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ELECTROMAGNETIC PLASMA WAVE EMISSIONS FROM THE AURORAL FIELD LINES

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Figure 4

IONOSPHERIC ELECTRONS

"INVERTED V" ELECTRONS

"AURORA HISS" GENERATION REGION

SATELLITE

"SAUCER" (UPGOING)

"AURORAL HISS" (DOWNGOING)

VLF SPECTROGRAM

INCREASING LATITUDE
Electromagnetic Plasma Wave Emissions
from the Auroral Field Lines

by

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ABSTRACT

Several types of electromagnetic waves are known to be emitted by charged particles on the auroral field lines. In this paper we review the most important types of auroral radio emissions, both from a historical perspective as well as considering the latest results. Particular emphasis is placed on four types of electromagnetic emissions which are directly associated with the plasma on the auroral field lines. These emissions are (1) auroral hiss, (2) saucers, (3) ELF noise bands, and (4) auroral kilometric radiation. Ray tracing and radio direction finding measurements indicate that both the auroral hiss and auroral kilometric radiation are generated along the auroral field lines relatively close to the Earth, at radial distances from about 2.5 to 5 Re, probably in direct association with the acceleration of auroral particles by parallel electric fields. The exact mechanism by which these radio emissions are generated has not been firmly established. For the auroral hiss the favored mechanism appears to be amplified Cerenkov radiation. For the auroral kilometric radiation several mechanisms have been proposed, usually involving the intermediate generation of electrostatic waves by the precipitating electrons.
I. INTRODUCTION

For many years it has been known that certain types of electromagnetic emissions from the earth's magnetosphere are closely associated with the occurrence of auroras. As early as 1933, BURTON and BOARDMAN (1933) reported observations of bursts of very-low-frequency (VLF) "static" which were closely correlated with flashes of auroral light. Later investigations using ground based VLF radio receivers firmly established that auroral disturbances at high latitudes are often accompanied by intense bursts of broad-band radio noise at frequencies from a few hundred Hz to over 100 kHz (ELLIS, 1957; DUNCAN and ELLIS, 1959; DOWDEN, 1959; MARTIN et al., 1960; JØRGENSEN and UNGSTRUP, 1962; MOROZUMI, 1963; HARANG and LARSEN, 1964). Because of the close association of these radio emissions with aurora and their broad bandwidth these emissions came to be known as auroral hiss, following the classification scheme of HELLIWELL (1965). The first satellite observations of auroral hiss were reported by GURNETT (1966) who showed that auroral hiss is closely correlated with intense fluxes of precipitating electrons with energies less than 10 keV. Subsequent studies have firmly established that auroral hiss is generated along the auroral field lines by intense fluxes of electrons precipitating into the ionosphere with energies in the range from a few hundred eV to several keV (HARTZ, 1970; GURNETT and FRANK, 1972a; HOFFMAN and
LAASPERE, 1972). Since the auroral hiss emissions occur at frequencies below the local electron gyrofrequency these waves must be propagating in the whistler mode. Simple ray tracing considerations show that the auroral hiss appears to be propagating downward from a source at an altitude of about 5,000 to 10,000 km. Pointing flux measurements by MOSIER and GURNETT (1969) showed that another type of emission, called a saucer, also occurs on the auroral field lines, propagating upward from a source at altitudes of approximately 1,400 km. These VLF saucer emissions have been studied in greater detail by JAMES (1976). Other electromagnetic emissions have been observed along the auroral field lines at even lower frequencies, from 100 to 300 Hz, by GURNETT and FRANK (1972b). These emissions, which are called ELF noise bands, have a very narrow bandwidth and are also closely associated with the auroral electron precipitation.

