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DIAGONAL DOMINANCE FOR THE MULTIVARIABLE NYQUIST
ARRAY USING FUNCTION MINIMIZATION

by

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SUMMARY

A new approach to the design of multivariable control systems using the Multivariable Nyquist Array method has been developed. The technique utilizes a conjugate direction function minimization algorithm to achieve a diagonal dominant condition over the extended frequency range of the control system. The minimization is performed on the ratio of the moduli of the off-diagonal terms to the moduli of the diagonal terms of either the inverse or direct open loop transfer function matrix.

In addition to the ability to achieve diagonal dominance with a minimum of designer intervention, several new feedback design concepts and evaluative measures are introduced. These include:

1. Dominance control parameters for each control loop.
2. Compensator normalization to evaluate open loop conditions for alternative design configurations.
3. An interaction index to determine the degree and type of system interaction when all feedback loops are closed simultaneously.

This new design capability has been implemented on an IBM 360/75 in a batch mode but can be easily adapted to an interactive computer facility. The design method represents a significant contribution to the design and analysis of multivariable control systems in the frequency domain and has been applied to the Pratt and Whitney F100 turbofan engine with three inputs and three outputs.

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SECTION 1.

INTRODUCTION

Over the past several decades, considerable effort has been expended in the development and synthesis of linear multivariable feedback control theory and its application to the design of multivariable control systems.

Initially the analysis of automatic control systems utilized time functions usually expressed in differential or integral form. Maxwell, more famous for his work in field theory, presented the first mathematical treatment of a control mechanism in 1868 [1]. During the two decades preceding World War II, important contributions took place in aviation, electronics and circuit theory. For example, Nyquist's classic work in 1932 [2] on stability of linear feedback systems was prompted not by stability problems in control theory but by a desire to better understand the characteristics of certain communication networks. During World War II these concepts were rediscovered by control people and have since played an important role in the control field.

By 1945 the theory of linear servomechanisms and the fundamentals of mathematical modeling were well developed. The concept of steady state transfer functions had been introduced by Harris [3] and incorporated into the earlier work of Nyquist to further the understanding of the dynamic behavior and design of servomechanisms. This mathematical concept was popularized by Gardner and Barnes [4] with the introduction of transform calculus. By the end of the forties, the analysis and synthesis of linear continuous systems was basically limited to trial and error methods.

Around 1950, Evans [5] introduced the Root Locus method, which for the first time provided a means for the direct synthesis of control systems.

These techniques have since been further developed and still represent one of the most useful synthesis techniques available for linear systems.

In the mid-1950's, control engineering sees an unprecedented growth rate. Analog, digital and hybrid computers have reached high levels of perfection and more important, become universally available. The single input single output frequency domain techniques of Bode, Nyquist and Evans can now be coded for computer generated plots and the world of computer aided system design is created.

With the computational speed and numerical accuracy now available and inspired by the work of Russian and American research teams, the field of control enters a new era referred to as "modern control theory" encompassing the general areas of optimal and adaptive control. Vector space methods provide the mathematical foundations for the time domain synthesis of multi input multi output control systems. In 1960, Kalman [6] provided a definitive treatment of the linear case with a quadratic cost function and showed that the optimal feedback control is determined by the unique positive semi definite solution of the matrix Riccati differential equation. A special issue of the IEEE Transactions on Automatic Control Theory (1971) on the Linear Quadratic Gaussian (LQG) problem reflects the tremendous breadth and depth of this field.

The modern control era is often characterized as the "algorithmic era". The system description, design goals and parameter constraints are in many cases manipulated and massaged until the problem format fits a description for which there is an algorithmic solution. The resulting controls are usually highly interactive, require full state feedback and generally result in low integrity systems. Often the problem has been so narrowly defined that the final control configuration is unique.

In general, the success of linear optimal control theory (and other pole placement algorithms), when viewed from the frequency domain, is that full state feedback ensures adequate gain and phase margins in each of the feedback loops. If all system states are not available for measurement, severe penalties in terms of phase lag may be incurred. Techniques have therefore been developed to provide estimates of the inaccessible states [7,8]. This, in turn, leads to a dynamic feedback controller.

In the late 1960's, it became clear to Rosenbrock, MacFarlane and others that the vector time response methods leading to the LQR problem and associated regulator solution methods were not the panacea long promised. Optimal control design techniques although suitable for multi input multi output system analysis did not possess many of the design capabilities of the classical methods. By a suitable generalization of the frequency response methods, originally introduced by Bode, Nyquist and Weiner, to the multi input multi output system, new dimensions in the classical design concepts were created.

An algebraic theory, based on Rosenbrock's work, defines the structural relationships in terms of which feedback systems may be manipulated into a variety of feedback forms. The generalization of Nyquist's fundamental criterion, the concept of integrity and Bode's sensitivity results were extended to vector forms. A survey of the major results in linear multivariable feedback theory from the vector frequency response viewpoint was outlined by MacFarlane [9-11]. Of those techniques presently available, the Inverse Nyquist Array [12] introduced by Rosenbrock and the Characteristic Locus Method [13] introduced by MacFarlane have surfaced as two of the most useful frequency domain design techniques for a wide range of practical multivariable feedback systems. Both methods require a computer-aided

design facility with an interactive graphic display unit upon which the appropriate loci are computed and displayed.

After an initial inquiry into the design capabilities, mathematical dependencies and computational requirements of the multivariable frequency domain techniques, it becomes apparent that the basic principles and the governing philosophy of the Inverse Nyquist Array (INA) provide for the maximum utility of the single loop classical design theories. The INA extends Nyquist's stability criterion to inverse polar plots and multi input multi output systems. It provides the mechanism to reduce system interaction to a degree wherein each feedback loop can be independently designed. It utilizes the theorems of Gershgorin and Ostrowski to delineate the bounds of the eigenvalues of the transfer matrix at each frequency and thereby define the degree of both the open loop and closed loop interactive effects. These basic concepts and the underlying components associated with an INA design are further outlined in section 2.

The single most detracting feature of the INA design philosophy is the unreasonably high degree of designer intervention to secure the condition of "diagonal dominance". Since this condition is crucial to the success of an INA design, a principal objective of this research project was to develop an alternative method to search for the dominant condition. In section 3, of this report, a new algorithm utilizing a conjugate direction function minimization technique is presented. In fact, the algorithm is sufficiently versatile so as to be appropriate to the design of a multi input multi output system using the Direct Nyquist Array (DNA) in addition to the INA.

In addition to the tremendous versatility and flexibility of the proposed algorithm, several new design concepts are introduced in section 3.

These new concepts respond to the concerns pertaining to the level of dominance required to complete the design on a single loop basis, the degree and type of closed loop interaction which will result upon simultaneous closure of all feedback loops when each loop has been designed independently and a method of comparing system compensators each of which produce the desired dominance condition.

In section 4, several examples are presented to demonstrate the computational efficiency and effectiveness of the proposed technique. The principal example of this section is the analysis and design for the Pratt and Whitney F100 turbofan engine at sea level static conditions.

SECTION 2

MULTIVARIABLE NYQUIST ARRAY

The Multivariable Nyquist Array (MNA) design method is herein proposed as the union of two mutually exclusive design techniques: the Inverse Nyquist Array (INA) and the Direct Nyquist Array (DNA) methods. Both methods have identical design objectives and are founded upon a common mathematical structure. The methods are mutually exclusive in the sense that the INA utilizes inverse polar plots while the DNA uses direct polar plots. The principal point of departure is the use and interpretation of the multivariable Nyquist stability criterion in achieving the final system design.

The fundamental objective of the MNA design methods is to decrease system interaction to such an extent that the closed loop system design problem reduces to a set of independent single loop design problems. Although simply stated, the actual reduction procedure proposed by Rosenbrock [12,14] requires a high degree of designer intervention and is fundamentally a trial and error procedure. The algorithm developed in subsequent sections of this report considerably reduces this designer dependency thus making the MNA design method a more viable design tool.

Historically, the first attempt to eliminate system interaction was proposed by Boksenbom and Hood [15]. Their procedure was to completely decouple system input output pairs through the appropriate design of pre and post compensator matrices. The resulting compensator forms are necessarily complicated and in many cases unstable and/or physically unrealizable. It did, however, provide for single loop closure on a completely non-interactive basis. Further attempts at system decoupling

are reported in [16] and [17].

The Multivariable Nyquist Array method adopts a considerably more sophisticated viewpoint in an attempt to achieve a similar design condition. The MNA recognizes that the extreme degree of decoupling theory is not really necessary to establish the desired design conditions. Compensator designs should be stable and realizable and preferably as simple as possible for ease of implementation. It further recognizes that some degree of system interaction may actually be desirable in the event of sensor or actuator failures.

In the remaining parts of this section, the mathematical foundations of the MNA are briefly introduced followed by an outline of the INA and DNA design methods. The section concludes with a discussion of the advantages and limitations of MNA design philosophy as originally proposed by Rosenbrock.

A. Mathematical Perspectives

In Figure 2.1, $G(s)$ is an $m \times m$ transfer matrix representing the coupling of the m inputs to the m outputs. The pre and post compensator

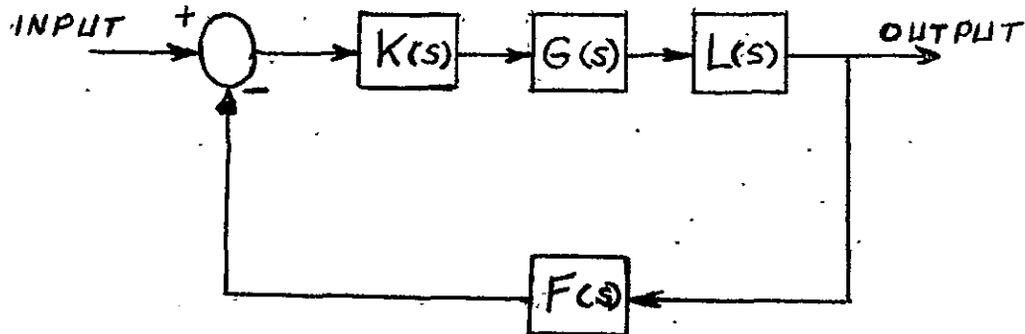


Figure 2.1 Multivariable System Configuration.

matrices, $K(s)$ and $L(s)$, respectively, are each of dimension $m \times m$. The feedback gain matrix $F(s)$ is assumed to be diagonal of similar dimensions. Clearly, if

$$Q(s) = L(s)G(s)K(s)$$

is diagonal, loop closure may proceed on an individual loop basis with a guarantee of zero system interaction. It is this premise upon which the MNA is based. However, the adherence to strict diagonalization is relaxed with the substitution and exploitation of the concept of diagonal dominant matrices.

Definition A matrix is said to be diagonal dominant when the moduli of its diagonal elements are greater than the sum of the moduli of the corresponding off-diagonal elements, taken by row or by column.

That is, if Z is a $m \times m$ complex matrix then

- a. Z is diagonally row dominant if

$$|z_{ii}| - \sum_{\substack{j=1 \\ i \neq j}}^m |z_{ij}| > 0 \quad \text{for all } i = 1, 2, \dots, m$$

- b. Z is diagonally column dominant if

$$|z_{ii}| - \sum_{\substack{j=1 \\ i \neq j}}^m |z_{ji}| > 0 \quad \text{for all } i = 1, 2, \dots, m.$$

It is important to note that to satisfy the definition Z must be entirely row dominant or entirely column dominant. The definition does not provide for a mixture of row and column dominance for different diagonal elements.

Quite apart from the concept of dominance is a theorem by Gershgorin which states that all eigenvalues of a complex matrix Z are located in the union of the circular discs defined by

$$|\lambda - z_{ii}| \leq r_i \quad i = 1, 2, \dots, m$$

with center at z_{ii} and radii given by

$$r_i = \sum_{\substack{j=1 \\ i \neq j}}^m |z_{ij}| \quad (\text{by rows})$$

or

$$c_i = \sum_{\substack{j=1 \\ i \neq j}}^m |z_{ji}| \quad (\text{by columns})$$

The latter radii follow from the fact that the eigenvalues of Z^T are equal to the eigenvalues of Z .

Now let Z be a function of the complex variable s , i.e.

$$Z(s) = \begin{bmatrix} z_{11}(s) & z_{12}(s) & \dots & z_{1m}(s) \\ z_{21}(s) & z_{22}(s) & \dots & z_{2m}(s) \\ \cdot & & \cdot & \\ \cdot & & \cdot & \\ \cdot & & \cdot & \\ z_{m1}(s) & z_{m2}(s) & \dots & z_{mm}(s) \end{bmatrix} \quad 2.1$$

At each value of s on a specified contour D in the s -plane, $Z(s)$ is a complex matrix and the preceding definition and theorems apply at each and every point on D . Thus the eigenvalues of $Z(s)$ are functions of s and the concept of diagonal dominance can be reformulated as

$$|z_{ii}(s)| - \sum_{\substack{j=1 \\ i \neq j}}^m |z_{ij}(s)| > 0 \quad (\text{row dominance}) \quad 2.2$$

and

$$|z_{ii}(s)| - \sum_{\substack{j=1 \\ i \neq j}}^m |z_{ji}(s)| > 0 \quad (\text{column dominance}) \quad 2.3$$

Let D be a large contour in the complex s -plane consisting of the

imaginary axis from $s = -jR$ to $s = +jR$ together with a semicircle of radius R in the right half plane as indicated in Figure 2.2. As s traverses the D contour in a clockwise direction, $z_{ii}(s)$ will generate a

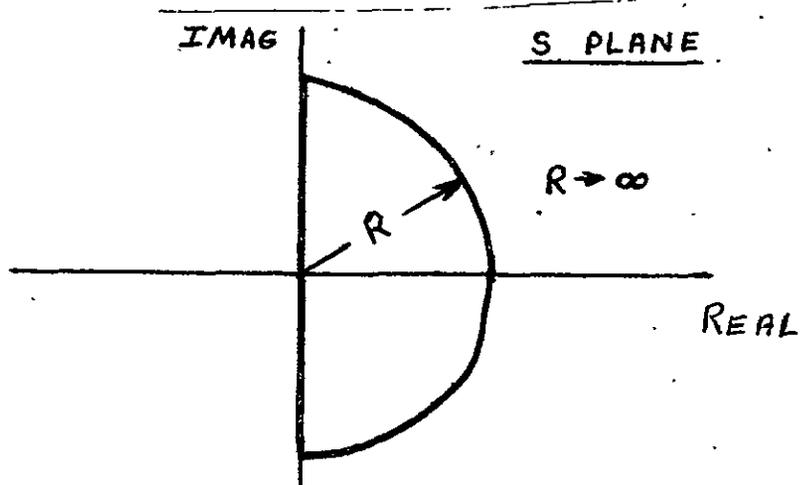


Figure 2.2 The D contour in the s -plane

curve Γ_i for $i = 1, 2, \dots, m$ in the complex plane. From the application of Gershgorin's theorem at each point on D , a band of circles centered about Γ_i will similarly be generated as in Figure 2.3. If a separate figure is constructed for each loci, then the collection of figures represent a set of "fuzzy" Nyquist (or inverse Nyquist) diagrams, one for each input-output pair.

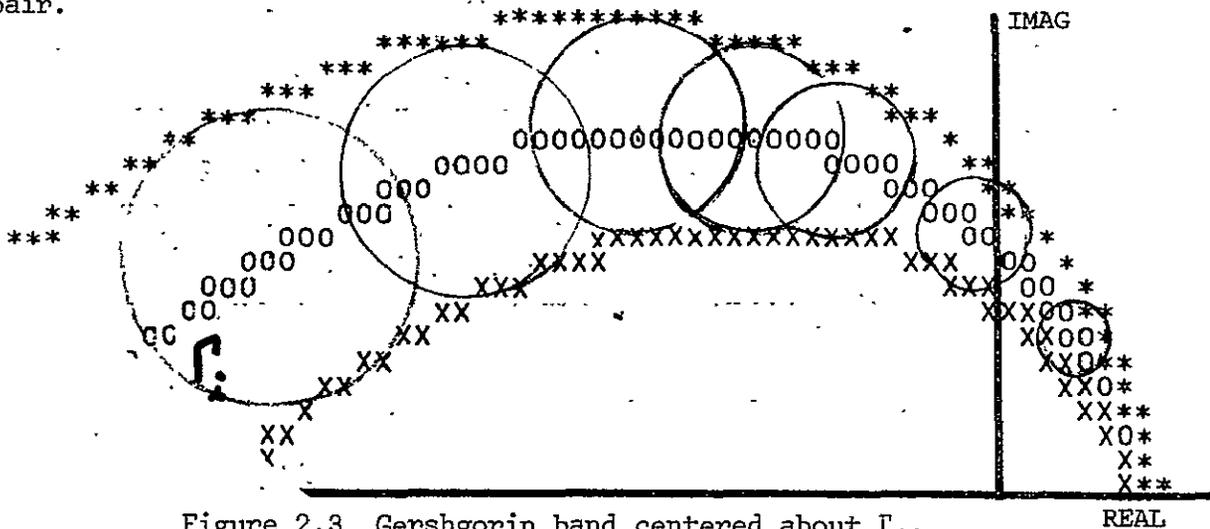


Figure 2.3 Gershgorin band centered about Γ_i .

Clearly, the eigenvalues of $Z(s)$ are captured within the union of the Gershgorin bands since the eigenvalues at each point lie within the corresponding union of discs. If the matrix $Z(s)$ is diagonal for all s on D , then the Gershgorin band reduces to zero width and the Γ_i are the characteristic loci of $Z(s)$. Thus the width of the Gershgorin band provides a qualitative measure of the departure from the diagonal condition. In a control system setting this departure would reflect the degree of open loop system interaction.

A further interpretation of Gershgorin's theorem states that the eigenvalues of $Z(s)$ lie in the intersection of the union of Gershgorin bands by row and by column. This follows immediately from the eigenvalues of $Z^T(s)$ being equal to those of $Z(s)$. Thus if the intersection excludes the origin then the determinant of $Z(s)$ cannot be zero, i.e., if $Z(s)$ is diagonal dominant then $\det|Z(s)| \neq 0$. With this corollary the following theorem results [14].

Theorem 2.1 Let $Z(s)$ be an $m \times m$ rational matrix and let D be a closed elementary contour having on it no pole of $z_{ii}(s)$, $i = 1, 2, \dots, m$. Let there exist $\epsilon > 0$ such that for each s on D either

$$|z_{ii}(s)| - \sum_{\substack{j=1 \\ i \neq j}}^m |z_{ij}(s)| > \epsilon \quad i = 1, 2, \dots, m$$

or

$$|z_{ii}(s)| - \sum_{\substack{j=1 \\ i \neq j}}^m |z_{ji}(s)| > \epsilon \quad i = 1, 2, \dots, m$$

Let $z_{ii}(s)$ map D into Γ_i , $i = 1, 2, \dots, m$, and let $\det|Z(s)|$ map C into Γ_z . Let Γ_i encircle the origin N_i times, and let Γ_z encircle the origin N_z times. Then

$$N_z = \sum_{i=1}^m N_i$$

This theorem provides for a number of stability conditions for the control problem depending upon which matrices are dominant. Further considerations are given in the individual discussions of the MNA methods.

In review, it is the Gershgorin theorem bounding the eigenvalues of a complex matrix and the concept of diagonal dominance which provide the necessary foundations for the MNA methods. Theorem 2.1 will form the basis for closed loop stability considerations. Final system design will then proceed on an individual loop basis.

B. Inverse Nyquist Array

To introduce the Inverse Nyquist Array, consider Figure 2.1 to represent a block diagram for a single input-single output feedback control system. For this system, the open loop transfer function is

$$Q(s) = L(s)G(s)K(s) \quad 2.4$$

Let $F(s)$ be a constant scalar feedback gain with closed loop system transfer function $H(s)$, given by

$$H(s) = \frac{Q(s)}{1+Q(s)F} \quad 2.5$$

The INA method uses inverse relationships for a variety of reasons, one of which is the simplification of equation 2.5, i.e.,

$$H^{-1}(s) = \frac{1+Q(s)F}{Q(s)} = Q^{-1}(s) + F \quad 2.6$$

Here the inverse closed loop transfer function is simply a linear translation in the complex plane of the inverse open loop transfer function.

For notational convenience and to avoid confusion at later stages of the development the following definitions are made

$$\hat{H}(s) = H^{-1}(s)$$

$$\hat{Q}(s) = Q^{-1}(s) \quad 2.7$$

$$\hat{K}(s) = K^{-1}(s)$$

Thus, (2.6) becomes

$$\hat{H}(s) = \hat{Q}(s) + F \quad 2.8$$

Now let $\hat{Q}(s)$ map the D contour (in the s-plane) into $\hat{\Gamma}_q$ which encircles the origin \hat{N}_q times clockwise. Also let $\hat{H}(s)$ in (2.8) map D into $\hat{\Gamma}_h$ which encircles the origin \hat{N}_h times clockwise. Here D is the usual Nyquist contour in the s-plane and is sufficiently large to include all finite poles and zeros of $\hat{Q}(s)$ and $\hat{H}(s)$ in the closed right half plane.

For the system of equation (2.8), \hat{N}_h is the number of times $\hat{\Gamma}_q$ encircles the point

$$(-F, 0)$$

and the following statement of the Nyquist Stability Criterion for the INA results:

Theorem 2.2 Let the open loop system, $Q(s)$, have ρ_0 poles in the closed right half plane. Then the closed loop system is asymptotically stable if and only if

$$\hat{N}_q - \hat{N}_h = \rho_0$$

Figure 2.4 represents a typical INA plot of $\hat{\Gamma}_q$. Each crossing of $\hat{\Gamma}_q$ by the critical point represents the entry of a pole of the closed loop system into the right half plane if the crossing is from "left to right". A "right to left" crossing represents the removal of a pole from the right half plane. All directional crossings are relative to an observer on $\hat{\Gamma}_q$ in the direction of increasing frequency.

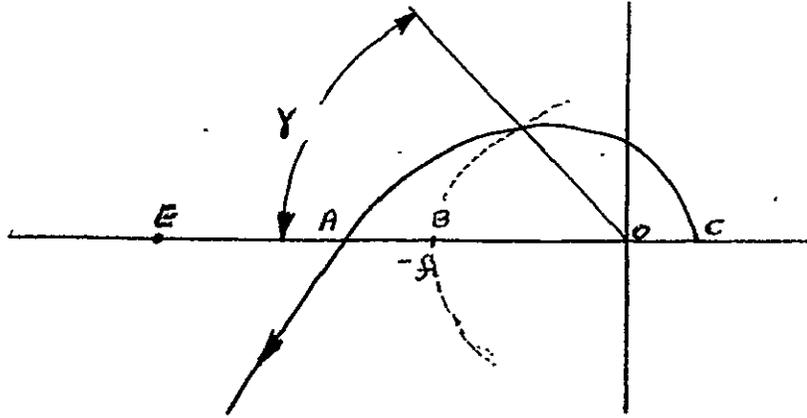


Figure 2.4 Typical INA plot of $\hat{\Gamma}_q$.

Some important observations regarding the INA plot of Figure 2.4 can now be made:

1. The INA begins at zero frequency and terminates at infinite frequency. This characteristic provides for an immediate display of low frequency information essential to most design problems.
2. With $F=0$, the origin $(0,0)$ represents the open loop situation. Thus if the system is open loop stable, then any gain in the vicinity of the origin which does not cross $\hat{\Gamma}_q$, represents a stable closed loop system operation.
3. The closed loop inverse transfer function $\hat{H}(s)$ is given by the same diagram with the origin shifted to $(-F,0)$.
4. Only positive frequency is plotted with the negative frequency range inferred. Thus if the feedback gain were such that the critical point was at point A, then two poles are about to cross into the right half plane at the gain crossover frequency ω_A .

5. The ratio OA to OB is the gain margin of the closed loop system with gain F.
6. The phase margin of the closed loop system is γ when a gain of F is introduced into the feedback path.
7. The steady state offset is given by the ratio $\hat{Q}(0)/\hat{H}(0)$ and is obtained from the INA plot directly as the ratio of OC to BC. As F increases, the offset decreases.
8. The effective bandwidth of the closed loop system is given by the value of ω for which

$$|\hat{H}(j\omega)| = 1.414|F(j\omega)|$$

9. Feedback compensation procedures for the INA are conceptually similar to the direct Nyquist array. For example, to provide more phase advance a phase lead compensator could be introduced into the feedback path. In Figure 2.4, this compensator would shift the INA plot up and away from the origin with gain crossover occurring at point E.

Clearly, the Inverse Nyquist Array for single-input single-output systems is at least as versatile a design tool as the direct Nyquist method. Many authors contend that the simplification of the closed loop transfer function coupled with the low frequency profile make the INA diagram a more useful design mechanism for single loop design. A more detailed study of inverse polar plots is contained in Rosenbrock [14] and Raven [18].

The design of feedback control units for multi input multi-output systems using the Inverse Nyquist Array is relatively straightforward once the condition of diagonal dominance has been achieved. Fundamentally, this condition suggests that system interaction has been reduced to such an extent that each control loop can be closed separately and independently from the remaining loops using single loop theory.

Let Figure 2.1 represent a feedback control system with m inputs and m outputs. The system transfer matrix $G(s)$ is $m \times m$ with each element $g_{ij}(s)$ consisting of a numerator and denominator polynomial representing the transmittance between input j and output i . In general, $G(s)$ or $\hat{G}(s)$ will not be dominant over the frequency range of interest and therefore must be modified to conform to the requirements of the multivariable array methods.

For the INA method, the pre and post compensator matrices (each $m \times m$) must be selected such that

$$\hat{Q}(s) = \hat{K}(s)\hat{G}(s)\hat{L}(s) \quad 2.9$$

is diagonally dominant over the D contour. Using the definitions of (2.2) and (2.3), $\hat{Q}(s)$ is diagonal dominant if

$$a. \quad \sum_{\substack{j=1 \\ i \neq j}}^m |\hat{q}_{ij}(s)| / |\hat{q}_{ii}(s)| < 1 \quad (\text{row dominant}) \quad 2.10$$

or

$$b. \quad \sum_{\substack{j=1 \\ i \neq j}}^m |\hat{q}_{ji}(s)| / |\hat{q}_{ii}(s)| < 1 \quad (\text{column dominant}) \quad 2.11$$

for all $i = 1, 2, \dots, m$. From Gershgorin's Theorem, all eigenvalues of $\hat{Q}(s)$ are captured within the union of the Gershgorin bands centered about $\hat{\Gamma}_i$ with radius as the sum of the moduli of the off diagonal terms taken by row or by column.

