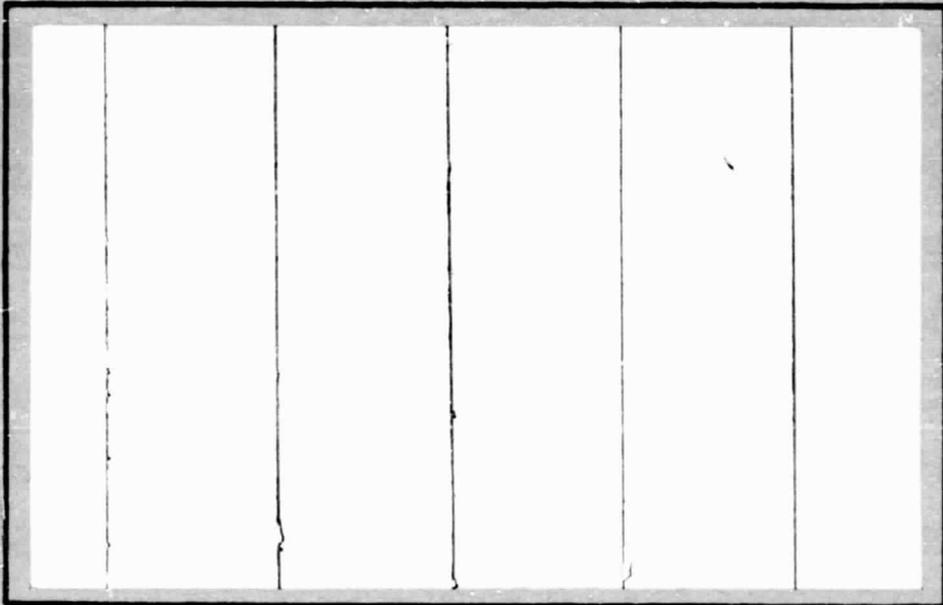


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(NASA-CR-154982) INFLUENCE OF  
TWO-DIMENSIONAL HYDROTHERMAL GRADIENTS ON  
INTERLAMINAR STRESSES NEAR FREE EDGES  
Interim Report (Virginia Polytechnic Inst.  
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Blacksburg, VA. 24061

VPI-E-77-26

September, 1977

Influence of Two-Dimensional Hygrothermal  
Gradients on Interlaminar Stresses Near Free Edges

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INFLUENCE OF TWO-DIMENSIONAL HYGROTHERMAL GRADIENTS  
ON INTERLAMINAR STRESSES NEAR FREE EDGES<sup>1</sup>

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ABSTRACT: Interlaminar stresses are determined for mechanical loading, uniform hygrothermal loading, and gradient moisture loading through implementation of a finite element computer code. Nonuniform two-dimensional hygroscopic gradients are obtained from a finite difference solution of the diffusion equation. It is shown that hygroscopic induced stresses can be larger than those resulting from mechanical and thermal loading, and that the distribution of the interlaminar normal stress may be changed significantly in the presence of a two-dimensional moisture gradient in the boundary layer of a composite laminate.

KEY WORDS: moisture, temperature, composites, interlaminar stresses, finite elements, finite difference, diffusion, boundary layer, hygrothermal.

<sup>1</sup> Supported by NASA Grant NGR 47-004-129.

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## INTRODUCTION

Aerospace vehicles are subjected to extremes in temperature and moisture (hygrothermal) conditions within their flight regimes. These extremes in hygrothermal conditions have resulted in a rather intensive research effort on the part of materials engineers to understand their effects on the internal stress distribution and resulting behavior of resin matrix composites. Recent finite element investigations [1-3] have shown the significance of a uniform temperature change on the interlaminar stress distribution in composite laminates with free edges. Pipes et al [4] performed a "unified" treatment of the hygrothermal problem with a gradient moisture condition imposed; however, since lamination theory was employed the interlaminar effects were not assessed.

The moisture diffusion work of Shen and Springer [5], using one-dimensional analysis, has shown the existence of transverse moisture concentration gradients in composite laminates. The works of Whitney [6] employing a three-dimensional analysis technique with average laminate properties and that of Farley and Herakovich [7] using a more general two-dimensional finite difference scheme have shown the possibility of large transverse ( $z$ ) and lateral ( $y$ ) moisture gradients, especially near free edge boundary layer regions.

In the present study, a detailed investigation of the effect of uniform and gradient hygrothermal loading on the interlaminar stress distributions in finite width symmetric composite laminates is presented. These stress distributions are compared with those produced by mechanical loads and the residual stresses induced during the curing process. Many cases were studied during the course of the investigation; however,

due to limitations of space only selected results will be presented to demonstrate the various phenomena. It will be shown that moisture induced stresses can be larger than the stresses due to thermal and mechanical loading, and that moisture gradients can significantly alter the stress distributions.

The method of analysis used in this investigation was a computer implementation (NOLACS) of the displacement formulation of the finite element method. The program NOLACS is an improved version of NONCOM [8] and NONCOM1 [9] which utilizes the full three-dimensional non-linear mechanical material properties including hygrothermal effects. A modular form of NOLACS utilizes low speed storage to greatly reduce the problem size restrictions of its predecessors.

#### PROBLEM DESCRIPTION

The problem under consideration is the stress analysis of a long, finite width, symmetric composite laminate subjected to uniform axial mechanical loading and gradient hygrothermal loading (Fig. 1). Individual layers in the laminate are considered to be homogeneous orthotropic materials, and all stresses and strains are assumed independent of the axial (x) coordinate. The constitutive equation for such material behavior may be written in condensed notation as

$$\{\sigma\} = [\bar{C}](\{\epsilon\} - \{\alpha\}T - \{\beta\}M) \quad (1)$$

where:

$[\bar{C}]$  is the transformed 6 x 6 stiffness matrix

$\{\sigma\}$  is a 6 x 1 stress vector

$\{\epsilon\}$  is a 6 x 1 strain vector

$\{\alpha\}$  is a 6 x 1 vector of thermal coefficients

$\{\beta\}$  is a 6 x 1 vector of moisture coefficients

T is temperature

M is moisture concentration

The finite element formulation of the problem will not be presented here; interested readers may find the detailed presentation in references 8 and 9. Although previous applications of this formulation have been concerned only with the case of uniform temperature or moisture distributions, the initial assumptions in the problem formulation do not preclude its application to gradient conditions.

Utilizing the formulation described above, several laminates have been examined using the finite element grid shown in Fig. 1 where symmetry has been invoked to limit the analysis to one-quarter of the laminate cross section. This model consists of 216 nodes with 376 constant strain triangular finite elements. The symmetry displacement boundary conditions along  $z=y=0$  are also shown in the figure as well as the stress free conditions along  $z=H$  and  $y=b$ . Although the stress free boundary conditions along the free edges will not be explicitly satisfied due to problem formulation (displacement method), low number of degrees of freedom and the order of the finite elements used (constant stress elements), a sufficiently adequate stress distribution for relative comparisons and general conclusions is obtained. The laminates studied were  $[0_2/90_2]_S$ ,  $[90/0_2/90]_S$ ,  $[0/\pm 45/90]_S$ , and  $[90/0/\pm 45]_S$ . They were chosen because they exhibit significant interlaminar stresses in regions where hygrothermal gradients may radically influence these distributions. Both idealized one-dimensional uniform moisture gra-

dients (Fig. 2), and two-dimensional nonuniform gradients representing actual moisture distributions (Fig. 3) were applied to the laminates studied.

The gradients representing actual moisture distributions were generated by a second order accurate two-dimensional hygrothermal finite difference program called HYDIP [7]. This finite difference program considers the problem of moisture diffusion where the diffusion coefficient  $D$  may be a function of temperature  $T(y,z)$ , moisture concentration  $M(y,z)$  and fiber orientation  $\theta$ , i.e.

$$D = D(T(y,z), M(y,z), \theta) \quad (2)$$

The governing field equation

$$\frac{\partial D}{\partial y} \frac{\partial M}{\partial y} + D_y \frac{\partial^2 M}{\partial y^2} + \frac{\partial D}{\partial z} \frac{\partial M}{\partial z} + D_z \frac{\partial^2 M}{\partial z^2} = \frac{\partial M}{\partial t} \quad (3)$$

is solved by the finite difference procedure subject to the symmetry boundary conditions

$$\left. \frac{\partial M}{\partial z} \right|_{z=0} = 0; \quad \left. \frac{\partial M}{\partial y} \right|_{y=0} = 0 \quad (4)$$

and the variable surface conditions

$$M = M(z) \Big|_{y=b}; \quad M = M(y) \Big|_{z=H}. \quad (5)$$

In generating moisture distributions the laminate was assumed initially dry with a uniform saturated ( $M = 1.4\%$ ) moisture condition imposed on the free surfaces.

