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IMPERFECTION ON THE BUCKLING LOAD AND MASS
OF GRAPHITE-EPOXY BLADE-STIFFENED PANELS
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EFFECT OF BOW-TYPE INITIAL IMPERFECTION ON THE BUCKLING LOAD AND MASS OF GRAPHITE-EPOXY BLADE-STIFFENED PANELS

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EFFECT OF BOW-TYPE INITIAL IMPERFECTION ON THE BUCKLING
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SUMMARY

Design studies are carried out with a structural synthesis computer code which accounts for first order effects of an initial bow and which can be used for sizing stiffened composite panels having an arbitrary cross section. The studies, which focus on graphite-epoxy blade-stiffened panels, examine the effect of a small initial bow on both the load carrying ability of panels and on the mass of panels designed to carry a specified load. Large reductions in the buckling load caused by a small initial bow emphasize the need for considering a bow when a panel is designed.

INTRODUCTION

Imperfections can have a profound effect on the stability of highly optimized structural systems if the imperfections are ignored during the structural synthesis (see, for example, ref. 1). To deal with this problem, a primary objective in structural design should be to account both for imperfections that are likely to occur during fabrication and for damage that is likely to occur during service. The effects of certain forms of imperfections on the design of stiffened compression panels made of metal are presented in reference 2. The effect of impact damage on the compressive strength of graphite-epoxy panels is discussed in reference 3.

In the present report, one type of imperfection discussed in reference 2 - an overall bow in a panel - is examined. The structure that is studied is a blade-stiffened compression panel made of graphite-epoxy material. The effect of a bow on the load carrying ability of a minimum-mass panel designed

for zero bow is presented. Mass-strength curves are presented for panels designed with and without an initial bow. The effect of extensional stiffness and shear stiffness requirements are investigated briefly.

SYMBOLS

Values are given in both SI and U.S. Customary Units. The calculations were made in U.S. Customary Units.

A	surface area of one period of stiffened panel, $2(b_1+b_2)L$
b_k	element lengths defined in figure 3
c	distance from neutral surface of panel to location where strain is to be calculated
D_x	smearred orthotropic bending stiffness
E_1, E_2	Young's modulus of composite material in fiber direction and transverse to fiber direction, respectively
ET	longitudinal extensional stiffness of panel
e	overall bow in panel, measured at panel midlength (see fig. 1)
G_{12}	shear stiffness of composite material in coordinate system defined by fiber direction
k	integer
L	panel length (see fig. 1)
M	bending moment caused by bow in panel

n	integer
N_x	applied longitudinal compressive loading per unit width of panel (see fig. 1)
$N_{x_{cr}}$	value of N_x that causes buckling
N_{x_E}	Euler buckling load of panel
$N_{x_{cr}}(e=0)$	value of N_x that causes buckling when no initial bow is present
$(N_x)_{design}$	value of N_x for which panel is designed
t_k	lamina thicknesses defined in figure 3.
W	mass of one period of stiffened panel
$\frac{W/A}{L}$	mass index
X, Y, Z	coordinate axes defined in figures 1 and 2
x, y, z	coordinate directions
ϵ_{max}	maximum allowable value of longitudinal or transverse strain in each lamina
ϵ_x	strain in x direction
μ_{12}	Poisson's ratio of composite material in coordinate system defined by fiber direction
λ	buckling half-wavelength
ρ	density

ANALYSIS-DESIGN PROCEDURE

The feasibility of including an accurate buckling analysis in a computer code for designing stiffened composite panels was demonstrated in reference 4. Although based on the same concepts as the code in reference 4 - that is, non-linear math programming with constraints expanded in Taylor series - the code used to obtain the present results has a greatly expanded capability. In addition to any combination of inplane loadings, the advanced code can treat lateral pressure, an overall bow, and temperature effects. Arbitrary panel cross sections are possible and can usually be generated with only three or four input cards. The computer code is described in detail in reference 5. Two significant aspects of the code are the manner in which the bow is accounted for and the buckling analysis. These two topics are discussed in the following two sections.

Effect of Initial Overall Bow

The approach used here to account for the effect of an initial bow in the panel is the same as that used in reference 2 (with appropriate changes to account for laminated walls). The panel is assumed to have the initial bow shown in figure 1. The stresses acting on the panel cross section are taken to be the sum of the stress from N_x and the stress resulting from the moment caused by the bow. In terms of the longitudinal strain ϵ_x , this gives

$$\epsilon_x = \frac{N_x}{ET} + \frac{M \cdot c}{D_x} \quad (1)$$

The moment varies over the length of the panel. At the midlength of the panel the moment is largest and is given by

$$M = \frac{N_x \cdot e}{1 - \frac{N_x}{N_{x_E}}} \quad (2)$$

in which N_{x_E} is the Euler or wide-column buckling load of the panel. The denominator in equation (2) gives the nonlinear effect of the deformation growing with the applied load. Except for one wavelength, the buckling calculations are made assuming that the midlength stresses from equations (1) and (2) are the stresses over the entire length of the panel. The exception is the buckling mode having a half-wavelength λ equal to the panel length L . For that case, the moment M is considered to be zero. The initial bow in the panel does not, therefore, directly affect the $\lambda = L$ buckling load.

Buckling Analysis

The buckling analysis used to obtain the results presented in this report is a stiffened panel buckling analysis code denoted VIPASA, which is described in references 4-7. The VIPASA analysis treats an arbitrary assemblage of plates with each plate loaded by an N_x , N_y , and N_{xy} loading. The response of each plate element making up the stiffened panel is obtained using an exact solution of the thin plate equations. The analysis connects these individual plate elements and maintains continuity of the buckle pattern across the intersection of neighboring plate elements. The stiffened panel is assumed to be uniform in the x direction (fig. 1) and simply-supported along the edges $x = 0$ and $x = L$. For these reasons the buckle patterns in the x direction are taken to be sine waves whose half-wavelengths are fractions $(1/n)$ of the panel length.