Assume, for the moment, that $\hat{Q}(s)$ is diagonal dominant by rows over the D contour as in Figure 2.5 for $m=2$. If the system is open loop stable then the closed loop system will be guaranteed asymptotically stable for all feedback gains on the real axis in the vicinity of the origin and bounded by the Gershgorin bands. This stability criterion (Theorem 2.1) can now be restated as follows:

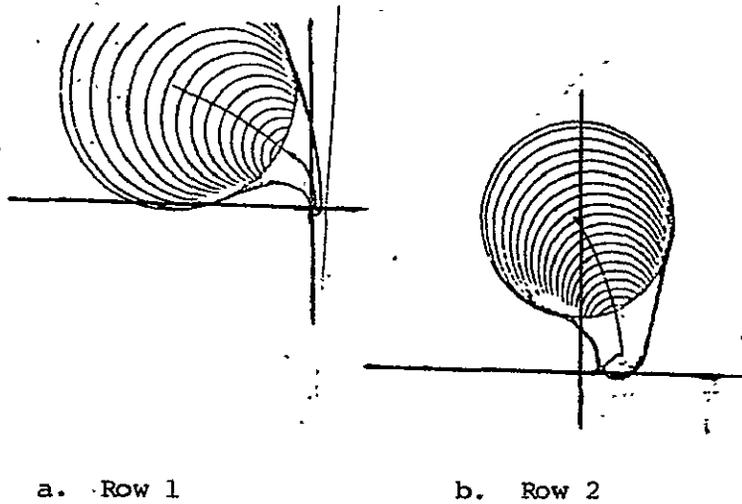


Figure 2.5 $\hat{Q}(s)$ diagonal dominant with $m=2$.

Theorem 2.3 Let each of the Gershgorin bands based on the diagonal elements $\hat{q}_{ii}(s)$ of $\hat{Q}(s)$ exclude the origin and the point $(-f_i, 0)$. Let these bands encircle the origin \hat{N}_{qi} times and encircle the point $(-f_i, 0)$, \hat{N}_{hi} times. Then the closed-loop system is asymptotically stable if and only if

$$\sum_{i=1}^m \hat{N}_{qi} - \sum_{i=1}^m \hat{N}_{hi} = \rho_o \quad 2.12$$

where ρ_o is the number of open loop poles of $\hat{Q}(s)$ in the right half plane. Here the Gershgorin bands are defined by the radii of (2.10) or (2.11).

Theorem 2.3 is stated in its most useful form for application purposes since \hat{N}_{qi} and \hat{N}_{hi} are evaluated from the same set of Gershgorin bands. The theorem could be stated in a more general form wherein new bands would be recalculated for each set of f_i .

Note that theorem 2.3 specifically states band encirclements and not just

encirclements of the $\hat{\Gamma}_i$. Thus the theorems and corollaries for the INA apply only for feedback gain values located outside the Gershgorin bands. The stability theorems simply do not apply for any gain values located within a band and no inferences regarding stability or instability can be made.

For the system in Figure 2.5, each feedback loop could be independently closed using the inner most envelope of the Gershgorin band for gain margin and phase margin assessment. In general, the Gershgorin band provides a conservative estimate of the stable gain space and is most useful as a preliminary design tool.

To provide further insight into the INA design mechanism, assume $\hat{Q}(s)$ diagonal dominant and all feedback gains f_i chosen in accordance with Theorem 2.3. Let $h_{ii}(s)$ be the transmittance from input i to output i when all feedback loops are closed i.e. $f_i \neq 0, i=1,2,\dots,m$. Define $r_i(s)$ as the transmittance from input i to output i when all feedback loops, except the i th loop, are closed, i.e. $f_i=0$. From standard feedback relationships

$$h_{ii}(s) = r_i(s)/(1+r_i(s)f_i) \quad 2.13$$

$$\hat{h}_{ii}(s) = \hat{r}_i(s) + f_i \quad 2.14$$

Hence, to complete a set of single loop designs by opening one feedback path at a time, it is the quantity $\hat{r}_i(s)$ which governs the system behavior and not $q_{ii}(s)$. Rosenbrock exploited this relationship to demonstrate that $\hat{r}_i(s)$ is located within the Gershgorin band for all stabilizing feedback gains. He further demonstrated that when all gains except f_i are specified, the transmittances $\hat{r}_i(s)$ for $i=1,2,\dots, m$ are located within a narrower set of bands. This new set of bands is based upon a theorem by Ostrowski and are appropriately labeled the Ostrowski bands. For the INA method, the Ostrowski bands are always located within the Gershgorin bands.

Theorem 2.4 (Ostrowski [19]) Let D be the closed Nyquist contour in the s-plane and let Z(s) be row dominant on D with no pole of $z_{ii}(s)$ $i=1,2,\dots,m$ on D. Then if s_0 is a point on D, Z(s) has an inverse Z(s) such that for $i=1,2,\dots,m$

$$|\hat{z}_{ii}(s_0) - z_{ii}^{-1}(s_0)| < \phi_i d_i(s_0) < d_i(s_0)$$

when

$$d_i(s_0) = \sum_{\substack{j=1 \\ j \neq i}}^m |z_{ij}(s_0)| \quad i = 1, 2, \dots, m$$

are the Gershgorin radii and ϕ_i are a set of "shrinking factors" defined by

$$\phi_i(s_0) = \max_{\substack{j \\ j \neq i}} \frac{d_j(s_0)}{|z_{jj}(s_0)|}$$

For the INA, $\hat{Z}(s) = \hat{H}(s) = \hat{Q}(s) + F$, hence

$$|h_{ii}^{-1}(s) - [f_i + \hat{q}_{ii}(s)]| < \phi_i(s) d_i(s) \quad 2.15$$

for row dominance, and

$$|h_{ii}^{-1}(s) - (f_i + \hat{q}_{ii}(s))| < \phi_i^1(s) d_i^1(s) \quad 2.16$$

for column dominance. The shrinking factors thus become

$$\phi_i(s) = \max_{j \neq i} \frac{d_j(s)}{|f_j + \hat{q}_{jj}(s)|} \quad 2.17$$

and

$$\phi_i^1(s) = \max_{j \neq i} \frac{d_j^1(s)}{|f_j + \hat{q}_{jj}(s)|} \quad 2.18$$

Using (2.14), (2.15) and (2.16) become

$$|r_i^{-1} - \hat{q}_{ii}(s)| < \phi_i(s) d_i(s) < d_i(s) \quad 2.19$$

for row dominant $\hat{H}(s)$ and

$$|r_i^{-1} - \hat{q}_{ii}(s)| < \phi_i^1(s) d_i^1(s) < d_i^1(s) \quad 2.20$$

for column dominant $\hat{H}(s)$.

The Ostrowski bands serve two useful functions. First they locate the transmittances \hat{r}_i within a narrower set of bands when all loops except the i th are closed. As the f_j , $j \neq i$ are varied, \hat{r}_i will also vary so that the i th Ostrowski band depends upon f_j , $j \neq i$. Second, they provide a more accurate measure of phase and gain margin for the i th control loop and thus reduce the problem to the design of f_i . An extremely important feature of theorem 2.4 is that once the f_i , $i \neq j$ are specified to obtain the Ostrowski band for loop j , the band for loop j will continue to shrink when the f_i , $i \neq j$ increase. That is, if the feedback control in loops $i \neq j$ are increased to improve the control in the respective loops, the feedback design for loop j is unaffected.

The Inverse Nyquist Array method consist of the following fundamental operations:

1. Design $\hat{K}(s)$ and $\hat{L}(s)$ such that $\hat{Q}(s)$ is dominant by row or column using (2.10) or (2.11).
2. Plot the Gershgorin Bands for each control loop.
3. Evaluate stability for the diagonal dominant $\hat{Q}(s)$ using theorem 2.3.
4. Finalize the design using the Ostrowski Bands and single loop control theory.

Clearly, the application of the INA method is predicated upon the ability of the system designer to achieve an adequate degree of system dominance. The degree of dominance attainable is primarily governed by two factors:

1. The structural sophistication and realization of pre and post compensator forms;

2. The ability of the designer to manipulate the compensator parameters to achieve, maintain or improve dominance in each row (or column) of $\hat{Q}(s)$.

Current methods available for compensator design require a high degree of designer interaction and are therefore best suited for interactive computer facilities. Often success in achieving dominance is based upon the experience and intuition of the system designer in the application of these trial and error methods. In addition the methods are restricted, for the most part, to the design of constant compensators although attempts at more sophisticated structural forms has been reported [14].

One of the first methods suggested beyond a total trial and error approach was to diagonalize $\hat{G}(s)$ at $s=0$ [12] by setting

$$\hat{K} = LG(0)$$

If $G(0)$ is non-singular, $\hat{Q}(s)$ will be dominant near the origin of the complex plane but not necessarily over the extended dynamic frequency range of the system. In either event, however, no general guidelines exist for improving dominance in any given row (or column) of $\hat{Q}(s)$ since any modification of the coefficients of \hat{K} (or \hat{L}) translate to vector addition when forming

$$\hat{Q}(s) = \hat{K}\hat{G}(s)\hat{L}$$

Similar conditions exist for high frequency diagonalization using

$$\hat{K} = LG(\infty)$$

Therefore diagonalization at either end of the frequency spectrum provides, at best, a starting point for the trial and error approach.

The pseudodiagonalization method developed by Hawkins [20] and generalized by Rosenbrock [14] formulates the dominance objective as an eigenvalue-eigenvector problem. The method is best suited for constant compen-

sator design but could be used for polynomial forms of $\hat{K}(s)$. Briefly, pseudodiagonalization determines the set of row elements of \hat{K} (for row dominance) or column elements of \hat{L} (for column dominance) which "most nearly" diagonalizes $\hat{Q}(s)$ at specified frequencies. For the eigenvalue-eigenvector problem this translates to minimizing the weighted sum of squares of the off diagonal elements of $\hat{Q}(s)$ at the N specified frequencies,

$$\text{Min } \sum_{r=1}^N \gamma_r \left\{ \sum_{\substack{j=1 \\ i \neq j}}^m |\hat{q}_{ij}(j\omega_r)|^2 \right\} \quad i = 1, 2, \dots, m \quad 2.21$$

where m is the dimension of $G(s)$ and γ_r are designer specified weighting factors. The minimization in (2.21) is subject to either a constraint on the elements (for row dominance)

$$\sum_{i=1}^m K_{ji}^2 = 1 \quad 2.22$$

or a constraint on the diagonal element

$$|\hat{q}_{jj}(j\omega)| = 1 \quad \omega \text{ specified} \quad 2.23$$

If the constraint in (2.22) is used, \hat{q}_{jj} may vanish since it does not appear in the problem formulation. If $N=1$ in (2.21) and constraint (2.23) is used, problems analogous to the case when $\omega=0$ may result. The additional feature here, however, is that the eigenvalue-eigenvector problem may be resolved at any frequency within the range of interest. The solution at the selected frequency is then tested at other frequencies for dominance. For $N \geq 2$, the eigenvalue-eigenvector problem resulting from the minimization of (2.21) subject to (2.22) (or (2.23)) further complicates the dominance issue since ω in (2.23), N values of ω_r and N values for γ_r in (2.21) must be selected a priori. If either pseudodiagonalization scheme yields domi-

nance over the frequency spectrum then the designer may resort to trial and error methods to improve the degree of dominance. However, if dominance does not prevail, conclusions concerning the non-existence of constant compensators may be erroneous. This would be the case, for example, when constant compensators exist which yield dominance over the D contour in the sense of (2.10) or (2.11) but did not satisfy (2.21) for any combination of ω_r , λ_r and N.

C. Direct Nyquist Array

The direct Nyquist Array (DNA) method for single-input single-output systems is well established in the control literature. Fundamentally, it is a polar plot of the open loop transfer function as s traverses the familiar Nyquist contour.

If Figure 2.2 represents a single input single output feedback system, then the open loop transfer function is

$$Q(s) = L(s)G(s)K(s) \quad 2.24$$

with the closed loop transfer function as

$$H(s) = \frac{Q(s)}{1+Q(s)F(s)} \quad 2.25$$

Let $\phi(s)$ be the characteristic polynomial of $H(s)$, i.e.,

$$\phi(s) = 1 + Q(s)F(s) \quad 2.26$$

and let $\phi(s)$ map D into a closed contour Γ_ϕ as s traverses D in a clockwise direction. From the Nyquist stability criterion, if Γ_ϕ encircles the origin N_ϕ times clockwise then the closed loop system is asymptotically stable if and only if

$$N_\phi = 0 \quad 2.27$$

An equivalent but more convenient form of the Nyquist criterion is available using (2.24). Let $Q(s)$ map D into the closed contour Γ_ϕ and let

$F(s)$ be a constant different from zero. Taking the usual precautions to insure that D encloses every finite pole and zero of $\phi(s)$ in the right half plane, (2.26) can be rearranged to

$$\psi(s) = F^{-1} + Q(s) \quad 2.28$$

and the following theorem results [14]

Theorem 2.5 Let F be constant and let the open loop system have ρ_0 poles in the closed right half plane. Let $Q(s)$ map D into Γ_q making N_q clockwise encirclements of the point $(-\frac{1}{F}, 0)$. Then the closed loop system of Figure 2.1 is asymptotically stable if and only if

$$N_q = -\rho_0 \quad 2.29$$

In theorem 2.5, (2.28) could be written as

$$N_q = Z_0 - \rho_0$$

where Z_0 is the number of finite zeros of $\psi(s)$ in the closed right half plane. However, the system is stable if and only if $Z_0=0$ thus (2.29) results. A typical Nyquist polar plot is indicated in Figure 2.6 for positive frequencies. The closed loop system design can now be completed using gain margin, phase margin, etc. as the criteria and $-\frac{1}{F}$ as the critical

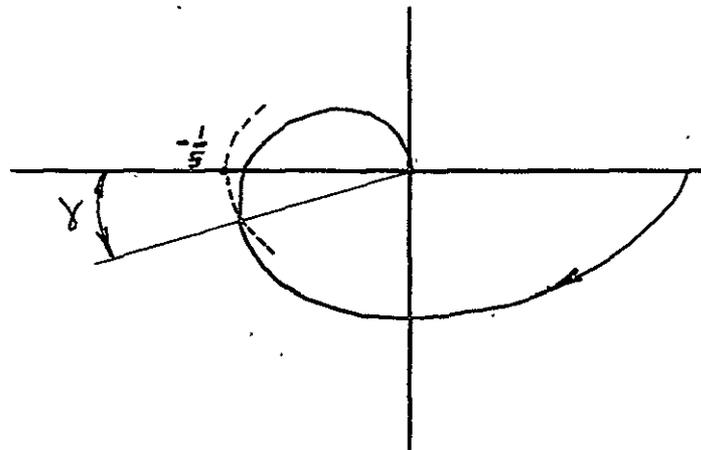


Figure 2.6 Nyquist polar plot.

point. Further design considerations for the single input single output case can be found in [18] and [21].

The extension of the above design concept and in particular theorem 2.5 to multi input multi output systems will require the diagonal dominant condition imposed on $Q(s)$. Therefore, let $Q(s)$ be diagonal dominant, i.e.,

$$|q_{ii}(s)| > \sum_{\substack{j=1 \\ i \neq j}}^m |q_{ij}(s)| \quad \begin{array}{l} \text{Row dominant} \\ i = 1, 2, \dots, m \end{array} \quad 2.30$$

or

$$|q_{ii}(s)| > \sum_{\substack{j=1 \\ i \neq j}}^m |q_{ji}(s)| \quad \begin{array}{l} \text{Column dominant} \\ i = 1, 2, \dots, m \end{array} \quad 2.31$$

for all s on D . Let F^{-1} (or equivalently F) be a diagonal constant matrix, then the off diagonal elements of $\psi(s)$,

$$\psi(s) = F^{-1} + Q(s) \quad 2.32$$

are equal to the off diagonal elements of $Q(s)$. The diagonal elements of 2.32 are

$$\psi_{ii}(s) = f_{ii}^{-1} + q_{ii}(s) \quad i = 1, 2, \dots, m \quad 2.33$$

Now let $f_{ii}^{-1} + q_{ii}(s)$ map D into a closed curve $\Gamma_{f+q_{ii}}$ and let $q_{ii}(s)$ map D into Γ_{q_i} . From Gershgorin's Theorem, all eigenvalues of $F^{-1} + Q(s)$ are captured within the union of the Gershgorin bands centered about $\Gamma_{f+q_{ii}}$ with radius equal to the sum of the moduli of the off diagonal terms taken by row or by column. But the Gershgorin radii for $F^{-1} + Q(s)$ are equal to the radii for $Q(s)$. Therefore exclusion of the origin by a band centered about $\Gamma_{f+q_{ii}}$ is equivalent to the exclusion of the point $(-f_i^{-1}, 0)$ by a band centered about Γ_{q_i} . The following theorem summarizes this thought [14].

Theorem 2.6 Let each of the Gershgorin bands centered about Γ_{q_i} exclude the point $(-f_{ii}^{-1}, 0)$ $i=1, 2, \dots, m$. Let these bands encircle the point $(-f_{ii}^{-1}, 0)$, N_i times, $i=1, 2, \dots, m$. Then the closed loop system is asymptotically stable if and only if

$$\sum_{i=1}^m N_i = -\rho_0 \quad 2.34$$

Theorem 2.6 provides the necessary criteria to complete the DNA design on a single loop basis. To improve the design capability through the reduction of the Gershgorin band, a theorem analogous to the Ostrowski theorem for the INA is available [14].

Theorem 2.7 Let F be a diagonal constant matrix, and let $F^{-1}+Q(s)$ be dominant on D . Let $h_{ii}(s)$ represent the transfer function from input i to output i when all feedback loops except the i th loop are closed. Then for each s on D ,

$$|q_{ii}(s) - h_i(s)| < \phi_i^1(s)d_i^1(s) < d_i^1(s) \quad 2.35$$

for row dominance and

$$|q_{ii}(s) - h_i(s)| < \phi_i^1(s)d_i^1(s) < d_i^1(s) \quad 2.36$$

for column dominance where

$$\phi_i = \max_{\substack{j \\ i \neq j}} \frac{d_j(s)}{|f_{jj}^{-1} + q_{jj}(s)|} \quad 2.37$$

$$\phi_i^1(s) = \max_{\substack{j \\ j \neq i}} \frac{d_j^1(s)}{|f_{jj}^{-1} + q_{jj}(s)|} \quad 2.38$$

and the $d_j(s)$ and $d_j^1(s)$ are the appropriate Gershgorin radii.

Extreme care must be exercised in using theorem 2.7 in a DNA application. This is evident, for example, in the row dominant conditions of

(2.35) and (2.37). Here it is observed that the shrinking factors in (2.37) decrease as the f_{jj} decrease, as they must since

$$|q_{ii}(s)| = |h_i(s)| \quad 2.39$$

in the open loop condition. Therefore, in contrast to the INA, the Ostrowski bands increase as the feedback gains in the remaining loops increase. This condition may cause some difficulty if the feedback design for loop j is based upon the set $\{f_i, i \neq j\}$ and some of the f_i increase as a result of later design modifications. The increase in value of some of the f_i will cause the Ostrowski band for loop j to increase which in some cases may be sufficiently large so as to encircle the design point in loop j . If this situation prevails, the entire DNA design is voided, since the critical design point in every loop must remain outside the corresponding Ostrowski band for Theorem 2.6 to be valid. Hence the feedback design in each loop must be reevaluated whenever significant positive increments are made in the remaining feedback loops.

The above situation is cited by Rosenbrock [14] as the single most determining factor for choosing the INA method over the DNA method. However, the graphical interpretation of theorem 2.7 is not quite as grim as Rosenbrock may suggest. The following reasons are cited:

1. For any specified set of feedback gains $[f_i, i \neq j]$, the transfer function from input j to output j is contained within the j th Gershgorin band (Theorem 2.6). Thus if the Gershgorin bands are sufficiently narrow then single loop closure can be completed without invoking theorem 2.7.
2. Theorem 2.7 can be used in precisely the same sense as theorem 2.4 was used in an INA design, if the feedback gains $\{f_i, i \neq j\}$ are usually large (and outside the respective Gershgorin Bands), then as

the f_i are reduced under single loop closure, the Ostrowski bands in theorem 2.7 will shrink.

3. Examination of (2.37) (or (2.38)) suggests that if all f_i $i \neq j$ are infinite, then the largest value of $\phi(s)$ (or $\phi^1(s)$) is given by

$$\max_{\substack{j \\ i \neq j}} \frac{d_j(s)}{|q_{jj}(s)|} \quad 2.40$$

But this expression is simply the maximum degree of dominance attained in the remaining Gershgorin bands. Therefore each Gershgorin band may be immediately reduced by the appropriate factor in (2.40). This important concept is used in the algorithm proposed in section 3 of this report.

Apart from the graphical interpretation and use of Ostrowski's Theorem, the DNA and the INA methods are similar in concept. For every theorem, definition and design concept pertaining to the INA method there exists an analogous theorem, definition and design concept for the DNA method. The methods are mutually exclusive in the sense that a design initiated using the INA method cannot be completed using DNA design concepts and vice versa. The main point of departure lies in the graphical interpretation of the fundamental theorems, wherein the DNA method utilizes polar plots to interpret the design objectives and the INA utilizes the inverse polar plot.

Clearly, the DNA method (as well as the INA method) is critically dependent upon the ability of the system designer to achieve a diagonal dominant condition. All methods appropriate to the INA method to achieve a dominant condition are also suitable to the DNA design method.

The DNA method can be summarized as follows:

1. Design $K(s)$ and $L(s)$ for diagonal dominance of $Q(s)$ by row or by column.
2. Plot the Gershgorin bands.
3. Evaluate system stability using theorem 2.6.
4. Complete the design using single loop control theory within the limitations of theorem 2.7.

C. Discussion

The multivariable Nyquist array is a very useful and versatile design technique for multi input multi output systems. The principal feature of the MNA is the utilization of the mathematical foundations of complex matrices to reduce system interaction to the extent that each feedback loop can be independently designed. The Gershgorin and Ostrowski theorems are easy to apply and the final closed loop design concepts are well documented in the control literature.

The utility of the MNA design philosophy is totally dependent upon the ability of the system designer to achieve an adequate degree of system dominance. It is therefore in the interests of the designer to have available new methods of achieving the dominant condition and thus provide greater flexibility in the use of the MNA design methods.

Ideally, any new dominance method should be sufficiently flexible to address the following issues:

1. Design of constant compensators for either row or column dominance.
2. Design of frequency dependent compensators with fixed structural form for row or column dominance.
3. Eliminate the need for designer oriented trial and error methods to improve dominance.

4. Provide a measure for the comparison of two or more compensator pairs each yielding a dominant condition.
5. Utilize the degree of dominance in the remaining rows or columns to improve the design in a particular row or column.
6. Be appropriate for both the INA and the DNA methods.

This list highlights several major concerns regarding the use of the MNA which has not been treated in the literature to date. The concerns are primarily directed toward the degree of dominance required in a particular design, the utilization of dominance to further improve the design base for any given row or column element and the degree and type of system interaction resulting from the simultaneous closure of all feedback loops.

A new technique to generate diagonal dominant compensator pairs is described in the following sections. The method addresses each of the concerns cited above and is suitable for implementation in either an interactive or batch computer mode.

SECTION 3

NEW DOMINANCE ALGORITHM

In this section, a new dominance algorithm for the Multivariable Nyquist Array is developed. The algorithm utilizes the explicit definition of diagonal dominance of section 2 to minimize a non-analytic function of the pre and post compensator matrix parameters. A conjugate direction function minimization technique applied over the dynamic frequency range of interest is used to achieve the desired dominant condition.

In its most general form, the new dominance algorithm is characterized by four specific phases:

- a. Parameter initialization
- b. Dominance evaluation
- c. Parameter optimization
- d. Design considerations

The most unique features of the algorithm are the non-interactive nature of phases b and c and the increased degree of flexibility and designer interaction in phase d. In addition, the algorithm is sufficiently general so as to be appropriate to the batch computer mode design of both the Inverse Nyquist Array and the Direct Nyquist Array.

The algorithm is developed in the following manner: First, the concept of dominance is reviewed from a computational and structural viewpoint. The main computational unit, which performs phase b and phase c above, is then introduced followed by the program control unit. Finally, closed loop system design concepts new to the MNA design philosophy are presented and discussed.

A. Dominance Observations

The general concept of diagonal dominance was presented in section 2. It is now of interest to carefully examine the detailed structure of the dominance condition and to identify the points of similarity for row and column dominance determination.

Consider a general complex matrix $Z(s)$ to be $m \times m$ and the square matrices A and B to be constant. Let $P(s)$ be the matrix product

$$P(s) = A Z(s) B \quad 3.1$$

with diagonal dominance of $P(s)$ defined as

$$\sum_{\substack{j=1 \\ i \neq j}}^m |p_{ij}(s)| / |p_{ii}(s)| < \theta_i \leq 1 \quad i=1,2,\dots,m \quad 3.2$$

for row dominance and

$$\sum_{\substack{j=1 \\ i \neq j}}^m |p_{ji}(s)| / |p_{ii}(s)| \leq \theta_i \leq 1 \quad i=1,2,\dots,m \quad 3.3$$

for column dominance. In (3.2) and (3.3), θ_i represent a specified set of constants contained in the semi-open interval $(0,1]$ and will be defined in part D of this section.

Using (3.2) as the definition of row dominance and assuming B to be specified, the following observations can be made:

OBS 1

Diagonal dominance for row i of $P(s)$ is determined exclusively by the elements of the i th row of A .

OBS 2

Dominance of row i of $P(s)$ is unaffected by a scaling of row i of A .

OBS 1 states that the elements of row i of the A matrix (a_{ij} $j=1,2,\dots,m$) do not enter into the consideration of dominance for any other row

$j \neq i$. Hence, only m parameters in the A matrix need be determined for each row dominant condition. OBS 2 suggests that the row elements of A can be multiplied by a common factor with a guarantee of dominance preservation. This observation is used in part D to provide a mode of comparison of compensation forms and is fundamentally a scaling or normalization operation.

OBS 3

Using (3.3) with B specified, column dominance of (3.1) may be obtained only if all elements of matrix A are manipulated simultaneously.

This observation follows immediately upon noting that all elements of A , a_{ij} , influences the behavior of every column of $P(s)$. As a result of OBS 3, the method of pseudodiagonalization cannot be used to search for column dominance when B is specified.

The transpose of $P(s)$ in (3.1) provides a similar set of conditions for the B matrix when A is specified. Specifically, let

$$P_1(s) = P^T(s) = B^T Z^T(s) A^T = A_1 Z_1(s) B_1 \quad 3.4$$

In (3.4), $A_1 = B^T$, $Z_1(s) = Z^T(s)$ and $B_1 = A^T$. Hence, observations 1-3 are now appropriate to (3.4) and are summarized as follows

OBS 4

If A is specified in (3.1), then column dominance of $P(s)$ may be attained via the manipulation of the elements of B wherein the elements of the j th column of B only influence the dominance condition in the j th column of $P(s)$.

OBS 5

If A is specified in (3.1), then row dominance of $P(s)$ may be obtained only if the m^2 elements of B can be simultaneously manipulated.

