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COMPUTER PROGRAM FOR FLAT SECTOR THRUST BEARING PERFORMANCE

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A versatile computer program is presented which achieves a rapid, numerical solution of the Reynolds equation for a flat sector thrust pad bearing with either compressible or liquid lubricants. Program input includes a range in values of the geometric and operating parameters of the sector bearing. Performance characteristics are obtained from the calculated bearing pressure distribution. These are the load capacity, center-of-pressure coordinates, frictional energy dissipation, and flow rates of liquid lubricant across the bearing edges. Two sample problems are described.
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SUMMARY

A versatile computer program is presented which achieves a rapid, numerical solution of the Reynolds equation for a flat sector thrust pad bearing with either compressible or liquid lubricants. Program input includes a range in values of the geometric and operating parameters of the sector bearing. Performance characteristics are obtained from the calculated bearing pressure distribution. These are the load capacity, center-of-pressure coordinates, frictional energy dissipation, and flow rates of liquid lubricant across the bearing edges. Two sample problems are described and solved, one each for gas and liquid lubricants.

INTRODUCTION

The computer program presented in this report calculates the performance characteristics of both gas and liquid lubricated flat sector pad thrust bearings. A typical pad configuration is shown in figure 1. The bearing consists of several pads each of which has an extent angle \( \beta \) and inner and outer radii \( r_i \) and \( r_o \), respectively. Each pad assumes both pitch and roll with respect to the rotating runner to provide a (generally) converging film thickness in the direction of rotation.

In a recent paper (ref. 1) it was shown that any pitch and roll of a sector shaped pad about a certain point can be transformed to a corresponding pure pitch about a certain radial line. This can be understood from figure 1 by visualizing a plane parallel to the runner that goes through the origin of the sector (point 0 in fig. 1). The radial pivot line is the intersection between this parallel plane and the plane of the tilted sector, and it can be either within or outside the sector boundaries.

Based on this observation the flat sector thrust pad has been analyzed for both compressible and incompressible lubricants in references 1 and 2.

The objects of this report are to: (1) describe the numerical analysis and solution of the basic Reynolds' equation, (2) document the resulting computer
programs FSTBP1 and FSTBP2 for compressible (gas) and incompressible (liquid) lubricants, respectively, and (3) serve as a users guide for these two programs.

Input data for the programs describe the physical characteristics and the geometry of the sector pad. These are inner-to-outer radius ratio, pad angle extent, pad pivot angle, the ratio of pad pitch to minimum film thickness, and bearing compressibility number (this is not specified for liquid films).

The computed results include the pad thrust loading, the frictional power loss coefficient, and the center-of-pressure coordinates. Additional calculated results are flow leakage from the downstream edges for the liquid film case. (For a complete bearing the results of the several pads are added.)

This report also contains six appendices which give complete details of the numerical methods of solution, two FORTRAN listings of the computer programs, and two sample problems with output listings.

STATEMENT OF THE PROBLEM

It is required to develop a computer program that numerically solves the lubrication boundary value problem defined physically by the Reynolds equation over a tilted sector pad.

The Reynolds equations are essentially diffusion equations for the lubricant film pressure. In cylindrical coordinates the equation for the isothermal compressible case with density proportional to pressure is

$$\frac{\partial}{\partial r} \left( \frac{r h^3 p}{\mu} \frac{\partial p}{\partial r} \right) + \frac{1}{r} \frac{\partial}{\partial \theta} \left( \frac{p h^3}{\mu} \frac{\partial p}{\partial \theta} \right) = \delta r \omega \frac{\partial (ph)}{\partial \theta}$$

and for the incompressible lubricant film

$$\frac{\partial}{\partial r} \left( \frac{r h^3}{\mu} \frac{\partial p}{\partial r} \right) + \frac{1}{r} \frac{\partial}{\partial \theta} \left( \frac{h^3}{\mu} \frac{\partial p}{\partial \theta} \right) = \delta r \omega \frac{\partial h}{\partial \theta}$$

The lubricant film thickness is expressed in terms of the independent variables $r$ and $\theta$. By considering the clearance $h_0$ along the pivot line as a reference, the film thickness at any point $(r, \theta)$ is given by

$$h = h_0 + \gamma r \sin(\theta_p - \theta)$$

where $\gamma$ is the amount of tilt or pitch about this line. All of the symbols used in these and the following equations are defined in appendix A.
The normalized (dimensionless) form of these equations are

\[
\frac{\partial}{\partial R} \left( H^3 R P \frac{\partial P}{\partial R} \right) + \frac{1}{R} \frac{\partial}{\partial \theta} \left( H^3 P \frac{\partial P}{\partial \theta} \right) = \Lambda H^2 R \frac{\partial H}{\partial \theta} \tag{4}
\]

for compressible films, and

\[
\frac{\partial}{\partial R} \left( RH^3 \frac{\partial P}{\partial R} \right) + \frac{1}{R} \frac{\partial}{\partial \theta} \left( H^3 \frac{\partial P}{\partial \theta} \right) = H^2 R \frac{\partial H}{\partial \theta} \tag{5}
\]

for an incompressible lubricant. From equation (3), the film thickness becomes

\[H = 1 + \epsilon R \sin(\theta_p - \theta)\]

Normalization of the variables leading to equations (4) to (6) is described in references 1 and 2.

The pressure distribution over the pad area is obtained by numerically solving equation (4) or (5) under boundary conditions which are defined in the next section. Further sector pad calculations are based on this pressure distribution.

The following bearing performance characteristics are then calculated:

1. Pad load \(W\)
2. Center-of-pressure coordinates \(R_{cp}, \theta_{cp}\)
3. Power loss coefficient (normalized coefficient of friction) \(\bar{F}/\bar{W}\)
4. Volumetric lubricant flow rates across the sector pad edges (liquids only) \(q_{le}, q_{te}, q_{so}, \text{ and } q_{si}\)

Inputs are in vector arrays with a range of design parameter values:

1. Pad dimensions -
   (a) Inner radius ratio \(R_1 = r_I/r_o > 0\)
   (b) Sector angle \(\beta\) in degrees
2. Pivot line angle ratio \(\theta_p/\beta\)
3. Compressibility factor (also called "bearing number") \(\Lambda\), for the gas film case
4. Ratio of pad slope to minimum pad-runner clearance \(\epsilon/H_2 > 0\)

METHOD OF SOLUTION

The two forms of the Reynolds equation are first transformed to the following boundary value problems, which are then solved numerically for specific boundary conditions,
Find a function \( u(R, \theta) \) that satisfies the equation

\[
L(u) = f(R, \theta) \tag{7}
\]

on the domain

\[
\mathcal{D} = \left( \theta, R \Big| 0 \leq \theta \leq \beta; \ R_1 \leq R \leq 1; \ \beta - \frac{\pi}{2} < \theta < \frac{\pi}{2} \right) \tag{8}
\]

In the compressible case, \( L \) is the nonlinear operator

\[
L = \frac{\partial^2}{\partial R^2} + \frac{1}{R^2} \frac{\partial^2}{\partial \theta^2} - \left( \frac{\partial}{\partial R} \ln \frac{H}{R} \right) \frac{\partial}{\partial R} - \left( \frac{1}{R^2} \frac{\partial}{\partial \theta} \ln \frac{H}{R} + \frac{\Lambda H^2}{H(Q)} \right) \frac{\partial}{\partial \theta}
\]

\[
u = Q = (PH)^2
\]

\[
f = 0
\]

and

\[
u = H^2 \quad \text{on boundary} \ \partial \mathcal{D}, \ (P = 1)
\]

In the incompressible case, \( L \) is the linear operator

\[
L = \frac{\partial^2}{\partial R^2} + \frac{1}{R^2} \frac{\partial^2}{\partial \theta^2} + \left( \frac{\partial}{\partial R} \ln \frac{RH^3}{H} \right) \frac{\partial}{\partial R} + \left( \frac{1}{R^2} \frac{\partial}{\partial \theta} \ln \frac{RH^3}{H} \right) \frac{\partial}{\partial \theta}
\]

\[
u = P
\]

\[
f = \left( \frac{H^2}{H^3} \right) \frac{\partial H}{\partial \theta}
\]

and

\[
u = 0 \quad \text{on boundary} \ \partial \mathcal{D}, \ (P = 0)
\]

Note again that on the pad boundary \( P = 1 \) (compressible) and \( P = 0 \) (incompressible). On the boundary \( \partial \mathcal{D} \) the pressure is fixed at the ambient value, \( p = p_a \), for both gas and liquid lubricant cases. But the definition of dimensionless pressure differs:
\[ P = \frac{P - P_a}{K} \quad \text{(incompressible)} \]

where

\[ K = \frac{6\mu r_0^2}{h_0^2} \]

Numerical solution of equation (9) or (10) is described in appendix B. These boundary value equations are approximated by finite difference equations on a polar mesh over the sector pad area as indicated in figure 2. In order to adopt Simpson Rule integration formula, the \( R \) and \( \theta \) intervals are divided into even numbers of increments.

As indicated in the previous section, the solution of the equations is in the form of the film pressure distribution over the pad area. From this distribution the thrust pad performance characteristics are obtained by Simpson Rule integration of the following expressions:

Normalized load capacity

\[ \bar{W} = \frac{W}{P_a r_0^2} = \int_{R_1}^{1} \int_{0}^{\beta} (P - 1) R \, d\theta \, dR \quad \text{(compressible)} \quad (12a) \]

or

\[ \bar{W} = \frac{W}{K r_0^2} = \int_{R_1}^{1} \int_{0}^{\beta} PR \, d\theta \, dR \quad \text{(incompressible)} \quad (12b) \]

Unit load

\[ UL = \frac{W}{P_a A} \quad \text{(compressible)} \]

\[ UL = \frac{W}{KA} \quad \text{(incompressible)} \]

where the \( \bar{W} \) is calculated from equation (12a) or (12b) depending on fluid type.
Center-of-pressure radial coordinate

\[ R_{cp} = \int_{0}^{\beta} \int_{R_{i}}^{1} \frac{(P - 1)R^2}{W} dR d\theta \quad \text{(compressible)} \quad (14a) \]

or

\[ R_{cp} = \int_{0}^{\beta} \int_{R_{i}}^{1} \frac{PR^2}{W} dR d\theta \quad \text{(incompressible)} \quad (14b) \]

Center-of-pressure angle coordinate

\[ \theta_{cp} = \sin^{-1} \left[ \frac{\int_{0}^{\beta} \int_{R_{i}}^{1} (P - 1)R^2 dR \sin \theta d\theta}{R_{cp} W} \right] \quad \text{(compressible)} \quad (15a) \]

or

\[ \theta_{cp} = \sin^{-1} \left( \frac{\int_{0}^{\beta} \int_{R_{i}}^{1} PR^2 dR \sin \theta d\theta}{R_{cp} W} \right) \quad \text{(incompressible)} \quad (15b) \]

Center-of-pressure distance from pivot line

\[ X_{cp} = R_{cp} \sin(\theta_{cp} - \phi) \quad (16) \]

Power loss coefficient

\[ \frac{F}{W} = \frac{1}{6W} \int_{0}^{\beta} \int_{R_{i}}^{1} \left( \frac{\Delta R^3}{H/H_2} + 3R \frac{H}{H_2} \frac{\partial P}{\partial \theta} \right) dR d\theta \quad (17) \]

Equation (17) holds for both compressible fluids (\( \Lambda \) variable) and incompressible fluids. In the incompressible case, the program puts \( \Lambda = 1 \).
Volumetric flow rates across the pad edges are:

Trailing edge:

\[ q_{te} = \frac{1}{2} \left( 1 - R_1^2 \right) + \frac{1}{3} \left( 1 - R_1^2 \right) \sin(\theta_p - \beta) - \int_{R_1}^{1} \left[ \frac{H^3}{H_2^3R^2} \left( \frac{\partial P}{\partial \theta} \right) \right] \, dR \quad (18) \]

Leading edge:

\[ q_{le} = \frac{1}{2} \left( 1 - R_1^2 \right) + \frac{1}{3} \left( 1 - R_1^2 \right) \sin \theta_p - \int_{R_1}^{1} \left[ \frac{H^3}{H_2^3R^2} \left( \frac{\partial P}{\partial \theta} \right) \right] \, dR \quad (19) \]

Outer arc:

\[ q_{so} = - \int_{0}^{\beta} \left[ \left( \frac{H}{H_2} \right)^3 \left( \frac{\partial P}{\partial R} \right) \right] \, d\theta \quad (20) \]

Inner arc:

\[ q_{si} = + R_1 \int_{0}^{\beta} \left[ \left( \frac{H}{H_2} \right)^3 \left( \frac{\partial P}{\partial R} \right) \right] \, d\theta \quad (21) \]

Flow is defined as positive in the direction of increasing \( R \) and \( \theta \).