The VIPASA analysis cannot treat panels that are curved in the x direction. The approach used in this report is to treat a panel as if it were straight, but to use a stress distribution (eqs. (1) and (2)) for a panel with a bow. Also, the value for N_{x_E} used in equation 2 is taken to be the lowest buckling load for $\lambda = L$.

DESIGN STUDIES

The loading considered in this report is a longitudinal compressive load, N_x , shown in figure 1. The lamina properties assumed for the graphite-epoxy material are given in table I. All calculations were made with a panel length

L equal to 76.2 cm (30 in.). The panel was modeled with four stiffeners and with symmetric boundary conditions along the unconnected longitudinal edges.

Configuration

The panel configuration considered is the blade-stiffened configuration shown in figure 2. The configuration is defined in terms of the element widths b_1 , b_2 , and b_3 , the thicknesses t_1 and t_4 of $\pm 45^\circ$ layers, and the thicknesses t_2 , t_3 , and t_5 of 0° layers. The 0° filaments are parallel to the X axis. All laminates are balanced and symmetric. No offsets were used in the modeling of the blade-stiffened panels. For that reason, the mathematical model is as shown in figure 3.

The three widths and the five thicknesses shown in figure 3 are the design variables in the structural synthesis. No upper or lower bounds were placed on the design variables. Except for the cases involving longitudinal extensional stiffness requirements, the lightest designs were designs with t_2 equal to zero.

In order to apply the moment to the panel cross section, the blade b_3 was divided into three elements of equal length. A longitudinal strain ϵ_x was calculated from equations (1) and (2) for each of these three elements and for the skin - a total of four values of ϵ_x . The resulting strain distribution on the panel cross section is shown in figure 4 for two different values of bending moment relative to axial load.

Effect of Bow on Buckling Load of Optimized Panel Designed for Zero Bow

A panel designed to support a loading of $N_x/L = 689 \text{ kPa (100 lbf/in}^2\text{)}$ with zero bow was analyzed for various amounts of bow. The results in terms of nondimensional quantities are presented in figure 5. Positive values of e/L produce a moment that puts the tip of the blade in tension and the skin in compression. For a value of e/L of only 0.001, the buckling load is reduced by about 24 percent. For $e/L = 0.001$ the lowest buckling load occurs for $\lambda = L/24$. For $e/L = -0.001$ the lowest buckling load occurs for $\lambda = L/3$.

Although the reduction in the buckling load shown in figure 5 is almost symmetric with respect to $e/L = 0.0$, this is not always the case. In some designs for which extensional and shear stiffness requirements were imposed, the buckling load curves were very unsymmetric with respect to $e/L = 0.0$.

Effect of Bow on Panel Mass

The design data in this section are presented in the form of a structural efficiency diagram in which the mass index $\frac{W/A}{L}$ of minimum-mass panels is given as a function of the loading index N_x/L . Structural efficiency diagrams and scaling principles for panels are discussed in reference 8. Although reference 8 is based on a simplified buckling analysis, the scaling principles apply equally well for the type of analysis used in this report - including the effect of the bow.

Buckling requirements.- The data are presented in figure 6. The solid lines represent panels that have only a buckling requirement. No material strength requirements are imposed. (For comparison, a hat-stiffened panel line is also shown. The line is taken from reference 4 where material properties differ slightly from those used in this report.) The panels are designed for three values of the bow parameter: $e/L = 0.0$, ± 0.001 , and ± 0.003 . The curves for which $e/L = \pm 0.001$ and ± 0.003 represent acceptable designs for two load conditions - (1) the design loading with a positive value of e/L , (2) the design loading with a negative value of e/L . In other words, the panels carry the design load whether the bow is positive or negative. The increase in mass required to design for the bow varies with the loading. Lightly loaded panels ($N_x/L = 68.9 \text{ kPa (10 lbf/in}^2\text{)}$) with $e/L = \pm 0.003$ are about 27 percent heavier than panels with no bow. Heavily loaded panels ($N_x/L = 6890 \text{ kPa (1000 lbf/in}^2\text{)}$) are about 17 percent heavier.

Buckling and material strength requirements.- The dashed curves represent the effect of material strength requirements which, for these cases, are maximum allowable lamina strains. Two sets of allowable strains are considered. The ± 0.004 designation means that for these panels the longitudinal and transverse strains in any lamina do not exceed ± 0.004 . A similar definition applies to the ± 0.005 curves. The allowable shear strain is 0.01 for

both sets of allowable strains; however, for these studies, shear strain is not an active design requirement. For clarity, design curves incorporating material strength requirements are not shown for the $e/L = \pm 0.001$ case. An initial bow has a large effect on the load at which material strength considerations become important. Consider, for example, a maximum strain requirement of ± 0.004 . For $e/L = 0.0$ the material strength requirement begins to have an effect at N_x/L equal to about 2760 kPa (400 lbf/in²). For $e/L = \pm 0.003$ the material strength requirement begins to have an effect at N_x/L equal to about 689 kPa (100 lbf/in²).

Buckling and Stiffness requirements. - The two symbols in figure 6 at $N_x/L = 689$ kPa (100 lbf/in²) represent two panels that have stiffness requirements similar to requirements in a commercial aircraft wing panel (ref. 9). The requirements are:

$$\begin{aligned} \text{Longitudinal extensional stiffness } ET &= 368 \text{ MN/m } (2.10 \times 10^6 \text{ lbf/in}) \\ \text{shear stiffness } GT &= 61.3 \text{ MN/m } (3.50 \times 10^5 \text{ lbf/in}) \end{aligned}$$

Both panels have both stiffness requirements. The lighter panel is designed for $e/L = 0.0$; the heavier panel is designed for $e/L = \pm 0.003$. The panel designed for the initial bow is about 1.4 percent heavier than the panel designed for zero bow. However, if the panel designed for zero bow is analyzed as if it had a bow of $e/L = 0.003$, it buckles at about 53 percent of the design load.

