>
<td>19.8%</td>
</tr>
<tr>
<td>$26-30</td>
<td>15.4%</td>
</tr>
<tr>
<td>$31-49</td>
<td>14.2%</td>
</tr>
<tr>
<td>$50</td>
<td>13.6%</td>
</tr>
<tr>
<td>$51-99</td>
<td>9.2%</td>
</tr>
<tr>
<td>$100</td>
<td>8.4%</td>
</tr>
<tr>
<td>&gt;$100</td>
<td>3.2%</td>
</tr>
</tbody>
</table>

Source: Rosen and Haimes, [10], Table 3-2, p. 47.
Figure IV-3
The Estimation of Consumers' Surplus

axis of Figure IV-3) by the number of households. This relationship between price and quantity purchased is, by definition, the market demand curve.

Recalling from Section B that we require only the market demand curve in order to estimate economic benefits via the net
willingness-to-pay approach, it is evident that we have precisely
the information we require to use this approach. In order to simplify
the calculations, we have fitted the points shown in Figure IV-3 with
the following curve: 1/

\[ N = 94.84p^{-0.52} \quad 10 \leq p \leq 100 \] (1)

This curve is plotted in Figure IV-3 as the dotted line. Assuming that
there are \( H \) households in the nation and that each purchasing
household buys only one receiver, the demand curve for receivers
may be approximated as

\[ R = .9484Hp^{-0.52} \quad 10 \leq p \leq 100 \] (2)

where \( R \) is the number of receivers sold.

2. Some Estimates of Benefits Using Consumer Surplus

We have prepared several different estimates of benefits, each
based on slightly different assumptions from the others. Some assumptions

1/ In the regression, we eliminated the two end points and for those
demand estimates given for an interval, we used the average of
the two prices. The coefficient of determination \( (R^2) \) was .93.
and estimates are relatively unrealistic. They are presented nonetheless because they give some insight into the sensitivity of estimated benefits. All estimates are based on the assumption of a 5.75 percent discount rate (an assumption adopted by RH), that home receivers last forever, and that the number of households in the United States will forever be 70 million. Benefits are evaluated over a 20 year horizon. Home receiver prices are assumed to be constant at $25.

a. **Assuming Immediate System Activation and Immediate Purchase of Home Receivers**

The complicating factor in this example is the durability of the receiver which means that we have to look at benefits over a period of time. Therefore the area under the demand curve alone does not correspond to annual benefits. The market demand curve is obviously influenced by both the expected life of the receiver and the expected life of the program. Clearly, other things being equal, consumers' willingness-to-pay for a receiver varies directly with its expected life and with the expected life of the home warning program. Because of this multiple year accrual of benefits, the benefit calculation must include an annualizing feature that converts a lump sum of benefits to an annual flow.

In our analysis, consumer expectations with regard to the life of the program are assumed to be embodied in stated willingness-to-pay. Moreover, an infinite life for home receivers is assumed in computing annual benefits; a finite life would result in higher estimated benefits.

1/ Because of such assumptions, the results of this chapter cannot be compared to those of the previous chapter.

2/ If the receiver lasted only one year and this demand curve was still valid, then the area under the demand curve multiplied by the number of households is a measure of annual benefits. In the opinion poll itself, there was no explicit information given about either the expected life of the program or the receiver.
Consider the benefits for any one year. If the receiver has an infinite life and if the individual is willing to pay \( p^* \) dollars for it, he is willing-to-pay an amount

\[ x = 0.0575 \, p^* \]

in equivalent annual rental cost. Therefore, we must multiply the area under the demand curve by the annualizing factor of 0.0575 to obtain annual benefits. Thus, the annual benefits per household are:

\[
C = \int_{25}^{100} (0.0575 \times 0.9484) \, p^{-0.52} \, dp
\]

where the limits of integration (25 and 100) are designed to capture almost all of the consumers' surplus. Note that this results in an underestimate of the benefits since the form of the fitted equation implies a positive demand even at prices above $100. Also, we use a price of $25 since RH claim that "benefits are relatively insensitive to receiver costs in the retail unit cost range of $15-25" [10]. Thus, our estimates represent a lower bound in terms of receiver cost. The reason we use $100 is that there is no information about demand above that price. (Also, note that with no upper limit, the integral will fail to converge.)

The surplus per household is:

\[
CS = (0.0575 \times 0.9484) \int_{25}^{100} p^{-0.52} \, dp
\]

\[
= (0.0545) \left[ \frac{p^{0.48}}{0.48} \right]_{25}^{100}
\]

\[
= \$0.5035
\]

annually.

Note that this results in an underestimate of the benefits since the form of the fitted equation implies a positive demand even at prices above $100. Also, we use a price of $25 since RH claim that "benefits are relatively insensitive to receiver costs in the retail unit cost range of $15-25" [10]. Thus, our estimates represent a lower bound in terms of receiver cost. The reason we use $100 is that there is no information about demand above that price. (Also, note that with no upper limit, the integral will fail to converge.)
For 70 million households, the annual benefits are:

\[ CS = (70 \times 10^6) (\$ .5035) = $35.2 \times 10^6 \]

The net present value of the benefits over 20 years are computed as

\[ CS = $35.2 \times 10^6 \int_{0}^{20} e^{-0.0575t} \, dt \]

or

\[ CS = $35.2 \times 10^6 \left[ -\frac{e^{-0.0575t}}{0.0575} \right]_{0}^{20} = $418 \times 10^6 \]

b. **Assuming Immediate System Activation and Staged Market Penetration**

New products typically penetrate the market over a period of time rather than immediately as was assumed in our computations above. Let us assume that the percentage of households that will actually have bought a home receiver at time \( t \) is

\[ N(t) = N(1 - e^{-\lambda t}) \quad (3) \]

where \( N \) is given in equation (1) above.

The speed with which \( N(t) \) approaches \( N \) obviously depends upon the value of the parameter \( \lambda \). We have chosen two alternative values, one
designed to yield \( N(t)/N = .5 \) in five years and one designed to yield \( N(t)/N = .5 \) in 10 years. Respectively, these values are

\[
\lambda_5 = 0.1386 \\
\lambda_{10} = 0.0693
\]

Under these assumptions, estimated benefits are given by

\[
B = \int_{0}^{20} e^{-0.0575t} (0.0575) \int_{0}^{100} N(p) H(1-e^{-\lambda t}) dp dt
\]

\[
= \int_{0}^{20} e^{-0.0575t} (1-e^{-\lambda t}) (0.0575) (70 \times 10^6) \int_{0.9484}^{100} p^{-0.52} dp dt
\]

from equation (2).

Therefore

\[
B = \int_{0}^{20} e^{-0.0575t} (1-e^{-\lambda t}) (0.0575) (70 \times 10^6) (8.757) dt
\]

\[
= (35.2 \times 10^6) \int_{0}^{20} e^{-0.0575t} (1-e^{-\lambda t}) dt
\]

\[
= (35.2 \times 10^6) \left[ \frac{e^{-(\lambda+.0575)t}}{\lambda+.0575} - \frac{e^{-.0575t}}{.0575} \right]^{20}_{0}
\]

1/ RH assumed that home receiver market penetration would be about 50 percent complete by 10 years after introduction of the system. The Opinion Research Corporation estimates that this same level will be attained in 5 years.

2/ From the calculations above, we know

\[
\int_{25}^{100} (.9484)p^{-0.52} dp = 8.757
\]
Values for benefits are presented in Table IV-2.

| Table IV-2 |
|---|---|
| **Estimates of Present Value of 20 Year Benefits Under Alternative Penetration Rates** |
| Benefits | $242.4 \times 10^6$ | $162.7 \times 10^6$ |

\( \lambda_5 = 0.1386 \quad \lambda_{10} = 0.0693 \)

It is evident that stretching out the purchases of home receivers reduces the estimated benefits. This is clearly reasonable since the stream of services rendered by a receiver are not available until the receiver is in homes.

c. **Assuming System Activation in Five Years and Staged Market Penetration**

Some time will be required, regardless of the system adopted, to put the transmission system in place. Assuming that no receivers are purchased until the transmission system is in place and that this will take five years, estimated benefits may be calculated as follows:

\[
B = e^{-0.0575(5)} \cdot 35.2 \times 10^6 \int_{0}^{15} e^{-0.0575t}(1 - e^{-\lambda t}) dt
\]

IV-15
Values for benefits estimated using this expression are presented in Table IV-3.

<table>
<thead>
<tr>
<th>Table IV-3</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Estimates of Present Value of 20 Year Benefits Under Alternative Penetration Rates, System Active in Five Years</strong></td>
</tr>
<tr>
<td>$\lambda_5 = 0.1386$</td>
</tr>
<tr>
<td>Benefits</td>
</tr>
</tbody>
</table>

Under all assumptions, the costs against which these estimates of consumers' surplus must be compared are those of the transmitting system. For example, if the cost of the transmitting system were less than $88$ million\(^1\) and if the assumptions used here in the estimation of benefits were valid, a disaster warning system would have positive net benefits.

3. **Comparisons With RH's Estimates**

The estimates presented in Table IV-3 are based on assumptions most nearly comparable to those of the RH calculations. These estimates cannot be directly compared to the benefits in RH and reproduced in Table II-4 above because the estimates given above in Table IV-3 are estimates of willingness-to-pay over and above what is actually paid for home receivers. That is, in a full-scale benefit-cost analysis, the benefits estimated by RH would have to be reduced by the present value of the receiver cost; receiver costs have already been netted out in

\(^1\) Again, because we are employing different assumptions in the various examples, the examples cannot be compared.
the consumers surplus calculation. \(^1\) Even if a value of $1 million were placed on each life lost (which is significantly higher than values most often used), the net benefits (after subtracting out receiver costs), as estimated by RH are on the order of $20-30 million. This is clearly smaller than the estimated benefits using the consumer surplus measure.

It should be pointed out, in addition, that the following assumptions have been made that tend to bias downward the estimates of benefits:

(i) No decline in the price of home receivers with volume. (RH assume that price declines.)

(ii) No growth in the number of households. (RH assume growth in the number of households.)

(iii) Home receivers are infinitely durable. (RH's assumption is ambiguous.)

(iv) Demand is zero everywhere outside the price range \([10, 100]\).

(v) Zero elasticity of demand with respect to income. (In other words, as consumers become wealthier, they won't increase their willingness-to-pay.)

(vi) Purchasing households buy only one receiver.

All of these assumptions work in the direction of reducing estimated benefits.

\(^1\) These benefits cannot be compared to the costs not covered by user charges estimated in Chapter III because of the differences in certain assumptions (e.g. the discount rate) and the fact that the receivers in the ORC study are not necessarily the same as those driven by the transmission systems in Chapter III.
D. Remarks

As we have shown, both theoretically and with an actual example, the consumer surplus measure of benefits is greater than what might be termed the "cost savings" approach. The actual implementation of this method for benefits measurement has also been demonstrated.

The use of this measure of benefits can be easier to calculate than the cost-savings approach. The reason for this is that the consumer incorporates all the cost-saving benefits into his demand function implicitly. Therefore, the analyst need not spend time searching for both areas of cost-savings and the magnitude of cost savings.

The following steps depict the implementation of consumer surplus in practice:

first, obtain estimates of demand for a particular receiver. These estimates may come from market surveys, actual sales data, or other sources. The important thing is to have quantities associated with different prices.

second, estimate the demand function. The technique here was statistical regression analysis but other methods may be used. Because of the importance of this estimated demand curve, some care must be taken in the specification and fitting of the functional form.

third, given the expected (or actual) price of the receiver calculate the consumers' surplus as the integral of the demand curve from the selling price to a price that represents a reasonable "maximum" price. This maximum price should be such that it is reasonable that there would be only minimal sales above it.

1/ The analysis of benefits in this chapter has rested upon the knowledge of the demand function for home receivers. In Chapter V, we present an alternative method that can be used when the demand function is not known.
finally, compare the consumers' surplus to the cost of the transmission system only (since consumer costs have been netted out). If the surplus is greater, then the economic benefits are greater than the economic costs of the system.

It must be remembered that the use of consumers' surplus as a measure of benefits is only appropriate if the following axioms are accepted:

(1) the benefit of a project to an individual is what the individual is willing to pay to participate in the project,

and

(2) the total benefits of a project are equal to the sum of the individual benefits.
THE ESTIMATION OF BENEFITS—THEORETICAL FOUNDATIONS

A.1 Measuring Value for a Single Individual

As we saw in Section B, a consumer's surplus is loosely defined as the difference between the largest amount of money which he will freely pay and the amount he actually pays in order to consume some bundle of goods. This is a workable definition, but it is imprecise because it doesn't make clear what bundle the individual has before and after the payment is made or how prices and the income facing the consumer change.

We can think of characterizing an economic system as seen by a consumer, by simply listing all prices \( p_1, \ldots, p_n \) where \( p_i \) is the price of the \( i \)-th good, and the consumer's income, \( I \). Thus, in a specific period \( t \), we can use an \( n+1 \) dimensional vector \( (p_{1t}, \ldots, p_{nt}, I_t) = (p_t, I_t) \) to characterize the economic system for a specific consumer. We will refer to such a characterization as a "state" of the economic system. Consumers' surplus analysis aims at assigning a value to a change in state in which a public or private project, a set of taxes or subsidies, or some other activity changes the price vector facing consumers. That is, it values any change that moves the consumer from some initial state \( (p_0, I) \) to another state \( (p_1, I) \).

---

1/ The work in this Section was taken from several sections of [1]. The original use of consumers' surplus is due to Dupuit [2] and was given some mathematical meaning by Hotelling [6]. Sections a and f draw heavily on Willig [12] (see also Willig [11]), while the analysis in Section 3, originally due to Hicks [5] is based on the recent paper by Morning [8]. The compensating and equivalent variations are, of course, due primarily to Hicks [5].

2/ If the project consists of making available a previously unavailable good, we can view it as a move from a state with \( p_{10} = M \) where \( M \) is arbitrarily large to \( p_{11} \).
Since individuals may choose freely their consumption in either state, the usual assumptions of economic theory imply that they will select their consumption bundles, represented by a vector \( \mathbf{x} = (x_1, \ldots, x_n) \), so as to achieve a maximum of utility. The conditions for this maximization are (see, e.g., [4]):

\[
U_i - \lambda p_i = 0 \quad i = 1, \ldots, n
\]

\[
p_1 x_1 + \ldots + p_n x_n = I
\]

where the individual's utility function is \( U(\mathbf{x}) \) with first partial derivatives \( U_i = \frac{\partial U(\mathbf{x})}{\partial x_i} \), and where the Lagrange multiplier \( \lambda \) gives the marginal utility of income to the consumer. (The "value", in utility terms of an additional, infinitesimally small amount of income.)

These necessary conditions are then solved for the demands for goods in terms of the prices and the individual's income. These so-called derived demand functions can be written as a vector function \( \mathbf{x}(p, I) \), where each element of the vector is the quantity of the \( i \)th good consumed when the consumer is in state \( (p, I) \). Given the derived demands it is possible to write the value of the individual's utility function given his optimal choices \( \mathbf{x}(p, I) \). This is called the indirect utility function and is \( U(\mathbf{x}(p, I)) \). It depends only on the state \( (p, I) \).

