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SOLAR GENERATED QUASI-BIENNIAL GEOMAGNETIC VARIATION

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MARCH 1977

GODDARD SPACE FLIGHT CENTER
GREENBELT, MARYLAND
SOLAR GENERATED QUASI-BIENNIAL
GEOMAGNETIC VARIATION

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ABSTRACT

The existence of highly correlated quasi-biennial variations in the geomagnetic field and in solar activity is demonstrated. The analysis uses a numerical filter technique applied to monthly averages of the geomagnetic horizontal component and of the Zürich relative sunspot number. Striking correlations are found between the quasi-biennial geomagnetic variations determined from several magnetic observatories located at widely different longitudes, indicating a worldwide nature of the obtained variation. The correlation coefficient between the filtered Dst index and the filtered relative sunspot number is found to be -0.79 at confidence level >99% with a time-lag of 4 months, with solar activity preceding the Dst variation. The correlation between the unfiltered data of Dst and of the sunspot number is also high with a similar time-lag. Such a time-lag has not been discussed in the literature, and a further study is required to establish the mode of sun-earth relationship that gives this time delay.
CONTENTS

Abstract ........................................... iii

INTRODUCTION .................................... 1

BACKGROUND ..................................... 2

METHOD OF ANALYSIS .............................. 4

DATA ............................................. 5

QUASI-BIENNIAL VARIATION IN GEOMAGNETIC FIELD .... 5

QUASI-BIENNIAL VARIATION IN SOLAR ACTIVITY .... 6

CORRELATION BETWEEN OSCILLATIONS IN Dst SUNSPOT
NUMBER ........................................... 7

DISCUSSIONS ..................................... 10

CONCLUSIONS .................................... 13

REFERENCES ..................................... 15

APPENDIX ........................................ 19
INTRODUCTION

Oscillations in various geophysical quantities having periods of about two years, which are referred to as quasi-biennial variations in this paper, have received considerable attention in recent years. In the case of such a variation in the geomagnetic field its existence and its exact nature are not well established. This is mainly because of the difficulty in extracting long period, small amplitude variations from records that contain other changes of larger amplitudes such as the daily or seasonal variations and disturbances of all sizes. The limited availability of magnetic observatory records having a long time span and high accuracy is another factor for the lack of definitive studies. Nevertheless, the study of quasi-periodic variations in the geomagnetic field is of interest because it provides valuable information concerning the solar-terrestrial relationships on one hand, and about the earth's interior on the other. Quasi-periodic variations in meteorological parameters have recently been extensively investigated because of general interest in long-term climatic changes and in solar weather relationships (e.g., King, 1975; Mason, 1976; Landsberg and Kaylor, 1976).

The interactions between the sun and the earth's magnetosphere and atmosphere have been studied intensively since in situ measurements in space became possible by means of satellites. Because of the relatively short time span of this new space exploration era these studies have been almost exclusively on short term solar-
terrestrial interactions. However, the long term interactions between the sun and the earth's environment may be of nature different from interactions for short lived disturbances such as magnetospheric substorms or magnetic storms. Therefore, it is of interest to investigate long term variations, for instance, in the geomagnetic field and to seek the causal relationship with solar phenomena. Such a study may lead to an understanding of different modes of the solar influences on the earth's environment.

BACKGROUND

The first mention of a quasi-periodic geomagnetic variation with a period of about two years appears to have been made by Kalinin (1952, 1954). However, Kalinin's results were regarded as being inconclusive by Currie (1966) because Currie's power spectral analysis by means of the Blackman and Tukey (1958) autocorrelation method showed no spectral peak near two years of period. On the other hand, Mansurov (1960) derived the secular change for the period of 1957 to 1959 for the Soviet Antarctic station Mirny by the same analysis procedure as that employed by Kalinin, and showed the presence of about one and one-half cycles of a quasi-biennial variation. In a more recent power spectral analysis of geomagnetic data by means of the maximum entropy method, Currie (1973) identified the spectral bands of the double solar cycle (average period 21.4 years) and the solar cycle (10.6 years) in the power spectral densities in the geomagnetic H and Z components. He interpreted the spectral peak near 2.15 years as the ninth and fifth harmonics of the double solar cycle and the solar cycle, respectively. Fraser-Smith (1972) gave a negative result on the existence of a quasi-biennial variation in the geomagnetic activity index Kp.
With a stimulus from the discovery by Reed and Rogers (1962) of the "26-month" zonal wind oscillation in the equatorial stratosphere, Shapiro and Ward (1962) made a spectral analysis of the Zürich relative sunspot number and pointed out the possibility of the existence of a spectral peak at the period of 25 months. However, their result is not conclusive since this peak was shown to be significant only at 5 percent level when subjected to a standard statistical test. In their search for similar oscillations in the geomagnetic field, Stacey and Westcott (1962) analyzed data on the horizontal component, H, of the magnetic field at Huancayo (12° S geographic latitude), Alibag (19° N), Apia (14° S), and Eskdalemuir (55° N) by the same method as that used by Kalinin (1954), namely by taking the difference between the 12-month and 24-month sliding averages of the monthly means, and obtained similar results to those derived by Kalinin. However, Stacey and Westcott stated that the correlation between the three equatorial stations was not consistently good. They performed a power spectral analysis, which indicated power spectral peaks near 26 to 27 months for the Alibag and Apia data; but the corresponding power spectral peak for Huancayo was weak and ill-defined. Pursuing the same question, London and Matsushita (1963) found no spectral peak near 26 months in the monthly means of quiet day amplitude of H from Huancayo.

