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Produced by the NASA Center for Aerospace Information (CASI)
GEOS-3 DOPPLER DIFFERENCE TRACKING

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MARCH 1977

GODDARD SPACE FLIGHT CENTER
GREENBELT, MARYLAND
GEOS-3 DOPPLER
DIFFERENCE TRACKING

B. Rosenbaum
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Greenbelt, Maryland
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ABSTRACT

The Doppler difference method has been applied to track the GEOS-3 spacecraft. In this method a pair of NASA 2 GHz ground tracking stations simultaneously track a spacecraft beacon signal to generate an observable in which bias and instability of the carrier frequency cancel. The baselines are formed by the tracking sites at Bermuda, Rosman, and Merritt Island. Measurements were made to evaluate the effectiveness of the Doppler differencing procedure in tracking a beacon target with the high dynamic rate of the GEOS-3 orbit. Results indicate the precision of the differenced data to be at a level comparable to the conventional precise two-way Doppler tracking. The Doppler difference tracking method forms a complementary adjunct to the operational NASA two-way Doppler tracking system and is being utilized for orbit determination. The baseline technique has also been applied to track GEOS-3 utilizing a satellite-to-satellite link. Residual noise of the differenced data type is an order of magnitude less than that of the one-way beacon data. The tracking method appears to be feasible for the forthcoming Tracking and Data Relay Satellite System and would add a capability to the System for precise beacon signal tracking.
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The technique of Doppler tracking is inherently precise since the signals are processed in a narrow bandwidth phase locked loop and individual Doppler cycles are counted. In the beacon mode however Doppler measurements are biased and noisy due to frequency instability in the carrier signal transmitted by the spacecraft. The bias is usually corrected by inclusion as a "solve for" parameter in an orbit estimation scheme. Although this procedure eliminates bias, the noise level of the observations is undiminished.

The Doppler difference technique is also a beacon tracking method but one that compensates for both bias and noise in the transmitted carrier signal. The method utilizes a baseline tracking technique whereby two stations simultaneously record one-way Doppler on the spacecraft beacon signal. The tracking records of both stations are subsequently brought together in a central data processing facility and measurements with the same time tag are differenced. The concept involved is that noise and bias in the beacon signal frequency are common to simultaneous observations and cancel in the differencing procedure. The result is a precise data type which can be applied for orbit determination.

This tracking technique is a new method first evaluated using the C-band beacon on the ATS-6 communications satellite as a target (Ref. 1). The bias of the carrier frequency was, in equivalent range rate, on the order of 100 m/sec and the noise level amounted to several cm/sec. When simultaneous one-way observations at the ATS-R stations (Rosman and Madrid) were differenced the bias canceled and the noise level was effectively reduced to a limit imposed by atmospheric propagation effects and fundamental instrumental factors.

Whereas ATS-6 is a high altitude satellite tracked along a near fixed line-of-sight (drifting several degrees per day) GEOS-3 is a low earth orbiting spacecraft tracked by the STDN S-band stations. The orbit is near circular with a nominal altitude of 843 km and inclination of 115°. The spacecraft is instrumented with a transponder which can operate in a beacon mode. The signal is being utilized as a target to assess and evaluate the effectiveness of the Doppler difference technique in tracking the high dynamic rate of the GEOS-3 orbit. The tracking data are further being used to assist in evaluating the S-band Doppler system as a tool for geodesy and precise orbit determination. The present document reports on a preliminary evaluation of the Doppler difference technique and utilization of the data type in orbit determination. The work is an
ongoing effort conducted as a part of GEOS-3 project investigations. The present effort is in addition for the purpose of demonstrating a low cost accurate technique of orbit determination, one that is applicable to platform location missions such as that associated with Search and Rescue and buoy tracking.

With the forthcoming advent of the Tracking and Data Relay Satellite System (TDRSS) there is increased interest in the tracking capabilities and utilization of satellite-to-satellite tracking (SST). The TDRS System will have two relay satellites in geostationary orbit, 130° apart in longitude. The configuration could be applied for simultaneous tracking of a spacecraft beacon signal in low earth orbit. With this in mind we have conducted a Doppler difference tracking test on the GEOS-3 S-band beacon utilizing the ATS-6/GEOS-3 SST link.

In this report we describe Doppler tracking system configurations, evaluate precision of the differenced data type, and apply the data in orbit determination.

DOPPLER DIFFERENCE TRACKING

The S-band STDN stations have in recent years been undergoing modifications to incorporate the Multifunctional Receiver (MFR) and STDN Ranging Equipment (SRE). The system is designed for tracking the high Doppler rate of low earth orbiting spacecraft and ranging to lunar distances. The SRE in conjunction with the MFR can perform two-way range and range rate tracking of high precision in an extended receiver bandwidth, 2200 MHz to 2300 MHz.

The GEOS-3 spacecraft has been tracked by this system using the Doppler difference method as well as the conventional two-way operations. The differenced data type can be generated in either of two operating modes (Figure 1). In the one-way beacon mode the target spacecraft transmits a 2247 MHz signal which is tracked by two or more stations. The new data type is formed from the difference of simultaneous Doppler observations. If there are n stations tracking then n-1 baseline observables can be formed.

