RESEARCH ON UNSTEADY, NONLINEAR TRANSONIC FLOW

Semi-Annual Progress Report
NASA GRANT NUMBER NSG 1219
9 August 1977

Dr. Samuel R. Bland, Head
Aeroelastic Analysis Section
NASA Technical Officer
National Aeronautics and Space Administration
Langley Research Center
Hampton, Virginia 23665

RE: Semi-Annual Progress Report
    NASA Grant Number NSG 1219

Dear Dr. Bland:

Enclosed please find, in triplicate, the semi-annual progress report for
NASA Grant NSG 1219, covering the period January 1977 - July 1977.

Sincerely,

Woodrow Whitlow, Jr.
Research Assistant

cc: Professor W. L. Harris, MIT
    Ms. Patricia Greer, OSP, MIT
    NASA Scientific and Technical Information Facility
    Baltimore-Washington International Airport
Previous work included solving the small perturbation equation for nonlifting steady flow, in parameter space, using the Alternating Direction Implicit method (ADI). These solutions were obtained on a coarse grid with 10 points on the airfoil ($\Delta x = .1$). This relatively large grid spacing in the region near the airfoil leads to some irregularities in the computed surface pressure coefficients ($C_p$) near the airfoil leading and trailing edges. To alleviate this problem, it was necessary to reduce the grid spacing in the vicinity of the airfoil.

In order to obtain a better definition of the flow in the region near the airfoil, a coordinate stretching was introduced. The stretching, which was introduced by Carlson, is symmetrical and divides the flow field into three regions. Those regions are illustrated in Figure 1. The relationships between the stretched and unstretched coordinate systems are

$$ z = a_1 \tan \left( \frac{\pi}{2} \eta \right) \quad (1) $$

$$ x_1 = -x_4 + a_2 \tan \left[ \frac{\pi}{2} (\xi + \xi_4) \right] + a_3 \tan \left[ \frac{\pi}{2} (\xi + \xi_4)^3 \right] \quad (2a) $$

$$ x_{II} = \xi (a_4 + b \xi^2) \quad (2b) $$

$$ x_{III} = x_4 + a_2 \tan \left[ \frac{\pi}{2} (\xi - \xi_4) \right] + a_3 \tan \left[ \frac{\pi}{2} (\xi - \xi_4)^3 \right] \quad (2c) $$

where $a_1$, $a_2$, $a_3$, $a_4$, $b$, $x_4$, and $\xi_4$ are constants which determine the geometry of the stretched coordinate system. The effect of the coordinate stretching is to increase the number of grid points near the airfoil and to map the infinite physical plane, $-\infty \leq z \leq \infty$, $-\infty \leq x \leq \infty$, onto the finite computational plane, $-1 \leq \eta \leq 1$, $-(1 + \xi_4) \leq \xi \leq (1 + \xi_4)$.

The steady-state small perturbation equation has been approximated by finite differences, in stretched coordinates, and solved using column
relaxation. To help in choosing a relaxation factor ($\omega$), the maximum error ($E$) in the entire flow field, after 250 iterations, has been plotted as a function of $\omega$. $E$ is defined as

$$E = \frac{g_{n,m}^{k+1} - g_{n,m}^k}{\omega}$$

(3)

where $g_{n,m}^k$ is the value of $g$ at the $k^{th}$ iteration. Figure 2 summarizes $E$ versus $\omega$.

Results were obtained for subcritical flow about a nonlifting parabolic arc airfoil at free stream Mach numbers ($M_{\infty}$) of .806, .825, .85, and .9. The solutions were considered converged when $\frac{g_{n,m}^{k+1} - g_{n,m}^k}{\omega}$ was less than $2 \times 10^{-5}$ everywhere in the flow field. The results are summarized in Table I. Figure 3 shows computed values of $C_p$ at $M_{\infty} = .825$ with the airfoil thickness ratio ($\tau$) ranging from .01 to .06.

Current efforts include the calculation of steady flows with shock waves and the calculation of two-dimensional unsteady flows.

Using the method of parametric differentiation and the integral equation method, a formulation of the small perturbation, unsteady, non-linear transonic flow over thin, shockless airfoils has been developed. This formulation allows for a calculation of the flow field variables as well as the surface pressure distribution. Singularities of the kernel functions of both the steady and unsteady solutions have been removed at all points in the flow field save the leading and trailing edges. The integral equations for the steady and unsteady components of the solution for $g(\bar{x},t)$ have been written in such a manner that a classical iteration procedure (Picard's method) at each $\tau$ - level is
appropriate. This means that the typical unsatisfactory approximations of the variables constituting the integrands may be avoided.

Additional analysis to complete the evaluation of remaining integrals are in progress.
### TABLE I

**SUMMARY OF RESULTS**

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<th>( \tau )</th>
<th>( M_\infty = .806 )</th>
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REFERENCES


Figure 1. Stretched Coordinate System
Figure 2. Maximum Error versus Relaxation Factor

$I = 250$
$49 \times 20$
$\tau = 0.01$
$M_\infty = 0.806$