PLANS - A FINITE ELEMENT PROGRAM FOR NONLINEAR ANALYSIS OF STRUCTURES
VOLUME II - USERS' MANUAL

A. PIFKO, H. ARMEN, JR., A. LEVY, AND H. LEVINE

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Grumman Research Department Memorandum RM-633

PLANS — A FINITE ELEMENT PROGRAM FOR
NONLINEAR ANALYSIS OF STRUCTURES

VOLUME II — USERS' MANUAL

by

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Materials and Structural Mechanics

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Approved by:

Richard A. Scheuing
Director of Research
FOREWORD

The PLANS system of programs for the nonlinear analysis of structures was developed within the Grumman Research Department with the cooperation, guidance, and partial support of the NASA Langley Research Center.

The programs are an outgrowth of the work reported in a series of NASA Contractor Reports: CR-803, CR-1649, CR-2351, and CR-2568. The last-named document is the theoretical companion volume to this manual.

The principal developers of PLANS are Drs. A. Pifko, H. Armen Jr., H. Levine, and A. Levy. The successful development of a system as broad in scope and complexity as PLANS requires the efforts of many individuals. The principal developers gratefully acknowledge the contributions of J. S. Miller for his efforts associated with the initial programming. Special thanks also go to Ms. P. Ogilvie, P. Zirk, and E. Yander for their diligence and dedication to the programming effort.
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SECTION 1.

INTRODUCTION AND INPUT OVERVIEW
INTRODUCTION AND INPUT OVERVIEW

The PLANS system, rather than being one comprehensive computer program, is a collection of finite element programs used for the nonlinear analysis of structures. This collection of programs evolved and is based on the organizational philosophy in which classes of analyses are treated individually based on the physical problem class to be analyzed. On the basis of this concept, each of the independent finite element computer programs of PLANS, with an associated element library, can be individually loaded and used to solve the problem class of interest. A number of programs have been developed for material nonlinear behavior alone and for combined geometric and material nonlinear behavior. Table 1 summarizes the usage, capabilities, and element libraries of the current programs of the PLANS system. These include:

- **BEND** for the plastic analysis of built-up structures where bending and membrane effects are significant
- **HEX** for problems requiring a three-dimensional elastic-plastic analysis
- **REVBY** for the plastic analysis of bodies of revolution
- **OUT-OF-PLANE MG** for the material and geometric nonlinear analysis of built-up structures

Supplementing these is a SATELLITE program for data debugging and plotting of input geometries.
### TABLE 1 OPERATIONAL MODULES OF PLANS - CURRENT STATUS (5/77)

<table>
<thead>
<tr>
<th>PROGRAM</th>
<th>APPLICATION</th>
<th>ELEMENT LIBRARY</th>
<th># ELEM</th>
<th># MODE</th>
<th>THICK OF LOADING</th>
<th>MATERIAL BEHAVIOR</th>
<th>EQUILIB. CORRECT</th>
<th>SUB-INCREMENT</th>
<th>RESTART</th>
<th>FLATTENING</th>
<th>VISC. GENERATOR</th>
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<tbody>
<tr>
<td>EOCD</td>
<td>Analysis of 3-D thin-walled built-up structures where bending is significant Plastic analysis only</td>
<td>Beam (9 cross-sections)</td>
<td>Variable</td>
<td>Variable</td>
<td>Airborne</td>
<td>Isotropic-Galilean Fluvial Navigating</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
<td>Yes (see Satellite)</td>
<td>None</td>
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<tr>
<td></td>
<td></td>
<td>Stringer (CBS-LII)</td>
<td></td>
<td></td>
<td>Concentrated</td>
<td>Linear &amp; Nonlinear Pumping</td>
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<td>Triangle Membrane</td>
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<td>Family (CUT-LII)</td>
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<td>Thermal (Ex. Heat)</td>
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<td></td>
<td></td>
<td>Triangular Bending &amp; Membrane</td>
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<td>Proportional only</td>
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<tr>
<td>EOCD</td>
<td>Analysis of massive 3-D bodies Plastic analysis only</td>
<td>3-D Hexahedron</td>
<td>2500</td>
<td>2500</td>
<td>Surface</td>
<td>Isotropic Kinematic Hardening</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
<td>None</td>
<td>Limited to 8 node sets.</td>
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<tr>
<td></td>
<td></td>
<td>8 node - 20 node Isoparametric</td>
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<td></td>
<td>Concentrated</td>
<td>Linear &amp; Nonlinear Pumping</td>
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<tr>
<td>EREB</td>
<td>Axysymmetric analysis of bodies of revolution Plastic analysis only</td>
<td>Isoparametric Shell of Rev.</td>
<td>500</td>
<td>900</td>
<td>Surface</td>
<td>Same as GEO</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>Yes (see Satellite)</td>
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<tr>
<td></td>
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<td>Rev. Triangle &amp; Thin Ring</td>
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<td>Concentrated</td>
<td>Linear &amp; Nonlinear Pumping</td>
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<tr>
<td>EREB</td>
<td>Analysis of 3-D thin-walled built-up structures where bending is not significant Plastic &amp; Geometric Nonlinearity (Small strain-large deflec.)</td>
<td>Beam (9 cross-sections)</td>
<td>600</td>
<td>900</td>
<td>Surface</td>
<td>Same as GEO</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
<td>Yes (see Satellite)</td>
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<td>Linear &amp; Nonlinear Pumping</td>
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<td>Family (CUT-LII)</td>
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<td>Thermal (Ex. Heat) &amp; Non-Proportional</td>
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<td>CPLAN-GN</td>
<td>Analysis of 3-D thin-walled built-up structures where bending is not significant Plastic &amp; Geometric Nonlinearity (Small strain-large deflec.)</td>
<td>Beam (9 cross-sections)</td>
<td>600</td>
<td>900</td>
<td>Surface</td>
<td>Same as GEO</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
<td>Yes (see Satellite)</td>
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In addition, a number of spin-off special purpose modules have been developed: FAST for fracture analysis, AXSHEL for the nonlinear analysis of shells of revolution, and BENST for the buckling analysis of large built-up structures. A general purpose nonlinear dynamic analysis module, DYCAST, is in final stages of development. These modules are not included in this document.

The modules that analyze material nonlinearities alone employ the "initial strain" concept within an incremental procedure to account for the effect of plasticity and include the capability for cyclic plastic analysis. The "initial strain" approach does not require that the stiffness matrix be updated at each step in the analysis but rather, the effect of plasticity enters into the analysis as an effective load vector. The cyclic plastic behavior is accounted for by implementing the Prager-Ziegler kinematic hardening theory. Geometric nonlinearities are included in the PLANS system programs by making use of an "updated" or convected coordinate approach, which requires the reformation of the stiffness matrix due to changes in the geometry and stress field during the incremental procedure.

The theoretical foundations upon which the PLANS system programs are based can be found in a companion volume, "PLANS - A Finite Element Program for Nonlinear Analysis of Structures, Volume I - Theoretical Manual," NASA Contractor Report NASA CR-2568.

This present volume describes the input data preparation for the four principal modules of PLANS, BEND, HEX, REVBY, and OUT-OF-PLANE MG as well as the SATELLITE program for input checking. In keeping with the philosophy of the PLANS system, the description of the input for each of the modules is in a self-contained section. Also presented is a section that shows the input data decks for a representative number of sample problems.
The input for all the modules begins with a title card that allows for any 80 character title (specified in columns 1-80). This title serves as a page heading for subsequent computer output. The input data following this card is divided into a number of functional groups, each describing a specific type of input information. These groups are briefly described below and schematically shown in Fig. 1. Each input group must be read in the specific order shown in the figure. In general, each group is delineated with a specific section end card. This is the alphanumeric SEND, left justified on the input card, in columns 1 through 4 or a blank card. The input groups are as follows:

I  Title Card
II Program Control Parameters and Options
III Node Specification
   This section defines an allowable set of node point identification numbers
IV  Element Connectivity
   Defines each element by specifying its type (i.e., beam, triangle, ..., etc.), identification number and connecting node points
V - VII Node Point Coordinates
   Defines the location of each node point in a global cartesian coordinate system
VIII Node Point Single and Multipoint Constraints
   Defines boundary conditions and nodal constraint equations
IX  Element Material and Section Properties
   Section properties include element thickness, area and moment of inertia where applicable.
Fig. 1 Order of Groups of Input Data
Material properties include such quantities as Young's modulus, Poisson's ratio, and quantities defining the plastic response such as yield stress and hardening parameters.

X

Applied Load

Defines spatial distribution of applied load

The last card in the deck is an alphanumeric STOP or END left justified on an input card in columns 1 through 4. STOP indicates that the job is complete and END indicates another problem deck follows.

Some general rules have been used in designing the input. These rules are listed below:

- Some of the data sections make use of a "keyword" of up to four characters, left justified in their appropriate field to specify an item of data. For example, cards specifying element connectivity for membrane triangles begin with TRIM, and material properties by MAT1. All sections that make use of keywords end with a section end keyword SEND.

- Sections that specify data without keywords end with a blank card section delimiter.

- The data deck ends with one of two keywords. If END is used, the program reads a new problem data deck. If STOP is used, as will probably be the case for most problems, the job ends.

- Two formats for input are used, E15.7 for floating point input (fields of 15) and I5 for integer or fixed point input (fields of 5). The fixed point
(integer) data must be right justified. The floating point data can be written in several forms. For example, 10.0 can be input as:

- 10.0 any place in the field, or
- 1.0E+01, 1.0E+1, 1.0E1, where the field is right justified

There are a number of places in the program where applicable node points or elements must be specified with a set of data. In these cases the nodes or members are specified by entering the appropriate number on the input cards in fields of five. However, for this purpose the user can also utilize a shorthand form of the input. That is, specifying \( m \) and \(-n\) consecutively is the equivalent of the specification of nodes or elements \( m, m+1, m+2, \ldots, n \) and specifying \( m, -p, \) and \(-n\) consecutively is the equivalent of the specification of nodes (elements) \( m, m+p, m+2p, \ldots, m+kp \) where \( m+kp \) is the highest integer of the form less than or equal to \( n \). For example, the specification of nodes 1 through 100 is written as 1-100 and nodes 1, 3, 5, ..., 99 as 1, -2, -99. This card input appears in fields of 5 (I5 format) with 16 items per card. Any number of continuation cards may be used. A blank I5 field ends the specification.
INSTRUCTIONS FOR USE OF BEND

A Program for the Elastic-Plastic Analysis
of Built-up Structures
INSTRUCTIONS FOR USE OF BEND

BEND is a finite element program to treat the elastic, elastic-plastic or elastic-cyclic plastic response of arbitrary built-up thin walled structures where bending and membrane effects are equally important. The finite element library consists of the following elements:

- Three-node uniform strain triangle
- Six-node linearly varying strain triangle
- Four- and five-node hybrid triangles to be used as transition elements between three- and six-node elements (see Fig. 1a)
- Two-node uniform strain stringer
- Three-node linearly varying strain stringer
- Beam with various cross sections subjected to bending about two planes as well as torsion
- Higher order triangular plate element with bending and membrane capability

The program is capable of treating the elastic and elastic-ideally plastic, linear strain hardening, and nonlinear strain hardening behavior of orthotropic materials. Further, the kinematic hardening theory of plasticity is used (Refs. 1-3) so that provision for cyclic loading conditions involving reversed plastic deformation is included.

The input to the program is categorized in the following sections:

I. Problem Title FORMAT (20A4)
   Any 80-character title describing the problem.

II. NPNTC, NPRNT, IRESRT, NUTAP, INPRT FORMAT (515)
   $0 \leq \text{NPNTC} \leq 63$:
NPNTC is the sum of the following integers corresponding to the option desired.

If NPNTC = 0  No intermediate printout
    = 1  Print the load vector
    = 2  Print element stiffness matrix
    = 16 Print each element stiffness matrix entry
to be stacked with its stacking index
    = 32 Print the total stiffness matrix

For example, if it is desired to print the load vector and the total stiffness matrix, NPNTC = 1 + 32 = 33.

NPRNT:

If < 0, perform elastic analysis only
If > 0, perform plastic analysis, printing output every NPRNT increments of load

IRESRT:  (See Section XVIII.)

0, Elastic (and/or) plastic run. Do not generate a restart tape
1, Elastic (and/or) plastic (cyclic) run. Build a new restart tape
2, Plastic run with elastic values from previously created restart tape
3, Plastic restart run starting at some specified load level
4, Cyclic restart run

NUTAP:

Applicable only if restarting from a restart tape

0, No new tape written
1, Complete new tape created and additional restart data written
INPRT:

0, Write restart tape only at $P = P_{\text{MAX}}$ (i.e., at maximum load)

N, Write restart tape every $N$ increments of load

III. Node Specification (1615)

This section defines an allowable set of external node point numbers. The maximum node number that can be used is 9999. The program uses this information in two ways. First to set up a table of allowable node points that is used to check all subsequent node point input. Secondly, the program converts each external node number to an internal number consecutively in the order that the node appears on the input card. Consequently the order of the input of external node numbers is completely arbitrary and need not be increasing monotonically. In practice the node numbers should be numbered so as to minimize the bandwidth. Once the input is read the program operates with the internal numbers which are now numbered from 1 through the number of nodes in the model. In this manner the node ordering and therefore the bandwidth of the stiffness matrix can be easily changed and nodes can be inserted or deleted by changing the external node specification.

The input is specified by entering the appropriate number on the input cards in fields of five. However, for this purpose the user can also utilize a shorthand form of the input. That is, specifying $m$ and $-n$ consecutively is the equivalent of the specification of nodes $m, m+1, m+2, \ldots, n$ and specifying $m, -p,$ and $-n$ consecutively is the equivalent of the specification of nodes $m, m+p, m+2p, \ldots, m+kp$ where $m+kp$ is the highest integer of the form less than or equal to $n$. For example, the specification of nodes 1 through 100 is written as 1-100 and 1, 3, 5, \ldots, 99 as 1-2-99. This card input appears in fields of 15 (15 Format) with 16 items per card. Any number of continuation cards may be used. A zero or blank 15 field ends the specification.
IV. Member Connectivity (Node Numbers of Each Member)

The first alphanumeric field defines the element type:

TRIM - Triangular membrane element (Ref. 3)
BEAM - Beam element
STRG - Stringer element (Ref. 4)
TRIP - Triangular plate bending element (with membrane effects) (Ref. 5)

The first integer field designates the member number. The next integer fields designate the connecting nodes as follows:

TRIM - The nodes for the triangular family of elements are specified around the perimeter beginning with a major (vertex) node, and followed by a minor (midside) node and then alternatively major and minor as shown in Fig. 1. The absence of a minor node must be indicated by a zero or blank field in the proper position.

BEAM - Three node specifications are necessary for the beam element. Nodes i and j (Fig. 2) which designate the element end points and a third node k, defining the normal to the beam axis about which the section properties are specified. This additional node may be a node of the structural idealization or it may be a "fictitious node" specified just for the purpose of defining the beam section properties. This is shown in Fig. 2.

STRG - Three node specifications are necessary for the stringer element. Nodes i and j, connecting the end points and if desired an additional node designating a midpoint node. A zero or blank for the midpoint node specifies a two-node stringer. The midside node is the third node specified.
Fig. 1a Triangular Family of Finite Elements Used

Fig. 1b Elements Topology
Fig. 2 Beam Element

Fig. 3 Stringer Element

Fig. 4 Triangular Plate Element - Bending and Membrane
TRIP - Three nodes are used to specify the plate element L, j, k as shown in Fig. 4.

If a planar structure is being analyzed using triangular plate and membrane elements, stress and strain output can be calculated with respect to the global coordinate system rather than the element local system by specifying one of the following cards:

- **GLXY** - Structure in the x-y plane
- **GLXZ** - Structure in the x-z plane
- **GLYZ** - Structure in the y-z plane

SEND - Ends the section.

V. **X-Coordinates of Nodes** FORMAT (E15.7, 13I5)

The x-coordinates of the nodes appearing in the I5 fields are set to the value in the E15.7 field. Any number of continuation cards may be used; their first fifteen columns are ignored. A zero or blank I5 field terminates the card scan for a given x-coordinate. A zero or blank first I5 field (columns 16-20) on a noncontinuation card ends the section. Both shorthand representations of Section III are allowed.

VI. **Y-Coordinates of the Nodes. Same as Section V.**

VII. **Z-Coordinates of the Nodes. Same as Section V.**
VIII. Boundary Conditions FORMAT (12I1, 3X, 13I5, /, 15X, 13I5)

The first twelve fields give the boundary conditions specifications in the order: \( u, v, w, \theta_x, \theta_y, \theta_z, \epsilon_{x'}, \epsilon_{y'}, \epsilon_{xy}, \kappa_x, \kappa_y, \kappa_{xy} \); where \( u, v, w \) are global displacements in the \( x, y, z \) directions, respectively; \( \theta_x, \theta_y, \theta_z \) are global rotations with respect to the \( x, y, z \) axes; \( \epsilon_{x'}, \epsilon_{y'}, \epsilon_{xy} \) are membrane components of the local membrane strain tensor; \( \kappa_x, \kappa_y, \kappa_{xy} \) are components of the local curvature tensor. The last six fields for the local membrane strains and curvatures (higher order degrees of freedom) are only used if a plate element is attached to the node. Both tensors are calculated in the local coordinate system (see Fig. 4) of the plate element. This local coordinate system is defined by the first plate element connecting that node that appears in the input stream. For example,

\[
\begin{align*}
\text{TRIP} & \quad 3 & 6 & 3 & 2 \\
\text{TRIP} & \quad 4 & 3 & 6 & 7 \\
\text{TRIP} & \quad 5 & 7 & 4 & 3 \\
\end{align*}
\]

Node 3 is not listed previously in the element input

The tensor quantities at node 3 are all defined in the local coordinate system of element 3. See Appendix IV for examples of plate boundary conditions.

The rotational degrees of freedom \( \theta_x, \theta_y, \theta_z \) are applicable only for beam and plate elements. If nonzero values are given to rotational or higher order degrees of freedom at a node and beam and/or plate elements are not attached to the node then a singular stiffness matrix will result.
Zero denotes a fixed degree of freedom
One denotes a free degree of freedom
Two will result in the application of a unit generalized displacement, or a corresponding card may be included in Section IX (dependent degrees of freedom) designating the magnitude of the applied generalized displacement.

The 13I5 fields give the applicable nodes for the boundary condition specifications, with both shorthand notations of Section III permitted. Any number of continuation cards may be used for a given specification. However, only the 13I5 fields are used on a continuation card. A zero or blank I5 field terminates the card scan for a given boundary condition specification. Note: If the last field of a card (columns 76-80) is the last specification, an additional blank card (continuation card) must follow. A zero or blank first I5 field (columns 16-20) on a noncontinuation card ends the section. If a node's boundary conditions are not specified in this section, all the degrees of freedom are assumed to be free. To change this default condition, the first card of this section should be set to the desired default (i.e., in the absence of beams and plates, 111000000000) with all nodes used in the problem specified. In the absence of plate elements the default is automatically 111111000000.

IX. Dependent Degrees of Freedom FORMAT (2I5, 2(5X, 2I5, E15.7))

This section designates the input for both single and multipoint constraints as well as applied displacements of the form:

1) \[ \delta_i = \alpha_1 \delta_{j1} + \alpha_2 \delta_{j2} + \cdots + \alpha_n \delta_{jn} \]

where \( \delta_i \) is a dependent degree of freedom,
\( \delta_{j1} \cdots \delta_{jn} \) are independent degrees of freedom,
and \( \alpha_1, \alpha_2 \cdots \alpha_n \) are coefficients.
2) Rotation of displacements at a node

\[ \delta_i = \alpha_1 \tilde{\delta}_i + \alpha_2 \tilde{\delta}_j + \alpha_3 \tilde{\delta}_k \]

\[ \delta_j = \beta_1 \tilde{\delta}_i + \beta_2 \tilde{\delta}_j + \beta_3 \tilde{\delta}_k \]

\[ \delta_k = \gamma_1 \tilde{\delta}_i + \gamma_2 \tilde{\delta}_j + \gamma_3 \tilde{\delta}_k \]

where the \( \alpha, \beta, \gamma \)'s are the direction cosines of the rotation, \( \tilde{\delta}_i, \tilde{\delta}_j, \tilde{\delta}_k \) are the displacements with respect to the original global directions and \( \tilde{\delta}_i, \tilde{\delta}_j, \tilde{\delta}_k \) are the components of displacements at the node with respect to the new coordinate axes. An example of this capability is given in Appendix I.

3) Applied generalized displacement

\[ \delta_i = \alpha_1 \]

where the coefficient \( \alpha_1 \) is the applied generalized displacement.

The first two fields designate a node number and a degree of freedom (i.e., 1-12). The dependency is defined in the following three fields. The two integer fields designate the node number and degree of freedom number and the coefficient by the floating point field. If there is another dependency for the node, it is designated in a similar fashion in the next three fields. Any number of continuation cards can be used with the first two fields blank. The section is ended by a blank or zero in the third integer field (blank card). Examples of the use of multipoint constraints are in Appendix I.
X. Material and Section Properties

The format for this input is dependent upon the member type. Each type of input begins with a word of up to four characters.

MAT1 Material properties for plane stress, necessary with membrane triangles (TRIM) and bending triangles (TRIP). For use with initially isotropic materials which exhibit perfectly plastic or strain hardening behavior and initially orthotropic materials that exhibit perfectly plastic behavior.

MAT2 Material properties for plane stress, necessary with membrane triangles (TRIM) and bending triangles (TRIP) with initially isotropic or orthotropic behavior that require orthotropic kinematic hardening theory.

MBM Material properties for a beam element

MSTG Material and section properties for a stringer

THIK Member thickness for triangular membrane (TRIM) and plate (TRIP) elements

MBET Angle between local axes and principal directions of orthotropy for TRIM and TRIP elements (see Figs. 1b and 4)

NLRS Layer and layer output information for TRIP elements

SREC Beam section properties for a solid rectangular section
<table>
<thead>
<tr>
<th>Code</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>SCIR</td>
<td>Beam section properties for a solid circular section</td>
</tr>
<tr>
<td>ZSEC</td>
<td>Beam section properties for a Z-section</td>
</tr>
<tr>
<td>ISEC</td>
<td>Beam section properties for an I-section</td>
</tr>
<tr>
<td>HCIR</td>
<td>Beam section properties for a hollow circular section</td>
</tr>
<tr>
<td>HREC</td>
<td>Beam section properties for a hollow rectangular section</td>
</tr>
<tr>
<td>LSEC</td>
<td>Beam section properties for an L-section</td>
</tr>
<tr>
<td>TSEC</td>
<td>Beam section properties for a T-section</td>
</tr>
<tr>
<td>CSEC</td>
<td>Beam section properties for a Channel section</td>
</tr>
<tr>
<td>SEND</td>
<td>Ends the section</td>
</tr>
</tbody>
</table>
MAT1 - Plane Stress Material Properties - FORMAT (A4, 1X, 5E15.7, /, 2E15.7, /, 4E15.7, /, 5E15.7, /, (1615))

The first four cards specify material properties, as follows:

CARD 1: MAT1

EONE = Young's modulus in principal property axis (1)
ETWO = Young's modulus in principal property axis (2)
BETA = No longer used. Set equal to zero.
GONTO = Shear modulus in (1)-(2) principal property plane
VONTO = Poisson's ratio, $v_{12}$

Note: $\gamma_1 = \frac{\sigma_1}{E_1} - \frac{v_{12}}{E_2} \frac{\sigma_2}{E_2}$

$\gamma_2 = \frac{v_{21}}{E_1} \sigma_1 + \frac{\sigma_2}{E_2} \sigma_2$

$\gamma_{12} = \frac{E_{12}}{G_{12}}$

CARD 2: TALF-1 = Coefficient of thermal expansion in 1-axis direction, $\alpha_1$
TALF-2 = Coefficient of thermal expansion in 2-axis direction, $\alpha_2$

CARD 3: SIGOX = Yield stress in principal 1-direction
SIGOY = Yield stress in principal 2-direction
SIGOZ = Yield stress in principal 3-direction
SIGXY = Shear yield stress in principal 1-2 plane
CARD 4:  RMOSE  =  If RMOSS ≠ 0;  RMOSS = a, the Ramberg-Osgood parameter used in Ramberg-Osgood representation of stress-strain behavior
If RMOSS = 0;  RMOSS = \overline{a}  the slope of the linear strain-hardening stress-strain representation, i.e., \overline{a} = E_T/E where E_T is the tangent modulus

RMOSS =  If RMOSS ≠ 0;  RMOSS = Ramberg-Osgood parameter a_{0.7} (see Ref. 7)

Note 1: If RMOSE = 0 and RMOSS = 0, the material for the element(s) is assumed to be elastic-ideally plastic

RMOSE = Ramberg-Osgood parameter E (Young's modulus)
YLDST = Yield stress in tension
YLDSC = Yield stress in compression

Input for the yield stress in tension and compression has been maintained in order to accommodate materials that exhibit initial anisotropic plastic behavior. In this case an initial translation of the yield surface is made consistent with the kinematic hardening theory.

Note: Only initially isotropic materials can be treated when considering linear or nonlinear strain hardening using this property card.

Succeeding cards give applicable members; both shorthands of Section III are permitted. Any number of continuation cards may be used for a given specification. A zero or blank I5 field ends each member listing.

