ACOUSTOELASTICITY.

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PRINCETON UNIVERSITY

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ACOUSTOELASTICITY

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<tr>
<td>A</td>
<td>wall area</td>
</tr>
<tr>
<td>a</td>
<td>cavity length; panel length</td>
</tr>
<tr>
<td>$a_n$</td>
<td>velocity potential generalized coordinate; see equation III-26</td>
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<tr>
<td>b</td>
<td>cavity width; panel width</td>
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<tr>
<td>$C_{nr}$</td>
<td>see equation III-8A</td>
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<tr>
<td>$c_0$</td>
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<tr>
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<tr>
<td>$F_n$</td>
<td>$n^{th}$ acoustic natural mode</td>
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<tr>
<td>$I_A$</td>
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</tr>
<tr>
<td>$I_P$</td>
<td>plate action integral</td>
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<td>$L_{nm}$</td>
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<td>$\lambda$</td>
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<tr>
<td>$n$</td>
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<tr>
<td>$P_n$</td>
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<td>$T_P$</td>
<td>kinetic energy of plate</td>
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<td>time</td>
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</table>
\( \mathbf{u} \) - fluid particle displacement vector

\( V_A \) - acoustic potential energy

\( V_P \) - strain energy of plate

\( V \) - volume

\( W \) - work

\( W \) - wall amplitude

\( W_n \) - \( n^{th} \) acoustic mode wall deflection; see equation III-7

\( w \) - wall deflection

\( x, y, z \) - cartesian coordinates

\( Z \) - impedance

\( \xi \) - displacement of fluid

\( \phi \) - velocity potential

\( \gamma \) - ratio of specific heats

\( \lambda \) - see equation VI-33

\( \psi_m \) - \( m^{th} \) (structural) wall natural mode

\( \rho \) - density

\( \zeta_m \) - \( m^{th} \) structural mode (critical) damping ratio

\( \zeta_n^c \) - \( n^{th} \) acoustic cavity mode (critical) damping ratio

\( \omega \) - frequency

\( \Omega_m^E, \Omega_m^A \) - see equation VI-33

Subscripts

\( o \) - equilibrium

- perturbation
A  - absorbent wall; acoustic
F  - flexible wall
H  - Helmholtz resonator
m,n,r,s - modal numbers
R  - rigid wall

Superscripts
A  - acoustic
a,b,c - cavity a,b,c, respectively
E  - external
c  - acoustic cavity
*  - complementary
### Approximate Conversions to Metric Measures

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### Temperature (exact)

- **Fahrenheit**
  - 5°F (after freezing) = -15°C
  - 32°F (freezing) = 0°C

- **Celsius**
  - 0°C (freezing) = 32°F
  - 100°C (boiling) = 212°F

- **Temperature**
  - Convert: °C x 9/5 = °F - 32
I INTRODUCTION

We consider internal sound fields, i.e. sound or pressure variations inside of bounded enclosures. We proceed from the simplest geometry to the more complex. Specifically the interaction between the (acoustic) sound pressure field and the (elastic) flexible wall of an enclosure will be considered. One might think of this as the field of "acoustoelasticity". A good introduction to this subject is given in "Sound, Structures and their Interaction" by Junger and Feit\textsuperscript{1}. Also the author has briefly discussed this subject in his book, "Aeroelasticity of Plates and Shells"\textsuperscript{2}.

From the point of view of applications, such problems frequently arise when the vibrating walls of a transportation vehicle induce a significant internal sound field. The walls themselves may be excited by external fluid flows. Cabin noise in various flight vehicles and the internal sound field in an automobile are representative examples.

The first physical model to be considered is a simple one; but one which is famous in the acoustic literature.
II HELMHOLTZ RESONATOR

The reader may also wish to consult Kinsler & Frey, Fundamentals of Acoustics, pp. 187-207, on this topic.

Consider a "small" volume (any linear dimension small compared to the acoustic wavelength) so that we may consider all properties constant within the volume. For example, consider a box with one vibrating wall.

We wish to relate the wall motion to the cavity pressure change and compute the coupled structural-acoustic natural frequency.

From the isentropic gas law
\[ p = k \rho^\gamma \]  \hspace{1cm} (1)

and conservation of mass
\[ \rho V = \text{constant} \]  \hspace{1cm} (2)

Consider small changes denoted by \( \hat{\cdot} \) from an equilibrium condition denoted by \( \cdot_0 \):
\[ \rho = \rho_0 + \hat{\rho} \]
\[ p = p_0 + \hat{p} \]
\[ V = V_0 + \hat{V} \]  \hspace{1cm} (3)
Substituting (3) into (1) and (2), and linearizing in (\( \dot{\gamma} \)),

\[
\frac{\rho}{\rho_0} = \gamma \frac{\hat{p}}{\rho_0}
\]  

(4)

\[
\frac{\dot{\rho}}{\rho_0} + \frac{\dot{V}}{V_0} = 0
\]  

(5)

Combining (3) and (4)

\[
\frac{\dot{p}}{p_0} = -\gamma \frac{\dot{V}}{V_0}
\]  

(6)

Now

\[
V_0 = Ad \quad \text{where} \ A \ \text{is wall area}
\]

\[
\dot{V} = \int \int \ \text{wdx} \text{dy}
\]

Thus

\[
\frac{\dot{p}}{p_0} = -\gamma \int \int \ \text{wdx} \text{dy} \quad \frac{ad}{Ad}
\]  

(7)

This is essentially the "Helmholtz resonator" approximation. The equation of motion of the wall is thus

\[
m \frac{\partial^2 w}{\partial t^2} = -\frac{\gamma p_0}{Ad} \int \int \ \text{wdx} \text{dy}
\]  

(8)

We have ignored the stiffness of the wall (which is valid for large acoustical stiffness, e.g., small depths) and also ignored acoustical inertia (which is valid for low frequencies, e.g., large wavelengths).
One can approximately solve (8) using Galerkin's method.

Assume

\[ w = W \sin \frac{\pi x}{a} \sin \frac{\pi y}{b} \quad \text{A} = ab \]

Substitute above into (8), multiply by \( \sin \frac{\pi x}{a} \sin \frac{\pi y}{b} \) and integrate over the plate area. The result is

\[ m \frac{d^2 W}{dt^2} + \gamma P_0 \frac{W}{\pi^4} = 0 \quad \text{(10)} \]

Substituting \( W = \bar{W} e^{i\omega t} \), we may compute the natural frequency as

\[ \omega^2 = \frac{\gamma P_0}{m} \frac{64}{\pi^4} \quad \text{(11)} \]

Now the problem we have just analyzed is not the one that Helmholtz analyzed. Helmholtz considered a bottle with a neck as in the followings sketch:

![Diagram of a bottle with a neck](image)

Denote by \( \xi \) the displacement of the fluid in the neck of the bottle. This fluid is considered incompressible and is treated as a rigid body.

The equation of motion is
\[ \rho_o A^2 \frac{d^2 \xi}{dt^2} = \left[ -\gamma \frac{\rho_o A}{V_0} \right] \xi \]  
(12)

The natural frequency is

\[ \omega^2 = \frac{\gamma \rho_o A}{V_0 \rho_o \ell} \]  
(13)

\[ = \frac{c^2 A}{V_0 \ell} \]

It is this problem which is usually termed the "Helmholtz resonator".

One can also consider the problem of an applied external pressure, \( p^E \), acting on the neck. The equation of motion is

\[ \rho_o \ell^2 \frac{d^2 \xi}{dt^2} = -\gamma \frac{\rho_o A}{V_0} \xi + p^E \]  
(14)

For the particular case of simple harmonic motion where

\[ p^E = \overline{p^E} e^{i\omega t} \]

\[ \xi = \overline{\xi} e^{i\omega t} \]

one may obtain from (14),

\[ \frac{p^E}{\overline{\xi}} = \frac{-\rho_o \ell \omega^2 + \frac{\gamma \rho_o A}{V_0}}{1} \]

Thus an "impedance" may be defined as
\[ z_H \equiv \frac{F}{i \omega C} = \rho_0 \omega \Im + \frac{\gamma p_0 A}{V_0 i \omega} \]  

(15)

where \( i \omega C \) \equiv velocity amplitude and \( z_H \) is the impedance of the Helmholtz resonator.

The internal cavity pressure, \( p^c \), can be computed from the above for a sinusoidal external pressure as

\[
\frac{p^c}{p} = \frac{1}{\frac{\rho_0 \omega^2 V}{\gamma p_0 A} - 1}
\]

or

\[
\frac{1}{\frac{\omega^2 V}{A c_0^2} - 1}
\]

(16)

A similar expression can be obtained for the membrane example first considered. The significance of (16) for our purposes is that it shows that for

\[
\frac{\omega^2 V}{A c_0^2} < 2 \text{ then } \left| \frac{p^c}{p} \right| > 1
\]

Hence for low frequency excitations the internal cavity pressure will exceed the external pressure level.

It is emphasized that the perturbation pressure is assumed not to vary within the volume in this model. This is reasonable so long as the acoustic wavelength, \( \lambda = \frac{c_0}{\omega} 2\pi \), is much larger than any characteristic dimension of the volume. Also the fluid properties are assumed not to vary within the neck.
III COUPLED FLUID-STRUCTURAL MOTION OF AN ACOUSTIC CAVITY
WITH A FLEXIBLE AND/OR ABSORBING WALL.

We consider again the enclosed volume which was considered in connection with the Helmholtz resonator problem.

A direct and concise derivation of the acoustic pressure in a closed cavity due to arbitrary motion of the flexible cavity walls will be given. The pressure is expanded in terms of the normal modes of the rigid-walled cavity. The result, which is valid for any cavity geometry, is given in the form of a set of linear ordinary differential equations for the response of each normal mode. For the sake of completeness the equations of motion of the flexible wall are also derived in terms of structural normal modes and hence the complete coupled fluid-structural equations of motion are given.

Most studies of the acoustic pressure within a cavity due to motion of a flexible wall have considered only simple harmonic motion, although implicitly the more general case of arbitrary motion can be handled via the Fourier integral theorem. However, a very simple result for arbitrary wall motions can be obtained directly from the acoustic equations using Green's Theorem without resorting to transform methods.

The end result of the present method is an expansion for the acoustic pressure within the cavity due to motion of the walls, in terms of the normal modes of the cavity with its walls assumed to be rigid. The derivation applies equally to all cavity geometries, and as such is a generalization of the normal mode expansion (or "guided wave" expansion,
in the terminology used there) derived less directly and for rect-
angular cavities only in [7].

Acoustical Problem

Let the cavity occupy a volume \( V \), and be surrounded by a wall surface \( A \), of which the portion \( A_F \) is flexible, while the remainder \( A_R \) is rigid. If the fluid within the cavity is at rest prior to motion of the wall, the fluid pressure \( p \) satisfies the familiar wave equation, and associated boundary condition:

\[
\nabla^2 p - \frac{1}{c_0^2} \frac{\partial^2 p}{\partial t^2} = 0 \quad (1)
\]

\[
\frac{\partial p}{\partial n} = -\rho_0 \frac{\partial^2 w}{\partial t^2} \quad \text{on } A_F
\]

\[
= 0 \quad \text{on } A_R \quad (2)
\]

In these equations \( \rho \) and \( c \) are the equilibrium fluid density and acoustic velocity within the cavity, and \( w \) is the displacement of the flexible portion of the wall in the normal direction \( n \) (positive outward).