Although both programs NOLACS and HYDIP have the capability to allow for hygrothermal dependent material properties and the finite element program has the additional capability of nonlinear material

behavior, the present study was limited to linear material behavior with the mechanical properties independent of hygrothermal conditions. All results were obtained using typical properties for graphite-polyimide.

They are:

$$\begin{aligned}
 E_{11} &= 19 \times 10^6 \text{ psi} & E_{22} = E_{33} &= 1.8 \times 10^6 \text{ psi} \\
 &(131.0 \text{ MPa}) & &(12.4 \text{ MPa}) \\
 G_{12} = G_{13} = G_{23} &= 0.85 \times 10^6 \text{ psi} \\
 &(5.86 \text{ MPa}) \\
 \nu_{12} = \nu_{13} = \nu_{23} &= 0.21 \\
 \alpha_{11} &= 0.3 \times 10^{-5} / ^\circ\text{F} & \alpha_{22} = \alpha_{33} &= 0.13 \times 10^{-4} / ^\circ\text{F} \\
 &(0.167 \times 10^{-5} / ^\circ\text{K}) & &(0.072 \times 10^{-4} / ^\circ\text{K}) \\
 \beta_{11} &= 0.0 / \% \text{Wt} & \beta_{22} = \beta_{33} &= 0.667 / \% \text{Wt} \\
 D_{11} &= 0.155 \times 10^6 \text{ in}^2 / \text{sec} & D_{22} = D_{33} &= 0.031 \times 10^6 \text{ in}^2 / \text{sec} \\
 &(1 \times 10^{-4} \text{ mm}^2 / \text{sec}) & &(0.2 \times 10^{-4} \text{ mm}^2 / \text{sec})
 \end{aligned}$$

where the diffusion coefficients are given for 440°K and the subscripts 1, 2 and 3 correspond to the lamina material principal directions.

## RESULTS AND DISCUSSION

Normalized stress profiles in the boundary layer region are presented in Figs. 4-14. for a variety of laminates and loading conditions. The loading conditions have been coded for clarity as described in Table 1 and all stresses have been "normalized" with respect to 1000 psi (6.895 MPa).

### *Uniform Hygrothermal and Mechanical Loading*

The results described in this section show comparisons between the

interlaminar stresses in  $[0_2/90_2]_s$  and  $[90/0/\pm 45]_s$  laminates for uniform hygrothermal loading ( $M=1.4\%$ ,  $T=280^\circ\text{F}$ ), extensional strain loading ( $\epsilon_x=1.0\%$ ) and a curing temperature ( $\Delta T=-200^\circ\text{F}$ ). From Figs. 4-6 it is evident that elevated hygrothermal conditions can produce stresses which are larger in magnitude than those produced by other types of loading. For the laminates studied, the maximum interlaminar hygrothermal shear stress  $\tau_{yz}$  is larger than the maximum interlaminar normal stress  $\sigma_z$  with the largest hygrothermal stresses being those of the bi-directional  $[0_2/90_2]_s$  laminate (Fig. 4). As indicated in the figures, the results are a function of the interface in question as well as the material properties and the laminate stacking sequence. Scaling the results in Figs. 4-6 indicates that a uniform moisture concentration of approximately 0.5% tends to compensate for residual curing stresses associated with a  $200^\circ\text{F}$  temperature drop. Similarly, for the material properties used and the laminates examined, a moisture concentration of 1.0% is approximately equivalent to a temperature increase of  $400^\circ\text{F}$ . As these results show, moisture effects are a significant environmental consideration and the remainder of this paper will be directed toward the moisture problem. Thermal trends can be estimated from the moisture results since the problem formulations are the same.

#### *One-Dimensional Uniform Gradient Moisture Loading*

A uniform temperature condition in a laminate can be reached in a reasonably short period of time because of the relatively high coefficient of thermal diffusivity; uniformly saturated moisture conditions can, however, take years to be attained at room temperature [5]. This leads one to believe that gradient moisture conditions are generally present even in controlled laboratory conditions.

1

The results presented in Figs. 7-10 show interlaminar stress distributions for four moisture gradients which are uniform over the laminate. They are designated  $\pm Y$  and  $\pm Z$  and depicted in Fig. 2. The interlaminar stress distributions associated with uniform gradient moisture loadings are compared with uniform saturated moisture loading in the figures. The  $+Y$  and  $+Z$  moisture gradients can be considered to simulate moisture absorption while the  $-Y$  and  $-Z$  gradients can represent desorption. By considering these  $Y$  and  $Z$  gradient conditions individually, one can more readily understand the basic phenomena.

The results shown in the figures indicate that the effects of the uniform gradient are localized in the boundary layer region with only a small shift in stress distribution from the uniform saturated condition. It is also apparent that absorption is more critical with respect to interlaminar stresses. This is because of the higher moisture concentration in the boundary layer region. Though not shown, it was noticed that during  $-Z$  gradient loading (desorption) a change in sign of the interlaminar shear stress occasionally occurred. These uniform gradient results are idealized cases which can, at best, yield trends that further the understanding of hygroscopic interlaminar stresses. As shown in references 6 and 7, actual moisture distributions have steep gradients in both the  $y$  and  $z$  directions. The effect of two-dimensional gradients are examined in the remainder of this paper.

#### *Two-dimensional Nonuniform Gradient Moisture Loading*

A series of moisture concentrations generated by HYDIP were applied to several laminates and interlaminar stress distributions were determined for each of three moisture concentration cases. The moisture contours are shown in Fig. 3 and the resulting stress distributions in Figs. 11-

14. Comparing these stress distributions to those previously examined and noting the location of the steep moisture gradients, it can be concluded that the stress distributions are significantly affected by the relative location of the steep gradients in the boundary layer region. After the steep gradients have passed through the boundary layer region, as in gradient case III, the interlaminar stress distributions resemble those of the uniform moisture case. These results also indicate that the interlaminar stress distributions do not appreciably change for average moisture contents equal to or greater than eighty percent of the uniform saturated condition. This later observation may be especially important for experimental work since it indicates that the interlaminar stress distribution is essentially fully established at an average moisture content which is eighty percent of the saturated condition.

It is apparent from the figures that the interlaminar normal stress distribution is influenced more by the location of the two-dimensional gradient than is the shear stress distribution. The sinusoidal variation of the normal stress for case II is unlike the distribution for any other loading condition previously investigated. Such a distribution could have important implications for crack arrest should there be a positive negative variation in the sign of the normal stress; such variations have been observed during this study. It is also apparent from the figures that the magnitude of the stress away from the free edge is generally lower for case II, and that the sinusoidal variation of the normal stress occurs when both  $y$  and  $z$  moisture gradients are present in the boundary layer.

## CONCLUSIONS

It has been shown that elevated hygrothermal conditions can be a significant factor in the determination of interlaminar stresses in the boundary layer region of composite laminates. Pertinent trends demonstrating the individual contributions of lateral and transverse gradients have been shown. By utilizing the diffusion program HYDIP, moisture distributions representing actual conditions have been analyzed. These results show that rather dramatic changes in the stress distributions may occur when two-dimensional gradients are present in the boundary layer region. After steep gradients have passed through the boundary layer (even though the entire laminate has not reached moisture equilibrium) the interlaminar stresses are very similar to the uniform hygrothermal loading condition.

The results presented here show that hygrothermal as well as mechanical loading should be included in any stress analysis of composite laminates with free edges. Further, these results which have been determined for constant materials properties indicate that additional study is needed in the area of hygrothermal dependent material properties and nonlinear material behavior in order to more accurately assess the load carrying capacity of composites.