FORTRAN PROGRAM

General Description

The foregoing analysis has resulted in two thrust pad computer programs: FSTBP1 for compressible fluid films, and FSTBP2 for incompressible lubricant films. The FORTRAN listing for each program is given in appendix C, and flow chart diagrams are presented in appendix D. The dictionary of the FORTRAN symbols used in the computer programs is appendix E.
Both programs have identical structure in the number and function of the subprograms, and in the general format of the input data.

The first flow diagram in appendix D, that of the supervisory module MAIN2, presents a compact overview of the logical sequence which the computer programs follow in producing pad performance characteristics from input operating conditions.

The first part of the computer program, which includes the first four subroutines called by the module MAIN2, accomplishes the numerical solution of the Reynolds equation using a Gauss-Seidel iterative method with under- or over-relaxation. Upon convergence of the iterations, the calculated film pressure distribution under the sector pad is passed to the second part of the program which performs the numerical double integrations on equations (12) to (17) resulting in the bearing pad performance characteristics.

A brief explanation of the function of each of the program modules follows:

(1) MAIN2 is the executive routine for processing multiple cases, and has primary control of logical flow throughout the complete program. The executive routine also controls the printing of the results.

(2) EUCLID creates the mesh $\Delta R$, $\Delta \theta$, and converts all angles to radians.

(3) XBEGN2 tests the given angular position of the pad pivot line against the pad coordinates, and then calculates the minimum film thickness between pad and runner, the pad slope, and the maximum-to-minimum film thickness ratio.

(4) COEFF generates the values of the nodal coefficients for the finite difference representation of the Reynolds equation. It also provides the initial values of the dependent variable for the first iteration of the Gauss-Seidel process.

(5) RELAX is the basic working routine for solving the Reynolds equation by Gauss-Seidel iterative method with a choice of relaxation parameter.

(6) TABULT is an integration subroutine completing the calculation of the pad loading, center-of-pressure coordinates, friction power loss, and lubricant flow rates.

(7) RSIMP uses the Simpson Rule to integrate tabulated functions along the radii at each angle mesh position.

No special effort was made to determine the optimum relaxation factor for the program with given input data. Initially the sequence for $\Omega_k = k/4$, where $k$ was incrementeded from 1 to 7, was tested for iteration efficiency. It was soon determined that $\Omega_5 = 1.25$ was generally superior to other values tried, and that value was used for most calculations. As compressibility number increased to 100 the optimum value of $\Omega$ decreased to 0.75.
Using the Program

The program always starts by reading in a three-card input deck. The contents of these cards is now described (all symbols are repeated in appendix E).

Card 1 (FORMAT 3I6, F8.2, 2E8.1)
NR number of radial increments (even)
NA number of angular increments (even)
INTRMX maximum number of iterations allowed if convergence is not achieved in the solution.
OMEGA relaxation parameter in the solution procedure.
HALT cutoff value for minimum film thickness. There is an error return and a warning printout if the calculated HMIN is less than HALT.
RESIDL convergence criterion; test on maximum change in dependent variable between iterations in subroutine RELAX.

Card 2 (FORMAT 4L6)
Values on this card control logical switches.
DEBUG when TRUE the values of the dependent variable are printed for all mesh points for each iteration. To avoid printout use FALSE.
TABOUT when TRUE the film pressure array over all mesh points is printed after convergence of the iteration routine. The value FALSE stops this printout.
OLDQ a TRUE value saves the dependent variable array after convergence of the iterative procedure so that this array can be used as the starting value for a new iterative calculation when multiple input cases are being calculated. A value of FALSE defines initial pressure as ambient over pad area.
VARGRD a TRUE value creates a variable mesh grid over the sector pad (fig. 2). Only needed for liquid lubricant calculations.

Card 3 (FORMAT 5I10)
This card sets the size indices for parameter arrays which are read in immediately after this card through namelist "VARBLE,"
The parameter data whose array sizes are defined by the values on card 3 are read into the MAIN2 subprogram through the namelist 'VARIBLE' input, utilizing as many cards as is necessary. These are without format description, and are now listed.

TRATIO (I) array of pivot line angles \( \theta / \beta \)
I=1, NTHETA

ERATIO (I) array of input parameters \( \gamma r_o / h_2 = \epsilon / H_2 \)
I=1, NRATIO

VRI (J) array of inner to outer radius ratios, \( r_i / r_o \)
J=1, NUMRI

VBETA (J) array for pad angle size \( \beta \)
J=1, NUMBET

VLMBDA (J) for compressible films, the array of bearing numbers \( \Lambda \)
J=1, NUMLMB

Documented description of the data input decks are provided at the end of each of the two programs FSTBP1 and FSTBP2 in appendix C.

CONCLUDING REMARKS

We have described a numerical method and computer program for solving two forms of the Reynolds equation within a circular sector region. These two forms of the equation are for compressible and incompressible fluid films.

The numerical method uses a curvilinear cell at each mesh point to derive the finite difference analog to the Reynolds equation. This represents a system of nonlinear equations with prescribed constant boundary values. Two computer programs were developed to solve the finite difference systems representing the compressible and incompressible fluid cases. The programs use a Gauss-Seidel iterative method with relaxation capability.
Input flexibility was built into the programs to allow a wide variation in sector geometry and operating conditions. This allows the program user to quickly evaluate alternative thrust bearing design configurations in order to find the optimum cases.

The programs are easy to use, and have enough generality to be used as the "black box" in situations where steady state solutions to the Reynolds equation are required for circular sector regions.

A rather coarse mesh of about 100 points is sufficient for the bearing load and moment calculations (8 radial divisions and 12 angular divisions). Edge leakage calculations which utilize pressure gradients at the boundaries require a finer mesh (14 radial and 20 angular divisions).

Each case required at most 40 to 50 iterations to satisfy the convergence criterion, and averaged 1 to 2 seconds total time on the UNIVAC 1100/42 computer.
APPENDIX A

MATHEMATICAL SYMBOLS

\[ A \quad \text{pad area, } \beta \left( \frac{r_0^2 - r_1^2}{2} \right) \]

\[ r \quad \text{closed domain over pad area} \]

\[ \partial \quad \text{boundary of pad domain} \]

\[ F \quad \text{friction power loss} \]

\[ \overline{F} \quad \text{nondimensional power loss, } \frac{F}{\rho_B \omega h_2 r_0^2} \text{ (incompressible)} \]

\[ \overline{F}/\overline{W} \quad \text{power loss coefficient, } \frac{F}{W \omega h_2} \]

\[ f \quad \text{forcing function in eq. (7)} \]

\[ H \quad \text{nondimensional film thickness, } \frac{h}{h_0} \]

\[ H_1 \quad \text{maximum film thickness ratio, } \frac{h_1}{h_0} \]

\[ H_2 \quad \text{minimum film thickness ratio, } \frac{h_2}{h_0} \]

\[ h \quad \text{film thickness} \]

\[ h_0 \quad \text{constant film thickness along the pivot line} \]

\[ h_1, h_2 \quad \text{maximum and minimum film thickness, respectively} \]

\[ K \quad 6 \mu \omega r_0^2 / h_2^2 \]

\[ L \quad \text{operator in Reynolds equation (eq. (7))} \]

\[ P \quad \text{nondimensional pressure, } \frac{p}{p_a} \text{ (compressible); or } \frac{(p - p_a)}{K} \text{ (incompressible)} \]

\[ p \quad \text{pressure} \]

\[ p_a \quad \text{ambient pressure} \]

\[ Q \quad (PH)^2 \]

\[ q \quad \text{volumetric edge flow rates, eqs. (18) to (21)} \]

\[ R \quad \text{nondimensional radius, } \frac{r}{r_0} \]

\[ R_{cp} \quad \text{nondimensional radius to center of pressure} \]

\[ R_i \quad \text{nondimensional inner radius, } \frac{r_i}{r_0} \]
radial coordinate

$\bar{r}$  

pad inner radius

$\bar{r}_i$  

pad outer radius

$\bar{r}_o$  

dependent variable at node $(i, j)$ in difference eq. (E-3)

$U_{ij}$

unit load (eq. 11.3) = $2\bar{W}/\beta\left(1 - \bar{R}_i^2\right)$

$UL$

dependent variable in Reynolds equation (eq. (7))

$W$

pad load capacity

$W$

nondimensional load, $W/p_a\bar{r}_o^2$ (compressible); or $W/K\bar{r}_o^2$ (incompressible)

$\bar{W}$

tilt about a tangent line (roll)

$\alpha_\theta$

tilt about a radial line (pitch)

$\alpha_r$

angular extent of pad

$\beta$

pitch about pivot line

$\gamma$

finite difference increment

$\epsilon$

tilt parameter, $\gamma\bar{r}_o/h_o$

$\epsilon/H_2$

clearance parameter, $\gamma\bar{r}_o/h_2$

$\theta$

angular coordinate

$\Delta$

bearing compressibility number, $6\mu\omega\bar{r}_o^2/p_a h_2^2$

$\Omega$

relaxation factor

$\omega$

bearing shaft speed
APPENDIX B

NUMERICAL ANALYSIS

Numerical solutions of the Reynolds equations (7) to (10) are obtained by discretizing the partial differential equations and solving the resulting set of difference equations by an iterative technique over the closed region $\Sigma$ of the sector pad area.

The sector pad is partitioned into a polar mesh (fig. 2(a)) with an even number of increments in both coordinates, resulting in an odd number of nodes. A variable mesh capability is provided for the liquid lubricant cases. The finest mesh starts at the pad boundary (fig. 2(b)). The enlarged view of the central difference mesh is shown in figure 3.

In central difference formulations, the dependent variable $U_{i,j}$ at node $(i,j)$ is a function of $U$ at the four surrounding nodes. The central difference operators for the Reynolds equations (7) to (10) are

\[
\frac{\partial^2 U}{\partial R^2} \sim \frac{U_{i+1,j} - 2U_{ij} + U_{i-1,j}}{(\Delta R)^2}
\]

\[
\frac{1}{R^2} \frac{\partial^2 U}{\partial \theta^2} \sim \frac{U_{i,j+1} - 2U_{ij} + U_{i,j-1}}{R^2(\Delta \theta)^2}
\]

\[
\frac{\partial U}{\partial R} \sim \frac{U_{i+1,j} - U_{i-1,j}}{2 \Delta R}
\]

\[
\frac{\partial U}{\partial \theta} \sim \frac{U_{i,j+1} - U_{i,j-1}}{2 \Delta \theta}
\]

where

\[
\Delta \theta = \frac{\beta}{NA}
\]

\[
\Delta R = \frac{1 - RI}{NR}
\]

(B-2)

The errors in the finite difference operators (B-1) are all of the order $(\Delta R)^2$ and $(\Delta \theta)^2$. 
When these finite differences are substituted for the partial derivatives in the \( L \)-operator of equation (7), and the coefficient functions of the first order derivatives are expanded, the solution for \( U_{ij} \) in terms of the \( U \)'s at the four surrounding nodes is obtained:

\[
U_{ij} = \frac{A_{ij} \cdot U_{i+1, j} + B_{ij} \cdot U_{i-1, j} + C_{ij} \cdot U_{i, j+1} + D_{ij} \cdot U_{i, j-1} + F_{ij}}{E_{ij}}
\]  

(B-3)

where the coefficients are given in table B-1.

The solution of equation (B-3) proceeds from an initial estimate of the \( U_{ij}^{(1)} \) over the mesh \( 1 \leq i \leq NR + 1, 1 \leq j \leq NA + 1 \). It is always safe, if not efficient, to start with ambient pressure across the film mesh, that is,

\[
\begin{align*}
\rho_{ij} &= 1 \\
U_{ij}^{(1)} &= \frac{H_{ij}^2}{J} \quad \text{(compressible)} \\
U_{ij}^{(1)} &= 0 \quad \text{(incompressible)}
\end{align*}
\]  

(B-4)

Subsequent calculations of the \( U_{ij}^{(n+1)} \) continue from the preceding values of the \( n \)-th estimates \( U_{ij}^{(n)} \), using equation (B-3). In the Gauss-Seidel iterative procedure, the \( U_{ij}^{(n)} \) are immediately replaced in the storage array by \( U_{ij}^{(n+1)} \) upon calculation, which modifies equation (B-3) to

\[
U_{ij}^{(n+1)} = \frac{A_{ij} U_{i+1, j}^{(n)} + B_{ij} U_{i-1, j}^{(n+1)} + C_{ij} U_{i, j+1}^{(n)} + D_{ij} U_{i, j-1}^{(n+1)} + F_{ij}}{E_{ij}}
\]  

(B-5)

over the domain \( 2 \leq i \leq NR, 2 \leq j \leq NA \).