Equation (1) has normal mode solutions \( F_n e^{i\omega t} \), \( n = 0, 1, 2, \ldots \) with the following properties

\[
\nabla^2 F_n = -\left( \frac{\omega_n}{c_0} \right)^2 F_n \quad (3)
\]

\[
\frac{\partial F_n}{\partial n} = 0 \quad \text{on } A \quad (4)
\]

*This is the perturbation pressure \( \hat{p} \), of course. The \( \hat{\cdot} \) is dropped subsequently for convenience.
\[ \frac{1}{V} \int_V F_n F_r dv = 0 \quad r \neq n \quad (5) \]

\[ \omega_n^A is the n\textsuperscript{th} acoustical natural frequency and \( F_n^A \) its related natural mode. Superscript \( A \) is to distinguish \( \omega_n^A \) and \( M_n^A \) from their structural counterparts to be discussed in a later section.

Note that equation (3) has the solution \( \omega_0^A = 0, F_0^A = 1 \). All other frequencies \( \omega_n^A, n = 1, 2, \ldots \) are positive and non-zero, however.

A brief review of acoustical normal mode theory is given in Appendix A.

The wave equation (1) can be transformed into a set of ordinary differential equations in time by using Green's Theorem in the form:

\[ \int_V (p \nabla^2 F_n - F_n \nabla^2 p) dv = \int_A \left( \frac{\partial F_n}{\partial n} - F_n \frac{\partial p}{\partial n} \right) dA \quad (6) \]

By defining

\[ P_n \equiv \frac{1}{\rho V} \int_V p F_n dv \quad (7) \]

\[ W_n \equiv \frac{1}{A_F} \int_A \frac{\omega F_n}{A_F} dA \]

and making use of the fact that \( p \) and \( F \) satisfy equations (1) and (3), and boundary conditions (2) and (4), the following ordinary differential equations for the acoustic modes are obtained from (6):
\[ \ddot{P}_n + \omega_n^2 P_n = - \frac{\dot{W}_n}{\dot{V}} \]  

A dot (') denotes differentiation with respect to time. The quantities \( P_m(t) \) are the coefficients in an acoustical normal mode expansion for the pressure:

\[ P = \frac{p}{p_0 c_0^2} = \sum \frac{P_n}{M_n^A} F_n \]  

Since the normal modes \( F_n \) satisfy the homogeneous boundary condition (4) on the entire wall surface \( A \), the normal derivative of expression (9) does not converge uniformly on the flexible portion \( A_F \) of the wall surface. Expression (9) is suitable, however, for calculating the pressure itself throughout the cavity and everywhere on the wall surface, including the flexible portion.

When one of the walls of the cavity is highly absorbent it is usual to model it through a simple point-impedance model where it is assumed that

\[ P = Z_A \dot{w}_A \]  

on \( A_A \)

where the \( A \) subscript is used to refer to the absorbent wall characteristics, e.g.

\( w_A \) - absorbent wall displacement

\( z_A \) - absorbent wall impedance.

*Note that for \( n = 0 \), we have the Helmholtz resonator result, i.e.

\[ F_0 = 1 \]

\[ W_0 = \frac{\int w dA}{A_F} \]

\[ P_0 = - \frac{W_0 A_F}{\dot{V}} \]

\[ M_0^A = 1 \]

\[ P = \rho_0 c_0^2 p_0 = - \rho_0 c_0^2 \frac{\int w dA}{V} \]
The boundary condition
\[
\frac{\partial p}{\partial n} = -\rho \frac{\partial v}{\partial n} \quad \text{on } A_A
\] (2)
still applies, of course. Using (10), (2) may be written for the absorbing wall as
\[
\frac{\partial p}{\partial n} = -\rho \frac{\partial v}{\partial n} \quad \text{on } A_A
\] (2a)
Using the above in (6) along with (9), (8) becomes
\[
p_n + \omega_n A^2 p_n + A \rho c_0^2 \frac{\partial p}{\partial r} \frac{r}{r} \frac{C_{nr}}{M_r^A} \frac{\partial w}{\partial r} = -A_F w_n \quad (8a)
\]
where
\[
C_{nr} \equiv \int \int_{A_A} P_{n r} \frac{F_{F r} dA}{z_A} \frac{\partial w}{\partial r} \bigg|_{A_A}
\]
The effect of an absorbent wall is to couple all of the (rigid wall) acoustic modes. Of course, as we will see explicitly in what follows, the flexible wall also couples all of the acoustic modes as well.

**Structural Modal Expansion**

In many technical applications the flexible portion $A_f$ of the cavity wall may be a structural element, such as a plate or shell. In such cases the wall deflection $w$ is often expressed as a series of the form
\[
w = \sum_{m} q_m w_m \quad (11)
\]
in which the modal functions $\psi_m$ are defined over the region $A_F$, their properties being determined by structural considerations. Using (11) a set of structural modal equations describing the wall motion are derived. As will be shown subsequently, in these equations the cavity acoustic pressure, $p$, appears in the form of generalized forces $Q_m$:

$$Q_m = \int_{A_F} \psi_m p \, dA$$

(12)

First, however, the quantities $W_m$ appearing in (8) and (8a) can be expressed using (7) and (11) as

$$W_n = \sum_m L_{nm} q_m$$

(13)

where

$$L_{nm} = \frac{1}{A_F} \int_{A_F} F_n \psi_m \, dA$$

(14)

Hence, (8a) becomes the acoustic modal equation

$$\ddot{P}_n + \omega^2 P_n + A_F \rho_0 c^2 \sum_{r} \frac{P_r C_r}{M_r} \frac{m}{V} L_{nm} q_m$$

(15)

These can then be solved in conjunction with the companion structural equations to be derived in the next section.

**Structural Considerations**

When the structure may be represented by a linear structural model the total fluid-structural interaction may be treated in a simple way. Let the structure be represented by a linear (partial, differential) equation.
\( S w + m \frac{3^2 w}{\partial t^2} = p^c - p^E \)  \hspace{1cm} (16)

S is a linear differential operator, representing structural stiffness. For example, for an isotropic, flat plate, \( S = \nabla^4 \), where \( \nabla^4 \) is the biharmonic operator. The second term on the left-hand side is the structural inertia, \( m \) being structural mass per unit area. On the right-hand side we have two pressure loadings, the first due to the cavity acoustics, the second due to some specified "external" agent.

For simplicity, we assume that the structural modes, \( \psi_m \), (cf. (11)) are normal structural modes satisfying an eigenvalue equation

\[ S \psi_m - m \psi_m \omega_m^2 = 0 \]  \hspace{1cm} (17)

and associated orthogonality condition

\[ \int_{A_F} m \psi_m \psi_r \, dA = M_m \quad \text{for} \ m = r \]
\[ = 0 \quad \text{for} \ m \neq r \]  \hspace{1cm} (18)

\( \omega_m \) is the \( m \)th structural natural frequency and \( \psi_m \) its associated normal mode.

Substituting (11) into (16), using (17), gives

\[ \sum_m \left[ \frac{d^2 q_m}{dt^2} + \omega_m^2 q_m \right] \psi_m = p^c - p^E \]  \hspace{1cm} (19)

Multiplying through by \( \psi_n \) and integrating over \( A_F \) gives, using (18),

\[ \frac{d^2 q_m}{dt^2} + \omega_m^2 q_m \right] M_m = Q^c_m + Q^E_m \]  \hspace{1cm} (20)

where (recall (12))

\[ Q^c_m = \int_{p^c} \psi_m \, dA. \]  \hspace{1cm} (21)
From (9) and (21),

\[ Q_m^c = A_p \rho_0 c_0^2 \sum P_n \frac{L_{nm}}{M_n} \]  

Hence the structural modal equations, (20), may be rewritten using (23) as

\[ M_m \left[ q_m'' + \omega_m^2 q_m \right] = \rho_0 c_0^2 \sum P_n \frac{L_{nm}}{M_n} + Q_m^E \]  

Summarizing the key relations are (15) and (24) with \( Q_n^E \) determined from its definition, (22). (15) and (24) are coupled acoustical-structural ordinary differential equations which may be solved by any standard method. Since we normally are dealing with systems under forced motion, \( Q_n^E \), the initial conditions on \( q_m \) and \( P_n \) will usually be the trivial ones.

\[ q_m = \frac{dq_m}{dt} = P_n = \frac{dP_n}{dt} = 0 \quad \text{at} \ t = 0 \]

Of course, any physically meaningful initial conditions, could be accommodated.

It should be noted that in applications, the use of normal structural modes may not always be convenient and hence direct structural coupling may occur. Of course, this will always be true for a nonlinear structural model [7] since the concept of normal modes loses much of its significance there.

Finally, although the acoustic pressure is obviously the physical variable of the greatest interest, some additional insight into the nature of the structural-acoustic interaction can be obtained by
considering instead the acoustic velocity potential. The two are related through Bernouilli's equation

\[ p = - \rho_0 \dot{\phi} \]  

(25)

If \( \phi \) is expanded in its natural modes (they are obviously the same modes as for \( p \) from the above)

\[ \phi = \sum a_n F_n \]  

(26)

one can from (9), recall

\[ p = \rho_0 c_0^2 \sum \frac{P_n F_n}{M_n^A} \]  

(9)

(25) and (26), determine that

\[ c_0^2 \frac{P_n}{M_n^A} = - \frac{\ddot{a}_n}{M_n} \]  

(27)

Using (27) in (15) and (24), one obtains

\[ VM_n^A \left[ a_n + \omega_n^A \dot{a}_n \right] + A \rho_0 c_0^2 \sum r_i \dot{a}_r C_{nr} = c_0^2 A_F \sum_{nm} L_{nm} \dot{q}_m \]  

(15)*

\[ M_m [\ddot{q}_m + \omega_m^2 q_m] = - \rho_0 A_F \sum \dot{a}_n L_{nm} + Q_m \]  

(24)*

The underlined terms are the structural-acoustic coupling terms.

The coefficients are anti-symmetric (to within a multiplicative constant) and this characteristic, combined with the first time derivatives of
$q_m$ and $a_n$, means that the equations possess gyroscopic coupling. Thus it is known that (if $A_A \to 0$ so that there is no absorbing wall) the coupled structural - acoustic natural frequencies are real, i.e. the system without absorbing walls is an undamped resonator. Moreover Meirovitch\textsuperscript{8} (in a different physical context) has shown how one may use conventional matrix eigenvalue methods to determine the natural frequencies of gyroscopically coupled systems.
IV A VARIATIONAL FORMULATION

The same results may be obtained through a variational formulation. Previously Gladwell and later Craggs have considered variational statements in the context of finite element representations for the coupled structural (wall) - acoustic (cavity) problem. Craggs' work has been carried furthest and we use it as a point of departure for our discussion. Even though we shall obtain the same results as before, the variational statement in its present form is somewhat awkward. Hence the author's preference is for the previous formulation using Green's theorem for the cavity. Others may feel differently, however, or perhaps be inspired to develop a more elegant and/or simpler variational statement. Hence the following brief discussion. Following Craggs, consider an action integral for the plate

$$ I_p = \int_{t_1}^{t_2} \left[ T_p - V_p + (W_E - W_A) \right] \, dt $$

(28)

where

$$ T_p = 1/2 \int \int m \left( \frac{\partial w}{\partial t} \right)^2 \, dx \, dy \quad \text{Plate Kinetic Energy of Flexible Wall} $$

$$ V_p = \frac{D}{2} \int \int \left[ \left( \frac{\partial^2 w}{\partial x^2} \right)^2 + \left( \frac{\partial^2 w}{\partial y^2} \right)^2 + 2\nu \left( \frac{\partial^2 w}{\partial x \partial y} \right)^2 + 2(1-\nu) \left( \frac{\partial^2 w}{\partial x^2} \right) \right] \, dx \, dy \quad \text{Strain Energy of Wall} $$

$$ \delta W^E = - \int \int p^E \delta w \, dx \, dy \quad \text{Virtual work due to External pressure} $$

$$ \delta W^A = \int \int p^A \delta w \, dx \, dy \quad \text{Virtual work due to Acoustic Cavity pressure} $$

(29)

or

$$ \delta W^A = - \int \rho_0 \phi \delta w \, dx \, dy $$

Bernoulli's relation has been used to obtain the last line, i.e.

$$ \gamma^c = - \rho_0 \phi $$
Using Hamilton's Principle on \( I_p \) (with \( p^E \) and \( p^C \) not varied) one obtains

\[
m \ddot{w} + D \nabla^4 w = -p^E + p^C \quad (30)
\]

with appropriate boundary conditions on the plate edges involving \( w \) and its derivatives.

