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F. W. STECKER
F. C. JONES

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GODDARD SPACE FLIGHT CENTER
GREENBELT, MARYLAND

THE GALACTIC HALO QUESTION:
NEW SIZE CONSTRAINTS FROM GALACTIC γ-RAY DATA

F. W. STECKER

F. C. JONES

Theoretical Studies Group
NASA/Goddard Space Flight Center

Abstract:

The recent acquisition of data on the distribution of galactic γ-ray emission has made possible a reconsideration of the long-standing controversy concerning the existence and extent of the galactic halo. Analysis of the implications of the SAS-2 γ-ray data, making use of recent CO line emission and other data for determining the large-scale distribution of galactic gas, implies that there is a nonuniform distribution of cosmic rays in the galaxy. This fact rules out large trapping halo models and particularly the recently proposed closed halo models in the same way that it rules out extragalactic origin models.

We also consider detailed models of diffusion halos of various sizes perpendicular to the galactic plane. In such models, the scale perpendicular to the plane has a strong effect in determining the radial distribution of cosmic rays. Such radial distributions are calculated for cylindrical coordinate models. The implied γ-ray longitude distributions are then calculated and compared with the SAS-2 data for goodness-of-fit. Assuming the sources to be supernova remnants or pulsars, cosmic ray nucleon halo models with scale heights greater than 3 kpc are found to provide a poor fit to the γ-ray longitude data (probability of 6% or less). Thin halo, or source dominated diffusion models are found to provide a good fit to the γ-ray data, with an upper limit scale height of $\sim$ 3 kpc.

Consideration of the γ-ray latitude data gives a half thickness L_e of 2 $\pm$ 2 kpc for the cosmic ray electron halo. A best-fit value of the product $L_e\ (I_e/I_\theta)$ at the galactic center, based on an estimate of the Compton production in the central region gives $[L_e(I_e/I_\theta)]_{G.C.} = 0.5$ kpc.

I Introduction:

The concept of a large quasispherical region of galactic cosmic rays surrounding our Galaxy, i.e. a "halo" or "corona," has become the subject of much controversy in the last decade. Most of the discussion has centered upon the question of the existence of an extensive halo of galactic radio emission with some discussion centering upon the question of the effective "lifetime" or trapping time of cosmic rays in the galaxy.

Shklovskii (1952) showed that the observed high latitude distribution of nonthermal radio emission could be naturally explained by postulating an extensive halo of radio emissivity surrounding the galaxy. Pikel'ner (1953) suggested that there should exist an accompanying gaseous halo and magnetic field of $\sim 3\times10^{-6}$ G in order to retain the cosmic rays producing the radio emission. Observations by Baldwin (1955) appeared to confirm the existence of an extensive radio halo, this halo having a radial dimension of the order of 20 kpc (Baldwin 1967). A somewhat flattened halo model with an axial ratio of $\sim$1.5 was suggested by Mills (1959). Biermann and Davis (1958) pointed out that an extensive cosmic ray halo would naturally account for the observed isotropy of cosmic rays. An electron intensity for the halo similar to that in the galactic disk was deduced by Felten (1966). However, the existence of the radio halo was already being questioned by Burke (1967). Some of the early radio work may have been affected by sidelobe contamination and the additional effects of radio spurs and loops (nearby features), which became apparent with higher resolution. An extragalactic background also complicates the analysis. More recent analyses have questioned the existence of an extensive halo (Wielibinski and Peterson 1968, Yates 1968, Razin 1971, Ilovaisky and Lequeux 1972, Price 1974), or favored a weak non-confining halo (Webster 1975) or a very thin halo or thick disk of a few kpc extent (Illovaisky and Lequeux 1972, Baldwin 1976). The opposite point of view in support of an extensive radio halo has been taken by Daniel and Stephens (1975 and references therein) and Bulanov Syravatskii and Dogiel (1976) and Ginzburg and Ptuskin (1976 and references therein, see also Ginzburg and Syrovatskii 1964).

There have also been searches for radio halos associated with edge-on spiral galaxies. Baldwin and Pooley (1973, see also Van der Kruit and Allen 1976) have found that a flat halo (or flat disk) of half thickness $\sim$ 2 kpc around NGC 891. Ekers and Sancisi (1977) have found a more extensive but still flattened halo around NGC 4631, however, these authors point out that NGC 4631 may not be a good analogue of our Galaxy since (1) it has an unusually bright central emission region and (2) it is surrounded by large concentrations of HI and (3) it has a nearby disturbed companion galaxy. Searches for halos in other edge-on spirals have so far given no positive results (e.g. Mills 1967) and the question of the existence of a radio halo around M31 has been clouded in interpretational difficulties (van der Kruit and Allen 1976). Further complicating the situation is the fact that the lack of an extensive radio halo is not a conclusive argument against the existence of a large halo in which cosmic rays are confined (Ginzburg and Ptuskin 1976).

Measurements of the ratio of Be/B and the fraction of ^{10}Be in the cosmic radiation have been made in order to determine the age of the cosmic rays from the survival of ^{10}Be produced by cosmic ray spallation (Webber et al 1973, O'Dell et al 1973, Hagen, Fisher and Ormes 1977, Garcia-Munoz 1975). Studies of cosmic ray secondaries (spalled nuclei and positrons) indicate that cosmic rays travel on the average through about 5 g/cm^2 of matter (Shapiro, Silberberg and Tsao 1975) A cosmic ray age of $\tau \lesssim 10^7$ yr would imply that the cosmic rays have traveled through a medium of average atomic density $\gtrsim 0.4$ cm^{-3} and therefore favor a disc containment model or one in which cosmic rays diffuse rapidly in the halo so that once they leave the disk have little chance of returning in a diffusion model. On the other hand a determination of $\tau \gg 10^7$ yr would favor containment in an extensive halo ($n \sim 10^{-2}$ cm^{-3}). The measurements quoted above appear as yet to provide an inconclusive or borderline test as the range of overlap at present appears to be $\sim (1\text{-}2) \times 10^7$ yr. The data of the Chicago group (Mason 1977) at energies of $\sim$ 100 MeV/nucleon give an age of $\sim 1.7 \times 10^7$ yr from ^{10}Be measurements while the age in the range of several GeV/nucleon (more typical of "cosmic rays" and also the range which is important for π-decay γ-ray production (Stecker 1973)) is given by O'Dell et al. (1975) to have a 1 σ upper limit of 10^7 yr.

In view of the situation with the radio data, and the additional present uncertainty regarding the age of galactic cosmic rays (O'Dell et al. 1973, Hagen et al 1975, Garcia-Munoz et al. 1975), it would seem desirable to have an independent test of the existence and extent of a cosmic ray halo. Such a test is furnished by the recent observations of the distribution of galactic 100 MeV γ-rays and their interpretation (Stecker 1976a,1977).

II. Definition of a Halo

Subsequent to their initial suggestion of a galactic halo, Pikel'ner and Shklovskii (1957, 1958, 1959) developed a model for the halo which related the spherical system of older (population II) stars, a gaseous halo, a cosmic ray halo and the radio halo. The halo of population II stars certainly exists. However, for the purposes of detailed consideration of the observational consequences and in view of the difficulties in interpretation, it may perhaps be better to separate out these various types of halo. Energy losses by synchrotron and Compton radiation deplete the cosmic ray electron component over 10^7-10^8 yr and thus shrink the observable radio halo while still allowing for an extensive cosmic ray nucleon halo (see, e.g. Bulanov, Syrovatskii and Dogiel 1976, Ginzburg and Ptuskin 1976 and references therein). Thus, evidence of an extensive radio halo may be taken as strong support for an extensive nucleon halo but the absence of such evidence may not with certainty rule out the existence of a nucleon halo. Such evidence makes cosmic ray trapping unlikely (Webster 1975) but would not rule out diffusive type models. We are faced then with a situation of uncertainty with regard to the putative nucleonic cosmic ray halo. It is a discussion of this nucleonic halo (hereafter to be called the "cosmic ray halo" or "halo") to which to restrict ourselves by addressing the γ-ray data. Since cosmic-ray nucleons do not suffer significant energy losses in a halo of fairly extensive size, evidence against an extensive cosmic-ray halo also argues against an extensive radio halo.

III. The Uniform or Closed Halo Models

Two types of halo models are usually considered, viz. (a) the uniform or closed type in which cosmic rays are trapped for long periods in the halo and can reenter the disk many times and (b) the diffusion halo. The closed halo model has been recently revived by Rasmussen and Peters (1976). The diffusion halo models have been extensively examined by various theorists at the Lebedev Institute (Bulanov, Syrovatskii and Dogiel, 1976, Ginzburg and Ptuskin 1976 and references therein). In addition, the uniform and diffusion halo models appear to have been confused in the literature in the past as has been pointed out by Ginzburg and Ptuskin (1976). The reader should keep the difference in mind in the subsequent discussion.