It is noted from table B-1 that for the compressible films the coefficients \( C_{ij} \) and \( D_{ij} \) contain the nonlinear term \( \sqrt{U_{ij}} \) by way of \( G_{ij} \). This creates no problem as the preceding iterate \( U_{ij}^{(n)} \) is always used in the \( G_{ij} \) for equation (B-5).

Relaxation of the basic iterative numerical procedure is used in this report to hasten convergence of the calculations. The relaxation method modifies equation (B-5) to the form
where the relaxation factor $\Omega$ is between 0 and 2. For $\Omega = 1$, the regular Gauss-Seidel form of equation (B-5) results.

Between successive iterations the change in the solution of equation (B-6) is denoted by

$$\text{TEST} = \left| U_{ij}^{(n+1)} - U_{ij}^{(n)} \right|$$

The largest value of TEST in one complete solution of equation (B-6) across the domain $\Omega$ is stored as STRERR. At the completion of each solution on $\Omega$, the maximum deviation or error between iterations is tested against a convergence criterion called RESIDL. When $\text{STRERR} < \text{RESHL}$ the numerical solution of equation (B-6) is defined as accomplished.

The pressure distribution over the pad domain $\Omega$ is determined from the last solution $U_{ij}$ after convergence using the definitions in equations (9) and (10):

$$P_{ij} = \frac{\sqrt{U_{ij}}}{H_{ij}} \quad \text{(compressible)}$$

$$P_{ij} = U_{ij} \quad \text{(incompressible)}$$

The bearing performance characteristics $\bar{W}$, $R_{cp}$, and $\theta_{cp}$ are expressed by equations (11), (13), and (14), respectively. The numerical integrations are accomplished using Simpson's 1/3 Rule, and are in two steps. The first carries out the radial integration

$$X_{F,j,K} = \frac{\Delta R}{3} \left\{ (P_{i,j} - 1)R_1^K - (P_{NR+1,j} - 1) 
+ \sum_{i=1}^{NR/2} \left[ 4(P_{2i,j} - 1)R_{2i}^K + 2(P_{2i+1,j} - 1)R_{2i+1}^K \right] \right\} \quad K = 1, 2$$

(B-9)
The circumferential integration step completes the calculation:

\[ \text{INT}_K = \frac{\Delta \theta}{3} \left[ X F_{1,K} - X F_{\text{NA}+1,K} + \sum_{i=1}^{\text{NA}/2} (4X F_{2i,K} + 2X F_{2i+1,K}) \right] \]

where

\[ \overline{W} = \text{INT}_{K=1} \]

and

\[ R_{cp} = \text{INT}_{K=2} \quad (B-10) \]

The \( \theta_{cp} \) calculation requires first

\[ AA = \frac{\Delta \theta}{3} \left[ -X F_{\text{NA}+1,2} \sin \beta + \sum_{i=1}^{\text{NA}/2} (4X F_{2i,2} \sin \theta_{2i} + 2X F_{2i+1,2} \sin \theta_{2i+1}) \right] \]

\[ (B-11) \]

and finally

\[ \theta_{cp} = \sin^{-1} \left( \frac{AA}{(W \cdot R_{cp})} \right) \quad (B-12) \]

The terms \( (P_{ij} - 1) \) in equation (B-9) are correct only for the compressible case, and represent gage pressures. These terms must be just \( P_{ij} \) for the incompressible solution as is derived from the second expression in equation (B-8).

The power loss coefficient, equation (17), and the leakage integrals, equations (18) to (21), require the evaluation of the first derivatives of pressure along the domain boundary \( \partial \Omega \). In order to achieve the same order of accuracy for the finite difference approximations on the boundaries as for the central difference expressions in equations (B-1), the following forward difference operation at the leading edge and inside radius are used:

\[ \left( \frac{\partial P}{\partial \theta} \right)_{\theta=0} \approx \frac{-3P_{i,1} + 4P_{i,2} - P_{i,3}}{2 \Delta \theta} \quad (B-13) \]

\[ R_i < R < 1 \]
At the trailing edge, and along the outside radius of the pad, the backward difference operators are used:

\[
\frac{\partial P}{\partial R} \bigg|_{\theta=\theta_j} \approx \frac{-3P_{i,1} + 4P_{2,j} - P_{3,j}}{2 \Delta R} \quad \text{(B-14)}
\]

As with the central difference operators (B-1), the forward and backward difference operators (B-13) to (B-16) are of the order \((\Delta R)^2\) and \((\Delta \theta)^2\) in error.

Since the pressures in the gradient expressions can be gage pressures, and since gage \(P_{ij} = 0\) on \(\partial \Sigma\), equations (B-13) to (B-16) can be further simplified to

\[
\frac{\partial P}{\partial \theta} \bigg|_{\theta=\theta_j} \approx \frac{4P_{i,2} - P_{i,3}}{2 \Delta \theta} \quad \text{on } R_1 < R < 1
\]

\[
\frac{\partial P}{\partial R} \bigg|_{\theta=\theta_j} \approx \frac{4P_{2,j} - P_{3,j}}{2 \Delta R} \quad \text{on } R = R_1
\]

\[
\frac{\partial P}{\partial \theta} \bigg|_{\theta=\theta_j} \approx \frac{P_{i,NA-1} - 4P_{i,NA}}{2 \Delta \theta} \quad \text{on } R_1 < R < 1
\]

\[
\frac{\partial P}{\partial R} \bigg|_{\theta=\theta_j} \approx \frac{P_{NR-1,j} - 4P_{NR,j}}{2 \Delta R} \quad \text{on } R = 1
\]
### TABLE B-1. COEFFICIENTS FOR CENTRAL DIFFERENCE APPROXIMATION OF REYNOLDS EQUATION (EQ. (B-3))

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Compressible films</th>
<th>Incompressible films</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_{ij}$</td>
<td>( \frac{1}{(\Delta R)^2} + \frac{1}{2 \cdot R \cdot H_{ij} \cdot \Delta R} )</td>
<td>( \frac{1}{(\Delta R)^2} + \frac{1}{2 \cdot R \cdot \Delta R} \left( 4 - \frac{3}{H_{ij}} \right) )</td>
</tr>
<tr>
<td>$B_{ij}$</td>
<td>( \frac{1}{(\Delta R)^2} - \frac{1}{2 \cdot R \cdot H_{ij} \cdot \Delta R} )</td>
<td>( \frac{1}{(\Delta R)^2} - \frac{1}{2 \cdot R \cdot \Delta R} \left( 4 - \frac{3}{H_{ij}} \right) )</td>
</tr>
<tr>
<td>$C_{ij}$</td>
<td>( \frac{1}{R_i^2 (\Delta \theta)^2} - \frac{\varepsilon \cdot \cos(\theta_p - \theta_j)}{2 \cdot R \cdot H_{ij} \cdot \Delta \theta} - G_{ij} )</td>
<td>( \frac{1}{R_i^2 (\Delta \theta)^2} - \frac{3 \cdot \varepsilon \cdot \cos(\theta_p - \theta_j)}{2 \cdot R \cdot H_{ij} \cdot \Delta \theta} )</td>
</tr>
<tr>
<td>$D_{ij}$</td>
<td>( \frac{1}{R_i^2 (\Delta \theta)^2} - \frac{\varepsilon \cdot \cos(\theta_p - \theta_j)}{2 \cdot R \cdot H_{ij} \cdot \Delta \theta} + G_{ij} )</td>
<td>( \frac{1}{R_i^2 (\Delta \theta)^2} + \frac{3 \cdot \varepsilon \cdot \cos(\theta_p - \theta_j)}{2 \cdot R \cdot H_{ij} \cdot \Delta \theta} )</td>
</tr>
<tr>
<td>$E_{ij}$</td>
<td>( 2 \cdot \left[ \frac{1}{(\Delta R)^2} + \frac{1}{R_i^2 (\Delta \theta)^2} \right] )</td>
<td>( 2 \cdot \left[ \frac{1}{(\Delta R)^2} + \frac{1}{R_i^2 (\Delta \theta)^2} \right] )</td>
</tr>
<tr>
<td>$F_{ij}$</td>
<td>0</td>
<td>( \frac{R_i \cdot H_{ij}^2 \cdot \varepsilon \cdot \cos(\theta - \theta_j)}{H_{ij}^3} )</td>
</tr>
<tr>
<td>$G_{ij}$</td>
<td>( \frac{\Lambda \cdot H_{ij}^2}{2 \cdot H_{ij} \cdot \sqrt{U_{ij}} \cdot \Delta \theta} )</td>
<td>0</td>
</tr>
<tr>
<td>$H_{ij}$</td>
<td>( 1 + \varepsilon \cdot R_i \cdot \sin(\theta_p - \theta_j) )</td>
<td>( 1 + \varepsilon \cdot R_i \cdot \sin(\theta_p - \theta_j) )</td>
</tr>
</tbody>
</table>
APPENDIX C

FORTRAN PROGRAMS
Flat Sector Pad Thrust Bearing Program Number 1 -- gas lubricant.

C MAIN EXECUTIVE FOR GAUSS-SEIDEL ITERATION
C
C REAL LAMBDA, OMEGA
LOGICAL DEBUG, TABOUT, OLDQ, VARGRD

C COMMON /BLOGIC/DEBUG, TABOUT, OLDQ, VARGRD
COMMON /GEOM/ ANGR, THPR, DR, DA
DIMENSION EXIT(5), TRATIO(15), ERATIO(26)
DIMENSION VR(10), VBETA(10), VLMBDA(10)
NAMELIST /VARBLE / TRATIO, ERATIO, VR, VBETA, VLMBDA

1 FORMAT(3I6, F8.2, 2E8.1)
2 FORMAT(4L6)
3 FORMAT(5I1)
4 FORMAT(2X, 12H NO. OF ROWS = , I3, 16H NO. OF COLS = , I3, 26H MAX NO. 1 OF ITERATIONS = , I4, 25H RELAXATION PARAMETER = , F4.2, 19H RESIDUAL ERROR = , F10.7, 2X, 36H SMALLEST ALLOWED FILM THICKNESS HALT = , 3G10.4//)
5 FORMAT(2X, 39H THE MINIMUM FILM THICKNESS IS LESS THAN, 6G10.4//)

C
C READ (5, 1) NR, NA, ITERMX, OMEGA, HALT, RESIDL
READ (5, 2) DEBUG, TABOUT, OLDQ, VARGRD
READ (5, 3) NTHETA, NRATIO, NUMRI, NUMBET, NUMLB
READ (5, VARBLE)

C WRITE (6, 4) NR, NA, ITERMX, OMEGA, RESIDL, HALT
WRITE (6, 900) "HEADING PRINTOUT..." 
WRITE (6, 910) "INITIALIZE PRINT SWITCH..."

C DO 500 NRI = I, NUMRI
RI = VR(NRI)

C DO 400 NBE = I, NUMBET
BETA = VBETA(NBET)

C DO 300 NLMB = I, NUMLB
LAMBDA = VLMBDA(NLMB)

C DO 200 NTHETA = I, NTHETA
XXTHTP = TRATIO(NTHETA)
CALL EUCLID(NR, NA, RI, BETA, XXTHTP) ; CALL RETURNS GEOMETRIC PARAMS. KOUNT = 0

C IF (IPRINT.GT.0) GO TO 8
WRITE (6, 910)
GO TO 10

8 WRITE (6, 915) ; PRINT START TO TOP OF PAGE...
10 WRITE (6, 920)
WRITE (6, 930) RI, BETA, LAMBDA, XXTHTP
WRITE (6, 940)
WRITE (6, 950)

C DO 100 NFILM = I, NRATIO

C
CALL BEGIN2(EPS,HALI,HMIN,HRAT,ERAT,RI,S12) @FILM THICKNESS CALCS.. GO TO 15

12 WRITE(6,5)HMIN @ ERROR EXIT.. GO TO 100

15 HLMBDA=LAMBDA*HMIN**2 IF(KOUNT.GT.P)GO TO 21
CALL ARRAYS(NR,NA,EPS,KOUNT,RI,HLMBDA) @FINITE DIFF. COEFFS.
CALL RELAX(NR,NA,OMEGA,ITERMX,RESIDL) @SOLVE REYNOLDS EQUATION.
CALL TABULT(NR,NA,RI,EXIT,LAMBDA,HMIN) @INTEGRALS OF PRESSURE
OVER PAD AREA..