Recently, Sugiura (1976) published a preliminary study (hereafter referred to as Paper 1), in which he showed evidence for the existence of quasi-biennial variations in the geomagnetic field and in solar
activity as represented by the Zürich relative sunspot number, and further indicated a statistically significant correlation between these variations. He suggested the possibility that quasi-biennial variations observed in the geomagnetic field, cosmic rays, and stratospheric and meteorological parameters are produced by a common cause on the sun. In Sugiura's analysis, annual averages were used both in the solar and geomagnetic data; therefore, the phase and amplitude of the quasi-biennial variations were not accurately determined. In the present paper the analysis is greatly improved by using numerically filtered monthly averages of solar and geomagnetic data. It is shown that the correlation between the quasi-biennial variation in the geomagnetic field and that in the relative sunspot number is indeed extremely high with a time lag that suggests a different mode of solar-terrestrial interaction than those presently known.

METHOD OF ANALYSIS

To obtain the amplitudes and phases of the quasi-biennial variations, time series consisting of the monthly averages of the relevant solar and geomagnetic data were passed through a band-pass filter. This numerical filter is the same as that used by Landsberg et al. (1963) for their analysis of the surface temperature. Characteristics of the filter are briefly described in the Appendix. A comparison of the present technique with the method of analysis used in Paper 1 is also made in the Appendix. This comparison is of value because both techniques offer respective advantages. For instance, while the former method provides detailed information on the wave form, the latter technique utilizing the second order time derivative of annual averages reduces the required numerical analysis by a large factor, and still preserves the coherency.
between data from different observatories.

DATA

The magnetic observatories whose data are analyzed in this paper, their coordinates, and the periods for which monthly averages were used in the analysis are given in Table 1. The records from Cheltenham and Fredericksburg were regarded as a single time series by subtracting a constant value from the Fredericksburg data so as to make the long-term variations as continuous as possible. Likewise, whenever an observatory moved its location, a constant value is added or subtracted to form a single time series.

QUASI-BIENNIAL VARIATION IN GEOMAGNETIC FIELD

Figure 1 presents filtered data of the horizontal component, H, of the magnetic field obtained at San Juan, Cheltenham-Fredericksburg, Tucson, Honolulu, and Kakioka, clearly demonstrating the presence of persistent, coherent quasi-biennial oscillations. The periods of these oscillations are about 2 years, but there are considerable variations in the time interval between neighboring peaks, ranging from about 1½ years to 3 years. In some cases, the apparent change in the peak-to-peak interval or an irregular wave form is due to a superposition of two trains of waves with a phase lag. However, the similarities among the oscillations observed at these observatories are, on the whole, striking. Indeed, the similarities are more impressive than are shown in Paper 1 on the basis of the second time derivatives of annual averages.

To show the high correlations between the data from pairs of observatories, the correlation coefficients were computed, the results of which are given in Table 2. In the table, "Student's t" and the number
of degrees of freedom, $n$, are provided besides the correlation coefficient to enable one to judge statistical significance. On the basis of the standard $t$-test, the probability that the obtained correlation should arise by random sampling from an uncorrelated population is much less than 0.001 in all cases; see, e.g., Fisher and Yates (1957) for the "Student's" $t$ distribution. There can be no question about the presence of coherent quasi-biennial oscillations at these observatories.