---

1/ We will assume that \( U(\mathbf{x}) \) is strictly concave, so that the conditions in (1) do define a unique maximum of utility, and also that the derived demand functions can be found when some of the optimal \( x_1 \)'s are zero.
To find out how much a consumer values the change from state \((p_0, I)\) to state \((p_1, I)\) we can ask him (1) how much he is willing to pay to keep the new prices \(p_1\) or (2) how little he will accept to be removed to the old prices \(p_0\). Either question calls for the consumer to find the change in his income which makes him indifferent, in utility terms, between the offered states. Given his indirect utility function, the income changes can be written as the solution to algebraic equations. Thus, the answer to question (1): the least amount which the consumer is willing to accept in order to be indifferent to a change from the prices \(p_0\) to \(p_1\), (traditionally designated \(C\), is defined by:

\[
U\left(x(p_0, I)\right) = U\left(x(p_1, I+C)\right)
\]

(2)

Similarly, the answer to question (2): the income change which will make the consumer's utility with prices \(p_1\) the same as his utility at the old prices is \(E\), the solution to:

\[
U\left(x(p_0, I-E)\right) = U\left(x(p_1, I)\right)
\]

(3)

---

1/ This is not equal to the change in prices multiplied by the quantity of the goods consumed. A change in prices changes relative as well as total consumption because of (1) changes in relative prices (the substitution effect) and (2) changes in real income (the income effect). See the following subsection for an illustration of the ideas presented here.
Both \( C \) and \( E \) are measures of a consumer's surplus. The signs of the income changes in Eqs. (2) and (3) have been chosen so that \( C \leq 0 \) and \( E \leq 0 \) as \( P_0 \geq P_1 \). If prices increase, both \( C \) and \( E \) are positive because consumers require compensation for their loss in welfare. If prices fall some amount must be paid (i.e., some income foregone) if utility is to be unchanged.

Notice that the technique used in (2) and (3) is a particular example of a general method of defining the change in income which will just make a consumer indifferent between two states of the economy. This compensation depends on the initial and final prices \( P_0 \) and \( P_1 \) and on the income \( I \) of the consumer. We may write this implicit function as \( Y(P_1; P_0, I) \) and call it the income compensation function \(^1\). It is defined by

\[
U\left( x(P; Y(P; P_0, I)) \right) = U\left( x(P_0, I) \right)
\]

for any price vector \( P \). Notice that \( Y(P_0; P_0, I) = I \) and that the two measures of value can be defined using the income compensation function to be:

\(^1\) When we wish to remind the reader that the income compensation function depends on prices, we will write the first price vector without a subscript. A subscript will be used when a particular final price is intended.
That is, as we have said before, \( E \) is the change in money income equivalent in utility to a change from prices \( p_0 \) to \( p_1 \) and \( C \) is the maximum amount a consumer will pay (or the least he will accept) for the change from \( p_0 \) to \( p_1 \).

A.2 Example

Since many of the concepts developed above may be unfamiliar, the following example may provide intuitive understanding of what is taking place. Assume we have a world with two goods, \( x_1 \) and \( x_2 \) with prices $2 and $5 respectively. The individual is assumed to have an income of $100 and a utility function of the form:

\[
U(x_1, x_2) = x_1x_2
\]

That is, the individual's utility depends on the combination of the two goods consumed. From the necessary conditions (1) above we know that:

\[\text{[4]}\]

The basis for this example comes from [4].
\[
U_1 (x_1, x_2) - \lambda p_1 = 0
\]

\[
U_2 (x_1, x_2) - \lambda p_2 = 0
\]

\[
p_1x_1 + p_2x_2 = 100
\]

In this case, we have

\[
x_2 - 2\lambda = 0
\]

\[
x_1 - 5\lambda = 0
\]

\[
2x_1 + 5x_2 = 100
\]

Solving for \(x_1\) and \(x_2\), we see that

\[
x_1 = 25
\]

and

\[
x_2 = 10.
\]

These are the individual's derived demands and depend on prices and income only. His utility is seen to be

\[
U(x_1, x_2) = x_1x_2 = 250.
\]
Suppose now that a project is proposed that will result in the price
of the second good being lowered from $5 to $4. We want to calculate \( C \)
and \( E \), the compensating and equivalent variations. For the compensating
variation we see that at the new set of prices, the necessary conditions are
(the superscripts indicate new period consumption):

\[
\begin{align*}
    x^1_2 - 2\lambda &= 0 \\
    x^1_1 - 4\lambda &= 0 \\
    2x^1_1 + 4x^1_2 &= 100 + C
\end{align*}
\]

plus the requirement that utility be unchanged, or,

\[
x^1_1 x^1_2 = x^0_1 x^0_2 = 250.
\]

This system of four equations in four unknowns can be solved to obtain.\(^1/\)

\[
\begin{align*}
    \lambda &= 5.59 \\
    C &= -10.56 \\
    x^1_1 &= 22.36 \\
    x^1_2 &= 11.18
\end{align*}
\]

and

\[
U(x^1_1 x^1_2) = 250.
\]

\(^1/\) The marginal utility of income \( \lambda \), is positive given the assumption
of non-satiation (essentially that more is preferred to less).
Thus, the individual would be willing to give up $10.56 in income to avoid returning to the old prices. Similarly, we could solve for the equivalent variation and find that it is equal to -$11.80.

The important thing to note in this example is that measures of "cost savings" understate the value, to the consumer, of the change in prices. We saw that originally the individual consumed 10 units of the second good. In terms of cost savings, he "saved" $1 for ten units for a total savings of $10. In terms of his willingness-to-pay, he was "better off" by $10.56. This is because a change in prices affects real income as well as relative prices and the individual reallocates his income to maximize his utility.

A. 3 Measures of Value and Measures of Welfare

The welfare meaning of the C and E measures of value depends on the ideas of Pareto optimality and of compensation. A particular state of the economy is said to be Pareto optimal if no change in state can be made which makes no one worse off and at least one consumer better off. The compensation principle has been proposed as an extension of the Pareto optimality. The idea behind the compensation principle is that we can judge the value of an act which changes prices by looking at the Pareto improvement which would occur if the change in state is made and incomes are costlessly redistributed after the change in state.
In particular, \( C \), which is called the "compensating variation" in income, has important implications for cost-benefit analysis under the compensation principle. Suppose some project will change the economy so as ultimately to change the prices faced by consumers. After the change is made (i.e., at price \( p_1 \)), the value of \( C \) tells us how much a consumer's income would have to be altered for him to be indifferent between the old and new states of the economy. The compensation principle states that if the algebraic sum of all the \( C \)'s of all the consumers in the economy is greater than the cost of the project, then it should be undertaken, because after the change it would be possible to pay the costs and to compensate fully all consumers by a redistribution of their incomes in such a way as to make at least one consumer better off.

As its name suggests, \( C \) may be thought of as the compensation (positive or negative) which a consumer requires to make him indifferent between living in the old state of the world \((p_0, I)\) and the new state \((p_1, I+C)\). The so-called "equivalent variation" in income, which we have symbolized by \( E \), has a similar interpretation. \( E \) is the amount of income which a consumer must forego if he is to be indifferent between the new state of the economy \((p_1, I)\), and the old state \((p_0, I-E)\). The value of \( E \) is thus the income change, relative to the old prices \( p_0 \) which is equivalent to the utility change induced by a change in prices to \( p_1 \). The equivalent variation is also sometimes interpreted as the maximum amount a consumer is willing to pay to avoid the price change.
The concepts of compensating and equivalent variation can also be illustrated graphically by reference to a standard indifference-curve diagram. Consider figure IV-A-1 which is based on a figure in Mishan [7]. There income, I, is measured along the vertical axis and the quantity of the new good (or the good whose price is being reduced) is measured along the horizontal axis. The indifference curves (U₀ and U₁) represent combinations of income and the amount of good x between which the individual is indifferent.

When a new good is introduced (or its price lowered) the new price lines are represented by the lines I₀ and I₀'. (The slope of these lines indicate the amount of income that must be given up in order to consume an additional unit of good x). If the consumer originally has income I₀ with no opportunity to consume x, he is on indifference curve U₀. When good x becomes available, he trades down the price line to point Q₁ where

![Graphical Depiction of Equivalent and Compensating Variations](image)

Figure IV-A-1
Graphical Depiction of Equivalent and Compensating Variations

Source: [7]
he consumes $M_1$ units of $x$.

The compensating variation is the amount of income he would pay to be able to make this trade. From Figure IV-A-1 we see this is just the income equal to $I_0 - I_1$. For if the individual gives up $(I_0 - I_1)$ units of income he will then be at point $I_1$. Trading down the price line $I_1, I_1'$ (which is, of course, parallel to $I_0, I_0'$) he stops at $Q_0$ which is on indifference curve $U_0$. Therefore he is just as well off as before the trading opportunity. By similar reasoning, the equivalent variation is equal to the distance $I_2 - I_0$.

The equivalent variation is useful when a change in state which affects only prices or incomes is being considered. For instance, it is a relevant measure to use when evaluating the effects of a tax or a subsidy. The compensating variation is useful in different circumstances, where the incomes and prices will both be changed; and it is the compensating variation which is most often meant in benefit-cost analysis when one speaks loosely of "willingness-to-pay" or of the consumer's surplus. This is because $C$ measures the value between a proposed change and the present state, while $E$ is used to compare different proposed changes. Nevertheless, it should be clear that different measures of value are appropriate in different circumstances. The analyst must take care that he specifies completely the change in state which he proposes before he tries to evaluate it.
A. 4 The Marginal Value of a Price Change

The measures C and E derived above are defined for any price change, but it is useful to derive the measure of value appropriate to an infinitesimal change in a price. That is, what is the value of the partial derivative of \( Y(p, p_0, I) \) with respect to an element of \( p \), evaluated at \( p = p_0 \)?

To find this derivative we begin by finding the conditions which must hold if a consumer's utility is to remain constant after an infinitesimal change in price has occurred, accompanied by a compensating adjustment in income. The derivative of utility with respect to a price \( p_j \) can be found by differentiating the indirect utility function \( U(x(p, I)) \). It must be zero if utility is held constant. Hence:

\[
\frac{\partial U}{\partial p_j} = \sum_{i=1}^{n} U_i \frac{\partial x_i}{\partial p_j} = 0 \quad j = 1, \ldots, n \quad (7)
\]

The term \( \frac{\partial x_i}{\partial p_j} \) includes the change in consumption of the \( i^{th} \) good due to the change in price (the substitution effect) and due to the compensating change in income. Now, by the necessary conditions for a maximum of the consumer's utility, Eq. (1), the condition in Eq. (7) becomes:

\[
\lambda \sum_{i=1}^{n} p_i \frac{\partial x_i}{\partial p_j} = 0 \quad j = 1, \ldots, n \quad (8)
\]
This holds if, as prices are changed, income is adjusted to keep the consumer on his original indifference curve. Since \( \lambda \neq 0 \) (unless the consumer is completely satiated) we can conclude that the summation in (8) will always be zero when compensation is provided.

The compensating change in income is now found by differentiating the consumer's budget constraint and applying this result:

\[
\frac{\partial Y(p; p_0, I)}{\partial p_j} = \frac{\partial I}{\partial p_j} = x_j + \sum_{i=1}^{n} p_i \frac{\partial x_i}{\partial p_j} = x_j \quad j = 1, \ldots, n \quad (9)
\]

That is, the marginal value (measured as a change in income) due to a change in the \( j^{th} \) price is just equal to the quantity demanded of the \( j^{th} \) good.

The meaning of (9) will perhaps be clearer if we realize that the income compensation function \( Y(p; p_0, I) \) is really an argument of \( x_j \) on the right-hand side of (7) because income is continuously adjusted to make (8) hold. Equation (9) can be rewritten in the following form \( 1/ \):

\[
\frac{\partial Y}{\partial p_j} \bigg|_{U=\text{const}} = x_j(p, Y(p)) \quad j = 1, \ldots, n \quad (10)
\]

\( 1/ \) In the remainder of this chapter, we drop the subscripts following the semi-colon in the income compensation function \( Y \). It is assumed to be \( (p_0, I) \). Thus, \( Y(p; p_0, I) = Y(p) \).
where \( x_j(p, Y(p)) \) is the demand for the \( j^{th} \) commodity when the consumer is compensated for price changes relative to the base state \((p_0, I)\), or the "compensated demand" for short.

What makes (10) interesting is that it serves as an alternative definition of the income compensation function because it is a system of partial differential equations for the unknown function \( Y(p) \) with the boundary condition \( I = Y(p_0) \). Equation (10) thus forms the basis for the common definition of consumers' surplus in terms of the area under the demand curve for a good, which we will now discuss.

A.5 Consumer's Surplus as the Area under a Demand Curve

Instead of proceeding from utility functions, the consumers' surplus measure is often derived by applying the following, hypothetical algorithm. A monopolist offers a consumer the opportunity to consume a particular good, say the \( j^{th} \) one, at a very high price. At first the consumer will buy nothing but as the price is lowered to \( p_{0j} \) he begins to buy some of the good. The monopolist continues to lower the price in infinitesimal increments of the \( j^{th} \) good. As the price changes, the consumer is compensated according to (10). Finally, at some price \( p_{1j} \) the process stops.

The total amount paid by the consumer is the area under the inverse demand curve

\[ \int \]

If the demand curve is \( x_j(p, Y(p)) \) the inverse demand curve gives the maximum price which the consumer will pay for the last, infinitesimal unit of the \( j^{th} \) good when he is consuming a quantity \( x_j \). It can be written:

\[ p_j(x_j; p_1, \ldots, p_{j-1}, p_{j+1}, \ldots, p_n, I). \]

The demand curve used in Section A above was the ordinary (uncompensated) demand curve.
of his income compensated demand curve for the $j^{th}$ good. The consumer's surplus is this area less the cost of the final amount purchased $p_1 x_j$. This is the shaded area in Figure IV-A-2.

Now if the price of the $j^{th}$ good was the only price to change during this bargaining process, we can write this area as the integral of the consumer's compensated demand function for the $j^{th}$ good.

$$C = Y(p_1) - Y(p_0) = \int_{p_{0j}}^{p_{1j}} x_j(p, Y(p)) dp_j$$

(11)
We can see that (11) is correct by differentiating with respect to the price $p_j$:

$$\frac{\partial C}{\partial p_j} = \frac{\partial Y}{\partial p_j} \bigg|_{p_1} = x_j(p_1, Y(p_1))$$

which is the same as (10). Since this derivative relation (which we know to be correct) holds by construction at every price between $p_{0j}$ and $p_{1j}$, the result in (11) must hold by the fundamental theorem of the integral calculus.

Equation (11) gives a method of valuing a change from one price to another for a single price change $\frac{1}{j}$, but it contains a circularity because it solves for $Y(p)$ by assuming that it is already known. It is thus impractical to use (11) to find the income compensation function (and hence to evaluate $C$ and $E$). A solution which is often adopted is to find the area under the ordinary demand curve (i.e., the demand curve which is not compensated for changes in utility). This procedure defines a function $A(p; p_0, I)$ which is:

$$A(p; p_0, I) = \int x_j(p, I) dp_j. \quad (12)$$

A more general form of (11) can be derived for changes in the price vector. This is

$$C = Y(p_1) - Y(p_0) = \int \sum_{i=1}^{n} x_i(p, Y(p_0)) \ dp_i.$$

We won't discuss this case further because we are concerned with single price changes in the disaster warning problem.
A. 6 The Relation between $A^\frac{1}{I}$, $E$ and $C$

The effect of using the ordinary demand curve can be analyzed by looking at the income elasticity of demand, $\eta$. We can see how the income elasticity affects $A$ by considering the case of a change in the price of good $j$, where the income elasticity of demand is assumed to be constant. The definition of the income elasticity of demand is:

$$\eta = \frac{\delta x_j}{\delta I} \frac{I}{x_j}$$

(This is, roughly, the percentage change in the demand for good $j$, given a one percent change in income). Then the demand in the final and initial states of the economy is related to the incomes by

$$\frac{x_j(P_1, Y(P_1))}{x_j(P_0, I)} = \left[ \frac{Y(P_1)}{I} \right]^{\eta}$$

(14)

Rearranging (14) and integrating both sides with respect to price gives the relation

$$A/I = \begin{cases} \frac{1}{1-\eta} \left[ \frac{Y(P_1)}{I} \right]^{1-\eta} - \frac{1}{1-\eta} & \text{if } \eta \neq 1 \\ \ln \left[ \frac{Y(P_1)}{I} \right] & \text{if } \eta = 1 \end{cases}$$

(15)

1/ For brevity, we will use $A$ instead of $A(p_0,p_1,I)$.