GO TO 22'

21 CALL RARRAY(EPS,KOUNT,HLMBDA) @ENTRY TO SUBROUTINE ARRAYS.
CALL RRELAX @EDIT TO THE GAUSS-SEIDEL ROUTINE.
CALL RTAR(EXIT,LAMBDAP,HMIN) @ENTRY TO INTEGRAL CALLS..

RR=EXIT(1)
AA=EXIT(2)
WW=EXIT(3)
FF=EXIT(4)
WUNIT=EXIT(5)

XCP=RR*SIN(ANGB*AA-THPR)
HCP=1.-EPS*XCP
H2CP=HMIN/HCP
HINVRS=1./HMIN
R3=ERAT
R7=FF/WW

WRITE(6,960)HRAT,HINVRS,R3,WW,WUNIT,FF,R7,H2CP,RR,AA,XCP

KOUNT=KOUNT+1

CONTINUE
IPRINT=IPRINT+1
CONTINUE
CONTINUE
CONTINUE
CONTINUE
CONTINUE

900 FORMAT(1H1/*40X,40HNUMERICAL RESULTS - GAS BEARING ANALYSIS/40X,
128HLEWIS RESEARCH CENTER (NASA)/**/) 
910 FORMAT(50X,17HSYSTEM PARAMETERS//15X,4H(P1),25X,4H(P2),28X,4H(P3),
127X,4H(P4)//)
915 FORMAT(1H1/*50X,17HSYSTEM PARAMETERS//15X,4H(P1),25X,4H(P2),28X,
14H(P3),27X,4H(P4)//)
920 FORMAT(10X,17HINNER/OUTER RADII,14X,17HBETWEEN PAD ANGLE,14X,14HBE
ARING NUMBER,14X,17HPIVOT ANGLE RATIO//14X,5HR1/RO,22X,14HBETA (DE
2GRETS),19X,7HLAMBDAP ,20X,11HTHE TAP/BETA//)
940 FORMAT(50X,17HNUMERICAL RESULTS//5X,4H(R1),8X,4H(R2),8X,4H(R3),8X,
      14H(R4),8X,4H(R5),8X,4H(R6),8X,4H(R7),8X,4H(R8),8X,4H(R9),8X,5H(R10
      2),7X,5H(R11))
950 FORMAT(5X,5H1/H2,7X,4H1/H2,5X,10HEPSILON/H?,3X,9HLOAD,WBAR,3X,9HU
      INIT-LOAD,2X,10HFRICTION,F,4X,6HF/WBAR,5X,8HH2/H(CP),5X,5HR(CP),5X,
      210HTHETA(CP)//,4X,5HX(CP)/117X,4HBETA//)
960 FORMAT(11(2X,G10.5)/)
C
STOP
END
SUBROUTINE EUCLID(NR, NA, RI, BETA, THRAT)

COMMON GEOM/ANGR, THPR, DR, DA

FACT=6.2831853/360.  @ 2 PI RADIANS/360 DEGREES...
ANGB=BETA*FACT  @ BETA IN RADIANS...
THPR=THRAT*ANGB  @ THETAP (PIVOT ANGLE) IN RADIANS...
DR=(1.-RI)/FLOAT(NR)  @ RADIAL INCREMENT...
DA=ANGB/FLOAT(NA)  @ ANGLE INCREMENT IN RADIANS...

RETURN

END
SUBROUTINE BEGIN2(E,HALT,HMIN,HRATIO,ERATIO,RI,S)

C
COMMON/GEOM/BTR,THPR,DR,DA
THETAP=THPR
BETA=BTR
RAD90=1.5707963  90 DEGREES IN RADIANS.
C
C..CALCULATE MINIMUM FILM RATIO AND BEARING NUMBER.
C.
C..REGION NUMBER 1....PIVOT ANGLE = THETAP .LT. 0 ...
10 IF(THETAP.GE.0) GO TO 20
   EPS=ERATIO/(1.-ERATIO*SIN(THETAP-BETA))
   HMIN=EPS/ERATIO
   HRATIO=(1.+EPS*RI*SIN(THETAP))/HMIN
   IF(HMIN.LT.HALT) RETURN 7
   GO TO 100
C
C..REGION NUMBER 2....THETAP .GE. ZERO AND .LE. BETA ...
20 IF(THETAP.GT.BETA) GO TO 30
   EPS=ERATIO/(1.-ERATIO*SIN(THETAP-BETA))
   HMIN=EPS/ERATIO
   HRATIO=(1.+EPS*SIN(THETAP))/HMIN
   IF(HMIN.LT.HALT) RETURN 7
   GO TO 100
C
C..REGION NUMBER 3....THETAP .GT. BETA AND .LT. 90 DEGREES ...
30 IF(THETAP.GT.RAD90) GO TO 40
   EPS=ERATIO/(1.-ERATIO*RI*SIN(THETAP-BETA))
   HMIN=EPS/ERATIO
   HRATIO=(1.+EPS*RI*SIN(THETAP))/HMIN
   IF(HMIN.LT.HALT) RETURN 7
   GO TO 100
C
C..REGION NUMBER 4....THETAP .GT. 90 DEGREES ...
40 EPS=ERATIO/(1.-ERATIO*RI*SIN(THETAP-BETA))
   HMIN=EPS/ERATIO
   HRATIO=(1.+EPS)/HMIN
   IF(HMIN.LT.HALT) RETURN 7
C
100 CONTINUE
E=EPS
RETURN
C
END
SUBROUTINE ARRAYS(NR,NA,EPS,KK,RI,HLMBDA)
C
C..SUBPROGRAM CALCULATES VALUES OF THE NODAL COEFFICIENTS AND THE INITIAL
C..VALUES OF THE DEPENDENT VARIABLE FOR THE FIRST ITERATION
C
LOGICAL DEBUG,OLDQ
C
COMMON/GEOM/ANGR,THPR,DR,DA
COMMON/BLKA/A(15,21)/BLKB/B(15,21)/BLKC/C(15,21)/BLKD/D(15,21)
COMMON/BLKE/E(15,21)/BLKG/G(15,21)/BLKH/H(15,21)/BLKQ/Q(15,21)
COMMON/BLKR/R(15)/BLKTH/TH(21)
COMMON/BLOGIC/DEBUG,TABOUT,OLDQ,VARGND
C
LASTR=NR+1
LASTA=NA+1
CR1=.5/DR
CR2=1./DR**2
CA1=.5/DA
CA2=1./DA**2
GO TO 5
C
ENTRY RARRAY(EPS,KK,HLMBDA)

5

DO 2 JA=1,LASTA

RAD=RI
TH(JA)=THETA
ANG=(THPR-THETA)" PIVOT LINE IN RADIANS
STRIG=EPS*SIN(ANG)
CTRIG=EPS*COS(ANG)

DO 1 JR=1,LASTR

R(JR)=RAD
HRA=1.*RAD*STRIG

IF(KK.EQ.0) GO TO 6
IF(.NOT.OLDQ) GO TO 6
IF(JA.EQ.0) OR JA.EQ.NAP1) GO TO 6
IF(JR.EQ.0) OR JR.EQ.NRPI) GO TO 6

HRATIO=HRA/H(JR,JA)
Q(JR,JA)=Q(JR,JA)*HRATIO**2
GO TO 7

6

Q(JR,JA)=HRA**2
H(JR,JA)=HRA
FRST=1./RAD-STRIG/HRA
SCND=CTRIG/(RAD*HRA)
CA2RAD=CA2/RAD**2

A(JR,JA)=CR2+CR1*FRST
B(JR,JA)=CR2-CR1*FRST
C(JR,JA)=CA2RAD+CA1*SCND
D(JR,JA)=CA2RAD-CA1*SCND
E(JR,JA)=2.*(CR2+CA2RAD)

GO TO 5


```plaintext
G(JR, JA) = HLMDBA*CA1/HRA

C
IF(.NOT.DEBUG) GO TO 4
WRITE(6, 3) JR, JA, THETA, RAD  @BEGIN DEBUG SEARCH
WRITE(6, 3) JR, JA, STRIG, CTRIG
WRITE(6, 3) HRA, Q(JR, JA), A(JR, JA), B(JR, JA), C(JR, JA), D(JR, JA), E(JR, JA), G(JR, JA)

1 CONTINUE
RAD = RAD + DR
THETA = THETA + DA
CONTINUE

3 FORMAT( ) @FORMAT-FREE WRITE STATEMENT FOR DEBUG
RETURN
END
```
SUBROUTINE RELAX(NR, NA, OMEGA, ITERMX, RESIDL)

C GAUSS-SEIDEL ITERATION WITH CONVERGENCE WHEN LARGEST DIFFERENCE BETWEEN succeed for any matrix element is less than the initialized RESIDL

COMMON/BLKA/145,21)/BLKB/B(15,21)/BLKC/C(15,21)/BLKD/D(15,21)
COMMON/BLKE/E(15,21)/BLKG/G(15,21)/BLKH/H(15,21)/BLKQ/Q(15,21)
COMMON/BLKP/P(15,21)/BLKR/R(15)/BLKTH/T(21)
COMMON/BLOGIC/DFBUG,TABOUT,OLDQ,VARGRD

LOGICAL DEBUG,TABOUT
REAL OMEGA

C FORMAT FREE WRITE FOR DEBUG
C FORMAT(140X,23HRADIUS FROM INNER RADIUS//22X,11F10.3//)
C FORMAT(1HO,IX,6HTHETA=,F10.4t3X,2HP=,11F10.4//) C GO TO 40

ENTRY RRELAX

DO 300 KK=1,ITERMX
    STRERR=0.0
    DO 200 JJ=2,NA
        DO II=2,NR
            STORQ=O(II,JJ) ! Q-VALUE AT LAST ITERATION.
            GSQT=G(II,JJ)/SQRT(STORQ)
            CCOF=C(II,JJ)-GSQT
            DCOF=D(II,JJ)+GSQT
            Q(II,JJ)=(1.-OMEGA)*STORQ*OMEGA*(A(II,JJ)*Q(II+1,JJ)+
               B(II,JJ)*Q(II-1,JJ)+CCOF*Q(II,JJ+1)+
               DCOF*Q(II,JJ-1))/E(II,JJ)
            ERROR=Q(II,JJ)-STORQ
            IF(.NOT.DEBUG)GO TO 50 ! DEBUG SWITCH
            WRITE(6,3C)II,JJ,STORQ,ERROR,ST'RERR ! DEBUG TRACE
            TEST=ABS(ERROR)
            IF(TEST.LE.STRERR)GO TO 100 ! UPDATE LARGEST RESIDUAL
                STRERR=TEST
                ISTORE=II
                JSTORE=JJ
                QSTORE=Q(II,JJ)
                STRERR=TEST
                ISTORE=II
                JSTORE=JJ
                QSTORE=Q(II,JJ)
                CONTINUE
            CONTINUE
            CONTINUE
            IF(STRERR.LT.RESIDL)GO TO 400 ! UPDATE LARGEST RESIDUAL
            CONTINUE

C IF(.NOT.DEBUG)GO TO 400
WRITE(6,3C)KK,STRERR,QSTORE,ISTORE,JSTORE,RESIDL

400 CONTINUE

400 LASTR=NR+1
LASTA=NA+1
IF(DEBUG)WRITE(6,30)KK,STRERR,QSTORE,ISTORE,JSTORE @DATA AT END OF 
@ITERATIONS...

DO 460 JA=1,LASTA 
DO 450 JR=1,LASTR 
P_NORM(JR,JA)=SQRT(Q(JR,JA)/(Q(JR,JA)+H(JR,JA)**2))-1. @NORMALIZED PRESSURE 
@RELATIVE TO ATMOSPHERE

450 CONTINUE
460 CONTINUE

C

IF(.NOT.TABOUT)GO TO 500 @SKIP NORMALIZED 
@PRESSURE PRINTOUT

WRITE(6,35)(R(K),K=1,LASTR)

C

DO 470 L=1,LASTA 
WRITE(6,36)(TH(L),(PNORM(M,L),M=1,LASTR))