It has been previously noted that the SAS-2 observations of $\sim$ 100 MeV γ-radiation in the galaxy imply a nonuniform cosmic-ray distribution in the galaxy which argues against extragalactic cosmic ray origin (Stecker 1975, Dodds et al 1975, Stecker et al 1975). The implied existence of a nonuniform galactic cosmic-ray distribution puts new restrictions on halo models for propagation and containment of cosmic rays. If cosmic rays are produced primarily as a result of supernova explosions or in pulsars, the striking correlation of the galactic cosmic ray distribution and the supernova remnant distribution (Stecker 1975a) and pulsar distribution as given by Seiradakis (1976) implies source dominated diffusion (or convection, see Jokipii 1976) of cosmic rays out of the galaxy. This argues against trapping of cosmic rays in a large halo with multiple reflections of cosmic rays at the boundaries. Such trappings would produce a more uniform distribution of cosmic rays in the galaxy than the γ-ray observations imply (Stecker 1976a, 1977). The large halo model with a long ($\sim 10^8$ yr) trapping time has been advocated in the past (see, e.g. Ginzburg and Syrovatskii 1964). An attractive feature of the large halo model has been that it eases the requirements for isotropization of cosmic rays in the solar galactic neighborhood without invoking extra-galactic cosmic ray origin (Biermann and Davis 1958). Recent analysis of the high galactic latitude radio data indicates however, that the magnetic field in the halo region may not be strong enough to trap cosmic rays (Webster 1975).

The existence of any type of closed galaxy model with long term ($\gtrsim 10^8$ yr) confinement of cosmic rays is brought into question by the γ-ray results. This includes the closed galaxy model (Rasmussen and Peters 1976) where cosmic rays have undergone spallation through $X = \rho ct \sim 100$ g/cm^2. The isotopic measurements also argue against the uniform confinement halo model. We will therefore devote the rest of our discussion to an analysis of the diffusion halo models using the γ-ray data. These models appear not to be definitely ruled out by either the radio data or the age data (Bulanov, Syrovatskii and Dogiel 1976, Ginzburg and Ptuskin 1976).

IV. The Diffusion Calculation

The model we shall employ to investigate the effect of a diffusion halo on the cosmic ray distribution is often referred to as the flat diffusion model (Ptuskin 1974). In this model the cosmic-ray sources are assumed to be confined to a thin disk of half thickness L_0; the cosmic rays subsequently diffuse in a (possibly larger) disk of half thickness L ($L>L_0$). We assume cylindrical symmetry about the z axis which is perpendicular to the disk surface and the diffusing volume is taken to be of infinite extent in the radial direction r. The cosmic ray particles are free to diffuse in z and in r until they reach the surfaces $z = \pm L$ at which point they escape freely; the appropriate boundary conditions for the particle density are therefore $n(z=\pm L, r) = 0$.

In previous applications of this model (Ginzburg and Syrovatskii, 1971; Pacheco 1971, Ptuskin 1974, Bulanov et al. 1972, Guet and Pacheco 1973, Ginzburg and Ptuskin 1976 and refs.) the source is taken to be uniform for $-L_0 \leq z \leq L_0$ and for all r. We, on the other hand are specifically interested in radial variations of the cosmic ray density since it is the radial dependence of the cosmic ray flux which, when folded with the observed distribution of interstellar gas, gives the observed galactic longitude variation of the cosmic γ-ray flux.

In the following we shall assume a constant scalar diffusion coefficient for the sake of simplicity. Different values of the diffusion coefficient in the z and r direction could be simply accommodated by using a different scale for the z and r distances but since the scale factors would be proportional to the square root of the diffusion coefficient values our results will not be too sensitive to this assumption.

The diffusion equation may be written in cylindrical coordinates as

$$\frac{\partial^2 n_{cr}}{\partial z^2} + \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial n_{cr}}{\partial r}\right) = S(r,z) / D \qquad (1)$$

where S(r,z) is the source strength density, D is the diffusion coefficient and we have made use of the assumed cylindrical symmetry. Since we will ultimately normalize the solution of (1) to the observed density of cosmic ray particles at the position of the solar system (r=10 kpc) the magnitudes of S and D will not enter and we may set

$$S(r,z)/D = q(r) \quad \text{for } |z| \le L_o$$
$$= 0 \text{ otherwise}$$

where various forms of q(r) will be tried and discussed later.

In the appendix we show that the Green's function appropriate to (1) is given by

$$G(z,z';r,r') = -\frac{1}{L}\sum_{n=0}^{\infty}\cos(k_n z)\cos(k_n z')I_o(k_n r_<)K_o(k_n r_>) \qquad (2)$$

where $k_n = \frac{(2n+1)\pi}{2L}$ and $r_<$ and $r_>$ represent the smaller and the larger of the two variables r and r'.

Since we are interested only in source functions that are uniform in z between $\pm L_o$ we may integrate out the z' variable immediately to obtain for the cosmic ray density

$$n(z,r) = \sum_{n=0}^{\infty}\cos(k_n z)\, f_n(r) \qquad (3)$$

where

$$f_n(r) = \frac{-4\sin(k_n L_o)}{(2n+1)\pi}\int_0^{\infty} dr' r' q(r') I_o(k_n r_<) K_o(k_n r_>)$$

$$= \frac{-4\sin(k_n L_o)}{(2n+1)\pi}\left\{K_o(k_n r)\int_0^{r} dr' r' q(r')\, I_o(k_n r') \right. \qquad (4)$$

$$\left. + I_o(k_n r)\int_r^{\infty} dr' r' q(r')\, K_o(k_n r')\right\}.$$

The radial source functions, $q(r)$, will be discussed in detail in the next section. They are mainly of the form

$$q(r) = r^A \exp(-Br) \tag{5}$$

where A and B have been determined by fitting to experimental data. Once a particular form for $q(r)$ has been chosen, the integrals in (4) are performed numerically to obtain the functions $f_n(r)$ and the series in (3) is summed until a satisfactory convergence is obtained.

As was previously stated the cosmic ray density function $n_{cr}(z,r)$ obtained in this manner is normalized to the proper solar system value at $z = 0$, $r = 10$ kpc. This solar system value is obtained from the cosmic ray flux observed at earth by demodulation.

V. The Cosmic Ray Source Distribution

We consider various possible source distributions for cosmic rays to be connected with the formation of population I stars and their evolution to pulsars and supernovae. In this case, the distributions to be considered will be functions of galactocentric distance r (Burton 1976, Stecker 1976b). This distinguishes our diffusion halo models from those considered by Ptuskin (1974) and other workers at the Lebedev Institute (Ginzburg and Ptuskin 1976) who assume a uniform cosmic ray distribution throughout the galactic disk. We choose for examination the following radial distributions obtained from observational studies and analyses:

A) Supernova remnant distribution of Ilovaisky and Lequeux (1972).

B) Supernova remnant distribution of Kodaira (1974).

C) Pulsar distribution of Seiradakis (1976) (weighted and unweighted).

D) Distribution of molecular clouds as given by Burton et al (1975).

E) Distribution of ionized gas as given by Lockman (1976).

In all cases, we consider the sources to be uniformly distributed in the z direction (perpendicular to galactic plane) over a disk of thickness 100 pc. Our results are insensitive to source width since, for any reasonable population I source distribution, the thickness of the source disk is much smaller than the halo thickness.

The various experimentally determined radial distributions were fit with curves of the form

$$q(r) = r^A \exp(-Br).$$

The fit was a standard least squares fit to the data points, and in all cases but one the points were given equal weighting. In the case of the pulsar distribution two fits were obtained: one with equal weighting and one where the points were weighted inversely as the square of the reported experimental error. As can be seen from the pulsar distribution and curves shown in figure 1 and from Table 1, including the experimental uncertainty in the fitting procedure can make a significant difference in the result. It should be noted that including the errors in fitting the pulsar data of Seiradakis (1976) yields a distribution that is almost identical to that obtained for the supernova remnants by Kodiara (1974).

TABLE I. Source Distribution Parameters

Source Distribution	Parameters for $q(r) \propto r^A \exp^{-Br}$ A	B
Supernova Remnants (Ilovaisky and Lequeux 1972)	0.64	2.35
Supernova Remnants (Kodaira (1974)	1.20	3.22
Pulsars; weighted (Seiradakis 1976)	1.38	3.50
Pulsars; unweighted (Seiradakis 1976)	6.19	12.40
CO Clouds (Burton et al 1975)	7.41	13.92
Ionized Gas (Lockman 1976)	14.80	27.30

As an additional model, we approximate the SN distribution of Kodaira by the broken linear relation

$$q_{Br} = \begin{cases} 1 + 2.9\,r & 0 \le r \le .5 \\ 3.9 - 2.9\,r & .5 \le r \le 1.15 \\ 1.54 - 0.85\,r & 1.15 \le r \le 1.5 \\ 0 & r > 1.5 \end{cases}$$

and for which $q(0) \neq 0$. In fact, the other source distributions which are characteristic of Population I class objects (Stecker 1976b) involving the formation and evolution of new stars, drop off inside of 4 kpc except for the small nuclear region at the galactic center. Because the gas density is low in this central region and the path lengths are relatively small, the calculation of the γ-ray longitude distribution, as discussed in this section is not very sensitive to the exact value of q in the innermost regions of the Galaxy.