The action integral for the acoustic system with a prescribed motion at the cavity surface is

\[
I_A = \int_{t_1}^{t_2} \left( T_A - V_A - W_A^* \right) dt \quad (31)
\]

where \( \mathbf{u} \) - (vector) particle fluid displacement

\[
T_A = \frac{1}{2} \iiint \rho_0 \dot{\mathbf{u}} \cdot \mathbf{u} \, dx \, dy \, dz \quad \text{Acoustic Kinetic Energy}
\]

\[
V_A = \frac{1}{2} \iiint \frac{p^2}{\rho_0 c_0^2} \, dx \, dy \, dz \quad \text{Acoustic Potential Energy}
\]

\[
\delta W_A^* = \iint_\Omega \frac{\partial W_A^*}{\partial \delta p} \, dx \, dy
\]

\( \delta W_A^* \) is the complementary work done by the virtual forces (acting through the real displacements) at the structural-acoustic interface.

Introducing the velocity potential, \( \phi \),

\[
\dot{\mathbf{u}} = \nabla \phi
\]

and recalling Bernoulli's relation

\[
p^c = -\rho_0 \dot{\phi}
\]

the above may be written as

\[
T_A = \frac{1}{2} \iiint \rho_0 (\nabla \phi \cdot \nabla \phi) \, dx \, dy \, dz
\]
\[ V_A = \frac{1}{2} \iiint \frac{\rho_0}{c_0^2} \left( \phi^2 \right) \, dx \, dy \, dz \]  

\[ \delta W_A^* = - \iint w \rho_0 \delta \phi \, dx \, dy \]

Using Hamilton's Principle with respect to \( I_A \) (with \( w \) not varied) one obtains

\[ \nabla^2 \phi - \frac{1}{c_0^2} \dot{\phi} = 0 \]  

with the boundary condition

\[ \frac{\partial \phi}{\partial z} = \dot{w} \]  

on \( z = 0 \) (flexible wall)

As the attentive reader will already have noted, our use of cartesian coordinates is a matter of convenience and the results readily carry over to cavities and flexible walls of arbitrary shape.

The necessity for considering both virtual work, \( \delta W_A \), and complementary virtual work \( \delta W_A^* \) makes this variational statement appear somewhat cumbersome to the author and it is for this reason that the earlier approach using Green's Theorem is preferred.

It is worth noting that an absorbing wall in the cavity can also be included, specifically in \( \delta W_A^* \). We add a subscript, \( A \), to all quantities to denote absorbent. The complementary virtual work due to the absorbing wall is

\[ \delta W_{AA}^* = - \iint w_A \rho_0 \delta \phi \, dx \, dy \]

Realizing that in Hamilton's Principle the above will appear as
one may integrate by parts to obtain

\[ \int_{t_1}^{t_2} \delta W_{AA} \, dt = \iiint_A \rho_0 \delta \phi \, dx \, dy \, dt \]

Further if we model the absorbing wall through a simple point-impedance model then

\[ P = z_A w_A \]

or

\[ - \rho_0 \dot{\phi} = z_A w_A \quad \text{on} \quad \partial A \]

where \( z_A \) - absorbent wall impedance

\[ \frac{\rho_0^2}{z_A} \]

Thus

\[ \int_{t_1}^{t_2} \delta W_{AA} \, dt = -\iiint_A \frac{\rho_0^2}{z_A} \dot{\phi} \, dx \, dy \, dt \]

Using the above and proceeding through the formalism of Hamilton's Principle one obtains for the boundary condition on the absorbing surface (e.g. say \( z = d \))

\[ \frac{\partial \phi}{\partial z} = w_A \]

or

\[ \frac{\partial \phi}{\partial z} = - \frac{\rho_0^2}{z_A} \quad \text{on} \quad z = d \]

Modal Equations

Modal expansions are now made for \( w \) and \( \phi \).
\[ \omega = \sum q_m(t) \psi_m(x,y) \]
\[ \phi = \sum a_n(t) F_n(x,y,z) \]

It is simplest conceptually and generally most efficient computationally to choose the in vacuo plate modes for \( \psi_m \) and the rigid wall modes for \( F_m \). Other possibilities exist, of course, but they shall not be pursued here. Using the above choice, one has

\[ T_p = \frac{1}{2} \sum_{m} M_m \ddot{q}_m^2 \]
\[ V_p = \frac{1}{2} \sum_{m} M_m \omega_m^2 q_m^2 \]

where \( M_m = \int \int \psi_m^2 \, dx \, dy \) generalized plate mass

\[ \omega_m = m^{th} \text{ in vacuo natural plate frequency} \]

Here we have used the orthogonality of the \( \psi_m \), i.e.

\[ \int \int \psi_m \psi_n \, dx \, dy = 0 \quad \text{for} \quad \omega_m \neq \omega_n \]

Also to obtain \( V_p \) an integration by parts has been used along with the fact that \( \psi_m, \omega_m \) satisfy the Euler-Lagrange equation

\[ D^4 \psi_m - \omega_m^2 m \psi_m = 0 \]

and appropriate boundary equations for \( \psi_m \) on the plate edges.

One also has

\[ \delta W_E = \int \int \sum E_m q_m \delta q_m \]

where

\[ Q^E_m = - \int \int p^E \psi_m \, dx \, dy \]
and $\delta W_A = - A_F \rho \sum_{nm} \delta q_m L_{nm}$

where $L_{nm} = \iint_{A_F} F_n \psi_m \, dx \, dy$

$\int_{A_F} \, dx \, dy$

$A_F = \text{total area of (flexible) structural wall.}$

Using Hamilton's Principle one obtains, in the usual way, the structural modal equations

$-M_m [q_m + \omega_m^2 q_m] + Q_m^E - A_F \rho_0 \sum_{n} \delta a_n L_{nm} = 0 \tag{41}$

Turning now to the determination of the acoustic modal equations, one may construct

$T_A = \frac{1}{2} \sqrt{\rho_0 \omega_n^2 M_n a_n^2 c_n^2} \tag{42}$

$V_A = \frac{1}{2} \sqrt{\rho_0 \omega_n^2 M_n a_n^2 c_n^2}$

where $M_n^A = \iint_{V} F_n^2 \, dx \, dy \, dz$

$V = \text{total cavity volume, } \omega_n^A = \text{th natural (rigid wall) cavity frequency}$

and $\delta W^* = - \rho_0 A_F \sum_{nm} q_m \delta a_n^* L_{nm}$

$\int_{t_1}^{t_2} \delta W^*_{AA} \, dt = - A_F \rho_0 \sum_{n} \delta a_n C_n \delta a_n \, dt$
where \( C_{nr} \equiv \iint_{A_r} F_n \frac{F_r}{Z_A} \, dx dy \) on \( A_r \)

and \( A_A \equiv \) total area of absorbing surface

In constructing \( T_A \) and \( V_A \), we have used the orthogonality of \( F_m \), i.e.

\[
\iint_{A_r} F_r F_n \, dx dy dz = 0 \quad \text{for} \quad \omega_n^A \neq \omega_r^A
\]

and the fact that \( F_n, \omega_n^A \) satisfy the Euler-Lagrange equation,

\[
\nabla^2 F_n + \frac{\omega_n^A}{c_0^2} F_n = 0
\]

and appropriate boundary conditions on the cavity walls.

Using Hamilton's Principle one obtains

\[
\sum_{m} A_m \left[ a_n^m + \omega_n^A a_n^m \right] - c_0^2 A_m \sum_{m} \dot{q}_m \dot{L}_{nm} + \rho_0 c_0^2 A_A \sum_{r} \dot{a}_r C_{nr} = 0
\]

Equations (41) and (43) are the same as equations (24)* and (15)* which were obtained previously.

V ACOUSTIC NATURAL MODES IN MULTIPLY CONNECTED CAVITIES

For definiteness we consider two rectangular cavities although it will be clear that the basic method is applicable to fairly general geometries. See sketch.

Our interest here is in the acoustic natural modes of the two cavities connected by an opening of area $A_F$. In this opening there is, in general, a flexible structural member which deforms (as indicated by the dashed line) and thus permits sound transmission from one cavity to the next. All other portions of the cavity walls are rigid. As before the structural member has a displacement, $w$, with a modal expansion

$$w = \sum_m q_m(t) \psi_m(y,z)$$  \hspace{1cm} (1)

and the acoustical pressures in cavities a and b have modal expansions

$$p_a = \rho_0 c_0^2 \sum_n \left[ \frac{2 \sum_n p_n^a f_n^a(x,y,z)}{M_n^a} \right]$$

$$p_b = \rho_0 c_0^2 \sum_n \left[ \frac{\sum_n p_n^b f_n^b(x,y,z)}{M_n^{Ab}} \right]$$  \hspace{1cm} (2)
The modal equations of motion for the acoustic pressures are then

\[ p_n^a + \omega a^2 p_n^a = - \frac{A_F}{V_a} \sum m q_m L_n^a \]

\[ p_n^b + \omega b^2 p_n^b = + \frac{A_F}{V_b} \sum m q_m L_n^b \]

and for the structural wall

\[ M_m \left[ q_m + \omega^2 q_m \right] = \rho_0 c_0^2 A_F \sum n p_n^a L_n^a \]

\[ - \rho_0 c_0^2 A_F \sum n p_n^b L_n^b \]

\[ M_{ab} \]

The + sign in the second of (3) and the - sign in the right hand side of (4) is a result of the normal to the opening being outward from cavity a and inward to cavity b.

As in Sections III and IV one may introduce a velocity potential through Bernoulli's equation

\[ p_{a,b} = - \rho_0 \phi_{a,b} \]

with a modal expansion

\[ \phi_{a,b} = \sum n a_{a,b}^n \phi_n \]

(3) and (4) then become
(7) and (8) are in standard form for use of Meirovitch's algorithm\(^8\) to determine the natural frequencies (eigenvalues) and modes (eigenvectors) of the two coupled cavities.