470 CONTINUE
500 CONTINUE

C

RETURN

END
SUBROUTINE TABULT(NR, NA, RI, EXIT, LAMBDA, HMIN)
C
C EXECUTIVE FOR INTEGRAL CALCULATIONS TRYING THE SIMPSON 1/3 RULE
C FOR ANGULAR COORDINATES...
C
REAL LAMBDA
COMMON/GEOM/ANGP, THPR, DR, DA
COMMON/BLKXXX/XF(21,3) ARRAY OF INTEGRATION RESULTS FROM SUBR. RSIMP
DIMENSION ODD(4), EVEN(4), END(4), EXIT(5)
C
NA2=NA/2-1
LASTA=NA+1
GO TO 1
C
ENTRY RTAB(EXIT, LAMBDA, HMIN)
1 CALL RSIMP(NR, NA, LAMBDA, HMIN) @RADIAL INTEGRATION SUBROUTINE
C
DO 5 I=1,4 @SET STORAGE VECTORS TO ZERO..
   END(I)=0.0
   EVEN(I)=0.0
   ODD(I)=0.0
5 CONTINUE
C
   END(1)=END(1)+XF(1,1)+XF(LASTA,1)
   END(2)=END(2)+XF(1,2)+XF(LASTA,2)
   END(3)=END(3)+XF(LASTA,2)*SIN(ANGB)
   END(4)=END(4)+XF(1,3)+XF(LASTA,3)
C
DO 10 JA=1, NA2 INTEGRATION ON INTERIOR ORDS.
   EVEN(1)=EVEN(1)+XF(2*JA,1)
   EVEN(2)=EVEN(2)+XF(2*JA,2)
   EVEN(3)=EVEN(3)+XF(2*JA,2)*SIN(2*JA-1)
   EVEN(4)=EVEN(4)+XF(2*JA,3)
   ODD(1)=ODD(1)+XF(2*JA+1,1)
   ODD(2)=ODD(2)+XF(2*JA+1,2)
   ODD(3)=ODD(3)+XF(2*JA+1,2)*SIN(2*JA)
   ODD(4)=ODD(4)+XF(2*JA+1,3)
10 CONTINUE
C
   EVEN(1)=EVEN(1)+XF(NA,1) BLAST EVEN ORDNATE
   EVEN(2)=EVEN(2)+XF(NA,2)
   EVEN(3)=EVEN(3)+XF(NA,2)*SIN(NA)
   EVEN(4)=EVEN(4)+XF(NA,3)
C
   WW=DA*(END(1)+4.*EVEN(1)+2.*ODD(1))/3. @LOAD INTEGRAL.
   RR=DA*(END(2)+4.*EVEN(2)+2.*ODD(2))/3. @RADIAL MOMENT.
   AA=DA*(END(3)+4.*EVEN(3)+2.*ODD(3))/3. @ANGLE MOMENT.
   FF=DA*(END(4)+4.*EVEN(4)+2.*ODD(4))/6. @FRICTION INTEGRAL.
C
   AA=(ASIN(AA/RR))/ANGB @ANGULAR C.P. COORDINATE.
   RR=RR/WW @RADIAL C.P. COORDINATE.
   WUNIT=2.*WW/(ANGB*(1.-RI**2)) @UNIT LOAD.
C
EXIT(1)=RR
EXIT(2)=AA
EXIT(3)=WW
EXIT(4) = FF
EXIT(5) = WUNIT

C
RETURN
END
SUBROUTINE RSIMP(NR, NA, LAMBDA, HMIN)

C***SIMPSON INTEGRATION METHOD ALONG PAD RADI.

REAL LAMBDA

COMMON/GEOM/ANGP,THPR,DR,DA
COMMON/BLKP/P(15,21)/BLKR/R(15)/BLKH/H(15,21)
COMMON/BLKX/ XF(21,3)
DIMENSION END(3), EVEN(3), ODD(3), CC(5, 3)
DATA CC = -1, 4, -3, 3, 0, 0, 1, 0, 0, -1, 3, 0, 0, 3, -4, 1

C***USAGE OF FUNCTION DEFINITIONS ..
F INTEGRAND FOR LOADS AND MOMENTS,
H2 RATIO OF FILM THICKNESS TO MINIMUM FILM THICKNESS,
X INTEGRAND FOR FRICTION MOMENT CALCULATIONS ..

DEFINE H2(JP, JA) = H(JR, JA)/HMIN
DEFINE F(K, JR, JA) = P(JR, JA)*R(JR)**K
DEFINE X(M, JR, JA) = (CC(1,M)*P(JR, JA+2)+CC(2,M)*P(JR, JA+1)+
1 CC(3,M)*P(JR, JA)+CC(4,M)*P(JR, JA-1)+
2 CC(5,M)*P(JR, JA-2))*R(JR)*H2(JR, JA)/(2.*DA+
3 LAMBDA*R(JR)**3/(3.*H2(JR, JA))

NEND = NR/2-1
LASTR = NR+1
LASTA = NA+1
M = 1
DO 50 JA = 1, LASTA
IF(JA.EQ.LASTA) M = 3
DO 10 I = 1, 3 SET STORAGE VECTORS TO ZERO..
END(I) = 0.0
EVEN(I) = 0.0
ODD(I) = 0.0
10 CONTINUE

END(1) = END(1) + F(1, I, JA) + F(1, LASTR, JA)
END(2) = END(2) + F(2, I, JA) + F(2, LASTR, JA)
END(3) = END(3) + X(M, I, JA) + X(M, LASTR, JA)

DO 40 JJ = 1, NEND
EVEN(1) = EVEN(1) + F(1, 1, JJ, JA)
EVEN(2) = EVEN(2) + F(2, 1, JJ, JA)
EVEN(3) = EVEN(3) + X(M, 1, JJ, JA)
ODD(1) = ODD(1) + F(1, 2, JJ+1, JA)
ODD(2) = ODD(2) + F(2, 2, JJ+1, JA)
ODD(3) = ODD(3) + X(M, 2, JJ+1, JA)
40 CONTINUE

EVEN(1) = EVEN(1) + F(1, NR, JA) LAST EVEN ORDINATE
EVEN(2) = EVEN(2) + F(2, NR, JA)
EVEN(3) = EVEN(3) + X(M, NR, JA)
DO 30 II=1,3
   XF(JA,II)=DR*(END(II)+4.*EVEN(II)+2.*ODD(II))/3.
30   CONTINUE

C
M=2
50   CONTINUE
C
RETURN
END
TPF$10$.ELT

SAMPLE DATA INPUT AS READ BY SUBROUTINE "MAIN"

*CARD 1 - FORMAT 3I6,F8.2,2E8.1
NR - NUMBER RADIAL MESH INCREMENTS,
NA - NUMBER ANGULAR MESH INCREMENTS,
ITERMX - MAXIMUM ITERATIONS IN GAUSS-
SEIDEL ROUTINE IF CONVERGENCE
FAILS,
OMEGA - RELAXATION FACTOR IN GAUSS-
SEIDEL METHOD,
HALT - MIN LUBRICANT FILM THICKNESS,
RESIDL - CONVERGENCE CRITERION - TEST ON
MAXIMUM CHANGE IN Q-VARIABLE
BETWEEN ITERATIONS.

<table>
<thead>
<tr>
<th>NR</th>
<th>NA</th>
<th>ITERMX</th>
<th>OMEGA</th>
<th>HALT</th>
<th>RESIDL</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>12</td>
<td>400</td>
<td>1.25</td>
<td>.1E-3</td>
<td>.1E-4</td>
</tr>
</tbody>
</table>

*CARD 2 - FORMAT 4L6
DEBUG (*TRUE = PRINTOUT OF COMPLETE ARRAY OF
DEPENDENT Q-VARIABLE FROM
GAUSS-SEIDEL ROUTINE AT EACH
ITERATION),
TABOUT (*TRUE = PRINTOUT OF PRESSURE ARRAY
OVER PAD AREA AT
CONVERGENCE),
OLDQ (*TRUE = USE CONVERGED VALUES OF Q-
VARIABLE AS STARTING ESTIMATES
FOR NEW CALCULATIONS WITH
DIFFERENT INITIAL CONDITIONS),
VARGRD (*TRUE = CREATES VARIABLE MESH GRID
OVER SECTOR PAD FOR LIQUID
FILM CALCULATIONS).

'BUG TAB' OLD' VAR'D
FALSE FALSE TRUE TRUE

*CARD 3 - FORMAT 5I10
**INDICES FOR PARAMETER ARRAYS IN NAMELIST
INPUT "VARBLE"...

<table>
<thead>
<tr>
<th>NTHETA</th>
<th>NRATIO</th>
<th>NUMRI</th>
<th>NMBET</th>
<th>NUMLMB</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>20</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

*CARDS 4,5,...
**DATA INPUT FOR NAMELIST "VARBLE"...
***DESCRIPTION OF INPUT PARAMETER ARRAYS...

NUMBER OF PARAMETER PARAMETER DESCRIPTION
DATA ARRAY


| NTHETA | TRATIO | PAD PIVOT ANGLE, THETAP/BETA |
| NRATIO | ERATIO | PAD SLOPE FACTOR, EPSILON/HMIN |
| NUMRI | YRI | PAD INNER RADIUS, RI |
| NUMBET | VBETA | PAD ANGLE DIMENSION, BETA |
| NUMLMB | VLMDBA | BEARING NUMBER, LAMBDA |

---

### Variable

$\text{TRATIO}=1.0$, $\text{VPI}=0.5$, $\text{VBETA}=45.0$, $\text{VLMDBA}=50.0$

$\text{ERATIO}=5.1,1.5,2.5,3.5,4.5,5.5,6.5,7.5,8.5,9.5,10.0$

SEND
FLAT SECTOR PAD THRUST BEARING PROGRAM NUMBER 2 -- LIQUID LUBRICANT

**TPFS(0).MAIN2**

C** MAIN EXECUTIVE FOR GAUSS-SEIDEL ITERATION..LIQUID LUBRICANT...**

C LOGICAL DEBUG, TABOUT, OLDQ, VARGRD

C COMMON/BLOGIC/DFBUG, TABOUT, OLDQ, VARGRD
COMMON/GEOM/ANGB, THPR, DR, DA
DIMENSION EXIT(9), TRATIO(15), ERATIO(26)
DIMENSION VRI(10), VBETA(10), VLMBOA(10)

C NAMELIST/ VARNLE/TRATIO, ERATIO, VRI, VBETA, VLMBOA

C 1 FORMAT(3I6, F8.2, 2E8.1)
2 FORMAT(4L6)
3 FORMAT(5I1G)
4 FORMAT(2X, 12H NO. OF ROWS =, I3, 16H NO. OF COLS =, I3, 26H MAX NO.
OF ITERATIONS =, I4, 2H RELAXATION PARAMETER =, F4.2, 19H RESIDUAL
ERROR =, F10.7, 38H SMALLEST ALLOWED FILM THICKNESS HALT =, 3G10.4//)
5 FORMAT(2X, 39HTHE MINIMUM FILM THICKNESS IS LESS THAN, G10.4//)

C C
C READ(5, 1) NR, NA, ITERMX, OMEGA, HALT, RESID
READ(5, 2) DEBUG, TABOUT, OLDQ, VARGRD
READ(5, 3) NTHETA, NRATIO, NUMRI, NUMBET
READ(5, VARNLE)

C WRITE(6, 4) NR, NA, ITERMX, OMEGA, RESID, HALT @ NUMERICAL SOLUTION
PRINTOUT...
WRITE(6, 900) @ A HEADING PRINTOUT...
IPRINT = 0 @ A INITIALIZE PRINT SWITCH...

C DO 500 NRI = 1, NUMRI @ INCREMENT INNER RADIUS VALUES...
RI = VRI(NRI)

C DO 400 NBETA = 1, NUMBET @ INCREMENT BETA VALUES... DEGREES.
BETA = VBETA(NBETA)

C DO 200 NANGLE = 1, NTHETA @ INCREMENT THETAP/BETA RATIO
XXTHP = TRATIO(NANGLE)
CALL EUCLID(NR, NA, RI, BETA, XXTHP, VARGRD) @ CALCULATES GEOMETRIC
PARAMETERS...

C COUNT = 0

IF(IWRITE(6, 910) GO TO 8
GO TO 10

8 WRITE(6, 915) @ PRINT START TOP OF PAGE...
WRITE(6, 920)
WRITE(6, 930) PI, BETA, XXTHP
WRITE(6, 940)
WRITE(6, 950)

C DO 100 NFILM = 1, NRATIO @ INCREMENT EPS/H2 RATIO
ERAT = ERATIO(NFILM)
CALL BEGIN2(EPS,HALT,HMIN,HRAT,ERAT,RI,%12) \*FILM THICKNESS \*CALCULATIONS.
GO TO 10

WRITE(6,5)HMIN \*ERROR EXIT.
GO TO 100

IF(KOUNT.GT.0)GO TO 21

THREE SUBROUTINES TO SET UP AND SOLVE THE FINITE DIFFERENCE
REYNOLDS EQUATION, AND RETURN INTEGRALS OF PRESSURE OVER PAD AREA.
CALL ARRAYS(NR,NA,EPS,KOUNT,RI,HMIN)
CALL RELAX(NP,NA,OMEGA,ITERMX,RESIDL,VARGRD)
CALL TABULT(NR,NA,HMIN,RI,EXIT,VARGRD,EPS)
GO TO 22

CALL RARRAY(EPS,KOUNT,HMIN) \*ENTRY TO SUBROUTINE ARRAYS.
CALL RRELAX \*DITTO FOR GAUSS-SEIDEL ROUTINE..
CALL RTAR(HMIN,EXIT) \*ENTRY TO INTEGRAL CALCS.