VI. The γ-Ray Line Flux Longitude Distribution

The cosmic ray specific intensity in the galactic gas disk is given by the solution of equation (1) as

$$I_{cr}(r) = \frac{n_{cr}(r,\ z = 0)\ c}{4\ \pi} \tag{6}$$

It is convenient to express our length scales so that the distance between the sun and the galactic center ($R_o \approx 10$ kpc) is taken equal to 1. The γ-ray flux from π^o-decay can then be expressed by

$$I_\gamma(\ell) = \frac{1}{4\pi}\int ds\ q_\gamma(s) \tag{7}$$

where s is the distance along the line of sight and

$$q_\gamma(s) = \sum_{j,k} 4\pi\ n_j(s)\int dE\ \sigma_{j,k}(E)\ I_{cr,k}(E,\ s) \tag{8}$$

(see, e.g., Stecker 1971) where j and k represent the type of target and cosmic ray nuclei.

For a cosmic mixture of H and He interstellar gas as target nuclei, and a value $\xi \equiv I_{cr}/I_{cr,\theta}$, $I_{cr,\theta}$ being the demodulated flux of cosmic rays in the solar neighborhood ($I_{cr,\theta} = I_{cr}(r{=}1,\ z{=}0)$) as given by Gloeckler and Jokipii (1967). Stecker (1970) gives a value for the γ-ray production rate in the solar neighborhood for energy greater than E.

$$q^{>E}_{\gamma,\ \theta\pi} \equiv q^{>E}_{\theta\pi}\ n_H(r{=}1) \tag{9}$$

where $q^{>0}_{\theta\pi} = (1.3 \pm 0.2) \times 10^{-25}\ s^{-1}$

(Stecker 1973, using the data of Comstock et al. (1972) gives an upper limit of $q^{>0}_{\theta\pi} \leq 1.51 \times 10^{-25}\ s^{-1}$). Using the variables defined in Figure 2, and the assumed distribution $\xi(r,\theta,z) = \xi(r,z=0) \equiv \xi(r)$,

$n_H(r,\theta,z) = n_H(r,z=0) \equiv n_H(r)$

equation (7) becomes

$$I_\gamma(\ell) = \frac{R_\Theta q_\Theta}{4\pi} \int ds\, n_H(r)\, \xi(r) \tag{10}$$

where $r = (s^2 - 2s\cos\ell + 1)^{\frac{1}{2}}$

In this particular case, we wish to calculate the γ-ray flux above 100 MeV as would be seen by the SAS-2 detector integrated over galactic latitudes $-10^o \leq b \leq 10^o$. We consider the hydrogen gas to be made up of two disks, an atomic hydrogen disk with a half-width $h_1 = 1.1 \times 10^{-2} R_\Theta$ (110 pc) and a molecular hydrogen disk with $h_2 = 5 \times 10^{-3} R_\Theta$ (see, e.g. Burton 1976). In the outer galaxy ($1 \leq r \leq 1.5$), this disk width is larger by a correction factor $w(r) = 1.8r - 0.8$ (Baker and Burton 1975) which we take account of by using $w(r)$ as a weighting factor in the integral (with $w(r) = 1$ for $0 \leq r \leq 1$. With a mean atomic hydrogen density $n_{HI}(r)$ as given by Burton et al. (1975) and mean molecular density $n_{H_2}(r)$ as given by two determinations: (a) from Burton et al. (1976) and (b) from Scoville and Solomon (1975) normalized by Stecker et al. (1975). We then obtain the formula for the line intensity

$$I_\gamma(\ell) = \frac{R_\Theta q_{\Theta,T}^{>100}}{4\pi} \int_{-10^0}^{10^0} db \left[\int_0^{h_1 \cot b} ds\, n_{HI}(r)\xi(r)w(r) + 2\int_0^{h_2 \cot b} ds\, n_{H_2}(r)\xi(r)w(r) \right] \tag{11}$$

where $q_{\Theta,T}^{>100}$ is the total γ-ray production rate per H atom above 100 MeV,

$$q_{\Theta,T}^{>100} = q_{\Theta\pi}^{>100} + q_{\Theta,\text{Bremstrahlung}}^{>100} + q_{\Theta,\text{ Compton}}^{>100} \tag{12}$$

$$\simeq 1.33 \times 10^{-25}\ s^{-1} \quad \text{(Stecker 1977)}$$

(Strictly speaking, the Compton γ-ray production rate does not scale with n_H but this contribution is only about 5% of the total production rate so that only a small error is introduced.)

The theoretical fluxes calculated using equation (11) for varius diffusion models and source functions were compared with the most recent SAS-2 data (Kniffen, Fichtel and Thompson 1977) with point sources subtracted out. In each case, a χ^2 test for goodness of fit was made and the statistical probability of obtaining the SAS-2 longitude distribution was obtained, assuming the model represented the real γ-ray longitude distribution. The tests were made over the longitude range $10^{\circ} \leq \ell \leq 90^{\circ}$, since detailed CO surveys of the molecular cloud distribution are only available in the range $0^{\circ} \leq \ell \leq 90^{\circ}$ and the range $0^{\circ} \leq \ell \leq 10^{\circ}$ is expected to contain a significant contribution from Compton produced γ-rays which is not included here (see Stecker 1977 and Section VIII). Indeed all smooth models of the large scale galactic distribution of γ-rays do not reproduce features in the observed longitude distributions for $\ell > 120^{\circ}$ which must be produced by nearby fluctuations. This is supported in the SAS-2 contour maps for the anticenter region (Hartman et al. 1976).

VII. Goodness of Fit of the Theoretical Longitude Distributions - χ^2 Test

After the longitude distributions of γ-ray flux were calculated as described in the previous section they were compared with the fluxes observed by SAS-2 (Kniffen et al. 1977) to see which, if any, gave reasonable fits to the data. We determined the goodness of fit by use of the χ^2 statistic of K. Pearson (Cramér, 1946). The advantage of the χ^2 statistic over a simple sum of squares of the residuals is that it enables an absolute measure of goodness-of-fit to be given for a particular theoretical distribution.

If the data are broken up into R bins of width 2.5 degrees of longitude, theory gives the expected number of photons observed in that bin to be $N_{ei} = \alpha_i F_i$ where F_i is the predicted flux in that bin and α_i is an instrumental factor made up of such things as detector sensitivity, look time, etc. The SAS-2 data, on the other hand, are reported in terms of a flux f_i so we may assume that the observed number of photons was $N_{oi} = \alpha_i f_i$ with the same set of α_i's. The reported errors are primarily counting errors so that $\delta f_i = \frac{1}{\alpha_i} \delta N_{oi} \approx \frac{1}{\alpha_i} \sqrt{N_{oi}}$. We may eliminate the α_i to obtain

$$N_{oi} = \left(\frac{f_i}{\delta f_i}\right)^2 \tag{13}$$

and $\alpha_i = \frac{N_{oi}}{f_i} = \frac{f_i}{(\delta f_i)^2}$

We may now compute the sum

$$s^2 = \sum_{c=1}^{R} \frac{(N_{oi} - N_{ei})^2}{N_{ei}} = \sum_{c=1}^{R} \frac{(\alpha_i f_i - \alpha_i F_i)^2}{\alpha_i F_i} = \sum_i \alpha_i \frac{(f_i - F_i)^2}{F_i} \tag{14}$$

$$= \sum_{c=1}^{R} \frac{f_i}{F_i} \left(\frac{f_i - F_i}{\delta f_i}\right)^2$$

where the final form is written soley in terms of the theoretical fluxes, the observed fluxes, and the experimental errors for the fluxes.

If the observed counts N_{oi} are normally distributed about the "true" expected value N_{ei} then it can be shown that the quantity s^2 of (3) is distributed with a χ^2-distribution with R degrees of freedom (Cramér, 1946). In fact the N_{oi} have a Poisson distribution but if the mean value N_{ei} is large enough the Poisson distribution is well approximated by a normal distribution so one would expect the sum s^2 to have approximately the χ^2-distribution. From tables of the χ^2-distribution one can read the probability that the sum given in (3) would have a value greater than or equal to any given value. One can test the hypothesis that a given theoretical model represents the true expected flux values by noting the probability that one would obtain a value of χ^2 as large as the one actually obtained upon comparison of the model with the SAS-2 data. If this probability is too small e.g. $< 5\%$ then it is usual to assume that the hypothesis is unlikely to be correct and the theoretical curve is rejected.