In Paper 1, the amplitudes of the quasi-biennial variations derived from Kakioka data for disturbed, all, and quiet days were compared (Figure 2 in that paper). The amplitudes so obtained were then found to diminish in the order: disturbed days, all days, and quiet days. If the source of the quasi-biennial variation lies in the interior of the earth, one would expect the amplitude to be independent of magnetic activity which is of origin external to the earth. Therefore, it was concluded that the observed oscillations are caused by an external source. It was then shown that the second time derivative of the Dst index and that of the Zürich relative sunspot number were correlated at a statistically significant level (Figure 3 in Paper 1). It is of interest to examine this correlation with numerically filtered monthly averages of these activity indices because of the improved time resolution achieved by this technique. Before we make such a comparison we first demonstrate the existence of a quasi-biennial variation in solar activity.

QUASI-BIENNIAL VARIATION IN SOLAR ACTIVITY

By an application of the band-pass filter to the monthly averages of the Zürich sunspot number for the years 1900 to 1976, the existence of persistent quasi-biennial oscillations in this solar activity index was indicated as clearly as in the geomagnetic field. (Examples are
shown in Figures 2 and 3). As in the geomagnetic data the period of oscillation and the amplitude vary somewhat, but the regularity of the oscillations was striking. When the same filter is applied to the Ottawa 10.7 cm solar flux, we again obtain evidence of a quasi-biennial variation. In fact, as can be seen in Figure 2, the quasi-biennial oscillations in 10.7 cm solar flux are so coherent with those in the Zürich relative sunspot number that the two curves are almost identical. This result is not surprising because the relative sunspot number and 10.7 cm solar flux themselves are highly correlated. With the data for 1947 to 1975 the correlation coefficient between the monthly averages of the Zürich relative sunspot number and of the Ottawa 10.7 cm solar flux was found to be 0.983 with $n=345$ and $t=100.7$, where $n$ and $t$ are the number of degrees of freedom and "Student's" $t$. The correlation coefficient between the filtered data of these parameters is 0.949 with $n=291$ and $t=51.1$. (The number of pairs is less for this set because of the loss of the data points at both ends arising from the application of the numerical filter). These correlations are significant at $>99.9\%$ on the basis of $t$-tests.

**CORRELATION BETWEEN OSCILLATIONS IN Dst AND SUNSPOT NUMBER**

The magnetic activity index Dst represents the axially symmetric component of the geomagnetic field variations primarily caused by the sun. It mainly represents the field from the magnetospheric equatorial current system which exists even at magnetically quiet times (Sugiura, 1973; Sugiura and Poros, 1973). There are smaller contributions from the magnetopause current and the tail current.
The correlation between the numerically filtered monthly averages of Dst and those of the relative sunspot number is found to be even better than the correlation between annual averages of those parameters shown in Paper 1. Plots of the filtered Dst and sunspot data are presented in Figure 3, which shows a remarkably good anticorrelation; namely, the phases of high (or low) solar activity correspond to those of low (or high) Dst level. It was further found by visual inspection that this correlation is better when the Dst curve is advanced in time by about 4 months, i.e., when Dst is correlated with the sunspot number 4 months earlier. To study this phase relation quantitatively we present in Figure 4 the correlation coefficient between Dst and the relative sunspot number, R, and "Student's" $t$ as functions of lag time. The maximum absolute value of the correlation coefficient occurs at a time-lag of about 4 months, the correlation coefficient being -0.79. The value of $t$ is a maximum at this time-lag, meaning that the probability that the obtained correlation might occur by a random sampling from an uncorrelated population is a minimum for this time-lag. For the 4-month time-lag, such a probability is less than 0.001. (The degrees of freedom for the plotted points are between 158 and 172). Hence the result obtained here can be regarded as being statistically significant. However, the value of 4 months was determined statistically involving about eight cycles of oscillation and the phase differences for individual cycles fluctuate slightly, but the plots in Figure 3 and the results shown in Figure 4 indicate that the time-lag determined here is not a mere average of widely varying values, but represents a meaningful average value.

In Figure 4, we further observe that the next maximum in the correlation comes near a lag of +8 months; the correlation coefficient is +0.75,
again coinciding with a peak in $t$. The time interval of 12 months between the two maximums represents one half of the period, confirming that the oscillations we are dealing with has a period of about 2 years.

