QUESTIONS OF THE ANALYSIS OF MILLIMETER-WAVE FREQUENCY CONVERTERS ON DIODES WITH A SCHOTTKY BARRIER

G. S. Bordonskiy

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<td>16. Abstract</td>
<td>Analysis is conducted of millimeter-wave frequency converters on a diode with a Schottky barrier. The analysis includes investigation of the effect of the variable capacitance of the diode's elements on the frequency converters. Specifically, the transmission, impedance, and noise characteristics of the frequency converters are examined.</td>
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QUESTIONS OF THE ANALYSIS OF MILLIMETER-WAVE FREQUENCY CONVERTERS ON DIODES WITH A SCHOTTKY BARRIER

G. S. Bordonskiy

In the millimeter wave range, superheterodyne receivers with a frequency converter at the input have received widespread use.

Given in this study are the results of the analytic investigation of millimeter-wave frequency converters on a diode with a Schottky barrier. A model of a diode with a variable barrier layer capacitance is examined.

Introduction

One of the most important trends in the development of millimeter receiving equipment in recent years has been the improvement of superheterodyne receiving systems with a frequency converter at the input [1-4]. With such a structure, the characteristics of the frequency converter determine the noise temperature of the system, and special attention has been given to the development of the frequency converters. Substantial improvement of the characteristics has been achieved by using non-linear elements (NE) with quite small parasitic parameters, as well as by means of cooling the converters to nitrogen temperatures [2,3]. The achieved conversion loss magnitudes in 3 mm waves is 5 dB, and the intrinsic noise temperature is 300 K [3].

In addition, in spite of the successes, there have also been difficulties associated with the problems of the further reduction of the diameters of the junctions of the non-linear

*Numbers in the margin indicate pagination in the foreign text.
elements and the improvement of the characteristics of the frequency converters at higher frequencies. Reported in study [2] was the use of junctions in the form of a cross, with a band width of 0.25 μ and a 2 μ side of the cross, and burning out of the junctions was noted during attempts to increase the power input of the heterodyne, which did not make it possible to obtain optimal conversion losses. The achieved noise temperature of the cooled frequency converter for a 3 mm wave (300 K) is substantially higher than the calculated temperature given in study [5]. In addition, during practical operation of the radiometric system, difficulties occur with the heterodyne generators, as well as in the achievement of maximum characteristics because of considerable losses in the input circuit [6].

It should be noted that the questions of frequency conversion downwards on the widely-used diode with a Schottky barrier (DBSh) have been insufficiently dealt with. For example, studies are practically non-existent where calculation of the variable capacitance of the barrier of a non-linear element was carried out. A number of questions, such as conversion at the harmonics of a heterodyne or matching with transmission lines, require further treatment.

Carried out in the present study is the theoretical investigation of the transmission, impedance, and noise characteristics of frequency converters on a diode with a Schottky barrier, with regard for the variable nature of the capacitance of the diode's barrier. The characteristics of the harmonic frequency converters and cooled frequency converters are also investigated.

1. Transmission and Impedance Characteristics of Frequency Converters on a Diode with a Schottky Barrier

A frequency converter on a diode with a Schottky barrier can be represented by the following equivalent schematic—figure
1. $C_b$ is the variable capacitor of the barrier, $R_b$ is the variable resistor of the barrier, $R_s$ is the series resistor of the diode, and $L$ is the conductivity of the contact needle. For calculations of the characteristics of the frequency converter, the inductivity of the contact needle can be attributed to the external circuits, and its effect viewed separately; therefore, we will examine the frequency converter at the terminals of the diode.