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Table 1 Stress Codes

Superscript - denotes type of loading

$\epsilon$  - mechanical strain  $\epsilon_x = 1.0\%$

M - uniform moisture distribution  $M = 1.4\%$

C - curing process  $\Delta T = -200^\circ\text{F}$

T - uniform temperature distribution  $\Delta T = 280^\circ\text{F}$

$\pm Y, \pm Z$  - one-dimensional uniform moisture gradients

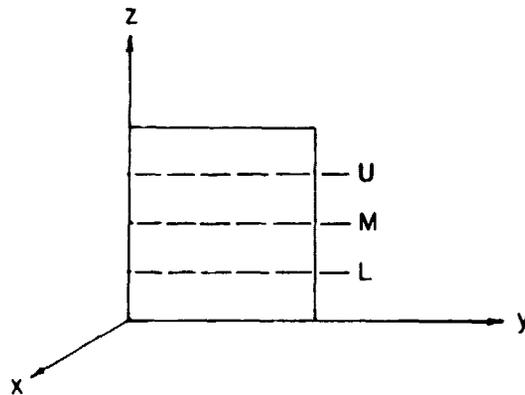
I, II, III - two-dimensional moisture distributions

Subscript - denotes interface as shown in sketch

U - upper - first interface from top surface

M - middle - second interface from top surface

L - lower - third interface from top surface



## LIST OF FIGURES

No.	Title
1	Finite Element Grid with Stress Free and Displacement Boundary Conditions
2	One-Dimensional Uniform Moisture Gradients
3	Two-Dimensional Nonuniform Moisture Gradients
4	Interlaminar Stress Profiles for a $[0_2/90_2]_s$ Laminate
5	Interlaminar Normal Stress Profiles for a $[90/0/\pm 45]_s$ Laminate
6	Interlaminar Shear Stress Profiles for a $[90/0/\pm 45]_s$ Laminate
7	Interlaminar Shear Stress Profiles for Uniform Moisture Gradients in a $[0_2/90_2]_s$ Laminate
8	Interlaminar Normal Stress Profiles for Uniform Moisture Gradients in a $[0_2/90_2]_s$ Laminate
9	Interlaminar Shear Stress Profiles for Uniform Moisture Gradients in a $[90/0_2/90]_s$ Laminate
10	Interlaminar Normal Stress Profiles for Uniform Moisture Gradients in a $[90/0_2/90]_s$ Laminate
11	Interlaminar Stress Profiles for Two-Dimensional Moisture Gradients for a $[0_2/90_2]_s$ Laminate
12	Interlaminar Shear Stress Profiles for Two-Dimensional Moisture Gradients for a $[0/\pm 45/90]_s$ Laminate -- Lower Interface
13	Interlaminar Shear Stress Profiles for Two-Dimensional Moisture Gradients for a $[0/\pm 45/90]_s$ Laminate -- Middle Interface
14	Interlaminar Normal Stress Profiles for Two-Dimensional Moisture Gradients for a $[0/\pm 45/90]_s$ Laminate

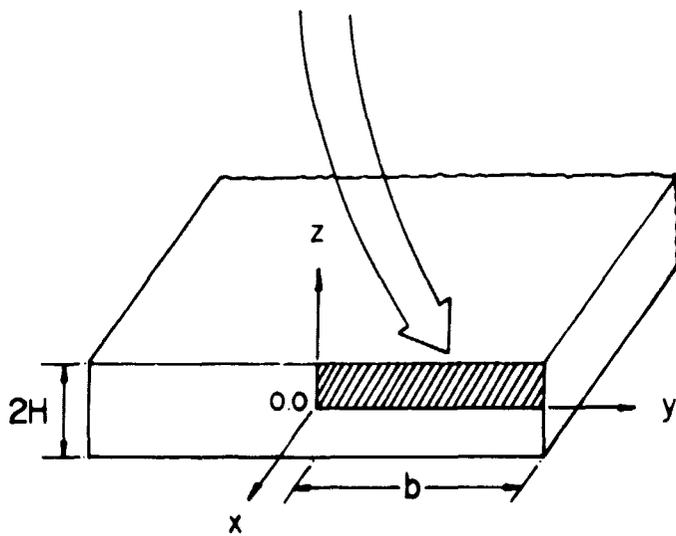
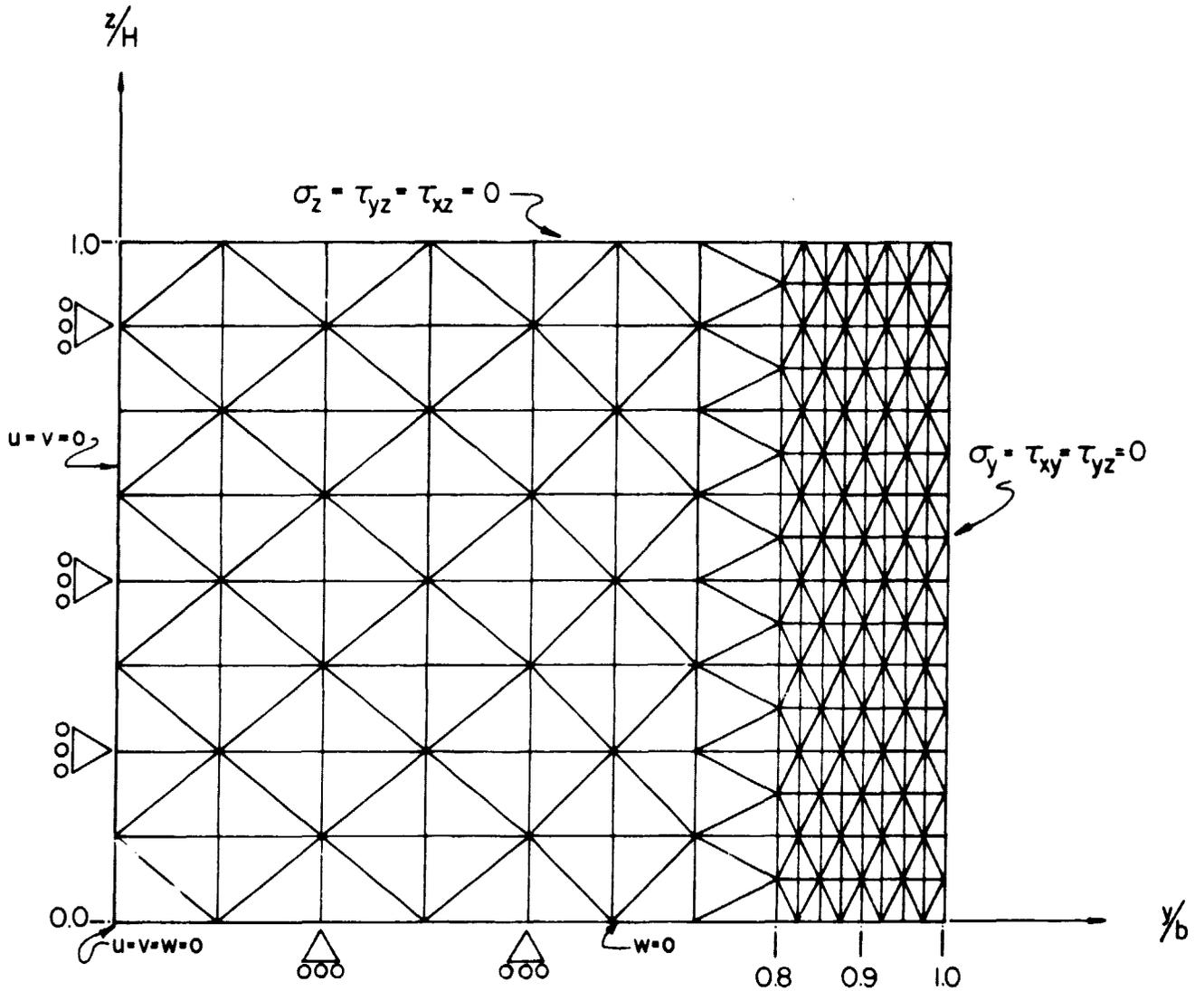
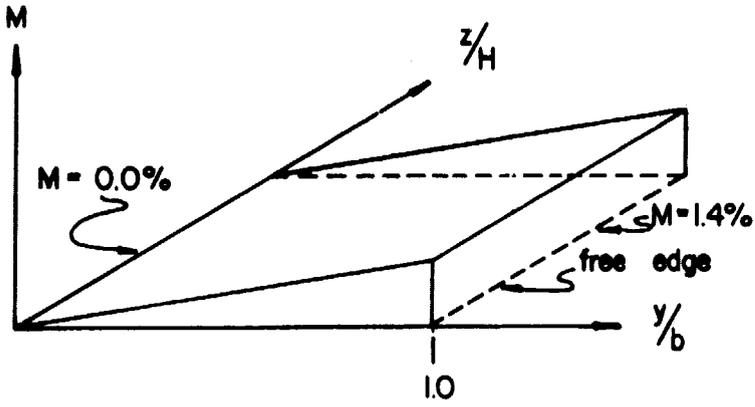
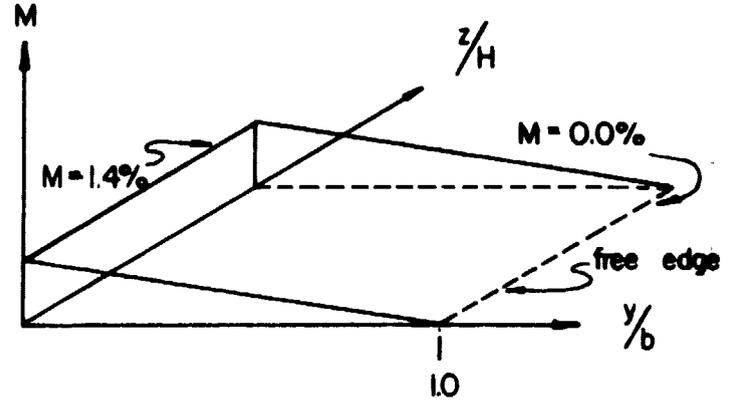


