SENSITIVITY ANALYSIS AND COMPARISON OF SALT DEPOSITION MODELS FOR COOLING TOWERS

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ABSTRACT

This paper is directed toward those individuals in industry, consulting firms and regulatory agencies who must make estimates of the impact of salt deposition from evaporative cooling towers using salt water. These towers emit small droplets, called drift, which contain the dissolved salts in the cooling water. If the deposition of salt from these droplets is sufficiently high, damage to vegetation may occur. At the present there has been little information published on the sensitivity of deposition models to the input variables or comprehensive comparisons of various models. In this paper, two basic models for estimating deposition from natural draft cooling towers are compared. The first is a simple trajectory model that ignores turbulent dispersion and the second is a more complex model incorporating atmospheric turbulence and stability. The major conclusions of the paper are: the trajectory model gives comparable results compared to the more sophisticated model, and the deposition calculations are exceedingly sensitive to the initial droplet size distribution.

INTRODUCTION

In addition to the huge amounts of water vapor and tiny fog droplets which are nearly pure, condensed water, a cooling tower emits larger droplets, called drift, containing the dissolved salts and bacteria present in the cooling water. Initially, these droplets will rise in the plume's updraft, but due to their high settling velocity, they will fall free of the plume. Then in the drier ambient air, they will evaporate, settle downward and be dispersed by atmospheric turbulence. Eventually they will impact on the ground or on vegetation in the area downwind of the tower. If the cooling tower uses brackish or sea water, or if toxic biocides or corrosion inhibitors are added to the water, the deposition of these droplets may injure the vegetation around the tower. To avoid this potential damage, design specifications for modern salt water towers typically include the requirement that the drift emission rate be less than 0.002% of the total flow of the circulating water which corresponds to emissions on the order of tens of liters of drift water per minute.

To ascertain if these emissions can cause an appreciable impact on the surroundings, some investigators have developed mathematical models to
estimate the salt deposition, and others have embarked on field measure-
ments of deposition and vegetation damage around large cooling towers such
as the extensive study at Chalk Point, Maryland [1]. Only at the conclu-
sion of such studies and careful analyses of the data can this question be
answered and the results extrapolated to future sites.

The mathematical deposition models play an important role in both inter-
preting the ongoing field measurements and in assessing the impact of
future towers. Since the development of the first model by Hosier, Pena
and Pena [2], a succession of models have been presented including those
by Roffman and Grimsby [3], Laskowski [4], Hanna [5], and Israel and Over-
camp [6]. They all share many similarities, but each new model takes
some unique approaches to the problem and their estimates can be signifi-
cantly different. Since there is no comprehensive set of deposition
measurements at the present, none of the models can be verified. This
leaves consultants, electric utilities, and regulatory agencies in a quan-
dary about the appropriateness of these or any other models in the envi-
ronmental assessment of a large cooling tower.

One approach to resolve some of these differences is to choose a model and
adapt it to test the various assumptions proposed by other modelers. This
has the advantage that all other details of the basic model are the same
so that any differences in the predictions are the result of the assump-
tion being tested. If the predictions differ significantly, field experi-
ments can be designed to test this assumption.

This study is an attempt at such an inter-model comparison. The basic
model chosen was an updated version of the Israel - Overcamp model [6]
applied to natural draft towers. It was modified to allow computation of
the deposition by either the ballistic or trajectory method introduced by
Hosier et al. [2], or the Gaussian diffusion approach of others. Several
different models for the rise of a droplet above the tower were included.
There were run for various meteorological and cooling tower conditions
including two different droplet distributions reported in the literature.
The deposition predictions were compared and certain cases are presented
in this paper. The major conclusions are: the trajectory model can
give comparable results to the equivalent Gaussian diffusion-deposition
model; the deposition in the first several kilometers is very sensitive
to the emission droplet size distribution; and the deposition estimates
are sensitive to the model chosen for the effective height of emission
of the droplet.