2/ Consider a demand curve $x_j = I^\eta$. Then the income elasticity of demand is

$$\frac{\delta x_j}{\delta I} \frac{I}{x_j} = \eta I^{\eta-1} \frac{I^{-1}}{\eta} = \eta$$

which is constant.

3/ A more detailed derivation of (15) and its successors is presented in [1].
Using (5), (6) and (15) it is possible to solve for the values of $C$ and $E$ in terms of $\eta$, $A$ and the consumer's income $I$.

$$
\frac{C}{I} = \begin{cases} 
\frac{1}{1-\eta} \left[1+(1-\eta)\frac{A}{I}\right]^{-1} & \text{if } \eta \neq 1 \\
e^{A/I} & \text{if } \eta = 1
\end{cases} \quad (16)
$$

$$
\frac{E}{I} = \begin{cases} 
\frac{1}{1-\eta} \left[1-\left(1-\eta\right)\frac{A}{I}\right]^{-1} & \text{if } \eta \neq 1 \\
1 - e^{-A/I} & \text{if } \eta = 1
\end{cases} \quad (17)
$$

It is interesting to note that the difference between either $C/I$ or $E/I$ and $A/I$ is small whenever the income elasticity is small. This can be seen from the Taylor's series expansions of (16) and (17) to second order, which are:

$$
\frac{C}{I} \approx \frac{A}{I} + \frac{\eta}{2} \left(\frac{A}{I}\right)^2 \quad (18)
$$

$$
\frac{E}{I} \approx \frac{A}{I} - \frac{\eta}{2} \left(\frac{A}{I}\right)^2 \quad (19)
$$

Equations (18) and (19) show that $A/I$ lies between the compensating and equivalent variations, and that all the differences between $C/I$, $E/I$ and $A/I$ are small if either the income elasticity $\eta$ or $A/I$, the magnitude of the measured surplus relative to income, are small.
It is possible to extend this kind of analysis to the case where the income elasticity of demand is not constant or where more than one price changes (see e.g. [13] or [16]). In these cases, it is possible to bound the values of $C/I$ and $E/I$ between two limits which depend on $A/I$ and the income elasticities of the goods. These limits can be shown to be small when the elasticities are near zero or when $A/I$ is small as was the case here (see [16]). These results all support the use of $A$ as an approximation to $C$ or $E$ when the demand for a good is income inelastic or when the change in welfare is small relative to income. This latter assumption certainly seems reasonable when dealing with a home warning receiver whose value to the individual generally isn’t large relative to his income.

The fact that $A$ is a measurable quantity if demands can be measured is probably the principal reason why the method of consumers' surplus is used in economics. Other methods are certainly more general or more satisfying theoretically. But the possibility of measuring the benefits or losses in welfare from the demand curves which consumers possess is an attractive, practical way of obtaining the information. Viewed in this light, the approximations made, for example, when $\eta$ was assumed to be constant become merely another source of error or uncertainty with which an analyst must deal.

A.7 Summary and Remarks

Consumers' surplus is a methodology for measuring the value of a change in the state of an economic system by determining the payment required to compensate an individual for the change in utility he experiences as a result of the change in state. Two important measures of this value are the equivalent variation and the compensating variation. The compensating
variation is the measure appropriate to cost-benefit analysis because it gives the change in money income which must accompany a change in price if a consumer is to be indifferent between his final and initial states. The equivalent variation, which compares the consumer's utility in situations where either prices or income change but not both, is a useful tool when policies changing only one of these are considered.

The measures are all related to one another by the income compensation function which gives the income $Y(p; p_0, I)$ required to make a consumer facing prices $p$ indifferent between that state of the economy and a base state $(p_0, I)$. The most important property of the income compensation function is that its derivative is equal to the consumer's derived demand, Eq. (10). To this differential equation there corresponds an integral equation which is often interpreted as the area under the consumer's derived demand curves. This path-independent line integral is often used as an alternate definition of the income compensation function.

It is possible to approximate this integral by the integral under the consumer's ordinary (i.e., uncompensated) demand curves. If the income elasticity of demand is constant and if only one price changes this integral can be solved for either the compensating or equivalent variation. In other cases, it is possible to derive upper and lower bounds on the variations. The bounds are close when the income elasticities of the goods are near zero and the surplus small relative to total income.

Individual changes in compensated incomes are usually added algebraically to determine the benefits to society of a proposed change in state. This procedure is acceptable under the compensation principle, which implicitly assumes that the marginal social utility of income is the same for all consumers. If equity considerations lead to a different specifi-
cation of the social welfare function, the appropriate aggregation procedure will differ and the area under an aggregate demand curve will not necessarily be the correct measure of social benefits.

However the consumers' surplus is aggregated, we must usually resolve the problem of finding out how much prices change by using the economic model. When it is impractical to take a general equilibrium approach, a partial equilibrium one, which models only some of the goods in our economy, must be used. This necessarily means that price changes in other sectors of the economy are not modeled and their effect on consumers' surplus therefore is unknown.

However, if goods in the excluded sector experience only a small price change, and if these changes have only a small effect on the demand for those goods in the included sector then it can be shown that the effect on consumers' surplus will also be small.

Despite the assumptions, approximations and value judgments which are a part of the methodology of consumers' surplus, the technique is often a useful one in economic analysis. It contains a clear connection with the basic ideas of efficiency at the core of welfare economics. Furthermore, it is often analytically easier to use consumer's surplus than to resort to the treatment of individual utilities. Finally, it is sometimes possible to measure the benefits of a project in a real setting by means of consumers' surplus, at least to within some known approximation error. This makes the technique important in cost-benefit analysis and in other areas of policy analysis which must use some criterion to choose among alternative acts.
CHAPTER IV
BIBLIOGRAPHY


CHAPTER V:
ASSESSING POTENTIAL CONSUMER DEMAND FOR HOME RECEIVERS:
THE STATISTICAL DECISION THEORY APPROACH

A. Overview

A disaster warning system (DWS) provides information about impending disasters, including hurricanes, tornadoes, and floods. This information, if acted upon, can result in substantial reductions in the loss of life and property. In a disaster's aftermath, a DWS may also be used to provide information in support of post-disaster recovery effort.

As we noted in Chapter I, the assessment of alternative warning systems must take account of the fact that the purchase of a home receiver is a private decision. However, it is only as a result of these private decisions that the benefits of disaster warning systems are realized. The evaluation of systems requiring purchase by individuals must proceed in two steps. First, the aggregate benefits derived by individuals must be estimated net of the costs of the home receiver itself. Then, these net benefits must be compared to the costs of those portions of the DWS not covered by user charges to determine whether or not the costs are less than the benefits. This two-part nature of the warning problem is important, for example, in analyzing the NOAA Weather Radio System (NWRS) and the Disaster Warning Satellite System (DWSS). Both these systems require the purchase
of radio equipment to receive warning broadcasts. As we have shown in Chapter III, home receiver costs for the NWRS and the DWSS are three times as great as the cost in the transmitting portion of either of the two systems, assuming these receivers are purchased by twenty percent of the nation's households. Clearly, information about the market for home receivers is important for determining the potential effectiveness and costs of the system.

An earlier study [7] used survey techniques to try to measure consumers demand for a home receiver. In this chapter we take a different approach. Our objective is, (i) to provide an alternative method for estimating the extent to which consumers will be willing to acquire the home receivers, (ii) to apply this method to a particular type of disaster -- namely tornadoes -- to illustrate the use of this method, and (iii) to infer from the analysis the characteristics of a DWS that may have important influences on the acquisition decision.

Briefly, we propose to compare the costs of alternative DWS receivers with the value of the information they provide to the individual. This approach is based on statistical decision theory. As defined by Raiffa and Schlaiffer: [10]

[Statistical decision theory deals] with the logical analysis of choice among courses of action when (a) the consequences of any course of action will depend upon the "state of the world," (b) the true state is as yet unknown, but (c) it is possible at a cost to obtain additional information about the state.

1/ See Chapter III for the other assumptions used.
The "states of the world" that concern us are, naturally, the effects of natural disasters, which differ in their intensity, duration and geographical extent. The decision maker, in this context, is the individual consumer. He decides, given information about the occurrence of a natural disaster, not only whether or not to buy a receiver, but what, if any, preventative action to take. Finally, the cost of information is the cost of the home receiver. (Since we assume that the transmission portion of a DWS will be installed regardless of the individual's decision concerning the purchase of a receiver, his tax payments that are used for the transmitting portion are sunk and do not enter his analysis.) Note that it is the information provision aspects of home receivers that make this method particularly suitable for this demand assessment. The individual is assumed to value the receiver only to the extent that it provides information and not for the receiver itself.

In Section B we present the basic elements of a decision problem and illustrate its solution in terms of a decision problem that arises more often (hopefully) than disaster warnings. One specific problem of decision analysis -- that of valuing the outcomes when they are not explicitly stated in monetary terms (e.g., lost lives) is discussed in Section C. As will become clearer in Section B, valuation of outcomes is absolutely necessary in order to make a logically consistent choice among system alternatives. In Section D of this paper, we use actual (albeit incomplete) data to examine a "representative" household's decision on whether or not to purchase a demutable home receiver using the techniques of statistical decision theory. The sensitivity of the results of our analysis to the assumptions used are discussed in Section E. Another study, related to the analysis performed here, provides a valuable yardstick for comparison and is described as well in Section E. We use the model to derive a demand curve for home receivers in Section F and discuss extensions and observations in Section G.
B. The Elements of Decision Analysis

1. The Framework

There are six basic elements to every problem of choice under uncertainty. They are:

(i) a set of alternative experiments which may be undertaken to gain more information about unknown parameters,
(ii) a sample space, which describes the possible outcomes of all experiments under consideration,
(iii) a set of possible actions (or decisions) from among which a choice is to be made,
(iv) a set of unknown parameters which govern the outcome of the decision to be taken,
(v) A payoff evaluation, which assigns a payoff to performing a particular experiment, observing a given outcome, taking a particular action, and then discovering the particular values of parameters that actually result,
(vi) a probability assessment, which assigns a joint probability distribution to unknown parameters and the outcomes of experiments to find out about them.

The six elements of a decision problem may be usefully described as a game between the decision maker and a make-believe player we call "Lady Luck." There are four basic moves in the game. First

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1/ The reader familiar with statistical decision theory can skip directly to Section C without loss of continuity.
the decision maker (DM) chooses an experiment. Then Lady Luck chooses an outcome to the experiment. Third, the DM selects an action. Finally, Lady Luck chooses the value of unknown parameters. The DM then receives a payoff corresponding to the outcomes of the various moves of the game.

In Figure V-1, we have translated these six elements into an example decision tree for an individual faced with the decision of whether or not to go on a picnic. In the tree, we can find the six elements referred to. The set of experiments consists of calling (or not calling) the weather forecaster. It is an experiment in that the individual expects to obtain additional information about the true state of the world (e.g., whether or not there will be rain). The sample space (outcomes) will be the information received, i.e., a "fair" or a "rain" forecast. The set of actions consists of taking the picnic or staying home. The unknown parameter in this case is whether or not it will rain. The payoff evaluation is how the individual values the combination of weather and activity (e.g., being on a picnic when it rains) The values given in the tree are only meant to provide an indication of the relative valuation of the various outcomes and activity decisions. Obtaining weather information is assumed to cost $5 so the outcomes on the lower branches are reduced correspondingly. Finally, the probability assessment determines the probability that a particular branch will be followed when leaving the chance mode.

1/ While two approaches to actual quantitative analysis of the decision problem are common in the literature, we consider only one in this paper; the so-called "extensive form," which is most easily illustrated by reference to a decision tree. The second approach, the normal form, leads to the same conclusions and is described in [9].
Figure V-1
DM's Decision Tree

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Experiment Outcome</th>
<th>Action Decision</th>
<th>Unknown Parameter</th>
<th>Payoff</th>
</tr>
</thead>
<tbody>
<tr>
<td>Go On Picnic</td>
<td>Fair</td>
<td></td>
<td></td>
<td>100</td>
</tr>
<tr>
<td>Stay Home</td>
<td>Rain</td>
<td></td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>Go On Picnic</td>
<td>Fair</td>
<td></td>
<td></td>
<td>25</td>
</tr>
<tr>
<td>Stay Home</td>
<td>Fair</td>
<td></td>
<td></td>
<td>50</td>
</tr>
<tr>
<td>Go On Picnic</td>
<td>Fair</td>
<td></td>
<td></td>
<td>95</td>
</tr>
<tr>
<td>Stay Home</td>
<td>Rain</td>
<td></td>
<td></td>
<td>-5</td>
</tr>
<tr>
<td>Go On Picnic</td>
<td>Fair</td>
<td></td>
<td></td>
<td>20</td>
</tr>
<tr>
<td>Stay Home</td>
<td>Fair</td>
<td></td>
<td></td>
<td>45</td>
</tr>
<tr>
<td>Go On Picnic</td>
<td>Rain</td>
<td></td>
<td></td>
<td>95</td>
</tr>
<tr>
<td>Stay Home</td>
<td>Rain</td>
<td></td>
<td></td>
<td>-5</td>
</tr>
<tr>
<td>Go On Picnic</td>
<td>Fair</td>
<td></td>
<td></td>
<td>20</td>
</tr>
<tr>
<td>Stay Home</td>
<td>Rain</td>
<td></td>
<td></td>
<td>45</td>
</tr>
</tbody>
</table>

= decision node
= chance node
In the construction of the tree, note that the decision maker has different amounts of information available to him at different decision nodes. For example, in the picnic problem, he has no information about what the weather will be like on the day of the picnic (although he may have an idea of how often it rains, for example) when he must decide about obtaining a forecast. When he makes the decision about the picnic itself, however, he may have more information about the weather (in the form of a forecast, if he made the decision to obtain one) than he had originally. Thus, the nodes in the tree can ordered in terms of what must logically come first. In the picnic example, it would not make sense to decide to go on the picnic and only then obtain the weather forecast. By assumption, the only reason for obtaining the forecast was to aid in making the go/no go decision.

Table V-1 introduces a standard notation for the elements of the problem depicted in the decision tree. As indicated, the payoff of the decision is dependent on the experiment (because of experiment costs), any action taken (because of action costs), and the value of the unknown parameter. The probabilities that are required are the conditional probabilities (conditional on the value of the unknown parameter) since most experiments do not provide perfect information and the probabilities of the unknown parameters (i.e., the probability of rain). The conditional probabilities are termed "likelihoods" and the probabilities of the unknown parameters are termed "prior probabilities," i.e., prior to any revision resulting from new information.
TABLE V-1

NOTATION FOR THE "REPRESENTATIVE" HOUSEHOLD'S DECISION PROBLEM

(i) **Experiment:**
   - $e_0$: Do not call weather forecaster
   - $e_1$: Call weather forecaster

(ii) **Outcome of experiment:**
   - $z_0$: No information received
   - $z_1$: "Fair" forecast
   - $z_2$: "Rain" forecast

(iii) **Actions:**
   - $a_0$: Go on Picnic
   - $a_1$: Stay home

(iv) **Unknown parameters:**
   - $\theta_0$: Fair
   - $\theta_1$: Rain

(v) **Payoffs:**
   - $f(e_i, a_k, \theta_l)$
   \[ i, k, l = 0, 1 \]

(vi) **Probabilities:**

- Probability of outcome conditional on experiment and unknown parameter
  \[ \Pr(z_j | e_i, \theta_l) \quad i, j, l = 0, 1 \]

- Probability of unknown parameter
  \[ \Pr(\theta_l) \quad l = 0, 1 \]
To find the best decision, we work backward from the right hand side of the decision tree shown in Figure V-1. Instead of starting by finding the optimal experiment to be chosen, we start by finding the optimal action to take for each possible experiment and outcome. By optimal, we mean to find that decision which maximizes expected utility. To find this optimal decision, we need to investigate what probabilities the decision maker would attach to unknown parameters given the experiment performed and the result obtained.