In interpreting the 4-month time delay it is important to know whether this time-lag is peculiar to the quasi-biennial variation or the same time-lag exists in the correlation between the original, unfiltered sunspot data and Dst. Thus the correlation coefficient was calculated between the unfiltered monthly averages of the Zürich relative sunspot number and those of Dst with varying time-lag, $\tau$, at intervals of 1 month from $\tau = -12$ months to $\tau = +7$ months and then at 6-month intervals outside this range. The results are shown in Figure 5, where $\tau$ is now in units of years instead of months as in Figure 4. The maximum correlation is found at $\tau = -3$ months, where $t$ is also a maximum. The number of degrees of freedom is 226 in all cases, and for the region of $\tau$ giving high correlations the confidence level is well above 99%. The high (positive) correlation near $\tau = -6$ years is obviously from the out-of-phase correlation of the (negative) correlation for the 11-year solar cycle change. The correlation coefficient varies slowly from $\tau = -3$ to -4 months; hence it is not certain whether or not the difference between the 3-month time-lag in this correlation and the 4-month time-lag in the correlation of the filtered data is significant. However, it is likely that the cause for the 4-month time-lag in the quasi-biennial variation in Dst relative to that in the sunspot number is the same as the cause by which the variations in the monthly average Dst lag behind those in the monthly average relative sunspot number.

The results in Figure 5 are somewhat unexpected because it is known that the annual mean intensity of magnetic storms peaks 2 or 3 years
after the sunspot maximum (e.g., Sugiura and Chapman, 1960). The present results mean that statistically, the integrated 'Dst effect' lags behind solar activity represented by the relative sunspot number by 3 or 4 months. A similar result has also been obtained by using the geomagnetic activity index Ap instead of Dst. This subject, which is outside the scope of this study, is now being investigated in more detail.

DISCUSSIONS

1. In Figure 1 we have plotted the San Juan data only after 1930, though the filtered data for this observatory begins from April 1927. The omission of data for the first 2 3/4 years was made because of the presence of waves of unusually large amplitude reaching values several times the normal amplitudes. We believe that these large amplitude oscillations are from artificial sources that existed in the early phase of the operation after the observatory moved from Vieques Island to San Juan; see the footnote in Table 1. Filtered data were obtained for Vieques Island, but were not included in Figure 1 because there was nothing unusual in the short stretch of data from this station. While preparing Paper 1 it was noticed that the second order time derivatives of annual mean H values indicated extremely large amplitude oscillations for some observatories (not shown in Paper 1), especially for the early years, say before 1940. It appears that the deduction of quasi-biennial oscillations provides a convenient means of testing the quality of observatory magnetic data. The application of the analysis method of Paper 1 to the vertical component, Z, indicated that the measurements by the conventional variometers are generally much less accurate for Z than for H and that the accuracies in the Z
data from many observatories seemed inadequate for the study of the quasi-biennial variations.

2. The oscillations seen in the numerically filtered data must be viewed with caution because of the application of the narrow band-pass filter (described in the Appendix). However, the existence of geomagnetic field oscillations having periods of a few years appears to be real. This is because the analyses made by Kalinin (1954) and Sugiura (1976) and in this paper utilizing entirely different techniques have all indicated the presence of such oscillations. In addition, the existence of a spectral peak or band near 2 years in the power spectral density distributions derived from various sets of geomagnetic data supports the above view (e.g., Currie, 1973). Our own power spectral analysis of the geomagnetic and solar data used in this paper by means of the maximum entropy method (e.g., Ulrych and Bishop, 1975) and also by the method of Blackman and Tukey (1958) indicates the existence of a band in the period range roughly 2 to 3 years. Figure 6 shows an example of such results; namely in this case, the power spectral density in the relative sunspot number calculated both by the maximum entropy method and by the Blackman and Tukey method. The frequency range for which the filter transfer function is greater than one half the maximum is indicated by a small box drawn in the figure; crudely speaking, oscillations having frequencies in this range are passed by the filter used in this analysis. Figure 6 clearly shows the presence of power in a band near 2 years. A detailed study of the power spectral densities in solar and geomagnetic activities is outside the scope of this paper and will be discussed in a separate report.

The correlation with high statistical significance between the relative sunspot number and the geomagnetic H component (or Dst) cannot
be questioned. However, the exact nature of the signals transmitted from the sun to the earth and giving a high correlation is not as yet clear. A visual comparison of the unfiltered sunspot and geomagnetic data does not offer any clue on this point, and at the present stage of the study the filtered data shown in this paper seem to provide a good means of presenting highly correlated solar and geomagnetic signals. The higher maximum correlation of (-)0.79 between the filtered solar and Dst data compared with the maximum correlation coefficient of (-)0.52 for the unfiltered data indicates a significance of the quasi-biennial variations.
CONCLUSIONS

By means of a numerical filter technique we have shown the existence of highly correlated quasi-biennial variations in solar activity and in the geomagnetic field. The following conclusions have been drawn from the present analysis:

(i) The quasi-biennial variations in the H component determined from several observatories at different longitudes, ranging from 140°E to 294°E, are found to have a strong linear dependence among themselves.