Of special interest is the case with a pure opening between the two cavities, i.e. one with zero mass and stiffness. In this case substantial further simplifications are possible. Thus

\[ M_m = \omega_m = 0 \] and (4) becomes (one could deal alternatively with (7) and (8))

\[ \sum_n \frac{p_n^a}{M_a} L_{nm}^a - \sum_n \frac{p_n^b}{M_b} L_{nm}^b = 0 \] (9)

To determine the natural frequencies, one assumes

\[ q_m = q_m e^{i\omega t} \]
\[ p_n^{a,b} = p_n^{a,b} e^{i\omega t} \] (10)

Thus from (3) we may determine
\[ \frac{p_{a,b}}{n} = \frac{A}{v_{a,b}} \sum m (-\omega^2 \omega_{m a}^a, b \omega_{m n}^b) \]  

Substitution of (10) and (11) into (9) gives (before substitution one should replace the summation index, m, in (11) by r to avoid confusion with the m in (9))

\[ \sum m Q_m = 0 \quad r = 1, 2, 3, \ldots \]

where

\[ Q_m = \frac{1}{v} \sum n \left( \frac{L_{n r}^a L_{n m}^a}{M_n^a \omega \omega_m^a} + \frac{L_{n r}^b L_{n m}^b}{M_n^b \omega \omega_m^b} \right) \quad (12) \]

The natural frequencies are determined by the condition that the determinant of coefficients must vanish

\[ |Q_m| = 0 \quad (13) \]

(13) is a non-standard eigenvalue problem because of the form that \( \omega^2 \) takes in \( Q_m \), see (12). However, it has one overwhelming advantage as compared to the standard (modified for gyroscopic coupling) eigenvalue approach embodied in (7) and (8): The size of the matrix in (13) will be much smaller than that in (7) and (8). This is because the number of two-dimensional opening modes, \( \psi_m \), may be much smaller than the number of three-dimensional cavity modes, \( F^a_n, F^b_n \), to achieve a given level of accuracy. This advantage will persist even when the opening is a structural member of finite stiffness and mass, but the
mathematics are a bit more cumbersome.

In closing this section we quote, without detailed derivations, two generalizations of the above results.

For an external wall of cavity \( a \) whose motion, \( w^E_a \), is prescribed (sinusoidal motion is assumed for simplicity)

\[
\omega^E_a = \sum_m q_m \psi_m
\]  

(14)

The equation of motion is

\[
\sum_m q_m Q_{rm} + Q_r = 0
\]  

(15)

where

\[
Q_r = \frac{A_{Ea}}{V_a} \sum_k q_k \frac{L_{nk} L_{nr}}{M_m [-\omega^2 + \omega_m^2]}
\]

Forced motion will be discussed in much greater depth in Section VI.

For three cavities, \( a, b, \) and \( c \), which are interconnected by openings (all other walls rigid) the counterpart of equation (12) becomes

\[
\sum_m q_m Q_{ab}^{bc(ab)} = 0
\]

\[
\sum_m q_m Q_{bc}^{bc} = 0
\]

where

\[
Q_{ab}^{bc(ab)} = \frac{A_{F}^{ab}}{V_a} \sum_{n} L_{nr}^{a(b)} L_{nm}^{a(b)} \frac{M_{n}^{a(b)}}{M_{n}^{a(b)} (-\omega^2 + \omega_m^2)}
\]

\[
Q_{bc}^{bc} = \frac{A_{F}^{ab}}{V_b} \sum_{n} L_{nr}^{b(a)} L_{nm}^{b(a)} \frac{M_{n}^{ab}}{M_{n}^{ab} (-\omega^2 + \omega_m^2)}
\]
where $A_F^{ab}$ is the area of common opening to cavities a and b, etc.

The generalization to any number of interconnected cavities should now be clear.

The relationship of this analysis to other methods also deserves brief mention. Morse and Ingard have considered two coupled cavities employing a Green's Function approach. The final result is the same as that obtained here, (12). See Appendix B for details. Alternatively, in the context of the variational formulation, as far as computing acoustical natural frequencies of multiply connected cavities with pure openings and otherwise rigid walls is concerned, the $q_m$ may be considered a Lagrange multiplier which enforces the constraint that the pressures in cavities a and b must be equal at their common opening. Hence the present analysis may be thought of as a component mode synthesis
where the components are the individual cavities. The results are entirely analogous to those previously obtained in structural vibrations, except the roles of force and deflection are interchanged\textsuperscript{14}. In structural applications, the Lagrange multipliers are forces of constraint enforcing common deflections where two structural components are connected; in acoustical applications, the Lagrange multipliers are deflections of constraint which enforce common forces (acoustical pressures) where two acoustical components (cavities) are connected.

With the combined, equivalent single cavity modes determined by the method described in this section, it is sufficient to consider a single cavity in the subsequent discussion of forced response in Section VI.
Basic Model

Here a single cavity is considered; multiply connected cavities can be treated by first determining their combined single cavity natural modes as in Section V.

The total pressure loading on the structural wall is now the sum of the internal acoustic cavity pressure and an external (prescribed) pressure loading. Hence, the structural modal equations are

\[
M_m \ddot{q}_m + 2\tau_m \omega_m \dot{q}_m + \omega_m^2 q_m = Q_m
\]

where \( Q_m \equiv \iint_A p \psi_m \, dx \, dy \)

\[ p = p^c + p^E \]

and \( p^c = \rho_0 c_0^2 \sum_n \frac{p_n \psi_n}{M_n} \)

\( p^E \), the prescribed external pressure, is the new element in the theoretical model. \( Q_m \) may be written

\[
Q_m = \rho_0 c_0^2 A_p \sum_n \frac{p_n L_{nm} + Q_m^E}{M_n} \]

where \( Q_m^E \equiv -\iint_A p^E \psi_m \, dx \, dy \)

Combining (1) and (3),

\[
M_m \ddot{q}_m + 2\tau_m \omega_m \dot{q}_m + \omega_m^2 q_m = \rho_0 c_0^2 A_p \sum_n \frac{p_n L_{nm} + Q_m^E}{M_n} \]
The cavity modal equations are

\[ p_n + \omega_n^2 p_n + \frac{A}{M} \rho_0 c_0^2 \sum_{r} \frac{\dot{p}_{r,n}}{r} - \frac{A_F}{V} \sum_{m} L_{nm} q_m \]  

(6)

One can write (5) and (6) in an attractive matrix form by again introducing the velocity potential, \( \phi \). Bernoulli's equation is

\[ p = -\rho_0 \dot{\phi} \]  

(7)

Using an expansion for \( \phi \),

\[ \phi = \sum_n a_n F_n \]  

(8)

(5) and (6) may be written

\[
\begin{bmatrix}
M \\
G \\
C \\
K \\
\end{bmatrix}
\begin{bmatrix}
\ddot{a} \\
\dot{q} \\
\dot{a} \\
q \\
\end{bmatrix}
=
\begin{bmatrix}
0 \\
E \\
0 \\
0 \\
\end{bmatrix}
\]  

(9)

where

\[
M = \begin{bmatrix}
M_m & 0 \\
0 & VM_n \end{bmatrix} \\
G = \begin{bmatrix}
0 & \rho_0 A_F L_{nm} \\
-c_0 A_F L_{nm} & 0 \\
\end{bmatrix} \\
C = \begin{bmatrix}
2M_m r_m \omega_m & 0 \\
0 & A_A \rho_0 c_0^2 C_{nr} \\
\end{bmatrix} \\
K = \begin{bmatrix}
M_m \omega_m^2 & 0 \\
0 & VM_n \omega_n A^2 \\
\end{bmatrix}
\]

These equations can be solved numerically by standard methods.8

In the remainder of this section, simplifying assumptions are considered which allow useful but approximate analytical solutions to be obtained. These should suffice for a rough estimate and also serve as a guide for more accurate and more elaborate numerical
solutions. In all cases sinusoidal excitation is assumed, i.e.

$$p^E = \frac{P^E}{P} e^{i \omega_c t}$$

where $\omega_c$ is the excitation frequency.

Because our model is linear any other time history can be considered through superposition of sinusoidal excitation. Random excitation may be considered using the sinusoidal excitation results and power spectra methods.$^{15}$
Simplified Models

One may anticipate the largest response will occur when the excitation frequency is near a structural or cavity resonant frequency.

(1) Exciting Frequency = Structural Wall Resonant Frequency.

If $\omega^E \approx \omega_s$, where $s$ denotes a structural resonant (natural) mode, then only $q_s$ will be important and all other $q_n$ may be neglected. Then, from (6)

$$ F_n = -A_F \frac{L_{ns} q_s}{V} \frac{\omega_s^2 - \omega_n^2}{-\omega_s^2 + \omega_n^2} $$

and from (10) and (1)

$$ M_s [q_s + 2 \tau_s \omega_s q_s + \omega_s^2 q_s] = -\rho_0 c^2 \frac{A^2}{F_n} \sum_{ns} q_s + Q^E e^{i \omega E t} $$

From the right hand side of (11) it is seen that if $\omega_n^2 > \omega_s^2$, then the $n^{th}$ cavity mode gives rise to equivalent mass, while if $\omega_n^2 < \omega_s^2$, then the $n^{th}$ cavity mode contributes an equivalent stiffness.

Note that the $n = 0$ cavity mode always contributes equivalent stiffness.

In many practical examples the structural wall resonant frequencies will be unchanged by the cavity per se, e.g. the sum on the right hand side of (11) can be neglected. The circumstances under which this is not true will be considered in Section VI, part (2). Making this assumption for now, from (11) one computes as
From (12), (10) and (2), the cavity pressure may be computed

\[ p^c = \frac{\rho_0 c_0^2 A_F}{2\tau_s i V} \sum_{n=1}^{N} \frac{F_n}{M_n A} L_{ns} \frac{Q^E_S}{A_F} \frac{Q^E_S}{M_s/A_F} \left[ -\omega_s^2 + \omega_n^2 \right] \]

(13)*

If \( \omega_s^2 \ll \) lowest non-zero acoustical resonance, then only \( n = 0 \) need be considered in (13). Hence (13) simplifies to

\[ p^c = \frac{\rho_0 V}{m A_F} \frac{c_0^2}{\omega_s^2} \left( \frac{V}{A_F} \right)^2 \frac{1}{2\tau_s i} \int \frac{\psi_s dA}{A_F} \int \frac{p^E_s dA}{A_F} \]

(14)*

Representative numbers are

\[ \frac{\rho_0 V}{m A_F} = 0.1, \quad \frac{c_0}{\omega_s V} = 1, \quad \tau_s = 0.1 \]

\[ \int \frac{\psi_s dA}{A_F} = \frac{2}{\pi}, \quad \int \frac{\psi_s^2 dA}{A_F} = 1/2 \]
Thus \( p_c \approx 5 p_E \), i.e. typically the sound pressure in the cavity will be larger than the external sound pressure, if \( \omega_s < \omega_A \). For \( \omega_s > \omega_A \), one must compute \( p_c \) from (13) and typically \( p_c < p_E \).

(2) Structural Resonant Frequency Changed by Coupling with Cavity Modes.

In most cases where there is a significant change in a structural resonant frequency due to the cavity, only the \( n = 0 \) cavity mode will be important. Hence (11) may be simplified to

\[
\begin{align*}
M_s & \left[ \ddot{q}_s + 2 \zeta_s \omega_s \dot{q}_s + \omega_s^2 q_s \right] \\
& = - \rho_0 c_0^2 A_F^2 \frac{I_s^2}{V} q_s + \frac{p_E}{V} e^{i \omega_E t} \\
& \text{(15)}
\end{align*}
\]

The effect of the cavity (within our approximations) is to modify the structural stiffness term, i.e. the total term is now

\[
M_s \omega_s^2 + \rho_0 c_0^2 A_F^2 \frac{I_s^2}{V} q_s
\]

Hence (15) may be written in the more compact form

\[
M_s [\ddot{q}_s + 2 \zeta_s \omega_s \dot{q}_s + \omega_s^2 q_s] = Q_s E e^{i \omega_E t} \text{(16)}
\]

where the coupled structural-cavity natural frequency is

\[
\omega_s^2 = \omega_s^2 + \rho_0 c_0^2 A_F^2 \frac{I_s^2}{M_s V} \text{(17)*}
\]

(17) generalizes an earlier result by Dowell and Voss for structural-cavity coupling. Using (17) the results in Section VI, part 1, e.g. (13), can now be generalized to include structural-cavity coupling. By
examining the ratio of the two terms in (17), one can assess when cavity effects on structural resonances are important.

\[
\left( \frac{\rho_0 c_0^2 A_F}{m V \omega_s^2} \right) \left[ \frac{\iint \psi_s^d dA}{A_F} \right]^2 \left[ \frac{\iint \psi_s^2 dA}{A_F} \right]^2
\]

Using our previous estimates for the integrals, the ratio becomes

\[
\left( \frac{\rho_0 c_0^2 A_F}{m V \omega_s^2} \right) \frac{8}{\pi^2}
\]

Clearly the lowest frequency structural mode will be most affected by the cavity. In extreme cases 5 more than one structural mode may be significantly affected.

