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FINAL REPORT

KU-BAND SIGNAL DESIGN STUDY

Prepared for

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1.0 SUMMARY

This report develops results pertinent to three areas which apply to the Shuttle Orbiter Ku-Band high data rate communication link. This includes carrier synchronization strategies which are best suited to the Orbiter Ku-band forward link, for example, the Costas and squaring loop phase detectors. This report develops the acquisition/tracking performance of a practical squaring loop in which the times two multiplier is mechanized as a limiter/multiplier combination, this squaring approach serves to produce the absolute value of the arriving signal as opposed to the perfect square law action which is required in order to render acquisition and tracking performance equivalent to that of a Costas loop.

The Ku-Band Orbiter signal design for the forward link has been accessed and acquisition time results as well as acquisition and tracking thresholds are summarized. Finally, a tradeoff study which pertains to bit synchronization techniques for the high rate Ku-Band channel is given and an optimum selection is made based upon the appropriate design constraints.

2.0 KU BAND SIGNAL ACQUISITION AND TRACKING THRESHOLDS FOR THE SHUTTLE FORWARD LINK

System threshold performance for the acquisition and tracking modes of the Shuttle Ku-Band communication receiver have been
established for the S-band PN despraver and carrier tracking loop configuration typical to the Shuttle S-Band transponder reported on in Reference 1. From the system specification a carrier-to-noise ratio, \( C/N_0 \), of 60.2 dB-Hz is assumed and this gives rise to an ideal bit error probability of \( 10^{-2} \) when the channel bit rate is 72 kbps (216 ksp). The PN code chip rate is taken to be 3.028 mcps as opposed to the 11.232 mcps code used on the Shuttle S-Band forward link. In addition, the PN code length has been shortened to 1023 chips as opposed to the longer length of 2047 chips used for Shuttle S-Band communications; however, for this investigation the 2047 length was used (worst case) in the computer calculations presented herein because of the uncertainty in the code length at the time that this work was being accomplished. The total incoming Doppler shift on the carrier, after correction at the TDRS, is assumed to be \( \pm 75 \) kHz. This number includes precorrection uncertainty at the ground station and the uncertainty in the Ku-band oscillators on board the Shuttle. It is also noted that this Doppler uncertainty band could be significantly increased without causing a problem as far as PN acquisition time is concerned. The analysis shows that the time to acquire the PN code with any one of the several algorithms investigated is less than the 10 second value given in the system specification. This assumes a
code search step size of one-half chip and individual cell search times of one or two code periods. The results obtained are not particularly sensitive to the number of code periods over which energy is collected. It appears that one or two is more than adequate. A reasonable value for the code acquisition time would seem to be a few seconds (perhaps 5) at the worst case C/N₀.

Because of the signal levels involved the acquisition time is not very sensitive to increases in C/N₀. (see supporting computer printout.) In the locked or tracking mode, the PN code acquisition/tracking control circuit thresholds are set such that the false alarm probability is on the order of 0.5. A threshold sensitivity analysis has been performed and it was found that no problem should arise due to reasonable (less than 10%) acquisition and tracking threshold drifts. The code acquisition analysis assumes a one dB correlation loss due to filtering, etc., and a 2.5 dB loss due to code misalignment of 0.25 chip. Code acquisition thresholds are set to give false alarm probabilities varying from 10⁻² to 10⁻⁴ without causing a significant change in code acquisition time. A reasonable PN code tracking threshold C/N₀ value is on the order of 57 dB-Hz.

Since the incoming code and carrier are coherent, the carrier loop can be locked quickly and the data related false lock problem appears to be avoidable in an appropriately designed receiver.
In fact it appears that the carrier loop can be locked in less than one second. The bit synchronizer (assuming a data-transition tracking type loop) can be locked in a few milliseconds. The frame synchronizer should also lock in a few milliseconds. The bit synchronizer acquisition threshold is well below 57 dB-Hz and it would appear that the same is true for the frame synchronizer. The antenna acquisition time and angle track thresholds are presently being addressed.

It is concluded that a "threshold-value" of \( C/N_0 = 60.2 \text{ dB-Hz} \) is adequate for meeting signal acquisition time specifications provided in the Hughes proposal. On the other hand a \( C/N_0 \) of two to three dB lower is considered reasonable for defining receiver tracking thresholds.

3.0 SYNCHRONIZATION TECHNIQUES AND TRADEOFFS

3.1 Introduction

Implementation of a carrier recovery circuit for a suppressed carrier binary phase-shift keyed signal poses the interesting question of which approach to take. This paper develops the tracking performance of a practical squaring loop in which the times two multiplier is mechanized as a limiter/multiplier combination; this "squaring" approach serves to produce the absolute value of the arriving signal as opposed to the perfect square law action which
is required in order to render acquisition and tracking performance equivalent to that of a Costas loop. The absolute value type circuit appears to be the more practical circuit to build when such things as low-signal-to-noise ratios, a wide dynamic range of signal level and temperature variations are considered. In the signal-to-noise ratio region of interest, it is shown that an absolute value type "square law" circuit degrades the input C/N₀ by 0.5 to 0.8 dB over that of an ideal squaring loop. This also says that the tracking performance of a Costas loop is better, by 0.5 to 0.8 dB, than that of a squaring loop implemented with a limiter/multiplier combination for times two multiplication. At high SNR it is shown that the tracking performance of the two mechanizations is identical. In addition, the beat note level and phase detector gain are nonlinear functions of the signal-to-noise ratio at the input to the limiter/multiplier. This is of concern as far as maintaining the design point loop bandwidth and damping as the signal level varies. Finally, the weak signal suppression factor is derived and plotted as a function of input signal-to-noise ratio.

3.2 Analysis of Squaring Circuit Mechanizations in Costas and Squaring Loops

It is well known that the Costas loop of Figure 3-1a and the squaring loop of Figure 3-1b have the same theoretical noise immunity in both the acquisition and tracking modes. However, in
Figure 3-1a. Block Diagram of a Perfect Squaring Loop.

Figure 3-1b. Block Diagram of a Costas Loop.
the implementation of a squaring loop, mechanization of the times
two multiplier is an important consideration insofar as system
performance at low signal-to-noise ratio is concerned. Con-
siderations which must be accounted for in the choice of the "squaring"
approach include a wide dynamic range with respect to the signal
level, good thermal stability, and accurate square law response
over the dynamic range of input signal and temperature levels of
interest. The performance with an analog multiplier usually
indicates degraded signal-to-noise performance relative to
theoretical. In an attempt to overcome the degrading effects at
low signal-to-noise ratios, an alternate approach to the implementa-
tion of a squaring circuit was considered. The alternate mechaniza-
tion of the loop is illustrated in Figure 3-2. As far as the authors
can tell the tracking performance of this limiter/multiplier imple-
mentation of a squaring circuit incorporated in a PLL has not been
addressed in the open literature.

The purpose of this paper is to investigate the performance of
this implementation as compared to the known performance of the
ideal squaring loop case, Reference 2. In particular, relative
comparisons are made on weak signal suppression factor on the
phase error signal in the second implementation, error signal SNR
as a function of the input SNR, effective loop SNR and squaring
Figure 3-2. Squaring Loop with Limiter/Multiplier Implementation for Squaring.
losses as functions of input filter bandwidths and input SNR’s. From this computation the degradations on loop performance due to this limiter/multiplier implementation from the ideal squaring loop case is established.

3.3 Preliminaries and Signal Models

Figure 3-2 shows the limiter/multiplier type of squaring loop under consideration. In Figure 3-2 $h_1(t)$ is a bandpass filter with center frequency $\omega_0$ and equivalent single-sided noise bandwidth $B_i$ ($B_i << \omega_0$), given by

$$B_i = \frac{1}{2\pi} \int_0^\infty \frac{|H(j\omega)|^2}{|H(j\omega_0)|^2} d\omega$$ (1)

where $H$ is the transfer function of the filter. The input signal to the bandpass limiter is defined to be of the form

$$x(t) = s(t) + n(t)$$ (2)

with

$$s(t) = \sqrt{2} A \sin (\omega_0 t + \theta(t))$$ (3)

where $\theta(t)$ is the information bearing signal, and $n(t)$ is the narrow band noise represented by

$$n(t) = 2 [n_c(t) \cos \omega_0 t - n_s(t) \sin \omega_0 t]$$ (4)
where \( n_c(t), n_s(t) \) are zero mean uncorrelated Gaussian processes with

\[
\begin{align*}
E[n_c^2(t)] &= E[n_s^2(t)] = \sigma_n^2/2, \\
\sigma_n^2 &= E[n^2(t)] = N_0/2, \\
E[n_c(t)n_s(t + \tau)] &= 0
\end{align*}
\]

(5)

In (5) \( N_0 \) is the one sided spectral density at the input to the BPF \( h_1 \).