The above observations are clarified upon examination of $P(s)$ in (3.1) when $m=2$

$$P(s) = \begin{vmatrix} a_{11}b_{11}z_{11}(s) + a_{12}b_{11}z_{21}(s) & a_{11}b_{12}z_{11}(s) + a_{12}b_{12}z_{21}(s) \\ + a_{11}b_{21}z_{12}(s) + a_{12}b_{21}z_{22}(s) & + a_{11}b_{22}z_{12}(s) + a_{12}b_{22}z_{22}(s) \\ a_{21}b_{11}z_{11}(s) + a_{22}b_{11}z_{21}(s) & a_{21}b_{12}z_{11}(s) + a_{22}b_{12}z_{21}(s) \\ + a_{21}b_{21}z_{12}(s) + a_{22}b_{21}z_{21}(s) & + a_{21}b_{22}z_{12}(s) + a_{22}b_{22}z_{21}(s) \end{vmatrix}$$

3.5

Further examination of (3.5) yields

OBS 6

If the elements of both A and B are unspecified, no obvious pattern for obtaining diagonal dominance of $P(s)$ is identifiable beyond the simultaneous manipulation of the $2m^2$ parameters. In addition, the elements of B are, in a sense, competing with the elements of A in the attempt to secure dominance of $P(s)$.

In view of OBS 6, it is apparent that designer intervention may be required to establish a hierarchical structure to the dominance evaluation procedure. This particular situation has not been automated in the proposed algorithm and remains a subject for future research. Several guidelines are presented in the discussion of the examples of section 4.

In the conditions imposed above, it was implicitly assumed that constant compensators were the desired form of system compensation. This would certainly be true from an implementation perspective and is therefore the initial assumption in any MNA design attempt. However, in some applications, constant compensator matrices may not yield the dominant condition or satisfactorily meet the closed loop design specifications.

The alternative is therefore dynamic compensation.

The observations made above for constant compensators can be extended to include the design of dynamic compensators. To illustrate, consider the objective to be row dominance of $P(s)$ in (3.1) with a specified post-compensator matrix $B(s)$. $A(s)$ in (3.1) can be generalized to

$$A(s) = \begin{vmatrix} a_{11}f_{11}(s) & a_{12}f_{12}(s) & a_{1m}f_{1m}(s) \\ a_{21}f_{21}(s) & a_{22}f_{22}(s) & a_{2m}f_{2m}(s) \\ \vdots & \vdots & \vdots \\ a_{m1}f_{m1}(s) & a_{m2}f_{m2}(s) & a_{mm}f_{mm}(s) \end{vmatrix} \quad 3.6$$

where $f_{ij}(s)$ are specified by the system designer and the a_{ij} are to be determined for dominance of $P(s)$. With respect to the a_{ij} 's, OBS 1-6 are retained. A special case of (3.6) is the pre-compensator form

$$A(s) = [A_0 + A_1/s] \quad 3.7$$

OBS 7

In an INA design using dynamic compensation, it may be desirable from an implementation viewpoint to structurally define the compensator in the inverse domain.

This observation is explored more fully in part D, but serves as a reminder that if $A(s)$ in (3.7) is the precompensator form for the INA, then $A^{-1}(s)$ could have poles and zeros in the open right half plane. This situation arises from the general observation that

$$a_{ij} f_{ij}(s) \neq [\hat{a}_{ij} \hat{f}_{ij}(s)]^{-1} \quad 3.8$$

and thus could be avoided with the precompensator form

$$A(s) = [A_0 + A_1/s]^{-1} \quad 3.9$$

Here OBS 3 would be imposed for both row and column dominance attempts.

With this insight into the structural and computational form of dominance evaluation, it is quite evident that (3.1) could represent either the INA or the DNA open loop design format. Thus a computational unit devoted to the search for diagonal-dominant-forming compensator parameters can be developed.

B. Generalized Optimization Unit

The main unit of the dominance algorithm is constructed in a generalized setting and is founded upon the observations in part A.

Let C represent a vector formed by consolidating the unknown compensator parameters into a single array. In some applications, such as row dominance of P(s) with B specified, C will be an m vector representing the unspecified values in the jth row of A. For other situations, reflected in OBS 6, C may be of dimension $2m^2$. Assume further that for any given C vector, P(s) can be properly reconstructed and evaluated for all s. The main unit then performs the following functions:

1. Accepts proper coding to identify the MNA design form and select the desired performance measure for dominance evaluation.
2. Adjust the elements of C to minimize the performance measure selected using a conjugate direction function minimization technique.

The performance measure selected is dependent upon the form of dominance desired. Fundamentally, there are two specific forms from which the selection is made:

$$a. \quad J_i(C, d_i) = \max_{\omega \in \Omega} \sum_{\substack{j=1 \\ i \neq j}}^m |p_{ij}(\omega)| / |p_{ii}(\omega)| \quad 3.10$$

$$b. \quad J(C, \theta_i) = \text{Max}_{\omega \in \Omega} \text{Max}_i \left[\sum_{\substack{j=1 \\ j \neq i}}^m |p_{ij}(\omega)| / |p_{ij}(\omega)| - \theta_i \right] \quad 3.11$$

The measure in (3.10) is selected whenever the unspecified coefficients can be subdivided into mutually exclusive sets. When this situation occurs m calls to the main unit will transpire, one call for each value of i . Alternatively, the measure in (3.11) is used whenever the dominance seeking coefficients must be simultaneously manipulated. Details are postponed to the following subsections.

The success of any numerical optimization technique in locating the extrema of a function of many variables is highly dependent upon the shape of the contours of the function to be extremized and the convergence properties of the optimization technique employed. Clearly, these concerns are imbedded in the successful evaluation of the compensator parameters in (3.10) or (3.11).

Examination of (3.11) reflects the interesting computational and numerical aspects of the performance measure. Each of the $p_{ij}(\omega)$'s are complex functions of the compensator parameters from which the definition of dominant matrices is composed. At each frequency, the rows (or columns) are scanned to identify the largest ratio. This array is then scanned (over ω) to determine its maximum value. If the largest ratio is less than θ_i , then the trial elements generating $P(s)$ yield dominance for all rows (or columns) over the frequency range of concern. If the ratio is greater than θ_i , then the elements of C must be adjusted to create the desired condition.

The majority of numerical optimization techniques require some form of localized gradient or second variational calculation. If the performance measure is a well defined analytical function, then a gradient

dependent algorithm is often the most efficient route to pursue. This is the premise upon which the method of pseudodiagonalization is based. However, gradient calculations for (3.10) or (3.11) could result in significant numerical difficulties due, in part, to the extremization over ω . For this reason a numerical optimization technique which does not explicitly depend on localized gradient calculations is preferred.

The method selected for implementation is the Zangwill-Powell [22,23] optimization technique. This method is known to be effective when sharp ridges and narrow valleys are present in the performance contours and is suitable for application to problems with a large set of variables to be optimized [22,24,25]. Other methods which do not require gradient calculations and therefore might be appropriate for the optimization of (3.10) or (3.11) are those of Swann [26] (an extension of Rosenbrock [27]), Smith [28] and Wood [29]. Of the optimization methods which do not require derivatives, Fletcher's study [30] suggests that Powell's method may be computationally the most efficient.

Fundamentally, the optimization unit of the dominance algorithm performs as follows:

1. Upon receipt of an initial guess for the C vector, $P(s)$ is evaluated to determine the ratio of the Gershgorin radii to the moduli of the corresponding diagonal element in (3.10) or (3.11) for all $\omega \in \Omega$.
2. The dominance ratio is appropriately scanned over ω to identify the largest ratio. The numerical values of $J(C, \theta_i)$ at subsequent evaluations are used by the conjugate direction minimization algorithm to adjust the components of C.
3. Step 2 is repeated until either the desired degree of dominance

is achieved or internal checks within the optimization method indicate no further improvement in $J(C, \theta_i)$ is likely with successive adjustments of C .

This optimization unit forms the nucleus of the dominant seeking algorithm. Access to and control of this unit is the function of the program supervisor.

C. Program Supervisor

The program supervisor performs the task of accepting information from the system designer and properly coding the optimization unit and the design unit (see next subsection) to perform the requested design. The supervisor will accept the following data for each design attempt:

1. INA or DNA design?
2. System dimension
3. Frequency range
4. Frequency increment (equal spacing)
5. Row or column dominance?
6. Identify fixed compensator
 - i. Precompensator
 - ii. Postcompensator
 - iii. None
 - iv. Both
7. Numerator and denominator coefficients for each element of $G(s)$
8. Dominance control parameters, θ_i , $i = 1, 2, \dots, m$
9. Plot options
 - i. No plot
 - ii. Row or column elements superimposed by rows only

- iii. Gershgorin band only
 - iv. (ii) and (iii) only
 - v. Gershgorin and Ostrowski bands
 - vi. Ostrowski only
10. Diagonal feedback gain elements for Ostrowski bands
11. Precompensator specifications
- i. Dynamic or constant?
 - ii. If dynamic, specify coefficients of $f_{ij}(s)$
 - iii. Initialize coefficients
 - a. Set to identity matrix
 - b. Diagonalize at $\omega = \omega_0$
 - c. Elements to be read in
12. Post compensator specifications
- i. Dynamic or constant?
 - ii. If dynamic, specify coefficients of $f_{ij}(s)$
 - iii. Initialize coefficients
 - a. Set to identity matrix
 - b. Diagonalize at $\omega = \omega_0$
 - c. Elements to be read in
13. Advance design control parameters

With the above information the MNA design is completely specified and a search for dominance using the optimization unit may be implemented. In the following subsection, design concepts new to the MNA design philosophy are introduced. Section 4 illustrates the use of these concepts in applications.

D. New Design Concepts

The Inverse Nyquist Array as conceived by Rosenbrock and implemented on a PDP-10 digital computer by Munro at the University of Manchester was intended to be used in an interactive computer mode. In this mode, the system designer is an integral part of the computational and evaluative phase of dominance determination. As a result of this high degree of designer intervention, final design considerations are based upon trial and error methods which, in most cases, are not systematized. It is the intent of this subsection to introduce new computer aided design techniques to the multivariable Nyquist array.

1. Dominance Control Parameters

Theoretically, any degree of system dominance is sufficient for the application of the MNA design method. The degree of system dominance may range from the marginally dominant condition (performance measure less than but near unity) to the decoupled condition (performance measure near zero). Correspondingly, if the level of system interaction is interpreted in terms of the width of the Gershgorin bands then there is a direct correlation between system interaction and dominance.

From a practical viewpoint, the degree and type of system interaction is an important design consideration in the selection of input-output pairs and corresponding compensator structures. For this reason it may be desirable to reduce open loop system interaction before the feedback loops are closed. This is accomplished through a specification of the "dominance control parameters", θ_i , $i=1, \dots, m$ in the system performance measures of (3.2) and (3.3) (or alternatively (3.10) and (3.11)).

In application, the system designer will specify each θ_i for $i = 1, 2, \dots, m$ where

$$0 < \theta_i \leq 1 \quad i=1,2,\dots,m \quad 3.12$$

The unspecified parameters in the compensator matrices are then adjusted by the optimization unit in an attempt to meet this degree of dominance. The θ_i (selected in accordance with (3.12)) is fundamentally a request to the optimization unit to make the largest Gershgorin radius smaller than $100 \theta_i$ percent of the corresponding diagonal element. If the optimization unit can satisfy the θ_i request in each row (or column), then the prescribed degree of dominance has been achieved.

An interesting observation regarding Ostrowski's theorem can be made when the dominance control specifications are satisfied. Recall that the shrinking factors in Ostrowski's theorem are

$$(INA) \quad \phi_i(s) = \max_{j \neq i} \frac{d_j(s)}{|f_j + z_{jj}(s)|} \quad 3.13$$

and

$$(DNA) \quad \phi_i(s) = \max_{j \neq i} \frac{d_j(s)}{\left| \frac{1}{f_j} + z_{jj}(s) \right|}$$

Let $f_j = 0$ in an INA design and $f_j = \infty$ in a DNA design, then

$$\phi_i(s) = \max_{j \neq i} \frac{d_j(s)}{|z_{jj}(s)|} \quad 3.15$$

Define

$$\partial_i = \text{Max}_s \phi_i(s) = \text{Max}_s \text{Max}_{j \neq i} \frac{d_j(s)}{|z_{jj}(s)|} \quad 3.16$$

hence

$$\phi_i(s) \leq \partial_i \quad \text{for all } i = 1, 2, \dots, m.$$

But the ∂_i are nothing other than the largest degree of dominance obtained for $j \neq i$, therefore

$$\partial_i = \max_{j \neq i} \theta_j \quad 3.17$$

Thus for any MNA design, the Gershgorin band may be immediately reduced by the corresponding ∂_i factor.

2. Interaction Index [31]

Once a suitable degree of dominance has been obtained, each feedback control loop may be independently closed using single loop classical control theory as outlined in section 2 of this report. Application of Ostrowski's theorem can then be used to further reduce the Gershgorin bands for each design loop. The corresponding Ostrowski bands can thus be used as a conservative estimate of the design parameters to improve the overall closed loop system design.

It is important to re-emphasize that each loop is designed independently. Furthermore, the width of the final set of Ostrowski bands, in a broad sense, reflect the degree of closed loop system interaction in the finalized design. However, no information regarding the type of interaction is available from the Ostrowski plots. For this information, the "Interaction Index" developed by Davison [31] is employed.

Briefly, the interaction index assumes that m linear time invariant proportional feedback control loops have been independently designed and are separately applied to the system. Davison considers the question of how much interaction will occur when all m control loops are to be applied simultaneously to the system.

Consider the linear time invariant system

$$\dot{x}(t) = A x(t) + B u(t) \quad 3.18$$

$$y(t) = C x(t) \quad 3.19$$

In an MNA format, the control law $u(t)$ is

$$\mu(t) = -K F L C x(t) + \mu^o(t) \quad 3.20$$

where K and L are the dominance producing compensators, the feedback gains f_j have been determined from the Ostrowski plots and $\mu^o(t)$ are the input disturbances to the system.

Under the assumption that m control laws have been found so that the resultant systems are stable and the jth controller satisfactorily controls output y_j , the following index of performance is chosen

$$J_j = \max_{x_o} \int_0^{\infty} y_j^2(t) dt \quad j = 1, 2, \dots, m \quad 3.21$$

$$x_o^T x_o = 1$$

Now, define the interaction index as

$$I_j = \frac{J_j^* - J_j}{J_j} \quad j = 1, 2, \dots, m \quad 3.22$$

Here J_j is the value of (3.21) resulting from the application of μ_j only and J_j^* is the value of (3.21) when all loops have been closed simultaneously. The index in (3.22) thus provides a measure of the relative change in the control of $y_j(t)$ when all feedback loops are simultaneously closed compared with the jth control law applied independently.

The problem defined by (3.18), (3.19), (3.20) and (3.21) can be reformulated in a Lyapunov function:

$$J_j = \max_{x_o} x_o^T Q_j x_o = \lambda_{\max}(Q_j) \quad 3.23$$

$$x_o^T x_o = 1$$

where $\lambda_{\max}(Q_j)$ is the largest eigenvalue of Q_j and Q_j is the solution to

$$[A - BKF_jLC]^T Q_j + Q_j [A - BKF_jLC] = -C_j C_j^T \quad 3.24$$

The C_j in (3.24) are the corresponding rows of matrix C . The matrix F_j in (3.24) has zeros in all positions except for f_{jj} , which is the feedback gain for loop j . To evaluate J_j^* , F_j is diagonal with each diagonal element set to the corresponding design gain.

Using (3.23), the interaction index becomes

$$I_j = \frac{\lambda_{\max}(Q_j^*)}{\lambda_{\max}(Q_j)} - 1 \quad 3.25$$

and will lie between the bounds

$$-1 < I_j < \infty \quad 3.26$$

$$\text{a. } \underline{-1 < I_j < 0}$$

This situation occurs when the performance index for J_j^* is less than J_j and is construed to be a favorable form of system interaction. That is, the control of output y_j is improved when all feedback loops are simultaneously closed when compared to the control of y_j using only input u_j . The ultimate limit of -1 is obtained when a high degree of system interaction exists such that J_j^* is near zero, i.e.

$$J_j^* \ll J_j \quad 3.27$$

$$\text{b. } \underline{I_j = 0}$$

This situation occurs when

$$J_j^* = J_j \quad 3.28$$

and thus implies that the closure of the remaining feedback loops has no effect upon the control of y_j . This case corresponds to a decoupled condition.

$$\text{c. } \underline{I_j > 0}$$

Here loop closure has a deleterious effect upon the control of y_j when

compared to the single loop closure of μ_j . Clearly, this form of system interaction is undesirable when I_j becomes large. As I_j tends toward infinity, the interaction becomes more severe and implies a tendency toward instability ($I_j = \infty$).

The interaction index thus becomes an important evaluative measure for closed loop system design and can be applied directly to the MNA.

3. Comparison of Compensator Designs

In the application of the Multivariable Nyquist Array design method, alternate structural forms and different initial guesses for the unspecified compensator parameters may yield (after optimization) different compensator designs, each of which satisfy the dominant conditions. It is therefore of interest to have a means for the comparison of compensators which may have been generated by different design forms.

In particular, consider an INA design with \hat{L} specified and \hat{K} constrained to be constant. For row dominance of $\hat{Q}(s)$, each row of \hat{K} is mutually exclusive of the elements of the remaining rows. Thus the i th row of \hat{K} may be normalized about the diagonal parameter. This procedure applied to alternate designs for \hat{K} will provide a direct comparison of dominant compensator forms and therefore may be used to determine the suitability and/or superiority of the alternate pre-compensator design forms.

As an illustration, consider an m th order $\hat{G}(s)$ with $\hat{L} = I$. Assume that m initial guesses for \hat{K} yield m different normalized compensators \hat{K}_r $r = 1, 2, \dots, m$ each yielding dominance for all rows of the corresponding $\hat{Q}_r(s)$. For each row the best design is selected and the corresponding row of \hat{K}_r identified. The composite \hat{K} matrix is then used to finalize the design.

This procedure could also be used for other MNA design forms and thus provide the desired mode for comparison.

For dissimilar structural forms each yielding an acceptable design, the Interaction Index discussed above could be used as the mode for comparison. Here the comparison of row versus column designs or designs using different input-output pairs could be made. Thus any final design decision could ultimately be based upon the level of system interaction and actuator and/or sensor failure accomodation.

E. Discussion

In this section, a new algorithm to obtain the diagonal dominant condition for the multivariable Nyquist array has been developed. The algorithm is compatible to both the INA and DNA design philosophy with either constant or dynamic compensation.

Fundamentally, the algorithm is based upon the observations and generalization of the concept of diagonal dominance. The characterization of dominance as a function minimization problem provides for a tremendous degree of design flexibility and eliminates a substantial portion of the trial and error aspects of previously used methods. In addition, the proposed algorithm may find dominance conditions when other techniques are inappropriate or have previously failed. This is certainly true whenever a design attempt is made for which pseudodiagonalization cannot be used, i.e., INA design for column dominance with specified L matrix.

Several advanced design concepts new to the MNA design philosophy have been introduced. These concepts are based upon the availability of a fast and efficient method to generate the dominance condition. The dominance control parameters are used in an attempt to secure a specified level of dominance in each control loop. They are used in the generalized optimi-

zation unit and correspond to the minimization of the largest Gershgorin radii within the frequency range specified. Since the frequency range is determined by the system designer it could vary from a single isolated point to a fixed interval in the frequency spectrum to the entire spectrum. The total impact of this degree of flexibility has yet to be realized.

Utilization of the interaction index and compensator normalization methods suggest a means by which the system designer can evaluate a proposed closed loop design in addition to providing a comparison of competing designs. These techniques could also be used in the assessment of dynamic feedback components and thus provide a quantitative measure of closed loop system response characteristics and interaction.

In the next section, the results obtained by the dominance algorithm are compared with previously reported applications. In addition, an analysis is performed on the F100 turbofan engine using an INA format. In each case the primary goal is to either verify the reported results and suggest new alternatives or to simply obtain the dominance condition with little effort devoted to attaining a final design.

SECTION 4

APPLICATIONS

The principal objective of this section is to demonstrate the efficiency of the algorithm in section 3 and to substantiate the suitability of the MNA design philosophy of section 2 as a viable closed loop design alternative for air breathing propulsion systems. In each of the applications considered herein, the principal concern has been to achieve the diagonal dominant condition with little attention devoted to any final design considerations.

The section is subdivided into three subsections:

- A. Previous applications of MNA
- B. F-100 Turbofan Engine
- C. Discussion

In part A, the algorithm is applied to several test cases with new and interesting results to be reported. Part B is the application of the MNA to a linearized operating point model of the Pratt and Whitney F100 series 2 engine at sea level static conditions. This represents the first attempt at an MNA design for a sixteenth order F100 model. Part C provides an analysis of the application areas and suggests new dimensions for closed loop design of air breathing propulsion systems.

A. Previous applications of MNA

At the present time, the control literature contains approximately seven or eight reported applications of the Inverse Nyquist Array. The design applications using the INA have been primarily conducted at the University of Manchester Institute of Science and Technology (UMIST) under the auspices of Prof. Rosenbrock and performed by his colleagues (Munro, Rutherford, etc.) and students. Little activity regarding the INA has been reported outside

UMIST due in part to the dependency of the INA on computer-aided graphic facilities. Since the algorithm of section three reduces this dependency, all previously reported design applications served as test cases for the new design algorithm. In every case, the algorithm operating in the appropriate mode for evaluation purposes either confirmed the reported design compensator forms and their indicated numerical values, suggested alternate dominant forms or modified the reported compensator values to extend the dominant condition to a larger frequency range.

The results of the trial cases were generally obtained in one pass through the algorithm using $\hat{K} = I$ or $\hat{K} = LG(0)$ as the initial guess with the specified frequency range subdivided into N equally spaced points. Depending upon the dynamic frequency range of interest, N could be selected as any integer value between one and one thousand. On the average, each test case required 100 CPU seconds on a batch mode IBM 360/75 to achieve a dominant condition and plot the indicated Gershgorin and/or Ostrowski bands.

This is in direct contrast to the typical two weeks to nine man-months of effort required to achieve dominance using the UMIST computer aided design suite.

In each of the following test cases, the dominance algorithm of section 3 provided new and interesting results regarding the specific applications.

1. Rosenbrock [14] - Boiler Furnace

In this application, the objective is the control of a boiler furnace with four inputs and four outputs as represented by

$$G(s) = \begin{vmatrix} \frac{.1}{1+4s} & \frac{.7}{1+5s} & \frac{.3}{1+5s} & \frac{.2}{1+5s} \\ \frac{.6}{1+5s} & \frac{1}{1+4s} & \frac{.4}{1+5s} & \frac{.35}{1+5s} \\ \frac{.35}{1+5s} & \frac{.4}{1+5s} & \frac{1}{1+4s} & \frac{.6}{1+5s} \\ \frac{.2}{1+5s} & \frac{.3}{1+5s} & \frac{.7}{1+5s} & \frac{.1}{1+4s} \end{vmatrix} \quad 4.1$$

For this $G(s)$, Rosenbrock has determined that the system is diagonal dominant in the INA sense ($\hat{G}(s)$) when

$$\hat{K} = \hat{L} = I \quad 4.2$$

It is further demonstrated that the dominance condition is improved if

$\hat{L} = I$ and

$$\hat{K} = G(0) = \begin{vmatrix} 1 & .7 & .3 & .2 \\ .6 & 1 & .4 & .35 \\ .35 & .4 & 1 & .6 \\ .2 & .3 & .7 & 1 \end{vmatrix} \quad 4.3$$

Using pseudo-diagonalization at $\omega = 0.9$ for all rows with $\hat{L} = I$, the following precompensator is obtained

$$K = \begin{vmatrix} 1.469 & -9.44 & -.148 & .050 \\ -.654 & 1.814 & -.249 & -.229 \\ -.229 & -.249 & 1.818 & -.654 \\ .050 & -.148 & -.944 & 1.469 \end{vmatrix} \quad 4.4$$

The effect of (4.3) is to reduce the Gershgorin band to the point where the system is essentially non-interacting.

Using the algorithm of section 3, the following runs were made in one submission of the computer program:

- a. $K=I$; $L=I$; INA; No optimization
- b. Same as (a) except $\hat{K}=G(0)$
- c. Same as (a) except \hat{K} was set equal to Rosenbrock's solution
- d. $K=I$; $L=I$; INA; optimize with $\theta_i=0$, for $i=1,2,3,4$; set all feedback gains to zero and plot Gershgorin and Ostrowski Bands
- e. Same as (d) except $\hat{K}=G(0)$
- f. Same as (d) except $\hat{K} = \text{Rosenbrock Solution}$

In runs d and e, the dominance control parameters were set to zero to determine the set the precompensator values which will yield a condition of maximum dominance. Runs a through c were made to verify the reported conditions in [14].

For each run, system dominance was obtained. The following table provides a comparison of the degree of dominance achieved in each case:

Table 4.1

Degree of Dominance Achieved	Run a	Run b	Run c	Run d	Run e	Run f
θ_1	.88041	.24899	.25402	.13118	.13134	.13117
θ_2	.81731	.28357	.24924	.13841	.14184	.13865
θ_3	.81731	.28357	.24924	.13814	.13846	.13835
θ_4	.88041	.24899	.25402	.13684	.13340	.13471

A comparison of columns a through c verify the reported conditions namely that pseudodiagonalization (Run c) reduced the degree of system interaction

to a significantly lower level. However, an overview of Table 4.1 indicates that the proposed algorithm was able to further improve the degree of dominance by approximately fifty percent (runs d and e). The final set of precompensator values are indicated below:

Run d: INA, $\hat{K}=I$, $L=I$, $\theta_i=0.0$ desired

$$\hat{K}_d = \begin{vmatrix} .91320 & .58078 & .25031 & .16693 \\ .49444 & .91399 & .33017 & .28856 \\ .28259 & .32363 & .90150 & .48944 \\ .16965 & .24860 & .58459 & .90534 \end{vmatrix} \quad 4.5$$

Run e: INA, $K=G(0)$, $L=I$, $\theta_i=0.0$ desired

$$\hat{K}_e = \begin{vmatrix} 1.10040 & .70007 & .30000 & .20000 \\ .54478 & 1.01650 & .37337 & .32247 \\ .34419 & .39708 & 1.10420 & .59983 \\ .19154 & .28148 & .65139 & 1.0143 \end{vmatrix} \quad 4.6$$

Run f: INA, \hat{K} = Rosenbrock solution, $L=I$, $\theta_i = 0.0$ desired

$$\hat{K}_f = \begin{vmatrix} .84900 & .53952 & .23280 & .15500 \\ .43153 & .79200 & .28384 & .24560 \\ .23436 & .26755 & .74215 & .40167 \\ .16510 & .24714 & .59242 & .92996 \end{vmatrix} \quad 4.7$$

It is clear from table 4.1 that any one of the precompensators corresponding to runs d, e or f would be adequate for this application. However, in other applications it may be necessary to form a composite compensator based upon the results obtained from different initial guesses for the unknown parameters. This is caused primarily by the inability of the optimization technique to detect and correct for the presence of local minima

during the optimization phase of the program.