R1= ERAT \*RATIO EPS/HMIN
R2= HRAT \*RATIO H1/H2
R3= EXIT(7) \*UNIT LOAD
R4= EXIT(3)/EXIT(1) \*FRICITION/LOAD
R5= EXIT(4) \*RO SIDE LEAKAGE
R6= EXIT(5) \*RI SIDE LEAKAGE

**R7 IS TRAILING EDGE LEAKAGE,**
**R8 IS FLOW INTO LEADING EDGE...**
R7= EXIT(9)
R8= EXIT(8)

R9=(EXIT(2)—PI)/(1*-RI)
R10= EXIT(6) \*THETA C.P.
R11= EXIT(2)*SIN(ANGB*RI-THPR)

WRITE(6,960)R1,R2,R3,R4,R5,R6,R7,R8,R9,R10,R11

KOUNT=KOUNT+1
CONTINUE
IPRINT=IPRINT+1
CONTINUE

CONTINUE
CONTINUE
CONTINUE
CONTINUE
FORMAT(1H1//40X,43HNUMERICAL RESULTS - INCOMPRESSIBLE ANALYSIS/
140X,28HELVIS RESEARCH CENTER(NASA)///)
FORMAT(50X,17HSYSTEM PARAMETERS//15X,4H(P1),25X,4H(P2),28X,4H(P3),
127X,4H(P4)///)
FORMAT(1H1//50X,17HSYSTEM PARAMETERS//15X,4H(P1),25X,4H(P2),28X,
14H(P3),27X,4H(P4)///)
FORMAT(10X,17HINNER/OUTER RADII,14X,17HBEARING PAD ANGLE,14X,17HPI
I
VOT ANGLE RATIO//14X,HRI/RO,22X,14HBETA (DEGREES),19X,11HTHETAP/B
2ETA//)

930 FORMAT(15X,610.5,20X,610.5,20X,610.5//)
940 FORMAT(50X,17HNUMERICAL RESULTS//5X,4H(R1),8X,4H(R2),8X,4H(R3),8X,
14H(R4),8X,4H(R5),8X,4H(R6),8X,4H(R7),8X,4H(R8),8X,4H(R9),8X,5H(P10
2),7X,5H(R11)///)
950 FORMAT(1X,1HHEPSILON/H2,5X,5H1/H2,5X,9HUNIT-LOAD,4X,9HFRICTION/,
14X,7HR0 SIDE,4X,7HR1 SIDE,6X,4HEDGE,8X,6HINSIDE,3X,11H(R(CP)-RI)/,
22X,10HTHETA(CP)/,4X,5H(X(CP))/44X,4HLOAD,4X,7LEAKAGE,4X,7LEAKAGE,
35X,7LEAKAGE,7X,4HFLOW,7X,6H(1-RI),10X,4HBETA///)
960 FORMAT(11(2X,G1R.5)/)
970 FORMAT(3G15.5)

C
STOP
END
SUBROUTINE EUCLID(NR, NA, RI, BETA, THRAT, VARGRD)

C SUBPROGRAM PRODUCES COMMON BLOCK CONTAINING GEOMETRIC PARAMETERS...

C LOGICAL VARGRD
 COMMON/GEOM/ANGP, THPR, DR, DA, BLKR/R(23), DELR(23)
 COMMON/BLKTH/TH1(25), DELTH(25)/BLKX/XP(25), TSIN(25), TCOS(25)
 COMMON/INDEX/LN(23), LS(23), LE(25), LW(25)
 INTEGER HALFR, HALFA

C FACT=6.2831853/360.0  @ 2 PI RADIANS/360 DEGREES...
 ANGB=BETA*FACT  @BETA IN RADIANS...
 THPR=THRAT*ANGB  @THETAP (PIVOT ANGLE) IN RADIANS...
 DR=(1.-RI)/FLOAT(NR)  @RADIAL INCPEMENT...
 DA=ANGB/FLOAT(NA)  @ANGLE INCREMENT IN RADIANS...

C INTEGR = 1
 IF(VARGRD)INTEGR=9
 LASTR=NR+INTEGR
 LASTA=NA+INTEGR
 HALFR=NR/2+1
 HALFA=NA/2+1

C IF(VARGRD)GO TO 1
   DELTH(1) = DA
   DELTH(LASTA) = DA
   DELR(1) = DR
   DELR(LASTR) = DP
 GO TO 2

1   DR4 = .25*DR
    DA4 = .25*DA
    DR2 = 2.*DR4
    DA2 = 2.*DA4
    DELR(1) = DR4
    DELTH(1) = DA4
    DELR(LASTR) = DR4
    DELTH(LASTA) = DA4

2   R(1)=RI
    R(LASTR)=1
    TH(1)=0.0
    TH(LASTA)=ANGB
    XSIN(1) =0.0
    XSIN(LASTA)=SIN(ANGB)
    TSIN(1) = SIN(THPR)
    TSIN(LASTA)=SIN(THPR-ANGB)
    TCOS(1)=COS(THPR)
    TCOS(LASTA)=COS(THPR-ANGB)

C INDICES FOR VARIABLE MESH DIFFERENCE EQUATIONS...
 KPLUS=1
 MINUS=LASTR

C LE(1)=1
 LW(LASTA)=LASTA
LS(1) = 1
LN(LASTR) = LASTR

C DO 100 J = 1, HALFR
K = 1
DDR = DR
IF(.NOT.VARGRD) GO TO 30
IF(J .GT. 2) GO TO 30
GO TO (100, 200), J

C 10
K = 3
DDR = DR4
NOW = 1
GO TO 30

C 20
K = 2
NOW = 2
DDR = DR2

C DO 50 L = 1, K
IF(J .EQ. 3) NOW = 2
KPLUS = KPLUS + 1
MINUS = LASTR + 1 - KPLUS
LS(KPLUS) = KPLUS + NOW
LN(KPLUS) = KPLUS + 1
LS(MINUS) = MINUS - 1
LN(MINUS) = MINUS + NOW
DELR(KPLUS) = DDR
DELR(MINUS) = DDR
R(KPLUS) = R(KPLUS - 1) + DELR(KPLUS - 1)
R(MINUS) = R(MINUS + 1) - DELR(MINUS + 1)
NOW = 1
GO TO 50
CONTINUE

C 100 CONTINUE

C KPLUS = 1
MINUS = LASTA

C DO 200 J = 1, HALFA
K = 1
DDA = DA
IF(.NOT.VARGRD) GO TO 130
IF(J .GT. 2) GO TO 130
GO TO (110, 120), J

C 110
K = 3
DDA = DA4
NOW = 1
GO TO 130

C 120
K = 2
DDA = DA2
NOW = 2

C 130 DO 150 L = 1, K
IF(J.EQ.3)NOW=2
KPLUS=KPLUS+1
MINUS=LASTA+1-KPLUS
LW(KPLUS)=KPLUS-NOW
LE(KPLUS)=KPLUS+1
LE(MINUS)=MINUS+NOW
LW(MINUS)=MINUS-1
DELTTh(KPLUS) = DDA
DELTTh(MINUS) = DDA
TH(KPLUS) = TH(KPLUS-1)+DELTTh(KPLUS-1)
TH(MINUS) = TH(MINUS+1)-DELTTh(MINUS+1)
TEMP1 = TH(KPLUS)
TEMP2 = TH(MINUS)
XSIN(KPLUS)=SIN(TEMP1)
XSIN(MINUS)=SIN(TEMP2)
TSIN(KPLUS)=SIN(THPR-TEMP1)
TSIN(MINUS)=SIN(THPR-TEMP2)
TCOS(KPLUS)=COS(THPR-TEMP1)
TCOS(MINUS)=COS(THPR-TEMP2)
NOW=1

150 CONTINUE
C
200 CONTINUE
C
RETURN
END
*TPFS(0)\xbeg12
SUBROUTINE BEGIN2(E,HALT,HMIN,HRATIO,ERATIO,R1,S)
C
COMMON/GEOM/ANGB,THPR,DR,DA
THETAP=THPR
BETA=ANGB
RAD90=1.5707963  390 DEGREES IN RADIANS...
C
C..CALCULATE MINIMUM FILM RATIO AND BEARING NUMBER..
C
C..REGION NUMBER 1....PIVOT ANGLE = THETAP .LT. 0 ... 10  IF(THETAP.GE.0.0) GO TO 20
EPS=ERATIO/(1.-ERATIO*SIN(THETAP-BETA))
HMIN=EPS/ERATIO
HRATIO=(1.+EPS*R1*SIN(THETAP))/HMIN
IF(HMIN.LT.HALT) RETURN 7.
GO TO 100
C..REGION NUMBER 2....THETAP .GE. ZERO AND .LE. BETA ... 20  IF(THETAP.GT.BETA) GO TO 30
EPS=ERATIO/(1.-ERATIO*SIN(THETAP-BETA))
HMIN=EPS/ERATIO
HRATIO=(1.+EPS*R1*SIN(THETAP))/HMIN
IF(HMIN.LT.HALT) RETURN 7.
GO TO 100
C..REGION NUMBER 3....THETAP .GT. BETA AND .LT. 90 DEGREES ... 30  IF(THETAP.GT.RAD90)GO TO 40
EPS=ERATIO/(1.-ERATIO*R1*SIN(THETAP-BETA))
HMIN=EPS/ERATIO
HRATIO=(1.+EPS*R1*SIN(THETAP))/HMIN
IF(HMIN.LT.HALT) RETURN 7.
GO TO 100
C..REGION NUMBER 4.... THETAP .GT. 90 DEGREES .... 40  EPS=ERATIO/(1.-ERATIO*R1*SIN(THETAP-BETA))
HMIN=EPS/ERATIO
HRATIO=(1.+EPS)/HMIN
IF(HMIN.LT.HALT) RETURN 7.
C
100 CONTINUE
E=EPS
RETURN
C
END
SUBROUTINE ARRAYS(NR, NA, EPS, KK, RI, HMIN)

C..SUBPROGRAM CALCULATES VALUES OF THE NODAL COEFFICIENTS AND THE INITIAL
C..VALUES OF THE DEPENDENT VARIABLE FOR THE FIRST ITERATION. LIQUID LUBE
C
LOGICAL OLDQ, VARGRD
C
COMMON/BLKA/A(23, 25)/BLKB/B(23, 25)/PLKC/C(23, 25)/BLKD/D(23, 25)
COMMON/BLKE/E(23, 25)/BLKF/F(23, 25)/BLKH/H(23, 25)/BLKQ/Q(23, 25)
COMMON/BLKR/R(23), DELR(23)/BLKG/DLOGIC/DE6, TAB, OLDQ, VARGRD
COMMON/BLKE/T(25), DELT(25)/BLKIN/XSIN(25), TSIN(25), TCOS(25)
C
C
INTEGR=1
IF(VARGRD)INTEGR=9
LASTA=NA + INTEGR
LASTR=NR + INTEGR
GO TO 5
C
C
ENTRY RARRAY(EPS, KK, HMIN)
S
DO 2 JA=1, LASTA
DA=DELTH(JA)
CAI=.5/DA
CA2=1./DA**2
STRIG=EPS*TSIN(JA)
CTRIG=EPS*TCOS(JA)
C
DO 1 JR=1, LASTR
RAD=R(JR)
DR=DELR(JR)
CR1=.5/DR
CR2=1./DR**2
RAD2=RAD**2
HRA=1.*RAD*STRIG
C
IF(KK.EQ.0) GO TO 6
IF(.NOT.OLDQ) GO TO 6
IF(JA.EQ.1.OR.JA.EQ.LASTA) GO TO 6
IF(JR.EQ.1.OR.JR.EQ.LASTR) GO TO 6
GO TO 7
C
* 6 Q(JR, JA)=0.0
7 M(JR, JA)=HRA
RH = RAD*HRA
FRST = CR1*(4.*HRA-3.)/RH
SCND = 3.*CTRIG*CA1/RH
CA2RAD = CA2/RAD2
C
A(JR, JA)=CR2+FRST
B(JR, JA)=CR2-FRST
C(JR, JA)=CA2RAD-SCND
D(JR, JA)=CA2RAD+SCND
E(JR, JA)=2.*ICR2+CA2RAD
F(JR, JA)=CTRIG*RAD*HMIN**2/HRA**3
C

1 CONTINUE
2 CONTINUE
C

RETURN
END
SUBROUTINE RELAX(NR,NA,OMEGA,ITERMX,RESIDL,VARGRO)

