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COMPARISON OF EFFECTS OF UNSTEADY LIFT AND SPANWISE AVERAGING IN FLIGHT THROUGH TURBULENCE

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Summary

Gust loads on airplanes flying through random atmospheric turbulence are attenuated at high frequencies by two effects. One of these effects, called the unsteady lift effect, accounts for the penetration of the gust by the wing and the buildup of circulation. The other effect, called spanwise averaging, results from variations of angle of attack across the span as well as along the flight path in a random turbulence field. The purpose of the present report is to make a comparison of the magnitudes of the two effects. This comparison utilizes existing theoretical results for spanwise averaging and for the unsteady lift of finite span wings. In addition, a simple expression is derived which closely approximates the theoretical relation for the spanwise averaging effect in the high frequency range. The results of this analysis confirm previously published findings that for unswept wings of moderately high aspect ratio the attenuation effects due to spanwise averaging in flight through isotropic turbulence and those due to unsteady lift effects are of approximately equal magnitude. The present analysis also provides a direct theoretical confirmation of the conclusion that for small values of the ratio of wingspan to scale of turbulence, the attenuation effects due to spanwise averaging are independent of the ratio of wingspan to scale of turbulence. These effects may be expressed as a function only of a reduced frequency based on wingspan.

Introduction

Gust loads on airplanes flying through random turbulence are attenuated at high frequencies by two effects. One of these effects, called the unsteady lift effect, accounts for the penetration of the gust by the wing and the buildup of circulation. The other effect, called spanwise averaging, results from variations of angle of attack across the span as well as along the flight path in a random turbulence field. Both these effects have been recognized as important in calculating airplane fatigue life, passenger comfort, or in the development of gust-alleviation systems. Theories for both effects have been developed, but the relative magnitudes of the two effects are difficult to determine on the basis of existing data, because the results of the original analyses have been plotted as functions of different parameters.
An approximate comparison of the relative magnitudes of the two effects is included in reference 1. This comparison is based on the asymptotic behavior of the functions describing the spanwise averaging effects at low and high frequencies, and on the use of an approximation to the two-dimensional unsteady lift function. The purpose of the present report is to make a comparison of the two effects based on a different approach. The analysis uses previously derived theoretical results for spanwise averaging throughout the frequency range, and unsteady lift effects for finite-span wings. In addition, a simple expression is derived which closely approximates the theoretical relation for the spanwise averaging effect throughout the high frequency range. These results are limited to the consideration of lift variations on rigid wings in incompressible flow and are influenced by certain simplifying approximations in the original analyses on which the comparison is based.

LIST OF SYMBOLS

A, B  real and imaginary parts of lift due to sinusoidal gusts
b  wingspan
C_Lg  lift coefficient due to penetration of gust
c  mean wing chord
c_r  root chord, for elliptical wing
K, A_1, A_2, A_3  parameters in formula for indicial lift (formula 6)
K_0  integral of K_0; \int_0^x K_0(x)dx
K_0, K_1  modified Bessel functions of the second kind, order 0 and 1
k  reduced frequency based on wingspan, \( \frac{wb}{U} \)
k'  reduced frequency based on scale of turbulence \( \left( \frac{\omega L^*}{U} \right) \)
k_1  reduced frequency based on semichord, \( \frac{\omega C}{2U} \)
L^*  scale of atmospheric turbulence
R  aspect ratio, \( \frac{b}{c} \)
s'  nondimensional time based on semichords travelled \( \frac{2U}{c} t \)
**ANALYSIS**

