VIBRATION ANALYSIS OF ROTOR BLADES WITH AN ATTACHED CONCENTRATED MASS

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V.R. Murthy

P.S. Barna, Principal Investigator

Technical Report

Prepared for the National Aeronautics and Space Administration Langley Research Center Hampton, Virginia

Under NASA Grant NSG 1143 June 1 - August 15, 1976 David Kershner, Technical Monitor Flight Instrumentation Division

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SUMMARY

The objective of this study is to determine the effect of an attached concentrated mass on the dynamics of helicopter rotor blades. The transmission matrix method is used to determine the natural vibrational characteristics (natural frequencies and mode shapes) of rotor blades. The problems treated are coupled flapwise bending, chordwise bending, and torsional vibration of a twisted nonuniform blade and its special subcase pure torsional vibration. The point transmission matrix for the attached concentrated mass at any spanwise and chordwise locations is derived. The orthogonality relations that exist between the natural modes of rotor blades with an attached concentrated mass are derived. Completely automated computer programs for determination of the natural vibrational characteristics are developed. For computational efficiency and users' convenience the following three separate programs are developed:

1. For determining the natural vibrational characteristics of twisted nonuniform rotor blades undergoing coupled flapwise bending, chordwise bending, and torsional vibration with the provision of an attached point mass at any spanwise and chordwise locations;

2. For determining the natural vibrational characteristics of twisted nonuniform rotor blades undergoing coupled flapwise bending, chordwise bending, and torsional vibration without the provision of an attached mass; and

3. For determining the natural vibrational characteristics of rotor blades undergoing pure torsional vibrations.

The effect of the following parameters on the collective, cyclic, scissor, and pure torsional modes of a seesaw rotor blade is determined:

1. Effect of collective pitch,
2. Effect of rotation,
3. Effect of magnitude of point mass,
4. Spanwise location of point mass, and
5. Chordwise location of point mass.

The listings of the computer programs developed and a sample output are given.
Helicopters operate in a severe vibrational environment. The vibrations result from mass unbalance, dynamic runout of rotors, torsional vibrations of branched systems, critical shaft conditions, whirl vibrations, etc. The vibrations also result in helicopters due to the fuselage controls and appendages in addition to the complex aerodynamically induced vibrations. Further, the rotor blades are subject to aeroelastic instability problems like divergence and flutter. These problems are becoming more and more important since rotor blades are becoming larger and thinner. It is important to maintain a low level of vibration in helicopters for the comfort of the crew and passengers, to minimize maintenance problems, and to increase the fatigue life of the blades. The determination of natural vibrational characteristics (natural frequencies and associated mode shapes) is an important part of the vibration analyses of helicopters. They are required to eliminate the resonant responses of the blade, and they are also widely used in the series solutions of the response problems. Furthermore, the natural vibration characteristics are of extreme importance in flutter problems and are the basis of nearly all practical flutter analyses.

In this report the natural vibrational characteristics of rotor blades with an attached point mass (at any chordwise and spanwise locations) are determined using the transmission matrix approach. For instance, the attached mass could be a sensor for measuring the angle of attack at any spanwise station. The effect of an attached mass on the natural frequencies is determined. For this purpose the continuous model of a twisted nonuniform blade with coupled flapwise bending, chordwise bending, and torsional degrees of freedom is considered. Completely automated computer programs for determination of the natural vibrational characteristics are developed. For the computational efficiency and users' convenience the following three separate computer programs are developed:

1. For determining the natural vibrational characteristics of twisted nonuniform rotor blades undergoing coupled flapwise bending, chordwise bending, and torsional vibrations with the provision of an attached point mass at any spanwise and chordwise locations;
2. For determining the natural vibrational characteristics of twisted nonuniform rotor blades undergoing coupled flapwise bending, chordwise bending, and torsional vibrations without the provision of an attached mass; and

3. For determining the natural vibrational characteristics of rotor blades undergoing pure torsional vibrations.

Holzer originally employed the transfer matrix method for approximate solutions of differential equations governing the torsional vibrations of rods, and the method is generally known as the Holzer's method (ref. 1). A method quite analogous to the Holzer's method was originated by Myklestad for the treatment of beams (ref. 2). The applicable equations were rearranged and simplified by Thomson to permit a systematic tabular computation and to extend the applicability of the method to more general problems (ref. 3). One of the earliest applications of the transfer matrix method was also the steady-state description of four terminal electrical networks, in which case the method is commonly designated as "four-pole parameters." Molloy (ref. 4) was one of the first to systematically apply four-pole parameters to acoustical, mechanical, and electromechanical vibrations. Pestel and Leckie (ref. 5) have catalogued transfer matrices for uniform elastomechanical elements up to twelfth-order. Rubin (refs. 6, 7) has extended the application through a completely general treatment. Transfer matrices have been applied to a wide variety of engineering problems by a number of researchers, including Targoff (refs. 8, 9), Isakson (refs. 10, 11), Lin (refs. 12, 13), Mercer (ref. 14), Mead (refs. 15, 16), Henderson (refs. 17, 18), McDaniel (refs. 19, 20), and Murthy (refs. 21 to 25) for application to rotor blades, stiffened beams, plates, shells and stiffened rings, etc. Most of the literature except references 21 to 25 deals with the discrete models of the continuous systems.

In the present report, the transmission matrix method is used to obtain the natural vibrational characteristics of the rotor blades with an attached point mass at any spanwise and chordwise locations. The point transmission matrix for the concentrated mass on the rotor blade at any spanwise and chordwise locations is derived. The transmission matrix for the continuous system is obtained by using the procedure described in reference 25. For completeness the derivation of the orthogonality relations that exist between
the natural modes which is given in reference 25 is included in the present report with a modification to account for an attached concentrated mass.

NOMENCLATURE

\[ [A] \]
 coefficient matrix of first-order differential equations

\( B_1, B_2 \)
 section constants (defined in reference 26)

\( b_0 \)
 semichord at the root

\( d \)
 chordwise distance of the attached concentrated mass from the shear center, positive towards leading edge

\( E I_1 \)
 flapwise bending stiffness

\( E I_2 \)
 chordwise bending stiffness

\( e \)
 distance between mass and elastic axes, positive when mass axis lies ahead of shear center

\( e_A \)
 distance between area centroid of tensile member and elastic axis

\( e_0 \)
 distance at root between elastic axis and axis about which blade is rotating

\( G J \)
 torsional stiffness

\( k_A \)
 polar radius of gyration of cross-sectional area effective in carrying tensile stresses about elastic axis

\( k_m \)
 polar radius of gyration of cross-sectional mass about elastic axis

\[ k_m^2 = k_{m1}^2 + k_{m2}^2 \]

\( k_{m1} \)
 mass radius of gyration of the cross section about the chord

\( k_{m2} \)
 mass radius of gyration of the cross section about an axis perpendicular to the chord passing through the shear center

\( M \)
 magnitude of the attached concentrated mass

\( M_x', M_y', M_z' \)
 resultant cross-sectional moments about \( x, y, \) and \( z \) directions respectively, \( M_x' \) = twisting moment, \( M_y' \) = flapwise bending moment, \( M_z' \) = chordwise bending moment
m mass per unit span
R span of the rotor (from axis of rotation to the tip of the blade)
T tension in the blade
[T] transmission matrix
t time

\( V_y, V_z \) shear force in \( y \) and \( z \) directions respectively

\( v \) amplitude of simple harmonic lateral displacement in the plane of rotation, positive in the positive \( y \)-direction

\( w \) amplitude of simple harmonic lateral displacement normal to the plane of rotation, positive in the positive \( z \)-direction

\( X_M \) spanwise location of the attached concentration mass

\( x, y, z \) right-handed Cartesian coordinate system which rotates with the blade such that the \( x \)-axis lies along the undeformed position of the elastic axis, \( y \) and \( z \) are cross-sectional axes, \( y \)-axis is positive towards the leading edge, \( z \)-axis is positive vertically upwards

\( \{z\} \) state vector

\( \beta \) blade twist prior to deformation

\( \nu \) slope of the deflection curve in the plane of the rotation

\( \phi \) amplitude of simple harmonic torsional deformation, positive leading edge upwards

\( \psi \) slope of the deflection curve normal to the plane of the rotation

\( \Omega \) angular velocity of rotation

\( \omega \) frequency of vibration

**SUPERSCRIP**

\( / \) differentiation with respect to argument

4
DERIVATION OF TRANSMISSION MATRIX OF CONTINUOUS SYSTEMS

For linear systems, the state vector satisfied a differential equation of the following form:

\[ \frac{d}{dx} \{z(x)\} = [A(x)] \{z(x)\} \]  \hspace{1cm} (1)

By definition of the backward transmission matrix

\[ \{z(x)\} = [T(x)] \{z(0)\} \]  \hspace{1cm} (2)

Differentiating this equation with respect to \( x \) gives

\[ \frac{d}{dx} \{z(x)\} = \frac{d}{dx} [T(x)] \{z(0)\} \]  \hspace{1cm} (3)

From equation (2) it is obvious that

\[ \{z(0)\} = [T(x)]^{-1} \{z(x)\} \]  \hspace{1cm} (4)

and the inverse of a transmission matrix always exists since the determination of a transmission matrix is unity. Substituting equation (4) into equation (3), the following relation is obtained

\[ \frac{d}{dx} \{z(x)\} = \frac{d}{dx} [T(x)] [T(x)]^{-1} \{z(x)\} \]  \hspace{1cm} (5)
Equating equations (1) and (5) yields

\[
\begin{align*}
[A(x)] \{z(x)\} &= \frac{d}{dx} [T(x)] \left[ T(x) \right]^{-1} \{z(x)\}
\end{align*}
\]

or

\[
\left( \frac{d}{dx} [T(x)] \left[ T(x) \right]^{-1} - [A(x)] \right) \{z(x)\} = \{0\}
\]

since equation (6) must be satisfied for all values of \( x \) and for all values of \( z \), it follows that \( [A(x)] = \frac{d}{dx} [T(x)] \left[ T(x) \right]^{-1} \). Then postmultiplying both sides by \( [T(x)] \) gives

\[
\frac{d}{dx} [T(x)] = [A(x)] [T(x)]
\]  

(7)

Therefore, the transmission matrix is given directly by the solution to equation (7). By letting \( x \) go to zero in equation (2), the required initial condition becomes

\[
[T(0)] = [I], \text{ the identity matrix}
\]

(8)

If equation (7) is solved as a coupled set of first-order differential equations, then equation (8) provides the sufficient number of initial conditions.

**BASIC EQUATIONS**

The basic differential equations of motion for combined flapwise bending, chordwise bending, and torsion of twisted nonuniform rotor blades are derived in reference 26. Using the transmission matrix formulation of the general case, the natural frequencies of subcases can be determined. These subcases arise if some degree of freedom is decoupled from the combined flapwise bending, chordwise bending, and torsion by virtue of some parameters being
zero. Unfortunately, the mock shapes of the subcases cannot be determined from the formulation of the general case, so the subcases must be formulated separately. The differential equations of motion for simple harmonic free vibrations with frequency, $\omega$, are listed below with

$$k_a = B_1 = B_2 = e_A = e_0 = 0$$

Case I: Combined flapwise bending, chordwise bending, and torsion

$$-(GJ\phi')' + \Omega^2mx(-v' \cos \beta + w' \sin \beta) + \Omega^2me \sin \beta v$$

$$+ \Omega^2m(k_2^2 - k_1^2) \cos 2\beta \phi - \omega^2mk^2 \phi$$

$$+ \omega^2me(v \sin \beta - w \cos \beta) = 0$$

$$[(EI_1 \cos^2 \beta + EI_2 \sin^2 \beta)v'' + (EI_2 - EI_1)\sin \beta \cos \beta v'']''$$

$$- (Tw')' - (\Omega^2mx \phi \cos \beta)'$$

$$- \omega^2m(w + e \phi \cos \beta) = 0$$

$$[(EI_2 - EI_1) \sin \beta \cos \beta w'' + (EI_1 \sin^2 \beta + EI_2 \cos^2 \beta)v'']'$$

$$- (Tw')' + (\Omega^2mx \phi \sin \beta)' + \Omega^2me \phi \sin \beta$$

$$- \omega^2m(v - e \phi \sin \beta) - \Omega^2mv = 0$$

$$T' + \Omega^2mx = 0$$

Case II: Pure torsion ($e = \beta = 0$)

$$-(GJ\phi')' + \Omega^2m(k_2^2 - k_1^2)\phi - \omega^2mk^2 \phi = 0$$

For the determination of the transmission matrix, it is required to reduce the governing differential equations of motion to a set of first-order differential equations. The following state vectors are chosen for this purpose.
Case I: Combined flapwise bending, chordwise bending, and torsion.

\[
\{z\}^T = \begin{bmatrix}
  w, v, \psi, \nu, M_x, M_z, M_y, -V_y, -V_z
\end{bmatrix}
\]

Case II: Pure torsion

\[
\{z\}^T = \begin{bmatrix}
  \phi, M_x
\end{bmatrix}
\]

The components of the state vector, \( \{z\} \), can be chosen in several ways, but they are chosen here such that they represent the physical quantities of deflections, slopes, moments, and shears. This is not absolutely required, but highly preferable for the application of transmission matrices to obtain the natural vibration characteristics. For simplification of the numerical computation the differential equations of motion are nondimensionalized as shown below

\[
\begin{align*}
\ddot{x} &= \frac{x}{R} \\
\ddot{\beta} &= \beta \\
\ddot{\phi} &= \frac{\phi}{b_0} \\
k_{m_1}^{-2} &= \frac{k_{m_1}^2}{b_0^2} \\
k_{m_2}^{-2} &= \frac{k_{m_2}^2}{b_0^2} \\
k_m^{-2} &= \frac{k_m^2}{b_0^2}
\end{align*}
\]
The nondimensional elements of the state vectors are defined as shown below.

\[ \overline{\Omega^2} = \frac{\omega^2 m_0 R^4}{E I_{10}} \quad \text{for Case I} \]

\[ \overline{\Omega^2} = \frac{\omega^2 m_0 R^4}{G J_0} \quad \text{for Case II} \]

\[ \overline{\omega^2} = \frac{\omega^2 m_0 R^4}{E I_{10}} \quad \text{for Case I} \]

\[ \overline{\omega^2} = \frac{\omega^2 m_0 R^4}{G J_0} \quad \text{for Case II} \]

\[ \overline{m} = \frac{m}{m_0} \]

\[ \overline{w} = \frac{w}{b_0} \]

\[ \overline{v} = \frac{v}{b_0} \]

\[ \overline{\Psi} = \frac{\Psi R}{b_0} \]

\[ \overline{\nu} = \frac{\nu R}{b_0} \]

\[ \overline{\phi} = \frac{\phi}{b_0} \]

\[ M_x = \frac{M R^3}{E I_{10} b_0^2} \quad \text{for Case I} \]
The resulting first-order nondimensional equations are given below.

Case I: Combined flapwise bending, chordwise bending, and torsion.

\[
\frac{d\bar{v}_x}{dx} = \bar{\psi} \tag{14a}
\]

\[
\frac{d\bar{v}_y}{dx} = \bar{\nu} \tag{14b}
\]

\[
\frac{d\bar{v}_z}{dx} = -c_2EI_{10}\bar{M}_z + c_1EI_{10}\bar{M}_y \tag{14c}
\]

\[
\frac{d\bar{v}_y}{dx} = c_3EI_{10}\bar{M}_z - c_2EI_{10}\bar{M}_y \tag{14d}
\]

\[
\frac{d\phi}{dx} = \frac{EI_{10}bq^2}{GJR^2} \bar{M}_x \tag{14e}
\]
\[ \frac{dM}{dx} = -\omega^2 \bar{m}_e \cos \beta \bar{w} + (\omega^2 + \Omega^2) \bar{m}_e \sin \beta \bar{v} \]

\[ + \Omega^2 \bar{m}_e \cos \beta \bar{\psi} - \Omega^2 \bar{m}_e \sin \beta \bar{v} \]

\[ + [\Omega^2_m (k_{m_2}^2 - k_{m_1}^2) \cos 2\beta - \omega^2_m k_m^2] \bar{\phi} \quad (14f) \]

\[ \frac{dM}{dx} = \frac{d}{dx} \left( \bar{m}_e \bar{v} \right) - \Omega \bar{m}_e \sin \beta \bar{\phi} - \bar{v} \bar{y} \]

\[ (14g) \]

\[ \frac{dM}{dx} = \frac{d}{dx} \left( \bar{m}_e \bar{v} \right) - \Omega \bar{m}_e \cos \beta \bar{\phi} \]

\[ (14h) \]

\[ \frac{dM}{dx} = (\omega^2 + \Omega^2) \bar{m}_v - (\omega^2 + \Omega^2) \bar{m}_e \sin \beta \bar{\phi} \]

\[ (14i) \]

\[ \frac{dM}{dx} = \omega^2 \bar{m}_v + \omega^2 \bar{m}_e \cos \beta \bar{\phi} \]

\[ (14j) \]

**Case II: Pure torsion**

\[ \frac{d\phi}{dx} = \frac{GJo \bar{b}^2}{GJR^2} \frac{M_x}{M} \]

\[ (15a) \]

\[ \frac{dK_x}{dx} = [\Omega^2_m (k_{m_2}^2 - k_{m_1}^2) - \omega^2_m k_m^2] \bar{\phi} \]

\[ (15b) \]

where \( C_1 = \frac{A_{22}}{D} \)

\[ C_2 = \frac{A_{12}}{D} \]
\[ C_3 = \frac{A_{11}}{D} \]

\[ D = A_{11}A_{22} - A_{12}^2 \]

\[ A_{11} = EI_1 \cos^2 \beta + EI_2 \sin^2 \beta \]

\[ A_{12} = (EI_2 - EI_1) \sin \beta \cos \beta \]

\[ A_{22} = EI_1 \sin^2 \beta + EI_2 \cos^2 \beta \]

\[ \frac{d}{T} = \frac{TR^2}{EI_{10}} \]

Equations (14) and (15) will define the elements of matrix \([A]\) in equation (7) for the evaluation of the transmission matrices of Cases I and II respectively. The required initial conditions are given by equation (8). The nondimensional form of equation (12) is as shown below

\[
\frac{dT}{dx} + \bar{\Omega}^2 mx = 0 \quad \text{(16)}
\]

**DERIVATION OF THE POINT TRANSMISSION MATRIX FOR A CONCENTRATED MASS ON A ROTOR BLADE**

A concentrated mass is assumed to be attached by a rigid massless bar along the chordline of an airfoil section of the blade as shown in figure 1.
Figure 1. Airfoil section with attached mass.

where

\[ M = \text{attached mass} \]

\[ 0 = \text{shear center of the cross-section} \]

\[ d = \text{distance of the mass from shear center along the chordline} \]

\[ \beta = \text{twist of the blade} \]

\[ y, z = \text{undeformed coordinate system} \]

Inertial Accelerations

Let \( x, y, z \) be undeformed coordinates and \( x_1, y_1, z_1 \) be deformed coordinates, and these are related by

\[ x_1 = x + u - y \frac{dv}{dx} - z \frac{dw}{dx} \]  \hspace{1cm} (17a)
\[ y_1 = y + v - z \phi \]  \hspace{1cm} (17b) \\

\[ z_1 = z + w + y \phi \]  \hspace{1cm} (17c)

Let \( X, Y, Z \) be a nonrotating coordinate system and \( x, y, z \) be a rotating coordinate system, and \( i, j, k \) be unit vectors along \( X, Y, Z \) directions respectively. The position vector \( \mathbf{r} \) can be expressed in terms of the rotating coordinates and nonrotating unit vectors as shown below (see figure 2 also).

\[
\begin{align*}
X &= x_1 \cos \Omega t - y_1 \sin \Omega t \\
Y &= x_1 \sin \Omega t + y_1 \cos \Omega t
\end{align*}
\]

Figure 2. Rotating and nonrotating coordinate systems.

\[ \mathbf{r} = iX + jY + kZ \]

or

\[ \mathbf{r} = i(x_1 \cos \Omega t - y_1 \sin \Omega t) + j(x_1 \sin \Omega t + y_1 \cos \Omega t) + kZ_1 \]

Differentiating this equation with respect to \( t \) gives
\[ \ddot{r} = i(\ddot{x}_1 \cos \Omega t - 2\Omega \dot{x}_1 \sin \Omega t - \Omega^2 x_1 \cos \Omega t - \dot{y}_1 \sin \Omega t \]
\[ - 2\Omega \dot{y}_1 \cos \Omega t + \Omega^2 y_1 \sin \Omega t) \]
\[ + j(\ddot{x}_1 \sin \Omega t + 2\Omega \dot{x}_1 \cos \Omega t - \Omega^2 x_1 \sin \Omega t + \dot{y}_1 \cos \Omega t \]
\[ - 2\Omega \dot{y}_1 \sin \Omega t - \Omega^2 y_1 \cos \Omega t) + k \ddot{z}_1 \]

Acceleration components with respect to the rotating coordinate system (x, y, z) are given by substituting \( \Omega t = 0 \) in the above equation. Denoting these components by \( a_x, a_y, \) and \( a_z \) in x, y, z directions respectively, the following relations can be obtained

\[ a_x = \ddot{x}_1 - \Omega^2 x_1 - 2\Omega \dot{y}_1 \]
\[ a_y = 2\Omega \dot{x}_1 + \dot{y}_1 - \Omega^2 y_1 \]
\[ a_z = \ddot{z}_1 \]

Substituting equation (17) in these acceleration components yields

\[ a_x = \ddot{u} - y\ddot{v} - z\ddot{w} - \Omega^2(x + u - yv' - zw') - 2\Omega(\dot{v} - \dot{z}\phi) \]
\[ a_y = 2\Omega(\ddot{u} - y\ddot{v} - z\ddot{w'}) + \ddot{v} - \ddot{z}\phi - \Omega^2(y + v - z\phi) \]
\[ a_z = \ddot{w} + y\dot{\phi} \]

By neglecting the small components of usual helicopters the following acceleration components are obtained

\[ a_x = -\Omega^2 x \] \hspace{1cm} (18a)
\[ a_y = \ddot{v} - z\ddot{\phi} - \Omega^2(v + y - z\phi) \] \hspace{1cm} (18b)
\[ a_z = \ddot{w} + y\dot{\phi} \] \hspace{1cm} (18c)
From geometry of figure 1, it is obvious that

\[ y = d \cos \beta \]  \hspace{1cm} (19a)
\[ z = d \sin \beta \]  \hspace{1cm} (19b)

Substituting equation (19) into equation (18), the following acceleration components acting on the concentrated mass are obtained

\[ a_x = -\Omega^2 x \]  \hspace{1cm} (20a)
\[ a_y = \ddot{v} - d \sin \beta \ddot{\phi} - \Omega^2 (v + d \cos \beta - d \sin \beta \phi) \]  \hspace{1cm} (20b)
\[ a_z = \ddot{w} + d \cos \beta \ddot{\phi} \]  \hspace{1cm} (20c)

**Force Equilibrium Equations**

The free-body diagram of a small element across the concentrated mass is shown in figure 3. For clarity purpose only forces are shown.