C
C..GAUSS-SEIDEL ITERATION WITH CONVERGENCE WHEN LARGEST DIFFERENCE BETWEEN
C..SUCCESSIVE ITERATIONS FOR ANY MATRIX ELEMENT IS LESS THAN THE INITIALIZED
C..PARAMETER "RESIDL"
C
COMMON/BLKE/E(23,25)/BLKF/F(23,25)/PLKF/F(25,25)/BLKH/H(23,25)/BLKQ/Q(23,25)
COMMON/INDEX/LN(23),LS(23),LE(25),LW(25)
C
LOGICAL VARGRO
REAL OMEGA
C
INTEGR=1
IF(VARGRO)INTEGR=9
LASTR=NR+INTEGR-1
LASTA=NA+INTEGR-1
C
GO TO 40
C
ENTRY RRELAX
40 DO 300 KK=ITERMX
STRERR=0.0
DO 200 JJ=2,LASTA
IF=LE(JJ)
IW=LW(JJ)

DO 100 II=2,LASTR
IN=LN(II)
IS=LS(II)
STORQ=Q(II,JJ)
FIRST=(1.-OMEGA)*STORQ
RADIAL=A(II,JJ)*Q(IN,JI)+B(II,JJ)*Q(IS,JJ)
ANGULR=C(II,JJ)*O(II,IE)+D(II,JJ)*Q(IW,JJ)
Q(II,JJ)=FIRST+OMEGA*(RADIAL+ANGULR+F(II,JJ))/E(II,JJ)
ERROR=Q(II,JJ)-STORQ

100  TEST=ABS(ERROR)

C
IF(TEST.LE.,STRERR)GO TO 100
STERR=TEST
ISTORE=II
JSTORE=JJ
QSTORE=Q(II,JJ).
C
100  CONTINUE
200  CONTINUE
IF(STRERR.LT.,RESIDL)GO TO 400
C
300  CONTINUE
400  CONTINUE
C
RETURN
END
SUBROUTINE TABULT(NR, NA, HMIN, RI, EXIT, VARGRD, EPS)

C EXECUTIVE FOR INTEGRAL CALCULATIONS TRYING THE SIMPSON 1/3 RULE
C FOR ANGULAR COORDINATES...

C

COMMON/GEOM/ANGP, THPR, DR, DA
COMMON/BLKXXX/XF(25,6), INTEGRATION RESULTS FROM SUBR. RSIMP
COMMON/BLKTH/TH(25), DTH(25)/BLKXSIN/XY(25), TSIN(25), TCOS(25)
COMMON/INDEX/LN(23), LS(23), LE(25), LW(25)
DIMENSION EXIT(9), SUMMA(7)

LOGICAL VARGRD

FUNCTION DEFINITION FOR SIMPSON INTEGRATION RULE...

DEFINE AREA(J,K) = DTH(J+1) * (XF(J,K) + 4 * XF(J+1,K) + XF(J+2,K))
DEFINE TRIG(J) = DTH(J+1) * (XF(J,2) * XSIN(J) + 4 * XF(J+1,2) * XSIN(J+1) + XF(J+2,2) * XSIN(J+2))

INTEGR = 1
IF(VARGRD) INTEGR = 9
LASTA = NA + INTEGR
LASTR = NR + INTEGR
QUIT = (LASTA - 1) / 2
ANGULAR MIDPOINT...
FAC1 = 1 - RI**2
FAC2 = (1 - RI**3) * EPS

ENTRY RTAB(HMIN, EXIT)

GO TO 1

CALL RSIMP(LASTR, LASTA, HMIN)

DO 1 = 1, 7
SET STORAGE VECTORS TO ZERO...
SUMMA(I) = 0.0
CONTINUE

INDEX LIST FOR STORAGE VECTORS AND FOR RETURN VECTOR "EXIT"....
(1) CALCULATIONS FOR TOTAL LOAD...
(2) RADIAL MOMENT AND C.P. COORDINATE...
(3) PAD FRICTION...
(4) LEAKAGE FROM OUTER PAD ARC (R=1)...
(5) INNER " " (R=RI)...
(6) ANGULAR MOMENT AND C.P. COORDINATE...
(7) UNIT LOAD...
(8) FLOW INTO LEADING EDGE...
(9) TRAILING EDGE LEAKAGE...

DO 10 ISUM = 1, QUIT
SUM OVER PAIRS OF INTERVALS...
JFRD = 2 * ISUM - 1
ODD NODES...
DO 9 K = 1, 6
TEMPORARY STORAGE...
SUMMA(K) = SUMMA(K) + AREA(JFRD, K)
CONTINUE

SUMMA(7) = SUMMA(7) + TRIG(JFRD)
CONTINUE
DO 12 KKK=1,2
   LL=4*(KKK-1)
   MM=LL+1
   NN=MM+2
   DO 11 ILK=MM,NN
      ILKO=ILK-(KKK/2)
      EXIT(ILKO) = SUMMA(ILK)/3.
      11 CONTINUE
   12 CONTINUE
EXIT(6)=(ASIN(EXIT(6)/EXIT(2)))/ANGB
EXIT(2)=EXIT(2)/EXIT(1)
EXIT(3)=EXIT(3)/6.
EXIT(7)=2.*EXIT(1)/(ANGB*FAC1)
EXIT(8)=FAC3+FAC4*XSIN(LASTA)-XF(LASTA,4)
EXIT(9)=FAC3+FAC4*XSIN(LASTA)-XF(LASTA,4)
RETURN
END
SUBROUTINE RSIMP(LASTR,LASTA,HMIN)
C
C*SIMPSON INTEGRATION METHOD ALONG PAD RADII, LIQUID CASE*
C
COMMON/BLK0/P(23,25)/BLKH/H(23,25)/BLKR/R(23),DR(23)
COMMON/BLKTH/H(25)/BLKHT/H(25),DT(25)
COMMON/INDEX/LN(23),LS(23),LE(25),LW(25)
DIMENSION SUMMA(4),CC(5,3)
DATA CC/-1994.,-3.,-3*0.0,1.,0.0,-1.,3*0.0,3.,-4.,1.0/
C
C*USAGE OF FUNCTION DEFINITIONS*
C	F	INTEGRAND FOR LOAD AND MOMENTS,
C	H2	RATIO OF FILM THICKNESS TO MINIMUM FILM THICKNESS,
C	Y	PARTIAL DERIVATIVE OF PRESSURE WITH RESPECT TO THETA,
C	X	INTEGRAND FOR FRICTION MOMENT CALCULATIONS,
C	Z	"CALCULATION OF LEADING EDGE & TRAILING EDGE
C	FLUID LEAKAGE"
C
DEFINE H2(JR,JA)=H(JR,JA)/HMIN
DEFINE F(K,JR,JA)=P(JR,JA)*R(JR)**K
DEFINE Y(M,JR,JA,JE,JW)=CC(1,M)*P(JR,JE+1)+CC(2,M)*P(JR,JE)+
1 CC(3,M)*P(JR,JA)+CC(4,M)*P(JR,JW)+
2 CC(5,M)*P(JR,JW-1))/(2.*DTH(JA))
1 R(JR)**3/H2(JR,JA)
C
DEFINE AREA1(K,JR,JA)= DR(JR+1)*(F(K,JR,JA)+4.*F(K,JR+1,JA)+
1 F(K,JR+2,JA))
DEFINE AREA2(H,JR,JA,JE,JW)=DR(JR+1)*X(H,JR,JA,JE,JW)+
1 4.*X(H,JR+1,JA,JE,JW)+
2 X(H,JR+2,JA,JE,JW))
DEFINE AREA3(M,JR,JA,JE,JW)=DR(JR+1)*(Z(M,JR,JA,JE,JW)+
1 4.*Z(M,JR+1,JA,JE,JW)+
2 Z(M,JR+2,JA,JE,JW))
IQUIT=(LASTR-1)/2
M=1
DO 50 JA=1,LASTA
IF(JA.EQ.LASTA) M=3
C
DO 10 I=1,4
10 SUMMA(I)=0.0
C
CONTINUE
JE=LE(JA)
JW=LW(JA)
DO 20 ISUM=1, IQUIT
20 SUMA(1)=SUMMA(1)+AREA1(K,JR,JA)
C
CONTINUE
JR=2*ISUM-1
DO 15 K=1,2
15 SUMMA(K)=SUMMA(K)+AREA1(K,JR,JA)
49

SUMMA(3) = SUMMA(3) + AREA2(3, JR, JA, JE, JW)
IF(M.EQ.1. OR M.EQ.3) SUMMA(4) = SUMMA(4) + AREA3(M, JR, JA, JE, JW)
20 CONTINUE
C
DO 40 II = 1, 4
   XF(JA, II) = SUMMA(II) / 3
40 CONTINUE
C
XF(JA, 5) = 5 * (P(LASTR-1, JA) - P(LASTR-2, JA)) * H2(LASTR, JA) ** 3 / DR(LASTR-1)
1 XF(JA, 6) = 5 * P(1) * (-P(3, JA) + 4 * P(2, JA)) * H2(1, JA) ** 3 / DR(1)
C
C
M = 2
50 CONTINUE
C
C
RETURN
END
SAMPLE DATA INPUT AS READ BY SUBROUTINE 'MAIN'...

**CARD 1**-FORMAT 3I6,F8.2,2E8.1

NR —NUMBER RADIAL MESH INCREMENTS,
NA —NUMBER ANGULAR MESH INCREMENTS,
ITERMX —MAXIMUM ITERATIONS IN GAUSS-SEIDEL ROUTINE IF CONVERGENCE FAILS,
OMEGA —RELAXATION FACTOR IN GAUSS-SEIDEL METHOD,
HALT —MIN LUBRICANT FILM THICKNESS,
RESIDL —CONVERGENCE CRITERION — TEST ON MAXIMUM CHANGE IN Q-VARIABLE BETWEEN ITERATIONS.

<table>
<thead>
<tr>
<th>NR</th>
<th>NA</th>
<th>ITERMX</th>
<th>OMEGA</th>
<th>HALT</th>
<th>RESIDL</th>
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</thead>
<tbody>
<tr>
<td>14</td>
<td>16</td>
<td>400</td>
<td>1.25</td>
<td>1E-3</td>
<td>1E-4</td>
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</table>

**CARD 2**-FORMAT 4L6

DEBUG (.TRUE. = PRINTOUT OF COMPLETE ARRAY OF DEPENDENT Q-VARIABLE FROM GAUSS-SEIDEL ROUTINE AT EACH ITERATION),
TABOUT (.TRUE. = PRINTOUT OF PRESSURE ARRAY OVER PAD AREA AT CONVERGENCE),
OLDQ (.TRUE. = USE CONVERGED VALUES OF Q-VARIABLE AS STARTING ESTIMATES FOR NEW CALCULATIONS WITH DIFFERENT INITIAL CONDITIONS),
VARGRD (.TRUE. = CREATES VARIABLE MESH GRID OVER SECTOR PAD FOR LIQUID FILM CALCULATIONS).

*BUG TAB OLD VAR*D
FALSE FALSE TRUE TRUE

**CARD 3**-FORMAT 5I10

..INDICES FOR PARAMETER ARRAYS IN NAMELIST INPUT "VARBLE" ..

<table>
<thead>
<tr>
<th>NTHETA</th>
<th>NRATIO</th>
<th>NUMRI</th>
<th>NUMBET</th>
<th>NUMLMB</th>
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<tr>
<td>1</td>
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**CARDS 4, 5...**

..DATA INPUT FOR NAMELIST "VARBLE" ...

..DESCRIPTION OF INPUT PARAMETER ARRAYS ..