MAT2 - Plane Stress Material Properties - FORMAT (A4, 1X, 5E15.7, /,
2E15.7, /, 4E15.7, /,
5E15.7, /, E15.7, /,
(1615))

The first five cards specify material properties, as follows:

CARD 1: MAT2

EONE = Young's modulus in principal property axis (1)
ETWO = Young's modulus in principal property axis (2)
BETA = No longer used. Set equal to zero.
CONTO = Shear modulus in (1)-(2) principal property plane
VONTO = Poisson's ratio, \( \nu_{12} \)

Note:
\[
\varepsilon_1 = \frac{\sigma_1}{E_1} - \frac{\nu_{12}}{E_2} \sigma_2
\]
\[
\varepsilon_2 = -\frac{\nu_{21}}{E_1} \sigma_1 + \frac{\sigma_2}{E_2}
\]
\[
\gamma_{12} = \frac{1}{G_{12}} \tau_{12}
\]

CARD 2: TALF-1 = Coefficient of thermal expansion in 1-axis direction, \( \alpha_1 \)

TALF-2 = Coefficient of thermal expansion in 2-axis direction, \( \alpha_2 \)

CARD 3: SIGOX = Yield stress in principal 1-direction
SIGOY = Yield stress in principal 2-direction
SIGOZ = Yield stress in principal 3-direction
SIGXY = Shear yield stress in principal 1-2 plane
CARD 4: \text{RMONX} \equiv \text{If } \text{RMOSX} \neq 0; \text{RMONX} = n, \text{ the shape parameter used in the Ramberg-Osgood representation of the } \sigma_1 - \varepsilon_1 \text{ component of the stress-strain behavior.}

\text{If } \text{RMOSX} = 0; \text{RMONX} = \overline{\sigma}_1 \text{ the slope of the linear strain-hardening stress-strain representation in the } 1\text{-direction, i.e., } \overline{\sigma}_1 = \frac{E_{T1}}{E_1} \text{ where } E_{T1} \text{ is the tangent modulus of the } \sigma_1 - \varepsilon_1 \text{ curve.}

\text{RMOSX} \text{  If } \text{RMOSX} \neq 0; \text{RMOSX} = \text{Ramberg-Osgood parameter } \sigma_{0.7} \text{ in principal } 1\text{-direction}

\text{RMONY} \text{  Same as RMONX in principal 2-direction}

\text{RMOSY} \text{  Same as RMOSX in principal 2-direction}

\text{RMONXY} \text{  Same as RMONX for shear component in 1-2 plane; } \tau_{12} - \gamma_{12} \text{ stress-strain curve}

CARD 5: \text{RMOSXY} \text{  Same as RMOSX for shear component in 1-2 plane; } \tau_{12} - \gamma_{12} \text{ stress-strain curve}

Note: \text{No component of the stress-strain curves may be ideally plastic, i.e., RMONX or RMONY or RMONXY = 0 is not allowed. If either is zero all must be zero and a MAT1 card should be used.}

The Ramberg-Osgood representation of the stress-strain curve is given by

2-16
\[ \sigma = \frac{\sigma}{7E} + \frac{3\sigma}{7E} \left( \frac{\sigma}{\sigma_{0.7}} \right)^{n-1} \]

\[ \gamma = \frac{\tau}{G} + \frac{3\tau}{7G} \left( \frac{\tau}{\tau_{0.7}} \right)^{n-1} \]

Succeeding cards give applicable members; both shorthands of Section III are permitted. Any number of continuation cards may be used for a given specification. A zero or blank 15 field ends each member listing.
MBM - Beam Material Properties - FORMAT (A4, 1X, 5E15.7, /, E15.7, /, (16I5))

CARD 1:  MBM

E = Young's modulus
ANU = Poisson's ratio
RM OSS = If not equal to zero, RMOSS equals Ramberg-Osgood parameter, \( \sigma_{0.7} \)
RM OSN = If RMOSS \( \neq 0 \); RMOSN = n, the shape parameter used in Ramberg-Osgood representation of stress-strain behavior. If RMOSS = 0; RMOSN = \( \bar{\alpha} \), the slope of the linear strain-hardening stress-strain representation, i.e., \( \bar{\alpha} = E_T / E \) where \( E_T \) is the tangent modulus
YLDST = Yield stress

CARD 2:  TALF = Coefficient of thermal expansion

Succeeding cards give applicable members; both shorthands of Section III are permitted. Any number of continuation cards may be used for a given specification. A zero or blank I5 field ends each member listing.
MSTG - Stringer Properties - FORMAT (A4, 1X, 5E15.7, / E15.7 /, (16I5))

CARD 1:  MSTG

E = Young's modulus
A = Cross-sectional area
RMOSN = If RMOSS ≠ 0; RMOSN = n, the shape parameter used in Ramberg-Osgood representation of stress-strain behavior.
   If RMOSS = 0; RMOSN = α, the slope of the linear strain-hardening stress-strain representation, i.e., \( \bar{\alpha} = \frac{E_T}{E} \) where \( E_T \) is the tangent modulus

RMOSS = If RMOSS ≠ 0; RMOSS = Ramberg-Osgood parameter \( \alpha_{0.7} \)

Note 1: If RMOSN = 0 and RMOSS = 0, the material for the element(s) is assumed to be elastic-ideally plastic

YLDST = Yield stress

CARD 2:  TALF = Coefficient of thermal expansion

Succeeding cards give applicable members; both shorthands of Section III are permitted. Any number of continuation cards may be used for a given specification. A zero or blank I5 field ends each member listing.
THIK - Element Thicknesses - FORMAT (A4, 1X, E15.7, /, (16I5))
(Necessary with membrane elements TRIM) and plate elements (TRIP))

CARD 1: THIK
THICK = Element thickness
Succeeding cards give applicable members; both shorthands of Section III are permitted. Any number of continuation cards may be used for a given specification. A zero or blank 15 field ends each member listing.

MBET - Orientation of Axes of Material Anisotropy - FORMAT (A4, 1X, E15.7, /, (16I5))

CARD 1: MBET
BETF = Angle $\beta$ in degrees between local x-axis and principal 1-axis for material orthotropy. See Figs. 1b and 4. Only applicable for TRIM and TRIP elements

Note: This card is an optional card. The default is BETF = 0.0 for all members. It should be used with initially orthotropic materials or those exhibiting orthotropic strain-hardening characteristics.

Succeeding cards give applicable members; both shorthands of Section III are permitted. Any number of continuation cards may be used for a given specification. A zero or blank 15 field ends each member listing.
NLRS - Layer and Print Option for TRIP Elements - FORMAT (A4, 1X, /, 2I5)

CARD 1: NLRS

CARD 2: NLRS = Number of layers through the thickness at which stresses are calculated for TRIP elements. NLRS must be an even number \( \leq 20 \)

NPLT = Print out option:
- = 0; Print stresses, strains, etc. only at top and bottom surfaces
- = 1; Print stresses, strains, etc. at each layer through the thickness as well.

Note: This is an optional card. The defaults are NLRS = 10 and NPLT = 0. If one is changed both should be specified. This card controls option for all TRIP elements.
Beam Section Properties - FORMAT (A4, 1X, 5E15.7, /, 3E15.7, /, formats for Cards 3 and 4 (see below), /, (1615))

The cards specifying beam section properties all start with the following information:

CARD 1: 
SREC 
SCIR 
ISEC 
ZSEC Defined at the beginning of the section. 
HCIR The different cross sections are shown 
HREC in Tables 1 and 2. 
LSEC 
TSEC 
CSEC

A = cross-sectional area 
Iyy = moment of inertia about y axis (see Fig. 5) 
Izz = moment of inertia about z axis (see Fig. 5) 
Iyz = product of inertia 
J = torsional rigidity

If the quantities, A, Iyy, Izz, Iyz are input as zero or blank then these quantities are calculated automatically.

Fig. 5 Definition of Coordinate Axes in Cross Section of Beam Element

2-22
CARD 2:  \( Y_0 = \) Eccentricity of attachment point in the \( y_o \) direction
\( Z_0 = \) Eccentricity of attachment point in the \( z_o \) direction
\( \beta = \) angle defining the transformation of the \( I_y, I_z, Iyz \) to another reference axis (see Fig. 5)

Additional cards are required according to which section is specified. The notation for each section is shown in Table 1.

SREC - FORMAT \((2E15.7)\)
CARD 3:  \( A = \) Width
\( B = \) Depth

SCIR - FORMAT \((E15.7)\)
CARD 3:  \( R = \) Radius

ZSEC, ISEC, and CSEC - FORMAT \((3E15.7, /, 3E15.7)\)
CARD 3:  \( A_1 = \) Dimension of upper flange (parallel to \( y \)-axis)
\( A_2 = \) Dimension of web (parallel to \( z \)-axis)
\( A_3 = \) Dimension of lower flange (parallel to \( y \)-axis)

CARD 4:  \( T_1 = \) Thickness of upper flange
\( T_2 = \) Thickness of web
\( T_3 = \) Thickness of lower flange

HCIR - FORMAT \((2E15.7)\)
CARD 3:  \( R = \) Outer radius
\( T = \) Thickness
HREC, LSEC, and TSEC - FORMAT (4E15.7)

CARD 3:  

A1 = Width (parallel to y-axis)  
A2 = Depth (parallel to z-axis)  
T1 = Thickness of upper and/or lower flanges  
T2 = Thickness of vertical webs

Succeeding cards give applicable members; both shorthands of Section III are permitted. Any number of continuation cards may be used for a given specification. A zero or blank 15 field ends each member listing.

SEND Ends the section.
<table>
<thead>
<tr>
<th>Section</th>
<th>Keyword</th>
<th>Input</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solid Rectangle</td>
<td>SREC</td>
<td>Area, moments of inertia, $I_{yy}$, $I_{zz}$, $I_{yz}$, $J$ eccentricities, $y_0$, $z_0$; transformation angle $\beta$. Section width and height, a, b.</td>
</tr>
<tr>
<td>Solid Circular</td>
<td>SCIR</td>
<td>Area, moments of inertia, $I_{yy}$, $I_{zz}$, $I_{yz}$, $J$ eccentricities, $y_0$, $z_0$; transformation angle $\beta$. Section radius, r.</td>
</tr>
<tr>
<td>Hollow Circular Section</td>
<td>HCIR</td>
<td>Area, moment of inertia, $I_{yy}$, $I_{zz}$, $I_{yz}$, $J$ eccentricities, $y_0$, $z_0$; transformation angle $\beta$. Outer radius, r, thickness, t.</td>
</tr>
<tr>
<td>Section</td>
<td>Keyword</td>
<td>Input</td>
</tr>
<tr>
<td>-------------------</td>
<td>---------</td>
<td>-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>Hollow Rectangular Section</td>
<td>HREC</td>
<td>Area, moment of inertia, $I_{yy}$, $I_{zz}$, $I_{yz}$, J eccentricities, $y_o$, $z_o$, transformation angle $\beta$. Width, $a_1$, depth, $a_2$, thickness of upper and lower flanges, $t_1$, thickness of vertical webs, $t_2$</td>
</tr>
<tr>
<td>L-Section</td>
<td>LSEC</td>
<td>Area, moment of inertia, $I_{yy}$, $I_{zz}$, $I_{yz}$, J eccentricities, $y_o$, $z_o$, transformation angle $\beta$. Dimension of flange and web $a_1$, $a_2$, thickness flange, and web, $t_1$, $t_2$.</td>
</tr>
<tr>
<td>T-Section</td>
<td>TSEC</td>
<td>Same as for L-Section</td>
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<tr>
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<td>-----------------------------------------------------------------------</td>
</tr>
<tr>
<td>Z-Section</td>
<td>ZSEC</td>
<td>Area, moments of inertia, $I_{yy}$, $I_{zz}$, $I_{yz}$, $J$ eccentricities, $y_o$, $z_o$, transformation angle $\beta$. Dimension of upper and lower flange, $a_1$, $a_3$. Dimension of web, $a_2$. Thickness of upper and lower flange, $t_1$, $t_3$. Thickness of web, $t_2$. <strong>NOTE:</strong> $a_1 = a_3$; $t_1 = t_3$.</td>
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<tr>
<td>I-Section</td>
<td>ISEC</td>
<td>Same as Z-section</td>
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<td>$a_1 \neq a_3$; $t_1 \neq t_3$</td>
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<tr>
<td>Channel Section</td>
<td>CSEC</td>
<td>Same as Z-Section</td>
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<td>$a_1 = a_3$; $t_1 = t_3$</td>
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<tr>
<td>Section</td>
<td>Keyword</td>
<td>Number Stress Points</td>
</tr>
<tr>
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Shear Center at Centroid
<table>
<thead>
<tr>
<th>Section</th>
<th>Keyword</th>
<th>Number Stress Points</th>
<th>Location of Stress Points</th>
<th>Location of Stress Points</th>
</tr>
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<tbody>
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<td>Hollow</td>
<td>HCIR</td>
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<tr>
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<td>2 a 7.2</td>
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<td>3 a 323.4</td>
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<td>4 a 36.6</td>
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</tr>
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<td></td>
<td>5 a 274.6</td>
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<td>6 a 85.4</td>
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<td>7 a 213.0</td>
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<td>8 a 147.0</td>
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<tr>
<td>Hollow</td>
<td>HREC</td>
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<td>3 $0$ $0.5a_2$</td>
<td></td>
</tr>
<tr>
<td></td>
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<td>7 $0.5a_1$ $0.77459a_2$</td>
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</tr>
<tr>
<td></td>
<td></td>
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<td>9 $0.5a_1$ 0</td>
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</table>

\[ a_1 = a_1 - t_2 \]
\[ a_2 = a_2 - t_1 \]
<table>
<thead>
<tr>
<th>Section</th>
<th>Keyword</th>
<th>Number Stress Points</th>
<th>Location of Stress Points</th>
<th>Location of Stress Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>L-Section</td>
<td>LSEC</td>
<td>6</td>
<td>$y \quad z$</td>
<td>$\bar{a}_1 = a_1 - \frac{t_2}{2}, ; \bar{a}_2 = a_2 - \frac{t_1}{2}$</td>
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<td>0.11271$a_1$</td>
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<td>0.0</td>
<td>0.50000$a_1$</td>
<td></td>
</tr>
<tr>
<td>T-Section</td>
<td>TSEC</td>
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<td>$y \quad z$</td>
<td>$\bar{a}_2 = a_2 - \frac{t_1}{2}$</td>
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<td>I</td>
<td>1</td>
<td>0.77459$a_1$</td>
<td>0.0</td>
<td>Axes at Shear Center</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>-0.77459$a_1$</td>
<td>0.0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.0</td>
<td>0.0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>0.0</td>
<td>0.88730$a_2 + \frac{t_1}{2}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>0.0</td>
<td>0.11271$a_2 + \frac{t_1}{2}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>0.0</td>
<td>0.50000$a_2 + \frac{t_1}{2}$</td>
<td></td>
</tr>
</tbody>
</table>
### TABLE 2  NUMBER AND LOCATION OF STRESS POINTS (CONT)

<table>
<thead>
<tr>
<th>Section</th>
<th>Keyword</th>
<th>Number of Stress Points</th>
<th>Location of Stress Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>Z-Section</td>
<td>ZSEC</td>
<td>9</td>
<td>y $a_2$, z $a_2$</td>
</tr>
<tr>
<td>1</td>
<td>-0.88730$a_1$</td>
<td>2</td>
<td>-0.11271$a_1$</td>
</tr>
<tr>
<td>3</td>
<td>-0.5000$a_1$</td>
<td>4</td>
<td>0.88730$a_1$</td>
</tr>
<tr>
<td>5</td>
<td>0.11271$a_1$</td>
<td>6</td>
<td>0.5000$a_1$</td>
</tr>
<tr>
<td>7</td>
<td>0.0</td>
<td>0.77459$a_2^{'}$</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>0.0</td>
<td>0.77459$a_2^{'}$</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>0.0</td>
<td>0.0</td>
<td></td>
</tr>
</tbody>
</table>

$a_2 = (a_2 - t_3) / 2$, $t_1 = t_3$

$a_2^{'} = (a_2 - t_1)$, $a_1 = a_3$

Shear Center at Centroid
### Table 2: Number and Location of Stress Points (Cont)

<table>
<thead>
<tr>
<th>Section</th>
<th>Keyword</th>
<th>Number Stress Points</th>
<th>Location of Stress Points</th>
<th>Location of Stress Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>Channel</td>
<td>CSEC</td>
<td>9</td>
<td>y</td>
<td>z</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1 0.88730(a_1) + e_s (0.5a_2)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>2 0.11271a_1 + e_s (0.5a_2)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>3 0.5000a_1 + e_s (0.5a_2)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>4 0.88730a_1 + e_s (-0.5a_2)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>5 0.11271a_1 + e_s (-0.5a_2)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>6 0.5000a_1 + e_s (-0.5a_2)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>7 e_s (0.77459a_2)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>8 e_s (-0.77459a_2)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>9 e_s (0.0)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\(e_s\) - distance from shear center to web center line

\[
e_s = \frac{1/2(a_1 - t_2)}{1 + 1/6\left(\frac{t_2(a_2 - t_1)}{t_1(a_1 - t_2)}\right)}\]

\(a_2 = a_2 - t_1\)

**NOTE:** \(t_1 = t_3\) ; \(a_1 = a_3\)
### TABLE 2 NUMBER AND LOCATION OF STRESS POINTS (CONT)

<table>
<thead>
<tr>
<th>Section</th>
<th>Keyword</th>
<th>Number of Stress Points</th>
<th>Location of Stress Points</th>
<th>Location of Stress Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>I-Section</td>
<td>ISEC</td>
<td>9</td>
<td>( y )</td>
<td>( z )</td>
</tr>
<tr>
<td>1</td>
<td>( 0.774569a_1 )</td>
<td>( b_1 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>(-0.77459a_1)</td>
<td>( b_1 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.0</td>
<td>( b_1 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>( 0.77459a_3 )</td>
<td>( b_2 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>( 0.77459a_3 )</td>
<td>( b_2 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>0.0</td>
<td>( b_2 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>0.0</td>
<td>( 0.8873b_1 + 0.11271b_2 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>0.0</td>
<td>( 0.11271b_1 + 0.8873b_2 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>0.0</td>
<td>( 0.5b_1 + 0.5b_2 )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[
\begin{align*}
\bar{a}_2 &= a_2 - 0.5(t_1 + t_3) \\
\bar{b}_1 &= b_1 - 0.5t_1 \\
\bar{b}_2 &= b_2 - 0.5t_3
\end{align*}
\]

Axes at Shear Center
XI. Applied Loads

Many different types of loading are currently admissible – concentrated loads at nodes, distributed loads on an edge of a triangular member, distributed surface loads, distributed line loads on beams, and thermal loads. Each loading situation is designated by one of the following four-character key words:

- CONC (Concentrated force or moment at a node)
- TRIA (Distributed edge load, TRIM elements)
- SURF (Distributed surface loads, TRIP element)
- EDG1 (Distributed edge load, side 1, TRIP element)
- EDG2 (Distributed edge load, side 2, TRIP element)
- EDG3 (Distributed edge load, side 3, TRIP element)
- BMLO (Distributed line load, beam element)
- TMPU (Temperatures at upper surface for TRIP element or temperatures for TRIM element)
- TMPM (Temperatures at middle surface for TRIP element)
- TMPL (Temperatures at lower surface for TRIP element)
- SEND (Section end)

Note: The keywords for the section on applied loads are used as section headings. The keyword appearing on an input card designates that the input to follow is associated with a particular type of applied loading. A blank card (as described in succeeding sections) delimits the input and indicates that the next card contains a different keyword. For example, CONC indicates input that follows is for concentrated loads and TRIA; distributed loads on an edge of a membrane triangle. Thus the input is as follows:
CONC
data
for
concentrated
loads
blank card

TRIA
data
for
distributed
loads
'blank card

SEND

It should be noted that the keywords can appear in any order and may be specified more than one time in the applied load section.

TRIA - Distributed edge load (TRIM elements) - FORMAT (A4, /, 4I5, 3E15.7, /, 15X, 3E15.7)

Each card gives the load components applied at a member side (in units of force) as follows:

CARD 1: TRIA

CARD 2: First I5 field: Number of node (m)
Second I5 field: Number of minor node on the side, zero, or blank if there is no minor node
Third I5 field: 
Fourth I5 field: 
First E15.7 field: 
Second E15.7 field: 
Third E15.7

CARD 3: First E15.7 field: 
Second E15.7 field: 
Third E15.7 field:

Number of other end point node (n)
NODR - Reference node number
x load component at node m
y load component at node m
z load component at node m
x load component at node n
y load component at node n
z load component at node n

Note: If NODR = 0 then x, y, z are global directions.
If NODR ≠ 0 then the x-direction is parallel to the element edge specified by m - n; the y-direction is perpendicular to x in the plane defined by the three points, m, n, NODR; the z-axis is perpendicular to the x-y plane.

The program allows for a linearly varying distributed load from node to node. As many load components as desired may be specified. A blank card (i.e., zero or blank first I5 field) ends the specification of distributed edge loads for TRIM elements.
CONC - Concentrated loads - FORMAT - (A4, /, I5, 3E15.7, /, 5X, 3E15.7)

Each card gives the load components in global directions at a specified node.

CARD 1: CONC

CARD 2: I5 field: Node number
First E15.7 field: Force component $F_x$
Second E15.7 field: Force component $F_y$
Third E15.7 field: Force component $F_z$

CARD 3: First E15.7 field Moment component $M_x$
Second E15.7 field Moment component $M_y$
Third E15.7 field Moment component $M_z$

A blank card (i.e., zero or blank first I5 field) ends the specification of concentrated loads.

SURF - Distributed Surface Loads (TRIP element) - FORMAT (A4, 1X, 3E15.7, /, 3E15.7, /, 3E15.7, /, 16I5))

CARD 1: SURF

PXI Value of distributed surface load in force/unit area in local element x-direction at node i of element (see Fig. 4).
PYI Value of distributed surface load in force/unit surface area in local y-direction at node i of element
PZI Value of distributed surface load in force/unit surface area in local z-direction at node i of element
CARD 2: PXJ  Same as PXI but at node  j  
PYJ  Same as PYI but at node  j  
PZJ  Same as PZI but at node  j  
CARD 3: PXK  Same as PXI but at node  k  
PYK  Same as PYI but at node  k  
PZK  Same as PZI but at node  k  

Note: A linear variation of the surface loads in the plane of the element is assumed.

Succeeding cards give applicable members; both shorthands of Section III are permitted. Any number of continuation cards may be used for a given specification. **A zero or blank I5 field ends each member listing.**

BMLO - Distributed Line Load, Beam Element - FORMAT (A4, /, 4E15.7, /, (16I5))

Card 1: BMLO
CARD 2: PYI  Force/unit length in local y-direction (see Fig. 5) at node i  
PYJ  Force/unit length in local y-direction at node j  
PZI  Force/unit length in local z-direction at node i  
PZJ  Force/unit length in local z-direction at node j  

Note: A linear variation of the distributed load between nodes is assumed.

Succeeding cards give applicable members; both shorthands of Section III are permitted. Any number of continuation cards may be used for a given specification. **A zero or blank I5 field ends each member listing. A blank card ends this section.**
EDGI - Distributed Edge Load; Side 1, TRIP Element - FORMAT (A4, 1X, 3E15.7, /, 3E15.7, /, (16I5))

This card inputs the distributed edge or line loads along side 1 of a TRIP element. Side 1 is the side connecting nodes i and j (see Fig. 4).

CARD 1: EDGI

PXI Local x-component of force/unit length at ith node
PYI Local y-component of force/unit length at ith node
PZI Local z-component of force/unit length at ith node

CARD 2: PXJ Same as PXI but at node j
PYJ Same as PYI but at node j
PZJ Same as PZI but at node j

Note: A linear variation of the edge loads is assumed between nodes i and j.

Succeeding cards give applicable members; both shorthands of Section III are permitted. Any number of continuation cards may be used for a given specification. A zero or blank 15 field ends each member listing.

EDG2 - Distributed Edge Load, Side 2, TRIP Element - FORMAT (A4, 1X, 3E15.7, /, 3E15.7, /, (16I5))

This section inputs the distributed edge or line loads along side 2 of a TRIP element. Side 2 is the side connecting nodes j and k (see Fig. 4). All input is the same as in the EDGI section with EDG2 replacing EDGI as the alphanemic clue word and "j" replacing "i" and "k" replacing "j" in the description.
EDG3 - Distributed Edge Load, Side 3. TRIP Element - FORMAT (A4, 1X, 3E15.7, /, 3E15.7, /, (16I5))

This section inputs the distributed edge or line loads along side 3 of a TRIP element. Side 3 is the side connecting nodes \( k \) and \( i \) (see Fig. 4). All input is the same as in the EDG1 section with EDG3 replacing EDG1 as the alphabetic clue word and "k" replacing "l" and "i" replacing "k" in the description.
TMPU - Nodal Temperatures At Upper Surface for TRIP Element or
Nodal Temperatures for TRIM Element - FORMAT (A4, /, E15.7,
1315, /, (15X, 1315))

This section is used to input temperatures at nodes of TRIM
elements and at the upper surface (i.e., \( z = + h/2 \), where \( h \) is
the plate thickness) of TRIP plate elements. For TRIP elements a
parabolic temperature distribution through the thickness is assumed.
To input temperatures at the lower surface and middle surface use
the TMPL and TMPM cards, respectively.

CARD 1: TMPU

CARD 2: E15.7 field
Temperature at node (TRIM)
or temperature at upper surface
of node (TRIP)

I5 Integer fields
Applicable nodes

The temperatures of the nodes appearing in the I5 fields are
set to the value in the E15.7 field. Any number of continuation
cards may be used; their first fifteen columns are ignored. A zero
or blank I5 field terminates the card scan for a given temperature.
A zero or blank first I5 field (columns 16-20) on a noncontinua-
tion card ends the TMPU input. Both shorthand representations of
Section III are allowed.