Since the auroral hiss, saucers, and ELF noise bands consist of internally trapped plasma wave modes these emissions cannot escape from the earth's magnetosphere. At higher frequencies, above the local characteristic frequencies of the plasma, radio emissions of auroral origin have been observed escaping from the earth. Since the ionosphere effectively blocks all radiation at frequencies below the local electron plasma frequency from reaching the earth's surface, escaping electromagnetic emissions of this type can only be observed by a satellite above the ionosphere. The first evidence of intense auroral-related radio emissions escaping from the earth's magnetosphere was obtained from the Elektron 2 and 4 satellites by BENEDIKTOV et al.
(1965, 1968). These observations showed that bursts of radio noise at 725 kHz and 2.3 MHz were originating from the earth in close association with geomagnetic storms. Later DUNCKEL et al. (1970) reported similar bursts of radio noise, also associated with high-latitude magnetic disturbances, at frequencies below 100 kHz. The first complete determination of the spectrum of these radio emissions was provided by the IMP-6 spacecraft which showed that the maximum intensities are in the frequency range from about 100 to 500 kHz (STONE, 1973; BROWN, 1973) and that the total power radiated from the earth is sometimes as large as $10^9$ watts (GURNETT, 1974), comparable to the decametric (3.0 to 30 MHz) radio emission from Jupiter. It was also determined that the intense radio bursts from the earth are directly associated with the occurrence of discrete auroral arcs and that the angular distribution of the escaping radiation is consistent with generation along the auroral field lines on the night side of the earth. Because of the close association of these radio emissions with auroral processes and the kilometer wavelength of the radiation, KURTH et al. (1975) have referred to this radiation as auroral kilometric radiation, which is the terminology that will be used in this paper. Other names for this radiation include "high pass noise" (DUNCKEL et al., 1970), "mid-frequency noise" (BROWN, 1973), and "terrestrial kilometric radiation" (GURNETT, 1974; ALEXANDER and KAISER, 1976).

These four types of electromagnetic emissions (auroral hiss, saucers, ELF noise bands, and auroral kilometric radiation) constitute all of the known electromagnetic plasma wave emissions from the auroral
field lines. The purpose of this paper is to review the present state of knowledge of these electromagnetic plasma wave emissions and to discuss the mechanisms by which these waves are thought to be generated.
II. AURORAL HISS, SAUCERS AND ELF NOISE BANDS

A typical example of an auroral hiss event detected by a low-altitude polar-orbiting satellite is shown in Figure 1. This event illustrates the primary identifying characteristics of auroral hiss, consisting of (1) the very broad frequency range of the emission, usually from a few kHz to several tens of kHz, (2) the large electric field intensities, often exceeding 1 mV m⁻¹ broad band electric field strength, (3) the occurrence in a narrow latitudinal band centered on the auroral zone, typically only 5° to 10° wide, and (4) the close association with intense fluxes of low-energy, 100 eV to 1 keV, electrons. The spatial distribution of the auroral hiss over the polar region is illustrated in Figure 2 (from HUGHES et al., 1971), which shows the frequency of occurrence of VLF magnetic field intensities greater than 10⁻¹² gamma² Hz⁻¹ at 9.6 kHz. The maximum occurrence of auroral hiss closely follows the auroral oval, varying from about 80° invariant latitude on the day side of the earth to about 72° invariant latitude on the night side of the earth. A pronounced dawn-dusk asymmetry is clearly evident, with a distinct minimum in the auroral hiss occurrence in the local morning from about 2 to 8 hours magnetic local time. This dawn-dusk asymmetry is probably related to the dawn-dusk asymmetry in the spectrum and intensity of the precipitating auroral electrons, in particular to the low intensity and diffuse
character of inverted-V electron precipitation events in the local morning (FRANK and ACKERSON, 1972). The relationship of the auroral hiss emission to the spectrum of the auroral electron precipitation is illustrated in further detail by the event in Figure 3 (from GURNETT and FRANK, 1972a), which shows that the auroral hiss emission occurs in direct association with an intense inverted-V electron precipitation event of the type first discussed by FRANK and ACKERSON (1971). A VLF saucer emission is also evident near the low latitude boundary of the electron precipitation region. Both the auroral hiss and the saucer emissions are characterized by a V-shaped frequency-time structure. This characteristic frequency-time variation is a spatial effect caused by the frequency dependent limiting ray direction of the whistler mode at large wave normal angles (MOJIER and GURNETT, 1969). Essentially the source illuminates a region along the magnetic field with a beam-width which increases with increasing frequency. As the spacecraft approaches the auroral field lines the highest frequencies are encountered first, since these rays can propagate at the largest angle to the magnetic field. The essential distinction between the auroral hiss and the saucer emissions is the direction of propagation, which is downward for the auroral hiss and upward for the saucers. Although the particles responsible for the downward propagating auroral hiss have been identified as inverted-V electrons, the particles responsible for the saucer emissions have not yet been established. It seems most likely that the upward propagating saucer emissions are produced by upward streaming ionospheric electrons of very low energy, \sim f eV, which
constitute the return current for the nearby inverted-V electron precipitation. This relationship is illustrated schematically in Figure 4. For observing altitudes in the range from about 1500 to 3000 km the latitudinal width of the saucer emissions, 10 to 100 km, is usually somewhat smaller than the width of the auroral hiss, 100 to 500 km. The saucers also have very sharp spectral structure, indicating a very small source region, whereas the auroral hiss is much more diffuse, indicating a broader more extended source.