The noise process \( n(t) \) can be further written as

\[
n(t) = \sqrt{2} N_i(t) \cos (\omega_0 t + \theta_i(t))
\]

(6)

where \( N_i, \theta_i \) are the noise envelope and phase respectively given by

\[
N_i(t) = \sqrt{n_c^2(t) + n_s^2(t)}
\]

and

\[
\theta_i(t) = \tan^{-1} \left( \frac{n_s(t)}{n_c(t)} \right)
\]

Combining (6) and (3) through some trigonometric identities we can rewrite the input signal \( x(t) \) in the form:

\[
x(t) = \sqrt{2} [(A-N_s(t)) \sin \phi(t) + N_c(t) \cos \phi(t)]
\]

(7)

where

\[
\begin{align*}
\phi(t) &= \omega_0 t + \phi(t) \\
N_c(t) &= N_i(t) \cos (\theta_i(t) - \phi(t)) \\
N_s(t) &= N_i(t) \sin (\theta_i(t) - \phi(t))
\end{align*}
\]

(8)
The noise processes $N_c$, $N_s$ involve the information bearing signal $\theta(t)$. To obtain the statistical properties of $N_c N_s$ we can rewrite (8) as follows:

\[ N_c(t) = n_c(t) \cos \theta(t) + n_s(t) \sin \theta(t) \]

\[ N_s(t) = n_s(t) \cos \theta(t) - n_c(t) \sin \theta(t) \]

which gives directly the following, assuming the noise process and the information bearing signal are statistically independent

\[ E[N_c(t)] = E[N_s(t)] = 0 \]

\[ E[N_c^2] = E[N_s^2] = \sigma_n^2 / 2 \]

\[ E[N_c(t)N_s(t)] = 0 \]

If in addition to (5) we define the normalized input noise autocorrelation function to be

\[ R_n(\tau) = \sigma_n^2 \gamma_n(\tau) \cos \omega_0 \tau \]

where $r_n(\tau)$ is low pass and has the properties

\[ R_n(0) = 1, |r_n(\tau)| < 1, \int_{-\infty}^{\infty} r_n(\tau) \, d\tau = \frac{1}{B_1} \]

then the auto- and cross-correlation functions for $N_c(t)$ and $N_s(t)$ can be written in terms of $r_n(\tau)$ and $\sigma_n^2$ as:
$E[N_{s}N_{s}] = E[N_{c}N_{s}] = \frac{\sigma_{n}^{2}}{2} r_{n}(\tau) \cos\Delta\theta_{\tau}$

$E[N_{s}N_{c}] = -E[N_{c}N_{s}] = \frac{\sigma_{n}^{2}}{2} r_{n}(\tau) \sin\Delta\theta_{\tau}$

where

$\Delta\theta_{\tau} = \theta(\tau) - \theta(\tau+\tau)$.

3.4 Bandpass Limiter Output Signal

The ideal limiter function is defined such that

$$y(t) = \text{sgn}(x(t)) = \begin{cases} +1 & \text{if } x(t) > 0 \\ -1 & \text{if } x(t) < 0 \end{cases}$$

(13)

The input process $x(t)$ can be written, from (7), as follows

$$x(t) = \sqrt{2} v(t) \cos (\omega(t) - \gamma(t))$$

(14)

where

$$v \equiv \sqrt{(A-N_{c})^{2} + N_{s}^{2}}$$

$$\gamma \equiv \tan^{-1} \left( \frac{A - N_{s}}{N_{c}} \right)$$

(15)

Using the complex Fourier transform approach of relating the limiter output time function to that of the input, one obtains [3]

$$y(t) = \frac{1}{2\pi j} \int_{c} G(j\lambda) \exp [j\lambda x(t)] d\lambda$$

(16)

$$= \frac{1}{2\pi} \int_{c} \frac{d\lambda}{\lambda} \exp [j\lambda v(t) \cos (\omega(t) - \gamma(t))]$$
with \( G(j\lambda) = 2/\lambda \), the transfer characteristic of the hard limiter \([2]\),
and \( C \) the dynamic path of integration. Expanding the exponential
function in (16) by the Jacobi-Anger formula

\[
e^{z \cos \theta} = \sum_{k=0}^{\infty} e_k I_k(z) \cos k\phi
\]

where \( e_k = 2 \) for all \( k > 1 \) and \( e_0 = 1 \), and where \( I_k(z) \) are modified
Bessel functions, the limiter output \( y(t) \) can be integrated to be

\[
y(t) = \frac{4}{\pi} \sum_{k=0}^{\infty} \frac{(-1)^k}{2k + 1} \cos \left\{ (2k + 1) \left[ \phi(t) - \gamma(t) \right] \right\}
\]  

(17)

It is clear from (17) that only odd harmonics are present in the
limiter output.

3.5 Squarer Output

To obtain suppressed carrier tracking, the data modulation \( \delta(t) \)
[bi-phase modulation assumed] has to be removed to create a cw
signal which is tracked by the PLL. This is accomplished by
multiplying the limiter output (a stream of \( \pm 1 \)'s) to the incoming
signal, and then obtaining the harmonics around \( 2\omega_0 \) by passing the
multiplier output through the zonal filter \( h_2 \). The multiplier output
can be written as:

\[
x(t) \cdot y(t) = \sqrt{2} v(t) \cos (\phi(t) - \gamma(t)) \times
\]

\[
\frac{4}{\pi} \sum_{k=0}^{\infty} \frac{(-1)^k}{2k + 1} \cos \left\{ (2k + 1) \left[ \phi(t) - \gamma(t) \right] \right\}
\]  

(18)
The second harmonic term can be selected from (18) to be the following:

\[ z(t) = \frac{4\sqrt{2}}{3\pi} V(t) \cos \left[ 2 \left( \phi(t) - \gamma(t) \right) \right] \]  

(19)

Note then that in the above discussion the filter \( h_2 \) is assumed a mathematical entity which selects only the second zone component. In reality, a physical bandpass filter can only approximate this condition since the spectrum of \( \cos [2(\hat{\phi}(t) - \gamma(t))] \) extends on all \( \omega \); however, because of the assumed narrow band \( \left( B_1 \ll \omega_0 \right) \) approximation, the error is small. Notice also from (19) that since bi-phase modulation is assumed \( z(t) \) is actually a CW signal at twice the carrier frequency \( \omega_0 \) when the noise effect \( \gamma(t) \) is negligible.

3.6 Weak Signal Suppression

To obtain the loop error signal, the squarer output \( z(t) \) is mixed with the local reference signal

\[ r(t; \phi) = \sqrt{2} \sin 2[\omega_0 t + \phi] \]  

(20)

where \( \phi \) is the phase difference between \( z(t) \) and \( r(t; \phi) \). Omitting \( 4\omega_0 \) terms the error signal that the PLL tracks is then
It is of interest to compare this error signal with that of the perfect squaring loop (see [2], page 59, Equation 2-69)

$$\epsilon_{sq}(t) = K_1 K_m \left\{ \frac{(A-N_s)^2 - N_c^2}{\sqrt{N_c^2 + (A-N_s)^2}} \sin 2\phi + \frac{2 N_c (A-N_s)}{\sqrt{N_c^2 + (A-N_s)^2}} \cos 2\phi \right\}$$

(22)

in which $K_1$, $K_m$ are rms VCO output in volts and multiplier gain respectively. Disregarding the multiplicative constants both $\epsilon(t)$ and $\epsilon_{sq}(t)$ can be written in the form

$$\epsilon(t) = \alpha \sin 2\phi + \beta \cos 2\phi$$

$$\epsilon_{sq}(t) = \alpha' \sin 2\phi + \beta' \cos 2\phi$$

(23)

where the $\alpha$ and $\beta$ are random processes. For the perfect squaring loop case (equation 22) the following equations are readily verified from (10)

$$E \left[ (A-N_s)^2 - N_c^2 \right] = A^2$$

$$E \left[ N_c(A-N_s) \right] = 0$$

(24)
Thus for the perfect squaring loop the mean error signal \( \varepsilon_{sq}(t) \) is directly proportional to \( A^2 \sin 2\phi \), and the term \( \beta \cos 2\phi \) can be considered as a zero mean equivalent noise process. Notice then in the perfect squaring loop case the mean amplitude of the error signal \( e(t) \) does not depend on \( \rho_i \). In addition, the error signal SNR can readily be computed from the first two moments of \( \varepsilon_{sq}(t) \) to be the following

\[
\text{SNR}_{\varepsilon_{sq}} = \frac{\text{E}[\varepsilon^2]}{\text{Var} (\varepsilon)} = \frac{\frac{A^4}{2\sigma_n^2 + \sigma_n^4}}{\frac{A^2}{\sigma_n^2}} = \frac{A^2}{2\sigma_n^2 + \sigma_n^4}
\]

This can then be written in terms of the input SNR as

\[
\text{SNR}_{\varepsilon_{sq}} = \frac{A^2}{1 + 2\rho_i}
\]

where \( \rho_i \) is defined as

\[
\rho_i = \frac{A^2}{\sigma_n^2} = \frac{A^2}{N_0 B_i}
\]