![Free-body diagram for forces.](image)

Figure 3. Free-body diagram for forces.
For force equilibrium the following relations must be satisfied

\[ T^R = T^L + M \alpha_{x} \]

\[ v^R_y = v^L_y + M \alpha_{y} \]

\[ v^R_z = v^L_z + M \alpha_{z} \]

Substituting equation (20) in the above equations and assuming simple harmonic free vibrations with frequency, \( \omega \), the following equations can be obtained

\[ T^R = T^L - M \Omega^2 x \] \hspace{1cm} (21a)

\[ v^R_y = v^L_y - (\omega^2 + \Omega^2) M \alpha_y + (\omega^2 + \Omega^2) M \alpha_{y} \sin \beta \phi \] \hspace{1cm} (21b)

\[ v^R_z = v^L_z - \omega^2 M \alpha_y - \omega^2 M \alpha_{y} \cos \beta \phi \] \hspace{1cm} (21c)

\[ \text{Inertial Moments} \]

From figure 4 the following relations for inertial force vector and position vector can be written

\[ \text{Figure 4. Inertial moments and position vector.} \]
The moment vector about the shear center \( \mathbf{\Omega} \) is given by

\[
\mathbf{M}_{\text{SC}} = r \mathbf{F} = iM_1 + jM_2 + kM_3
\]

where

\[
M_1 = -M_d[(\cos \beta - \phi \sin \beta) a_z - (\sin \beta + \phi \cos \beta) a_y]
\]

\[
M_2 = -M_d(\sin \beta + \phi \cos \beta)a_x
\]

\[
M_3 = M_d(\cos \beta - \phi \sin \beta)a_x
\]

Substituting equation (20) in the above inertial moment components and neglecting the nonlinear terms the following equations are obtained for simple harmonic free vibrations with frequency, \( \omega \).

\[
M_1 = \omega^2 M_d \cos \beta \omega - (\omega^2 + \Omega^2) M_d \sin \beta \omega \\
\quad \cdot (\omega^2 M_d^2 - \Omega^2 M_d^2 \cos 2\beta) \phi
\]

\[
M_2 = \Omega^2 M_d \cos \beta \times \phi
\]

\[
M_3 = \Omega^2 M_d \sin \beta \times \phi
\]
Moment Equilibrium Equations

The free-body diagram of a small element across the concentrated mass is shown in figure 5. For clarity purpose only moments are shown.

![Free-body diagram for moments](image)

**Figure 5.** Free-body diagram for moments.

For moment equilibrium the following relations must be satisfied

\[
\begin{align*}
M_R^x & = M_x^L - M_1 \\
M_R^y & = M_y^L + M_2 \\
M_R^z & = M_z^L - M_3
\end{align*}
\]

Substituting equation (22) in the above moment equilibrium equations

\[
\begin{align*}
M_R^x & = M_x^L - \omega^2 M_d \cos \beta w + (\omega^2 + \Omega^2) M_d \sin \beta \psi \\
& + (-\omega^2 M_d^2 + \Omega^2 M_d^2 \cos 2\psi) \phi \\
M_R^y & = M_y^L + \Omega^2 M_d \cos \beta \psi
\end{align*}
\]

(23a)          

(23b)
Point Transmission Matrix

Equations (21) and (23) can be put into a transmission matrix form across the concentrated mass by noting that the deflections and slopes are continuous across the mass. The resulting transmission matrix after nondimensionalization is as shown on the following page [eq. (24)] when

\[
\begin{align*}
\dot{x} &= \frac{d}{b_0} \\
M &= \frac{M}{m_0R} \\
\bar{x} &= \frac{x}{R} = \text{spanwise location of the mass.}
\end{align*}
\]

Case II: For this case the point transmission matrix reduces to the following equation:

\[
\begin{pmatrix}
\ddot{\bar{\phi}} \\
\dot{\bar{M}}_x
\end{pmatrix}
= \begin{bmatrix}
1 & 0 \\
(-\omega^2 + \Omega^2)\bar{M}d^2 & 1
\end{bmatrix}
\begin{pmatrix}
\bar{\phi} \\
\bar{M}_x
\end{pmatrix}
\]  

(25)

NATURAL VIBRATION CHARACTERISTICS

Natural Frequencies

The overall transmission matrix of the blade without the attached mass is obtained by integrating the differential equations given by equation (7) together with the initial conditions given by equation (8). The integration proceeds from \( \bar{x} = 0 \) to \( \bar{x} = 1 \). The matrix \([A]\) in equation (7) is the coefficient matrix of the first-order differential equations of motion, and it is obtained from equation (14). The coefficient \( \bar{T} \) appearing in...
\[
\begin{align*}
\begin{bmatrix}
\dot{\mathbf{w}} \\
\dot{\mathbf{v}} \\
\dot{\psi} \\
\dot{\phi} \\
\dot{\mathbf{M}_x} \\
\dot{\mathbf{M}_z} \\
\dot{\mathbf{M}_y} \\
\dot{\mathbf{V}_y} \\
\dot{\mathbf{V}_z}
\end{bmatrix}
&= 
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
-\omega^2\mathbf{M}_d \cos \beta & (\omega^2 + \Omega^2)\mathbf{M}_d \sin \beta & 0 & 0 & (-\omega^2 + \Omega^2 \cos 2\beta)\mathbf{M}_d^2 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -\Omega^2\mathbf{M}_d \sin \beta & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & \Omega^2\mathbf{M}_d \cos \beta & 0 & 0 & 1 & 0 \\
0 & (\omega^2 + \Omega^2)\mathbf{M}_d & 0 & 0 & -(-\omega^2 + \Omega^2)\mathbf{M}_d \sin \beta & 0 & 0 & 0 & 1 \\
\omega^2\mathbf{M}_d & 0 & 0 & 0 & \omega^2\mathbf{M}_d \cos \beta & 0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
\mathbf{w} \\
\mathbf{v} \\
\psi \\
\phi \\
\mathbf{M}_x \\
\mathbf{M}_z \\
\mathbf{M}_y \\
\mathbf{V}_y \\
\mathbf{V}_z
\end{bmatrix}
\end{align*}
\]
equation (14) is obtained by solving equation (16) together with the initial condition \( T(\overline{x} = 1) = 0 \). The overall transmission matrix of the blade when a concentrated mass is attached can be obtained as described below.

Let \( \overline{x}_M \) be the spanwise location of the attached mass. Equation (7) is integrated from \( \overline{x} = 0 \) to \( \overline{x} = \frac{x_M}{R} \) utilizing the initial conditions given by equation (8). Let \([T_1]\) be the transmission matrix up to the point \( \overline{x}_M \) and represent the transmission properties of the system from \( \overline{x} = 0 \) to \( \overline{x} = \frac{x_M}{R} \). The point transmission matrix of the attached mass is computed from equation (24), and let it be denoted by \([T_M]\). Equation (7) can again be integrated from \( \overline{x} = \frac{x_M}{R} \) to \( \overline{x} = 1 \) utilizing the initial conditions given by equation (8). Let \([T_2]\) be the resulting transmission matrix and represent the transmission properties of the system from \( \overline{x} = \frac{x_M}{R} \) to \( \overline{x} = 1 \). Then by using the product rule of the backward transmission matrix, the overall transmission matrix of the system can be obtained from the following equation.

\[
[T] = [T_2] [T_M] [T_1]
\]

While integrating equation (7) from \( \overline{x} = 0 \) to \( \overline{x} = \frac{x_M}{R} \) to obtain transmission matrix \([T_1]\), coefficient \( \overline{T} \) is obtained as usual by integrating equation (16). But this coefficient should be increased by a constant amount \( \frac{\Omega^2 x M}{R} \) to account for the tension in the blade due to the attached mass. While integrating equation (7) from \( \overline{x} = \frac{x_M}{R} \) to \( \overline{x} = 1 \) this increment in the tension will not be required. This fact is reflected in equation (21a) which is not utilized in obtaining the point transmission matrix given by equation (24). A similar procedure can be used for Case II also when a concentrated mass is attached to the blade. Having obtained the overall transmission matrix of the system either with or without the concentrated mass the frequency determinant can subsequently be obtained as discussed below.

The frequency determinant is dependent on the boundary conditions of the system, and the following three sets of root boundary conditions are assumed for collective, cyclic, and scissor modes of a seesaw rotor blade.

\[
M_x = M_y = M_z = V_y = V_z = 0 \text{ at } \overline{x} = 1 \text{ (tip of the blade) (26a)}
\]
Collective Modes

\[ w = v = \psi = M_z = 0, \quad M_x = -k_\phi \phi \text{ at } \bar{x} = 0 \]

(root of the blade) \hspace{1cm} (26b)

Cyclic Modes

\[ w = v = v = M_y = 0, \quad M_x = -k_\phi \phi \text{ at } \bar{x} = 0 \]

(root of the blade) \hspace{1cm} (26c)

Scissor Modes

\[ w = v = \psi = v = 0, \quad M_x = -k_\phi \phi \text{ at } \bar{x} = 0 \]

(root of the blade) \hspace{1cm} (26d)

By definition of the backward transmission matrix one can write

\[
\begin{bmatrix}
\bar{v} \\
\bar{\psi} \\
\bar{\phi} \\
\bar{M}_x \\
\bar{M}_y \\
\bar{M}_z \\
\bar{v}_y \\
\bar{v}_z \\
\end{bmatrix}
= [T_{ij}]
\begin{bmatrix}
\bar{w} \\
\bar{v} \\
\bar{\psi} \\
\bar{\phi} \\
\bar{M}_x \\
\bar{M}_y \\
\bar{M}_z \\
\bar{v}_y \\
\bar{v}_z \\
\end{bmatrix}
\]

(27)
where $T_{ij}$ represents the $(i, j)$ th nondimensional element of the transmission matrix from $\bar{x} = 0$ to $\bar{x} = 1$. The frequency determinant corresponding to collective modes is obtained by substituting the tip boundary conditions given by equation (26a) into the output state vector ($\bar{x} = 1$) and the root boundary conditions into the input state vector ($\bar{x} = 0$). This substitution yields five homogeneous equations, and the determinant of the coefficient matrix of these equations must vanish for nontrivial solutions. The resulting frequency determinant is given by the following equation

$$\begin{bmatrix}
T_{64} & T_{65} - \bar{k}_\phi T_{66} & T_{68} & T_{69} & T_{610} \\
T_{74} & T_{75} - \bar{k}_\phi T_{76} & T_{78} & T_{79} & T_{710} \\
T_{84} & T_{85} - \bar{k}_\phi T_{86} & T_{88} & T_{89} & T_{810} \\
T_{94} & T_{95} - \bar{k}_\phi T_{96} & T_{98} & T_{99} & T_{910} \\
T_{104} & T_{105} - \bar{k}_\phi T_{106} & T_{108} & T_{109} & T_{1010}
\end{bmatrix} = 0$$

where $\bar{k}_\phi$ is nondimensional control system spring rate defined by

$$\bar{k}_\phi = \frac{k_\phi R^3}{EI_{10} b_0^2}.$$ This represents the resistance to unit torsional deformation due to control systems. By using a similar procedure the frequency determinants corresponding to the cyclic and scissor modes can be obtained, and they are given below

For cyclic modes

$$\begin{bmatrix}
T_{63} & T_{65} - \bar{k}_\phi T_{66} & T_{67} & T_{69} & T_{610} \\
T_{73} & T_{75} - \bar{k}_\phi T_{76} & T_{77} & T_{79} & T_{710} \\
T_{83} & T_{85} - \bar{k}_\phi T_{86} & T_{87} & T_{89} & T_{810} \\
T_{93} & T_{95} - \bar{k}_\phi T_{96} & T_{97} & T_{99} & T_{910} \\
T_{103} & T_{105} - \bar{k}_\phi T_{106} & T_{107} & T_{109} & T_{1010}
\end{bmatrix} = 0$$
For scissor modes

\[
\begin{bmatrix}
T_{65} - k_\phi T_{66} & T_{67} & T_{68} & T_{69} & T_{610} \\
T_{75} - k_\phi T_{76} & T_{77} & T_{78} & T_{79} & T_{710} \\
T_{85} - k_\phi T_{86} & T_{87} & T_{88} & T_{89} & T_{810} \\
T_{95} - k_\phi T_{96} & T_{97} & T_{98} & T_{99} & T_{919} \\
T_{105} - k_\phi T_{106} & T_{107} & T_{108} & T_{109} & T_{1010}
\end{bmatrix}
= 0
\]

For pure torsional vibrations the boundary conditions, the transmission equation, and frequency determinants are given below.

**Boundary conditions**

\[ M_x = 0 \text{ at } \bar{x} = 1 \]

\[ M_x = -k_\phi \phi \text{ at } \bar{x} = 0 \]

**Transmission equation**

\[
\begin{bmatrix}
\phi \\
\bar{M}_x
\end{bmatrix}
_{\bar{x}=1} =
\begin{bmatrix}
T_{11} & T_{12} \\
T_{21} & T_{22}
\end{bmatrix}
\begin{bmatrix}
\phi \\
\bar{M}_x
\end{bmatrix}
_{\bar{x}=0}
\]

**Frequency equation**

\[ T_{21} - k_\phi T_{22} = 0 \]

The frequency determinant is a function of frequency \( \omega \), the solution of which yields the natural frequencies.

**Mode Shapes**

Having determined the natural frequencies, the associated mode shapes can be obtained subsequently in a straightforward manner. To determine the
natural frequencies the overall transmission matrix (i.e., $\bar{x} = 0$ to $\bar{x} = 1$) is required. For determination of the mode shapes the transmission matrix as a function of $\bar{x}$ is required. The transmission matrix as a function of $\bar{x}$ for a blade without the attached mass can be obtained by integrating equation (7) together with the initial conditions. If a numerical scheme is used for this purpose, the transmission matrix is known at various stations along the span. The transmission matrix as a function of $\bar{x}$ for a rotor blade with an attached concentrated mass can be obtained as described below.

Let $x_M$ be the spanwise location of the attached mass. Equation (7) is integrated from $\bar{x} = 0$ to $\bar{x} = x_M$ together with the initial conditions given by equation (8). Let $\bar{x}[T_1]_0$ be the transmission matrix from $\bar{x} = 0$, to $\bar{x}(\bar{x} < x_M)$. The matrix $\bar{x}[T_M][T_1]_0$ will be the same as matrix $[T_1]$ defined under Natural Frequencies. Let $[T_M]$ be the point transmission matrix of the attached mass. Equation (7) can again be integrated from $\bar{x} = x_M$ to $\bar{x} = 1$ utilizing the initial conditions given by equation (8). Let $\bar{x}[T_2]_{x_M}(x_M < x > 1)$ be the resulting transmission matrix. Then the transmission matrix as a function of $\bar{x}$ for the blade with an attached concentrated mass is given by the following equations

\[
[T(\bar{x})] = \frac{x}[T_1]_0 \quad 0 \leq \bar{x} \leq \bar{x}_M.
\]

\[
[T(\bar{x})] = \frac{x}[T_2]_{x_M} \left[ \frac{T_1}{T_M} \right] \quad x_M < \bar{x} < 1
\]

A similar procedure is used to obtain the transmission matrix as a function of $\bar{x}$ for Case II also.

**Case I:**

**Collective Modes**

From the definition of the transmission matrix, one can write the following equation

26
By substituting the root boundary conditions given by equation (26b) into the input state vector \((\bar{x} = 0)\) of equation (28) and extracting the first, second, and fifth rows and arranging in a matrix form:

\[
\begin{bmatrix}
\dot{\bar{w}} \\
\dot{\bar{v}} \\
\dot{\phi} \\
\bar{M}_x \\
\bar{M}_y \\
\bar{M}_z \\
\bar{V}_y \\
\bar{V}_z \\
\end{bmatrix} = \begin{bmatrix}
T_{14}(\bar{x}) & T_{15}(\bar{x}) & \bar{k}_\phi T_{16}(\bar{x}) & T_{18}(\bar{x}) & T_{19}(\bar{x}) & T_{110}(\bar{x}) \\
T_{24}(\bar{x}) & T_{25}(\bar{x}) & \bar{k}_\phi T_{26}(\bar{x}) & T_{28}(\bar{x}) & T_{29}(\bar{x}) & T_{210}(\bar{x}) \\
T_{54}(\bar{x}) & T_{55}(\bar{x}) & \bar{k}_\phi T_{56}(\bar{x}) & T_{58}(\bar{x}) & T_{59}(\bar{x}) & T_{510}(\bar{x}) \\
\end{bmatrix} \begin{bmatrix}
\bar{w} \\
\bar{v} \\
\bar{M}_x \\
\bar{M}_y \\
\bar{M}_z \\
\bar{V}_y \\
\bar{V}_z \\
\end{bmatrix} \quad \bar{x} = 0
\]

By substituting the tip boundary conditions given by equation (26a) into the output state vector \((\bar{x} = 1)\) and the root boundary conditions given by equation (26b) into the input state vector of equation (27) and extracting first and sixth to ninth rows of the equation while assigning \(\bar{w}(\bar{x} = 1) = 1\), the following equation is obtained:

\[
\begin{bmatrix}
\dot{\bar{w}} \\
\dot{\bar{v}} \\
\dot{\phi} \\
\bar{M}_x \\
\bar{M}_y \\
\bar{M}_z \\
\bar{V}_y \\
\bar{V}_z \\
\end{bmatrix} = \begin{bmatrix}
T_{14}(\bar{x}) & T_{15}(\bar{x}) & \bar{k}_\phi T_{16}(\bar{x}) & T_{18}(\bar{x}) & T_{19}(\bar{x}) & T_{110}(\bar{x}) \\
T_{24}(\bar{x}) & T_{25}(\bar{x}) & \bar{k}_\phi T_{26}(\bar{x}) & T_{28}(\bar{x}) & T_{29}(\bar{x}) & T_{210}(\bar{x}) \\
T_{54}(\bar{x}) & T_{55}(\bar{x}) & \bar{k}_\phi T_{56}(\bar{x}) & T_{58}(\bar{x}) & T_{59}(\bar{x}) & T_{510}(\bar{x}) \\
\end{bmatrix} \begin{bmatrix}
\bar{w} \\
\bar{v} \\
\bar{M}_x \\
\bar{M}_y \\
\bar{M}_z \\
\bar{V}_y \\
\bar{V}_z \\
\end{bmatrix} \quad \bar{x} = 0
\]
The mode shapes can now be computed from equation (32). If the differential equations of motion (eqs. (9), (10), and (11)) are decoupled by virtue of some parameters being zero, then the frequency equation will turn out to be the product of frequency equations of the uncoupled systems so that the eigenvalues of the uncoupled systems can be obtained from the coupled formulation. But then the coefficient matrix in equation (30) will be
singular and hence cannot be inverted. As a result, the mode shapes of the uncoupled modes cannot be determined from the coupled formulation.