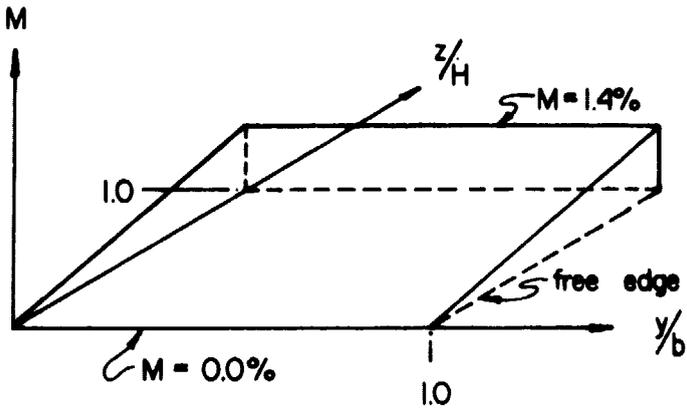
FIG. 1 FINITE ELEMENT GRID WITH STRESS FREE AND DISPLACEMENT BOUNDARY CONDITIONS



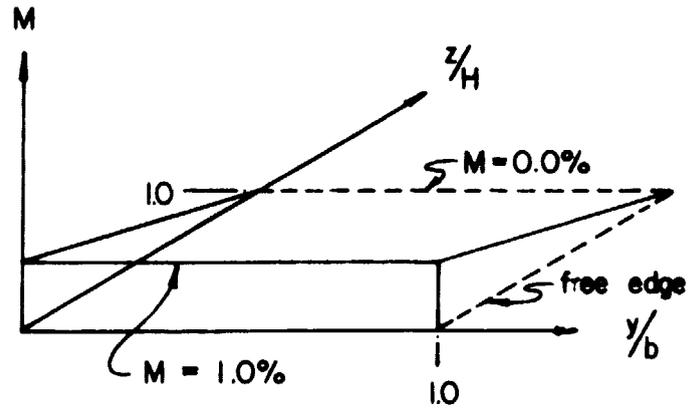
+ Y MOISTURE GRADIENT



- Y MOISTURE GRADIENT

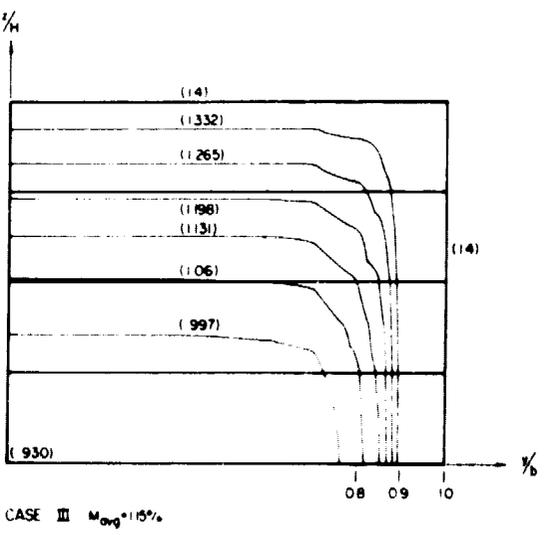
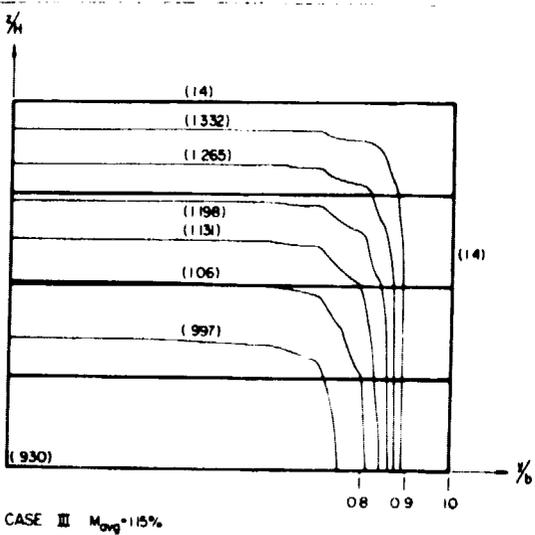
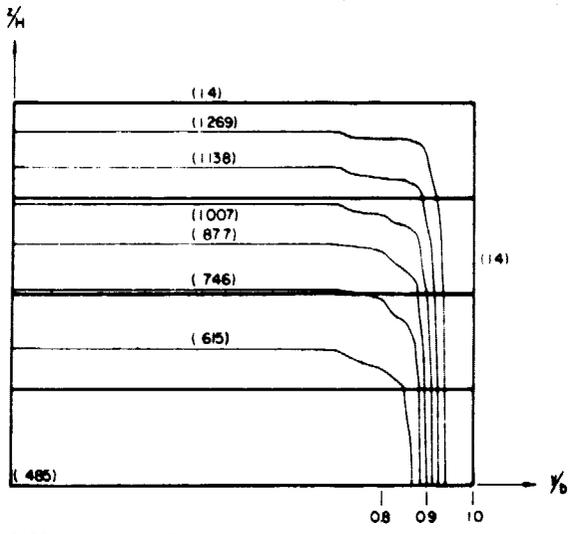
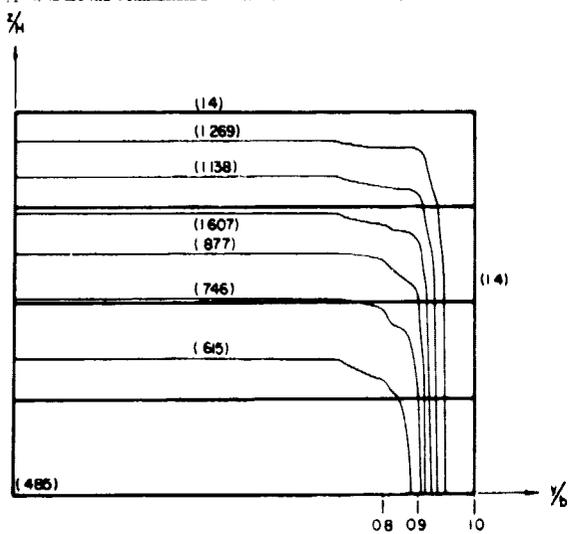
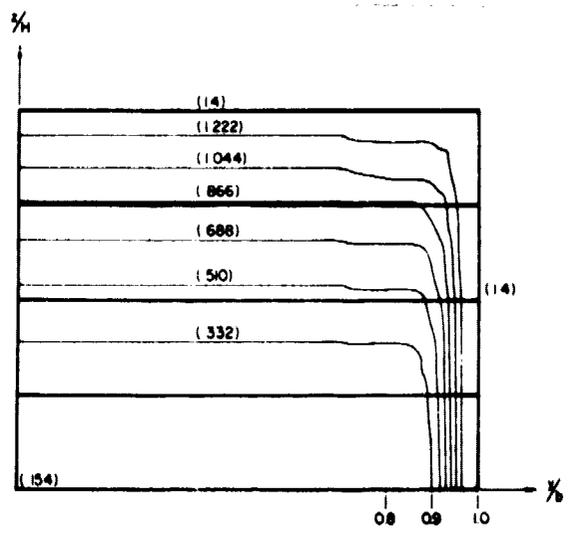
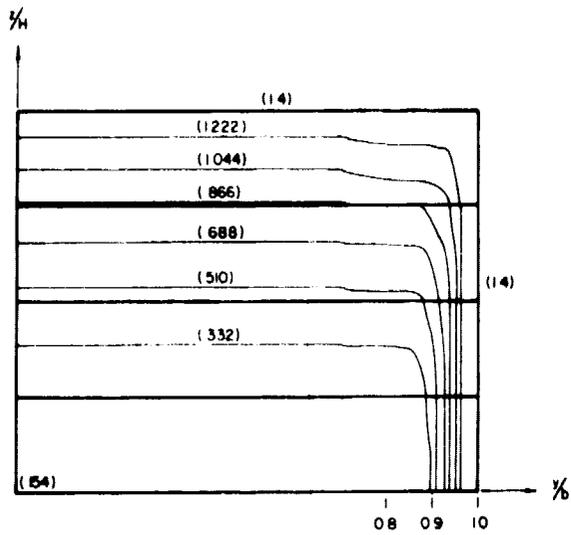