The ratio of \( \text{SNR}_{\varepsilon_{sq}} \) to \( \rho_i \) is given by

\[
\frac{\text{SNR}_{\varepsilon_{sq}}}{\rho_i} = \frac{\rho_i}{1 + 2\rho_i}
\]

which approaches \( 1/2 \) as \( \rho_i \to \infty \), and approaches \( \rho_i \) as \( \rho_i \to 0 \). Figure 3-4 shows a plot of this ratio.
However, in the case of limiter-multiplier implementation of the squaring loop, there is a signal suppression on $e(t)$, which is a function of input SNR $\rho_i$. Suppression in $e(t)$ will affect the loop bandwidth and tracking performance. However, if this can be compensated for by some control of the multiplier or loop gain, then signal suppression on $e(t)$, by itself, will not affect loop performance. The actual performance, of course, will depend on how much the noise power is suppressed relative to the signal power, which is discussed in the next section. To evaluate the SNR for $e(t)$ it is necessary to compute the first two moments of the terms $\alpha$, $\beta$ are concerned $N_c$ and $N_s$ occurs at the same time and thus they are independent Gaussian with variances $\sigma_n^2/2$, and the evaluations of the expectations are reasonably straightforward. The results are

\[
E \left[ \frac{(A-N_s)^2 - N_c^2}{\sqrt{N_c^2 + (A-N_s)^2}} \right] = \frac{3}{8} \sqrt{\pi} \frac{A}{\sqrt{\rho_i}} e^{-\rho_i} F \left[ \frac{5}{2}; 3; \rho_i \right]
\]

(29)

\[
E \left[ \frac{N_c(A-N_s)}{\sqrt{N_c^2 + (A-N_s)^2}} \right] = 0
\]

(30)
\[
E \left[ \frac{(A-N_s)^2 - N_c^2}{\sqrt{N_c^2 + (A-N_s)^2}} \right]^2 = \frac{\sigma_n^2}{2} (1 + \rho_i) + \frac{A^2}{8} \rho_1 e^{-\rho_i} F[4; 5; \rho_i]
\]

\[
E \left[ \frac{N_c (A-N_s)}{\sqrt{N_c^2 + (A-N_s)^2}} \right]^2 = \frac{\sigma_n^2}{16} \left\{ 2(1+\rho_i) - \frac{\rho_1^2}{2} e^{-\rho_i} F[4; 5; \rho_i] \right\}
\]

where \(\rho_i = \frac{A^2}{\sigma_n^2}\), and \(F\) is the confluent hypergeometric function \([6]\)

\[
F[\alpha, \gamma, x] = \sum_{v=0}^{\infty} \frac{(\alpha; 1; v)}{v! (\gamma; 1; v)} x^v
\]

with

\[(m; d; v) = m (m + d) \cdots (m + (v-1)d)\]

and

\[(m; d; 0) = 1\]

For (33) to converge, in general, it is required that \(|x| < 1\) and \(\gamma \neq 0, -1, -2, \ldots\). However, for the cases of interest in Equations (29) to (32) the particular confluent hypergeometric functions involved can be written in terms of simple functions and modified Bessel functions (see [4], p. 1073)

\[
F \left[ \frac{5}{2}; 3; \rho_i \right] = \frac{4}{3} e^{\frac{\rho_1}{2}} \left[ I_0 \left( \frac{\rho_1}{2} \right) + \left( 1 - \frac{1}{\rho_1} \right) I_1 \left( \frac{\rho_1}{2} \right) \right]
\]
\[ F \left[ 4; 5; \rho_i \right] = \frac{4}{\rho_i^4} \left[ e^{\rho_i} \left( \rho_i^3 - 3\rho_i^2 + 6\rho_i - 6 \right) + 6 \right] \] (35)

Written in such forms then it is easy to see that the convergence of (29)-(32) actually does not depend on the value of \( \rho_i \).

Since for large \( \rho_i/2 \)

\[ \Im \left( \frac{\rho_i}{2} \right) = \frac{\rho_i/2}{\sqrt{2\pi \rho_i/2}} \]

thus the mean amplitude of the error signal \( \epsilon(t) \), as \( \rho_i \to \infty \), is given from (21) with the multiplicative constant \( 4/3\pi \) neglected).

\[
\lim_{\rho_i \to \infty} E \left[ \epsilon(t) \right] = \lim_{\rho_i \to \infty} E \left[ \frac{(A-N_s)^2 - N_c^2}{\sqrt{N_c^2 + (A-N_s)^2}} \right] \\
= \lim_{\rho_i \to \infty} 3 \sqrt{\pi} A \sqrt{\rho_i} e^{-\rho_i} \frac{\rho_i}{2} \left[ I_0 \left( \frac{\rho_i}{2} \right) + (1-\frac{1}{\rho_i}) I_1 \left( \frac{\rho_i}{2} \right) \right]
\]

\[ = A \]

(36)

The weak signal suppression factor on \( E[\omega(t)] \) as a function of \( \rho_i \) can be given as

\[
E \left[ \frac{\epsilon(t)}{A} \right] = \sqrt{\frac{\pi}{2}} \sqrt{\rho_i} e^{-\rho_i} \left[ I_0 \left( \frac{\rho_i}{2} \right) + (1-\frac{1}{\rho_i}) I_1 \left( \frac{\rho_i}{2} \right) \right] \] (37)
This relationship is illustrated in Figure 3-3.

Similarity to Equation (28) we can compute the ratio between the SNR of the error signal $e(t)$ and the input SNR $\rho_1$. The following result follows directly from (34) and (35):

\[
\frac{\text{SNR}_e}{\rho_1} = \frac{\left(\frac{3}{8}\sqrt{\pi} \sqrt{\rho_i} e^{-\rho_i} I_0\left(\frac{5}{2}; 3; \rho_i\right)\right)^2}{\frac{1}{4} \left\{2(1 + \rho_i) - \frac{\rho_i^2}{2} e^{-\rho_i} I_0\left(4; 5; \rho_i\right)\right\}}
\]

\[
= \frac{\pi \rho_i e^{-\rho_i} \left[I_0\left(\frac{\rho_i}{4}\right) + (1 - \frac{1}{\rho_i}) I_1\left(\frac{\rho_i}{4}\right)\right]^2}{2 \left\{(1 + \rho_i) - \frac{e^{-\rho_i}}{\rho_i^2} \left[e^{-\rho_i} \left(\rho_i^3 - 3 \rho_i^2 + 6 \rho_i - 6\right) + 6\right]\right\}}
\]

This ratio is compared to that of the perfect squaring loop in Figure 3-4. It is of interest to observe the following limiting cases for the above ratio:

\[
\frac{\text{SNR}_e}{\rho_1} \rightarrow \frac{1}{2} \quad \text{as} \quad \rho_i \rightarrow \infty \quad (39)
\]

\[
\frac{\text{SNR}_e}{\rho_1} \rightarrow \frac{9\pi}{32} \rho_1 \quad \text{as} \quad \rho_i \rightarrow 0 \quad (40)
\]
Figure 3-3. Weak Signal Suppression Factor on Loop Error Signal

\[ \rho_i = \frac{2A^2}{N_0 W_i} \text{, Input SNR, dB} \]
Figure 3-4. Comparisons on Loop Error Signal's SNRs between Two Squaring Loop Implementations.
As far as this ratio is concerned the limiter-multiplier implementation is equivalent to the perfect squaring loop as $\rho_1 \to \infty$ and is 
\[ \sim 0.54 \text{ dB } (9\pi/32) \] worse than the perfect squaring loop as $\rho_1 \to 0$.

3.7 Effective Loop SNR and Squaring Losses

As far as loop performance is concerned the essential factor is the effective loop SNR (see [5]) which depends on the noise power spectral density $N_{\text{eq}}$ of the equivalent noise process in the error signal $e(t)$, around zero frequency. $N_{\text{eq}}$ is defined to be [2]

\[ N_{\text{eq}} = 2 \int_{-\infty}^{\infty} R_{n_{\text{eq}}} (t) \, dt \]  \hfill (41)

Where $n_{\text{eq}}$ is equivalent to zero mean noise processes in $e(t)$ (equations (21) or (22)). For the perfect squaring loop case the auto-correlation function $R_{n_{\text{sq}}}(t)$ of the equivalent noise process in (22) has been computed by Lindsey and Simon [2] to be

\[ R_{n_{\text{sq}}} (t) = 4 [A^2 R_{n_c}(t) + R_{n_c}^2(t)] \]  \hfill (42)

where $R_{n_c}(t)$ is as given in (12). From this the equivalent noise power spectral density around zero frequency $N_{\text{sq}}$ is computed to be

\[ N_{\text{sq}} = 4 A^2 N_0 \mathcal{A}_L^{-1} \]  \hfill (43)
where $N_0$ is the input noise spectral density and $J_L$ is defined to be the circuit squaring loss

$$J_L^{-1} = 1 + \frac{2}{SN_0} \int_{-\infty}^{\infty} n_c^2(\tau) \, d\tau$$  \hspace{1cm} (44)$$

The effective loop SNR $\rho_{\text{eff}}$ for the perfect squaring loop is given \cite{2} in terms of $J_L$ and the equivalent signal-to-noise ratio in the loop bandwidth ($B_L$) of a second order PLL $\rho = A^2/N_0 B_L$ by

$$\rho_{\text{eff}} \text{ (perfect squaring)} = \frac{\rho}{4} J_L$$  \hspace{1cm} (45)$$

