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GRAVITY MODEL IMPROVEMENT USING GEOS-3 (GEM 9 & 10)

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GRAVITY MODEL IMPROVEMENT USING GEOS-3 (GEM 9 AND 10)

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ABSTRACT

The spaceborne altimeter missions of GEOS-3 (50 cm accuracy) and the future SEASAT (10 cm accuracy) require precise knowledge of the radial position of the spacecraft to be most effective. Though errors in previous gravity models have produced large uncertainties in the orbital position of GEOS-3, significant improvement has been obtained with new geopotential solutions, Goddard Earth Models 9 and 10. Using least squares collocation GEM 9 was derived by combining laser data from GEOS-3, LAGEOS and Starlette, S-Band measurements on LANDSAT 1, together with data from 26 other satellites used in previous solutions. GEM 10 is a combination solution containing a global set of surface gravity anomalies along with the data in GEM 9. Radial errors of GEOS-3 for 5 day arcs have been reduced from about 5 m to 1 m based upon orbital intercomparisons, station navigations and analyses employing crossover points from passes of altimetry.

The use of collocation has permitted GEM 9 to be a larger field than previous derived satellite models, GEM 9 having harmonics complete to 20 x 20 with selected higher degree terms. The satellite data set has approximately 840,000 observations, of which 200,000 are laser ranges taken on 9 satellites equipped with retroreflectors. GEM 10 is complete to 22 x 22 with selected higher degree terms out to degree and order 30 amounting to a total of 592 coefficients. Comparisons with surface gravity and altimeter data indicate a substantial improvement in GEM 9 over previous satellite solutions; GEM 9 is in even closer agreement with surface data than the previously published GEM 6 solution which contained surface gravity. In particular the free air gravity anomalies calculated from GEM 9 and a surface gravity solution by Rapp (1977) are in excellent agreement for the high degree terms ($13 \leq \ell \leq 22$).

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The mass constant of the Earth, GM, has been estimated from the laser data as 398600.64 ± .02 km^3/sec^2, a value which is principally determined from LAGEOS. The speed of light used was 299792.5 km/sec. Geocentric station positions were determined for approximately 150 stations in GEM 10. These station coordinates, their mean sea level heights and altimetry data provide an estimate for the mean radius of the earth of a_e = 6378140 ± 1 m. Accuracy estimates derived for the potential coefficients have been verified with independent data sets. These produce commission errors in geoid heights of 1.9 m and 1.5 m (global RMS values) respectively for GEM 9 and 10.

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SECTION I. .... INTRODUCTION

I. INTRODUCTION

The Earth and Ocean Dynamics Applications Program (EODAP) of the National Aeronautics and Space Administration calls for knowledge of the global geoid to sub-meter levels of accuracy. While final realization of these goals will rely strongly on GEOS-3 and SEASAT altimetry, progress continues to be made toward comprehensive gravity modeling using conventional satellite tracking systems and surface gravimetry.

At Goddard Space Flight Center (GSFC) the emphasis has been on using as much of the precise satellite data as possible. Precise laser tracking such as the International Satellite Geodesy Experiment (ISAGEX, Brachet, 1970) laser systems of 1970 have yielded a substantial improvement in geopotential sensitivity and accuracy over the last few years (e.g., Wagner, et al 1977). However, the accuracy of the GSFC, SAO and French laser systems (with 5 cm noise levels for GSFC systems now deployed) on the new GEOS-3 and Starlette orbits could not be realized without continued geopotential improvement. In the case of GEOS-3, effective use of the altimeter data required a very significant improvement in radial orbit determination accuracies beyond the capabilities of existing gravity models. Improvement of GEOS-3 orbit determination by reduction of geopotential uncertainties was a major objective of Goddard Earth Models (GEM) 9 and 10.

GEM 9 is a gravity model based solely on optical, laser, and electronic observations taken on 31 satellites. GEM 10 combines the GEM 9 satellite data with surface gravimetry. (GEM 10 and other solutions which are derived from both satellite and surface observations (e.g., SAO 4.3, GRIM 2) are referred to as "combination" solutions.)

GEM 9 and 10 incorporate a number of significant changes in technique over previous GEM solutions. The extension of the GEM 9 satellite solution to 20 x 20 (complete in degree and order) was accomplished through the use of least squares collocation (Moritz, 1972). This technique is discussed in Section 3.2 and is also used by King-Hele (1974) and Anderle (private communication, 1977) in their gravity work. The adjustment of the earth's mass (GM) is another advancement (Section 3.3). A significant improvement was obtained in GEM 10 by now including the truncation of the gravity field (as well as the accuracy of the data) as an error source in weighting the gravimetry observations (Section 3.4). GEM 9 and 10 will be used as the base fields for other solutions being planned which will extensively use the altimeter data available on GEOS-3.

Many of the data systems for the GEOS-3 mission have been used to evaluate the GEM 9 and 10 models. While the satellite-to-satellite doppler relay (SSL) and the altimeter ranging data have not been included in these latest GEM models, these data have been used to assess the overall global improvement of the models. The laser tracking has also been used to test the models. These studies are included within this report and provide a strong demonstration of the high level of accuracy which has been achieved in GEM 9 and 10.

SECTION 2. . . . . DATA [redacted]

2. DATA

2.1 SATELLITE TRACKING DATA

A brief summary of the 840,000 satellite tracking measurements utilized in GEM 9, is given in Table 1. The main feature of the data in the new solution is the large amount of laser data employed totaling about 200,000 observations. Because of the sensitivity of the laser system to satellite perturbations (down to 5 cm), contributions of the laser observations have been computed complete through degree and order 22 for the harmonics, whereas the harmonics were computed complete only through degree and order 16 for the other types of data. The ISAGEX laser data have been used in previous solutions, but in these, the harmonics were computed complete only through degree 16.

A description of the satellites employed and their data distribution is given in Tables 2, 3A, and 3B. These tables respectively describe (2) satellite orbital characteristics and types of data employed, (3a) the distribution of data on satellite arcs containing optical data only, and (3b) the distribution of data on satellite arcs containing a variety of tracking systems consisting of electronic, laser, and additional optical observations.

Characteristics of the data among the various tracking systems are summarized in Table 4. Summaries by tracking network consist of the number of stations, observations, and satellites observed including accuracies and weights used for sigmas of the data in the solution. There are 561,000 measurements which have been used previously in GEM 7 and these are distributed among 9 different tracking networks. The table also shows the data which are unique to GEM 9, totaling 278,400 observations for Laser, S-Band, and NWL Doppler tracking systems.

TABLE 1.

GEM 9 SATELLITE TRACKING DATA

TYPE	NO. OBS.	NO. SATELLITES	NO. ARCS	HARMONICS (Complete)
OPTICAL	150,000	24	287	16 x 16
ELECTRONIC	477,000	11	97	16 x 16
LASER	213,000	9	127	22 x 22

LASER DATA DISTRIBUTION

GEOS–3	94,000 obs.	38 arcs
STARLETTE	28,000	26
LAGEOS*	25,000	11
BE–C	3,000	4
7 ISAGEX SATELLITES (BE–B, BE–C, D1–C, D1–D, GEOS–1, GEOS–2, PEOLE)	63,000	48

*LAGEOS USED FOR ESTIMATING GM AND STATIONS ONLY (SEE SECTION 3.3).

TABLE 2. SATELLITE ORBITAL CHARACTERISTICS USED IN GEM 9 AND 10

SATELLITE NAME	A (KILOMETERS)	E	I (DEGREES)	MEAN MOTION (REV/DAY)	PRIMARY RESONANT PERIOD DAYS	DATA TYPE**
AGENA–RB*	7297.	0.0010	69.91	13.92	5.0	O
ANNA–1B	7501.	0.0082	50.12	13.37	4.8	O, RR
BE–B	7354.	0.0135	79.69	13.76	3.0	L, RR, O
BE–C	7507.	0.0257	41.19	13.35	5.6	L, RR, O
COURIER	7469.	0.0161	28 31	13.46	3.8	O
DI–C	7341.	0.0532	39.97	13.81	2.5	L, O
DI–D	7622.	0.0848	39.46	13.05	8.4	L, O
ECHO–1RB	7966.	0.0118	47.21	12.21	11.9	O
GEOS–1	8075.	0.0719	59.39	11.96	7.0	L, RR, O
GEOS–2	7711.	0.0330	105.79	12.82	5.7	L, R, RR, O
GEOS–3	7226.	0.0008	114.98	14.13	4.5	L
GRS	7239.	0.0598	49.76	14.10	10.7	O
INJUN	7316.	0.0079	66.82	13.87	3.8	O
LANDSAT–1	7286.	0.0013	99.10	13.99	18.0	RR
LAGEOS	12273.	0.0038	109.85	6.39	2.7	L
MIDAS–4	9995.	0.0112	95.83	8.69	3.0	O
OGO–2	7341.	0.0752	87.37	13.79	3.8	O
OSCAR–7	7411.	0.0224	89.70	13.60	2.2	O
OVI–2	8317.	0.0184	144.27	11.45	2.2	O
PEOLE	7006.	0.0164	15.01	14.82	2.1	L, M
SAS	6923.	0.0035	3.04	15.09	4.6	M
SECOR–5	8151.	0.0793	69.22	11.79	3.4	O
STARLETTE	7331.	0.0204	49.80	13.83	2.8	L
TELSTAR	9669.	0.2429	44.79	9.13	14.9	O
TIROS–9	8024.	0.1173	96.41	12.07	19.5	M
TRANSIT–4A	7322.	0.0076	66.82	13.85	3.5	O
VANGUARD–2RB	8496.	0.1832	32.92	11.09	294.3	O
VANGUARD–2	8298.	0.1641	32.89	11.49	2.7	O
VANGUARD–3	8508.	0.1901	33.34	11.07	187.6	O
5BN–2	7462.	0.0058	89.95	13.46	2.4	O

*RB = Rocket Body

**L – Laser Range, R – Range, RR – Range Rate, O – Optical, M – Minitrack

TABLE 3A. DISTRIBUTION OF DATA FOR SATELLITE ARCS USING OPTICAL DATA ONLY

287 WEEKLY OPT. ARCS (PRIMARILY SAO BAKER-NUNN)

SATELLITE NAME	SATELLITE ID	NO. ARCS	NO. OBS.
AGENA–RB	640011	7	1005
ANNA–1B	620601	40	4183
BE–B	640841	4	469
BE–C	650321	22	4947
COURIER–1B	600131	12	3375
DI–C	670111	4	902
DI–D	670141	9	6386
ECHO–IRB	600092	18	2240
GEOS–I	650891	28	40855*
GEOS–II	680021	24	25315*
GRS	630261	5	369
INJUN–1	610162	9	768
MIDAS–4	610281	20	14879
OGO–2	650811	7	461
OSCAR–7	660051	4	1780
OVI–2	650781	4	910
SECOR–5	650631	4	290
TELSTAR–1	620291	16	1946
TRANSIT–4A	610151	14	1316
VANGUARD–2RB	590012	11	379
VANGUARD–2	590011	5	615
VANGUARD–3	590071	15	996
5BN–2	630492	5	355
	TOTALS	287	114700

*MOTS/SPEOPTS OBS.: GEOS–1 – 22100, GEOS–II – 22000 PLUS 2100 OBS. FROM INTERNATIONAL CAMERAS.

TABLE 38 DISTRIBUTION OF DATA FOR SATELLITE ARCS USING ELECTRONIC AND LASER TRACKING SYSTEMS OR COMBINED SYSTEMS INCLUDING CAMERAS

SATELLITE	BAKER-NUNN	MOTS/ SPEOPTS	INTER-NATIONAL CAMERAS	NWL DOPPLER	PRE-ISAGEX LASER	GRARR(G) S-BAND(S)	C-BAND(C) MINI-TRACK (M)	NO. ARCS*	ISAGEX LASER NO. OBS.	ISAGEX LASER NO. ARCS*	NEW LASER NO. OBS.	NEW LASER NO. ARCS**
	NUMBER OF OBSERVATIONS											
ANNA-1B				12300				4				
BE B				10600				6	2500	3		
BE C	700			13200	1000 22000			6 4	12500	8	3000	4
DI C	1000				700			2	7600	4		
DI D	2100				100			1	12200	7		
GEOS 1	6900	10500		116200	1800	4400 (G)		14	14900	10		
GEOS 2	4600	11000	1900	29500 86800	8000	137400 (G)	3800(C)	13 13	9000	10		
GEOS 3											94000	38
LAGEOS											25000	11
LANDSAT 1						19600 (S)		11				
PEOLE					200		1000 (M)	4	4300	6		
SAS							3400 (M)	5				
STARLETTE											28000	26
TIROS 9							1600 (M)	14				
TOTALS	15300	21600	1900	268600	33800	141800 (G) 19600 (S)	3800 (C) 6000 (M)	97	63000	48	150000	79

* 7-DAY LONG ORBITAL ARCS (1-DAY ARCS FOR LANDSAT-1)

** 5-DAY LONG ORBITAL ARCS

TABLE 4. DATA CHARACTERISTICS OF TRACKING SYSTEMS IN GEM 9

TRACKING SYSTEMS	AGENCY	DATA TYPE	ACCURACY (σ)	σ USED FOR WEIGHTING[1]	NO. OF STATIONS	NO. OF SATELLITES	NO. OF OBS.
BAKER–NUNN CAMERAS	SAO/ NASA	OPTICAL	2" (arcsec)	2"	27	24	83800
MOTS/SPEOPTS CAMERAS	GSFC/ NASA	OPTICAL	2"	2"	31	2	65600
INTERNATIONAL CAMERAS	SAO/	OPTICAL	2"	2"	6	1	4000
MINITRACK	GSFC/ NASA	DIRECTION COSINES	.0003	.0003	12 MOTS	4	6000
NWL DOPPLER	NWSC	RANGE RATE	4 cm/sec	4 cm/sec	32	5	181800
GRARR	GSFC/ NASA	RANGE, R–RATE	5 m 2 cm/sec	10 m 4 cm/sec	5	2	141800
C–BAND	USAF, NASA	RANGE	5 m	10 m	7	1	3800
PRE–ISAGEX LASERS (1967 – 70)	GSFC, SAO	RANGE	1 m	6 m	3	5	11800
ISAGEX LASERS (1970)	GSFC, SAO/ FRENCH	RANGE	50 cm 1 m	6 m 6 m	10[2]	7	63000
DATA UNIQUE TO GEM 9							
POLAR MOTION LASER (BE–C, 1970)	GSFC/ NASA	RANGE	50 cm	6 m	2	1	22000
S–BAND (LANDSAT–1)	GSFC/ NASA	RANGE RATE	2 cm/sec	4 cm/sec	14	1	19600
NWL DOPPLER (GEOS–2)	NWSC	RANGE RATE	4 cm/sec	4 cm/sec	(12) ABOVE	1	86800
NEW LASERS (1974–77)	GSFC, SAO/FR.	RANGE	5 cm 40 cm	–[4]	10[3]	4	150000
				TOTALS	146	31 UNIQUE	840000

[1] IN ADDITION, WEIGHTING WAS ADJUSTED ACCORDING TO RESIDUALS ON INDIVIDUAL ARCS (LERCH, et al., 1974, pg. 22).

[2] ISAGEX LASERS: 2 – GSFC, 5 – SAO, 3 – FRENCH. [3] NEW LASERS: 3 – GSFC, 5 – SAO, 2 – FRENCH.

[4] 3 M FOR BE–C AND GEOS–3, 1 M FOR LAGEOS, 6 M FOR STARLETTE.

2.2 SURFACE GRAVITY DATA

A set of 1654 equal area 5° mean gravity anomalies (Rapp, 1977), have been used along with the satellite tracking data in the combination solution GEM 10. The data is based upon approximately 38,000 1° mean gravity anomalies (Figure 13). Accuracy estimates for the 5° mean anomalies are depicted in Figure 1. Of the 1654 5° mean anomalies, 1507 were based directly on the 1° anomalies while the remaining 147 5° means were obtained by interpolation. The distribution of the number (N) of 1° anomalies within a 5° block is shown in Table 5 along with accuracy estimates of the 5° means. Only 625 of the 5° blocks contain a full set of 1° mean (observed) anomalies.

Figure 1. Rapp 5° (equal area) Anomalies and Their Uncertainties

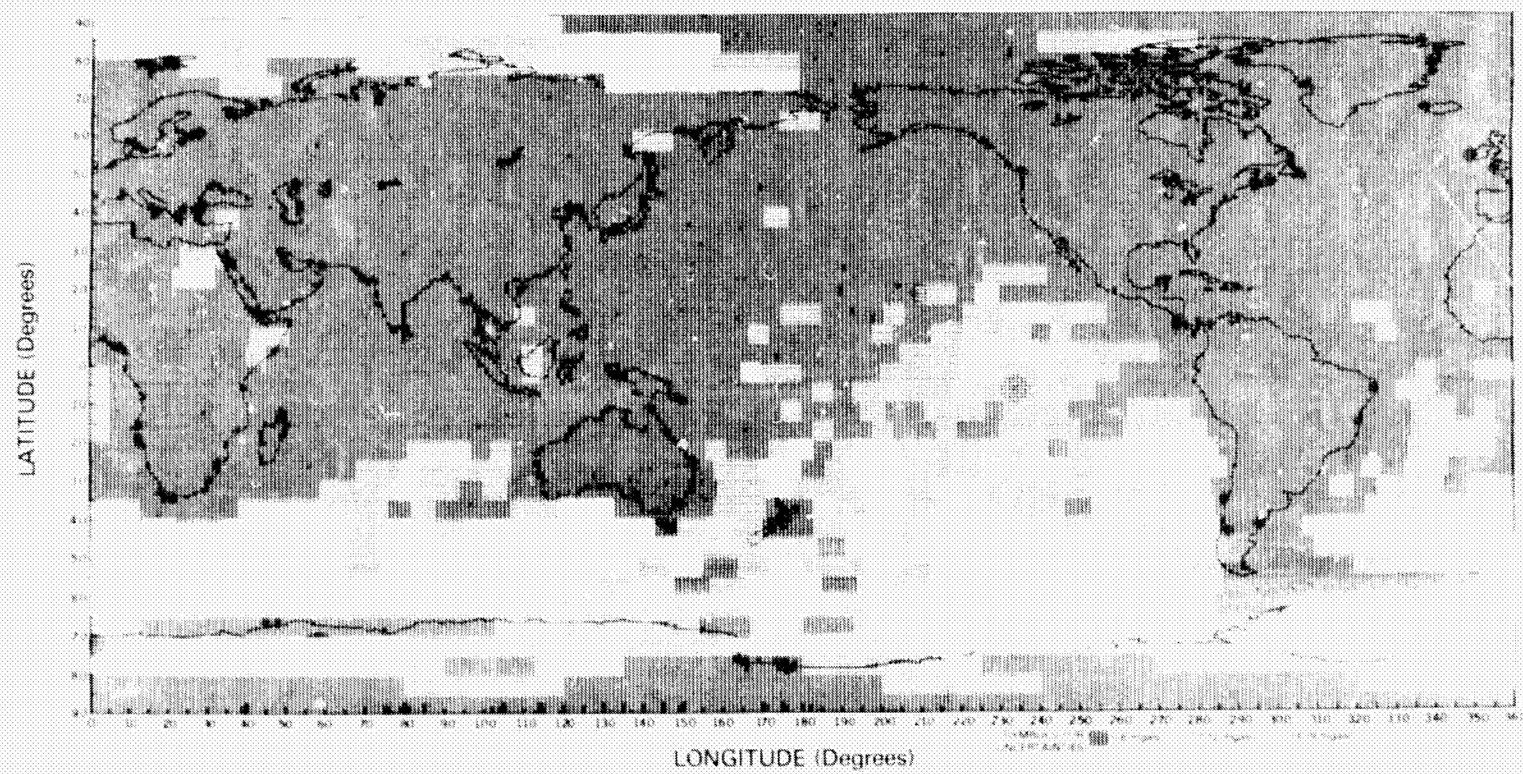

TABLE 5. AVERAGE ACCURACY OF 5° MEAN ANOMALIES COMPARED TO THE NUMBER (N) OF 1° ANOMALIES WITHIN THE 5° BLOCK

N	NUMBER OF 5° MEANS	AVERAGE ACCURACY (MGALS)
25	625	2.5
20 – 24	310	3.5
15 – 19	177	5.3
10 – 14	151	7.2
5 – 9	144	10.0
1 – 4	100	14.0
0	147*	17.0
TOTAL	1654	

*INTERPOLATED FROM NEIGHBORING 5° ANOMALIES.

SECTION 3. .... MODELING TECHNIQUES AND RESULTS

3. MODELING TECHNIQUES AND RESULTS

The basic modeling techniques employed for the orbital, geopotential, and station solutions are given in detail by Lerch et al, 1974. In this section we present the GEM 9 and 10 solutions for the potential coefficients and station coordinates along with a discussion of the new techniques employed. We also present and discuss solutions for fundamental geodetic reference parameters: the mean radius of the earth (a_e), the gravitational constant (GM), and mean equatorial gravity (g_e).

3.1 GEOPOTENTIAL

The gravitational potential was modeled in terms of spherical harmonics as follows:

$$V = \frac{GM}{r} \left\{ 1 + \sum_{\ell=2}^{\infty} \sum_{m=0}^{\ell} \left(\frac{a_e}{r}\right) \bar{P}_{\ell m} (\sin \phi) \left| C_{\ell m} \cos m\lambda + S_{\ell m} \sin m\lambda \right| \right\}$$

where GM is the earth's gravitational constant including the atmosphere, a_e is the earth's mean equatorial radius, $\bar{P}_{\ell m}$ is the fully normalized associated Legendre function of degree ℓ and order m (e.g., Kaula, 1966, p. 7) and r, ϕ, λ are the distance from the center of mass, latitude and longitude. The normalized potential coefficients ($C_{\ell m}$, $S_{\ell m}$) for GEM 9 and 10 are listed in Tables 6 and 7. Using these potential models in Brun's formula (Heiskanen and Moritz, 1967) geoids are computed and presented in Figures 2 and 3.

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[redacted] GEOPOTENTIAL COEFFICIENTS

TABLE 6. GODDARD EARTH MODEL 9
NORMALIZED COEFFICIENTS (UNITS OF 10^{-6})

ZONALS

INDEX N	M	VALUE	INDEX N	M	VALUE	INDEX N	M	VALUE	INDEX N	M	VALUE	INDEX N	M	VALUE
2	0	−484.16555	3	0	0.95848	4	0	0.54154	5	0	0.06844	6	0	−0.15121
7	0	0.09331	8	0	0.05095	9	0	0.02733	10	0	0.05284	11	0	−0.04862
12	0	0.03794	13	0	0.04408	14	0	−0.02111	15	0	0.00136	16	0	−0.00846
17	0	0.01619	18	0	0.01117	19	0	0.00133	20	0	0.02562	21	0	−0.00128
22	0	−0.00448	23	0	−0.01837	24	0	−0.00190	25	0	−0.00016	26	0	0.00404
27	0	0.01022	28	0	−0.02063	29	0	−0.00915						

SECTORIALS AND TESSERALS

INDEX N	M	VALUE C	VALUE S	INDEX N	M	VALUE C	VALUE S	INDEX N	M	VALUE C	VALUE S
2	1	−0.00021	−0.00406	3	1	2.02826	0.25244	4	1	−0.53287	−0.46512
5	1	−0.04910	−0.09225	6	1	−0.07543	0.01587	7	1	0.26724	0.09607
8	1	0.02483	0.06327	9	1	0.16065	0.01676	10	1	0.08238	−0.13553
11	1	0.00023	−0.01356	12	1	−0.06304	−0.05415	13	1	−0.03419	0.03644
14	1	−0.01764	0.04976	15	1	−0.01848	−0.00236	16	1	0.02543	−0.01232
17	1	−0.00353	−0.02223	18	1	0.00227	−0.00242	19	1	−0.03465	−0.00698
[illegible]	1	0.00129	−0.02215	21	1	−0.01238	0.00749	22	1	0.00094	0.00554
23	1	0.00248	0.00907	24	1	−0.00567	−0.00119	25	1	0.01176	−0.00442
2	2	2.43400	−1.39786	3	2	0.89198	−0.62241	4	2	0.35259	0.66272
5	2	0.64975	−0.32488	6	2	0.04812	−0.34854	7	2	0.32786	0.10505
8	2	0.07315	0.05121	9	2	0.02445	−0.02467	10	2	−0.08429	−0.00375
11	2	0.03947	−0.09951	12	2	0.00390	−0.00931	13	2	0.02214	−0.03804
14	2	−0.03632	0.04590	15	2	0.00864	−0.02976	16	2	−0.00524	0.01566
17	2	−0.00985	0.01244	18	2	0.00649	0.02368	19	2	0.00877	−0.00714
20	2	−0.00395	0.00673	21	2	0.01467	0.00653	22	2	−0.00147	−0.00964

TABLE 6. GODDARD EARTH MODEL 9 (continued)

INDEX N	INDEX M	VALUE C	VALUE S	INDEX N	INDEX M	VALUE C	VALUE S	INDEX N	INDEX M	VALUE C	VALUE S
3	3	0.70256	1.41140	4	3	0.98849	−0.20316	5	3	−0.47030	−0.20788
6	3	0.05910	0.00138	7	3	0.23987	−0.21778	8	3	−0.00992	−0.08830
9	3	−0.17360	−0.09715	10	3	−0.02071	−0.16867	11	3	−0.02883	−0.13577
12	3	0.06412	0.03136	13	3	−0.04313	0.07611	14	3	0.01271	−0.01504
15	3	0.02086	0.00562	16	3	−0.00379	−0.02645	17	3	−0.00821	0.00715
18	3	−0.00006	−0.00023	19	3	0.00058	0.00094	20	3	−0.01561	0.01130
21	3	0.00407	0.00361	22	3	−0.00149	0.01117	4	4	−0.19661	0.29894
5	4	−0.29049	0.05037	6	4	−0.10458	−0.46086	7	4	−0.28553	−0.12689
8	4	−0.24624	0.07539	9	4	−0.01134	0.01072	10	4	−0.10099	−0.06889
11	4	−0.04503	−0.06082	12	4	−0.07375	−0.01663	13	4	−0.02511	0.00559
14	4	−0.01831	0.00804	15	4	−0.04391	−0.00720	16	4	0.03351	0.01158
17	4	0.00095	−0.00601	18	4	0.02206	0.01095	19	4	−0.00396	−0.00992
20	4	0.00072	−0.00797	21	4	0.00313	−0.00155	22	4	−0.00543	0.00422
5	5	0.16123	−0.66181	6	5	−0.25597	−0.53881	7	5	0.01667	0.04367
8	5	−0.01251	0.08245	9	5	0.00490	−0.06410	10	5	−0.06216	−0.03550
11	5	0.04545	0.07889	12	5	0.04569	0.01287	13	5	0.06892	0.04606
14	5	0.01940	−0.02183	15	5	−0.01243	−0.00533	16	5	−0.01371	−0.00269
17	5	−0.02428	0.00761	18	5	0.01637	0.01262	19	5	−0.00436	−0.00110
20	5	0.00153	0.00204	6	6	0.00188	−0.24196	7	6	−0.36330	0.13170
8	6	−0.07471	0.32546	9	6	0.03737	0.21837	10	6	−0.03888	−0.08597
11	6	0.00256	0.03819	12	6	0.00273	0.03705	13	6	−0.03329	0.00278
14	6	−0.00684	−0.00702	15	6	0.02539	−0.04249	16	6	−0.00697	−0.02266
17	6	−0.00165	−0.00974	18	6	0.01215	−0.00964	19	6	0.00006	0.00323
20	6	−0.00620	−0.00024	7	7	−0.00924	0.02663	8	7	0.06778	0.07712
9	7	−0.10449	−0.07411	10	7	−0.00022	0.01616	11	7	0.01076	−0.08216
12	7	−0.01282	0.04908	13	7	−0.00490	−0.00121	14	7	0.01804	−0.03207
15	7	0.06506	0.01133	16	7	0.00412	−0.00553	17	7	0.01583	0.01255
18	7	−0.00308	−0.00071	19	7	−0.00856	0.00852	20	7	−0.01072	−0.00353
8	8	−0.12123	0.12712	9	8	0.19418	−0.01555	10	8	0.03989	−0.07159
11	8	0.00878	0.02269	12	8	−0.02429	0.02380	13	8	−0.02066	−0.00321