In the two-way, three-way mode an uplink signal is transmitted to the spacecraft and the transponded signal is returned to the transmitting station. At the same time one or more other stations having mutual view of the spacecraft can be utilized to track the downlink signal in the so-called three-way Doppler mode. These observations like the one-way Doppler can be applied to generate Doppler difference data. It should be noted that the three-way data are in their own a precise data type since their signal processing is controlled by independent Cesium atomic frequency standards at the transmitting and receiving stations.
The S-band stations utilized in the Doppler difference tracking are those positioned in or near the altimeter calibration area. The data acquired in our scheduling are largely from Rosman, Bermuda, and Merritt Island. These stations are within a proximity of one another to permit tracking of GEOS simultaneously for intervals of 5 to 10 minutes and more. For some trajectories through the area three stations can mutually view the satellite. The station at Quito has also been utilized but its geographical location with respect to other stations limits baseline tracking to about five minutes or less.

During the period of our tracking operations Rosman and Quito were instrumented with the SRE equipment and an Interim Tracking Data Processor (ITDP). The stations at Bermuda and Merritt Island were instrumented with the JPL Wide Bandwidth Doppler Extractor (WBDE).

**S-Band Doppler Signal Processing**

Doppler measurement is a coherent operation in which the reference frequencies utilized in signal processing are derived from a 5 MHz atomic standard. Noise in the Doppler signal is filtered in a tracking receiver and Doppler cycles are detected by an analog to digital converter.

The system is comprised of the MFR and SRE equipment (Figure 2). These units are preceded by a parametric amplifier which is positioned between the antenna and receiver to minimize system noise temperature.

The signal arriving at the antenna is a sum of the 2247 MHz S-band carrier frequency and a Doppler shifted component \( f_d \). At the receiver the signal is down converted by mixing with the 1800 MHz reference signal provided by the local oscillator. The 447 MHz + \( f_d \) output is fed to the MFR which filters the Doppler signal in a narrow bandwidth phase locked loop. There are six selectable bandwidths from 30 Hz to 3,000 Hz. The selection — 300 Hz for GEOS-3 — is based on spacecraft dynamics. The MFR also provides a further down conversion of the signal frequency. The filtered output 132.4 MHz - \( f_d \) is presented to the SRE. In the Doppler Extractor three reference frequencies are processed and mixed with the input signal to generate the 370 MHz + \( f_d \). The IF signal then undergoes several operations in the Doppler Multiplier. There is first a frequency translation to 1 MHz by mixing with the 69 MHz reference signal and then there follows a 250 multiplication in a PLL loop and frequency translation again by mixing with the 190 MHz reference signal. Dual signals 90° apart in phase are then supplied to the Doppler Counter in the form 60 MHz + 250 \( f_d \). The two signals at quadrature allow 1/4 cycle resolution for cycle counting. This combined with a Doppler multiplier of 250 provides a resolution in steps of 0.001 cycles. The counter continuously accumulates cycles and has
Figure 2. Simplified Block Diagram for MFR/SRE Doppler Signal Processing
a non-destruct readout capability. The average Doppler frequency is computed from accumulated cycles $N = N_1 - N_2$ counted over the sample interval $T = T_1 - T_2$ by the formula

$$f_d = \frac{N}{T} - f_b$$

where $f_b$ is a bias frequency inserted in signal processing to allow for a negative Doppler frequency shift.

Measurement Precision. Random noise processes are a limiting factor for Doppler precision in SRE equipment. Various effects in signal processing contribute to tracking errors. The Doppler Counter errors are due to phase jitter in analog to digital conversion and the quantization effect. The frequency instability of reference signals in equipment components is a source of added phase noise and jitter in Doppler measurements. Another factor is jitter at the output of the VCO Multiplier phase locked loop. In the Extractor the various mixing operations also contribute to Doppler output errors. The noise budget (Ref. 2) of these effects for the random count error amounts to $9.4 \times 10^{-3}$ cycles RMS or $0.94$ mHz when averaged over an integration period of ten seconds.

The Doppler precision being realized in tracking operations is governed more by the prevailing receiver (MFR) VCO phase noise $\sigma_\theta$. The figure depends on the receiver tracking loop bandwidth and the signal-to-noise ratio. The equations for determining the Doppler frequency precision are:

One-Way Doppler

$$\sigma_f = \frac{\sqrt{2} \, \sigma_\theta}{2 \, \pi \, T}$$

Two-Way Doppler

$$\sigma_f = \frac{\sqrt{2} \, \sigma_\theta}{4 \, \pi \, T}$$

Doppler Difference

$$\sigma_f = \frac{\sigma_\theta}{\pi \, T}$$

and the corresponding range rate precision is
\[ \sigma_R = \frac{C}{f} \sigma_f \]

where

\[ T = \text{Doppler integration period} \]
\[ C = \text{Speed of light} \]
\[ \sigma_f = \text{Doppler measurement precision} \]
\[ \sigma_R = \text{Range rate measurement precision} \]

Table 1 summarizes the measurement precision in Doppler frequency and equivalent range rate of the various Doppler modes. The assumed receiver VCO phase noise in GEOS tracking is \( \sigma_\theta = 0.3 \) radians.