**Explanation of Terms**

An explanation of the terms spanwise averaging effect and unsteady lift effect as used in this report is desirable to clarify the comparisons made in the subsequent analysis. When a wing moves through turbulent air, the vertical gust velocity may be measured by an instrument mounted at a single point on the span (say on the center line). If the resulting angle of attack were constant across the span, the wing would experience certain lift variations. These variations would differ from those obtained by simply multiplying the angle of attack by a constant (the value of $C_L$) because of the unsteady lift effect. This effect takes into account the penetration of the gust by different points along the chord and the lag in buildup of circulation. The unsteady lift effect attenuates the lift as the gust frequency increases, and may be considered as a transfer function relating the lift to the angle of attack. For a given pass through turbulence, the lift for the case of angle of attack constant across the span is deterministically related to the angle of attack. Because the turbulence is a statistical phenomenon, however, a given run cannot be repeated. For most purposes, therefore, the angle of attack variation is considered in terms of its power spectrum (the so-called point spectrum, since it is measured at a point traversing the atmosphere) and the power spectrum of the response is obtained. The ratio of the power spectra of the response and of the input at each frequency is the square of the transfer function at that frequency.
If the wing flies through an actual field of random turbulence, the vertical gust velocity as measured at a single point on the span may still be used to characterize the turbulence spectrum, but the value of vertical gust velocity at any given time is not constant across the span. If the total lift were measured as a function of time, the result would differ from that based on the assumption of constant velocity across the span. The lift in this case is not deterministically related to the measured angle of attack, because the spanwise variation of angle of attack varies in a random manner. In this case, therefore, the ratio of the power spectrum of the lift to the power spectrum of the measured angle of attack is the only information on the response that can be determined. This ratio includes both the unsteady lift effect and a further effect which is called the spanwise averaging effect.

In some analyses, the spanwise averaging effect has been taken into account by modifying the input gust spectrum to represent the averaged effect of the gust variations across the span. The resulting spectrum is called an effective or averaged spectrum. The ratio of the averaged spectrum to the point spectrum of turbulence is therefore a measure of the spanwise averaging effect. The spanwise averaging effect may also be considered as a function which attenuates the lift which would be produced by a wing flying through a one-dimensional turbulence field with the angle of attack constant across the span. This viewpoint is used in the present report. In general, the spanwise averaging effect further attenuates the lift at high frequency. In the subsequent analysis, the magnitude of the attenuation caused by the spanwise averaging effect is compared with that caused by the unsteady lift effect.

Spanwise Averaging Effect

The effects of spanwise variation in gust velocity on the lift of a wing in flight through random isotropic turbulence (or, more strictly, random axisymmetric turbulence) have been analyzed by previous investigators from two different viewpoints. In one, references 2 and 3, the turbulence field is considered as a superposition of sinusoidal waves of various frequencies oriented in all possible directions in the horizontal plane. In the second, references 4 and 5, gust-velocity correlation functions are used which are functions only of the distance between two points in the horizontal plane and not on the direction of the line joining the two points. In both cases, the statistical properties of the disturbances of an airplane flying through the turbulence field are independent of the direction of flight. Both approaches should give the same results for the power spectra of the lift variations. In the present report, the correlation function approach of references 4 and 5 is used, inasmuch as results are given in terms of known functions for several cases of interest.

In the report of reference 5, the problem of unsteady lift effects is considered separately from that of spanwise averaging. The justification for this assumption is given in reference 4. This approach is convenient for the present report because it makes possible the comparison of the magnitudes of the two effects.
In reference 5, the effect of spanwise variations in gust intensity on the lift due to atmospheric turbulence is analyzed for several assumed analytic expressions for the point spectra of turbulence, and for several span loadings. The point spectrum of gust velocity, as noted previously, is defined as the power spectrum of gust velocity as measured at a point traversing the atmosphere. Of the various spectra assumed, the so-called Dryden spectrum (case 1 of ref. 5) is the one most representative of actual atmospheric turbulence and still simple enough to permit closed-form solutions for the spectra of the lift. The point spectrum of vertical gust velocity for this case is given by the expression:

\[
\frac{\phi_w(k')}{\omega^2 L/U} = \frac{1}{\pi} \frac{1 + 3k'^2}{(1 + k'^2)^2}
\]