To compare the two implementations it is necessary to compute the $N_{eq}$ for the equivalent noise process in (21) and to define the effective squaring loss in the second implementation through Equation (45) by computing the effective loop SNR $\rho_{\text{eff}}$. To obtain $N_{eq}$ in this case, requires computation of the autocorrelation function of equivalent noise term in (21).

$$n_{eq} = 2 \left( \frac{N_0 \cdot (A-N_s)}{\sqrt{N_c^2 + (A-N_s)^2}} \right) \cos 2\phi$$

Assuming $\phi \approx 0$ (for tracking) the autocorrelation $R_{n_{eq}}(\tau)$ is found from
The random variables $N_c$, $N_s$, $N_{cT}$, $N_{sT}$ are jointly Gaussian, zero mean, with covariance matrix (see equation (12))

$$
A = \frac{\sigma_n^2}{2} \begin{pmatrix}
1 & 0 & \gamma_n(\tau) \cos \Delta \theta_{\tau} & \gamma_n(\tau) \sin \Delta \theta_{\tau} \\
0 & 1 & -\gamma_n(\tau) \sin \Delta \theta_{\tau} & \gamma_n(\tau) \cos \Delta \theta_{\tau} \\
\gamma_n(\tau) \cos \Delta \theta_{\tau} - \gamma_n(\tau) \sin \Delta \theta_{\tau} & 1 & 0 \\
\gamma_n(\tau) \sin \Delta \theta_{\tau} & \gamma_n(\tau) \cos \Delta \theta_{\tau} & 0 & 1
\end{pmatrix}
$$

For all practical considerations the correlation times of the input noise process $n(t)$ is much shorter than the modulation process $\theta(t)$, then the actual covariance matrix $A$ of $N_c$, $N_s$, $N_{cT}$, $N_{sT}$ is given by, for all practical considerations, the following:

$$
A = \frac{\sigma_n^2}{2} \begin{pmatrix}
1 & 0 & \gamma_n(\tau) & 0 \\
0 & 1 & 0 & \gamma_n(\tau) \\
\gamma_n(\tau) & 0 & 1 & 0 \\
0 & \gamma_n(\tau) & 0 & 1
\end{pmatrix}
$$
For simplicity in notation, define

\[ x_1 = N_c, \quad x_2 = N_{cT} \]

\[ y_1 = A - N_s, \quad y_2 = A - N_{sT} \]

Then the joint density of \( x_1, y_1, x_2, y_2 \) are given by

\[
p(x_1, y_1, x_2, y_2) = \frac{1}{4\pi^2|\Lambda|^{1/2}} \exp \left( -\frac{1}{2|\Lambda|^{1/2}} \right) \cdot \frac{\sigma_n^2}{2} \left[ (x_1^2 + x_2^2 + (y_1 - A)^2 + (y_2 - A)^2) \right. \\
\left. - 2 \gamma_n(\tau) (x_1 x_2 - (y_1 - A) (y_2 - A)) \right]
\]

Where \(|\Lambda|^{1/2} = \sigma_n^4 / 4 \left[ 1 - \gamma_n^2(\tau) \right]\). When \( A \neq 0 \) the computation of the expectation in (46) involves quite complicated four fold integrals and numerical integration seems to be the only possible method of solution.

If \( A = 0 \) (which is a good approximation to small input SNR cases), the expectation (46) can be evaluated exactly. In terms of the noise envelopes and random phase angles:

\[ V_i = \sqrt{x_i^2 + y_i^2}, \quad \theta_i = \tan^{-1} \left( \frac{y_i}{x_i} \right), \quad i = 1, 2 \]

the expectation (46) can be computed from the following integral:
\[ R_{\text{neq}}(t) = \frac{1}{4\pi^2|\Lambda|^{1/2}} \int \int d\nu_1 d\nu_2 \nu_1^2 \nu_2^2 e^{-\frac{\sigma_n^2(\nu_1^2 + \nu_2^2)}{4|\Lambda|^{1/2}}} \]

\[ \int \int d\varphi_1 d\varphi_2 \sin 2\varphi_1 \sin 2\varphi_2 e^{\frac{\sigma_n^2 r_n(t) \nu_1 \nu_2 \cos(\varphi_2 - \varphi_1)}{-2|\Lambda|^{1/2}}} \]

\[ (51) \]

The double integral on \( \varphi_1 \) and \( \varphi_2 \) can be evaluated directly to be

\[ \int \int d\varphi_1 d\varphi_2 \sin 2\varphi_1 \sin 2\varphi_2 e^{\frac{\sigma_n^2 \gamma_n(t) \nu_1 \nu_2 \cos(\varphi_2 - \varphi_1)}{-2|\Lambda|^{1/2}}} \]

\[ = 2\pi^2 I_2 \left( \frac{\sigma_n^2 \gamma_n(t)}{|\Lambda|^{1/2}} \nu_1 \nu_2 \right) \]

\[ (52) \]

With this simplification then (51) can be evaluated (see [6]) to be

\[ R_{\text{neq}}(t) = \frac{3\sqrt{2\pi}}{8} \cdot \frac{\sigma_n^2}{2} \cdot \frac{\gamma_n^2(t)}{\left(1 - \gamma_n^2(t)\right)^{3/2}} \]

\[ \times \left\{ 4 F \left[ \frac{5}{4}; \frac{7}{4}; 1; \chi \right]^2 + 5 F \left[ \frac{7}{4}; \frac{9}{4}; 2; \chi \right]^2 \right\} \]

\[ \left(53\right) \]
where we have defined

$$\chi = \frac{\gamma_n^2(\tau)}{4 \gamma_n^2(\tau)}$$  \hspace{1cm} (54)$$

and where the confluent hypergeometric function $F$ is defined through

$$F[a; \beta; \gamma; x] = \sum_{v=0}^{\infty} \frac{(a;1;\nu)(\beta;1;\nu)}{\nu!(\gamma;1;\nu)} x^v$$  \hspace{1cm} (55)$$

where $(x;y;k)$ is defined in (33). The effective loop SNR can then be computed, with this approximation for small SNR cases, from (37) and (53) to be

$$\sigma_{\text{eff}}' = A^2 \frac{\sqrt{\pi}}{2} \sqrt{\frac{\rho_i}{2}} e^{-\frac{\rho_i}{2}} \left[ I_0(\frac{\rho_i}{2}) + (1 - \frac{1}{\varphi_i}) I_1(\frac{\rho_i}{2}) \right]$$

$$\frac{2}{\nu_{\text{eq}} B_L}$$  \hspace{1cm} (56)$$

where $B_L$ is the one-sided loop bandwidth and $\nu_{\text{eq}}$ is the equivalent noise spectral density computed from $R_{\text{eq}}^{\nu}(\tau)$. Writing $\sigma_n^2/2$ as $N_0 W_i/4$ (53) can be written as

$$R_{\nu_{\text{eq}}}(\tau) = N_0 W_i g(\tau)$$  \hspace{1cm} (57)$$

where
\[ g(\tau) = \frac{3\sqrt{\pi}}{32} \gamma_n^2(\tau) \frac{(1 - \gamma_n^2(\tau))^2}{(2 - \gamma_n^2(\tau))^{5/2}} \cdot \{ 4 F \left[ \frac{5}{4}; \frac{7}{4}; 1; \chi^2 \right] + 5 \chi \ F \left[ \frac{7}{4}; \frac{9}{4}; 2; \chi^2 \right] - F \left[ \frac{5}{4}; \frac{7}{4}; 2; \chi^2 \right] \} \] (58)

and

\[ N_{eq} = 2N_0 \int_{-\infty}^{\infty} g(t) d(W_1 t) \] (59)

Then the effective loop SNR can be written in the same form as (45):

\[ \rho_{eff}' = \frac{A^2}{N_0 B_L} \cdot \frac{1}{4} \cdot \mathcal{E}'_L \]

where the equivalent squaring loss \( \mathcal{E}'_L \) can be written as

\[ \mathcal{E}'_L = \frac{\pi}{4} \rho_i e^{-\rho_i} \int \left[ I_0 \left( \frac{\rho_i}{2} \right) + (1 - \frac{1}{\rho_i}) I_1 \left( \frac{\rho_i}{2} \right) \right]^2 g(t) d(W_1 t) \] (60)

where \( g \) is defined in (58).

3.8 Numerical Results

As an example, consider a bandpass filter with an RC transfer function. The equivalent low pass spectrum for \( N_c(t) \) or \( N_s(t) \) has correlation functions:

\[ R_{Nc}(t) = R_{Ns}(t) = \frac{N_0 W_1}{4} \exp (-W_1 |t|) \] (61)
Assuming signal distortion due to filtering is negligible, then the squaring loss for an ideal squaring loop for this $R_{N_c}(t)$ is computed [3] to be:

$$\mathcal{L}_i = \frac{1}{1 + \frac{1}{4\rho_1}}$$

(62)

For the same correlation function the equivalent circuit squaring loss for the limiter/multiplier implementation can be computed from (60), where the integration on $g(t)$ can be performed numerically. This result is plotted as a function of $\rho_1$ together with equation (62) on Figure 3-5. It is noted that the limiter/multiplier implementation has relatively more squaring loss than the ideal squaring loop for low input SNR cases, which is expected. However, it is interesting to note that as $\rho_1 \to 0$ the difference between the two squaring losses asymptotically approaches $\approx 0.8$ dB.