Table 1
Doppler Measurement Precision
(Receiver (MFR) VCO phase noise: 0.3 radians)

<table>
<thead>
<tr>
<th>Measurement Type</th>
<th>One-Way Mode</th>
<th>Two-Way Mode</th>
<th>Doppler Difference Mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>Doppler (mHz)</td>
<td>6.8</td>
<td>3.4</td>
<td>9.6</td>
</tr>
<tr>
<td>Range Rate (mm/sec)</td>
<td>0.90</td>
<td>0.45</td>
<td>1.27</td>
</tr>
</tbody>
</table>

Integration period: 10 seconds.

**Transponder.** The GEOS-3 transponder is instrumented with two crystal oscillators. A voltage controlled crystal oscillator (VCXO) is used in the transponding operation and an auxiliary oscillator controls the frequency of the spacecraft beacon signal. The transmission of the beacon is initiated on ground command and is used both for acquisition and for one-way Doppler tracking. When an uplink signal is transmitted and phase lock is established on the spacecraft, a control switch disables the beacon and activates the transmission of the transponded signal (Figure 3).
Figure 3. GEOS-3 S-Band Transponder
In the one-way tracking mode the auxiliary oscillator operates as a free-running frequency standard. The oscillator provides a reference frequency of \(4 f_0 (= 74.9 \text{ MHz})\) which goes through a successive chain of multiplication to \(120 f_0 (= 2,247 \text{ MHz})\) for transmission as the beacon carrier signal. The frequency instability in the oscillator is the source of bias and noise in the carrier signal that Doppler difference tracking seeks to cancel.

In the two-way Doppler mode the spacecraft receives a Doppler shifted uplink carrier of 2069.112 MHz and transponds the signal with a turn-around frequency ratio of 240/221. The coherence of the signal processing operation is maintained by a phase locked tracking loop. The received signal is downconverted in two successive stages of frequency translation utilizing mixing frequencies, 108 \(f_0\) and 2 \(f_0\), that are harmonics of the VCXO output. This is followed by a \(\div 2\) divisor in the PLL loop intended to prevent self-lock of the feedback loop (Ref. 3). The upconverted frequency is 120 times the VCXO output.

Phase noise generated in the transponding operation is a source of perturbation for the carrier signal frequency. The effect on Doppler tracking error \(\sigma_f\) is determined by

\[
\sigma_f = \frac{\sqrt{2} \sigma_\theta}{T \times 360^\circ}
\]

where \(\sigma_\theta\) is the phase noise. Design specifications for the transponder require that the output phase noise be less than 3.5\(^\circ\). The corresponding Doppler error for \(T = 10\) seconds is 1.4 mHz equivalent to 0.09 mm/sec for two-way tracking. Laboratory measurements (Ref. 4) on the GEOS-3 flight back-up transponder indicate a VCXO phase noise output of 1\(^\circ\) which is well within design specifications.

**Signal Propagation Delay**

The preceding sections concerned tracking errors common to the various Doppler modes. The measurement precision for the Doppler difference mode depends in addition on the effectiveness of the differencing procedure in canceling the effects of beacon signal frequency instability. The noise patterns of the signal being observed simultaneously at two stations are essentially replicas of one another, but arrive offset in time by their propagation delay from the spacecraft. This offset can degrade noise cancellation in the differenced data. The purpose here is to derive an expression for dependence of residual noise on the beacon frequency instability and the time delay offset.
Let the simultaneously received signal frequency at Stations 1 and 2 be denoted by $f_r^{(1)}(t)$ and $f_r^{(2)}(t)$. These are respectively modeled as a sum of components as follows:

$$f_r^{(1)}(t) = f + f_{d}^{(1)}(t) + f_b + f_n(t - \tau_1)$$

$$f_r^{(2)}(t) = f + f_{d}^{(2)}(t) + f_b + f_n(t - \tau_2)$$

where

- $f$ = station receiver reference frequency
- $f_{d}^{(1)}(t)$, $f_{d}^{(2)}(t) = $ Doppler frequency
- $f_b$ = beacon frequency bias
- $f_n(t)$ = beacon signal frequency instability
- $t_s$ = spacecraft signal transmission time

and

- $\tau_1, \tau_2 = $ signal propagation delay time

The recorded Doppler frequency at the stations are respectively

$$\overline{f_{ob}^{(1)}}(t, T) = \frac{1}{T} \int_{t-T}^{t} \left[ f_r^{(1)}(t) - f \right] dt$$

(1)

$$\overline{f_{ob}^{(2)}}(t, T) = \frac{1}{T} \int_{t-T}^{t} \left[ f_r^{(2)}(t) - f \right] dt$$

where $T$ is the Doppler integration period. Accordingly the average Doppler difference measurement can be written as

$$\overline{f}_{dd}(t, T) = \overline{f_{ob}^{(1)}}(t, T) - \overline{f_{ob}^{(2)}}(t, T)$$

or

10
\[
\bar{f}_{dd}(t, T) = \frac{1}{T} \int_{t-T}^{t} \left[ f_d^{(1)}(t) + f_b + f_n(t - \tau_1) \right] dt \\
- \frac{1}{T} \int_{t-T}^{t} \left[ f_d^{(2)}(t) + f_b + f_n(t - \tau_2) \right] dt
\]

(2)

An immediate consequence of the differencing process is a cancellation of the bias \( f_b \).