TABLE 6. GODDARD EARTH MODEL 9 (continued)

INDEX N	M	VALUE C	VALUE S	INDEX N	M	VALUE C	VALUE S	INDEX N	M	VALUE C	VALUE S
14	8	−0.04572	−0.00017	15	8	−0.01726	0.02563	16	8	−0.01005	0.00567
17	8	0.01245	0.00300	18	8	0.00676	−0.01226	19	8	0.00813	−0.01796
20	8	−0.00306	−0.00465	9	9	−0.05810	0.09172	10	9	0.12430	−0.05053
11	9	−0.02893	0.03349	12	9	0.04154	0.01083	13	9	0.02678	0.04194
14	9	0.03880	0.01116	15	9	0.01303	0.04074	16	9	−0.01885	−0.03781
17	9	−0.01075	−0.01928	18	9	−0.02487	0.02529	19	9	0.00493	0.01924
20	9	0.00534	0.02217	21	9	0.00467	−0.00175	22	9	0.00507	−0.00136
10	10	0.10563	−0.01962	11	10	−0.06709	−0.00324	12	10	0.00028	0.05133
13	10	0.03154	−0.03227	14	10	0.05791	0.01384	15	10	0.02896	0.01124
16	10	0.01564	0.00788	17	10	0.01069	0.01173	18	10	−0.00246	−0.01030
19	10	−0.01507	−0.00637	20	10	−0.01150	−0.00746	21	10	−0.00048	0.00268
22	10	0.00216	0.00094	11	11	0.04047	−0.07862	12	11	0.01603	−0.00483
13	11	−0.04459	−0.01686	14	11	0.01920	−0.03660	15	11	0.00047	−0.00421
16	11	0.02186	0.00936	17	11	−0.04212	−0.00404	18	11	−0.00447	0.01734
19	11	−0.00497	0.02493	20	11	0.01074	−0.00997	21	11	−0.00506	−0.01019
22	11	−0.00501	−0.02392	12	12	−0.00764	−0.01178	13	12	−0.03230	0.09061
14	12	0.00835	−0.03024	15	12	−0.03347	0.01659	16	12	0.01770	0.00977
17	12	0.03148	0.02126	18	12	−0.03891	−0.02304	19	12	−0.00752	0.00613
20	12	−0.00911	0.02576	21	12	0.00330	0.02249	22	12	−0.01577	−0.01916
23	12	0.02040	0.01049	24	12	−0.00257	−0.01462	25	12	−0.00189	0.00732
13	13	−0.06009	0.06977	14	13	0.02803	0.04223	15	13	−0.02282	−0.00224
16	13	0.01223	−0.00750	17	13	0.01471	0.01921	18	13	−0.01200	−0.02737
19	13	−0.01235	−0.03066	20	13	0.02319	0.00394	21	13	−0.01641	0.01387
22	13	−0.03006	0.00774	23	13	−0.00217	−0.00165	24	13	0.00610	−0.00728
25	13	0.01522	−0.00880	26	13	−0.00323	−0.00820	27	13	−0.00770	−0.01073
28	13	0.02064	0.00875	29	13	−0.01083	−0.00896	14	14	−0.05119	−0.00591
15	14	0.00392	−0.02460	16	14	−0.01943	−0.03787	17	14	−0.01589	0.01083
18	14	−0.00851	−0.00934	19	14	−0.00552	−0.01273	20	14	0.01232	−0.01019
21	14	0.01955	0.01031	22	14	0.00954	0.00665	23	14	0.00881	−0.00568
24	14	−0.01659	0.00317	25	14	−0.02360	0.01669	26	14	0.00688	0.00148

TABLE 6. GODDARD EARTH MODEL (continued)

INDEX N	INDEX M	VALUE C	VALUE S	INDEX N	INDEX M	VALUE C	VALUE S	INDEX N	INDEX M	VALUE C	VALUE S
27	14	0.02350	0.00612	28	14	−0.00531	−0.01147	29	14	−0.00974	0.00956
15	15	−0.02108	−0.00640	16	15	−0.01320	−0.02607	17	15	0.00159	0.00393
18	15	−0.05460	−0.01615	19	15	−0.02174	−0.01616	20	15	−0.02311	0.00947
21	15	0.01066	0.01209	22	15	0.02425	0.00033	23	15	0.01267	0.00258
24	15	0.00487	−0.00157	25	15	−0.00289	−0.00148	16	16	−0.02546	0.01402
17	16	−0.02368	0.01308	18	16	0.00824	0.01840	19	16	−0.03536	−0.01327
20	16	−0.01433	−0.00130	21	16	0.00027	−0.00542	22	16	−0.00372	−0.00436
17	17	−0.01299	−0.00090	18	17	0.01983	0.00931	19	17	0.02971	−0.00531
20	17	−0.01435	−0.00674	21	17	−0.00288	−0.00243	22	17	−0.00225	−0.00325
18	18	0.00198	−0.01267	19	18	0.03539	−0.02120	20	18	−0.00275	0.02895
21	18	0.01417	−0.01220	22	18	0.01045	−0.01016	19	19	0.00865	−0.00143
20	19	0.01586	0.00864	21	19	0.01202	−0.00849	22	19	−0.01636	−0.02256
20	20	−0.00510	−0.00508	27	26	−0.01133	−0.00526	28	26	0.00754	−0.00232
29	26	−0.01043	−0.01602	27	27	−0.00100	−0.00332	28	27	−0.00861	0.00306
28	28	0.00487	0.00612	30	28	−0.01820	−0.03494				

TABLE 7. GODDARD EARTH MODEL 10
NORMALIZED COEFFICIENTS (UNITS OF 10^{-6})

ZONALS

INDEX N	M	VALUE	INDEX N	M	VALUE	INDEX N	M	VALUE	INDEX N	M	VALUE	INDEX N	M	VALUE
2	0	−484.16544	3	0	0.95838	4	0	0.54112	5	0	0.06862	6	0	−0.15070
7	0	0.09312	8	0	0.05021	9	0	0.02754	10	0	0.05261	11	0	−0.04857
12	0	0.03862	13	0	0.04400	14	0	−0.02299	15	0	0.00143	16	0	−0.00728
17	0	0.01675	18	0	0.01001	19	0	0.00020	20	0	0.02370	21	0	0.00010
22	0	−0.00249	23	0	−0.01909	24	0	−0.00553	25	0	0.00044	26	0	0.00562
27	0	0.01087	28	0	−0.02127	29	0	−0.00963						

SECTORIALS AND TESSERALS

INDEX N	M	VALUE C	VALUE S	INDEX N	M	VALUE C	VALUE S	INDEX N	M	VALUE C	VALUE S
2	1	0.00104	−0.00243	3	1	2.02855	0.25197	4	1	−0.53521	−0.46926
5	1	−0.05117	−0.09379	6	1	−0.07293	0.02316	7	1	0.27044	0.10196
8	1	0.02111	0.05504	9	1	0.15846	0.00831	10	1	0.08885	−0.13023
11	1	0.00554	−0.00242	12	1	−0.06945	−0.05344	13	1	−0.03593	0.02592
14	1	−0.00545	0.04033	15	1	−0.00281	0.00274	16	1	0.01965	0.00367
17	1	−0.01550	−0.02197	18	1	0.00297	−0.01471	19	1	−0.03022	−0.00623
20	1	0.00100	−0.01367	21	1	−0.01609	0.01570	22	1	0.00204	0.00134
23	1	0.00523	0.01281	24	1	−0.00656	0.00033	25	1	0.00499	−0.00213
2	2	2.43404	−1.39907	3	2	0.89272	−0.62346	4	2	0.35208	0.66404
5	2	0.65146	−0.32769	6	2	0.04935	−0.35387	7	2	0.32437	0.10813
8	2	0.07082	0.05462	9	2	0.02731	−0.03571	10	2	−0.08538	−0.01315
11	2	0.03107	−0.09117	12	2	0.00181	−0.00415	13	2	0.02761	−0.05666
14	2	−0.03616	0.03335	15	2	0.00332	−0.02052	16	2	−0.00941	0.02558
17	2	−0.01788	0.01258	18	2	0.00498	0.01623	19	2	0.01621	−0.00123
20	2	−0.00541	0.01019	21	2	0.00989	−0.00067	22	2	−0.00183	−0.00943

TABLE 7. GODDARD EARTH MODEL 10 (continued)

INDEX N	INDEX M	VALUE C	VALUE S	INDEX N	INDEX M	VALUE C	VALUE S	INDEX N	INDEX M	VALUE C	VALUE S
3	3	0.70028	1.41250	4	3	0.98850	−0.20179	5	3	−0.46712	−0.20298
6	3	0.05697	0.00332	7	3	0.23109	−0.21615	8	3	−0.01136	−0.08558
9	3	−0.16196	−0.09053	10	3	−0.01879	−0.16064	11	3	−0.05093	−0.12921
12	3	0.05913	0.02586	13	3	−0.02174	0.07047	14	3	0.03262	−0.00677
15	3	0.02250	0.02413	16	3	−0.01106	−0.01984	17	3	−0.00791	0.00178
18	3	−0.00136	−0.00400	19	3	0.00172	−0.00958	20	3	−0.00934	0.01568
21	3	0.00901	0.00527	22	3	−0.00504	0.00789	4	4	−0.19531	0.29883
5	4	−0.28754	0.04990	6	4	−0.10089	−0.46157	7	4	−0.28455	−0.12984
8	4	−0.24419	0.07507	9	4	−0.00977	0.01357	10	4	−0.09717	−0.07666
11	4	−0.04829	−0.07397	12	4	−0.08039	−0.02156	13	4	−0.01603	−0.00226
14	4	−0.00637	−0.00028	15	4	−0.04109	−0.00341	16	4	0.03198	0.03151
17	4	−0.00708	0.01038	18	4	0.03212	0.00546	19	4	0.00323	−0.01652
20	4	−0.00491	−0.01402	21	4	0.00253	0.00928	22	4	0.00025	0.01193
5	5	0.15617	−0.65983	6	5	−0.25833	−0.53730	7	5	0.01498	0.04312
8	5	−0.01608	0.08245	9	5	−0.00723	−0.05617	10	5	−0.06518	−0.03159
11	5	0.04723	0.07119	12	5	0.04429	0.00653	13	5	0.05961	0.05044
14	5	0.01997	−0.01693	15	5	0.00061	0.00058	16	5	−0.01064	−0.00135
17	5	−0.02296	0.00653	18	5	0.01421	0.01153	19	5	−0.00868	−0.00204
20	5	−0.00029	0.00158	21	5	0.00312	−0.00679	22	5	0.00581	0.00660
6	6	0.00271	−0.24213	7	6	−0.36170	0.13055	8	6	−0.07484	0.31977
9	6	0.03974	0.21681	10	6	−0.03877	−0.08487	11	6	−0.00350	0.03074
12	6	−0.00211	0.02615	13	6	−0.03671	0.00301	14	6	−0.00949	−0.00179
15	6	0.02390	−0.04516	16	6	−0.00635	−0.02898	17	6	−0.01310	−0.01936
18	6	0.00935	−0.01118	19	6	0.00876	0.01212	20	6	0.00697	0.00340
21	6	0.00047	0.00113	22	6	0.00318	−0.00024	7	7	−0.00717	0.01688
8	7	0.06632	0.07700	9	7	−0.10317	−0.07477	10	7	0.00388	0.01801
11	7	0.01382	−0.08498	12	7	−0.01897	0.04645	13	7	−0.00118	−0.00334
14	7	0.02282	−0.02217	15	7	0.06538	0.01653	16	7	−0.00237	−0.00768
17	7	0.01600	−0.00201	18	7	0.00254	−0.00389	19	7	0.00258	0.00405
20	7	−0.01390	−0.00599	21	7	−0.01030	0.00763	22	7	−0.00271	0.00169

TABLE 7. GODDARD EARTH MODEL 10 (continued)

INDEX N	INDEX M	VALUE C	VALUE S	INDEX N	INDEX M	VALUE C	VALUE S	INDEX N	INDEX M	VALUE C	VALUE S
8	8	−0.12262	0.12887	9	8	0.19939	−0.01222	10	8	0.04349	−0.06948
11	8	0.01085	0.02482	12	8	−0.02617	0.02585	13	8	−0.01830	−0.00381
14	8	−0.04525	−0.00434	15	8	−0.01598	0.02921	16	8	−0.01900	0.00834
17	8	0.01833	−0.00385	18	8	0.01756	−0.01142	19	8	0.01824	−0.01045
20	8	−0.00195	0.00990	21	8	0.00006	0.00021	22	8	−0.00781	0.00262
9	9	−0.05541	0.09097	10	9	0.12372	−0.04851	11	9	−0.03343	0.03788
12	9	0.03710	0.01169	13	9	0.02050	0.04507	14	9	0.03246	0.00995
15	9	0.01081	0.03356	16	9	−0.01781	−0.04536	17	9	−0.01019	−0.02609
18	9	−0.01934	0.02044	19	9	0.00906	0.00639	20	9	0.01785	0.01354
21	9	0.00450	0.00453	22	9	0.00850	0.00662	10	10	0.10112	−0.02228
11	10	−0.06487	−0.00122	12	10	−0.00466	0.05053	13	10	0.03104	−0.03120
14	10	0.04695	0.01273	15	10	0.01858	0.00283	16	10	0.00904	−0.00214
17	10	0.00982	0.01107	18	10	0.00142	−0.00712	19	10	−0.01995	−0.00809
20	10	−0.01252	−0.00787	21	10	−0.00168	−0.00024	22	10	0.00017	0.00707
11	11	0.04886	−0.07019	12	11	0.01619	−0.00381	13	11	−0.03917	−0.01086
14	11	0.02094	−0.03716	15	11	0.00008	0.00068	16	11	0.02278	0.00634
17	11	−0.02975	−0.00543	18	11	−0.00004	0.01638	19	11	0.00203	0.02191
20	11	0.01921	−0.01048	21	11	−0.00221	−0.01288	22	11	−0.00244	−0.02165
12	12	−0.00554	−0.01268	13	12	−0.03220	0.09102	14	12	0.00975	−0.03103
15	12	−0.03355	0.01650	16	12	0.01839	0.00875	17	12	0.03036	0.01899
18	12	−0.03557	−0.02274	19	12	−0.01017	0.00343	20	12	−0.00592	0.02306
21	12	0.00111	0.01673	22	12	−0.01396	−0.01715	23	12	0.01572	0.00483
24	12	0.00934	−0.01530	25	12	−0.00738	0.00359	13	13	−0.05954	0.06972
14	13	0.02809	0.04223	15	13	−0.02225	−0.00224	16	13	0.01213	−0.00625
17	13	0.01525	0.01986	18	13	−0.01153	−0.03674	19	13	−0.01165	−0.02961
20	13	0.02321	0.00395	21	13	−0.01557	0.01460	22	13	−0.02989	0.00838
23	13	−0.00118	−0.00074	24	13	0.00553	−0.00469	25	13	0.01504	−0.00764
26	13	−0.00235	−0.00538	27	13	−0.00821	−0.00785	28	13	0.02063	0.01063
29	13	−0.01165	−0.00576	14	14	−0.05122	−0.00543	15	14	0.00391	−0.02457
16	14	−0.01881	−0.03793	17	14	−0.01586	0.01086	18	14	−0.00797	−0.01007

TABLE 7. GODDARD EARTH MODEL 10 (continued)

INDEX N	INDEX M	VALUE C	VALUE S	INDEX N	INDEX M	VALUE C	VALUE S	INDEX N	INDEX M	VALUE C	VALUE S
19	14	−0.00581	−0.01257	20	14	0.01308	−0.01055	21	14	0.01969	0.01034
22	14	0.00971	0.00623	23	14	0.00839	−0.00532	24	14	−0.01797	0.00413
25	14	−0.02345	0.01681	26	14	0.00739	0.00105	27	14	0.02294	0.00612
28	14	−0.00724	−0.01113	29	14	−0.01098	0.01017	15	15	−0.02075	−0.00447
16	15	−0.01278	−0.02604	17	15	0.00235	0.00520	18	15	−0.05475	−0.01538
19	15	−0.02100	−0.01490	20	15	−0.02512	0.00549	21	15	0.00999	0.01154
22	15	0.02191	−0.00125	23	15	0.01389	0.00392	24	15	0.00191	−0.00401
25	15	−0.00780	−0.00435	16	16	−0.02579	0.00774	17	16	−0.02359	0.01142
18	16	0.01002	0.01677	19	16	−0.03212	−0.01255	20	16	−0.01244	−0.00331
21	16	0.00311	−0.00678	22	16	−0.00152	−0.00491	17	17	−0.02227	−0.00279
18	17	0.01587	−0.00260	19	17	0.02725	−0.00939	20	17	−0.00669	−0.00943
21	17	−0.00380	−0.00399	22	17	0.00342	−0.01193	18	18	−0.01066	−0.01451
19	18	0.03935	−0.01591	20	18	−0.00527	0.01647	21	18	0.02197	−0.00589
22	18	0.01463	−0.01305	19	19	0.00282	0.00122	20	19	0.00682	0.00621
21	19	−0.00146	−0.00573	22	19	−0.00439	−0.00815	20	20	0.00071	−0.00618
21	20	−0.03716	−0.00157	22	20	0.00466	0.01305	21	21	0.00375	−0.01106
22	21	−0.00529	0.00013	22	22	−0.00133	0.00515	27	26	−0.01078	−0.00566
28	26	0.00755	−0.00235	29	26	−0.00981	−0.01668	27	27	−0.00111	−0.00199
28	27	−0.00909	0.00262	28	28	0.00499	0.00602	30	28	−0.01810	−0.03480

Figure 2. Geoid Surface Computed from the GEM 9 Model (Height in Meters Above the Mean Ellipsoid, f = 1/298.256)

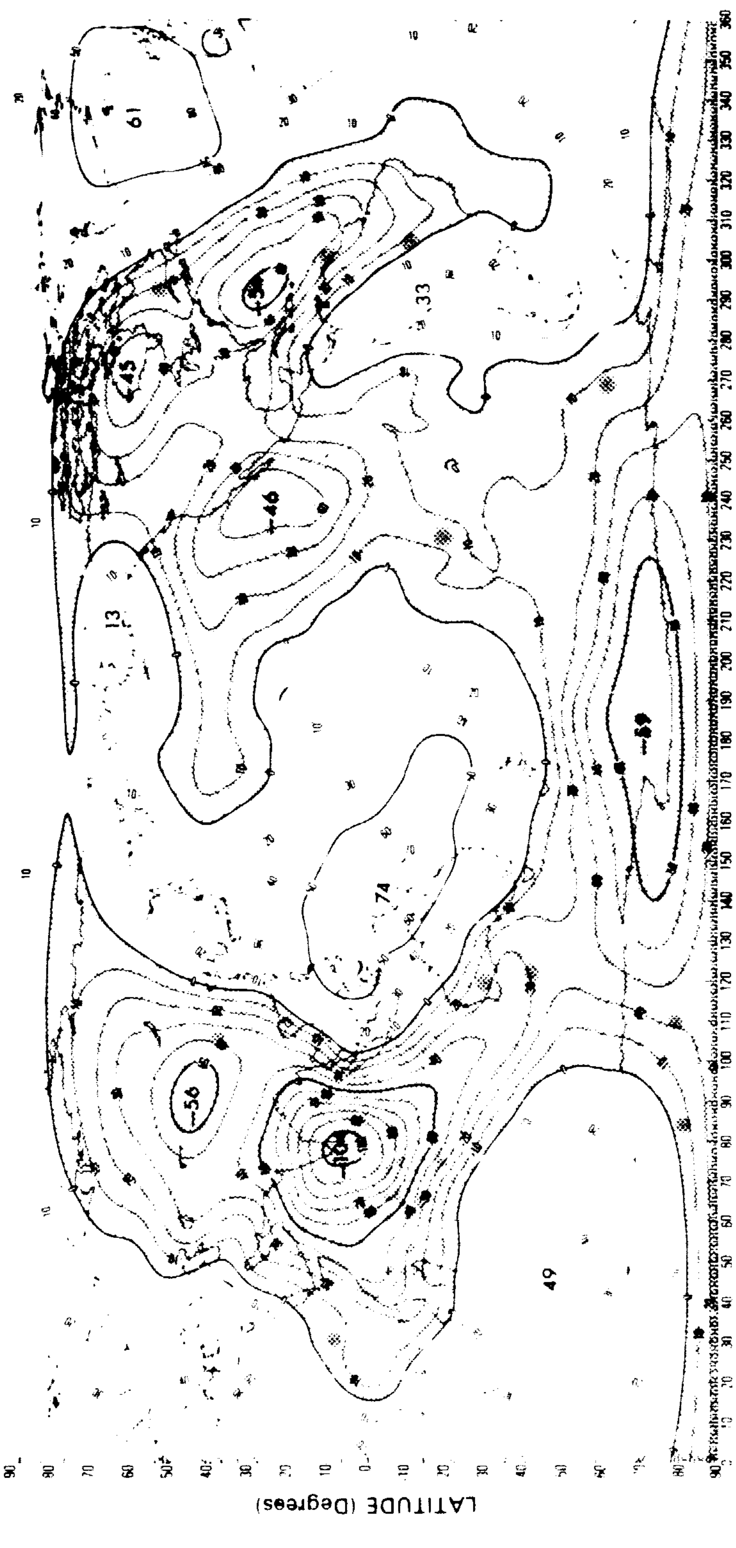
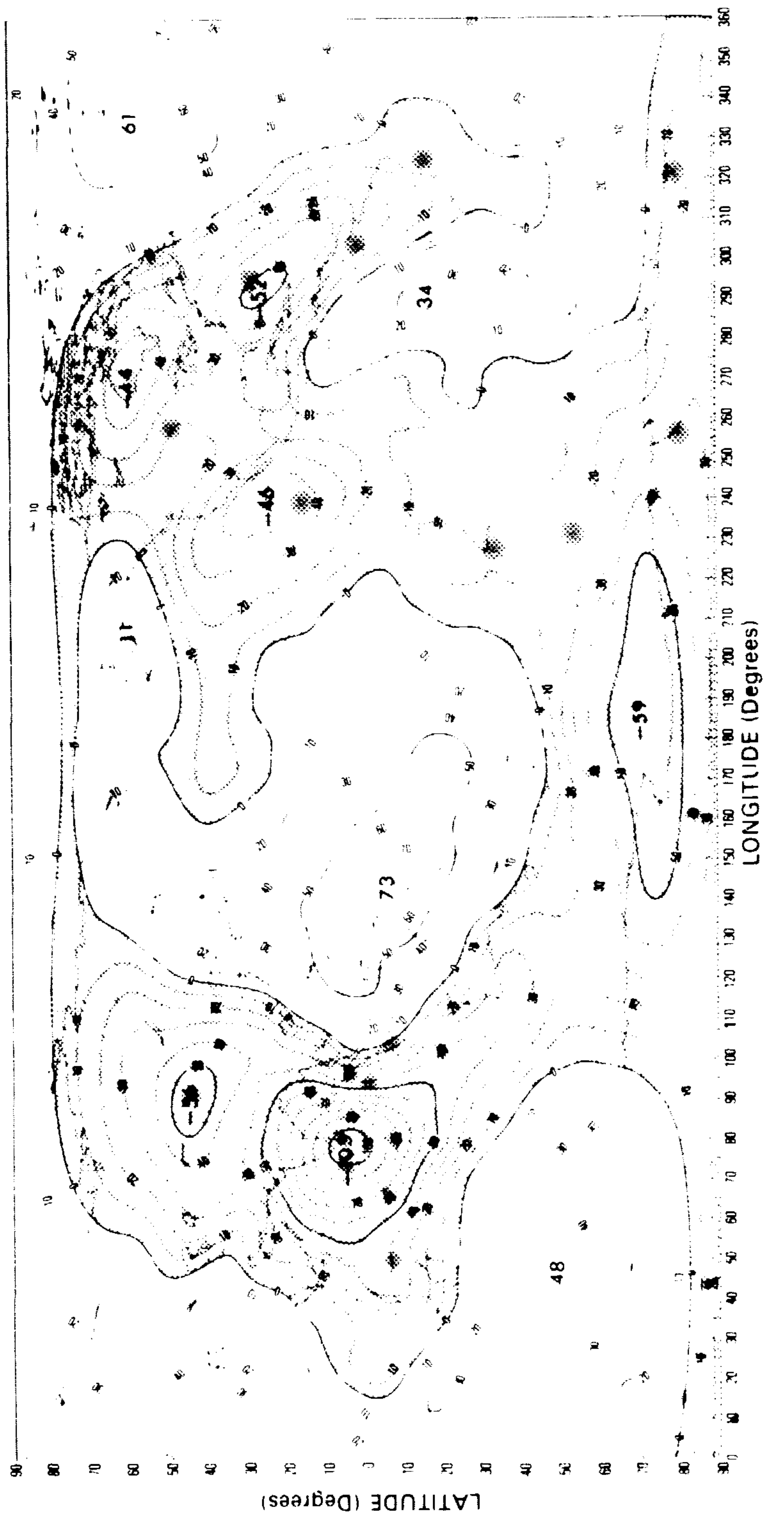


Figure 3. Geoid Surface Computed from the GEM 10 Model (Height in Meters Above the Mean Ellipsoid, f = 1/298.255)



3.2 COLLOCATION

The major innovation in GEM 9 over previous Goddard Earth Models was the use of "least squares collocation" (Moritz, 1972), which allowed the extension of the satellite field to 20 x 20. In this procedure we employed an approach similar to that of Rapp (1973; eq. 13). Conventional least squares simply minimizes the observation residuals (noise); the results of such an approach are described graphically in Figure 4 for gravity model recovery. In this figure the solution without any constraints (simple least squares) diverges at high degrees from the independent surface gravity data used to test it. The high correlation between certain high degree and order coefficients is the problem which causes an excessive adjustment of the coefficients in the solution. Least squares collocation essentially minimizes both the signal (e.g., harmonic coefficients) and the noise (observation residuals), thus controlling the excessive adjustment. First, we present the technique. The result of its application for GEM 9 and the result of other tests shown in Figure 4 are then discussed.