Whereas a theoretical model that yields a higher probability than an alternative model is usually to be preferred, one must apply this rule with caution. If one finds for example that there is a 99% chance of having a χ^2 as big or bigger than the one obtained, this indicates a very close fit of the data to the theoretical model; yet it also indicates that there was only a 1% chance of obtaining this close a fit to the "correct" model. This is in itself quite suspect. The general rule to be applied in interpreting these probabilities is that any values near 50% should be considered a good fit, for this is what one would expect most of the time with the "true" model.

Probabilities sensibly smaller than this (i.e. $\lesssim$ 10%) should be considered as poorer fits and probabilities less than 1% are quite unlikely and indicate that the model being tested is probably not correct.

In the next section we shall present the probability values obtained for the various models we have considered and discuss what we believe are the implications as to the existence and extent of a significant cosmic-ray halo.

VIII Results and Discussion

Tables 2 and 3 show the statistical probability that the γ-ray data will be produced with the observed intensities and errors in the SAS-2 data in the $10^{o} < \ell < 90^{o}$ range assuming that each theoretical model gives the "true" longitude distribution. These probabilities are obtained from the χ^2 test. The models are specified by the source functions described in Section V and the L values given in the tables. In the one dimensional approximation (diffusion perpendicular to an infinite plane (d^2n/dz^2=0, $n_{cr}(1) = 0$),

$$Icr(z) = I_{cr}(0)\ (L-z)\ (z < L) \tag{15}$$

Thus, the "size" of each halo model is smaller than L if one speaks in terms of a 1/e half thickness (0.63L) or a half-width half maximum (HWHM 0.5L).

TABLE 2. SAS-2 DATA PROBABILITY OF FIT (%) TO DIFFUSION HALO MODELS*

Source Model	No Halo (L=0)	L=5kpc	L=10 kpc	L=20 kpc
SN (Ilovaisky and Lequeux 1972)	32;16	3;0.06	<< 0.1	<< 0.1
SN (Kodaira 1974, Broken Linear Fit)	45;67	16;3	<< 0.1	<< 0.1
SN (Kodaira 1974)	42;39	6;0.25	<< 0.1	<< 0.1
Pulsar (Weighted, Seiradakis 1976)	44;47	7.3;4	~0.1; << 0.1	<< 0.1
Pulsar (Unweighted)	<< 0.1	~0.1;22	45;69	30;12
CO Cloud Distribution (Burton et al. 1976)	<< 0.1	1;38	48;62	25;8
Ionized Gas Distribution (Lockman 1976)	<< 0.1	<< 0.1	3;50	51;73

*First number is obtained using N_{H_2} from Burton et al. (1976). Second number is obtained using N_{H_2} from Scoville and Solomon (1975) as normalized by Stecker et al. (1975), E.g. B;S.

TABLE 3

PROBABILITIES OF FIT (%) USING THE KODAIRA SOURCE FUNCTION

L	0	1 kpc	3 kpc	5 kpc	10 kpc
with Burton et al. n_{H_2}	42	40	19	6	<< 0.1
with Scoville and Solomon n_{H_2}	39	31	4	0.25	<< 0.1

It can be seen from the tables that if one assumes a source distribution as given by the supernova remnant or pulsar (weighted) distribution, fits with probabilities greater than $\sim$5% are found only for models with small halos (L < 5 kpc or, from (15), a "scale height" < 3 kpc) with the most probable models being the ones with very thin halos or no halos (L=0). This is also consistent with the radio data (Baldwin 1976) and some isotope measurements (Hagen et al. 1976), although the radio and isotope measurements do not in themselves rule out extensive diffusion halos (Ginzburg and Ptuskin 1976; see also section II).

If one wishes to save the extensive halo hypothesis, it appears to be necessary to relate the source of cosmic rays to a sharper distribution such as that of molecular clouds or ionized gas rather than the presently favored supernova-pulsar distribution. One such hypothesis which has been discussed is that of large scale first order Fermi acceleration associated with the density wave theory of star formation (Stecker, et al 1974, Puget and Stecker 1974).

It can also be seen from the tables that it makes no qualitative difference to the results whether one adopts the molecular cloud distribution given by Burton et al (1976) or that given by Scoville and Solomon (1975) in computing the γ-ray fluxes.

Finally we have also examined with the χ^2 test, some galactic γ-ray models discussed in the literature. The model considered by Stecker (1975) is found to have a 39% probability in the longitude range $10^o \leq \ell \leq 90^o$. The more elaborate model of Kniffen, Fichtel and Thompson (1977) based on spiral arm enhancements, leads to only a 5% probability, reflecting a lack of detailed correlation between the arm features of the model and the peak in the observed data in the $10^o \leq \ell \leq 90^o$ range.

Consideration of other longitude ranges may show a better correlation, however. Extrapolating from models which we have investigated with large L, the constant cosmic ray model of Fuchs, Schlickeiser and Thielheim (1976) would provide a poor fit (probability $<<1\%$) to the data. This is expected as it would correspond to the case $L \to \infty$.

Figure 3 shows the cosmic ray distribution in the plane of galaxy obtained using the broken linear fit to the Kodaira SN distribution for diffusion halos with various values of L (all normalized to 1 at 10 kpc (r=1)). Figure 4 shows contours of constant cosmic ray intensity in r and z obtained from this model for L=1, 3 and L=10 kpc. The L=1 and 3 kpc cases can fit the γ-ray data, but the L=10 kpc case is ruled out by the γ-ray data.

Figures 5 and 6 show the γ-ray longitude distributions calculated using the weighted pulsar distribution for a source function with no halo (L=0) and a 10 kpc halo. The Scoville and Solomon n_{H_2} was used in these models. They are compared with the latest SAS-2 longitude distribution with point sources removed (Kniffen, Fichtel and Thompson 1977). The L=0 case has a probability of 47%. The L=10 Kpc case has a probability of fit of $\sim 10^{-8}$.

Figure 7 shows the model of Figure 5 with an additional contribution from Compton interactions as calculated from the model of Stecker (1977) with $h\xi$ = 500 pc. The value of $h\xi$ used was obtained by a minimum χ^2 fit, normalizing the theoretical Compton longitude profile. In this notation, h would correspond to the half-thickness of the electron disk (L_e) and $\xi = I_e/I_0$ is the normalized cosmic ray electron flux in the galactic center region, i.e., if $\xi = 1$, L_e = 0.5 kpc at the galactic center.

One may attempt to determine the thickness of the cosmic ray electron halo directly from the latitude observations of galactic γ-rays, as has been attempted by Schlickeiser and Thielheim (1977). These authors have estimated a value for L_e of 3.2 kpc. They obtained this value by taking the SAS-2 data, subtracting out an estimated extragalactic component and an estimated galactic component and then assuming that the remainder is due to Compton interactions of cosmic-ray halo radiation with the blackbody and starlight radiation fields. However, it is not clear how these authors obtained a value for the extragalactic component and separated out a halo component. Unfortunately there are also large uncertainties in all of the estimates involved using this method. Stecker (1977) has pointed out that the SAS-2 data at high galactic latitudes is consistent with the sum of galactic and extragalactic contributions with no significant halo contribution. However, some halo contribution is certainly also consistent with the SAS-2 data, so that we examine this question in more detail here.

We assume the SAS-2 high latitude flux to be made up of three components, a flux from the galactic disk, a Compton flux from a thick disk or halo of half-thickness L_e, and an extragalactic component:

$$I_{SAS} = I_{gal} + I_C + I_{ex} \tag{16}$$

In our case, we take the extragalactic flux to be determined by an E^{-3} power-law fit in accordance with cosmological pion-decay theories of the diffuse background (Stecker 1975b). We then obtain, for γ-ray energies above 100 MeV.

$$\begin{aligned} I_{SAS} &= (1.9 \pm 0.4) \times 10^{-5} && \text{(Fichtel et al. 1975)} \\ I_{gal} &= 0.6 \times 10^{-5}\ \mathrm{cm^{-2}s^{-1}sr^{-1}} && \text{(Stecker 1977)} \\ I_{ex} &= 0.8 \times 10^{-5} && \text{(Stecker 1977)} \end{aligned} \tag{17}$$

Thus, by combining (16) and (17), one obtains a value for I_C of $(5 \pm 4) \times 10^{-6} cm^{-2} s^{-1} sr^{-1}$. The uncertainty given here is a lower limit, not reflecting the uncertainty in the theoretical estimates but only the uncertainty in the data. The value for L_e is then given by

$$L_e = \frac{4\pi}{q_{\odot,C}^{>100}} I_C \sin b \qquad (18)$$

where b is the mean latitude at which the SAS-2 flux was observed.