DEPOSITION MODELS

Natural draft cooling towers emit a wide size range of droplets with a
maximum diameter in the order of 1000μm and a mass median diameter less
than 100μm [7]. The terminal velocities of these droplets range up to
4 m/s. When emitted they rise up in the moist plume, but they will even-
tually break away from its updraft, settle downward, evaporate and be
dispersed by atmospheric turbulence.
Most deposition models share certain features. They all divide the droplet emission size distribution into discrete size intervals and usually treat the droplets in each interval independently. All models have some method of estimating the break-away point or effective height of emission of the droplets. The models use a variety of approaches for computing the evaporation, settling, and dispersion of the droplets ranging from simple ballistic or trajectory models [2] to those which simultaneously integrate three dimensional diffusion equations for evaporating droplets [3].

The basic model chosen is a modified version of that described by Israel and Overcamp [6]. For this study, it is put in a sector-averaged form such as would be used for climatological estimates of long-term salt deposition. The model has been adapted to use several sub-models for calculating the effective height of emission of the drops and to use either the trajectory or diffusion approaches to computing deposition.

Effective Height of Emission

Three possible mechanisms for droplets breaking away from the plume's updraft are gravitational settling, centrifugal forces due to the swirling motion in the counter-rotating vortices of a bent-over plume, and diffusive forces ejecting droplets from the plume. At this time, no satisfactory model exists for the diffusive mechanism [8]. Estimates can be made about the relative magnitudes of gravitational and centrifugal forces on the droplet. The gravitational acceleration is \( g \), whereas the centrifugal is \( \frac{v^2}{R} \) in which \( v \) is a characteristic tangential velocity and \( R \) is the local radius of curvature. For an order of magnitude estimate, let \( v \) be the exit velocity and \( R_0 \) be the tower's exit radius. Then the ratio of centrifugal to gravitational forces on the droplet, \( S \) is*

\[
S = \frac{v^2}{gR_0}
\]  

(1)

For a hyperbolic tower, \( v \) is of the order of 5 m/s and \( R_0 \) is of the order of 30 m. Therefore, \( S \) is about 0.1 or less which implies that for the plume as a whole gravitational forces dominate. On the other hand, at high wind speeds, a small vortex pair has been observed to form at the lip of the tower as the plume is partially pulled into the tower's wake [9]. In this case centrifugal forces may dominate and eject drops from the vortices near the top of the tower.

If it is assumed that its gravitational settling is the primary mechanism for a droplet to breakaway from the plume, it is reasonable to further assume that this will occur when the droplet's settling velocity exceeds the local updraft velocity in the plume. Hosler et al. [2] used this approach with the assumption that the vertical velocity in the plume, \( w \),

* The \( S \) parameter or separation is commonly used in the analysis of the relative importance of centrifugal to gravitational forces in a cyclone separator used in the control of particulate emissions.
varied linearly from its value at the tower exit, \( W_Q \), to zero at the ultimate rise of the plume. Israel and Overcamp [6] and also Hanna [5] used the "2/3 rise law" of a buoyant plume in a neutral atmosphere to derive an expression for the characteristic vertical velocity within the plume. Equating the local vertical velocity to the droplet's terminal velocity, \( v_0 \), the distance above the tower where the droplet breaks free from the plume is

\[
\Delta H_D = \frac{2}{3 \beta^2} \frac{F_0}{u v_D^2}
\]

in which \( u \) is the mean wind speed, \( \beta \) is the entrainment coefficient, and \( F_0 \) is the initial buoyancy flux which is defined as:

\[
F_0 = \omega w o R o^2 \frac{T_{pv} - T_{av}}{T_{pv}}
\]

\( R_o \) is the exit radius of the tower, and \( T_{pv} \) and \( T_{av} \) are the virtual temperatures at the exit and in the ambient [10].