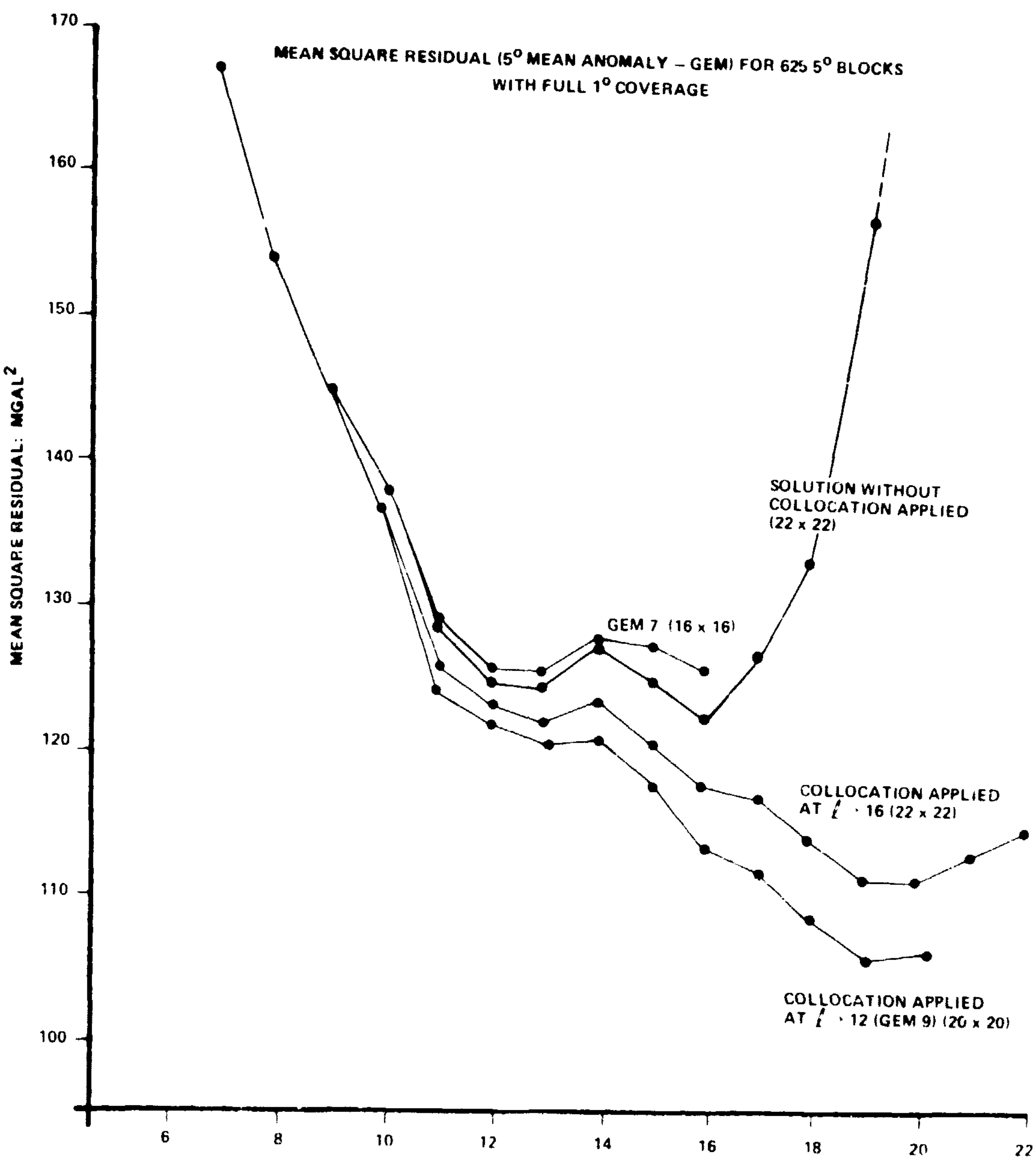
The principle of collocation is to minimize

$$r^T W r + s^T \bar{W} s \equiv Q \qquad (1)$$

with respect to the unknowns x, where

- x - geopotential, station and orbit parameters
- r - satellite observation residuals
- W - diagonal weight matrix for satellite observation residuals
- s - signal, harmonic (potential) coefficients representing a subset of **x**
- $\bar{W}$ - diagonal weight matrix with elements $1/\sigma_s^2$ where $\sigma_s(\ell,m) = 10^{-5}/\ell^2$

FIGURE 4. COMPARISON OF TRUNCATED GEM 9 SATELLITE DERIVED FIELDS USING DIFFERING LEVELS OF COLLOCATION WITH SURFACE GRAVITY DATA



Let s represent a subset of the potential coefficients with the partition

$$x = \begin{bmatrix} y \\ s \end{bmatrix} \qquad \Delta x = \begin{bmatrix} \Delta y \\ \Delta s \end{bmatrix}, \tag{2}$$

and using the linearized forms from Taylor's series

$$\begin{aligned} r &= r_o - A\Delta y - B\Delta s \qquad \text{(where A and B are matrices of partial derivatives)} \\ s &= s_o + \Delta s, \end{aligned} \tag{3}$$

then minimizing Q in (1) above gives the normal equations

$$\begin{bmatrix} A^T WA & A^T WB \\ B^T WA & B^T WB + \bar{W} \end{bmatrix} \begin{bmatrix} \Delta y \\ \Delta s \end{bmatrix} = \begin{bmatrix} A^T Wr_o \\ B^T Wr_o - \bar{W}s_o \end{bmatrix} \tag{4}$$

Allowing for a scale factor w to adjust the relative weighting between W and $\bar{W}$ above, we have

$$W = w\, W_o \qquad (W_o \text{ is the formal weight matrix}) \tag{5}$$

$$w = \bar{f}^2/f^2 \tag{6}$$

$$\sigma = f\sigma_o$$

$$\bar{\sigma} = \bar{f}\ \sigma_s$$

where f is an estimate for scaling up the standard deviations (σ_o) of the potential coefficients implicit in the satellite normal equations and $\bar{f}$ is a corresponding estimate for scaling the rms size coefficient (σ_s) as given by Kaula's rule. Based upon the size of the coefficients and the scaling of their standard errors in GEM 7, we used $\bar{f} = \sqrt{1/2}$ and $f = \sqrt{10}$ giving w = .05 in GEM 9.

TABLE 8. RATIO OF DIAGONAL TERMS (d) OF THE SATELLITE NORMAL MATRIX IN GEM 9 TO THE DIAGONAL TERMS (d) OF THE SIGNAL MATRIX

$$d = (10^{-5}/\ell^2)^{-2} \text{ FOR DEGREE } \ell.$$

m*	ℓ = 12	ℓ = 16	ℓ = 20
0	1,000,000	250,000	20,000
1	80,000	6,000	630
2	13,000	2,500	630
3	10,000	2,000	200
4	5,000	1,000	200
5	8,000	800	160
6	2,500	1,300	160
7	10,000	300	160
8	4,000	1,300	80
9	13,000	800	250
10	20,000	1,600	80
11	20,000	2,500	400
12	100,000	25,000	4,000
13		40,000	16,000
14		630,000	63,000
15		100,000	2,500
16		600	1,600
17			800
18			310
19			80
20			25

*C and S tesseral terms are essentially the same.

Since the signal matrix contains only diagonal terms which were added to the data matrix it is interesting to compare their relative sizes. As seen from Table 8, the satellite unscaled normal equations ($B^T W_o B$) have considerably larger diagonal terms (even out to degree 20) than the signal matrix W, which is still true even after w = .05 is applied. This demonstrates that our application of collocation can have a significant effect only by indirectly controlling ill-conditioned vectors (correlation effects) in the system.

Collocation was applied to the coefficient subset for degree $\ell > 12$ except for resonant terms of order 12, 13, and 14.

Figure 4 shows the improvement when collocation is applied to terms above degree 12 compared to those when applied above degree 16. The former solution was chosen as GEM 9. Interestingly, when collocation was applied to terms above degree 8, the results were almost the same indicating that this method was unnecessary for the lower part of the recovered geopotential. The results for GEM 7 in Figure 4 show that simple least squares can provide a reasonable satellite solution complete to 16 x 16. A solution similar to GEM 7 (16 x 16, no collocation) was obtained using the GEM 9 data, giving results comparable to GEM 7 within one $mgal.^2$ Hence, without controlling matrix ill-conditioning it is unlikely that worldwide geopotential improvement would have resulted using the new data. GEM 10 was also derived using the collocation technique applied to the coefficients above degree 12. GEM 10 is complete to 22 x 22. It is important to note (as mentioned above) that collocation was not applied to the resonance terms (m = 12, 13 and 14).

3.3 DETERMINATION OF GM

The simultaneous determination of GM with the geopotential and station positions was performed. Table 9 describes additional tests which were made to

TABLE 9.
GM DERIVED FROM SATELLITE LASER DATA

SATELLITE	GM (km^3/sec^2)	NO. OF 5–DAY ARCS	NO. OF OBS.
LAGEOS	398600.64	11	25,000
STARLETTE	0.70	26	28,000
GEOS–3	0.84	38	94,000
COMBINED	398600.64		

SUBSETS	GM	NO. OF ARCS	NO. OF OBS.
STARLETTE	398600.44	9	12,000
STARLETTE	0.87	9	10,000
STARLETTE	0.73	8	6,000
GEOS–3	0.65	18	33,000
GEOS–3	0.92	16	54,000
LAGEOS	0.64	5	16,000
LAGEOS	0.65	6	9,000

LAGEOS SUBSETS	DETERMINED GM (km^3/sec^2)	FORMAL STANDARD ERROR (km^3/sec^2)	NO. OF OBS.
LAGEOS INDIVIDUAL ELEVEN ARCS			
1	.635	.014	2,037
2	.567	.054	651
3	.538	.020	2,830
4	.682	.095	1,167
5	.664	.011	2,203
6	.687	.020	2,037
7 *	.493	.431	1,676
8	.641	.032	1,492
9	.602	.016	6,021
10	.647	.019	1,481
11	.829	.020	3,634
ARCS 1 THROUGH 6 (6 arcs)	.648	.007	10,880
ARCS 7 THROUGH 11 (5 arcs)	.647	.009	14,259
WORST CASE (from LAGEOS)			
6 HIGH GM ARCS	.661	.007	11,960
6 LOW GM ARCS	.611	.008	13,179
ALL DATA	398600.638	.005	25,139

*Inadvertantly, this was only a 10 hour arc.

evaluate solutions for GM where only station coordinates and orbit parameters are solved simultaneously with GM. The speed of light used was 299792.5 km/sec. The estimation of GM was exclusively from laser tracking data. The presence of LAGEOS dominated the determination, and the GM results obtained were repeated using the LAGEOS data by itself. The value of GM is 398600.64 km^3/sec^2 for all 11 LAGEOS arcs. LAGEOS not only dominated the combination solution, but in the subset solutions it also gave much more consistent results than either GEOS-3 or Starlette. This is because the high altitude of LAGEOS provides good geometry and dynamics for estimating GM with separability for station coordinates. The individual LAGEOS arcs shown in Table 9 were recombined taking the highest 6 determined values for one solution and the remaining 5 lowest values in a second solution. These two "worst case" solutions were both within .03 of the above value of GM, whereas a typical set of arcs (first 6 and last 5) are within .008 of this value. Based upon these results and the formal uncertainty of .005 for the total solution value (398600.64), the value of .02 was selected as a conservative estimate of the uncertainty for GM.

3.4 MODIFIED TREATMENT OF THE SURFACE GRAVITY DATA FOR INCLUSION INTO GEM 10

Another major innovation over previous GEM solutions is in the treatment of the surface gravimetry. In GEM 10 the surface gravity data has less overall weight than in previous GEM combination solutions. For the GEM 10 solution, an additional 5 mgal was added to each individual observation uncertainty. This 5 mgal uncertainty was used to represent the unmodeled truncation error for 5° mean anomalies when solving for a 22 x 22 field. This weighting scheme had the benefit of making the data quality more uniform over the globe than in previous models. The result was a solution which agreed with the gravimetry over the oceans about as well as over land. More importantly, agreement with worldwide altimetry was superior with this more uniform weighting (see later Section 4.4).

3.5 STATION COORDINATES AND GEODETIC REFERENCE PARAMETERS

GEM 9 and 10 simultaneously determined the center of mass positions for 146 tracking stations. These station coordinates for GEM 10 are presented in Table 10. Table 11 compares the GEM 10 station positions with those estimated by Marsh (1977) for the Calibration Area lasers. The geocentric station coordinate differences are seen to be about 1 m in these results. These results indicate the highly accurate laser station coordinates have been obtained, with uncertainties being significantly less than 3 m given in Lerch et al, 1974.

Three methods were used to derive a mean value of the semi-major axis, a_e, of the earth's reference ellipsoid, all of which agree to within one meter of $a_e = 6378140$ m. These results made use of reference parameters such as GM, equatorial gravity (g_e) and ellipsoidal flattening. These parameters are all compared in Section 3.5.4 with the set adopted by a special study group of the IAG in 1975.

3.5.1 a_e Derived from the GEM 10 Station Coordinates and Their Mean Sea Level Surveyed Heights

Station coordinates and mean sea level heights from survey were used and gave $a_e = 6378139.9 \pm 1.5$ m (Table 12). Subset solutions presented in Table 12 for the different tracking systems all agree to within one meter of the mean value except for the Baker-Nunn sites which differ by 2.9 m. These results are based upon the following formula:

$$a_e - a_e\,(\text{reference}) = \sum_{i=1}^{L} \frac{h_i - \text{MSL } H_i - N_i}{L}$$

TABLE 10. STATION COORDINATES OF GEM 10 (a_e = 6378145m., 1/f = 298.255)

STATION NAME	STATION NUMBER	LATITUDE DD	LATITUDE MM	LATITUDE SS.SSS	LONGITUDE DDD	LONGITUDE MM	LONGITUDE SS.SSS	HEIGHT METERS
1BPOIN	1021	38	25	49.815	282	54	49.087	-38.8
1FTMYR	1022	26	32	53.238	278	8	4.616	-25.6
1OOMER	1024	-31	23	24.997	136	52	15.624	131.3
1SATAG	1028	-33	8	58.433	289	19	53.929	719.1
1MOJAV	1030	35	19	47.928	243	5	59.673	893.9
1JOBUR	1031	-25	53	0.858	27	42	26.638	1546.6
1NEWFL	1032	47	44	29.389	307	16	46.791	70.1
1COLEG	1033	64	52	18.279	212	9	37.760	174.2
1GFORK	1034	48	1	21.325	262	59	20.154	224.2
1WNKFL	1035	51	26	45.911	359	18	9.023	111.5
1ULASK	1036	64	58	37.120	212	28	31.481	301.7
1ROSMN	1037	35	12	7.220	277	7	41.811	873.4
1ORORL	1038	-35	37	32.020	148	57	14.996	944.5
1ROSMA	1042	35	12	7.185	277	7	41.570	872.3
1TANAN	1043	-19	0	31.966	47	17	59.667	1371.6
MADGAR	1122	-19	1	14.815	47	18	11.514	1382.3
MADGAS	1123	-19	1	14.366	47	18	11.066	1383.4
ROSRAN	1126	35	11	45.562	277	7	26.832	837.6
ULASKR	1128	64	58	19.343	212	29	13.468	349.1
CARVON	1152	-24	54	10.570	113	42	59.670	12.0
BDA3	1302	32	21	4.781	295	20	31.728	-16.9
CYI3	1304	27	45	51.725	344	21	58.376	193.5
CRO3	1308	-24	54	23.279	113	43	32.077	16.5
GDRTKS	1312	22	7	34.712	200	20	5.478	1158.1
GDSA	1314	35	20	29.838	243	7	37.302	924.4
TEX3	1316	27	39	17.723	262	37	30.991	-34.3
MADR	1323	40	27	19.824	355	49	53.930	826.8
GWM3	1324	13	18	38.148	144	44	12.928	136.6
HSK8	1325	-35	34	59.662	148	58	40.499	1138.5
GDSB	1328	35	20	29.879	243	7	34.870	931.6
MIL3	1371	28	30	29.588	279	18	23.868	-30.2
ACN3	1375	-7	57	17.311	345	40	22.557	551.5
ETC3	1377	38	59	54.935	283	9	26.064	10.0
ETCA	1391	38	59	54.233	283	9	29.094	16.3
HOWARD	2001	39	9	48.478	283	6	7.823	119.3
NEWMEX	2003	32	16	53.574	253	14	53.601	1165.5
SANHES	2008	-23	13	3.712	314	7	49.686	595.6
MISAWA	2013	40	43	14.120	141	19	51.729	45.5

TABLE 10. (continued)

ANCHOR	2014	61	17	0.052	210	10	29.747	66.0
TAFUNA	2017	-14	17	50.191	189	17	3.411	35.7
THULEG	2018	76	32	19.932	291	13	53.664	57.9
MCMURD	2019	-77	50	51.667	166	40	25.699	-19.8
AUSTIN	2092	30	17	17.432	262	16	5.217	156.9
WAHIWA	2100	21	31	15.383	202	0	10.710	403.4
LACRES	2103	32	16	44.153	253	14	46.220	1166.1
LASHAM	2106	51	11	9.141	353	58	25.656	222.3
APLMND	2111	39	7	48.273	283	6	12.323	100.9
SMITHL	2112	-34	40	26.262	138	39	17.124	27.7
PRETOR	2115	-25	56	48.170	28	20	52.011	1597.0
ASAMOA	2117	-14	19	50.257	139	17	3.354	40.3
SANMIG	2121	14	59	16.402	120	4	21.378	58.7
WALDOP	2203	37	51	51.793	284	29	33.055	-32.1
CANTON	2706	-2	47	35.334	188	20	4.890	27.3
MAHE	2717	-4	40	13.748	55	28	46.830	548.6
ASCENS	2722	-7	58	10.006	345	35	40.876	92.1
COCOS	2723	-12	11	44.932	96	50	3.582	-22.9
MOSLAK	2738	47	11	7.535	240	39	43.542	338.2
SHEMAL	2739	52	42	55.761	174	6	40.301	43.4
BELTSV	2742	39	1	39.845	283	10	28.358	8.0
STNVIL	2745	33	25	31.969	269	5	10.453	9.3
CARGIL	2809	-46	24	43.756	168	18	13.551	-0.3
PARIBO	2815	5	26	53.104	304	47	42.609	-6.8
MESHED	2817	36	14	26.218	59	37	44.326	967.7
FRTLMY	2822	12	7	53.901	15	2	6.953	312.0
NATLDP	2837	-5	54	57.951	324	49	56.214	30.0
APLTWO	2911	39	9	48.290	283	6	14.799	112.2
ETRPRE	4050	-25	56	37.039	28	21	29.238	1573.8
ETRMRT	4082	28	25	29.533	279	20	6.371	-34.9
NBER34	4740	32	20	53.441	295	20	46.251	-29.6
NBER05	4760	32	20	53.662	295	20	46.081	-26.3
NWAL18	4840	37	50	29.394	284	30	51.933	-36.4
NWAL13	4860	37	51	37.739	284	29	24.704	-37.6
WOOR38	4946	-30	49	5.327	136	50	16.993	68.3
1UNDAK	7034	48	1	21.267	262	59	20.298	221.5
1EDINB	7036	26	22	46.646	261	40	7.997	29.1
1COLBA	7037	38	53	36.121	267	47	41.524	236.0
1BERMD	7039	32	21	49.581	295	20	35.618	-4.9
1PURIO	7040	18	15	28.704	294	0	24.056	1.9
1GSFCP	7043	39	1	15.530	283	10	21.096	13.2
1DENVR	7045	39	38	47.994	255	23	39.299	1766.7
GODLAS	7050	39	1	14.072	283	10	19.337	12.9
ROSLAS	7051	35	11	47.130	277	7	27.050	848.3

TABLE 10. (continued)

WALLAS	7052	37	51	35.886	284	29	24.034	-28.2
MOBLAS	7053	39	1	15.661	283	10	19.604	20.6
CRMLAS	7054	-24	54	15.095	113	42	58.595	20.8
GMISLS	7060	13	18	33.753	144	44	14.244	134.3
STALAS	7063	39	1	13.348	283	10	20.135	14.6
ML0302	7065	39	1	14.402	283	10	19.338	14.1
BDILAI	7067	32	21	13.767	295	20	38.264	-27.2
GRKLAS	7068	21	27	37.756	288	52	5.346	-22.7
1JUM24	7071	27	1	13.666	279	53	13.276	-18.5
1JUM40	7072	27	1	14.262	279	53	13.294	-21.9
1JUPC1	7073	27	1	14.508	279	53	14.199	-30.1
1JUBC4	7074	27	1	14.714	279	53	14.047	-16.8
1SUDBR	7075	46	27	21.259	279	3	11.097	243.2
1JAMAC	7076	18	4	34.574	283	11	27.548	427.9
1GSFCN	7077	38	59	57.123	283	9	38.352	12.5
WALNOT	7078	37	51	47.304	284	29	28.725	-36.0
1CARVN	7079	-24	54	23.164	113	43	16.716	-1.3
HAULAS	7809	43	55	56.436	5	42	45.087	701.2
DAKLAS	7820	14	46	2.757	342	35	28.141	42.6
GRASSE	7842	43	45	7.893	6	54	10.914	1306.3
ORGLAS	7901	32	25	24.488	253	26	47.296	1618.8
OLILAS	7902	-25	57	35.841	28	14	52.921	1566.2
ARELAS	7907	-16	27	56.733	288	30	24.993	2486.0
HOPLAS	7921	31	41	3.187	249	7	19.193	2344.5
NATLAS	7929	-5	55	40.146	324	50	7.610	32.7
GRELAS	7930	38	4	42.172	23	55	57.910	506.2
DELFTH	8009	52	0	5.139	4	22	16.311	28.6
ZIMWLD	8010	46	52	36.422	7	27	53.846	931.6
MALVRN	8011	52	8	35.792	358	1	54.174	196.0
HAUTEP	8015	43	55	57.017	5	42	45.175	697.4
NICEFR	8019	43	43	32.432	7	17	59.461	420.0
MUDONI	8030	48	48	21.325	2	13	46.859	177.8
1ORGAN	9001	32	25	24.822	253	26	49.217	1623.7
1OLFAN	9002	-25	57	36.021	28	14	52.788	1566.9
WOOMER	9003	-31	6	2.082	136	47	3.407	161.2
1SPAIN	9004	36	27	46.550	353	47	37.389	67.6
1TOKYO	9005	35	40	22.781	139	32	16.897	91.3
1NATAL	9006	29	21	34.541	79	27	27.713	1877.4
1QUIPA	9007	-16	27	56.766	288	30	24.889	2489.6
1SHRAZ	9008	29	38	13.776	52	31	11.825	1587.8
1CURAC	9009	12	5	25.002	291	9	44.866	-17.1
1JUPTR	9010	27	1	13.991	279	53	13.788	-16.3
1VILDO	9011	-31	56	34.740	294	53	36.808	632.5
1MAUIO	9012	20	42	25.944	203	44	34.363	3048.0

TABLE 10. (continued)

HOPKIN	9021	31	41	2.961	249	7	19.151	2347.8
AUSBAK	9023	-31	23	25.802	136	52	43.906	142.1
DODAIR	9025	36	0	19.853	139	11	31.582	896.6
DEZEIT	9028	8	44	50.995	38	57	33.831	1912.6
NATALB	9029	-5	55	40.443	324	50	7.608	39.1
COMRIV	9031	-45	53	12.521	292	23	9.807	197.2
JUPGEO	9049	27	1	14.039	279	53	13.662	-1.0
AGASSI	9050	42	30	21.398	288	26	30.638	147.5
ATHENG	9051	37	58	36.289	23	46	40.689	225.2
MALVRN	9080	52	8	36.017	358	1	54.137	166.1
GREECE	9091	38	4	44.283	23	55	59.482	503.9
COLDLK	9424	54	44	34.007	249	57	23.449	674.7
EDWAFB	9425	34	57	50.636	242	5	8.208	754.7
OSLONR	9426	60	12	39.069	10	45	3.968	617.2
JOHNST	9427	16	44	39.021	190	29	9.635	26.6
OLISAO	9902	-25	57	35.894	28	14	53.011	1565.5
ARESAO	9907	-16	27	56.753	288	30	24.923	2491.6
HOPSAO	9921	31	41	3.189	249	7	19.156	2346.5
NATSAO	9929	-5	55	40.154	324	50	7.562	35.0
GRESAO	9940	38	4	42.412	23	55	58.004	503.5

TABLE 11. A COMPARISON OF THE CALIBRATION AREA LASER STATIONS DETERMINED BY MARSH ET AL, 1977 WITH THOSE OF GEM 10

STATION COORDINATE DIFFERENCES (Marsh Minus GEM 10)

STATION	NUMBER	LOCATION	LATITUDE (arc sec)	LONGITUDE* (arc sec)	HEIGHT (meters)
STALAS	7063	GODDARD SPACE CENTER	–.006	0.004	–1.008
BDALAS	7067	BERMUDA	–.032	– .005	–0.951
GRTLAS	7068	GRAND TURK	.002	.002	–1.260

*A constant longitude rotation of :"387 was removed.

STATION BASELINE DIFFERENCES

	GEM 10 BASELINE	BASELINE DIFFERENCE (Marsh Minus GEM 10)
STALAS TO BDALAS	1322741.59 m	0.50 m
STALAS TO GRTLAS	2012725.10 m	0.00 m
BDALAS TO GRTLAS	1364265.99 m	–0.83 m

RELATIVE STATION HEIGHT DIFFERENCES

	GEM 10 RELATIVE HT.	MARSH MINUS GEM 10 RELATIVE HT.
STALAS – BDALAS	41.79 m	–.06 m
STALAS – GRTLAS	37.24 m	.25 m
BDALAS – GRTLAS	–4.55 m	.31 m

TABLE 12. THE MEAN EQUATORIAL RADIUS OF THE EARTH (a_e) DETERMINED FROM TRACKING STATION COORDINATES

TRACKING SYSTEM	NUMBER OF STATIONS	ESTIMATED a_e (meters)
MOTS/SPEOPTS CAMERAS	31	6378140.8
BAKER–NUNN CAMERAS	27	6378142.8
LASERS	16	6378139.0
DOPPLER	23	6378139.4
ALL SYSTEMS	114	6378139.9

where

h - height of station above reference ellipsoid

L - number of stations

MSLH - mean sea level height of station from survey

N - geoid height of station.

A detailed gravimetric geoid model based upon GEM 10 was used for estimating the geoid heights. The short wavelength features of this geoid provided predominantly positive anomalies of 3 to 10 meters at a number of the stations, particularly those situated on Islands. The geoid was obtained privately from Marsh, and the method (which employed Stokes' function) is described in Marsh et al, 1976.

3.5.2 a_e Inferred from GEOS-3 Intensive Mode Altimetry

GEOS-3 intensive mode altimetry was utilized for estimating the mean equatorial radius of the earth. The altimetry data set was selected for a 5^o gridded distribution. These data were reduced in five day orbital arcs in which both laser and altimetry contributed to the determination of the orbit. GEM 10 was used for the orbit and geoid computation. A single altimeter range bias was estimated from the altimeter residuals for each of these arcs. This altimeter bias contains all altimeter system biases along with the average error in the mean equatorial radius of the ellipsoid being used to compute the altimeter residuals. Martin (1977) has calibrated the intensive mode altimeter and finds it to measure short by 5.3 meters with a small uncertainty of 20 cm. Using this value for the system bias in the altimetry, the a_e implied from ten five day arcs of altimetry is 6378141.0 m. These results are summarized in Table 13. A second important result to be noted in Table 13 is the exceptionally good fit to the altimeter data obtained using the GEM 10 geoid. GEM 10 did not use altimetry in its solution so this result can be viewed as a calibration of GEM 10.

TABLE 13. ESTIMATION OF THE MEAN EQUATORIAL RADIUS OF THE EARTH FROM GEOS–3 INTENSIVE MODE ALTIMETRY

ARC EPOCH	ARC LENGTH (DAYS)	NO. OF ALT. OBS.	ALTIMETER RMS OF FIT (METERS)	a_e IMPLIED BY RECOVERED ALT. BIAS 6378000 + m
750516	5	2305	2.27	140.5
750521	5	5454	2.57	140.8
750527	5	2139	2.22	140.5
750601	5	549	2.67	142.1
750701	5	328	1.64	141.8
750716	5	2251	2.78	140.7
750730	5	3465	2.04	140.9
750803	5	3555	2.50	141.8
750815	5	2154	2.55	140.4
750825	5	2229	2.63	140.5
TOTAL/ AVERAGE		24429	2.46	141.0

3.5.3 a_e Inferred From Mean Equatorial Gravity (g_e) and GM

A third method for estimating a_e is based upon a new value of equatorial gravity g_e, derived in GEM 10 from surface gravity data, and new value of GM. From the simple relation

$$g_e = \frac{GM}{a_e^2}$$

the variational relationship becomes

$$\frac{\delta g_e}{g_e} = \frac{\delta GM}{GM} - 2\frac{\delta a_e}{a_e}$$

or

$$\delta a_e = \frac{a_e}{2}\left(\frac{\delta GM}{GM} - \frac{\delta g_e}{g_e}\right).$$

By using the old and new values of the reference parameters and by removing the atmospheric mass ($\frac{\Delta GM}{GM} = .87 \times 10^{-6}$) from the new satellite derived value of GM (398600.64 km^3/sec^2), the adjustment for a_e is derived from the above equation. The result for a_e and associated reference parameters are given below in Table 14. The parameters refer to the old speed of light (C).