By adopting a similar procedure for cyclic and scissor modes of Case I and for Case II, the following equations can be obtained for the determination of the mode shapes

Cyclic Modes:

\[
\begin{bmatrix}
\vec{w} \\
\vec{v} \\
\vec{\phi}
\end{bmatrix}
= \begin{bmatrix}
T_{13}(x) & T_{15}(x) & -k_\phi T_{16}(x) & T_{17}(x) & T_{19}(x) & T_{110}(x) \\
T_{23}(x) & T_{25}(x) & -k_\phi T_{26}(x) & T_{27}(x) & T_{29}(x) & T_{210}(x) \\
T_{53}(x) & T_{55}(x) & -k_\phi T_{56}(x) & T_{57}(x) & T_{59}(x) & T_{510}(x)
\end{bmatrix}
\begin{bmatrix}
a_{12} \\
a_{22} \\
a_{32} \\
a_{42} \\
a_{52}
\end{bmatrix}
\]

where

\[
\begin{bmatrix}
a_{12} \\
a_{22} \\
a_{32} \\
a_{42} \\
a_{52}
\end{bmatrix}
= \begin{bmatrix}
T_{13} & T_{15} & -k_\phi T_{16} & T_{17} & T_{19} & T_{110} \\
T_{63} & T_{65} & -k_\phi T_{66} & T_{67} & T_{69} & T_{610} \\
T_{73} & T_{75} & -k_\phi T_{76} & T_{77} & T_{79} & T_{710} \\
T_{83} & T_{85} & -k_\phi T_{86} & T_{87} & T_{89} & T_{810} \\
T_{93} & T_{95} & -k_\phi T_{96} & T_{97} & T_{99} & T_{910}
\end{bmatrix}^{-1}
\begin{bmatrix}
1 \\
0 \\
0 \\
0 \\
0
\end{bmatrix}
\]

Scissor Modes:

\[
\begin{bmatrix}
\vec{w} \\
\vec{v} \\
\vec{\phi}
\end{bmatrix}
= \begin{bmatrix}
T_{15}(x) & T_{17}(x) & T_{19}(x) & T_{110}(x) \\
T_{25}(x) & T_{27}(x) & T_{29}(x) & T_{210}(x) \\
T_{55}(x) & T_{57}(x) & T_{59}(x) & T_{510}(x)
\end{bmatrix}
\begin{bmatrix}
a_{13} \\
a_{23} \\
a_{33} \\
a_{43} \\
a_{53}
\end{bmatrix}
\]

29
where

\[
\begin{align*}
\alpha_{13} &= \begin{bmatrix} T_{15} - \frac{k}{\phi} T_{16} & T_{17} & T_{18} & T_{19} & T_{110} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \\
\alpha_{23} &= \begin{bmatrix} T_{65} - \frac{k}{\phi} T_{66} & T_{67} & T_{68} & T_{69} & T_{610} \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \\
\alpha_{33} &= \begin{bmatrix} T_{75} - \frac{k}{\phi} T_{76} & T_{77} & T_{78} & T_{79} & T_{710} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \\
\alpha_{43} &= \begin{bmatrix} T_{85} - \frac{k}{\phi} T_{86} & T_{87} & T_{88} & T_{89} & T_{810} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \\
\alpha_{53} &= \begin{bmatrix} T_{95} - \frac{k}{\phi} T_{96} & T_{97} & T_{98} & T_{99} & T_{910} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}
\end{align*}
\]

Case II:

\[
\bar{\phi}(x) = T_{11}(x) - \frac{k}{\phi} T_{12}(x)
\]

ORTHOGONALITY OF NATURAL MODES

The governing differential equations of motion given by equations (9), (10), and (11) can be expressed in operator notation as follows

\[
\begin{align*}
L_{11}[\phi] + L_{12}[v] + L_{13}[w] &= \omega^2 mk \frac{d^2 \phi}{dx^2} + \omega^2 \text{me sin } \beta v \\
&\quad + \omega^2 \text{me cos } \beta w \\
L_{21}[\phi] + L_{22}[v] + L_{23}[w] &= \omega^2 \text{me cos } \beta \phi + \omega^2 \text{me} \\
L_{31}[\phi] + L_{32}[v] + L_{33}[w] &= -\omega^2 \text{me sin } \beta \phi + \omega^2 \text{me}
\end{align*}
\]

where

\[
\begin{align*}
L_{11} &= -(GJ) \frac{d}{dx} - GJ \frac{d^2}{dx^2} + \Omega^2 \text{m(}k_2 - k_1\text{)} \cos 2\beta \\
L_{12} &= -\Omega^2 \text{mxe sin } \beta \frac{d}{dx} + \Omega^2 \text{me sin } \beta
\end{align*}
\]
Let $\omega_r$ and $\omega_s$ be two distinct eigenvalues and $(\phi_r, v_r, w_r)$ and $(\phi_s, v_s, w_s)$ be the corresponding eigenfunctions resulting from the solution of the problem described in equations (33), (34), and (35). The eigenvalue problems can be written as

$$L_{11}[\phi_r] + L_{12}[v_r] + L_{13}[w_r] = \omega_r^2 m k_m^2 \phi_r - \omega_r^2 m e \sin \beta v_r + \omega_r^2 m e \cos \beta w_r$$

(36)
\[ L_{21}[\phi_r] + L_{22}[v_r] + L_{23}[w_r] = \omega_r^2 \text{me} \cos \beta \phi_r + \omega_r^2 \text{mv}_r \]  

(37)

\[ L_{31}[\phi_r] + L_{32}[v_r] + L_{33}[w_r] = -\omega_r^2 \text{me} \sin \beta \phi_r + \omega_r^2 \text{mv}_r \]  

(38)

\[ L_{11}[\phi_s] + L_{12}[v_s] + L_{13}[w_s] = \omega_s^2 \text{mk} \frac{2}{m} \phi_s - \omega_s^2 \text{me} \sin \beta v_s + \omega_s^2 \text{me} \cos \beta w_s \]  

(39)

\[ L_{21}[\phi_s] + L_{22}[v_s] + L_{23}[w_s] = \omega_s^2 \text{me} \cos \beta \phi_s + \omega_s^2 \text{mv}_s \]  

(40)

\[ L_{31}[\phi_s] + L_{32}[v_s] + L_{33}[w_s] = -\omega_s^2 \text{me} \sin \beta \phi_s + \omega_s^2 \text{mv}_s \]  

(41)

Multiplying equation (36) by \( \phi_s \) and equation (39) by \( \phi_r \), subtracting one result from the other, and integrating both sides of the equation between 0 to \( R \) yields

\[
\int_0^R (\phi_s L_{11}[\phi_r] + \phi_s L_{12}[v_r] + \phi_s L_{13}[w_r]) \\
- \phi_r L_{11}[\phi_s] - \phi_r L_{12}[v_s] - \phi_r L_{13}[w_s] \, dx \\
= (\omega_r^2 - \omega_s^2) \int_0^R \text{mk} \frac{2}{m} \phi_r \phi_s \, dx \\
+ \int_0^R (-\omega_r^2 \text{me} \sin \beta v_r \phi_r + \omega_r^2 \text{me} \cos \beta w_r \phi_r) \, dx \\
+ \omega_s^2 \text{me} \sin \beta v_s \phi_s - \omega_s^2 \text{me} \cos \beta w_s \phi_r \, dx \\
\]

(42)

Using a similar procedure on equations (37) and (40) and on equations (38) and (41) the following two equations can be obtained
\[
\int_0^R \left( w_s L_21[\phi_r] + w_s L_22[v_r] + w_s L_23[w_r] \\
- w_r L_21[\phi_s] - w_r L_22[v_s] - w_r L_23[w_s] \right) dx \\
= (\omega_r^2 - \omega_s^2) \int_0^R mw_r w_s dx \\
+ \int_0^R (w_r^2 \omega_s \cos \beta \phi_r - w_s^2 \omega_s \cos \beta \phi_s v_r) dx \\
\] (43)

\[
\int_0^R \left( v_s L_31[\phi_r] + v_s L_32[v_r] + v_s L_33[w_r] \\
- v_r L_31[\phi_s] - v_r L_32[v_s] - v_r L_33[w_s] \right) dx \\
= (\omega_r^2 - \omega_s^2) \int_0^R mv_r v_s dx \\
+ \int_0^R (-w_r^2 \omega_s \sin \beta \phi_r v_s + w_s^2 \omega_s \sin \beta \phi_s v_r) dx \\
\] (44)

Adding equations (42), (43), and (44) yields

\[
\int_0^R \left( \phi_s L_{11}[\phi_r] + \phi_s L_{12}[v_r] + \phi_s L_{13}[w_r] \\
- \phi_r L_{11}[\phi_s] - \phi_r L_{12}[v_s] - \phi_r L_{13}[w_s] \\
+ w_s L_21[\phi_r] + w_s L_22[v_r] + w_s L_23[w_r] \\
- w_r L_21[\phi_s] - w_r L_22[v_s] - w_r L_23[w_s] \\
+ v_s L_31[\phi_r] + v_s L_32[v_r] + v_s L_33[w_r] \\
- v_r L_31[\phi_s] - v_r L_32[v_s] - v_r L_33[w_s] \right) dx \\
\] (continued)
Integrating by parts the integrals on the left-hand side of the above equation and simplifying the following equation is obtained:

\[
(\omega_r^2 - \omega_s^2) \int_0^R \left[ m k^2 \phi_r \phi_s + \omega_r \omega_s + \nu \nu_r \phi_s \phi_r \right] dx + e \sin \beta (\phi_r \omega_s + \phi_s \omega_r) + e \cos \beta (\phi_r \nu_s + \phi_s \nu_r) dx = \{GJ(\phi_r \omega_s - \phi_s \omega_r) \}
\]

\[
+ \{-v_r (A_w \nu + A'w \nu + C'v \nu + \Omega^2 m \nu \sin \beta \nu) \}
\]

\[
+ \{-w_r (B_s \nu + B'w \nu + A_v \nu + A'v \nu - \Omega^2 m \nu \cos \beta \nu) \}
\]

\[
+ \{v_r (A_w \nu + C'v \nu) \} + \{w_r (B_w \nu + A'v \nu) \}
\]

\[
+ \{v_s (A_w \nu + A'w \nu + C'v \nu + \Omega^2 m \nu \sin \beta \nu) \}
\]

\[
+ \{w_s (B_w \nu + B'w \nu + A_v \nu + A'v \nu - \Omega^2 m \nu \cos \beta \nu) \}
\]

\[
+ \{-w_s (B_w \nu + A'v \nu) \} + \{-v_s (A_w \nu + C'v \nu) \}
\]

If the boundary conditions of the system are such that the right-hand side of equation (45) is zero, then the differential eigenvalue problem as defined by equations (33), (34), and (35) is said to be a self-adjoint differential eigenvalue problem. The boundary conditions corresponding to fixed, hinge and free ends, etc. make the differential eigenvalue problem of equations (33), (34), and (35) self-adjoint, and the following orthogonality relationship can be identified.
The eigenfunctions \((\phi_r', \nu_r', \omega_r)\) and \((\phi_s', \nu_s', \omega_s)\) corresponding to the distinct eigenvalues \(\omega_r\) and \(\omega_s\) respectively are orthogonal in the following fashion

\[
\int_0^R \left[ \frac{k^2}{m} \phi_r' \phi_s' + \nu_r w_s + \nu_s w_r - e \sin \beta \left( \phi_r' \nu_s' + \phi_s' \nu_r' \right) \right. \\
+ e \cos \beta \left( \phi_r' w_s + \phi_s' w_r \right) \right] dx = 0
\]

The vanishing of the right-hand side of equation (45) can be demonstrated, for example, by considering the boundary conditions of the scissor modes. The boundary conditions given by equations (26a) and (26d) can be expressed in terms of the deflections as shown below

\[
w = v = \nu = \omega = 0, \quad GJ\phi' = -k\phi \quad \text{at} \quad x = 0
\]

\[
\begin{align*}
M_x &= GJ\phi' = 0 \\
M_y &= Bw'' + Av'' = 0 \\
M_z &= Aw'' + Cv'' = 0 \\
-\nu_y &= Aw''' + A'w'' + Cv''' + C'v'' + \Omega^2 \text{mex} \sin \beta \phi \\
-\nu_z &= Bw''' + B'w'' + Av''' + A'v'' - \Omega^2 \text{mex} \cos \beta \phi
\end{align*}
\]

By substituting equations (47) and (48) into the right-hand side of equation (45), it is obvious that it vanishes by the fact that the eigenfunctions satisfy all the boundary conditions.

When a concentrated mass \(M\) is attached to the blade at the spanwise location \(x = x_M\), the orthogonality relation given by equation (46) can be rewritten as
\[
\int_0^R \left\{ [m + \delta (x - x_M)]M \{k m^2 \phi_r s + w_r s + v_r s
\right.
\]
\[
- \epsilon \sin \beta (\phi_r s + \phi_s v) + \epsilon \cos \beta (\phi_r s + \phi_s v) \} \right\} dx = 0
\]

or

\[
\int_0^R \left\{ m^2 \phi_r s + w_r s + v_r s - \epsilon \sin \beta (\phi_r s + \phi_s v)
\right.
\]
\[
+ \epsilon \cos \beta (\phi_r s + \phi_s v) \right\} dx + M\{d^2 \phi_r s + w_r s
\]
\[
+ v_r s - d \sin \beta (\phi_r s + \phi_s v)
\]
\[
+ d \cos \beta (\phi_r s + \phi_s v) \right\} \bigg|_{x=x_M} = 0
\] (49)

The orthogonality relations given by equations (46) and (49) are valid as long as boundary conditions are self-adjoint and independent of eigenvalues.

**NUMERICAL RESULTS AND DISCUSSION**

The natural frequencies and associated mode shapes of the Bell Helicopter OH-58A, 206A-1 seesaw rotor blade are determined by using the transmission matrix method. The effects of the following parameters on the natural frequencies corresponding to the collective, cyclic and scissor modes with coupled flapwise bending, chordwise bending, and torsional degrees of freedom are determined:

1. Effect of the collective pitch,
2. Effect of the rotation,
3. Effect of the spanwise location of the concentrated mass,
4. Effect of the chordwise location of the concentration mass, and
5. Effect of the magnitude of the concentrated mass.
As a special subcase the effect of parameter 5 on the pure torsional frequencies of the blade is determined. The properties of the Bell Helicopter OH-58A, 206A-1 are given in table 1. The natural frequencies obtained by varying parameters 1 to 5 are presented in tables 2 to 6 for collective, cyclic and scissor modes. The mode shapes with and without the concentrated mass are given in tables 7 to 9 for collective, cyclic, and scissor modes. The pure torsional frequencies are given in table 10, and the corresponding mode shapes are shown in table 11. The percentage effects of various parameters are given in table 12.

Effect of Collective Pitch

The collective pitch significantly alters the predominantly bending natural frequencies corresponding to the collective and cyclic modes. The effect is more significant on the cyclic mode frequencies compared to the collective mode frequencies. The predominantly torsion and rigid body mode frequencies are not affected by the variation of collective pitch. The scissor mode frequencies are not at all affected by collective pitch. However, the mode shapes including the scissor modes will be altered significantly by the variation of collective pitch. A particular mode is considered as predominant in this report (whether it is a predominantly flapwise bending or chordwise bending or torsion) by comparing the following quantities in a given mode:

1. Maximum torsional deformation in radians,
2. Maximum nondimensional flapwise deflection with respect to the semichord \((w/b_0)\), and
3. Maximum nondimensional chordwise deflection with respect to the semichord \((v/b_0)\).

Effect of Rotational Speed

The rotational speed significantly changes the natural frequencies and mode shapes as expected.
Effect of Magnitude of Attached Mass

The attached point mass significantly affects the predominantly torsional mode frequencies. As the magnitude of the mass is increased, the predominantly bending mode adjacent to a torsional mode will be altered to an additional torsional mode, and in such cases the frequencies and mode shapes are also affected significantly.

Effect of Spanwise Location

In this case predominantly torsional frequencies are also affected. The effect is increased as the mass is moved towards the tip. When a mass of 1.5 lb is attached at midspan, the effect on the predominantly bending frequencies is insignificant, and the first predominantly torsional mode frequency is altered by approximately 5 percent. But when the same mass is moved to the tip, in addition to the significant effect of torsional mode frequency (20%), the bending mode adjacent to the original torsional mode is altered to an additional predominantly torsional mode.

Effect of Chordwise Location

If the mass is attached nearer to elastic axis (quarter chord distance behind the leading edge), the effect on the frequencies including the torsional frequency is insignificant. When the mass is attached right at the leading edge, the effect on the bending frequencies is negligible and the torsional frequency is affected by four to five percent. When the mass is attached at semichord distance away from the leading edge, in addition to the significant effect on torsional frequency (20%), the bending mode adjacent to the original torsional frequency is altered to an additional torsional mode.

Effect of Mass on Pure Torsional Frequencies

The pure torsional frequencies are affected significantly by the addition of a concentrated mass as can be seen from table 10. The effect on the first mode frequency by addition of a 2.0 lb mass at the tip of the blade at a semichord distance away from the leading edge is 28 percent. The percentage effect decreases with the increasing number of mode.
Generalized Masses

The generalized masses due to the addition of a concentrated mass are affected due to the following two reasons

1. Due to the changes in the natural frequencies and mode shapes, and
2. Due to the additional concentrated mass, in which case the new orthogonality relation given by equation (49) should be used instead of the one given by equation (46).

The changes in the generalized masses could be significant. Hence, in the response and stability analyses using the modal analysis, the natural vibration characteristics should be recomputed accounting for the additional mass.

Finally, the description and features of the computer programs developed are given in Appendix A. The listings of the programs and sample output are given in Appendix B.
Some Pertinent Data of the Blade

1. Twist of the blade = -10.6 deg (linear)
2. Collective pitch range = 8 to 22 deg
3. Normal operating rotational speed = 354 RPM
4. Blade chord = 13 in. (uniform)
5. Span of the blade
   (Axis of rotation to the tip of the blade) = 211.8 in.
6. Control system spring rate = 225000 in.-lb/Rad
7. Distance of the blade from the axis of rotation = 18.5 in.
Table 1. Elastic properties of the blade.

<table>
<thead>
<tr>
<th>No.</th>
<th>Station (in.)</th>
<th>( m \times 10^{-2} ) (lb·sec(^2)/in.(^2))</th>
<th>( EI_1 \times 10^8 ) (lb·in.(^2))</th>
<th>( EI_2 \times 10^8 ) (lb·in.(^2))</th>
<th>( GJ \times 10^8 ) (lb·in.(^2))</th>
<th>( e ) (in.)</th>
<th>( \frac{mk^2}{m_1} \times 10^{-2} ) Through Center of Gravity</th>
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<th>( EI_1 \times 10^8 ) (lb-in.^2)</th>
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<th>( GJ \times 10^8 ) (lb-in.^2)</th>
<th>( e ) (in.)</th>
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Table 2. Effect of collective pitch on natural frequencies, $\Omega = M = 0$.

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<th>III Mode</th>
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<td>51.4956 FB</td>
<td>153.0226 FB</td>
<td>220.4402 CB</td>
<td>295.7369 FB</td>
<td>329.0796 T</td>
</tr>
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<td>36.3463 CB</td>
<td>51.4956 FB</td>
<td>153.0226 FB</td>
<td>220.4402 CB</td>
<td>295.7369 FB</td>
<td>329.0796 T</td>
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<td>295.7369 FB</td>
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Table 3. Effect of rotational speed on natural frequencies, collective pitch = 15°, M = 0.

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<th>IV Mode</th>
<th>V Mode</th>
<th>VI Mode</th>
<th>VII Mode</th>
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<td>270</td>
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<td>95.7667 FB</td>
<td>169.7463 CB</td>
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<td>117.0175 FB</td>
<td>179.5617 CB</td>
<td>250.7197 FB</td>
<td>330.0171 T</td>
<td>394.2283 FB</td>
<td>541.7232 FB</td>
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<td>51.4956 FB</td>
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<td>253.7052 FB</td>
<td>330.0189 T</td>
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Table 4. Effect of magnitude of attached mass on natural frequencies, $x_M = 211.8$ in., $d = 9.7$ in., $\Omega = 354$ RPM, collective pitch $= 15^\circ$.

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<th>Magnitude of Attached Mass (lb)</th>
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<th>III Mode</th>
<th>IV Mode</th>
<th>V Mode</th>
<th>VI Mode</th>
<th>VII Mode</th>
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<td>179.5617 CB</td>
<td>250.7197 FB</td>
<td>330.0171 T</td>
<td>394.2283 RB</td>
<td>541.7232 FB</td>
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<td>200.2380 FB</td>
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<td>117.4931 FB</td>
<td>228.0937 CB</td>
<td>237.0499 T</td>
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Table 5. Effect of spanwise location of concentrated mass on natural frequencies, $M = 1.5$ lb, $\Omega = 354$ RPM, collective pitch $= 15^\circ$, chordwise distance of mass from leading edge $= 6.5$ in.

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<tr>
<th>Nature of the Modes</th>
<th>Spanwise Location and Chordwise Location From Elastic Axis (in.)</th>
<th>Natural Frequencies (Rad/sec), FB = Flapwise Bending, CB = Chordwise Bending, T = Torsion, RB = Rigid Body Mode</th>
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<tbody>
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<td>$x_M = 158.85$</td>
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<td>$d = 9.7$</td>
<td>$x_M = 158.85$</td>
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<td>$x_M = 158.85$</td>
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<td>Scissor Modes</td>
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<td>$d = 9.7$</td>
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<td>$d = 9.7$</td>
<td>$x_M = 158.85$</td>
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<td>$d = 9.7$</td>
<td>$x_M = 158.85$</td>
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Table 6. Effect of chordwise location of concentrated mass on natural frequencies, $M = 1.5$ lb, $\Omega = 354$ RPM, $x_M = 211.8$ in.

<table>
<thead>
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<th>Nature of the Modes</th>
<th>Chordwise Locations From Shear Center and Leading Edge (in.)</th>
<th>Natural Frequencies (Rad/sec), $FB = $ Flapwise Bending, $CB = $ Chordwise Bending, $T = $ Torsion, $RB = $ Rigid Body Mode</th>
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<td>$43.1161$ FB, $117.4469$ FB, $176.5766$ CB, $237.0735$ T, $264.9626$ T, $393.2712$ FB, $539.0953$ FB</td>
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Table 7. Collective mode shapes, Ω = 354 RPM, collective pitch = 15°, x_M = 211.8 in., d = 9.7 in., d_M = 6.5 in.

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(continued)
Table 7. Collective mode shapes, $\Omega = 354$ RPM, collective pitch = $15^\circ$, $x_M = 211.8$ in., $d = 9.7$ in., $d_{\&e} = 6.5$ in. (continued).

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Table 7. Collective mode shapes, \( \Omega = 354 \) RPM, collective pitch = 15°, \( x_M = 211.8 \) in., \( d = 9.7 \) in., \( d_e = 6.5 \) in. (continued).

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(continued)
Table 7. Collective mode shapes, $\Omega = 354$ RPM, collective pitch $= 15^\circ$, $x_M = 211.8$ in., $d = 9.7$ in., $d_{xe} = 6.5$ in. (concluded).

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Table 8. Cyclic mode shapes, \( \Omega = 354 \) RPM, collective pitch = 15°, \( x_M = 211.8 \) in., \( d = 9.7 \) in., \( \bar{d}_{ae} = 6.5 \) in.

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Table 8. Cyclic mode shapes, $\Omega = 354$ RPM, collective pitch = $15^\circ$, $x_M = 211.8$ in., $d = 9.7$ in.,
$d_{2e} = 6.5$ in. (continued).

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(continued)
Table 8. Cyclic mode shapes, $\Omega = 354$ RPM, collective pitch $= 15^\circ$, $x_M = 211.8$ in., $d = 9.7$ in., $d_e = 6.5$ in. (continued)

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Table 8. Cyclic mode shapes, $\Omega = 354$ RPM, collective pitch = $15^\circ$, $x_M = 211.8$ in., $d = 9.7$ in., $d_e = 6.5$ in. (concluded).

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Table 9. Scissor mode shapes, $\Omega = 354$ RPM, collective pitch = $15^\circ$, $x_M = 211.8$ in., $d = 9.70$ in., $d_{fe} = 6.5$ in.

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(continued)
Table 9. Scissor mode shapes, $\Omega = 354$ RPM, collective pitch $= 15^\circ$, $x_M = 211.8$ in., $d = 9.70$ in., $d_{ve} = 6.5$ in. (continued).

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Table 9. Scissor mode shapes, \( \Omega = 354 \text{ RPM} \), collective pitch = 15\(^\circ\), \( x_M = 211.8 \text{ in.} \), \( d = 9.70 \text{ in.} \), \( \Delta x_e = 6.5 \text{ in.} \) (continued).