For this case, Table 4.1 would suggest the utilization of row 1 from \hat{K}_f in (4.7), rows 2 and 3 from \hat{K}_d in (4.5) and row 4 from \hat{K}_e in (4.6) to yield the composite form

$$K = \begin{vmatrix} .84900 & .53952 & .23280 & .15500 \\ .49444 & .91399 & .33017 & .28856 \\ .28259 & .32363 & .90150 & .48944 \\ .19154 & .28148 & .65139 & 1.01430 \end{vmatrix} \quad 4.8$$

with corresponding dominance levels as

$$\theta = \begin{vmatrix} .13117 \\ .13841 \\ .13814 \\ .13684 \end{vmatrix} \quad 4.9$$

From Ostrowski's theorem, the transmittance from input i to output i when all feedback loops are closed (except for loop i), can be located within a narrower set of bands within the Gershgorin bands using the shrinking factors

$$\phi_i(s) = \text{Max}_{j \neq i} \frac{d_j(s)}{|f_j + q_{jj}(s)|} \quad 4.10$$

where $d_j(s)$ are the Gershgorin radii, f_j the feedback gain in loop j and $q_{jj}(s)$ the diagonal element in $Q(s)$. For all $f_j=0$ the shrinking factors are bounded by

$$0 \leq \text{Max}_{j \neq i} \frac{d_j(s)}{|q_{jj}(s)|} \leq \text{Max}_{j \neq i} \theta_i = \theta_i$$

where θ_i are the corresponding dominance levels. In this application, the θ_i are

$$\theta = \begin{vmatrix} .13841 \\ .13814 \\ .13841 \\ .13841 \end{vmatrix}$$

which implies that the Gershgorin band may be immediately reduced by approximately 86.2 percent. The Ostrowski band in each loop will shrink further when the feedback gains are increased.

The results presented above were obtained in one batch mode submission of the computer program with a total expenditure of 30 CPU seconds including program compile time.

2. Munro [32] - Aircraft Autostabilization

In this application, the control unit for a two input two output model of a delta-winged aircraft is developed using the Inverse Nyquist Array method with constant pre and post compensation matrices. Here the authors restricted the post-compensator to the form

$$L_a = \begin{vmatrix} k_{11}^a & 0 \\ 0 & 1 \end{vmatrix} \quad \text{or} \quad L_b = \begin{vmatrix} 1 & 0 \\ 0 & k_{22}^b \end{vmatrix} \quad 4.11$$

Using the L_b form, row dominance was obtained using pseudo diagonalization with the following results for $0 \leq \omega \leq 4.0$

$$\hat{K} = \begin{vmatrix} -12.08 & -34.19 \\ -4.39 & 2.31 \end{vmatrix} \quad 4.12$$

and

$$L_b = \begin{vmatrix} 1 & 0 \\ 0 & 6.67 \end{vmatrix} \quad 4.13$$

It should be noted that pseudodiagonalization does not assist in the selection of the elements of the postcompensator matrix when row dominance of $\hat{Q}(s)$ is to be obtained via the manipulation of the \hat{K} elements. Thus to obtain (4.12) and (4.13) requires a hierarchical approach to parameter selection. The results indicated above correspond to a set of dominance control parameters of [.86, .39].

With the algorithm of section 3, the following six cases were programmed for one batch mode submission:

$$a. \quad \hat{K} = I \quad \ell_{22}^b = 1, 5, 10$$

$$b. \quad \hat{K} = LG(0) \quad \ell_{22}^b = 1, 5, 10$$

In each case, the L_b form was selected for comparison with (4.13). Identical results would have been achieved if the L_a form had been used with

$$\ell_{11}^a = 1/\ell_{22}^b$$

The following results were obtained with $\theta_i = 0.0$:

Table 4.2

Initial \hat{K}	ℓ_{22}^b		
	1	5	10
I	Dominance Row 2 only	Dominance Row 2 only	Dominance Row 1 only
LG(0)	Dominance Row 2 only	Dominance Row 2 only	Dominance Row 2 only

From Table 4.2, it is clear that the desired condition is obtained from $\ell_{22}^b=10$, using a composite precompensator formed using the appropriate dominance producing row of \hat{K} as follows

$$a. \quad \hat{K}_{\text{initial}} = I \quad \hat{K}_{\text{final}} = \begin{vmatrix} 1.0 & -.8388(10^6) \\ -1.8616 & .99938 \end{vmatrix}$$

$$\theta_a = [1.4244 \quad .57973]$$

$$b. \quad \hat{K}_{\text{initial}} = LG(0) \quad \hat{K}_{\text{final}} = \begin{vmatrix} -1.4625 & -3.0661 \\ -.37956 & -.62166 \end{vmatrix}$$

$$\theta_i = [.55472 \quad 1.8911]$$

c. Composite precompensator

$$\hat{K}_{\text{final}} = \begin{vmatrix} -1.4625 & -3.0661 \\ -1.8616 & .99938 \end{vmatrix}$$

$$\theta_1 = [.55472 \quad .57973]$$

$$L = \begin{vmatrix} 1 & 0 \\ 0 & 10 \end{vmatrix}$$

This application serves to demonstrate that for some systems, efforts to achieve the dominance condition for $\hat{Q}(s)$ may be dependent upon the initial starting guess for \hat{K} . To explore this further, a parametric study was performed on ℓ_{22}^b for the starting conditions of Table 4.2. The results of this study are reflected in figures 4.1 and 4.2. For each value of ℓ_{22}^b , the dominance control parameters were set to zero and the precompensator values optimized. The level of dominance achieved for each case reflects a local minimum in the performance index and is indicated in the figures for each row of $\hat{Q}(s)$.

From figures 4.1 and 4.2 it is clear that a composite precompensator matrix yielding the desired compensator values can be obtained for any ℓ_{22}^b in the range

$$5.9 \leq \ell_{22}^b \leq 14$$

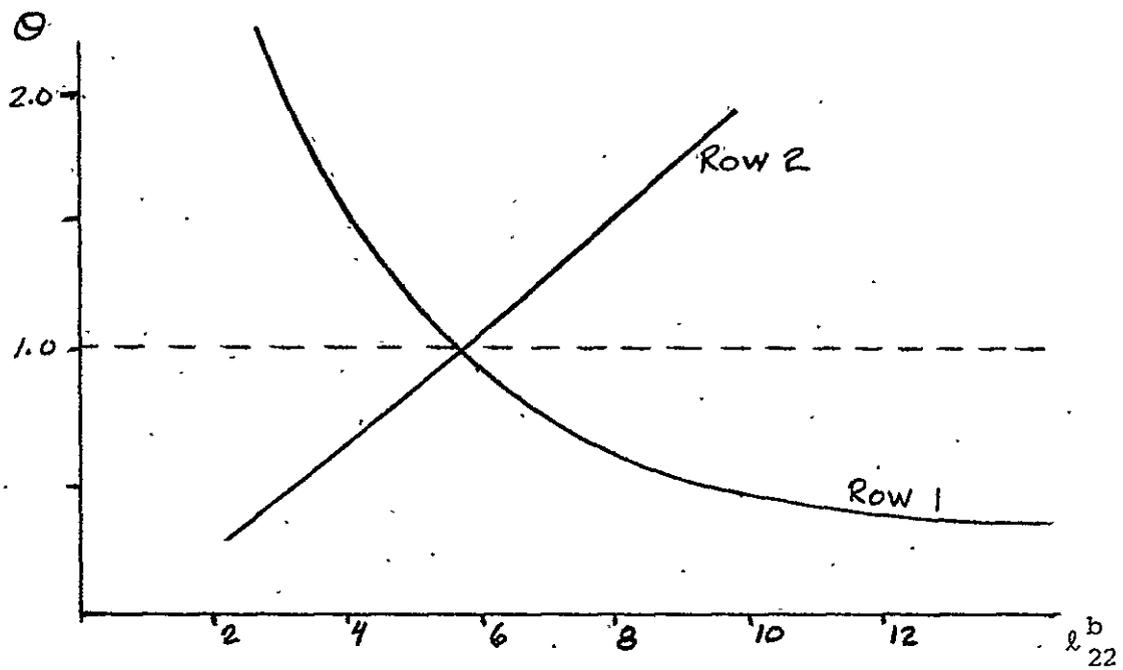


Figure 4.1 $\hat{K}_{\text{INITIAL}} = \text{LG}(0)$

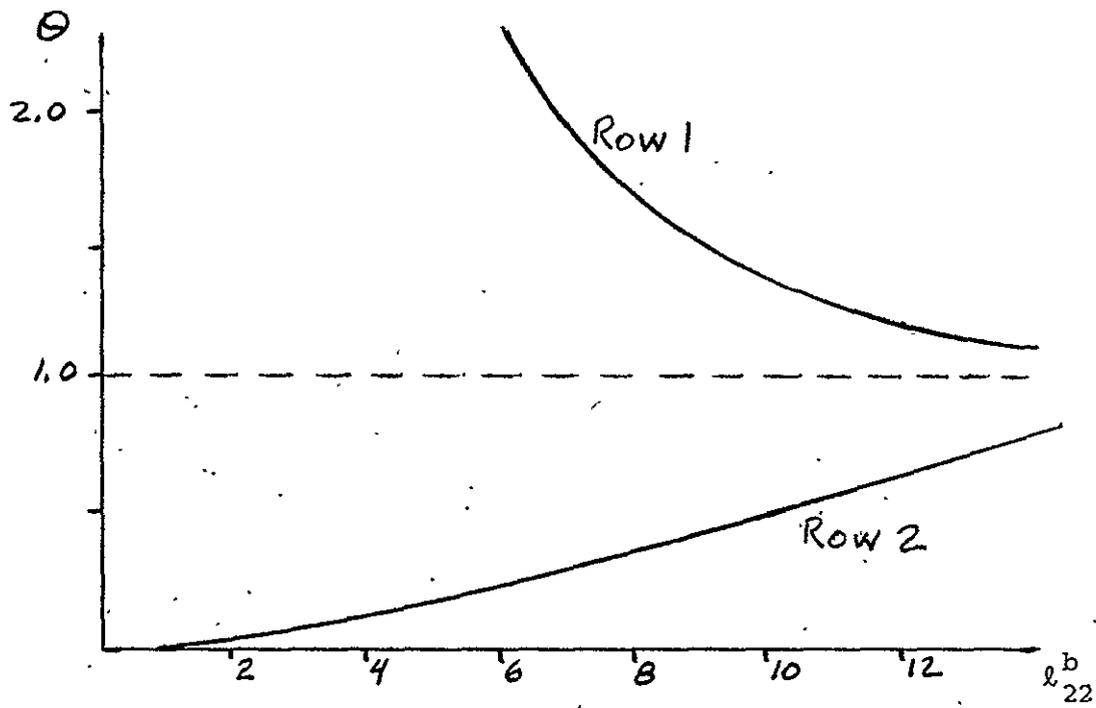


Figure 4.2 $\hat{K}_{\text{INITIAL}} = \text{I}$

No attempt was made to identify the upper limit for ℓ_{22}^b since this value can be deduced from the Gershgorin plots.

Using L_b in (4.13) the results of Munro are confirmed using the procedure indicated above and may be improved to the dominance levels of [.8317 .3867] with

$$\hat{K} = \begin{vmatrix} -14.59 & -30.59 \\ -4.303 & 2.31 \end{vmatrix}$$

A review of the printout corresponding to $\hat{K} = \hat{L} = I$ with no optimization reflects the greatest departure from dominance occurring in both rows near $\omega = 1$. Selecting $\ell_{22}^b = 10$ and the initial guess for the precompensator as

$$\hat{K}_{\text{initial}} = R_e[G(j1)]$$

dominance was obtained (via the algorithm) for both rows simultaneously with

$$\hat{K}_{\text{final}} = \begin{vmatrix} -.13171 & -.27622 \\ -1.0447 & .56083 \end{vmatrix}$$

and dominance levels of

$$\theta = [.55475 \quad .57973]$$

This result demonstrates that alternate methods of selecting initial parameter values based upon the systems characteristics are available.

3. Munro [33] - Automotive Gas Turbine

In this application of the INA a two input two output transfer matrix is used to describe the dynamic characteristics of an automotive

$$G(s) = \begin{vmatrix} \frac{.806s + 2.64}{s^2 + 1.15s + 2.202} & \frac{-(15s + 1.42)}{s^3 + 12.8s^2 + 13.6s + 2.36} \\ \frac{1.95s^2 + 2.12s + 4.9}{s^3 + 9.15s^2 + 9.39s + 1.62} & \frac{7.14s^2 + 25.8s + 9.35}{s^4 + 20.8s^3 + 116.4s^2 + 111.6s + 188} \end{vmatrix}$$

gas turbine engine over the frequency range $0 \leq \omega \leq 25$. From [33] column diagonal dominance is obtained with $L = I$ and

$$\hat{K} = \begin{vmatrix} 1.15 & -.5175 \\ 1.30 & .415 \end{vmatrix}$$

Column dominance for $\hat{Q}(s)$ with a fixed postcompensator requires continuous monitoring of both column dominance indices when any parameter in \hat{K} is adjusted. This form of the algorithm is outlined in section 3 and the above results were confirmed with an initial guess for \hat{K} as the identity matrix for $L = I$ and $0 \leq \omega \leq 25$.

To examine the effectiveness of the algorithm in section 3, the following input data was provided to the algorithm:

- a. Row dominance desired
- b. $L = I$ specified
- c. $K = I$ as initial guess
- d. $0 \leq \omega \leq 25$ subdivided into two hundred equally spaced points

The following row dominant results were obtained within 30 CPU seconds on the IBM 360/75:

$$L = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$$

$$\hat{K} = \begin{vmatrix} 2.9730 & -4.2414 \\ 12.507 & 1.0247 \end{vmatrix}$$

with

$$\theta_1 = .38007$$

$$\theta_2 = .42557$$

representing the maximum degree of system dominance in the respective rows. With this degree of dominance, the Gershgorin bands can be immediately reduced by 57.443% and 61.993%, respectively, with zero feedback gains.

Recalling the definition of dominance from section 2, it is clear that if the system is both row and column dominant at a particular frequency, then the smaller radius can be used for each row or column element in determining the appropriate Gershgorin radius. Examination of the computer listing corresponding to the row dominant conditions above indicates that $\hat{q}_{11}(s)$ is both row and column dominant over the range $0 \leq \omega < .5$ and $4.0 < \omega \leq 25$, and $\hat{q}_{22}(s)$ is both row and column dominant over $0 \leq \omega \leq 25$. This condition implies that the smaller radius can be used for both elements everywhere except $.5 \leq \omega \leq 4.0$. The remarks above concerning band reduction will still apply to the new radii.

The Gershgorin and Ostrowski bands for the row dominant configuration possess the same shape and form as those reported in [33] for column dominance.

4. Sain [34] - Turbofan Engine

In [34], the two input two output transfer matrix representing the dynamic characteristics of a turbofan engine is presented as

$$G_{11}(s) = \frac{.018s^5 + .145s^4 - 92.05s^3 - 396.9s^2 + 29801s + 95,491}{\Delta(s)}$$

$$G_{12}(s) = \frac{.546s^5 + 71.9s^4 + 2247s^3 - 1943s^2 - 16885s - 12495}{\Delta(s)}$$

$$G_{21}(s) = \frac{.086s^5 + 31.63s^4 + 3321.5s^3 + 25500s^2 + 76068.3s + 78277}{\Delta(s)}$$

$$G_{22}(s) = \frac{.13s^5 - .437s^4 + 68.2s^3 + 1703.3s^2 + .742.9s - 3532.2}{\Delta(s)}$$

$$\Delta(s) = s^5 + 140.7s^4 + 5337.6s^3 + 38691s^2 + 119690s + 133389$$

Although the Gershgorin bands are indicated for a column dominant $\hat{Q}(s)$ in [34], the corresponding compensator matrices were not provided. However, the authors suggest that column dominance was obtained during a search for row dominance using pseudodiagonalization about $\omega = 1$. Using this method to achieve dominance they conclude that "though our examples are ... introductory, they do serve to show that typical jet engine models do not yield trivial dominance questions".

With the algorithm of section 3 and $G(s)$ above, row dominance was obtained for

$$L = \begin{vmatrix} 0 & 1 \\ \frac{1}{3} & 0 \end{vmatrix}$$

$$K = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$$

The corresponding dominance levels are

$$\theta = [.848051 \quad .406645]$$

which can be reduced to

$$[.75329 \quad .36597]$$

using

$$L = \begin{vmatrix} 0 & 1 \\ \frac{1}{3} & 0 \end{vmatrix}$$

$$\hat{K} = \begin{vmatrix} 1.2755 & -.016011 \\ -.86064 & .77641 \end{vmatrix}$$

Setting the feedback gains to zero, the Ostrowski factors are

$$\sigma = [.36597 \quad .75329]$$

wherein the appropriate Gershgorin band can be immediately reduced. A plot of the corresponding Ostrowski band indicates an essentially decoupled system as all lines are coincident over most of the frequency range considered.

For this application, six hundred points were used over $0 \leq \omega \leq 200$. Beyond 200 radians little dynamic activity occurs and thus was not used in the dominance evaluations. However the above compensator matrices retain the dominant condition for $\omega \geq 200$.

5. Other Cases

The dominance algorithm of section 3 has been used to examine the form and level of dominance in numerous examples in Rosenbrock's text and the current literature. In each case, the algorithm confirmed the reported results and improved the level of dominance using the dominance control parameters. In cases where the system was not dominant beyond a specified frequency, the algorithm adjusted the compensator values to secure dominance over the entire spectrum.

B. F-100 Turbofan Engine

The engine under consideration is a Pratt and Whitney F100-PW-100 afterburning turbofan. The F100 is a low bypass ratio, twin spool

axial flow engine with the following components:

1. Three stage fan driven by a two stage turbine
2. Ten stage compressor driven by an air cooled two stage turbine
3. Main burner with an annular chamber
4. Variable area exhaust nozzle

Using the non-linear dynamic simulation of the F100 engine and the offset derivative method, a set of linear dynamic equations in state variable form for each of the thirty seven operating points is reported in [35]. For this study the sea level static (SLS) intermediate point was selected. This is in correspondence with zero Mach number, zero altitude and a power level angle of 83° .

The linear model at SLS intermediate is a sixteenth order system with the following state variables:

- x_1 = Fan Speed
- x_2 = Compressor Speed
- x_3 = Compressor Discharge Pressure
- x_4 = Interturbine Volume Pressure
- x_5 = Augmentor Pressure
- x_6 = Fan Inside Diameter Temperature
- x_7 = Duct Temperature
- x_8 = Compressor Discharge Temperature
- x_9 = Burner Exit Fast Response Temperature
- x_{10} = Burner Exit Slow Response Temperature
- x_{11} = Burner Exit Total Temperature
- x_{12} = Fan Turbine Inlet Fast Response Temperature
- x_{13} = Fan Turbine Inlet Slow Response Temperature

x_{14} = Fan Turbine Exit Temperature

x_{15} = Duct Exit Temperature, T_{t6c}

x_{16} = Duct Exit Temperature, T_{t7m}

The engine inputs and outputs used for this study are:

a. Two Input - Two Output Model

Inputs: U_1 = Main Burner Fuel Flow

U_2 = Nozzle Jet Area

Outputs: y_1 = Fan Speed

y_2 = Compressor Speed

b. Three Input - Three Output Model

Inputs: U_1 = Main Burner Fuel Flow

U_2 = Nozzle Jet Area

U_3 = Inlet Guide Vane Position

Outputs: y_1 = Fan Speed

y_2 = Compressor Speed

y_3 = Augmentor Pressure

The A and B matrices corresponding to the above models using

$$\dot{x} = A x + B u$$

$$y = C x$$

are contained on Page 65 and 66 of [35]. Application of Danielevski's method for computing $G(s)$ yields the following set of transfer functions:

$$\begin{aligned} G_{11}(s) = & [- .0457s^{15} + 54.789s^{14} + 45.(10^5)s^{13} \\ & + .1384(10^8)s^{12} + .2211(10^{10})s^{11} \\ & + .20558(10^{12})s^{10} + .11944(10^{14})s^9 \\ & + .4534(10^{15})s^8 + .11516(10^{17})s^7 \end{aligned}$$

$$\begin{aligned}
& + .197(10^{18})s^6 + .2246(10^{19})s^5 \\
& + .1657(10^{20})s^4 + .75166(10^{20})s^3 \\
& + .1923(10^{21})s^2 + .2323(10^{21})s \\
& + .91419(10^{20})] / \Delta(s)
\end{aligned}$$

$$\begin{aligned}
G_{12}(s) = & [- 451.6s^{15} - .3095(10^6)s^{14} + .2062(10^8)^{13} \\
& + .1886(10^{11})s^{12} + .30163(10^{13})s^{11} \\
& + .2589(10^{15})s^{10} + .1441(10^{17})s^9 \\
& + .5592(10^{18})s^8 + .1547(10^{20})s^7 \\
& + .3035(10^{21})s^6 + .4115(10^{22})s^5 \\
& + .3672(10^{23})s^4 + .1994(10^{24})s^3 \\
& + .5782(10^{24})s^2 + .7456(10^{24})s \\
& + .2988(10^{24})] / \Delta(s)
\end{aligned}$$

$$\begin{aligned}
G_{13}(s) = & [- .1058(10^3)s^{15} - .1135(10^6)s^{14} \\
& - .3974(10^8)s^{13} - .6970(10^{10})s^{12} \\
& - .7280(10^{12})s^{11} - .4933(10^{14})s^{10} \\
& - .2269(10^{16})s^9 - .7254(10^{17})s^8 \\
& - .1622(10^{19})s^7 - .2521(10^{20})s^6 \\
& - .2663(10^{21})s^5 - .1835(10^{22})s^4 \\
& - .7703(10^{22})s^3 - .1771(10^{23})s^2 \\
& - .1919(10^{23})s - .6876(10^{22})] / \Delta(s)
\end{aligned}$$

$$\begin{aligned}
G_{21}(s) = & [.1111s^{15} + 42.91s^{14} - .4241(10^4)s^{13} \\
& - .2208(10^7)s^{12} - .8216(10^8)s^{11} \\
& + .2254(10^{11})s^{10} + .2870(10^{13})s^9 \\
& + .1579(10^{12})s^8 + .5023(10^{16})s^7 \\
& + .1003(10^{18})s^6 + .1285(10^{19})s^5 \\
& + .1044(10^{20})s^4 + .5152(10^{21})s^3 \\
& + .1408(10^{21})s^2 + .1784(10^{21})s \\
& + .7423(10^{20})]/\Delta(s)
\end{aligned}$$

$$\begin{aligned}
G_{22}(s) = & [- 546.1s^{15} - .4005(10^6)s^{14} - .7166(10^8)s^{13} \\
& - .523(10^{10})s^{12} - .1456(10^{12})s^{11} \\
& + .4209(10^{13})s^{10} + .7393(10^{15})s^9 \\
& + .5099(10^{17})s^8 + .21934(10^{19})s^7 \\
& + .5989(10^{20})s^6 + .1032(10^{22})s^5 \\
& + .1103(10^{23})s^4 + .7002(10^{23})s^3 \\
& + .2399(10^{24})s^2 + .3529(10^{24})s \\
& + .1594(10^{24})]/\Delta(s)
\end{aligned}$$

$$\begin{aligned}
G_{23}(s) = & [- .06575s^{15} - 4420.3s^{14} - .8978(10^6)s^{13} \\
& - .9572(10^8)s^{12} - .6854(10^{10})s^{11} \\
& - .3658(10^{12})s^{10} - .13769(10^{14})s^9 \\
& - .3029(10^{15})s^8 - .1933(10^{16})s^7 \\
& + .6354(10^{17})s^6 + .1416(10^{19})s^5
\end{aligned}$$

$$\begin{aligned}
& + .7046(10^{19})s^4 - .5499(10^{20})s^3 \\
& - .5667(10^{21})s^2 - .1230(10^{22})s \\
& - .6916(10^{21})] / \Delta(s) \\
G_{31}(s) = & [.0099s^{15} + 10.77s^{14} + 3779.8s^{13} \\
& + .6525(10^6)s^{12} + .6614(10^8)s^{11} \\
& + .4313(10^{10})s^{10} + .1901(10^{12})s^9 \\
& + .5820(10^{13})s^8 + .12475(10^{15})s^7 \\
& + .1860(10^{16})s^6 + .1885(10^{17})s^5 \\
& + .1244(10^{18})s^4 + .4989(10^{18})s^3 \\
& + .1097(10^{19})s^2 + .1134(10^{19})s \\
& + .3845(10^{18})] / \Delta(s) \\
G_{32}(s) = & [- 98.39s^{15} - .1032(10^6)s^{14} - .3561(10^8)s^{13} \\
& - .6111(10^{10})s^{12} - .6197(10^{12})s^{11} \\
& - .4051(10^{14})s^{10} - .1789(10^{16})s^9 \\
& - .5461(10^{17})s^8 - .1161(10^{19})s^7 \\
& - .1706(10^{20})s^6 - .1691(10^{21})s^5 \\
& - .10817(10^{22})s^4 - .4162(10^{22})s^3 \\
& - .8719(10^{22})s^2 - .8677(10^{22})s \\
& - .2935(10^{22})] / \Delta(s) \\
G_{33}(s) = & [.5069s^{15} + 532.9s^{14} + .1898(10^6)s^{13} \\
& + .3319(10^8)s^{12} + .3389(10^{10})s^{11} \\
& + .2208(10^{12})s^{10} + .9625(10^{13})s^9
\end{aligned}$$

$$\begin{aligned}
& + .2873(10^{15})s^8 + .5909(10^{16})s^7 \\
& + .8299(10^{17})s^6 + .7754(10^{18})s^5 \\
& + .4594(10^{19})s^4 + .1607(10^{20})s^3 \\
& + .3071(10^{20})s^2 + .2841(10^{20})s \\
& + .9280(10^{19})] / (s)
\end{aligned}$$

where

$$\begin{aligned}
\Delta(s) = & s^{16} + 1063.8s^{15} + .3780(10^6)s^{14} \\
& + .6691(10^8)s^{13} + .7021(10^{10})s^{12} \\
& + .4777(10^{12})s^{11} + .2215(10^{14})s^{10} \\
& + .7195(10^{15})s^9 + .1658(10^{17})s^8 \\
& + .2715(10^{18})s^7 + .3125(10^{19})s^6 \\
& + .2474(10^{20})s^5 + .1297(10^{21})s^4 \\
& + .4257(10^{21})s^3 + .7998(10^{21})s^2 \\
& + .7430(10^{21})s + .2411(10^{24})
\end{aligned}$$

1. Two Input - Two Output Model

For the sixteenth order state model with the two inputs and two outputs indicated above, the transfer matrix becomes

$$G(s) = \begin{vmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{vmatrix}$$

Using the dominance algorithm in an INA mode over the frequency range

$0 \leq \omega \leq 200$ with 200 equally spaced increments, system dominance was achieved for

$$L = \begin{vmatrix} .555 & 0 \\ 0 & 1.0 \end{vmatrix}$$

$$\hat{K}_1 = \begin{vmatrix} 0 & 3840 \\ 1.0 & 0 \end{vmatrix}$$

This result was obtained when the dominance control parameters were set to unity. For $\theta_i = 0.0$, dominance levels of [.74024, .96677] were obtained for rows 1 and 2, respectively, with

$$\hat{K}_2 = \begin{vmatrix} .816 & 2289.0 \\ 1.0995 & -47.39 \end{vmatrix}$$

Since the Gershgorin circles provide no information apart from a bound on the eigenvalues of $\hat{Q}(s)$ (or $Q(s)$ in a DNA), it is only necessary to compute the envelope of the Gershgorin and/or Ostrowski bands. For this purpose, the numerical method developed by Crossley [36] is used to calculate the envelopes centered about the corresponding diagonal element.