TABLE 14. GEODETIC REFERENCE PARAMETERS

PARAMETER	OLD	NEW	ACCURACY
GM*	398600.8	398600.29	± .02 km^3/sec^2
g_e	978031.0	978031.5	± .5 mgal
a_e	6378145	6378139.3	± 1.5 m

*Excludes the atmospheric mass and refers to c = 299792500 m/s.

3.5.4 Comparison of Fundamental GEM 10 Reference Parameters With Those Adopted by the IAG (1975)

The GEM 10 reference parameters described in Section 3.5.3 are in remarkable agreement with the set established by a special study group of the IAG (Moritz, 1975). The GEM 10 values are adjusted to the IAG system by including in GM the atmospheric mass and the new speed of light (c = 299792458 m/sec). The GEM 10 adopted a_e is the composite value obtained from Sections 3.5.1, 3.5.2, and 3.5.3 (a_e = 6378140.). The GEM 10 values in the IAG system are compared to those adopted by the IAG in Table 15. The differences shown in Table 15 are very consistent with the uncertainties which have been stated for the parameters derived in the GEM 10 solution.

TABLE 15. COMPARISON OF THE IAG 1975 AND GEM 10 GEODETIC PARAMETERS

PARAMETER	IAG 75	GEM 10	UNITS
GM*	398600.5	398600.47	km^3/sec^2
g_e	978031.8	978031.8	mgal
1/f	298.257	298.255	---
a_e	6378140	6378140	m
c	299792458	299792458	m/sec

*Includes atmospheric mass (ΔGM = .35) and new speed of light (ΔGM = –.17).

SECTION 4. .... EVALUATION OF THE GRAVITY FIELD

4. EVALUATION OF THE GRAVITY FIELD

4.1 ERROR ESTIMATES OF THE POTENTIAL COEFFICIENTS

It is always of interest to know the accuracy of a computed physical quantity as distinct from its formal precision measured by an experiment. In the case of comprehensive gravity model solutions, it has been shown by Lerch et al (1974) and Wagner (1976) that the formal uncertainties obtained from the solutions can be scaled to obtain reasonable estimates of the true uncertainties for the individual coefficients themselves. As indicated in equation 6 of Section 3.2 a scale factor, $f = \sqrt{10}$, was applied to the system of normal equations of GEM 9 and 10 in order to provide for realistic standard errors. We wish to test these error estimates here. Table 16 presents the coefficient errors for the GEM 9 solution; the values in Table 16 represent the scaled error estimates (normalized) for the GEM 9 harmonics. Table 17 presents the estimated errors for the harmonics in the GEM 10 model. These error estimates were tested in three separate studies. We especially wished to confirm that a truly significant improvement has been obtained over previous GEM solutions for terms above degree 12 (which is indicated by significantly smaller uncertainties in GEM 9 and 10).

Rapp (1977) has estimated the terrestrial potential solely from surface data. Therefore, his model is completely independent of GEM 9 which was derived exclusively from satellite tracking data. Rapp's model was used to calibrate the formal errors ascribed to the GEM 9 and 10 solutions.

Figure 5 presents the estimated uncertainties from Rapp and GEM 9 compared to the size of the coefficients from "Kaula's rule," and those computed from these two solutions themselves.

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TABLE 16. ESTIMATED ERRORS FOR GEM 9 (UNITS = 10^{-9} FOR FULLY NORMALIZED HARMONICS)

DEGREE \ ORDER	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28
2	1	3	3																										
3	1	5	8	9																									
4	1	5	5	4	6																								
5	2	7	9	10	9	13																							
6	2	7	8	7	8	6	8																						
7	2	9	13	13	12	12	10	16																					
8	2	9	12	10	11	8	10	7	10																				
9	2	13	15	18	16	15	13	14	11	9																			
10	2	10	17	14	15	12	12	10	11	7	10																		
11	3	15	17	22	21	19	17	16	13	10	10	16																	
12	3	11	18	19	18	17	17	12	13	7	10	5	6																
13	4	14	18	21	22	19	19	17	14	10	11	11	3	3															
14	4	14	15	20	18	18	18	16	14	9	11	6	5	2	2														
15	5	13	19	19	20	20	19	19	17	11	13	11	4	2	1	5													
16	4	13	16	16	19	16	19	18	15	11	12	9	5	3	2	5	12												
17	5	14	17	19	19	20	19	19	18	14	15	11	5	3	1	5	5	12											
18	3	12	15	18	16	19	18	18	18	15	14	11	6	3	2	6	12	17	21										
19	4	14	17	16	18	17	18	17	17	15	15	11	9	4	1	4	5	11	11	15									
20	4	12	14	15	16	16	16	16	16	14	15	11	7	4	2	7	11	12	14	16	17								
21	4	15	14	15	15					14	15	12	9	3	2	5	8	10	9	9									
22	5	11	13	14	14					13	14	11	9	4	3	9	11	11	11	11									
23	5	12											15	4	3	6													
24	6	11											21	6	5	10													
25	6	11											23	7	7	8													
26	7													6	5														
27	7													10	9												7	11	
28	9													10	6												1	7	7
29	8													10	8												7		
30																													2

TABLE 17. ESTIMATED ERRORS FOR GEM 10 (UNITS = 10^{-9} FOR FULLY NORMALIZED HARMONICS)

DEGREE \ ORDER	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28
2	1	2	3																										
3	1	5	8	9																									
4	1	4	4	4	6																								
5	2	6	9	9	9	12																							
6	2	7	7	6	7	6	8																						
7	2	8	11	12	11	11	10	15																					
8	2	8	10	9	9	8	9	7	9																				
9	2	11	13	15	14	13	12	12	10	9																			
10	2	9	13	12	13	10	11	9	10	6	9																		
11	3	12	14	17	16	16	14	14	11	9	9	13																	
12	3	9	14	14	15	14	14	11	11	6	9	5	5																
13	4	12	14	16	16	15	15	14	12	9	10	9	3	3															
14	4	11	12	14	14	15	14	13	12	8	10	6	4	2	2														
15	5	10	14	15	15	16	14	15	13	9	11	10	3	2	1	5													
16	4	10	12	12	15	13	15	14	12	9	10	8	5	3	2	5	10												
17	4	12	13	15	15	15	15	15	14	11	12	9	5	3	1	4	5	11											
18	3	10	12	14	13	14	14	14	14	12	11	9	6	3	2	5	10	13	15										
19	4	11	13	12	14	13	13	13	13	12	12	10	7	4	1	4	4	9	10	12									
20	4	10	11	12	13	12	13	13	12	11	12	10	6	4	2	6	9	10	11	12	13								
21	4	12	11	12	12	12	12	12	12	11	11	10	8	3	2	4	7	8	7	8	11	12							
22	5	9	10	11	11	11	11	11	11	11	11	9	8	3	3	8	9	9	9	9	10	11	11						
23	5	10											11	4	3	5													
24	6	9											13	5	5	8													
25	5	9											13	6	6	7													
26	7													6	5														
27	7													8	8												7	11	
28	9													9	6												1	7	7
29	8													10	7												7		
30																													2
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28

ORDER

FIGURE 5.
ROOT MEAN SQUARE POTENTIAL
COEFFICIENT VARIATION (V_ℓ) AND
ERROR ESTIMATES ($\bar{\sigma}_\ell$)

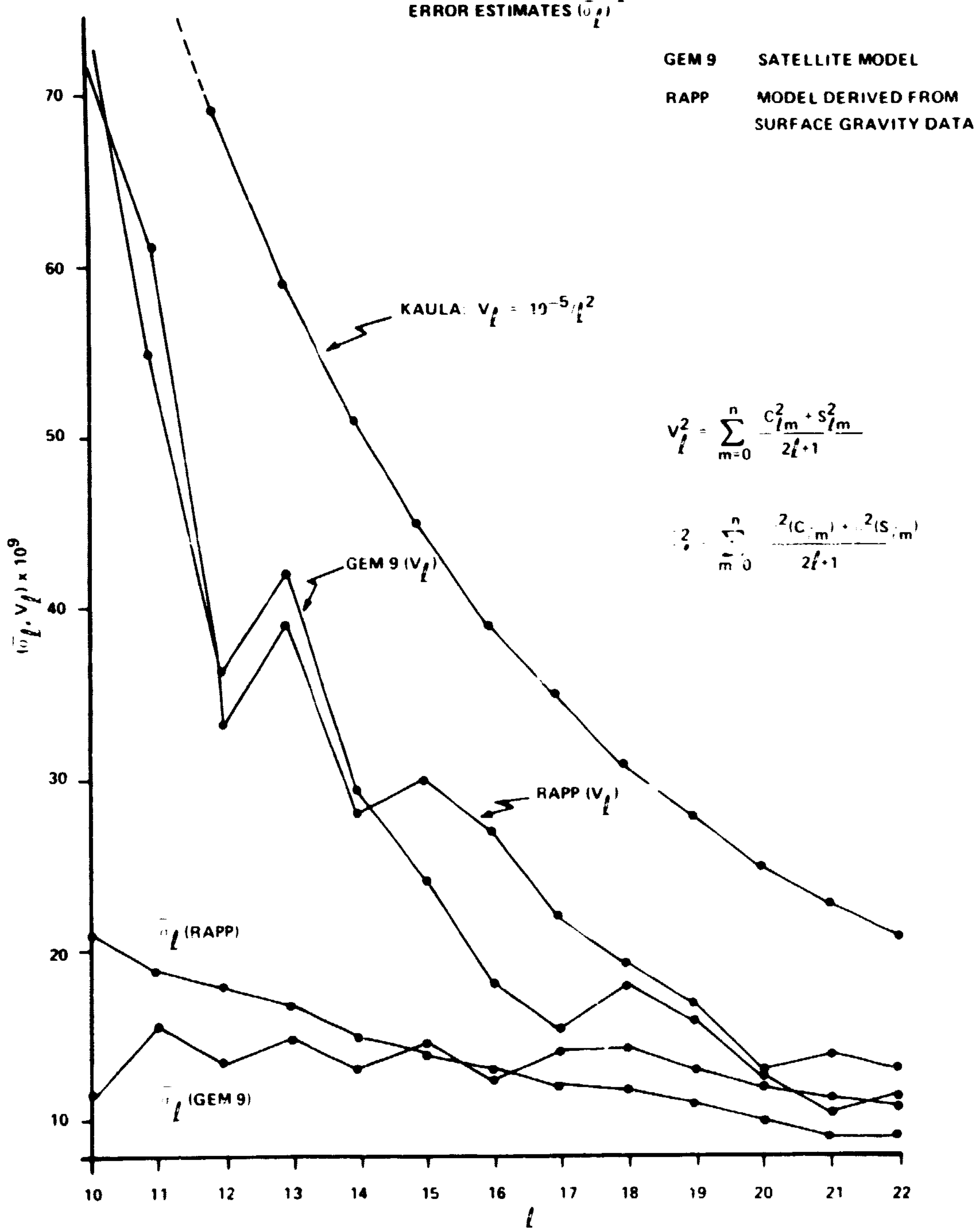


FIGURE 6.
COMPARISON OF COEFFICIENTS AND STANDARD ERRORS BETWEEN MODELS OF RAPP AND GEM 9 AND 10

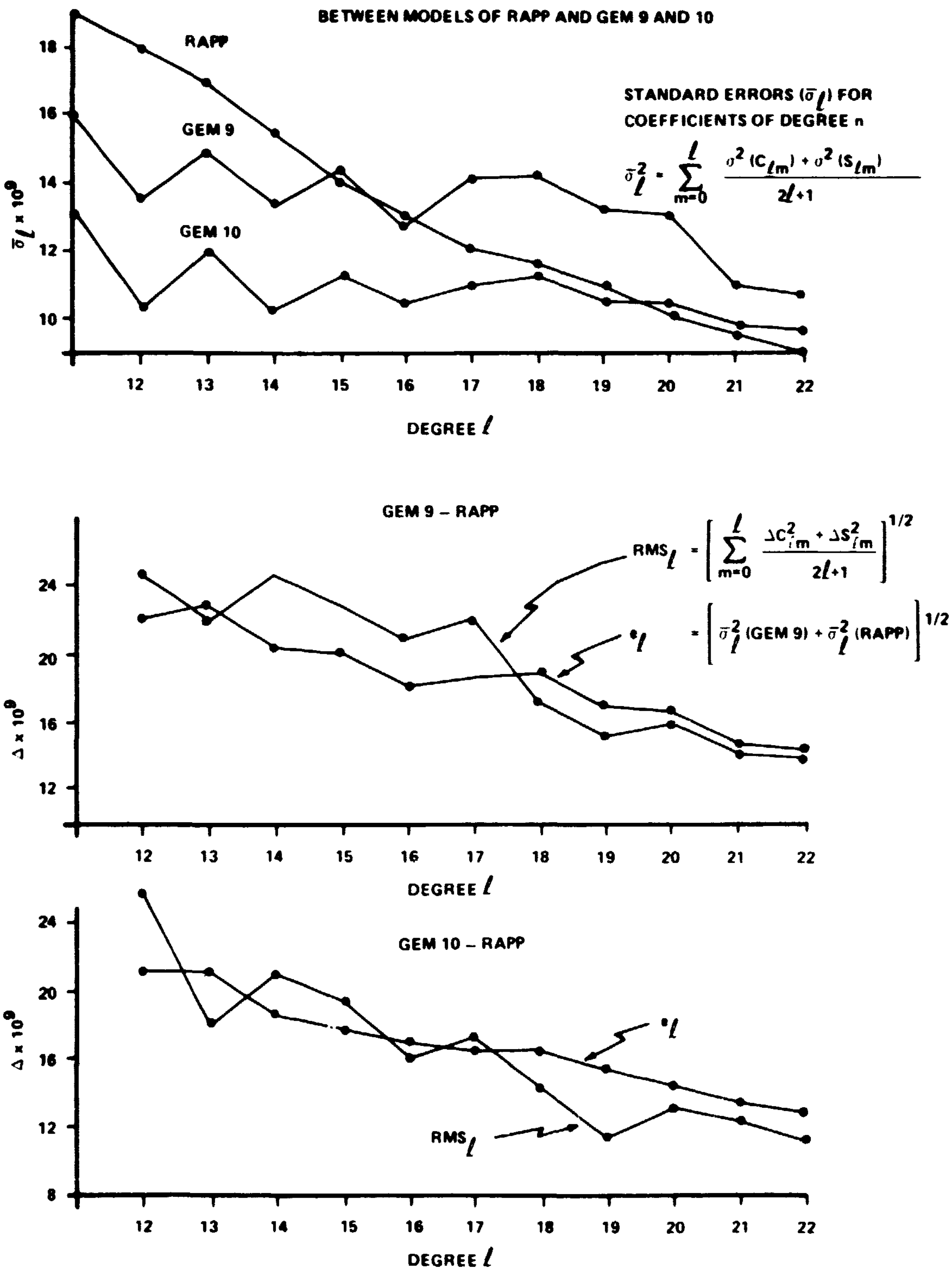


Figure 6 is a calibration of the actual coefficient differences between the Rapp and GEM models compared to their estimated uncertainties. The low degree and order terms ($\ell < 12$) in the Rapp model are not well determined from the gravimetry. The level of agreement between the uncertainties of the coefficients and the actual coefficient differences for the Rapp and GEM 9 solutions (as exhibited in Figure 6) is remarkably good. It shows that the error estimates for the high degree terms (Table 16) are realistic.

Surface gravity data (the 5° mean anomalies employed in GEM 10) were used as a second method to test the standard deviations of GEM 9. Commission errors (σ_s) of gravity anomaly due to errors in the GEM 9 model were derived from the gravity data based upon Kaula's statistics, (Kaula, 1966a). A scale factor f was computed to calibrate the standard deviations in GEM 9 as follows:

$$\sigma_s^2 = f^{1/2} \left[\sum_{\ell=2}^{30} \sum_{m=0}^{\ell} \gamma^2 (\ell-1)^2 \left(\sigma^2 (c_{\ell m}) + \sigma^2 (s_{\ell m}) \right) \right]^{1/2}$$

where γ = 978000 mgal. Results are given in Table 18 which are consistent for the various subsets of data and they verify the GEM 9 standard errors within a 20% tolerance. The commission error of gravity anomaly based upon the GEM 9 standard errors are plotted in Figure 7 as a function of the harmonics complete through degree ℓ.

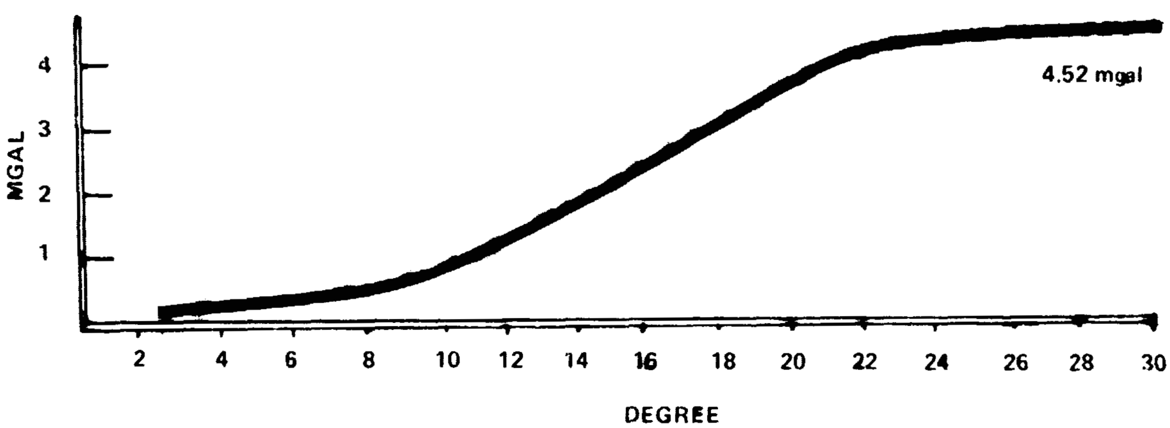
A third approach using laser residuals was employed for testing the standard errors for the coefficients. The ORbital ANalysis Program (ORAN, Martin, 1970) was used to integrate these coefficient errors as a gravity error model. The total estimated gravity error was propagated into simulated Grand Turk laser observations contained within a five day orbital reduction. High correlation in the errors of the zonal and resonance terms (m = 0, 13 and 14) required the elimination of their effects from the experiment. All other terms were included in the GEM 10 error model.

TABLE 18. CALIBRATION FACTOR (f) FOR GEM 9 STANDARD ERRORS BASED UPON COMMISSION ERRORS (σ_s) FROM 5° MEAN GRAVITY ANOMALIES

NO. OF 5° MEAN ANOMALIES	N ≥ *	σ_s (mgal)	f
622	25	4.8	1.1
932	20	5.0	1.1
1109	15	5.2	1.1
1260	10	5.3	1.2
1404	5	5.6	1.2

*N is the number of 1° observed anomalies used in computing the 5° mean gravity anomaly.

FIGURE 7. COMMISSION ERROR OF GRAVITY ANOMALY
BASED UPON GEM 9 STANDARD ERRORS



A five day orbit was computed using GEM 10 fitting the laser observations from August 4th to 9th, 1975. In this orbit, the laser data from Grand Turk (station No. 7068) was given zero weight and thereby did not contribute to the solution. The RMS (Root Mean Square of the residuals) fit to the Grand Turk observations, although unweighted in the solution, yielded an RMS of only 82 cm. In all, there were ten passes (2243 observations) of Grand Turk data.

The estimated RMS predicted by ORAN for the GEM 10 gravity error contribution for all the Grand Turk measurement residuals was 78 cm. Gravity model error is the dominant error source in this test. The agreement between the ORAN simulation and the actual orbital fit to the Grand Turk data indicates that the standard errors for the coefficients are reasonable. An analysis similar to the above for estimating a gravity error model using ORAN is found in Martin and Roy, 1972.

4.2 EVALUATION USING SURFACE GRAVIMETRY

Surface gravity measurements are an important source of independent information for evaluating a global comprehensive gravity field. GEM 9 and 10 have been extensively studied using surface gravimetry. Figure 8 shows a comparison of recent GEM models with surface gravity. The GEM 9 field is in closer agreement with the independent surface gravity than any previous GEM satellite field. GEM 9 is in even closer agreement with this new surface gravimetry than the GEM 6 (Lerch, et al, 1974) combination solution. GEM 10 also out-performs GEM 8. This is encouraging given the lower weight for the surface data (as discussed earlier) in GEM 10. Figure 9 compares recent surface gravity data sets with GEM 9. Quite clearly, the agreement between satellite and surface information is improving with time.

FIGURE 8.
COMPARISON OF GEM MODELS WITH SURFACE GRAVITY

MEAN SQUARE DIFFERENCES (5° ANOMALY – GEM)

1250 ANOMALIES (N · 10)

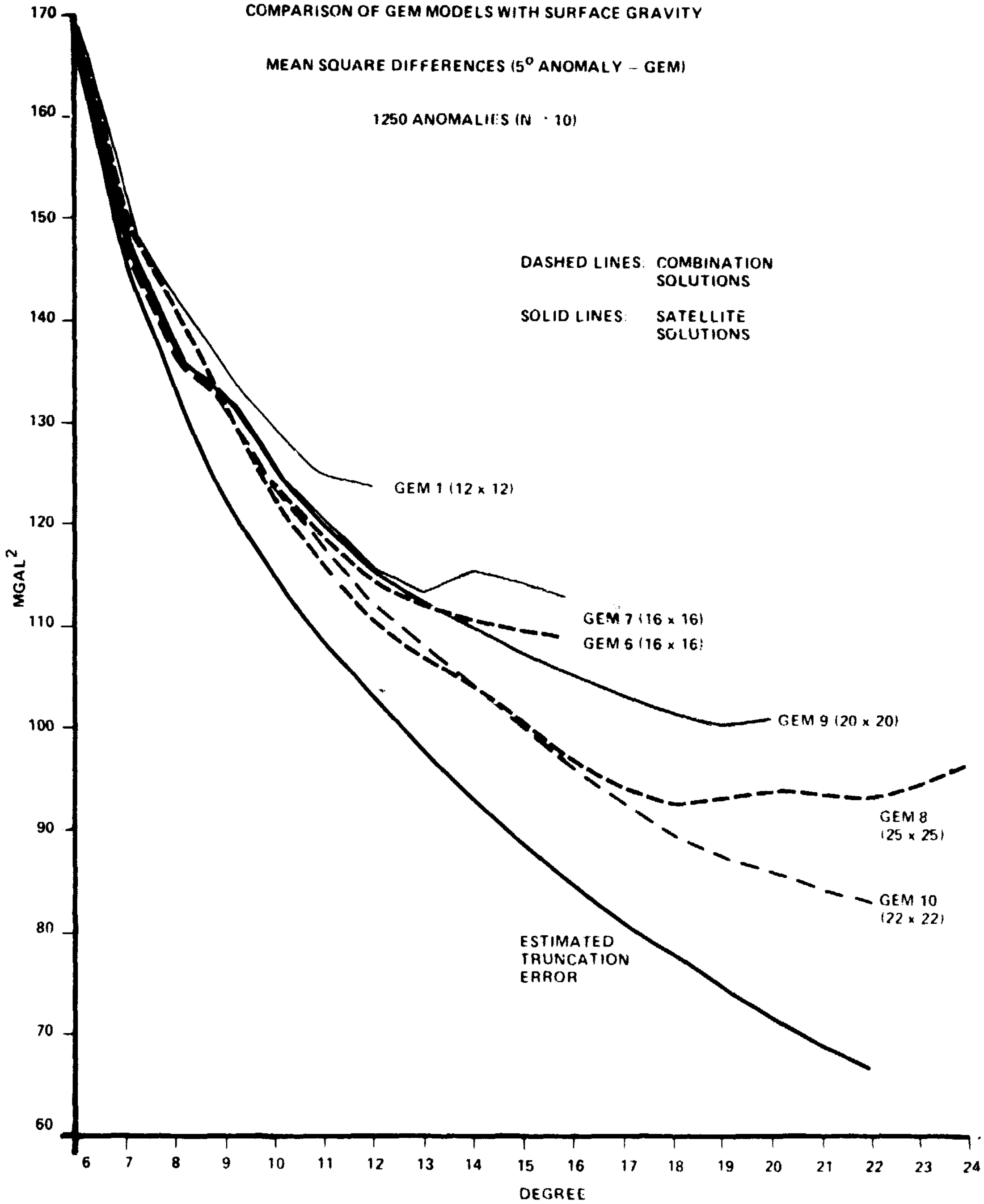
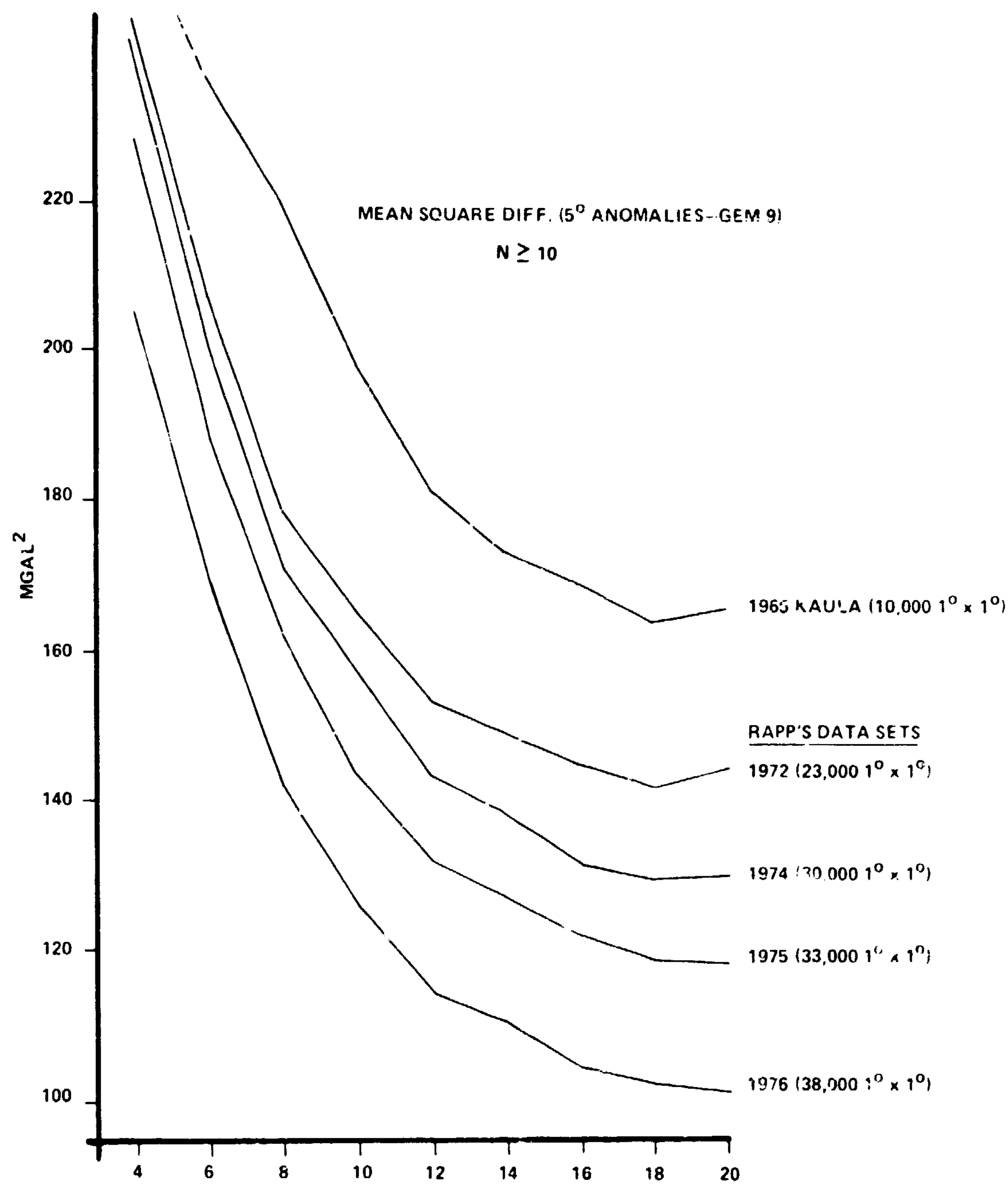


FIGURE 9.