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<td>0.2719</td>
<td>-0.0063</td>
<td>0.2642</td>
<td>-0.3385</td>
<td>-0.3224</td>
<td>-0.3385</td>
<td>-0.3224</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.2873</td>
<td>-0.7704</td>
<td>0.0007</td>
<td>0.2393</td>
<td>0.0624</td>
<td>0.0419</td>
<td>0.0624</td>
<td>0.0419</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.0306</td>
<td>-0.2573</td>
<td>0.3925</td>
<td>0.3354</td>
<td>-0.0651</td>
<td>-0.1389</td>
<td>-0.0651</td>
<td>-0.1389</td>
</tr>
</tbody>
</table>

(continued)
Table 9. Scissor mode shapes, $\Omega = 354$ RPM, collective pitch $= 15^\circ$, $x_M = 211.8$ in., $d = 9.70$ in., $d_{x\theta} = 6.5$ in. (concluded)

<table>
<thead>
<tr>
<th>$\frac{X}{R}$</th>
<th>1. $w/b_0$</th>
<th>2. $v/b_0$</th>
<th>V Mode $M = 0$</th>
<th>V Mode $M = 2.0$ lb</th>
<th>VI Mode $M = 0$</th>
<th>VI Mode $M = 2.0$ lb</th>
<th>VII Mode $M = 0$</th>
<th>VII Mode $M = 2.0$ lb</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.6</td>
<td>1</td>
<td>-0.2356</td>
<td>-0.3005</td>
<td>-0.0130</td>
<td>-0.1799</td>
<td>-0.6291</td>
<td>-0.6602</td>
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</tr>
<tr>
<td></td>
<td>2</td>
<td>0.3458</td>
<td>-0.5884</td>
<td>0.0022</td>
<td>0.2615</td>
<td>0.1380</td>
<td>0.1360</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.0430</td>
<td>-0.4077</td>
<td>-0.0892</td>
<td>0.5151</td>
<td>-0.0892</td>
<td>-0.1962</td>
<td></td>
</tr>
<tr>
<td>0.7</td>
<td>1</td>
<td>-0.8320</td>
<td>-0.8314</td>
<td>0.0037</td>
<td>-0.5518</td>
<td>-0.0630</td>
<td>-0.1374</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.3151</td>
<td>-0.2804</td>
<td>0.0001</td>
<td>0.2078</td>
<td>0.0641</td>
<td>0.0839</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.0525</td>
<td>-0.5493</td>
<td>-0.1120</td>
<td>0.6773</td>
<td>-0.1120</td>
<td>-0.2451</td>
<td></td>
</tr>
<tr>
<td>0.8</td>
<td>1</td>
<td>-0.8808</td>
<td>-0.9040</td>
<td>0.0291</td>
<td>-0.5479</td>
<td>0.5705</td>
<td>0.5310</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.1477</td>
<td>0.0658</td>
<td>-0.0034</td>
<td>0.0545</td>
<td>-0.0359</td>
<td>-0.0056</td>
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<tr>
<td></td>
<td>3</td>
<td>0.0565</td>
<td>-0.6812</td>
<td>-0.1285</td>
<td>0.8153</td>
<td>-0.1285</td>
<td>-0.2792</td>
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</tr>
<tr>
<td>0.9</td>
<td>1</td>
<td>-0.1730</td>
<td>-0.2406</td>
<td>-0.0199</td>
<td>-0.1020</td>
<td>0.3072</td>
<td>0.3658</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>-0.1465</td>
<td>0.3860</td>
<td>0.0012</td>
<td>-0.1799</td>
<td>-0.0423</td>
<td>-0.0134</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.0551</td>
<td>-0.7999</td>
<td>-0.1318</td>
<td>0.9248</td>
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<td>-0.2902</td>
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</tr>
<tr>
<td>1.0</td>
<td>1</td>
<td>0.7545</td>
<td>1.0000</td>
<td>-0.2017</td>
<td>0.5505</td>
<td>-0.5594</td>
<td>-0.7009</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>-0.4282</td>
<td>0.6675</td>
<td>0.0190</td>
<td>-0.4385</td>
<td>0.0334</td>
<td>0.0614</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.0565</td>
<td>-0.8873</td>
<td>-0.1371</td>
<td>1.0000</td>
<td>-0.1371</td>
<td>-0.2931</td>
<td></td>
</tr>
</tbody>
</table>
Table 10. Effect of magnitude of concentrated mass on torsional frequencies, $\Omega = 354$ RPM, $d = 9.70$ in., $x_M = 211.8$ in.

<table>
<thead>
<tr>
<th>Magnitude of the Mass</th>
<th>I Mode</th>
<th>II Mode</th>
<th>III Mode</th>
<th>IV Mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>335.2502</td>
<td>913.6214</td>
<td>1494.7750</td>
<td>2036.0238</td>
</tr>
<tr>
<td>0.5</td>
<td>303.0671</td>
<td>845.0031</td>
<td>1388.6847</td>
<td>1909.5994</td>
</tr>
<tr>
<td>1.0</td>
<td>277.4304</td>
<td>798.8885</td>
<td>1329.8973</td>
<td>1852.5323</td>
</tr>
<tr>
<td>1.5</td>
<td>256.8390</td>
<td>768.4413</td>
<td>1297.7905</td>
<td>1825.8265</td>
</tr>
<tr>
<td>2.0</td>
<td>240.0246</td>
<td>747.5134</td>
<td>1278.4021</td>
<td>1810.9621</td>
</tr>
</tbody>
</table>
Table 11. Pure torsional mode shapes, $\Omega = 354$ RPM, $d = 9.7$ in., $x_M = 211.8$ in.

<table>
<thead>
<tr>
<th>Station $x_R$</th>
<th>Mode I $M = 0$</th>
<th>Mode I $M = 2.0$ lb</th>
<th>Mode II $M = 0$</th>
<th>Mode II $M = 2.0$ lb</th>
<th>Mode III $M = 0$</th>
<th>Mode III $M = 2.0$ lb</th>
<th>Mode IV $M = 0$</th>
<th>Mode IV $M = 2.0$ lb</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>-0.2758</td>
<td>-0.2423</td>
<td>-0.3990</td>
<td>-0.3941</td>
<td>0.3767</td>
<td>0.3726</td>
<td>-0.4141</td>
<td>-0.4076</td>
</tr>
<tr>
<td>0.1</td>
<td>-0.2695</td>
<td>-0.2368</td>
<td>-0.3897</td>
<td>-0.3851</td>
<td>0.3676</td>
<td>0.3637</td>
<td>-0.4035</td>
<td>-0.3975</td>
</tr>
<tr>
<td>0.2</td>
<td>-0.2226</td>
<td>-0.1976</td>
<td>-0.2786</td>
<td>-0.2914</td>
<td>0.1870</td>
<td>0.2166</td>
<td>-0.0988</td>
<td>-0.1438</td>
</tr>
<tr>
<td>0.3</td>
<td>-0.0513</td>
<td>-0.0593</td>
<td>0.2022</td>
<td>0.1019</td>
<td>-0.5575</td>
<td>-0.4164</td>
<td>0.8996</td>
<td>0.7895</td>
</tr>
<tr>
<td>0.4</td>
<td>0.1770</td>
<td>0.1246</td>
<td>0.7467</td>
<td>0.5891</td>
<td>-0.9313</td>
<td>-0.9091</td>
<td>0.5508</td>
<td>0.8170</td>
</tr>
<tr>
<td>0.5</td>
<td>0.3959</td>
<td>0.3050</td>
<td>0.9996</td>
<td>0.9207</td>
<td>-0.3831</td>
<td>-0.7277</td>
<td>-0.7391</td>
<td>-0.2955</td>
</tr>
<tr>
<td>0.6</td>
<td>0.5922</td>
<td>0.4766</td>
<td>0.8374</td>
<td>0.9937</td>
<td>0.5691</td>
<td>0.0273</td>
<td>-0.7048</td>
<td>-0.9761</td>
</tr>
<tr>
<td>0.7</td>
<td>0.7587</td>
<td>0.6361</td>
<td>0.3642</td>
<td>0.8185</td>
<td>1.0000</td>
<td>0.7720</td>
<td>0.4836</td>
<td>-0.3710</td>
</tr>
<tr>
<td>0.8</td>
<td>0.8876</td>
<td>0.7795</td>
<td>-0.2406</td>
<td>0.4434</td>
<td>0.5125</td>
<td>0.9863</td>
<td>0.9058</td>
<td>0.7147</td>
</tr>
<tr>
<td>0.9</td>
<td>0.9720</td>
<td>0.9031</td>
<td>-0.7554</td>
<td>-0.0403</td>
<td>-0.4430</td>
<td>0.5167</td>
<td>-0.1117</td>
<td>0.8571</td>
</tr>
<tr>
<td>1.0</td>
<td>1.0000</td>
<td>1.0000</td>
<td>-0.9481</td>
<td>-0.5096</td>
<td>-0.9081</td>
<td>-0.3537</td>
<td>-0.8792</td>
<td>-0.2628</td>
</tr>
</tbody>
</table>
Table 12. Percentage effects on the natural frequencies.

<table>
<thead>
<tr>
<th>Variable Parameter</th>
<th>Range of the Parameter</th>
<th>Constant Parameters</th>
<th>Nature of the Mode Shapes</th>
<th>I Mode</th>
<th>II Mode</th>
<th>III Mode</th>
<th>IV Mode</th>
<th>V Mode</th>
<th>VI Mode</th>
<th>VII Mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>Collective Pitch</td>
<td>0, 22 deg.</td>
<td>Ω = 0, H = 0</td>
<td>Collective</td>
<td>0.0</td>
<td>4.7</td>
<td>1.4</td>
<td>-5.5</td>
<td>3.8</td>
<td>-0.5</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Cyclic</td>
<td></td>
<td>0.0</td>
<td>-28.0</td>
<td>10.3</td>
<td>19.6</td>
<td>-5.4</td>
<td>7.8</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Scissor</td>
<td></td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Rotational Speed</td>
<td>0, 354 RPM</td>
<td>M = 0</td>
<td>Collective</td>
<td>---</td>
<td>1338.3</td>
<td>251.6</td>
<td>71.9</td>
<td>97.2</td>
<td>33.6</td>
<td>64.6</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Cyclic</td>
<td></td>
<td>---</td>
<td>80.4</td>
<td>152.0</td>
<td>90.3</td>
<td>25.3</td>
<td>30.9</td>
<td>0.3</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Scissor</td>
<td></td>
<td>367.0</td>
<td>23.9</td>
<td>127.3</td>
<td>15.1</td>
<td>11.6</td>
<td>17.0</td>
<td></td>
</tr>
<tr>
<td>Magnitude of the Attached Mass</td>
<td>0; 2.0 lb</td>
<td>( x_{\text{H}} = 211.8 \text{ in.} )</td>
<td>Collective</td>
<td>-0.5</td>
<td>0.5</td>
<td>-2.3</td>
<td>-9.4</td>
<td>-20.8</td>
<td>-0.3</td>
<td>-0.5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \Delta_{\text{x}} = 6.5 \text{ in.} )</td>
<td>Cyclic</td>
<td>-2.1</td>
<td>-1.0</td>
<td>0.8</td>
<td>-1.5</td>
<td>-2.4</td>
<td>-20.1</td>
<td>-8.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \Omega = 354 \text{ RPM} )</td>
<td>Scissor</td>
<td>-2.6</td>
<td>-1.1</td>
<td>0.5</td>
<td>-3.9</td>
<td>-9.3</td>
<td>-19.5</td>
<td>-0.1</td>
</tr>
<tr>
<td>Spanwise Location of the Attached Mass</td>
<td>105.9, 211.8 in.</td>
<td>( d_{\text{x}} = 6.5 \text{ in.} )</td>
<td>Collective</td>
<td>-0.4</td>
<td>2.1</td>
<td>-0.4</td>
<td>-4.9</td>
<td>-16.0</td>
<td>0.1</td>
<td>-0.4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( M = 1.5 \text{ lb} )</td>
<td>Cyclic</td>
<td>-1.4</td>
<td>-0.7</td>
<td>1.8</td>
<td>-0.9</td>
<td>0.1</td>
<td>-15.0</td>
<td>-4.2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \Omega = 354 \text{ RPM} )</td>
<td>Scissor</td>
<td>-1.8</td>
<td>-0.8</td>
<td>2.1</td>
<td>0.0</td>
<td>-5.9</td>
<td>-14.9</td>
<td>0.2</td>
</tr>
<tr>
<td>Chordwise Location of the Attached Mass from the Leading Edge</td>
<td>-3.25, 6.5 in.</td>
<td>( x_{\text{H}} = 211.8 \text{ in.} )</td>
<td>Collective</td>
<td>0.0</td>
<td>-0.1</td>
<td>-0.3</td>
<td>-4.2</td>
<td>-19.4</td>
<td>0.5</td>
<td>1.4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( M = 1.5 \text{ lb} )</td>
<td>Cyclic</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>-0.7</td>
<td>-0.3</td>
<td>-15.3</td>
<td>-7.8</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \Omega = 354 \text{ RPM} )</td>
<td>Scissor</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>-0.3</td>
<td>-5.3</td>
<td>-18.4</td>
<td>0.6</td>
</tr>
<tr>
<td>Magnitude of the Mass</td>
<td>0, 2.0 lb</td>
<td>( x_{\text{H}} = 211.8 \text{ in.} )</td>
<td>Pure Torsion</td>
<td>-20.4</td>
<td>-18.2</td>
<td>-14.5</td>
<td>-11.1</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( d_{\text{x}} = 6.5 \text{ in.} )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \Omega = 354 \text{ RPM} )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
REFERENCES


APPENDIX A

DESCRIPTION OF THE COMPUTER PROGRAMS
Fortran Program I:

Computes the natural frequencies and associated mode shapes of a nonuniform, pretwisted rotor blade with combined flapwise bending, chordwise bending, and torsional degrees of freedom.

I Data Card: IBC, ISTAGE, NS

    FORMAT I!, I!, I3

IBC: Variable indicates the nature of modes required

IBC = 1, implies collective modes
IBC = 2, implies cyclic modes
IBC = 3, implies scissor modes

ISTAGE: Program performs two functions

ISTAGE = 1, computes the values of the frequency determinants only
ISTAGE = 2, computes the natural frequencies and mode shapes

NS = Number of stations at which data is provided

II Data Card: SPAN, OMEGA, B, TSR, RPITCH

    FORMAT 5 E 14.7

SPAN = Span of the blade (inches), distance between axis of rotation and tip of the blade
OMEGA = Rotational speed of the blade (rotations per minute)
B = Semichord at the root (inches)
TSR = Control system spring rate (in.-lb/rad)
RPITCH = Collective pitch setting at the root (degrees)

III Data Card: DEB

    FORMAT E 14.7

DEB = Distance of the blade from the axis of rotation
The following data should be provided in the order

STA, MASS, EI1, EI2, GJ, E, BETA, KMIS, KM2S

FORMAT 5E 14.7 on each card

STA = Station locations (inches)

The first station should correspond to the axis of rotation, and the last station should correspond to the tip of the blade. The distance between the stations need not be equal. For accuracy it is preferable to choose the stations such that the variation of structural properties in between can be approximated by a linear relationship.

MASS = Mass per unit span (lb-sec²/in.²)

EI1 = Flapwise bending stiffness (lb-in.²)

EI2 = Chordwise bending stiffness (lb-in.²)

GJ = Torsional stiffness (lb-in.²)

E = Distance between mass and elastic axes (inches) positive when mass axis lies ahead of elastic axis

BETA = Twist of the blade not including the collective pitch (degrees)

KMIS = Mass moment of inertia of the cross-sectional mass about the chord (lb-sec²)

KM2S = Mass moment of inertia of the cross-sectional mass about an axis perpendicular to the chord passing through the shear center

If the data is provided say at 10 stations, each of the above variables take 2 cards and a total of 18 cards.

If the computer variable ISTAGE = 1 (see the 1st Data Card) then the following card is the last card, otherwise it is last but one card

H1, H2, H3

FORMAT 3 E 14.7

H1 = Starting frequency (rad/sec)

H2 = Frequency increment (rad/sec)

H3 = Ending frequency (rad/sec)
Case 1. ISTAGE = 1.

In this case frequency determinants are only computed for various frequencies. It starts with frequency $H_1$ and increments by $H_2$ and goes up to the value $H_3$. For each of these values it computes the nondimensional frequency determinant and prints. It prints three columns:

I column: Frequency (rad/sec)

II column: Nondimensional frequency $\left(\sqrt{\frac{M}{EI_{10}}} \cdot \text{Frequency}\right)$

III column: Nondimensional frequency determinant value

Case II. ISTAGE = 2.

In this case the natural frequencies and mode shapes are computed. The natural frequencies are computed by frequency scanning technique. Sign changes in the values of frequency determinant are detected starting from value $H_1$ at steps of $H_2$ till the required sign changes are detected or the value $H_3$ is reached. So the required number of frequencies that lie between $H_1$ and $H_3$, whichever is less, are computed. If two frequencies lie closer than increment $H_2$, then there is a chance of missing those frequencies, so $H_2$ has to be chosen such that no two frequencies are closer than $H_2$. This can be estimated by looking at the frequency versus frequency determinant values, which can be obtained when the program is executed under ISTAGE = 1.

The output under ISTAGE = 2 prints the natural frequencies and mode shapes. The natural frequencies are given in (rad/sec), Hertz, and in nondimensional units $\left(\frac{m g R_t}{EI_{10}}\right)$, and the mode shape deflections are given by the following:

- Flapwise deflection $w/b_0$ (nondimensional)
- Chordwise deflection $v/b_0$ (nondimensional)
- Torsional deflection $\phi$ (radians)

The following card is required when the program is executed under ISTAGE = 2.

NF, ITER, BLANK, DOT, STAR, INC

FORMAT I2, I1, 3A1, I2

NF = Number of frequencies required not exceeding 10
ITER = Number of subdivisions required before interpolation, usually ITER = 1 is sufficient. If ITER = 1, after the frequency is detected within the interval H2, then interval H2 is subdivided and the frequency is detected within the interval of H2/10 before computing the actual frequency by interpolation. If ITER = 2, the interval is reduced to H2/100.

BLANK = one blank space

DOT = .

STAR = *

INC = increment in the mode number in the normal execution of the program INC = 0. If the first two frequencies are separated by at least 1.0 rad/sec and the third frequency onwards are separated by at least 20 rad/sec, it is not economical (computation wise) to scan the entire frequency range by incrementing at 1.0 rad/sec. In such a case the first two frequencies are detected by scanning with H2 = 1.0 in one execution and the third frequency onwards are computed in a second execution with H2 = 20.0. In the second execution INC = 2 must be fed to keep track of the correct mode number.

Total number of data cards

ISTAGE = 1

Integer greater than \( (-\frac{NS}{5}) + 4 \)

ISTAGE = 2

Integer greater than \( (-\frac{NS}{5}) + 5 \)

Where NS = number of stations
Fortran Program II:

Computes the natural frequencies and associated mode shapes of a nonuniform, pretwisted rotor blade with combined flapwise bending, chordwise bending, and torsional degrees of freedom with an attached concentrated mass at any spanwise and chordwise locations.

I Data Card: IBC, ISTAGE, NS

   FORMAT I1, I1, I3

Similar to Program I

II Data Card: SPAN, OMEGA, B, TSR, PMASS

   FORMAT 5 E 14.7

SPAN, OMEGA, B, TSR are defined in Program I

PMASS = Magnitude of the attached mass (lb)

III Data Card: CLP, SLP, RPITCH, DEB

RPITCH, DEB are defined in Program I.

CLP = Chordwise location of the mass from shear center (inches), positive forward of shear center

SLP = Spanwise location of the mass (inches)

Rest of the data is same as in Program I.
Fortran Program III

Computes the natural frequencies and associated mode shapes of a nonuniform rotor blade with pure torsional degree of freedom with an attached concentrated mass at any spanwise and chordwise locations.

I Data Card: ISTAGE, NS

    FORMAT Il, I3

Defined in Program I.

II Data Card: SPAN, B, TSR, OMEGA

    FORMAT 4 E 14.7

Defined in Program I.

III Data Card: PMASS, CLP, SLP

    FORMAT 3 E 14.7

Defined in Program II.

IV Data Card: onwards

STA, MASS, GJ, KMIS, KM2S

    FORMAT 5 E 14.7 on each card

Described in Program I.