Figures 4.3 and 4.4 display the Gershgorin bands for

$$\hat{Q}(s) = \hat{K}_1 \hat{G}(s) \hat{L}$$

over the frequency range $0 \leq \omega \leq 200$.

Using precompensator K_2 , the Gershgorin bands in Figures 4.5 and 4.6 are obtained. These bands can be immediately reduced using the Ostrowski shrinking factors with zero feedback gains and are presented in Figures 4.7 and 4.8. Note the high degree of band reduction between Figures 4.6 and 4.8 which can be further reduced by increasing the feedback gain in the second loop.

To further examine the Ostrowski band for \hat{q}_{22} , the frequency range was reduced to $0 \leq \omega \leq 50$ and is presented in Figure 4.9. From this figure it

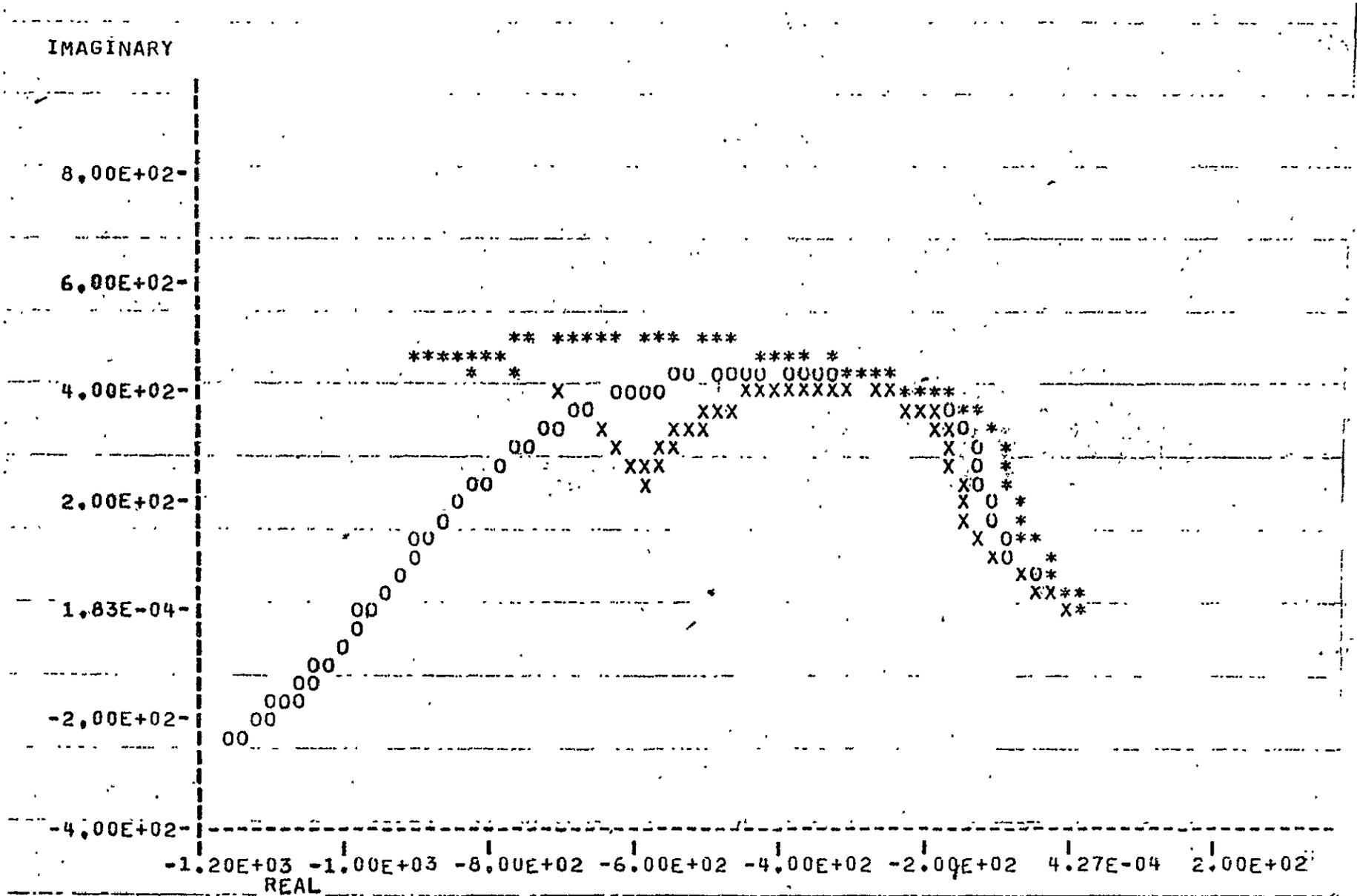


Figure 4.3 Gershgorin Band for Row 1 using K_1 $0 \leq \omega \leq 200$

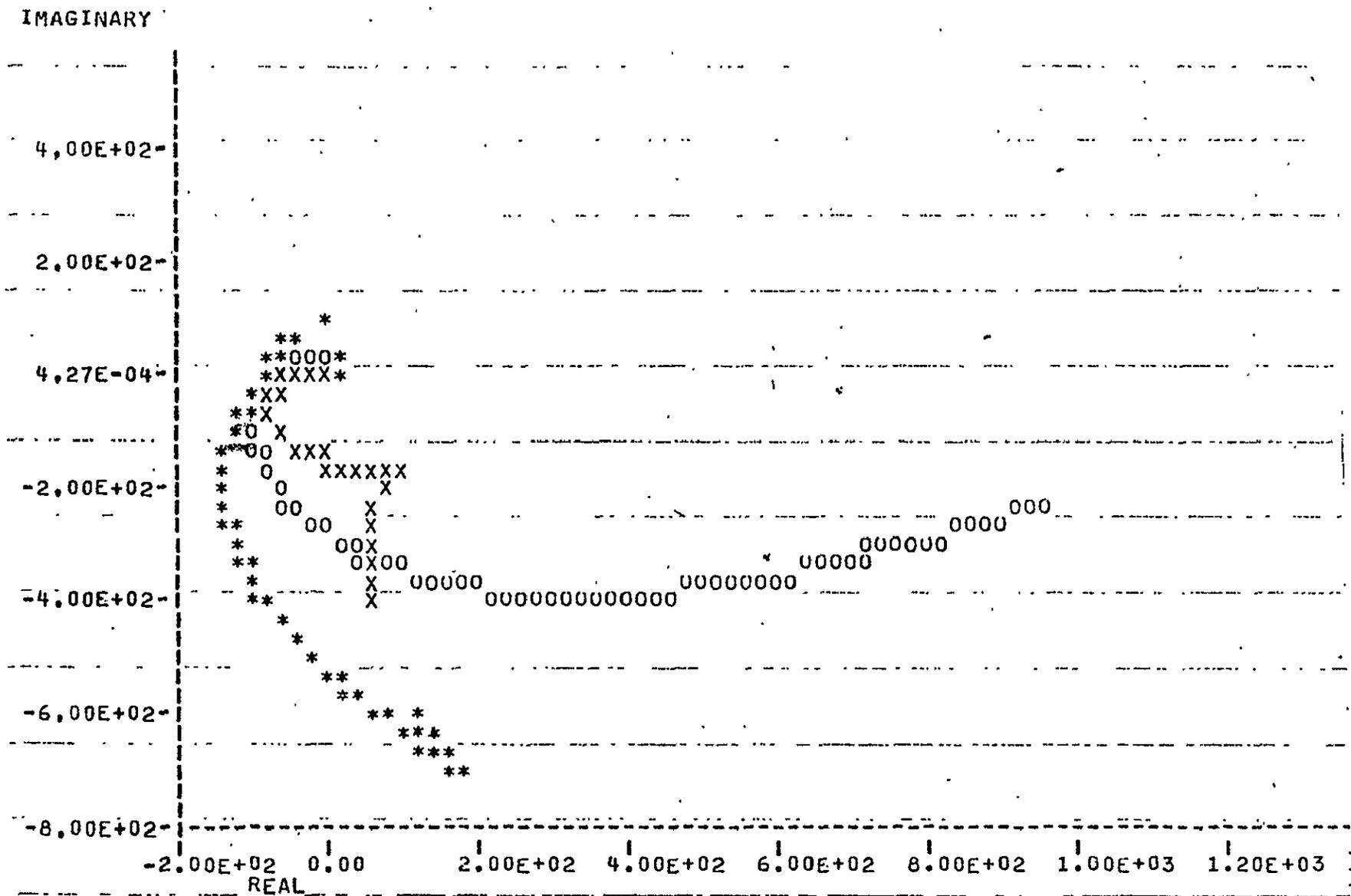


Figure 4.4 Genshgorin Band for Row 2 using K_1 $0 \leq \omega \leq 200$

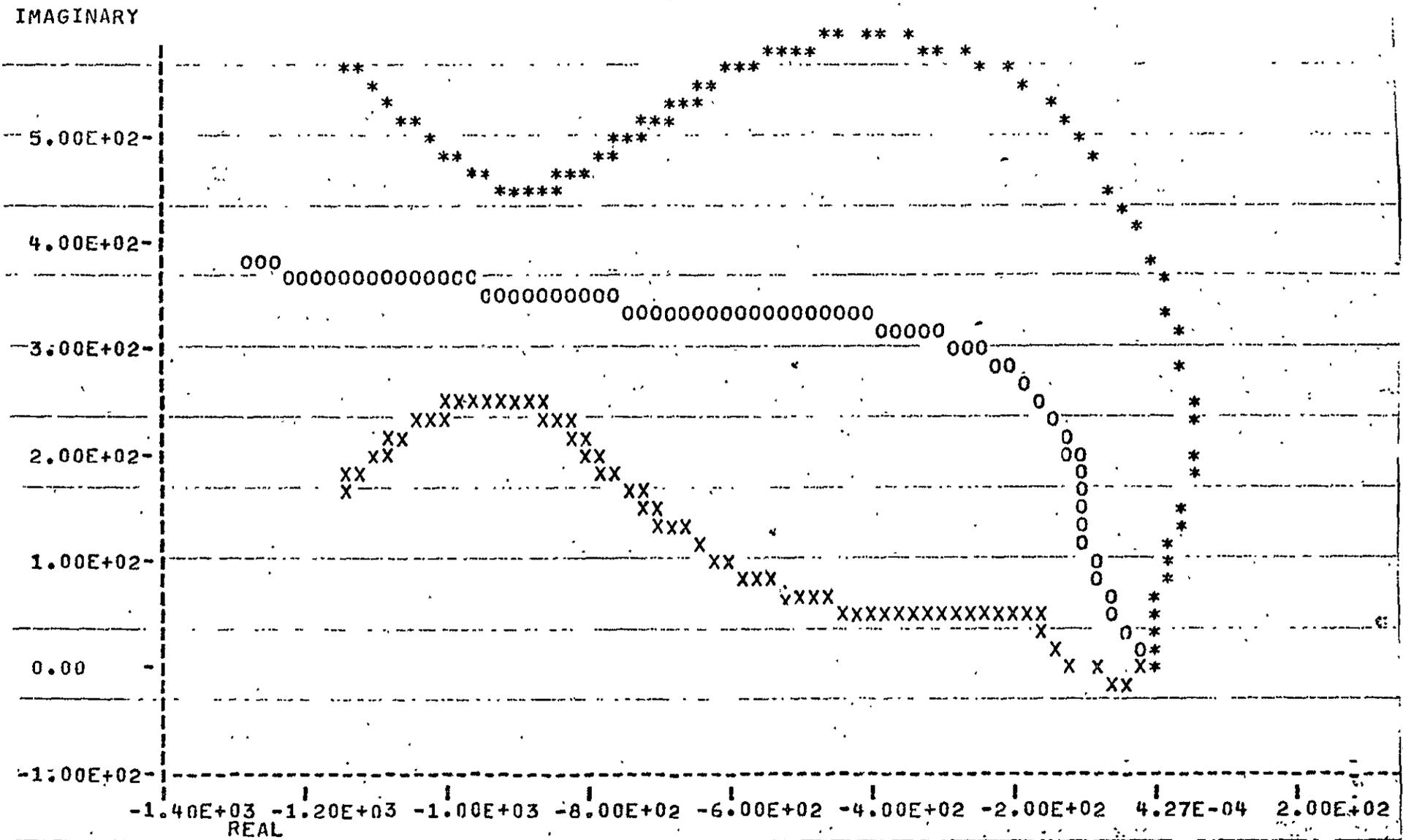


Figure 4.5 Gershgorin Band for Row 1 using \hat{K}_2 $0 \leq \omega \leq 200$

IMAGINARY

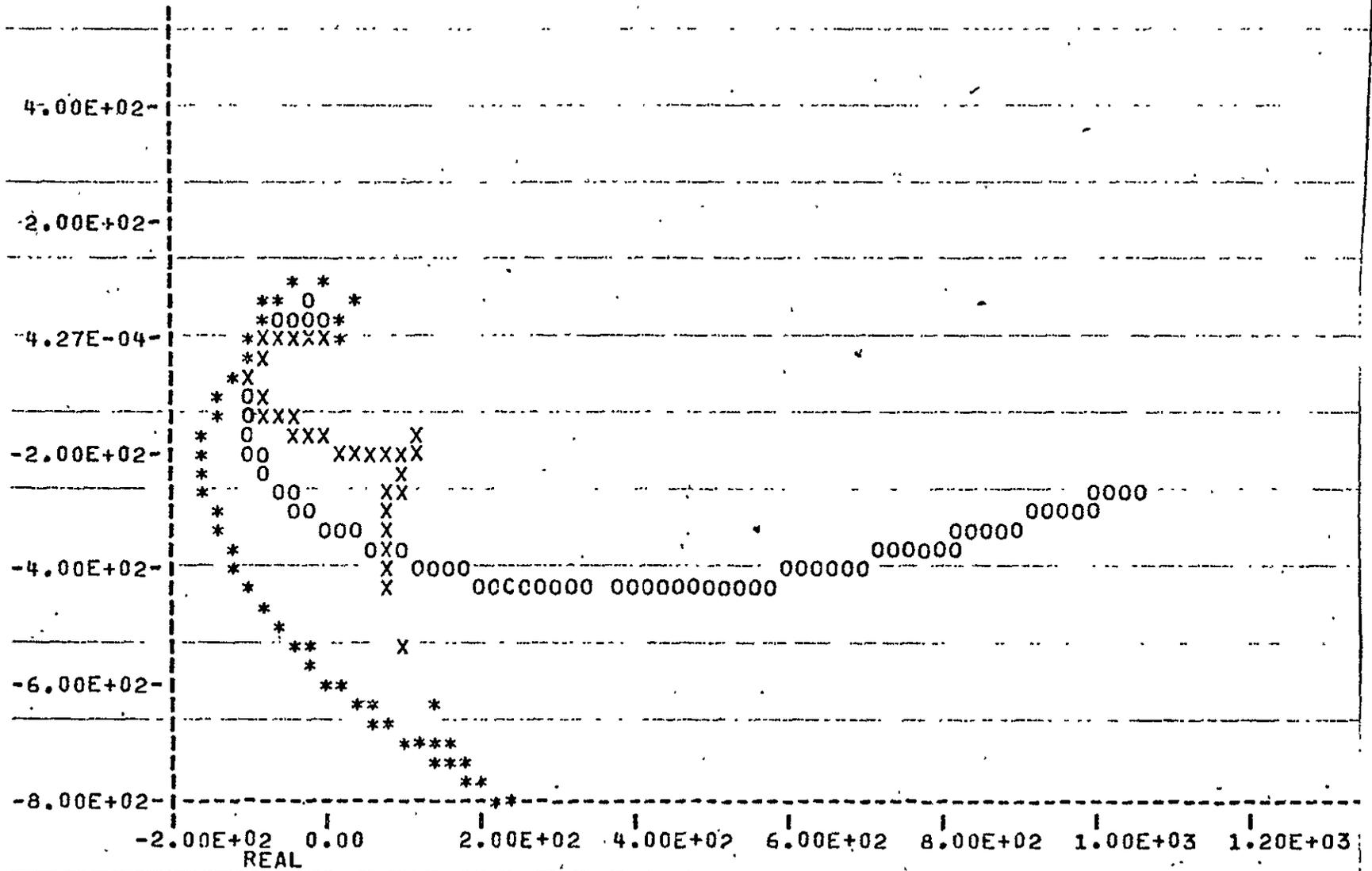


Figure 4.6 Gershgorin Band for Row 2 using K_2 $0 \leq \omega \leq 200$

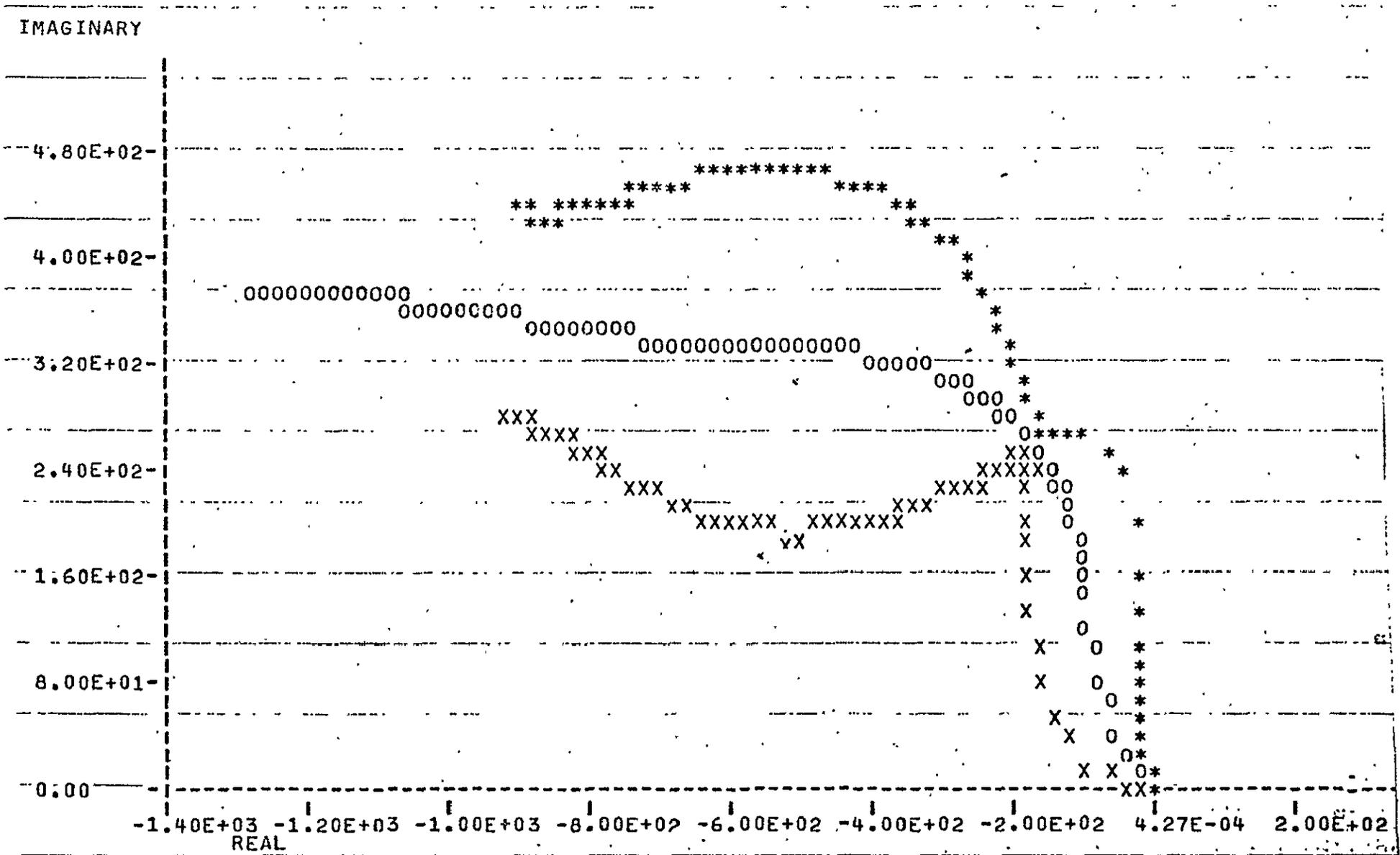


Figure 4.7 Ostrowski Band for Row 1 using K_2 $0 \leq \omega \leq 200$

IMAGINARY

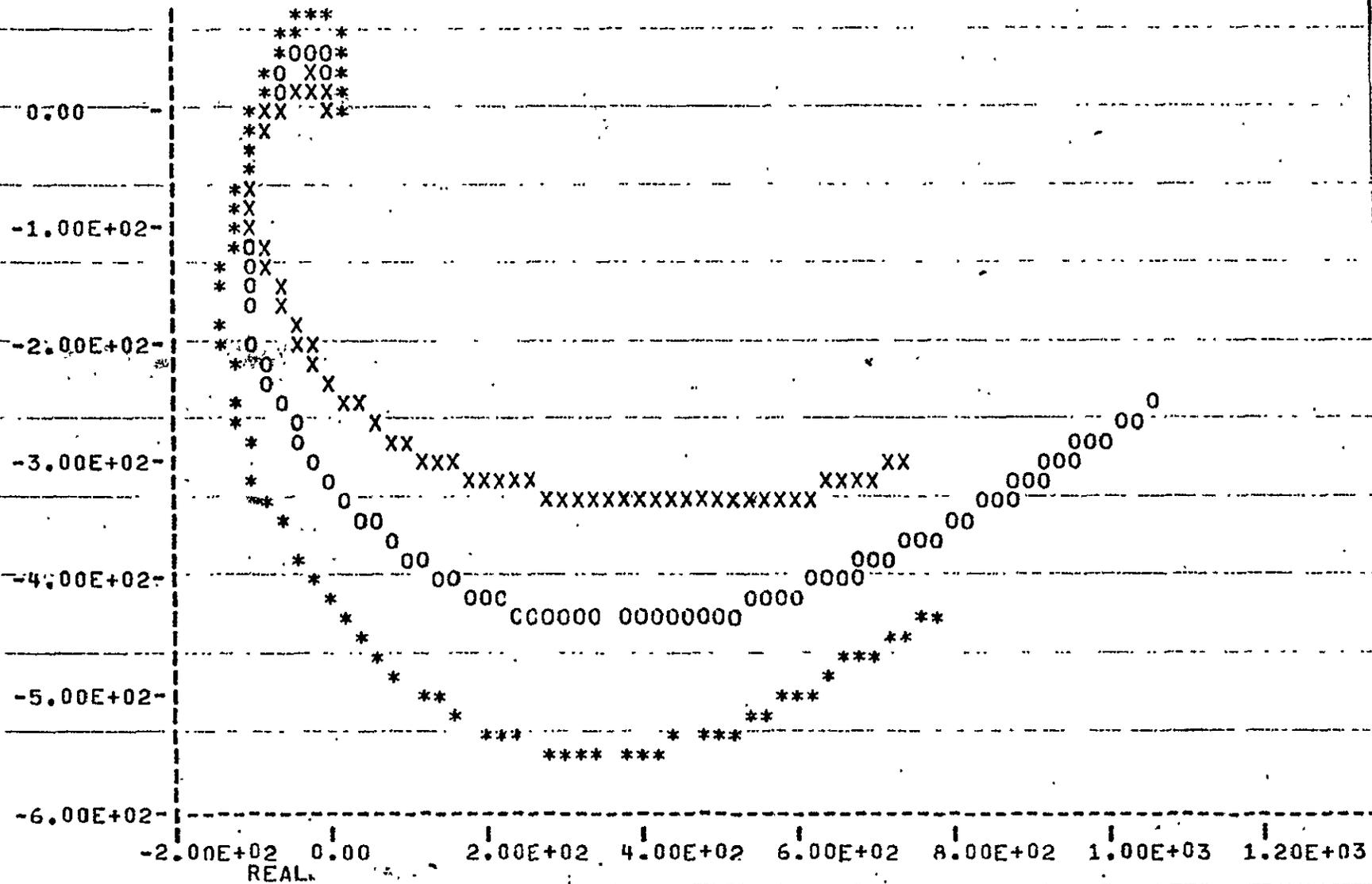


Figure 4.8 Ostrowski Band for Row 2 using K_2 $0 \leq \omega \leq 200$

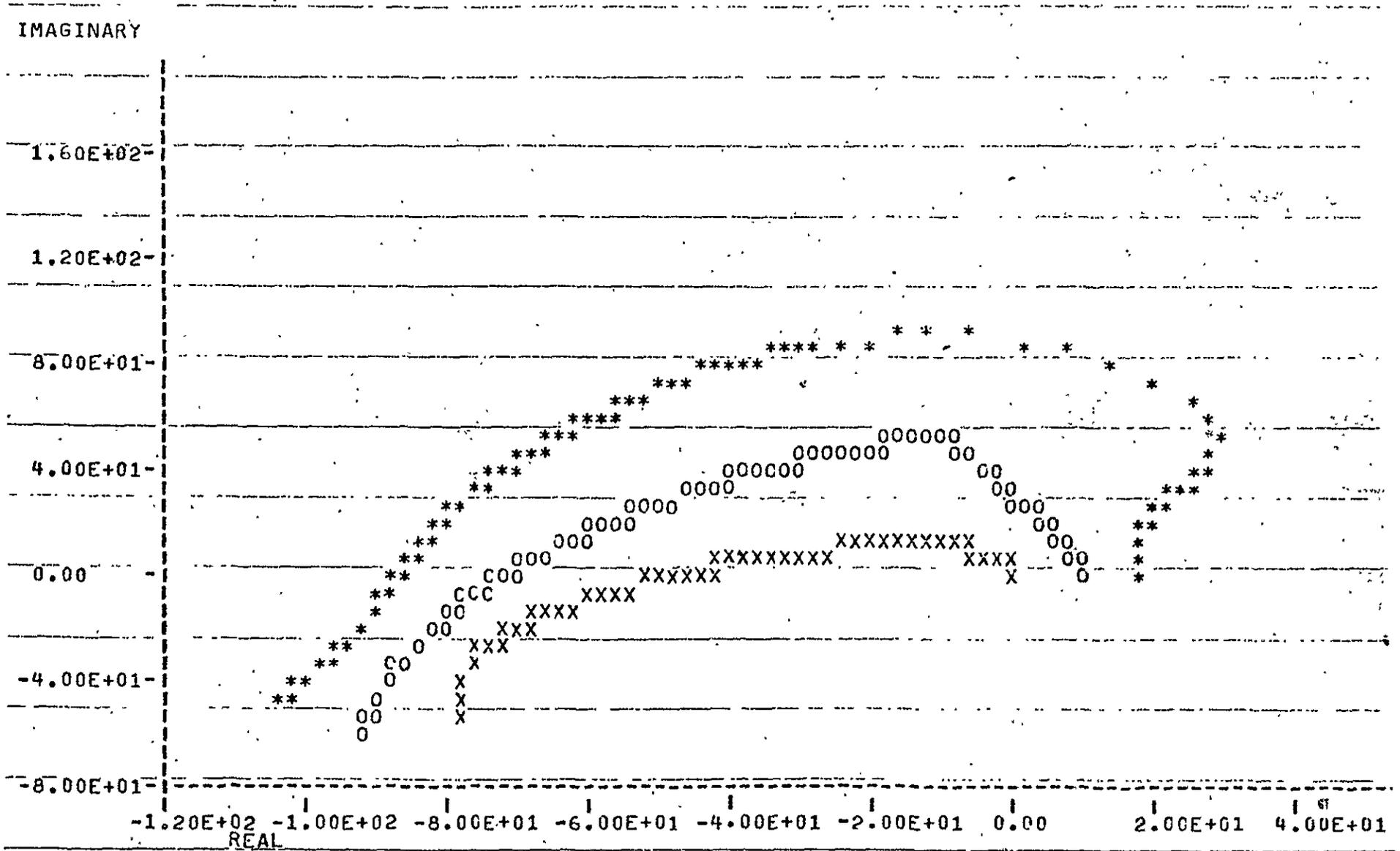


Figure 4.9 Ostrowski Band for Row 2 using K_2 $0 \leq \omega \leq 50$

is evident that the corresponding feedback gain is restricted to $0 \leq f_2 \leq 40$ for stability in the sense of Rosenbrock.