Effective vertical gust velocity is defined as the gust velocity, constant across the span, which would give the same lift at any frequency as that on the actual wing. The power spectra of effective gust velocity are given in reference 5 for wings with several span loadings, including elliptical and rectangular. Subsequently in this report, the spanwise averaging effects for these two cases are compared. Also, the spanwise averaging effect is compared with the unsteady lift effect for elliptical wings. The case of rectangular loading is the only one for which a closed-form expression for the power spectrum of lift is given in reference 5. This expression is:

\[
\frac{\phi_w(k')}{\omega^2 L^*/U} = \frac{2}{\pi} \frac{1}{\beta^2(1 + k'^2)^3} \left\{ 3k'^2 \beta \sqrt{1 + k'^2} \right. \\
+ (1 - 3k'^2) \left[ 2 - 2\beta \sqrt{1 + k'^2} K_1 \left( \beta \sqrt{1 + k'^2} \right) \right] \\
- \beta^2(1 + k'^2) K_0 \left( \beta \sqrt{1 + k'^2} \right) \right\} K_0 \left( \beta \sqrt{1 + k'^2} \right)
\]

Plots of this expression are given in reference 5 as a function of the dimensionless frequency \( k' = \frac{\omega L^*}{U} \) for various values of \( \beta = \frac{b}{L^*} \). The value \( \beta = 0 \) corresponds to the point spectrum, formula (1). In actual atmospheric turbulence, the scale of turbulence \( L^* \) has been found to exceed 300 m whereas the value of \( b \) ranges from about 9 m to 45 m. The value of \( \beta \) for actual airplanes is therefore in the range from less
than .03 to .15. The smallest increment of $\beta$ plotted in reference 4, however, is 0.25. The effect of spanwise averaging on actual airplanes is, therefore, difficult to determine from the plots given in reference 5.

A derivation is now given for a simpler expression which is approached by formula (1) in the range of small values of $\beta$ and high frequencies. The search for such an expression was motivated in part by the high numerical accuracy required to obtain accurate results from formula (1) in this range of variables. For the high frequency or short wave-length components of turbulence, the averaging effects would be expected to depend on the ratio of wave-length to span rather than on the ratio of span to the scale of turbulence. Formula (2) was therefore expressed where possible in terms of a reduced frequency based on wingspan,

$$ k = \frac{\omega b}{U} = \frac{\omega L}{U} b^* = k^*\beta $$

The resulting expression is

$$ \frac{\phi_{we}(k^*)}{\omega L^* U} = \frac{2}{\pi} \frac{k^2}{k^2(1 + k^2)^3} \left[ 3 k k^2 \sqrt{1 + k^2} \right] _0 \left( k \sqrt{1 + k^2} \right) $$

$$ + (1 - 3k^2) \left[ 2 - 2 k \sqrt{1 + k^2} \right] _1 \left( k \sqrt{1 + k^2} \right) $$

$$ - k^2 \left( \frac{1 + k^2}{k^2} \right) _0 \left( k \sqrt{1 + k^2} \right) \right] $$

If the relatively short wavelengths (say less than 3 times the span) are considered, then $k = \frac{\omega b}{U} = \frac{2\pi b}{\lambda}$ is greater than 2, and for $\beta$ less than 0.1, the value of $k^* = \frac{k}{\beta}$ is greater than 20. As a result, the combination $\frac{1 + k^2}{k^2}$ occurring in formula (3) is very close to 1.0, and may be dropped.
The values \( K_1 \) and \( K_0 \) represent modified Bessel functions of the second kind of orders 1 and 0, and the parameter \( K_{10} = \int_0^\infty K_0(x)dx \). The argument of these functions in formula (3) then reduces to \( k \). The variations of these Bessel functions with \( k \) is shown in figure 1. Both \( K_1 \) and \( K_0 \) approach zero exponentially and are close to 0 for \( k \) greater than 4.0, and the value of \( K_{10} \) is almost at its final value, which is \( \frac{\pi}{2} \) (see ref. 6, pg. 486, formula 11.4.23). Thus, for values of \( k > 4 \), formula 3 may be greatly simplified as follows:

\[
\phi_{w_e}(k') = \frac{2}{\pi} \frac{k'^2}{k^2(1 + k'^2)^3} \left[ \frac{3\pi}{2} k k'^2 + 2(1 - 3k'^2) \right] \tag{4}
\]