Another less frequent case where the structural frequency may be changed due to acoustic action is when a structural mode frequency and acoustic mode frequency are close together. Clearly this is an undesirable situation, since large sound levels would be anticipated at this double resonance. This special case is considered in (4) below.

(3) Exciting Frequency = Cavity Resonant Frequency.

When \( \omega^E + \omega^A \), then \( P_n \) will be the dominant pressure mode.

But (in the absence of absorbent wall damping) if \( P_n \) is finite, then from (6) one concludes that

\[
\sum \Sigma L_{nm} q_m = 0 \quad (18)
\]

The structural equation, (5), becomes

\[
M_m [a_m + 2 \tau_m \omega_m^2 \omega_{nm}^2] = \rho_0 c_0^2 A_F \frac{P_n L_{nm} + Q_m^E}{M_n^A} \quad (19)
\]
Solving (19) for \( q_m \) gives

\[
\bar{q}_m = \rho_0 c_0^2 A_F \frac{F_n L_{nm}}{M_m^A} + \bar{Q}_m^E \frac{M_m^A}{M_m[-\omega_m^2 + 2\tau_m \omega_m i\omega_n + \omega_n^2]}
\]

(20)

(20) into (18), gives

\[
\begin{bmatrix}
\sum_n L_{nm}^2 \\
- \omega_m^2 + 2\tau_m \omega_m i\omega_n + \omega_n^2
\end{bmatrix}
\begin{bmatrix}
F_n \\
M_n^A
\end{bmatrix}
\rho_0 c_0^2 A_F
\]

\[
\begin{bmatrix}
- \omega_m^2 + 2\tau_m \omega_m i\omega_n + \omega_n^2
\end{bmatrix}
\begin{bmatrix}
Q_m^E L_{nm} \\
M_m[-\omega_m^2 + 2\tau_m \omega_m i\omega_n + \omega_n^2]
\end{bmatrix}
= 0
\]

(21)

Solving (21) for \( F_n \),

\[
F_n = -M_n^A \sum_m \frac{Q_m^E L_{nm}}{\rho_0 c_0^2 A_F M_m[-\omega_m^2 + 2\tau_m \omega_m i\omega_n + \omega_n^2]} \]

\[
- \omega_m^2 + 2\tau_m \omega_m i\omega_n + \omega_n^2
\]

(22)

For the special case where \( \omega_n^A = \omega_m \) for some \( m \), then the sums can be well approximated by a single term and the above relationship simplifies. Even when \( \omega_n^A \neq \omega_m \) for any \( m \), there may be a predominant structural mode in which case the sum could be approximated by a single term. First, however, we digress to consider another important limiting case.
If $P_n$ and $P_E$ are constants over $A_F$, then a very interesting result obtains, namely

$$Q_m^E = -P_E \int \psi_m dA, \quad L_{nm} = P_n \int \frac{\psi_m dA}{A_F}$$

the ratio of sums simplifies and using (2) and (22),

$$\frac{p_c}{\rho_0 c_0^2} = \frac{\sum \frac{P_n F_n}{M_n A}}{\rho_0 c_0^2} = \frac{P_E}{\rho_0 c_0^2}$$

Clearly, if $p_c|_{A_F} = P_E$ then $w = 0$!

To see this in detail recall that,

$$\frac{p_c}{\rho_0 c_0^2} = \frac{\sum P_n F_n}{M_n A}$$

which can be simplified to

$$\frac{p_c}{\rho_0 c_0^2} = \frac{P_n F_n}{M_n A}$$

(for $n^{th}$ cavity mode dominant)

Using (4) and (22) in (2) one concludes

$$p_c|_{A_F} = p_E + P_E$$

and $w \equiv 0$ q.e.d.

Note that if
\[ p^E = C F_n \left|_{\text{on } A_F} \right. \]

then the same result holds, namely

\[ p^C \left|_{\text{on } A_F} \right. = p^E \quad \text{and } w = 0 \]

If \( F_n \) is constant, but \( p^F \neq \text{constant} \), then a similar calculation gives

\[
\frac{p^c}{\rho^0 C^* 2} \left|_{\text{on } A_F} \right. = \frac{1}{\rho_0^0 C^* 2} \sum \frac{\int p^E \psi_m dA \int \psi_m dA}{M_m [\ldots]} \quad \text{and } w \neq 0
\]

\[
\frac{\Sigma [\int \psi_m dA]^2}{M_m [\ldots]} \quad \text{(23)}
\]

Note that \( p^c \) is constant over \( A_F \). Only if there is a single dominant structural mode will this simplify further. Now return to (22).

Single Dominant Structural Mode:

For a single dominant resonant structural mode, \( m \), (22) becomes

\[
\bar{P}_n = -M_n A \frac{Q^E}{\rho^0 C^* 2 A_F L_{nm}}
\]

\[
(24)
\]

and

\[
p^c = F_n \int p^E \psi_m dA \quad \text{(25)*}
\]

\[
\int F_n \psi_m dA
\]

If \( p^E = C F_n \mid_{\text{on } A_F} \), then \( p^c = p^E \) on \( A_F \) and \( w = 0 \) as before.

Note that, within reason, \( \psi_m \) may be selected as an appropriate linear combination of natural modes. Hence a dominant structural mode assumption
is quite useful.

The central conclusion to be drawn from the above is that, in simple terms, we may think of the cavity as acting as a vibration absorber for the structural wall when the external exciting frequency is equal to the cavity resonance frequency. Hence, at most, the internal cavity pressure will be equal to the external pressure and indeed, if \( \omega_m \gg \omega_n^A \) for all \( m \), and/or \( p^E \) varies significantly over the flexible wall, then \( p^c \ll p^E \).

(3a) Effect of Cavity Wall Absorption (Damping).

In our model the absorption characteristics are modeled by an impedance, \( z_A \). For representative acoustical materials \(^{16}\)

\[
z_A = 1 - 10 \\
\rho_0 c_0
\]

Again we shall assume a single cavity mode dominates. Then an equivalent damping ratio of the usual type may be defined

\[
2 \xi_n^C = \frac{\rho_0 c_0^2 A_A C_{nn}}{V^2} \frac{A_A}{\omega_n^A M_n^A}
\]

or

\[
2 \xi_n^C = \frac{\rho_0 c_0^2}{V^2} \frac{A_A}{\omega_n^A} \frac{A_A}{\omega_n^A} \frac{\int \int F_n F_n \, dA}{\int \int \int F_n^2 \, dV}
\]

Typically \( \xi_n^C \) would be \( .03 - .3 \).

Using an equivalent \( \xi_m^C \), (6) becomes

\[
\ddot{p}_n + 2 \xi_n^C \omega_n^A \dot{p}_n + \omega_n^A p_n^2 = - A_F \sum \frac{L_{nm} \dot{q}_m}{V} \quad (6a)
\]
As \( \omega + \omega_n \), (6a) becomes

\[
2 \cdot \xi_n \cdot \frac{\bar{P}_n}{V} = A_F \cdot \frac{\sum_{m} L_{nm} \bar{q}_m}{V}
\]

(26)

Using (20) in (26),

\[
\frac{V}{A_F} \cdot 2 \cdot \xi_n \cdot \frac{\bar{P}_n}{V} = \rho_0 c_0^2 A_F \left[ \sum_{m} \frac{L_{nm}^2}{M_m} \right] \cdot \frac{\bar{p}_n}{M_n} + \sum_{m} Q_0 \cdot \frac{E_{nm}}{M_m} \cdot \frac{M_m}{L_{nm}} \cdot \left[ -\omega_n^2 + 2 \tau_m \omega_m^A \omega_n + \omega_m^2 \right]
\]

(27)

For \( \omega_n \approx \omega_m \), retain only one term in sums and thus

\[
-4\omega_n^2 \tau_m \cdot \frac{c}{A_F} \cdot \frac{V}{M_m} \cdot L_{nm} = \rho_0 c_0^2 A_F \cdot \frac{L_{nm}^2 \bar{P}_n}{M_n} + \sum_{m} Q_0 \cdot \frac{E_{nm}}{M_m} \cdot \frac{M_m}{L_{nm}}
\]

(28)

Solving,

\[
\bar{P}_n = -\frac{E}{\sum_{m} L_{nm}} \cdot \rho_0 c_0^2 A_F \cdot \frac{L_{nm}^2}{M_n} \cdot \frac{1}{\sum_{m} Q_0 \cdot \frac{E_{nm}}{M_m} \cdot \frac{M_m}{L_{nm}}} \cdot \frac{1}{\rho_0 c_0^2 A_F \cdot \frac{L_{nm}^2}{M_n}}
\]

(29)

Hence effect of cavity damping is to decrease internal pressure field.
However, the numerical effect is typically small.

(4) Exciting Frequency = Structural Wall and Cavity Resonant Frequencies in Close Proximity.

For simplicity we ignore any absorbing wall and consider a single dominant structural wall mode and also a single dominant cavity mode whose natural frequencies are very near each other. From (9) the equations of motion are

\[ - M_m \ddot{q}_m + 2 \tau_m \omega_m \dot{q}_m + \omega_m^2 q_m + Q_m^E - A_F \rho \dot{\alpha}_n \dot{L}_{nm} = 0 \]  

(30)

\[ V M_n \left[ a_n + \omega_n^2 a_n \right] - c_0^2 A_F \dot{q}_m L_{nm} = 0 \]  

(31)

For a single cavity mode

\[ p^c = - \rho \dot{\alpha}_n F_n \]  

(32)

(30), (31) and (32) may be solved to obtain

\[ \frac{p^c}{l_{nm} A_F} = \frac{\lambda \Omega_m^E F_n}{\sqrt{\left\{ \Omega_m^E + 2 \tau_m \Omega_m^E i + 1 \right\} \left[ -\Omega_m^E + \Omega_m^A \right] - \lambda \Omega_m^E}} \]  

(33)

where

\[ \Omega_m^E = \omega / \omega_m \]  

\[ \Omega_m^A = \omega / \omega_m \]  

\[ \lambda = \rho c_0^2 A_F^2 l_{nm}^2 / \sqrt{V M_n A M_m \omega_n^2} \]
Note that

\[ \lambda \approx \frac{\rho_0 V}{m A_F} \left[ \frac{A_F}{V} \frac{c_0}{\omega_m} \right]^2 \]

From (33) the coupled structural-acoustic resonant frequencies may be determined and the associated cavity pressure. For the special case of

\[ \Omega_m^A = 1 \]

and

\[ \lambda \ll 1 \quad \text{(typically} \lambda \ll 1) \]

particularly simple results may be obtained. The resonant frequencies are then

\[ \Omega_m^E \approx 1 \pm \lambda^{1/2} \quad (34) \]

and the cavity pressure at resonance

\[ \frac{\Delta c}{p} = \frac{\lambda^{1/2}}{2 \zeta_m} \frac{F_n}{L_{nm} A_F} \quad (35) \]

The left hand side of (35) is approximately \( p^c / p^E \). Hence for typical \( \zeta_m \) and \( \lambda \) not too small, \( p^c / p^E > 1 \) when a cavity and structural wall mode coincide in natural frequencies.