Note: For the TRIM element the
nodal temperatures are
averaged to obtain one
element temperature. For
the TRIP element a linear
in-plane variation of the
temperatures from node to
node is assumed.
TMPM - Nodal Temperatures at Middle Surface for TRIP Element -
FORMAT (A4, /, E15.7, 13I5, /, (15X, 13I5))

This section is used to input temperatures at nodes at the middle surface (z = 0) of TRIP plate elements. For TRIP elements a parabolic temperature distribution through the thickness is assumed. To input temperatures at the upper and lower surface use the TMPU and TMPL cards, respectively.

The input formats and information for the card are identical to the TMPU card with TMPM replacing TMPU.

TMPL - Nodal Temperatures at Lower Surface for TRIP Element -
FORMAT (A4, /, E15.7, 13I5, /, (15X, 13I5))

This section is used to input temperatures at nodes at the lower surface (z = - h/2, h is the plate thickness) of TRIP elements. For TRIP elements a parabolic temperature distribution through the thickness is assumed. To input temperatures at the upper and middle surface use the TMPU and TMPM cards, respectively.

The input formats and information for this card are identical to the TMPU card with TMPL replacing TMPU.

A card with SEND in the first four columns ends the section for applied loading.

XII. Members to be Printed - FORMAT (16I5)

Specifies the members whose strains and stresses are to be printed. Both shorthands of Section III are allowed. A blank card or card with only zero entries ends the section.
XIII. Nodes to Be Printed - FORMAT (1615)

Nodes whose displacements are to be printed as per Section XII.

XIV. Parameters for Plastic Analysis - FORMAT (2E15.7)

\[
P_{\text{MAX}} = \text{Maximum load to be applied for this half cycle}
\]

\[
P_{\text{PPCT}} = \text{Load increment as a decimal multiple of yield load.}
\]

XV. Parameters for Succeeding Load Cycles - FORMAT (I5; 2E15.7)

\[
N_{\text{PRNT}} = \text{If equal to zero, no additional load cycle and end of problem. If greater than zero, print output every } N_{\text{PRNT}} \text{ increments.}
\]

\[
P_{\text{MAX}} = \text{Maximum load for this load cycle}
\]

\[
P_{\text{PPCT}} = \text{Load increment as percentage of yield load.}
\]

Note: \(P_{\text{PPCT}}\) should be equal to one-half the original value since the plastic range is twice the elastic range for initial loading.

XVI. Change Plastic Material Properties for New Load Cycle

The input for this section begins with a four-character word as follows:

- **MAT1** New material properties for plane stress membrane triangles (TRIM) or plate (TRIP) elements (see Section X for applicability)
- **MAT2** New material properties for orthotropic strain-hardening materials (see Section X for applicability)
- **MBM** New material properties for beam elements
- **MSTG** New material properties for a stringer element
MAT1 - FORMAT (A4, 1X, I5, /, 4E15.7, /, 5E15.7)

CARD 1: MAT1
  IMEM One member number from the group that is to be changed

CARD 2: SIGOX
  SIGOY Defined in Section X
  SIGOZ
  SIGXY

CARD 3: RMOSN
  RMOS  Defined in Section X
  RMOSE
  YLDST
  YLDSC

One set of three cards is required for each MAT1 group to be changed. A MAT1 group may be changed to a MAT2 group after a half load cycle.

MAT2 - FORMAT (A4, 1X, I5, /, 4E15.7, /, 5E15.7, /, E15.7)

CARD 1: MAT2
  IMEM One member number from the group that is to be changed

CARD 2: SIGOX
  SIGOY Defined in Section X
  SIGOZ
  SIGXY

CARD 3: RMONX
  RMOS  Defined in Section X
  RMOSX
  RMONY
  RMSOSY
  RMONXY

2-44
CARD 4: RMOSXY Defined in Section X

One set of four cards is required for each MAT2 group to be changed. A MAT2 group may be changed to a MAT1 group after a half load cycle.

MBM or MSTG - FORMAT (A4, 1X, I5, /, 3E15.7)

CARD 1: MBM
or
MSTG
IMEM (One member number from the group that is to be changed)

CARD 2: RMOSN (Ramberg-Osgood parameters defined in Section X)
RM O S S
YLDST

The section is ended with a section end card, SEND.

Note: If there is no change in material properties, a SEND card must appear in the appropriate place in the input stream.

XVII. Problem End - FORMAT (A4)

Each problem's input must be ended with a card reading "ENDb" where "b" denotes a blank, in columns 1-4. The last problem in a run should end with a card reading "STOP" in column 1-4 instead of "ENDb."
XVIII. Restarting a Problem

The initial restart tape is created on Unit 21. Subsequent restart jobs mount the restart tape as Unit 21 and if desired (NUTAP > 0) copies and continues the restart tape on Unit 22.

CARD 1: Problem Title - FORMAT (20A4)
         As in Section I

CARD 2: NPNTC, NPRNT, IRESRT, NUTAP, INPRT - FORMAT (5I5).
         As in Section II. Here NPNTC is ignored.

If IRESRT is equal to 2

Restart from an elastic critical load using a previously created restart tape

CARD 3: PMAX, PPCT - FORMAT (2E15.7)
         PMAX = Maximum load to be applied for this half cycle
         PPCT = Load increment as percentage of yield load

Input is then continued from Section XV

If IRESRT is equal to 3

Continuation from some previously plastic load. The job can be restarted from a previous maximum load or from any intermediate load level.

CARD 3: PMAX, NRSRT - FORMAT (E15.0, I5)
         If starting from a previous maximum load NRSRT can be ignored. If starting from some intermediate load level, NRSRT is a unique number obtained as output from the job that generated the restart tape. This output is of the form:
PLASTIC ANALYSIS VALUES FOR RESTART TAPE HAVE BEEN WRITTEN AT $P = 1.5000000E+03$ FOR PRINT INTERVAL 30

In this case NRSRT is equal to 30 and the load is not necessarily the last maximum value.

Note: For cases when the job ends abnormally (for example when time or lines are exceeded) the job can be restarted with a value for NRSRT obtained from the output. In this case the last value of NRSRT should not be used. This will insure that a complete set of data will be accessed from the restart tape.

If IRESRT is equal to 4

Restart from a previous maximum load level followed by a reversal of a previous load distribution (CYCLE). A reversal of a previous load distribution can be placed on the structure from any previously generated and saved state of stress.

CARD 3: PMAX, NRSRT - FORMAT (E15.0, I5)

If starting from a previous maximum load NRSRT can be ignored. If starting from some intermediate load level, NRSRT is a unique number obtained as output from the job that generated the restart tape. This output has the form shown in the preceding discussion for IRESRT equal to three. The value of PMAX is ignored.

Succeeding cards follow as in Sections XV through XVII.
XIX. Alternate Input Method

All or part of the input may be read from a tape. This tape is mounted as Unit 23 with the appropriate job control cards. The tape is accessed by specifying the keyword, TAPE (FORMAT A4), in the proper place in the card input stream. Input will then be read from the tape until a SEND (A4) is encountered on the tape. This returns the input reader to the card input stream.

Note: All SEND formats used to end sections must be in card form as the SEND encountered on the tape only sends the reader back to the card input stream. Also blank cards which end sections are best not put on the tape but rather on cards as this allows the user to add card input to tape input, i.e., if a blank ends a section and is encountered on the tape format then the section ends and no new data may be added, but if the blank does not occur on tape the section is still open and can be closed with a blank card in the card input stream.

Example:

Input for member connectivity, Section IV, and coordinates have been generated on a tape in the prescribed manner.
Input cards are as follows:

**TITLE**

0 5

1 - 100

**TAPE** (reads data from a tape)

**TRIM**

. (additional elements if desired)

. .

**SEND**

**TAPE** (reads X-coordinates from tape)

(Blank card to end section for X-coordinates)

**TAPE** (reads Y-coordinates from tape)

(Blank card to end section for Y-coordinates)

5.0 (card input for Z-coordinates)

. .

(Blank card ends section for Z-coordinates)

If no additional tape input, remaining card input as usual.
REFERENCES


APPENDIX I — MULTIPOINT CONSTRAINTS

EXAMPLE 1

Node 1 and 2 are fixed. There is an applied displacement at node 3 in the negative y direction. The distance between nodes 3 and 4 remains constant.

Boundary Conditions

<table>
<thead>
<tr>
<th>Card Number</th>
<th>Node Selection</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>011000000000</td>
<td>1-4</td>
<td>default condition</td>
</tr>
<tr>
<td>000000000000</td>
<td>1 2 4</td>
<td></td>
</tr>
<tr>
<td>021000000000</td>
<td>3</td>
<td></td>
</tr>
</tbody>
</table>

Blank card

Dependencies

<table>
<thead>
<tr>
<th>Node</th>
<th>Dof</th>
<th>Node</th>
<th>Dof</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>2</td>
<td>3</td>
<td>2</td>
<td>-.050</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>3</td>
<td>2</td>
<td>1.0</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>1.0</td>
</tr>
</tbody>
</table>

Note: In the boundary condition cards for node 4, degrees of freedom 2 and 3 must be specified 0 because they are effectively eliminated from the solution (they are dependent degrees of freedom).
EXAMPLE 2 Rotation of Displacements at a Boundary

For this problem we set the displacement $\bar{u}$ equal to zero and $\bar{v}$ free. This corresponds to the normal displacement along $\bar{x}$ equal to zero and the tangential displacement (in $\bar{y}$ direction) free. The global coordinates of the problem are $x$, $y$. Thus:

$$u = \bar{u} \cos \phi - \bar{v} \sin \phi$$

$$v = \bar{u} \sin \phi + \bar{v} \cos \phi$$

The multipoint constraint cards are:

<table>
<thead>
<tr>
<th>25</th>
<th>1</th>
<th>25</th>
<th>1</th>
<th>.86602540E00</th>
<th>25</th>
<th>2</th>
<th>-.50E00</th>
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</table>

Note: On the boundary condition cards, since $\bar{u} = 0.0$ node 25 degree of freedom 1 is specified to be 0 but node 25 degree of freedom 2 is specified 1 or free since it remains an independent degree of freedom (although rotated). If the normal displacement $\bar{u}$ were not fixed (and the transformation performed merely to apply a normal load), it would have a 1 boundary condition specified.
Problem

Solve $AX = Y$, where

$$A = \begin{bmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{bmatrix}$$

is a banded positive definite symmetric matrix; $X$ is the desired solution vector; and $Y$ is the known right side (load vector).

PODSYM is the user interface and supervisory routine. It uses the Cholesky algorithm which factors the total stiffness matrix into $LL^T$ (where $L$ is a lower triangular matrix) and then solves a pair of triangular sets of equations.
The factorization is carried out by subroutine QFACT which supervised the storage and data set allocation and subroutine QCHOL which generates the lower triangular L-matrix. QCHOL implements the Cholesky algorithm to factor the positive definite symmetric \( A \) matrix as the product of a lower triangular matrix with its transpose:

\[
A = LL^T
\]

A straightforward argument establishes the possibility of decomposing any positive definite matrix in this fashion. Once \( L \) has been obtained, it is not difficult to solve the system of linear equations \( AX = Y \) by calculating \( Z \) as the solution of the lower triangular system \( LZ = Y \), and then \( X \) as the solution of the upper triangular system \( L^TX = Z \). The forward solution \( (LZ = Y) \) is accomplished by subroutines QFSOL and QFOR and the back solution \( (L^TX = Z) \) by subroutines QBSOL and QBAC. Before the call to QBSOL, subroutine REVERS is called, which reverses the rows of \( L \) so that the last row becomes the first row, etc. This is accomplished in order to sequentially access \( L^T \) during the back solution.

The above algorithm is noteworthy for its assured stability and general efficiency. It is possible to carry out an error analysis of the procedure as it is represented on a digital computer; such analysis shows that the computed \( L \) matrix satisfies an equation of the form

\[
A + E = LL^T
\]

with bounds on the elements of \( E \) which show that \( E \) is small compared to \( A \). The effect of rounding errors made in the subsequent solution of \( LZ = Y \) and \( L^TS = Z \) may then be taken into
account by (implicitly) introducing an additional perturbation $F$ into $A$, and it is then concluded that the computed solution $X_0$ exactly satisfies the equation

$$(A = E + F)X_0 = Y$$

Since $E + F$ can be shown to be small, one would like to infer that $X_0$ is almost an exact solution of the original equations, and unless $A$ is too nearly singular, such a conclusion is indeed warranted. But if $A$ is very ill-conditioned, no such result can be guaranteed, and $X_0$ may be far from the mathematically correct solution; in this event single-precision computation will not suffice for the calculation of an accurate solution, and since the solution will be very sensitive to small errors in $A$, it is unlikely that even a high-precision computation will yield satisfactory results unless $A$ and $Y$ are known (and supplied) to more than single-precision accuracy. The PODSYM subroutines make a fairly realistic attempt to detect and report pathological conditions of this sort.

The large positive definite matrices that occur in practical work very often contain a large number of zero entries and the program seeks to benefit from the presence of these elements by modifying the standard Cholesky formulae.

$$\ell_{kk} = \left[ A_{kk} - \sum_{j=1}^{k-1} \ell_{kj}^2 \right]^{\frac{1}{2}}$$

and

$$\ell_{ik} = \left[ A_{ik} - \sum_{j=1}^{k-1} \ell_{ij} \ell_{kj} \right] / \ell_{kk} \text{ for } i > k$$
to read instead

\[ \ell_{kk} = \left[ A_{kk} - \sum_{j=\nu(k)}^{k-1} \ell_{kj}^2 \right]^{\frac{1}{2}} \]

and

\[ \ell_{ik} = \left[ A_{ik} - \sum_{j=\mu(i,k)}^{k-1} \ell_{ij} \ell_{kj} \right] \bigg/ \ell_{kk} \]
APPENDIX III — CHANGING THE NUMBER OF NODES AND MEMBERS IN BEND

The number of nodes and members allowed for a BEND problem is variable. The following changes must be made in the MAIN program for a successful up-dimensioning or down-dimensioning of nodes and/or members.

- Reset MXMEM equal to the maximum number of members desired
- Reset MXNOD to the maximum number of nodes desired
- Redimension C the main working area so that it is greater than the greater of 10.5 * MXNOD + 6.5 * MXMEM or 24 * MXNOD + 1975
- Set NCORE equal to the dimension of the C array
- For maximum efficiency set MHCON equal to a prime number approximately 1/3 more than MXMEM
- For maximum efficiency set NHCON equal to a prime number approximately 1/3 more than MXNOD. A table of prime numbers appears in Ref. 6, page 870.
- For CDC versions, dimension IRAC (random access array) to MXMEM+1.

The following limitations on the number of property specifications exist in BEND and can only be changed through redimensioning within the program:
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<tr>
<th>Property or Section</th>
<th>Current Limitation</th>
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<td>Number of MBM property types</td>
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<tr>
<td>Number of beam section property types</td>
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<tr>
<td>Number of types of edge loads for EDG1, EDG2, EDG3 arrays</td>
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<td>Number of types of surface loads applied (SURF)</td>
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APPENDIX IV — EXAMPLES OF PLATE BOUNDARY CONDITIONS

SIMPLY SUPPORTED EDGES NOT RESTRAINED AGAINST MOVEMENT IN THE X-Y PLANE.

LATERAL IN-PLANE

SECTION A-A

SECTION B-B

BOUNDARY CONDITION: 1 = FREE; 0 = FIXED

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ALL ZERO
IF NO IN-PLANE LOAD

NOTES:

(1) = 0 FOR ZERO OR UNIFORM DIRECT IN-PLANE LOADS
   = 1 FOR NON-UNIFORM IN-PLANE LOADS
(2) = 0 FOR ZERO IN-PLANE SHEAR LOAD
   = 1 FOR IN-PLANE SHEAR

NODE 5 COMPLETELY FREE
CLAMPED - EDGES RESTRAINED AGAINST MOVEMENT IN THE X-Y PLANE

BOUNDARY CONDITION: 1 - FREE; 0 - FIXED

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NODE 5 COMPLETELY FREE

2-60
**BOUNDARY CONDITION:** 1 - FREE; 0 = FIXED

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**NOTES**

(1) = 0 FOR ZERO OR UNIFORM DIRECT IN-PLANE LOAD

(2) = 0 FOR ZERO IN-PLANE SHEAR LOAD

NODE 5 COMPLETELY FREE
**SIMPLY-SUPPORTED:** EDGES RESTRAINED

**LATERAL**

**SECTION A-A**

**SECTION B-B**

**BOUNDARY CONDITION:** 1 - FREE; 0 = FIXED

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**NOTES:**

1. WHEN DISPLACEMENTS ARE NOT RESTRAINED IN THE DIRECTION OF THE EDGE
2. WHEN DISPLACEMENTS ARE FULLY RESTRAINED
   NODE 5 COMPLETELY FREE
INSTRUCTIONS FOR USE OF HEX

A Program for the Elastic-Plastic Analysis of Three Dimensional Solids
INSTRUCTIONS FOR THE USE OF HEX

HEX is a finite element program to treat the elastic, elastic-plastic or elastic-cyclic plastic response of arbitrary three dimensional solid structures. The program uses a family of isoparametric hexahedra elements (Refs. 1 and 2) consisting of eight-node hexahedra and hexahedra with up to 12 additional midside nodes as shown in Fig. 1. Sample problems from the present program can be found in Ref. 3.

The program is capable of treating the elastic and the elastic-ideally plastic response of orthotropic materials. In addition, consideration is given to isotropic materials exhibiting elastic-ideally plastic, linear strain hardening, or nonlinear strain hardening behavior. Further, the kinematic hardening theory of plasticity is used (Refs. 4-6) so that provision for cyclic loading conditions involving reversed plastic deformation is included.

The input to the program is categorized in the following sections:

I. **Problem Title** FORMAT (20A4)
   
   Any 80-character title describing the problem.

II. **NPNTC, NPRNT, IRESRT, NUTAP, INPRT** FORMAT (515)

   \[ 0 \leq \text{NPNTC} \leq 63: \]

   NPNTC is the sum of the following integers corresponding to the option desired.

   If NPNTC = 0  No intermediate printout
   = 1  Print the load vector
   = 2  Print element stiffness matrix
NPNTC = 16  Print each element stiffness matrix entry to be stacked with its stacking index = 32  Print the total stiffness matrix

For example, if it is desired to print the load vector and the total stiffness matrix, NPNTC = 1 + 32 = 33.

NPRNT:
If $\leq 0$, perform elastic analysis only.
If $> 0$, perform plastic analysis, printing output every NPRNT increments of load.
IRESRT: (See Section XVIII)
0, Elastic (and/or) plastic run. Do not generate a restart tape
1, Elastic (and/or) plastic (cyclic) run. Build a new restart tape
2, Plastic run with elastic values from previously created restart tape
3, Plastic restart run starting at some specified load level
4, Cyclic restart run

NUTAP:
Applicable only if restarting from a restart tape.
0, No new tape written
1, Complete new tape created and additional restart data written

INFRT:
0, Write restart tape only at \( P = P_{\text{MAX}} \) (i.e., at maximum load)
N, Write restart tape every \( N \) increments of load

III. Node Specification (1615)

This section defines an allowable set of external node point numbers. The maximum node number that can be used is 9999. The program uses this information in two ways. First to set up a table of allowable node points that is used to check all subsequent node point input. Secondly, the program converts each external node number to an internal number consecutively in the order that the node appears on the input card. Consequently the order of the input of external node numbers is completely arbitrary and need not be increasing monotonically. In practice the node numbers should be numbered so as to minimize the bandwidth. Once the input is read the
program operates with the internal numbers which are now numbered from 1 through the number of nodes in the model. In this manner the node ordering and therefore the bandwidth of the stiffness matrix can be easily changed and nodes can be inserted or deleted by changing the external node specification.

The input is specified by entering the appropriate number on the input cards in fields of five. However, for this purpose the user can also utilize a shorthand form of the input. That is, specifying $m$ and $-n$ consecutively is the equivalent of the specification of nodes $m, m + 1, m + 2, \ldots, n$ and specifying $m, -p,$ and $-n$ consecutively is the equivalent of the specification of nodes $m, m + p, m + 2p, \ldots, m + kp$ where $m + kp$ is the highest integer of the form less than or equal to $n$. For example, the specification of nodes 1 through 100 is written as 1 - 100 and 1, 3, 5, ..., 99 as 1-2-99. This card input appears in fields of 5 (I5 Format) with 16 items per card. Any number of continuation cards may be used. A zero or blank 15 field ends the specification.

IV. Member Connectivity (Node Numbers of Each Member)

FORMAT (A4, 6X, 9I5)

The first alphanumeric field defines the element type as follows

- HEX8 - eight node hexahedra element
- HX20 - hexahedra element with up to twelve additional midside nodes
- MSGN - mesh generator for eight-node hexahedra
- SEND - ends the section

The first integer field designates the member number. The next integer field designates the connecting nodes as follows:

- HEX8 - The eight corner nodes are specified in the order shown on Fig. 2.
Fig. 2 Order of Corner Nodes

Fig. 3 Order of Midside Nodes
HX20 - The eight corner nodes are specified first in the order shown in Fig. 2. The additional midside nodes are specified on a second card (FORMAT (1215)) according to the order shown in Fig. 3. An element can be constructed with any combination of midside nodes. If a midside node is not specified, a zero is placed in its corresponding position on the card. If a midside node is specified for an element, a midside node should also be specified for every element coincident with that side.

Note: The hexahedra elements with midside nodes can have curved boundaries. The shape of any side is described by a quadratic polynomial through the three node points. If a side containing a midside node is straight, the coordinates of the midside node can be read in as zero. In this case the nodal coordinates are calculated within the program.

MSGN - The nodal connectivity is automatically generated for eight-node hexahedra according to the convention in Fig. 2. The first card for this input option requires the four-character identifier MSGN. A second card is required with FORMAT (5I5) which specifies the number of element subdivisions in three coordinate directions. The integers in the five fields are
the subdivisions in the $x_1$, $x_2$, $x_3$ directions, and the first node number and element number in the mesh. The coordinate system must be right handed. The mesh generator assumes that the nodes are numbered consecutively along the $x_1$ direction specified in the first 15 field then incremented on the $x_1 - x_3$ face until that face is completely specified. The mesh is then incremented in the $x_2$ direction and then consecutively numbered on the $x_1 - x_3$ plane. The element numbers are numbered consecutively starting at the specified starting value in the positive $x_1$ direction. The mesh generating capability still requires that the nodes be defined in Section IV and that the nodal coordinates be explicitly defined in Section VI. An example of the convention for a $4 \times 3 \times 2$ mesh in Cartesian coordinates with the starting —node and element equal to one is shown in Fig. 4.

V. X-Coordinates of Nodes FORMAT (E15.7, I3I5)

The global $x$-coordinates of the nodes appearing in the I5 fields are set to the value in the E15.7 field. Any number of continuation cards may be used; their first fifteen columns are ignored. A zero or blank I5 field terminates the card scan for a given $x$-coordinate. A zero or blank first I5 field (columns 16-20) on a noncontinuation card ends the section. Both shorthand representations of Section III are allowed.

VI. Y-Coordinates of the Nodes. Same as Section V.

VII. Z-Coordinates of the Nodes. Same as Section V.
VIII. Boundary Conditions FORMAT (I3, 2I1, 5X, 14I5)

The first three fields give the boundary conditions specifications in the order: \( u, v, w \); where \( u, v, w \) are global displacements in the global \( x, y, z \) directions, respectively.

- Zero—denotes a fixed degree of freedom
- One denotes a free degree of freedom
- Two will result in the application of a unit generalized displacement, or a corresponding card may be included in Section IX (dependent degrees of freedom) designating the magnitude of the applied generalized displacement.

The 14I5 fields give the applicable nodes for the boundary condition specifications, with both shorthand notations of Section III permitted. Any number of continuation cards may be used for a given specification. However, only the 14I5 fields are used on a continuation card. A zero or blank I5 field terminates
the card scan for a given boundary condition specification. Note: if the last field of a card (columns 76-80) is the last specification, an additional blank card (continuation card) must follow. A zero or blank first I5 field (columns 11-15) on a noncontinuation card ends the section. If a node's boundary conditions are not specified in this section, all the degrees of freedom are assumed to be free. To change this default condition, the first card of this section should be set to the desired default with all nodes used in the problem specified. Note: Maximum number of nodes is currently 2500.

IX. Dependent Degrees of Freedom FORMAT (2I5, 2(5X, 2I5, E15.7))

This section designates the input for both single and multipoint constraints as well as applied displacement of the form:

1) \[ \delta_i = \alpha_1 \delta_1 + \alpha_2 \delta_2 + \cdots + \alpha_n \delta_n \]
   where \( \delta_i \) is a dependent degree of freedom,
   \( \delta_1, \cdots, \delta_n \) are independent degrees of freedom,
   and \( \alpha_1, \alpha_2, \cdots, \alpha_n \) are coefficients.

2) Rotation of displacements at a node
   \[ \delta_i = \alpha_1 \tilde{\delta}_i + \alpha_2 \tilde{\delta}_j + \alpha_3 \tilde{\delta}_k \]
   \[ \delta_j = \beta_1 \tilde{\delta}_i + \beta_2 \tilde{\delta}_j + \beta_3 \tilde{\delta}_k \]
   \[ \delta_k = \gamma_1 \tilde{\delta}_i + \gamma_2 \tilde{\delta}_j + \gamma_3 \tilde{\delta}_k \]
   where the \( \alpha, \beta, \gamma \)'s are the direction cosines of the rotation, \( \delta_i, \delta_j, \delta_k \) are the displacements with respect to the original global directions and \( \tilde{\delta}_i, \tilde{\delta}_j, \tilde{\delta}_k \) are the components of displacements at the node with respect to the new coordinate axes. An example of this capability is given in Appendix I.
3) Applied generalized displacement

\[ \delta_i = \alpha_1 \]

where the coefficient \( \alpha_1 \) is the applied generalized displacement.