(ii) The quasi-biennial variations deduced from the Zürich relative sunspot number and the Ottawa 10.7 cm solar flux are virtually identical.

(iii) There is a strong linear dependence between the quasi-biennial variation in the relative sunspot number (or 10.7 cm flux) and that in the geomagnetic Dst index which represents variations in the axially symmetric external field. The correlation between these quantities is a maximum at a time-lag of 4 months with solar activity preceding the Dst variation.

(iv) A similar time-lag is obtained in the correlation between the unfiltered data for the Dst and for the sunspot number.

(v) The origin of the 4-month time-lag in these correlations is not known. Since the relative sunspot number is used merely as a parameter expressing solar activity, this time-lag may not represent the true time delay in the causal relationship.

(vi) Estimates of the power spectral density of the relative sunspot number indicate the presence of a band of signals near 2
years period. However, the exact nature of the highly correlated solar and geomagnetic oscillations is not as yet understood.

Acknowledgements. We wish to thank Dr. H. E. Landsberg, Dr. J. W. King, and Dr. T. Yukutake for valuable discussions and Dr. J. London for calling our attention to his and Dr. Matsushita's earlier study. We are indebted to W. B. Schar for his assistance in the data analysis. We also wish to express our thanks to: Dr. K. Yanagihara and Dr. M. Kawamura, respectively the former and present Director of the Kakioka Magnetic Observatory for providing the year books and additional data, and the World Data Center-A for Solar-Terrestrial Physics in Boulder for supplying the geomagnetic and solar data used in this analysis.
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Table 1. Magnetic observatories, their coordinates, and the years for which monthly average data were used.

<table>
<thead>
<tr>
<th>Observatory</th>
<th>Geographic Latitude</th>
<th>Geographic E. Longitude</th>
<th>Dipole Latitude</th>
<th>Data Period (month/year)</th>
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</thead>
<tbody>
<tr>
<td>Vieques Island*</td>
<td>18.2°</td>
<td>294.5°</td>
<td>29.6°</td>
<td>4/1903 to 10/1924</td>
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<tr>
<td>San Juan</td>
<td>18.2</td>
<td>293.8</td>
<td>29.6</td>
<td>1/1926 to 12/1974</td>
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<tr>
<td>Cheltenham**</td>
<td>38.7</td>
<td>283.2</td>
<td>50.1</td>
<td>1/1902 to 12/1955</td>
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<tr>
<td>Fredericksburg</td>
<td>38.2</td>
<td>282.6</td>
<td>49.5</td>
<td>1/1956 to 12/1974</td>
</tr>
<tr>
<td>Tucson</td>
<td>32.2</td>
<td>249.2</td>
<td>40.5</td>
<td>1/1910 to 12/1974</td>
</tr>
<tr>
<td>Honolulu</td>
<td>21.3</td>
<td>202.0</td>
<td>21.2</td>
<td>1/1902 to 12/1974</td>
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<tr>
<td>Kakioka</td>
<td>36.2</td>
<td>140.2</td>
<td>26.1</td>
<td>1/1924 to 12/1972</td>
</tr>
</tbody>
</table>

* The observatory on Vieques Island, Puerto Rico was terminated in October 1924, and the recording at the new site in San Juan began in January 1926.

** The magnetic observatory at Cheltenham, Maryland was moved to its new location at Fredericksburg, Virginia in the end of 1955.
Table 2. The correlation coefficient, $\rho$, "Student's" $t$, and degrees of freedom, $n$, for each pair of observatories; the data used consist of filtered monthly averages of the H component.

<table>
<thead>
<tr>
<th></th>
<th>FR</th>
<th>TU</th>
<th>HO</th>
<th>KA</th>
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<tbody>
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<td></td>
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APPENDIX: Numerical Filters Used in This and Previous Analyses

The numerical filter used is a band-pass filter with 55 weights. As can be seen in its transfer function shown in Figure 7, the filter very effectively passes only those signals having periods near 2 years. The transfer function, $Y$, is a maximum at 25 months in period, and is reduced to one half the peak value at about 37 and 18 months; at 50 and 15 months in period, $Y$ is down to one-tenth the peak. The side lobes are less than 3.4% of the maximum $Y$. Since the weights are symmetric, no phase shift is introduced. The numerical values of the weights are given in Landsberg et al. (1963).