The expression for \( \bar{f}_{dd}(t, T) \) is now given as a sum of an actual Doppler difference \( \bar{f}_d^{(1)}(t, T) - \bar{f}_d^{(2)}(t, T) \) and a residual \( \bar{f}_n(t, T) \). Thus

\[
\bar{f}_{dd}(t, T) = \bar{f}_d^{(1)}(t, T) - \bar{f}_d^{(2)}(t, T) + \bar{f}_n(t, T)
\]

(3)

where

\[
\bar{f}_d^{(1)}(t, T) = \frac{1}{T} \int_{t-T}^{t} f_d^{(1)}(t) dt
\]

\[
\bar{f}_d^{(2)}(t, T) = \frac{1}{T} \int_{t-T}^{t} f_d^{(1)}(t) dt
\]

From Equations 1, 2, and 3 we obtain

\[
\bar{f}_n(t, T) = \frac{1}{T} \int_{t-T}^{t} f_n(t - \tau_1) dt - \frac{1}{T} \int_{t-T}^{t} f_n(t - \tau_2) dt
\]

(4)

The expressions on the right side of the equation are, for the respective stations, an average of beacon signal frequency instability taken over the Doppler integration period. The residual denoted by \( \bar{f}_n(t, T) \) is a measure of the effect of Doppler differencing procedure for canceling beacon signal disturbances. The form of the cancellation is clarified as follows.
We rewrite the second term on the right side of Equation (4)

\[ \bar{f}_n(t, T) = \frac{1}{T} \int_{t-T}^{t} f_n(t - \tau_1) \, d\tau - \frac{1}{T} \int_{t+T+\tau_1-\tau_2}^{t+\tau_1-\tau_2} f_n(t - \tau_1) \, d\tau \]  

(5)

Then rearranging the integration limits and using

\[ \tau = |\tau_1 - \tau_2| \]

we have

\[ \bar{f}_n(t, T; \tau) = \frac{1}{T} \int_{t-T}^{t-\tau} f_n(t - \tau_1) \, d\tau - \frac{1}{T} \int_{t}^{t+\tau} f_n(t - \tau_1) \, d\tau \quad \tau_1 > \tau_2 \]  

(6a)

or

\[ \bar{f}_n(t, T; \tau) = \frac{1}{T} \int_{t-T-\tau_2}^{t} f_n(t) \, d\tau - \frac{1}{T} \int_{t-\tau_1}^{t-\tau_2} f_n(t) \, d\tau \quad \tau_1 > \tau_2 \]  

(6b)

The residual of the differenced Doppler, \( \bar{f}_n(t, T; \tau) \), is a sample difference of \( f_n(t) \) integrated over the interval \( \tau \) and averaged over the period \( T \). The cancellation of the beacon signal noise will then depend primarily on the ratio \( \tau/T \) and the variance of beacon noise.

The drift in the beacon frequency carrier signal also generates a residual bias in the differenced data. Using Equation 6 it can be shown that the bias is given by \( \tau D \) where \( D \) is the frequency drift rate. The relative propagation delay time \( \tau \) is determined from the spacecraft-baseline geometry, but the drift rate of the beacon signal frequency must be estimated from an analysis of one-way Doppler data. In the next section beacon Doppler data is evaluated.

**S-Band Doppler Difference Data**

GEOS-3 has been tracked in the one-way Doppler mode simultaneously by pairs of S-band stations located in the calibration area to obtain the precise differenced data type. GEOS has also been tracked in the same area by the GSFC laser system. We have generated reference orbits from the highly accurate ranging data.
to evaluate the Doppler observations. Residuals have been formed between the observed (O) one-way data and the computed (C) value derived from the laser reference orbit. Figure 4a shows the O-C residuals determined for a tracking pass at the Merritt Island station covering a near ten minute time span. The residuals exhibit a large systematic bias which is predominantly due to offset in the beacon carrier signal frequency. The frequency bias is not only large but varies in a wide range from about 4 kHz to 6 kHz. The bias in range rate is on the order of -1 km/sec, varying over a span of 300 m/sec. The systematic variation in the bias indicates a drift in the spacecraft beacon frequency. The magnitude of the effect is large enough so that the drift rate can be evaluated from the difference of consecutive residuals. Figure 5a is a plot of the bias drift rate. The data have a near uniform trend from about (1 m/sec)/10 sec to (7 m/sec)/10 sec at which the rate reaches a plateau. The fluctuations are due to noise derived from the spacecraft beacon carrier signal.

The signal was also tracked simultaneously at Rosman where the bias and noise characteristics are essentially a replica of that for Merritt Island. The noise level has been evaluated for the one-way Doppler at both stations and for their differenced Doppler using a least squares polynomial curve fitting technique. Table 2 summarizes the standard deviation of the residuals computed in intervals of two minutes. The residual noise for the one-way observations is in a range between 100 mHz and 1 Hz. Although the variation is appreciated from interval to interval, the stations have a similar noise level for common intervals. This is attributed to the beacon signal noise which is the same for both stations. The noise level in the differenced Doppler is reduced from the level of the beacon data by more than an order of magnitude. The standard deviation varies between 15 mHz and 47 mHz corresponding to 2.0 mm/sec and 6.3 mm/sec respectively in equivalent range rate.