+ Z MOISTURE GRADIENT



- Z MOISTURE GRADIENT

FIG. 2 ONE-DIMENSIONAL UNIFORM MOISTURE GRADIENTS



$[0.7/90.2]_s$

$[0.7/4590]_s$

FIG. 3 TWO-DIMENSIONAL NONUNIFORM MOISTURE GRADIENTS

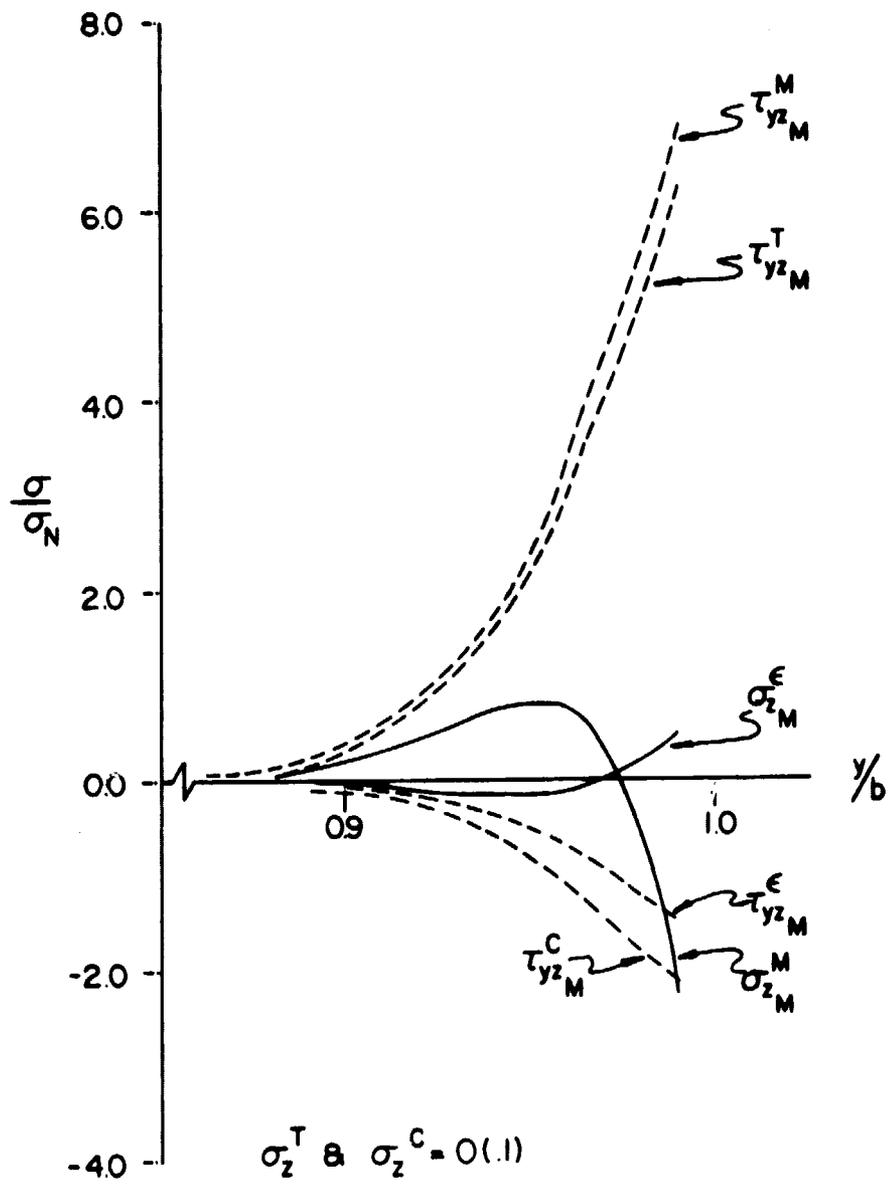


FIG. 4 INTERLAMINAR STRESS PROFILES FOR A  $[0_2/90_2]_s$  LAMINATE

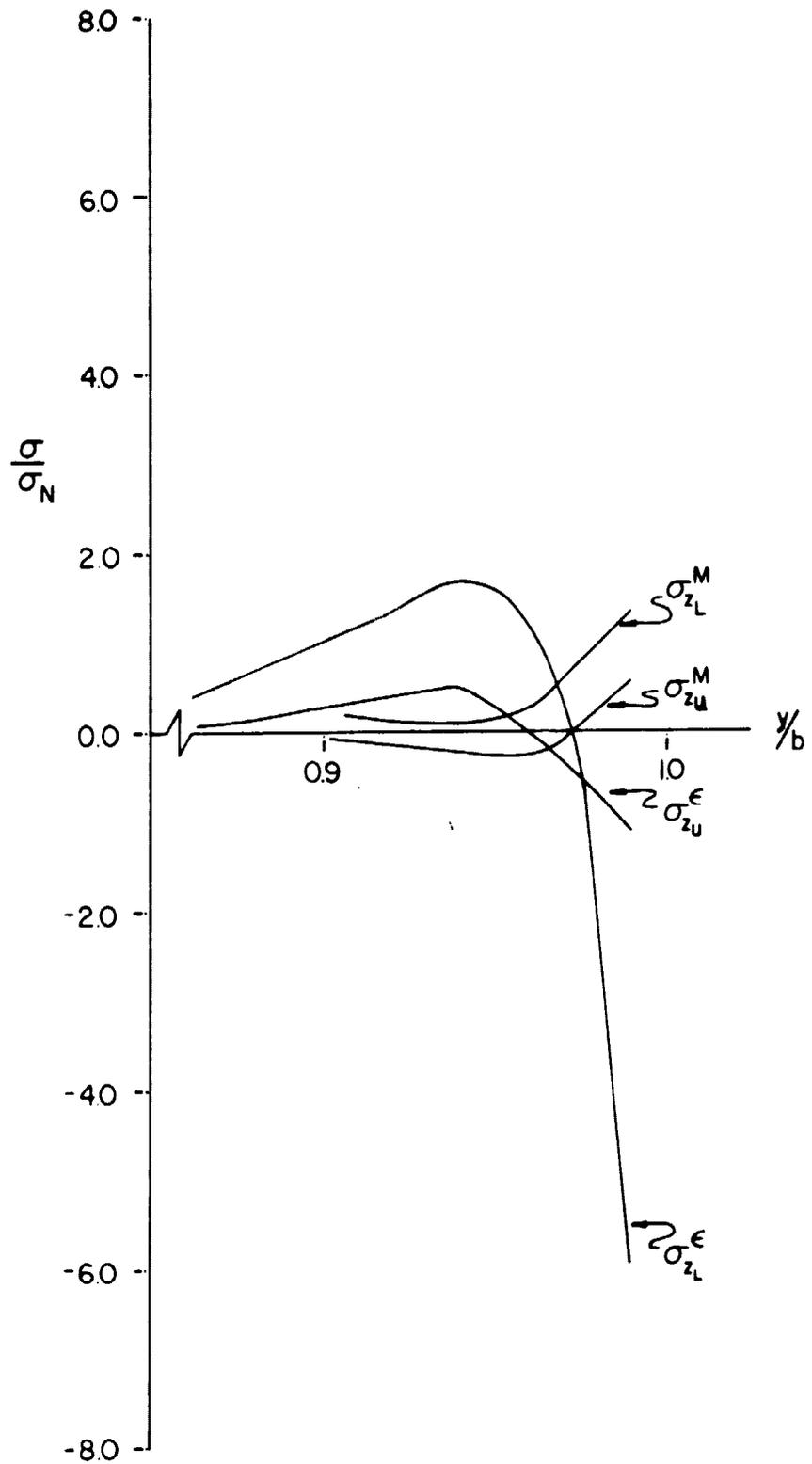


FIG. 5 INTERLAMINAR NORMAL STRESS PROFILES FOR A  
 $[90/0/\pm 45]_s$  LAMINATE

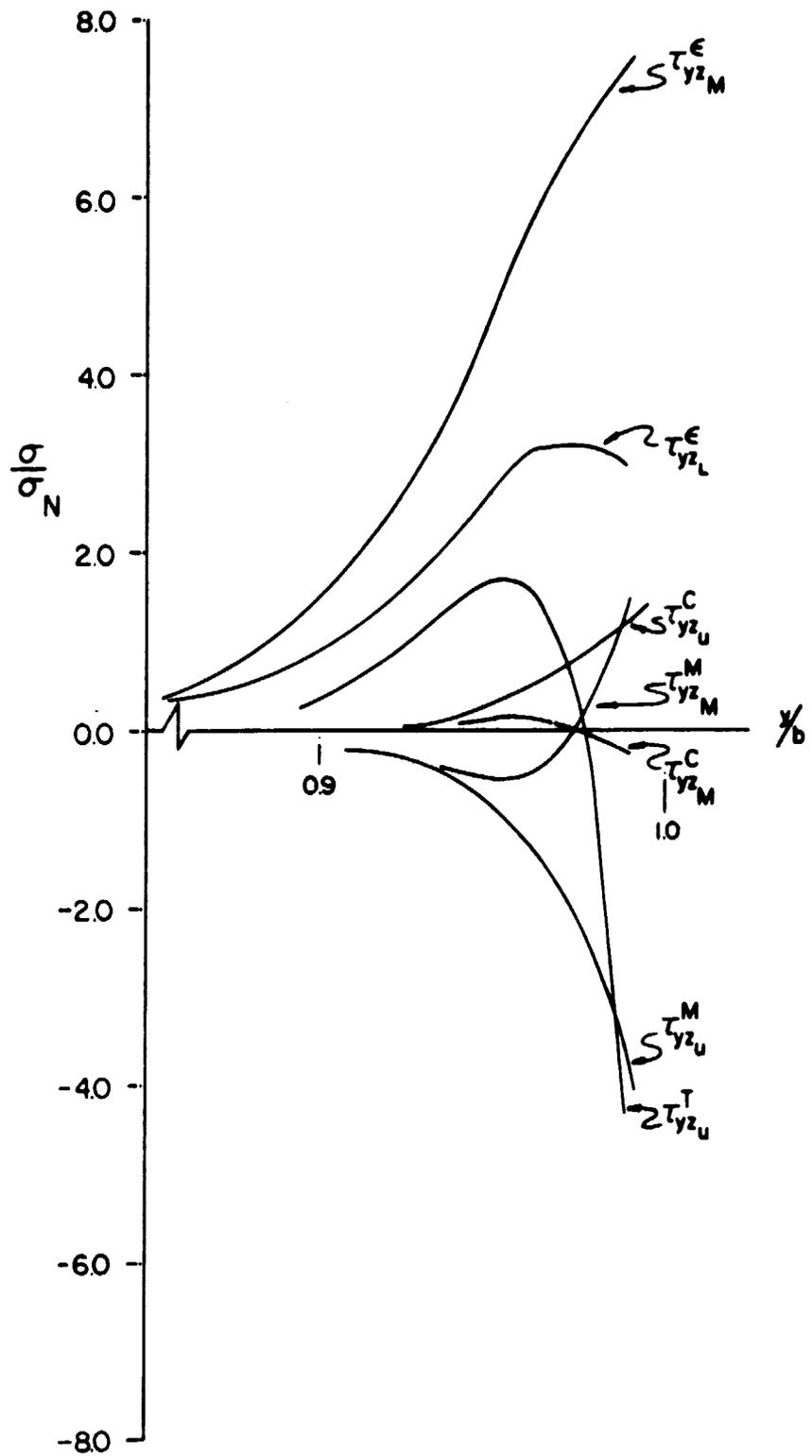