As $\rho_1$ becomes large, the $A \approx 0$ approximation is no longer valid. However, it is seen from the definitions of definition of $N_{eq}(t)$ in (21) that

$$N_{eq} \to 2N_c(t) \quad \text{as} \quad \rho_1 \to \infty$$

and

$$R_{eq}(\tau) \to 4R_{Nc}(\tau) \quad \text{as} \quad \rho_1 \to \infty$$

$$N_{eq} \to 4N_0 \quad \text{as} \quad \rho_1 \to \infty$$
Figure 3-5. Comparison on Squaring Losses Between Two Squaring Loop Implementations

Assume RC Filter with

\[ R_N(\tau) = \frac{N^2 W_i}{4} \exp(-W_i |\tau|) \]
On the other hand, since the signal suppression factor approaches unity as to \( \rho_i \rightarrow \infty \), the effective loop SNR approaches, as \( \rho_i \rightarrow \infty \):

\[
\rho'_{\text{eff}} = \frac{2}{\rho} = \frac{\frac{A^2}{4N_{0B_L}}}{4} \text{ as } \rho_i \rightarrow \infty
\]

From (63), it is clear that \( \rho'_{\text{eff}} \) approaches unity, as was in the case of the ideal squaring loop case (eq. (62)). Therefore, we conclude that the loops have identical tracking performance at high signal-to-noise ratios.

4.0 SYMBOL SYNCHRONIZER TRADEOFFS

In this section of the report various symbol synchronization techniques are discussed and tradeoffs made. The classes of symbol synchronization systems covered in the discussions are:

- Harmonic generating/tracking symbol synchronizers
- MAP symbol synchronizers
- A class of early-late gate type symbol synchronizers
  (absolute value, least of squares, I-Q loops)
- I-Q loop symbol synchronizers
- Digital data transition tracking loop synchronizers
- Symbol sync systems operate in two distinct modes: the signal or sync acquisition mode and the synchronous or tracking mode

(Reference 2, Chapter 10). The signal acquisition mode relates to
system performance during the time the clock signal is being established, while the tracking mode relates to system performance as data detection is being accomplished. Each mode has fundamental physical restrictions and characteristics. The best performance is achieved when these two modes of operation are as independent as possible. Performance indices are different for the two modes. Those considered in the following discussion are:

<table>
<thead>
<tr>
<th>Sync Acquisition Mode</th>
<th>Synchronous or Tracking</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Sync acquisition range</td>
<td>• Symbol sync jitter</td>
</tr>
<tr>
<td>• Sync acquisition time</td>
<td>• Symbol slippage rate</td>
</tr>
<tr>
<td>• Probability of sync acquisition</td>
<td>• Symbol error probability degradation</td>
</tr>
<tr>
<td>• Cost/Complexity of hardware</td>
<td>• Static phase behavior</td>
</tr>
<tr>
<td></td>
<td>• Cost/complexity of hardware</td>
</tr>
</tbody>
</table>

Symbol synchronizers can be implemented with either analog or digital circuits. The performance of each has been compared and it can be shown to be approximately equal, with the digital implementation exhibiting only a very small degradation in accuracy, Ref. 4. The advantages of digital mechanization are stability, accuracy, and reliability.

4.1 Harmonic Generating-Tracking Symbol Synchronizers

Perhaps the oldest approach to the problem of producing symbol synchronization from the information bearing signal is
illustrated in Figure 4-1. Since symbol sync information is carried in the data transitions, a nonlinear operation (e.g., a differentiating circuit followed by a rectifier) can be used to generate a clock component whose strength depends upon the data transition density, the signal data power, and the bit rate. For Manchester encoded data, the ratio of the power in the clock component to the total data power is given by

$$\frac{P_c}{P_{total}} = \frac{31(1-p(1-p))^2}{R^2}$$

where $p$ is probability of occurrence of a "1" in the data stream and $R$ is the bit rate. The unfortunate feature of such a system is that the differentiation of a noisy signal produces a noisier signal and also generates a sizable self-jamming/self-noise component whose power is also proportional to the bit rate squared. For high input SNR's and fixed data rate signals, such as high-rate QPSK where $E_b/N_0 = 5$ to $10$ dB and the data rate is fixed, the
thermal effects and the self-noise generated can be simultaneously tolerated. However, at low input SNR's, $E_b/N_0 < 3$ dB, the self-noise term and thermal noise term are of the same order and marked degradation (6 dB) in system performance results. Thus at high noise levels relative to that of the data signal, the data transitions are erroneously generated, and the phase-locked oscillator tends to follow the noise rather than the clock component.

An alternate harmonic tracking symbol synchronizer is illustrated in Figure 4-2. The noise performance of this type of symbol

![Image](image)

Figure 4-2. Symbol Synchronizer for Digital Data and High SNR.

synchronizer is known to be superior to that of Figure 4-1 as the operation does not depend on the data transitions of the signal component directly. The synchronizer does, however, exhibit a sharp performance threshold for low input SNR's due to the product of the noise with its delayed version. Superior noise performance can be obtained from the symbol synchronizers discussed below, and no further reference will be made to this class.
4.2 Symbol Synchronizer Based Upon MAP Estimation Theory

The process of extracting the necessary timing from the information-bearing waveform for coherent detection of the digital stream has been referred to as "data derived sync" or "self-sync". The problem of estimating symbol sync from a received signal-plus-noise can be optimally handled using the theory of maximum likelihood estimation for an unknown parameter in Gaussian noise (Reference 2, Chapter 9). In this theory one assumes complete knowledge of an observed sequence of $K$ data bits and proceeds to arrive at the maximum likelihood estimate of bit sync. A practical interpretation of this in the time domain suggests correlating the received signal with a stored replica of the basic received pulse shape and taking the log hyperbolic cosine of this result and accumulating these values over all possible bit sync possibilities. The value which yields the largest accumulated value is then declared the best symbol sync estimate.

Unfortunately, this approach to symbol sync suffers from two difficulties. First, the values for the symbol sync epoch take on a continuum of values over the bit duration. To practically apply this theory to the development of a practical PCM bit sync system, one must quantize the interval $[0, T]$ into a number of levels and perform a parallel search procedure over these allowable bit sync positions.
 Needless to say, a parallel search is prohibitive of equipment while a serial search is prohibitive in terms of bit sync acquisition time. The second disadvantage of applying this theory to the development of a practical symbol sync system has to do with the fact that it is open loop, i.e., no provision is made in the theory for operation in the tracking mode. The symbol sync jitter performance of the optimum maximum likelihood estimate synchronizer in terms of the ratio of energy per bit to noise spectral density (ST/N₀) for various values for K and various pulse waveform shapes is given in Reference 2, Chapter 9, pp. 420-429.

To include the tracking function into the requirements of a symbol sync system, various authors (Reference 2, Chapter 9) propose tracking the symbol sync with a closed loop system whose error signal, generated from the open loop maximum likelihood estimate, is used to control a voltage control oscillator (VCO). We now discuss a large class of symbol sync systems which are based on estimation theory.

The functional diagram of a closed loop symbol synchronizer based on MAP estimation theory is illustrated in Figure 4-3. The phase of the timing pulse generator, which controls the start and termination of the integrate-and-dump circuits, is bumped every T sec by an amount proportional to the magnitude of the gradient of the a posteriori estimate of the bit sync epoch $\xi$ and in a direction
based on its sign as computed for the previous KT sec. Notice the nonlinearity \( \tanh x \) in the in-phase channel. We note for large SNR's that \( \tanh x = \text{sgn} x \), while for small SNR's, \( \tanh x = x \). For high SNR's the synchronizer is reminiscent of the classical I-Q bit synchronizer.

Other symbol synchronizers based on MAP estimation theory are of the early-late gate type. The functional diagram for an early-late gate type symbol synchronizer which destroys the data modulation by taking absolute values to generate a tracking S-curve is illustrated in Figure 4-4. In Figure 4-5 an early-late gate symbol synchronizer with square-law type of nonlinearity is illustrated. The detailed analysis of performance can be found in Reference 2, Chapter 9, pp. 458-466.

4.3 Symbol Synchronizers Motivated by Optimum Design of the Phase Detector

There are several approaches to the problem of selecting an
Figure 4-4. Functional Block Diagram of an Early-Late Gate Type of Symbol Synchronizer with Absolute Value Type of Nonlinearity.