The bias in the differenced data due to drift in the beacon frequency can be calculated from data in Figure 5a. At the time 20$^{th}$ 55$^{m}$ 09$^{s}$ UTC, for example, D = (4.8 m/sec)/10 sec and τ = 1.3 milliseconds. The bias is then $\tau D = 0.62$ mm/sec, a relatively small correction term.

In the one-way Doppler tracking passes analyzed the frequency bias varies over a range on the order of 10 kHz (Figure 4a-c). During each pass the rate of change of the bias (in Hz/sec) is negative. Figures 5a, b, c for example show the very similar pattern in the beacon bias frequency drift rate for three passes.

The Doppler difference data have also been processed for orbit determination. Figure 6a is a plot of residuals for the range rate difference observations. The RMS of residuals is 3.8 mm/sec.
Figure 4a. GEOS-3 S-Band Beacon Frequency Bias
Figure 4b. GEOS-3 S-Band Beacon Frequency Bias
Figure 4c. GEOS-3 S-Band Beacon Frequency Bias
Figure 5b. GEOS-3 Beacon Frequency Drift Rate

Station: Merritt IS.
Frequency: 2247 MHz
Drift Rate: \( D(t) = \frac{B(t) - B(0)}{t} \)

Time from February 24, 1976 21h 05m 00s UTC, min

Bias Drift Rate (m/sec) / 10 sec
BIAS DRIFT RATE, (m/sec /10 sec

STATION: ROSMAN
FREQUENCY: 2247 MHz
DRIFT RATE: \( D(t) = \frac{B(t+T) - B(t)}{T} \)

TIME FROM MARCH 3, 1976 20\(^{th}\) 49\(^{th}\) 00\(^{th}\) UTC, min

Figure 5c. GEOS-3 Beacon Frequency Drift Rate
Table 2
Standard Deviation of Doppler Residuals

<table>
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<tr>
<th>Time Interval February 25, 1976</th>
<th>One-Way Doppler Mode mHz</th>
<th>Difference Doppler mHz</th>
<th>Range Rate Difference mm/sec</th>
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<td>Rosman</td>
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<td>922</td>
<td>923</td>
<td>15</td>
</tr>
<tr>
<td>01′50″</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Another example for orbit determination is provided by differenced beacon data on February 24, 1976 observed at the Rosman and Merritt Island stations. The residuals in range rate difference are 2.5 mm/sec RMS (Figure 6b). For purposes of intercomparison in orbit determination we utilize differenced data generated from two-way, three-way observations at Bermuda and Merritt Island. The residuals are 3.2 mm/sec RMS (Figure 6c).

The Doppler data types generated in baseline tracking then appear to have comparable precision. The Doppler difference data of the one-way beacon mode are being applied for orbit determination.

Beacon Oscillator Drift. The frequency of the S-band beacon carrier signal varied (Figure 4c) over a range of 3 kHz in a ten minute pass corresponding to a drift rate $\Delta f/f = 1.4 \times 10^{-7}$ per minute. The rate being observed during beacon signal tracking is more than an order of magnitude greater than the design limit.

The oscillator frequency stability is a sensitive function of temperature and could be disturbed by the thermal environment. Figure 7 presents data on the pre-flight calibration of the S-band frequency output versus temperature.
SATCHEL: GEOS-3
FREQUENCY: 2247 MHz
BEACON DOPPLER MODE

ELEVATION ANGLE

<table>
<thead>
<tr>
<th></th>
<th>35°</th>
<th>53°</th>
<th>16°</th>
</tr>
</thead>
<tbody>
<tr>
<td>MERRITT IS</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ROSMAN</td>
<td>15°</td>
<td></td>
<td>44°</td>
</tr>
<tr>
<td></td>
<td></td>
<td>30°</td>
<td></td>
</tr>
</tbody>
</table>

RESIDUAL RMS = 0.38 CM/SEC

Figure 6a. Range Rate Difference Residuals
SATELLITE: GEOS-3
FREQUENCY: 2247 MHz
BEACON DOPPLER MODE

ELEVATION ANGLE

<table>
<thead>
<tr>
<th>MERRITT IS.</th>
<th>30°</th>
<th>33°</th>
<th></th>
<th>15°</th>
</tr>
</thead>
<tbody>
<tr>
<td>ROSMAN</td>
<td>15°</td>
<td></td>
<td></td>
<td>29°</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>23°</td>
</tr>
</tbody>
</table>

RESIDUAL RMS = 0.25 cm/sec

Figure 6b. Range Rate Difference Residuals
SATellite: GEOS 3
FREQUENCY: 2247 MHz
2-w/3-w DOPPLER MODE

BERMUDA

\[
\begin{array}{ccc}
34^\circ & 40^\circ & 15^\circ \\
\end{array}
\]

ROSMA

\[
\begin{array}{ccc}
31^\circ & 35^\circ & 30^\circ \\
\end{array}
\]

RESIDUAL RMS = 0.32 CM/SEC

Figure 6c. Range Rate Difference Residuals
frequency was measured at four temperatures between -7°C and +41°C for a corresponding frequency range -27.4 kHz to +3.2 kHz. The frequency measurements in Figures 4a and b are also beyond the calibration interval.