(5) Convection Effects on External Sound Sources and Internal Sources.

The results discussed to this point are for a stationary external sound source. Here a few brief (cautionary) comments are offered with respect to other sound sources.

If substantial convection of an external sound source is present,
hydrodynamic coincidence effects may occur and also radiation
damping in the external flow may be important\textsuperscript{17}.

For sound sources inside the cavity itself, the wall absorption
will play a much more important role than for the external sound
sources studied here\textsuperscript{16}.

(6) Summary of Key Relations from Simplified Models to Determine Cavity
Pressure.

\begin{align*}
\text{Exciting frequency} & = \text{structural wall resonant frequency.} \\
\text{For } & \omega_{E} + \omega_{s}, (13)^* \text{ or } (14)^* \text{ modified by } (17)^* \text{ as necessary.} \\
\text{Exciting frequency} & = \text{cavity resonant frequency.} \\
\text{For } & \omega_{E} + \omega_{n} A, \text{(single dominant structural mode)} (25)^* \\
\text{Exciting frequency} & = \text{structural wall and cavity resonant frequencies in close proximity.} \\
\text{For } & \omega_{E} + \omega_{s} = \omega_{n} A, \quad (35)^*.
\end{align*}

When the external excitation frequency is well separated from all
structural or cavity resonant frequencies, there may not be a single
dominant structural and/or cavity mode. However, some simplification
may still be possible by neglecting the interaction between structural
wall and cavity, i.e. one may first determine the structural wall
motion and then use that result to determine the internal cavity acoustic
pressure. In this approximation the effect of the cavity sound pressure
on the structural wall motion is omitted.

It is also worth emphasizing that for off-resonant exciting fre-
quencies, although the numerics may be more elaborate due to the
necessity of accounting for multiple modes, the basic theoretical model
may be more accurate because the uncertainty with respect to structural
and/or acoustic damping values will be less important.
VII NUMERICAL RESULTS AND COMPARISONS WITH EXPERIMENTS

For a single cavity with a flexible wall and an external sound source, the theoretical model has been verified experimentally by several authors 4,5,18,19,20,21.

Here we assess the capability of the model to describe accurately the acoustic natural modes in multiply connected cavities. Once the combined natural modes of the multiply connected cavities are determined and verified experimentally, they may be treated as one single cavity. Hence the earlier work for a single cavity 4,5,18,19,20,21 then may be taken as experimental verification for the forced excitation of multiply connected cavities as well.

Acoustic Natural Modes in Multiply Connected Cavities.

The experimental studies were conducted by Smith 22 who has considered several geometrical arrangements for two acoustic cavities with rigid walls and a partial opening between them, see Fig.1. In Fig.2 the lowest found longitudinal resonant frequencies from theory (Section V) and experiment are shown vs partition size, $\gamma_c$, normalized by cavity height, $d$. As may be seen the agreement is good with the exception of the $10^*$ mode for $\Omega < .4$ where a mechanical resonance of the loudspeaker used to excite the acoustic modes contaminates the data. In practical terms this is unavoidable as both theory and experiment show that $\Omega_{10} = .5 \rightarrow 0$ as the partition is closed, $\gamma_c/d = 0 \rightarrow .5$. In these experiments, $c_0 = 1177 \text{ ft/sec}$, $a = d = 10''$ and the width dimension was 4'' to provide two-dimensional conditions in the frequency range of interest. The thickness of the partition (assumed

*10 means there is 1 longitudinal nodeline and 0 lateral notelines, etc.
zero in the theoretical calculations) is .5" as is the thickness of all external walls. The cavity is constructed from plexiglass.

In Figure 3 for a constant aperture (opening) size, \( \gamma_a/d \), the effect of aperture location is studied. Both theory and experiment show a small effect.

A second configuration studied by Smith consists of two acoustic cavities, one twice the dimensions of the other, with rigid walls and a partial opening between them. In Figure 4 the lowest four longitudinal resonant frequencies from theory and experiment are shown vs. normalized partition size, \( \gamma_c/d \). Again the agreement is good. In Figure 5, the longitudinal pressure distributions (along with their resonant frequencies) are shown for the first seven (symmetric with respect to height) acoustical modes with a full opening between cavities. The agreement between theory and experiment is very good.

Altogether one concludes the theoretical results are a faithful description of the physical model. For more detail the reader should consult Reference 22.

**Forced Response of a Cavity with a Flexible Wall**

*Experimental Arrangement:*

For this discussion, Gorman's work \(^{18,19}\) is used; however, also see Reference 4,5,20 and especially 21. The experimental arrangement is shown in Figure 6. The flexible wall panel was a 10" x 20" x 0.05" aluminum alloy plate that was bonded onto a rectangular frame consisting
of aluminum channel members welded together at their ends. By bonding the plate to the cavity in this way, a clamped edge boundary condition was approximated. A sealed cavity, also 10" x 20", was constructed beneath the panel in such a way that the cavity depth could be varied in 2 inch increments from 12" to 2" deep. The cavity itself was made of 0.5" thick plexiglass and was supported by four plexiglass "feet".

The panel was excited acoustically by a Wolverine LS15, 20 watt loudspeaker driven by a B & K Beat Frequency Oscillator, type 1022. The external sound field was set at 100 db at the mid-point on the plate surface for all measurements. By using a single speaker, an external field distribution that was variable in space was obtained. See Figure 7.

Initially, there were two basic measurements that were felt to be important: the measurement of panel amplitude and the measurement of cavity pressures due to a sinusoidal driving force. As work progressed, however, the need arose for two more measurements: the measurement of panel and cavity damping ratios. The reasons for these latter measurements will become apparent later.

Panel Amplitude Measurement:

The panel motion was measured by the use of a Bently Nevada motion pickup, Model 302, that was mounted on an aluminum frame located above the panel. The frame allowed movement of the pickup to any point on the surface of the panel, and also allowed variation of the distance between the pickup and the panel. As the panel oscil-
lated, the voltage generated by the motion pickup was fed through an amplifier and recorded on a amplitude vs. frequency plot. Such a plot is shown in Figure 8, for a cavity depth of 12".

For the measurement depicted in Figure 8, the motion pickup was positioned at the center of the panel. By positioning the pickup in this way, one may obtain deflection measurements for the symmetric panel modes, i.e., the modes which have a peak at the panel center, but not for the antisymmetric modes, i.e., those modes with a node at the panel center. The dominant features of the plot are the three resonant peaks, corresponding to the first, third and fifth panel modes, occurring at 113 cps, 210 cps and 410 cps. Modes above the fifth mode have an amplitude that is negligible compared to the first three symmetric modes. Above 500 cps, the panel is essentially motionless. The dominant panel response is at the panel fundamental mode, as is expected.

Cavity Pressure Measurement:

The sound pressure level within the cavity, when the panel has been excited by the loudspeaker, was measured using a B & K 1/4" microphone, Type 4136 with a Type 2615 cathode follower with Type UA 0035 connector. This microphone was installed in holes that were drilled in the side of the cavity. As with the panel motion pickup, the voltage from the microphone due to excitation of the cavity was recorded on an amplitude vs. frequency plot. Such a plot is shown in Figure 9, for a cavity depth of 12".

For the measurement depicted in Figure 9, the microphone was positioned in the hole located at the 3" cavity depth level. Since all theoretical calculations involve the cavity pressure at a point just beneath the panel, the 3" depth level was chosen to place the
microphone as close to the undersurface of the panel as possible.

In Figure 9, the cavity pressure is plotted against frequency, where this pressure is the difference between the dB level inside the cavity and the dB level outside the cavity on the upper surface of the panel. Again, the dominant features are the three primary resonant peaks occurring at 113 cps, 210 cps and 518 cps. The first two resonances correspond to the first and third panel modes, and thus indicate that the panel is driving the cavity at these frequencies. The resonance at 518 cps is the fundamental cavity mode. Note that this mode occurs above the frequency at which the panel becomes motionless (500 cps), and thus, in effect, the cavity is acting as a rigid cavity with no flexible walls. Theoretically, since the panel is motionless, the pressure level difference between the external and internal measurements should be zero. This is practically the case in Figure 9, the slight variation from zero due to the very slight motion of the panel at this frequency. Note also, that, as with the panel amplitude, the greatest cavity response occurs at the panel fundamental frequency, with the second greatest response occurring at the cavity fundamental frequency.

Damping Ratio Measurement:

The damping ratios of the panel and cavity can be measured experimentally in two different ways: the first, by using the "peak method", and the second, by using the "decay method". The decay method was the method used to determine experimentally damping ratios.
The actual measurement of damping ratios was as follows: using an oscillograph, a plot of amplitude vs. time was obtained. As the oscillograph was recording, the voltage to the loudspeaker was cut off, thus eliminating the external sound field, and the classic exponential decay plot was obtained.

Figure 10, plots the damping ratio of the panel with a 12" deep cavity as a function of frequency. The experimental points were measured at the first, third and fifth panel modes for this panel - cavity configuration. The curve obtained is consistent with a theoretical model, which says that for damping that is proportional to panel velocity, the damping ratio varies as the inverse of the frequency. The slight variation from theory at the higher panel frequencies is probably due to coupling between the panel and cavity modes.

Figure 11, plots the damping ratio of the cavity (12" depth) as a function of frequency. The experimental points were measured at the first, second and third cavity depth modes. The curve obtained suggests that the cavity damping ratio at a constant cavity depth varies as the inverse of the frequency squared. This result is inconsistent with Sheshadri, concerning the damping of Helmholtz resonators, and also does not agree with the variation of panel damping with frequency at a constant cavity depth. As yet, no explanation can be given for this damping result, other than to conclude that such a result seems to indicate a more complex damping mechanism than that of the panel. For the panel, the damping was assumed to be proportional to the
velocity of the panel. For the cavity, such a simple relationship cannot
be assumed.

Results:

Panel Frequency Response

Since the panel-cavity system being discussed is an integrated
system, one must investigate the coupling effects between the panel
and cavity. It is well known that the cavity effect on the panel will
become more pronounced as the cavity depth decreases. In
Figure 12, the panel deflection, normalized about the mid-point of
the panel, has been plotted against the panel length, normalized
with the total panel length, for the fundamental mode of the panel
backed up by a 12" and a 2" deep cavity. Figure 13, plots a similar
response, only normalized about the one-third point of the panel, for
the third panel mode. These graphs depict the effect of intermodal
coupling at shallow cavity depths. The panel response at the 12"
depth for both the first and third panel modes is very close to the
"in-vacuo" curve given by Dowell and Voss, inter alia, indicating
that a 12" deep cavity approximates an infinite cavity fairly effectively
for the panel size investigated here.

For a cavity depth of 2", the mode shapes vary considerably. In
the case of the panel fundamental mode, the response is more concen-
trated about the center of the panel than for the 12" depth case.
The response can be seen to be approaching the shape of the third
mode, indicated intermodal coupling at this shallow cavity depth.
Indeed, if the cavity was made more shallow, the panel fundamental
mode would take on the modal pattern of the third mode by having two
nodal points\textsuperscript{4,21}. For the third panel mode at the 2" cavity depth, the change in modal shape is less dramatic. However, there is some effect, especially around the midpoint of the panel, and this measured effect corresponds to the results of Dowell and Voss. Again, if the cavity were made more shallow, the effect would be more dramatic with the probable elimination of all nodal points, indicating intermodal coupling between the first and third panel modes.