The value of the entrainment coefficient for the bent-over plume, \( \beta \), has been reported to have a wide range of values in the literature. In a recent review on plume rise, Briggs [11] finds that 0.5 gives the best fit for the visible depth of the plume and \( \beta = 0.6 \) is best for the radius of the effective mass of the plume. In this study a value of 0.5 was chosen as opposed to the 0.7 used by Israel and Overcamp [6]. With a smaller value of \( \beta \), the Equation 2 predicts a higher rise above the tower. In principle, for this application \( \beta \) should actually be chosen on the basis of either fitting measurements of vertical velocities to the plume rise law, or, if possible, by fitting actual measurements of the breakaway point of droplets to Equation 2.

Equation 2 assumes that the plume originates as a point source of buoyancy with no vertical velocity. Since a natural draft tower has an exit diameter on the order of 60m and an exit velocity of around 5 m/s, this assumption may be questionable. The basic entrainment equations for a bent-over plume can be integrated for a source with finite initial size and vertical velocity. Appendix A describes one such derivation and gives a set of equations that can be solved for the height above the tower, \( \Delta H_D \), where the plume's updraft velocity equals the droplet's settling velocity. Using either Equation 2 or the solution in Appendix A for \( \Delta H_D \), the effective height of emission for a droplet, \( H_D \), is the sum of its rise above the tower, \( \Delta H_D \), and the height of the tower, \( H_s \),

\[
H_D = H_s + \Delta H_D
\]

Since either atmospheric turbulence or stability will eventually limit the effective rise of plume, Equation 2 is not valid for small droplets with low settling velocities. An upper bound can be set as the ultimate rise of the plume. This model uses the dry plume rise equations of Briggs [12]. For a stably stratified atmosphere the rise of the plume is
\[ \Lambda H_e = 2.9 \left( \frac{0}{u_s} \right)^{1/3} \]  

(5)

in which \( s \) is \((g/T)/(\partial \theta/\partial z)\) and \( \theta \) is the potential temperature. For unstable or neutral conditions the plume rise is

\[ \Lambda H_e = 1.6 \left( \frac{F_0}{u} \right)^{1/3} (3.5x^*)^{2/3} \]  

(6)

in which \( x^* = 34 F_0^{0.4} \) with \( x^* \) in meters and \( F_0 \) in \( \text{m}^3/\text{s} \). In a recent paper, Wigley [13] argues that for a stable atmosphere, the final plume rise be lowered by approximately \( (R_0/\beta) \) to account for the finite initial size of the plume. This is also suggested in Equation A5 for a plume in the neutral case, but has not been incorporated in this model.

An example will show the effect of these plume models in estimating \( H_0 \). For a cooling tower with a height of 125m, exit diameter of 60m, and initial buoyancy flux of 2000 \( \text{m}^3/\text{s}^3 \) and with a wind speed of 4 m/s and neutral stability, Figure 1 shows the effective height of emission, \( H_0 \), for the linear model similar to that used by Hosler et al. [2], the point source plume model of Equation 2, and the finite source plume model of Appendix A. For large droplets, the linear model predicts a higher rise than either the point or finite source models. The point source prediction is greater than for the finite source model, but the difference is significant for only the largest droplets.

**Evaporation and Settling**

In the model for this study, it is assumed that droplet begins evaporation when it breaks away from the moist plume. The evaporation equation used is similar to that given by Mason with the incorporation of the ventilation factor to account for the increased evaporation due to the droplet's motion [14,6]. The settling velocities use Stokes law or empirical equations fitted to the data of Gunn and Kinzer [15] on the terminal velocity of drops.

The two basic approaches to computing deposition are the ballistic or trajectory approach of Hosler et al. [2] that ignores atmospheric turbulence or a downward-sloping Gaussian plume that attempts to account for turbulence.