The first two I5 fields designate a node number and a degree of freedom (i.e., 1-3). The dependency is defined in the following three fields. The two integer fields designate the node number and degree of freedom number and the coefficient by the floating point field. If there is another dependency for the node, it is designated in a similar fashion in the next three fields. Any number of continuation cards can be used with the first two fields blank. The section is ended by a blank or zero in the third integer field (blank card). Examples of the use of multipoint constraints are in Appendix I.

X. Material Properties

The input for material properties consists of cards as follows:

CARD 1: FORMAT (A4, 1X, 5E15.7)

- MAT1 = four-character identifier
- EONE = Young's modulus in principal property axis (1)
- ETWO = Young's modulus in principal property axis (2)
- ETEE = Young's modulus in principal property axis (3)
- VONTO = Poisson's ratio, \( \nu_{12} \)
- VTOTE = Poisson's ratio, \( \nu_{23} \)

CARD 2: FORMAT (4E15.7)

- VTETO = Poisson's ratio, \( \nu_{32} \)
- GONTO = Shear modulus, \( G_{12} \)
- GTOTE = Shear modulus, \( G_{23} \)
- GONTE = Shear modulus, \( G_{13} \)
CARD 3: FORMAT (3E15.7)
BETA1 = \{ Angles between 1, 2, 3 material axes and local element \( x, y, z \) axes (defined in the note at the end of the section).
BETA2 =
BETA3 =

CARD 4: FORMAT (3E15.7)

TALF-1 Coefficient of thermal expansion in the principal 1 direction
TALF-2 Coefficient of thermal expansion in the principal 2 direction
TALF-3 Coefficient of thermal expansion in the principal 3 direction

CARD 5: FORMAT (5E15.7)
SIGOX = Yield stress in principal 1 direction
SIGOY = Yield stress in principal 2 direction
SIGOZ = Yield stress in principal 3 direction
SIGOXY = Shear yield stress in principal 1-2 plane
SIGOYZ = Shear yield stress in principal 2-3 plane

CARD 6: FORMAT (5E15.7)

SIGOZX = Shear yield stress in principal 3-1 direction
RMOSN = If \( RMOSS \neq 0 \); \( RMOSN = n \), the shape parameter used in Ramberg-Osgood representation of stress-strain behavior.

If \( RMOSS = 0 \); \( RMOSN = \alpha \) the slope of the linear strain-hardening stress-strain representation, i.e., \( \alpha = E_T/E \) where \( E_T \) is the tangent modulus.
RMOSS = If RMOSS ≠ 0; RMOSS = Ramberg-Osgood parameter $a_{0.7}$

Note (1): If RMOSN = 0 and RMOSS = 0, the material for the element(s) is assumed to be elastic-ideally plastic.

YLDST = Yield stress (for isotropic materials)

Note (2): Only initially isotropic materials can be treated when considering linear or nonlinear strain-hardening.

RMOSE = Ramberg-Osgood parameter $E$ (Young's modulus)

Succeeding cards give applicable members; both shorthands of Section III are permitted. Any number of continuation cards may be used for a given specification. A zero or blank I5 field ends each member listing.

Note: The transformation between the 1, 2, 3 material axes and the local element $x, y, z$ axes are defined by path dependent angular rotations $\beta_1, \beta_2, \beta_3$. First a rotation about the 1 axis, $\beta_1$, then a rotation about the 2 axis in the transformed system and then a rotation, $\beta_3$, about the 3 axis in the transformed system.
ACUM - Table describing yield stress versus accumulated plastic strain relationship

    FORMAT (A4/A4, 1X, I5, /, A4, 1X, 2E15.7, /, (1615))

CARD 1:

    ACUM - four character identifier

CARD 2:

    ACUM - four character identifier
    NUMP - number of pairs of points used to represent the yield stress versus accumulated plastic strain

CARD 3:

    ACUM - four character identifier
    VARSIG - yield stress
    SUMEPS - accumulated plastic strain

Card 3 is repeated "NUMP" times where NUMP is the number of pairs in the table. A maximum of 20 pairs is allowed.

Succeeding cards give applicable members; both shorthands of Section III are permitted. Any number of continuation cards may be used for a given specification. A zero or blank I5 field ends each member listing.
GAUSS or LOBATTO - Designation of stress points within an element

CARD 1: FORMAT (A4, 1X, 3115)

\[
\begin{align*}
\text{GAUSS} &= \text{five letter character identifier} \\
\text{or} \\
\text{LOBAT} &= \text{five letter character identifier} \\
\text{NX} &= \text{number of stress points in local } x \text{ direction} \\
\text{NY} &= \text{number of stress points in local } y \text{ direction} \\
\text{NZ} &= \text{number of stress points in local } z \text{ direction}
\end{align*}
\]

CARD 2: Applicable members; both shorthands of Section III are permitted. Any number of continuation cards may be used for a given specification. A zero or blank 15 field ends each member listing.

SEND in columns 1-4 ends the section.

Note: Up to 8 Gauss or 10 Lobatto points are permitted in any one direction and a total of 50 points are permitted in any one element. For local coordinate definitions see Figure 2. For a definition of these integration points see Ref. 7.
XI. **Applied Loads**

Three different types of loading are currently admissible — concentrated loads at nodes, distributed loads on a surface of a hexahedron member, and thermal loads. Each loading situation is designated by one of the following four-character key words:

- **CONC** (Concentrated load)
- **SURF** (Surface distributed load)
- **TEMP** (Temperatures for thermal loading)

**Distributed load -**

**CARD 1:** FORMAT (A4)

```
Key word  SURF
```

**CARD 2:** FORMAT (I5, 5X, 8I5)

- **First I5 field:** number of nodes on the loaded surface. There is a minimum of four nodes per surface and a maximum of eight in order to accommodate midside nodes.

- **Second through fifth I5 fields:** These four fields contain the four corner node numbers in counterclockwise or clockwise order.
Sixth through ninth I5 fields: These four fields contain the midside node numbers (if any). If a particular midside node is omitted, a zero or blank is placed in the appropriate field on the input card.

The input order for a given surface is shown below for counterclockwise order.

```
       1  2  3  4  5
       \  / \  /  \\
        \ /  /   \\
         \ /   \\
          \   \\
           \  \\
            \ \\
             \ \\
              \  \\
               \ \\
                \ \\
```

The magnitude of the surface tractions at each node specified above follow: There is one card for each node specified.

**SUCCEEDING CARDS:** FORMAT (I5, 3E15.7)

- **NUM** = Node number specified on CARD 2 (I5 field)
- **TX** = Surface traction in global x-direction
- **TY** = Surface traction in global y-direction
- **TZ** = Surface traction in global z-direction

A blank card ends the input for surface loads.

**Note:** The keyword SURF appears only once at the beginning of the specification of surface loads. Additional input for surface loads begin with card 2.
Concentrated load -

CARD 1: FORMAT (A4)

Key word CONC

CARD 2: FORMAT (I5, 3E15.7)

NUM = Node number at point of application of load (I5 field)
FX = Load component in the global x-direction (first E15.7 field)
FY = Load component in the global y-direction (second E15.7 field)
FZ = Load component in the global z-direction (third E15.7 field)

As many load components as desired may be specified. A zero or blank first I5 field ends the specification of concentrated loads.

Note: The keyword CONC appears only once at the beginning of the specification of concentrated loads. Additional input for concentrated loads begin with card 2.

Thermal load -

Each card after the first designates the temperature increment from some reference temperature followed by its applicable nodes. The input format is identical to that described for the input of nodal coordinates in Section V.

CARD 1: FORMAT (A4, 1X, E15.7)

Key word TEMP

TREF = Reference temperature in E15.7 field

The succeeding cards are as in Section V for nodal coordinates. That is, the nodal temperature in the first E15.7 field followed by the applicable nodes. A blank card ends the specification of nodal temperatures.
Note: The nodal temperature specified at the corner nodes of an individual hexahedra are interpolated to obtain the temperature used in calculating stresses and strains at the stress points.

SEND in columns 1-4 ends the section for applied loading.

XII. Members to be Printed - FORMAT (16I5)

Specify the members whose strains and stresses are to be printed. Both shorthands of Section III are allowed. A maximum of 2500 members may be specified. Members in excess of 2500 and undefined member numbers are ignored. A blank card or card with only zero entries ends the section.

XIII. Nodes to be Printed - FORMAT (16I5)

Up to 2500 nodes whose displacements are to be printed for the analysis, as per Section XII. A blank card ends the section.

XIV. Parameters for Plastic Analysis - FORMAT (2E15.7)

PMAX = Maximum load to be applied for this half cycle

PPCT = Load increment as percentage of yield load

XV. Parameters for Succeeding Load Cycles - FORMAT (A5, I5, 2E15.7)

CARD 1: FORMAT (A5)

Keyword defining the load type as follows:

CYCLE = Same load distribution as last half cycle

NEW = New load distribution to be specified

If any other word (or a blank card) is used the analysis is complete.

CARD 2: FORMAT (I5, 2E15.7)

NPRNT = If equal to zero, no additional load cycle and end of problem. If greater than zero, print output every NPRNT increments.

PMAX = New maximum load for this load cycle
PPCT = Load increment as percentage of yield load.

Note: When CYCLE is specified, the original load distribution is maintained. In this case PPCT should be set equal to one-half the original value since the elastic range for subsequent cycles is twice the elastic range for initial loading.

XVa. If NEW is used in Section XV a new load must be specified. This is accomplished with input identical to Section XI - Applied Loads. If CYCLE is specified in Section XV this section is omitted.

XVI. Change Plastic Material Properties When Cycle is Specified

CARD 1: FORMAT (A4, 1X, I5)

    MAT1 = Key word
    IMEM = One member number from the group that is to be changed.

CARD 2: FORMAT (5E15.7)

    SIGOX
    SIGOY
    SIGOZ
    SIGOXY
    SIGOYZ

Defined in Section X

CARD 3: FORMAT (5E15.7)

    SIGOXZ
    RMOSN
    RMOSS
    YLDST
    RMOSE

Defined in Section X

The section is ended with a section end card, SEND.
Note: If there is no change in material properties, a SEND card must appear in the appropriate place in the input stream. Succeeding load reversals (CYCLE) or new loads (NEW) are specified with additional input beginning with Section XV. A blank card (or any alphabetic other than CYCLE or NEW) branches to Section XVII to end the job.

XVII. Problem End - FORMAT (A4)

Each problem's input must be ended with a card reading "ENDb" where "b" denotes a blank, in columns 1-4. The last problem in a run should end with a card reading "STOP" in column 1-4 instead of "ENDb."

XVIII. Restarting a Problem

The initial restart tape is created on Unit 21. Subsequent restart jobs mount the restart tape as Unit 21 and if desired (NUTAP > 0) copies and continues the restart tape on Unit 22.

CARD 1: Problem Title - FORMAT (20A4)

As in Section I

CARD 2: NPNTC, NPRNT, IRESRT, NUTAP, INPRT - FORMAT (5I5)

As in Section II. Here NPNTC is ignored.

If IRESRT is equal to 2

Restart from an elastic critical load using a previously created restart tape

CARD 3: PMAX, PPCT - FORMAT (2E15.7)

\[ \text{PMAX} = \text{Maximum load to be applied for this half cycle} \]
\[ \text{PPCT} = \text{Load increment as percentage of yield load} \]

Input is then continued from Section XV
If IRESRT is equal to 3

Continuation from some previously plastic load. The job can be restarted from a previous maximum load or from any intermediate load level.

CARD 3: PMAX, NRSRT - FORMAT (E15.0, I5)

If starting from a previous maximum load NRSRT can be ignored. If starting from some intermediate load level, NRSRT is a unique number obtained as output from the job that generated the restart tape. This output is of the form,

* * * * * 
PLASTIC ANALYSIS VALUES FOR RESTART TAPE HAVE BEEN WRITTEN AT P = 1.5000000E+03 FOR PRINT INTERVAL 30
* * * * *

In this case NRSRT is equal to 30 and the load is not necessarily the last maximum value.

Note: For cases when the job ends abnormally (for example when time or lines are exceeded) the job can be restarted with a value for NRSRT obtained from the output. In this case the last value of NRSRT should not be used. This will insure that a complete set of data will be accessed from the restart tape.

If TRESRT is equal to 4

Restart from a previous maximum load level followed by application of a new load (NEW) or reversal of a previous load distribution (CYCLE). A new load or a reversal of a previous load distribution can be placed on the structure from any previously generated and saved state of stress.

3-21
CARD 3:  PMAX, NRSRT - FORMAT (E15.0, I5)

If starting from a previous maximum load NRSRT can be ignored. If starting from some intermediate load level, NRSTR is a unique number obtained as output from the job that generated the restart tape. This output has the form shown in the preceding discussion for IRESRT equal to three. The value of PMAX is ignored.

Succeeding cards follow as in Sections XV through XVII.

XIX. Alternate Input Method

All or part of the input may be read from a tape. This tape is mounted as Unit 23 with the appropriate job control cards. The tape is accessed by specifying the keyword, TAPE (FORMAT A4), in the proper place in the card input stream. Input will then be read from the tape until a SEND (A4) is encountered on the tape. This returns the input reader to the card input stream.

Example:

Input for member connectivity, Section IV, and coordinates have been generated on a tape in the prescribed manner.

Input cards are as follows:

TITLE

0  5

1 - 100

TAPE  (reads data from a tape)
HEX8
.
.
SEND
TAPE (reads X-coordinates from tape)
(Blank card to end section for X-coordinates)
TAPE (reads Y-coordinates from tape)
(Blank card to end section for Y-coordinates)
5.0 (card input for Z-coordinates
.
.
(Blank card ends section for Z-coordinates)

If no additional tape input, remaining card input as usual.

Note: All SEND formats used to end sections must be in card form as the SEND encountered on the tape only sends the reader back to the card input stream. Also blank cards which end sections are best not put on the tape but rather on cards as this allows the user to add card input to tape input, i.e., if a blank ends a section and is encountered on the tape format then the section ends and no new data may be added, but if the blank does not occur on tape the section is still open and can be closed with a blank card in the card input stream.
REFERENCES


3-24
APPENDIX I — MULTIPOINT CONSTRAINTS

EXAMPLE 1

Node 1 and 2 are fixed.
There is an applied displacement at node 3 in the negative y direction. The distance between nodes 3 and 4 remains constant.

Boundary Conditions

<table>
<thead>
<tr>
<th>Node Dof</th>
<th>Node Dof</th>
<th>Boundary Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>011000</td>
<td>1-4</td>
<td>default condition</td>
</tr>
<tr>
<td>001000</td>
<td>1 2 4</td>
<td></td>
</tr>
<tr>
<td>021000</td>
<td>3</td>
<td>blank card</td>
</tr>
</tbody>
</table>

Dependencies

<table>
<thead>
<tr>
<th>Node Dof</th>
<th>Node Dof</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 2</td>
<td>3 2</td>
<td>-.050</td>
</tr>
<tr>
<td>4 2</td>
<td>3 2</td>
<td>1.0</td>
</tr>
<tr>
<td>4 3</td>
<td>3 3</td>
<td>1.0</td>
</tr>
</tbody>
</table>

Note: In the boundary condition cards for node 4, degrees of freedom 2 and 3 must be specified 0 because they are effectively eliminated from the solution (they are dependent degrees of freedom).
EXAMPLE 2 Rotation of Displacements at a Boundary

For this problem we set the displacement $\bar{u}$ equal to zero and $\bar{v}$ free. This corresponds to the normal displacement along $\bar{x}$ equal to zero and the tangential displacement (in $\bar{y}$ direction) free. The global coordinates of the problem are $x, y$. Thus:

$$u = \bar{u} \cos \varphi - \bar{v} \sin \varphi$$

$$v = \bar{u} \sin \varphi + \bar{v} \cos \varphi$$

The multipoint constraint cards are:

```
25 1 25 1 .86602540E00 25 2 -.50E00
25 2 25 1 .50E00 25 2 .8660254E00
```

Note: On the boundary condition cards, since $\bar{u} = 0.0$ node 25 degree of freedom 1 is specified to be 0 but node 25 degree of freedom 2 is specified 1 or free since it remains an independent degree of freedom (although rotated). If the normal displacement $\bar{u}$ were not fixed (and the transformation performed merely to apply a normal load), it would have a 1 boundary condition specified.
APPENDIX II — SOLUTION PACKAGE

PODSYM — Solution of Symmetric Positive Definite Banded Matrix Equations

Problem

Solve $AX = Y$, where

\[
A = \begin{bmatrix}
\vdots & \vdots & \vdots \\
\vdots & \ddots & \vdots \\
\vdots & \vdots & \ddots \\
\end{bmatrix}
\]

is a banded positive definite symmetric matrix; $X$ is the desired solution vector; and $Y$ is the known right side (load vector).

PODSYM is the user interface and supervisory routine. It uses the Cholesky algorithm which factors the total stiffness matrix into $LL^T$ (where $L$ is a lower triangular matrix) and then solves a pair of triangular sets of equations.
The factorization is carried out by subroutine QFACT which supervised the storage and data set allocation and subroutine QCHOL which generates the lower triangular $L$ matrix. QCHOL implements the Cholesky algorithm to factor the positive definite symmetric $A$ matrix as the product of a lower triangular matrix with its transpose:

$$A = LL^T$$

A straightforward argument establishes the possibility of decomposing any positive definite matrix in this fashion. Once $L$ has been obtained, it is not difficult to solve the system of linear equations $AX = Y$ by calculating $Z$ as the solution of the lower triangular system $LZ = Y$, and then $X$ as the solution of the upper triangular system $L^TX = Z$. The forward solution ($LZ = Y$) is accomplished by subroutines QFSOL and QFOR and the back solution ($L^TX = Z$) by subroutines QBSOL and QBAC. Before the call to QBSOL, subroutine REVERS is called, which reverses the rows of $L$ so that the last row becomes the first row, etc. This is accomplished in order to sequentially access $L^T$ during the back solution.

The above algorithm is noteworthy for its assured stability and general efficiency. It is possible to carry out an error analysis of the procedure as it is represented on a digital computer; such analysis shows that the computed $L$ matrix satisfies an equation of the form

$$A + E = LL^T$$

with bounds on the elements of $E$ which show that $E$ is small compared to $A$. The effect of rounding errors made in the subsequent solution of $LZ = Y$ and $L^TX = Z$ may then be taken into
account by (implicitly) introducing an additional perturbation \( F \) into \( A \), and it is then concluded that the computed solution \( X_0 \) exactly satisfies the equation

\[(A = E + F)X_0 = Y\]

Since \( E + F \) can be shown to be small, one would like to infer that \( X_0 \) is almost an exact solution of the original equations, and unless \( A \) is too nearly singular, such a conclusion is indeed warranted. But if \( A \) is very ill-conditioned, no such result can be guaranteed, and \( X_0 \) may be far from the mathematically correct solution; in this event single-precision computation will not suffice for the calculation of an accurate solution, and since the solution will be very sensitive to small errors in \( A \), it is unlikely that even a high-precision computation will yield satisfactory results unless \( A \) and \( Y \) are known (and supplied) to more than single-precision accuracy. The PODSYM subroutines make a fairly realistic attempt to detect and report pathological conditions of this sort.

The large positive definite matrices that occur in practical work very often contain a large number of zero entries and the program seeks to benefit from the presence of these elements by modifying the standard Cholesky formulae

\[
\ell_{kk} = \left[ \frac{A_{kk} - \sum_{j=1}^{k-1} \ell_{kj}^2}{\ell_{kk}} \right]^{\frac{1}{2}}
\]

and

\[
\ell_{ik} = \left[ A_{ik} - \sum_{j=1}^{k-1} \ell_{ij}^2 \ell_{kj} \right] / \ell_{kk} \text{ for } i > k
\]
to read instead

\[ \ell_{kk} = \left[ A_{kk} - \sum_{j=v(k)}^{k-1} \ell_{kj}^2 \right]^{\frac{1}{2}} \]

and

\[ \ell_{ik} = \left[ A_{ik} - \sum_{j=\mu(i,k)}^{k-1} \ell_{ij} \ell_{kj} \right] / \ell_{kk} \]
INSTRUCTIONS FOR
THE USE OF REVBY

A Program for the Elastic-Plastic
Analysis of Bodies of Revolution
REVBY is a finite element program to treat the elastic, elastic-plastic, or elastic-cyclic plastic response of orthotropic axisymmetric solids of revolution under axisymmetric loadings. Currently, three different types of elements are available for the analysis of thick and thin bodies of revolution:

1. Revolved Triangular Element

The revolved triangular element used is similar to the element presented in Ref. 1, restricted to axisymmetric loading, i.e., independent of the circumferential direction, but including orthotropic material properties (see Fig. 1).

2. Thin Shell Element

The thin shell element used was developed at Grumman and the theoretical derivations are presented in Refs. 2 and 3. Cubic Hermitian polynomials represent the three displacement components. Again, only axisymmetric loading is allowed but orthotropic material properties are included (see Fig. 2).

3. Thin Ring Stiffener Element

The thin ring stiffener element is similar to the one presented in Ref. 4 with the exception that the ring may be attached with arbitrary eccentricities. No warping of the cross section is allowed and the shear center and centroid must coincide. Five different cross-sectional geometries are available.
Displacement Assumption:

\[ u_r = a_1 + a_2 r + a_3 z \]
\[ u_z = a_4 + a_5 r + a_6 z \]
\[ u_\zeta = a_7 + a_8 r + a_9 z \]

Initial Strain Distribution:

\[ \varepsilon_{ij}(r,z) = \text{constant} \]

Initial Strain Distribution:

\[
\begin{bmatrix}
\varepsilon_s \\
\varepsilon_\zeta \\
\gamma_{s\theta}
\end{bmatrix} = (1 - \xi) \begin{bmatrix}
\varepsilon^i_s(\zeta) \\
\varepsilon^i_\zeta(\zeta) \\
\gamma^i_{s\theta}(\zeta)
\end{bmatrix} + \xi \begin{bmatrix}
\varepsilon^j_s(\zeta) \\
\varepsilon^j_\zeta(\zeta) \\
\gamma^j_{s\theta}(\zeta)
\end{bmatrix}
\]

Fig. 1 Axisymmetric Revolved Triangle

Fig. 2 Axisymmetric Thin Shell Element
The plasticity theory used is presented in Refs. 5, 6, and 7. The input to the program is categorized in the following sections.

I. Problem Title (20A4)

Any 80-character title describing the problem.

II. NPNTC and NPRNT (215)

NPNTC: is for selective printing. NPNTC is the sum of the following integers corresponding to the options desired.

\[
\begin{align*}
\text{NPNTC} = 0, & \quad \text{no intermediate output} \\
\text{NPNTC} = 1, & \quad \text{print total load vector} \\
\text{NPNTC} = 8, & \quad \text{print nonzero element stiffness matrix entries with their stacking indices} \\
\text{NPNTC} = 16, & \quad \text{print the total stiffness matrix}
\end{align*}
\]

NPRNT: If \( \leq 0 \), perform elastic analysis only

If \( > 0 \), perform elastic and plastic analysis, printing the results of the plastic analysis every NPRNT load increments

III. Nodes by Partition (16I5)

This section defines an allowable set of external node point numbers. The maximum node number that can be used is 9999. The program uses this information in two ways. First to set up a table of allowable node points that is used to check all subsequent node point input. Secondly, the program converts each external node number to an internal number consecutively in the order that the node appears on the input card. Consequently the order of the input of external node numbers is completely arbitrary and need not be increasing monotonically. In practice the node numbers should be numbered so as to minimize the bandwidth. Once the input is read the
program operates with the internal numbers which are now numbered from 1 through the number of nodes in the model. In this manner the node ordering and therefore the bandwidth of the stiffness matrix can be easily changed and nodes can be inserted or deleted by changing the external node specification.

The input is specified by entering the appropriate number on the input cards in fields of five. However, for this purpose the user can also utilize a shorthand form of the input. That is, specifying \( m \) and \(-n\) consecutively is the equivalent of the specification of nodes \( m, m+1, m+2, \ldots, n \) and specifying \( m, -p, \) and \(-n\) consecutively is the equivalent of the specification of nodes \( m, m+p, m+2p, \ldots, m+kp \) where \( m+kp \) is the highest integer of the form less than or equal to \( n \). For example, the specification of nodes 1 through 100 is written as 1 - 100 and 1, 3, 5, \ldots, 99 as 1-2-99. This card input appears in fields of 5 (I5 Format) with 16 items per card. Any number of continuation cards may be used. A zero or blank I5 field ends the specification.

IV. Node Numbers for Members (A4,6X,4I5)

The first field defines the element type:

TRIR - triangular element designation (Ref. 1)
SHEL - shell element designation (Refs. 2, 3)
RING - ring element designation (Ref. 4)

The first integer field designates a member number and the three following fields give node numbers for the member.

For triangular members, three nodes are specified. The \( i, j, k \) nodes are given in counterclockwise order. For shell members, two nodes are specified, \( i \) and \( j \), in the positive \( s \) direction. For ring members, only one node is specified. SEND in the first field ends the section.

Note: Maximum number of elements (members) allowed is 600 currently.
V. **Global Coordinates (E15.7,13I5)**

The coordinates are read in a specified order, \( r \) first, then \( z \), and last the PHI coordinates. PHI is the angle in radians between the \( z \)-axis and the normal to the shell surface, measured positive clockwise (Fig. 2) from the \( z \)-axis. The floating point number designates a coordinate value. Following are the applicable node numbers as in Section III.