NUMBER OF PARAMETER DATA ARRAY PARAMETER DESCRIPTION
<table>
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<tr>
<td>NTHETA</td>
<td>Pad pivot angle, $\theta_{p/beta}$</td>
</tr>
<tr>
<td>NRATIO</td>
<td>Pad slope factor, $\epsilon_{s/\min}$</td>
</tr>
<tr>
<td>NUMRI</td>
<td>Pad inner radii, $R_i$</td>
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<tr>
<td>NUMBET</td>
<td>Pad angle dimension, $\beta$</td>
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<tr>
<td>NUMLMB</td>
<td>Bearing number, $\Lambda$</td>
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<tr>
<td>VRI</td>
<td>Pad inner radii, $R_i$</td>
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<tr>
<td>VBETA</td>
<td>Pad angle dimension, $\beta$</td>
</tr>
<tr>
<td>VLMBDA</td>
<td>Bearing number, $\Lambda$</td>
</tr>
</tbody>
</table>

**INPUT follows...**

```plaintext
$\text{VARBLE} \text{ TRATIO}=1.0, \text{ VRI}=5, \text{ VBETA}=45., \text{ VLMBDA}=50.,$
$\text{ ERATIO}=5, 1.5, 2, 2.5, 3, 3.5, 4, 4.5, 5, 5.5, 6, 6.5, 7, 7.5, 8,$
$8.5, 9, 9.5, 10.$

$\text{END}$
APPENDIX D

FLOW CHARTS

All FORTRAN symbols used in these flow charts are defined in appendix E.
Executive Program MAIN2

- **ENTRY**
- **READ DATA**
  - Defines operating conditions and pad dimensions
- **Call EUCLID**
  - Establishes finite difference mesh over pad area
- **Call BEGIN2**
  - Determines film lubricant dimensions
- **Call ARRAYS**
  - Calculates coefficient arrays for finite difference equation, and initial values for difference equation
- **Call RELAX**
  - Solves finite difference (Reynolds) equation for film pressures
- **Call TABULT**
  - Final numerical integration of double integrals for finding pad loading, friction drag, center-of-pressure coordinates, edge flows for liquid lubricants
- **Call RSIMP**
  - First (radial) numerical integration of double integrals over pressure field
BEGIN2

Test pivot angle relative to pad leading edge

Calculate geometric parameters:
\( \varepsilon, \frac{h_1}{h_2}, \frac{h_2}{h_0} \)

Is \( \frac{h_2}{h_0} \) less than a preset positive limiting value?

Yes
Write error message
ERROR RETURN

No
RETURN
EUCLID

No

Variable grid?

Yes

Generate uniform mesh nodes

Calculate: $\Delta R, \Delta \theta$
& arrays: $R_k$, $\theta_m$

Calculate central difference mesh indices for:
$\partial^2 P/\partial R^2$ & $\partial P/\partial R$,
$\partial^2 P/\partial \theta^2$ & $\partial P/\partial \theta$,
$R_1 \leq R_k \leq 1$,
$0 \leq \theta_m \leq \beta$

RETURN

Generate variable mesh nodes

Calculate: $\Delta R_k$
& array: $R_k$

Calculate central difference indices with variable mesh for:
$\partial^2 P/\partial R^2$ & $\partial P/\partial R$,
$R_1 \leq R_k \leq 1$

Calculate: $\Delta \theta_m$
& array: $\theta_m$

Calculate central difference indices with variable mesh for:
$\partial^2 P/\partial \theta^2$ & $\partial P/\partial \theta$,
$0 \leq \theta_m \leq \beta$
ARRAYS

Initialize pad dimensions & mesh nodes (variable grid?)

ENTRY RARRY

For:

\[ R_1 \leq R_k \leq 1; \quad 0 \leq \theta_m \leq \beta \]

Calculate local film thickness

\[ HRA = 1 + \varepsilon R_k \cdot \sin (\theta_p - \theta_m) \]

1st entry into subroutine?

Yes

1st guess at dependent variable:

- \( U_{km} = HRA^2 \) (gas)
- \( U_{km} = 0 \) (liquid)

No

Are we on pad boundaries?

Yes

\( U_{km} = HRA^2 \) (gas)
\( U_{jm} = 0 \) (liquid)

No

\( U_{km} \) from last case used as first guess in present case:

\[ U_{km} = \frac{(HRA/H_{km})}{U_{km}^{\text{last}}} \]
\[ U_{km} = U_{km} \] liquid

Calculate coefficient arrays:

- \( A_{km}, B_{km}, C_{km}, D_{km}, E_{km}, F_{km}, G_{km} \)

Update film thickness array:

\( H_{km} = HRA \)

RETURN
RELAX

Initialize pad dimensions & mesh nodes (variable grid?)
Reset iteration count KK=0

ENTRY RRELAX

For:
\[ R_{ik} \leq R \leq 1; \quad 0 \leq \theta_m \leq \beta \]

Gas film?

Yes

Linearize coefficients \( C_{km} \) & \( D_{km} \) at KKth iteration:
\[
CCOF = C_{km} - \frac{C_{km}}{u_{km}^{KK-1}}
\]
\[
DCOF = P_{km} + \frac{G_{km}}{u_{km}^{KK-1}}
\]

No

Solve finite difference Reynolds Equation at KKth iteration by Gauss-Seidel Method with relaxation factor for dependent variable:
\[ u_{km}^{KK} \]

Calculate the maximum error between iterations:
\[
ERROR = \max(u_{km}^{KK} - u_{km}^{KK-1})
\]

Is \( \text{abs}(ERROR) \) less than a preset test convergence value?

Yes

Calculate & store pressure array:
\[
P_{km}^\text{gas} = \frac{u_{km}^{KK}}{h_{km}}
\]
\[
P_{km}^\text{liquid} = u_{km}^{KK}
\]

RETURN

No

Proceed with next iteration
Initialize pad dimensions & mesh nodes (variable grid?)

ENTRY RTAB

Call RSIMP
Retrieve numerical values of radial integrands at $\theta_m$ nodes in arrays:

\[ X^K_{m} \]

Simpson numerical integrations:

\[ \text{EXIT}_K = \int_{0}^{\theta_m} X^K_{m} \, d\theta \]

for $K=1$: pad loading
2: radial center-of-pressure coordinate
3: angle center-of-pressure coordinate
4: friction dissipation

Yes

Liquid film?

Calculate edge flow rates at $R=R_\perp$ & $R=1$

No

RETURN
Initialize pad dimensions & mesh nodes

For:

\[ R_1 \leq R_m \leq 1; \ 0 \leq \theta_m \leq \beta \]

Simpson integration along radii; parameters \( \theta_m \)

\[ X_F^k_m = \int_{R_1}^{R_m} P_{km} R^n dR \]

- \( n=1 \) for pad loading
- \( n=2 \) for moment about origin

\[ X_F^3_m = \int_{R_1}^{R_m} (\text{shear stress}) dR \]

Liquid film?

Yes

\( X_F^0 \) & \( X_F^4 \) for edge leakage (flow)

at leading & trailing edge, resp.

\( X_F^5 \) & \( X_F^6 \) = radial pressure gradients

at \( R=R_1 \) & \( R=1 \), resp., for leakage flows at those radii

No

RETURN
APPENDIX E

FORTRAN SYMBOLS

A(15, 21)  coefficient array for $U_{i+1, j}$
AA        integral for $\theta_{op}$ calculation
ANG       difference $\theta_p - \theta$
ANGB      angle $\beta$ in radians
B(15, 21)  coefficient array for $U_{i-1, j}$
BETA      sector dimension $\beta$
C(15, 21)  coefficient array for $U_{i, j+1}$
CC(5, 3)   coefficient array for $\partial P/\partial \theta$ in fluid shear calculation
D(15, 21)  coefficient array for $U_{i, j-1}$
DA        $\Delta \theta$ of mesh
DEBUG     extensive printout logical switch; TRUE causes detailed output of iterative solution routine. FALSE prevents printout.
DR         $\Delta R$ of mesh
E(15, 21)  coefficient array for $U_{i, j}$
END(4)     storage array in numerical integration procedure
EPS        current value of clearance parameter $\epsilon$
ERATIO(25) input array of $\epsilon/H_2$ values
ERROR      difference between $U_{ij}$ values at $k^{th}$ iteration and $(k-1)^{st}$ iteration
EVEN(4)    storage array in numerical integration procedure
EXIT(5)    array of integration results from performance calculations
F(15, 21)  array of values for right hand side of nonhomogeneous Reynolds equation (liquid film)
F(K, I, J)  a DEFINE (local) function in radial integration subroutine for load (K=1) and moment (K=2) calculation
FF         integral of frictional energy dissipation
G(15, 21) array of factors for nonlinear part of coefficients to $U_{i, j+1}$ and $U_{i, j-1}$

H(15, 21) array of values of dimensionless film thickness $h/h_0$

HALT test value for smallest HMIN

HLMBDA $\Lambda H^2$

HMIN minimum film thickness ratio, $h_2/h_0$

HRATIO maximum-to-minimum film thickness ratio $h_1/h_2$

H2(I, J) a DEFINE function in RSIMP for ratio $h/h_2$

ISTORE \(i^{th}\) mesh position where occurs the maximum $U_{ij}$ change between successive iterations

ITERMX input value for maximum allowed number of iterations

JSTORE \(j^{th}\) mesh position where occurs the maximum $U_{ij}$ change between successive iterations

K index in DEFINE function $F(K, I, J)$

LAMBDA compressibility factor, $\Lambda$; sometimes called the "bearing number"

LASTA trailing edge boundary node

LASTR outer radial boundary node

LN(I), LS(I) finite difference indices for $\partial P/\partial R$ and $\partial^2 P/\partial R^2$ at $R(I)$; for variable mesh coding

LE(J), LW(J) finite difference indices for $\partial P/\partial \theta$ and $\partial^2 P/\partial \theta^2$ at $\theta(J)$; for variable mesh coding

M index in DEFINE function $X(M, I, J)$

NA number of angular mesh increments

NR number of radial mesh increments

NRATIO number of $\epsilon/H_2$ values input in ERATIO array

NTHETA number of $\theta_p/\beta$ values input in TRATIO array

NUMBET number of $\beta$ values input in VBETA array

NUMLMB number of $\Lambda$ values input in VLMBDA array
NUMRI  number of radius ratio values in VRI array
ODD(4)  storage array in numerical integration procedure
OLDQ  logical switch for use of previous numerical solution
array of \( u_{ij} \) as first guess for new calculation.
FALSE returns ambient pressure as initial guess.
OMEGA  iteration relaxation factor, \( \Omega \)
P(I, J), PNORM(I, J)  normalized lubricant film pressure relative to ambient
pressure
Q(I, J)  the working storage array for the \( u_{ij} \) in the compressible
gas program
QSTORE  value of the maximum change of \( u_{ij} \) for one iteration
over the entire sector pad domain
R(15)  array of values \( r/r_0 \) at the radial mesh nodes
\( I = 1(1) \text{LASTR} \)
RESIDL  input error limit on maximum allowed difference of \( u_{ij} \)
between successive iterations
RI  current value of radius ratios from VRI array
RR  integral of first moment for \( R_{cp} \)
STRERR  storage value of ERROR during search for maximum
error within one iteration pass
TH(21)  array of \( \theta_j \) values for all \( j \)-mesh
THPR  nodes \( J = 1(1) \text{LASTA} \)
TRATIO(10)  input array of \( \theta_p/\beta \) values
VARGRD  variable mesh logical switch; TRUE causes finer finite
difference mesh next to pad boundaries; FALSE utilizes
a constant mesh increment in either coordinate
VLMBDI(10)  input array of \( \Lambda \) values
VRI(10)  input array of radius ratio values, RI
WUNIT  unit load capability of bearing, i.e., total load WW per
unit area
WW  integral for bearing load capability
X(M, I, J)  a DEFINE (local) function in radial integration subroutine RSIMP for friction dissipation. M = 1 at J = 1, M = 2 for J = 2(1)LASTA-1, and M = 3 at J = LASTA

XF(21, K)  array of three functions from radial integration subroutine with values at each \( \theta_j \) mesh point. K = 1, 2, 3 correspond to load, center of pressure, and friction calculations, respectively. K = 4, 5, 6 correspond to edge leakage.
APPENDIX F

SAMPLE PROBLEMS

Computer output listings are presented for two representative runs, one for a compressible lubricant case and one for liquid lubricant. Input data cards for both cases are shown with the computer program listings in appendix C.

In both runs $\beta=45^\circ$, $\theta_p/\beta=1$, $R_1=.5$, and 20 values of $\epsilon/H_2=.5(.5)10$. In addition $\Lambda=50$, for the compressible case.

The execution of all cases by the compiled programs required less than 1 minute of computer time on the UNIVAC 1100/42.
### NUMERICAL RESULTS - GAS BEARING ANALYSIS
LEWIS RESEARCH CENTER (NASA)

#### SYSTEM PARAMETERS

<table>
<thead>
<tr>
<th>RI/RO</th>
<th>BEARING PAD ANGLE</th>
<th>BEARING NUMBER</th>
<th>PIVOT ANGLE RATIO</th>
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#### NUMERICAL RESULTS

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<tr>
<td>RI/RO</td>
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#FIN
### Numerical Results - Incompressible Analysis

**Lewis Research Center (NASA)**

#### System Parameters

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<th>Value 3</th>
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#### Numerical Results

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Note: The table continues with similar entries for other values of RI/RO.
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**FREE LIQUID.**
REFERENCES


Figure 1. - Geometry of sector pad.
Figure 2. Sector Pad Mesh Definition for Finite Difference Scheme.
Figure 3. Central Difference Mesh for Partial Derivatives.