After the structural data one or two cards are to be introduced depending on the value of ISTAGE, as discussed under Program I.
APPENDIX B

LISTINGS OF THE COMPUTER PROGRAMS AND SAMPLE OUTPUT
C
C
C NATURAL VIBRATION CHARACTERISTICS; COUPLED FLAPWISE BENDING, CHORD-WISE BENDING AND TORSION; SEE-SAW ROTOR, COLLECTIVE, CYCLIC AND SCISSOR MODES. USES FUNCTION DET AND SUBROUTINES INTPOL, PLOT.
C NATURE AND SHAPES
C
C
C
C ------------------------------------------
C DECLARATION AND COMMON STATEMENTS
C
REAL M1S, KM1S, KM2S
DIMENSION E(101), EI1(101), EI2(101), G1(101), KMS(101), KM1S(101),
1KM2S(101), M1S(101), B1A(101), D1(101), D2(101), D3(101), D4(101),
2D5(101), D6(101), D7(101), D8(101), D9(101), D10(101), D11(101),
3D12(101), STA(101), PHI(51), W(51), V(51), SL(51), FREQEN(10)
COMMON/X1/FREQEN,H1,H2,H3,ITER, IJK
COMMON/X2/J1,PP,FRE,HERTZ,SL,BLANK, DOT, STAR
COMMON/X3/STA,SPAN
COMMON/X4/IBC
COMMON/X5/D1,D2,D3,D4,D5,D6,D7,D8,D9,D10,D11,D12,N1,N2
COMMON/X6/TSR
COMMON/X7/NS
COMMON/X8/OMEGAN
COMMON/X11/IND
C
C
C THIS SECTION READS THE DATA OF THE SYSTEM
C
READ(5,5) IBC, ISTAGE, NS
READ(5,10) SPAN, OMEGA, B, TSR, RPITCH, DEB
READ(5,10) (STA(J), J=1, NS), (M1S(J), J=1, NS), (EI1(J), J=1, NS), (EI2(J)
1,J=1,NS), (G(J), J=1, NS), (E(J), J=1, NS), (B1A(J), J=1, NS), (KMS(J), J=
21, NS), (KM2S(J), J=1, NS)
READ(5,10) H1, H2, H3
IF(ISTAGE.EQ.2) READ(5,15) NF, ITER, BLANK, DOT, STAR, INC
C
C THIS SECTION PRINTS THE DATA OF THE SYSTEM
C
WRITE(6,20)
WRITE(6,25)
IF(IBC.EQ.1) WRITE(6,30)
IF(IBC.EQ.2) WRITE(6,35)
IF (IBC .EQ. 3) WRITE(6, 40)
IF (ISTAGE .EQ. 2) WRITE(6, 45) NF
WRITE(6, 135) H1
WRITE(6, 150) H2
WRITE(6, 50) H3
IF (ISTAGE .EQ. 2) WRITE(6, 155) INC
WRITE(6, 55) SPAN
WRITE(6, 60) OMEGA
WRITE(6, 65) B
WRITE(6, 70) TSR
WRITE(6, 190) RPI
WRITE(6, 195) DE
WRITE(6, 90) NS
WRITE(6, 95)
WRITE(6, 100) STA(J), J = 1, NS
WRITE(6, 105)
WRITE(6, 100) MASS(J), J = 1, NS
WRITE(6, 110)
WRITE(6, 100) (EI1(J), J = 1, NS)
WRITE(6, 115)
WRITE(6, 100) (EI2(J), J = 1, NS)
WRITE(6, 120)
WRITE(6, 100) (E(J), J = 1, NS)
WRITE(6, 125)
WRITE(6, 100) (BETA(J), J = 1, NS)
WRITE(6, 130)
WRITE(6, 100) (Kt(J), J = 1, NS)
WRITE(6, 140)
WRITE(6, 100) (KMIS(J), J = 1, NS)
WRITE(6, 145)
WRITE(6, 100) (KM2S(J), J = 1, NS)
WRITE(6, 25)

THIS SECTION CALCULATES THE SYSTEM PROPERTIES AT THE REQUIRED STATIONS BY INTERPOLATION AND PRINTS THE INTERPOLATED VALUES

CALL INTPOL(MASS)
CALL INTPOL(EI1)
CALL INTPOL(EI2)
CALL INTPOL(E)
CALL INTPOL(BETA)
CALL INTPOL(KMIS)
CALL INTPOL(KM2S)
WRITE(6, 20)
WRITE(6, 25)
WRITE(6,175)
WRITE(6,125)
WRITE(6,100)
WRITE(6,110)
WRITE(6,115)
WRITE(6,120)
WRITE(6,100)
WRITE(6,125)
WRITE(6,130)
WRITE(6,100)
WRITE(6,140)
WRITE(6,100)
WRITE(6,145)
WRITE(6,100)
WRITE(6,25)

------------------------------------------------------------------------
THIS SECTION NON-DIMENSIONALIZES THE DATA AND COMPUTES THE
COEFFICIENTS OF THE FIRST-ORDER DIFFERENTIAL EQUATIONS WHICH ARE
NOT DEPENDENT ON THE FREQUENCIES

DO 205 J=1,101
KMLS(J)=KMLS(J)/MASS(J)
KMS(J)=KMS(J)/MASS(J)+EI(J)*EI(J)

205 CONTINUE

DO 206 I=NZ,101
BETA(I)=BETA(I)+RPITCH
PI=4.0*ATAN(1.0)
OMEGA=OMEGA*PI/30.0
OMEGAS=OMEGA*OMEGA
SPANS=SPAN*SPAN
BS=8*8
TSR=TSR*SPAN*SPAN/(EI1(1)*BS)
FACT=SQRT(MASS(1)*SPANS*SPAN/EI1(1))
OMEGAN=FACT*FACT*OMEGAS
FL=BS/SPANS
N1=3
N2=7
IF(IBC.EQ.2) N1=4
IF(IBC.EQ.2) N2=8
IF(IBC.EQ.3)N2=4
DO 210 J=1,101
210 D5(J)=MASS(J)/MASS(1)
X=0.0
H5=0.01/24.0
DO 215 J=1,101
D2(J)=D5(J)*H5*X
215 X=X+0.01
D12(101)=0.0
D12(100)=9.0*D2(101)+19.0*D2(100)-9.0*D2(99)+D2(98)
DO 220 J=2,99
J=101-J
220 D12(J)=D12(J+1)-D2(J+2)+13.0*(D2(J+1)+D2(J))-D2(J-1)
D12(1)=D12(2)+D2(4)-5.0*D2(3)+19.0*D2(2)+9.0*D2(1)
X=0.0
DO 230 J=1,101
BETA(J)=PI*ABS((J)/180.0)
C=COS(BETA(J))
S=SIN(BETA(J))
C*S*S*S
A11=EI1(J)*C*S+EI2(J)*S
A12=(EI2(J)-EI1(J))*C*S
A22=EI1(J)*S*S+EI2(J)*C*S
D=A11*A22-A12*A12
D1(J)=E11(1)*A12/D
D2(J)=E11(1)*A22/D
D3(J)=EI1(1)*A11/D
D4(J)=E11(1)*F1/GJ(J)
D5(J)=D5(J)*E1/GJ(J)
D6(J)=D6(J)*S
D6(J)=D6(J)*C
D8(J)=D8(J)*X*OMEGAN
D9(J)=D7(J)*X*OMEGAN
D10(J)=OMEGAN*D5(J)*((KM2S(J)-KMI1(J))*C*S*S)/B
D11(J)=D5(J)*KMS(J)/B
D12(J)=D12(J)*OMEGAN
X=X+0.01
H1=H1*FACT
H2=H2*FACT
H3=H3*FACT
IF(ISTAGE.EQ.2) GO TO 240

WRITE(6,20)
WRITE(6,25)

---

THIS SECTION CALCULATES THE FREQUENCY DETERMINANTS OF THE SYSTEM---

---
WRITE(6,180)
WRITE(6,29)
P=HI*H1
IF(HI.GT.H3) GO TO 265
FR=HI/FACT
F=DET(P)
WRITE(6,185)FR,H1,F
H1=H1+H2
GOTO 235

THIS SECTION CALCULATES THE NATURAL FREQUENCIES AND THE ASSOCIATED MODAL FUNCTIONS OF THE SYSTEM. MODAL FUNCTIONS ARE NORMALIZED WITH RESPECT TO THE MAXIMUM DEFLECTION OF THE PREDOMINANT MODE.

CALL NATFRE(NF)
SL(1)=0.0
DO 245 J=1,50
   SL(J+1)=SL(J)+0.02
   IF(IJK.EQ.0) GO TO 261
   J1=J+INC
   PP=FREQEN(J)
P=PP*PP
   IF((J1+18C).EQ.2.AND.OMEGA.LE.0.0) GO TO 246
   CALL SHAPES(P,W,V,PHI)
   GO TO 248
246  DO 247 I=1,51
   V(I)=SL(I)
   W(I)=0.0
   PHI(I)=0.0
   GO TO 256
248  IF(IND.EQ.1) GO TO 256
   AMAX=W(1)
   DO 250 I=1,51
      IF(ABS(AMAX).LT.ABS(W(I)))AMAX=W(I)
      IF(ABS(AMAX).LT.ABS(V(I)))AMAX=V(I)
   250  IF(ABS(AMAX).LT.ABS(PHI(I)))AMAX=PHI(I)
   DO 255 I=1,51
      W(I)=W(I)/AMAX
      V(I)=V(I)/AMAX
      PHI(I)=PHI(I)/AMAX
   255  CONTINUE
   FRE=PP/FACT
   HERTZ=FRE/(2.0*PI)
   CALL PLOT(W,1)
CALL PLOT(V,2)  
CALL PLOT(PHI,3)  
CONTINUE  
IF(JK .LT. NP) WRITE(6,160) JK  
IF(ISTAGE .EQ. 1) WRITE(6,25)  
C FORMAT  
C FORMAT(I1,11,I3)  
C FORMAT(5E14.7)  
C FORMAT(I2,I1,3A12)  
C FORMAT(1H1)  
C FORMAT(/2X,IX,FORMAT(***************  
C FORMAT(/35X,"NATURE OF THE MODES",19X,"COLLECTIVE MODES")  
C FORMAT(/35X,"NATURE OF THE MODES",19X,"CYCLIC MODES")  
C FORMAT(/35X,"NATURE OF THE MODES",19X,"SCISSOR MODES")  
C FORMAT(/5X,"NUMBER OF FREQUENCIES REQUIRED",=",15)  
C FORMAT(/5X,"FREQUENCY INCREMENT(RAD/SEC)",=",E14.7)  
C FORMAT(/5X,"LENGTH OF THE BLADE (INCHES)",=",E14.7)  
C FORMAT(/5X,"ROTATIONAL VELOCITY OF THE BLADE (RPM)",=",E14.7)  
C FORMAT(/5X,"SEMI-CHORD OF THE BLADE (INCHES)",=",E14.7)  
C FORMAT(/5X,"CONTROL SYSTEM SPRING RATE (IN-LB/RAD)",=",E14.7)  
C FORMAT(/5X,"NUMBER OF DATA POINTS",=",15)  
C FORMAT(/5X,"STATION LOCATIONS (INCHES)")  
C FORMAT(/7(4X,"E14.7)"))  
C FORMAT(/5X,"MASS PER UNIT LENGTH (LB-SEC**2/IN**2)")  
C FORMAT(/5X,"FLAPWISE BENDING STIFFNESS (LB-IN**2)")  
C FORMAT(/5X,"CHORDWISE BENDING STIFFNESS (LB-IN**2)")  
C FORMAT(/5X,"TORSIONAL STIFFNESS (LB-IN**2)")  
C FORMAT(/5X,"DISTANCE BETWEEN MASS AND ELASTIC AXIS(INCHES)")  
C FORMAT(/5X,"TWIST OF THE BLADE NOT INCLUDING THE COLLECTIVE PITCH (DEGREES)")  
C FORMAT(/5X,"STARTING FREQUENCY (RAD/SEC)",=",E14.7)  
C FORMAT(/5X,"MASS MOMENT OF INERTIA ABOUT THE CHORD (LB-SEC**2)")  
C FORMAT(/5X,"MASS MOMENT OF INERTIA ABOUT AN AXIS PERPENDICULAR TO THE CHORD THROUGH THE CENTER OF GRAVITY (LB-SEC**2)")  
C FORMAT(/5X,"ENDING FREQUENCY (RAD/SEC)"
INCREMENT IN THE MODE NUMBER = E14.7

THE NUMBER OF FREQUENCIES DETECTED WITHIN THE RANGE ARE ONLY

THE FOLLOWING ARE THE INTERPOLATED VALUES AT 101 EQUAL DISTANT STATIONS

THE FOLLOWING COLUMNS ARE 1. FREQUENCY (RAD/SEC) 2. NONDIMENSIONAL FREQUENCY 3. VALUE OF THE FREQUENCY DETERMINANT RESPECTIVELY

COLLECTIVE PITCH (DEGREES) = E14.7

DISTANCE OF THE BLADE FROM THE ROOT (INCHES) = E14.7

STOP

END
SUBROUTINE NATFRE(N)

C THIS SUBROUTINE SCANS THE FREQUENCY DETERMINANT WITH RESPECT TO
C THE FREQUENCY TILL THE SPECIFIED NUMBER OF SIGN CHANGES ARE
C DETECTED STARTING FROM ZERO FREQUENCY. USES FUNCTION DET

C

DIMENSION FREQEN(10), JKL(10)
COMMON/X1/FREQEN,H1,H3,ITER,IJK
IJK=0
ITRN=0
DO 5 J=1,N
5 JKL(J)=0
PP=H1
P=PP*PP
F=DET(P)
IF(ABS(F).GT.0.0001)GO TO 10
IJK=IJK+1
JKL(IJK)=1
FREQEN(IJK)=PP
PP=PP*H
P=PP*PP
F=DET(P)
10 F=SIGN(1.0,F)
5 PP=PP*H
IF(P-P*H).GT..H GO TO 30
P=PP*PP
G=DET(P)
IF(ABS(G).GT.0.0001)GO TO 20
IJK=IJK+1
JKL(IJK)=1
FREQEN(IJK)=PP
IF(IJK.EQ.N)GO TO 30
PP=PP*H
P=PP*PP
F=DET(P)
F=SIGN(1.0,F)
GO TO 15
20 G=SIGN(1.0,G)
IF(F*G.GT.0.0)GO TO 25
IJK=IJK+1
FREQEN(IJK)=PP-H
IF(IJK.EQ.N)GO TO 30
F=G
GO TO 15
30 ITRN=ITRN+1
IF(ITRN.GT.ITER)GO TO 55
IF(IJK.EQ.0) GO TO 55
H=H/10.0
DO 50 J=1,IJK
IF(JKL(J).EQ.1)GO TO 50
PP=FREQEN(J)
F=DET(P)
F=SIGN(1.0,F)
35 PP=PP+H
F=PP*PP
G=DET(P)
IF(ABS(G).GT.0.0001)GO TO 40
J=1
FREQEN(J)=PP
GO TO 50
40 G=SIGN(1.0,G)
IF(F*G.GT.0.0)GO TO 45
FREQEN(J)=PP-H
GO TO 50
45 F=G
GO TO 35
50 CONTINUE
GO TO 30
55 DO 60 J=1,IJK
IF(JKL(J).EQ.1)GO TO 60
PP=FREQEN(J)
P=PP*PP
F=DET(P)
PP=PP+H
F=PP*PP
G=DET(P)
DIFF=G-F
FREQEN(J)=PP-G*H/DIFF
60 CONTINUE
65 CONTINUE
RETURN
END
SUBROUTINE PLOT(A,N)

THIS SUBROUTINE PRINTS THE NATURAL_FREQUENCIES AND MODE SHAPES AND
PLOTS THE MODE SHAPES

REAL LINE
DIMENSION A(51),SL(51),LINE(51)
COMMON/X1,PP,FRE,HERTZ,SL,BLANK,DOT,STAR
COMMON/X11/IND
WRITE(6,10)
WRITE(6,20)
WRITE(6,30)J1,FRE,HERTZ,PP
WRITE(6,20)
IF(N.EQ.1)WRITE(6,40)
IF(N.EQ.2)WRITE(6,50)
IF(N.EQ.3)WRITE(6,60)
WRITE(6,70)
IF(IND.EQ.0) GO TO 75
WRITE(6,150)
RETURN
75 DO 80 J=1,6
80 WRITE(6,90)SL(J),A(J),SL(J+9),A(J+9),SL(J+18),A(J+18),SL(J+27),A(J+27),SL(J+36),A(J+36),SL(J+45),A(J+45))
WRITE(6,90)SL(7),A(7),SL(16),A(16),SL(25),A(25),SL(34),A(34),SL(43),A(43))
WRITE(6,90)SL(8),A(8),SL(17),A(17),SL(26),A(26),SL(35),A(35),SL(44),A(44))
WRITE(6,90)SL(9),A(9),SL(18),A(18),SL(27),A(27),SL(36),A(36),SL(45),A(45))
WRITE(6,20)
WRITE(6,10)
DO 100 J=1,51
100 LINE(J)=DOT
J=25.*A(1)+1.*1.5
LINE(J)=STAR
WRITE(6,110)LINE(J),J=1,51
DO 120 J=1,51
120 LINE(J)=BLANK
LINE(26)=DOT
DO 130 JJ=3,51,2
J=25.*A(JJ)+1.*1.5
LINE(J)=STAR
WRITE(6,140)LINE(J),J=1,51
LINE(J)=BLANK
130 LINE(26)=DOT
10  FORMAT(1H1)
20  FORMAT(//5X,"****************************************************************
2**************")
30  FORMAT(//5X," MODE NUMBER =",I2,8X,"FREQ. RAD/SEC =",F10.4,8X,"FRE
10. Hertz =",F10.4,8X," NON-DIMEN. FREQ. =",F10.4)
40  FORMAT(//50X,"FLAPWISE DEFLECTION/SEMICHORD")
50  FORMAT(//50X,"CHORDWISE DEFLECTION/SEMICHORD")
60  FORMAT(//50X,"TORSIONAL DEFLECTION(RADIANS)")
70  FORMAT(6(4X,"STA X/Y",4X,"DEFLN"))
90  FORMAT(//12(2X,F8.4))
110  FORMAT(//:////40X,51A1)
140  FORMAT(40X,51A1)
150  FORMAT(:////5X,"THE MATRIX TO BE INVERTED IN THE MODE SHAPES COMP
111  UTATIONS IS SINGULAR AND HENCE COMPUTATIONS ARE ABANDONED")
99  RETURN
100  END
SUBROUTINE INTPOL(A)

C THIS SUBROUTINE INTERPOLATES THE REQUIRED VALUES.

DIMENSION A(101), STA(101), TABLE(101,1), B(101)
COMMON /X3/ STA, SPAN
COMMON /X7/ NS
IN=0
DO 5 J=1,NS
5 IF(ABS(A(J)).LE.O.0) IN=IN+1
IF(IN.NE.NS) GO TO 15
DO 10 J=1,101
10 A(J)=0.0
RETURN
15 A(101)=A(NS)
   NH1=NS-1
   DO 20 I=1,NH1
20   TABLE(I,1)=(A(I+1)-A(I))/(STA(I+1)-STA(I))
   H=SPAN/100.0
   XARG=H
   DO 35 I=2,100
35   CONTINUE
   IF(J.EQ.NS. OR. XARG.LE.STA(J)) GO TO 30
   DO 25 J=1,NS
25   IF(J.EQ.NS. OR. XARG.LE.STA(J)) GO TO 36
   XARG=XARG+H
   XARG=XARG+H
   IF(MAX.LE.2) MAX=2
   ISUB=MAX-1
   YEST=TABLE(ISUB,1)
   B(I)=YEST*(XARG-STA(ISUB))+A(ISUB)
   35 CONTINUE
   30 MAX=J
   ISUB=MAX-1
   YEST=TABLE(ISUB,1)
   B(I)=YEST*(XARG-STA(ISUB))+A(ISUB)
   35 XARG=XARG+H
   DO 40 J=2,100
40 A(J)= B(J)
RETURN
END
FUNCTION DET(P)

C THIS FUNCTION CALCULATES THE VALUE OF THE FREQUENCY DETERMINANT
C USES SUBROUTINE TRAMAT

DIMENSION TF(10,10), A(6,6)
COMMON/X4/IBC
COMMON/X6/TSR
COMMON/X9/TF
COMMON/X12/DETER
CALL TRAMAT(P)
DO 5 J=9,10
DO 5 I=1,5
5 A(I,J)=TF(I+5,J)
IF(IBC-2)10,20,30
10 DO 15 I=1,5
15 A(I,1)=TF(I+5,4)
A(I,2)=TF(I+5,5)-TF(I+5,6)*TSR
A(I,3)=TF(I+5,8)
GOTO 40
20 DO 25 I=1,5
25 A(I,1)=TF(I+5,3)
A(I,2)=TF(I+5,5)-TF(I+5,6)*TSR
A(I,3)=TF(I+5,7)
GOTO 40
30 DO 35 I=1,5
35 A(I,1)=TF(I+5,5)-TF(I+5,6)*TSR
A(I,2)=TF(I+5,7)
A(I,3)=TF(I+5,8)
40 CONTINUE
CALL SOLUTN(A,1)
DET=DETER
RETURN
END
SUBROUTINE TRAMAT(P).

INTEGRATION OF DIFFERENTIAL EQUATIONS, USES THE FUNCTION RUNGE

DIMENSION O1(101), O2(101), O3(101), O4(101), O5(101), O6(101), O7(101),
O8(101), O9(101), O10(101), O11(101), O12(101), V1(10, 10), V(10), T(10),
2T(10, 10, 10), TF(10, 10).