Only the integer fields are used on continuation cards. A zero or blank node field other than the first field on a noncontinuation card ends the card scan. On a continuation card, the floating field is left blank. A zero or blank first node field on a noncontinuation card ends each section for \( r \), \( z \), and PHI. If there are no shell elements, PHI is set to zero by default and need not be specified.

VI. **Boundary Conditions (6I1,4X,14I5)**

The first six fields give a boundary condition specification, in the order:

1) \( u_r, u_z, u_\theta \), if the node is attached to a triangular member. The last three fields for such a node must be zero.

2) \( u, w, \chi, \epsilon_s, v, \gamma_{s\theta} \), if the node is attached to a shell element. See Fig. 2 for convention. \( u \) is the meridional displacement of the shell middle surface, \( w \) is the normal displacement, \( \chi \) is the rotation of a normal to the middle surface, \( \epsilon_s \) is the meridional membrane strain, \( v \) is the circumferential displacement, and \( \gamma_{s\theta} \) is the shear strain. At a pole, the condition \( u_r = 0, \chi = 0, v = 0, \gamma_{s\theta} = 0 \), must be prescribed.
3) \( u, w, \chi \) if the node is attached to a ring member.
   The last three fields must be zero.

   Zero - denotes a fixed degree of freedom

   One - denotes a free degree of freedom

   Two - denotes an applied displacement and a corresponding card must be included in Section VII, designating the magnitude of the applied displacement.

   The integer fields are the applicable node numbers as in Section III. On continuation cards, only the integer fields are used. If a node's boundary conditions are not specified in this section, all six degrees of freedom are assumed free. To change the default, the first card should be set to the desired default (i.e., for triangles 111000) with all nodes used in the problem then specified.

   Blank card ends section.

Note: Maximum number of nodes currently allowed is 900.

VII. Dependent Degrees of Freedom (215,2(5X,215,E15,7))

The first two fields designate a node number and its degree of freedom. The dependency is defined in the following three fields. The two integer fields are node number and degree of freedom. The coefficient is contained in the following field. If there is another dependency for the node, it would be designated in the last three fields.

   If there is yet another dependency, it would appear on a continuation card with the first two integer fields blank. Both single and multipoint constraints of the form \( \delta_i = \alpha_1 \delta_{j1} + \alpha_2 \delta_{j2} + \ldots + \alpha_n \delta_{jn} \) may be considered as well as applied displacements of the form \( \delta_i = \alpha_1 \), e.g.,

   Blank card ends section.
1. Linking Up a Shell and a Triangle for a Cylinder

From shell theory restrictions

\[ u_{r27} = u_{r28} = u_{r29} = w_{26} \]

\[ u_{z27} = -u_{26} - \frac{h}{2} x_{26} \]

\[ u_{z28} = -u_{26} \]

\[ u_{z29} = -u_{26} + \frac{h}{2} x_{26} \]
The input cards, for this case, would appear as (let \( h = 0.1 \) inches):

27 1 26 2 1.0 E00
28 1 26 2 1.0 E00
29 1 26 2 1.0 E00
27 2 26 1 -1.0 E00 26 3 - 0.05 E00
28 2 26 1 -1.0 E00
29 2 26 1 -1.0 E00 26 3 + 0.05 E00

Note: In the boundary condition cards for nodes 27, 28, and 29, degrees of freedom 1 and 2 must be fixed (specified \( = 0 \)) because they are effectively eliminated from the solution (they are dependent degrees of freedom).

2. Rotation of Displacements at Boundary

![Rotation of Displacements at Boundary](image)

For this problem, we set the displacement normal to the boundary equal to zero. We have:

\[
\begin{align*}
    u_1 &= u_r \cos \varphi + u_z \sin \varphi \\
    u_2 &= -u_r \sin \varphi + u_z \cos \varphi
\end{align*}
\]
Let us assume that the node in question is node 25 and $\varphi = 30$ degrees. The multipoint constraint cards are:

```
25 1 25 1 0.86602540 E00 25 2 0.50 E00
25 2 25 1 - 0.50 E00 25 2 0.86602540 E00
```

Note: On the boundary condition cards, since $u_1 = 0$, node 25 degree of freedom 1 is specified to be 0 but node 25 degree of freedom 2 is specified (or free) since it remains an independent degree of freedom (although rotated). If the normal displacement $u_1$ were not fixed (and the transformation performed merely to apply a normal load) it would have a 1 boundary condition specified.

3. Applied Displacement

Suppose for a shell we wish to apply a unit lateral ($w$) displacement at node 25. We have the multipoint constraint card as

```
25 2 25 2 1.0 E00
```

Note: A 2-boundary condition for node 25 degree of freedom 2 is specified on the boundary condition card. The end of this section is designated by a blank or zero in the third integer field (e.g., blank card).
VIII. Material Properties

The format of this input is dependent upon the member type:

A. Triangle Elements (4(5E15.7),3E15 7,/,16(15))

<table>
<thead>
<tr>
<th>Card</th>
<th>Property Type</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>E1</td>
<td>Young's modulus in 1 direction (if $\beta = 0$, (\theta)-direction)</td>
</tr>
<tr>
<td>2</td>
<td>E2</td>
<td>Young's modulus in 2 direction (if $\beta = 0$, (z)-direction)</td>
</tr>
<tr>
<td>3</td>
<td>E3</td>
<td>Young's modulus in 3 direction (if $\beta = 0$, (r)-direction)</td>
</tr>
<tr>
<td></td>
<td>V12</td>
<td>Poisson's ratio in 1-3 directions</td>
</tr>
<tr>
<td></td>
<td>V23</td>
<td>Poisson's ratio in 2-3 directions</td>
</tr>
<tr>
<td></td>
<td>V31</td>
<td>Poisson's ratio in 3-1 directions</td>
</tr>
<tr>
<td></td>
<td>G12</td>
<td>Shear modulus in 1-2 plane</td>
</tr>
<tr>
<td></td>
<td>G23</td>
<td>Shear modulus in 2-3 plane</td>
</tr>
<tr>
<td></td>
<td>G13</td>
<td>Shear modulus in 3-1 plane</td>
</tr>
<tr>
<td></td>
<td>$\beta$</td>
<td>Orientation of material properties axis with respect to (\theta)-(z) coordinates (in degrees). This is a rotation about the (r)-axis and $\beta$ is positive clockwise.</td>
</tr>
</tbody>
</table>

4-10
<table>
<thead>
<tr>
<th>Card</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>SIGOX: Tensile (or compressive) yield stress in ( \theta )-direction</td>
</tr>
<tr>
<td></td>
<td>SIGOY: Tensile (or compressive) yield stress in ( z )-direction</td>
</tr>
<tr>
<td></td>
<td>SIGOZ: Tensile (or compressive) yield stress in ( r )-direction</td>
</tr>
<tr>
<td></td>
<td>SIGXY: Shear yield stress in ( r-\theta ) plane</td>
</tr>
<tr>
<td></td>
<td>RMOSN: Ramberg-Osgood shape parameter ( n ), ( \varepsilon = \sigma/E + (3\sigma/7E)(\sigma/\sigma_{0.7})^{n-1} ). If set = 0.0 material for the group of members is assumed to elastic ideally plastic. If ( \text{RMOS} = 0.0 ) and ( \text{RMOSN} \neq 0 ), ( \text{RMOSN} ) is ratio of slope of stress-strain curve for linear strain hardening to Young's modulus ( (\alpha = E_T/E \text{ where } E_T \text{ is the tangent modulus}) ).</td>
</tr>
<tr>
<td></td>
<td>RMOSS: Ramberg-Osgood parameter ( \sigma_{0.7} ). If zero and ( \text{RMOSN} ) not zero have linear strain hardening.</td>
</tr>
<tr>
<td>4</td>
<td>YLDST: Yield stress in tension (or compression)</td>
</tr>
<tr>
<td></td>
<td>RMOSE: Ramberg-Osgood parameter ( E ) (Young's modulus)</td>
</tr>
<tr>
<td></td>
<td>SIGYZ: Shear yield stress in ( \theta-z ) plane</td>
</tr>
<tr>
<td></td>
<td>SIGZX: Shear yield stress in ( z-r ) plane</td>
</tr>
<tr>
<td>5</td>
<td>TALPH (1): 3 thermal coefficients of expansion in ( 1, 2, 3 ) directions ( (\theta, z, r \text{ if } \beta = 0^\circ) )</td>
</tr>
</tbody>
</table>

Note: Orthotropic material properties should not be used for strain hardening problems in the current version of REGBY.
Succeeding cards give applicable members as in Section III. A zero or blank E1 ends the section. If there are no triangular members, no card is required.

B. Shell Elements \(((4E15.7,\,5E15.7,\,3E15.7,\,2E15.7)/(,16I5))\)

1) Material Properties

- **E1**: Young's modulus in meridional direction
- **E2**: Young's modulus in circumferential direction
- **G33**: Shear modulus in plane of shell median surface
- **V12**: Poisson's ratio in plane of shell median surface
- **SIGOX**: Tensile or compressive yield stress in meridional direction
- **SIGOY**: Tensile or compressive yield stress in circumferential direction
- **SIGOZ**: Tensile or compressive yield stress in direction normal to shell surface
- **SIGXY**: Shear yield stress in plane of shell median surface
- **RMOSN**: (See Card 3 for triangular members)
- **RMOSS**: (See Card 4 for triangular members)
- **YLDST**
- **RMOSE**

- **TALPH (1)**: Two thermal coefficients of expansion in meridional and circumferential direction, respectively
Succeeding cards give applicable members as in Section III. A zero or blank El ends the section. If there are no shell elements, no blank card is required.

2) Thickness for Shell Elements (follows the material properties. All nodes must have a thickness specified.)

\[(E_{15.7},/(1615))\]

First field designates thickness value. Applicable nodes follow as in Section III. Section ends with zero or blank in first field.

3) NG and NLRS (215). For shell elements, one card. NG designates the number of Gauss points used in integration scheme for shell element, and the second field, NLRS, designates number of layers through the thickness at which plastic strains, stresses, etc., are calculated. NG \(\leq 8\), NLRS must be even, and \(\leq 30\).

C. Ring Elements

\[\text{Card 1} \quad E_{15.7},/2E_{15.7},/A4.6X,3E_{15.7},/3E_{15.7},/
3E_{15.7},/E_{15.7},(1615)\]

- E : Young's modulus
- A : Cross sectional area of ring
- IX : Moment of inertia of ring about centroidal x'-'axis (see diagram below)
- IXY : Product of inertia of ring about centroidal axis
- IY : Moment of inertia of ring about centroidal y'-'axis
ZO

Distance from shell middle surface to ring centroid (normal to shell middle surface).
Plus in negative z-direction.

Card 2

ECCEN(Y)
Distance along meridian from ring centroid to point of attachment to shell middle surface.
Plus in negative s-direction.

SREC
Solid rectangular cross section

SCIR
Solid circular cross section

Card 3

ZSEC
Z-section

ISEC
I-section

HCIR
Hollow circular cross section

Remaining three fields designate section dimensions as shown in the diagrams below.

[Diagrams showing various cross sections with dimensions labeled.]
**INPUT**

<table>
<thead>
<tr>
<th>SREC</th>
<th>SCIR</th>
<th>ZSEC</th>
<th>ISEC</th>
<th>HCIR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Card 3</td>
<td>2a</td>
<td>a</td>
<td>a</td>
<td>2a</td>
</tr>
<tr>
<td></td>
<td>2b</td>
<td>2h</td>
<td>2h</td>
<td>t</td>
</tr>
<tr>
<td></td>
<td></td>
<td>b</td>
<td>b</td>
<td></td>
</tr>
<tr>
<td>Card 4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>t_1</td>
<td>t_1</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>t_2</td>
<td>t_2</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>t_3</td>
<td>t_3</td>
<td></td>
</tr>
</tbody>
</table>

Note: For ZSEC, a = b, t_1 = t_3; for ISEC, 2a = 2b, t_1 = t_3.

Card 4 If section name is ZSEC or ISEC, this card is required for three additional dimensions, otherwise this card is omitted.

Card 5

SIGOX Yield stress in tension or compression

Card 6

TALPH (1) Thermal coefficient of expansion

Succeeding cards give applicable members as in Section III. A zero or blank E ends the section. If there are no ring elements, no card is required.
IX. Load Vector Components

This format is also dependent on member type.

A. Triangular Members (I5,3E15.7,/,I10,3E15.7)

The integer field gives node number (i-j pairs); the floating-point fields give components at the respective nodes as follows:

\[ p_r = 1.0 \] corresponds to an applied unit pressure (force/unit area) in the radial direction at node i

\[ p_z = 1.0 \] corresponds to an applied unit pressure (force/unit area) in the z-direction at node i

\[ p_\theta = 1.0 \] corresponds to an applied unit pressure (force/unit area) in the \( \theta \)-direction at node i

The load between any two adjacent nodes is assumed to vary linearly. Therefore, the load vector is specified by two cards, the first specifying the load at node i and the second, the load at node j. A zero or blank first integer field ends the section. If there are no uniform loads, a blank card is required. If there are no triangular elements, no card is required.

B. Shell Members (3E15.7,/,I6I5)

The three fields designate the meridional component, \( p_s \), normal component, \( p_z \), and the circumferential component, \( p_\theta \), of applied stresses at nodes. Applied stresses are assumed to vary linearly from node to node. The applicable nodes follow as in Section III. A blank card ends the section. If there are no uniform loads, a blank card is required. If there are no shell elements, no card is required.
C. Concentrated Loads (4E15.7,/(1615))

For triangle component, specify $F_r$, $F_z$, and $F_\theta$ in $r$, $z$, $\theta$ directions, and for the shell component specify $N_s$, $Q_s$, $M_s$, and $N_{s\theta}$ in $u$, $w$, $\chi$, and $v$ directions, respectively, in lb/in. These are forces per unit length of circumference, except at $r = 0$, where $F_r = F_\theta = 0$ and $N_s = M_s = N_{s\theta} = 0$ and $F_z$ or $Q_s$ represent an actual force (lbs).

Applicable nodes follow as in Section III. A blank or zero card ends the concentrated loads specification section.

X. Nodal Temperatures (A4,E15.7,/(A4,3E15.7,/(1615))

If the first field is SEND, there are no temperatures and this section ends. If the first field is blank, there are thermal loads and the floating number field designates the reference temperature value ($T_{ref}$), which is only for the user's information.

The next cards contain $\Delta T = T - T_{ref}$ at each node (up to three per node depending on the element to which it is attached) in the floating point fields. For a triangular element, one temperature is specified per node. For a shell element, three temperatures are specified, one at the top surface ($+h/2$), $T_1$, one at the median surface ($z = 0$), $T_2$, and one at the bottom surface ($-h/2$), $T_3$. The program assumes a parabolic distribution of temperature through the thickness (if $T_2 = (T_1 + T_3)/2$, this reduces to a linear variation). For a ring element, three temperatures are specified. A parabolic distribution is assumed through the ring depth. $T_1$ is the temperature at $\tilde{x} = +a$ or $+h$. $T_2$ is the temperature at the centroid, and $T_3$ is the temperature at $\tilde{x} = -a$ or $-h$ (see diagram in Section VIII).

Applicable nodes follow as in Section III.
XI. Members Whose Results are to be Printed for Elastic Solution (1615) As in Section III
   Blank cards end section.

XII. Nodes Whose Results are to be Printed for Elastic Solution (1615) As in Section III
    Blank cards end section.

XIII. Same as Section XI for Plasticity, if applicable.
    Blank cards end section.

XIV. Same as Section XII for Plasticity, if applicable.
    Blank cards end section.

XV. \textbf{PMAX and PPCT (2E15.7)}
    
    PMAX - Maximum load to be applied for this half cycle
    PPCT - Load increment as a percentage of yield load

XVI. \textbf{NPRNT, PMAX, PPCT (15,2E15.7)}

    For succeeding load cycles, one card giving new NPRNT, PMAX, and PPCT. Zero NPRNT signifies no new load cycle and end of problem. For succeeding half-cycles, PPCT equal to one-half the original value should be used since the elastic range is twice the elastic range for initial loading.

XVII. Change in Material Properties (A4)

    To change properties of any group of members, read indicator for new material properties (A4).

A. If RING is input, it indicates a change in the RING element material properties. A card with a member number belonging to the group whose properties are to be changed and the new quantities are read in (15,4E15.7).
Member Number  Member of group to be changed
SIGOX
RMOSN  New Values
RMOS S
TALPH

B. NEWM signifies new material properties for either triangle or shell elements. The new values are read in \((I5,5E15.7,/,5E15.7,/,3E15.7)\)

Member Number  Member of group to be changed
SIGOX
SIGOX
SIGOX
SIGOZ  New Values
SIGXY
RMOSN
RMOS S
YLDST
RMOS E  New Values
SIGYZ
SIGZ X

Card 3  TALPH (1)
TALPH (2)  New Values
TALPH (3)

C. SEND as input indicates no new material properties or end of this section.
XVIII. Each Problem's Input Must Be Ended with a Card Reading "ENDb" Where "b" Denotes a Blank, in Columns 1-4. The Last Problem in a Run Should End with a Card Reading "STOP" in Column 1-4 Instead of "ENDb."

References


INSTRUCTIONS FOR USE OF
OUT-OF-PLANE GM

A Program for the Nonlinear
Analysis of Built-Up Structures
OUT-OF-PLANE GM is a finite element program for the nonlinear analysis of arbitrary built-up thin walled structures. This program considers the sheet material to be in states of membrane stress. The finite element library consists of the following elements:

- Three-node uniform stress triangle
- Six-node linearly varying stress triangle
- Four- and five-node hybrid triangles to be used as transition elements between three- and six-node elements (see Fig. 1a)
- Two-node uniform stress stringer
- Three-node linearly varying stress stringer
- Beam with various cross sections subjected to bending about two planes as well as torsion

The program is capable of treating the elastic and the elastic-ideally plastic response of orthotropic materials. In addition, consideration is given to isotropic materials exhibiting elastic-ideal plastic, linear strain hardening, or nonlinear strain hardening behavior. Further, the kinematic hardening theory of plasticity is used.

The effects of geometric nonlinear behavior is treated by considering a convected coordinate system that accounts for changes in geometry and by the introduction of an initial stress stiffness matrix. The solution strategy implemented by the program is outlined in a companion volume, "PLANS - A Finite Element Program for Nonlinear Analysis of Structures, Volume I - Theoretical Manual," NASA Contractor Report NASA CR-2568, p. 31.

The input to the program is categorized in the following sections.
I. Problem Title FORMAT (20A4)

Any 80-character title describing the problem.

II. Load and Print Control Parameters FORMAT (6I5./4E15.7)

CARD 1:

NPNTC is the sum of the following integers corresponding to the option desired.

If NPNTC = 0 No intermediate printout
    = 1 Print the load vector
    = 2 Print element stiffness matrix
    = 16 Print each element stiffness matrix entry to be stacked with its stacking index
    = 32 Print the total stiffness matrix

For example, if it is desired to print the load vector and the total stiffness matrix, NPNTC = 1 + 32 = 33.

NPRNT ≥ 0 Print output every NPRNT increments of load.
If NPRNT equals zero the output is printed every load step.

NFORM > 0 Reform elastic stiffness matrix to account for changing geometry every NFORM increments of load.
The effect of the geometric (initial stress) stiffness matrix is still introduced as an "effective load" in this case.

IREST:
IREST = 0 Do not generate a restart tape
IREST = 1 Build a restart tape
IREST = 2 Run continuing from a created restart tape

5-2
NUTAP: Applicable only if restarting from a restart tape. When IREST = 2 and NUTAP = 0 no additional restart data is written. If NUTAP = 0 restart tape is continued.

INPRT: INPRT = 0 Write restart tape only at P = PMAX (i.e., at maximum load)
INPRT = N Restart data written every INPRT time steps

NRSRT: Applicable only if restarting from a restart tape. NRSRT is a unique number that is printed in the job that generated the restart tape for each load that data is written on the restart tape. This number defines the starting load for the continuation job (see Section XVII).

CARD 2:

PMAX: Maximum load factor. The load factor multiplies the vector of applied load. If the applied loading is normalized to unity then PMAX represents the magnitude of the largest component of the applied load. If the maximum desired load is input then PMAX is equal to one and each load step represents a percentage of the total load.

DELP: Number of load increments. The load increment ΔP is specified by PMAX/DELP.

PTAN: Load at which full tangent modulus method is used, i.e., the stiffness matrix explicitly includes the initial stress stiffness matrix. If the "tangent modulus" method is to be used for the entire analysis, PTAN = 0.0. In this case the stiffness matrix is reformed in every load step. If the "effective
load" method is to be used for the entire analysis, $PTAN = PMAX$. In this case the stiffness matrix includes only the elastic stiffness matrix and is reformed every NFORM load steps. When $0 < PTAN < PMAX$ then the program uses the "effective load" method up to $PTAN$ and then implements the "tangent modulus" until the maximum load is reached.

**EQMULT:** Factor multiplying the equilibrium correction term. $EQMULT = 0.0$ indicates no equilibrium correction and $EQMULT = 1.0$ indicates full equilibrium correction.

### III. Node Specification (1615)

This section defines an allowable set of external node point numbers. The maximum node number that can be used is 9999. The program uses this information in two ways. First to set up a table of allowable node points that is used to check all subsequent node point input. Secondly, the program converts each external node number to an internal number consecutively in the order that the node appears on the input card. Consequently the order of the input of external node numbers is completely arbitrary and need not be increasing monotonically. In practice the node numbers should be numbered so as to minimize the bandwidth. Once the input is read the program operates with the internal numbers which are now numbered from 1 through the number of nodes in the model. In this manner the node ordering and therefore the bandwidth of the stiffness matrix can be easily changed and nodes can be inserted or deleted by changing the external node specification.

The input is specified by entering the appropriate number on the input cards in fields of five. However, for this purpose the
user can also utilize a shorthand form of the input. That is, specifying \( m \) and \(-n\) consecutively is the equivalent of the specification of nodes \( m, m+1, m+2, \ldots, n \) and specifying \( m, -p, \) and \(-n\) consecutively is the equivalent of the specification of nodes \( m, m+p, m+2p, \ldots, m+kp \) where \( m+kp \) is the highest integer of the form less than or equal to \( n \). For example, the specification of nodes 1 through 100 is written as 1-100 and 1, 3, 5, ..., 99 as 1-2-99. This card input appears in fields of 5 (I5 Format) with 16 items per card. Any number of continuation cards may be used. A zero or blank I5 field ends the specification.

IV. Member Connectivity (Node Number of Each Member)
FORMAT (A4, 6X, 9I5)

The first alphanumeric field defines the element type:

- TRIM - Triangular membrane element (Ref. 3)
- BEAM - Beam element
- STRG - Stringer element

The first integer field designates the member number. The next integer fields designate the connecting nodes as follows:

- TRIM - The nodes for the triangular family of elements are specified around the perimeter beginning with a major (vertex) node. Then a minor (midside) node and then alternatively major and minor as shown in Fig. 1. The absence of a minor node must be indicated by a zero or blank field in the proper position.

- BEAM - Three node specifications are necessary for the beam element.
Fig. 1a Triangular Family of Finite Elements Used

Fig. 1b Elements Topology
Nodes $i$ and $j$ (Fig. 2) which designate the element end points and a third node $k$, defining the normal to the beam axis about which the section properties are specified. This additional node may be a node of the structural idealization or it may be a "fictitious node" specified just for the purpose of defining the beam section properties. This is shown in Fig. 2.

STRG - Three node specifications are necessary for the stringer element. Nodes $i$ and $j$, connecting the end points, and if desired an additional node designating a midpoint node. This is shown in Fig. 3. A zero or blank for the midpoint node specifies a two-node stringer. The midside node is the third node specified.

SEND Ends the section

V. **X-Coordinates of Nodes** FORMAT (E15.7, 13I5)

The $x$-coordinates of the nodes appearing in the 15 fields are set to the value in the E15.7 field. Any number of continuation cards may be used; their first fifteen columns are ignored. A zero or blank 15-field terminates the card scan for a given $x$-coordinate. A zero or blank first 15 field (columns 16-20) on a noncontinuation card ends the section. Both shorthand representations of Section III are allowed.

VI. **Y-Coordinates of the Nodes.** Same as Section V.

VII. **Z-Coordinates of the Nodes.** Same as Section V.

VIII. **Boundary Conditions** FORMAT (611, 9X, 13I5)

The first six fields give the boundary conditions specifications in the order: $u$, $v$, $w$, $\theta_x$, $\theta_y$, $\theta_z$; where $u$, $v$, $w$ are global displacements in the $x$, $y$, $z$ directions, respectively, and $\theta_x$, $\theta_y$, $\theta_z$ are rotations with respect to the $x$, $y$, $z$ axes. The last three fields for $\theta_x$, $\theta_y$, $\theta_z$ are only used when using a beam element.
Fig. 2 Beam Element

Fig. 3 Stringer Element
Zero denotes a fixed degree of freedom

One denotes a free degree of freedom

Two will result in the application of a unit generalized displacement, or a corresponding card may be included in Section IX (dependent degrees of freedom) designating the magnitude of the applied generalized displacement.

The 1315 fields give the applicable nodes for the boundary condition specifications, with both shorthand notations of Section III permitted. Any number of continuation cards may be used for a given specification. However, only the 1315 fields are used on a continuation card. A zero or blank I5 field terminates the card scan for a given boundary condition specification. Note: If the last field of a card (columns 76-80) is the last specification, an additional blank card (continuation card) must follow. A zero or blank first I5 field (columns 16-20) on a noncontinuation card ends the section. If a node's boundary conditions are not specified in this section, all the degrees of freedom are assumed to be free. To change this default condition, the first card of this section should be set to the desired default (i.e., in the absence of beams, 111000) with all nodes used in the problem specified. Note: Maximum number of nodes is currently 900.