The Compton production rate for the solar vicinity for energies greater than 100 MeV, $q_{\odot,C}^{>100}$, is given from recent estimates as

$$q_{\odot,C}^{>100} = \begin{cases} 6 \times 10^{-27} & \text{(Stecker 1976a)} \\ 5.5 \times 10^{-27} cm^3 s^{-1} & \text{(Piccinotti and Bignami 1976)} \\ 8.5 \times 10^{-27} & \text{(Stecker 1977)} \end{cases} \qquad (19)$$

The uncertainties depend on various estimates of the optical and infrared photon fields and on the exact shape of the electron spectrum. We adopt here a value of $q_{\odot,C}^{>100} = (7 \pm 1.5) \times 10^{-27}\ cm^{-3} s^{-1}$, again probably underestimating the uncertainty. We also take for a mean latitude, $b = 40^o$. We then obtain from equation (18) an estimate for the mean half-thickness of the electron halo of

$$L_e = 2 \pm 2 \text{ kpc.}$$

The radio measurements and the cosmic ray age measurements also tend to favor, or at least be consistent with a thick disk or thin halo with $L \simeq (1\text{-}3)$ kpc (Ilovaisky and Lequeux 1972, Baldwin 1976, Stecker 1977, see also section I). Our present results on the cosmic ray nucleon halo, using a supernova-pulsar type source distribution are also perfectly consistent with this thick disk model, as can be seen from Table 3.

One can use the values obtained for L from the diffusion model with a supernova-pulsar type source distribution, i.e. $L < 5$ kpc in order to obtain values for the diffusion coefficient D and the mean-free path for diffusion λ. These quantities can be evaluated from the relations

$$\begin{aligned} D &\simeq L^2/2\tau \\ \lambda &= 3\ D/c \end{aligned} \tag{20}$$

where τ is the cosmic ray age (Ginzburg and Syrovatskii 1964).

Estimates of D and λ obtained from equation (20) are shown in Tables 4 and 5.

Table 4. Values for D ($cm^2 s^{-1}$)

	$\tau(10^6 yr)$		
L(kpc)	3	10	20
1	4.5×10^{28}	1.5×10^{28}	7.5×10^{27}
2	1.8×10^{29}	6.0×10^{28}	3.0×10^{28}
4	4.1×10^{29}	1.4×10^{29}	6.8×10^{28}
5	1.1×10^{30}	3.8×10^{29}	1.9×10^{29}

Table 5. Values for λ (pc)

	$\tau(10^6$ yr)		
L(kpc)	3	10	20
1	1.5	0.5	0.25
2	6.0	2.0	1.0
3	13.5	4.5	2.25
5	37.5	12.5	6.25

Values of L greater than 5 kpc are considered to be highly unlikely (see Table 3).

APPENDIX

Consider the eigenvalue equation

$$\frac{\partial^2 n}{\partial z^2} + \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial n}{\partial r}\right) - \lambda n = 0 \tag{A1}$$

The above equation is separable in z and r and the solutions that fit the appropriate boundary condictions ($n=0$ at $z = \pm L$, integrable in r) are

$n_\gamma(z,r) = \cos(k_n z)\ J_o(k,r)$

where $k_n = \frac{(2n+1)\pi}{2L}$

and $\lambda = -(k_n^2 + k^2)$

Since the diffusion operator is self adjoint, the eigensolutions form a complete set and it is straightforward to verify that

$$\frac{1}{L}\sum_{n=0}^{\infty} \cos(k_n z)\cos(k_n z') = \frac{1}{2}\left(\delta(z-z') + \delta(z+z')\right) \tag{A2}$$

(Since we are interested only in models that are symmetric in z, we have employed only the symmetric eigenfunctions resulting in the symmetric delta function.)

$$\int_0^\infty dk\ k\ J_o(kr)\ J_o(kr') = \frac{1}{r}\delta(r-r') \tag{A3}$$

If one has a complete set of orthonormal eigensolutions of the equation

$$L\ U_\lambda = \lambda U_\lambda \tag{A4}$$

Such that

$$\int_\lambda U_\lambda(x)\ U_\lambda(X') = \delta(x-x') \tag{A5}$$

Then the Green's function defined by

$$L\ G(x-x') = \delta(x-x')$$

is constructed from the eigensolutions.

$$G(x-x') = \int_{\lambda} \frac{U_{\lambda}(x)U_{\lambda}(x')}{\lambda} \tag{A6}$$

(Cushing 1975, Jones 1970).

For our problem we thus have

$$G(z,z',r,r') = -\frac{1}{L}\sum_{n=0}^{\infty} \cos(k_n z)\ \cos(k_n z')\int_0^{\infty} \frac{dk\ k\ J_0(kr)J_0(kr')}{k^2+k_n^2} \tag{A7}$$

The integral over k may be performed to yield

$$G(z,z',\ r,r') = -\frac{1}{L}\sum_{n=0}^{\infty}\ \cos(k_n z)\ \cos(k_n z')I_0(k_n r_<)\ K_0(k_n r_>) \tag{A8}$$

where $r_<$ $(r_>)$ is the lessor (greater) of r and r' and I_0 and K_0 are the modified Bessel functions of the first and second kinds respectively.

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Figure Captions

Figure 1 The pulsar distribution in the Galaxy obtained by Seiradakis (1976) together with best fits to a distribution function of the form $r^A \exp(-Br)$ both unweighted (curve A) and weighted according to the error bars as shown (curve B).

Figure 2 Definition of distance variables used in the γ-ray flux equations. (G.C. = galactic center)

Figure 3 Cosmic ray distribution in the galactic plane (z=0) using a broken linear fit to the supernova remnant distribution of Kodaira (1974) and various diffusion halo sizes L (in units of $R_\odot$ = 10 kpc).

Figure 4 Contours of constant cosmic ray intensity in the r-z plane for the source model of Figure 3 with L=1,3 and 10 kpc.

Figure 5 The γ-ray longitude distribution obtained using the weighted pulsar source model with L=0 (no halo source dominated model) compared with the SAS-2 data. (Probability of fit equals 47%). The distribution of n_{H_2} was obtained from Scoville and Solomon (1975) and Stecker et al. (1975). Using the Burton et al. (1975) distribution for n_{H_2} or the supernova remnant source distribution of Kodaira (1974) also gives a good fit (see Table 2).

Figure 6 The γ-ray longitude distribution obtained using the weighted pulsar source model with an L=10 kpc diffusion halo compared with the SAS-2 data. (Probability of fit $\sim 10^{-8}$).

Figure 7 The γ-ray longitude distribution obtained using the same model as in Figure 5, but with an additional Compton γ-ray flux as discussed in the text. The SAS-2 data are again shown for comparison.