Figure 4-5. An Early-Late Gate Type of Bit Synchronizer with Square-Law Type of Nonlinearity.

optimum phase detector characteristic. What is optimum depends on the performance measure chosen to represent system behavior. For example, during the signal acquisition mode, the performance
measures are acquisition time and range, and probability of acquisition. After symbol sync has been acquired, attention is focused on the tracking mode where measures such as mean squared bit sync jitter, mean time to first loss of sync, and bit slip rate become significant. In the past, several approaches have been addressed:

- **Acquisition mode**
  - Choice of the phase detector characteristic to maximize acquisition range (Reference 7, Chapter 10)
  - Choice of the phase detector characteristic to minimize acquisition time (Reference 7, Chapters 10 & 11)

- **Tracking Mode**
  - Minimization of the area under the tail of the symbol sync error probability density function (Reference 2, Chapter 10)
  - Minimization of the $k^{th}$ absolute central moment of the symbol sync error pdf

### 4.4 Digital-Data Transition Tracking Symbol Synchronizers

A symbol sync system originally proposed by Lindsey and Tausworthe, Reference 1 (and now operational in the Deep Space Network and elsewhere) for demodulation of coded telemetry signals is the **digital data transition tracking loop** illustrated in Figure 4-6. This was based on an optimum design of the phase detector in the tracking mode.

In this design, the input noise-free signal $s(t, \epsilon)$ is a random pulse train characterized specifically as the rectangular pulse...
Figure 4-6. The Digital Data Transition Loop (DTTL).

defined by \( p_s(t) = \sqrt{S} \) for \( 0 \leq t \leq T \), \( p_s(t) = 0 \) for all other \( t \). The sum of this signal pulse noise \( n(t) \) is passed through two parallel branches, which are triggered by a timing pulse generator according to a digitally filtered version of an error signal formed from the product of the branch outputs. Furthermore, two branches are held at a fixed phase relationship with one another by the timing generator. Basically, the in-phase branch monitors the polarity of the actual transitions of the input data and the mid-phase branch obtains a measure of the lack of sync. The particular way in which these two pieces of information are derived and combined to synchronize the loop is described in detail in Reference 3, Chapter 9, pp. 422-458.
4.5 Performance Comparison for Various Symbol Synchronizers

4.5.1 Tracking Mode/RMS Symbol Sync Jitter

In comparing the performance of several different bit sync configurations, one must choose a fixed operating condition that is common to all. For the comparison we set the SNR $R_d = \frac{E_b}{N_0} = -5 \text{ dB} (-2 \text{ dB in a one-half bit rate bandwidth})$, i.e., the Shuttle threshold design point, and considered the normalized rms bit sync jitter $\sigma_x$ as the performance measure. Since it is known (Reference 2) that the difference of square loop performance is inferior to the absolute value type synchronizer as well as the digital data transition loop synchronizer, the performance comparison is only made here between these latter two. Furthermore, it has been shown (Reference 2) that the optimum setting of the earlier integrator relative to the late integrator in the absolute value synchronizer is $\Delta_0 \simeq \frac{1}{4}$.

Table 4-1 compares the rms jitter performance at the Shuttle brassboard design point of the digital data transition synchronizer to that of the absolute value synchronizer for various normalized loop bandwidths; that is, the loop bandwidth times the bit duration $B_L T$; and two different window widths, $\varepsilon_0$, in the digital data transition loop synchronizer. It is significant that:
Table 4-1. Comparison of the Absolute Value Synchronizer to DTTL Synchronizer for Two Different Window Widths

<table>
<thead>
<tr>
<th>ρ₀</th>
<th>0.05</th>
<th>0.0167</th>
<th>0.0143</th>
<th>0.0111</th>
<th>0.0100</th>
<th>0.50</th>
<th>0.25</th>
<th>0.05</th>
<th>0.0167</th>
<th>0.0143</th>
<th>0.0111</th>
<th>0.0100</th>
</tr>
</thead>
<tbody>
<tr>
<td>AV; δ₀ = 1/4</td>
<td>DTTL; ϵ₀ = 1/2</td>
<td>DTTL; ϵ₀ = 1/4</td>
<td>AV (δ₀ = 1/4)</td>
<td>DTTL (δ₀ = 1/2)</td>
<td>DTTL (δ₀ = 1/4)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>R_d = E/R_0</td>
<td>B_4 TLs</td>
<td>δ₀</td>
<td>δ₀</td>
<td>ϵ₀</td>
<td>ϵ₀</td>
<td>360δ₀, degrees</td>
<td>AV (δ₀ = 1/4)</td>
<td>DTTL (δ₀ = 1/2)</td>
<td>DTTL (δ₀ = 1/4)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-5dB</td>
<td>1.0</td>
<td>19.8855</td>
<td>17.566</td>
<td>11.94</td>
<td>43</td>
<td>2.2 dB</td>
<td>0.53 dB</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-5dB</td>
<td>0.50</td>
<td>14.0470</td>
<td>12.405</td>
<td>8.44</td>
<td>30</td>
<td>2.2 dB</td>
<td>0.53 dB</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-5dB</td>
<td>0.25</td>
<td>9.9327</td>
<td>8.79</td>
<td>5.97</td>
<td>22</td>
<td>2.2 dB</td>
<td>0.53 dB</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-5dB</td>
<td>0.05</td>
<td>4.4420</td>
<td>3.93</td>
<td>2.67</td>
<td>9.6</td>
<td>2.2 dB</td>
<td>0.53 dB</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-5dB</td>
<td>0.0167</td>
<td>2.5566</td>
<td>2.27</td>
<td>1.54</td>
<td>5.5</td>
<td>2.2 dB</td>
<td>0.53 dB</td>
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AV = Absolute value synchronizer
DTTL = Digital data transition loop synchronizer

The absolute value synchronizer rms bit sync jitter is 2.2 dB inferior to that of the digital data transition loop synchronizer with \( \Delta_0 = \xi_0 = \frac{1}{4} \) for \( R_d = -5 \text{ dB} \).

The absolute value synchronizer loop rms bit sync jitter is 0.53 dB inferior to that of the digital data transition loop synchronizer with \( \Delta_0 = \frac{1}{4}, \xi_0 = \frac{1}{2} \) for \( R_d = -5 \text{ dB} \).

Therefore, on the basis of this performance measure the digital data transition tracking loop synchronizer is the recommended mechanization and was selected for the hardware simulation.

4.6 Acquisition Performance Comparisons for the DTTL and Absolute Value Type Symbol Synchronizers

This section compares the acquisition performance of DTTL and absolute value synchronizers. Since the theory of signal
acquisition in the presence of noise is not complete, we present here the key results obtained by computer simulation. Here acquisition time was determined as a local clock frequency. The time to acquire was defined as the time to stop slipping cycles plus the time for the loop to settle in phase to 5% of a bit period. Figure 4-7 plots the acquisition performance of the two bit synchronizers under consideration for three different input conditions. These are with a noise free square wave input, with a noise free NRZ input, and with noise random NRZ with an $E_\text{n}/N_0 = 2.5$ dB. In addition,

![Graph of Acquisition versus Frequency Offset to Bandwidth Ratio]

Figure 4-7. Acquisition versus Frequency Offset to Bandwidth Ratio. the acquisition performance of a continuous wave PLL as taken from Lindsey (Reference 7) is shown. Several conclusions should be noted:
• The amount of frequency offset that can be tolerated is much less than one would predict using the standard sinusoidal PLL theory.

• At low SNR's, acquisition times can be excessive and the acquisition range is considerably degraded.

• The acquisition time, for a given input frequency offset, is strongly dependent upon whether the input data is periodic, random, noise-free, or noisy.

• The DTTL loop is superior to the early-late gate absolute value synchronizer under all input conditions.

• With random NRZ, the DTTL can tolerate nearly four times more frequency uncertainty than that of the absolute value synchronizer.

• The acquisition time is highly dependent on the transition density in the data stream.

On the basis of these results, the digital data transition tracking loop synchronizer is again the recommended approach for hardware simulation as it is capable of outperforming the absolute value type in both the acquisition mode and tracking mode.

4.7 Bit Error Probability Degradation Due to Symbol Sync Jitter

Because the effects of bit sync jitter on the bit error probability remain an unsolved theoretical problem in systems which employ convolutional codes, we present here the effects of bit sync jitter...
on the performance of uncoded coherent communications which use Manchester coding. These curves, illustrated in Figure 4-8 represent plots of the bit error probability versus $R_d = ST/N_0$ for various values of the normalized rms bit sync jitter $\sigma_\alpha$.

![Figure 4-8. BER versus SNR for Various Values of Jitter.](image)

### 4.7.1 Summary of Selected Symbol Synchronizer Configurations

Based on the foregoing discussions, symbol synchronization in the presence of frequency offset and low SNR's is best accomplished by window-optimized digital data transition tracking loop
synchronizer. This synchronizer outperforms a class of early-
late gate type symbol synchronizers which is illustrated in
Table 4-2. Based upon the tradeoffs given in this section LinCom
has selected the DTL for hardware simulation.

5.0 POWER DIVISION ANALYSIS OF A THREE CHANNEL
UNBALANCED QPSK SIGNAL OUT OF A BANDPASS
LIMITER

5.1 Summary

This part of the report is concerned with the Shuttle Ku Band
transmitter design which three channels are proposed for use in
transmitting data. Since a three channel signal gives rise to a
noncoherent signal envelope, it is of interest (due to AM to PM
conversion effects which may occur in the TDRS repeater) to process
the combined signal via a bandpass limiter prior to transmission;
thus a constant envelope signal results.