Although no further design attempts were made for the two input case, it is apparent from the Ostrowski bands that dynamic compensation in the feedback loop may be required to achieve a step response with minimal overshoot.

2. Three Input - Three Output Model

Using the $G(s)$ matrix as

$$G(s) = \begin{vmatrix} G_{11}(s) & G_{12}(s) & G_{13}(s) \\ G_{21}(s) & G_{22}(s) & G_{23}(s) \\ G_{31}(s) & G_{32}(s) & G_{33}(s) \end{vmatrix}$$

an INA design with constant compensators was initiated. When the post-compensator matrix was set to the identity matrix, diagonal dominance could not be obtained in all rows simultaneously with the algorithm of section 3. The next effort was to constrain the L matrix to the form

$$L = \begin{vmatrix} 0 & l_{12} & 0 \\ l_{11} & 0 & 0 \\ 0 & 0 & l_{33} \end{vmatrix}$$

symbolizing the desire to control the individual outputs rather than linear combinations of the system outputs. With this structural form, row dominance was obtained in two passes through the basic algorithm with the results

$$L = \begin{vmatrix} 0 & .05 & 0 \\ .01 & 0 & 0 \\ 0 & 0 & 1.0 \end{vmatrix}$$

and

$$\hat{K} = \begin{vmatrix} 1.000 & 1636.43 & -1.3583 \\ .0001379 & 1.000 & -.087418 \\ .02308 & -206.685 & 1.000 \end{vmatrix}$$

For this case, the dominance control parameters were selected as

$$\theta_1 = 0.75 \quad \theta_2 = 0.40 \quad \theta_3 = 0.10$$

The corresponding Gershgorin bands are contained in Figures 4.8 to 4.10 over the frequency range of $0 \leq \omega \leq 50$.

Selecting the feedback parameters as

$$F = \begin{vmatrix} 150 & 0 & 0 \\ 0 & 5.0 & 0 \\ 0 & 0 & 5.0 \end{vmatrix}$$

the Ostrowski bands of Figures 4.10 to 4.12 were obtained.

Using the K, L and F matrices indicated above, the interaction indices become

$$\begin{aligned} I_1 &= -0.2703 \\ I_2 &= +0.0402 \\ I_3 &= -.00180 \end{aligned}$$

The first index suggests a moderate degree of system interaction in loop 1 resulting from the closure of the remaining loops. This level of interaction might have been anticipated from the level of dominance requested ($\theta_1 = 0.75$). The negative sign for I_1 implies a constructive form of interaction in that closure of loops 2 and 3 augment the design efforts of loop 1 in the control of output $y_1(t)$. This design information can not be obtained from the Gershgorin or Ostrowski plots.

The interaction index for output $y_2(t)$ suggests a low level of interaction upon closure of loops 1 and 3. The plus sign for I_2 , although not

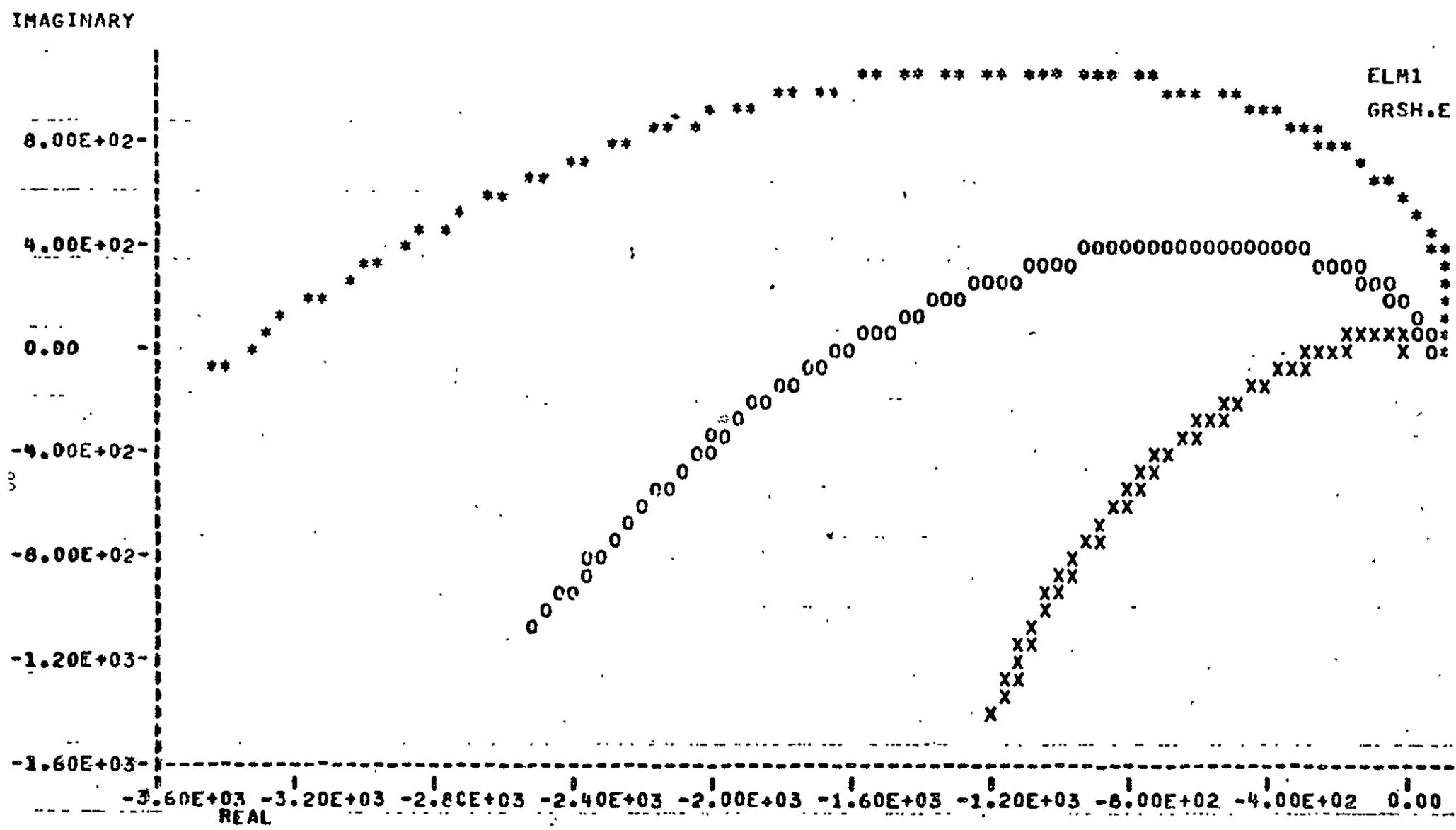


Figure 4.10 Gershgorin Band for Row 1 $0 \leq \omega \leq 50$

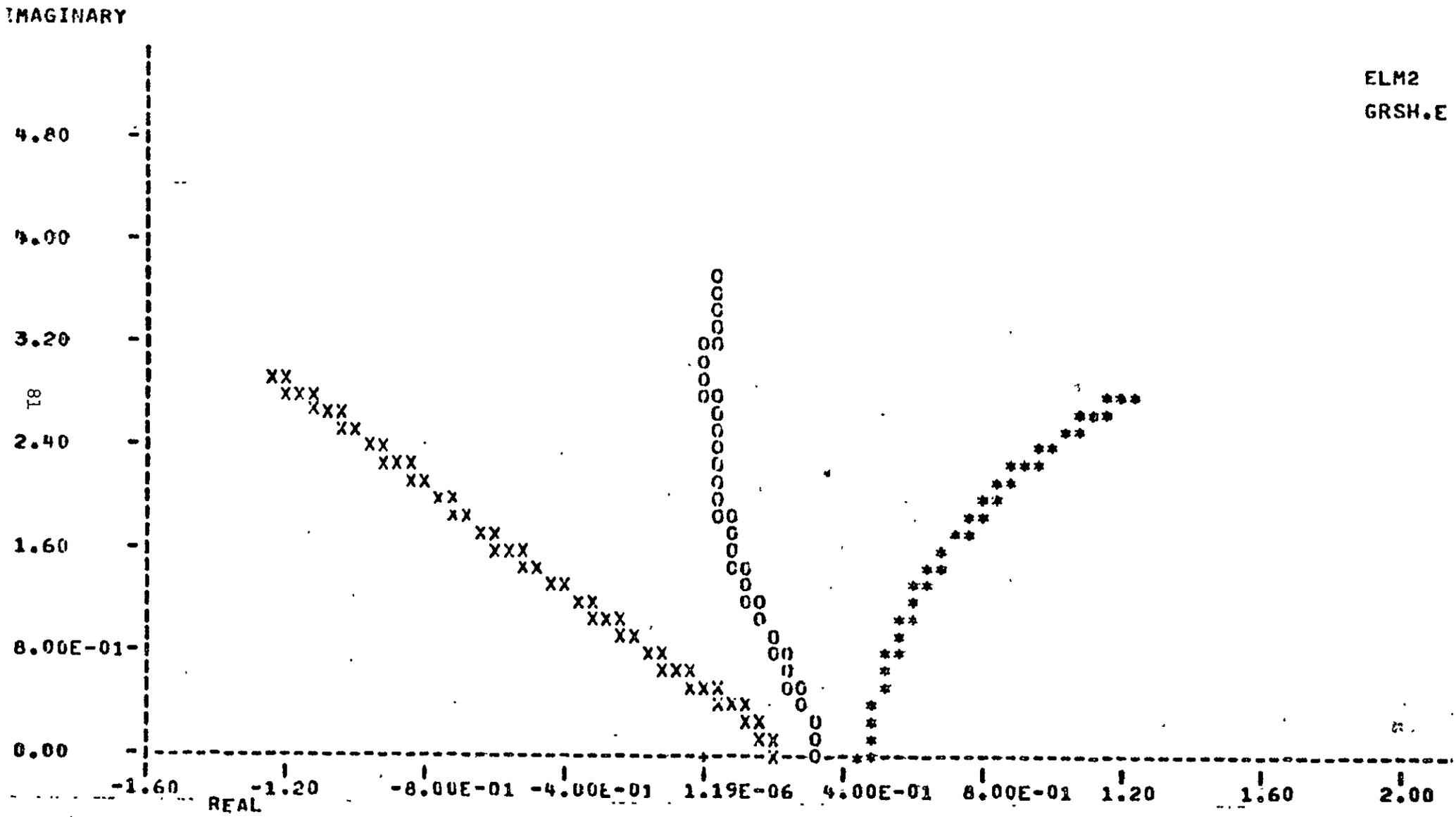


Figure 4.11 Gershgorin Band for Row 2 $0 \leq \omega \leq 50$

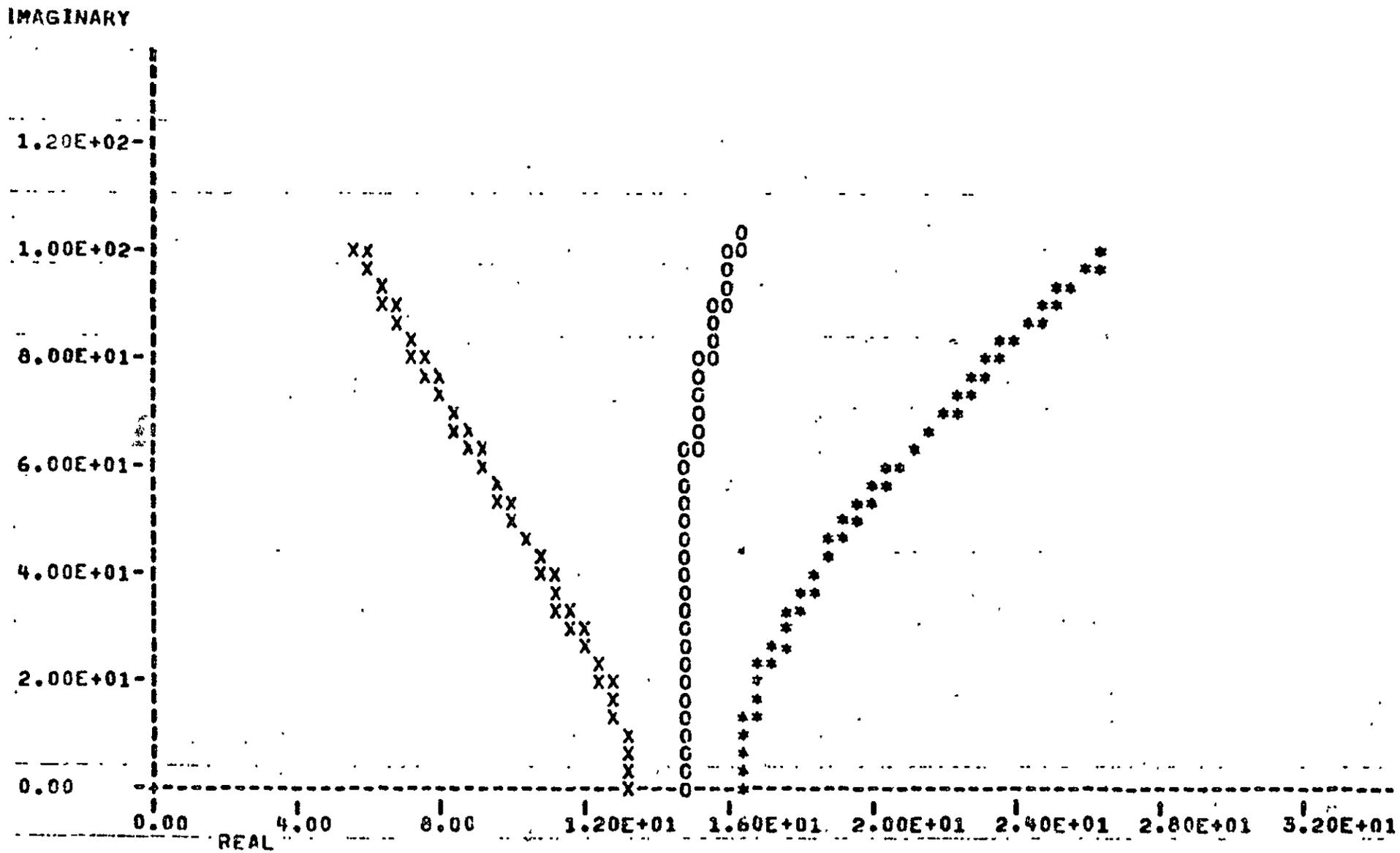


Figure 4.12 Gershgorin Band for Row 3 $0 \leq \omega \leq 50$

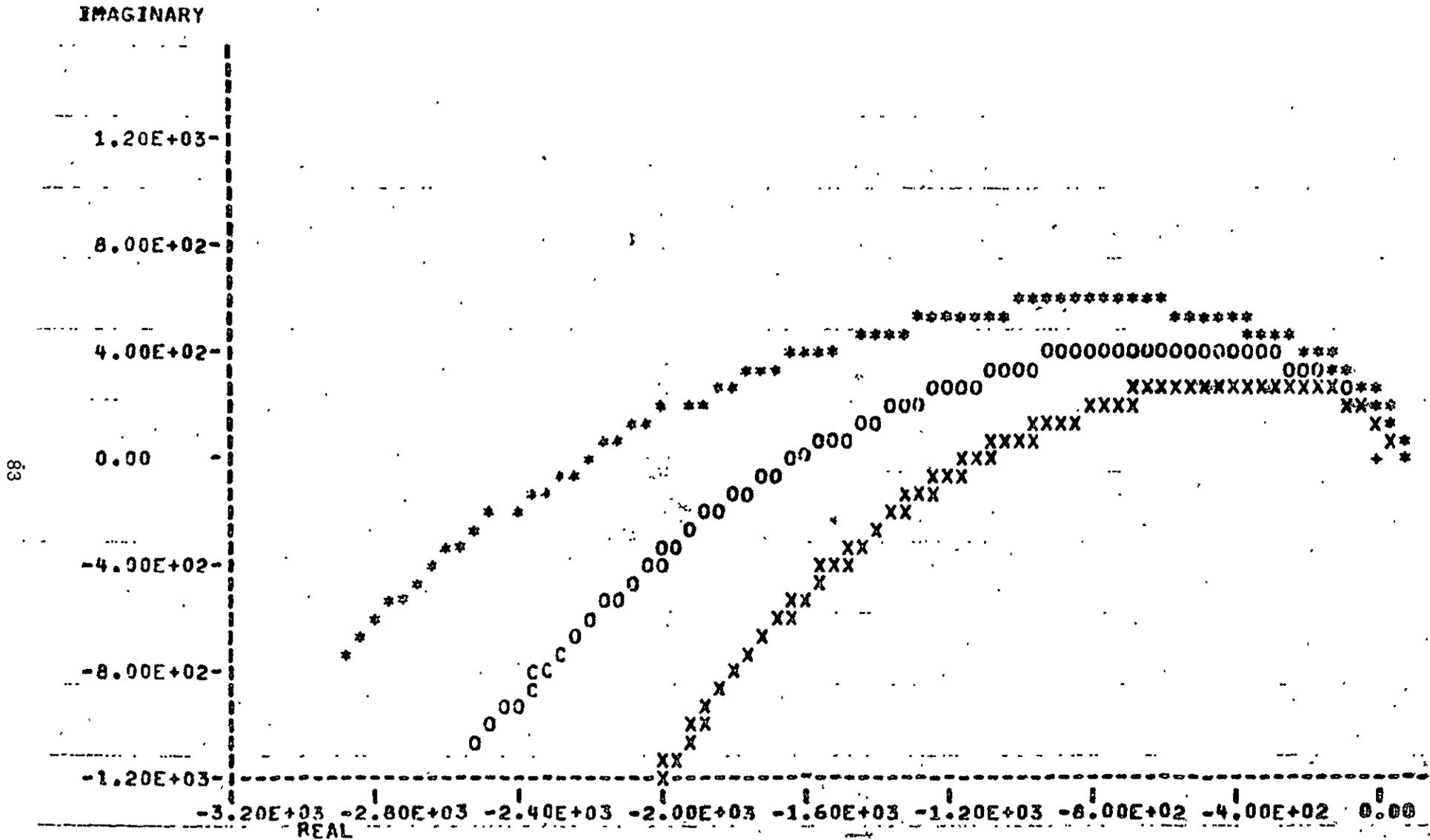


Figure 4.13 Ostrowski Band for Row 1 $0 \leq \omega \leq 50$

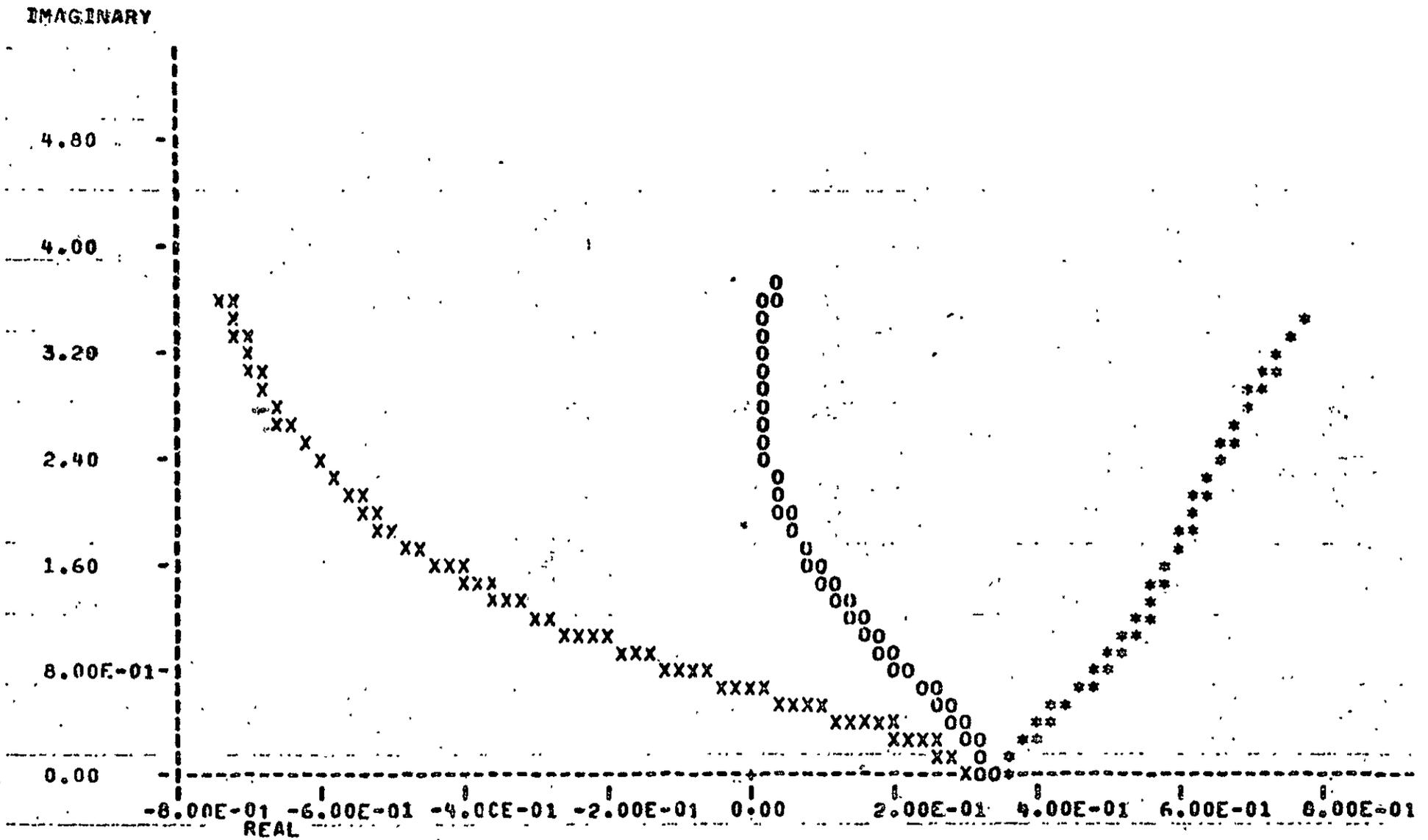


Figure 4.14 Ostrowski Band for Row 2 $0 < \omega < 50$

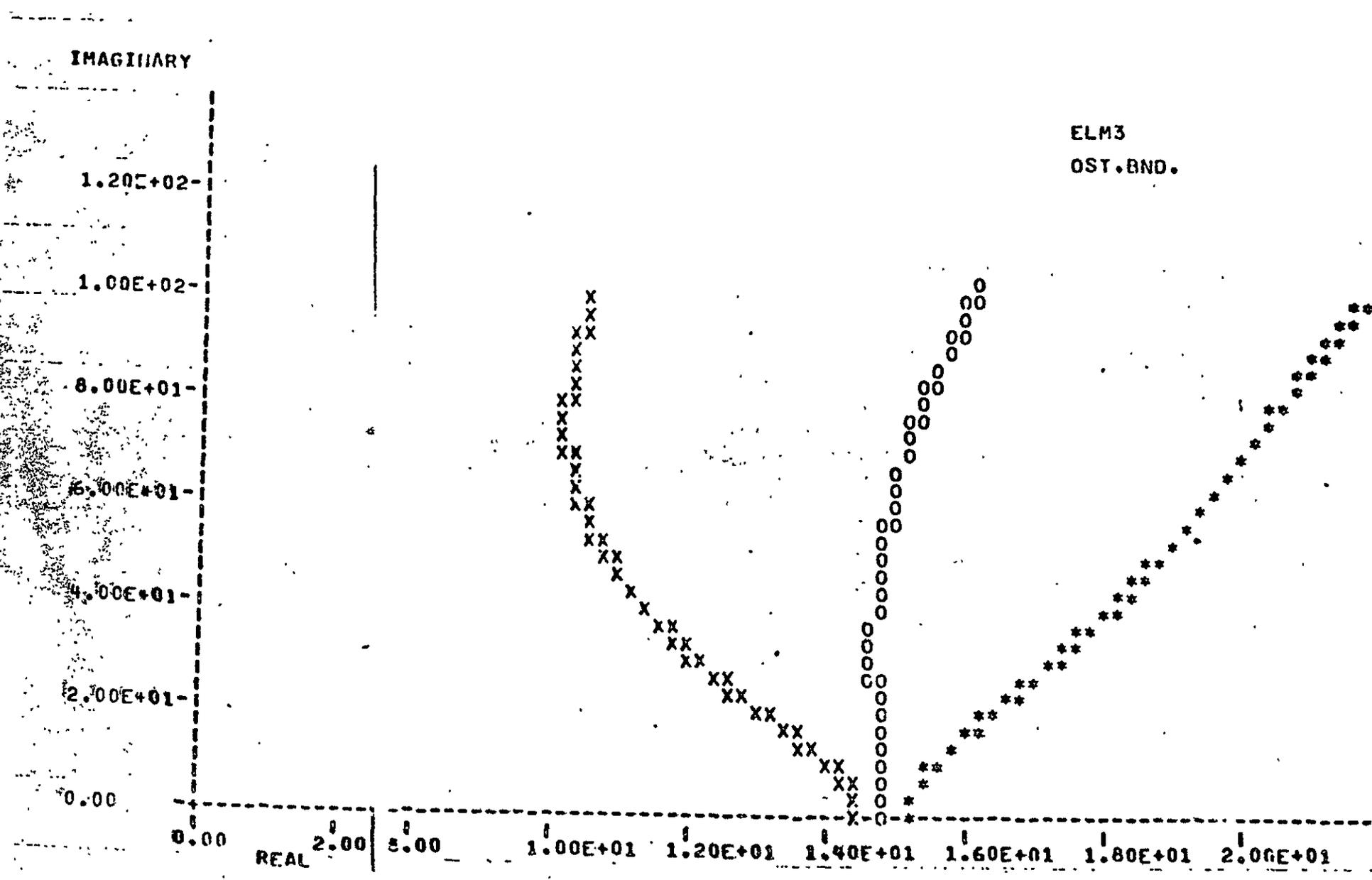


Figure 4.15 Ostrowski Band for Row 3 $0 \leq \omega \leq 50$

significant in this case, suggests that loop closure could have a deleterious effect on the control of $y_2(t)$. Thus, if the magnitude of I_2 had suggested a higher level of system interaction, design efforts to decouple $y_2(t)$ could be considered. In this example, an alternate approach might be to restructure the post compensator matrix and re-examine dominance using the proposed algorithm.