The quantity which shows how the square of the lift is attenuated by the spanwise averaging effect is the ratio of the power spectrum of the effective gust angle at any frequency to the power spectrum of the angle at that same frequency given by the point spectrum. This ratio may be interpreted as the square of the expected ratio of lift for a given frequency to the lift which would occur if the angle of attack at that frequency were constant across the span. This ratio, in the range of values of interest, may be determined by dividing formula (4) by the formula for the point spectrum formula (1).

The result is

\[
\frac{\phi_{w_e}}{\phi_{w}} = \frac{\pi}{k} - \frac{4}{k^2} \tag{5}
\]

(k > 4, \( k' > 20 \))

Thus for short wavelengths and small values of the ratio of wingspan to scale of turbulence, the attenuation of lift due to spanwise averaging is a function only of the reduced frequency \( k = \frac{\omega b}{U} \).
This fact is also observed in reference 1, but frequency in that report is expressed in terms of the parameter $k_1 = \frac{\omega C}{2U}$. In terms of the parameter $k$ used herein, $k_1 = \frac{\omega C}{2U} = \left(\frac{\omega b}{U}\right)\left(\frac{c}{2b}\right) = \frac{k}{2R}$. In reference 1, therefore, the spanwise averaging effect is stated to be a function of the parameter $k_1$ and the aspect ratio. When the results are expressed as a function of $k$, the reduced frequency based on wingspan, the aspect ratio disappears from the formula.

Unsteady Lift Effect

The two-dimensional theory for unsteady-lift effects on a wing penetrating a gust was first given in reference 7. The effect of finite aspect ratio was analyzed in reference 8. More complete analyses using lifting surface theory are possible using high-speed computers, but the results given in reference 8 are used herein because they are believed to be sufficiently accurate for unswept wings of moderately high-aspect ratio and elliptic lift distribution. Expressions for the indicial response of the lift penetrating a step gust for wings of aspect ratio 3, 6, and \( \infty \) are given in the form

\[
C_L(s') = K \left(1 - A_1 e^{-s'a_1} - A_2 e^{-s'a_2} - A_3 e^{-s'a_3}\right)
\]

where \( s' = \frac{2U}{c} t \)

Values of the coefficients $K$, $A_i$, and $a_i$ for the wings of aspect ratios 3, 6, and \( \infty \) are given in table I.

The expressions may be converted to give the frequency response to sinusoidal gusts. First take the Laplace transform of formula (6) and convert it to a transfer function by dividing by the transform of a step, \( \frac{1}{s} \). The result is

\[
C_L(s) = K \left(1 - \frac{A_1 s}{s + a_1} - \frac{A_2 s}{s + a_2} - \frac{A_3 s}{s + a_3}\right)
\]

where $s$ is the Laplace variable.

Then by substituting $s = ik_1$, where $k_1 = \frac{\omega C}{2U}$, the frequency response function of lift due to sinusoidal gusts is obtained. The resulting expression is
\[
\frac{C_{Lq}(k_1)}{K} = A(k_1) + iB(k_1)
\]

where

\[
A = 1 - \frac{A_1k_1^2}{a_1^2 + k_1^2} - \frac{A_2k_1^2}{a_2^2 + k_1^2} - \frac{A_3k_1^2}{a_3^2 + k_1^2}
\]

\[
B = -\frac{a_1A_1k_1}{a_1^2 + k_1^2} - \frac{a_2A_2k_1}{a_2^2 + k_1^2} - \frac{a_3A_3k_1}{a_3^2 + k_1^2}
\]

The square of the ratio of the amplitude of the response at any value of \( k_1 \) to the amplitude at zero frequency is given by

\[
\left| \frac{C_{Lq}}{C_{Lg}(0)} \right|^2 = A^2 + B^2
\]

These results are a function of \( k_1 = \frac{\omega C_p}{U} \) and should be converted to a function of \( k = \frac{\omega b}{U} \) for comparison with the spanwise averaging results. This conversion can be made only in the case of finite aspect ratio.