The event in Figure 3 also illustrates the occurrence of ELF noise bands in the same region as the inverted-V electron precipitation and the auroral hiss. The electromagnetic character of these narrow band emissions is clearly indicated by their detection with both the electric and magnetic antennas. Poynting flux measurements indicate the presence of both upgoing and downgoing components. These emissions occur near but below the local proton gyrofrequency. Since the polarization of these waves has never been measured it is not known whether these emissions are propagating in the whistler mode or the ion cyclotron mode.
III. AURORAL KILOMETRIC RADIATION

The intense radio emissions escaping outward from the earth's auroral regions at frequencies above the local electron plasma frequency are characterized by a very intense peak in the frequency spectrum from about 100 to 500 kHz. Figure 5 shows the median power flux spectrums of this radiation at various local times around the earth, as measured from the IMP-6 spacecraft at radial distances greater than 25 $R_e$ (Kaiser and Alexander, 1977). The intensity of this noise is highly variable and is closely correlated with the auroral electrojet index, AE. During geomagnetically quiet times the radiation intensity at 25 $R_e$ is often completely below the galactic background, whereas at other times the intensity can be as much as six to eight orders of magnitude above the galactic background.

Power fluxes as large as $10^{-14}$ watts m$^{-2}$ Hz$^{-1}$ have been observed at 30 $R_e$, with even larger intensities closer to the earth (Gurnett, 1974). The occurrence of intense bursts of kilometric radiation is closely associated with the occurrence of auroral arcs. This association is illustrated in Figure 6, which shows the occurrence of intense bursts of kilometric radiation at 178 kHz during periods (passes 1094 and 1095) when discrete auroral arcs are present and no radiation during periods (passes 1093, 1095 and 1097) when no discrete arcs are present. The association with discrete arcs provides
substantial evidence that the generation of auroral kilometric radiation is closely associated with inverted-V events, since the inverted-V electron precipitation is associated with discrete arcs. Both direction finding measurements (KURTH et al., 1975; KAISER and STONE, 1975) and lunar occultation measurements (ALEXANDER and KAISER, 1976) show that the most intense bursts of kilometric radiation come from the auroral field lines on the night side of the Earth at radial distances ranging from 2 to 5 Re. These results are illustrated in Figure 7, which shows a series of source positions obtained by occultation measurements from the RAE-2 spacecraft in orbit around the moon. The moon in this case was in the dusk meridian plane (magnetic local time ~ 17.5 hours) so that the day-night position of the source can be resolved. The occultation measurements at 1255, 1635 and 2020 UT indicate the occurrence of multiple sources located at various points along a magnetic field line at about 70 to 75° invariant latitude. Occasionally dayside sources are also observed (ALEXANDER and KAISER, 1976), apparently associated with the dayside polar cusp region. The dayside sources occur less frequently and are less intense than the nightside sources. Angular distribution measurements by GREEN et al. (1977) are also consistent with a nightside, high latitude, source for the intense auroral kilometric radiation. A typical angular distribution is illustrated in Figure 3, which shows the frequency of occurrence of auroral kilometric radiation intensities above a preset threshold at 178 kHz as a function of the magnetic latitude and magnetic local time of the observing point. A sharp low latitude
boundary is evident, varying from about 45° on the dayside of the earth to near the equator on the night side of the earth. At large distances from the earth this cutoff forms a cone-shaped boundary, with the axis of the cone tipped toward local evening (≈ 22 hours, magnetic local time) by about 20°. The radiation is confined almost entirely to the poleward side of this boundary. Symmetrical cone-shaped boundaries are present in the northern and southern hemispheres, evidently corresponding to sources in the northern and southern auroral regions, respectively. Detailed studies have shown that these cone-shaped boundaries are strongly frequency dependent, with the solid angle of the emission region increasing with increasing frequency. Multiple satellite studies have also shown that the entire region poleward of the cone-shaped boundary is illuminated with comparable radiation intensities.