FIG. 6 INTERLAMINAR SHEAR STRESS PROFILES FOR A  
 $[90/0 + 45]_s$  LAMINATE

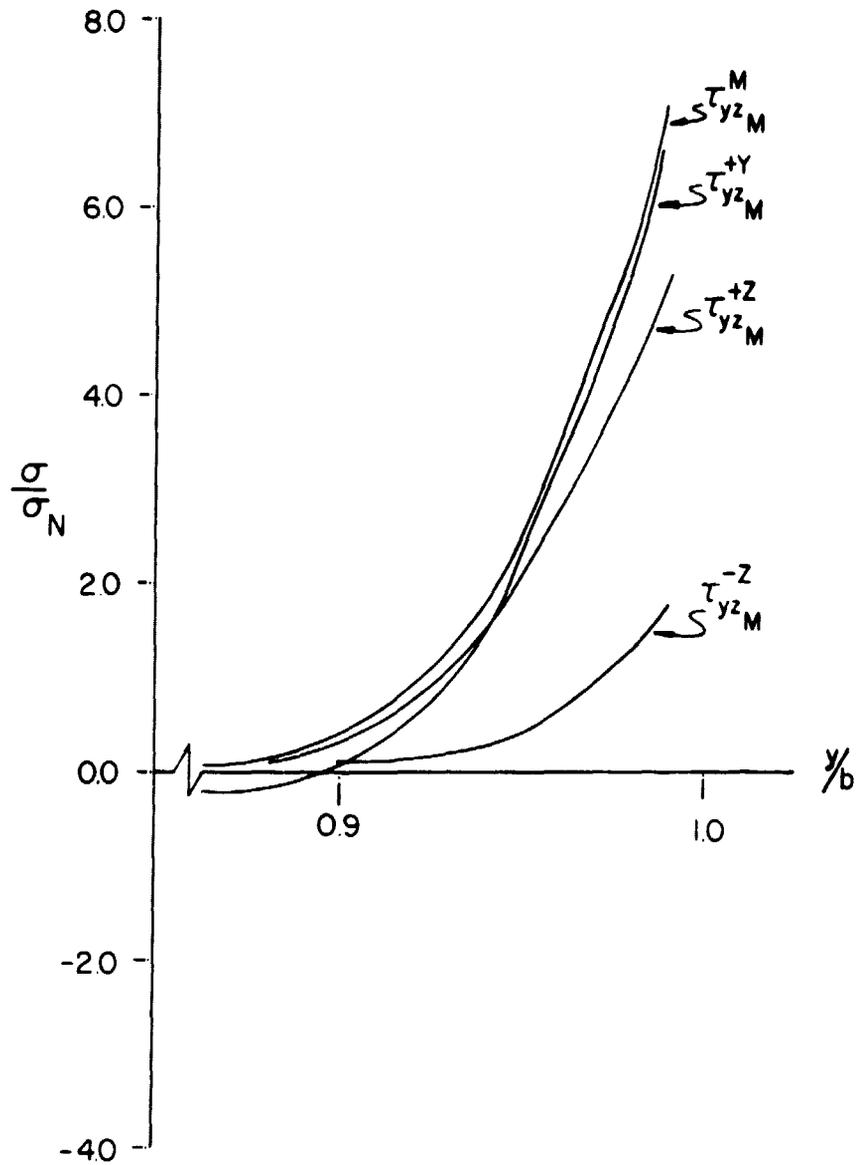


FIG. 7 INTERLAMINAR SHEAR STRESS PROFILES FOR  
 UNIFORM MOISTURE GRADIENTS IN A  $[0_2/90_2]_s$  LAMINATE

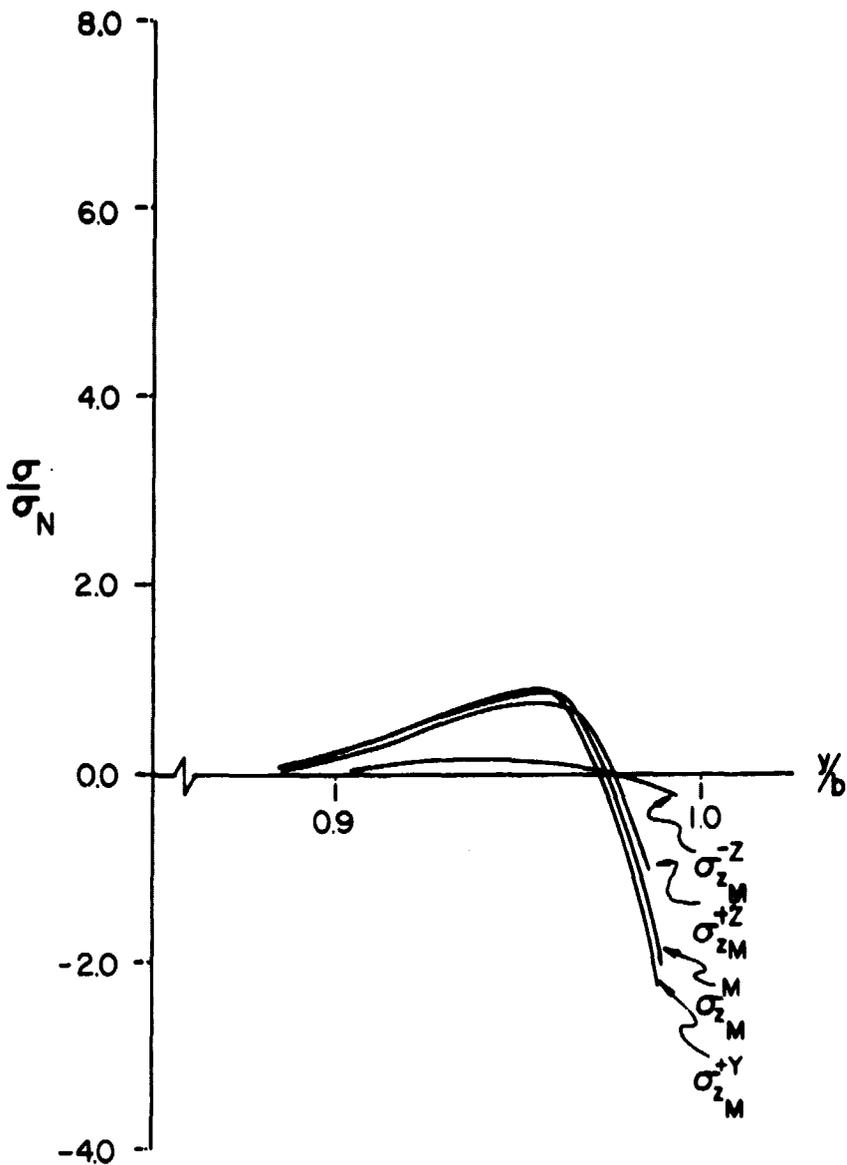


FIG. 8 INTERLAMINAR NORMAL STRESS PROFILES FOR UNIFORM MOISTURE GRADIENTS IN A  $[0_2/90_2]_8$  LAMINATE

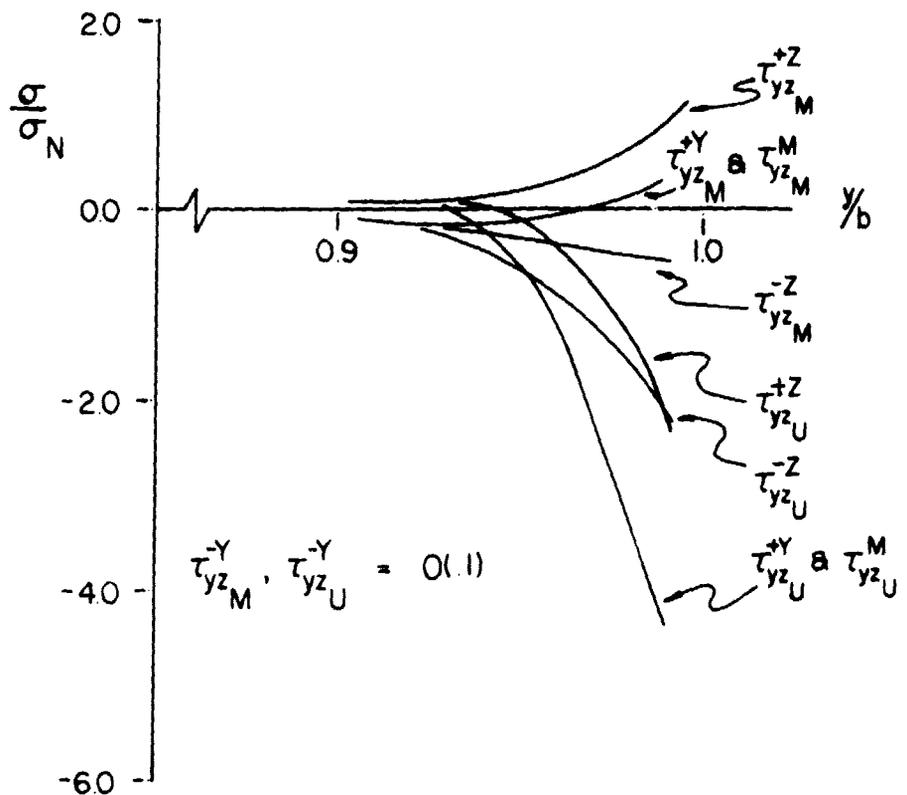