Figure 1

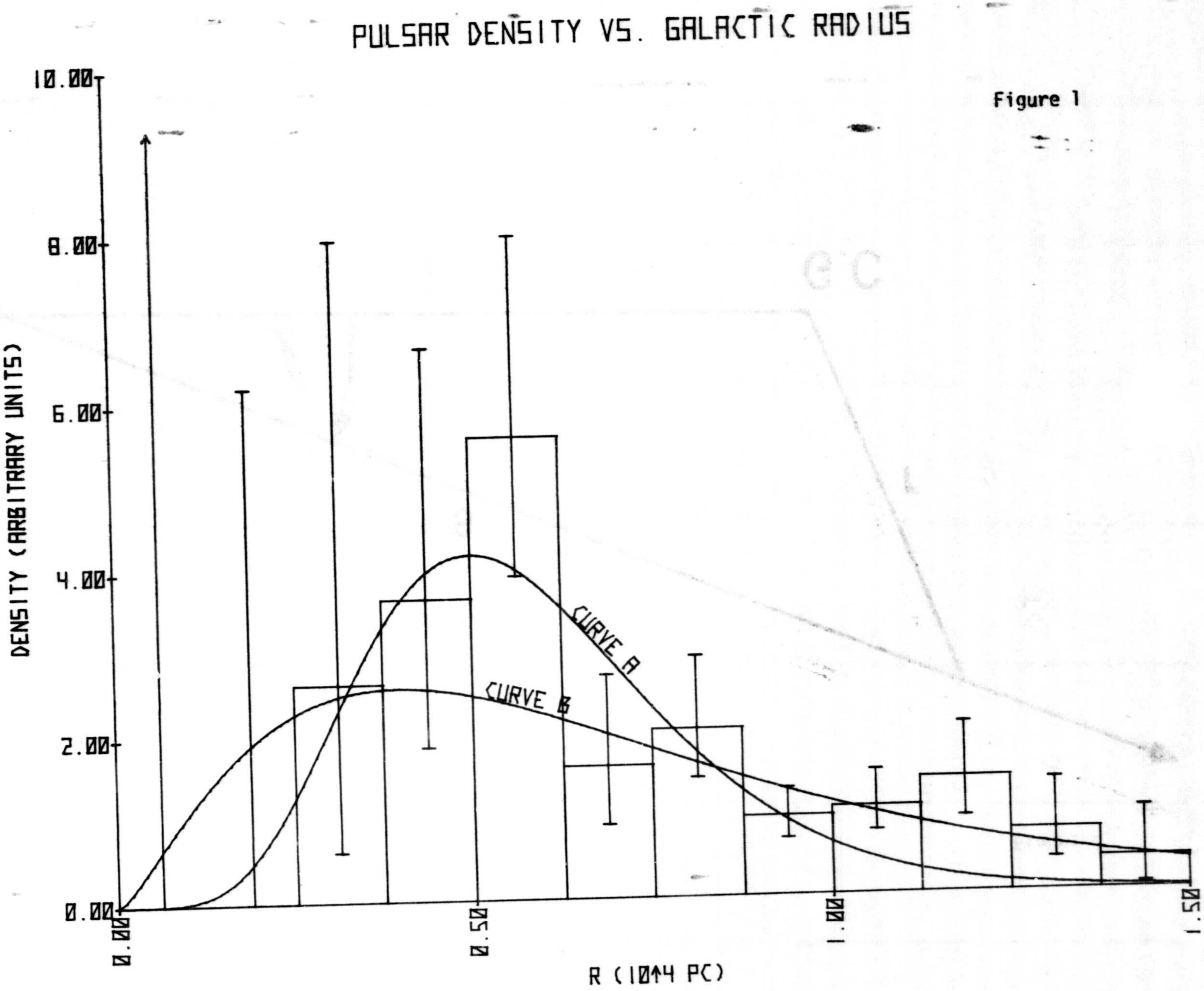

PULSAR DENSITY VS. GALACTIC RADIUS
DENSITY (ARBITRARY UNITS)
10.00
8.00
6.00
4.00
2.00
0.00
0.00
0.50
1.00
1.50
R (10↑4 PC)
CURVE A
CURVE B

Figure 2

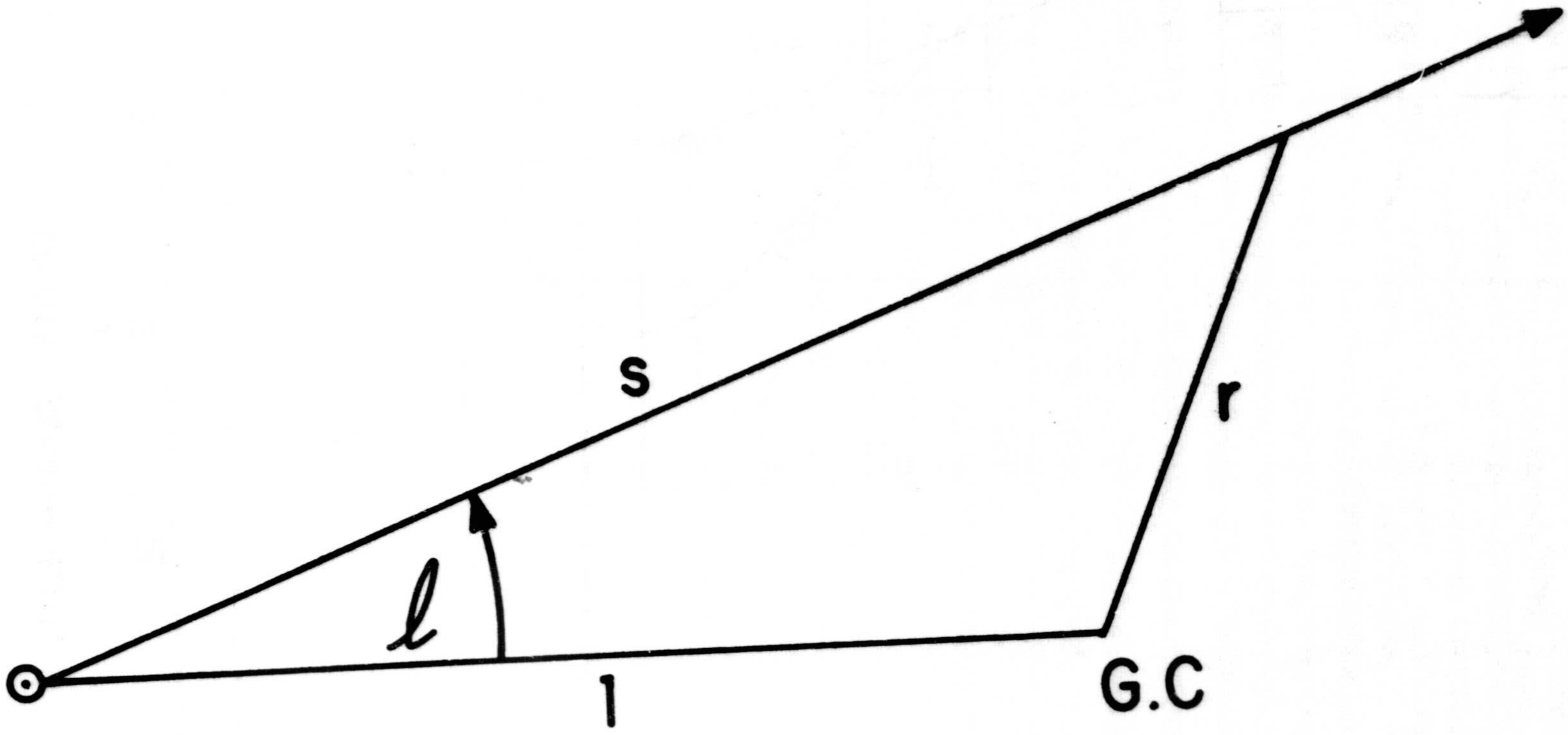
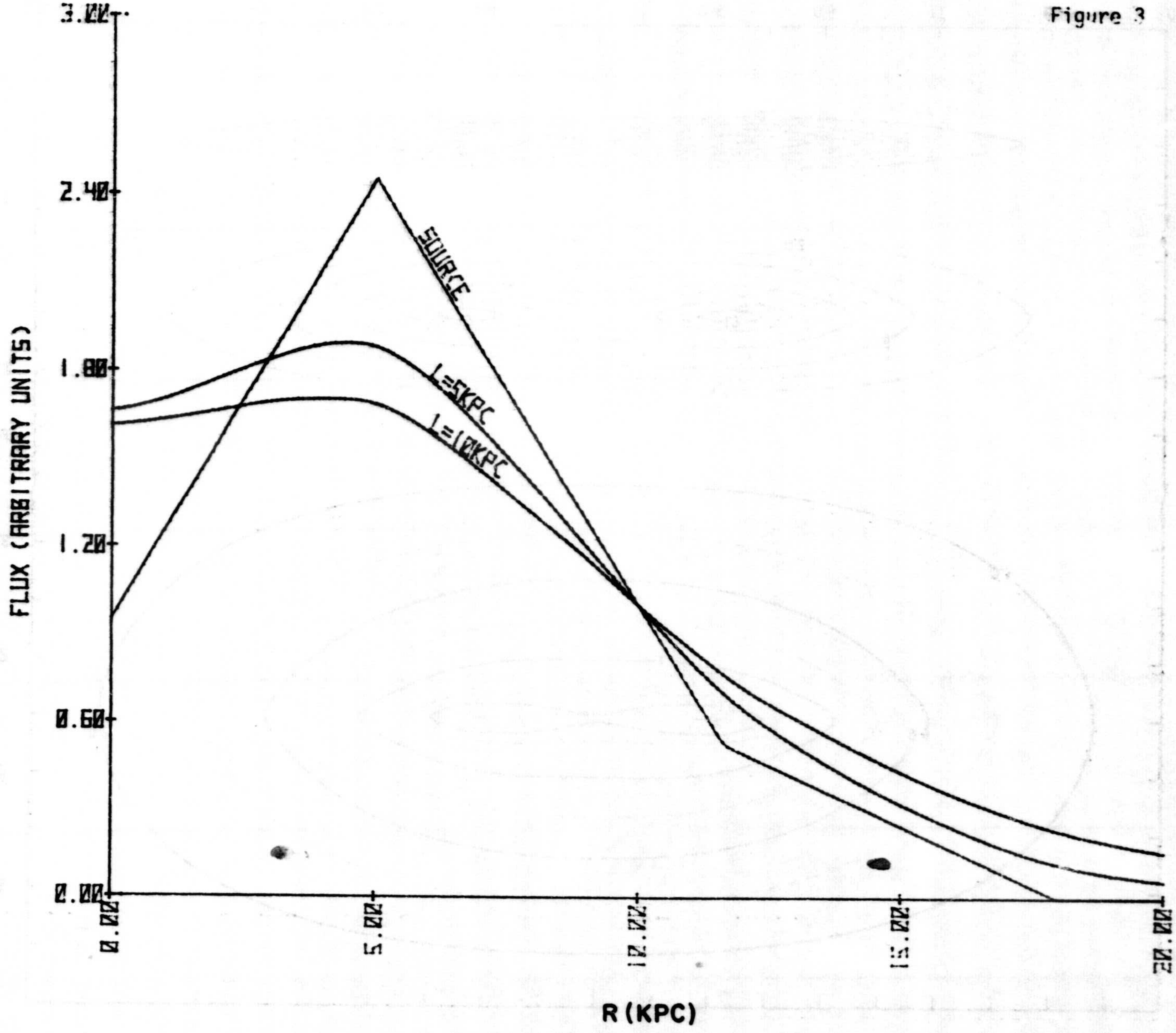