Ray tracing studies show that the general features of the angular distribution of the auroral kilometric radiation can be explained by relatively simple propagation considerations if the radiation is generated by a small localized source at about 2 to 3 Re along an auroral field line on the night side of the earth. Some typical ray tracing results are illustrated in Figure 9, which shows the distribution of ray paths at various frequencies for a representative model of the polar ionosphere. Because the index of refraction increases with decreasing altitude the general effect is for the ray paths to be refracted upward away from the ionosphere, thus producing a low-latitude cutoff in the region accessible to the radiation.
Detailed calculations of the distribution of intensity within the accessible region cannot be performed without a more detailed understanding of the generation mechanism. However, the overall features of the angular distribution can be understood from propagation considerations of this type. At the present time the polarization of the auroral kilometric radiation has not been directly measured. Several indirect methods indicate that the polarization is right hand with respect to the magnetic field in the generation region. For example, comparison of ray paths for the right and left hand modes of propagation, as in Figure 9, with observed angular distributions, as in Figure 10, give the best fit and consistency if the radiation is right hand polarized. Furthermore, recent observations by Gurnett and Green (1977) of a cutoff in the spectrum of the auroral kilometric radiation at the local electron gyrofrequency are consistent with the expected propagation cutoff of the right-hand polarized extraordinary (R-X) mode. Right hand polarization is also consistent with the polarization of the decametric radiation from Jupiter (Warwick, 1967), which is thought to be fundamentally similar to the terrestrial kilometric radiation.
When comparing these various auroral electromagnetic emissions one cannot help but be impressed by the close similarity of the spatial regions within which the auroral hiss and the auroral kilometric radiation are generated. In both cases the radiation is believed to be generated in a relatively small region along the auroral field lines at about 3.0 $R_e$ radial distance. Evidence that the source region is relatively small and localized is provided by the distinct V-shaped spectral features often observed in the auroral hiss and by the small angular size of the auroral kilometric radiation source obtained from the direction finding and lunar occultation measurements. Although no evidence exists showing that these two types of radiation originate from exactly the same region the evidence does strongly suggest that the two source regions overlap to a substantial extent. Since both the auroral hiss and the auroral kilometric radiation are closely associated with the inverted-V electron precipitation it seems most likely that the generation of these radio emissions is closely related to the acceleration of these electrons. Substantial evidence now exists (Haerendel et al., 1976; Mozer et al., 1977) showing that precipitating auroral electrons are accelerated by parallel electric fields at altitudes ranging from 5,000 to 15,000 km, in the same general region where these radio emissions are produced.
Although both the auroral hiss and the auroral kilometric radiation appear to originate from a common spatial region, distinctly different mechanisms are required to explain these two types of radiation since they are propagating in different plasma wave modes. As shown in Figure 10, the whistler-mode auroral hiss must be generated at frequencies below the local electron gyrofrequency, whereas the auroral kilometric radiation must be generated at frequencies above the local electron plasma frequency. For many years it has been suggested that auroral hiss is produced by incoherent Cerenkov radiation from the precipitating auroral electrons (ELLIS, 1957; JØRGENSEN, 1968; LIM and LAASPERE, 1972; TAYLOR and SHAWHAN, 1974). Although the Cerenkov mechanism has many desirable features, it is generally concluded that this mechanism produces power fluxes which are several orders of magnitude too low to explain the observed auroral hiss intensities. At the present time the best possibility for explaining the observed auroral hiss intensities appears to be the mechanism proposed by MAGGS (1976), in which incoherent Cerenkov radiation generated by the precipitating electron beam is amplified to higher intensities by a whistler-mode instability. Similar mechanisms also appear to be responsible for the generation of saucer emissions (JAMES, 1976), although the details have not been investigated as thoroughly as for the auroral hiss. Cerenkov radiation has also been considered as a mechanism for generating the auroral kilometric radiation and again the computed power fluxes are much too small. Rough estimates indicate that an overall efficiency of 0.1 to 1% is required for the generation
of this radiation. Such high conversion efficiencies can only be produced by a coherent plasma instability. Instability mechanisms which have been proposed include the generation of escaping electromagnetic radiation by interactions between electrostatic waves at the upper hybrid resonance (BENSON, 1975; BARBOSA, 1977), by interactions of the precipitating electron beam with ion turbulence (PALWADESSO et al., 1975), by electrostatic waves produced by energetic ions streaming outward from the auroral regions (BOSWELL, 1977), and by direct conversion via coherent electron cyclotron emission from the precipitating electrons (MELROSE, 1975). At the present time, no consensus exists about which, if any, of these mechanisms can account for the intense kilometric radiation from the earth's auroral regions.
V. CONCLUSIONS