Aside from affecting the mode shapes of the panel, the cavity also affects the natural frequencies of the panel due to the stiffness and virtual mass effects discussed previously. In Figure 14, the panel fundamental frequency is plotted against cavity depth. Note that, as the cavity depth decreases, the panel fundamental frequency is an excellent example of the stiffness effect, for, as the cavity depth is decreased, the cavity becomes stiffer, thus raising the panel natural frequency. Figure 15, plots the panel frequencies vs. cavity depth for the third and fifth panel modes. Note that there is apparent a slight stiffness effect on the third mode, and the lack of any stiffness effect on the fifth mode. In fact, the fifth panel mode displays features of the virtual mass effect, for as the cavity depth decreases, the natural frequencies decrease also. This fact also indicates that the virtual mass effect is present in symmetric panel modes, and not just in the antisymmetric modes. The frequency response for the third and fifth panel modes is consistent with theory, which states that the stiffness effect becomes less for the higher symmetric modes and that the virtual mass effect is present in all modes.\textsuperscript{4,5,21}
In Figure 16, a comparison between theory and experiment is made. The ratio of panel frequency (modified by coupling with the cavity) to "in-vacuo" panel frequency is plotted against panel length to cavity depth ratio, a/d. The "in-vacuo" panel frequencies were computed from Warburton's theory, and the panel frequencies' variation with cavity depth were computed from Dowell and Voss' theory, which is an earlier version of the present analysis. There is excellent agreement between theory and experiment at the large cavity depths, with some variation from theory occurring at shallow cavity depths. One or two panel modes plus the cavity Helmholtz mode were used in the calculation. This agreement between theory and experiment seems to indicate that below an a/d of 10, for a panel with similar size, thickness and material properties as the one investigated here, a one-term panel mode approximation to the panel natural frequencies may be used with good accuracy. For a/d > 10, higher term approximations must be employed. More generally, for $\lambda_c a/d > 10,000$ higher term approximations must be used to compute panel natural frequencies for a panel length to width ratio, a/b, of two.

Cavity Frequency Response

The cavity also responds in certain characteristic modes. Theoretically, at low frequencies below the cavity fundamental, the cavity pressure should be constant over the cavity depth since the cavity is responding in its Helmholtz mode. Figure 17, plots the cavity pressure, normalized with the external pressure, along the cavity depth at the panel fundamental frequency. Apparent is the fact that the cavity pressure distribution is not exactly constant, but is approximately so.
Figure 18, plots a similar normalized pressure distribution at the cavity fundamental depth frequency. Theory predicts that the cavity should respond in a cosine mode, with the internal and external pressures equal at the top and bottom of the cavity, and with the response undergoing a phase shift with cavity depth. Note that the experimental pressure distribution in the cavity at the cavity depth fundamental follows the theoretical cosine curve fairly well except at the deepest part of the cavity.

Panet Displacement and Damping Effect

There are three types of damping that will be referred in this discussion: constant damping, frequency damping, and experimental damping. Constant damping is the value measured for \( d = 12'' \) and assumes that there is no variation of panel modal damping ratio with cavity depth. Frequency damping allows for variation of damping ratio with frequency and employs the data measured at a 12'' cavity depth for various panel resonances. Thus, the only effect changing this type of damping is the variation of panel modal frequency with cavity depth. Experimental damping is the damping ratio measured for the exact conditions under investigation.

Figure 19, plots the panel damping ratio due to the effect of increasing frequency alone, and from experiment. Note that the frequency effect forces the panel damping ratios lower, whereas, the experimental damping ratios are higher, as the cavity depth is reduced. The reasons for this behavior not readily apparent; however, it is felt that, if there were some leakage from the cavity thus creating more losses in the system, the effect would be to make.
the panel respond less ideally, i.e., to increase the damping ratio. Experiments show that the cavity pressure increases with a decrease in cavity depth. Thus, if there were a leak present, the effect would be greatest at the shallow cavity depths. This reasoning is consistent with the experimental results depicted in Figure 19.

With the panel damping ratios determined, the theory may be used to compute a one-term solution for the panel amplitude, for three cases: constant damping, frequency damping and experimental damping. These results have been plotted in Figure 20, along with the measured panel amplitudes. The constant damping and frequency damping cases predict the correct panel amplitudes only at large cavity depths; however, the experimental damping ratios predict the panel amplitudes well throughout the range of cavity depths tested. From this, one may conclude, that, given the proper damping ratios, a one-term approximation for the panel amplitudes is accurate within the range of cavity depths tested.

A similar result was seen for panel amplitudes for the third and fifth panel modes. Again, the experimental damping ratio predicted the panel amplitudes more accurately than the frequency damping effect.

Cavity Pressure and Damping Effects

As with the panel amplitude, the theory may be used to compute cavity pressures. Figure 21, plots the variation of cavity pressure with cavity depth for constant damping effect, frequency damping, and experimental damping. The exciting external frequency is equal to the panel resonant frequency. Recall that the damping ratios used in these calculations are the effective panel damping ratios and not
those of the cavity; the latter were neglected. Even though cavity damping has not been considered, there is excellent agreement between experiment and the experimental damping case. This indicates that, for the fundamental panel mode, and the range of cavity depths tested, the use of one-term panel theory, the Helmholtz cavity mode and only panel damping ratios will predict cavity pressures accurately.

Similar results have been obtained for random external pressure excitation\textsuperscript{19}. Pretlove\textsuperscript{5} has made measurements of panel natural frequency variation with cavity depth; Guy and Bhattacharya\textsuperscript{21} have measured cavity pressures and panel natural frequencies. Generally good agreement with theory has also been shown in Ref. 5, 19 and 21.
VIII CONCLUDING REMARKS

A comprehensive theoretical model is developed for interior sound fields which are created by flexible wall motion resulting from exterior sound fields. Including in the model are the mass, stiffness and damping characteristics of the flexible wall as well as the mass, stiffness, and damping (due to absorbing interior walk) of the acoustic cavity. Full coupling between the wall and cavity is permitted although detailed analysis, numerical results and experiment suggest that it is the exceptional (though perhaps occasionally important) case when the structural wall dynamics characteristics are significantly modified by the cavity.

Based upon the general theory, an efficient computational method is proposed and used to determine acoustic natural frequencies of multiply connected cavities.

Simplified formulae are developed for interior sound levels in terms of in-vacuo structural wall and (rigid wall) acoustic cavity natural modes.

Comparisons of theory with experiment show generally good agreement. The principal uncertainty remains the structural and/or cavity damping mechanisms. For external sound excitation, cavity damping is demonstrated to be generally unimportant; however, it may be of substantial importance for interior sound sources. No systematic experimental data are available for cavity damping to assess the validity of the present theoretical
model in this respect.

The results of the present work, as well as those of Wolf, Nefske and Howell\textsuperscript{25} using finite element techniques and Howlett and Morales\textsuperscript{26} (based upon earlier work by Cockburn and Jolly\textsuperscript{27}), suggest that effective analytical models are available subject to the uncertainties concerning damping previously discussed.

For a further entree to the literature one may consult Reference 25 and additional references cited therein, as well as the recent work by Petyt, Lea and Koopman.\textsuperscript{28}
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APPENDIX A

ACOUSTIC NORMAL MODES OF A CAVITY WITH RIGID WALLS.

Here we consider a representative, but simple, calculation of acoustical normal modes.

Normal Modes of a Rectangular Acoustic Cavity

We shall seek non-trivial (i.e. non-zero) solutions for a cavity with rigid walls. As usual the equation of motion is

\[ \nabla^2 p - \frac{1}{c_0^2} \frac{\partial^2 p}{\partial t^2} = 0 \]

(1)

with the boundary condition

\[ \frac{\partial p}{\partial n} = 0 \text{ on all (rigid) walls} \]

(2)

where \( n \) is coordinate normal to any wall. For a rectangular cavity with dimensions and coordinates shown below,

the boundary conditions are

\[ \frac{\partial p}{\partial x} = 0 \text{ on } x = 0, \ a \]

\[ \frac{\partial p}{\partial y} = 0 \text{ on } y = 0, \ b \]

\[ \frac{\partial p}{\partial z} = 0 \text{ on } z = 0, \ d \]
We assume simple harmonic motion

\[ p(x,y,z,t) = F(x,y,z) e^{i\omega t} \]  

(3)

and seek the frequencies, \( \omega = \omega_m \), for which non-trivial solutions are possible. These are called the normal mode (or natural) frequencies. Mathematically speaking they are the eigenvalues of the problem. The corresponding solutions for \( F, F = F_m(x,y,z) \) are the normal or natural modes or eigenfunctions. Assuming a separable solution for \( F \),

\[ F(x,y,z) = X(x) Y(y) Z(z) \]  

(4)

we may substitute (3) and (4) into (1) to obtain (subscripts denote differentiation with respect to that variable)

\[ \frac{X_{xx}}{X} + \frac{Y_{yy}}{Y} + \frac{Z_{zz}}{Z} + \left( \frac{\omega}{c_0} \right)^2 = 0 \]  

(5)

Introducing separation constants, \( k_x, k_y, \) (5) becomes

\[ \frac{X_{xx}}{X} = -k_x^2 \]

\[ \frac{Y_{yy}}{Y} = -k_y^2 \]  

(6)

\[ \frac{Z_{zz}}{Z} = k_x^2 + k_y^2 \left( \frac{\omega}{c_0} \right)^2 - k_z^2 \]  

(This defines \( k_z \) in terms of \( \omega \) and vice versa):
Solving (6)

\[ X = A \cos k_x x + B \sin k_x x \]
\[ Y = C \cos k_y y + D \sin k_y y \]
\[ Z = E \cos k_z z + F \sin k_z z \]

(7)

To determine the constants, A, B, etc., \( k_x \), etc., we satisfy the boundary conditions (2). In terms of \( X, Y, Z \), these are

\[ X_x = 0 \quad \text{on } x = 0, \ a \]
\[ Y_y = 0 \quad \text{on } y = 0, \ b \]
\[ Z_z = 0 \quad \text{on } z = 0, \ d \]

(8)

Using (7) and the first of these we have

\[ k_x [-A \sin k_x x + B \cos k_x x] = 0 \quad x = 0, \ a \]

or \[ k_x B = 0 \]

(9)

and \[ k_x [-A \sin k_x a + B \cos k_x a] = 0 \]

Thus \( B = 0 \) and either

\[ A = 0 \quad \text{or} \]

(10)

\[ \sin k_x a = 0 \quad \Rightarrow k_x a = 0, \pi, 2\pi, 3\pi, \ldots \]

The latter is the alternative which corresponds to non-trivial solutions. The corresponding \( X \) is

\[ X = A \cos k_x x = A \cos \frac{m_x \pi x}{a}, \text{ where } m_x = 0, 1, 2, 3, \ldots \]

(11)
Similarly for $Y$ and $Z$

$$k_{y}b = m_{y}\pi, \quad m_{y} = 0, 1, 2, \ldots$$

$$Y = C \cos \frac{m_{y}\pi y}{b}$$

$$k_{z}d = m_{z}\pi, \quad m_{z} = 0, 1, 2, \ldots$$

$$Z = E \cos \frac{m_{z}\pi z}{d}$$

Hence from (4) and (6), using (10), (11), (12),

$$F_{m}(x,y,z) \sim \cos \frac{m_{x}\pi x}{a} \cos \frac{m_{y}\pi y}{b} \cos \frac{m_{z}\pi z}{d}$$

(13)

and

$$\frac{\omega_{m}^{2}}{c_{0}^{2}} = \pi^{2} \left( \frac{m_{x}}{a} \right)^{2} + \left( \frac{m_{y}}{b} \right)^{2} + \left( \frac{m_{z}}{d} \right)^{2}$$

Note $F_{m}$ is only determined to within an arbitrary constant. Also we use for shorthand a single modal subscript, $m$, on $F_{m}$ and $\omega_{m}$. Of course, each $m$ corresponds to a single triplet combination of integers, $m_{x}, m_{y}, m_{z}$.