FIG. 9 INTERLAMINAR SHEAR STRESS PROFILES FOR  
 UNIFORM MOISTURE GRADIENTS IN A  $[90/0_2/90]_s$  LAMINATE

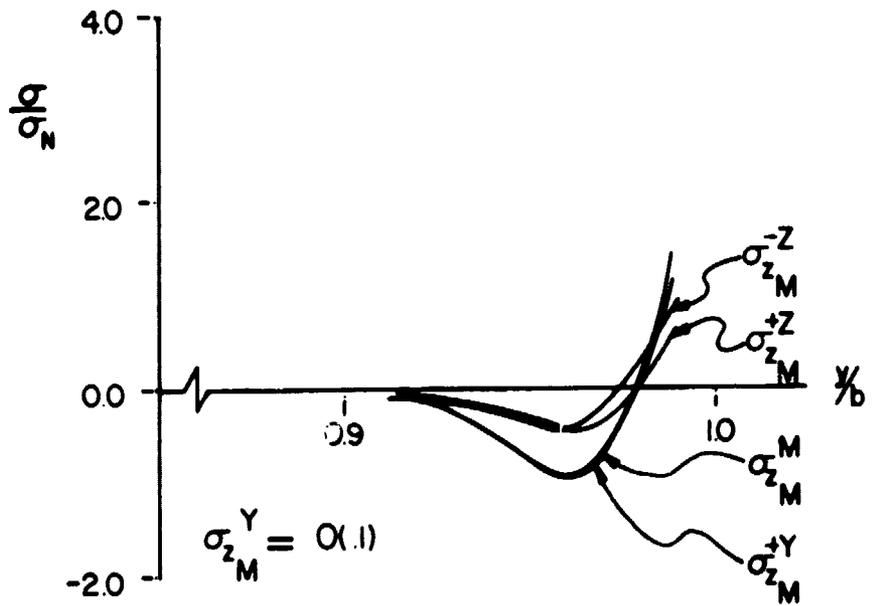


FIG. 10 INTERLAMINAR NORMAL STRESS PROFILES FOR UNIFORM MOISTURE GRADIENTS IN A  $[90/0_2/90]_s$  LAMINATE

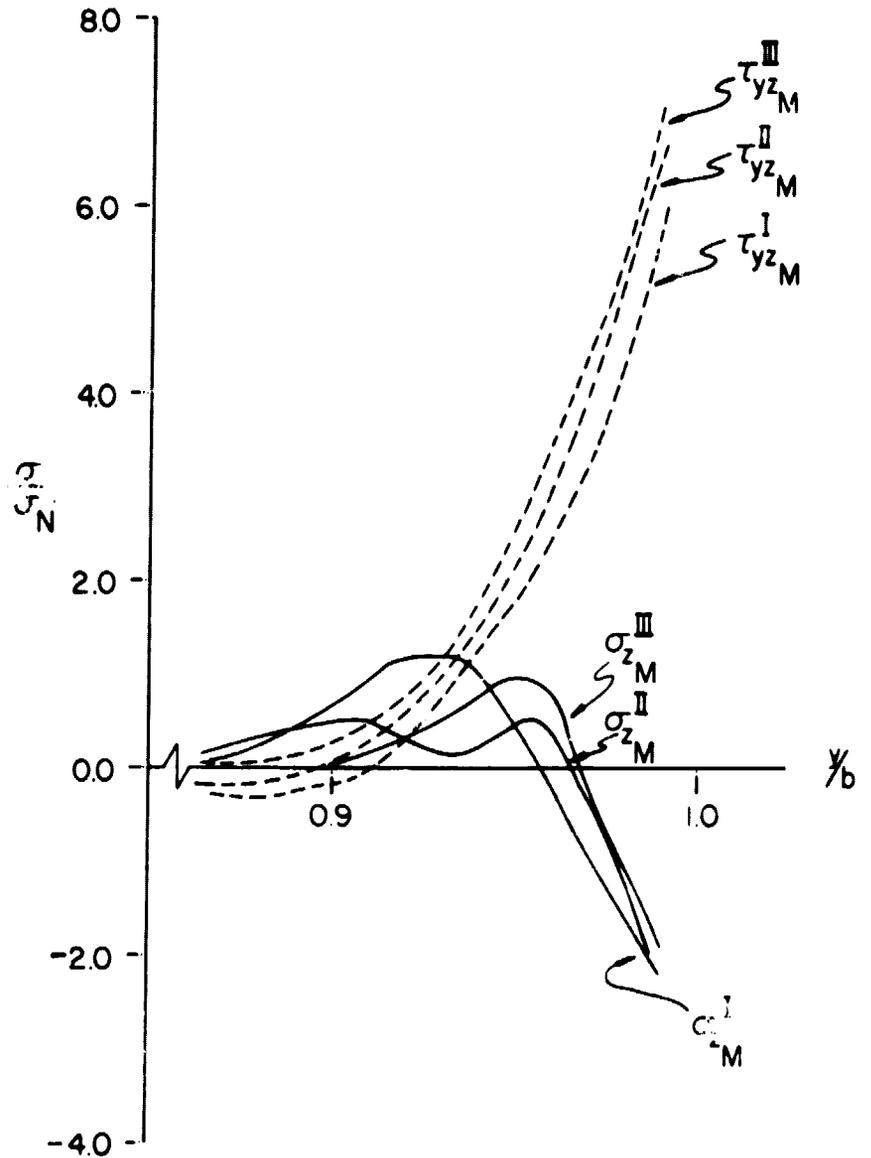


FIG. II INTERLAMINAR STRESS PROFILES FOR TWO-DIMENSIONAL MOISTURE GRADIENTS IN A  $[0_2/90_2]_s$  LAMINATE

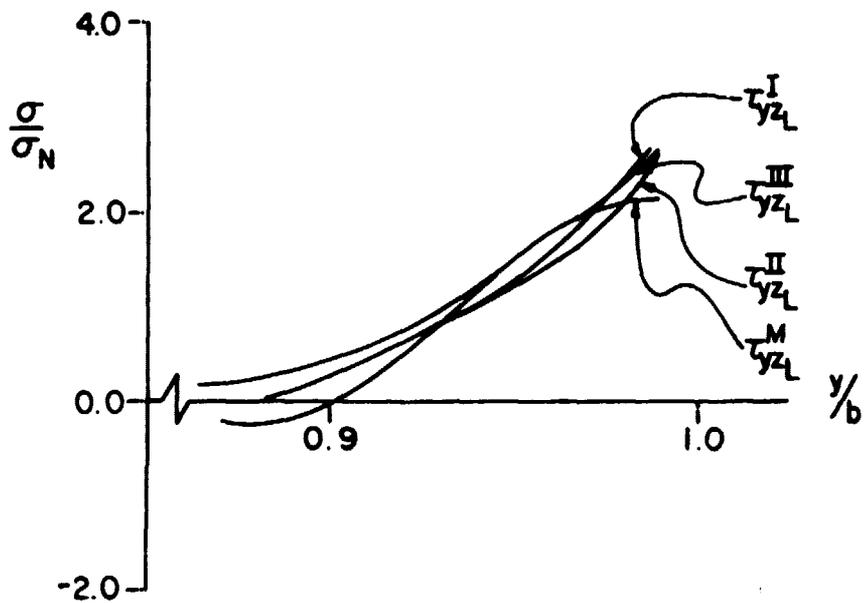