It is evident that the auroral particle distributions produce a variety of complex and very interesting electromagnetic emissions. At the present time the mechanisms by which these electromagnetic emissions are generated are rather poorly understood and the full explanation of these radio emission processes represents a significant challenge to both the theorists and the experimentalists. It should, however, be possible to arrive at a reasonably clear understanding of how these radio emissions are generated since a great deal is already known about the charged particle distributions and processes which occur along the auroral field lines and our understanding of these processes is advancing rapidly. Because a comparable detailed knowledge of the charged particle distribution will probably never be known for most other radio sources in the universe, the study of these terrestrial radio emissions provides a unique opportunity to extend our understanding of similar electromagnetic plasma wave emissions from other planets and astronomical objects.
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FIGURE CAPTIONS

Figure 1  An example of an intense broad band auroral hiss emission associated with intense fluxes of low energy, few hundred eV, auroral electrons. The auroral hiss is indicated by the enhanced 7.35 and 70.0 kHz electric field intensities from about 1546 to 1549 UT, slightly poleward of the E > 45 keV electron trapping boundary.

Figure 2  The frequency of occurrence distribution of auroral hiss at 9.6 kHz as a function of invariant latitude and magnetic local time. Only events with magnetic field intensities greater than 10^-12 gamma^2 Hz^-1 are counted (from HUGHES et al., 1971).

Figure 3  High resolution frequency-time spectrograms of electric and magnetic fields detected by the Injun 5 spacecraft at an altitude of about 2500 km showing the occurrence of auroral hiss, saucer emissions and an ELF noise band in close association with an inverted-V electron precipitation event (from GURNETT and FRANK, 1972a).

Figure 4  A schematic illustration showing the spatial relationships between the downward propagating auroral hiss and
the upward propagating saucer emissions for events such as in Figure 3. The association of the saucer emissions with upward streaming ionospheric electrons has not yet been confirmed.

Figure 5

Median spectrums of auroral kilometric radiation observed at various local times around the earth by the IMP-6 spacecraft at radial distances \( R > 25 \, R_e \). The three spectrums in each plot are for various ranges of the auroral electrojet, AE, index. The kilometric radio emissions from the earth are closely correlated with auroral zone currents as indicated by the AE index (from KAISER and ALEXANDER, 1977).

Figure 6

The intensity auroral kilometric radiation at 178 kHz observed about \( 30 \, R_e \) from the earth and a sequence of photographs of the aurora taken by a low altitude satellite over the northern polar region. The intense bursts of auroral kilometric radiation are seen to be closely correlated with the occurrence of discrete auroral arcs.

Figure 7

Source positions for auroral kilometric radiation as determined by lunar occultation measurements with the RAE 2 spacecraft in orbit around the moon (from ALEXANDER and KAISER, 1976).
Figure 8  The angular distribution of auroral kilometric radiation with intensities above a given threshold as a function of magnetic latitude and magnetic local time.

Figure 9  Ray paths computed for various initial wave normal angles for a point source located at various points along an auroral field line at 70° invariant latitude. The cone shaped latitudinal cutoff evident in Figure 8 is the result of refraction of the radiation upward away from the ionosphere in the region near the source (from GREEN et al., 1977).

Figure 10  Both the auroral hiss and auroral kilometric radiation appears to originate from a common region at about 2.0 to 5.0 Re along the auroral field line. This region is believed to correspond to the region of parallel electric fields responsible for the auroral electron acceleration.
Figure 1

SOUTHERN HEMISPHERE
ORBIT 6713, FEB. 11, 1970

ELECTRIC FIELD
70.0 KHz
7.35 KHz

AURORAL HISS

ELECTRONS
160 ≤ E ≤ 280 eV, α = 90°
155 ≤ E ≤ 275 eV, α = 0°
325 ≤ E ≤ 570 eV, α = 90°

ELECTRONS > 45 keV
TRAPPING BOUNDARY

ELECTRONS
1160 ≤ E ≤ 2040 eV
α = 90°
Figure 3
Figure 4
Figure 5

MEDIAN FLUX DENSITY AT 25 RE (W m^-2 Hz^-1)

06-09 HR

03-06 HR

09-12 HR

09-03 HR

12-15 HR

21-24 HR

15-18 HR

18-21 HR

FREQUENCY (kHz)

A<75

0<75<A<200

A>200

A<75

0<75<A<200

A>200
Figure 6
18 FEB. 1975
17.5 MLT

- 0915 U.T.
- 1255
- 1635
- 2020

Figure 7
VLF AURORAL HISS

REGION OF PARALLEL ELECTRIC FIELDS AND INVERTED "V" ELECTRON ACCELERATION

\[ f_{pe} = \text{ELECTRON PLASMA FREQUENCY} \]
\[ f_{ge} = \text{ELECTRON GYRO FREQUENCY} \]

Figure 10