2. Probability Assignment

Prior to obtaining experimental evidence, we know from experience (e.g., historical evidence, or some other source), the probabilities of the unknown parameters. If no experiment is performed, there would be no basis to revise these prior probabilities, Pr(θ), and so the decision maker would have to make his decision on the basis of these prior probabilities. For our simple picnic problem, we assume that it rains, on average, once every five days so that the probability of rain, Pr(θ₁), is .2 and the probability of fair weather, Pr(θ₀), is .8.

If an experiment is performed, the decision maker would naturally want to consider how the information derived from it can be used to sharpen his estimate of the probabilities of these events. That is, we want to find the probabilities of θ₀ and θ₁ conditional upon the outcome

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1/ By using utility rather than monetary returns, we can allow for behavior often characterized as risk averse, a very desirable feature for any analysis involving potentially large losses. It is also more general and can incorporate maximization of monetary returns as a special case. The use of utility functions is discussed in Section C below.
of the experiment (the forecast). Before we do this, we must determine the likelihoods (in this case the forecasting accuracy). Therefore, we assume that:

\[
\Pr(\text{forecaster says rain given it does rain}) = \Pr(z_2 | \theta_1) = .6
\]

and

\[
\Pr(\text{forecaster says fair given it is fair}) = \Pr(z_1 | \theta_0) = .75
\]

The probability of the complementary events (i.e., erroneous forecasts) are then just 1 minus these values. The revision of the prior probabilities is made via use of the classic result in probability theory known as Bayes' Theorem. To obtain the revised probabilities, we combine the likelihoods (conditional probabilities) with the priors (probabilities for the unknown parameter) using the standard laws of probability. For example, we determine the joint probability of \( \theta_0 \) and \( z_1 \) (fair weather and a "fair" forecast), denoted \( \Pr(\theta_0, z_1) \) by the multiplication rule:

\[
\Pr(\theta_0, z_1) = \Pr(z_1 | \theta_0) \Pr(\theta_0) = (.75)(.8) = .6
\]

For our example problem, the joint probabilities are shown in Table V-2.

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See any book on probability theory or statistical decision theory (e.g., [9]).
Table V-2

Joint Probabilities for the Picnic Decision

<table>
<thead>
<tr>
<th>Probability of and</th>
<th>No Weather Information Sought ($e_0$)</th>
<th>Weather Information Sought ($e_1$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>No Forecast ($z_0$)</td>
<td>&quot;Fair&quot; Forecast ($z_1$)</td>
</tr>
<tr>
<td>Day is Sunny ($\theta_0$)</td>
<td>.8</td>
<td>.6</td>
</tr>
<tr>
<td>Day is Rainy ($\theta_1$)</td>
<td>.2</td>
<td>.08</td>
</tr>
<tr>
<td>Marginal Probability for Forecasts</td>
<td>1.0</td>
<td>.68</td>
</tr>
</tbody>
</table>

The second step in determining the revised probability is to compute the probability of $\theta_0$ and $\theta_1$ conditional upon $z_0$ and $z_1$, using Bayes' Rule:

$$Pr(\theta_i | z_j) = \frac{Pr(\theta_i, z_j)}{Pr(z_j)}$$

for $i, j = 0, 1$ (2)

where

$$Pr(z_j) = \sum_i Pr(\theta_i, z_j)$$ (3)

For example, $Pr(\theta_0 | z_1) = Pr(\theta_0, z_1)/Pr(z_1) = .6/.68 = .88$ is the probability of fair weather given a "fair" forecast and $Pr(z_1) = Pr(\theta_0, z_1) + Pr(\theta_1, z_1) = .6 + .08 = .68$ is the probability of a "fair" forecast.
These revised probabilities are frequently termed 'posterior' probabilities because they reflect revisions of beliefs about the relative probability of the values taken by unknown parameters based upon experimental evidence. Notice that our posterior probabilities are affected by the 'accuracy' of the experiment. Because the experiment may be 'unreliable,' we remain somewhat uncertain about the true value of the unknown parameter. The probability values assigned to the right hand most branches are exactly those just calculated, i.e., the $Pr(\theta_i \mid z_j)$ which are given in Table V-3.

Probabilities appear in one more segment of the decision tree -- that which corresponds to the outcome of the experiment. We have already computed the required probabilities (called 'pre-posterior' probabilities) in Equation 3 above. Accordingly, we assign the probabilities $Pr(z_j)$ (the marginal probabilities) to the appropriate branches of the tree. Note that these probabilities depend both upon the accuracy of the experiment (as reflected in the likelihood we assign to the outcome of the experiment) and prior probabilities concerning the value of unknown parameters. Figure V-2 presents the decision tree for the picnic problem with the appropriate probabilities entered. (The labels on the nodes are discussed below.)

The probability values are summarized in Table V-3. Recall that prior probabilities are the probabilities attached to the unknown parameters prior to any experimentation. The likelihoods are the probability of an experiment outcome given the value of the unknown parameter. Both of these (the priors and the likelihoods) are known before the formal decision analysis. Computed probabilities are the joint probabilities (the probability
Table V-3

Summary of Probabilities for the Picnic Decision

1. Probabilities Known Before Analysis
   a. Prior Probabilities: \( \Pr(\theta_i) \)
      \[
      \begin{align*}
      \Pr(\text{fair weather}) &= \Pr(\theta_0) = 0.8; \\
      \Pr(\text{rain}) &= \Pr(\theta_1) = 0.2
      \end{align*}
      \]
   b. Likelihoods: \( \Pr(z_i | \theta_j) \)

<table>
<thead>
<tr>
<th>Probability Forecaster says</th>
<th>Fair</th>
<th>Rain</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fair Weather</td>
<td>.75</td>
<td>.25</td>
</tr>
<tr>
<td>Rainy Weather</td>
<td>.4</td>
<td>.6</td>
</tr>
</tbody>
</table>

2. Computed Probabilities
   a. Joint Probabilities: \( \Pr(\theta_i, z_j) = \Pr(z_j | \theta_i) \cdot \Pr(\theta_i) \)
   b. Marginal Probabilities \( \Pr(z_j) = \sum_{i=0}^{1} \Pr(\theta_i, z_j) \)

<table>
<thead>
<tr>
<th>Probability of and</th>
<th>No Weather Information Sought ( (z_0) )</th>
<th>Weather Information Sought ( (z_1) )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>No Forecast ( (z_0) )</td>
<td>'Fair' Forecast ( (z_1) )</td>
</tr>
<tr>
<td>Day is Sunny ( (\theta_0) )</td>
<td>.8</td>
<td>.6</td>
</tr>
<tr>
<td>Day is Rainy ( (\theta_1) )</td>
<td>.2</td>
<td>.08</td>
</tr>
<tr>
<td>Marginal Probability for Forecasts</td>
<td>( \Pr(z_0) = 1.0 )</td>
<td>( \Pr(z_1) = 0.68 )</td>
</tr>
</tbody>
</table>

   c. Posterior Probabilities: \( \Pr(\theta_i | z_j) = \Pr(\theta_i, z_j) / \Pr(z_j) \)

<table>
<thead>
<tr>
<th>Probability of given</th>
<th>Fair ( (\theta_0) ) Weather</th>
<th>Rainy ( (\theta_1) ) Weather</th>
</tr>
</thead>
<tbody>
<tr>
<td>No Forecast ( (z_0) )</td>
<td>.8</td>
<td>.2</td>
</tr>
<tr>
<td>'Fair' Forecast ( (z_1) )</td>
<td>.38</td>
<td>.12</td>
</tr>
<tr>
<td>'Rain' Forecast ( (z_2) )</td>
<td>.62</td>
<td>.38</td>
</tr>
</tbody>
</table>
### Figure V-2

**Assignment of Probabilities**

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Experiment Outcome</th>
<th>Action Decision</th>
<th>Unknown Parameter</th>
<th>Payoff</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>No Information</td>
<td>Stay Home</td>
<td>Fair .8</td>
<td>25</td>
</tr>
<tr>
<td></td>
<td>No Information</td>
<td>Stay Home</td>
<td>Rain .2</td>
<td>50</td>
</tr>
<tr>
<td></td>
<td>Go On Picnic</td>
<td></td>
<td>Fair .5</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Rain .5</td>
<td>0</td>
</tr>
</tbody>
</table>

- **Call Weather Forecaster**
- **No Weather Information Sought**
- **No Information Received**
- **Marginal Probability**
- **Posterior Probability**

- **Node**
- **Decision Node**

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V-14
that a particular unknown parameter and experiment outcome both occur),
the marginals, or pre-posteriors, (the probabilities of each of the 
experiment outcomes), and the posteriors (the probability of the unknown 
parameters given the experiment outcome). In the picnic problem,
remember, the unknown parameters are "rain" and "sun" while the 
experiment outcomes are "rainy forecast" and sunny forecasts."

3. Backward Calculation

We conclude the analysis of our illustrative decision problem 
in extensive form by first computing expected payoffs for each possible 
action, conditional upon performing a given experiment and observing a 
particular outcome (nodes A in Figure V-2). The optimal action given an 
experiment and outcome is that which produces the highest expected 
payoff. (Recall that payoffs need not be in monetary terms.) We then 
compute the expected payoff conditional on the choice of an experi-
ment (node B in Figure V-2). The optimal experiment is that with the 
highest expected payoff.

This process can be seen in Figure V-3. There, the numbers 
above each of the nodes represent the expected value to the decision-
maker as a result of being at that node. As expected value is the 
weighted average of all possible values that the decision maker could 
receive as a result of his decision. The weights are simply the 
probabilities of receiving that payoff. A simple example is the case of 
rolling a die and receiving $30 if the number 1 or 2 appears and nothing 
otherwise. In this case, the expected value is just:
Figure V-3
The Decision Computation

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Experiment Outcome</th>
<th>Action Decision</th>
<th>Unknown Parameter</th>
<th>Payoff</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>Go On Picnic</td>
<td>Fair, Rain 0.2</td>
<td></td>
<td>100</td>
</tr>
<tr>
<td>80</td>
<td>No Information</td>
<td>Stay Home</td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>90</td>
<td>No Information</td>
<td>Fair, Rain 0.8</td>
<td></td>
<td>25</td>
</tr>
<tr>
<td>1.0</td>
<td>1.0</td>
<td>Fair, Rain 0.2</td>
<td></td>
<td>50</td>
</tr>
</tbody>
</table>

= decision node
= chance node
\[
EV = \frac{2}{6} (\$30) + \frac{4}{6} (\$0) = \$10
\]

For the picnic problem, the expected value at node (1) in Figure V-3 is

\[
EV = 0.62 (\$20) + 0.38 (\$45) = 29.50
\]

and at node (2), it is

\[
EV = 0.68 (\$83) + 0.32 (\$57) = 75
\]

The double slashes indicate non-optimal decisions. Therefore, the optimal decisions are (1) to go on the picnic and (2) not to obtain weather information.

The reason for this second decision is clear. Given that weather information is costly ($5) and given that it is not used (decision (1)), the information costs more than it is worth.

4. **The Expected Value of Perfect Information**

Suppose now that the decision maker was offered the following opportunity: he could buy the services of a "clairvoyant" who would tell him, with no possible mistake, what the weather would be on a specific day. The question becomes, how much would the decision maker be willing to pay for the clairvoyant's services? This amount is called the "expected value of perfect information." This number is important because it provides an upper bound on the amount of money the decision maker will be willing to spend for weather forecasts that are less than perfect.
The answer to the question can be determined by forming a new tree representing this decision. Consider the tree in Figure V-4.

Figure V-4

The Decision Tree For Clairvoyance

\[
\begin{align*}
\text{Take Picnic} & \quad 100 \\
\text{Stay Home} & \quad 50 \\
\end{align*}
\]

At the first node, chance decides what the weather will be. This is just \( \Pr(\theta_0) \) and \( \Pr(\theta_1) \) for the probabilities of sunshine and rain, respectively. If the individual finds out that it will be sunny, he will go on the picnic. The payoff associated with this event is 100. If he finds out that it will rain, he will stay home and the payoff will be 50. The expected payoff, \( \pi_C \), with clairvoyance then is:

\[
\pi_C = \Pr(\theta_0)(100) + \Pr(\theta_1)(50) = .8(100) + .2(50) = 90
\]

Without clairvoyance, but with the prior information discussed in the previous section, the decision maker faces an expected payoff of 80.
The increase in expected payoff due to the clairvoyant, or the expected value of perfect information is then, just:

$$\text{EVPI} = \pi_C - V = 90 - 30 = 10$$

The EVPI represents the maximum amount the decision maker would be willing-to-pay for better information about the weather. The reason is simple. If the forecast cost the individual, say, 15, he knows that even if the forecast is perfect, his expected payoff is

$$\pi_C - 15 = 75$$

which is less than the expected payoff if he always went on the picnic (30). Therefore, he would not even consider obtaining a forecast. However, since the actual cost of the forecast is only 5, we know that for some improvement in the forecast accuracy, the optimal decision will be to obtain weather information.

C. Valuing Payoffs

In assigning or determining the payoffs for a decision problem, three separate problems often arise. First, we must determine those costs that are relevant to the decision maker. Second, the payoffs and experiment costs must be made commensurate. Third, a way must be found to represent accurately the decision maker's relative valuation of different payoffs. The purpose of this section is to discuss these problems and possible treatments for them. In a properly performed statistical decision analysis, assumptions about the payoffs are explicit and the sensitivity of the optimal decision to assumptions about the payoffs can be analyzed.
The first step in assigning payoffs is the determination of the relevant costs and benefits. Thus, it is important that a distinction be made in the analysis of which costs and/or benefits are to be considered. If the decision maker is the government, the costs and benefits to be considered would be social costs and benefits. That is, they would include private costs and benefits and also the costs and benefits of externalities associated with the decision. If, on the other hand, the decision maker is a private individual the relevant costs and benefits would be his own private costs and he would not consider costs borne by others as a result of his actions. All costs must, of course, represent true opportunity costs. For example, sunk costs (costs which result from immutable past decisions) should not be considered.

Second, for the analysis to be meaningful, the payoffs and the experiment costs must be commensurate. This is particularly difficult when analyzing disaster warning systems since loss of life is included in the payoffs. For loss of life to be on comparable terms with monetary costs, some estimate of how the decision-maker values changes in his survival probability, even if implicit, must be made. Objections are often raised that this value is infinite (or at least very, very great). Casual observation of human behavior suggests that this is not so. We take risks continually ranging from driving to not spending three continuous months during the tornado season in the cellar, and such risk taking would constitute irrational behavior if people truly placed an infinite value on increasing their chances for survival.
Given that there is such a finite value, how could (or should) we go about measuring it? The methods that have been suggested include both explicit and implicit approaches. Methods of the explicit type include the discounted future earnings approach, the loss to society approach, and the insurance approach. These methods might be expected to lead to an underestimate of the value an individual places on changes in his survival probability since market transactions reflect only a part (although it could be a major part) of the individual's life.