RESULTS AND DISCUSSIONS

The ratio of the lift of a wing in flight through random isotropic turbulence to the lift with angle of attack constant across the span is shown as a function of \( k = \frac{\omega b}{U} \) in figure 2. In this and subsequent plots, the square root of the ratio of the power spectra is shown in order to represent the attenuation of the lift rather than that of the square of the lift. The values for the elliptic span loading were obtained by numerical integration of the appropriate formulas in reference 5 and those for the rectangular span loading were calculated from formula (2). The results obtained from the approximate formula for rectangular span loading (formula 5) are also shown in figure 2. These results are seen to be indistinguishable from those given by the exact formula at values of \( k \) greater than 4.0. The results of figure 2 were calculated for various
Values of \( \beta = \frac{L}{W} \), the ratio of wingspan to scale of turbulence, in the range of interest for full-scale airplanes flying in the atmosphere. In this range of values of \( \beta \), the results are practically independent of \( \beta \) except at very low frequencies. An enlargement of the portion of the curve near \( k = 0 \) also is shown in figure 2. A slightly greater reduction of lift is shown for the wings with larger values of \( \beta \). This reduction, however, is less than 1.5 percent at \( \beta = 0.25 \). This slight decrease in the expected value of lift as \( k \) approaches 0 for wings with the larger values of \( \beta \) is believed to be caused by the probability of occasionally encountering a gust which is not constant across the span and which, therefore, reduces the average lift. This probability may be increased for wings of larger \( \beta \) because a larger area of the atmosphere is sampled. This slight reduction of lift at \( k = 0 \) becomes very small for wings with \( \beta < 0.1 \) and is neglected in subsequent discussions.

The conclusion of the present study that the attenuation of lift at small values of \( \beta \) is essentially independent of \( \beta \) is in agreement with the result of reference 1, which is based on a somewhat different representation of the spanwise averaging effects. Reference 1 also points out that the average rate of zero crossings of the normal acceleration can be determined using this result, whereas an attempt to calculate the rate of zero crossings using the Dryden spectrum without accounting for the attenuation effects leads to an infinite frequency of zero crossings.

The rolling moments acting on a wing in flight through random turbulence, like the lift, may be shown to depend only on the value of \( k \), the reduced frequency based on wingspan, and not on the value of \( \beta \), at all except very low frequencies. This result is pointed out in reference 9.

A comparison of the attenuation of lift due to spanwise averaging on an elliptic wing with the attenuation due to unsteady lift effects for wings of aspect ratio 3 and 6 is shown in figure 3. The curves show a similar shape even though they are based on entirely different theoretical considerations.

The analysis of reference 8, though it was based on the assumption of an elliptical lift distribution, did not take into account the exact planform of the leading edge which would determine the time of penetration of the gust at various points along the span. Also, the steady-state lift was based on lifting-line theory rather than lifting-surface theory. As a result of these assumptions, the shape of the curves for the unsteady lift effect may be subject to some inaccuracy. The main purpose of the comparison on figure 3 is to show the relative magnitudes of the spanwise averaging and unsteady lift effects throughout the frequency range. In reference 1, the conclusion is reached that the attenuation effect due to spanwise averaging is approximately equal to that due to unsteady lift on a wing of aspect ratio 10. This result appears to be in reasonable agreement with that of the present analyses. In the analysis of reference 1, the unsteady lift effects are based on an approximation to the unsteady lift function based on two-dimensional airfoil theory. The attenuations due to unsteady lift effects and due to spanwise averaging used in reference 1 are both somewhat greater than those in the present analysis, because of the different approximations used in estimating these effects.
In the case of an actual wing in flight through turbulence both attenuation effects will be present. The product of the two effects must therefore be used in calculating the overall attenuation of lift in flight through random turbulence. This combined alleviation effect of the unsteady lift and spanwise averaging effect is shown in figure 4 for elliptical wings of aspect ratios of 3 and 6. These curves show the actual ratio of the lifts rather than the square of lift which would be indicated if the power spectra were compared.