The current through the diode is determined as

$$i = i_1 + i_2 = g(t) V + \frac{d}{dt} \left[ C(t) V \right]. \quad (1)$$

In the case of a weak signal, that is with $U_k < U_h$ (where $U_k$ is the signal voltage, $U_h$ is the heterodyne voltage), the time changes of the parallel-connected differential conductivity $g(t)$ and capacitance $C(t)$ are determined by the heterodyne voltage, are actual functions of time, and are represented as a Fourier series in the following form

$$g(t) = g_0 + \frac{1}{2} \sum_{k=1}^{\infty} g_k e^{j k \omega_H t}, \quad (2)$$

$$C(t) = C_0 + \frac{1}{2} \sum_{k=1}^{\infty} C_k e^{j k \omega_H t}. \quad (3)$$

The filters of the step-down frequency converter divide the low voltages of the combination frequencies into the corresponding loads $Y_k$:

$$\mathcal{U}_k = \frac{1}{2} \sum_c \left( \mathcal{U}_k e^{j \omega_k t} + \mathcal{U}_k e^{-j \omega_k t} \right). \quad (4)$$

For values of $\omega/\omega_c \geq 0.1$ ($\omega_c = 1/R_s C_s$), it is not necessary
to consider all of the combination frequencies \[?\]. We will limit the examination to three frequencies: signal \(\omega_i\), image \(\omega_{-i}\), and intermediate, and we will consider all the rest closed at the terminals of the non-linear elements. By substituting (2-4) into (1), we obtain a system of equations which associates the currents and voltages of the combination frequencies.

\[
\begin{bmatrix}
i_k \\
i_o \\
i_{-k}
\end{bmatrix} = \begin{bmatrix}
g + j\omega_k C_o & g + j\omega_k C_k & g + j\omega_k C_{-k} \\
g - j\omega_k C_k & g + j\omega_o C_o & g + j\omega_o C_k \\
g - j\omega_o C_{-k} & g - j\omega_{-k} C_k & g + j\omega_{-k} C_{-k}
\end{bmatrix} \times \begin{bmatrix}
u_k \\
u_o \\
u_{-k}
\end{bmatrix},
\]

where \(k\) is the harmonic number at which frequency conversion takes place.

The conductivity matrix \([Y']\) in equation (5) does not take into account the effect of series resistance. In order to take \(R_s\) into account, we will convert the currents and voltages in the barrier layer to currents and voltages at the I-I terminals (see fig. 1) and find a new conductivity matrix \([Y]\).

\[
[Y] = \begin{bmatrix}
y_{dd} & y_{dp} & y_{dy}
y_{dp} & y_{pp} & y_{py}
y_{dy} & y_{yp} & y_{yy}
\end{bmatrix} = \left([1] + [Y'][Z]\right)^{-1} \times [Y'],
\]

where \([I]\) is an identity matrix, \([Z]=R_s[I]\).

Then, having established the conductivity at the image frequency \(Y_{-k}\), we arrive at a system which describes the equivalent quadripole
The equations which associate the coefficients of the matrices in (7) and (6) have the following form:

\[
\begin{align*}
Y_{dd} &= y_{dd} - \frac{y_{dx} y_{dx}}{y_{dx}^* + y_{k}} ; \\
Y_{dp} &= y_{dp} - \frac{y_{dx} y_{dp}}{y_{dx}^* + y_{k}} ; \\
Y_{pd} &= y_{pd} - \frac{y_{dx} y_{pd}}{y_{dx}^* + y_{k}} ; \\
Y_{pp} &= \text{Re}(y_{pp}) - \frac{y_{xx}^* y_{xp}}{y_{xx}^* + y_{k}}.
\end{align*}
\]  \( (8,9) \)

Knowing the coefficients of the conductivity matrix in (7), one can determine the minimum conversion losses and impedances of the frequency converter [8].

The minimum conversion losses \( L_{\text{min}} \) is

\[
L_{\text{min}} = \left| \frac{Y_{dp}}{Y_{pd}} \right| \frac{1 + \sqrt{1 - \varepsilon}}{1 - \sqrt{1 - \varepsilon}}  \tag{12}
\]

where

\[
\varepsilon = \frac{2 |Y_{dp} \cdot Y_{pd}|}{2 \text{Re}(Y_{dd}) \cdot \text{Re}(Y_{pp}) + |Y_{dp} \cdot Y_{pd}| - \text{Re}(Y_{dp} \cdot Y_{pd})}.
\]