The implicit methods are based on the concept that an individual reveals the value he places on small changes in survival probability through risky actions he voluntarily takes. Most recently applied by Conley [1], and Thaler and Rosen [11], this method is based on an individual's willingness-to-pay for marginal changes in the probability of survival. While these methods do not provide such straightforward measures for the value attached to changes in risk as do the explicit methods, Conley shows that, in general, the discounted future earnings approach underestimates it. As suggested above, the discounted earnings approach therefore places a floor on the estimate to be used in calculations.

The third problem is associated with attitudes toward risk on the part of the decision maker. Recognizing that many, if not most, individuals are averse to risk (i.e., they are willing to pay an actuarially unfair premium to insure against a "large" loss), there must be some way to incorporate this behavior into our analysis. Recall that in Section B we assumed that the individual acted to maximize expected utility. Therefore, we can postulate a utility function for the individual, translate the

1/ These methods are discussed in [5].
payoffs and information costs into "utils" and solve the decision problem in exactly the same way as before.

An individual who is "risk neutral" or a "risk preferer" can have a suitable utility function and, therefore, this approach is more general than an expected money value (EMV) approach. The effect of assuming a particular functional form can be illustrated with the curves shown in Figure V-5. There, three different functions have been drawn. The first, AA, exhibits risk aversion (we see why in a moment). The second, BB, is a utility function for an EMV'er, i.e., it exhibits risk neutrality. Finally, CC is the utility function of a risk preferer.

Figure V-5
Utility Functions and Risk Aversion

Returning now to AA, we can see what risk aversion means. Suppose the individual is faced with a 50-50 chance of incurring a loss of L (assumed to be large relative to his net worth). From the function AA,
we see that his loss in utility would be $U(-L)$ if this loss occurred. His expected loss is $(-L/2)$ with utility $U(-L/2)$. However, his expected utility loss is $U(-L/2)/2$ which is greater than $U(-L/2)$. (We can assume, without loss of generality, that $U(0) = 0$.) Since the individual is assumed to maximize expected utility, he will be willing to pay a premium to avoid the risk. This premium can be found by finding that amount of money whose utility is $U(-L/2)$. We find, from AA, that this is $(-L/2) - P$ and taking the difference, the premium is equal to $P$. In other words, the individual is willing to pay an amount $P$ in excess of his expected loss in order to avoid the risk of incurring the total loss. If we did the same thing for functions BB and CC, we would find that, for the EMV'er (with function BB) the risk premium, $P$, would be zero. For the risk preferer, $P$ would actually be negative, i.e., the individual is willing to pay to be given the gamble.

The application of this concept to decisions which include outcomes with large wealth consequences is clear. Facing the risk of large property damages, where the probabilities are small, the "average" individual is likely to be willing to pay in excess of the expected loss (which may be negligible) in order to lower the probability of such a loss. The utility concept is not so easily applied empirically when the outcome involves death or other consequences resulting in changes in an individual's well-being. In this case, the individual's utility function will involve two arguments, wealth and "health." (The value of a given amount of wealth varies depending on the individual's well-being.) We know of no attempts to estimate functions of this type that have been reported in the literature.
Therefore, in the example application of Section D (in which death is the outcome) we do not use a utility function. However, we provide an example of its use for outcomes involving wealth losses in the appendix.

D. An Illustration of the Methodology

1. Introduction

In this section, we apply the statistical decision theory described above to analyze the decision-making process an individual might go through (at least implicitly) when deciding whether or not to purchase a demutable receiver. Although the receiver would, of course, transmit all warnings, we restrict this analysis to tornadoes. Further, we consider only deaths resulting from tornadoes. This allows us to illustrate the application of the methodology without making the calculations extremely complicated. This application uses data currently available to the extent possible, but it is not intended to provide accurate demand forecasts. There are several reasons why this is so:

- Most areas of the country are subject to more than one natural disaster. We consider a single household exposed to one type of disaster -- tornadoes.
- We have made several simplifying assumptions about the probabilities of different events, both through time and over large geographic areas.
- Our estimates of the payoffs are open to question for many of the reasons discussed in Section C.
- No allowance is made for the provision of daily weather forecasts, which are also a service obtained from a
demutable receiver that can receive NOAA Weather Radio transmissions. (For a description of the NOAA Weather Radio System, see Chapter II above.)

The sections are arranged in the same sequence in which an actual analysis would be performed. The sections therefore provide a step-by-step guide to actual implementation of the method.

In Section 2, the decision problem facing a representative individual is described. In Section 3, we develop estimates of the event probabilities in a form in which the available data can be used. The likelihoods (those probabilities dealing with forecasting accuracy) are calculated in Section 4. The calculation formulae are developed to extract the maximum amount of information from the available data.

In Section 5, we calculate the joint and posterior probabilities that are derived from the priors and the likelihoods. Section 6 is concerned with developing cost estimates for the experiment, the action costs and the payoffs. The optimal decision is determined in Section 7.

2. The Decision Problem

Our decision maker is faced with the following problem. On any particular day he must decide whether or not to rent a home receiver designed to receive a warning signal from a DWS.\(^1\) With this receiver, we assume that the decision maker is certain to receive any warnings that are issued. Without the receiver, other sources of information (e.g., radio and television) are available to him, but he may not be listening.

\[^1\] The reason for using rental instead of purchase as the relevant decision is solely to simplify the probability calculations.
Therefore, if he does not have a DWS receiver, the probability that he will receive any one warning is less than one. We also know that weather forecasting is an imprecise science so that there are cases of both issuing warnings when there is no tornado (false alarms) and issuing no warning when there is a tornado (failure to warn). The use of a receiver clearly does not affect the quality of the forecast.

When our decision maker receives a warning, he decides whether or not to take some form of preventative action. The preventative action he takes (whatever its form) is assumed to alter the probability of death in his family given that there is a fatal tornado. \(^1\)\(^2\)

The decision tree representing this mode is shown in Figure V-6. The structure of the tree is most easily understood by looking at a representative node at each stage. First, at node A, the decision maker decides whether or not to rent the demutable receiver. If he chooses to rent \((e_1)\), there is a cost to that decision as noted by the single slash through the branch. At node B, a chance node, we find the probabilistic event of the issuance of a (tornado) warning. (In this application we do not make a distinction between watches and warnings since the NWS demutes for both.) Although the issuance of a warning is not affected by the previous receiver decision, we will see shortly the respective conditional probabilities of the receipt of a warning are indeed affected. At node C, following the

---

1/ A fatal tornado is defined here as the joint event "tornado occurs" and "at least one death results" where the one death need not be in the decision maker's family.

2/ In the general decision problem, one form of action might be to obtain more information. Because we are using tornadoes in this example, the swiftness with which the disaster strikes is assumed to preclude this option.
Figure V-6

The Decision Tree

<table>
<thead>
<tr>
<th>Receiver Decision</th>
<th>Warning Received</th>
<th>Action Decision</th>
<th>Fatal Tornado?</th>
<th>Death in Household?</th>
<th>No. of Deaths in Household</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- No
- Yes

Note: (see below)
issuance of a warning, the decision maker decides whether or not to take action. Notice that this decision only arises if the warning is issued. (It is possible to think of the decision maker taking action without any warning, but such behavior would appear uncommon and we ignore the possibility here.) Again, there is a cost to making the decision in favor of action. At node D, the event of interest is the occurrence of a fatal tornado. Although realism would require all tornadoes (perhaps weighted by the degree of severity) be included, suitable data is readily available only for fatal tornadoes. Further, as we shall see below, the payoffs are in terms of deaths. Therefore, it seems that little is lost (and much simplicity is gained) by restricting ourselves to fatal tornadoes.

Node E represents the chance event that at least one person in the decision maker's family is killed as a result of the fatal tornado. Note that if there is no fatal tornado, there can be no death in the decision maker's family. At node F, chance determines how many individuals in the family are killed (M represents the size of the family). The payoffs and the probabilities associated with each of the chance events will be added to the decision tree after we discuss some of the data available for this problem.

Table V-4 provides the notation for this example. Note that we have separated the occurrence of a fatal tornado from that of a death in the decision-maker's family. The reason for this is that the data on forecasting is related to deaths in the forecast box 1/ and not in an individual's family. This addition provides a link between warnings and deaths in the family.

1/ See Table V-4.
Table V-4

Notation for the Application

(i) **Experiment:** \(e_0\); Do not rent receiver
\[e_1\]; Rent receiver

(ii) **Outcome of experiment:** \(z_0\); No warning received
\[z_1\]; Warning received

(iii) **Action:** \(a_0\); No action taken
\[a_1\]; Preventative action taken

(iv) **Unknown parameters:** \(\phi_0\); No fatal tornadoes in the forecast box
\[\phi_1\]; Fatal tornadoes in the forecast box
\[\theta_0\]; No deaths in decision maker's household
\[\theta_1\]; At least one death in household

(v) **Payoffs:** \(f(e_i, a_k, m)\)

(vi) **Additional Notation:** \(M\) = number of individuals in the decision maker's household
\(m\) = number of individuals killed in the household
\(N\) = population of a forecast box (an area of approximately 25,000 square miles for which the warning is issued)
\(n\) = number of individuals killed in a forecast box
3. **Event Probabilities**

a. **Introduction**

The first problem is the calculation of the event probabilities. In this application, this process is more complicated than in the example provided in Section B. Referring back to Figure V-6, we see that there are two events of interest, namely:

- the occurrence of a fatal tornado ($\phi_1$)
- the occurrence of a death in the household ($\theta_1$)

We will now develop the calculation formulae for each of these events and, with the existing data, estimate these probabilities. (Sections b and c).

In Section d, we develop conditional probabilities for the "household" events (e.g., the probabilities of a death in the household given a fatal tornado.)

b. **The Occurrence of a Fatal Tornado**

The occurrence of a fatal tornado can be viewed in the following way. By some means, chance assigns each individual in a population of $N$ individuals either a "1" with probability $p$ or a "0" with probability $(1 - p)$.

These numbers are assigned to each individual independently. The probability law describing this process is represented by the binomial distribution,

$$
\binom{N}{n} = \frac{N!}{n!(N-n)!}
$$

1/ One often uses the Poisson probability law in discussions about the occurrence of accidents. That is usually done as an approximation for the binomial law when $N$ is large and $p$ is small because the binomial requires calculation of, e.g., $N! \cdot (N-1) \cdots (2)(1)$. In this case, however, the algebra works out so nicely that the binomial law can be used.

2/ By definition,

$$
\binom{N}{n} = \frac{N!}{n!(N-n)!}
$$
\[
\Pr(n) = \binom{N}{n} p^n (1 - p)^{N - n}
\]  

(4)

If we think of the assignment of "1" to be "killed by a tornado," then the probability of \( n \) deaths in a population of size \( N \) is exactly described by (4). (Recall that these are prior probabilities, i.e., prior to any other information such as the fact that there is a tornado.)

The event "no fatal tornado occurs" \( (\phi_0) \) is equivalent to the event "no deaths" \( (n = 0) \). Therefore,

\[
\Pr(\phi_0) = \Pr(n = 0) = \binom{N}{0} p^0 (1 - p)^{N - 0} = (1 - p)^N
\]  

(5)

and

\[
\Pr(\phi_1) = 1 - \Pr(\phi_0)
\]  

(6)

In order to derive an estimate of \( p \), we require data on the number of tornado fatalities and the population. Because of the way in which the problem was modeled, we must be careful about the time period we select for use in this estimation process. The NWS began issuing tornado forecasts in 1952 \[2\]. Thus, after that date systematic warnings were available about the occurrence of a tornado. The probabilities we are attempting to measure are the unconditional probabilities of fatal tornadoes in the absence of warnings. Therefore, we want to use data prior to 1952.
On the other hand, improvements in building construction, the overall educational level, etc., may tend to lower this probability by reducing fatalities. Therefore, we would want to collect data from recent periods. In an arbitrary selection, we chose the period 1936-1955 as the period over which to average tornado deaths and 1945 as the year for which population was measured.

The population used in the estimation of the probability is the population of the forecast box. Since the forecast box is approximately 25,000 square miles [3] and assuming the population is spread uniformly over the entire area, we estimate \( N \) as the total population divided by the number of forecast boxes (total area divided by 25,000).

Finally, we restrict the area of concern to those states which are "prone to" tornadoes. We use as a discriminator the average annual number of deaths for the years 1953-1971 and a cutoff of one. Thus, any state with an average of more than one tornado-related death per year is included. There are 21 of these states and they are listed in Table V-5. The total population in 1945 is estimated (based on the average of 1940 and 1950 Census data [3]) to be 76,074,000 with a land area of 1,364,400 square miles. The NWS statistics show an average of 182.55 deaths per year during the period 1936-1955 [14]. This was revised to 178 using data in [14] to reflect the fact that we are concerned only with 21 states.

---

\(^1/\) The reason for using this later period to determine "tornado-prone" states is that if there has been a geographical shift in the incidence of tornadoes we want to take our population and land area data from the states currently affected.
Table V-5
States With Tornado Threat 1/

<table>
<thead>
<tr>
<th>States</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ohio</td>
</tr>
<tr>
<td>Indiana</td>
</tr>
<tr>
<td>Illinois</td>
</tr>
<tr>
<td>Michigan</td>
</tr>
<tr>
<td>Wisconsin</td>
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<tr>
<td>Minnesota</td>
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<tr>
<td>Iowa</td>
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<tr>
<td>Missouri</td>
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<tr>
<td>Nebraska</td>
</tr>
<tr>
<td>Alabama</td>
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<tr>
<td>Massachusetts</td>
</tr>
<tr>
<td>Georgia</td>
</tr>
<tr>
<td>Florida</td>
</tr>
<tr>
<td>Kentucky</td>
</tr>
<tr>
<td>Tennessee</td>
</tr>
<tr>
<td>Alabama</td>
</tr>
<tr>
<td>Mississippi</td>
</tr>
<tr>
<td>Arkansas</td>
</tr>
<tr>
<td>Louisiana</td>
</tr>
<tr>
<td>Oklahoma</td>
</tr>
<tr>
<td>Texas</td>
</tr>
</tbody>
</table>

1/ Annual average tornado deaths greater than one person 1953-1971 [14].
In Table V-6 we perform the relevant calculations. In line a, the number of forecast boxes is calculated as described above. The average population in a forecast box is computed in line b. In line c, we divide the tornado deaths in the forecast box by the population in a forecast box and then divide this quotient by 365 days to reflect daily deaths. From line d, we see that the probability of a fatal tornado in a forecast box on any one day is estimated to be .009.