FIG. 12 INTERLAMINAR SHEAR STRESS PROFILES FOR TWO-DIMENSIONAL MOISTURE GRADIENTS IN A  $[0/\pm 45/90]_s$  LAMINATE --- LOWER INTERFACE

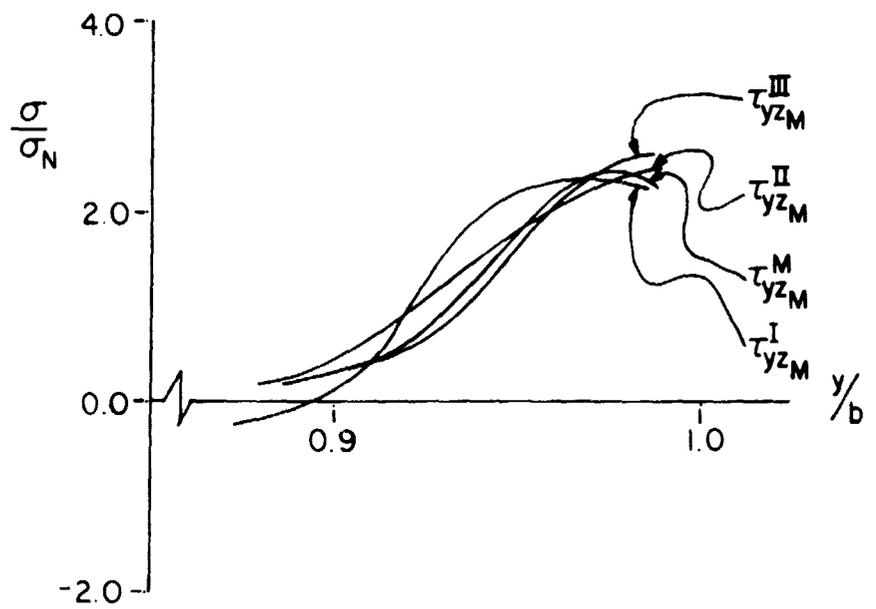


FIG. 13 INTERLAMINAR SHEAR STRESS PROFILES FOR TWO-DIMENSIONAL MOISTURE GRADIENTS IN A  $[0\pm45/90]_5$  LAMINATE --- MIDDLE INTERFACE

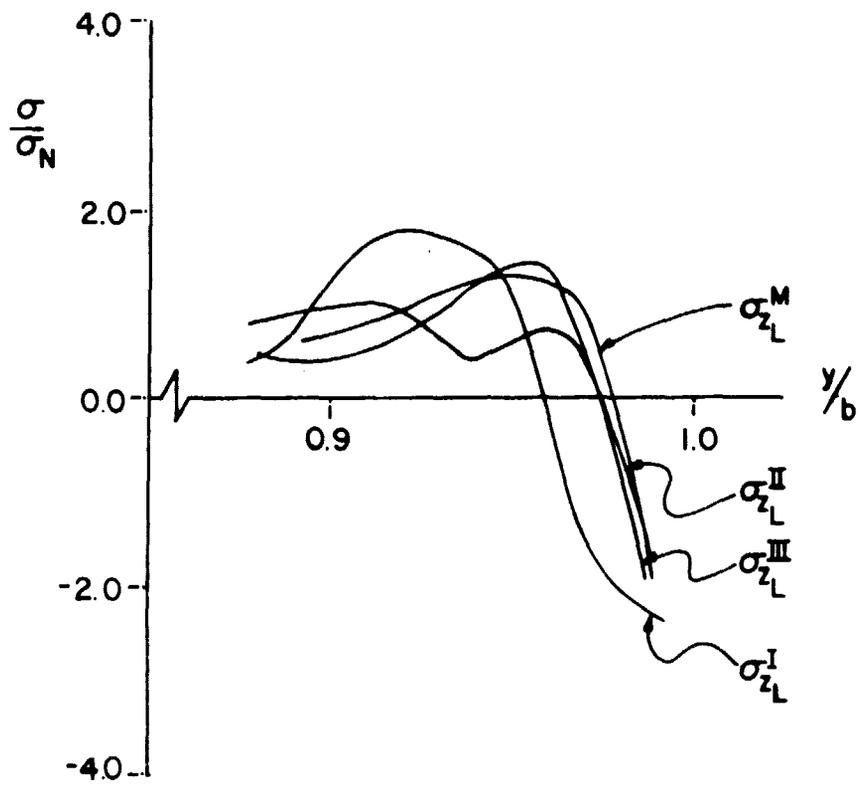


FIG. 14 INTERLAMINAR NORMAL STRESS PROFILES FOR TWO-DIMENSIONAL MOISTURE GRADIENTS IN A  $[0/\pm 45/90]_s$  LAMINATE