This section derives the exact power split for arbitrary
modulation indices which is obtained in the high rate, medium
rate, low rate and intermodulation channels after limiting the
three channel signal. For the 80/20 power split, the exact
values of power division is computed and summarized. For this
special case it is concluded that the percentage of power in the
high and medium rate channels remains largely in tact after
limiting and that signal suppression in the low rate channel
takes place (approximately 1.5 dB). The power in the inter-
The modulation term is .50% of the total power transmitted. It is demonstrated that the intermodulation signal component is in phase with the signal in the high rate channel.

5.2 Introduction

At the present time, data transmission from the Shuttle orbiter via the Ku band link will employ two modulation schemes. One involves an all digital transmission scheme in which high data rates are involved. The other scheme involves transmission of low-rate digital data combined with analog TV data via analog FM modulation. In either case, transmission of the digital data will employ unconventional quadrature phase shift keyed (QPSK) modulation scheme in which the two quadrature components are unbalanced with respect to data rate and power.

This report is concerned with the first modulation scheme in which three channels are to be used to transmit the data. The functional model of the proposed Shuttle Ku band orbiter transmitter is depicted in Figure 5.1. As noted from this figure, three data sources \( d_1(t) \), \( d_2(t) \) and \( d_3(t) \) serve to modulate the carrier oscillation \( \cos \omega_c t \). Data source \( d_1(t) \) is presumed to be the high data rate source (on the order of 100 megasymbols/sec), \( d_2(t) \) represents a medium rate source, (on the order of a few 2 to 4 megasymbol/sec) and \( d_3(t) \) a low rate source (on the order of 200 kilosymbols/sec). At point 1 in Figure 5.1 an unbalanced QPSK
signal emerges. This signal is then used to phase modulate the quadrature component of the carrier oscillation \( \cos \omega_s t \). Data source \( d_1(t) \) is used to phase modulate the carrier so that the two quadrature RF oscillations at points 2 and 3 in Figure 5.1 are summed and gain adjusted to give the desired power split and signal at point 4. Since the envelope of the signal at point four is not constant and since AM to PM conversion is likely to take place in the transmitters power amplifier or in the TDRS itself, it is of interest to study the deleterious effects created by insertion of a bandpass limiter/zonal filter in the signal path. This will serve to create a constant envelope signal at the transmitter output. In particular, it is of interest to:

1. study the three channel power split out of the limiter versus the input power split;
2. to determine the power in the intermodulation distortion term as well as its phase relationship with respect to the three data components. In addition it is of interest to look at a Costas type receiver structure which can be used to demodulate and synchronize to the received signal.

5.3 Power Division Analysis Out the Bandpass Limiter

Consider the modulator block diagram illustrated in Fig. 5-1 whose purpose is to generate a signal for simultaneous transmission of three channels of information. For example, two of the channels represent independent data channels (one having a data rate up to
Figure 5.1. Shuttle Orbiter Transmitter Functional Model.
100 Msps while the other has a rate up to 2 Mbps) and the third
channel consists of operational data at a rate of 192 Kbps. The
structure of the modulator is such as to form an unbalanced quadri-
phase signal wherein the high rate data signal $\sqrt{P_1}d_1(t)$ is biphase
modulated on the in phase carrier $\sqrt{2}\cos w_c t$ and the sum of the
two lower rate signals $\sqrt{P_2}d_2(t)$ and $\sqrt{P_3}d_3(t)$ after being modulated
onto separate square-wave subcarriers $S_{q2}(t)$ and $S_{q3}(t)$, are added
and biphase modulated onto the quadrature carrier $\sqrt{2}\sin w_c t$. The
sum of the in-phase and quadrature modulated carriers, i.e., the
unbalanced and quadriphase signal $s(t)$ is then power amplified
where the power amplifier is assumed to have a bandpass, hard-
limiting gain characteristic.

In view of the above comments the signal generated at the
transmitter output, point 5 in Figure 5-1, can be conveniently
modeled by

$$s(t) = \sqrt{2} \left[ \sqrt{P_1}d_1(t) \cos w_c t + \right] \frac{\text{High Data Rate Component}}{
\left[ \sqrt{P_2}S_{q2}(t)d_2(t) + \sqrt{P_3}S_{q3}(t)d_3(t) \right] \sin w_c t} \frac{\text{Medium Rate Component}}{ \text{Low Rate Component}} \frac{\text{High Data Rate Component}}{\text{Medium Rate Component}} \frac{\text{Low Rate Component}}{1}$$

This signal can be viewed as a doubly unbalanced QPSK trans-
mission in that the term in the brackets in (1) represents an
unbalanced QPSK signal at the subcarrier level while the sum of the two terms in (1) represent an unbalanced QPSK signal at the RF level. It is convenient to define the terms

\[ \begin{align*}
C(t) & \triangleq \sqrt{P_1}d_1(t) \triangleq \sqrt{P_1}s_1(t) \\
S(t) & \triangleq \sqrt{P_2}S_2(t)d_2(t) + \sqrt{P_3}S_3(t)d_3(t) \\
& \triangleq \sqrt{P_2}s_2(t) + \sqrt{P_3}s_3(t)
\end{align*} \]

(2)

and \( s_1(t) \), \( s_2(t) \) and \( s_3(t) \) are \( \pm 1 \) binary waveforms. Thus (1) can be rewritten in the form

\[ s_1(t) = \sqrt{2}[C(t) \cos \omega c t + S(t) \sin \omega c t] \]

(3)

Alternately, in polar coordinates (amplitude and phase), Eq. (1) can be rewritten as

\[ s_1(t) = \sqrt{2}V(t)\sin(\omega_c t + \gamma(t)) \]

(4)

where

\[ V(t) \triangleq \sqrt{C^2(t) + S^2(t)} \]

\[ \phi(t) \triangleq \tan^{-1} \frac{C(t)}{S(t)} \]

(5)

It is well known, Ref. 7, Appendix of Chapter 4, passing \( s(t) \) of Eq. (4) through a bandpass, hard limiter preserves the phase \( \phi(t) \).
Thus, the resultant first zone output of the limiter is given by

\[ s_1(t) = \sqrt{2P} \sin(\omega t + \gamma(t)) \]  

(Ref. 7)

where \( P \) is the total power in the first zone after amplification.

In terms of in-phase and quadrature components, Eq. (6) can be rewritten as

\[ s_1(t) = \sqrt{2P} \left[ \frac{S(t)}{V(t)} \sin \omega t \sqrt{\frac{C(t)}{V(t)}} \right] \cos \omega t \]

(7)

From (3) and (5), we have that

\[ V(t) = \sqrt{P_1s_1(t) + P_2s_2(t) + P_3s_3(t) + 2\sqrt{P_2P_3}s_2(t)s_3(t)} \]

(8)

Let \( P_T = P_1 + P_2 + P_3 \) denote the total input power. Then,

\[ V(t) = \sqrt{P_T \left( 1 + 2\sqrt{\frac{P_2}{P_T}}\frac{P_3}{P_T} \frac{s_2(t)s_3(t)}{V(t)} \right)} \]

(9)

or equivalently

\[ \frac{1}{V(t)} = \frac{1}{\sqrt{P_T}} \left\{ \frac{1}{2} \left[ \frac{1}{\sqrt{1 + 2\sqrt{\frac{P_2}{P_T}}\frac{P_3}{P_T}}} + \frac{1}{\sqrt{1 - 2\sqrt{\frac{P_2}{P_T}}\frac{P_3}{P_T}}} \right] - s_2(t)s_3(t) \frac{1}{2} \left[ \frac{1}{\sqrt{1 - 2\sqrt{\frac{P_2}{P_T}}\frac{P_3}{P_T}}} - \frac{1}{\sqrt{1 + 2\sqrt{\frac{P_2}{P_T}}\frac{P_3}{P_T}}} \right] \right\} \]

(10)
Introducing the following definitions:

\[
C_1 \triangleq \frac{1}{2} \left[ \frac{1}{\sqrt{1+2\sqrt{\frac{P_2}{P_T}} \sqrt{\frac{P_3}{P_T}}} + \frac{1}{\sqrt{1-2\sqrt{\frac{P_2}{P_T}} \sqrt{\frac{P_3}{P_T}}}} \right] \tag{11}
\]

\[
C_2 \triangleq \frac{1}{2} \left[ \frac{1}{\sqrt{1-2\sqrt{\frac{P_2}{P_T}} \sqrt{\frac{P_3}{P_T}}} - \frac{1}{\sqrt{1+2\sqrt{\frac{P_2}{P_T}} \sqrt{\frac{P_3}{P_T}}}} \right]
\]

(11)

Into (2) and (11) into (7) and (10) yields the signal model into the bandpass limiter of Fig. 5-1, viz.,

\[
s_i(t) = \sqrt{2P} \left[ C_1 \sqrt{\left( \frac{P_1}{P_T} \right) - C_2 \sqrt{\frac{P_3}{P_T}}} s_2(t) + C_1 \sqrt{\left( \frac{P_3}{P_T} \right) - C_2 \sqrt{\frac{P_2}{P_T}}} s_3(t) \sin \omega t \right]
\]

+ \sqrt{2P} \left[ C_1 \sqrt{\frac{P_1}{P_T}} s_1(t) - C_2 \sqrt{\frac{P_1}{P_T}} s_1(t) s_2(t) s_3(t) \cos \omega t \right] \text{ (12)}