IX. Dependent Degrees of Freedom FORMAT (215, 2(5X, 215, E15.7))

This section designates the input for both single and multipoint constraints as well as applied displacement of the form:

1) \[ \delta_i = \alpha_1 \delta_{j1} + \alpha_2 \delta_{j2} + \ldots + \alpha_n \delta_{jn} \]

where \( \delta_i \) is a dependent degree of freedom,
\( \delta_{j1}, \ldots, \delta_{jn} \) are independent degrees of freedom
and \( \alpha_1, \alpha_2, \ldots, \alpha_n \) are coefficients
2) Rotation of displacements at a node

\[
\delta_i = a_1 \delta_i + a_2 \delta_j + a_3 \delta_k
\]

\[
\delta_j = \beta_1 \delta_i + \beta_2 \delta_j + \beta_3 \delta_k
\]

\[
\delta_k = \gamma_1 \delta_i + \gamma_2 \delta_j + \gamma_3 \delta_k
\]

where the \( a, \beta, \gamma \)'s are the direction cosines of the rotation, \( \delta_i, \delta_j, \delta_k \) are the displacements with respect to the original global directions, and \( \delta_i, \delta_j, \delta_k \) are the components of displacements at the node with respect to the new coordinate axes. An example of this capability is given in Appendix I.

3) Applied generalized displacement

\[
\delta_i = \alpha_i
\]

where the coefficient \( \alpha_i \) is the applied generalized displacement.

The first two fields designate a node number and a degree of freedom (i.e., 1-6). The dependency is defined in the following three fields. The two integer fields designate the node number and degree of freedom number and the coefficient by the floating point field. If there is another dependency for the node, it is designated in a similar fashion in the next three fields. Any number of continuation cards can be used with the first two fields blank. The section is ended by a blank or zero in the third integer field (blank card). Examples of the use of multipoint constraints are in Appendix I.

X. Material and Section Properties

The format for this input is dependent upon the member type. Each type of input begins with a word of up to four characters.
MAT1 Material properties for plane stress, necessary with membrane triangles (TRIM)

MBM Material properties for a beam element

MSTG Material and section properties for a stringer

THIK Member thickness for triangular membrane (TRIM) elements

MBET Angle between local axes and principal directions of orthotropy for TRIM elements

SREC Beam section properties for a solid rectangular section

SCIR Beam section properties for a solid circular section

ZSEC Beam section properties for a Z-section

ISEC Beam section properties for an I-section

HCIR Beam section properties for a hollow circular section

HREC Beam section properties for a hollow rectangular section

LSEC Beam section properties for an L-section

TSEC Beam section properties for a T-section

CSEC Beam section properties for a channel section

SEND Ends the section

MAT1 - Plane Stress Material Properties - FORMAT (A4, 1X, 5E15.7, /, 2E15.7, /, 4E15.7, /, 5E15.7, /, (16I5))

The first four cards specify material properties, as follows:
CARD 1: MAT1

EONE = Young's modulus in principal property axis (1)
ETWO = Young's modulus in principal property axis (2)
BETA = No longer used. Set equal to zero.
GONTO = Shear modulus in (1)-(2) principal property plane
VONTO = Poisson's ratio, $v_{12}$

Note: $\epsilon_1 = \frac{\sigma_1}{E_1} - \frac{v_{12}}{E_2} \sigma_2$

$$\epsilon_2 = -\frac{v_{21}}{E_1} \sigma_1 + \frac{\sigma_2}{E_2}$$

$$\gamma_{12} = \frac{\tau_{12}}{G_{12}}$$

CARD 2: TALF-1 = Coefficient of thermal expansion in 1-axis direction, $\alpha_1$
TALF-2 = Coefficient of thermal expansion in 2-axis direction, $\alpha_2$

CARD 3: SIGOX = Yield stress in principal 1-direction
SIGOY = Yield stress in principal 2-direction
SIGOZ = Yield stress in principal 3-direction
SIGXY = Shear yield stress in principal 1-2 plane

CARD 4: RMOSN = If RMOSS \neq 0; RMOSN = n, the shape parameter used in Ramberg-Osgood representation of stress-strain behavior

If RMOSS = 0; RMOSN = $\bar{\alpha}$ the slope of the linear strain hardening stress-strain representation, i.e., $\bar{\alpha} = E_T / E$ where $E_T$ is the tangent modulus
If RMOSS ≠ 0; RMOSS = Ramberg-Osgood parameter $\sigma_{0.7}$

Note 1: If RMOSN = 0 and RMOSS = 0, the material for the element(s) is assumed to be elastic-ideally plastic.

RMOSE = Ramberg-Osgood parameter E (Young's modulus)

YLDST = Yield stress in tension

YLDSC = Yield stress in compression

Input for the yield stress in tension and compression has been maintained in order to accommodate materials that exhibit initial anisotropic plastic behavior. In this case an initial translation of the yield surface is made consistent with the kinematic hardening theory.

Note: Only initially isotropic materials can be treated when considering linear or nonlinear strain hardening using this property card.

Succeeding cards give applicable members; both shorthands of Section III are permitted. Any number of continuation cards may be used for a given specification. A zero or blank 15 field ends each member listing.

MBET - Orientation of Axes of Material Anisotropy - FORMAT (A4, 1X, E15.7, /, (1615))

CARD 1: MBET

BETF = Angle $\beta$ in degrees between local x-axis and principal 1-axis for material orthotropy. See Fig. 4. Only applicable for TRIM elements.

Note: This card is an optional card. The default is BETF = 0.0 for all members. It should be used with initially orthotropic materials.

Fig. 4 Orientation of Element Local and Material Axes

Succeeding cards give applicable members; both shorthands of Section III are permitted. Any number of continuation cards may be used for a given specification. A zero or blank I5 field ends each member listing.

MSTG - Stringer Properties - FORMAT (A4, 1X, 5E15.7, /E15.7/, (16I5))

CARD 1: MSTG

E = Young's modulus
A = Cross sectional area
RMOSN = If RMOSS ≠ 0; RMOSN = n, the shape parameter used in Ramberg-Osgood representation of stress-strain behavior.
If \( RMOSS = 0; \) \( RMOSS = \alpha, \) the slope of the linear strain hardening stress-strain representation, i.e., \( \alpha = E_T/E \) where \( E_T \) is the tangent modulus.

\[ \text{RM OSS} = \text{If } \text{RM OSS} \neq 0; \text{RM OSS} = \text{Ramberg-Osgood parameter } \sigma_{0.7} \]

Note 1: If \( RMOSS = 0 \) and \( RMOSS = 0, \) the material for the element(s) is assumed to be elastic-ideally plastic.

\[ \text{YLDST} = \text{Yield stress} \]

**CARD 2:** \( \text{TALF} = \text{Coefficient of thermal expansion} \)

Succeeding cards give applicable members; both shorthands of Section III are permitted. Any number of continuation cards may be used for a given specification. A zero or blank I5 field ends each member listing.

**THIK - Element Thicknesses - FORMAT (A4, 1X, E15.7)**

Necessary with membrane elements.

**CARD 1:** \( \text{THIK} \)

\[ \text{THICK} = \text{Element thickness} \]

Succeeding cards give applicable members; both shorthands of Section III are permitted. Any number of continuation cards may be used for a given specification. A zero or blank I5 field ends each member listing.

**MBM - Beam Material Properties - FORMAT (A4, 1X, 5E15.7, /, E15.7, /, (16I5))**

**CARD 1:** \( \text{MBM} \)

\[ \text{E} = \text{Young's modulus} \]
ANU = Poisson's ratio

RMOSS = If not equal to zero, RMOSS equals Ramberg-Osgood parameter, \( \sigma_{0.7} \)

RMOSN = If RMOSS \( \neq 0 \); \( \sigma_{0.7} \), the shape parameter used in Ramberg-Osgood representation of stress-strain behavior.

If RMOSS = 0; \( \sigma_{0.7} \), the slope of the linear strain hardening stress-strain representation, i.e., \( \bar{\sigma} = E_T/E \) where \( E_T \) is the tangent modulus

YIDST = Yield stress

CARD 2: TALF = Coefficient of thermal expansion

Succeeding cards give applicable members; both shorthands of Section III are permitted. Any number of continuation cards may be used for a given specification. A zero or blank 15 field ends each member listing.

Beam Section Properties - FORMAT (A4, 1X, 5E15.7, /, 3E15.7, /, 3E15.7, /, 1615))

The cards specifying beam section properties all start with the following information:

CARD 1: SREC
    SCIR
    ISEC
    ZSEC Defined at the beginning of the section.
    HCIR The different cross sections are shown
    HREC in Tables 1 and 2.
    LSEC
    TSEC
    CSEC

5-16
A = cross-sectional area
I_{yy} = moment of inertia about y axis (see Fig. 5)
I_{zz} = moment of inertia about z axis (see Fig. 5)
I_{yz} = product of inertia
J = torsional rigidity

CARD 2: Y_0 = Eccentricity of attachment point in the y_0 direction
Z_0 = Eccentricity of attachment point in the z_0 direction
\bar{\beta} = angle defining the transformation of the I_y, I_z, I_{yz} to another reference axis (see Fig. 5)

Additional cards are required according to which section is specified. The notation for each section is shown in Table 1.

Fig. 5 Definition of Coordinate Axes in Cross Section of Beam Element
SREC - FORMAT (2E15.7)

CARD 3:  A  = Width
         B  = Depth

SCIR - FORMAT (E15.7)

CARD 3:  R  = Radius

ZSEC, ISEC, and CSEC - FORMAT (3E15.7, /, 3E15.7)

CARD 3:  A1  = Dimension of upper flange
         A2  = Dimension of web
         A3  = Dimension of lower flange

CARD 4:  T1  = Thickness of upper flange
         T2  = Thickness of web
         T3  = Thickness of lower flange

HCIR - FORMAT (2E15.7)

CARD 3:  R  = Outer radius
         T  = Thickness

HREC, LSEC, and TSEC - FORMAT (4E15.7)

CARD 3:  A1  = Width (parallel to y-axis)
         A2  = Depth (parallel to z-axis)
         T1  = Thickness of upper and/or lower flanges
         T2  = Thickness of vertical webs

Succeeding cards give applicable members; both shorthands of Section III are permitted. Any number of continuation cards may be used for a given specification. A zero or blank I5 field ends each member listing.

SEND      Ends the section.
<table>
<thead>
<tr>
<th>Section</th>
<th>Keyword</th>
<th>Input</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solid Rectangle</td>
<td>SREC</td>
<td>Area, moments of inertia, $I_{yy}$, $I_{zz}$, $I_{yz}$, J eccentricities, $y_0$, $z_0$, transformation angle $\beta$. Section width and height, $a$, $b$.</td>
</tr>
<tr>
<td>Solid Circular</td>
<td>SCIR</td>
<td>Area, moments of inertia, $I_{yy}$, $I_{zz}$, $I_{yz}$, J eccentricities, $y_0$, $z_0$, transformation angle $\beta$. Section radius, $r$.</td>
</tr>
<tr>
<td>Hollow Circular</td>
<td>HCIR</td>
<td>Area, moment of inertia, $I_{yy}$, $I_{zz}$, $I_{yz}$, J eccentricities, $y_0$, $z_0$, transformation angle $\beta$. Outer radius, $r$, thickness, $t$.</td>
</tr>
</tbody>
</table>
### Table 1: Section Input and Geometry (Cont)

<table>
<thead>
<tr>
<th>Section</th>
<th>Keyword</th>
<th>Input</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hollow Rectangular Section</td>
<td>HREC</td>
<td>Area, moment of inertia, $I_{yy}$, $I_{zz}$, $I_{yz}$, $J$ eccentricities, $y_o$, $z_o$, transformation angle $\beta$. Width, $a_1$, depth, $a_2$, thickness of upper and lower flanges, $t_1$, thickness of vertical webs, $t_2$.</td>
</tr>
<tr>
<td>L-Section</td>
<td>LSEC</td>
<td>Area, moment of inertia, $I_{yy}$, $I_{zz}$, $I_{yz}$, $J$ eccentricities, $y_o$, $z_o$, transformation angle $\beta$. Dimension of flange and web $a_1$, $a_2$, thickness flange, and web, $t_1$, $t_2$.</td>
</tr>
<tr>
<td>T-Section</td>
<td>TSEC</td>
<td>Same as for L-Section</td>
</tr>
<tr>
<td>Section</td>
<td>Keyword</td>
<td>Input</td>
</tr>
<tr>
<td>--------------</td>
<td>---------</td>
<td>----------------------------------------------------------------------</td>
</tr>
</tbody>
</table>
| Z-Section    | ZSEC    | Area, moments of inertia, I<sub>yy</sub>, I<sub>zz</sub>, I<sub>yz</sub>,
|              |         | J eccentricities, y<sub>0</sub>, z<sub>0</sub>, transformation     |
|              |         | angle \( \beta \). Dimension of upper and lower flange, \( a_1 \), \( a_3 \).
|              |         | Dimension of web, \( a_2 \), thickness of upper and lower flange, \( t_1 \), \( t_3 \),
|              |         | thickness of web, \( t_2 \).                                          |
|              |         | NOTE: \( a_1 = a_3 \); \( t_1 = t_3 \)                              |
| I-Section    | ISEC    | Same as Z-section \( a_1 \neq a_3 \); \( t_1 \neq t_3 \)             |
| Channel      | CSEC    | Same as Z-Section \( a_1 = a_3 \); \( t_1 = t_3 \)                  |

![Diagram of section geometry](image)
### Table 2: Number and Location of Stress Points

<table>
<thead>
<tr>
<th>Section</th>
<th>Keyword</th>
<th>Number Stress Points</th>
<th>Location of Stress Points</th>
<th>Location of Stress Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solid</td>
<td>SREC</td>
<td>16</td>
<td>± y</td>
<td>± z</td>
</tr>
<tr>
<td>Rectangle</td>
<td></td>
<td></td>
<td>1: 0.430568a 0.430568b</td>
<td>5: 0.169991a 0.430568b</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>3: 0.430568a 0.169991b</td>
<td>7: 0.169991a 0.430568b</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>9: 0.169991a 0.430568b</td>
<td>13: 0.169991a 0.430568b</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>11: 0.169991a 0.169991b</td>
<td></td>
</tr>
<tr>
<td>Solid</td>
<td>SCIR</td>
<td>18</td>
<td>r ± θ°</td>
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<tr>
<td>Circular</td>
<td></td>
<td></td>
<td>2: 0.887a 12.1</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>4: 0.887a 60.98</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>6: 0.887a 137.05</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>8: 0.1127a 12.1</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>10: 0.1127a 60.98</td>
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</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>12: 0.1127a 137.05</td>
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</tr>
<tr>
<td></td>
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<td>14: 0.500a 12.1</td>
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</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>16: 0.500a 60.98</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>18: 0.500a 137.05</td>
<td></td>
</tr>
</tbody>
</table>

Shear Center at Centroid

---

*Note: \(r\) represents the radial distance from the center, and \(\theta\) represents the angle.*
<table>
<thead>
<tr>
<th>Section</th>
<th>Keyword</th>
<th>Number Stress Points</th>
<th>Location of Stress Points</th>
<th>Location of Stress Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hollow Circular</td>
<td>HCIR</td>
<td>8</td>
<td>r</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1 a 352.8</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>2 a 7.2</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>3 a 323.4</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>4 a 36.6</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>5 a 274.6</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>6 a 85.4</td>
<td></td>
</tr>
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<td></td>
<td></td>
<td></td>
<td>7 a 213.0</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>8 a 147.0</td>
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<tr>
<td>Hollow Rectangular</td>
<td>HREC</td>
<td>12</td>
<td>±y ±z</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1 0.77459a₁ 0.5a₂</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>3 0 0.5a₂</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>7 0.5a₁ 0.77459a₂</td>
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</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>9 0.5a₁ 0 0</td>
<td></td>
</tr>
</tbody>
</table>

\[
\overline{a_1} = a_1 - t_2
\]

\[
\overline{a_2} = a_2 - t_1
\]
<table>
<thead>
<tr>
<th>Section</th>
<th>Keyword</th>
<th>Number of Stress Points</th>
<th>Location of Stress Points</th>
<th>Location of Stress Points</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>L-Section</strong></td>
<td>LSEC</td>
<td>6</td>
<td>1 (0.88730a_2, 0.0)</td>
<td>(y\text{-}z) axes at Shear Center</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>2 (0.11271a_2, 0.0)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>3 (0.50000a_2, 0.0)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>4 (0.0, 0.88730a_1)</td>
<td></td>
</tr>
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<td>5 (0.0, 0.11271a_1)</td>
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</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>6 (0.0, 0.50000a_1)</td>
<td></td>
</tr>
</tbody>
</table>

\[ a_1 = a_1 - \frac{t_2}{2}, \quad a_2 = a_2 - \frac{t_1}{2} \]

| **T-Section** | TSEC | 6 | 1 \(0.77459a_1, 0.0\) | \(y\text{-}z\) axes at Shear Center |
| | | | 2 \(-0.77459a_1, 0.0\) | |
| | | | 3 \(0.0, 0.0\) | |
| | | | 4 \(0.0, 0.88730a_2 + \frac{t_1}{2}\) | |
| | | | 5 \(0.0, 0.11271a_2 + \frac{t_1}{2}\) | |
| | | | 6 \(0.0, 0.50000a_2 + \frac{t_1}{2}\) | |

\[ a_2 = a_2 - t_1 \]
TABLE 2 NUMBER AND LOCATION OF STRESS POINTS (CONT)

<table>
<thead>
<tr>
<th>Section</th>
<th>Keyword</th>
<th>Number Stress Points</th>
<th>Location of Stress Points</th>
<th>Location of Stress Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>Z-Section</td>
<td>ZSEC</td>
<td>9</td>
<td>y z</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>1 -0.88730a_1</td>
<td>-a_2</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>2 -0.11271a_1</td>
<td>-a_2</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>3 -0.5000a_1</td>
<td>-a_2</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>4 0.88730a_1</td>
<td>-a_2</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>5 0.11271a_1</td>
<td>-a_2</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>6 0.5000a_1</td>
<td>-a_2</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>7 0.0</td>
<td>0.77459a_2</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>8 0.0</td>
<td>0.77459a_2</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>9 0.0</td>
<td>0.0</td>
<td></td>
</tr>
</tbody>
</table>

\[ \bar{a}_2 = \frac{(a_2 - t_1)}{2} , \ t_1 = t_3 \]

Shear Center at Centroid

\[ a_2' = (a_2 - t_1) , \ a_1 = a_3 \]
<table>
<thead>
<tr>
<th>Section</th>
<th>Keyword</th>
<th>Number Stress Points</th>
<th>Location of Stress Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>Channel Section</td>
<td>CSEC</td>
<td>9</td>
<td>y: 0.88730(a_1 + e_s), 0.5(a_2)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>2: 0.11271(a_1 + e_s), 0.5(a_2)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>3: 0.50000(a_1 + e_s), 0.5(a_2)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>4: 0.88730(a_1 + e_s), -0.5(a_2)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>5: 0.11271(a_1 + e_s), -0.5(a_2)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>6: 0.50000(a_1 + e_s), -0.5(a_2)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>7: (e_s), 0.77459(a_2)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>8: (e_s), -0.77459(a_2)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>9: (e_s), 0.0</td>
</tr>
</tbody>
</table>

\(e_s\) - distance from shear center to web center line

\[
e_s = \frac{1/2(a_1 - t_2)}{1 + 1/6 \left[ \frac{t_2(a_2 - t_1)}{t_1(a_1 - t_2)} \right]}, \quad a_2 = a_2 - t_1
\]

NOTE: \(t_1 = t_3\); \(a_1 = a_3\)
<table>
<thead>
<tr>
<th>Section</th>
<th>I-Section</th>
<th>ISBC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Keywrods</td>
<td>Number of Stress Points</td>
<td>Location of Stress Points</td>
</tr>
<tr>
<td>I-Section ISEC</td>
<td>9</td>
<td>y z</td>
</tr>
<tr>
<td>1</td>
<td>0.774569a1</td>
<td>I b1</td>
</tr>
<tr>
<td>2</td>
<td>-0.77459a1</td>
<td>F A-j</td>
</tr>
<tr>
<td>3</td>
<td>0.0</td>
<td>A2</td>
</tr>
<tr>
<td>4</td>
<td>0.77459a3</td>
<td>b2</td>
</tr>
<tr>
<td>5</td>
<td>0.77459a3</td>
<td>T3</td>
</tr>
<tr>
<td>6</td>
<td>0.0</td>
<td>b3</td>
</tr>
<tr>
<td>7</td>
<td>0.0</td>
<td>+0.8873h</td>
</tr>
<tr>
<td>8</td>
<td>0.0</td>
<td>+0.1127b2</td>
</tr>
<tr>
<td>9</td>
<td>0.0</td>
<td>+0.5(b1 + 0.5t)</td>
</tr>
</tbody>
</table>

Axes at Shear Center: a2 = a2 - 0.5(t1 + t3); b1 = b1 - 0.5t1; b2 = b2 - 0.5t2

Location of Stress Points:
- \( b_1 = \frac{t_3}{t_1} \) (a3)
- \( b_2 = \frac{t_1}{t_3} \) (a1)
- \( b_3 = b_1 + 0.5b_2 \)
- \( b_3 = b_2 + 0.5b_1 \)
- \( b_2 = 0.8873b_1 + 0.1127b_2 \)
- \( b_1 = 0.1127b_1 + 0.8873b_2 \)
XI. Applied Loads

Two different types of loading are currently admissible — concentrated loads at nodes, and distributed line loads on beams. Each loading situation is designated by one of the following four-character key words:

- CONC (Concentrated force or moment at a node)
- BMLO (Distributed line load, beam element)
- SEND (Section end)

Note: The keywords for the section on applied loads are used as section headings. The keyword appearing on an input card designates that the input to follow is associated with a particular type of applied loading. A blank card (as described in succeeding sections) delimits the input and indicates that the next card contains a different keyword. For example, CONC indicates input that follows is for concentrated loads and TRIA; distributed loads on an edge of a membrane triangle. Thus the input is as follows:

```
CONC
  data
  for
    concentrated
    loads
  blank card
BMLO
SEND
```

It should be noted that the keywords can appear in any order and may be specified more than one time in the applied load section.
CONC - Concentrated loads - FORMAT (A4, /, I5, 3E15.7, /, 5X, 3E15.7)

Each card gives the load components in global directions at a specified node.

CARD 1: CONC

CARD 2: I5 field: Node number
First E15.7 field: Force component $F_x$
Second E15.7 field: Force component $F_y$
Third E15.7 field: Force component $F_z$

CARD 3: First E15.7 field Moment component $M_x$
Second E15.7 field Moment component $M_y$
Third E15.7 field Moment component $M_z$

A blank card (i.e., zero or blank first I5 field) ends the specification of concentrated loads.

BMLO - Distributed Line Load, Beam Element - FORMAT (A4, /, 4E15.7)

CARD 1: BMLO

CARD 2: PYI Force/unit length in local y-direction (see Fig. 5) at node i
PYJ Force/unit length in local y-direction at node j
PZI Force/unit length in local z-direction at node i
PZJ Force/unit length in local z-direction at node j

Note: A linear variation of the distributed load between nodes is assumed.
Succeeding cards give applicable members; both shorthands of Section III are permitted. Any number of continuation cards may be used for a given specification. A zero or blank I5 field ends each member listing.

A card with SEND in the first four columns ends the section for applied loading.

XII. Members to be Printed — FORMAT (16I5)

Specify the members whose strains and stresses are to be printed. Both shorthands of Section III are allowed. A maximum of 600 members may be specified. Members in excess of 600 and undefined member numbers are ignored. A blank card or card with only zero entries ends the section.

XIII. Nodes to Be Printed — FORMAT (16I5)

Up to 900 nodes whose displacements are to be printed for the analysis, as per Section XII.

XIV. Parameters for Succeeding Load Cycles — FORMAT (3I5, /, 3E15.7)

CARD 1: NPRNT = If equal to zero, no additional load cycle, end of problem. If greater than zero, print output every NPRNT increments of load

FORM = Reform elastic stiffness matrix every NFORM increments of load

NEWL = If equal to zero, no new load distribution is to be input. If greater than zero, a new load distribution will be read in

CARD 2: PMAX = New maximum load factor

DELP = Number of load increments for new load cycle

PTAN = Load at which full tangent modulus is used

5-30
XV. **New Applied Loads**

New load distribution is read in if $\text{NNEWL} \neq 0$. Format is identical to Section XI.

XVI. **Problem End - FORMAT (A4)**

Each problem's input must be ended with a card reading "ENDb" where "b" denotes a blank, in columns 1-4. The last problem in a run should end with a card reading "STOP" in column 1-4 instead of "ENDb."
XVII. Restarting a Problem

The initial restart tape is created on Unit 21. Subsequent
restart jobs mount the restart tape as Unit 21 and if desired
(NUTAP > 0) copies and continues the restart tape on Unit 22.

CARD 1: Problem Title - FORMAT (2A4)
      As in Section I

CARD 2: NPNTC, NPRNT, IFORM, IREST, NUTAP, INPRT, NRSRT
      As in Section II. Here NPNTC is ignored and
      IREST = 2

NRSRT is a unique number obtained as output from the
job that generated the restart tape. This output is
of the form:

* * * * *

RESTART TAPE HAS BEEN WRITTEN FOR P = 7.100000E+04
NRSRT = 8

* * * * *

In this case NRSRT is equal to 8 and the job will be
restarted at a load factor of 71.0. A restart job
can be started at any intermediate load level for
which the restart tape has been written.