The total conductivity of the signal generator, which corresponds to the minimum conversion loss, is

\[
Y_a = \text{Re}(Y_{dd}) \left[ 1 - \frac{\text{Re}(Y_{dp} \cdot Y_{pd})}{\text{Re}(Y_{dd}) \cdot \text{Re}(Y_{pp})} - \frac{\text{Im}(Y_{dp} \cdot Y_{pd})}{2 \text{Re}(Y_{dd}) \cdot \text{Re}(Y_{pp})} \right]^2 \right]^{1/2} - \left(15\right)
\]

\[-j \left\{ \text{Im}(Y_{dd}) - \frac{1}{2} \text{Re}(Y_{dd}) \left[ \frac{\text{Im}(Y_{dp} \cdot Y_{pd})}{\text{Re}(Y_{dd}) \cdot \text{Re}(Y_{pp})} \right] \right\}.
\]
The corresponding total conductivity for the intermediate frequency is

\[ Y_{\text{tot}} = R_e(Y_{pp}) \{1 - \frac{R_e(Y_{dd}) R_e(Y_{pp})}{2 R_e(Y_{dd}) R_e(Y_{pp})} \left[ \frac{\text{Im}(Y_{dd}) \text{Im}(Y_{pp})}{\text{Re}(Y_{dd}) \text{Re}(Y_{pp})} \right]^2 \}^{1/2} \]

\[ + j \left\{ \text{Im}(Y_{pp}) - \frac{1}{2} \frac{R_e(Y_{pp})}{\text{Re}(Y_{pp})} \left[ \frac{\text{Im}(Y_{dd}) \text{Im}(Y_{pp})}{\text{Re}(Y_{dd}) \text{Re}(Y_{pp})} \right] \right\}. \] (14)

Then, we find the coefficients of expansion of the differential conductivity of the barrier and the capacitance into a Fourier series.

We write the volt-ampere characteristics of a diode with a Schottky barrier in the following form

\[ J = J_s \left( e^{\frac{V_0}{kT}} - 1 \right), \] (15)

where \( V_0 \) is a parameter determined by the mechanism of carrier transport through the barrier.

For thermoelectronic emission

\[ V_0 = \frac{n k T}{q}, \] (16)

\[ J_s = A \frac{m^*}{m} T^2 e^{-\frac{V_0}{kT}}, \] (17)

where \( n \) is the non-ideality factor, \( K \) is Boltzmann's constant, \( q \) is the electron charge, \( J_s \) is the saturation current, \( T \) is the physical temperature, \( A \) is Richardson's constant, \( m^*/m \) is the ratio of the effective mass of the electron to the physical
mass, and $\psi$ is the height of the barrier.

For an exponential volt-ampere characteristic (15), the coefficients of conductivity are determined by the following expressions

$$g_o = \frac{f}{V_o} e^{\frac{U_o}{V_o}} I_o \left(\frac{U_H}{V_o}\right), \quad (18)$$

$$g_k = \frac{f}{V_o} e^{\frac{U_o}{V_o}} I_k \left(\frac{U_H}{V_o}\right), \quad (15)$$

where $I_o$, $I_k$ are modified Besselian functions, $U_B$ is the bias voltage, and $U_H$ is the voltage amplitude of the heterodyne at the barrier.

Expressions (18,19) are correct for modulation of the non-linear elements by the sinusoidal voltage of the heterodyne. In general, the voltage at the barrier is not sinusoidal because of the presence of loads at the harmonics of the heterodyne. Noted in study [9] is an appreciable distortion of the shape of the heterodyne voltage because of the effect of the inductivity of the contact needle for the centimeter range, with values of capacitance and inductivity close to resonance values, and is apparently of limited interest for the short wave section of millimeter waves, in view of the low values of impedance of the barrier's capacitance and the substantially larger values of the impedance of the contact needle. Therefore, all calculations are carried out assuming modulations by the sinusoidal heterodyne voltage.