The interaction index for output $y_3(t)$ indicates a decoupled condition. This result is in correspondence with the level of the dominance control parameter ($\theta_3 = 0.1$) specified and achieved by the dominance algorithm.

To obtain the step response for the preliminary design above to commanded step changes in the system outputs, the block diagram of Figure 4.16 was used.

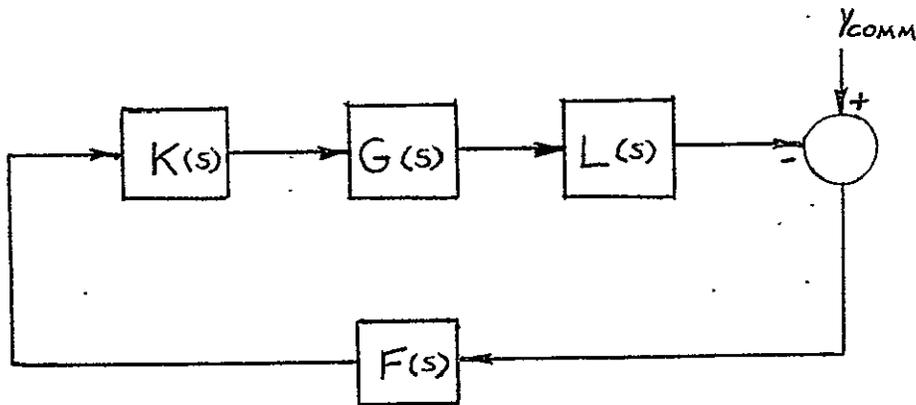


Figure 4.16 Closed loop system

In state variable form, Figure 4.16 becomes

$$\dot{X} = (A-BKFLC)X + BKF Y_{\text{COMM}}$$

$$y(t) = C X(t)$$

with Figures 4.17 to 4.19 representing the step responses corresponding to the following commanded input vector

$$Y_{\text{COMM}} = \begin{bmatrix} 10.0 \\ 10.0 \\ 1.0 \end{bmatrix}$$

In each figure, the step responses have been superimposed to demonstrate that system interaction has been significantly reduced.

To calculate steady state offsets in the step responses from the Ostrowski INA diagrams, Figure 2.4 of Section 2 may be used. From Figures 4.13-4.15

$$F = \begin{bmatrix} 150 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

the offsets are

$$\text{Row 1: } 100 \frac{\phi C}{CAB} \% = \frac{45}{195}(100)\% = 28.2\%$$

$$\text{Row 2: } 100 \frac{\phi C}{CAB} \% = \frac{0.3}{5.3}(100)\% = 5.66\%$$

$$\text{Row 3: } 100 \frac{\phi C}{CAB} \% = \frac{14.22}{19.22}(100)\% = 74\%$$

Thus the steady state values will be 71.8%, 94.34% and 26% of the commanded values. These values are easily verified from the step responses of Figures 4.17 to 4.19.

STEP RESP CLSD LOOP

88

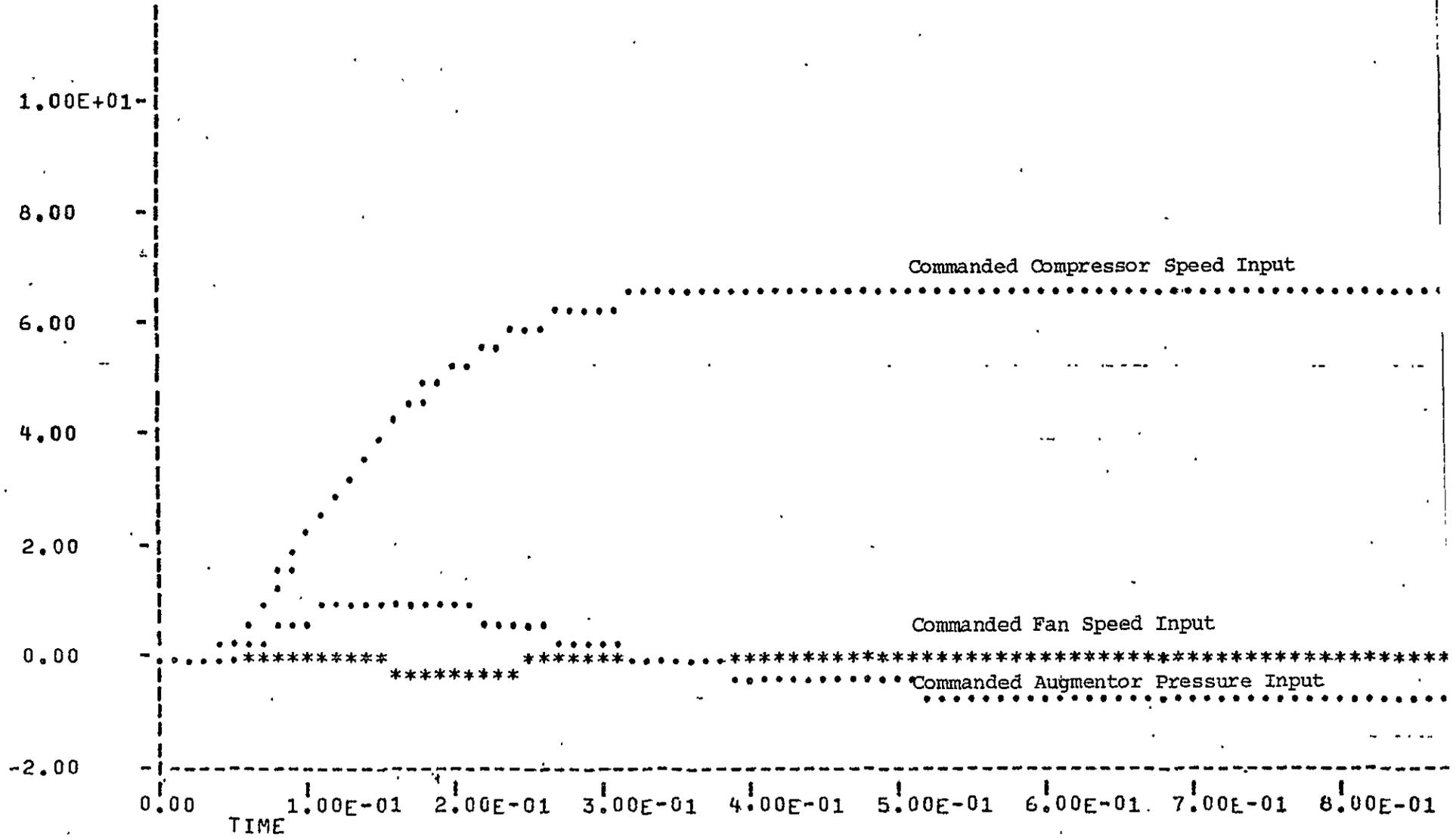


Figure 4.17. Step Responses for Compressor Speed to Commanded Inputs

STEP RESP CLSD LOOP

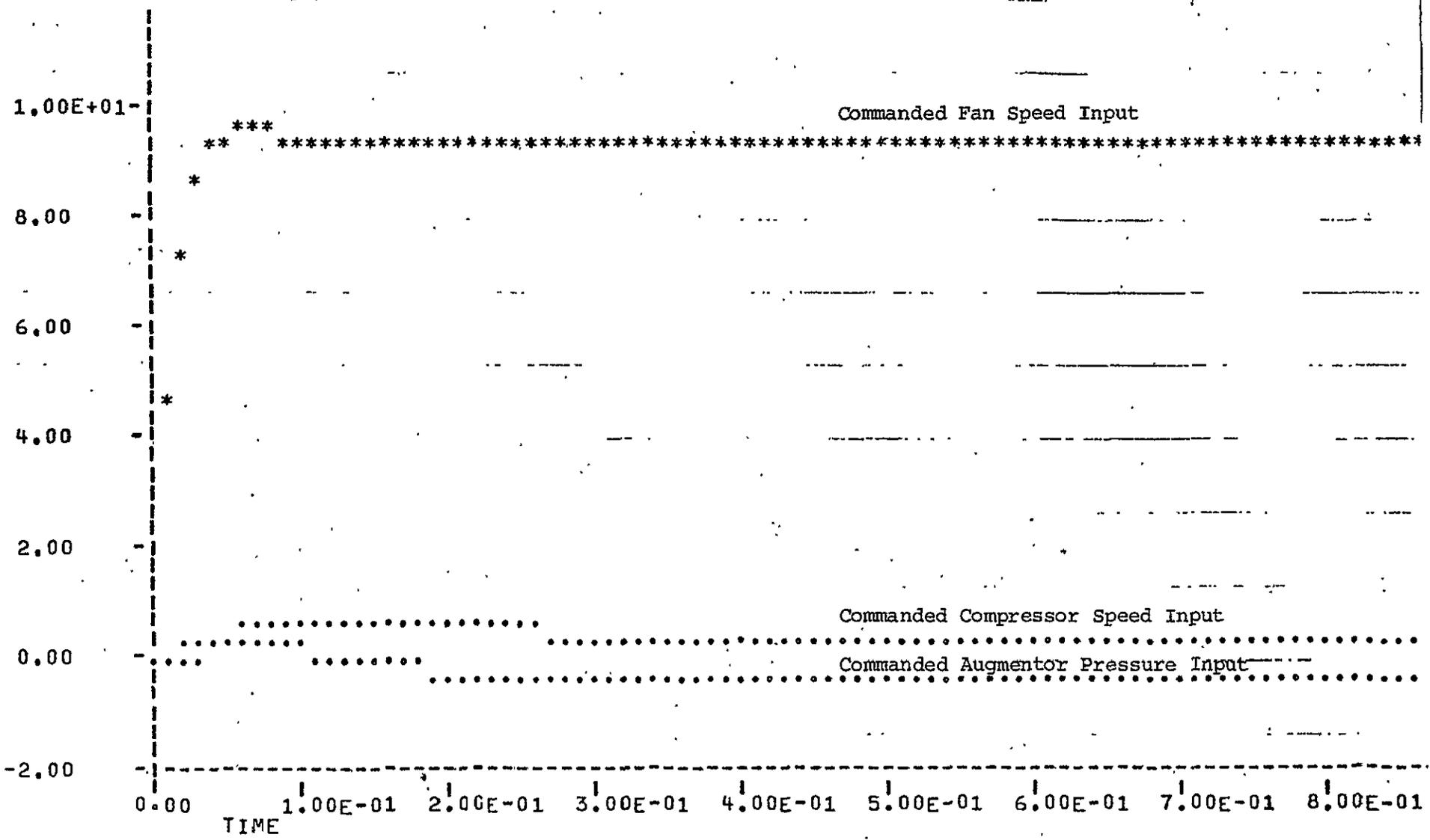


Figure 4.18. Step Response for Fan Speed to Commanded Inputs

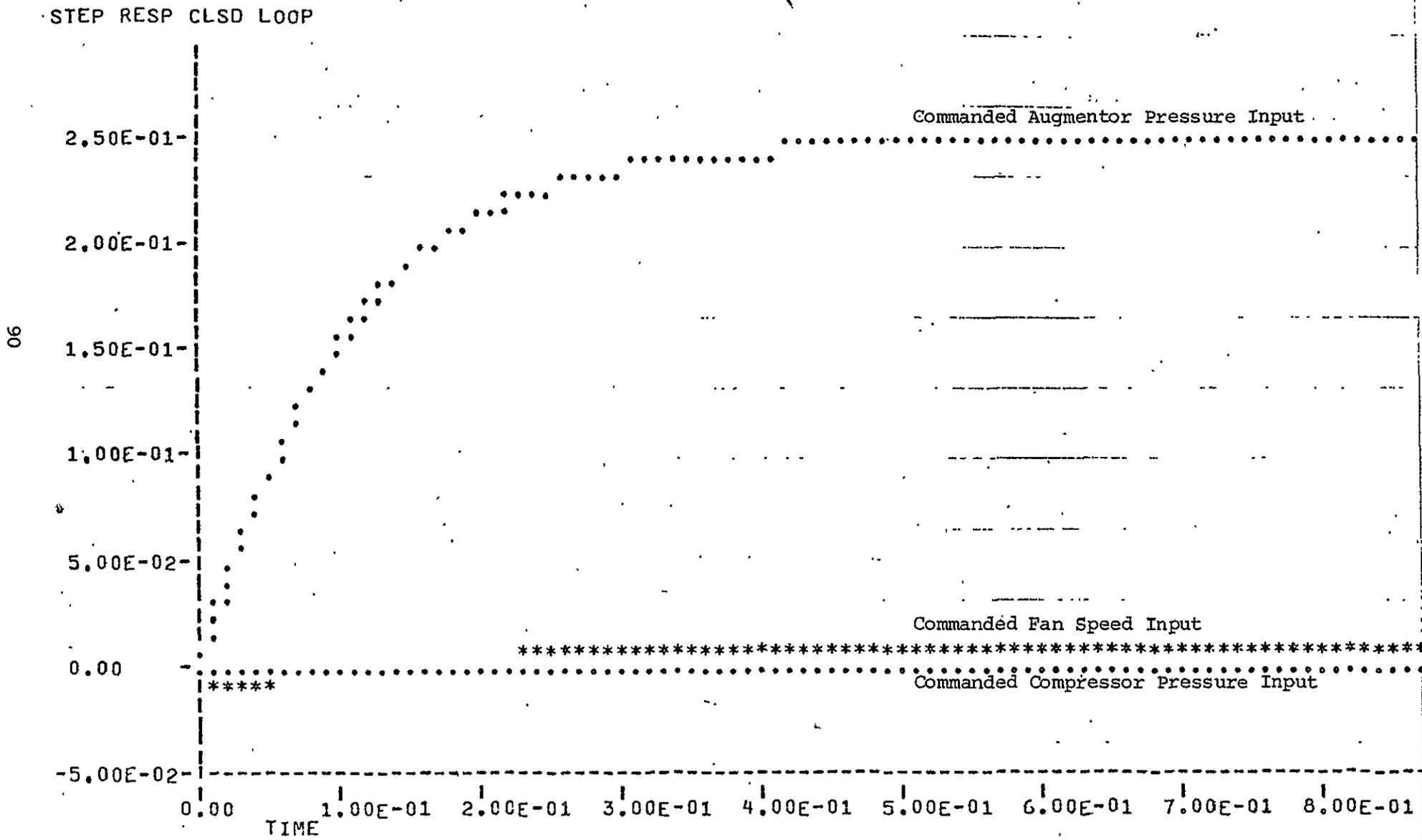


Figure 4.19. Step Responses for Augmentor Pressure to Commanded Inputs

To demonstrate that the above design for the F100 with three inputs and three outputs is not structurally unique, a new postcompensator was selected as

$$L = \begin{vmatrix} 0 & .05 & 0 \\ 0 & 0 & 2 \\ .01 & 0 & 0 \end{vmatrix}$$

The initial guess for the precompensator was selected as the identity matrix with $0 \leq \omega \leq 200$ as the frequency range of interest subdivided into two hundred equally spaced points.

Application of the algorithm provided diagonal dominance in all rows with

$$\theta = \begin{vmatrix} .50275 \\ .22111 \\ .30606 \end{vmatrix}$$

and

$$K = \begin{vmatrix} 2.9757 & 4834.2 & -8.7416 \\ .001416 & -13.906 & .0696 \\ .005564 & 21.099 & -1.5824 \end{vmatrix}$$

The Gershgorin bands for $0 \leq \omega \leq 200$ are contained in Figures 4.20 to 4.22. Figures 4.23 to 4.25 show the same bands over the frequency range $0 \leq \omega \leq 50$.

Once diagonal dominance has been obtained, Ostrowski's theorem will apply for any set of stable feedback gains. Using the information that the F100 is open loop stable, all feedback gains were set to zero. The corresponding Ostrowski bands are provided in Figures 4.26 to 4.28. From these figures it is evident that the Gershgorin bands have been significantly reduced. Using the dominance levels above, the following minimum levels of