In Paper 1 we obtained the second order time derivatives of annual averages of the solar and geomagnetic data. Because of the space limitation in that communication, the details of the analysis technique were not discussed. Therefore, we give a brief account of the method and compare it with the technique used in this analysis. The previous method also can be considered as being a numerical filter technique. The underlying idea is (a) to take annual averages to filter high frequency components and (b) to amplify signals by a factor $\omega^2$ near the period of 2 years. The latter comes from the factor $\omega^2$ in the second time derivative of a wave having an angular frequency of $\omega$. With the Nyquist frequency at 0.5 cycles per year, or 2 years in period, the technique can potentially be useful in extracting quasi-biennial variations. However, the effectiveness of the technique depends on the method of numerical differentiation. A numerical differentiation is usually made by a least squares fit of data to a parabola of second order (e.g., Lanczos, 1964). For instance, Yukutake (1965) applied this technique to his study of the solar cycle variation in the geomagnetic secular...
change. However, since the least squares fit, which could be made with any number of points above 3, smooths the data, the method is not suitable for the present purpose. Therefore, in Paper 1 an exact fit was made to a parabola, which was then differentiated twice. This procedure is equivalent to the simple 3-point difference scheme for the second derivative. The transfer function of the procedure is given in Figure 8. The figure shows that the filtering of low frequency signals is not very efficient. For comparison, the normalized transfer function, \((\omega/\pi)^2\), for a heuristic estimate is also shown in Figure 8 by broken lines. The slightly lower effectiveness of the actual numerical procedure compared with the ideal case is due to the use of annual averages which obviously does not provide adequate time resolution. This shortcoming counter-balances the filtering of the unwanted high frequency noise by taking annual averages.

When the results such as those shown in Figure 1 obtained by the present technique are compared with the more coarse data derived by the method of Paper 1 using annual averages, we find that the phase information is lost considerably by the latter method as can be expected from the nearness of the Nyquist frequency to the frequencies of the signals in question. However, the filter of Paper 1 affected the data from different stations and for the sunspot number in the same manner so that the correlation between them was preserved to a high degree. Nevertheless, for the study of quasi-biennial variations the use of annual means is not recommended except for a crude survey of the presence or absence of the oscillations without requiring detailed information on the waves.
Figure 1. Filtered H data from San Juan, Cheltenham-Fredericksburg, Tucson, Honolulu, and Kakioka, indicating strongly correlated quasi-biennial variations. Tick-marks along the vertical axes represent 5 gamma scales for H.
Figure 2. Filtered data for the Ottawa 10.7 cm solar flux and for the Zürich relative sunspot number.
Figure 3. Filtered data for Dst and for the Zürich relative sunspot number. The Dst plots are shifted to the left by 4 months.
Figure 4. The coefficient, $\rho$, of correlation between the filtered Dst and the filtered relative sunspot number and the "Student's" $t$, as functions of the time-lag, $\tau$. 

Equation: $R(T+\tau) \text{ vs } Dst(T)$
Figure 5. The coefficient, $\rho$, of correlation between the unfiltered Dst and the unfiltered relative sunspot number and the "Student's" $t$, as functions of the time-lag, $\tau$. 

$R(T+\tau) \text{ vs Dst}(T)$

1957—1975

$\tau (\text{YEARS})$

$\rho$

$t$

$-6 -5 -4 -3 -2 -1 0 1 2 3 0$

$-0.6 -0.4 -0.2 0 0.2 0.4 0.6$

$0 2 4 6 8 10$

$\rho$

$t$
Figure 6. The power spectral densities in the Zürich relative sunspot number, estimated by the maximum entropy method and by the Babcock and Tukey autocorrelation method. Periods (in years) corresponding to several spectral peaks are indicated. The width of the half maximum of the bandpass filter is shown by the rectangular window.
Figure 7. The transfer function, $Y$, for the band-pass filter as a function of frequency, $f$, normalized by the sampling frequency $f_s$. 
Figure 8. The equivalent transfer function, $Y$, for the analysis procedure used in Paper 1 as a function of frequency, $f$, normalized by the sampling frequency, $f_s$. A heuristic estimate of the transfer function of the same procedure is shown by broken lines for comparison.