One of the most interesting and useful properties of the $F_{m}$ is their "orthogonality", i.e.

$$\int \int \int F_{m} F_{n} \, dx \, dy \, dz = 0 \text{ for } m \neq n$$

(14)

This can be proved most directly using Green's Theorem for $F_{m}$ and $F_{n}$.

$$\int \int \int [F_{n} \nabla^{2} F_{m} - F_{m} \nabla^{2} F_{n}] \, dx \, dy \, dz$$

$$= \int \int \left[ \frac{\partial F_{m}}{\partial n} - \frac{\partial F_{n}}{\partial n} \right] \, dA$$

(15)
\[
\frac{\partial^2 F_m}{\partial n^2} = 0 \quad \text{from (2)}
\]

and
\[
\nabla^2 F_m = -\left(\frac{\omega_m}{c_0}\right)^2 F_m
\quad \text{from (1)}
\]
\[
\nabla^2 F_n = -\left(\frac{\omega_n}{c_0}\right)^2 F_n
\]

Hence (15) becomes
\[
(\omega_n^2 - \omega_m^2) \iiint F_n F_m \, dx \, dy \, dz = 0 \quad (16)
\]

and hence for \( m \neq n \),
\[
\omega_m \neq \omega_n
\]
and
\[
\iiint F_n F_m \, dx \, dy \, dz = 0
\]
q.e.d.
APPENDIX B

STANDING WAVES IN MULTIPLY CONNECTED CAVITIES

USING A GREEN'S FUNCTION APPROACH

An alternative description of the cavity sound field in terms of its influence or Green's function is sometimes useful. This will be briefly considered here. For further discussion, see Morse and Ingard, Chapter 9, pp. 554-564.

Green's Function Determination

Let: \[ w(x,y,z) = \delta(x-\xi) \delta(y-\eta) \delta(z-\zeta) e^{i\omega t} \] (1)

Then from II- (7), \[ W_n(t) = \frac{1}{A_F} F_n(\xi,\eta,\zeta) e^{i\omega t} \] (2)

and from II- (8), \[ P_n = \frac{(i\omega)^2 e^{i\omega t} F_n(\xi,\eta,\zeta)}{V [\omega^2 - \omega_0^2]} \] (3)

Thus from II- (9), \[ \frac{P}{\rho_0 c_0^2} = \sum \frac{F_n(\xi,\eta,\zeta)F_n(x,y,z)[-(i\omega)^2]e^{i\omega t}}{M_n[\omega_n^2 - \omega_0^2]} \] (4)

The summation

\[ \frac{- (i\omega)^2 \sum F_n(\xi,\eta,\zeta)F_n(x,y,z)}{V M_n[\omega_n^2 - \omega_0^2]} \] (5)

may be thought of as a Green's function giving the pressure at \( x, y, z \) due to a point harmonic displacement of unit amplitude at \( \xi, \eta, \zeta \).

Two Coupled Cavities

The Green's function may be used to treat two cavities connected by a opening. For definiteness we consider rectangular cavities, although it will be clean the basic
method is applicable to fairly general geometries.
Consider two volumes or cavities with their respective Green's functions and a common "wall area", \( A_F \).

\[
G_a = - (i\omega)^2 \sum_n F_n^a (\xi, n, \zeta) F_n^a (x, y, z) \frac{1}{\left( \omega^2 - \omega_n^2 \right)}
\]

\[
G_b = + (i\omega)^2 \sum_n F_n^b (\xi, n, \zeta) F_n^b (x, y, z) \frac{1}{\left( \omega^2 - \omega_n^2 \right)}
\]

There is a + sign on \( G_b \) because of the sign convention for normal to \( A_F \). Also note that only on \( A_F \) do both \( G_a \) and \( G_b \) apply.

Now

\[
\bar{p}_a (x, y, z = 0 \text{ on } A_F) = \iint_{\omega_a (\xi, n, \zeta = 0)} G_a (x, y, z = 0 \text{ on } A_F; \xi, n, \zeta = 0) \, d\xi d\eta
\]

and

\[
\bar{p}_b (x, y, z = 0 \text{ on } A_F) = \iint_{\omega_a (\xi, n, \zeta = 0)} G_b (x, y, z = 0 \text{ on } A_F; \xi, n, \zeta = 0) \, d\xi d\eta
\]

Now further we have on \( A_F \), the physical continuity conditions

\[
\bar{w} \equiv \bar{w}_a = \bar{w}_b \text{ and } \bar{p}_a = \bar{p}_b
\]

Thus using (8) - (10)

\[
\iint_{\omega} [G_a - G_b] \bar{w} \, d\xi d\eta = 0
\]
This is a homogenous integral equation for \( w \). It is an eigenvalue equation which can be solved in several ways, e.g. collocation method or modal methods, for the coupled cavities natural frequencies.

One may also consider further complications due to the wall in one cavity moving in a prescribed way. For example at \( z = -d_a \) in room \( a \), say, \( \bar{w} = \bar{w}_a^d(\xi, n) \). Note \( A_F \) now includes both walls, \( \zeta = 0 \) and \( \zeta = -d_a \). Then

\[
\bar{p}_a(x, y, z = 0) = \iint_{A_F} G_a(x, y, z = 0; \xi, n, \zeta = 0) \bar{w}_a(\xi, n) d\xi d\eta \\
+ \iint_{A_F} G_a(x, y, z = 0; \xi, n, \zeta = -d_a) \bar{w}_a^d(\xi, n) d\xi d\eta
\]

and from (9), (10), (12)

\[
\iint_{A_F} [G_a - G_b] \bar{w} d\xi d\eta + \iint_{A_F} G_a(x, y, z = 0; \xi, n, \zeta = -d_a) \bar{w}_a^d d\xi d\eta = 0 \quad (13)
\]

Given \( \bar{w}_a^d \), one may solve for \( \bar{w} \) and hence from (8) and (9), determine \( \bar{p} \) in both cavities. In particular if \( \bar{w} \) and \( \bar{w}_a^d \) are now expanded in terms of structural modes, \( \psi_m \), and one uses (6) and (7) in (11) and (13), (11) becomes (V-12) and (13) becomes (V-15). These same results are derived in a more compact way using the approach of Section V. Hence the present discussion will not be pursued further.
EXPERIMENTAL SET-UP

OSCILLOSCOPE

HP 200 CD WIDE-RANGE OSCILLATOR

McINTOSH MC 240 AMPLIFIER

HP 521C ELECTRONIC COUNTER

B & K TYPE 2603 MICROPHONE AMPLIFIER

MICROPHONE

LOUDSPEAKER

B & K TYPE 4002 STANDING WAVE APPARATUS

[CAVITY IS 20" x 10" x 4"]
CAVITY NATURAL FREQUENCIES VS. PARTITION SIZE
(2-D CASE, LONGITUDINAL (i.e. X-DIRECTION) MODE)

\[ \Omega = \frac{\omega a}{c\pi} = \frac{2af}{c} \]

--- = THEORETICAL RESULTS
\[ \Delta \Delta \Delta \text{ = EXPERIMENTAL RESULTS} \]

FIGURE 2
NATURAL CAVITY FREQUENCY VS. APERATURE LOCATION
(2-D CASE, LONGITUDINAL (i.e. X-DIRECTION) MODE)

\[ \Omega = \frac{\omega a}{c \pi} = \frac{2af}{c} \]

\[ y_0/d = 0.5 \]

--- THEORETICAL RESULTS
\[ \Delta \Delta \Delta \text{ = EXPERIMENTAL RESULTS} \]

**FIGURE 3**
\[ \Omega = \frac{\omega a}{c \pi} = \frac{2af}{c} \]

- \( \Omega \) = THEORETICAL RESULTS
- \( \Delta \Delta \Delta \) = EXPERIMENTAL RESULTS

CAVITY NATURAL FREQUENCY VS. PARTITION SIZE (2-D CASE, LONGITUDINAL MODES)

FIGURE 4
\[ \Omega = \frac{\omega a}{c \pi} \]

\( \Omega_{\text{THEO}} = 0.71 \)
\( \Omega_{\text{EXP}} = 0.72 \)

\( \Omega_{\text{THEO}} = 1.22 \)
\( \Omega_{\text{EXP}} = 1.24 \)

\( \Omega_{\text{THEO}} = 2.02 \)
\( \Omega_{\text{EXP}} = 2.00 \)

FIGURE 5
\[ \Omega_{\text{THEO}} = 2.00 \]
\[ \Omega_{\text{EXP}} = 2.04 \]

\[ \Omega_{\text{THEO}} = 2.23 \]
\[ \Omega_{\text{EXP}} = 2.23 \]

\[ \Omega_{\text{THEO}} = 2.65 \]
\[ \Omega_{\text{EXP}} = 2.62 \]

FIGURE 5 Continued
Amplitude Measurement
Pressure Measurement
Damping Measurement

Oscillator

Frequency Analyzer
Recorder

Oscilloscope

Oscillograph

EXPERIMENTAL ARRANGEMENT

FIGURE 6
**EXTERNAL PRESSURE FIELD**

\[
\overline{dB} = dB_x - dB_{x=0.5}
\]

**FIGURE 7**
PANEL RESPONSE TO SINUSOIDAL EXTERNAL FIELD

FIGURE 8
CAVITY RESPONSE TO SINUSOIDAL EXTERNAL FIELD

FIGURE 9
PANEL DAMPING VS. FREQUENCY

FIGURE 10
CAVITY DAMPING VS. FREQUENCY

Frequency (cps)

Damping Ratio

10000

1000

500

0.000

0.002

0.004

0.006

0.008

0.010

1/4π² Damping

Experiment
CAVITY EFFECT ON PANEL NORMAL MODE SHAPES
(Fundamental Mode)
FIGURE 12
CAVITY EFFECT ON PANEL NORMAL MODE SHAPES
(Third Mode)

FIGURE 13
FUNDAMENTAL PANEL FREQUENCY VS. CAVITY DEPTH

FIGURE 14
THIRD AND FIFTH PANEL FREQUENCY VS. CAVALITY DEPTH

FIGURE 15
1.3

One Term Theory

Two Term Theory

Experiment

FIRST MODE

CAVITY EFFECT ON PANEL NATURAL FREQUENCIES

FIGURE 16
PRESSURE DISTRIBUTION
(at panel fundamental)

FIGURE 17
PRESSURE DISTRIBUTION
(at cavity fundamental)

FIGURE 18
Constant Damping

Frequency Damping

Experiment

PANEL DAMPING RATIO VS. CAVITY DEPTH
(Panel Fundamental Mode)

FIGURE 19
FIGURE 20

PANEL DEFLECTION VS. CAVITY DEPTH
(Panel Fundamental Mode)
CAVITY PRESSURE VS. CAVITY DEPTH

FIGURE 21