The Concept of Modeling

Webster's New Collegiate Dictionary has the following definition for the word "model," in the sense in which we will be using it: "Model ... a system of postulates, data, and inferences presented as a mathematical description of an entity or a state of affairs." The reason that we are interested in modeling is that we would like to be able to predict an effect; namely, spacecraft charging. A model may be regarded as a mathematical representation of the link between cause and effect. If we can identify the cause, then the model provides a method for calculating (that is, predicting) the effects about which we are concerned.

There are different kinds of models, which can be conveniently arranged into three categories: (1) statistical models; (2) parametric models; and (3) physical models. A statistical model is usually the first kind of model that is arrived at in describing a phenomenon. For example, if it is cloudy, the probability of rain is increased, because we know that clouds and rain correlate. Statistical models using correlation can be useful because they can provide clues as to what is the cause for a phenomenon. But they can also be misleading because the correlation may be between two effects, or, the correlation may be coincidental. An example
of a correlation related to spacecraft charging is shown in Figures 1 and 2. Spacecraft anomalies occur more frequently between 0000 and 0600 hours in local time. Spacecraft charging events also show the same pattern. These figures are taken from papers by McPherson, Cauffman and Schober,\(^2\) and from Reasoner et al.\(^3\) The correlation between the two patterns provides evidence that the two phenomena – spacecraft anomalies and spacecraft charging – may be related.

The most useful kind of model is one that is based on understanding the physics of the actual processes that are involved in the phenomenon. Frequently, the processes are complicated or are only imperfectly understood. In such a case, a parametric model may be useful. Here, one or more physical parameters are selected which characterize the physical processes, and an approximate model is constructed based on these parameters. An example of such a model is shown in Figure 3 where the electron current to a sphere in a plasma has been calculated with the assumption of a spherically symmetric Debye potential distribution.\(^4\) The current depends upon the Debye length parameter, as well as upon the plasma density and temperature and the sphere radius. This calculation is not exact but it gives the correct qualitative behavior and is much easier to calculate than the exact current.

![Diagram](image)

**Figure 1.** Local Time Dependence of Circuit Upset for Several DoD and Commercial Satellites
2. MODELING SPACECRAFT CHARGING

There are four categories of models for spacecraft charging, and McPherson has identified these four categories with four regions in the spacecraft-environment configuration, as shown in Figure 4. Region 1 is the undisturbed plasma far away from the spacecraft. This region may be assumed to be free from fields due to sheath effects. It is the source region for the plasma particles which make up part of the spacecraft charging currents.

Region 2 is the plasma sheath region where there are quasistatic electric fields. These electric fields are caused by the local charge distributions (space charge and surface charges) and in turn affect the trajectories of the charged
FOUR REGIONS FOR MODELING

Region 1: Undisturbed Plasma
- Field Free
- Particle Source Boundary

Region 2: Plasma Sheath Region
- Quasi Static, Self Consistent Electric Fields
- Surface Charge Boundary

Region 3: Spacecraft Surface
- Component in Equivalent Circuit
- Nonlinear Properties

Region 4: Spacecraft Equivalent Circuit
- Return Currents and Radiative E.M. Fields
- Coupled Currents and Fields

Figure 4. Regions Involved in Modeling Spacecraft Charging

particles going through this region. Hence, a self-consistent solution must be obtained for the particle and field distributions.

Region 3 is the spacecraft surface which is characterized by different materials and hence different properties for absorbing, emitting, and conducting charged particles. These properties may be very nonlinear. The spacecraft surface with its charge and potential distributions is a boundary for region 2, and it is also a part of the spacecraft electric circuit.

Region 4 is the spacecraft equivalent circuit describing the paths for currents and coupling for electromagnetic fields between the spacecraft components. One of the ultimate objectives for the overall modeling is to understand and predict the response of spacecraft components to the charging phenomena.

The first session of the conference has discussed region 1, the geosynchronous environment. Later sessions will discuss regions 3 and 4; that is, characterization of spacecraft materials and response to charging events. In the remainder of this discussion, a review will be made of some of the approaches to modeling region 2, the sheath about the spacecraft, and the related charge and potential distributions on the spacecraft surfaces.

The objective of modeling the spacecraft sheath is to obtain accurate values for the various charging currents which traverse the sheath and help to determine
the spacecraft surface potential and charge distributions. In this case the cause is the undisturbed environment, the effect is the surface potentials and charges, and the link is the charging currents. There are a number of different kinds of charging currents, as shown in Figure 5. Electrons and ions from the plasma travel to the spacecraft. Photoelectrons, secondary electrons, and reflected and backscattered electrons can travel from the spacecraft to the plasma or to other parts of the spacecraft surface. All of the particle trajectories which are external to the spacecraft are both influenced by the sheath electric fields and contribute to their configuration.