C. The Probability of Death in the Household

Given the data in the previous section, we can develop the probability of death in the decision maker's household. Letting \( m \) denote the number of deaths in the household, we know, by definition, that

\[
Pr(m) = \sum_{n=m}^{N} Pr(m|n) \cdot Pr(n)
\]

(7)

The distribution of \( Pr(m|n) \) can be viewed as follows: given an area with a population of \( N \) individuals, select a sample of size \( M \) (family size). If there are \( n \) people dead in the population, what is the probability that there are \( m \) individuals that are dead in the sample? The answer can be found with the hypergeometric distribution. Thus,

\[
Pr(m|n) = \frac{\binom{n}{m} \cdot \binom{N - n}{M - m}}{\binom{N}{M}}
\]

(8)

Therefore, from (4), (7) and (8)

\[
Pr(m) = \sum_{n=m}^{N} \frac{\binom{n}{m} \cdot \binom{N - n}{M - m}}{\binom{N}{M}} \cdot \binom{N}{n} p^n (1 - p)^{N - n}
\]

(9)
Table V-6

Calculation of Fatal Tornado Probabilities

a. Number of Forecast Boxes:

\[ F = \frac{1,364,400}{25,000} = 54.6 \]

b. Forecast Box Population:

\[ N = \frac{76,074,000}{F} = 1.394 \times 10^6 \]

c. Probability of Individual Death (p):

\[ \frac{178}{54.6} \times \frac{1}{1.394 \times 10^6} = 0.009 \]

\[ \frac{1}{365} = 6.41 \times 10^{-9} \]

d. Probability of Tornados:

(1) "No Fatal Tornado"

\[ Pr(\phi_0) = Pr(n=0) = (1 - p)^N = 0.991 \]

and (2) "Fatal Tornado"

\[ Pr(\phi_1) = (1 - Pr\phi_0) = 0.009 \]
For \( n > N - M + m \), \( \Pr(m|n) = 0 \). (To see this, consider the case where \( n = N \), \( m < M \). Then, if the entire population is killed, all members of the family must be killed, so \( \Pr(m < M) = 0 \).) From this, \( (9) \) can be rewritten as

\[
\Pr(m) = \sum_{n=m}^{N-M+m} \frac{n!}{m!(M-m)!} \frac{(N-n)}{(N)} \frac{N^n}{n} p^n (1 - p)^{N-n}
\]

(10)

With some algebraic manipulation, we have,

\[
\Pr(m) = \frac{M!}{m!(M-m)!} p^m (1 - p)^{M-m} \left[ \sum_{k=0}^{K} \binom{K}{k} p^k (1 - p)^{K-k} \right]
\]

(11)

where \( k = n - m \) and \( K = N - M \). But the term in brackets is one since it is the sum over the range of a probability distribution. Thus,

\[
\Pr(m) = \binom{M}{m} p^m (1 - p)^{M-m}
\]

(12)

The probability of no deaths in the family (\( \theta_0 \)) is, therefore (assuming \( M = 4 \)),

\[
\Pr(\theta_0) = \Pr(m = 0) = (1 - p)^M = 1 - 2.56 \times 10^{-8} = .999999974
\]

and the probability of at least one death in the family is,

\[
\Pr(\theta_1) = 1 - \Pr(\theta_0) = 2.56 \times 10^{-8}
\]
d. **Conditional Event Probabilities**

From the decision tree, we see that we are also interested in some events of the type "death in the household \((\theta_1)\) given a fatal tornado \((\phi_1)\)." These probabilities can be easily calculated from the information above.

First note that

\[
\Pr(\theta_1) = \Pr(\theta_1|\phi_0)\Pr(\phi_0) + \Pr(\theta_1|\phi_1)\Pr(\phi_1)
\]

But the first term is zero because (by definition) tornado deaths can only occur if a fatal tornado occurs. Thus:

\[
\Pr(\theta_1|\phi_1) = \frac{\Pr(\theta_1)}{\Pr(\phi_1)} = 2.84 \times 10^{-6}
\]

is the probability of at least one death in the family given a fatal tornado.

While the conditional probabilities of \(m\) deaths given a death in the household could be calculated similarly, that approach would ignore the useful information that one death in the household makes it more likely that others are killed (i.e., the independence assumption is faulty for co-located groups). This merely says that we would expect that the information that one death has occurred in the family would cause the decision maker to revise the individual probability of death upwards. This is intuitively plausible since members of a household would normally be in the same geographic area when a tornado struck. In an attempt to correct for this, we make use of the fact that the average area of a tornado path is 2.5 square miles [3]. We use this to recompute the relevant population by looking at the average population in an area of 2.5 square miles, which is 139.4. Then a revised value for \(p\) (call it \(p'\)) is

\[
p' = 6.41 \times 10^{-5}.
\]
We could now compute the probability of \( m \) deaths in the household given at least one death in the family. To simplify the calculations we consider the conditional expectation of lives lost given at least one is lost. This is simply,

\[
E = \sum_{i=1}^{M} i \cdot \Pr(i | \theta_1) = \sum_{i=1}^{M} i \frac{\Pr'(i)}{\Pr'(\theta_1)} = 1 - (1 - p')^M \sum_{i=1}^{M} i \left(\frac{M}{i}\right)(p')^i(1-p)^{M-i}
\]

(15)

where the \( \Pr' \) represent these revised probabilities (i.e., using \( p' \) instead of \( p \)). For example, if \( M = 4 \), the expected number of lives lost in the household given at least one is lost is 1.0001.

Table V-7 provides a summary of the results that will be used in the remainder of the study.

<table>
<thead>
<tr>
<th>Table V-7</th>
<th>Event Probabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Pr(\text{No fatal tornados}) = \Pr(\Phi_0) ) = .991</td>
<td></td>
</tr>
<tr>
<td>( \Pr(\text{Fatal Tornado}) = \Pr(\Phi_1) ) = .009</td>
<td></td>
</tr>
<tr>
<td>( \Pr(\text{No deaths in household}) = \Pr(\Phi_0) = 1 - 2.56 \times 10^{-8} )</td>
<td></td>
</tr>
<tr>
<td>( \Pr(\text{At least one death in household}) = \Pr(\Phi_1) = 2.56 \times 10^{-8} )</td>
<td></td>
</tr>
<tr>
<td>( \Pr(\text{no deaths in household given no fatal tornado}) = \Pr(\Phi_0</td>
<td>\Phi_0) = 1.0 )</td>
</tr>
<tr>
<td>( \Pr(\text{at least one death in household given no fatal tornado}) = \Pr(\Phi_1</td>
<td>\Phi_0) = 0.0 )</td>
</tr>
<tr>
<td>( \Pr(\text{no deaths in household given fatal tornado}) = \Pr(\Phi_0</td>
<td>\Phi_1) = .9999972 )</td>
</tr>
<tr>
<td>( \Pr(\text{at least one death in household given fatal tornado}) = \Pr(\Phi_1</td>
<td>\Phi_1) = 2.34 \times 10^{-5} )</td>
</tr>
</tbody>
</table>

Expected lives lost in family given at least one life in family is

\[
\text{lost} = 1.0001
\]
4. The Likelihoods

The data that will be used to estimate the likelihoods concerns warning issuance, not receipt. To move from the probabilities concerning warning issuance to warning receipt, we assume that the two events are independent. Thus, the probability that a warning is issued and received is the product of the two probabilities. If the receiver is rented, we assume that the warning is received with certainty, i.e., the probability of receipt is 1.

In Table V-8, we provide the data required for the calculation of the probability of receipt of a warning given that the demutable receiver is not rented. Rosen and Haimes [8] provide data on radio and television audience. We assume: (1) that the radio and television audience is separate (i.e., total audience is the sum of the radio and television audience), and (2) that this percentage may be interpreted as the percentage of households that would receive a warning via radio and television if one were issued.

Table V-8

<table>
<thead>
<tr>
<th>Hour Beginning</th>
<th>Combined Adult TV</th>
<th>Probability of a Tornado (Y)</th>
<th>Expected Audience</th>
</tr>
</thead>
<tbody>
<tr>
<td>0000</td>
<td>19</td>
<td>.015</td>
<td>.29</td>
</tr>
<tr>
<td>0100</td>
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<td>.13</td>
</tr>
<tr>
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<td>.014</td>
<td>.11</td>
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<tr>
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<td>.10</td>
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</tr>
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<td>.038</td>
<td>1.99</td>
</tr>
<tr>
<td>2300</td>
<td>32</td>
<td>.035</td>
<td>1.12</td>
</tr>
</tbody>
</table>

\(Y\) From [4], p. 46. As a percentage.

\(Y\) From (14), p. 56.

Total Expected Audience 38.715
If we assume that a warning is issued in the same hour as the tornado occurs, then the expected audience size is the sum over the size of audience in any hour multiplied by the probability of a tornado in that hour. From Table V-6 we find that this expected value is 37 percent of the population. Thus, we use .37 as the probability of receipt of a warning via radio and television given one is issued.

The bias in this figure is not clear. Because the assumptions made above tend to overestimate the audience, this figure will tend to be high. On the other hand, there are other means of obtaining tornado warnings such as contacting the NWS directly, contacting the news media, etc. This would tend to increase the probability that any warning issued is received. Because of the ambiguity in the bias, we use .37 as the relevant probability.

To estimate the probability of warning receipt we use two pieces of data found in the literature. First, Galway [2] has found that the percentage of fatal tornadoes for which a warning had been issued is .56. Thus, in the case of the receiver being rented (e₁)

\[
1/ \text{If the tornado warning is issued earlier, the proper way to calculate the expected audience is to estimate the cumulative audience over the period from warning issuance to the occurrence of the tornado. For example, Galway [2] presents data showing the mean lead time from warning issuance to impact of the tornado is from 2.55 - 3.27 hours. Therefore, the expected audience is (roughly) the percentage of households that watch television or listen to radios some time in the period from t-3 to t multiplied by the probability of a tornado at time t and summed over all possible t's (i.e., all 24 hours). The available data cannot support this calculation. However, if we look at the one-time audience at t-3 and rework the calculation in the Table V-8 (i.e., audience at 0900 times probability of tornado at 1200+, etc.) the expected audience is 33.4%. The cumulative audience must be greater but by how much is unknown. Because of the uncertainty about cumulative audience size, we return to the original assumption about warning issuance and tornado impact occurring in the same hour.} \]
\[ \Pr(z_1|\phi_1, e_1) = .56 \]

which is the probability of warning given the occurrence of a fatal tornado. Therefore,

\[ \Pr(z_0|\phi_1, e_1) = 1 - \Pr(z_1|\phi_1, e_1) = .44 \]

Note that this latter number is the "failure to warn" probability.

Kessler [3] finds that "about 40 percent of affirmative predictions are correct, i.e., are followed by tornadoes somewhere in the forecast box." If we assume that prediction of fatal tornadoes is as accurate as for all, then the probability that a tornado occurs given the issuance of a warning is: \(1/\)

\[ \Pr(\phi_1|z_1, e_1) = .40 \]

and

\[ \Pr(\phi_0|z_1, e_1) = 1 - \Pr(\phi_1|z_1, e_1) = .60 \]

is the probability of no fatal tornado given a warning.

---

\(1/\) It would seem intuitive that fatal tornadoes, being "larger" may have a higher probability of being detected. If so, better information is received over both "normal" channels and a demutable receiver. The effect of better information on the decision is discussed in the sensitivity analyses of Section E.
If we combine these two pieces of data with the event probabilities on the occurrence of a fatal tornado \( \Pr(\phi_1) \) we will be able to compute all the likelihoods. This is accomplished by calculating the false alarm probability \( \Pr(z_1 | \phi_0) \). By Bayes' Rule (we drop the \( e_l \) notation for simplicity)

\[
\Pr(\phi_1 | z_1) = \frac{\Pr(\phi_1, z_1)}{\Pr(z_1)}
\]

where

\[
\Pr(z_1) = \Pr(z_1, \phi_0) + \Pr(z_1, \phi_1).
\]

Inserting (17) into (16) we can compute,

\[
\Pr(z_1 | \phi_0) = \frac{\Pr(z_1 | \phi_1) \Pr(\phi_1)[1 - \Pr(\phi_1 | z_1)]}{\Pr(\phi_1 | z_1) \Pr(\phi_0)}
\]

Substituting in the values for quantities on the right-hand side, we obtain

\[
\Pr(z_1 | \phi_0, e_1) = \frac{(0.36)(0.009)[1 - 0.4]}{(0.4)(0.991)} = 7.6 \times 10^{-3}
\]

and

\[
\Pr(z_0 | \phi_0, e_1) = 1 - 7.6 \times 10^{-3} = 0.9924
\]

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The likelihoods when the receiver is not rented \((e_0)\) are calculated using,

\[
Pr(z_1 | \phi_0, e_0) = Pr(z_1 | \phi_0, e_1) \cdot Pr(\text{receipt}) = 7.6 \times 10^{-3} \cdot .37 = 2.8 \times 10^{-3}
\]

and

\[
Pr(z_1 | \phi_0, e_0) = Pr(z_1 | \phi_1, e_1) \cdot Pr(\text{receipt}) = .56 \cdot .37 = .21
\]

The complementary probabilities are then one minus these results. The likelihoods are summarized in Table V-9.

<table>
<thead>
<tr>
<th>Given Probability of</th>
<th>Without Receiver ((e_0))</th>
<th>With Receiver ((e_1))</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>No Fatal Tornado ((\phi_0))</td>
<td>Fatal Tornado ((\phi_1))</td>
</tr>
<tr>
<td>No Warnings Received((z_0))</td>
<td>.9972</td>
<td>.79</td>
</tr>
<tr>
<td>Warning Received((z_1))</td>
<td>(2.8 \times 10^{-3})</td>
<td>.21</td>
</tr>
</tbody>
</table>
5. **Joint and Posterior Probabilities**

Given the prior probabilities and the conditional probabilities, we can now proceed directly with the calculation of the joint and posterior probabilities. Since this is done in the same way as described in Section B only the results are presented here. The joint probabilities are given in Table V-10 and the posterior probabilities in Table V-11.

<table>
<thead>
<tr>
<th>A. No Receiver Rented (e₀)</th>
<th>z₀ (No Warning Recv'd)</th>
<th>z₁ (Warning Recv'd)</th>
<th>Marginal Probabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>φ₀ (No Fatal Tornadoes)</td>
<td>.9883</td>
<td>2.8 x 10⁻³</td>
<td>.9911</td>
</tr>
<tr>
<td>φ₁ (Fatal Tornado)</td>
<td>7.03 x 10⁻³</td>
<td>1.87 x 10⁻³</td>
<td>.0089</td>
</tr>
<tr>
<td>Marginal Probabilities</td>
<td>.99536</td>
<td>4.64 x 10⁻³</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>B. Receiver Rented (e₁)</th>
<th>z₀ (No Warning Recv'd)</th>
<th>z₁ (Warning Recv'd)</th>
<th>Marginal Probabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>φ₀ (No Fatal Tornadoes)</td>
<td>.9836</td>
<td>7.5 x 10⁻³</td>
<td>.9911</td>
</tr>
<tr>
<td>φ₁ (Fatal Tornado)</td>
<td>3.9 x 10⁻³</td>
<td>5.0 x 10⁻³</td>
<td>.0089</td>
</tr>
<tr>
<td>Marginal Probabilities</td>
<td>.9875</td>
<td>1.25 x 10⁻²</td>
<td></td>
</tr>
</tbody>
</table>

1/ The numbers presented in this report were rounded only at the end of the computation. Rounding at each step will, of course, result in minor variations from these results.
Table V-11
Posterior Probabilities

A. No Receiver Rented (e₀)

<table>
<thead>
<tr>
<th>Probability of:</th>
<th>( \phi_0 ) (No Fatal Tornadoes)</th>
<th>( \phi_1 ) (Fatal Tornado)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( z_0 ) (No Warning Received)</td>
<td>.99293</td>
<td>( 7.06 \times 10^{-3} )</td>
</tr>
<tr>
<td>( z_1 ) (Warning Received)</td>
<td>.5977</td>
<td>.4023</td>
</tr>
</tbody>
</table>

B. Receiver Rented (e₁)

<table>
<thead>
<tr>
<th>Probability of:</th>
<th>( \phi_0 ) (No Fatal Tornadoes)</th>
<th>( \phi_1 ) (Fatal Tornado)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( z_0 ) (No Warning Received)</td>
<td>.99604</td>
<td>( 3.96 \times 10^{-3} )</td>
</tr>
<tr>
<td>( z_1 ) (Warning Received)</td>
<td>.6019</td>
<td>.3981</td>
</tr>
</tbody>
</table>
The effect of the receiver is seen in the probabilities conditioned on no warning \( z_0 \). Having the receiver leads to fewer errors of the second type (i.e., failure to receive a warning) and more errors of the first type (i.e., the receipt of false warnings). From this, it is clear that in the model, the decision as to whether to rent a receiver or not hinges on the relative costs of these two types of errors.