IMPROVEMENT IN RECENT SURFACE GRAVITY DATA SETS
(5° MEAN ANOMALIES) BASED UPON
COMPARISON WITH GEM 9 HARMONICS



Some recent combination solutions are: Goddard Space Flight Center - GEM 10, GEM 8, PGS 110 and GEM 8.1; the Smithsonian Astrophysical Observatory -SAO 4.3 (Gaposchkin, 1976); and GRIM 2 (Balmino, 1976). These are compared to the Rapp, 1977 surface gravimetry (Figure 10). GEM 8.1 is a repeat of the GEM 8 solution using the new approach (described in Section 3.4) for combining the surface data but maintained the GEM 8 weight for the total data set. PGS 110 is a repeat of the GEM 8 solution but complete to 30 x 30 instead of 25 x 25. While GEM 10 performs very well, the relative weight of the surface data in GEM 8.1 was larger than GEM 10. Therefore, GEM 8.1 as would be expected agrees better with the surface data than does GEM 10.

Table 19 shows the degree variances of the gravity anomalies from recent GEM solutions. The impact of the collocation (constraint) is noticeable in the loss of power in the high degree coefficients of GEM 9 and 10. The high degree coefficients in GEM 9 and 10 are somewhat smaller than their counterparts in recent GEM solutions.

4.3 EVALUATION OF THE FREE AIR GRAVITY ANOMALIES DERIVED FROM GEM 9 AND 10

A free air gravity anomaly map was computed from the complete GEM 9 and 10 sets of coefficients. These maps are presented in Figures 11 and 12 respectively. They are remarkably similar. Almost all gravity features are found in the same geographical location in these models, but there are occasional significant differences in the amplitudes for the indicated anomalies. Generally, when there is a significant difference in amplitude between the two fields, GEM 10 shows anomalies with larger peak amplitudes. This is due to the surface gravity data providing greater definition of localized features. An example of this can be found over the Andes Mountains in South America. Both fields show nearly identical placement for the anomaly high in the Andes region, but in GEM 10 the peak is about 5 mgals larger.

FIGURE 10.

COMPARISON OF COMBINATION MODELS WITH SURFACE GRAVITY DATA

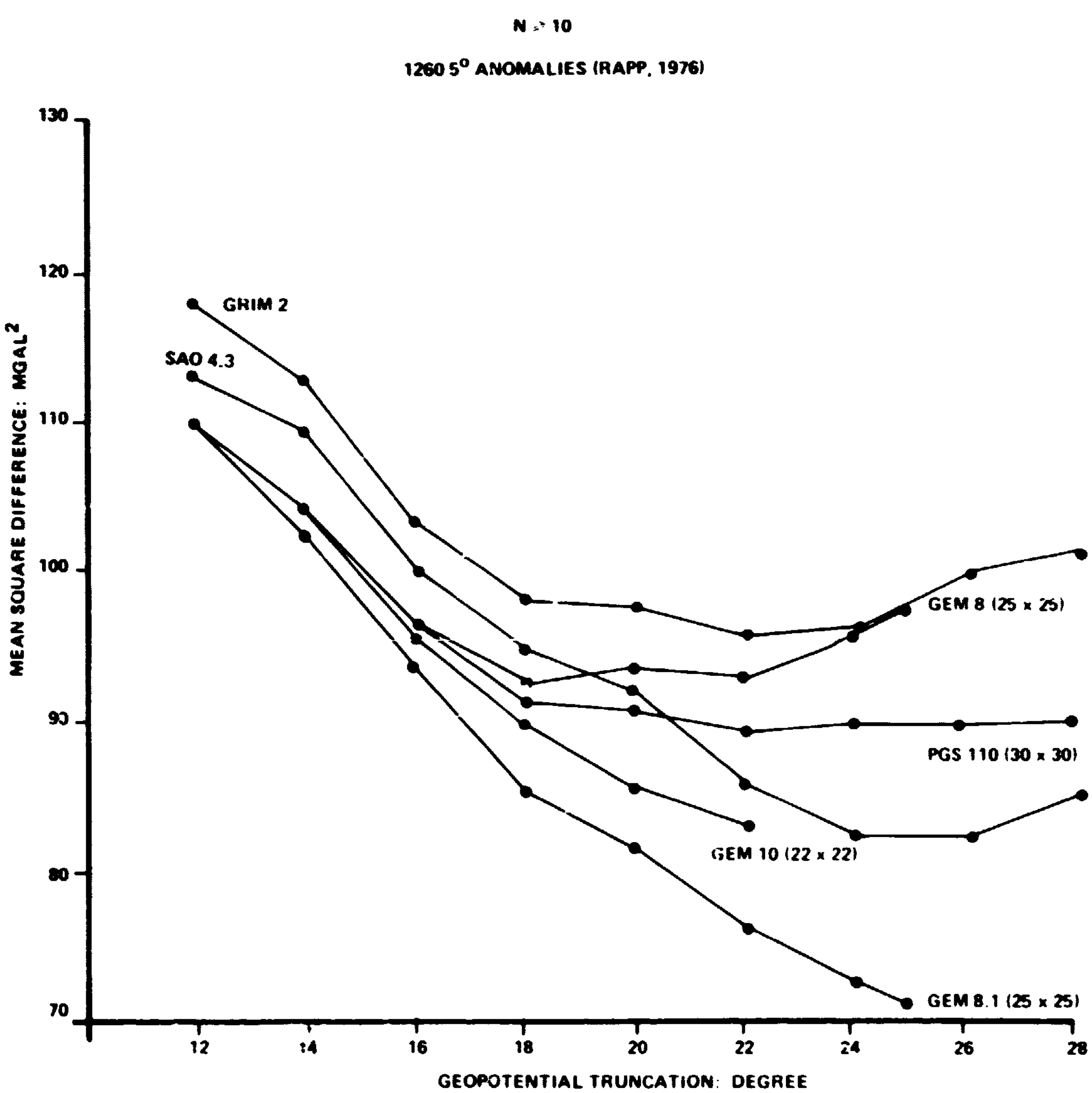


TABLE 19. DEGREE VARIANCES OF GRAVITY ANOMALIES IN MGAL^2

$$\gamma^2 (\ell - 1)^2 \sum_{m=0}^{\ell} (C^2_{\ell m} + S^2_{\ell m})$$

DEGREE (ℓ)	GEM7	GEM8	GEM8.1	GEM9	GEM10
3	33.6	33.7	33.7	33.5	33.5
4	19.5	19.6	19.5	19.5	19.6
5	21.1	21.1	20.9	20.7	20.6
6	18.8	19.1	19.0	18.9	19.0
7	19.4	18.3	18.5	19.3	19.1
8	11.2	10.2	10.2	11.6	11.4
9	11.4	11.2	10.9	11.4	11.1
10	10.1	9.8	10.2	10.0	9.7
11	7.7	7.3	7.1	6.7	6.6
12	3.5	3.2	3.6	3.6	3.6
13	11.1	6.5	7.3	6.5	6.2
14	6.2	3.3	3.4	4.0	3.4
15	5.4	4.5	3.9	3.2	3.0
16	5.7	3.5	3.4	2.3	2.6
17	1.7	6.9	5.4	2.0	2.1
18	1.7	5.0	3.3	3.3	3.1
19	1.1	9.4	4.6	2.9	2.8
20	1.3	8.4	3.6	2.2	2.0
21	1.0	5.9	3.7	1.1	1.8
22	0.5	5.9	3.4	1.8	1.7
23	0.2	7.1	3.1	0.5	0.5
24	0.2	8.7	3.8	0.3	0.4
25	0.6	6.9	3.3	0.7	0.7
26	0.7	0.2	0.4	0.0	0.0
27	1.0	1.6	0.9	0.6	0.6
28	0.6	3.8	3.5	0.9	0.9
29	2.1	2.8	2.0	0.6	0.6
30	0.0	0.0	0.0	1.2	1.2

Figure 11. Free Air Gravity Anomalies Computed from the GEM 9 Model: MGALS

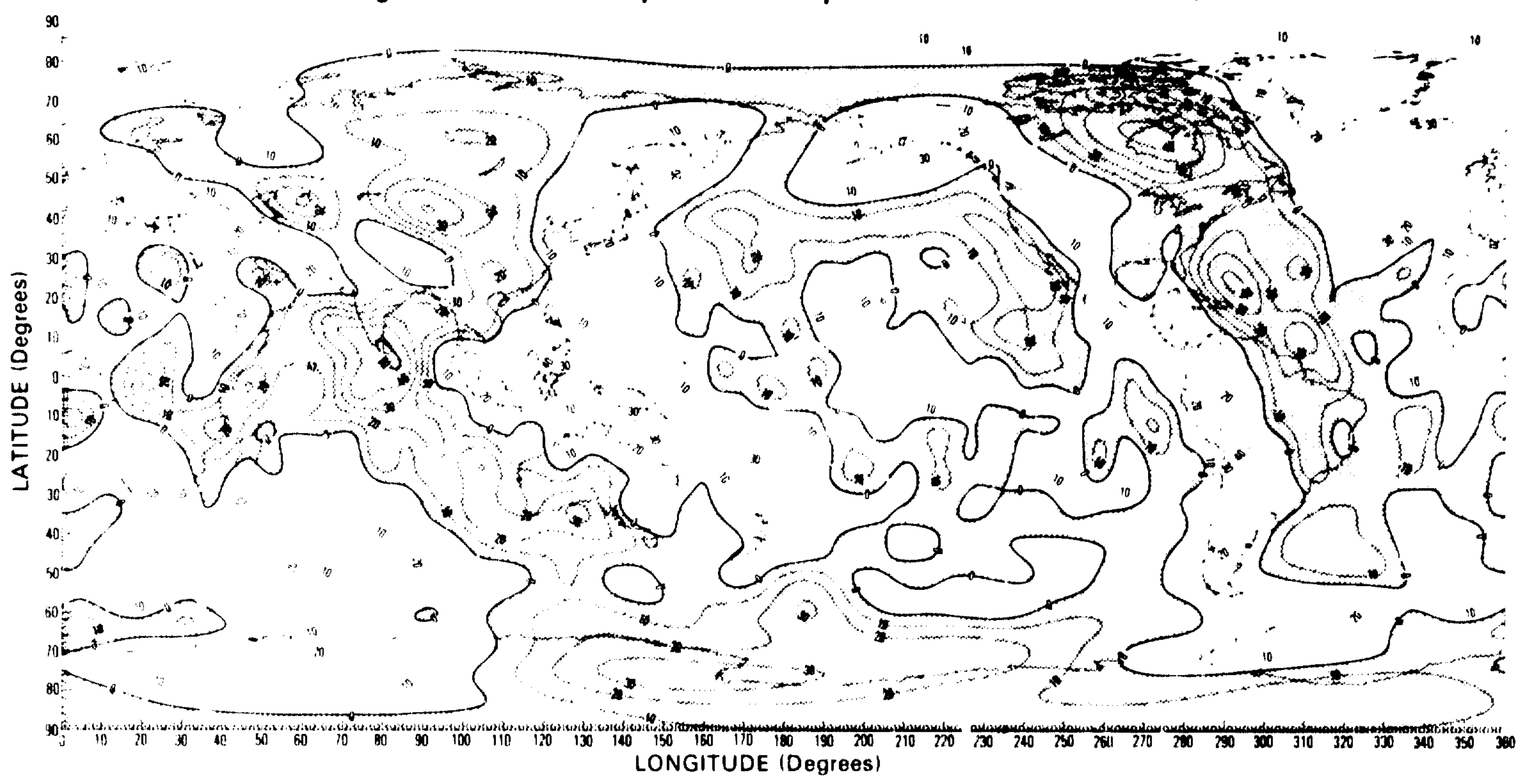
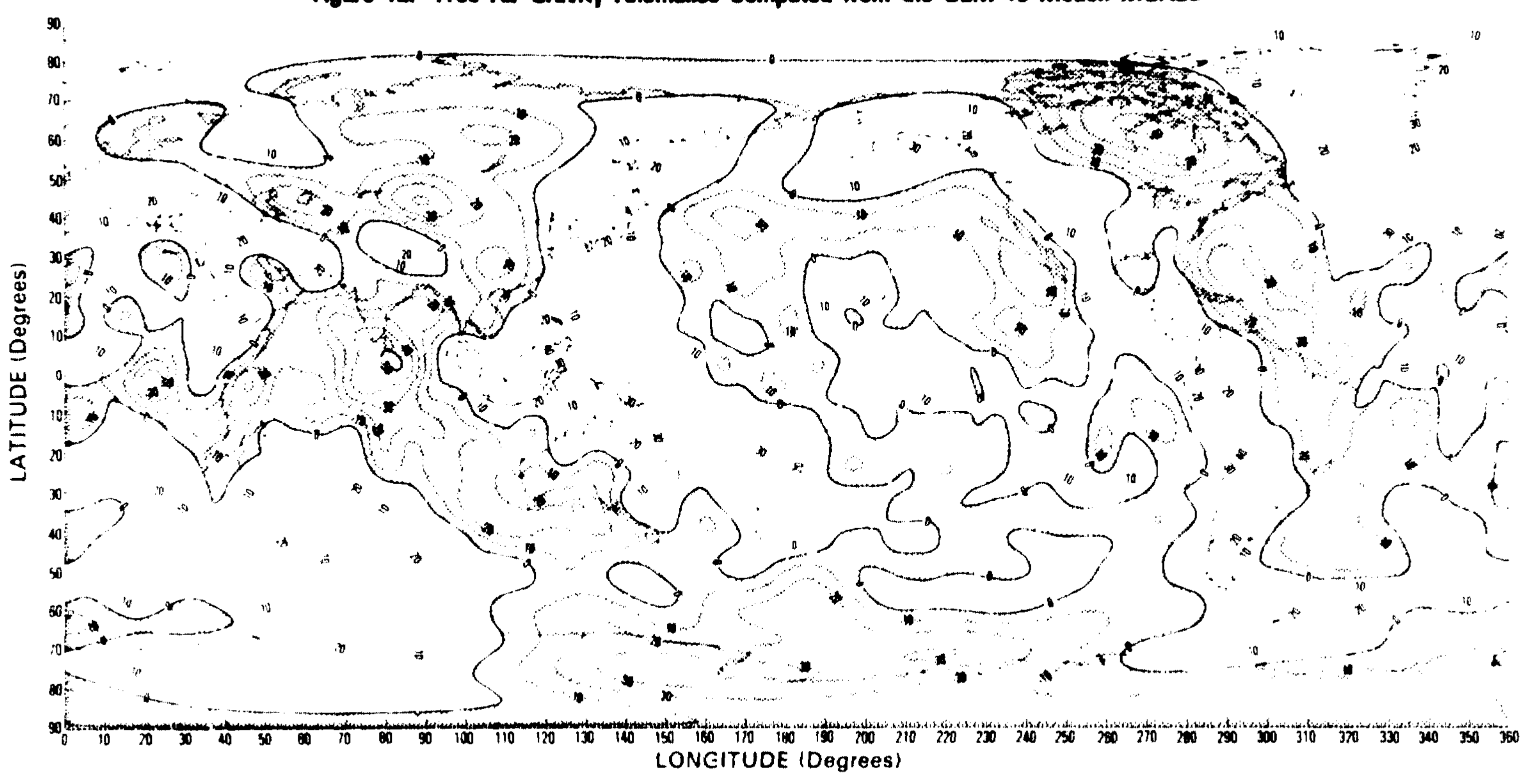


Figure 12. Free Air Gravity Anomalies Computed from the GEM 10 Model: MGALS



There are basically two ways to independently assess the accuracy of a given gravity model. A direct comparison can be made between the satellite gravity models over areas where detailed surface gravimetry exists, (such as North America, Western Europe and Australia). A second comparison can be made between the geoid computed from the gravity model and the geoidal profile directly measured by satellite altimeter experiments. This later approach is discussed in Section 4.4. The first approach is discussed below.

Since the low degree portion of the gravity fields are generally recognized as being accurately determined from satellite observations, we have concentrated our comparisons on the higher order terms in the model. Figure 13 presents the geographical distribution of the surface data in Rapp's (1977) potential model computed solely from surface data. Rapp's data set of $1^{\circ} \times 1^{\circ}$ free air anomalies cover approximately 68% of the earth's surface. Almost two-thirds of the measurements are in the northern hemispere, however.

Figure 14 presents a map of the Rapp free air anomalies computed for coefficients of degree 13 to 22 from his model. The contour interval is 4 mgals. The darker areas are those where the free air anomalies are less than -4 mgals. The lighter shaded regions are areas with small gravity signal at this wavelength being from -4 to +4 mgals. The white areas locate positive anomaly features being greater than 4 mgals. The half wavelength for this portion of the gravity field ranges from 1500 to 900 km.

Figure 15 is a similar free air anomaly map from GEM 9 for coefficients of degree 13 to 22. The GEM 9 model is completely independent from the model computed by Rapp since it uses only satellite tracking data. Figure 16 overlays the boundaries of the Rapp inferred anomalies onto the GEM 9 anomaly map. The agreement in terms of the geographical location of the anomalies is striking. Those areas which have good gravimetry show excellent agreement between the Rapp model and GEM 9. In almost all instances, the discrepancies in this comparison occur in those regions where Rapp does not have data (e.g., the

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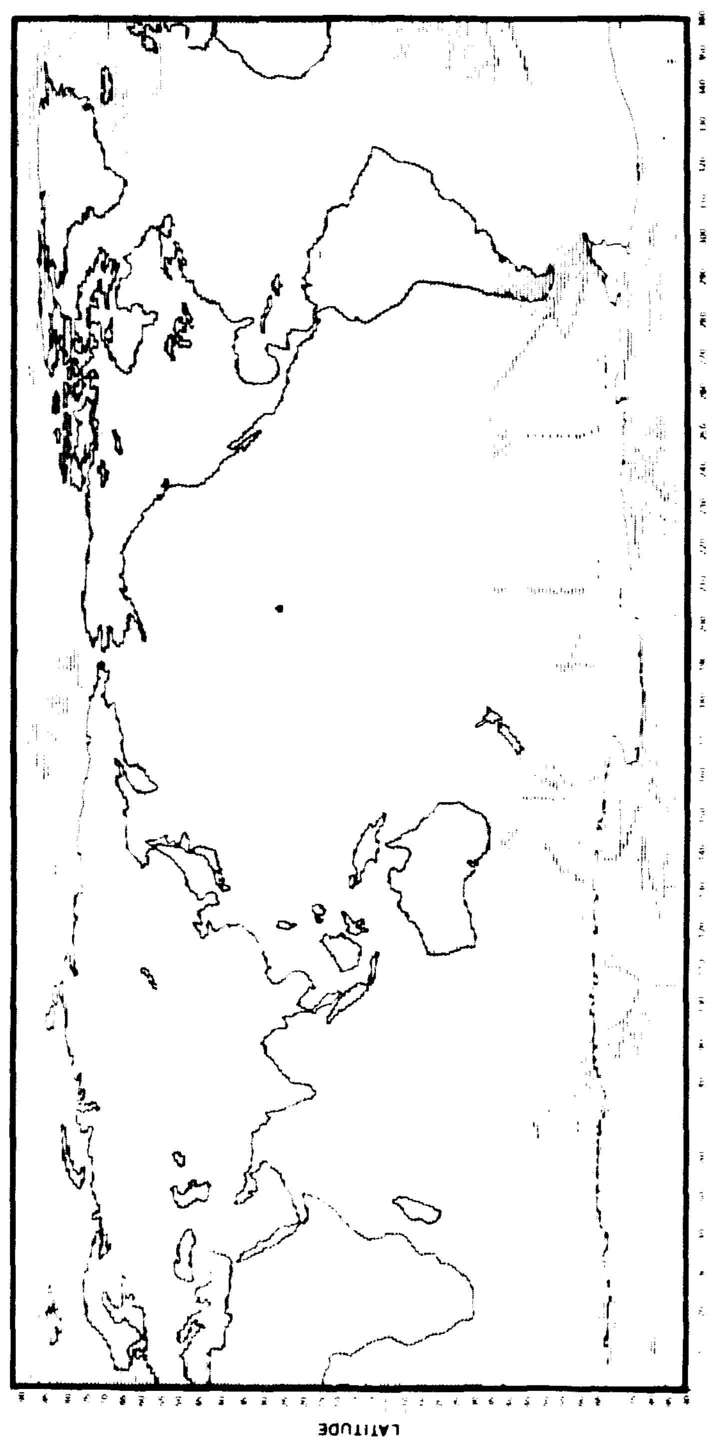