Figure 3



FLUX (ARBITRARY UNITS)
3.00
2.40
1.80
1.20
0.60
0.00
SOURCE
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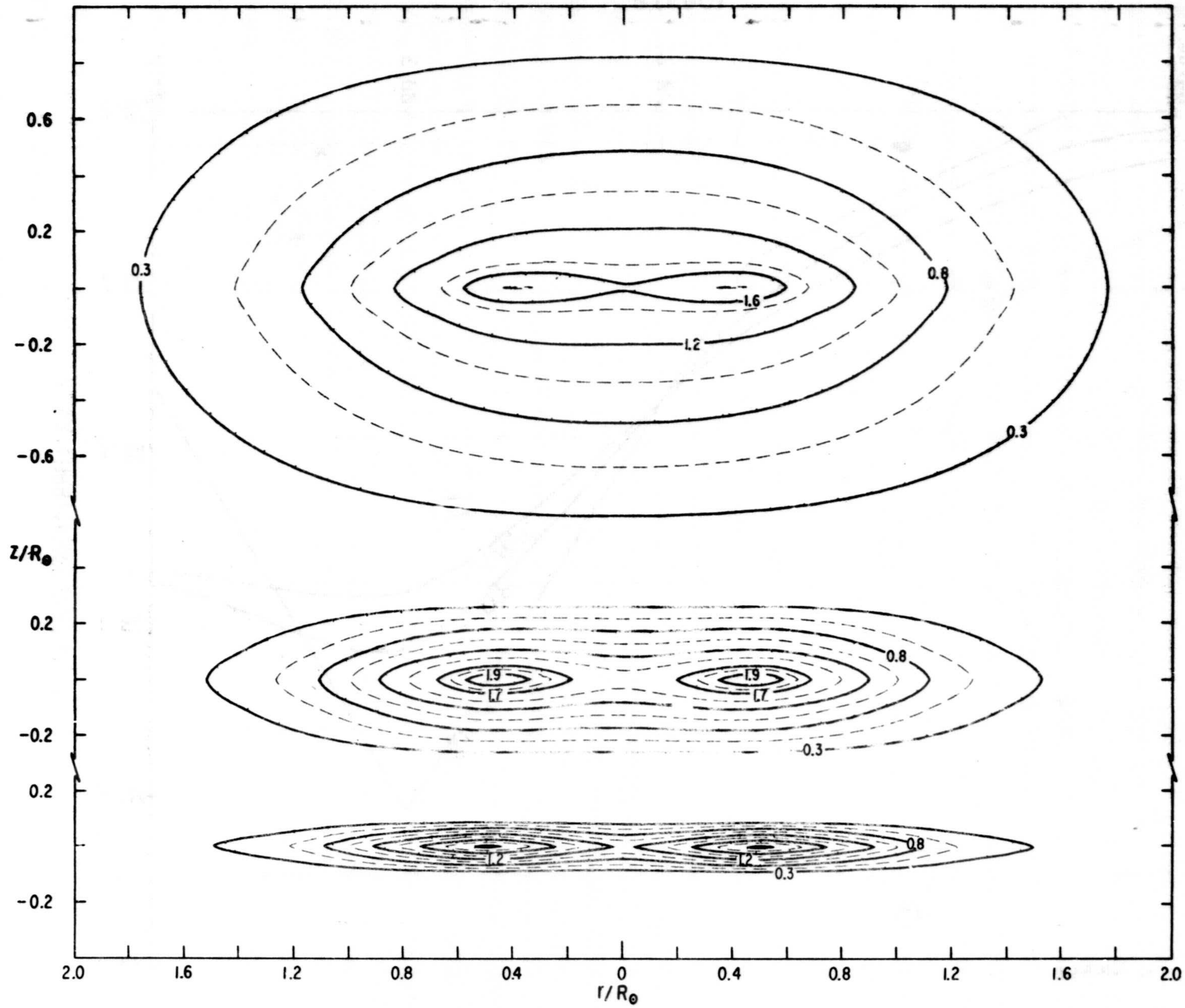
L=10KPC
0.00
5.00
10.00
15.00
20.00
R (KPC)