The capacitance of the barrier of a diode with a Schottky barrier can be represented in the following form [10]:

$$C_b(V) = \frac{C_0}{\sqrt{1 - \frac{V}{\psi}}}, \quad \text{with} \quad V > 0, \quad (20)$$
It is more convenient to approximate the capacitance of the diode with a Schottky barrier with simpler functions in order to obtain the analytic expressions for the coefficients of expansion into a Fourier series. We will adopt the approximating function in the form

\[ C_6(V) = \frac{C(0)}{\sqrt{1 - \frac{V}{V_2}}} \quad \text{with} \quad V < 0. \quad (21) \]

The coefficients \( A_1, A_2, \) and \( A_3 \) are selected so that the difference in capacitances would be appreciable only in the area of large direct currents, where the conductivity of the barrier is great and the magnitude of the capacitance does not have a substantial effect on the characteristics of the frequency converter.

After transformations, the solution is reduced to an integral, which is available in reference books

\[
\frac{\pi}{\alpha} \int_0^{\pi} \frac{\cos n \alpha \cos x}{1 + \alpha \cos x} \, dx = \frac{\pi}{\sqrt{1 - \alpha^2}} \left( \frac{\sqrt{1 - \alpha^2 - 1}}{\alpha} \right)^n \quad \text{for} \quad \alpha^2 < 1.
\]

(23)

The restriction \( \alpha^2 < 1 \) covers the majority of the conditions which are of interest. As a result, we obtain
The dependences of the minimum conversion losses, the optimal conductivity of the signal generator, and the output conductivity with conversion at the first, second, and third harmonics of the heterodyne are found according to the calculation relationships (12-14), using a computer.

The calculations are carried out for the currently widely-used n-GaAs diodes with a Schottky barrier, which have a barrier height of 0.9-0.95 V [1-3,7]. For millimeter diodes, the diameter of the junction is 1-5 \( \mu \). At room temperature, for these data, the saturation current is equal to \( 10^{-15} - 10^{-14} \) A. The value \( J_s = 3 \times 10^{-14} \) A is utilized for calculations for a diode with a junction diameter of 3 \( \mu \) and a value of \( V_o = 0.026 \) V, which is close to that for an ideal diode (0.025 V). Calculations are carried out for the wide-band conditions of the image frequency which are most interesting in the case of millimeter waves (\( Y^-_K = Y^-_{-K} \)).

Given in figure 2 are the graphs of the dependences of the minimum conversion losses for an ideal diode with conversion at the 1st, 2nd, and 3rd harmonics of the heterodyne (curves 1, 2, and 3, respectively), as a function of the current \( Y^-_o \), rectified
by the diode (to 10 mA). The bias voltage is equal to zero. As is evident, an increase is observed in the conversion losses by approximately 1 dB with an increase in the harmonic number by one. The dependences of the optimal resistance of the signal generator and output resistor, which correspond to these cases, are given in figure 3 and 4.

In the presence of parasitic parameters of the diode structure, an increase occurs in the conversion losses. Given in figure 5 are the dependences of $L_{m,n}$ for converters at the 1st, 2nd, and 3rd harmonics, with a ratio $\omega/\omega_c = 0.15$, a bias voltage equal to zero, and $R_s = 10$ ohms. The greatest increase in conversion losses is characteristic with conversion at the 3rd harmonic, equalling 1.5-2 dB, as compared with a frequency converter at the 2nd harmonic; the difference increases in proportion to the increase in the rectified current, whereas, with conversion at the basic frequency, the conversion losses result in saturation with currents greater than 2 mA. Given in figures 6 and 7 are the dependences of the optimal $R_H$ and $R_{out}$ for these conditions. We would note that, in all cases, the approximate equality $R_{out} \approx 2R_H$ is observed.