The probabilities just calculated are shown on the appropriate branches of the decision tree in Figure V-7. On the tree, we assume now that taking action \( a_1 \) results in a reduction of the probability of a death in the family by 90%, i.e.,

\[
\Pr(\phi_1 | \theta_1; a_1) = 0.1 \times \Pr(\phi_1 | \theta_1; a_0)
\]

for either decision \( e_i \) about the receiver. This is an arbitrary assumption that accounts for the motivation to take action while recognizing that such action is not a perfect guarantee against disaster.

6. **Payoffs**

With all of the probabilities calculated, the only thing that remains is the calculation of the payoffs on the decision tree. The first step is determining the equivalent daily rental for a receiver (Table V-12). This is done to facilitate the comparison between the payoffs, which occur daily and the experiment cost. Using an 8% rate of discount, \(^1/\) the factor by which to multiply the $15 receiver cost is calculated as:

\(^1/\) Again, the choice of a discount rate depends to a large degree on the particular problem or administrative guidelines. The 8% used here is designed to reflect a private individual's discount rate (after tax) in his cost benefit calculus.
Figure V-7
Probabilities of Following Alternative Paths Emanating From the Chance Nodes

---

Notes:

1/ From Table V-10.
2/ From Table V-11.
3/ From Table V-7 and assumption of 90 percent reduction in probability of death when action is taken.
\[ C = 15 \left( \frac{i}{1 - \left( \frac{1}{1 + i} \right)^n} \right) \]

where \( i \) is the daily equivalent of 8\% and \( n \) is the life of the receiver in days. (In this case, 2,555 days.) No allowance for maintenance has been included in the rental. Thus, our value of $0.0077 per diem underestimates the cost that would be incurred.

Table V-12

Receiver Cost (@ 8%/Year)

= \( 0.000512 \times 15 \) = $0.0077/day

7 years,
365 days

The action cost has already been arbitrarily set at $1. While some may feel that really there is no cost to taking action, observation of people's behavior in the face of disaster warnings again tends to suggest that there must be at least a minimal cost, for often no action is taken. If life were valued at all, rational behavior would always suggest costless action be taken when the probability of death is positive.

Assignment of costs to changes in survival probabilities is by far the most controversial step in the analysis. However, a recent empirical study by two economists provides us with a usable estimate that can be justified on theoretical grounds. The two economists, Richard Thaler and Sherwin Rosen [11], use data from the labor market to estimate the risk premium that workers in "risky" jobs require in order to be induced to enter the risky occupation.
This is more satisfying since people often make decisions over probability distributions that only alter their risks marginally. The number they arrive at is $176 per .001 change in the probability of death. \(^1\)

Thaler and Rosen's interpretation of this result is as follows:

Suppose 1,000 men are employed on a job entailing an extra death risk of .001 per year. Then, on average, one man out of 1,000 will die during the year. The regression indicates that each man would be willing to work for $176 per year less if the extra death probability were reduced from .001 to .0. Hence, they would together pay $176,000 to eliminate that death: the value of the life saved must be $176,000. Furthermore, it must also be true that those firms actually offering jobs involving .001 extra death probabilities must have to spend more than $176,000 to reduce the death probability to zero, because there is a clear-cut gain from risk reduction if costs were less than this amount.

If the figure is adjusted to 1976 dollars, the result is approximately $300,000. Incorporating this figure into our statistical decision framework is equivalent to assessing the changes in risk associated with various decisions and then checking to see whether or not the costs of the decision is greater than the individual's revealed willingness-to-pay. Therefore, our analysis uses these estimates on the individual's willingness-to-pay for small increments in the probability of survival adjusted to be comparable with the state of nature, i.e., death. For decisions that make marginal changes in the probability of survival, we also need not worry about utility functions since they have been incorporated by the estimates of willingness-to-pay.

In order to make the computations easier we will truncate the tree before the final node in the following way. Since the decision maker

\(^1\) In 1967 dollars.
maximizes expected value, the value at the final node (before the arcs indicating the number of deaths in the family) can be calculated from the expected lives lost multiplied by the value of increased survival probability. Letting $E_N$ be the expected value at the last node, we know,

$$E_N = \sum_{k=1}^{M} \Pr(m = k|\theta_1)(\$300,000)(k) = \$300,000 \sum_{k=1}^{M} k\Pr(m = k|\theta_1)$$

But the summation has already been calculated and is 1.0001. Therefore;

$$E_N = \$300,030$$

7. **Decision Computation**

With the information on probabilities and payoffs, we are now ready to proceed with the analysis. The decision maker's decision tree is presented in Figure V-8. As described in Section B, we begin on the right side of the tree and work backwards. We first calculate the expected loss given the decision on the receiver, the receipt or nonreceipt of a warning, and the decision on taking preventative action. Comparing the two cases where no warning had been received ($z_0$) and no action taken ($a_0$), the effect of the 100 percent "coverage" of the demutable receiver is apparent. Without such a receiver, the expected loss is -$.006$ while with the receiver the loss is -$.0034$. 
Figure V-8
The Decision Computation for the EMV'er

- Decision Cost
- Decision Node
- Chance Node
Moving one step back, we determine the optimal decision on taking action by minimizing the expected loss of that decision. To the values calculated in the previous paragraph, we add $1 if the decision to take action \( (a_1) \) is made. From Figure V-8, we see that if a warning is received \( (z_1) \), it is never optimal to take preventative action. (The double bar through an arc indicates a nonoptimal decision.)

Because the receipt of the warning message is probabilistic, we next calculate the expected loss given the receiver decision. Here, we have what may be a surprise -- the expected value is the same regardless of the receiver decision. The reason, naturally, is to be found at the action decision. Since receipt of the warning did not lead to any change in behavior, it's to be expected that the implicit value of the receiver is zero.

In the final (and now seen to be unnecessary) step, the receiver costs are added which makes the decision to rent the receiver more costly (in terms of expected losses) than the decision not to rent the receiver. 1/

E. Sensitivity Analyses

1. Results of the Sensitivity Analyses

The results of the analysis in the previous section indicate that a demutable receiver which receives information on warning alone (or over which other information, valueless to the decision-maker, is received), will not be purchased by our representative decision-maker.

1/ The fact that the receiver cost is the same as the expected values given the receiver decision is only coincidental.
In the analysis, many assumptions were made that directly affect the decision. In this section, we address the sensitivity of our results to the following assumptions:

- The size of radio and television audience that may receive any warning issued
- The failure-to-warn probabilities
- The false alarm probabilities
- The assumed value associated with increased survivability
- The number of individuals in a household
- The reduction in losses from taking action
- The revised probability given at least one family death
- The cost of action
- The cost of a demutable receiver
- The population assumed to be subject to tornadoes

Table V-13 presents the results of these analyses. Column A is the variable being tested. Column B is the value used in the base case reported in the previous section (and used for the other sensitivity analyses and Column C contains the new value (both high and low). The effect on the action decision (both with and without the receiver) is shown in Column D while the result on the receiver decision is in Column E.

1/ The perfect forecast test means both no cases of "failure to warn" or of "false alarms."

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Table V-13
Results of Sensitivity Analyses

<table>
<thead>
<tr>
<th>A Variable</th>
<th>B Base Case</th>
<th>C New Value</th>
<th>D Action No Receiver</th>
<th>E Decision With Receiver</th>
<th>F Receiver Decision</th>
</tr>
</thead>
<tbody>
<tr>
<td>Audience Size</td>
<td>0.369</td>
<td>0.75</td>
<td>None</td>
<td>None</td>
<td>Do Not Rent</td>
</tr>
<tr>
<td>Failure to Warn Probability</td>
<td>0.44</td>
<td>0.01</td>
<td>None</td>
<td>None</td>
<td>Do Not Rent</td>
</tr>
<tr>
<td>False Alarm Probability</td>
<td>7.5 x 10^-3</td>
<td>0</td>
<td>None</td>
<td>None</td>
<td>Do Not Rent</td>
</tr>
<tr>
<td>Perfect Forecasting</td>
<td>N.A.</td>
<td>(see text)</td>
<td>None</td>
<td>None</td>
<td>Do Not Rent</td>
</tr>
<tr>
<td>Value Associated with Increased Survival Probability</td>
<td>$300,000</td>
<td>100,000</td>
<td>None</td>
<td>None</td>
<td>Do Not Rent</td>
</tr>
<tr>
<td>Family Size</td>
<td>4</td>
<td>6</td>
<td>None</td>
<td>None</td>
<td>Do Not Rent</td>
</tr>
<tr>
<td>Reduction in Losses From Taking Action</td>
<td>90%</td>
<td>50%</td>
<td>None</td>
<td>None</td>
<td>Do Not Rent</td>
</tr>
<tr>
<td>Expected Lives Lost Given At Least 1 Death</td>
<td>1.0001</td>
<td>1.000</td>
<td>None</td>
<td>None</td>
<td>Do Not Rent</td>
</tr>
<tr>
<td>Action Cost</td>
<td>$1</td>
<td>$10</td>
<td>Take Action</td>
<td>Take Action</td>
<td>Do Not Rent</td>
</tr>
<tr>
<td>Receiver Cost</td>
<td>$15</td>
<td>$5</td>
<td>None</td>
<td>None</td>
<td>Do Not Rent</td>
</tr>
<tr>
<td>Population At Risk</td>
<td>26 million</td>
<td>30 million</td>
<td>None</td>
<td>None</td>
<td>Do Not Rent</td>
</tr>
</tbody>
</table>
As is evident from Table 11, the decision maker generally neither takes action nor rents the receiver. (Obviously, if he never takes action, it will not be in his interest to rent the receiver.) This suggests that the problem, in the sensitivity analysis here, is that the cost of action is the stumbling block (or, at least, the first of two). Therefore, a new set of sensitivity analyses were developed under the assumption that action costs nothing. The alternatives were not so extensive since we are no longer are concerned with the values for assumptions that only make the decision less favorable. We have also consolidated the false alarm and failure to warn alternatives and consider only the perfect forecasting option. In addition, we consider the effects when the expected lives lost is changed or the receiver cost is varied.

The results from this new set of analyses as shown in Table V-14 indicate that only in the case of the revised probability of an individual's death, given that at least one in the household is dead, is one. That is, when the expected number of lives lost (given that at least one lost) is exactly four, the receiver is rented. (Naturally, action is always taken when a warning is issued because that action is now considered "free".)

**Table V-14**

**Sensitivities Assuming Action Costs Nothing**

<table>
<thead>
<tr>
<th>Alternative</th>
<th>Value</th>
<th>Receiver Decision</th>
</tr>
</thead>
<tbody>
<tr>
<td>Perfect Forecasting</td>
<td>N. A.</td>
<td>Do Not Rent</td>
</tr>
<tr>
<td>Expected Lives Lost</td>
<td>4</td>
<td>Rent</td>
</tr>
<tr>
<td>Receiver Cost</td>
<td>$5</td>
<td>Do Not Rent</td>
</tr>
</tbody>
</table>
The net result of these sensitivity analyses is that given the assumptions and the data that were used in this chapter the demutable home receiver, providing tornado warnings only, would reach a small percentage of the total national market. We must repeat here that this analysis was designed to illustrate the method and not to be used as accurate estimate of the market for home receivers. Clearly, as the analysis in this part has shown, the method can be applied to this type of problem. In the following section, we examine another study related to our results to see if the method we have suggested has implications that are compatible.

2. Comparison With Other Results

The Opinion Research Corporation (ORC) conducted a survey [7] to determine the market for home warning devices. Using a market survey approach, they found:

As the detailed tabulations show, overt consumer interest in the various options, at or exceeding expected retail price levels, ranges from 12 percent of U.S. households for the basic automobile warning receiver to 21 percent of U.S. households for the home disaster/attack option (May 1974). Expressed interest in other options ranges from 17 percent to 21 percent (statistically similar within survey sampling tolerances).

It would, however, be foolhardy to accept this strong level of demand as manifest. As pointed out by Siri Nehevaajs, consumer demand measured through a personal or telephone interview possesses many of the characteristics of direct, individualized sales methods.

When a product such as the contemplated warning receiver is introduced, it is highly unlikely that such direct one-on-one techniques will be used...

1/ In [7], p. 10.
Viewed another way, the lowest percentage of respondents who indicate that they are very likely to buy a receiver is the 6 percent responding in this manner to the basic automobile device. Those stating that they are very likely to buy the receiver are at the 9 to 17 percent level for all home warning device options, and 14 percent for the automobile device with the accident/hazard option.

One interesting implication of the ORC study is that their addition of a non-natural disaster warning feature (in this case, warning of poor road conditions) significantly increases the market for the automobile receiver. If we consider the decision tree developed in this example again and visualize the addition of branches to indicate the provision of information of higher probability, lower cost events (such as poor road conditions) it is likely that the receiver may become worth the cost for a larger market.

F. Estimating the Demand Curve

Using the statistical decision theory approach, any one individual will, of course, decide either to buy or not to buy. Therefore, attempts to apply directly the results of our analysis in this part to the population would necessarily result in a conclusion that either the entire population would rent (purchase) the receiver or none would. Given the variation in human tastes for safety and the less than perfect market for tornado safety that currently exists, both of these extremes appear unlikely.

Such variations in taste can be used to estimate a demand curve for home receivers with the aid of the decision theory approach described in this chapter. This section illustrates the derivation of a demand curve for home receivers based on the variation in tastes for risks as expressed by individuals' willingness-to-pay to increase their
probability of survival. From the discussion of the Thaler and Rosen article [11] in section D.6 above, we know that there is a simple one-to-one relationship between this willingness-to-pay and the "value of life." Specifically, because of the way the Thaler and Rosen model was estimated, the value of life is one thousand times the willingness-to-pay figure. Thus, the $300,000 value of life used in the example of section D corresponds to a willingness-to-pay of $300 for an increase in the survival probability of .001. In this section, we continue using the "value of life" figure for consistency but the proper interpretation is in terms of willingness-to-pay.

With all other assumptions remaining the same as they were for the example analysis of section D, it can be shown \(^1\) that the minimum value of life required to make the individual just willing to buy the receiver (which is assumed to cost $15) is $2 million. Therefore, if this were the only characteristic that varied among individuals, all individuals who value their lives at more than $2 million would buy the receiver.

As the price of the receiver is allowed to vary, this minimum value of life will also change. For example, if the price of the receiver is lowered, it is reasonable to believe that it will take less (in terms of value of life) to induce the individual to acquire the receiver. We can use the model developed in section D to find the minimum value of life required to induce purchase for various receiver prices. In Table V-15, the minimum values of life associated with several different receiver prices are presented.

\[^1\] Using the decision analysis just illustrated.
Table V-15

Relationship Between Price of Home Receiver
and Willingness-to-Pay to Increase Probability of Survival

<table>
<thead>
<tr>
<th>Receiver Price</th>
<th>Minimum Values-of-Life ($M)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$15.00</td>
<td>$2.0</td>
</tr>
<tr>
<td>10.00</td>
<td>1.7</td>
</tr>
<tr>
<td>7.50</td>
<td>1.5</td>
</tr>
<tr>
<td>5.00</td>
<td>1.3</td>
</tr>
<tr>
<td>3.00</td>
<td>1.2</td>
</tr>
<tr>
<td>2.00</td>
<td>1.1</td>
</tr>
</tbody>
</table>

We observe that, as expected, at lower prices the minimum value of life required for receiver purchase is also lower. Now, since these are minimum values of life, individuals with a value of life sufficiently high for them to buy a receiver at, say, $15 will buy one at all lower prices. This suggests that in order to derive a demand curve for home receivers, there must be some way to estimate the proportion of households with sufficiently high values of life at different receiver prices.