The pre-flight calibration of the frequency was performed under conditions of temperature control whereas the in-flight operation indicates spurious environmental influence. Power for the beacon signal is turned on moments before beacon tracking is initiated. The similar pattern for the frequency rate (Figures 5a–c) then suggests that transient thermal disturbances are introduced during this operation.

The Doppler difference tracking considered thus far has involved a beacon target observed directly by two ground stations. We now consider a modified operation of this method utilizing satellite-to-satellite tracking (SST). The feature is to introduce a relay satellite in the radio link between spacecraft and ground stations. The system (Figure 8) is like that of two geostationary relay satellites
Figure 8. TDRSS Doppler Difference Tracking Configuration
of the TDRS System simultaneously tracking a beacon signal in low earth orbit. The target satellite in this configuration could be instrumented with an antenna array oriented for SST tracking like that of GEOS-3 and provide links to a pair of relay satellites at wide angular separation. The system of relay satellites can then simultaneously beacon track the signal in regions of mutual viewing. For the low earth orbit of GEOS-3 there would be coverage on a significant part of most revolutions. These considerations indicate that the TDRS System has a capability for a substantially broader operation in the Doppler difference mode then do the S-band ground stations which are limited as in the case of the GEOS-3 orbit to several short tracking passes per day.

A successful demonstration of precise two-way range and range-rate tracking on GEOS-3 via the ATS-6 relay satellite has been performed as a part of the Tracking and Data Relay Experiment (Ref. 5). The same tracking configuration can be readily utilized for one-way Doppler tracking on the GEOS-3 beacon signal though the more interesting matter is to demonstrate the effectiveness of the Doppler difference procedure for canceling the effects of frequency instability in a beacon carrier signal. This requires a second tracking link which can be provided by an S-band station. In the overall configuration there are then two stations recording Doppler data simultaneously, the STDN station observing an S-band carrier and the ATSR station observing a C-band carrier. The beacon tracking data at the two stations then can be compared and the improvement of the residual noise level for the Doppler differented data evaluated.

Figure 9 shows the tracking link geometry and the carrier frequencies. The S-band beacon signal (2247 MHz) transmitted by GEOS-3 is observed directly at a STDN station. The operation in this link is unchanged from that utilized in the previously described S-band baseline tracking. Along a second link the beacon signal is acquired by ATS-6, transponded to C-band (3947 MHz) and relayed to the Madrid ATSR site. Although Madrid does observe a Doppler shifted C-band signal derived from the beacon target the station also transmits an uplink C-band pilot tone (6150.000 MHz). This is to phase lock the ATS-6 transponder master oscillator and provide for coherent signal processing with the ground station.

The requirement for Doppler difference tracking is that the beacon signal observed at the two stations be derived from a common oscillator. GEOS-3 has two independent S-band antenna systems (Figure 10) which utilize the same transponder and auxiliary oscillator. One antenna is earth oriented and the other antenna system is the set of quadrant antennas space oriented. Both systems have broad antenna beams. Their parameters are given in Table 3.
Figure 9. C-Band/S-Band Beacon Tracking of GEOS-3
In the present SST experiment the GEOS-3 S-band beacon signal propagates along two links, one directed toward the earth and the other directed away from the earth. Although the spacecraft has dual antenna systems appropriately oriented, only one S-band antenna can radiate at any given time. The S-band ground station can only observe the earth oriented antenna. Fortunately, this antenna beam is sufficiently broad to allow ATS-6 to observe this beacon signal when GEOS-3 is near the horizon. From its geostationary position at 35° East longitude, ATS-6 and the STDN stations (Rosman, Bermuda, and Merritt Island) were able to track simultaneously when the spacecraft was in the vicinity of the calibration area.

The tracking of GEOS-3 by ATS-6 was performed by the high gain 9-meter diameter antenna (Table 3) which is programmed for pitch and roll maneuvers to track the target satellite.

Table 3
Antenna Beam Parameters

<table>
<thead>
<tr>
<th>Antenna</th>
<th>Beam Width</th>
<th>Gain (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>GEOS-3 S-Band</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Earth Oriented</td>
<td>150°</td>
<td>2.0</td>
</tr>
<tr>
<td>Space Oriented</td>
<td>120°</td>
<td>1.5</td>
</tr>
<tr>
<td>ATS-6 C-Band</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9-Meter Diameter</td>
<td>1.4°</td>
<td>36</td>
</tr>
</tbody>
</table>

Doppler Signal Processing Via ATS-6 (Ref. 6)

This section treats one-way Doppler signal processing for the tracking of the GEOS-3 beacon via the ATS-6 relay satellite. The analysis yields an expression for the Doppler observable at the ATSR station. This is applied to form the differenced observable with respect to the STDN station. The signal processing is a modification of that employed for two-way SST tracking (Ref. 6) and generates a new observable. The essential departure from the two-way operation is the deletion of the forward S-band transmission link from ATS-6 to GEOS-3.