COMMON/X5/O1, O2, O3, O4, O5, O6, O7, O8, O9, O10, O11, O12, N1, N2
COMMON/X8/OMEGAN
COMMON/X9/TF
COMMON/X10/TT

DO 10 J=1, 10
DO 5 I=1, 10

5 TT(I, J, J) = 0.0
5 TT(I, J, J) = 1.0

10 V(I, J, J) = 1.0
M = 0
DO 45 I=3, 10
IF(I .EQ. N1 .OR. I .EQ. N2) GO TO 45
DO 15 J=1, 10

15 V(I) = V(I, J, J)
J = 1
DO 40 L = 1, 50
20 K = RUNGE(V, T, J, M)
25 IF(K .NE. 1) GO TO 30
FACT = P + OMEGAN
T(1) = V(3)
T(2) = V(4)
T(3) = O1(J) * V(7) + O2(J) * V(8)
T(4) = O3(J) * V(7) + O4(J) * V(8)
T(5) = O5(J) * V(6)
T(6) = P * O6(J) * V(1) + FACT * O7(J) * V(2) + O8(J) * V(3) - O9(J) * V(4) + O10(J) *
V(5) - P * O11(J) * V(5)
T(7) = O12(J) * V(4) + V(9) - O9(J) * V(5)
T(8) = O12(J) * V(3) + O10(J) + O8(J) * V(9)
T(9) = FACT * O5(J) * V(2) - FACT * O7(J) * V(5)
T(10) = P * O5(J) * V(1) + P * O6(J) * V(5)
GO TO 20

30 DO 35 JJ=1, 10
35 TT(L+1, JJ, I) = V(JJ)
40 CONTINUE
44 TT(1, JJ, I) = TT(5, JJ, I)
45 CONTINUE
RETURN
END
FUNCTION RUNGE(Y, F, JJ, M)

C FOURTH-ORDER RUNGE-KUTTA METHOD

C

DIMENSION Y(10), F(10), PHI(10), SAVEY(10)

M = M + 1
GO TO(5, 10, 20, 30, 40), M

5 RUNGE = 1
RETURN

10 DO 15 JJ = 1, 10
SAVEY(JJ) = Y(JJ)
PHI(JJ) = F(JJ)
15 Y(JJ) = SAVEY(JJ) + 0.01 * F(JJ)
J = J + 1
RUNGE = 1
RETURN

20 DO 25 JJ = 1, 10
PHI(JJ) = PHI(JJ) + 2.0 * F(JJ)
25 Y(JJ) = SAVEY(JJ) + 0.01 * F(JJ)
RUNGE = 1
RETURN

30 DO 35 JJ = 1, 10
PHI(JJ) = PHI(JJ) + 2.0 * F(JJ)
35 Y(JJ) = SAVEY(JJ) + 0.02 * F(JJ)
J = J + 1
RUNGE = 1
RETURN

40 DO 45 JJ = 1, 10
45 Y(JJ) = SAVEY(JJ) + (PHI(JJ) + F(JJ)) / 300.0
M = 0
RUNGE = 0
RETURN
END
SUBROUTINE SHAPES(P,W,V,PHI)

C THIS SUBROUTINE CALCULATES THE MODE SHAPES. USES THE SUBROUTINE TRAMAT

DIMENSION h(51),V(51),PHI(51),A(10,6),TF(10,10),TT(51,10,10),X(5)
COMMON/X4/IBC
COMMON/X6/TSR
COMMON/X9/TF
COMMON/X10/TT
COMMON/X11/IND
COMMON/X13/X
CALL TRAMAT(P)
A(1,4)=TF(1,9)
A(1,5)=TF(1,10)
A(1,6)=1.0
DO 10 I=2,5
DO 5 J=4,5
5 A(I,J)=TF(I+J+5)
10 IF(IBC-2)15,25,35
15 A(I,1)=TF(I+4)
A(I,2)=TF(I+5)-TSR*TF(I,6)
A(I,3)=TF(I+8)
DO 20 I=2,5
A(I,1)=TF(I+4)
A(I,2)=TF(I+5)-TSR*TF(I+4,6)
20 A(I,3)=TF(I+8)
GO TO 45
25 A(I,1)=TF(I+3)
A(I,2)=TF(I+5)-TSR*TF(I,6)
A(I,3)=TF(I+7)
DO 30 I=2,5
A(I,1)=TF(I+4)
A(I,2)=TF(I+5)-TSR*TF(I+4,6)
30 A(I,3)=TF(I+7)
GO TO 45
35 A(I,1)=TF(I+5)-TSR*TF(I,6)
A(I,2)=TF(I+7)
A(I,3)=TF(I+8)
DO 40 I=2,5
A(I,1)=TF(I+4,5)-TSR*TF(I+4,6)
A(I,2)=TF(I+4,7)
40 A(I,3)=TF(I+4,8)
45 CALL SOLUTN(A,2)
IF(IND.EQ.1) GO TO 130
IF(IBC-2) 100,110,120

100   DO 105 J=1,51
*  W(J)=X(1)*TT(J,1,12)+X(2)*TT(J,2,5)-TSR*TT(J,1,6)+X(3)*TT(J,1,7)+
1X(4)*TT(J,1,9)+X(5)*TT(J,1,10)
*  V(J)=X(1)*TT(J,2,4)+X(2)*TT(J,2,5)-TSR*TT(J,2,6)+X(3)*TT(J,2,8)+
1X(4)*TT(J,2,9)+X(5)*TT(J,2,10)

105   PHI(J)=X(1)*TT(J,5,4)+X(2)*TT(J,5,5)-TSR*TT(J,5,6)+X(3)*TT(J,5,8)
1+X(4)*TT(J,5,9)+X(5)*TT(J,5,10)
RETURN

110   DO 115 J=1,51
*  W(J)=X(1)*TT(J,5,5)+X(2)*TT(J,5,7)+X(3)*TT(J,5,8)+
1X(4)*TT(J,5,9)+X(5)*TT(J,5,10)
*  V(J)=X(1)*TT(J,5,5)+X(2)*TT(J,5,7)+X(3)*TT(J,5,8)+
1X(4)*TT(J,5,9)+X(5)*TT(J,5,10)

115   PHI(J)=X(1)*TT(J,5,5)+X(2)*TT(J,5,7)+X(3)*TT(J,5,8)
1+X(4)*TT(J,5,9)+X(5)*TT(J,5,10)
RETURN

120   DO 125 J=1,51
*  W(J)=X(1)*TT(J,1,12)+X(2)*TT(J,1,7)+X(3)*TT(J,1,8)+
1X(4)*TT(J,1,9)+X(5)*TT(J,1,10)
*  V(J)=X(1)*TT(J,2,4)+X(2)*TT(J,2,5)-TSR*TT(J,2,6)+X(3)*TT(J,2,8)+
1X(4)*TT(J,2,9)+X(5)*TT(J,2,10)

125   PHI(J)=X(1)*TT(J,5,4)+X(2)*TT(J,5,5)-TSR*TT(J,5,6)+X(3)*TT(J,5,8)
1+X(4)*TT(J,5,9)+X(5)*TT(J,5,10)
RETURN

130   END
SUBROUTINE SOLUTN(INI)

DIMENSION A(6,6),X(5),IROW(5),JCOL(5),JORD(5)

COMMON/X11/IND

COMMON/X12/DETER...

COMMON/X13/X

IND=0

N=5

MAX=N

IF(N.EQ.2) MAX=N+1

DETER=1.0

DO 80 K=1,N

KM1=K-1

PIVOT=0.0

DO 60 I=1,N

DO 60 J=1,N

IF(K.EQ.1) GO TO 55

DO 50 ISCAN=1,KM1

DO 50 JSCAN=1,KM1

IF(I.EQ.IROW(ISCAN)) GO TO 60

IF(J.EQ.JCOL(JSCAN)) GO TO 60

50 CONTINUE

55 IF(ABS(A(I,J)).LE.ABS(PIVOT)) GO TO 60

PIVOT=A(I,J)

IROW(K)=I

JCOL(K)=J

CONTINUE

50 IF(ABS(PIVOT).GT.0.1E-20) GO TO 65

DETER=DETER*PIVOT

D

70 A(IROWK,J)=A(IROWK,J)/PIVOT

A(IROWK,JCOLK)=1.0/PIVOT

DO 80 I=1,N

AIJCK=A(I,JCOLK)

IF(I.EQ.IROWK) GO TO 80

A(I,JCOLK)=-AIJCK/PIVOT

70 CONTINUE

75 IF(J.NE.JCOLK) A(I,J)=A(I,J)-AIJCK*A(IROWK,J)

80 CONTINUE

DO 85 I=1,N

IROWI=IROW(I)

JCOLI=JCOL(I)

JORD(IROWI)=JCOLI

IF(NI.EQ.1) GO TO 85
X(JCOLI) = A(IROWI, MAX)

CONTINUE

INTCH = 0

NM1 = N - 1

DO 90 I = 1, NM1

IP1 = I + 1

DO 90 J = IP1, N

IF (JORD(J) .GE. JORD(I)) GO TO 90

JTEMP = JORD(J)

JORD(J) = JORD(I)

JORD(I) = JTEMP

INTCH = INTCH + 1

CONTINUE

90

IF (INTCH / 2 .NE. INTCH) DETER = -DETER

IF (ABS(DETER) .GT. 1.0E-30) GO TO 95

IND = 1

95

RETURN

END
FORTRAN PROGRAM II
NATURAL VIBRATION CHARACTERISTICS, COUPLED FLAPWISE BENDING, CHORD-WISE BENDING AND TORSION, SEE-SAW ROTOR, COLLECTIVE CYCLIC AND SCISSOR MODES. USES FUNCTION \( \text{DET} \) AND SUBROUTINES \( \text{INTPOL}, \text{PLOT} \) :

\( \text{NATFRE} \) AND SHAPES.

DECLARATION AND COMMON STATEMENTS

\[
\begin{align*}
\text{REAL } & \text{MASS, KMS, KMIS, KM2S} \\
\text{DIMENSION } & \text{E(101), EI1(101), EI2(101), GJ(101), KMS(101), KMIS(101),} \\
& \text{1KM2S(101), MASS(101), BETA(101), D1(101), D2(101), D3(101), D4(101),} \\
& \text{2D5(101), D6(101), D7(101), D8(101), D9(101), D10(101), D11(101),} \\
& \text{3D12(101), STA(101), PHT(51), W(51), V(51), SL(51), FREQEN(10)} \\
\text{COMMON} & \text{XI, FREQEN, H1, H2, H3, ITER, IJK} \\
\text{COMMON} & \text{X2/J, PP, FRE, HERTZ, SL, BLANK, DOT, STAR} \\
\text{COMMON} & \text{X3/STA, SPAN} \\
\text{COMMON} & \text{X4/IBC} \\
\text{COMMON} & \text{X5/D1, D2, D3, D4, D5, D6, D7, D8, D9, D10, D11, D12} \\
\text{COMMON} & \text{X6/TSR} \\
\text{COMMON} & \text{X7/NS} \\
\text{COMMON} & \text{X8/PMASS, CLP, SLP, CBETA, SBETA, CTBETA, N1, N2, JSAVE} \\
\text{COMMON} & \text{X11/IND} \\
\text{COMMON} & \text{X12/OMEGAN}
\end{align*}
\]

THIS SECTION READS THE DATA OF THE SYSTEM

READ(5,5) IBC, ISTAGE, NS
READ(5,10) SPAN, OMEGA, B, TSR, PMASS, CLP, SLP, R, PITCH, DEB
READ(5,10) (STA(J), J = 1, NS), (MASS(J), J = 1, NS), (EI1(J), J = 1, NS), (EI2(J), J = 1, NS), (GJ(J), J = 1, NS), (E(J), J = 1, NS), (BETA(J), J = 1, NS), (KMIS(J), J = 1, NS), (KM2S(J), J = 1, NS)
READ(5,10) H1, H2, H3
IF(ISTAGE.EQ.2) READ(5,15) NF, ITER, BLANK, DOT, STAR, INC

THIS SECTION PRINTS THE DATA OF THE SYSTEM

WRITE(6,20)
WRITE(6,25)
IF(IBC.EQ.1) WRITE(6,30)

PAGE INTENTIONALLY BLANK
IF(IBC.EQ.2)WRITE(6,35)
IF(IBC.EQ.3)WRITE(6,40)
IF(ISTAGE.EQ.2)WRITE(6,45)
WRITE(6,135)HI
WRITE(6,50)H2
WRITE(6,150)H3
IF(ISTAGE.EQ.2)WRITE(6,155)INC
WRITE(6,55)SPAN
WRITE(6,60)OMEGA
WRITE(6,65)B
WRITE(6,70)TSR
WRITE(6,190)RPITCH
WRITE(6,195)DEB
IF(PMASS.LE.0.0)GOTO 200
WRITE(6,75)PMASS
WRITE(6,80)CLP
WRITE(6,85)SLP
200
WRITE(6,90)NS
WRITE(6,95)
WRITE(6,100) (STA(J),J=1,NS)
WRITE(6,105)
WRITE(6,100) (MASS(J),J=1,NS)
WRITE(6,110)
WRITE(6,100) (EI1(J),J=1,NS)
WRITE(6,115)
WRITE(6,100) (EI2(J),J=1,NS)
WRITE(6,120)
WRITE(6,100) (GJ(J),J=1,NS)
WRITE(6,125)
WRITE(6,100) (E(J),J=1,NS)
WRITE(6,130)
WRITE(6,100) (BETA(J),J=1,NS)
WRITE(6,140)
WRITE(6,100) (KM1S(J),J=1,NS)
WRITE(6,145)
WRITE(6,100) (KM2S(J),J=1,NS)
WRITE(6,25)

---

THIS SECTION CALCULATES THE SYSTEM PROPERTIES AT THE REQUIRED STATIONS BY INTERPOLATION AND PRINTS THE INTERPOLATED VALUES
---

CALL INTPOL(MASS)
CALL INTPOL(EI1)
CALL INTPOL(EI2)
CALL INTPOL(GJ)
CALL INTPOL(E)
CALL INTPOL(BETA)
CALL INTPOL(KMIS)
CALL INTPOL(KM2S)
WRITE(6,20)
WRITE(6,25)
WRITE(6,175)
WRITE(6,25)
WRITE(6,105)
WRITE(6,100)(MASS(J),J=1,101)
WRITE(6,110)
WRITE(6,100)(EI1(J),J=1,101)
WRITE(6,115)
WRITE(6,100)(EI2(J),J=1,101)
WRITE(6,120)
WRITE(6,100)(GJ(J),J=1,101)
WRITE(6,125)
WRITE(6,100)(E(J),J=1,101)
WRITE(6,130)
WRITE(6,100)(BETA(J),J=1,101)
WRITE(6,140)
WRITE(6,100)(KM1S(J),J=1,101)
WRITE(6,145)
WRITE(6,100)(KM2S(J),J=1,101)
WRITE(6,25)

C
C-------------------------------------------------------
C THIS SECTION NON-DIMENSIONALIZES THE DATA AND COMPUTES THE
C COEFFICIENTS OF THE FIRST-ORDER DIFFERENTIAL EQUATIONS WHICH ARE
C NOT DEPENDENT ON THE FREQUENCIES
C-------------------------------------------------------
C
DO 205 J=1,101
KM1S(J)=KM1S(J)/MASS(J)
KM2S(J)=KM2S(J)/MASS(J)+E(J)*E(J)
205

NZTS=DEB/SPAN
NZ=NZTS+1

DO 206 I=NZ,101
BETA(I)=BETA(I)+RPITCH
PI=4.0*ATAN(1.0)
OMEGA=OMEGA*PI/30.0
OMEGAS=OMEGA*OMEGA
SPAN=SPAN*SPAN
BS=B*B
TSR=TSR*SPAN*SPAN/(EI1(1)*BS)
FACT=SQRT(MASS(1)*SPAN*SPAN/EI1(1))
OMEGAN=FACT*FACT*OMEGAS
IF(PMASS.LE.0.0) SLP=SPAN
PMASS*PMASS/(SPAN*MASS(1)*386.4)
CLP=CLP/B
SLP=SLP/SPAN
F1=BS/SPAN
N1=50.0*SLP+0.5
N2=N1+1
JSAVE=2*N1+1
H4=PMASS*SLP
SBE=BETA(JSAVE)
CBETA=COS(SBE)
SBETA=SIN(SBE)
CTBETA=CBETA*CBETA-SBETA*SBETA
DO 210 J=1,101
210 D5(J)=MASS(J)/MASS(1)
X=0.0
H5=0.01/24.0
DO 215 J=1,101
D2(J)=D5(J)*H5*X
215 X=X+0.0
DO 220 J=1,101
D12(101)=0.0
D12(100)=9.0*D2(101)+19.0*D2(100)-5.0*D2(99)+D2(98)
DO 220 J=1,101
J=101-J
220 D12(J)=D12(J+1)-D2(J+2)+13.0*(D2(J+1)+D2(J))-D2(J-1)
D12(1)=D12(2)+D2(4)-5.0*D2(3)+19.0*D2(2)+9.0*D2(1)
DO 225 J=1,JSAVE
225 D12(J)=D12(J)+H4
X=0.0
DO 230 J=1,101
BETA(J)*PI*BETA(J)/180.0
C=COS(BETA(J))
S=SIN(BETA(J))
CS=C*C
SS=S*S
A11=EI1(J)*CS+EI2(J)*SS
A12=(EI2(J)-EI1(J))*C*S
A22=EI1(J)*SS+EI2(J)*CS
D=A11*A22-A12*A12
D1(J)=-EI1(1)*A12/D
D2(J)=EI1(J)*A22/D
D3(J)=EI1(1)*A11/D
D4(J)=EI1(1)*F1/GJ(J)
D6(J)=D5(J)*E(J)/B
D7(J)=D6(J)*S
D8(J)=D6(J)*C
D9(J)=D6(J)*X*OMEGAN
D10(J)=OMEGAN*D5(J)*((KM2S(J)-KM1S(J))*C-CS*SS)/BS
D11(J) = D5(J) * KMS(J) / BS
D12(J) = D12(J) * OMEGAN
X = X + 0.01
H1 = H1 * FACT
H2 = H2 * FACT
H3 = H3 * FACT
IF (ISTAGE.EQ.2) GO TO 240

THIS SECTION CALCULATES THE FREQUENCY DETERMINANTS OF THE SYSTEM

WRITE(6,20)
WRITE(6,25)
WRITE(6,180)
WRITE(6,25)
P = H1 * H1
IF (H1.GT.H3) GO TO 265
FR = H1 / FACT
F = DET(P)
WRITE(6,185) FR, H1, F
H1 = H1 + H2
GOTO 235

THIS SECTION CALCULATES THE NATURAL FREQUENCIES AND THE ASSOCIATED
MODAL FUNCTIONS OF THE SYSTEM. MODAL FUNCTIONS ARE NORMALIZED WITH
RESPECT TO THE MAXIMUM DEFLECTION OF THE PREDOMINANT MODE

CALL NATFRE(NF)
SL(1) = 0.0
DO 245 J = 1, 50
SL(J+1) = SL(J) + 0.02
IF (IJK.EQ.0) GO TO 261
DO 260 J = 1, IJK
J1 = J + INC
PP = FREQEN(J)
P = PP*PP
IF ((J1+8C).EQ.2 .AND. OMEGA.LE.0.0) GO TO 246
CALL SHAPES(P, W, V, PHI)
GOTO 248
DO 247 I = 1, 51
V(I) = SL(I)
W(I) = 0.0
PHI(I) = 0.0
GOTO 256
IF(IND.EQ.1) GO TO 256
AMAX=W(I)
DO 250 I=1,51
IF(ABS(AMAX).LT.ABS(W(I))) AMAX=W(I)
IF(ABS(AMAX).LT.ABS(V(I))) AMAX=V(I)
IF(ABS(AMAX).LT.ABS(PHI(I))) AMAX=PHI(I)
DO 255 I=1,51
W(I)=W(I)/AMAX
V(I)=V(I)/AMAX
PHI(I)=PHI(I)/AMAX
CONTINUE
FRE=PP/FACT
HERTZ=FRE/(2.0*PI)
call PLOT(W,1)
call PLOT(V,2)
call PLOT(PHI,3)
CONTINUE
IF(IJK.LT.NF) WRITE(6,160) IJK
IF(ISTAGE.EQ.1) WRITE(6,25)

C FORMATS

5 FORMAT(I1,I1,I1)
10 FORMAT(5E14.7)
15 FORMAT(I2,I1,3A1,I2)
20 FORMAT(I1H1)
25 FORMAT(/2X, Mandatory format for output
1 ********************** Mandatory format for output
2 **********************
30 FORMAT(/35X,"NATURE OF THE MODES",19X,"COLLECTIVE MODES")
35 FORMAT(/35X,"NATURE OF THE MODES",19X,"CYCLIC MODES")
40 FORMAT(/35X,"NATURE OF THE MODES",19X,"SCISSOR MODES")
45 FORMAT(/5X,"NUMBER OF FREQUENCIES REQUIRED")
1 =",I5)
50 FORMAT(/5X,"FREQUENCY INCREMENT (RAD/SEC")
1 =",E14.7)
55 FORMAT(/5X,"LENGTH OF THE BLADE (INCHES")
1 =",E14.7)
60 FORMAT(/5X,"ROTATIONAL VELOCITY OF THE BLADE (RPM")
1 =",E14.7)
65 FORMAT(/5X,"SEMI-CHORD OF THE BLADE (INCHES")
1 =",E14.7)
70 FORMAT(/5X,"CONTROL SYSTEM SPRING RATE (IN-LB/RAD")
1 =",E14.7)
75 FORMAT(/5X,"WEIGHT OF THE SENSOR (LB")
1 =",E14.7)
80 FORMAT(/5X,"CHORDWISE LOCATION OF THE SENSOR (INCHES")

104
90 FORMAT(//5X,"NUMBER OF DATA POINTS"), I5)
105 FORMAT(//5X,"MASS PER UNIT LENGTH (LB-SEC**2/IN**2)")
110 FORMAT(//5X,"FLAPWISE BENDING STIFFNESS (LB-IN**2)")
115 FORMAT(//5X,"CHORDWISE BENDING STIFFNESS (LB-IN**2)")
120 FORMAT(//5X,"TORSIONAL STIFFNESS (LB-IN**2)")
125 FORMAT(//5X,"DISTANCE BETWEEN MASS AND ELASTIC AXIS (INCHES)")
130 FORMAT(//5X,"TWIST OF THE BLADE NOT INCLUDING THE COLLECTIVE PITCH (DEGREES)")
135 FORMAT(//5X,"STARTING FREQUENCY (RAD/SEC)"), E14.7)
140 FORMAT(//5X,"MASS MOMENT OF INERTIA ABOUT THE CHORD (LB-SEC**2)")
145 FORMAT(//5X,"MASS MOMENT OF INERTIA ABOUT AN AXIS PERPENDICULAR TO THE CHORD THROUGH THE CENTER OF GRAVITY (LB-SEC**2)")
150 FORMAT(//5X,"ENDING FREQUENCY (RAD/SEC)"), E14.7)
155 FORMAT(//5X,"INCREMENT IN THE MODE NUMBER"), I5)
160 FORMAT(//5X,"THE NUMBER OF FREQUENCIES DETECTED WITH IN THE RANGE ARE ONLY"), I5)
175 FORMAT(//5X,"THE FOLLOWING ARE THE INTERPOLATED VALUES AT 101 EQUIJO "
180 FORMAT(//5X,"THE FOLLOWING COLUMNS ARE 1. FREQUENCY (RAD/SEC) 2. NONDIMENSIONAL FREQUENCY 3. VALUE OF THE FREQUENCY DETERMINANT RESPECTIV "
190 FORMAT(//5X,"COLLECTIVE PITCH (DEGREES)")
195 FORMAT(//5X,"DISTANCE OF THE BLADE FROM THE ROOT (INCHES)"), E14.7)
STOP
END
SUBROUTINE NATFRE(N)

C
C THIS SUBROUTINE SCANS THE FREQUENCY DETERMINANT WITH RESPECT TO
C THE FREQUENCY TILL THE SPECIFIED NUMBER OF SIGN CHANGES ARE
C DETECTED STARTING FROM ZERO FREQUENCY. USES FUNCTION DET

C

DIMENSION FREQEN(10), JKL(10)
COMMON/X1/FREQEN,H1,H3,ITER, IJK

IJK = 0
ITRN = 0
DO 5 J = 1, N

5 JKL(J) = 0
PP = H1
P = PP * PP
F = DET(P).
IF (ABS(F) GT 0.0001) GO TO 10
IJK = IJK + 1
JKL(IJK) = 1
FREQEN(IJK) = PP
PP = PP + H
P = PP * PP
F = DET(P)

F = SIGN(1.0, F)

10 PP = PP + H
IF (PP GT H3) GO TO 30
P = PP + PP
G = DET(P)
IF (ABS(G) GT 0.0001) GO TO 20
IJK = IJK + 1
JKL(IJK) = 1
FREQEN(IJK) = PP
IF (IJK EQ N) GO TO 30
PP = PP + H
P = PP * PP
F = DET(P)
F = SIGN(1.0, F)
GO TO 15