CARD 3: PMAX, DELP, PTAN, EQMULT
      As in Section II

Succeeding cards follow as in Sections XII through XVI.

5-32
APPENDIX I — MULTIPLE GONSTRAINTS

EXAMPLE 1

Node 1 and 2 are fixed. There is an applied displacement at node 3 in the negative y-direction. The distance between nodes 3 and 4 remain constant.

Boundary Conditions

<table>
<thead>
<tr>
<th>Card</th>
<th>Node 1</th>
<th>Node 2</th>
<th>Node 3</th>
<th>Node 4</th>
<th>Default Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>011000</td>
<td>1-4 default condition</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>000000</td>
<td>1 2 4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>021000</td>
<td>3 Blank card</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Dependencies

<table>
<thead>
<tr>
<th>Node</th>
<th>Dof</th>
<th>Node</th>
<th>Dof</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>2</td>
<td>3</td>
<td>2</td>
<td>0.050 (specified displacement)</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>3</td>
<td>2</td>
<td>1.0   restraint condition that</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>1.0   3-4 remain rigid</td>
</tr>
</tbody>
</table>

NOTE: In the boundary condition cards for node 4, degrees of freedom 2 and 3 must be specified 0 because they are effectively eliminated from the solution (they are dependent degrees of freedom).
EXAMPLE 2

Rotation of Displacements at a Boundary

For this problem we set the displacement \( \bar{u} \) equal to zero and \( \bar{v} \) free. This corresponds to the normal displacement along \( \bar{x} \) equal to zero and the tangential displacement (in \( \bar{y} \) direction) free. The global coordinates of the problem are \( x, y \). Thus:

\[
\begin{align*}
    u &= \bar{u} \cos \varphi - \bar{v} \sin \varphi \\
    v &= \bar{u} \sin \varphi + \bar{v} \cos \varphi
\end{align*}
\]

The multipoint constraint cards are:

\[
\begin{array}{cccccccc}
25 & 1 & 25 & 1 & 0.866025400E00 & 25 & 2 & -0.50E00 \\
25 & 2 & 25 & 1 & 0.50E00 & 25 & 2 & 0.866025400E00 \\
\end{array}
\]

Note: On the boundary condition cards, since \( \bar{u} = 0.0 \) node 25 degree of freedom 1 is specified to be 0 but node 25 degree of freedom 2 is specified 1 or free since it remains an independent degree of freedom (although rotated). If the normal displacement \( \bar{u} \) were not fixed (and the transformation performed merely to apply a normal load), it would have a 1 boundary condition specified.
INSTRUCTIONS FOR USE
OF SATELLITE

A Preprocessing Program
for PLANS
INSTRUCTIONS FOR USE OF SATELLITE

SATELLITE is the preprocessing program associated with the PLANS system programs that checks and plots the undeformed finite element model. Additionally, it calculates the ordering of the external node numbers that leads to the minimum semibandwidth. SATELLITE is currently not operational for the HEX program.

The input for SATELLITE is as follows:

CARD 1: FORMAT (20A4)

Keywords left justified in fields of four. One of the following is required and indicates the program whose data follows.

- PLNE - PLANE program, In-Plane Membrane Program
  Used for Fracture Analysis
- BEND - BEND program, Elastic-Plastic Analysis of
  Built-Up Structures
- REVB - REVBY program, Elastic-Plastic Analysis of
  Bodies of Revolution
- OPGM - OUT-OF-PLANE GM, Nonlinear Analysis of
  Built-Up Structures
- DYCA - DYCAST program, Nonlinear Dynamic Analysis
  of Built-Up Structures

Additional optional keywords can be specified in fields of four. Any number of spaces can be skipped on the input card that are multiples of four.

- BAND - Specifies that the optimum semibandwidth is to be calculated. In this case the optimum order of the external node numbers for

6-1
Section III data will be printed and punched. These punched data are used with the input deck. If this keyword is not specified the bandwidth will not be optimized.

SCAN - Process and check input data but do not produce any plots

PRIN - Print transformed coordinates to be plotted

The input data deck follows Card 1. Input through the nodal co­ordinates is necessary. The remainder of the deck may be input although the cards are read and not processed until a STOP or END card is reached. Input for the SATELLITE program continues as follows.

CARD 2: FORMAT (5I5)

This card determines the number and type of plots. Up to five different labeled pictures can be obtained. These are specified with the fixed point numbers right justified in any of the fields of five as follows:

1 - labeled unconnected nodes
2 - labeled nodes, unlabeled members
3 - labeled nodes and members
4 - unlabeled nodes, labeled members
5 - unlabeled nodes and members

CARD 3: FORMAT (4E15.7)

ALPHA - rotation of structure about the global x-axis
BETA - rotation of structure about the global y-axis
GAMMA - rotation of structure about the global z-axis.
LENGTH - maximum width of picture in the paper transverse direction. LENGTH is a function of the plotter being used.

If ALPHA, BETA, GAMMA are zero then the picture is projected on the x-y plane with the y direction in the transverse paper direction. The angles lead to a path dependent transformation by rotating the body first about the global x then y and z axes, respectively. The resulting viewing plane is still the original global x-y plane. The viewing plane can be changed with the following card.

CARD 4: FORMAT (6A1)

This card specifies the viewing plane to be used by specifying the six alphanumeric characters XYZH-I. The first two characters specifying the viewing plane with the first indicating the length direction and the second the width of the paper. The last three characters can be the characters + or - and indicate that the normal to the plane is in the positive or negative coordinate direction. For example:

YZX++ indicates the viewing plane as the global YZ plane with the Z axis in the paper width direction and Y along the paper length

CARD 5: FORMAT (4(A4,1X))

Element types to be omitted. For BEND, OPGM, DYCAST:

BEAM - will omit all beam elements
TRIM - will omit all triangle membrane elements
STGR - will omit all stringer elements
TRIP - will omit all triangular plate elements

For REVBY:
TRIR - will omit all triangle elements
SHEL - will omit all shell elements
RING - will omit all ring elements

If there are no element types to be omitted a blank card must be specified.

CARD 6: FORMAT (16I5)

Nodes to be included in the plot. All the shorthand notations are allowed, i.e., 1 through 100 is input as 1-100 and 1, 3, ..., 99 as 1-2-99. A blank card or card with only zero entries ends the section.

If only a part of the structure is to be plotted, this section should include only the nodes that lie in the section. Only these nodes are considered when scaling the plot so that the subsection will be scaled up to the maximum allowable size.

CARD 7: FORMAT (16I5)

Elements to be included in the plot. Input is the same as for Card 6. All nodes associated with the elements to be plotted must be specified with Card 6.

Note: If all the elements and nodes are to be plotted it is sufficient to specify one negative number in the first I5 field whose magnitude is greater than the largest external element/node number in the finite element grid.
CARD 8: FORMAT (A4)

If this card is blank and

1. END was read at the end of the data deck a new data deck is read starting from Card 1

2. STOP was read at the end of the data deck, the job is complete

If this card is REPT the input will be repeated from Card 2 and the same structure with different plotting parameters will be plotted.
EXAMPLE INPUT
EXAMPLE INPUT

The following pages contain the input data decks for a number of sample problems. The principal intent in providing sample input is to aid the user in better understanding the discussion in the main body of the users' manual. Also provided with each problem is a brief discussion and representative data so that benchmark results are available for comparison. A detailed picture of the sample problem idealizations showing node and element labeling has not been provided. It is suggested that the input plotting program SATELLITE be used to generate computer plots of the input data, if desired.

Many sample problems that demonstrate the capability of the methods used in the PLANS program can be found in Refs. 1-3.
Example Problem No. 1

Program: BEND

Problem Title: Classical Truss - Axial Force Element

Comments: This problem uses the stringer element STGR in a simple 4 element truss. Symmetry conditions were used so that only half the truss was modeled.

The accompanying figure shows the applied load versus the force in the vertical element (element 3). These results were generated using a load step of $\Delta P = 848.5$ lbs or 3774.3 N (which is 0.01 of the initial yield load). Agreement with results from an exact analysis was quite good.
Bend Sample Problem #1 Classical Truss STRG Axial Force Elements
PROGRAM LISTING OF INPUT DATA CARDS

BEND SAMPLE PROBLEM #1 CLASSICAL TRUSS STRG AXIAL FORCE ELEMENTS

```
0  50
  1 -4
STRG  1  1  2
STRG  2  2  4
STRG  3  3  3
STRG  4  1  4
SEND
   0.0  1
   .5  2  3
   1.0  4
BLANK CARD
   0.0  1  4  3
   .5  2
BLANK CARD
   0.0  1  -4
BLANK CARD
  000000  1  3
  110000  2
  100000
BLANK CARD
BLANK CARD
MSTG  1.0  E&07  1.0  E&00  10.0  E&00  1.0  E&05  6.0  E&04
BLANK CARD
MSTG  1  -3  E&07  0.5  E&00  10.0  E&00  1.0  E&05  6.0  E&04
BLANK CARD
SEND
   4  0.5  E&00  0.0  E&00  0.0  E&00
BLANK CARD
BLANK CARD
SEND
   1  -4
BLANK CARD
   -4
BLANK CARD
  250000.0
BLANK CARD
STOP
```

STOP
Example Problem No. 2

Program: BEND

Problem Title: NASA Frame with an End Load - BEAM ELEMENTS

Comments: This problem demonstrates the use of beam elements in an open space framework. Fifty seven beam elements of various cross section (L-section, T-section, rectangular section) connecting 40 nodes are used in the model. Multipoint constraints were used to enforce symmetry conditions between the left and right hand side of the structure. Ideally plastic behavior was assumed with a yield stress of 50130 psi or 345.6 MPa.

The figure shows displacement in the vertical and axial directions at the first bulkhead at the centerline (node 108) and at the midpoint between the first two bulkheads in the upper stringer (node 1001). The results indicate a plastic collapse at a load of $P = 15000$ lbs or 66.7 KN.
Bend Sample Problem #2 NASA Frame with End Load

VERTICAL DEFLECTION
FIRST BAY FRAME
NODE 108

VERTICAL DEFLECTION
NODE 1001

DISPL. IN DIRECTION
OF THE LOAD AT
NODE 108

TOTAL LOAD
**PROGRAM LISTING OF INPUT DATA CARDS**

---

**BEND SAMPLE PROBLEM 2** NASA FRAME WITH AN END LOAD - BEAM ELEMENTS

<table>
<thead>
<tr>
<th>Beam</th>
<th>1</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>101</td>
<td>1001</td>
<td>102</td>
</tr>
<tr>
<td>Beam</td>
<td>Beam</td>
<td>Beam</td>
</tr>
<tr>
<td>101</td>
<td>1001</td>
<td>102</td>
</tr>
<tr>
<td>101</td>
<td>1001</td>
<td>102</td>
</tr>
<tr>
<td>101</td>
<td>1001</td>
<td>102</td>
</tr>
</tbody>
</table>
**PROGRAM LISTING OF INPJT DATA CARDS**

<table>
<thead>
<tr>
<th>BEAM</th>
<th>58</th>
<th>205</th>
<th>208</th>
<th>201</th>
</tr>
</thead>
<tbody>
<tr>
<td>BEAM</td>
<td>64</td>
<td>311</td>
<td>303</td>
<td>316</td>
</tr>
<tr>
<td>BEAM</td>
<td>66</td>
<td>311</td>
<td>316</td>
<td>303</td>
</tr>
<tr>
<td>BEAM</td>
<td>68</td>
<td>403</td>
<td>400</td>
<td>406</td>
</tr>
<tr>
<td>BEAM</td>
<td>72</td>
<td>411</td>
<td>403</td>
<td>406</td>
</tr>
<tr>
<td>BEAM</td>
<td>73</td>
<td>411</td>
<td>414</td>
<td>406</td>
</tr>
<tr>
<td>BEAM</td>
<td>75</td>
<td>1001</td>
<td>101</td>
<td>105</td>
</tr>
<tr>
<td>BEAM</td>
<td>76</td>
<td>201</td>
<td>1001</td>
<td>105</td>
</tr>
<tr>
<td>BEAM</td>
<td>77</td>
<td>1003</td>
<td>201</td>
<td>105</td>
</tr>
<tr>
<td>BEAM</td>
<td>78</td>
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<td>105</td>
</tr>
<tr>
<td>BEAM</td>
<td>79</td>
<td>1006</td>
<td>303</td>
<td>105</td>
</tr>
<tr>
<td>BEAM</td>
<td>80</td>
<td>403</td>
<td>1005</td>
<td>103</td>
</tr>
<tr>
<td>BEAM</td>
<td>81</td>
<td>105</td>
<td>3001</td>
<td>101</td>
</tr>
<tr>
<td>BEAM</td>
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-13, 1553125E00 205 206 208
16, 3263125E00 303 304
-16, 3203125E00 311 312 316
-8, 1615625E00 8002
20, 4375E00 406 408
-20, 4375E00 411 414 422
14, 7578125E00 2002 1003
18, 5790625E00 2004 1005
-11, 6484375E00 4000 3001 6000
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PROGRAM LISTING OF INPUT DATA CARDS

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PROGRAM LISTING OF INPUT DATA CARDS

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54  57  58  61  64  66  8  10  12  14  16  18  20  22  24  26  28  30

SREC  0.0E00  0.0E00  0.5E00  18.0E00  0.0E00  1.791

25  27

TSEC  4.474E00  0.048831  0.0729367  0.0  7.0293E-03

1.0  2.09

TSEC  4.474E00  0.048831  0.0729367  0.0  7.0293E-03

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LSEC  1.121E00  0.01099  0.01099  -0.07687  1.57675E-04

1.0  1.0  0.625  0.625

SEND

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101  -13

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102  -13

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105  -13

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106  -13

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106  -48

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SEND

29  -50

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-9000

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50000.0  0.01

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STOP
Example Problem No. 3

Program: BEND

Problem Title: 8 Element Square Plate

Comments: This problem represents a simple statically determinate case that uses all combinations of the triangular membrane element, TRIM. These range from 3-node constant strain triangles, the 6-node linear strain triangle, and the two interface cases with 4 and 5 nodes. A distributed edge load (using the TRIA feature) is applied on one edge.

The figure shows load versus longitudinal strain for a linear strain hardening material.
BEND SAMPLE PROBLEM #3 EIGHT ELEMENT SQUARE MEMBRANE

E = 1. x 10^7 PSI = 6.89 x 10^{10} Pa
\nu = .3
YIELD STRESS = 36000 PSI = 2.48 x 10^8 Pa
LINEAR HARDENING = \frac{E}{E_t} = .1
FRCGRAM LISTING OF INPUT DATA CARDS

SEND SAMPLE PROFILE = 3 8 ELEMENT SQUARE PLATE TRIM ELEMENTS

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BLANK CARD
FFCGRAM LISTING OF INPUT DATA CARDS

1. 2. 3. 4. 5. 6. 7. 8.
1234567890123456789012345678901234567890123456789012345678901234567890

SEND
-1000

BLANK CARD
55550.0

STOP

BLANK CARD

BLANK CARD
Example Problem No. 4

Program: BEND

Problem Title: Collapse of a Uniformly Loaded Circular Plate

Comments: This problem demonstrates the use of the plate element TRIP. One quadrant of a uniformly loaded, simply supported circular plate was modeled consisting of 50 elements, 36 nodes, and 151 degrees of freedom. The multipoint constraint feature was used to transform the edge rotations to their normal and tangential components in order to apply the simple support boundary conditions \( \theta_t = 0 \). To be noted also is that the order of the element input was set so that the local directions of the curvatures at each node (defined by the first element containing that node) line up with the plate boundaries and lines of symmetry. This was done to enforce boundary/symmetry conditions for the higher order degrees of freedom (see Appendix IV, Section 2).

A complete discussion of the plastic analysis of this plate as well as a number of other plate configurations is given in Ref. 2. The figure shown here is for the load versus central deflection up to the collapse load.
PERFECT PLASTICITY

\[ \sigma_o = 36000 \text{ psi} = 2.48 \times 10^6 \text{ Pa} \]
\[ E = 1.0 \times 10^7 \text{ psi} = 6.89 \times 10^{10} \text{ Pa} \]
\[ \nu = 0.24 \]
\[ h = 1.0 \text{ IN.} = 0.0254 \text{ M} \]
\[ a = 10.0 \text{ IN} = 0.254 \text{ M} \]

LOAD VERSUS CENTRAL DEFLECTION

Bend Sample Problem #4 Uniformly Loaded Simply Supported Circular Plate
PROGRAM LISTING OF INPUT DATA CARDS

PEND SAMPLE PROBLEM = 4 UNIFORMLY LOADED CIRCULAR PLATE

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|      | 4.0 | 5  |    |    |    |    |
|      | 3.74162 | 6  |    |    |    |    |
|      | 2.82844 | 7  | 14 | 23  | 34  |
|      | 6.0 | 10 |    |    |    |
|      | 5.83332 | 11 |    |    |    |
|      | 5.29146 | 12 |    |    |    |
|      | 4.24221 | 13 | 22 | 33  |
|      | 8.0 | 17 |    |    |    |
|      | 7.874 | 18 |    |    |    |

1234567890123456789012345678901234567890123456789012345678901234567890
### Program Listing of Input Data Cards

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**Blank Cards**

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- Card 11: 1.0
- Card 16: 1.0
- Card 21: 1.0
- Card 26: 1.0
- Card 31: 1.0
- Card 36: 1.0

**Values**

- 

---

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7.48328  19
6.78232  20
5.65688  21
10.0      26
9.89545  27
9.59166  23
9.05532  29
8.24621  30
7.6711   31

BLANK CARD
```

---

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0.0      1 -36
BLANK CARD
```

---

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C0100  C00010  1
C0111  C00110  2  5  10  17
C0111  C00110  4  9  16  25
C0100  C00001  27 -35
C0C0  C0000  26  36
```

---

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27  5  27  5  14142199
28  4  28  5  -95916595
28  5  28  5  28284397
29  4  29  5  -90553799
29  5  29  5  42426596
30  4  30  5  -62462101
30  5  30  5  56568844
31  4  31  5  -70711002
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32  4  32  5  -56568844
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35  4  35  5  -14142199
35  5  35  6  98994904
36  4  36  5  0.0
36  5  36  5  1.0
```

---

```
BLANK CARD
```

---

```
WAT1  1.0  F+07  1.0  F+07  0.0  4.032258E+06  .24
```

---

```
```

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PROGRAM LISTING OF INPUT DATA CARDS

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1.0 1.0

STOP

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1.0 1.0
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STOP

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1.0 1.0
1.0 1.0
1.0 1.0

STOP
Example Problem No. 5

Program: BEND

Problem Title: Thermoplastic Analysis of a Circular Disk

Comments: This problem demonstrates the thermal stress capability in conjunction with the triangular membrane element. To do this, an elastic plastic analysis of an annular disk subjected to a steady state axisymmetric radial temperature distribution was performed. Since the response is axisymmetric, only a slice of the disk was modeled with the multipoint constraint feature being used to enforce tangential and normal boundary conditions on one edge. Forty constant strain triangular elements connecting 32 nodes were used in the idealization. The steady state temperature distribution is applied at nodes according to the relation

\[ T = T_a - (T_b - T_a) \frac{\ln(r/a)}{\ln(b/a)} \]

where \( T_a \) and \( T_b \) are the inner and outer radius temperature, respectively.

Results shown in the figure are for dimensionless circumferential stress versus the radial coordinate. Comparison between these results and Ref. 4 are good.
Finite Element Model of a Sector of the Disk

\[ E = 1 \times 10^7 \text{ psi} = 6.89 \times 10^9 \text{ Pa} \]
\[ a = 1.0 \times 10^{-5} \text{ sec}^{-1} = 1.8 \times 10^{-5} \text{ C}^{-1} \]
\[ \sigma_{0.1} = 4000 \text{ psi} = 2.76 \times 10^7 \text{ Pa} \]
\[ n = 5 \]
\[ \sigma_0 = 2000 \text{ psi} = 138 \times 10^7 \text{ Pa} \]

DISTRIBUTION OF CIRCUMFERENTIAL STRESS

Bend Sample Problem #5 Thermoplastic Analysis of Circular Disk

7-24
BEND SAMPLE PROBLEM #5 THERMOPLASTIC ANALYSIS OF CIRCULAR DISK

| TRIM | 01 02 03 04 05 06 07 08 09 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 |
|------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|
| 0    | 01               | 02               | 03               | 04               | 05               | 06               | 07               | 08               | 09               | 10               | 11               | 12               | 13               | 14               | 15               | 16               | 17               | 18               | 19               | 20               | 21               | 22               | 23               | 24               | 25               | 26               | 27               | 28               | 29               | 30               | 31               | 32               | 33               | 34               | 35               | 36               | 37               | 38               | 39               | 40               |
| SEND | 1.0 | E 00 | 3 | 6 |
|      | 1.1 | E 00 | 9 | 12 |
|      | 1.2 | E 00 | 15 | 18 |
|      | 1.3 | E 00 | 21 | 24 |
|      | 1.4 | E 00 | 27 | 30 |
|      | 1.5 | E 00 | 33 | 36 |
|      | 1.6 | E 00 | 39 | 42 |
|      | 1.7 | E 00 | 46 | 48 |
|      | 1.8 | E 00 | 51 | 54 |
|      | 1.9 | E 00 | 57 | 60 |
|      | 2.0 | E 00 | 63 | 66 |

| 1.09452189E 00 | E 00 | 1 | 4 |
| 1.0939741 E 00 | E 00 | 7 | 10 |
| 1.0934262 E 00 | E 00 | 13 | 16 |
| 1.0928784 E 00 | E 00 | 19 | 22 |
| 1.0923306 E 00 | E 00 | 25 | 28 |
| 1.0917828 E 00 | E 00 | 31 | 34 |
| 1.0912350 E 00 | E 00 | 37 | 40 |
| 1.08814254 E 00 | E 00 | 43 | 46 |
| 1.0891394 E 00 | E 00 | 49 | 52 |
| 1.08895916 E 00 | E 00 | 55 | 58 |
| 1.0890437 E 00 | E 00 | 61 | 64 |

| BLANK CARD | 0.0 | E 00 | 3 | -3 | -66 |
| 0.0523359 | E 00 | 2 | 5 |
| 0.0575695 | E 00 | 8 | 11 |
| 0.0628031 | E 00 | 14 | 17 |
| 0.0680367 | E 00 | 20 | 23 |
| 0.0732703 | E 00 | 26 | 29 |
| 0.0785039 | E 00 | 32 | 35 |
| 0.0837375 | E 00 | 38 | 41 |
| 0.0889711 | E 00 | 44 | 47 |
| 0.0942047 | E 00 | 50 | 53 |
## Program Listing of Input Data Cards

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*1046712 E 00 62 65
*1045284 E 00 1 4
*1149613 E 00 7 10
*1254341 E 00 13 16
*1358870 E 00 19 22
*1463988 E 00 25 28
*1567929 E 00 31 34
*1672455 E 00 37 40
*1770983 E 00 43 46
*1881512 E 00 49 52
*1986040 E 00 55 58
*2090569 E 00 61 64

BLANK CARD

0.0 1 63

BLANK CARD

110000000000 1 -3 7 -9 13 -15 19 -21 25 -27 31 -33 37
100000000000 1 -6 61 3 61 63

BLANK CARD

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7 2 7 1 *1045284 7 2 *9945218
13 1 13 1 *9945218 13 2 -1045284
13 2 13 1 *1045284 13 2 *9945218
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19 2 19 1 *1045284 19 2 *9945218
25 1 25 1 *9945218 25 2 -1045284
25 2 25 1 *1045284 25 2 *9945218
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37 2 37 1 *1045284 37 2 *9945218
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49 2 49 1 *1045284 49 2 *9945218
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55 2 55 1 *1045284 55 2 *9945218
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61 2 61 1 *1045284 61 2 *9945218

BLANK CARD

MAT1 1.0 E 07 1.0 E 07 0.0 E 00 3.846153 E 06 .30000E 00
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**Note:** The above text is a program listing of input data cards, likely for a computer or a specific application. The values and format suggest it is related to numerical data processing or scientific calculations.
## Program Listing of Input Data Cards

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**THIK**

1 - 40

**SEND**

**TMP**

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**SEND**

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**BLANK CARD**

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**BLANK CARD**

| 200.0 | 0.05 |

**BLANK CARD**

**STOP**
Example Problem No. 6

Program: BEND

Problem Title: Elastic Plastic Analysis of an SAE Keyhole Specimen

Comments: The accompanying figures show an SAE keyhole specimen, the finite element idealization of half of the plate, and results for the strain at the notch versus load. The idealization uses 92 constant strain elements connecting 64 nodes that leads to 120 degrees of freedom.
19.56 - 12.7 - 2.4
+---ALL DIMENSIONS
CENTIMETERS

15.24

.9525 DIA

.9525 THICK

1.9

3.81

SAE KEYHOLE SPECIMEN

FINITE ELEMENT IDEALIZATION

Bend Sample Problem #6 Load versus Notch Strain for SAE Keyhole Specimen
NO  TYPE ANALYSIS
1  NEUBER'S RULE - PLANE STRESS
3  NASTRAN - PLANE STRESS
5  EXPERIMENTAL
Δ PLANS - PLANE STRESS