**KINDS OF CHARGING CURRENTS**
(in rough order of importance)

1. Electrons from plasma
2. Photoelectrons from spacecraft surfaces
3. Ions from plasma
4. Secondary electrons from surface --- from electron impact
5. Secondary electrons from surface --- from ion impact

**TRANSFER OF CHARGE CAN OCCUR:**

1. From environment to spacecraft
2. From spacecraft to environment
3. Between different spacecraft surfaces
   - via external trajectories
   - via internal paths

*Figure 5. Different Charging Currents which Affect Spacecraft Charging*

The condition for equilibrium — that is, a steady-state or quasistatic situation — is that the net current density vanishes at every point on the surface of insulating materials, or that the net current to every conducting element vanishes. In some situations — for example, when discharges occur — it is important to look at the time-dependent behavior of the charging currents, but for the most part the time
constants for charging are so short that the gross features of the sheath may be considered to be in quasistatic equilibrium.

Mathematically, the sheath problem may be described in terms of: (1) the Poisson equation, which tells one how to find the potential distribution for given space charge densities and boundary conditions; and (2) the Vlasov equation which tells one how to find the space charge densities for a given potential distribution. The solution of each of these equations depends upon knowing the solution to the other, so that a self-consistent procedure must be found for a solution. Usually, the procedure involves an initial guess and then successive improvements by iteration.

The various approaches that have been used in attempting to solve this problem have differed mainly in how the Vlasov equation has been solved. The Vlasov equation essentially states that the velocity distribution function (strictly speaking, the phase space density) for a given kind of particle is constant along the particle's trajectory. Consequently, a solution involves either a calculation (or approximation) of the particle trajectory, or else it must make use of some other physical relationship that provides equivalent information.

In table 2 of Parker, the various approaches to the Vlasov equation are arranged into three categories. This classification is taken from a review by Parker of theoretical work done on satellite sheaths and wakes. The following summary of the various treatments is also largely taken from Parker's report.

The inside-out method follows particle trajectories backwards in time from a point in the sheath at which the density is desired to the point of origin of the particles. At the origin of the trajectory the distribution function may be evaluated since the origin is either in the undisturbed plasma or at the spacecraft surface, where the distribution functions may be assumed to be known. The inside-out method is flexible since the points at which the density is to be evaluated may be chosen arbitrarily. Also, the method applies equally well to ions or electrons. The disadvantage of this method is that the information obtained about a trajectory is lost when one moves to the next point for obtaining density, and hence the computation can be time-consuming.

The inside-out method was developed by Parker, and has been used by Fournier to calculate the wake of a moving cylinder, and by Parker for calculating the steady-state plasma flow about an arbitrarily thick disk. This method was used by Grabowski and Fischer in conjunction with the assumption of quasineutrality, so that their treatment was not general. It was also used by Taylor for the wake of an infinitely long cylinder of rectangular cross-section, but the calculation was not carried beyond the first iteration, and is therefore not self-consistent. Parker and Whipple used the method for two-electrode probes on a satellite but did not self-consistently solve for the sheath potential distribution. Liu and
Illung\textsuperscript{13} used this method for the far-wake zone of a satellite to predict wave-like behavior. Parker\textsuperscript{14, 15} has also used the method for two-electrode rocket-borne and laboratory probe systems, and for the problem of a small probe in the sheath of a large electrode.

The outside-in method follows particle trajectories in the same direction as the actual particle motion. The main disadvantage of this method is that it is difficult to choose the trajectories in such a way that an accurate density can be obtained at an arbitrary point. In the special case where trajectories do not cross or reverse direction, the flux tube method of choosing trajectories may be used. This technique was used by Davis and Harris\textsuperscript{16} for a wake calculation assuming cold ions, by Call\textsuperscript{17} for the cold-ion wakes of both cylinders and spheres, by Martin\textsuperscript{18} for the cold-ion wakes of a strip and disk, and by McDonald and Smetana\textsuperscript{19} for the wake of an infinitely long cylinder in a drifting monoenergetic plasma. Another approach using the outside-in method is to divide the space into cells and to evaluate the density in each cell according to the time that the particle spends in it. This method is closely related to "particle-pushing" or simulation calculations, and can be readily adapted to time-dependent problems. Again, accurate calculations can be time-consuming since many trajectories are required to obtain good statistics within cells. This method was studied by Parker\textsuperscript{7} for mono-energetic-ion distribution with drift, and was used by Maslennikov and Sigov\textsuperscript{20} for the cold-ion wake of a sphere.