The trajectory method used in this study integrates the evaporation and droplet settling equations to determine the time for a droplet to fall a distance, \( H_0 \), its average settling velocity, \( \bar{v}_d \), and the downwind distance where it strikes the ground, \( x_D \), which is given by

\[ x_D = \frac{H_0 u}{\bar{v}_d} \]  

(7)

The model is applied by dividing the emission droplet distribution into
The sector-averaged deposition for the droplet class bounded by \( D_i \) and \( D_{i+1} \) is

\[
\omega_i = \frac{Q_{D_i}}{\frac{\pi}{16} (x_{D_{i+1}}^2 - x_{D_i}^2)}
\]

in which \( Q_{D_i} \), the total emission of salt in the interval, is spread over a \( 22^\circ \) sector between the distances \( x_{D_{i+1}} \) and \( x_{D_i} \).

In diffusion-deposition methods, a downward-sloping Gaussian plume is used to simulate the effects of atmospheric turbulence. In this model, the general equation for deposition developed by Overcamp [16] is used to compute the deposition due to the droplets of a given size:

\[
\omega_i(x) = \frac{\bar{v}'_D Q_{D_i} (1+\alpha)}{\sqrt{2\pi} u (\frac{\pi x}{8})} \exp \left[ -\frac{1}{2} \left( \frac{H_D - \bar{v}'_D x}{2\alpha z} \right)^2 \right]
\]

in which \( \alpha \) is the partial image coefficient, \( \bar{v}'_D \) is the average settling velocity and \( v'_D \) is the final or deposition velocity. A discussion of how these velocities are determined is given in Appendix B.

Unlike the trajectory method in which only one droplet class contributes to the deposition at any particular downwind distance, many droplet size classes can deposit at a location. The total deposition at any distance downwind is found by summing Equation 7 for all the droplet sizes.

MODEL COMPARISON

Prior to presenting model comparisons, it is important to recognize that it is unreasonable to expect that any deposition model can be verified to any greater degree than our present ability in predicting the \( \text{SO}_2 \) concentrations due to an elevated, buoyant release. For this gaseous case, it is often quoted that for any given situation, the Gaussian plume model "should be correct within a factor of 3" [17]. Therefore, it is unrealistic to anticipate that detailed verification of any salt deposition model could be made to within better than a factor of two or three or more without an exhaustive series of tests under repeatable meteorological conditions with simultaneous emission drop distribution measurements. Therefore, if two models give comparable results for the same input conditions, it may be futile to attempt to resolve any minor differences with field measurements. On the other hand, field experiments will be invaluable to determine if the predictions are correct as to the magnitude and location of the deposition.
In view of the above, a series of deposition calculations will be compared for the models discussed in the preceding section. For the calculation, the tower height is 125m and its exit diameter is 60m. The updraft velocity is 5 m/s and the buoyancy flux is 2000 m²/m³, and the atmospheric stability class for the diffusion model is Pasquill C. Except in one case, the relative humidity is 70%. The salt concentration of the droplets is assumed to be 35,000 ppm and to be sodium chloride. The total salt emission from the tower is 0.01 kg/s which roughly corresponds to a drift rate of 0.002% of the circulatory water flow. It should be noted that the measured drift rate at Chalk Point has been reported to be significantly lower [7].

The two different emission droplet distributions used are given in Table I. The first one, designated as the small distribution, was taken from an earlier paper [6]. It has droplet sizes ranging up to 450 µm in diameter. The second, designated as the large distribution, came from preliminary droplet measurements at Chalk Point, Maryland* and has been used in an early attempt at a model validation [18]. Its droplets range in size up to 800 µm. Later measurements at Chalk Point show that droplets over 1000 µm are emitted [7].

Figure 2 shows the salt deposition estimates using the sector-averaged, finite source plume, diffusion model for both the small and large droplet distributions. Although the total salt emissions are identical for both, the large droplet distribution case predicts a factor of 40 higher maximum deposition than does the small distribution. For the large case, there is a distinct maximum at 300m downwind of the tower. For the smaller case, there is a broad peak extending from 1 to 10 km downwind.