The variable capacitance has the most obvious effect on the frequency converter at the 1st harmonic. This is especially manifest for a $\omega/\omega_c$ on the order of several hundredths, which can be seen from figure 8, where conversion losses are given for $\omega/\omega_c = 0.036$, 0.15, and 0.2. For comparison, the dependences are given with broken lines for the case of a constant barrier capacitance, equal to the average for the period of effect of the heterodyne voltage. The bias voltage is equal to zero. There is the possibility of a decrease in conversion losses below the maximum value under wide-band conditions—3 dB for a resistive frequency converter. Calculation is carried out in study [11] for inverse bias voltages, and the possibility of
intensification with conversion downward is shown, which, however, was accompanied by a considerable worsening of the noise characteristics. ((8-10 words cut off))...phenomena disappear and the conversion loss curves have roughly the same appearance as for a constant barrier capacitance, although the variable capacitance somewhat reduces the conversion losses (~0.5 dB). A decrease in conversion losses leads to an appreciable increase in the optimal resistance of the signal generator and the output resistance (figs. 9, 10).

The attainable minimum conversion losses for harmonic frequency converters, as a function of $\omega/\omega_c$, are given in figure 11.

With conversion at the 2nd and 3rd harmonics, the effect of the capacitance's variability is expressed appreciably less. Compared in figure 12 are two cases with conversion at the 2nd harmonic and $\omega/\omega_c = 0.15$.

It is interesting to trace the effect of the bias voltage on the characteristics of a frequency converter. Given in figure 13 and 14 are two curves each for bias voltages of 0 and 0.5 V with conversion at the 1st and 2nd harmonics of the heterodyne, from which it follows that the bias input increases conversion losses, which is especially noticeable for a harmonic frequency converter, with deterioration by 2.5 dB and more.

An increase in the bias voltage is, to a certain extent, equivalent to an increase in the saturation current (here, however, it is necessary to remember that the diode's capacitance depends on the bias voltage). An increase in the saturation current leads to an increase in conversion losses, which is also most noticeable for a harmonic frequency converter, figure 15. With conversion at the 3rd harmonic, a change in
$I_5$ from $10^{-14}$ A to $10^{-8}$ A evokes a change in conversion losses by 4 dB, which may have been the cause of the considerably worse results with conversion at the harmonics for the point-contact diodes used earlier [12].

In addition to $C_b$, $R_b$, $I_5$, and $U_B$, the parameter $V_o$, which is determined by the physical temperature of the junction, is included among the initial conditions. With a decrease in temperature, the contribution of tunneling of thermally-excited carriers begins to increase appreciably [13], and the value of $V_o$ is determined as

$$V_o = \frac{E_{\infty}}{q} \frac{c \hbar}{kT},$$

(26)

where $N$ is the concentration of carriers in the semiconductor, $\hbar = h/2\pi$ is Planck's constant, $E_{\infty} = q\hbar (N/\varepsilon_1 m^*)$, $\varepsilon_1$ is the dielectric constant of the semiconductor.

The dependences of $V_o$ on the temperature and concentration of the carriers for gallium arsenide of the n-type are given in study [14], and figure 16. Values of $V_o$ are selected equal to 0.017 V and 0.012 V, and a saturation current of $\sim 10^{-25}$ A, for calculation of the characteristics of the frequency converter. The dependences of the conversion losses and impedances, with conversion at the 1st harmonic, are given in figures 17 and 18.

Thus, the given data indicate the substantial effect of variable barrier capacitance in the case of conversion at the basic frequency of the heterodyne. With conversion at the harmonics, an increase is observed in the conversion losses by roughly 1 dB with an increase in the harmonic number by one. Increases in the diode's saturation current and direct bias voltage lead to an increase in conversion losses, which is
especially manifest in converters at the harmonics. Cooling of the frequency converter to a temperature on the order of 80 K does not lead to a substantial change in the transmission characteristics.

2. Noise Characteristics of a Frequency Converter

The study conducted above of the transmission characteristics of frequency converters, with regard for the variable capacitance of the barrier of the non-linear elements, shows the possibility of achieving small conversion losses. Here, it is necessary to answer the question of the noise characteristics of frequency converters.