One method that can be used for this procedure is to consider the probability distribution that depicts the variation in people's tastes toward risk. For example, suppose that the underlying probability distribution is of the form shown in Figure V-16. The function Pr(v) is the probability density function that indicates the probability that an individual has a value of life in some interval. For example, the probability that the value of life, say Pr(v*), is between \( v_1 \) and \( v_2 \) is the integral,

\[
Pr(v^*) = \int_{v_1}^{v_2} Pr(v)dv
\]
Figure V-16

Estimating the Proportion of Households Acquiring Receiver Given the Value of Life

Now, because this is a probability distribution, we know that \( \int_{-\infty}^{\infty} \Pr(v) dv = 1 \)

Now, let \( v_p \) represent the minimum value of life required to induce an individual to purchase a receiver when the receiver price is \( p \). Then the probability that a random individual will have a value of life in excess of this minimum is just

\[
Pr(v_p^*) = \int_{v_p}^{\infty} Pr(v) dv
\]

which is the shaded area in Figure V-16. Thus, the proportion of households that would acquire the receiver is, according to this model, just

\[
Pr(v_p^*) .
\]

---

1/ Although the probability density in Figure V-16 starts at 0, we can assume that \( Pr(v) = 0 \) for \( v < 0 \).
The next step is to determine the proper probability function to use in the analysis since, to our knowledge, there have been no empirical studies of distaste toward risk. However, the following characteristics of such a probability distribution seem reasonable. First, the domain of the function is the positive real line. This assumes that no one is willing to pay to be exposed to a fatal tornado. The only other piece of information that we "know" is the average value of life as estimated by Thaler and Rosen [11]. With only these two bits of information, Tribus [12] has shown that the "maximum entropy" probability distribution is the exponential. The reason for using a maximum entropy distribution is that it incorporates the knowledge we do have while otherwise allowing for the most uncertainty about other information.\(^1\)

The exponential probability distribution has the form,

\[ p(x) = \lambda e^{-\lambda x} \quad \lambda, x \geq 0 \]

and is the distribution illustrated in Figure V-16 above. The mean of the distribution is \(1/\lambda\) and in our example the parameter \(\lambda\) is the reciprocal of the "average" survival probability value. Although we used $300 in our example, this is biased downward since the sample will have more "adventurous" individuals in it. Therefore, we assume that the average value is $500 so that \(\lambda = 2\) (in thousands of dollars). Then, the probability of an individual having a value of life greater than $2,000 is

---

\(^1\) The interested reader is referred to Tribus [12] for a formal discussion of the principle of maximum entropy. The idea, however, is easy to understand. Suppose we know only a few things about the random variable described by the distribution. (For example, in this case we know only the domain of definition and the mean value.) We would like to select a probability distribution that reflects our lack of knowledge by being as "spread-out" as possible. Tribus shows that a quantity called the entropy, measures how spread-out a probability distribution is. Thus, a "maximum-entropy" distribution has the highest level of entropy consistent with our knowledge. If we know only the domain and mean, as is the case here, then the exponential distribution has the maximum entropy.
\[
\Pr(x \geq 2) = \int_{2}^{\infty} e^{-\lambda x} \, dx = \left[ -e^{-\lambda x} \right]_{2}^{\infty} = \frac{e^{\lambda}}{2} = 0.0183
\]

Thus, an estimate of the percentage of households with a willingness-to-pay for increasing survival probability sufficiently great to induce them to buy a $15 home receiver is 1.83%. (Recall that in this example the only disaster considered is a tornado. With the addition of other disasters and local continuous weather forecasts, the estimate of the market would be greater.) For each of the values in Table V-15, we can compute a similar figure and the results are shown in Table V-17.

**Table V-17**

Estimated Market For Home Receivers at Different Prices

<table>
<thead>
<tr>
<th>Price</th>
<th>Market (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$15.00</td>
<td>1.83</td>
</tr>
<tr>
<td>10.00</td>
<td>3.33</td>
</tr>
<tr>
<td>7.50</td>
<td>4.98</td>
</tr>
<tr>
<td>5.00</td>
<td>7.43</td>
</tr>
<tr>
<td>3.00</td>
<td>9.07</td>
</tr>
<tr>
<td>2.00</td>
<td>11.08</td>
</tr>
</tbody>
</table>

(Note that the market consists of households in the twenty-one state area defined in Table V-5.) A demand curve is then estimated from the data in Table V-17. The estimated demand curve (and the associated consumers' surplus indicated by the shading) is shown in Figure V-17. The functional form of the demand curve illustrated in Figure V-17 is:

\[ N = 0.2381p^{-0.865} \]
where \( N \) is the fraction of households that will purchase a receiver when the price is \( p \). Using the same procedures described in Chapter IV, the annual consumers' surplus per household (assuming immediate system activation and immediate market penetration) is

\[
CS = \int_{15}^{100} \left( \frac{.08}{.2381} \right) p^{-1.865} \frac{c}{p} \, dp
\]

\[
= (.019048) \left[ \frac{p}{.135} \right]_{15}^{100}
\]

\[
= .019048 \left( 13.793238 - 10.676737 \right)
\]

\[
= .019048 \left( 3.116501 \right)
\]

\[
= .059363
\]

Total annual consumers' surplus is then found by multiplying the value per household by the number of households in the market. The number of households was estimated by using the ratio of population in the 21 states (76 million) to total population in 1945 \(^1\) (133 million) and Rosen and Haimes figure of 70 million households in the U. S. Then total annual consumers' surplus is

\[
CS = .059363 \times 70 \times 10^6 \times \frac{76}{133} = 2.37 \times 10^6
\]

Over a twenty year period, total consumers' surplus is

\[
CS = 2.37 \times 10^6 \int_{0}^{20} e^{-0.08t} \, dt
\]

\(^1\) The reason for using 1945 is discussed on page V-32.
\[ = 2.37 \times 10^6 \left( \frac{-e^{-0.08t}}{0.08} \right)^{20} \]

\[ = 2.37 \times 10^6 (9.976294) \]

\[ = $23.69 \times 10^6 \]

Incorporation of assumptions about system activation and market penetration can now be performed in exactly the same manner as in Chapter IV.

G. Remarks and Extensions

In this chapter we have presented a method that appears to suitable for analyzing the decision process of the customers of the information transmitted over a DWS. Further, we applied this method to actual data (where available) to examine the implications. Finally, we compared the results with other information on the market for DWS receivers and safety.

Two observations can be made as a result of this analysis. First, the statistical decision theory methodology can be successfully applied to the disaster warning receiver problem. Second, the results obtained are not contradicted by existing studies of related problems.

From the existing studies, two further remarks appear warranted. First, a fruitful avenue of investigation is a determination of other information that could be transmitted over the DWS (broadly conceived) and which is valued by the consumer. \(^1\) Second, the

\(^1\) A related problem is the determination of specific groups that might face sufficiently different circumstances that purchase of a receiver is worthwhile. Such a group might be the rural population in poor coverage areas.
incorporation of information concerning the distribution of individual characteristics (e.g., willingness-to-pay for safety) will provide a more detailed assessment of market potential (and benefit/cost considerations). Both of these should be investigated in any application of this methodology.

Finally, it should be observed that there are other disasters for which an individual may desire a means of obtaining warnings. Table V-17 lists property and life losses associated with three common types of disasters, tornados, floods and hurricanes. Of the three, tornados appear to be associated with the greatest loss of life (with a few exceptions). Floods and hurricanes, however, represent more extensive phenomena, that affect larger areas. A tornado appears, strikes, and is over quickly while floods (except flash floods) and hurricanes develop more slowly.

Table V-17

Statistics on Tornadoes, Floods, and Hurricanes

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Tornadoes, number</td>
<td>1,514</td>
<td>2,983</td>
<td>6,081</td>
<td>4,298</td>
<td>604</td>
<td>549</td>
<td>555</td>
<td>74</td>
</tr>
<tr>
<td>Lives lost, total</td>
<td>1,506</td>
<td>1,753</td>
<td>2,212</td>
<td>647</td>
<td>48</td>
<td>73</td>
<td>153</td>
<td>25</td>
</tr>
<tr>
<td>Most in single tornado</td>
<td>316</td>
<td>283</td>
<td>283</td>
<td>153</td>
<td>22</td>
<td>18</td>
<td>29</td>
<td>4</td>
</tr>
<tr>
<td>With property loss of $500,000 and over</td>
<td>56</td>
<td>130</td>
<td>174</td>
<td>156</td>
<td>19</td>
<td>15</td>
<td>33</td>
<td>20</td>
</tr>
<tr>
<td>Floods:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lives lost</td>
<td>583</td>
<td>508</td>
<td>587</td>
<td>603</td>
<td>297</td>
<td>135</td>
<td>74</td>
<td>(NA)</td>
</tr>
<tr>
<td>Property loss</td>
<td>1,484</td>
<td>3,350</td>
<td>2,721</td>
<td>2,247</td>
<td>903</td>
<td>225</td>
<td>228</td>
<td>(NA)</td>
</tr>
<tr>
<td>North Atlantic tropical cyclones and hurricanes:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number reaching U.S. coast</td>
<td>41</td>
<td>40</td>
<td>33</td>
<td>34</td>
<td>3</td>
<td>14</td>
<td>8</td>
<td>3</td>
</tr>
<tr>
<td>Hurricanes only</td>
<td>19</td>
<td>21</td>
<td>14</td>
<td>20</td>
<td>2</td>
<td>9</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>Living lost in U.S.</td>
<td>768</td>
<td>493</td>
<td>692</td>
<td>356</td>
<td>226</td>
<td>11</td>
<td>8</td>
<td>12</td>
</tr>
</tbody>
</table>

NA: Not available.

1 Tropical cyclones have maximum winds of 39 to 73 miles per hour; hurricanes have maximum winds of 74 miles per hour or higher.


Source: [14].
This suggests that in a comprehensive study of demand for home receivers using decision theoretic approaches, some modification of the decision tree may be necessary to take account of the differences in disasters. For example, with a hurricane sufficient warning time is almost always available. However, an individual may be unsure of the exact extent, location and time of the disaster. Therefore, the use of a home receiver may allow for postponement of an action decision (the individual can remain at home for a longer time) because he is "certain" to receive the latest forecast. The decision tree may then have "action" nodes that reflect such a postponement leading, ultimately, to lower action costs. Thus, for slowly developing disasters such as hurricanes, the value of a home receiver may be more in the area of better planning capability and not in the reduction of disaster losses.
In this appendix, we indicate first the theoretical foundations for employing specific utility functions and then show how the utility functions can be applied to calculate revised payoffs. We assume that the only impact of a disaster is on financial wealth. In order to minimize the burden on the reader's part, we use exactly the same model and assumptions as above but we now assume that $50,000 is the total financial wealth of the household and that 10 percent of that wealth will be lost in the event a disaster strikes. Thus, we assume that the payoff associated with a disastrous outcome (call it $\theta_1$) is $5,000.

The next step in the analysis is to determine an appropriate utility function with which to compute the utility equivalents of the experiment costs, the action costs, and the payoffs. In choosing a utility function for use in this analysis two criteria become important: (1) the choice of a functional form, and (2) the "reasonableness" of the parameters in the function.

As far as the functional form, utility functions can be generally described in terms of their first two derivatives. Thus, a reasonable restriction on the form is that:

$$U'(w) > 0$$

i.e., the marginal utility of wealth is positive. Simply put, each individual is assumed to prefer more to less wealth. Absolute risk aversion can be defined at a given wealth, w, as:

$$r(w) = - \frac{U''(w)}{U'(w)}$$

Positive risk aversion is assumed to obtain in this analysis which implies $U'(w) < 0$. Pratt [6] also defines proportional (or relative) risk aversion,
Two distinct classes of functions are often used in discussions of behavior under risk. These are constant (absolute) risk aversion and constant relative (proportional) risk aversion. These two concepts can be easily illustrated with the following examples. Let the degree of risk aversion be represented by the premium an individual is willing to pay to avoid a risky situation (the $P$ of Figure V-5 in Section C). Then, with constant absolute risk aversion and for risky situations involving an absolute wealth change (e.g., $100), the risk premium is unchanged regardless of the individual's wealth. Constant relative risk aversion implies that risk premium will be constant for risky situations involving constant proportional wealth changes (e.g., 10 percent of total wealth). Generally, decreasing absolute risk aversion is taken to be a desirable characteristic, since it is believed that such behavior is the rule.

The form of the utility function we use in this example application is a modification of a function used by Malkiel and Quandt [4]. Thus, we assume

\[ U(x) = \frac{1 - e^{-sx}}{e^s} + x, \quad s > 0 \]

where $x$ represents relative wealth loss. The utility function has been normalized so that

\[ U(0) = 0 \]

and

\[ U(-1) = -1 \]
Taking the appropriate derivatives,

\[ U'(x) = \frac{s e^{-sx} + 1}{e^s} \]

and

\[ U''(x) = \frac{-s^2 e^{-sx}}{e^s} \]

Thus

\[ r(x) = -\frac{U''(x)}{U'(x)} = \frac{s e^{-sx}}{s e^{-sx} + 1} \]

To see that this function exhibits decreasing absolute risk aversion, we calculate

\[ r'(x) = \frac{-s^3 e^{-sx} (s e^{-sx} + 1) + s^2 e^{-sx} (s e^{-sx})}{(s e^{-sx} + 1)^2} \]

\[ = \frac{-s^3 e^{-sx}}{(s e^{-sx} + 1)^2} < 0. \]

Given that the form of the utility function is reasonable, what can we say about the value of the parameter \( s \) that we use in deriving actual payoffs? Although our choice of 2.3 as the value of \( s \) in the utility function is due to Malkiel and Quandt [4], our functional form has been modified from theirs to obtain to be relevant in the range from \((-1, 0)\). Their use of 2.3 results (apparently) not from empirical work but from some other considerations not stated. (Others have attempted to estimate utility functions but these have been generally defined on consumption and not on wealth.) The sensitivity of the results to the particular choice of a para-
meter is lessened substantially by our use of relative wealth changes instead of absolute changes. Since all numbers are in the range -1 to 0, the results are all of comparable magnitude and depend directly on the degree of curvature (the parameter, s). The sensitivity of the results could be tested directly by varying the value used for s.

With the utility function completely specified, we now can compute the utility equivalents for each of the costs associated with the decision problem. These are shown in table A-1. With these numbers now in the relevant portions of the decision tree, the computation of the optimal decision proceeds in exactly the same way as for the EMV case. (We are not concerned here with the actual decisions since the preventable property damage in the case of tornadoes is quite small.)

Table A-1
Costs and Payoffs for a Risk Averter

Utility Function:

\[ U(w) = \frac{1 - e^{-sx} + x}{e^s} \]

\( w = \text{loss/total wealth} \)

\( s = 2.3 \)

(1) Receiver cost: \(-5.09 \times 10^{-8}\)

(2) Action cost: \(-6.62 \times 10^{-6}\)

(3) Payoffs: \((e_i, z_i, \theta_i) = -3.06 \times 10^{-2}\)
BIBLIOGRAPHY


14. ______, NOAA; *Climatological Data, National Summary* V26 (13) 1975.