CONCLUDING REMARKS

The results of the present analyses provide a direct theoretical confirmation of the previously published finding that for small values of the ratio of wing span to the scale of turbulence (typical of actual airplanes) the attenuation effects of spanwise averaging are independent of the ratio of wing span to scale of turbulence. These spanwise averaging effects may be expressed simply as a function of a reduced frequency based on wing span.

These results also confirm previously published findings that for unswept wings of moderately high aspect ratio the attenuation effects due to spanwise averaging in flight through isotropic turbulence and those due to unsteady lift effects have a similar variation with frequency and are of approximately equal magnitude. The results of the present analysis are believed to be somewhat more reliable than those published previously because the analysis employs spanwise averaging effects based on existing theories throughout the frequency range rather than simply an interpolation between low- and high-frequency asymptotes. In addition, the unsteady lift effects are based on a theory for finite span wings rather than on results for infinite aspect ratio.

In order to obtain the power spectrum of lift on a wing flying through turbulence, the attenuation due to both spanwise averaging and unsteady lift effect should be included.
REFERENCES


TABLE I.- VALUES OF PARAMETERS IN EXPRESSIONS
FOR LIFT IN PENETRATING A STEP GUST

(Formula 6)

<table>
<thead>
<tr>
<th>Aspect Ratio</th>
<th>$K$</th>
<th>$A_1$</th>
<th>$A_2$</th>
<th>$A_3$</th>
<th>$a_1$</th>
<th>$a_2$</th>
<th>$a_3$</th>
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</thead>
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<td>3</td>
<td>$1.2\pi$</td>
<td>.679</td>
<td>.227</td>
<td>0</td>
<td>.558</td>
<td>3.20</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>$1.5\pi$</td>
<td>.448</td>
<td>.272</td>
<td>.193</td>
<td>.290</td>
<td>.725</td>
<td>3.00</td>
</tr>
<tr>
<td>$\infty$</td>
<td>$2\pi$</td>
<td>.236</td>
<td>.513</td>
<td>.171</td>
<td>.058</td>
<td>.364</td>
<td>2.42</td>
</tr>
</tbody>
</table>
Figure 1.- Plot of the modified Bessel functions of the second kind of orders 1 and 0, $K_1$ and $K_0$, as functions of the argument $k$. 

![Graph showing the plot of modified Bessel functions $K_0(k)$ and $K_1(k)$ as functions of the argument $k$.]
Figure 2.- Comparison of attenuation of lift due to spanwise averaging in random turbulence on wings with elliptical and rectangular span load distribution. Inset shows enlargement of curves near $k = 0$. 

Reduced frequency, $k = \frac{\omega b}{U}$

Approximate curve: $\sqrt{\frac{\pi}{k} - \frac{4}{k^2}}$
Figure 3.- Comparison of attenuation of lift due to spanwise averaging on an elliptical wing with attenuation due to unsteady lift effects for wings of aspect ratio 3 and 6.
Figure 4.- Attenuation of lift due to combined effects of unsteady lift and spanwise averaging in flight through random turbulence for elliptical wings of aspect ratio 3 and 6.
A comparison is made of the attenuation of lift due to unsteady lift effects and spanwise averaging in flight through turbulence. Previously derived theoretical results for these two effects are placed in a form which makes direct comparison possible. The attenuation effects from these two causes are found to be of approximately equal magnitude for unswept wings of moderate aspect ratio. For small values of the ratio of wingspan to scale of turbulence, the attenuation due to spanwise averaging is a function only of reduced frequency based on wingspan and is independent of the ratio of wingspan to scale of turbulence.