To help explain these differences, Figure 3 shows the contribution of the 16 individual droplet size intervals to the total deposition for the large distribution. The droplets larger than 450 µm account for all of the deposition in the first kilometer. The peak is a result of this small fraction of the salt that is in the large droplets falling on such a small area. The sharpness of the deposition peak for the droplet size classes with the largest droplets is probably an artifact of the model. This model assumes that all the droplets in a particular size interval are emitted from a single point located at a height H₀ above the ground and directly over the center of the tower. The model could be modified to account for the finite size of the tower and distribution of the droplets across the tower as suggested by Hanna and Gifford [19]. Or a different approach could be to use a source function that distributed the emissions of a given size interval over a range of altitudes centered about the point H₀ above the ground. Either of these methods would broaden the individual deposition peaks and slightly lower the maximum at the expense of substantially increased computation time and probability for

*These size distributions were calculated from data supplied by Environmental Systems Corporation, Knoxville, TN.
VI-C-124

error.

Figure 4 shows a comparison between the total deposition predicted by the trajectory and diffusion method with both using the same model for computing the effective height of emission. Since the trajectory model predicts that only one size droplet can fall at a particular point, it is easy to relate each "step" of prediction with one droplet size interval. Comparing Figures 3 and 4, it can be seen that the location and magnitude of the deposition due to the larger drops is similar for both models. As the droplets get smaller and their settling velocities decrease, atmospheric turbulence will begin to dominate. For the smallest droplets, it will be the principal mechanism in bringing droplets near the ground. Figure 3 shows this effect. The maxima of the smaller droplet interval deposition curves are 8 to 15 km from the tower, and they all are quite broad in contrast to the distinct, narrow curves for the larger droplets. On the other hand, the trajectory model predicts the small droplets will fall only very far from the tower. Figure 4 shows that the 125 μm size interval droplets deposit between 13 and 38 km from the tower. The smaller ones fall still further out. This shortcoming of the trajectory approach has been noted by Pena and Hosler [20] and is implied in Hanna's [5] suggestion to use a trajectory approach for droplets with initial diameters greater than 200 μm and the normal Gaussian model for the smaller ones.

Figure 5 shows a similar comparison for the small droplet class. Again the agreement is reasonable within the context of a factor of two or three. But because of the trajectory model's inability to treat small droplets properly, the similarity of predictions beyond 10 km for both Figures 4 and 5 may be partially coincidental because they depend on the relative magnitudes of salt in the smaller size intervals.

Figures 6 and 7 show the effect on total deposition of the linear, point and finite source models for estimating the effective height of emission. For both the large and small distributions there is little difference between the point source and the finite source models. But for the large droplet distribution, there is nearly an order of magnitude lower peak deposition for the linear plume model than the other plume models. The explanation can be readily seen in Figure 1. The linear model predicts a substantially higher effective height of emission than the models based on a plume model. For the small distribution there is little significant difference among the models because the predictions for the effective height of emission are approximately the same.

The last comparison of this study shows the effect of humidity on the deposition patterns. Figure 8 gives the deposition for the finite source plume model for 90% and 70% relative humidity. Somewhat surprisingly, there is little difference in deposition for the large distribution case. This is because the largest droplets with settling speeds of in the order of several meters per second only stay in the air for 1 to 3 minutes and do not evaporate very much irrespective of the humidity. For the small distribution case, its biggest droplets are in the air for a longer time and do
evaporate. With low relative humidity, the droplet will evaporate faster, lower its settling velocity, and land farther out. This will lower the peak deposition.

CONCLUSIONS

This paper presents a comparison of various assumptions that have been proposed for salt deposition models for natural draft cooling towers. Deposition calculations are presented that show that the trajectory method can give very comparable results to the more sophisticated Gaussian diffusion model. These calculations are very sensitive to the emission drop size distribution spectrum and the model used for estimating the effective height of emission of the drift droplets.