20 G = SIGN(1.0, G)
IF (F*G GT 0.0) GO TO 25
IJK = IJK + 1
FREQEN(IJK) = PP - H
IF (IJK EQ N) GO TO 30

25 F = G
GO TO 15
30    ITRN=ITRN+1
     IF(ITRN.GT.ITER)GO TO 55
     IF(IJK.EQ.0) GO TO 65
     H=H/10.0
     DO 50 J=1,IJK
     IF(JKL(J).EQ.1)GO TO 50
     PP=FREQEN(J)
     P=PP*PP
     F=DET(P)
     F=SIGN(1.0,F)
     35    PP=PP+H
     P=PP*PP
     G=DET(P)
     IF(ABS(G).GT.0.0001)GO TO 40
     JKL(J)=1
     FREQEN(J)=PP
     GO TO 50
     40    G=SIGN(1.0,G)
     IF(F*G.GT.0.0)GO TO 45
     FREQEN(J)=PP-H
     GO TO 50
     45    F=G
     GO TO 35
     50    CONTINUE
     GO TO 30
     55    DO 60 J=1,IJK
     IF(JKL(J).EQ.1)GO TO 60
     PP=FREQEN(J)
     P=PP*PP
     F=DET(P)
     PP=PP+H
     P=PP*PP
     G=DET(P)
     DIFF=G-F
     FREQEN(J)=PP-G*H/DIFF
     60    CONTINUE
     65    CONTINUE
     RETURN
     END
SUBROUTINE PLOTCAN

C
C
C THIS SUBROUTINE PRINTS THE NATURAL FREQUENCIES AND MODE SHAPES AND
C PLOTS THE MODE SHAPES
C--------------------------------
C
REAL LINE
DIMENSION A(51), SL(51), LINE(51)
COMMON/X2/J1, PP, FRE, HERTZ, SL, BLANK, DOT, STAR
COMMON/X11/IND
WRITE(6,110)
WRITE(6,20)
WRITE(6,30)J1, FRE, HERTZ, PP
WRITE(6,20)
IF(N.EQ.1)WRITE(6,40)
IF(N.EQ.2)WRITE(6,50)
IF(N.EQ.3)WRITE(6,60)
WRITE(6,70)
IF(IND.EQ.0) GO TO 75
WRITE(6,150)
RETURN

75 DO 80 J=1,6
80 WRITE(6,90)(SL(J), A(J), SL(J+9), A(J+9), SL(J+18), A(J+18),
J=1,51) A(J+27), SL(J+36), A(J+36), SL(J+45), A(J+45))
WRITE(6,90)(SL(J), A(J), SL(J+16), A(J+16), SL(J+25), A(J+25),
J=1,51) A(J+34), SL(J+43), A(J+43))
WRITE(6,90)(SL(J), A(J), SL(J+26), A(J+26), SL(J+35), A(J+35),
J=1,51) A(J+44), SL(J+44))
WRITE(6,90)(SL(J), A(J), SL(J+36), A(J+36), SL(J+45), A(J+45))
WRITE(6,20)
WRITE(6,10)
DO 100 J=1,51
100 LINE(J)=DOT
J=25.0*(A(J)+1.0)+1.5
LINE(J)=STAR
WRITE(6,110)(LINE(J), J=1,51)
DO 120 J=1,51
120 LINE(J)=BLANK
LINE(26)=DOT
DO 130 JJ=3,51,2
J=25.0*(A(JJ)+1.0)+1.5
LINE(J)=STAR
WRITE(6,140)(LINE(J), J=1,51)
LINE(J)=BLANK
130 LINE(26)=DOT

108
10 FORMAT(IH1)
20 FORMAT(/2X, "********************************************************************************
 MODE NUMBER =", I2, 8X, "FREQ. RAD/SEC =", F10.4, 8X, "FREQ. HERTZ =", F10.4, 8X, "NON-DIMEN. FREQ. =", F10.4)
30 FORMAT(/5X, "FLAPWISE DEFLECTION/SEMICHRD")
40 FORMAT(/50X, "CHORDWISE DEFLECTION/SEMICHRD")
50 FORMAT(/55X, "TORSIONAL DEFLECTION")
60 FORMAT(/5X, "THE MATRIX TO BE INVERTED IN THE MODE SHAPES COMPUTATIONS IS SINGULAR AND HENCE COMPUTATIONS ARE ABANDONED")
70 FORMAT(6(4X, "STA X/L", 4X, "DEFLN"))
80 FORMAT(/12(2X, F8.4))
90 FORMAT(/40X, 51A1)
100 FORMAT(/40X, 51A1)
110 FORMAT(/5X, "THE MATRIX TO BE INVERTED IN THE MODE SHAPES COMPUTATIONS IS SINGULAR AND HENCE COMPUTATIONS ARE ABANDONED")
RETURN
END
SUBROUTINE INTPOL(A)

C THIS SUBROUTINE INTERPOLATES THE REQUIRED VALUES.

C DIMENSION A(IO1),STA(101),TABLE(101,1),B(101)
COMMON /X3/STA,SPAN
COMMON/X7/NS
IN=0
DO 5 J=1,NS
5 IF(ABS(A(J)) .LE. 0.0) IN=IN+1
IF(IN.NE.NS)GO TO 15
DO 10 J=1,101
10 A(J)=0.0
RETURN
15 A(IO1)=A(NS)
NM1=NS-1
DO 20 I=1,NM1
20 TABLE(I,1)=(A(I+1)-A(I))/(STA(I+1)-STA(I))
H=SPAN/100.0
XARG=H
DO 35 I=2,100
DO 25 J=1,NS
IF(J.EQ.NS .OR. XARG.LE.STA(J)) GO TO 30
25 CONTINUE
30 MAX=J
IF(MAX.LE.2) MAX=2
ISUB=MAX-1.
YEST=TABLE(ISUB,1)
B(I)=YEST*(XARG-STA(ISUB))+A(ISUB)
35 XARG=XARG+H
DO 40 J=2,100
40 A(J)= B(J)
RETURN
END
FUNCTION DET(P)

C THIS FUNCTION CALCULATES THE VALUE OF THE FREQUENCY DETERMINANT
C USES SUBROUTINE TRAMAT

DIMENSION TF(10,10), A(5,5), IROW(5), JCOL(5), JORD(5)
COMMON/X4/IBC
COMMON/X6/TSR
COMMON/X9 /TF
CALL TRAMAT(P)
DO 5 J=9,10
   DO 5 I=1,5
      A(I,J-5)=TF(I+5,J)
   10 IF(IBC-2)10,20,30
      DO 15 I=1,5
         A(I,1)=TF(I+5,4)
         A(I,2)=TF(I+5,5)-TF(I+5,6)*TSR
         A(I,3)=TF(I+5,8)
      GOTO 40
   20 DO 25 I=1,5
      A(I,1)=TF(I+5,3)
      A(I,2)=TF(I+5,5)-TF(I+5,6)*TSR
      A(I,3)=TF(I+5,7)
   30 DO 35 I=1,5
      A(I,1)=TF(I+5,5)-TF(I+5,6)*TSR
      A(I,2)=TF(I+5,7)
      A(I,3)=TF(I+5,8)
   40 CONTINUE
N=5
DET=1.0
DO 80 K=1,N
   KM1=K-1
   PIVOT=0.0
   DO 80 I=1,N
      DO 80 J=1,N
         IF(K.EQ.1) GO TO 55
      50 IF(ISCAN=1,KM1)
         DO 50 JSCAN=1,KM1
            IF(J.EQ.IROW(ISCAN)) GO TO 60
            IF(J.EQ.JCOL(JSCAN)) GO TO 60
   50 CONTINUE
55 IF(ABS(A(I,J)).LE.ABS(PIVOT)) GO TO 60
   PIVOT=A(I,J)
   IROW(K)=I
   60 CONTINUE
JCOL(K)=J

CONTINUE
IF(ABS(PIVOT).GT.0.1E-20) GO TO 65
DET=0.0
RETURN

65 IROWK=IROW(K)
JCOLK=JCOL(K)
DET=DET*PIVOT
DO 70 J=1,N

A(IROWK,J)=A(IROWK,J)/PIVOT
A(IROWK,JCOLK)=1.0/PIVOT

70 DETW0.O RETURN
IROWK-IROW(K)
JCCLK=JCOL(K)
DET=DET*PIVOT
DO 70 J=1,N
A(IROWK,J)=A(IROWK,J)/PIVOT
A(IROWK,JCOLK)=1.0/PIVOT

DO 80 I=1,N
AIJCK=A(I,JCOLK)
IF(I.EQ.IROWK) GO TO 80
A(I,JCOLK)=-AIJCK/PIVOT
DO 75 J=1,N

75 IF(J.NE.JCOLK) A(I,J)=A(I,J)-AIJCK*A(IROWK,J)

80 CONTINUE
DO 85 I=1,N
IROWI=IROW(I)
JCOLI=JCOL(I)

85 JCROI(IROWI)=JCOLI
INTCH=0
NM2=N-I
DO 90 I=1,NM1
IP1=I+1
DG 90 J=IP1,N
IF(JORD(J).GE.JORD(I)) GO TO 90
JTEMP=JORD(J)
JORD(J)=JORD(I)
JORD(I)=JTEMP
INTCH=INTCH+1

90 CONTINUE
IF(INTCH/2*2.NE.INTCH)DET=-DET
RETURN
END
SUBROUTINE TRAMAT(P)

------------------------------------------------------------------------
THIS SUBROUTINE COMPUTES THE TRANSMISSION MATRIX THROUGH THE SYSTEM
USES THE SUBROUTINES INTG AND MATMUL
------------------------------------------------------------------------

DIMENSION TF(10,10), TT(51,10,10), PT(10,10), A(10,10), B(10,10), C(10, 110)
COMMON/X8/PMASS,CLP,SLP,CBETA,SBETA,CBETA,N1,NZ,NJSAVE
COMMON/X9 /TF
COMMON/X10/TT
COMMON/X12/OMEGAN
DO 10 I=1,10
DO 9 J=1,10
TT(1,I,J)=0.0
PT(I,J)=0.0
TT(1,I,I)=1.0
PT(1,I)=1.0
PT(6,1)=P*PMASS*CLP*CBETA
PT(10,1)=P*PMASS
PT(6,2)=(P+OMEGAN)*PMASS*CLP*SBETA
PT(9,2)=(P+OMEGAN)*PMASS
PT(6,5)=(-P+OMEGAN*CTBETA)*PMASS*CLP*CLP
PT(7,5)=OMEGAN*PMASS*CLP*SLP
PT(8,5)=-PT(7,5)*CBETA
PT(7,5)=PT(7,5)*SBETA
PT(9,5)=-PT(6,2)
PT(10,5)=-PT(6,1)
IK=N1
CALL INTG(P,I,IK,1)
IF(N1.GE.50) GO TO 15
IJ=N2
IL=JSAVE
CALL INTG(P,15,50,IL)
DO 20 I=1,10
DO 9 J=1,10
A(I,J)=TT(N2,I,J)
CALL MATMUL(PT,A,B)
N=N2+1
IF(N.GT.51) GO TO 40
DO 35 I=N,51
DO 25 J=1,10
DO 25 K=1,10
A(J,K)=TT(I,J,K)
CALL MATMUL(10,8,C)
DO 30 J=1,10
DO 30 I=N,51
35 CONTINUE
30 CONTINUE
40 CONTINUE

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DO 30 K=1,10
   TT(I,J,K)=C(J,K)
30 CONTINUE
   GO TO 50
40 DO 45 I=1,10
   DO 45 J=1,10
45   TT(51,I,J)=B(I,J)
50 DO 55 I=1,10
   DO 55 J=1,10
55   TF(I,J)=TT(51,I,J)
RETURN
END
SUBROUTINE INTG(P,N1,NJ,NK)
C
C INTEGRATION OF DIFFERENTIAL EQUATIONS USES THE FUNCTION RUNGE
C
DIMENSION D1(IO1),D2(IO1),D3(IO1),D4(IO1),D5(IO1),D6(IO1),D7(IO1),
1D8(IO1),D9(IO1),D10(IO1),D11(IO1),D12(IO1),V1(10,10),V(10),T(10),
2TT(51,10,10)
COMMON/X5/D1,D2,D3,D4,D5,D6,D7,D8,D9,D10,D11,D12
COMMON/X10/TT
DO 10 J=1,10
DO 5 I=1,10
5 V(I,J)=0.0
10 V(J,J)=1.0
M=0
DO 45 I=1,10
DO 15 J=1,10
15 V(J)*V(J,I)
J=NK
K=RUNGE(V,T,J,M)
25 IF(K,NE.1) GO TO 30
FACT=P+OMEGAN
T(1)=V(3)
T(2)=V(4)
T(3)=D1(J)*V(7)+D2(J)*V(8)
T(4)=D3(J)*V(7)+D1(J)*V(8)
T(5)=D4(J)*V(6)
T(6)=P*D6(J)*V(1)+FACT*D7(J)*V(2)+D8(J)*V(3)-D9(J)*V(4)+D10(J)*
1V(5)-P*D11(J)*V(5)
T(7)=D12(J)*V(4)+V(9)-D9(J)*V(5)
T(8)=D12(J)*V(3)+V(10)+D8(J)*V(5)
T(9)=FACT*D5(J)*V(2)-FACT*D7(J)*V(5)
T(10)=P*D5(J)*V(1)+P*D6(J)*V(5)
GO TO 20
30 DO 35 JJ=1,10
35 TT(L+1,JJ,I)=V(JJ)
40 CONTINUE
45 CONTINUE
RETURN
END
FUNCTION RUNGE(Y,F,J,M)

C        ---------------------
C        FOURTH-ORDER RUNGE-KUTTA METHOD
C        ---------------------

DIMENSION Y(IO),F(IO),PHI(IO),SAVEY(IO)
M=M+1
GO TO(5,10,20,30,40),M
5  RUNGE=1
RETURN
10  DO 15 JJ=1,10
SAVEY(JJ)=Y(JJ)
PHI(JJ)=F(JJ)
15  Y(JJ)=SAVEY(JJ)+0.01*F(JJ)
J=J+1
RUNGE=1
RETURN
20  DO 25 JJ=1,10
PHI(JJ)=PHI(JJ)+2.0*F(JJ)
25  Y(JJ)=SAVEY(JJ)+0.01*F(JJ)
RUNGE=1
RETURN
30  DO 35 JJ=1,10
PHI(JJ)=PHI(JJ)+2.0*F(JJ)
35  Y(JJ)=SAVEY(JJ)+0.02*F(JJ)
J=J+1
RUNGE=1
RETURN
40  DO 45 JJ=1,10
45  Y(JJ)=SAVEY(JJ)+(PHI(JJ)+F(JJ))/300.0
M=0
RUNGE=0
RETURN
END
SUBROUTINE MATMUL(A, B, C)

C
C
C MATRIX MULTIPLICATION
C
C
DIMENSION A(10,10), B(10,10), C(10,10)
DO 5 I=1,10
DO 5 J=1,10
C(I,J)=0.0
DO 5 K=1,10
5 C(I,J)=C(I,J)+A(I,K)*B(K,J)
RETURN
END
SUBROUTINE SHAPES(PW, PHI)

C
C THIS SUBROUTINE CALCULATES THE MODE SHAPES. USES THE SUBROUTINE TRAMAT

C

DIMENSION W(51), V(51), PHI(51), A(6, 6), TF(10, 10), TT(51, 10, 10), X(5), IROV(5), JCOL(5), JORD(5)
COMMON/X4/IBC
COMMON/X6/TSR
COMMON/X9/TF
COMMON/X10/TT
COMMON/X11/IND
IND=0
CALL TRAMAT(P)
A(1,4)=TF(1,9)
A(1,5)=TF(1,10)
A(1,6)=0.0
DO 10 I=2,5
A(I,5)=TF(I-4,5)
A(I,6)=TF(I-4,6)
A(I,7)=0.0
DO 15 I=(4,5)
A(I,J)=TF(I,J+5)
DO 10 I=2,5
A(I,6)=TF(I-4,J+5)
A(I,7)=TF(I-4,8)
DO 15 J=4,5
A(I,J)=TF(I,J+5)
A(I,6)=TF(I,J+6)
A(I,7)=TF(I,J+8)
DO 10 J=4,5
A(I,6)=TF(I,J+5)
A(I,7)=TF(I,J+6)
A(I,8)=TF(I,J+8)
DO 10 J=4,5
A(I,9)=TF(I,J+5)
A(I,10)=TF(I,J+6)
A(I,11)=TF(I,J+8)