Let us now define the quantities:

\[
\tilde{P}_1 = \left( \frac{P}{P_T} \right) P_1 C_1^2 \text{ (High Rate)}
\]

\[
\tilde{P}_2 = \left( \frac{P}{P_T} \right) \left[ C_1 \sqrt{\frac{P_2}{P_T}} - C_2 \sqrt{\frac{P_3}{P_T}} \right]^2 \text{ (Medium Rate)}
\]

\[
\tilde{P}_3 = \left( \frac{P}{P_T} \right) \left[ C_1 \sqrt{\frac{P_3}{P_T}} - C_2 \sqrt{\frac{P_2}{P_T}} \right]^2 \text{ (Low Rate)}
\]

\[
\tilde{P}_d = \left( \frac{P}{P_T} \right) P_1 C_2^2 \text{ (Intermodulation)} \text{ (13)}
\]
In view of (13) we can rewrite (12) as

\[ s_0(t) = \sqrt{2} \left[ \sqrt{P_1} s_1(t) - \sqrt{P_d} s_1(t) s_2(t) s_3(t) \right] \cos \hat{\phi}(t) \]

**High Data Rate Term**

\[ \sqrt{2} \left[ \sqrt{P_2} s_2(t) + \sqrt{P_3} s_3(t) \right] \sin \hat{\phi}(t) \]

**Medium Rate Term**

**Low Rate Term**

where \( \hat{\phi}(t) = \omega_c t \). Thus, we see that under the assumptions of ideal alignment of the transmitter in-phase/quadrature channels, that the limiters first zonal output consists of four terms instead of the three which appeared at its input. In particular, the intermodulation term in (14) is in phase with the high data rate signal component and there is a corresponding adjustment of powers in the individual data signals which can be assessed exactly via (13).

### 5.3.1 The 80/20 Power Split Division After Limiting

If we arbitrarily set the output level of the power amplifier equal to the total input power level, i.e., \( P = P_T \), then for fixed values of \( P_1/P_T \), \( P_2/P_T \), and \( P_3/P_T \), one can compute from (13) the corresponding values of \( \bar{P}_1/P_T \), \( \bar{P}_2/P_T \), \( \bar{P}_3/P_T \), and \( \bar{P}_d/P_T \) for the transmitted unbalanced quadriphase signal of (12). For example, suppose the high rate channel contains 80% of the total power while the remaining 20% is split with 80% going to the next highest rate channel and 20% to the lowest rate channel. Equivalently
we have, into the limiter that,

$$\frac{P_1}{P_T} = .8 = -.969 \text{ dB (High Rate)}$$

$$\frac{P_2}{P_T} = .16 = -7.96 \text{ dB (Medium Rate)}$$

$$\frac{P_3}{P_T} = .04 = -13.98 \text{ dB (Low Rate)}$$

(15)

and out of the limiter we obtain from (11), (12) and (13)

$$\hat{\frac{P_1}{P_T}} = .8157 = -.885 \text{ dB}$$

$$\hat{\frac{P_2}{P_T}} = .1503 = -8.23 \text{ dB}$$

$$\hat{\frac{P_3}{P_T}} = .0287 = -15.4 \text{ dB}$$

$$\hat{\frac{P_d}{P_T}} = .00529 = -22.7 \text{ dB}$$

(16)

Upon comparing (15) with (16) we see that there is very little change

in the powers in the high rate and medium rate channels; however,

due to the weak signal effect inherent in the limiter, it has

suppressed the power in the low rate channel by approximately

1.5 dB. Moreover, the ratio of the individual powers of the

signal components at the input to the limiter to their output values is
\[ \frac{\hat{P}_1}{P_1} = .981 = -.0833 \text{ dB} \]
\[ \frac{\hat{P}_2}{P_2} = .939 = -.27 \text{ dB} \]
\[ \frac{\hat{P}_3}{P_3} = .7175 = -1.44 \text{ dB} \]
\[ \frac{\hat{P}_d}{P_1} = .0066 = .06\% \]

Therefore, the presence of the limiter suppresses the power in the weaker (low data rate) channel by 1.44 dB, the medium rate channel by 0.27 dB and the high rate channel by 0.08 dB. The power in the intermodulation product is 0.5% of the total power and it is in phase with the high rate data channel.
REFERENCES


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**PDU = 0.1398E 01**  
**PUC = 0.1354E 01**

**TOLERANCE = 0.100**

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<td>ALFA</td>
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<tr>
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</tr>
<tr>
<td>0.1000E-02</td>
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<tr>
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<td>0.0100E-02</td>
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**TOLERANCE**

**ALFA:**
- 0.3972E-01
- 0.3792E-01
- 0.3704E-01
- 0.3580E-01
- 0.3486E-01

**BETA:**
- 0.3972E-01
- 0.3792E-01
- 0.3704E-01
- 0.3580E-01
- 0.3486E-01
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TOLERANCE = $10.0$

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<tr>
<td>$(\hat{r})$</td>
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<tr>
<td>Beta</td>
<td>0.00E+00</td>
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| TOLERANCE = $2.5$ |

<table>
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<tr>
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<tr>
<td>Beta</td>
<td>0.00E+00</td>
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</tbody>
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| TOLERANCE = $5.0$ |

<table>
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<tbody>
<tr>
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<td>0.50E+00</td>
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<tr>
<td>Beta</td>
<td>0.00E+00</td>
<td>0.00E+00</td>
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| TOLERANCE = $10.0$ |

<table>
<thead>
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<th>$0.18E+02$</th>
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<tr>
<td>Beta</td>
<td>0.00E+00</td>
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| TOLERANCE = $100.0$ |

<table>
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<tbody>
<tr>
<td>$(\hat{r})$</td>
<td>1.00E+00</td>
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<td>0.50E+00</td>
</tr>
<tr>
<td>Beta</td>
<td>0.00E+00</td>
<td>0.00E+00</td>
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PLW: $0.10010-05$ TUD: $0.20593E+01$ TQD: $0.27008E+01$
### TOLERANCE = ±10

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### TOLERANCE = ±0.1

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### TOLERANCE = ±0.01

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<td>ALFA</td>
<td>BETA</td>
<td>TOL</td>
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**TOLERANCE** = 1.00E-06

**ALFA** = 1.00E+00, **BETA** = 1.00E+00

**TOLx10^-5** = 1.00E+05, **TOLx10^-10** = 1.00E+10, **TOLx10^-20** = 1.00E+20

**PLW** = 0.10010-05, **TOL** = 0.2859E 01, **TOL** = 0.2708E 01

**TOLERANCE** = 1.00E-06

**ALFA** = 1.00E+00, **BETA** = 1.00E+00

**TOLx10^-5** = 1.00E+05, **TOLx10^-10** = 1.00E+10, **TOLx10^-20** = 1.00E+20

**PLW** = 0.10010-05, **TOL** = 0.2859E 01, **TOL** = 0.2708E 01

**TOLERANCE** = 1.00E-06

**ALFA** = 1.00E+00, **BETA** = 1.00E+00

**TOLx10^-5** = 1.00E+05, **TOLx10^-10** = 1.00E+10, **TOLx10^-20** = 1.00E+20

**PLW** = 0.10010-05, **TOL** = 0.2859E 01, **TOL** = 0.2708E 01

**TOLERANCE** = 1.00E-06

**ALFA** = 1.00E+00, **BETA** = 1.00E+00

**TOLx10^-5** = 1.00E+05, **TOLx10^-10** = 1.00E+10, **TOLx10^-20** = 1.00E+20

**PLW** = 0.10010-05, **TOL** = 0.2859E 01, **TOL** = 0.2708E 01

**TOLERANCE** = 1.00E-06

**ALFA** = 1.00E+00, **BETA** = 1.00E+00

**TOLx10^-5** = 1.00E+05, **TOLx10^-10** = 1.00E+10, **TOLx10^-20** = 1.00E+20

**PLW** = 0.10010-05, **TOL** = 0.2859E 01, **TOL** = 0.2708E 01

**TOLERANCE** = 1.00E-06

**ALFA** = 1.00E+00, **BETA** = 1.00E+00

**TOLx10^-5** = 1.00E+05, **TOLx10^-10** = 1.00E+10, **TOLx10^-20** = 1.00E+20

**PLW** = 0.10010-05, **TOL** = 0.2859E 01, **TOL** = 0.2708E 01

**TOLERANCE** = 1.00E-06

**ALFA** = 1.00E+00, **BETA** = 1.00E+00

**TOLx10^-5** = 1.00E+05, **TOLx10^-10** = 1.00E+10, **TOLx10^-20** = 1.00E+20

**PLW** = 0.10010-05, **TOL** = 0.2859E 01, **TOL** = 0.2708E 01

**TOLERANCE** = 1.00E-06

**ALFA** = 1.00E+00, **BETA** = 1.00E+00

**TOLx10^-5** = 1.00E+05, **TOLx10^-10** = 1.00E+10, **TOLx10^-20** = 1.00E+20

**PLW** = 0.10010-05, **TOL** = 0.2859E 01, **TOL** = 0.2708E 01