For those who evaluate the environmental impact assessment for cooling towers using salt water, it is concluded that the droplet size distribution may be as important, or possibly more important, than the actual model used in the assessment especially if there is concern about the salt deposition within the first kilometer of the tower.

Finally, it is encouraged that other investigators will adopt this approach to the sensitivity of their model so that it can be determined whether the above conclusions can be applied to all models or if they are a peculiarity of the basic model chosen for this study.

APPENDIX A

The entrainment equations for a buoyant plume in a neutral atmosphere can be integrated with finite initial size and vertical velocity. The equations for conservation of mass, momentum and buoyancy in a neutral atmosphere are [11,21]

\[ u \frac{dR^2}{dx} = 28wR \]  
\[ u \frac{d}{dx} (R^2w) = \frac{F}{u} \]  
\[ F = F_0 \]  

These equations can be integrated with the initial conditions \( R = R_0 \) and \( w = w_0 \) to give an equation for the vertical velocity as a function of downwind distance

\[ w = \frac{R_0^2w_0 + F_0x/u^2}{[R_0^3 + 38R_0^2w_0 + 38Fw_0/u^2]^{2/3}} \]  

-9-  

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Another equation can be found for the rise above the tower

\[ z = \frac{1}{\beta} \left[ R_o^3 + \frac{3\beta R_o^2 w_0 x}{u} + \frac{3\beta F_x^2}{2u^3} \right]^{1/3} - \frac{R_o}{\beta} \]  \hspace{1cm} (A5)

By setting \( w \) equal to the droplet settling velocity, \( v_D \), Equations (A4) and (A5) can be solved to determine the height above the tower where the droplet breaks away from the plume, \( \Delta H_D \). This solution is analogous to Equation (2) for the point source plume.

**APPENDIX B**

Equation (9) has two droplet settling velocities: \( \bar{v}_D \), the average settling velocity, and \( v' \), the final or deposition velocity. For these evaporating droplets, both velocities should be a function of distance. But for computational simplicity, one set of these velocities is chosen for each droplet size interval. The settling velocity and evaporation are integrated for a droplet emitted from height \( H_D \). The integration continues until it strikes the ground, or if the droplet is small, until it is carried downwind a distance equal to \( x_{\text{max}} \), the point of maximum ground level concentration of gaseous emissions from height \( H_D \). The distance \( x_{\text{max}} \) is approximately given by

\[ \sigma_z (x_{\text{max}}) = \frac{H_D}{\sqrt{2}} \]  \hspace{1cm} (B1)

This latter condition recognizes that the maximum deposition of the droplets occurs no further away than does the maximum for gases. This prevents calculating very small values of average settling velocities for small droplets released from a great height as is the case with the trajectory model. It should give a good representation of the settling velocities near the maximum of its deposition curve.
REFERENCES


### TABLE I

**DROPLET SIZE DISTRIBUTIONS**

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Figure 1. Effective Height of Emission versus Droplet Diameter and Settling Velocity

Figure 2. Salt Deposition for the Finite Source, Plume, Diffusion - Deposition Model
Figure 3. Salt Deposition for the Finite Source, Plume, Diffusion Deposition Model Showing the Contribution of the Individual Droplet Classes

Figure 4. A Comparison Between the Trajectory and the Diffusion Deposition Model for the Large Droplet Class
Figure 5. A Comparison Between the Trajectory and the Diffusion-Deposition Model for the Small Droplet Class.

Figure 6. Salt Deposition for the Diffusion-Deposition Model Using the Finite, Point and Linear Plume Models for the Large Droplet Distribution.
Figure 7. Salt Deposition for the Diffusion-Deposition Model Using the Finite, Point and Linear Plume Models for the Small Droplet Distribution

Figure 8. The Effect of Humidity on the Salt Deposition for the Finite Source Plume, Diffusion-Deposition Model