FIGURE 13. 1° DATA DISTRIBUTION FOR THE RAPP, 1977 SURFACE GRAVIMETRY

FIGURE 14. FREE AIR GRAVITY ANOMALIES FOR THE RAPP, 1977 GRAVITY MODEL FOR COEFFICIENTS OF DEGREE 13 TO 22

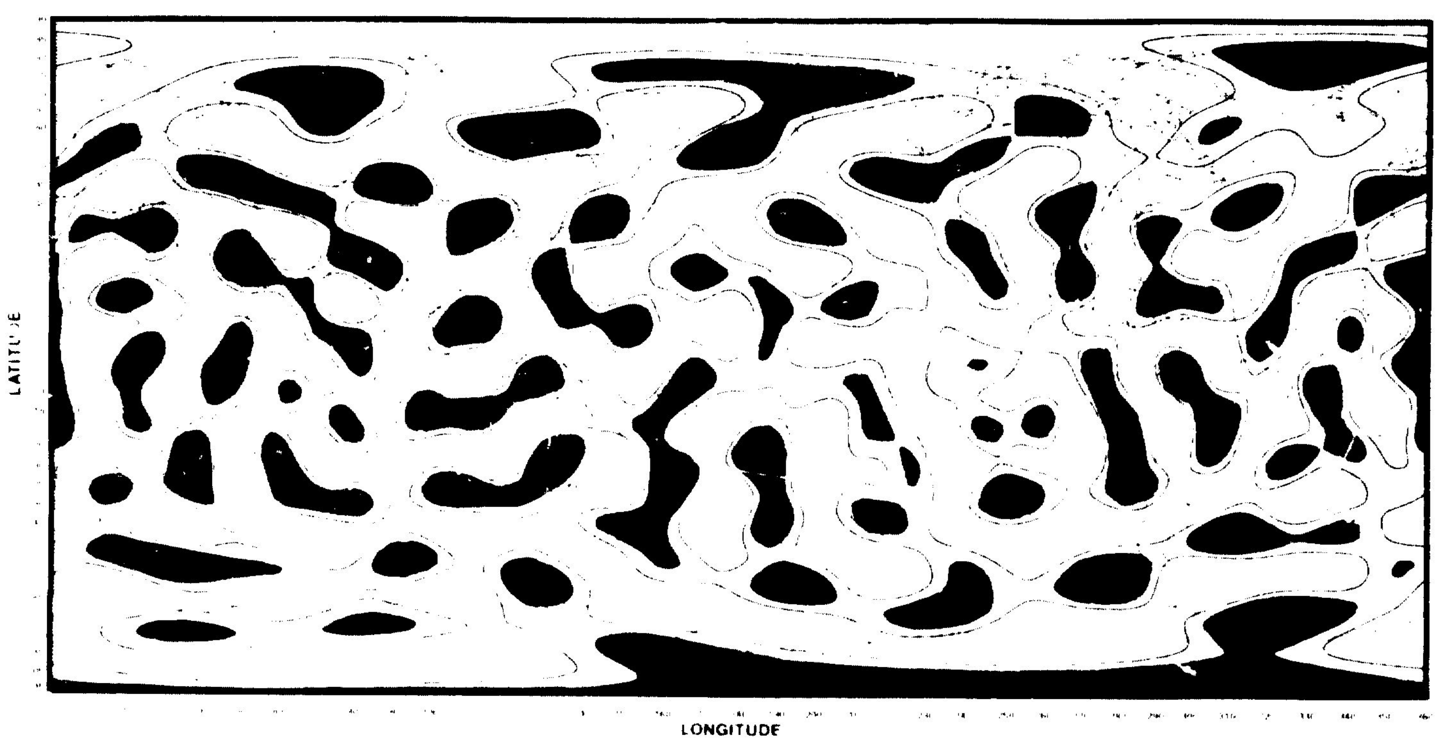


FIGURE 15. FREE AIR GRAVITY ANOMALIES FOR THE GEM 9 GRAVITY MODEL FOR COEFFICIENTS OF DEGREE 13 TO 22

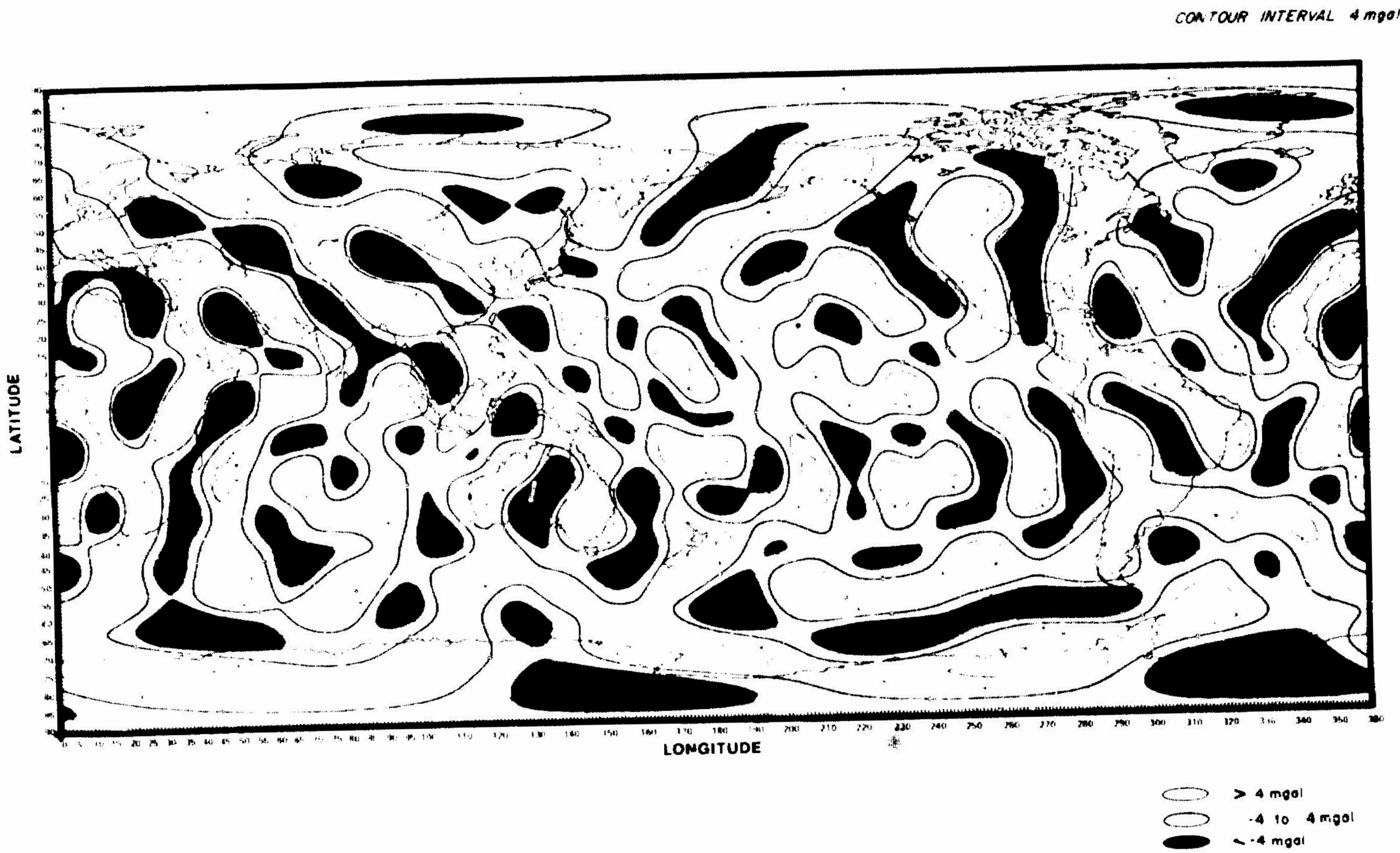
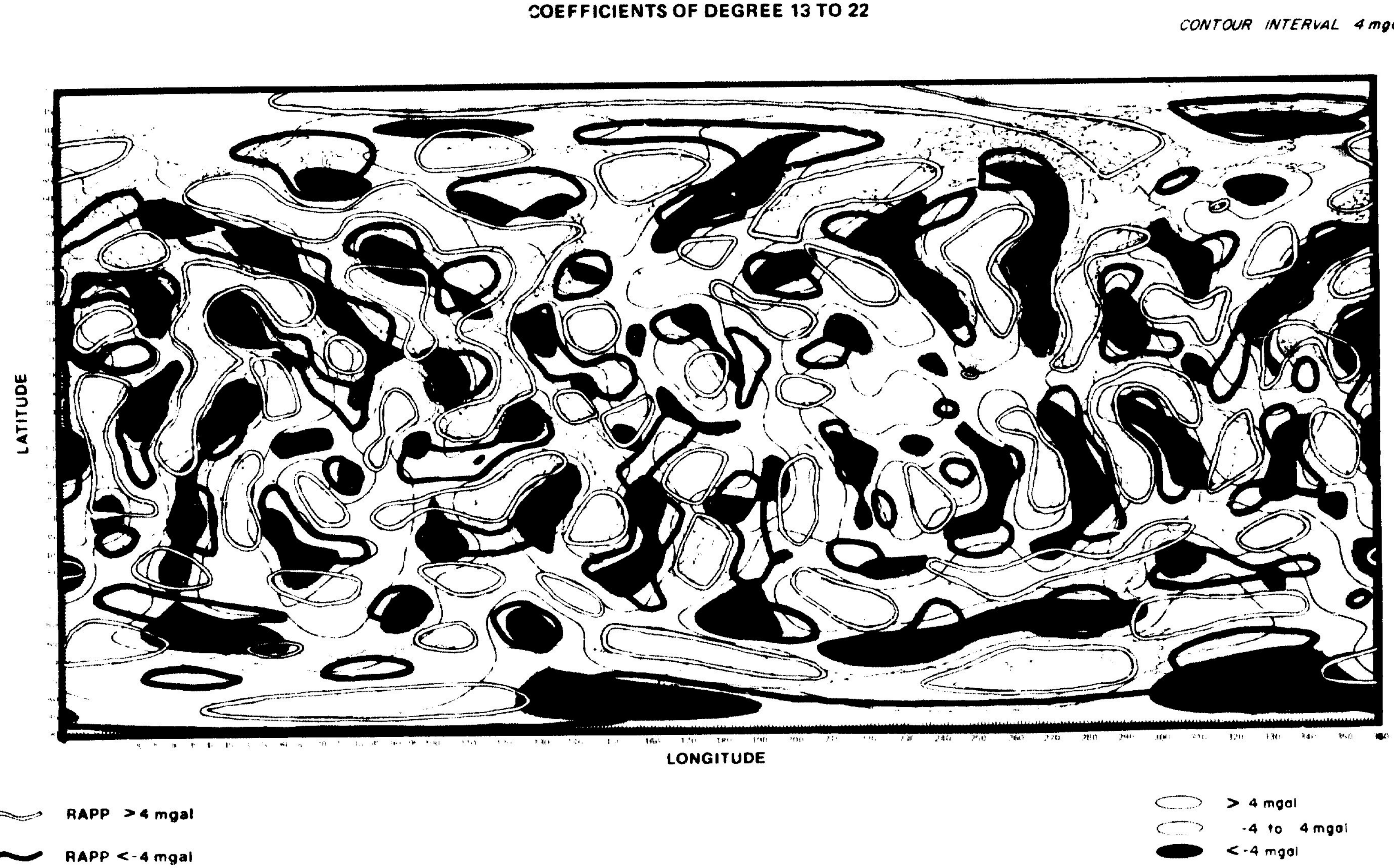


FIGURE 16. A COMPARISON OF THE FREE AIR GRAVITY ANOMALIES FROM THE RAPP, 1977 AND GEM 9 GRAVITY MODELS FOR COEFFICIENTS OF DEGREE 13 TO 22



southern oceans). This intercomparison demonstrates that the satellite derived gravity models are becoming increasingly more accurate in their ability to resolve relatively short wavelength gravitational features. The comparisons with altimetry presented in Section 4.4 confirm this conclusion.

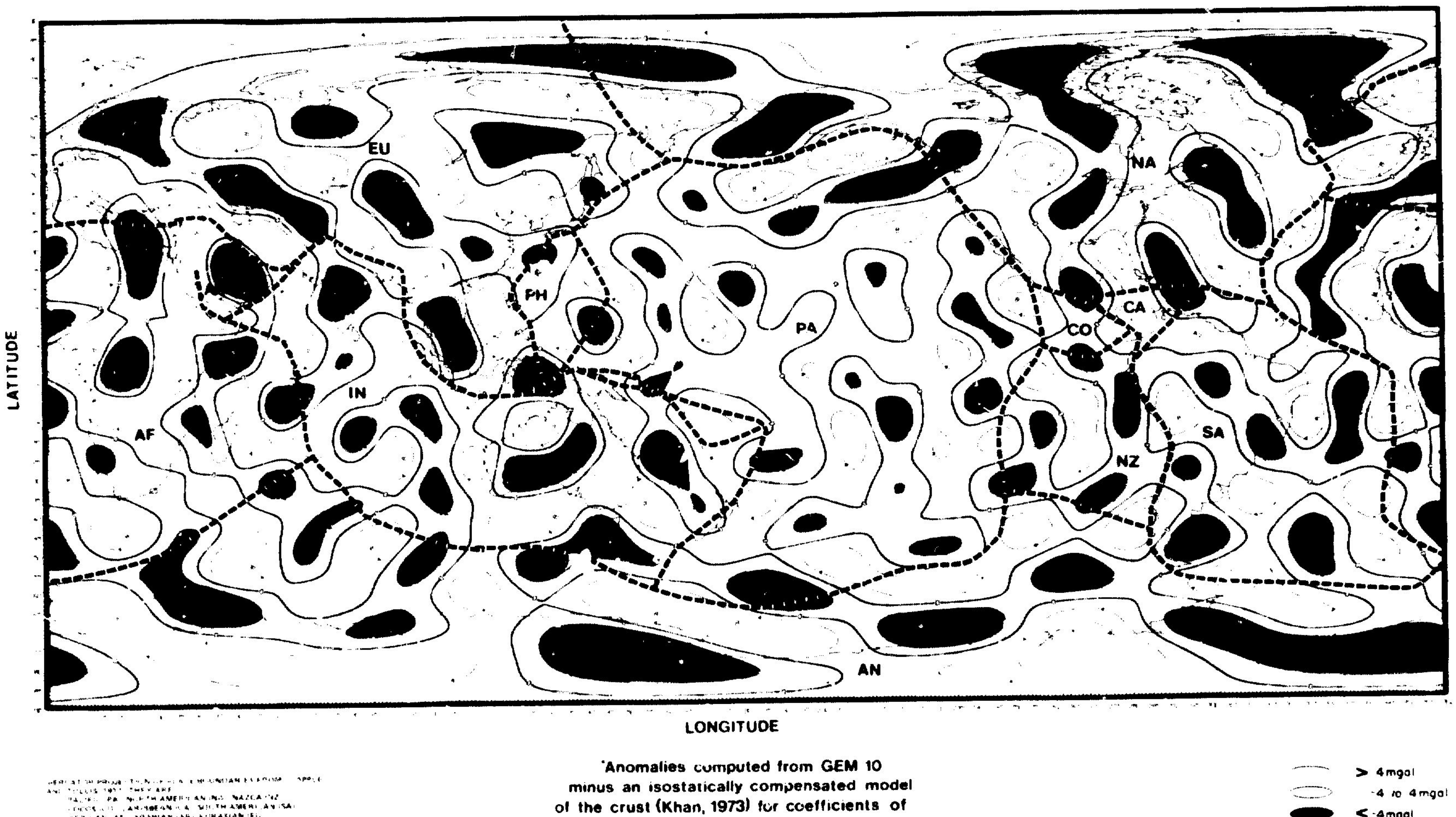
The relatively high degree portion (ℓ = 13 to 22) of the satellite derived gravity field is of geophysical interest. Therefore, we have prepared a map (Figure 17) of the estimated gravity anomalies of the upper mantle derived from GEM 10 with the crustal features removed (using the isostatic model of Khan (1973)). This map shows the estimated mantle gravity features of half wavelengths ranging from 1500 to 900 km. To facilitate an analysis of convective processes, we have indicated the tectonic plate boundries obtained from Chapple and Tullis, 1977.

4.4 EVALUATION OF GEM 9 AND 10 USING ALTIMETER DATA

4.4.1 Evaluation of GEM-9 and 10 Using the "Round the World" Data Taken from Skylab

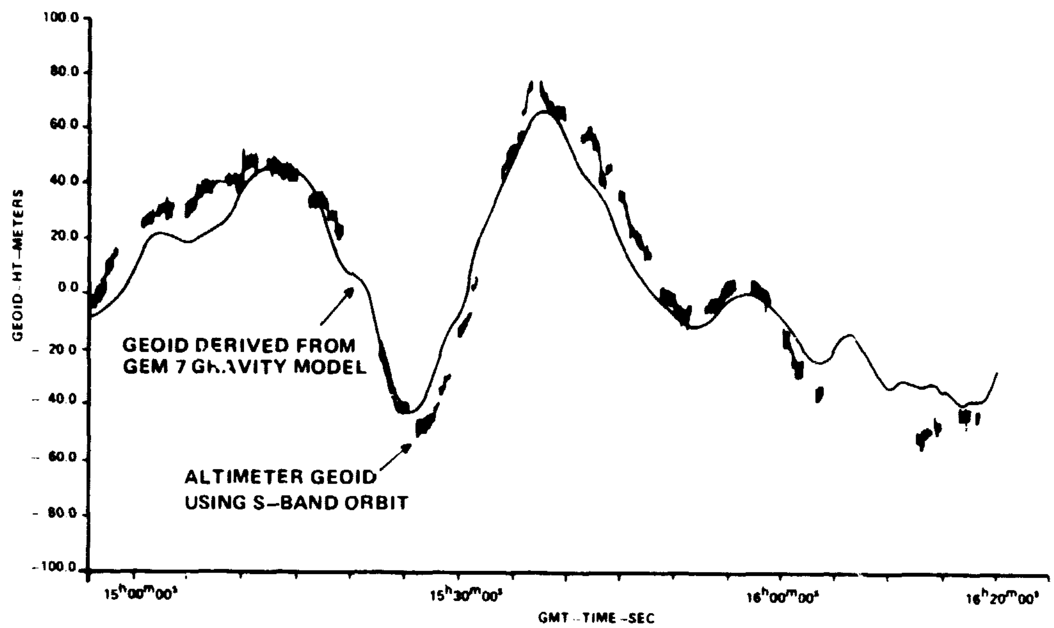
The SKYLAB-193 radar altimeter was operated nearly continuously around the world on January 31, 1974. This direct measurement of the sea surface topography provided for the first time an independent basis for the evaluation of a global geoid computed from satellite derived gravity models. The models considered were the Goddard Space Flight Center GEM (1-10) models; the Smithsonian Astrophysical Observatory SAO 4.3 model, and GRIM 2. This data has previously been used by Marsh et. al., 1975 for gravity model evaluation. The results obtained in our analysis differ somewhat from those of Marsh. A time tag error was discovered in the application of the SKYLAB Airlock Module Time. This error bias has been corrected in our tests. The "round the world" data consisted of 396 six second smoothed altimeter ranges which encircled the world. The RMS of fit to this data is shown in Figure 18. The 3.16 and 3.01 meter residual RMS from GEM 9 and 10, respectively, is quite satisfying. Contained within these residuals are:

FIGURE 17. ESTIMATE* OF THE FREE AIR GRAVITY ANOMALIES
DUE TO THE UPPER MANTLE



*Anomalies computed from GEM 10 minus an isostatically compensated model of the crust (Khan, 1973) for coefficients of degree 13 to 22

FIGURE 18. SKYLAB "ROUND THE WORLD" ALTIMETER RESIDUAL RMS BEFORE CORRECTION FOR A TIME TAG ERROR



SKYLAB "ROUND THE WORLD" DATA TAKE (RMS ~ 8 m)
31 JANUARY 1974

ALTIMETER RESIDUALS AFTER TIME TAG CORRECTION

MODEL	RMS OF 396 ALTIMETER OBS. IN METERS
SURFACE GRAVITY ONLY	6.25
SAO4.3	6.21
GRIM2	5.70
GEM 1	3.74
GEM 2	3.91
GEM 3	4.08
GEM 4	5.13
GEM 5	3.89
GEM 6	4.47
GEM 7	3.28
GEM 8	4.57
GEM 9	3.16
GEM 10	3.01

- commission error in the computation of the sea surface from the GEM 9 and 10 coefficients themselves,

- omission error in the same computation from the models due to their truncation,

- altimeter noise which for SKYLAB was assessed to be 1 to 2 meters, and

- orbital error in the radial positioning of SKYLAB.

The truncation error by itself is estimated to be nearly 2.5 meters.

4.4.2 Evaluation of GEM 10 using GEOS-3 Intensive Mode Altimeter Data

The GEOS-3 altimeter was generally operated over specific geographic areas during a specified period of a few weeks. These areas were varied over time so that a global data set could be compiled from the total complement of acquired GEOS-3 altimeter passes. This type of data accumulation does not lend itself to the global calibration of a gravity model. The time requiring precision orbit determination with all the data would be about a year. However, during February and March of 1976, the altimeter was operated in a more continuous fashion and a reasonable, although not completely global, distribution of altimetry is available.

A test to independently assess the accuracy of the geoid from some recent gravity models was designed. Two five-day orbital arcs were reduced during this concentrated tracking period. The first extended from February 29 to March 4, 1976 while the second was from March 10 to 15, 1976. Each orbit determination made use of all the laser and intensive mode altimeter data available during these intervals. Figure 19 shows the distribution of the altimeter passes which were employed in the two solutions.

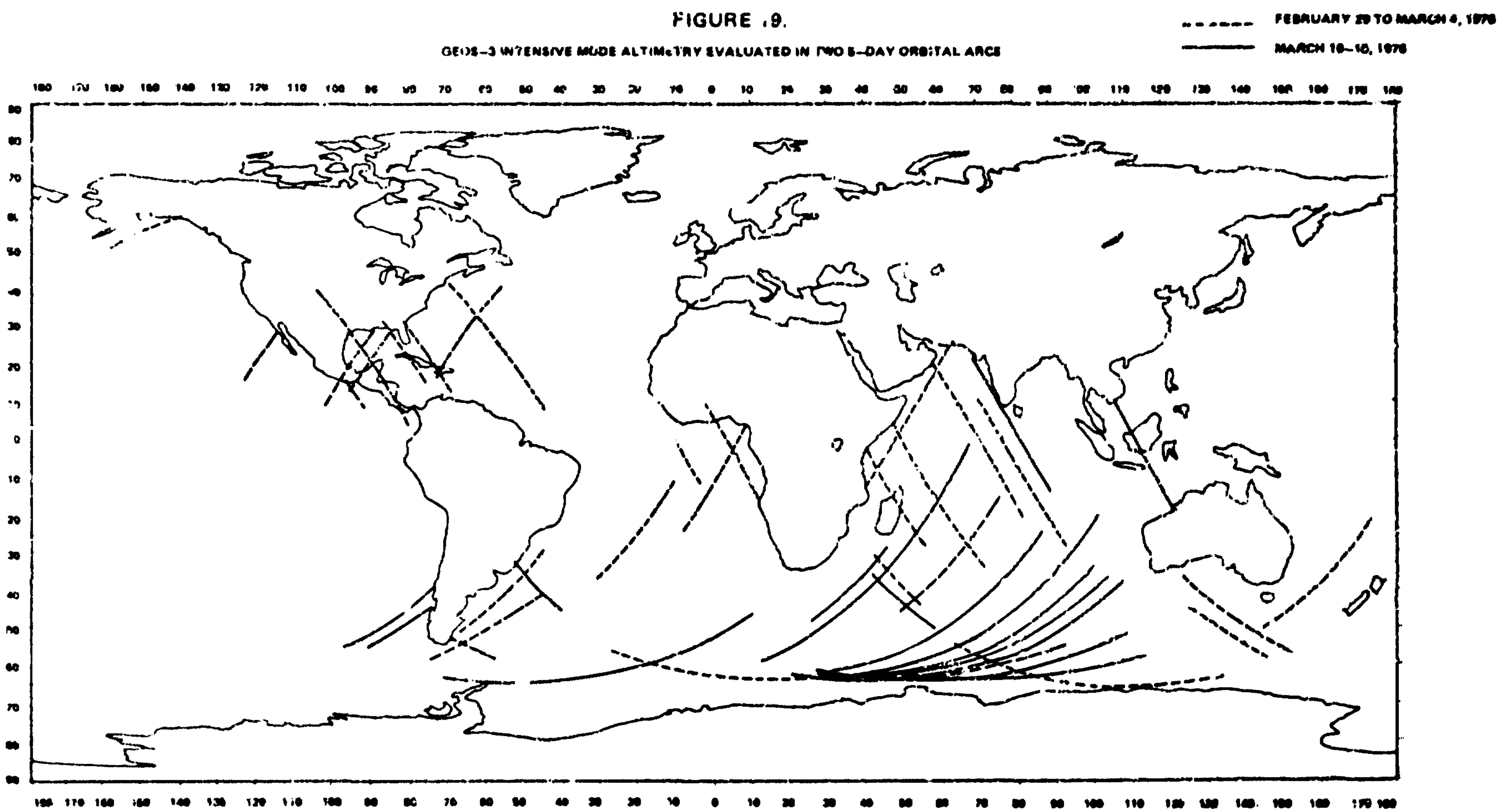


FIGURE 19.

GEOS-3 INTENSIVE MODE ALTIMETRY EVALUATED IN TWO 5-DAY ORBITAL ARCS

The five day orbits were determined using the complete GEM 10, SAO 4.3 and GEM 8 gravity models. The reference geoid used in computing the altimeter measurement residuals came from these models respectively. Only intensive mode altimetry was used; a single bias representing the mismodeling of a_e and instrument bias was solved for in each of the 5 day arcs.

Table 20 summarizes the RMS of fit to the 10750 altimeter observations contained within the two five day arcs. The GEM 10 results were excellent; they showed an even closer agreement between the GEM 10 geoid and the altimeter profiles than was seen in the SKYLAB comparisons (Section 4.4.1). The GEM 10 results are completely consistent with those presented in Section 3.5.2 for 10 five day arcs having less globally distributed data. The way this test was performed makes it difficult to attribute the poorer results obtained from SAO 4.3 and GEM 8 to geoid error. Orbital error is also contained within the residuals from the respective models and probably contributes a sizable amount to the total residual RMS obtained.

On the other hand, analysis of 42 short arcs (10-20 min./arc) data in which the orbital errors were removed empirically show significant improvement in the geoid from GEM 9 and 10 over GEM 7 and 8. Residuals of altimeter derived sea surface with geoid heights from these fields in these globally distributed arcs were 3.30 m for GEM 8, 2.85 m for GEM 7, 2.66 m for GEM 9 and 2.52 m for GEM 10. The improvement of GEM 9 over GEM 8 is especially gratifying since GEM 9 is a smaller field without the benefit of surface data.

In a second case, the GEM 10 field was truncated at twelfth degree and order for the computations of the geoid, while the orbit determined previously from the full GEM 10 field was retained. This variation of the test was made to assess what degradation, if any, would result in eliminating the contribution of the high degree and order terms to the GEM 10 geoid. The altimeter residual RMS (Table 12) increased by almost 1.5 meters when these higher degree and order terms were eliminated. The degradation due to truncation of GEM 10 to 12 x 12 can be estimated by:

TABLE 20.
GEOS–3 INTENSIVE MODE ALTIMETER
RESIDUAL RMS FOR TWO CONCENTRATED 5–DAY
DATA TAKES REDUCED IN 5–DAY ORBITAL ARCS

GRAVITY MODEL:		ALTIMETER RMS IN METERS	
ORBIT COMPUTATION	GEOID MODEL	FEBRUARY 29 TO MARCH 4, 1976 (5972 OBS.)	MARCH 10 TO MARCH 15, 1976 (4778 OBS.)
GEM 10	GEM 10	3.06	2.01
GEM 10	GEM 10 Truncated at 12x12	4.49	3.73
GEM 8	GEM 8	5.14	4.16
SAO 4.3	SAO 4.3	9.81	11.06

$$L = \left| (T_T)^2 - (T_F)^2 \right|^{1/2}$$

where

L is the loss of accuracy due to truncation

T_T is the total RMS from the truncated solutions (combined), and

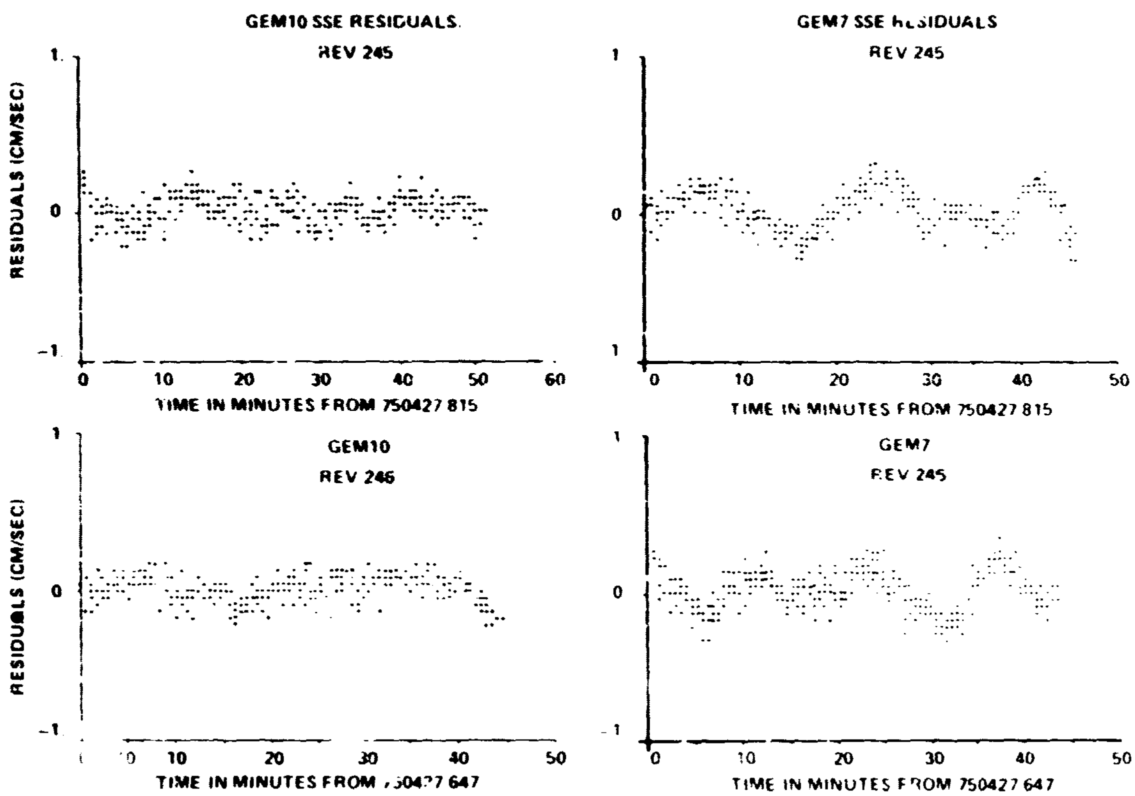
T_F is the total RMS from the solutions using the full model (combined).

Combining the results from the two cases, the estimated loss of accuracy due to the truncation of GEM 10 to 12 x 12 is 3.22 m. This is compelling evidence that the higher degree and order coefficients in GEM 10 contribute accurate information to the computation of a global geoid. This strengthens the conclusions made in Section 4.3. The worldwide contribution of the terms of degree 13 to 22 in GEM 10 is at least 2.5 meters (rms). This would further indicate that this portion of the GEM 10 model is highly accurate.

4.5 EVALUATION OF GEM 9 AND 10 USING ATS-6/GEOS-3 DOPPLER EXCHANGE DATA

GEOS-3 and ATS-6 performed a four-way doppler exchange experiment (Satellite to Satellite Experiment: SSE). The SSE data were not included in GEM 9 and 10. A two revolution orbit of GEOS-3 (revolutions 245 and 246) was reduced which had particularly strong laser ground tracking and two consecutive 45 minute SSE tracks. Figure 20 summarizes these results. This test was designed to evaluate the high frequency portion of the geopotential model. GEM 10 fit the data particularly well. The randomness (RND) of the SSE residuals is also listed. GEM 10 again was the superior solution though a small signal still remains in these residuals.

FIGURE 20



ATS-6/GEOS-3 SATELLITE TO SATELLITE EXCHANGE DATA RESIDUAL SUMMARY FOR GEOS-3 REVOLUTIONS 245 AND 246

MODEL	RMS OF RESIDUALS CM/SEC	RND (RANDOM NORMAL DEVIATE)
GEM9	096	19 7
GEM10	089	16 0
GEM7	131	54 8
GEM8	108	29 3
SAO 4 3	233	221 8
GRIM2	197	159 9

4.6 EVALUATION OF 13th ORDER HARMONICS USING RESONANT SATELLITE ORBITS

Klosko and Wagner (1975) used over 130 constraint equations developed from the analysis of deep resonance orbital passages, new shallow resonance harmonic determinations, and the frequency decomposition of existing satellite geopotential models to obtain improved values for the 13th order tesseral harmonics. In all, thirteen satellite orbits having inclinations from 28^{o} to retrograde were evaluated for this solution. The estimated harmonics were complete to the 32nd degree.

The 13th order coefficients obtained from this resonance solution are compared with GEM 7 and GEM 9 (Figure 21 and Table 21). GEM 9 is in closer agreement with the resonance information than was GEM 7. Term C28,13 seems to show large variation from solution to solution. When this term is removed from the comparison, the GEM 9 field has less than one half the RMS for coefficient differences when compared to the resonance solution than had GEM 7. This result is all the more surprising because the shallow resonance information from GEM 7 is a significant component of the Klosko and Wagner solution.