IMAGINARY

8.00E+04-

4.00E+04-

8.59E-02-

-4.00E+04-

-8.00E+04-

-1.20E+05-

-1.60E+05-

-1.60E+04 -8.00E+03 0.00 8.00E+03 1.60E+04 2.40E+04 3.20E+04 4.00E+04 4.80E+04
REAL

92

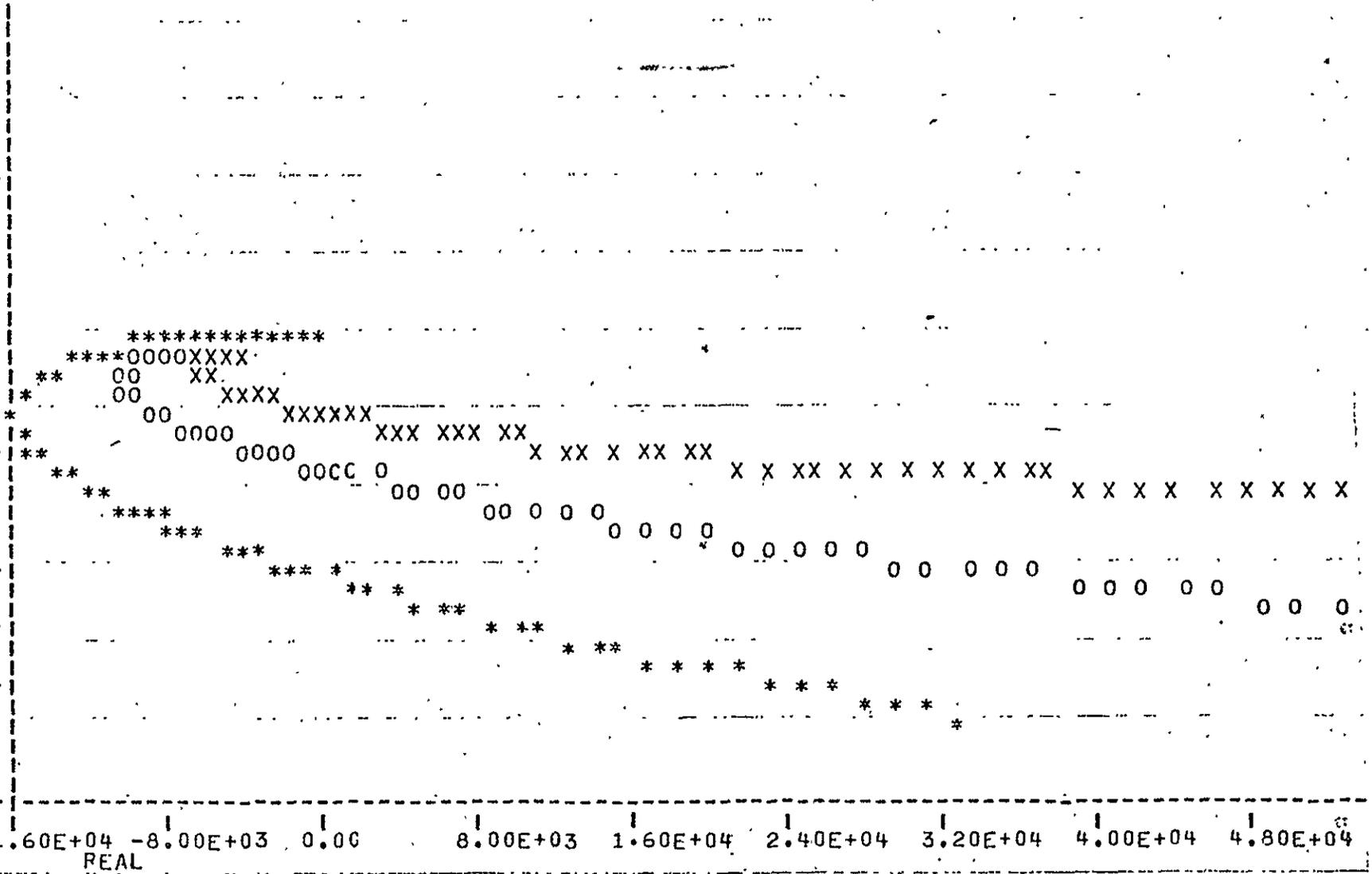


Figure 4.20 Gershgorin Band for Row 1 $0 \leq \omega \leq 200$

IMAGINARY

1.20E+01-

1.00E+01-

8.00

6.00

4.00

2.00

0.00

-2.40

-1.60

-8.00E-01

1.07E-06

8.00E-01

1.60

2.40

3.20

REAL

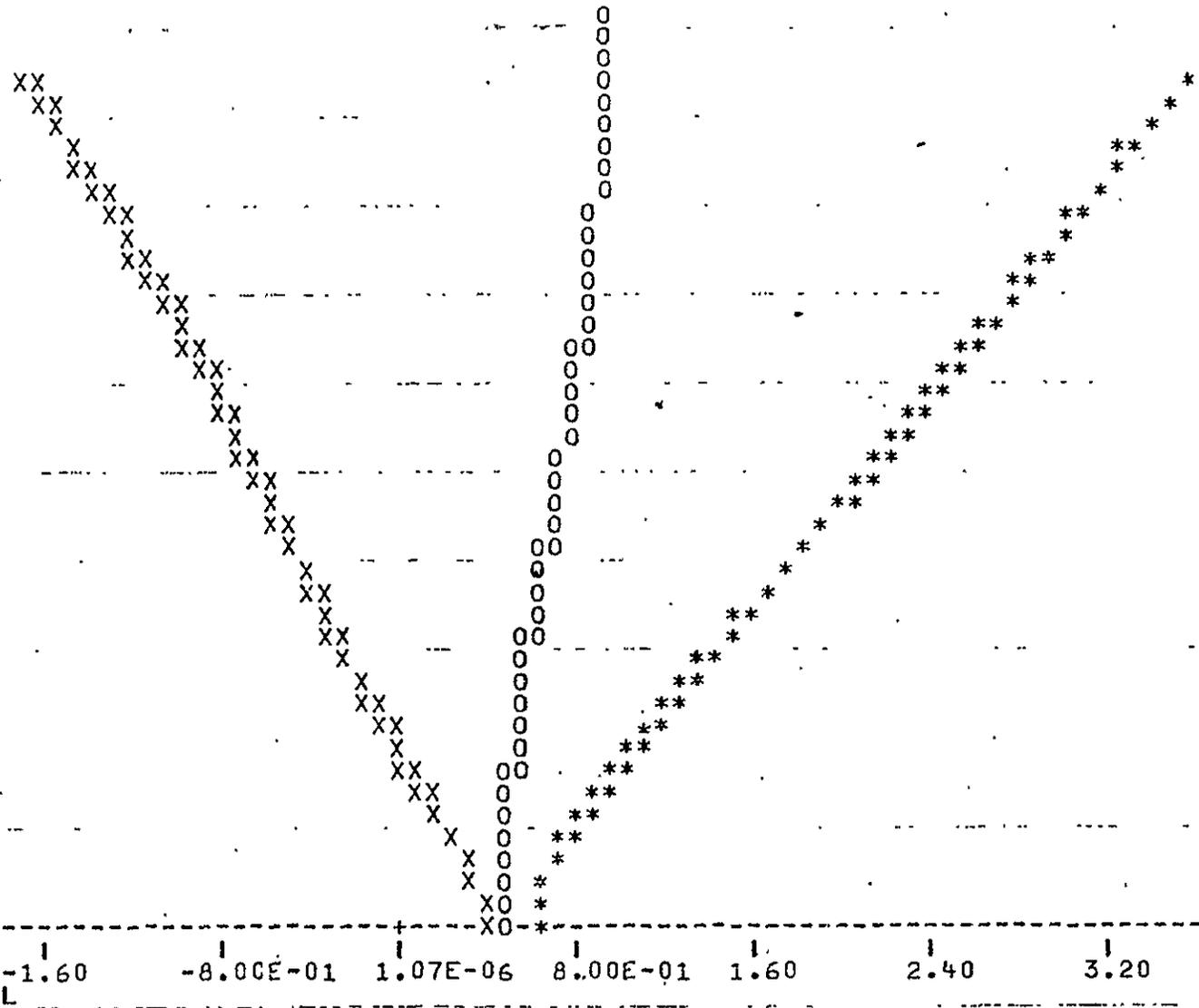


Figure 4.21 Gershgorin Band for Row 2 $0 \leq \omega \leq 200$

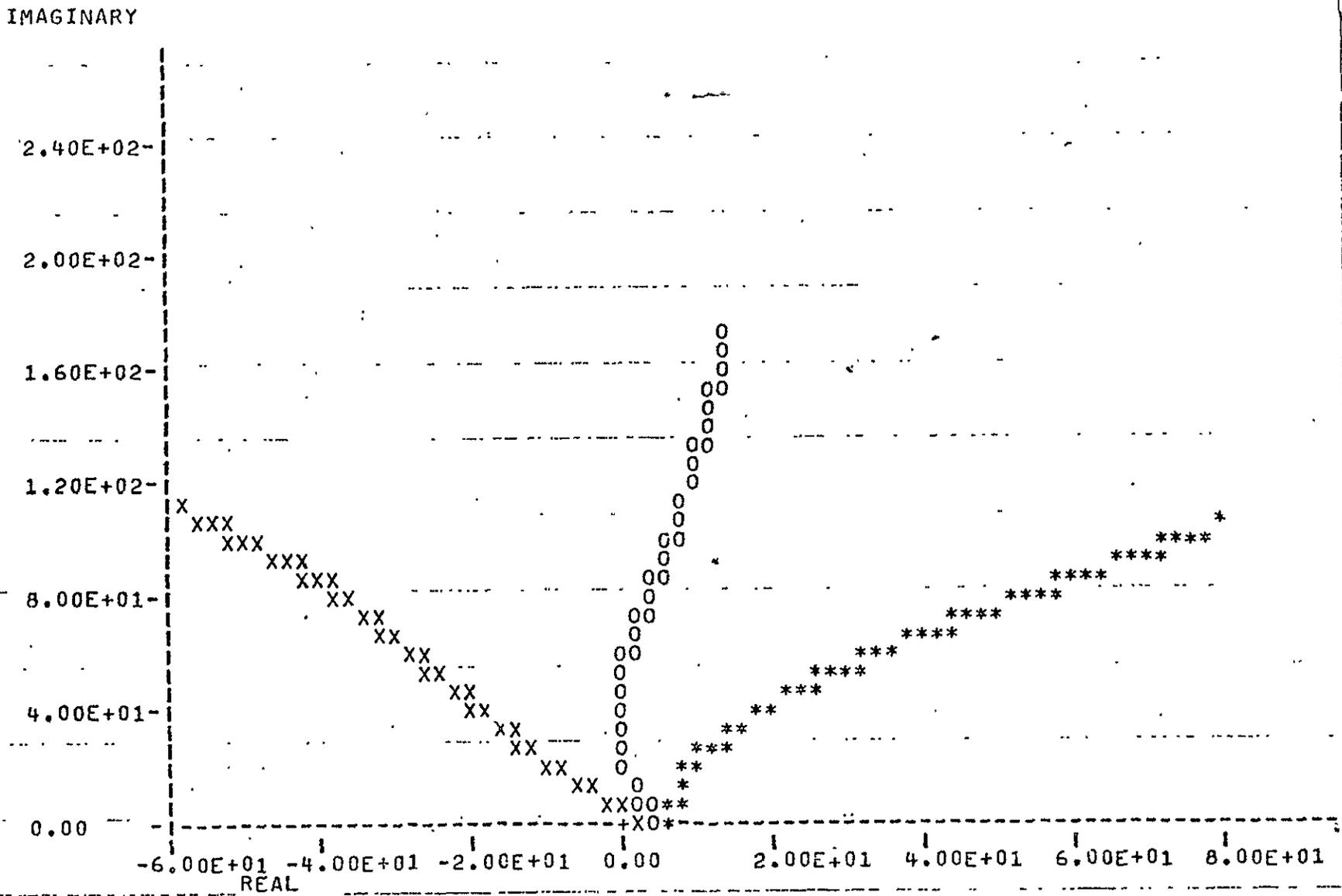


Figure 4.22 Gershgorin Band for Row 3 $0 \leq \omega \leq 200$

2-9

IMAGINARY

95

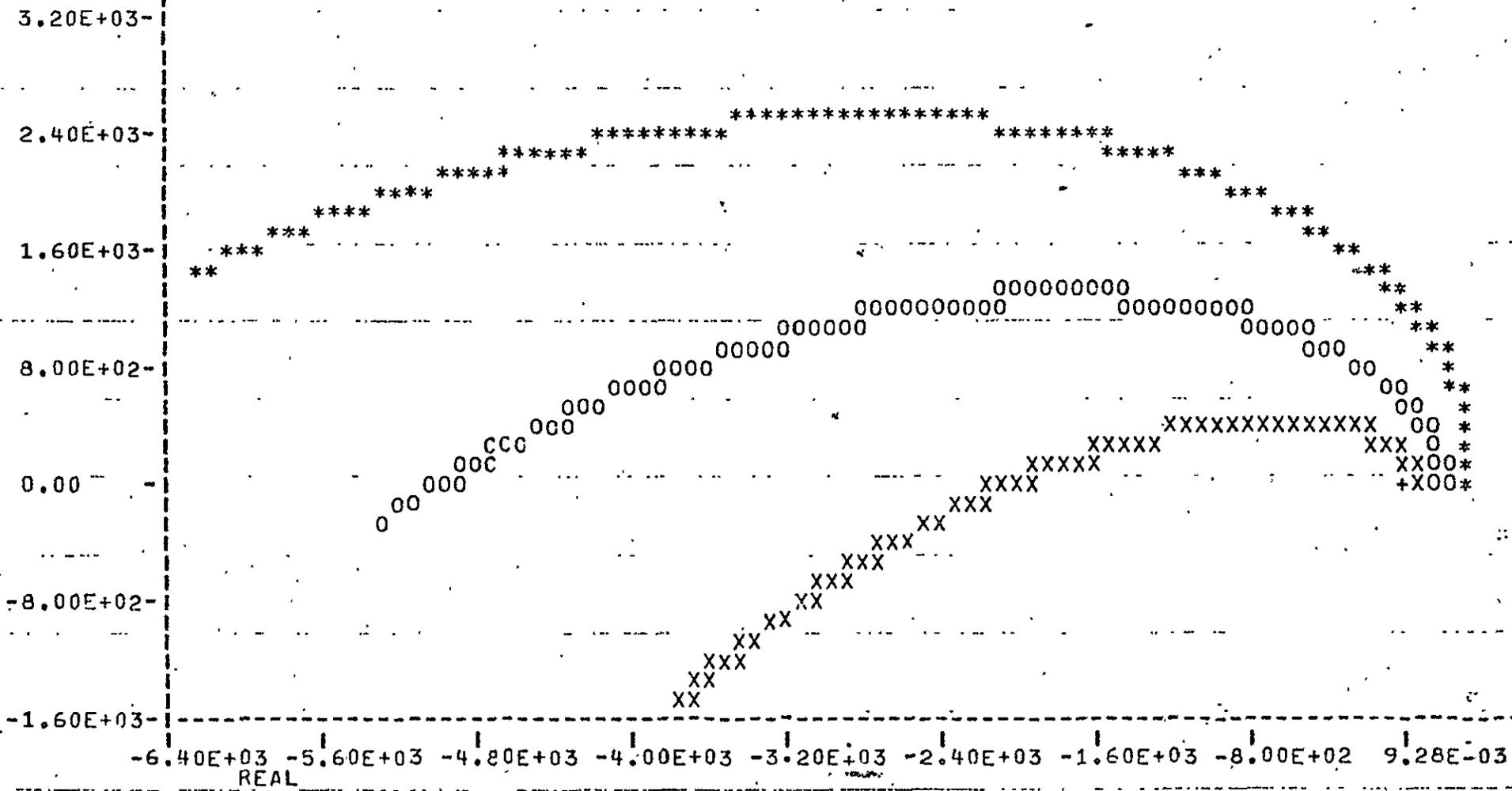
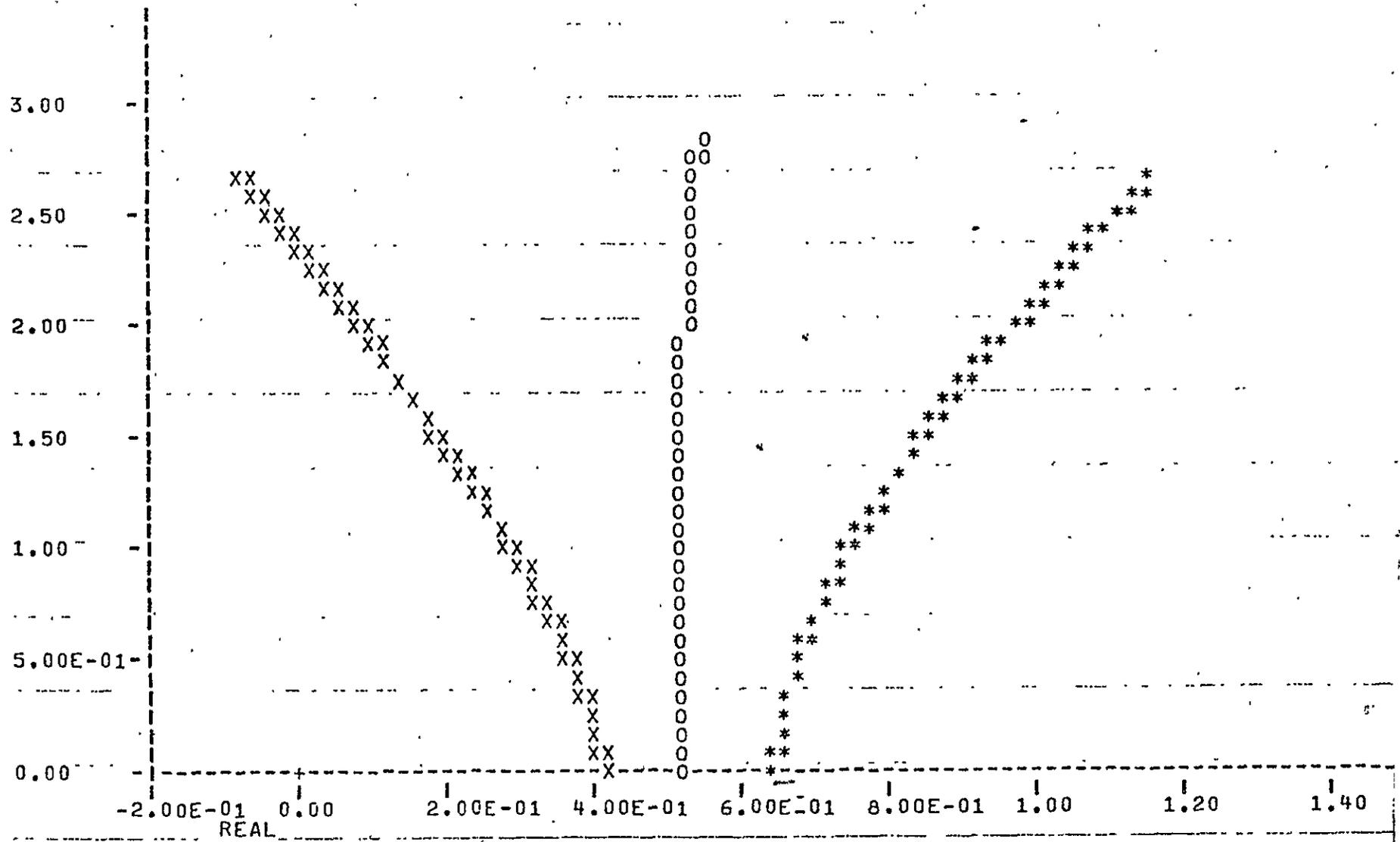


Figure 4.23 Gershgorin Band for Row 1 $0 \leq \omega \leq 50$

IMAGINARY

Figure 4.24 Gershgorin Band for Row 2 $0 \leq \omega \leq 50$

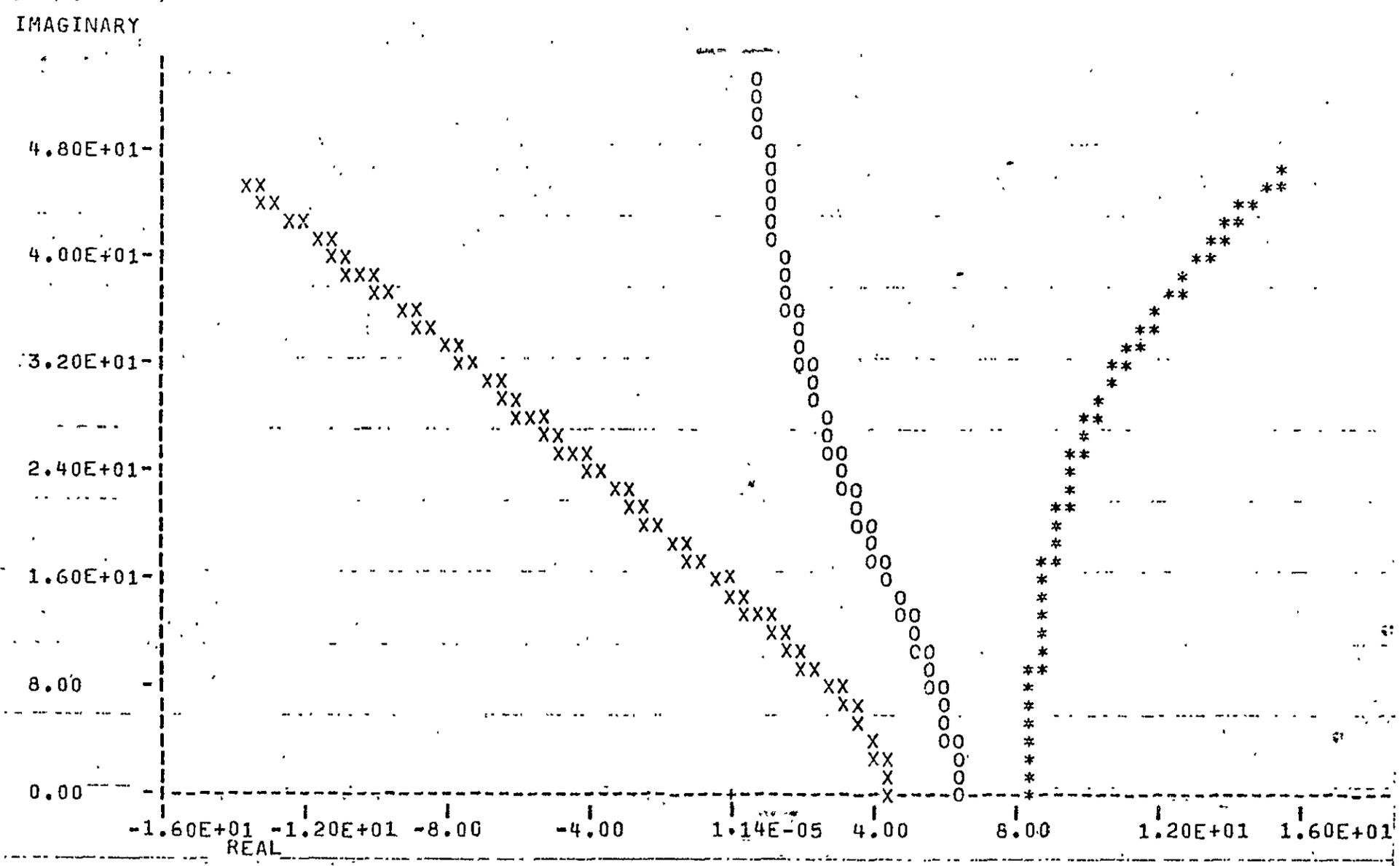


Figure 4.25 Gershgorin Band for Row 3 $0 \leq \omega \leq 50$

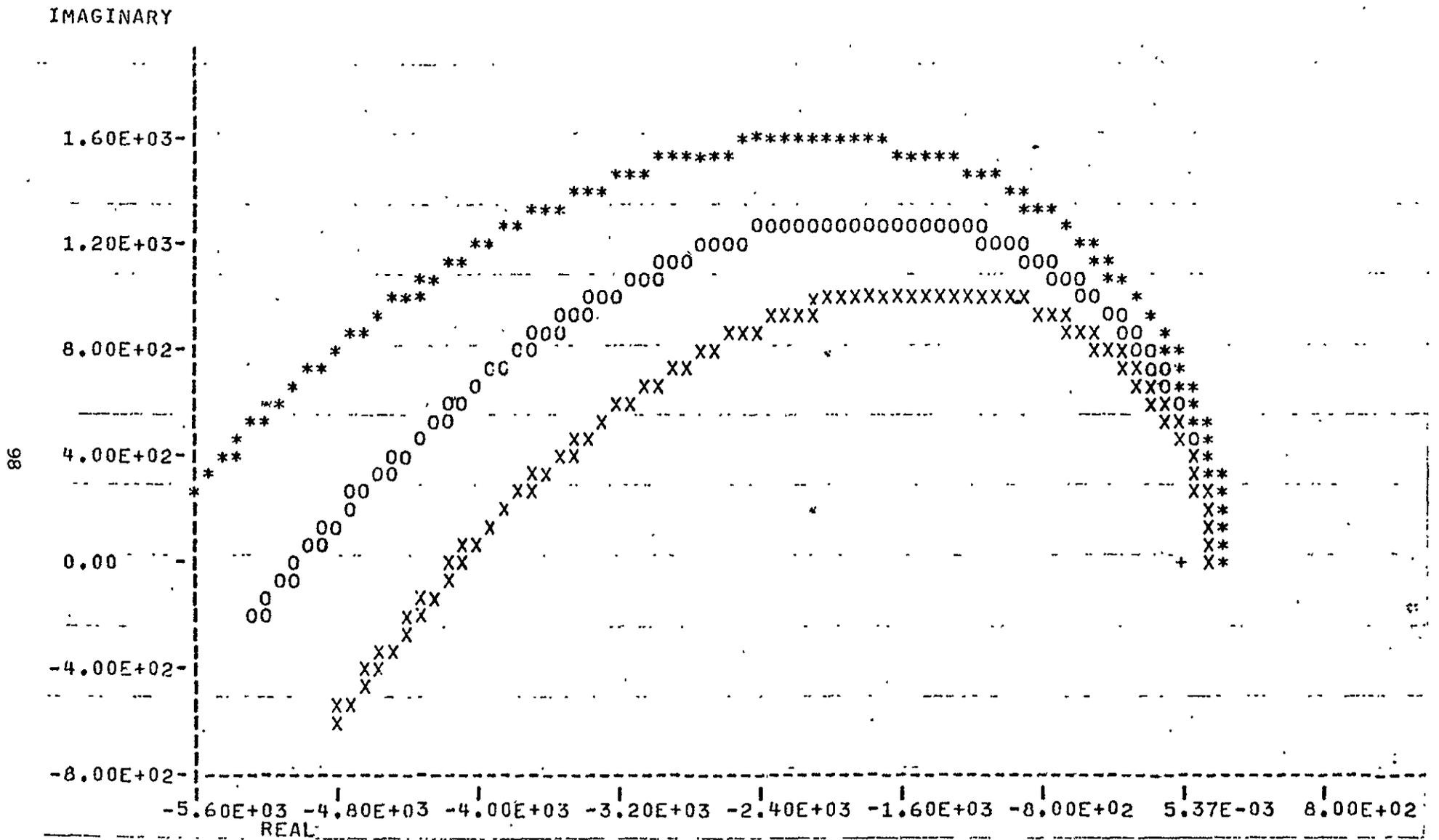


Figure 4.26 Ostrowski Band for Row 1 $0 \leq \omega \leq 50$

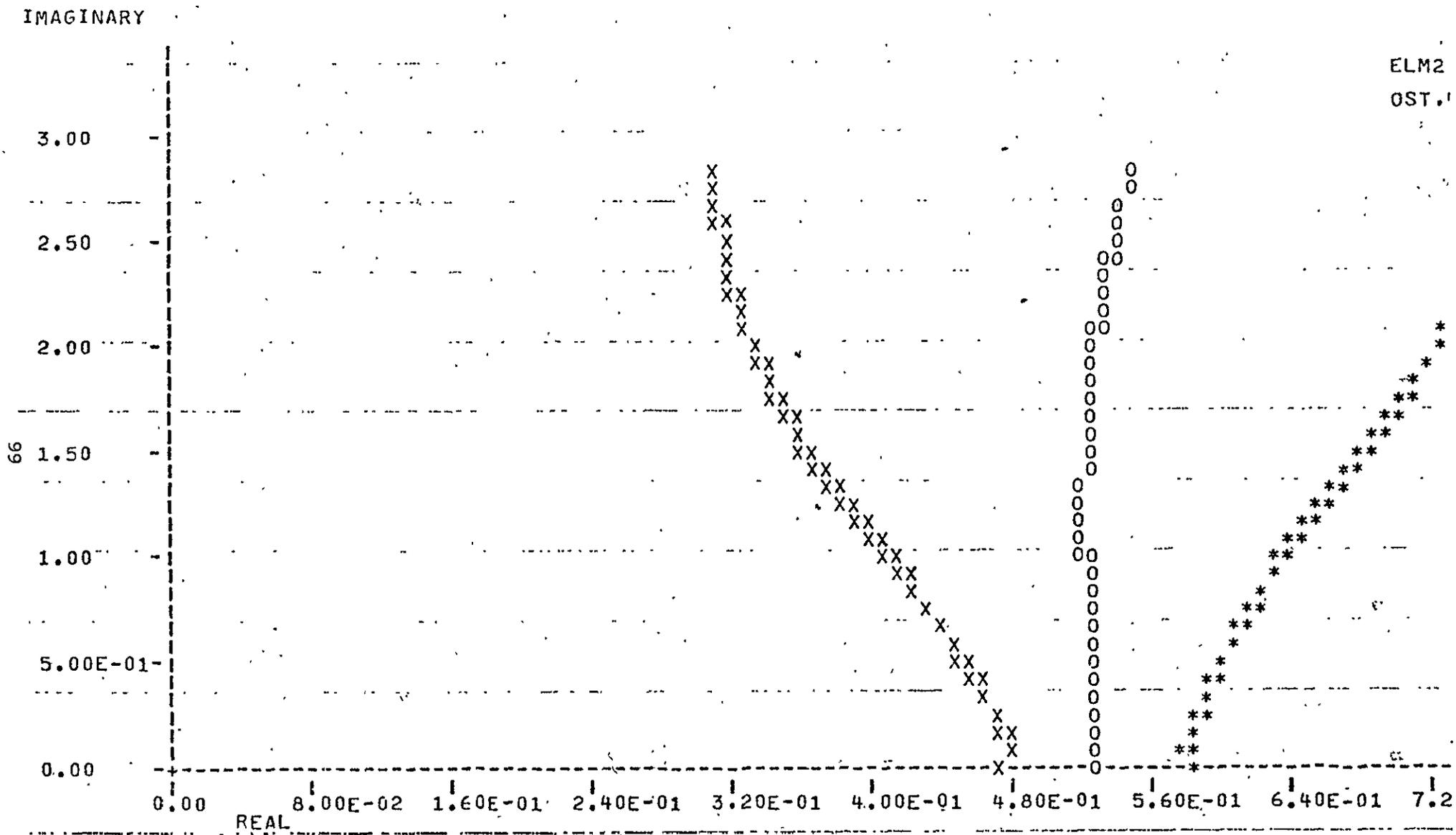


Figure 4.27 Ostrowski Band for Row 2 $0 \leq \omega \leq 50$

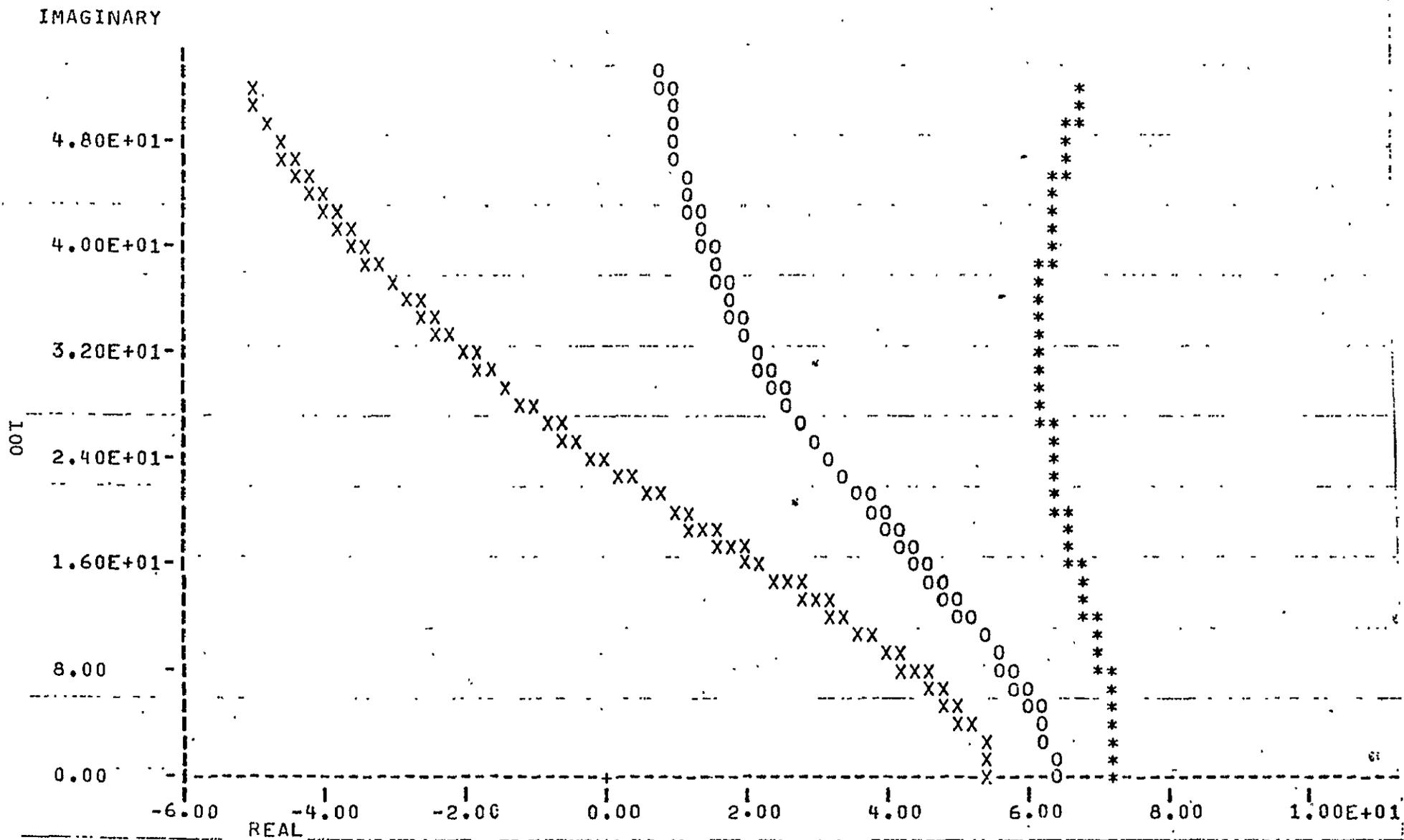


Figure 4.28 Ostrowski Band for Row 3 $0 \leq \omega \leq 50$

of reduction obtained are

69% reduction of Gershgorin Band # 1

50% reduction of Gershgorin Band # 2

50% reduction of Gershgorin Band # 3

The designer is now in a position wherein the Ostrowski bands can be further reduced by simply increasing the gains in each loop and recalculating the shrinking factors for the selected set of gains. To complete the design, eigenvalue checks and/or step responses may be used.

C. Discussion

The applications considered in this section demonstrate the versatility and effectiveness of the dominance algorithm described in section 3. The algorithm is computationally fast and efficient with most applications requiring 100 CPU seconds or less to achieve the dominance condition.

When the algorithm was tested against previously known results, two specific conditions were examined. First the reported results for the dominance producing compensators were implemented and verified for each case. In every instance, exact duplication of the gain space and Gershgorin and/or Ostrowski band was achieved. This condition established the accuracy of the algorithm in a non-optimization mode. The second condition ignored the reported parameter values and attempted to achieve diagonal dominance using alternate starting values for the compensators and the generalized optimization unit. Many new and interesting solutions were obtained and are reported in subsection A above.

As a final test for the dominance algorithm, the sixteenth order state model for the F100 turbofan engine was used to generate the appropriate transfer matrices corresponding to the two input and three input frequency

domain models. The results obtained clearly demonstrate the utility of the MNA design philosophy as a viable alternative for the design of feedback control units for the turbofan engine. Although the results presented are preliminary, it is apparent that acceptable system performance using the design philosophy of section 3 is easily achieved.

SECTION 5

CONCLUDING REMARKS

The Multivariable Nyquist Array as defined in this report is the union of two mutually exclusive design methods: the Inverse Nyquist Array and the Direct Nyquist Array. The two design methods are mutually exclusive in the sense that a design initiated in the inverse polar plane cannot in general be completed in the direct polar plane. This apparent inconsistency is due in part to the lack of duality between the definition of dominance for the INA and the corresponding definition for the DNA since

$$\hat{q}_{ij} \neq \hat{q}_{ij}^{-1}$$

However, the methods are structurally similar and thus provide the basis for the proposed design merger.

Exploiting the structural similarities between the two design methods, the dominance seeking algorithm of section 3 is appropriate for use in either the INA or the DNA design mode. In addition, constant or frequency dependent compensators can be evaluated for either row or column dominance with an indicated degree of preferred dominance. System interaction is easily assessed and a means of compensator comparison has been provided.

In its present form the MNA design algorithm performs in a batch computer mode. It is computationally efficient as demonstrated in section 4 and provides an effective alternative design for turbofan engine control systems. In addition, the dominance algorithm is ideally suited for implementation on an interactive computer network. In this computer mode, it is conceivable that a complete design via the MNA could be accomplished within one working day.

SECTION 6

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