Figure 4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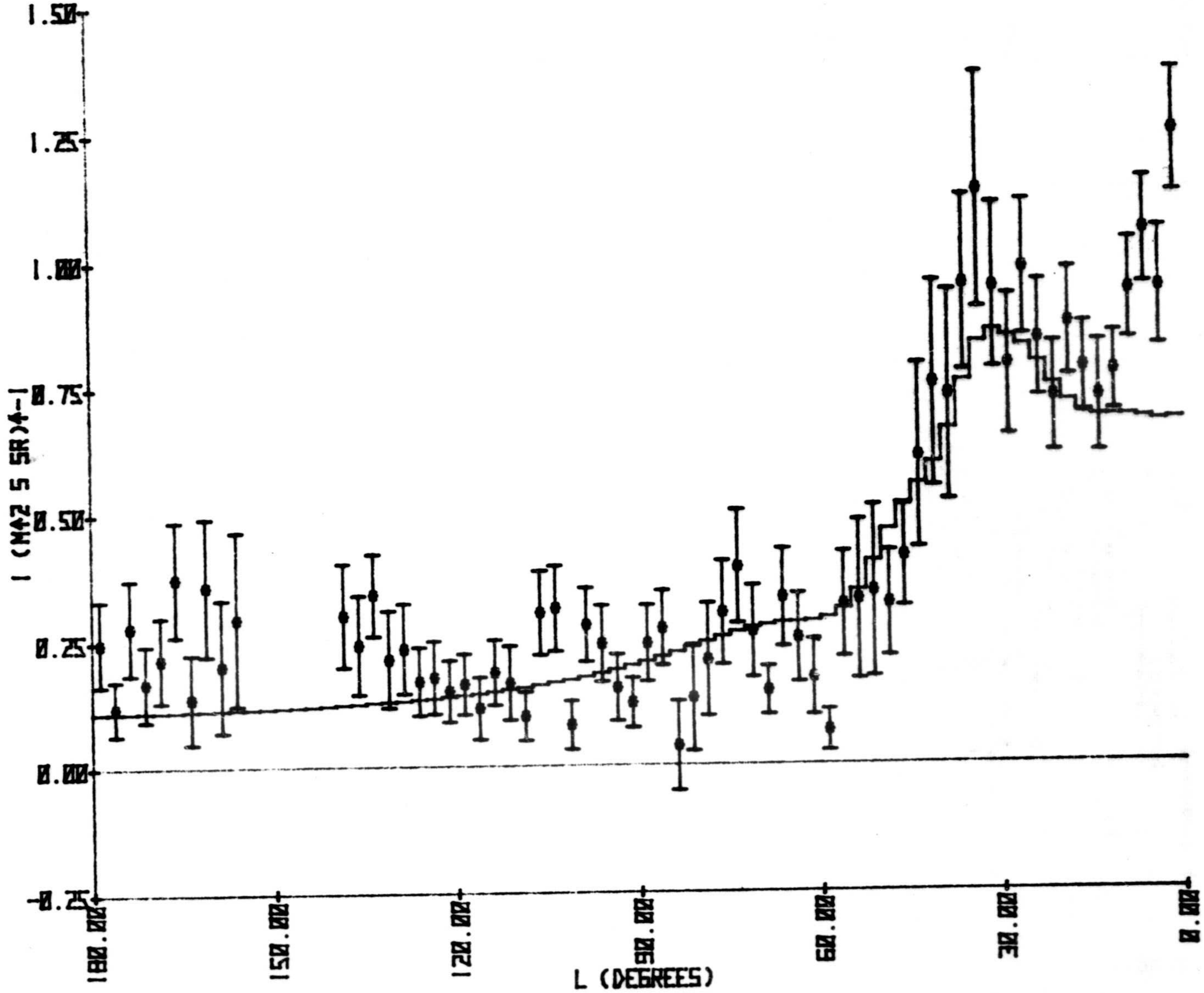
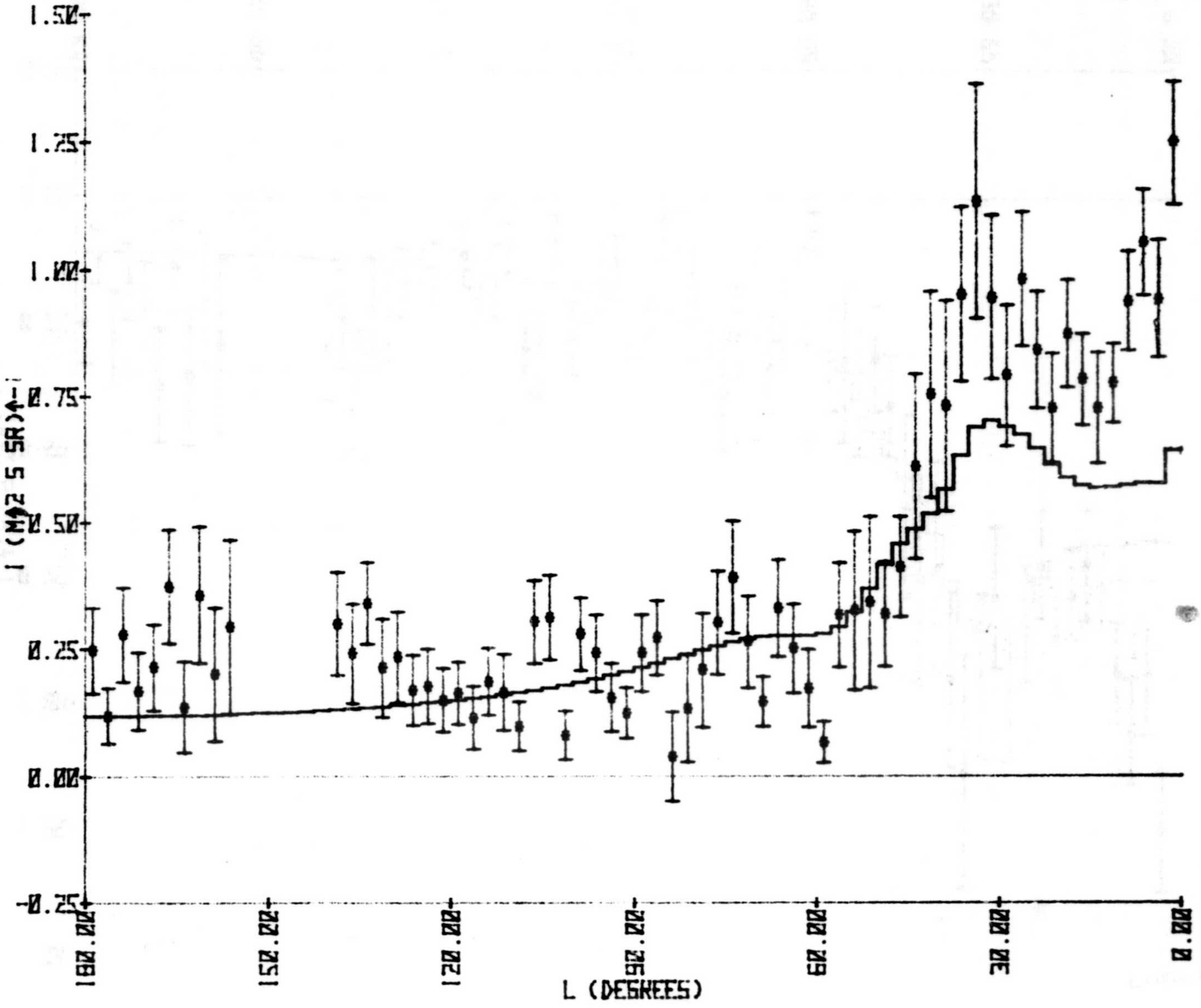
0.8
1.6
1.2
0.3
1.9
1.7
1.9
1.7
0.8
0.3
1.2
1.2
0.8
0.3
0.6
0.2
-0.2
-0.6
Z/R⊙
0.2
-0.2
0.2
-0.2
2.0
1.6
1.2
0.8
0.4
0
0.4
0.8
1.2
1.6
2.0
r/R⊙

Figure 5



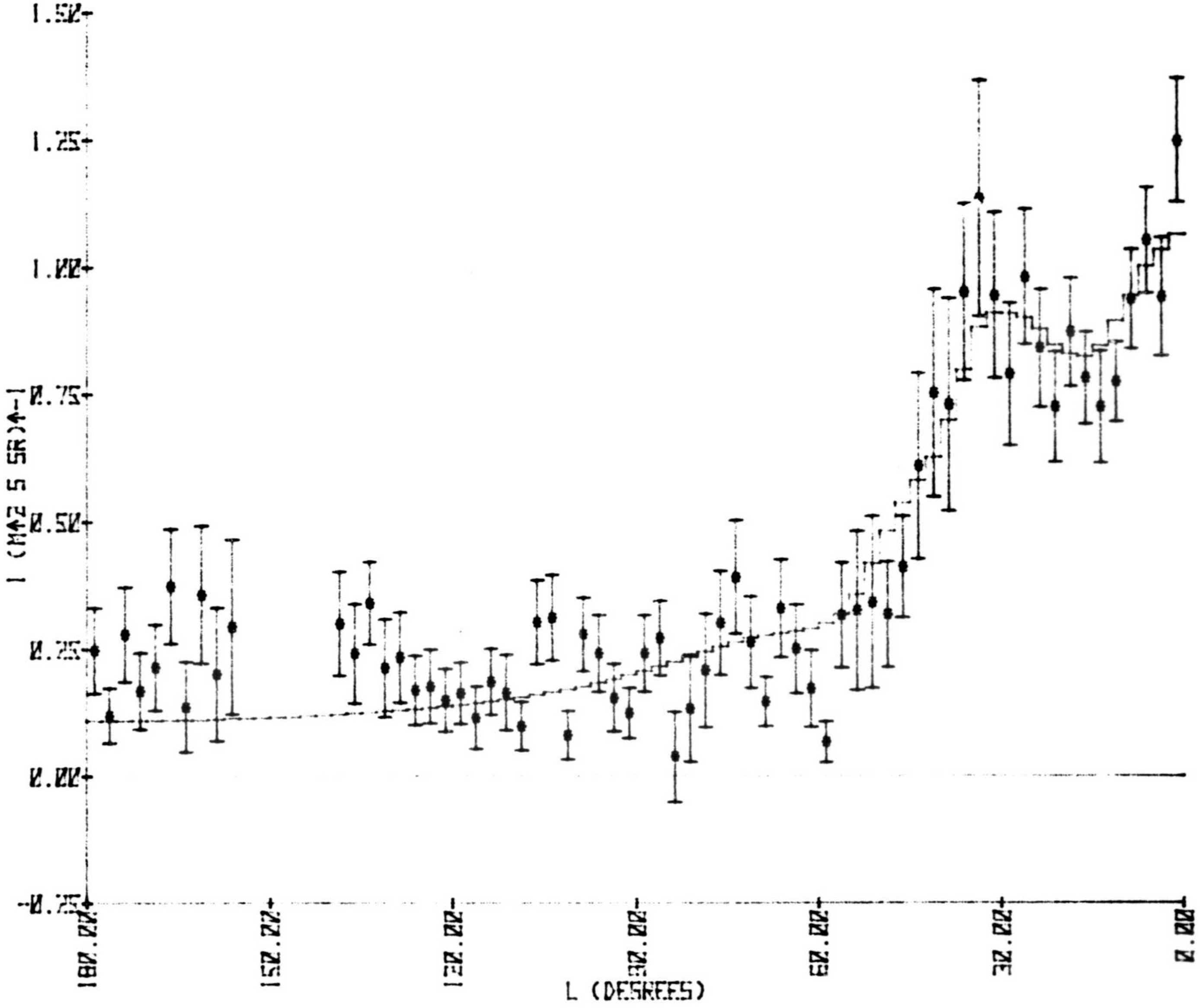
1.50
1.25
1.00
0.75
0.50
0.25
0.00
-0.25
I (M42 S SR)4-1
180.00
150.00
120.00
90.00
60.00
30.00
0.00
L (DEGREES)

Figure 6



1.50
1.25
1.00
0.75
0.50
0.25
0.00
-0.25
I (M↑2 S SR)↑-1
180.00
150.00
120.00
90.00
60.00
30.00
0.00
L (DEGREES)

Figure 7



1.50
1.25
1.00
0.75
0.50
0.25
0.00
-0.25
I (M↑2 S SR)↑-1
180.00
150.00
120.00
90.00
60.00
30.00
0.00
L (DEGREES)