Panels designed for a bow and for zero bow. - Another way of studying the effect of a bow on panel mass is to examine on a structural efficiency diagram the loading at which a panel designed for zero bow fails when an initial bow is present. That information is presented in figure 7. The solid lines are for panels designed for $e/L = \pm 0.003$. The dashed lines are for panels designed for zero bow. The long dashes are for panels analyzed for zero bow; the short dashes are for these same panels analyzed with a bow of $e/L = \pm 0.003$. The short curves are for panels with the extensional and shear stiffness requirements given earlier. The longer curves have no stiffness requirement. In all cases a maximum strain requirement of ± 0.004 is imposed.

The difference between the solid curve and the corresponding curve composed of short dashes indicates the mass saving provided by designing a

panel to meet a bow requirement rather than accepting a knockdown. In the case of the panels designed without a stiffness requirement the mass saving is small except in the heavily loaded region. In the case of the panels designed to meet the stiffness requirements, the mass savings are negligible in the left-hand portion of the data, where the stiffness requirements dominate the design; however, in the right hand portion of this same data, substantial differences can exist. The differences arise because the panels designed without a bow have various configurations of essentially the same mass. When these configurations are analyzed with a bow, the scatter in the load-carrying ability produces the cross-hatched area. The two symbols in the stiffness design data are an example of the reduction in the buckling load caused by a bow.

Buckling Response of Panels Designed for a Bow

Panels designed for a bow have a characteristic buckling response diagram. An example is shown in figure 8. This example is for the panel designed for $N_x/L = 689 \text{ kPa}$ (100 lbf/in^2) and $e/L = \pm 0.003$. The mass-strength data for this case is included in figures 6 and 7. As stated earlier, the panel carries the design load with either a positive or negative bow. The buckling load of this panel as a function of buckling half-wavelength is shown in figure 8 for the positive bow (0.003), negative bow (-0.003), and for zero bow.

Buckling response for $e/L = \pm 0.003$. - The circular symbols and square symbols represent the buckling response of the panel when it is analyzed with $e/L = -0.003$ and $e/L = +0.003$, respectively. For a positive bow - tip of blade in tension and skin in compression - the lowest buckling load occurs for $\lambda = L/19$. For a negative bow, the lowest buckling load occurs for $\lambda = L/2$, $L/3$, and $L/4$. It was explained earlier that the bow does not directly affect the buckling load for $\lambda = L$. For this reason, the panel has the same buckling load at $\lambda = L$ for both the positive, negative, and zero bow. As seen in figure 8, the buckling load for $\lambda = L$ is substantially higher than the design load. That large margin on overall buckling is characteristic of panels designed using the procedure described in this report for an initial bow. The large margin is, however, somewhat fictitious. If the

buckling load for $\lambda = L$ were reduced slightly, the panel would no longer carry the design load; it would immediately fail with one of the buckling half-wavelengths mentioned above - depending upon the sign of the initial bow in the panel.

Buckling response for $e/L = 0.0$.- The triangular symbols represent the buckling response of the panel when it is analyzed with $e/L = 0.0$. The data gives an indication of the margins that must be placed on the buckling loads at the various half-wavelengths to enable a panel to support the design load if a bow is present.

DESIGN IMPLICATIONS

Calculations presented in this report have shown that a slight bow can cause a substantial reduction in the buckling load of a graphite-epoxy blade-stiffened panel. Similar reductions probably exist for other configurations. The initial bow may be the major reason that the hat-stiffened panel of reference 10 failed at loadings less than the design load. (See table VI, ref. 10.)

Since a slight bow (like other imperfections) is virtually unavoidable, it is important to account for the possibility of a bow when the panel is designed. In some cases (see fig. 7) a panel designed for an initial bow may be only slightly lighter than a panel designed for zero bow but with a knock-down for the bow. In other cases the differences may be greater. Since designing for an initial bow is the straightforward approach and since it will always provide some mass saving, that approach is recommended.

In reference 2 it was found that well-built aluminum panels could be designed for $e/L = \pm 0.001$. At present, manufacturing techniques for graphite-epoxy panels are not as well developed as they are for aluminum panels. Additional data are required to determine the trade-offs between light, well-built graphite-epoxy panels that have a small e/L and heavier, cheaper graphite-epoxy panels that have a larger e/L .

CONCLUDING REMARKS

An analysis-design procedure which accounts for first order effects of an initial bow and which can be used for sizing stiffened composite panels subjected to combined loadings has been developed and exercised. The procedure is based on a rigorous buckling analysis and is applicable to panels having an arbitrary cross section. The procedure is described briefly in this report and is presented in detail in reference 5.

Design studies carried out with the procedure focus on graphite-epoxy blade-stiffened panels. These studies show that a slight initial bow can cause as much as a 47 percent reduction in the buckling load. Such a reduction emphasizes the need for considering a bow when a panel is designed. Structural efficiency diagrams indicate that there can be a substantial mass penalty for panels designed for a bow. Material strength requirements become active design requirements at much lower loads when a panel has a bow than when it has no bow.

The present report, together with reference 5, continues to demonstrate that a panel design procedure with a high quality buckling analysis and with complete generality of constraints is practical. Such procedures can be used to avoid premature failure from complex buckling modes and to determine mass and proportions of panels for multiple design load conditions and constraints.