Figure 11 is a simplified block diagram of the signal processing for the one-way SST configuration. The system is comprised of the GEOS-3 beacon transmitter,
Figure 11. One-Way Doppler SST Signal Processing
the ATS-6 relay transponder, and the Madrid ATSR station. For the analysis of the Doppler observable we will require the Doppler factors for the propagation links between these terminals. A detailed treatment of Doppler factors in SST tracking is given in Reference 7. We use the classical approximation:

<table>
<thead>
<tr>
<th>Propagation Link</th>
<th>Doppler Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Madrid to ATS-6</td>
<td>( a_1 = 1 - \frac{\dot{R}_1}{C} )</td>
</tr>
<tr>
<td>ATS-6 to Madrid</td>
<td>( a_2 = \frac{1}{1 + \frac{\dot{R}_1}{C}} )</td>
</tr>
<tr>
<td>GEOS-3 to ATS-6</td>
<td>( \beta_2 = \frac{1}{1 + \frac{\dot{R}_2}{C}} )</td>
</tr>
</tbody>
</table>

where

\( \dot{R}_1 = \text{ATS-6 to Madrid range rate} \)

\( \dot{R}_2 = \text{GEOS-3 to ATS-6 range rate} \)

The C-band pilot tone (6150.000 MHz) at Madrid is synthesized from a 5 MHz reference frequency standard and transmitted to ATS-6. The S-band beacon signal from GEOS is also acquired by ATS-6 and coherently processed in the relay transponder. The two signals received at the ATS-6 transponder are the Doppler shifted carrier frequencies 2247 \( \beta_2 \) MHz and 6150 \( a_1 \) MHz. The C-band signal drives a feedback loop whose nominal output, 25 MHz, at G is supplied to two multipliers. The products (S and U) are coherently mixed with the S-band signal in two successive stages of frequency translation yielding

\[
V = 2247 \beta_2 + 1700 a_1 \text{ MHz}
\]

where \( V \) is the ATS-6 transmitted signal frequency. \( V \) is the sum of the Doppler shifted S-band signal and the downconverted C-band signal.
The signal received at the Madrid site is

\[ W = 2247 a_2 \beta_2 + 1700 a_1 a_2 \]

or in the form of a C-band carrier plus Doppler

\[ W = 3947 \text{ MHz} + f_d(C) \]

where

\[ f_d(C) = 1700(a_1 a_2 - 1) + 2247(a_2 \beta_2 - 1) \text{ MHz} \]

This signal is heterodyned in the receiver by mixing with the 3,877 MHz local oscillator frequency to generate 70 MHz + f_d.

The IF signal is supplied to the Doppler Extractor which is configured in the PLL Doppler mode to extract the Doppler information from the carrier. The contents of the Doppler counter are read in a non-destruct mode at the rate of one per ten seconds. The average Doppler shift is computed from the formula

\[ \overline{f_d(C)}(t) = 10^{-2} \left\{ \frac{N(t) - N(t - T)}{T} - f_b \right\} \]

where \( N(t) \) is the integrated Doppler count, \( T \) the sample period, and \( f_b \) is a prescribed bias.

Utilizing the expression for the Doppler factors cited above the Doppler frequency shift \( \overline{f_d(C)} \) is

\[ \overline{f_d(C)} = - \frac{2 f_t}{C} \left\{ a_1 \overline{R_1} + a_2 (\overline{R_1} + \overline{R_2}) \right\} \]

where the bars are used to denote average range rate

\[ f_t = 2247 \text{ MHz} \]

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The corresponding Doppler observation at the S-band station is

\[ \overline{R}^{(s)} = - \frac{f_t}{c} \overline{R}_3 \]

where

\[ \overline{R}_3 = \text{GEOS-3 average range rate from the S-band station} \]

Then we have for the average Doppler difference

\[ \overline{f}_{dd} = \overline{f}_{d(C)} - \overline{f}_{d(s)} = - \frac{2f_t}{c} \left\{ (a_1 + a_2) \overline{R}_1 + a_2 (\overline{R}_2 - \overline{R}_3) \right\} \]

The observable accordingly depends on two range rates from ground stations and the satellite-to-satellite range rate. Written in numerical form the expression is

\[ \overline{f}_{dd} = \frac{1}{c} \left\{ (2 \times 1700 + 2247) \overline{R}_1 + 2247 (\overline{R}_2 - \overline{R}_3) \right\} \text{ MHz} \]

\( \overline{R}_2 - \overline{R}_3 \) is the range rate difference of GEOS-3 with respect to ATS-6 and the S-band ground station.

The above equation between an observed Doppler difference and range rate parameters applies to averaged components. We recall that an average range rate measures a range change over a Doppler count interval given by

\[ \overline{R}(t) = \frac{R(t) - R(t - T)}{T} \]

Range itself is a distance between two points linked by a tracking signal. When the observable is processed in an orbit estimation program the signal propagation
in each transmit-receive link must be accurately modeled in time and position coordinates.

Results for trajectory computation in two-way ATS-6/GEOS-3 tracking from a single ground station are described in Reference 5. Our new data type for beacon tracking in the SST mode is related to that utilized in the two-way mode and can be correspondingly modeled for orbit computation. The matter of time tagging the signal propagation at each terminal in SST tracking has been considered in detail in References 6 and 8.