The basic noise sources in diode frequency converters is the shot noise of the barrier and the thermal noise of the series resistor. The shot current, generated in the barrier, is proportional to the equivalent direct current $I_e$:

\[ \overline{i_{sh}^2} = 2q \oint J_A \Delta f, \quad \text{where} \quad J_A = J_0 + 2I_s. \quad (27) \]

In the absence of a heterodyne voltage, the shot current may be represented as the infinite sum of the noise currents

\[ i_{sh} = \sum_{m=1}^{\infty} i_m \cos (\omega_m t + \gamma_m). \quad (28) \]

The equivalent noise temperature of the barrier's differential resistor is found as

\[ T = \frac{q V_0}{2K} \left( \frac{R_b}{R_b + R_s} \right) + T' \left( \frac{R_s}{R_b + R_s} \right). \quad (29) \]

where $T'$ is the temperature of the series resistor $R_s$. 
In the presence of a heterodyne voltage on the barrier, the equivalent current \( J_e \) turns out to be a periodic function with a frequency \( \omega_H \)

\[
J_e = J_{es} + 2 \sum_{f=1}^{\infty} J_{ef} \cos \omega_H t + \ldots
\]  

As a result, the shot noise becomes amplitude modulated and the noise components are broken down into secondary components with frequencies \( \omega = n\omega_H \pm \omega_m \) \( (n=0,1,2,...) \)

\[
i_{sh} = \sum_{m=1}^{\infty} \left[ i_m \cos(\omega_m t + \gamma_m) \right] + \sum_{f=1}^{\infty} \sum_{e} \left[ \cos \left( (n\omega_H + \omega_m) t + \gamma_m \right) \right] \]

As a result, the noise sources at the signal, image, and intermediate frequencies prove to be correlated.

In order to find the noise characteristics of the frequency converter, it is necessary to sum up the contributions of the shot and thermal noises, converted from the signal and image frequencies, with the noises at the intermediate frequency. The noise temperature at the output of the frequency converter in the wide band mode for the image frequency is

\[
\frac{1}{T_{out}} = \frac{\overline{V_1^2} + \overline{V_2^2} + 2 \overline{V_3^2}}{4K \Delta f R_H \overline{V_{out}}} ,
\]  

where \( \overline{V_1^2}, \overline{V_2^2}, \overline{V_3^2} \) are the RMS values of the voltages of the noises at the output load, evoked by the shot and thermal noises of the series resistor and the signal generator resistor, respectively.

A similar problem is solved in study [11], but only the
case when thermoelectronic emission is the dominant mechanism of passage through the barrier is examined. As is common knowledge, the shot current does not depend on the barrier temperature, but is determined by the parameter of the volt-ampere characteristic $V_\theta$. The presence of physical temperature in the formulas in study [11] is explained only by the dependence of the current through the barrier on this value; therefore, by comparing the expressions for the volt-ampere characteristic of non-linear elements (15, 16, 26), we will substitute the product $\eta \cdot T$ for $q V_s / K$. As a result, it proves possible to calculate the noise characteristics of a cooled frequency converter, since the multiplier $\eta$, which is indeterminate with a change in temperature, disappears and the parameter $V_\theta$ is determined according to formula (26). We will also generalize the calculation expressions for the case of a harmonic frequency converter.

After the appropriate transformations of the expressions $V_1^2, V_2^2, V_3^2$, we obtain the following for the noise temperature at the output of the frequency converter:

$$T_{out} = \frac{1}{R_{\eta\cdot out}} \left[ \frac{qV_o}{K} \left\{ \frac{g_o}{2} \left| Z_{\phi d} \right|^2 + \frac{R_{p\phi}}{2} + g_{2k} \frac{R_{p\phi}}{R_{p\phi}} \frac{Re \left( Z_{\phi d} \right)}{\left| Z_{\phi d} \right|^2} \right\} \right] + T \cdot R_s \left\{ \frac{1}{2} \left| Z_{\phi d} \left( \frac{1}{Y_H} \right) \right|^2 \right\} + \frac{R_{p\phi}^2}{\left( R_s + R_{\eta\cdot out} \right)^2} + T \cdot Re \left[ \frac{1}{Y_H} \right] \left| Z_{\phi d} \left( \frac{1}{Y_H} \right) \right|^2 \right\}, \quad (33)$$

where

$$Z_{\phi d} = -\frac{g_k}{D} \left( g_o - j\omega C_o - g_{2k} + j\omega C_{2k} + Y_H', * \right),$$

$$D = \left( g_o + \frac{1}{R_s + R_{\eta\cdot out}} \right) \left( \left| g_o + j\omega C_o + Y_H' \right|^2 - \left| g_o + j\omega C_{2k} \right|^2 \right).$$
the index $k$ indicates the harmonic number at which conversion takes place, and $T_\phi$ is the temperature of the signal generator resistor.