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TABLE I. - PROPERTIES OF GRAPHITE-EPOXY MATERIAL
USED IN SAMPLE CALCULATIONS

Material Properties		
Symbol	Value in SI units	Value in U.S. Customary Units
ρ	1589 kg/m ³	.0571 lbm/in ³
E_1	131 GPa	19.00 X 10 ⁶ psi
E_2	13.0 GPa	1.89 X 10 ⁶ psi
G_{12}	6.41 GPa	.93 X 10 ⁶ psi
ν_{12}	.31	.31

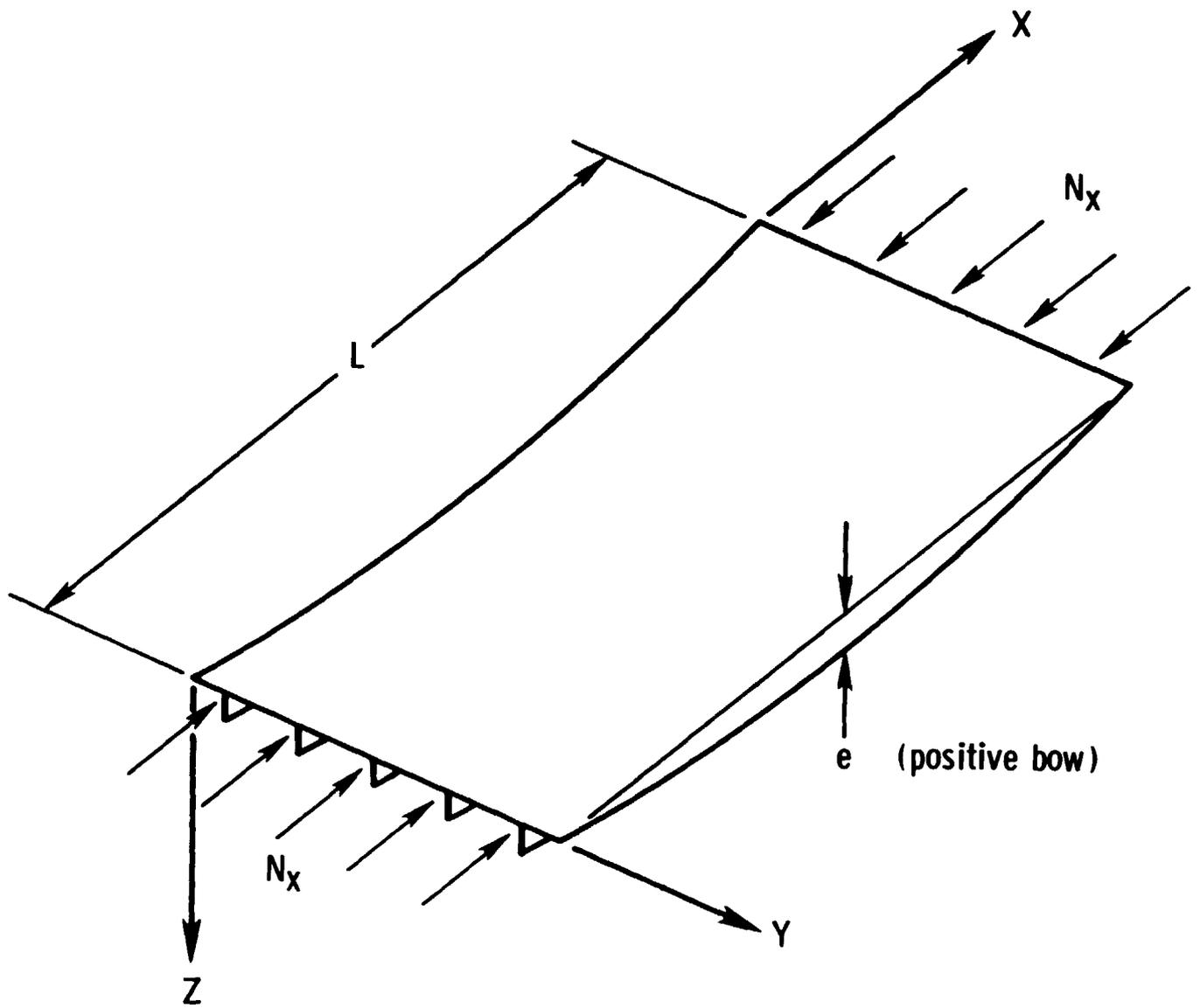


Figure 1. - Panel with initial bow

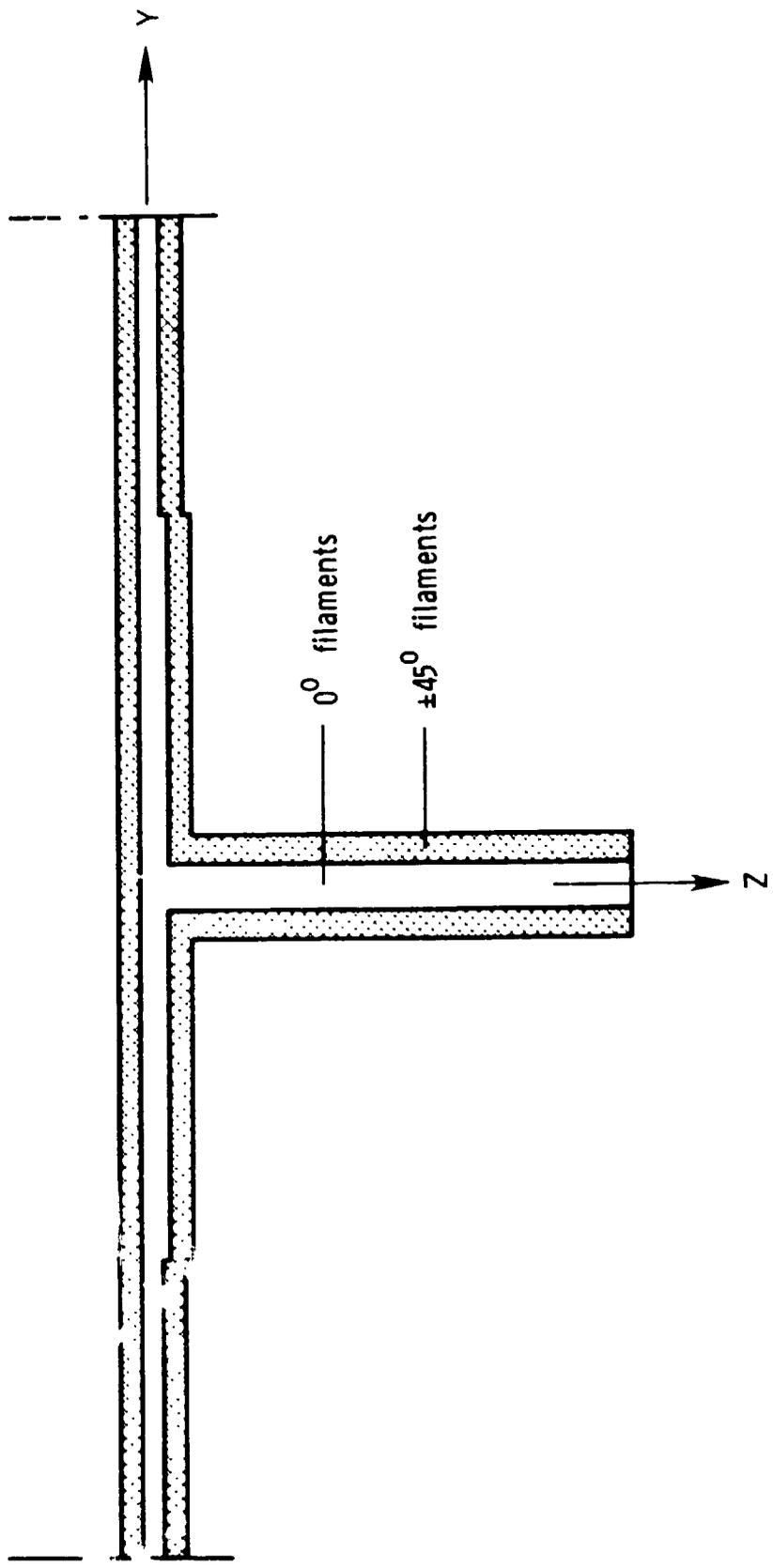


Figure 2.- Blade-stiffened panel configuration

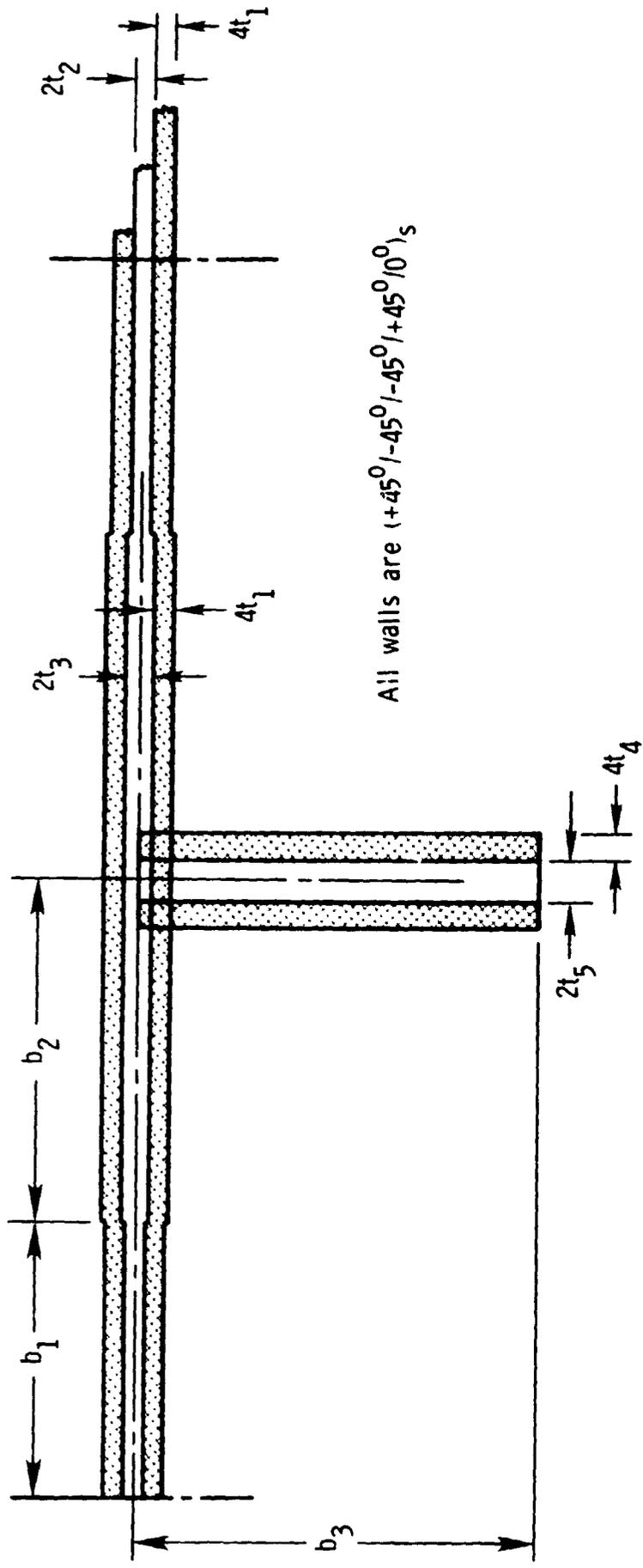
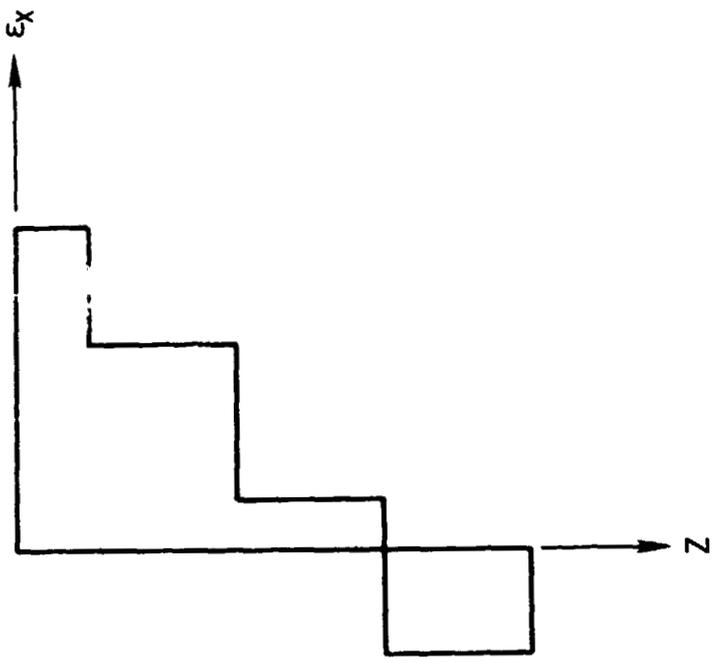
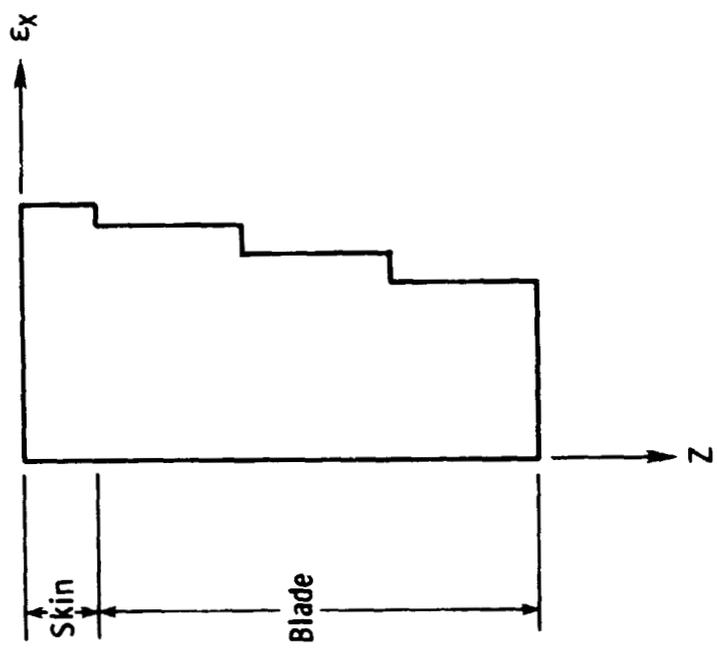