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```fortran
45      N=5
        MAX=N+1
        DETER=1.0
        DO 80 K=1,N
        KMI=K-1
        PIVOT=0.0
        DO 60 I=1,N
        DO 60 J=1,N
          IF(K.EQ.I)GO TO 55
        DO 50 ISCAN=1,KMI
        DO 50 JSCAN=1,KMI
          IF(I.EQ.IROW(ISCAN))
          IF(J.EQ.JCOL(JSCAN))
            CONTINUE
          GO TO 60
        GO TO 60
      55      IF(ABS(A(I,J)).LE.ABS(PIVOT)) GO TO 60
        PIVOT=A(I,J)
        IROW(K)=I
        JCOL(K)=J
      60      CONTINUE
        IF(ABS(PIVOT).GT.0.1E-20)GO TO 65
        IND=1
        RETURN
      65      IROWK=IROW(K)
        JCOLK=JCOL(K)
        DETER=DETER*PIVOT
        DO 70 J=1,MAX
          A(IROWK,J)=A(IROWK,J)/PIVOT
          A(IROWK,JCOLK)=1.0/PIVOT
        DO 80 I=1,N
          AIJCK=A(I,JCOLK)
          IF(I.EQ.IROWK) GO TO 80
          A(I,JCOLK)=-AIJCK/PIVOT
        DO 75 J=1,MAX
          IF(J.NE.JCOLK) A(I,J)=A(I,J)-AIJCK*A(IROWK,J)
        CONTINUE
        DO 85 I=1,N
          IROWI=IROW(I)
          JCOLI=JCOL(I)
          JORD(IROWI)=JCOLI
          X(JCOLI)=A(IROWI,MAX)
        INTCH=0
        NMI=N-1
        DO 90 I=1,NMI
          IP1=I+1
          DO 90 J=IP1,N
            IF(JORD(J).GE.JORD(I)) GO TO 90
            JTEMP=JORD(J)
            JORD(J)=JORD(I)
            JORD(I)=JTEMP
          CONTINUE
45      N=5
        MAX=N+1
        DETER=1.0
        DO 80 K=1,N
        KMI=K-1
        PIVOT=0.0
        DO 60 I=1,N
        DO 60 J=1,N
          IF(K.EQ.I)GO TO 55
        DO 50 ISCAN=1,KMI
        DO 50 JSCAN=1,KMI
          IF(I.EQ.IROW(ISCAN))
          IF(J.EQ.JCOL(JSCAN))
            CONTINUE
          GO TO 60
        GO TO 60
      55      IF(ABS(A(I,J)).LE.ABS(PIVOT)) GO TO 60
        PIVOT=A(I,J)
        IROW(K)=I
        JCOL(K)=J
      60      CONTINUE
        IF(ABS(PIVOT).GT.0.1E-20)GO TO 65
        IND=1
        RETURN
      65      IROWK=IROW(K)
        JCOLK=JCOL(K)
        DETER=DETER*PIVOT
        DO 70 J=1,MAX
          A(IROWK,J)=A(IROWK,J)/PIVOT
          A(IROWK,JCOLK)=1.0/PIVOT
        DO 80 I=1,N
          AIJCK=A(I,JCOLK)
          IF(I.EQ.IROWK) GO TO 80
          A(I,JCOLK)=-AIJCK/PIVOT
        DO 75 J=1,MAX
          IF(J.NE.JCOLK) A(I,J)=A(I,J)-AIJCK*A(IROWK,J)
        CONTINUE
        DO 85 I=1,N
          IROWI=IROW(I)
          JCOLI=JCOL(I)
          JORD(IROWI)=JCOLI
          X(JCOLI)=A(IROWI,MAX)
        INTCH=0
        NMI=N-1
        DO 90 I=1,NMI
          IP1=I+1
          DO 90 J=IP1,N
            IF(JORD(J).GE.JORD(I)) GO TO 90
            JTEMP=JORD(J)
            JORD(J)=JORD(I)
            JORD(I)=JTEMP
          CONTINUE
```
INTCH=INTCH+1
CONTINUE
IF (INTCH/2*Z.NE.INTCH) DETER = -DETER
IF (ABS(DETER).GT.1.0E-30) GO TO 95
IND=1
RETURN
95 IF (IBC-2) 100, 110, 120
100 DO 105 J=1,51
   W(J)=X(1)*TT(J,1,4)*X(2)*(TT(J,1,5)-TSR*TT(J,1,6))+X(3)*TT(J,1,8)+
       X(4)*TT(J,1,9)+X(5)*TT(J,1,10)
   V(J)=X(1)*TT(J,2,4)*X(2)*(TT(J,2,5)-TSR*TT(J,2,6))+X(3)*TT(J,2,8)+
       X(4)*TT(J,2,9)+X(5)*TT(J,2,10)
105 PHI(J)=X(1)*TT(J,5,4)*X(2)*(TT(J,5,5)-TSR*TT(J,5,6))+X(3)*TT(J,5,8)+
       X(4)*TT(J,5,9)+X(5)*TT(J,5,10)
RETURN
110 DO 115 J=1,51
   W(J)=X(1)*TT(J,1,3)*X(2)*(TT(J,1,5)-TSR*TT(J,1,6))+X(3)*TT(J,1,7)+
       X(4)*TT(J,1,9)+X(5)*TT(J,1,10)
   V(J)=X(1)*TT(J,2,3)*X(2)*(TT(J,2,5)-TSR*TT(J,2,6))+X(3)*TT(J,2,7)+
       X(4)*TT(J,2,9)+X(5)*TT(J,2,10)
115 PHI(J)=X(1)*TT(J,5,3)*X(2)*(TT(J,5,5)-TSR*TT(J,5,6))+X(3)*TT(J,5,7)+
       X(4)*TT(J,5,9)+X(5)*TT(J,5,10)
RETURN
120 DO 125 J=1,51
   W(J)=X(1)*TT(J,1,5)-TSR*TT(J,1,6))+X(2)*TT(J,1,7)+X(3)*TT(J,1,8)+
       X(4)*TT(J,1,9)+X(5)*TT(J,1,10)
   V(J)=X(1)*TT(J,2,5)-TSR*TT(J,2,6))+X(2)*TT(J,2,7)+X(3)*TT(J,2,8)+
       X(4)*TT(J,2,9)+X(5)*TT(J,2,10)
125 PHI(J)=X(1)*TT(J,5,5)-TSR*TT(J,5,6))+X(2)*TT(J,5,7)+X(3)*TT(J,5,8)+
       X(4)*TT(J,5,9)+X(5)*TT(J,5,10)
130 FORMAT(/5X,"THE MATRIX TO BE INVERTED IS SINGULAR IN THE MODE
ISHAPES COMPUTATIONS AND HENCE ABANDONED")
RETURN
END
C C
C NATURAL VIBRATION CHARACTERISTICS, PURE TORSION, USES FUNCTION DET
C AND SUBROUTINES INTPOL, PLOT, NATFRE AND SHAPES
C
REAL MASS, KM1S, KM2S, KMS
DIMENSION GJ(101), KM1S(101), KM2S(101), KMS(101), MASS(101), D1(101),
D2(101), D3(101), STA(101), PHI(51), SL(51), FREGEN(10)
COMMON/X1/FREQEN, H1, H2, H3, ITER, IJK
COMMON/X2/J1, PP, FRE, HERTZ, SL, BLANK, DOUT, STAR
COMMON/X3/STA, SPAN
COMMON/X4/CLS, D2, D3
COMMON/X5/TSR
COMMON/X6/NS
COMMON/X7/PMASS, CLP, SLP, N1, N2, JSAVE
COMMON/X11/OMEGAN

---------

THIS SECTION READS THE DATA OF THE SYSTEM
---------

READ(5,5) ISTAGE, NS
READ(5,10) SPAN, BLANK, DOUT, STAR
READ(5,10) PMASS, CLP, SLP
READ(5,10) (STA(J), J=1, NS), (MASS(J), J=1, NS), (GJ(J), J=1, NS),
1-KM1S(J), J=1, NS), (KM2S(J), J=1, NS)
READ(5,10) H1, H2, H3
IF (ISTAGE. EQ. 2) READ(5,15) NF, ITER, BLANK, DOUT, STAR, INC

---------

THIS SECTION PRINTS THE DATA OF THE SYSTEM
---------

WRITE(6,20)
WRITE(6,25)
IF (ISTAGE. EQ. 2) WRITE(6,30) NF
WRITE(6,35) H1
WRITE(6,40) H2
WRITE(6,45) H3
IF (ISTAGE. EQ. 2) WRITE(6,50) INC
WRITE(6,55) SPAN
WRITE(6,60) OMEGA
WRITE(6,65) B
WRITE(6,70) TSR
IF (PMASS. LE. 0.0) GO TO 200
WRITE(6,75) PMASS
WRITE(6,80) CLP
WRITE(6,85) SLP
THIS SECTION CALCULATES THE SYSTEM PROPERTIES AT THE REQUIRED STATIONS BY INTERPOLATION AND PRINTS THE INTERPOLATED VALUES

CALL INTPOL(MASS)
CALL INTPOL(GJ)
CALL INTPOL(BETA)
CALL INTPOL(KM1S)
CALL INTPOL(KM2S)
WRITE(6,20)
WRITE(6,25)
WRITE(6,125)
WRITE(6,25)
WRITE(6,105)
WRITE(6,100)(MASS(J),J=1,101)
WRITE(6,110)
WRITE(6,100)(GJ(J),J=1,101)
WRITE(6,115)
WRITE(6,100)(KM1S(J),J=1,101)
WRITE(6,120)
WRITE(6,100)(KM2S(J),J=1,101)
WRITE(6,25)

THIS SECTION COMPUTES THE NON-DIMENSIONAL COEFFICIENTS OF THE FIRST-ORDER DIFFERENTIAL EQUATIONS

BS=B*8
SPANS=SPAN*SPAN
F1=BS/SPANS
PI=4.0*ATAN(1.0)
OMEGA=OMEGA*PI/30.0
FACT = MASS(1) * SPANS * SPANS / GJ(1)

OMEGA = OMEGA * OMEGA * FACT

FACT = SQRT(FACT)

TSR = TSR * SPANS / (GJ(1) * BS)

IF (PMASS .LE. 0.0) SLP = SPAN

PMmass = PMass / (SPAN * MASS(1) * 386.4)

CLP = CLP / B

SLP = SLP / SPAN

N1 = 50.0 * SLP + 0.5

N2 = N1 + 1

JSAVE = 2 * N1 + 1

DO 205 J = 1, 101

Kms(J) = Kms(J) + KN2S(J)

D1(J) = GJ(1) * F1 / GJ(J)

D2(J) = OMEGA * (Kms(J) - Kms(J)) / (MASS(1) * BS)

D3(J) = Kms(J) / (MASS(1) * BS)

H1 = H1 * FACT

H2 = H2 * FACT

H3 = H3 * FACT

IF (ISTAGE .EQ. 2) GO TO 215

WRITE(6, 20)
WRITE(6, 25)
WRITE(6, 130)
WRITE(6, 25)

P = H1 + H1

IF (H1 .GT. H3) GO TO 245

FR = H1 / FACT

F = DET(P)

WRITE(6, 135) FR, H1, F

H1 = H1 + H2

GOTO 210

THIS SECTION CALCULATES THE NATURAL FREQUENCIES AND THE ASSOCIATED MODAL FUNCTIONS

CALL NATFRE(NF)

SL(1) = 0.0

DO 220 J = 1, 50

SL(J + 1) = SL(J) + 0.02

DO 240 J = 1, IJK

JL = J + INC
PP=FRE'EN(J)
PP=PP*PP
CALL SHAPES(P,PHI)
AMAX=PHI(I)
DO 225 I=1,51
225 IF(ABS(AMAX).LT.ABS(PHI(I)))AMAX=PHI(I)
DO 230 I=1,51
230 PHI(I)=PHI(I)/AMAX
FRE=PP/FACT
HERTZ=FRE/(2.0*PI)
235 CALL PLOT(PHI)
240 CONTINUE
IF(IJK.LT.NF)WRITE(6,140)
IF(ISTAGE.EQ.1)WRITE(6,25)
245 CONTINUE
C
C FORMATS
C
5 FORMAT(I1,I3)
10 FORMAT(5E14.7)
15 FORMAT(12,I3,3A1,I2)
20 FORMAT(1H1)
25 FORMAT(180)
30 FORMAT(/5X,"NUMBER OF FREQUENCIES REQUIRED","=",I5)
35 FORMAT(/5X,"STARTING FREQUENCY(RAD/SEC)","=",E14.7)
40 FORMAT(/5X,"FREQUENCY INCREMENT(RAD/SEC)","=",E14.7)
45 FORMAT(/5X,"ENDING FREQUENCY (RAD/SEC)","=",E14.7)
50 FORMAT(/5X,"INCREMENT IN THE MODE NUMBER","=",I5)
55 FORMAT(/5X,"LENGTH OF THE BLADE(INCHES)","=",E14.7)
60 FORMAT(/5X,"ROTATIONAL VELOCITY OF THE BLADE(RPM)","=",E14.7)
65 FORMAT(/5X,"semi-chord at the root(INCHES)","=",E14.7)
70 FORMAT(/5X,"CONTROL SYSTEM SPRING RATE(IN-LB/RAD)","=",E14.7)
75 FORMAT(/5X,"WEIGHT OF THE SENSOR(LB)","=",E14.7)
80 FORMAT(/5X,"CHORDWISE LOCATION OF THE PROBE(INCHES)","=",E14.7)
FORMAT(//5X,"SPANWISE LOCATION OF THE PROBE(INCHES) =",E14.7)
1
FORMAT(//5X,"NUMBER OF DATA POINTS =",I5)
95
FORMAT(//5X,"STATION LOCATIONS(INCHES)"
100
FORMAT(//7(4X,E14.7))
105
FORMAT(//5X,"MASS PER UNIT LENGTH(LB-SEC**2)"
110
FORMAT(//5X,"TORSIONAL STIFFNESS(LB-IN**2)"
115
FORMAT(//5X,"MASS MOMENT OF INERTIA ABOUT THE CHORD(LB-SEC**2)"
120
FORMAT(//5X,"MASS MOMENT OF INERTIA ABOUT AN AXIS PERPENDICULAR TO THE CHORD THROUGH THE CENTER OF GRAVITY(LB-SEC**2)"
125
FORMAT(//5X,"THE FOLLOWING ARE THE INTERPOLATED VALUES AT 101 EQUAL DISTANT STATIONS"
130
FORMAT(//5X,"THE FOLLOWING COLUMNS ARE 1: FREQUENCY(RAD/SEC) 2:NONDIMENSIONAL FREQUENCY 3: VALUE OF THE FREQUENCY DETERMINANT RESPECTI
135
140
FORMAT(//5X,"THE NUMBER OF FREQUENCIES DETECTED WITHIN THE RANGE ARE ONLY =",I5)
STOP
. END
SUBROUTINE NATFRE(N)

C THIS SUBROUTINE SCANS THE FREQUENCY DETERMINANT WITH RESPECT TO
C THE FREQUENCY TILL THE SPECIFIED NUMBER OF SIGN CHANGES ARE
C DETECTED STARTING FROM ZERO FREQUENCY. USES FUNCTION DET

DIMENSION FREQEN(10),JKL(10)
COMMON/X,FREQEN,H,F,H3,ITER,IJK
IJK=0
ITRN=0
DO 5 J=1,N
  JKL(J)=0
  PP=H1
  P=PP*PP
  F=DET(P)
  IF(ABS(F).LT.0.0001)GO TO 10
  IJK=IJK+1
  JKL(IJK)=1
  FREQEN(IJK)=PP
  PP=PP+H
  P=PP*PP
  F=DET(P)
  IF(ABS(F).LT.0.0001)GO TO 10
  F=SIGN(1.0,F)
  PP=PP+H
  P=PP*PP
  G=DET(P)
  IF(ABS(G).LT.0.0001)GO TO 20
  IJK=IJK+1
  JKL(IJK)=1
  FREQEN(IJK)=PP
  IF(IJK.EQ.N)GO TO 30
  IF(PP.GE.H3)GO TO 30
  PP=PP+H
  P=PP*PP
  F=DET(P)
  F=SIGN(1.0,F)
  GO TO 15
  G=SIGN(1.0,G)
  GO TO 20
  F=6
GO TO 15

ITRN=ITRN+1
IF(ITRN.GT.ITER)GO TO 55

H=H/10.0

DO 50 J=1, IJK
IF(JKLCJ).EQ.1)GO TO 50
PP=FREQEN(J)
P=PP*PP
F=DET(P)
F=SIGN(1.O, F)

35 PP=PP+H
P=PP*PP
G=DET(P)
IF(ABS(G).GT.0.0001)GO TO 40
JKL(J)=1
FREQEN(J)=PP
GO TO 50

40 G=SIGN(1.O, G)
IF(F*G.GT.0.0)GO TO 45
FREQEN(J)=PP-H
GO TO 50

45 F=G
GO TO 35

50 CONTINUE
GO TO 30

55 DO 60 J=1, IJK
IF(JKLCJ).EQ.1)GO TO 60
PP=FREQEN(J)
P=PP*PP
F=DET(P)
PP=PP+H
P=PP*PP
G=DET(P)
DIFF=G-F
FREQEN(J)=PP-G*H/DIFF

60 CONTINUE
RETURN
END
SUBROUTINE PLOT(A)
THIS SUBROUTINE PRINTS THE NATURAL FREQUENCIES AND MODE SHAPES AND PLOTS THE MODE SHAPES

REAL LINE

DIMENSION A(51), SL(51), LINE(51)
COMMON/X2, J, PP, FRE, HERTZ, SL, BLANK, DOT, STAR
WRITE(6, 10)
WRITE(6, 20)
WRITE(6, 30)
WRITE(6, 20)
WRITE(6, 60)
WRITE(6, 70)

DO 80 J=1, 6
WRITE(6, 90) (SL(J), A(J), SL(J+9), A(J+9), SL(J+18), A(J+18), SL(J+27), A(J+27), SL(J+36), A(J+36), SL(J+45), A(J+45))
WRITE(6, 90) (SL(7), A(7), SL(16), A(16), SL(25), A(25), SL(34), A(34), SL(43), A(43))
WRITE(6, 90) (SL(8), A(8), SL(17), A(17), SL(26), A(26), SL(35), A(35), SL(44), A(44))
WRITE(6, 90) (SL(9), A(9), SL(18), A(18), SL(27), A(27), SL(36), A(36), SL(45), A(45))
WRITE(6, 20)
WRITE(6, 10)

DO 100 J=1, 51
100 LINE(J)=DOT
       J=25.0*(A(J)+1.0)+1.5
       LINE(J)=STAR
       WRITE(6, 110) (LINE(J), J=1, 51)
       DO 120 J=1, 51
120 LINE(J)=BLANK
       LINE(26)=DOT
       DO 130 JJ=3, 51, 2
       J=25.0*(A(JJ)+1.0)+1.5
       LINE(J)=STAR
       WRITE(6, 140) (LINE(J), J=1, 51)
       LINE(J)=BLANK
130 LINE(26)=DOT

10 FORMAT(1H1)
20 FORMAT(/2X, "************************************************************")
30 FORMAT(/5X, "MODE NUMBER =", T2, 8X, "FREQ. RAD/SEC =", F10.4, 8X, "FREQ. HERTZ =", F10.4, 8X, "NON-DIMEN. FREQ. =", F10.4)
60 FORMAT(/5X, "TORSIONAL DEFLECTION")
SUBROUTINE INTPOL(A)

RETURN
END
THIS SUBROUTINE INTERPOLATES THE REQUIRED VALUES.

DIMENSION A(101), STA(101), TABLE(101, 1), B(101)
COMMON /X3/STA, SPAN
COMMON /X6/NS
IN=0
DO 5 J=1,NS
   IF(ABS(A(J)) LE 0.0) IN=IN+1
   IF(IN.NE.NS)GO TO 15
DO 10 J=1,101
10 A(J)=0.0
RETURN
5
15 A(101)=A(NS)
   NS1=NS-1
   DO 20 I=1, NS1
20 TABLE(I, 1)=(A(I+1)-A(I))/(STA(I+1)-STA(I))
   H=SPAN/100.0
   XARG=H
   DO 35 I=2, 100
   DO 25 J=1, NS
      IF(J.EQ.NS OR XARG.LE.STA(J)) GO TO 30
   25 CONTINUE
30 MAX=J
   IF(MAX.LE.2) MAX=2
   ISUB=MAX-1
   YEST=TABLE(ISUB+1)
   B(I)=YEST*(XARG-STA(ISUB))+A(ISUB)
35 XARG=XARG+H
   DO 40 J=2, 100
40 A(J)=B(J)
RETURN
END
FUNCTION DET(P)
THIS FUNCTION CALCULATES THE VALUE OF THE FREQUENCY DETERMINANT.

USES SUBROUTINE TRAMAT

DIMENSION TF(2,2)
COMMON/X5/TSR
COMMON/X9/TF
CALL TRAMAT(P)
DET=TF(2,1)-TSR*TF(2,2)
RETURN
END

SUBROUTINE TRAMAT(P)
THIS SUBROUTINE COMPUTES THE TRANSMISSION MATRIX THROUGH THE SYSTEM.

USES THE SUBROUTINES INTG AND MATHUL.

DIMENSION TF(2,2), PT(2,2), A(2,2), B(2,2), C(2,2)
COMMON/X7/PMASS, CLP, SLP, N1, N2, JSAVE
COMMON/X9 /TF
COMMON/X10/TT
COMMON/X11/OMEGAN
DO 10 I=1,2
DO 5 J=1,2
TT(1,I,J)=0.0
PT(I,J)=0.0
TT(1,I,I)=1.0
PT(I,I)=1.0
PT(2,1)=(-P+OMEGAN)*PM4SS*CLP*CLP
IK=N1
CALL INTG(P, IK, I)
IF(N1.GE.50) GO TO 15
IJ=N2
IL=JSAVE
CALL INTG(P, IJ, 50, IL)
10       DO 20 I=1,2
DO 20 J=1,2
20       A(I,J)=TT(N2, IJ)
CALL MATHUL(PT, A, B)
N=N2+1
IF(N.GT.51) GO TO 40
DO 35 I=N,51
DO 25 J=1,2
DO 25 K=1,2
25       A(J,K)=TT(I, J, K)
CALL MATHUL(A, B, C)
DO 30 J=1,2
DO 30 K=1,2
30       TT(I, J, K)=C(J, K)
35       CONTINUE
GO TO 50
40       DO 45 I=1,2
DO 45 J=1,2
DO 45 K=1,2
45       TT(51, I, J)=3(I, J)
50       DO 55 I=1,2
DO 55 J=1,2
DO 55 K=1,2
55       TF(I, J)=TT(51, I, J)
RETURN
SUBROUTINE INTG(P,NI,NJ,NK)
DIMENSION D1(101), D2(101), D3(101), V1(2,2), V(2), T(2), TT(51,2,2)
COMMON/X4/D1, D2, D3, COMMON/X10/TT
DO 10 J=1, 2
DO 5 I=1, 2
5 V1(I, J)=0.0
10 V1(J, J)=1.0
M=0
DO 45 I=1, 2
DO 15 J=1, 2
15 V(J)=V1(J, I)
J=NI
DO 40 L=NI, NJ
20 K=RUNGE(V, T, J, M)
25 IF(K.NE.1) GO TO 30
   T(I)=D1(J)*V(2)
   T(2)=D2(J)*V(1)-P*D3(J)*V(1)
GO TO 20
30 DO 35 JJ=1, 2
35 TT(L+1, JJ, I)=V(JJ)
40 CONTINUE
45 CONTINUE
RETURN
END

FUNCTION RUNGE(YF, J, M)

C INTEGRATION OF DIFFERENTIAL EQUATIONS USES THE FUNCTION RUNGE
FOURTH-ORDER RUNGE-KUTTA METHOD

DIMENSION Y(2), F(2), PHI(2), SAVEY(2)

M=M+1
GO TO(5, 10, 20, 30, 40), M

RUNGE=1
RETURN

DO 15 JJ = 1, 2
SAVEY(JJ) = Y(JJ)
PHI(JJ) = F(JJ)
15 Y(JJ) = SAVEY(JJ) + 0.01*F(JJ)
J = J + 1
RUNGE = 1
RETURN

DO 25 JJ = 1, 2
PHI(JJ) = PHI(JJ) + 2.0*F(JJ)
25 Y(JJ) = SAVEY(JJ) + 0.01*F(JJ)
RUNGE = 1
RETURN

DO 35 JJ = 1, 2
PHI(JJ) = PHI(JJ) + 2.0*F(JJ)
35 Y(JJ) = SAVEY(JJ) + 0.02*F(JJ)
J = J + 1
RUNGE = 1
RETURN

DO 45 JJ = 1, 2
45 Y(JJ) = SAVEY(JJ) + PHI(JJ)*F(JJ))/300.0
M = 0
RUNGE = 0
RETURN
END

SUBROUTINE MATMUL(A, B, C)
DIMENSION A(2,2), B(2,2), C(2,2)

DO 5 I=1,2
    DO 5 J=1,2
        C(I,J) = 0.0
    5 CONTINUE

DO 5 K=1,2
    C(I,J) = C(I,J) + A(I,K)*B(K,J)
5 CONTINUE

RETURN
END

SUBROUTINE SHAPES(P, PHI)
THIS SUBROUTINE CALCULATES THE MODE SHAPES. USES THE SUBROUTINE

TRAMAT

DIMENSION PHI(51), TT(51,2,2)
COMMON/X5/TSR
COMMON/X10/TT
CALL TRAMAT(P)
DO 5 J=1,51
   PHI(J) = TT(J,1,1) - TSR*TT(J,1,2)
5 RETURN
END
<table>
<thead>
<tr>
<th>NUMBER OF DATA POINTS</th>
<th>DISTANCE OF THE BLADE FROM THE ROOT (INCHES)</th>
<th>COLLECTIVE DEGREES</th>
<th>CONTROL SYSTEM SPRING RATE (LM/INCH)</th>
<th>ROTATIONAL VELOCITY OF THE BLADE (RPM)</th>
<th>INCREMENT IN THE FREQUENCY (RAD/SEC)</th>
<th>ENDING FREQUENCY (RAD/SEC)</th>
<th>FREQUENCY INCREMENT (RAD/SEC)</th>
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NATURE OF THE MUTES

COLLECTIVE MUTES

INCREMENTS

IN THE LENGTH OF THE BLADE (INCHES)

NUMBER OF FreqUENCIES REQUIRED

MUTU of frQuencies

STARTING FreqUency (RAD/SEC)

END OF FreqUency (RAD/SEC)

INCREMENT IN THE FREQUENCY (RAD/SEC)

END OF FREQUENCY (RAD/SEC)

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FREQUENCY INCREMENT (RAD/SEC)
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<th>FLAPWISE BENDING STIFFNESS (LA-IN**2)</th>
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<tbody>
<tr>
<td>7700000E+02</td>
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<td>2600000E+09</td>
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<td>4200000E+09</td>
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<th>CHORDWISE BENDING STIFFNESS (LA-IN**2)</th>
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<td>2340000E+09</td>
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<th>TORSIONAL STIFFNESS (LA-IN**2)</th>
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<tr>
<td>3070000E+09</td>
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<td>4500000E+07</td>
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<table>
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<tr>
<th>DISTANCE BETWEEN MASS AND ELASTIC AXISE (INCHES)</th>
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<tr>
<td>9240000E+09</td>
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<tr>
<td>1660000E+09</td>
</tr>
<tr>
<td>3120000E+09</td>
</tr>
<tr>
<td>6980000E+07</td>
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<tr>
<td>Mass Moment of Inertia About an Axis Perpendicular to the Chord Through the Center of Gravity (lb-ft(^2))</td>
</tr>
<tr>
<td>---</td>
</tr>
<tr>
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</tr>
<tr>
<td>0.000000E+00</td>
</tr>
<tr>
<td>1.100000E+00</td>
</tr>
<tr>
<td>2.000000E+00</td>
</tr>
<tr>
<td>5.000000E+00</td>
</tr>
<tr>
<td>1.000000E+01</td>
</tr>
</tbody>
</table>

**Notes:**
- The table entries are in scientific notation.
- The units for mass moment of inertia are lb-ft\(^2\) and for twist are deg/ft.
THE FOLLOWING ARE THE INTERPOLATED VALUES AT 101 EQUIDISTANT STATIONS

** FLAPWISE BENDING STIFFNESS (IN**2**/SEC**2**/IN**4**) 

<table>
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** FLAPWISE BENDING STIFFNESS (IN**2**/SEC**2**/IN**4**) 

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**Mode Number: 2**

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Non-Dimensional Freq. = 25.8569

Flapwise Deflection

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Non-Dimen, Freq. = 70.7782

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**FLAPWISE DEFLECTION**

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