Bend Sample Problem #6 (Continued)
## PROGRAM LISTING OF INPUT DATA CARDS

```
1.......... .2........ ...... 4
*..........
.. a.. *.. ...
*...........

BEND SAMPLE PROBLEM # 6  SAE KEYHOLE SPECIMEN

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1          -64
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TRIM    42  61  61  3
TRIM    43  69  61  1
TRIM    44  60  61  61
TRIM    45  60  57  62
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TRIM    47  67  63  62
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2.8000E 00 45
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2.4000E 00 49
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2.8500E 00 52
2.9500E 00 53
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2.7000F 00 64

BLANK CARD

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0.9000E 00 64

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BLANK CARD
BLANK CARD

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35.0000E 03 35.0000E 03 35.0000E 03
5.26320E 00 38.5890E 03 29.5000E 06
1 92

THIK
0.3750E 00
1 92

SEND
CUNC
15 0.0000E 00 -1.0000E 00 0.0000E 00
0.0000E 00 0.0000E 00
16 0.0000E 00 -1.0000E 00 0.0000E 00
0.0000E 00 0.0000E 00

BLANK CARD
BLANK CARD

SEND
1 92

BLANK CARD
1 -64

BLANK CARD
-7.5000E 33 0.0100E 00

STOP
Example Problem No. 7

Program: REVBY

Problem Title: Plastic Analysis of a Ring Stiffened Spherical Shell

Comments: This problem demonstrates the use of the shell and ring element. The structure consists of a uniformly loaded spherical shell with a stiffened circular hole at the shell apex. The problem is idealized using 26 axisymmetric shell elements and one ring element connecting 27 nodes.

The figure shows the normal displacement versus applied pressure at the ring shell interface and also results for an unstiffened shell. Sudden collapse occurs for the stiffened shell at $q\alpha^4/\chi^4 = 15000$.

Additional results for this problem are in Ref. 3.
LOAD-DEFLECTION CURVES FOR RING-STIFFENED SPHERICAL SHELL UNDER EXTERNAL PRESSURE

$\frac{w}{t}$

$E = 1.0 \times 10^7$ psi $= 6.90 \times 10^{13}$ Pa

$\nu = \frac{1}{3}$

$\sigma_0 = 30,000$ psi $= 2.07 \times 10^8$ Pa

Square Ring Stiffener

$E = 1.0 \times 10^7$ psi $= 6.90 \times 10^{10}$ Pa

$A = 0.030625 \text{ in.}^2 = 1.94 \times 10^{-6} \text{ m}^2$

$I_x = I_y = 0.78158 \times 10^{-4} \text{ in.}^4 = 3.253 \times 10^{-11} \text{ m}^4$

0 - Unstiffened Hole

$\Delta$ - Stiffened Hole

26 ELEMENTS

105 D.O.F.

(a) NORMAL DISPLACEMENT AT THE HOLE BOUNDARY

REVBY Sample Problem #1 Ring Stiffened Spherical Shell
PROGRAM LISTING OF INPUT DATA CARDS

REVBY SAMPLE PROBLEM 1 RING STIFFENED SPHERICAL SHELL

SHEL 1 1 1 2
SHEL 2 2 2 3
SHEL 3 3 3 4
SHEL 4 4 4 5
SHEL 5 5 5 6
SHEL 6 6 6 7
SHEL 7 7 7 8
SHEL 8 8 8 9
SHEL 9 9 9 10
SHEL 10 10 10 11
SHEL 11 11 11 12
SHEL 12 12 12 13
SHEL 13 13 13 14
SHEL 14 14 14 15
SHEL 15 15 15 16
SHEL 16 16 16 17
SHEL 17 17 17 18
SHEL 18 18 18 19
SHEL 19 19 19 20
SHEL 20 20 20 21
SHEL 21 21 21 22
SHEL 22 22 22 23
SHEL 23 23 23 24
SHEL 24 24 24 25
SHEL 25 25 25 26
SHEL 26 26 26 27
SHEL 27 27 27 28
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4 8724484 01 2
5 7448647 01 3
6 6170520 01 4
7 4891728 01 5
8 6517762 01 6
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- 2.76475 \( \times 10^1 \) 00 17
- 2.94904 \( \times 10^1 \) 00 18
- 3.13305 \( \times 10^2 \) 00 19
- 3.31678 \( \times 10^2 \) 00 20
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- 3.54604 \( \times 10^2 \) 00 22
- 3.66049 \( \times 10^2 \) 00 23
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- 3.91755 \( \times 10^2 \) 00 26
- 4.00013 \( \times 10^{-2} \) 02 27

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**SPEC**

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**BLANK CARD**

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PROGRAM LISTING OF INPJT DATA CARDS

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1.0000000E-05 27

BLANK CARD 0.0000000E+00 -1.0000000E+00 0.0000000E+00 1 -27

BLANK CARD

BLANK CARD

BLANK CARD 26 27

BLANK CARD -27

BLANK CARD 60.0 .01

BLANK CARD

STOP
Example Problem No. 8

Program: REVBY

Problem Title: Thick Tube Under Internal Pressure

Comments: This problem demonstrates the use of the axisymmetric revolved triangular element, TRIR. Five bays of four triangles each (20) are used to idealize thick tube under internal pressure.

A series of results are shown in the figure and compared with a solution in Ref. 5. Additional discussion of this problem is in Ref. 3.
PRESSURE $p$ VERSUS RADIAL
DISPLACEMENT $u$ (b) AND RADIUS
OF ELASTIC-PLASTIC BOUNDARY, $\rho$

DISTRIBUTION OF RADIAL STRESS

DISTRIBUTION OF CIRCUMFERENTIAL STRESS

REVBY Sample Problem #2 Thick Cylinder Under Internal Pressure
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*Note: Some data entries are missing.*
Example Problem No. 9

Program: REVBY

Problem Title: Thermoplastic Analysis of a Circular Disk

Comments: This problem is the same as sample problem 5 using the BEND program. Here 10 shell elements are used connecting 11 nodes leading to 22 degrees of freedom. Comparison made with the results of Ref. 4 indicate excellent agreement.
FINITE ELEMENT IDEALIZATION USING SHELL ELEMENTS

\[ E = 1.0 \times 10^7 \text{ psi} = 6.9 \times 10^{10} \text{ Pa} \]
\[ \alpha = 1.0 \times 10^{-5} \text{ F}^{-1} = 1.8 \times 10^{-8} \text{ C}^{-1} \]
\[ \sigma_{0.7} = 4000 \text{ psi} = 2.75 \times 10^7 \text{ Pa} \]
\[ n = 5 \]
\[ \sigma_o = 2000 \text{ psi} = 1.38 \times 10^7 \text{ Pa} \]

\[ \frac{\sigma_o}{E \alpha T_b} \]

\[ r/a \]

\[ \text{ANNULAR DISK} \]

DISTRIBUTION OF CIRCUMFERENTIAL STRESS

REVBY Sample Problem #3 Thermoplastic Analysis of a Circular Disk Shell Elements
REV3Y SAMPLE PROBLEM # 3 THERMOPLASTIC ANALYSIS OF CIRCULAR DISK - II

0 10
1 -11

SHEL 1 1 2
SHEL 2 2 3
SHEL 3 3 4
SHEL 4 4 5
SHEL 5 5 6
SHEL 6 6 7
SHEL 7 7 8
SHEL 8 8 9
SHEL 9 9 10
SHEL 10 10 11

SEND
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1.1000000E 00 2
1.2000000E 00 3
1.3000000E 00 4
1.4000000E 00 5
1.5000000E 00 6
1.6000000E 00 7
1.7000000E 00 8
1.8000000E 00 9
1.9000000E 00 10
2.0000000E 00 11

BLANK CARD
0.0000000E 00 1 -11
BLANK CARD
0.0000000E 00 1 -11

1001 & 0 1 -11

BLANK CARD
10.0000000E 06 10.0000000E 06 0.3840000E 07 0.3000000E 00
2.0000000E 03 2.0000000E 03 2.0000000E 03 1.547005E 03 0.5000000E 01
4.0000000E 03 2.0000000E 03 1.0000000E 05 1.0000000E-05
1 -10

BLANK CARD
0.1000000E 00 1 -11

BLANK CARD
0 16

BLANK CARD
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**Send**

BLANK CARD

BLANK CARD

BLANK CARD

BLANK CARD

STOP
PROGRAM LISTING OF INPUJ DATA CARD

REV 4 SAMPLE PROBLEM = 4 THERMOPLASTIC ANALYSIS OF CIRCULAR DISK - I
0 10
1 -32

TRIR 1 1 23 12
TRIR 2 1 2 23
TRIR 3 2 13 23
TRIR 4 3 12 24
TRIR 5 2 24 14
TRIR 6 2 3 24
TRIR 7 3 14 24
TRIR 8 14 13 24
TRIR 9 3 25 14
TRIR 10 3 4 29
TRIR 11 4 15 23
TRIR 12 15 14 23
TRIR 13 4 26 13
TRIR 14 4 5 26
TRIR 15 5 16 26
TRIR 16 16 15 26
TRIR 17 5 27 16
TRIR 18 5 6 27
TRIR 19 6 17 27
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TRIR 25 7 29 18
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TRIR 28 19 18 29
TRIR 29 9 30 19
TRIR 30 8 9 30
TRIR 31 9 20 30
TRIR 32 20 19 30
TRIR 33 9 31 20
TRIR 34 9 10 31
TRIR 35 10 21 31
TRIR 36 21 20 31
TRIR 37 10 32 21
TRIR 38 10 11 32
TRIR 39 11 22 32
TRIR 40 22 21 32
SEND
Example Problem No. 10

Program: REVBY

Problem Title: Thermoplastic Analysis of a Circular Disk

Comments: Same as sample problem 5 (BEND) and 8 (REVBY). Use here is made of 40 revolved triangular element connecting 32 nodes leading to 53 degrees of freedom. Results shown in the figure are for average stresses at the intersection of the diagonals of each bay. Again comparison with the results from Ref. 4 are good.
FINITE ELEMENT IDEALIZATION USING REVOLVE TRIANGULAR ELEMENTS

E = 1.0 $10^7$ psi = $6.89 \times 10^{10}$ Pa

$\alpha = 1.0 \times 10^{-5} \cdot \text{F}^{-1} = 1.8 \times 10^{-5} \cdot \text{C}^{-1}$

$\sigma_{0.7} = 4000 \text{ psi} = 2.76 \times 10^7 \text{ Pa}$

$n = 5$

$\sigma_0 = 2000 \text{ psi} = 1.38 \times 10^8 \text{ Pa}$

○ INNER RADIUS $\Delta T = 16 \text{ °F} = 8.88 \text{ °C}$

(ELASTIC)

△ INNER RADIUS $\Delta T = 200 \text{ °F} = 111.1 \text{ °C}$

ELASTIC-PLASTIC BOUNDARY

DISTRIBUTION OF CIRCUMFERENTIAL STRESS

REVBY Sample Problem #4 Thermoplastic Analysis of a Circular Disk
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PROGRAM LISTING OF INPUT DATA CARDS

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PROGRAM LISTING OF INPUT DATA CARDS

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3. -32
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</table>

**HEX MUDULF**

**PLASTIC-PLASTIC ANALYSIS**

**PREPARED BY**

**THE RESEARCH DEPARTMENT**

**OF GRUMMAN AEROSPACE CORPORATION**

**UNDER PARTIAL FUNDING FROM**

**THE NASA LANGLEY RESEARCH CENTER**
Example Problem No. 11

Program: HEX

Problem Title: Bar Subjected to an End Displacement

Comments: This is a simple statically determinate problem to demonstrate the use of the HEX element. The automatic mesh generation feature is used (MSGN) to specify a $1 \times 6 \times 4$ mesh of 24 elements. An applied edge displacement is imposed on one end.

The figure shows the axial stress versus applied end displacement.
APPLIED UNIFORM FIXED DISPLACEMENT

MPa

KSI

225

275

300

342

AXIAL STRESS

0.06

0.8

1

1.2

1.4

END DISPLACEMENT

HEX SAMPLE PROBLEM #1 BAR SUBJECTED TO AN END DISPLACEMENT

IN X 10^3

mm

30

32

34

36

38

40

42

44

46

7-61
PROGRAM LISTING OF INPUT DATA CARDS

HEX SAMPLE PROBLEM # 1 LONG BAR SUBJECTED TO AN APPLIED END DISPLACEMENT

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| | 5.00000000E-01 | 11 | -20 |   |   |
| | 8.75000000E-01 | 21 | -30 |   |   |
| | 1.25000000E 00 | 31 | -40 |   |   |
| | 1.50000000E 00 | 41 | -50 |   |   |
| | 1.75000000E 00 | 51 | -60 |   |   |
| | 2.00000000E 00 | 61 | -70 |   |   |

BLANK CARD

| |    |    |    |    |   |
| | -5.00000000E-01 | 1 | 2 | 11 | 12 | 21 | 22 | 31 | 32 | 41 | 42 | 51 | 52 | 61 |
| | -5.00000000E-01 | 62 | 2 | 4 | 13 | 14 | 23 | 24 | 33 | 34 | 43 | 44 | 53 | 54 | 63 |
| | -2.50000000E-01 | 3 | 4 | 13 | 14 | 23 | 24 | 33 | 34 | 43 | 44 | 53 | 54 | 63 |
| | -2.50000000E-01 | 64 | 2 | 4 | 13 | 14 | 23 | 24 | 33 | 34 | 43 | 44 | 53 | 54 | 63 |
| | 0.00000000E 00 | 5 | 6 | 15 | 16 | 25 | 26 | 35 | 36 | 45 | 46 | 55 | 56 | 65 |
| | 0.00000000E 00 | 66 | 1 | 3 | 6 | 9 | 12 | 15 | 18 | 21 | 24 | 27 | 30 | 33 |
| | 2.50000000E-01 | 7 | 8 | 17 | 18 | 27 | 28 | 37 | 38 | 47 | 48 | 57 | 58 | 67 |
| | 2.50000000E-01 | 68 | 2 | 4 | 13 | 14 | 23 | 24 | 33 | 34 | 43 | 44 | 53 | 54 | 63 |
| | 5.00000000E-01 | 9 | 10 | 19 | 20 | 29 | 30 | 39 | 40 | 49 | 50 | 59 | 60 | 69 |
| | 5.00000000E-01 | 70 |   |   |   |   |   |   |   |   |   |   |   |   |   |

BLANK CARD

| |    |    |    |    |   |
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| | 101 | 2 | 4 | 8 | 10 |
| | 000 | 5 |   |   |   |
| | 100 | 6 |   |   |   |
| | 121 | 61 | -70 |   |   |

BLANK CARD

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| | 62 | 2 | 70 | 2 | 1.0 |
| | 63 | 2 | 70 | 2 | 1.0 |
| | 64 | 2 | 70 | 2 | 1.0 |
| | 65 | 2 | 70 | 2 | 1.0 |
| | 66 | 2 | 70 | 2 | 1.0 |
PROGRAM LISTING OF INPUT DATA CARDS

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68  2  70  2  1.0
69  2  70  2  1.0
70  2  70  2  0.003114

BLANK CARD

MAT1  3.0 E 07  3.0 E 07  3.0 E 07  3.0 E 07  3.33333E 00  3.33333E 00
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      0.0  E 00  0.0  E 00  0.0  E 00
      0.0  E 00  0.0  E 00  0.0  E 00
      2.0784 E 04  2.0784 E 04  2.0784 E 04  2.0784 E 04
      1.5  E-24  1.5  E-24  1.5  E-24

SEND
SEND
-100
BLANK CARD
-100
BLANK CARD
1.5  .01
BLANK CARD
STOP
Example Problem No. 12

Program: HEX

Problem Title: Thermoplastic Analysis of a Circular Disk

Comments: This problem is the same as problems 5 (BEND), 8, and 9 (REVBY). Here the idealization of a slice of the disk is used with 10 hexahedra elements with midside nodes in the radial direction. Thirty Lobatto stress points are used in each direction, 10 in the radial, 3 in the circumference, and 1 in the thickness direction.
FINITE ELEMENT MODEL OF A SECTOR OF THE DISK

\[ E = 1.0 \times 10^7 \text{ psi} = 6.89 \times 10^{10} \text{ Pa} \]
\[ a = 1.0 \times 10^{-5} \text{ C}^{-1} = 1.8 \times 10^{-5} \text{ C}^{-1} \]
\[ \sigma_{0.7} = 4000 \text{ psi} = 2.76 \times 10^7 \text{ Pa} \]
\[ n = 5 \]
\[ \sigma_o = 2000 \text{ psi} = 1.38 \times 10^7 \text{ Pa} \]

\[ \text{INNER RADIUS } AT = 16^\circ F = 8.88^\circ C (ELASTIC) \]
\[ \text{INNER RADIUS } AT = 200^\circ F = 111^\circ C \]

DISTRIBUTION OF CIRCUMFERENTIAL STRESS

HEX Sample Problem #2 Thermoplastic Disk
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| BLANK CARD |
| ML  | 1-0 | E+07 | 1-6 | E+07 | 1-0 | E+07 | 1-3 | 3 |
| M  | 1-0 | E+07 | 1-6 | E+07 | 1-0 | E+07 | 1-3 | 3 |
| LUBA | 10 | 1 | 3 |
| SEND | TEMP | 0-9 | 1 | -6 |
| 1-0 | 9.2E+10 | 101 | -104 |
| 8.6E+04 | 7 | -13 |
| 7.6E+00 | 105 | -108 |
| 3.6E+00 | 13 | -18 |
| 6.7E+07 | 1.2 | -12 |
| 6.1E+03 | 1.9 | -24 |
| 5.0E+03 | 113 | -116 |
| 5.4E+03 | 2.5 | -30 |
| 4.6E+07 | 117 | -120 |
| 4.1E+07 | 31 | -36 |
| 3.6E+07 | 121 | -124 |
| 1.9E+01 | 57 | -42 |
| 2.7E+04 | 125 | -128 |
| 2.3E+05 | 43 | -48 |
| 3.1E+01 | 124 | -132 |
| 1.2E+02 | 49 | -54 |
| 1.1E+01 | 133 | -136 |
| 7.4E+00 | 55 | -68 |
| 6.5E+00 | 137 | -140 |
| 0.1 | -60 |

| BLANK CARD | 0-9 | | 0 |

| BLANK CARD | 0-9 | 1-10 |

| BLANK CARD | 0-9 | 1-10 |

| BLANK CARD | 0-9 | | 0 |

| BLANK CARD | 0-9 | 1-10 |

| BLANK CARD | 0-9 | 1-10 |

| BLANK CARD | 0-9 | | 0 |
PROGRAM LISTING OF INPUT DATA CARDS

STOP
Example Problem No. 13

Program: HEX

Problem Title: Collapse of a Simply Supported Uniformly Loaded Beam

Comments: Ten 20 node hexahedra elements are used to model half of a simply supported beam. A $1 \times 1 \times 8$ array of Lobatto points are used (8 through the thickness) to determine stresses within each element. Lobatto points were chosen in order to have stress points at the surface. Eight points were taken through the thickness in order to accurately define an elastic plastic boundary through the thickness.

The figure shows the central deflection versus load for an ideally plastic material. Results are in good agreement with Ref. 5.
LOAD VS. NON-DIMENSIONAL CENTER DEFLECTION \( (w_0) \) CORRESPONDS TO DISPLACEMENT AT INITIAL YIELD) OF A SIMPLY-SUPPORTED BEAM. HEX RESULTS CORRESPOND TO AN1X1X8 ARRAY OF LOBATTO POINTS.

HEX Sample Problem #3 Collapse of a Uniformly Loaded Beam
**PROGRAM LISTING OF INPUT DATA CARDS**

| I   | 2   | 1   | 101 | 102 | 7   | -16  | 103  | 104  | 17   | -26  | 105  | 106  | 27   | -36  | 107  | 108  |
|-----|-----|-----|-----|-----|-----|------|------|------|------|------|------|------|------|------|------|
| 37  | -46 | 109 | 110 | 47  | -56  | 111  | 112  | 57   | -66  | 113  | 114  | 67   | -76  | 115  | 116  |
| 77  | -86 | 117 | 118 |

**HEX SAMPLE PROBLEM #.3**

**COLLAPSE OF SIMPLY SUPPORTED UNIFORMLY LOADED BEAM**

| I   | 2   | 1   | 101 | 102 | 7   | -16  | 103  | 104  | 17   | -26  | 105  | 106  | 27   | -36  | 107  | 108  |
|-----|-----|-----|-----|-----|-----|------|------|------|------|------|------|------|------|------|------|
| 37  | -46 | 109 | 110 | 47  | -56  | 111  | 112  | 57   | -66  | 113  | 114  | 67   | -76  | 115  | 116  |
| 77  | -86 | 117 | 118 |

**SEND**

| I   | 2   | 1   | 101 | 102 | 7   | -16  | 103  | 104  | 17   | -26  | 105  | 106  | 27   | -36  | 107  | 108  |
|-----|-----|-----|-----|-----|-----|------|------|------|------|------|------|------|------|------|------|
| 37  | -46 | 109 | 110 | 47  | -56  | 111  | 112  | 57   | -66  | 113  | 114  | 67   | -76  | 115  | 116  |
| 77  | -86 | 117 | 118 |

---

**SEND**

| I   | 2   | 1   | 101 | 102 | 7   | -16  | 103  | 104  | 17   | -26  | 105  | 106  | 27   | -36  | 107  | 108  |
|-----|-----|-----|-----|-----|-----|------|------|------|------|------|------|------|------|------|------|
| 37  | -46 | 109 | 110 | 47  | -56  | 111  | 112  | 57   | -66  | 113  | 114  | 67   | -76  | 115  | 116  |
| 77  | -86 | 117 | 118 |

---

**SEND**

| I   | 2   | 1   | 101 | 102 | 7   | -16  | 103  | 104  | 17   | -26  | 105  | 106  | 27   | -36  | 107  | 108  |
|-----|-----|-----|-----|-----|-----|------|------|------|------|------|------|------|------|------|------|
| 37  | -46 | 109 | 110 | 47  | -56  | 111  | 112  | 57   | -66  | 113  | 114  | 67   | -76  | 115  | 116  |
| 77  | -86 | 117 | 118 |

---

**SEND**

| I   | 2   | 1   | 101 | 102 | 7   | -16  | 103  | 104  | 17   | -26  | 105  | 106  | 27   | -36  | 107  | 108  |
|-----|-----|-----|-----|-----|-----|------|------|------|------|------|------|------|------|------|------|
| 37  | -46 | 109 | 110 | 47  | -56  | 111  | 112  | 57   | -66  | 113  | 114  | 67   | -76  | 115  | 116  |
| 77  | -86 | 117 | 118 |

---

**SEND**

| I   | 2   | 1   | 101 | 102 | 7   | -16  | 103  | 104  | 17   | -26  | 105  | 106  | 27   | -36  | 107  | 108  |
|-----|-----|-----|-----|-----|-----|------|------|------|------|------|------|------|------|------|------|
| 37  | -46 | 109 | 110 | 47  | -56  | 111  | 112  | 57   | -66  | 113  | 114  | 67   | -76  | 115  | 116  |
| 77  | -86 | 117 | 118 |
**PROGRAM LISTING OF INPUT DATA CARDS**

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### PROCAM LISTING OF INPUT DATA CARDS

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</tbody>
</table>

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12345678901234567890123456789012345678901234567890123456789012345678901234567890
PROGRAM LISTING OF INPUT DATA CARDS

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6.3 -1.0
66 -1.0
112 -1.0
114 -1.0
8  63 73 76 66 68 116 70 114
63 -1.0
66 -1.0
66 -1.0
70 -1.0
73 -1.0
76 -1.0
70 -1.0
114 -1.0
116 -1.0
73 83 86 76 78 118 86 116
73 -1.0
76 -1.0
78 -1.0
83 -1.0
86 -1.0
116 -1.0
118 -1.0

BLANK CARD

SEND 1 -6 8
BLANK CARD 1 -86 171 -118
BLANK CARD 180.0 0.01
BLANK CARD

STOP
```
GEOMNL MODULE

ELASTIC–PLASTIC ANALYSIS

PREPARED BY

THE RESEARCH DEPARTMENT

OF GRUMMAN AEROSPACE CORPORATION

UNDER PARTIAL FUNDING FROM

THE NASA LANGLEY RESEARCH CENTER
Example Problem No. 14

Program: OPLANE-GM

Problem Title: Uniformly Loaded Restrained Beam

Comments: Ten beam elements are used to model half a simply supported restrained beam. A full tangent modulus method is used (PTAN = 0.0).

The figure shows the central deflection and internal axial force versus the total load.
10 EQUAL SIZE ELEMENTS IN ONE HALF BEAM
ΔP = 60 LB, = 266.9 N

LOAD DEFLECTION BEHAVIOR

VARIATION OF INTERNAL AXIAL FORCE WITH LOAD

OPLANE-MG Sample Problem #1 Uniformly Loaded Restrained Beam
PROGRAM LISTING OF INPUT DATA CARDS

UPLANEMG SAMPLE PROBLEM = 1 UNIFROMLY LOADED RESTRAINED BEAM

1 0 1 10 0.0 1.0
1 12
BEAM 1 1 2 12
BEAM 2 2 3 12
BEAM 3 3 4 12
BEAM 4 4 5 12
BEAM 5 6 12
BEAM 7 4 7 12
BEAM 8 5 12
BEAM 9 5 10 11 12
SEND 0 12

BLANK CARD 1 12
BLANK CARD 1 11
BLANK CARD 12
BLANK CARD 12
BLANK CARD 11
BLANK CARD

MBM 1.0 0.25
1 10
SRE 2 0.0 0.0 1.0 1.2
PROGRAM LISTING OF INPUT DATA CARDS

1 10

BLANK CARD

SEND 1 -5

BLANK CARD

BLANK CARD

BLANK CARD

END
REFERENCES