"Other" methods are defined as treatment which avoid explicit trajectory calculations and make use of other physical relationships. For example, configurations with inherent symmetry such as spheres or cylinders in an isotropic plasma may be treated by working with constants of the motion (that is, energy, angular momentum, etc.) which characterize the particle trajectories. These simple configurations are useful because solutions can serve as benchmarks for the numerical methods developed for more realistic problems. Also, they serve to illustrate the basic physical processes that may be involved in the charging phenomenon. Bernstein and Rabinowitz\textsuperscript{21} used this approach to treat the problem of a sphere in a plasma containing mono-energetic ions. Laframboise\textsuperscript{22} treated exactly the problems of both spheres and cylinders in Maxwellian plasmas. Chang and Blenkowski\textsuperscript{23} used this approach to treat the problem of a thin sheath when there is emission of electrons at the surface of a spherical probe in a plasma. Schroder\textsuperscript{24} and Whipple\textsuperscript{25} extended this to treat the case of a thick sheath. Parker\textsuperscript{26} has formulated a computer program which treats arbitrary sheath thicknesses for electron emitting spherical probes in an isotropic plasma with arbitrary velocity distributions.

Other approaches which avoid trajectory calculations have used various assumptions or approximations such as expressing the ion or electron density in
terms of the local potential by means of the Boltzmann factor, neglecting ion thermal velocities, assuming quasineutrality, etc. Liu \(^{27}\) and Jew \(^{28}\) assumed that the ion axial component of velocity is constant. They then determined limiting trajectories for the density integral by further approximations, namely, an additional assumed approximate constant of the motion, evaluated using the local field in the vicinity of the point in question. Kiel, Gey, and Gustafson \(^{29}\) treated the wake of a sphere, assuming straight-line paths for the ion trajectories, and also assuming approximate formulas for the electron densities. Gurevich et al. \(^{30}\) assumed quasineutrality, using the Boltzmann factor for both ions and electrons, and assumed in addition that the ion axial component of velocity was constant and that the ion thermal velocity was small.

3. MODEL VERIFICATION

A model cannot be considered to be reliable until it has been verified. Verification means comparing the prediction of the model with experimental results and finding agreement. As can be seen from the number of theoretical treatments of the spacecraft sheath problem, models are fairly easy to generate. It is much more difficult, in general, to perform the kind of experiment which will provide data for verification or nonverification of the model. This seems to be especially true in space physics, where there is such a long process involved in performing experiments on spacecraft. The process begins with a proposal, and then continues through the experiment design and construction, a great deal of testing, finally a launch which may or may not be completely successful, and then data acquisition, transmission through telemetry links and ground stations back to the experimenter, and finally reduction and analysis of the data by the experimenter. This cycle from the conception of the experiment until its analysis typically involves several years, and it is no wonder that in space physics the connection between a theoretical model and its experimental verification is frequently somewhat remote.

An example of how models can be rendered academic by the acquisition of data is provided by work that has been done on the photoelectron sheath about a spacecraft. A number of workers, beginning with Singer and Walker \(^{31}\) in 1962 have discussed the effect of photoelectrons on the plasma sheath surrounding a spacecraft. Various velocity distribution functions for the photoelectrons were treated in various geometries, but almost all of the treatments were for a conducting body. Guernsey and Fu \(^{32}\) and Fu \(^{33}\) showed that if the photoelectrons dominate the space charge near the satellite surface, it would be possible for a potential minimum to develop in the sheath so that the potential distribution would be nonmonotonic.
(See Whipple for a review of these treatments.) Such a potential minimum was found by analysis of the electron data obtained from the UCSB particle detectors on the ATS-6 satellite. However, it was shown that none of the models could adequately explain the data. The potential minimum which was inferred from the data was much too large to be explained in terms of the ordinary space charge limited effect. It is probable that the minimum must be explained in terms of differential charging of the spacecraft surfaces. Electrons are emitted from these differentially charged portions of the spacecraft surface providing the required negative space charge for the formation of the potential minimum. However, a quantitative model for this phenomenon has not yet been formulated.

Another way of verifying sheath models that has not been adequately exploited is through laboratory experiments. Although it is not possible to completely simulate the geosynchronous environment in a laboratory, it should be possible to study many of the individual processes. It is certainly possible to generate fluxes of particles in the appropriate energy ranges in the laboratory, and there should be no problem in simulating solar photoemission. It should be possible to learn a great deal about the charging process and especially about the interaction between various spacecraft elements by using realistic models of a spacecraft in such a laboratory environment.

Acknowledgments

I would like to thank Dr. Lee W. Parker for permission to use material from his reviews. This work was supported by NASA Grant NGL 05-005-007 and Air Force Contract F19628-76-C-0214.

References