FIGURE 21 COMPARISON OF GEM 7 & 9 13th ORDER COEFFICIENTS WITH THOSE DERIVED FROM RESONANCE INFORMATION

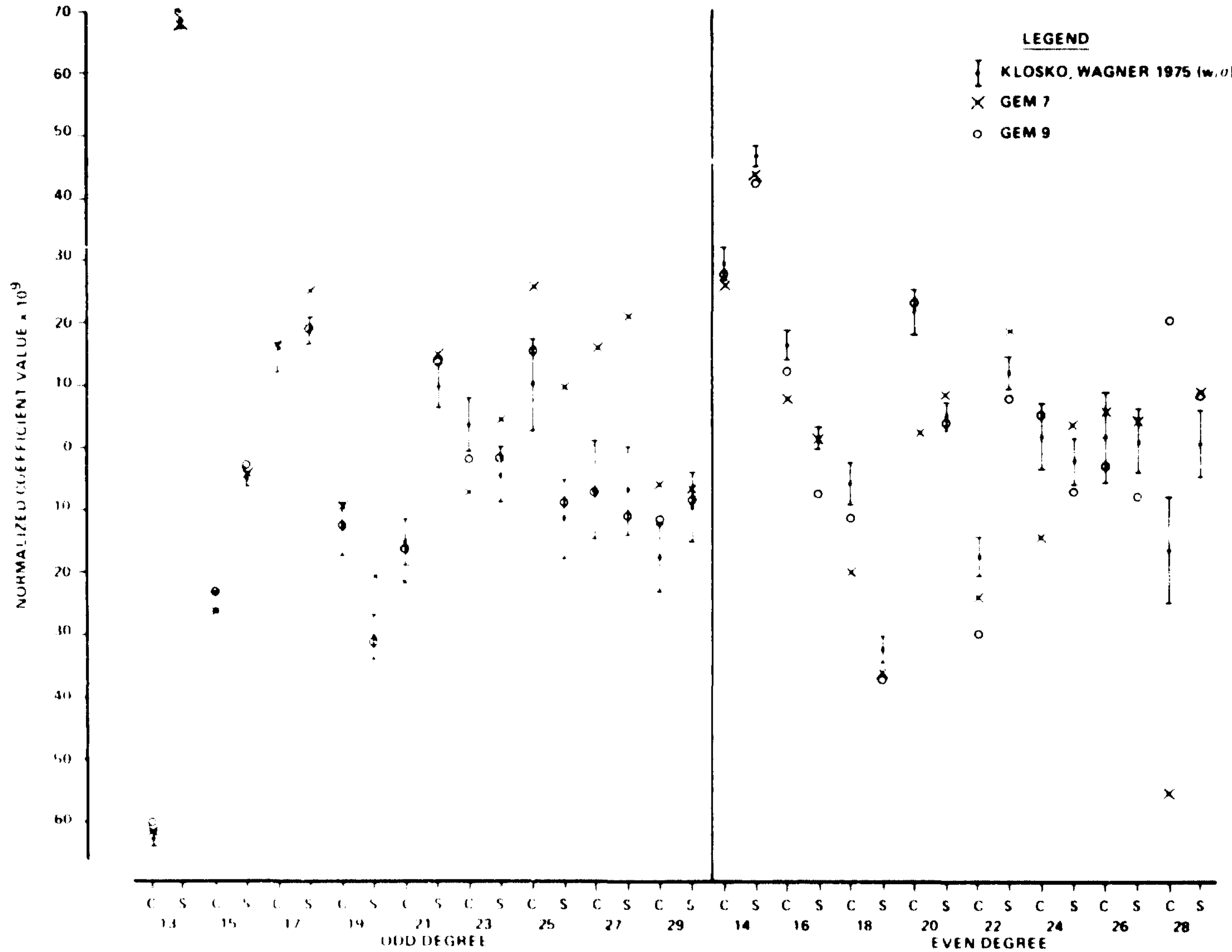


TABLE 21.
COMPARISON OF 13TH ORDER COEFFICIENTS FROM GEM MODELS WITH THOSE DERIVED FROM RESONANCE ANALYSIS (NORMALIZED VALUE x 10^9)

ℓ = m = 13	(1) KLOSKO, WAGNER 1975 NORM. VALUE	σ	(2) GEM7	(3) GEM9	1 − 2	1 − 3
C_{13}	−62.6	(.9)	−60.9	−60.1	− 1.7	− 2.5
S_{13}	68.6	(.9)	67.6	69.8	1.0	− 1.2
C_{14}	29.5	(2.1)	25.6	28.0	3.9	1.5
S_{14}	46.9	(1.1)	43.9	42.2	3.0	4.7
C_{15}	−24.6	(1.2)	−25.3	−22.8	0.7	− 1.8
S_{15}	− 4.9	(1.0)	− 3.1	− 2.2	− 1.8	− 2.7
C_{16}	16.7	(1.8)	8.4	12.2	8.3	4.5
S_{16}	1.7	(1.3)	1.6	− 7.5	0.1	9.2
C_{17}	14.4	(1.9)	16.8	14.7	− 2.4	− 0.3
S_{17}	18.7	(1.8)	24.9	19.2	− 6.2	− 0.5
C_{18}	− 5.8	(2.8)	−19.2	−12.0	13.4	6.2
S_{18}	−32.7	(1.7)	−35.8	−37.4	3.1	4.7
C_{19}	−12.7	(3.8)	− 9.4	−12.4		− 0.3
S_{19}	−30.6	(3.7)	−20.2	−30.7	−10.4	0.1
C_{20}	21.9	(3.2)	2.9	23.2	19.0	− 1.3
S_{20}	5.0	(2.3)	8.2	3.9	− 3.2	1.1
C_{21}	−15.0	(3.5)	−21.9	−16.4	6.9	1.4
S_{21}	9.7	(3.7)	16.2	13.9	− 6.5	− 4.2
C_{22}	−17.5	(3.0)	−24.3	−30.0	6.8	12.5
S_{22}	12.0	(2.5)	19.3	7.7	− 7.3	4.3
C_{23}	3.8	(4.0)	− 7.0	− 2.2	10.8	6.0
S_{23}	− 4.3	(4.1)	4.9	− 1.6	− 9.2	− 2.7
C_{24}	1.4	(4.9)	−14.9	6.1	16.3	− 4.7
S_{24}	− 2.1	(3.7)	3.8	− 7.3	− 5.9	5.2
C_{25}	10.1	(7.0)	25.1	15.2	−15.0	− 5.1
S_{25}	−11.6	(6.3)	11.1	− 8.8	−22.7	− 2.8

TABLE 21. (continued)

COMPARISON OF 13TH ORDER COEFFICIENTS FROM GEM MODELS WITH THOSE DERIVED FROM RESONANCE ANALYSIS (NORMALIZED VALUE x 10^9) (cont.)

ℓ = m = 13	(1) KLOSKO, WAGNER 1975 NORM. VALUE	σ	(2) GEM7	(3) GEM9	1 - 2	1 - 3
C_{26}	1.3	(7.5)	5.1	− 3.2	− 3.8	4.5
S_{26}	0.7	(5.5)	4.0	− 8.2	− 3.3	8.9
C_{27}	− 7.0	(7.7)	16.4	− 7.7	−23.4	0 7
S_{27}	− 6.8	(6.9)	21.2	−10.7	−28.0	3.9
C_{28}	−16.5	(8.0)	−55.9	20.6	39.4	−37.1
S_{28}	0.4	(5.2)	9.2	8.8	− 8.8	− 8.4
C_{29}	−17.4	(5.6)	− 6.5	−10.8	−10.9	− 6.6
S_{29}	− 9.5	(5.3)	− 7.4	− 9.0	− 2.1	− 0.5
C_{30}	9.9	(8.1)	---	---		
S_{30}	4.7	(4.0)	---	---		
		RMS OF RESIDUALS		w/C_{28}:	12.56	7.93
				w/out C_{28}:	10.75	4.80

SECTION 5. .... EVALUATION OF GEM 9 AND 10 FOR ORBIT DETERMINATION ACCURACY

5. EVALUATION OF GEM 9 AND 10 FOR ORBIT DETERMINATION ACCURACY

The ability to model accurately the gravitational forces on near earth satellites is one of the most important applications for improved geopotential models. GEM 9 and 10 have undergone extensive testing in this regard.

The GEOS-3 orbital accuracies were of paramount concern given the demands of altimeter support. But also of concern was the quality of the computed orbits for Beacon Explorer-C (BE-C) and LANDSAT. LANDSAT data was used for the first time in GEM 9 and 10.

BE-C is used extensively in the Laser Polar Motion and San Andreas Fault Experiments. As such, it has been extensively tracked by various laser systems. Table 22 presents the results obtained using laser data from BE-C. This laser data is not in the GEM 9 or 10 solutions, though other range data are used from the same stations to BE-C. Two station configurations -those on the East coast of the United States and those on the West coast - were tested. GEM 9 and 10 show considerable reduction in the overall fit to this laser data when compared to other available models. In the case of Starlette, Marsh and Williamson (1976) have extensively analyzed the orbital accuracies obtained from some preliminary GEM models.

The GEOS-3 spacecraft is in a nearly circular orbit at an altitude of approximately 840 km. The spacecraft is not extremely dense and has an area/mass ratio of $1.4365m^2/345.909$ kg (.004). At this altitude, the estimates of the atmospheric drag perturbations on GEOS-3 range from $12m/day^2$ to $20m/day^2$ (along track) when using the Jacchia (1971) Density Model. A drag perturbation of this magnitude requires extremely refined modeling to avoid prohibitively large orbital positioning errors. We account for the drag on GEOS-3 in a variety of ways depending on the length of the orbit to be determined; briefly:

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TABLE 22.

BEACON EXPLORER-C (BE-C)

LASER RESIDUAL RMS FROM THE SAFE EXPERIMENT

MODEL	EAST COAST OBSERVATIONS (3 SIX-HOUR ARCS) LASER RMS IN CM	WEST COAST OBSERVATIONS (2 FOUR-HOUR ARCS) LASER RMS IN CM
GEM 7	54	50
GEM 8	126	39
GEM 9	18	23
GEM 10	29	18
SAO 4.3	280	154
GRIM2	756	269

- When the orbit is less than 12 hours in length, a ballistic coefficient (C_D) is modeled at a fixed value of 2.5. Any small residual error is easily absorbed in the epoch parameters of the orbit.

- When the orbit is longer than 12 hours but shorter than 36 hours, C_D is allowed to adjust.

- Lastly, when the orbit is longer than 36 hours, a time varying as well as a constant C_D are adjusted to the data.

The GEODYN Program (T. Martin, 1972) is used for the orbital reductions. GEODYN uses Cowell type numerical integration techniques. For GEOS-3 orbital reductions, luni-solar gravitational perturbations, solar radiation pressure, BIH polar motion and UT1 data and atmospheric drag using the Jacchia 1971 Density Model are modeled. We also model solid earth tides (K_2 = .29) and the ocean tides using the diurnal lunar model of Hendershott (1970).

The orbits calculated for GEOS-3 were thoroughly tested. The radial accuracy of GEM 9 and 10 has been evaluated using intersecting GEOS-3 altimetry passes from independent and widely separated orbits (in time). The crossover points were differenced to estimate the radial error in the GEOS-3 orbits computed from GEM 9 and 10.

The altimetry residual for the Kth revolution is given by

$$res_K = a_K - (r_K - g_K - t_K)$$

where

a = altimeter range

r = satellite height above the reference ellipsoid

g = geoid height

t = tide height

Since the residuals are differenced over the same location, the geoid height cancels. Tides were modeled using Hendershott (1970) and small errors are present. Ignoring the tides (t), the difference of the satellite altimetry residuals for the Kth and Jth point at intersection K, J is

$$\Delta res_{K,J} = (a_K - a_J) - (r_K - r_J).$$

If the altimetry is assumed to be noiseless, and having a constant bias, then the altimeter crossover residual difference is a measure of radial orbital error.

Four one-day arcs spanning the altimeter measurements were computed from laser range data; the altimetry was not used in the determination of the orbit. There were 11 intersections in the Atlantic Ocean region and 28 intersections southwest of Australia as illustrated in Figure 22. The altimetry intersections in the Atlantic region involved at least one pass of altimetry in the global mode which Martin (1977) has shown to have varying off-nadir biases of from 1 to 3 meters. The Australia intersections were computed from altimetry which was all in the intensive mode. The intensive mode data has a known bias of a constant (-5.30 ± .21 meters) but is not noticeably affected by pointing errors. These data therefore should yield superior crossover results.

Table 23 lists the crossover results obtained from GEM 9 and 10 along with other representative fields. The RMS is given separately for intensive mode, global mode and the total set of 39 intersections. It is readily seen that the intensive mode is much more accurate than the global mode even though the Australia intersections are further from tracking stations. The radial error for GEM 9 and 10 in these tests appears to be less than 1 meter.

FIGURE 22.
LOCATION OF ALT.METER CROSSOVER POINTS
(39 POINTS)

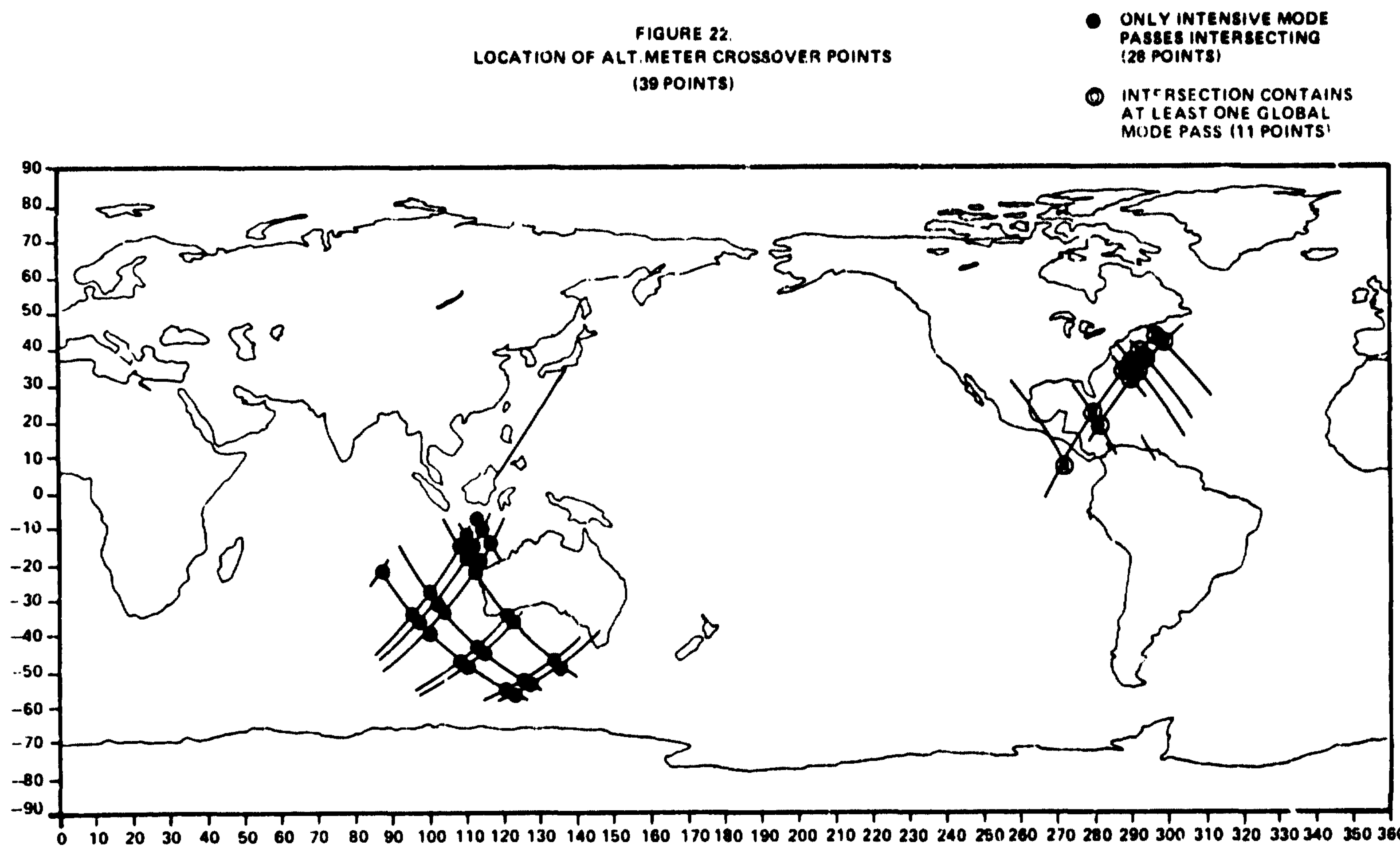
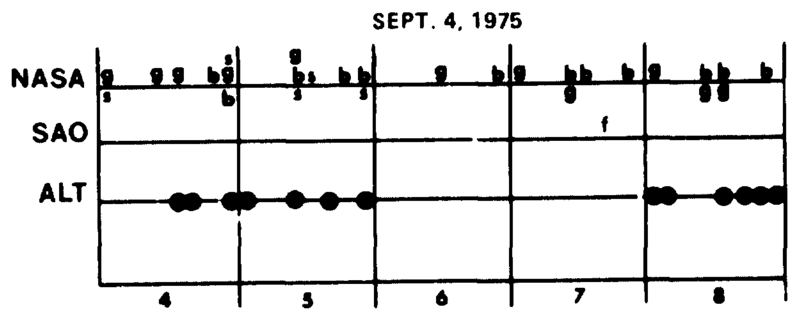


TABLE 23.
RMS OF ALTIMETER CROSS OVER RESIDUALS (METERS)

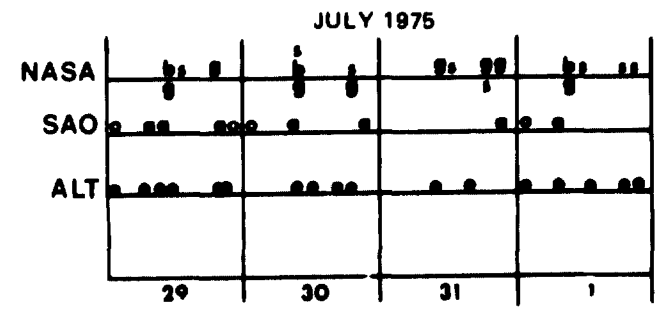
	GEM7	GEM8	GEM9	GEM10	SAO4.3	GRIM2	(PGS968)*
ONLY INTENSIVE MODE PASSES ARE INVOLVED IN CROSS OVER 28 POINTS	6.36	5.06	1.37	1.39	12.95	10.65	(1.09)
AT LEAST ONE GLOBAL MODE PASS INVOLVED IN CROSS OVER 11 POINTS	5.16	5.37	2.77	2.63	10.95	13.81	(2.52)
TOTAL 39 POINTS	6.05	5.14	1.87	1.82	12.42	11.63	(1.62)

*BEST EXPERIMENTAL RESULT.

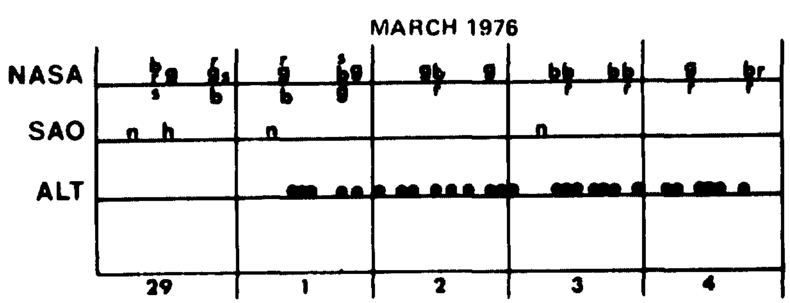
FIGURE 23.
DATA DISTRIBUTION OF FOUR ARCS USED IN GEM10 LONG ARC ANALYSIS



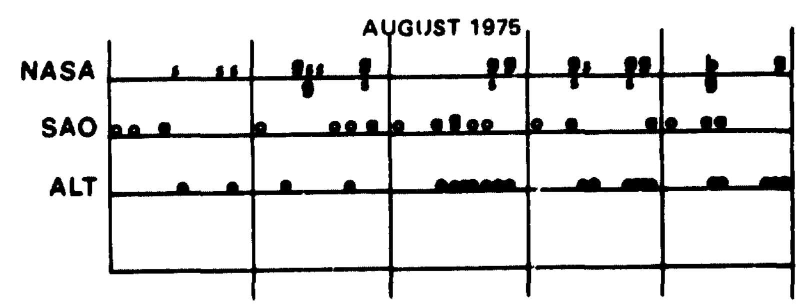
ARC ONE: CALIBRATION AREA (WITH THE EXCEPTION OF ONE SAFLAS PASS) TRACKING ALONE WAS USED TO DETERMINE THE ORBIT



ARC THREE: GLOBAL LASER DATA WAS USED



ARC TWO: MOSTLY CALIBRATION AREA DATA USED FOR ORBIT DETERMINATION. RAMLAS HAS TIMING ERROR ADJUSTMENT.



ARC FOUR: GLOBAL LASER DATA WAS USED

KEY: LASERS

NASA:		SAO:	
g	GRTLAS	o	OILLAS
s	STALAS	a	ARELAS
b	BDALAS	g	GRELAS
r	RAMLAS	n	NATLAS
		h	HOPLAS
		f	SAFLAS

A second altimeter cross over test was completed using the GEM 10 gravity model. In this study four long orbits (three 5 day and one 4 day arc lengths) were determined from available laser data. Figure 23 gives the details of the laser tracking data used in these trajectories. Arcs one and two used tracking almost exclusively from the NASA lasers in the GEOS-3 Calibration Area. Arc two was especially weak since the laser data at Patrick AFB (RAMLAS) had timing problems; RAMLAS timing biases had to be estimated from this data simultaneously with the orbit thereby further reducing the strength of the solution. Arcs three and four had a good distribution of NASA and SAO laser data. The altimeter data (intensive mode only) was not used in the orbit determinations.

Figure 24 shows the location of the 127 altimeter cross over points obtained by intercomparing all four of these arcs. The cross over distribution is still unbalanced, but it is nearly global for sampling different parts of the orbit especially those parts away from the tracking stations.

Figure 25 presents a histogram of the GEM 10 altimeter cross over residuals. Arc two has been segregated by itself and the results from this arc do show an anticipated degradation in radial orbital accuracy. The 80 intersections which do not involve data from arc two have a residual RMS of 1.31 meters. The total RMS (including arc two) for 127 cross overs is 1.60 meters. These results reflect numerous errors besides radial orbital errors.

An error budget for the crossover results is estimated by:

$$R^2 = 2(E^2) + G^2 + 2(T^2) + 2(\epsilon^2)$$

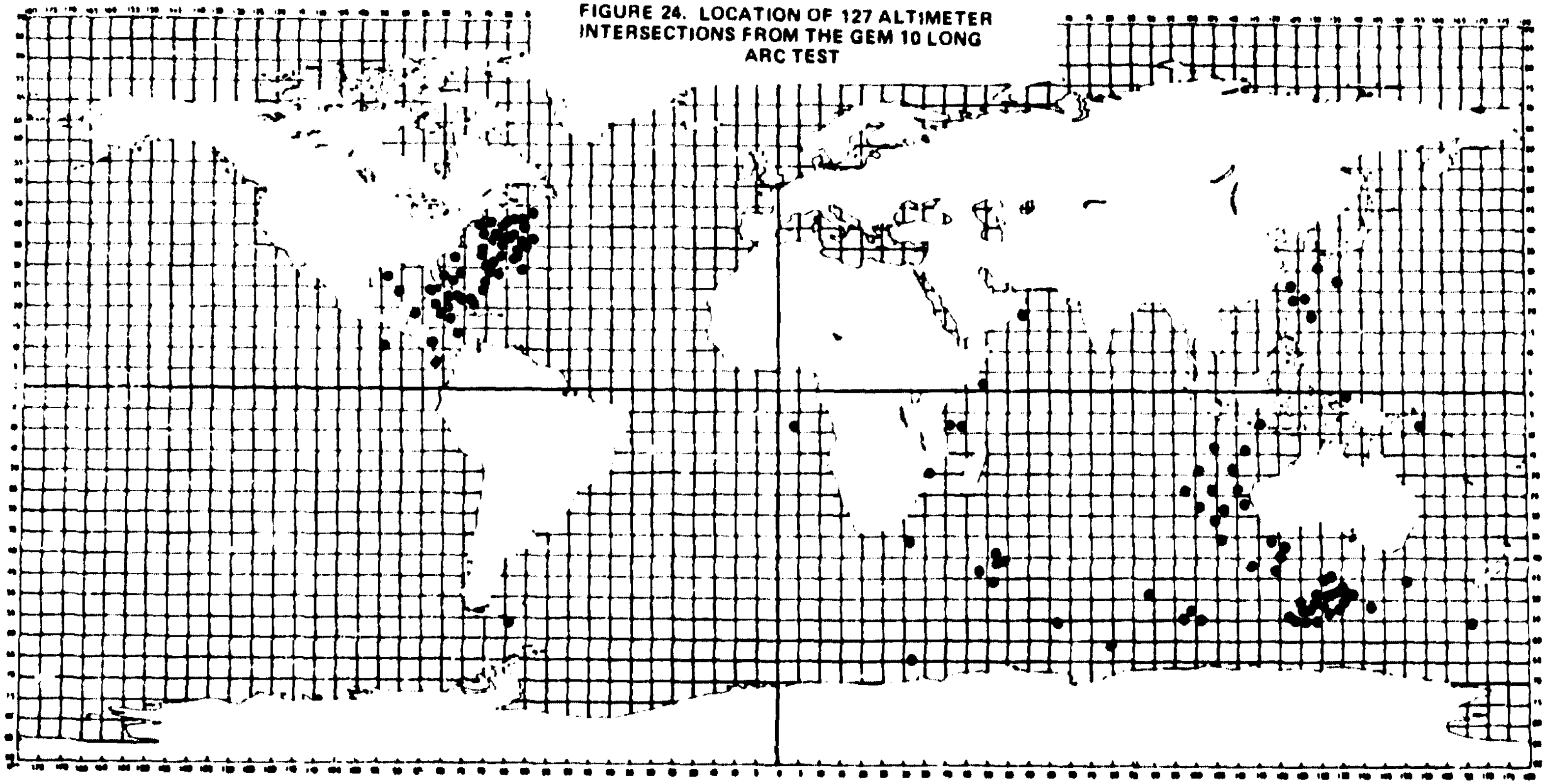
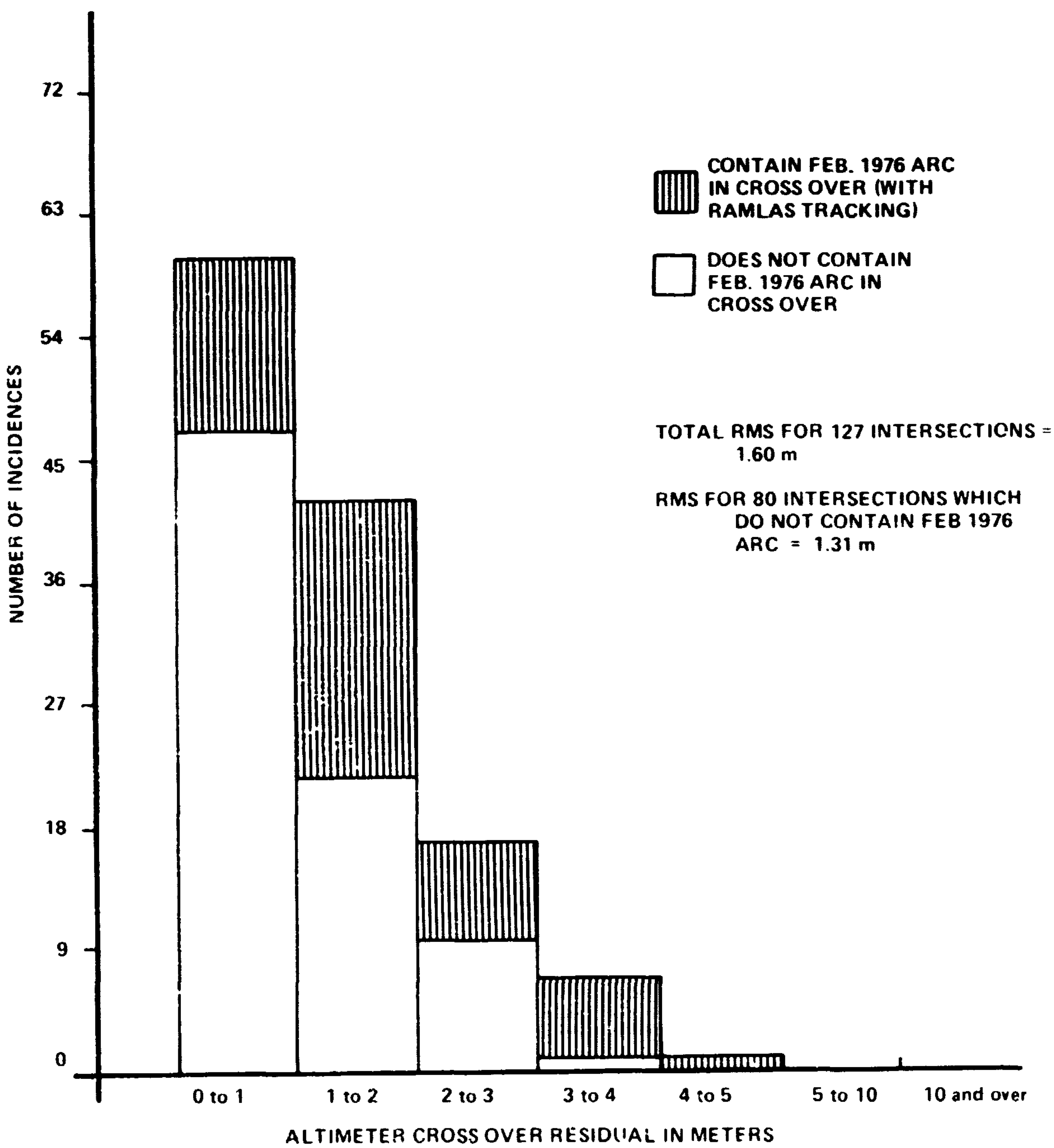


FIGURE 24. LOCATION OF 127 ALTIMETER INTERSECTIONS FROM THE GEM 10 LONG ARC TEST

FIGURE 25. HISTOGRAM OF GEM 10 LONG ARC ALTIMETER CROSS OVER TEST



where

R is the total residual altimeter cross over RMS; (1.31m)

G is the geoid height mismatch. The crossover data were compiled by hand and the altimetry was not interpolated to obtain a value at the precise intersection point. Rather, the closest points in the respective passes were used and these can be spacially separated by as much as 20 km. An estimate of this mismatch is .3 m on average.