Figure 3.- Mathematical model of blade-stiffened panel, including design variables



Large bow



Small bow

Figure 4. - Idealized longitudinal strain distributions on compression panels with bow

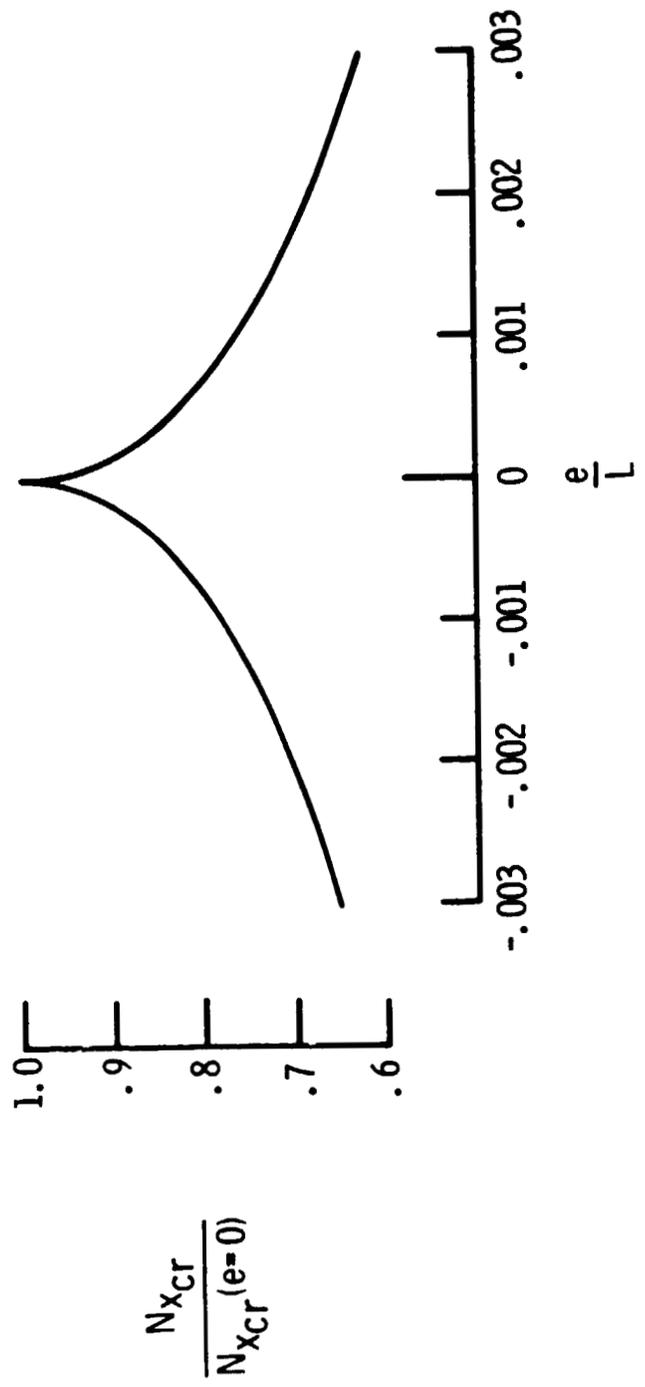


Figure 5.- Effect of bow on buckling load of blade-stiffened panel designed for zero bow