C-Band/S-Band Tracking Residuals

This section presents results on tracking noise in Doppler difference measurements utilizing an SST link. The one-way Doppler observables are $f_{d(C)}$ recorded at the C-band ATSR station tracking via the ATS-6 transponder and $f_{d(S)}$ recorded at the S-band station. In addition there is the derived Doppler observable $f_{dd} = f_{d(C)} - f_{d(S)}$ generated from the difference of simultaneous one-way observations.

To evaluate tracking noise in the observables we utilize the least squares curve fitting technique. The data are analyzed in intervals of 2 1/2 minutes. Table 4 summarizes results for several passes including one of ten minutes duration. The residuals for the one-way Doppler S-band and C-band data are in a range of 500 mHz to 1000 mHz, but for common time intervals there is relatively close agreement in residual noise between the two stations even though their frequency bands differ. The noise is attributed to the beacon signal frequency instability.

In the Doppler difference data the standard deviation of the residuals decreases to a range of 40 mHz to 85 mHz. The improvement over the one-way Doppler noise amounts to an order of magnitude. This indicates the effectiveness of the differencing technique for canceling a major part of the one-way Doppler beacon noise while tracking via the SST link.

The reduced noise though is higher than that realized in tracking directly with two S-band ground stations. The cancellation of beacon signal noise is somewhat degraded by the large offset in signal propagation delay time $\tau$ introduced by the addition of the SST link. By way of comparison we recall our other baseline tracking results.

The one-way residual noise in tracking GEOS-3 from S-band stations was determined to be on the order of several hundred millihertz and the corresponding differenced data had a noise level reduced to a range of 15 mHz to 47 mHz. The offset $\tau$ for the S-band baselines was typically about two milliseconds.
Table 4
Standard Deviation of S-Band/C-Band Doppler Residuals

<table>
<thead>
<tr>
<th>Time Interval</th>
<th>S-Band Station</th>
<th>S-Band MHz</th>
<th>C-Band MHz</th>
<th>Doppler Difference mHz</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nov. 3, 1975</td>
<td>Rosman</td>
<td>771</td>
<td>765</td>
<td>40</td>
</tr>
<tr>
<td>Nov. 7, 1975</td>
<td>Merritt Island</td>
<td>846</td>
<td>874</td>
<td>71</td>
</tr>
<tr>
<td></td>
<td></td>
<td>633</td>
<td>638</td>
<td>62</td>
</tr>
<tr>
<td></td>
<td></td>
<td>601</td>
<td>592</td>
<td>70</td>
</tr>
<tr>
<td></td>
<td></td>
<td>923</td>
<td>909</td>
<td>85</td>
</tr>
<tr>
<td>May 27, 1976</td>
<td>Merritt Island</td>
<td>829</td>
<td>854</td>
<td>40</td>
</tr>
<tr>
<td>May 28, 1976</td>
<td>Merritt Island</td>
<td>499</td>
<td>493</td>
<td>64</td>
</tr>
</tbody>
</table>

In the tracking of ATS-6 by the Doppler difference method (Ref. 1) the one-way Doppler noise was about 300 MHz to 400 mHz and the differenced data attained a level of 15 mHz. The propagation links to the C-band ground stations were of comparable distance and their offset time $\tau$ was about 10 milliseconds.

In both the S-band and C-band baseline tracking configurations the offset times were then small enough so that the residuals of the one-way differenced data were in the tracking system noise level. In the SST tracking via ATS-6 the offset time however is about 250 milliseconds which is well over an order of magnitude greater than that of the other configurations. This can account for the increased noise level.
We note here that the disparity in signal propagation time to the two ground stations is largely eliminated when the tracking configuration employs a pair of geostationary relay satellites such as those of the TDRS System (Figure 11). The signal paths of the two links are then of comparable dimension and system configuration is appropriate for precise tracking of a beacon target by the Doppler difference method.

CONCLUSION

This investigation has involved an evaluation of the Doppler difference method for tracking the high dynamic rate of the GEOS-3 orbit. The precision for beacon signal tracking was determined from residual noise generated by the application of least squares parameter estimation methods. When the data were processed in an orbit determination program the residuals for the differenced Doppler were at a level of several millimeters per second. This is comparable to that attained through S-band two-way and three-way Doppler tracking modes.

GEOS-3 orbital solutions have been generated from laser ranging data for the altimeter calibration area as a reference to calibrate and position S-band Doppler tracking stations. This investigation is continuing with the application of two-way and three-way Doppler data for the location of the Bermuda, Merritt Island, and Rosman stations. The Doppler difference tracking data generated in the one-way beacon mode are being applied for orbit determination.

A GEOS-3 type orbital mission could also be tracked in the Doppler difference mode by a pair of relay satellites in geostationary orbit. The TDRS System appears to be applicable for this operation. The System would have the advantage of broad tracking coverage compared to the limitations imposed by ground based tracking. Measurements of the effectiveness of the differencing technique in SST tracking indicate an order of magnitude improvement in precision over the one-way Doppler. With the TDRSS configuration the precision would be further enhanced and at a level near the system noise for two-way range rate.

ACKNOWLEDGMENTS

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3. Gardner, F. M., Phaselock Techniques, J. Wiley & Sons, Inc., 1966, Ch. 6