T is the ocean tidal error. This has been estimated to be .3 m in each arc, on average from the Hendershott Model.

ξ is the altimeter noise. In these tests, we used the uneditted major frame averages made available from Wallops Space Flight Center. Our noise estimate is .3 meters in each arc, and

E is the orbital error in each arc.

When this equation is solved, the estimated orbital error is .80 m from the three long arcs. With all four arcs, the estimated radial orbit error is 1.03 m.

The GEOS-3 orbital accuracies from GEM 9 and 10 have also been extensively tested on 3 revolution, 1 day and 5 day arcs estimated from laser range data. Appendix I presents these results.

Tests 1 through 9 show various methods employed in determining the accuracy of the fields. The basic approach has been to intercompare two different orbit trajectories. For example, an orbit is determined over a period of time. A shorter arc length within the first is selected and its trajectory is determined. We then compare the two solutions in their radial, crosstrack and along track component differences over their common interval. In this way we can evaluate the accuracy of the field in all three components.

In tests 1 through 5, five day orbital arcs are compared with one and two day arcs which all contain subsets of the same data. Test 6 navigates a station height by using separate passes of data not in the orbital solution. Since we allow the station to adjust only in height, it is a good evaluation of the radial accuracy of the field for high elevation passes. We chose one northern and two southern hemisphere stations to insure that we had a good global sample.

In tests 7 and 8 we did not include the data from the short arc in the longer one. This left a gap of from one day to 32 hours in the longer arc. Test 9 evaluates the RMS of fit for a 5 day orbital arc determined from laser data. All tests are described in detail on their individual summaries.

SECTION 6. . . . . SUMMARY [redacted]

6. SUMMARY

The major objectives of GEM 9 and 10 were achieved. GEOS-3 orbital accuracies from these models are about 1 m in their radial components for 5 day arc lengths. The new GEM 9 and 10 models yield significantly improved results when compared to the surface gravimetry, SKYLAB and GEOS-3 altimetry and highly accurate BE-C laser ranges than do previous GEM solutions. We believe that a genuine improvement has been achieved for the global representation of the terrestrial potential.

Additionally, a new value of GM has been determined dynamically from laser tracking. A consistent value of the mean equatorial radius of the earth was obtained from the estimated tracking station coordinates, the GEOS-3 altimeter data and the implied value of g_e. The average value of a_e was found to be nearly constant among the different techniques used to estimate this parameter. The set of recommended or adopted physical constants from this work are:

- GM = 398600.64 km^3/sec^2
- a_e = 6378140 m
- c = 299792.5 km/sec
- f = 1/298.255

The accuracies of the geopotential coefficients have been estimated and imply commission errors in geoid height of 1.9 m and 1.5 m (global RMS values) respectively for GEM 9 and 10. This error estimate was obtained from independent calibrations with the surface gravimetry, the GEOS-3 altimetry and an error propagation using a gravity model error model derived from these estimates.

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APPENDIX I .... GEOS-3 ORBITAL TESTS

APPENDIX 1.

GEOS 3 ORBITAL TESTS

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LASER DATA SET FOR GEOS-III
(APRIL THROUGH AUGUST 1975)

ID. 7502701 YEAR = 75

STATION NAME	STATION NUMBER	DAYS 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	TOT. NO. PASSES
MONTH = APRIL																																	
GTKMOL	7068	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	2	2	2	1	2	2	1	0		14
STALAS	7063	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	3	3	1	0	1	1	3	0	0	0		14
BDAMOL	7067	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0		1
TOTALS		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	2	3	4	3	3	3	2	5	2	1	0		
MONTH = MAY																																	
STALAS	7063	0	0	1	0	0	1	2	2	1	0	1	3	0	3	2	0	0	0	0	2	3	1	0	2	0	0	4	2	2	0	0	32
HOPLAS	9921	0	0	0	0	0	0	1	0	0	0	2	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	7
GTKMOL	7063	0	0	0	0	0	0	0	0	1	0	0	0	3	2	1	0	2	1	1	1	1	2	2	1	0	1	1	1	0	0	0	21
GRELAS	9940	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1	0	0	0	0	0	0	0	0	2	1	1	0	0	0	0	6
OLILAS	9902	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2
NATLAS	9929	0	0	0	0	0	0	0	0	0	0	1	1	0	0	2	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	6
ARELAS	9907	0	0	0	0	0	0	0	0	0	0	0	0	0	2	1	1	0	1	1	1	1	0	0	0	0	0	0	0	0	0	0	8
CANISL	7819	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0	0	1	0	1	5
BDAMOL	7067	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	2
TOTALS		0	0	1	0	0	1	3	2	2	1	5	5	3	8	7	3	2	2	3	4	5	3	3	4	4	3	8	3	3	0	1	89
MONTH = JUNE																																	
OLILAS	9902	1	0	1	0	2	2	0	0	1	1	2	3	3	1	1	1	3	1	1	2	3	4	0	1	2	3	1	3	2	3		48
STALAS	7063	0	2	1	2	0	0	0	0	0	0	0	0	2	2	3	1	1	1	2	1	3	2	3	1	0	0	0	0	1	1		29
GRELAS	9940	0	1	1	1	0	0	1	0	0	1	1	2	2	0	0	0	2	2	1	0	1	0	1	1	0	0	1	0	0	1		20
BDAMOL	7067	0	0	1	1	2	0	0	0	0	0	0	1	0	0	0	0	1	0	1	1	1	0	0	0	0	0	1	1	1	0		12
HOPLAS	9921	0	0	1	0	0	1	0	0	0	1	1	1	0	2	0	0	0	0	2	1	1	0	2	2	1	0	3	0	1	2		22
GTKMOL	7068	0	0	0	1	0	1	0	0	0	0	0	1	2	3	3	2	1	3	0	2	2	1	1	1	1	1	3	2	1	2		34
CANISL	7819	0	0	0	1	1	1	1	0	2	0	1	0	0	0	1	1	0	0	0	0	1	2	1	1	1	0	1	0	0	0		16
NATLAS	9929	0	0	0	1	2	2	2	2	0	2	0	0	0	0	0	0	0	0	2	2	1	2	1	0	0	1	0	1	0	0		21
ARELAS	9907	0	0	0	0	0	0	1	0	0	0	1	2	2	0	0	0	0	1	1	1	1	1	2	3	2	2	2	1	0	2		25
TOTALS		1	3	5	7	7	7	5	2	3	5	6	10	11	8	8	5	8	8	10	10	14	12	11	10	7	7	12	8	6	11		227
MONTH = JULY																																	
NATLAS	9929	2	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		3
ARELAS	9907	2	2	3	3	2	1	2	3	2	3	2	1	1	1	3	2	3	3	2	1	2	3	3	3	2	1	2	3	3	2	2	68
STALAS	7063	3	3	3	0	0	1	2	0	0	0	0	0	0	0	0	0	0	1	1	1	2	4	3	1	0	3	4	1	1	2	3	3[illegible]
HOPLAS	9921	1	0	0	0	0	0	0	0	0	2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	5
OLILAS	9902	2	2	2	1	3	3	3	2	3	3	3	3	1	3	2	2	1	1	3	1	2	1	1	1	1	3	1	2	1	2	1	60
GRELAS	9940	1	2	1	1	0	0	0	0	0	0	0	0	1	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	8
BDAMOL	7067	0	1	1	0	0	0	1	1	2	3	3	2	1	4	4	2	1	2	4	4	0	0	1	0	3	1	1	0	1	1	0	44
GTKMOL	7068	0	1	0	0	1	3	3	3	1	3	3	3	1	1	1	0	0	0	0	0	0	1	2	0	2	2	2	1	3	2	3	42
TOTALS		11	11	10	6	6	8	11	9	8	14	13	9	5	10	10	6	6	7	10	7	6	9	10	5	8	10	10	7	9	9	9	269
MONTH = AUGUST																																	
OLILAS	9902	1	2	4	3	1	1	0	3	1	1	2	2	3	1	1	1	1	3	2	2	2	0	1	2	2	3	2	0	0	0	2	49
ARELAS	9907	2	1	1	1	3	2	3	2	0	1	3	1	2	1	2	2	1	2	2	2	3	2	1	1	2	1	1	2	1	1	0	50
BDAMOL	7067	2	0	0	0	0	1	1	3	1	1	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	2	1	4	3	4	25
GTKMOL	7068	1	0	3	3	3	2	1	2	2	1	0	3	2	2	2	2	2	3	3	1	3	2	3	0	3	3	3	3	1	3	2	64
STALAS	7063	3	4	3	1	4	0	1	3	2	1	0	1	0	0	0	0	0	1	4	0	0	0	0	0	2	4	3	4	2	2	0	45
NATLAS	9929	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	1	0	0	0	3
TOTALS		9	7	11	8	11	7	6	13	6	5	5	7	7	4	5	5	4	9	11	5	8	6	6	3	9	11	12	11	8	9	8	236

May, days 18–22 (boxed): WEAK 5^{D} TEST PERIOD

August, days 2–6 (boxed): STRONG 5^{D} TEST PERIOD

August, days 18–22 (boxed): 3 REV. 5^{D} TEST PERIOD

TEST 1 DESCRIPTION: A five day arc is determined from laser data. Within the same data span, five one day arcs are also determined from the laser data. These orbital trajectories are then differenced every minute over their common time interval. The radial (R), cross track (C) and along track (A) position differences are statistically evaluated as an RMS difference in each of these ballistic components. These RMS differences for various test gravity models for each comparison (i.e., the 5 day arc versus each one day arc) are presented. The 5 day arc selected (May 18 to 23, 1975) was viewed by us as being very weak given the limited amount of tracking data available. Some of the one day arcs are also very weak.

ORBITAL COMPARISON: RV
5/18 to 23/75
5^D ARC VERSUS EACH ONE DAY ARC

GRAV. MODEL		5^D v. 1^D_1	5^D v. 1^D_2	5^D v. 1^D_3	5^D v. 1^D_4	5^D v. 1^D_5
	R	4.23	6.83	5.23	[illegible].40	11.32
GEM7	C	15.91	14.04	3.98	7.29	11.71
	A	13.44	16.18	42.84	23.45	29.32
	R	2.65	1.24	0.88	0.21	1.84
GEM9	C	9.04	0.42	1.52	1.43	1.83
	A	6.44	3.92	8.79	0.84	5.37
	R	2.64	0.69	0.79	0.11	1.74
GEM10	C	8.92	1.03	1.48	1.32	1.77
	A	6.44	2.15	8.35	0.72	4.26
NO. OF PASSES IN EACH 1^D ARC		2	3	4	5	3

D ≡ DAYS

TEST 2 DESCRIPTION: A five day arc is determined from laser data. Within the same data span, a two day arc, and two one day arcs are also determined from the laser data. These orbital trajectories are then differenced every minute over their common time interval. The radial (R), cross track (C) and along track (A) position differences are statistically evaluated as an RMS difference in each of these ballistic components. These RMS differences for various test gravity models for each comparison (i.e., 5 day versus 2 day arcs, 5 day versus 1 day arc No. 1, 5 day versus 1 day arc No. 2, etc), are presented. The 5 day arc selected (May 18 to 23, 1975) was viewed by us as being very weak given the limited amount of tracking data available. The second one day arc (5/22 to 23) had only 3 passes of data, none of which was past the 11th hour on this day while this orbit was differenced for a full 24 hour interval.

ORBITAL COMPARISON: RV

5/18 TO 23/75

5^D ARC; 2^D ARC (5/21 – 23); 1^D_1 (5/21); 1^D_2 (5/22)*

GRAV. MODEL		5^D v. 2^D	5^D v. 1^D_1	5^D v. 1^D_2	2^D v. 1^D_1	2^D v. 1^D_2	COMMENT
	R	4.70	8.40	11.32	3.93	11.71	
GEM7	C	6.63	7.29	11.71	1.43	15.05	
	A	23.58	23.44	29.32	10.15	39.94	
	R	0.79	2.24	4.96	1.62	5.15	
GEM8	C	4.68	2.26	8.19	3.47	3.79	
	A	22.64	7.92	61.51	4.16	31.32	
	R	4.79	5.36	28.97	0.59	32.18	
GRIM2	C	8.29	15.41	40.35	23.10	45.57	
	A	10.12	12.71	>50	5.26	>50	
	R	5.08	7.67	44.04	4.89	49.02	
SAO4.3	C	8.19	14.47	63.69	19.53	71.22	
	A	28.74	19.85	>50	12.95	>50	
	R	0.34	0.21	1.84	0.54	1.63	
GEM9	C	0.24	1.43	1.83	1.19	1.84	
	A	2.26	0.84	5.36	1.39	7.55	
	R	0.42	0.11	1.74	0.53	1.41	
GEM10	C	0.29	1.32	1.77	1.03	1.64	
	A	2.36	0.74	4.25	1.29	6.04	

*1^D_2 has only 3 passes of data to 11^h and 13^h predict.

TEST 3 DESCRIPTION: A five day arc (August 2–7, 1975) was determined from laser data. A one day arc (August 3–4, 1975) was determined from the same laser data. These orbital trajectories are then differenced for every minute over their common one day period and an RMS is computed for the difference in each ballistic component – (R) radial, (C) cross track, (A) along track. This arc was viewed by us as being exceptionally well tracked from the laser system and should yield strong orbit determination possibilities.

ORBITAL COMPARISON: RV

8/2 TO 7/75

5^D ARC; 1^D 8/3 – 4

GRAV. MODEL		5^D v. 1^D
GEM7	R	6.41
	C	3.05
	A	31.67
GRIM2	R	10.15
	C	2.26
	A	38.23
SAO4.3	R	7.56
	C	16.86
	A	36.34
GEM9	R	0.53
	C	0.43
	A	2.23
GEM10	R	0.47
	C	0.44
	A	2.03

*1^D has 11 passes of laser data.

TEST 4 DESCRIPTION: A five day arc (October 27 to November 1, 1975) was determined from laser data. A two day arc (October 29 to 31) and a one day arc (October 29) was also determined from the same laser data. These orbital trajectories are then differenced for every minute of their common periods and an RMS is computed for the difference in each ballistic component.

ORBITAL COMPARISON: RV
10/27 TO 31/75
5^D; 2^D (29 – 31); 1^D (29 – 30)

GRAV. MODEL		5^D v. 2^D	5^D v. 1^D	2^D v. 1^D
	R	7.40	7.31	
GRIM2	C	23.63	20.92	
	A	44.32	49.85	
	R	4.38	8.45	5.03
SAO4.3	C	40.20	42.37	5.15
	A	72.12	46.39	31.81
	R	1.99	3.59	3.47
GEM7	C	5.64	6.37	2.43
	A	9.88	26.42	20.72
	R	7.69	7.49	
GEM8	C	4.84	4.87	
	A	30.17	42.43	
	R	0.46	0.69	0.27
GEM9	C	1.25	1.47	0.23
	A	2.33	2.33	2.87
	R	0.49	0.82	0.35
GEM10	C	1.43	1.75	0.33
	A	2.40	2.84	3.40

TEST 5 DESCRIPTION: Two three day arcs are computed which overlap for one day. The orbits are determined from laser data. These orbits are then differenced over their common day and from these differences an RMS is computed for each of the ballistic components.

ORBITAL COMPARISON
TWO 3^D ARCS WITH ONE DAY OVERLAP, DIFFERENCED FOR ONE DAY

5/15 – 18 vs. 18 – 21

GRAV. MODEL		3^D v. 3^D
GEM7	R	3.07
	C	7.61
	A	41.45
GEM9	R	0.23
	C	3.19
	A	7.14
GEM10	R	0.32
	C	3.14
	A	7.05

6/10 – 13 vs. 13 – 16

GRAV. MODEL		3^D v. 3^D
GEM7	R	4.29
	C	3.93
	A	40.73
GEM9	R	0.60
	C	1.00
	A	5.02
GEM10	R	0.57
	C	1.02
	A	4.94

TEST 6 DESCRIPTION: These tests require two steps. First, a 5 day (August 2 to 7, 1975) laser orbit is determined with a station removed from the solution. This recovered orbit is then held fixed and each individual pass of this station's laser data is used to estimate a correction to the station height. These height (Δh) corrections are interpreted as an estimate of the radial orbit error in the five day arc. This is similar to the station navigation/orbital error estimates performed at NWL.

STATION NAVIGATIONS FOR 5^D ARC: 8/3 TO 8/8

1 Station is zero weighted and orbit is determined.

2 Orbit from 1 held fixed, and station height is adjusted for each individual pass of data.

STATION	TIME OF PASS	RMS	MAX. ELEV.	ΔH
GRTLAS	804 0959	1.42	57°.6	2.06
GRTLAS	804 1938	.37	34°.6	0.57
GRTLAS	805 0944	.82	87°.5	−1.12
GRTLAS	806 2044	.17	68°.3	0.18
GRTLAS	807 2031	.14	75°.0	−0.02
ARESAO	805 0813	2.21	60°.2	1.43
ARESAO	807 2158	1.58	74°.4	0.61
ARESAO	808 0909	1.97	58°.3	−0.60
ARESAO	808 2144	2.05	62°.5	1.25
OLISAO	804 0144	2.05	58°.2	1.15
OLISAO	805 0130	2.67	80°.4	2.01
OLISAO	806 0116	1.12	56°.8	−0.57
OLISAO	808 0227	1.97	43°.1	−2.44

TEST 7 DESCRIPTION: Five day laser arcs are recovered with a one day gap imposed on the data set during the middle day of the arc. A one day arc is determined over this deleted interval, using only the deleted data. The orbital trajectories for the 5^d and 1^d arcs are compared over this 1^d interval and their differences in each ballistic component are statistically evaluated as an RMS difference.

GRAVITY MODEL		DATE		RMS POSITION DIFFERENCE OVER COMMON 1^d		
5 DAY ARC	1 DAY ARC	5 DAY ARC	1 DAY ARC	RADIAL	CROSS TRACK	ALONG TRACK
GEM9	GEM9	6/14 – 23 1975	6/16 1975	0.81	1.09	2.10
GEM9	GEM9	7/4 – 9 1975	7/6 1975	0.76	0.87	3.55
GEM9	GEM9	8/2 – 7 1975	8/4 1975	0.45	1.09	1.48
GEM9	GEM9	2/11 – 16 1976	2/11 1976	0.31	0.87	2.27
GEM10	GEM10	6/14 – 19 1975	6/16 1975	0.80	1.38	2.27
GEM10	GEM10	7/4 – 9 1975	7/6 1975	0.78	0.96	3.64
GEM10	GEM10	8/2 – 7 1975	8/4 1975	0.42	1.08	1.38
GEM10	GEM10	2/11 – 16 1976	2/11 1976	0.34	0.62	3.29

TEST 8 DESCRIPTION: A five day laser arc is compared with a well tracked 3 revolution arc. The data contained in the three revolution segment of the five day arc is deleted from the five day arc recovery. The orbital trajectory differences over this common 3 revolution time span are statistically evaluated in each ballistic component as an RMS difference.

GRAVITY MODELS		DATE		RMS DIFFERENCES		
5^D	3 REV	5^D	3 REV	RADIAL	CROSS	ALONG
GEM9	GEM9	6/19 – 24 1975	6/21 15^h – 6/21 20^h	0.79	0.52	2.86
GEM10	GEM10	6/19 – 24 1975	6/21 15^h – 6/21 20^h	0.82	0.75	3.02
GEM9	GEM9	8/2 – 7 1975	8/3 8^h – 8/3 14^h	0.64	1.31	1.37
GEM10	GEM10	8/2 – 7 1975	8/3 8^h – 8/3 14^h	0.57	1.04	1.20

TEST 9 DESCRIPTION: Three laser arcs of five days length are computed from laser data. This test compares the RMS of fit to the laser ranges themselves.

	RMS OF FIT (METERS)		
MODEL	EPOCH 1 750622	EPOCH 2 750704	EPOCH 3 750729
GEM7	6.84	7.62	11.53
GEM8	7.47		6.89
SAO4.3	12.02		14.17
GRIM2	11.50	11.82	12.88
GEM9	1.46	1.92	1.25
GEM10	1.46	1.91	1.25

REFERENCES [redacted]

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REFERENCES

Balmino, G., B. Moynot, C. Reigber, "GRIM2 1976 Gravimetric Geoid Evaluation," paper presented at AGU, December 1976.

Brachet, G., "International Satellite Geodesy Experiment Plan," ISAGEX Rep 7, Centre Nat. Etud. Sci., Toulouse, France, 1970.

Chapple, W.M. and T.E. Tullis, "Evaluation of the Forces that Drive the Plates," Journal of Geophysical Research, Vol. 82, No. 14, May 1977.

Gaposchkin, M., "Gravity Field Determination Using Laser Observations," Center for Astrophysics preprint No. 548, presented to Royal Society of London, February 1976.

Hendershott, M., W. Munk, "Tides," Annual Review of Fluid Mechanics, 2, 1970.

Heiskanen, W. and H. Moritz, Physical Geodesy, W.H. Freeman and Company, San Francisco, 1967.

Jacchia, L.G., "Revised Static Models of the Thermosphere and Exosphere with Empirical Temperature Profiles," SAO Special Report No. 332, May 5, 1971.

Kaula, W.M., Theory of Satellite Geodesy, Blaisdell Publishing Company, Waltham, Mass., 1966.

Khan, M.A., "Earth's Isostatic Gravity Anomaly Field," Goddard Space Flight Center Document X-592-73-199, June 1973.

King-Hele, D.G., D.M.C. Walker, R.H. Gooding, "Evaluation of Harmonics in the Geopotential of Order 15 and Odd Degree," RAE Technical Report 74026, March 1974.

Klosko, S., C. Wagner, "13th Order Tesseral Harmonics from Analysis of Resonant Satellite Orbits," paper presented at the AGU, December 1975.

Lerch, F., C. Wagner, J. Richardson, and J. Brownd, "Goddard Earth Models (5 and 6)," GSFC Document X-921-74-145, 1974.

Marsh, J., B. Douglas, S. Vincent, and D. Walls, "Test and Comparisons of Satellite Derived GEOIDS with SKYLAB Altimeter Data," CSFC Document X-921-75-176, 1975.

Marsh, J.G., E.S. Chang, and T.D. Conrad, "Detailed Gravimetric Geoid Computations in Support of the GEOS-III Altimeter Experiment," paper presented at the Fall Annual Meeting of the AGU, San Francisco, 1976.

Marsh, J.G. and R.G. Williamson, "Precision Orbit Computation for Starlette," Goddard Space Flight Center Document X-921-77-25, December 1976.

Marsh, J.G., and T.D. Conrad, "Station Coordinates for the GEOS-3 Altimeter Calibration Area," presented at Spring Annual Meeting of AGU, Washington, D.C., May 1977.

Martin, C.F., "Calibration Results for the GEOS-3 Altimeter," Wolf Research and Development Group Report No. 006-77, April 1977.

Martin, C.F. and N.A. Roy, "Error Model for the SAO 1969 Standard Earth," The Use of Artificial Satellites for Geodesy, AGU Monograph, 1972.

Martin, C.F., "Mathematical Description of the Error Analysis of Satellite to Satellite Tracking Program," Wolf Research and Development Group Report on Contract NAS 5-11736 - MOD3, February 1970.

Martin, T.V., "GEODYN Systems Operation Description," Wolf Research and Development Group Final Report on Contract NAS 5-11736-149, February 1972.

Minster, J.B., T.H. Jordon, P. Molnar and E. Haines, "Numerical Modelling of Instantaneous Plate Tectonics," Geophysical Journal of the Royal Astr. Soc., 36, pg. 541-576, 1974.

Moritz, H., "Advanced Least Squares Methods," OSU Report No. 175, June 1972.

Moritz, H., "Report of Special Study Group No. 5.39 of IAG: Fundamental Geodetic Constants," presented at XVI General Assembly of IUGG/IAG, Grenoble, France, August 1975.

Rapp, R., "Numerical Results from Combination of Gravimetric and Satellite Data Using the Principal of Least Squares Collocation," OSU No. 200, March 1973.

Rapp, R., private communication, 1976.

Rapp, R., "Potential Coefficient Determinations from 5° Terrestrial Gravity Data," OSU No. 251, January 1977.

Richter, F.M., and B. Parsons, "On the Interaction of Two Scales of Convection in the Mantle," Journal of Geophysical Research, Vol. 80, No. 17, June 1975.

Wagner, C., F. Lerch, J. Brownd, and J. Richardson, "Improvement in the Geopotential Derived from Satellite and Surface Data (GEM 7 and 8)," Journal of Geophysical Research, Vol. 82, No. 5, February 1977.

Wagner, C.A., "The Accuracy of Goddard Earth Models," Goddard Space Flight Center Document X-921-76-187, June 1976.