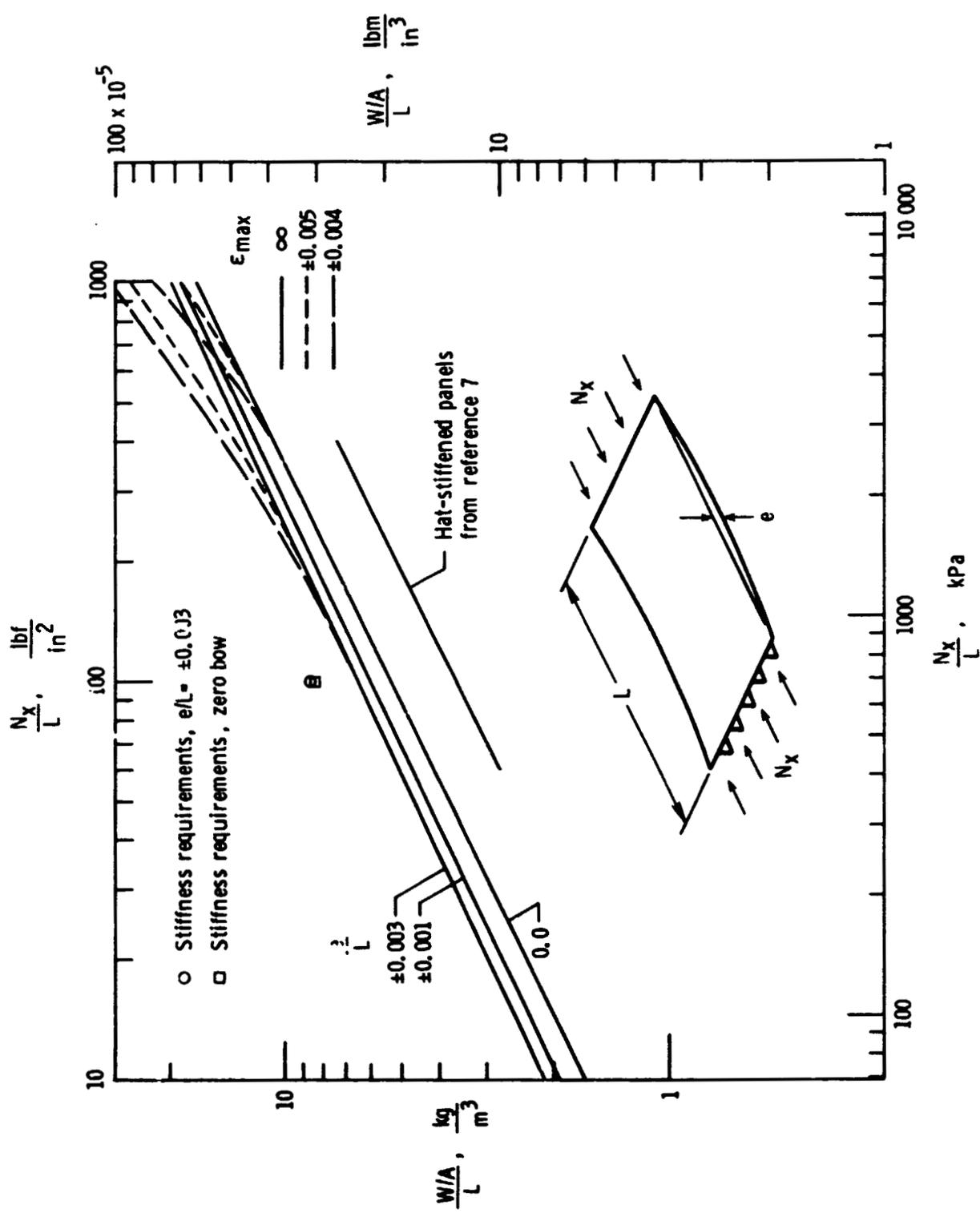


Figure 6. - Structural efficiency of graphite-epoxy blade-stiffened panels

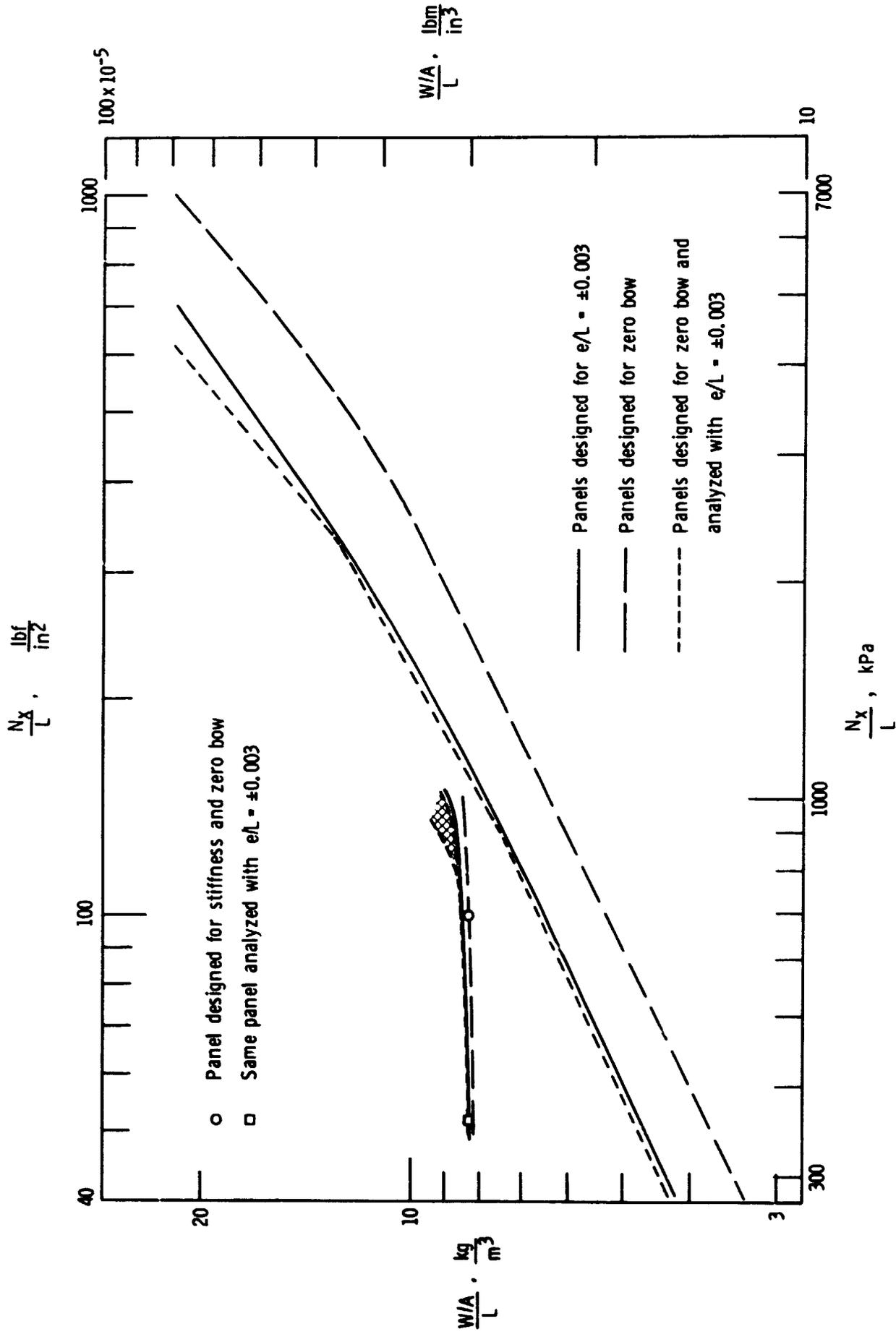


Figure 7.- Structural efficiency of graphite-epoxy blade-stiffened panels designed for a bow and for zero bow

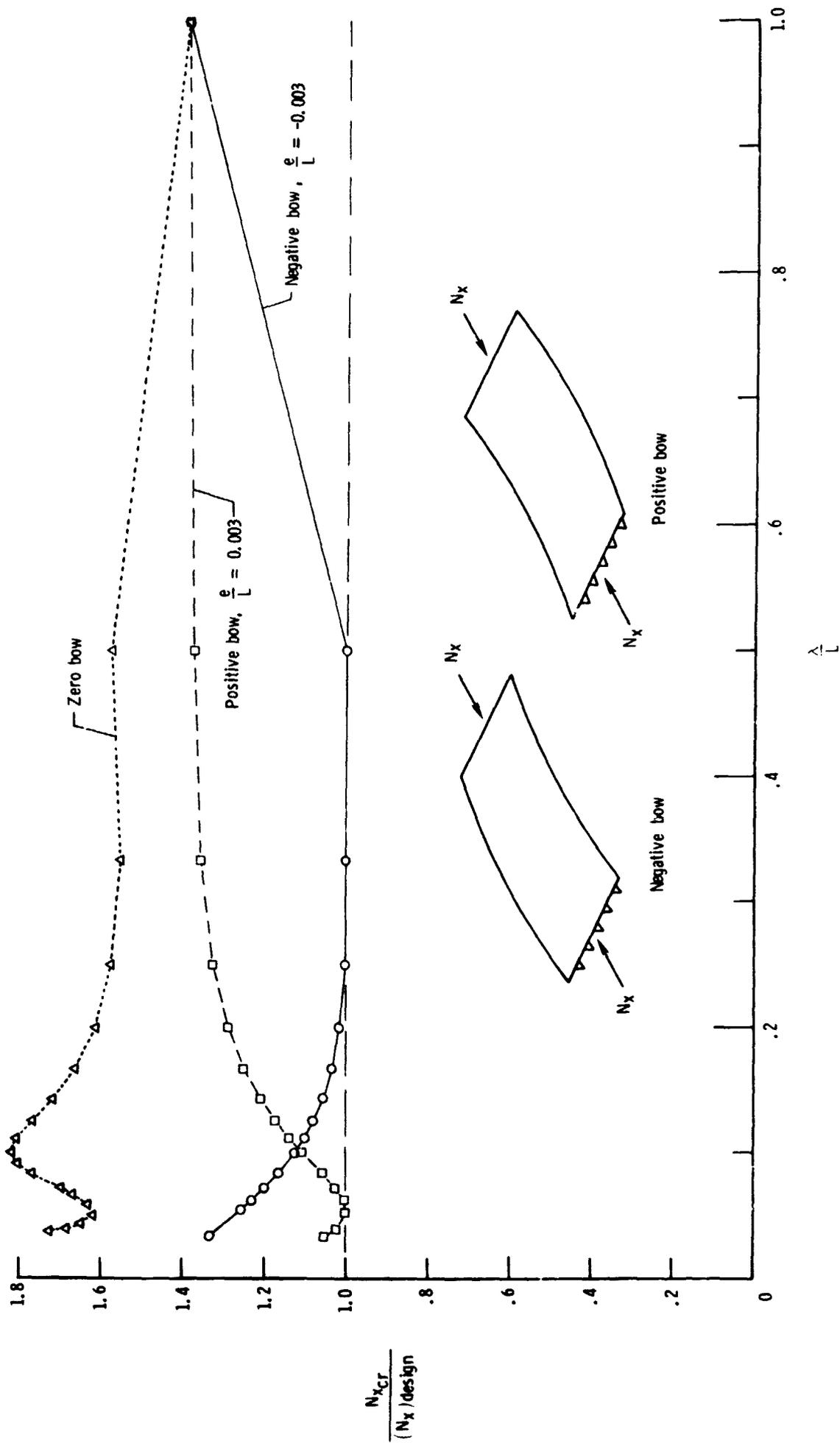


Figure 8.- Ratio of buckling load to design load as a function of buckling half-wavelength for a blade-stiffened panel designed for a loading of $N_x/L = 689 \text{ kPa}$ (100 lbf/in²) and for a bow of $e/L = \pm 0.003$

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16. Abstract Design studies are carried out with a structural synthesis computer code which accounts for first order effects of an initial bow and which can be used for sizing stiffened composite panels having an arbitrary cross section. The studies, which focus on graphite-epoxy blade-stiffened panels, examine the effect of a small initial bow on both the load carrying ability of panels and on the mass of panels designed to carry a specific load. Large reductions in the buckling load caused by a small initial bow emphasize the need for considering a bow when a panel is designed.			
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