The results of the calculations of the output noise temperature of the frequency converter at the 1st and 2nd harmonics are given in figure 19. For all cases, except the case of a cooled frequency converter, $T'$ and $T_\phi$ are taken as equal to 290 K. In general, it is interesting to find the characteristics of a frequency converter, optimized according to the minimum of the noise coefficient of a heterodyne system. Such a problem was solved in study [11] for a centimeter wave frequency converter (where the pronounced effect of the variable capacitance was observed), and the noise coefficient of the intermediate frequency amplifier was 1.5 dB. As it turned out, the difference in noise coefficients of the system, for cases of constant and variable capacitances, is insignificant; therefore, below, the impedance of the signal generator is taken as equal to the optimal impedance from the point of view of the transmission coefficient. The constants $V_o$ and $J_s$ have the values 0.026 V and $3 \cdot 10^{-12}$ A in those cases where they are not specified.

As should also have been expected, $T_{out}$ changes little in a broad range of rectified currents in an ideal diode. However, the presence of the parasitic parameters of the diode leads to
considerable changes in the noise temperature, especially with conversion at the first harmonic and a \( \omega/\omega_c \) on the order of several hundredths (fig. 20). The broken line corresponds to the case of constant barrier capacitance.

The effect of variable barrier capacitance disappears in proportion to the increase in the ratio \( \omega/\omega_c \) (0.1, 0.2), and for \( \omega/\omega_c > 0.2 \), it is practically absent (figs. 21, 22). With conversion at the 2nd harmonic, the noise temperature at the output increases appreciably only with \( \omega/\omega_c = 0.036 \) and currents less than 0.5 mA (fig. 23). Also represented in figure 24 are the dependences of the intrinsic single band noise temperature of the frequency converter \( T_{\text{intr}} = T_{\text{out}} - 2T_\rho \) with conversion at the basic heterodyne frequency. It is evident that the intrinsic noise temperature increases greatly with small \( \omega/\omega_c \) and rectified currents <2 mA, and proves roughly identical if \( \omega/\omega_c > 0.1 \).

The dependences of the output noise temperature for a frequency converter at the 1st and 2nd harmonics of a heterodyne, with input of the direct bias voltage to the diode, are given in figures 25 and 26. Because of the decrease in amplitude of the heterodyne voltage, the phenomena associated with modulation of the capacitance disappear, and with \( U_s = 0.7 \) V, the peak of the noise temperature in the area of relatively small currents is smoothed out (see fig. 25).

The results of the calculations for a cooled frequency converter on a diode with a Schottky barrier are given in figure 27. Utilized in the calculations are the constants \( \omega/\omega_c = 0.15 \), \( V_0 = 0.012 \) V and 0.017 V, \( J_s = 10^{-25} \) A, and \( T \rho = 0 \) K. For comparison, the graphs which correspond to constant capacitance of the diode with a Schottky barrier are given (broken lines). By comparing figures 22 and 27, we see that the differences in output noise...
temperatures for a converter with constant and variable capacitances of the non-linear element barrier prove roughly equal, both for cooled and uncooled cases. However, in an uncooled frequency converter, the relative increase in output noise is $\sim 20\%$ with $I_0 = 2$ mA and $\omega/\omega_c = 0.15$, while in a cooled frequency converter, it is $\sim 50\%$. With a decrease in $\omega/\omega_c$, one should expect an even greater difference in the output noise temperature. Nevertheless, if one finds the intrinsic noise temperature with cooling, then the difference in intrinsic noise temperatures with single-band receiving proves appreciably less (fig. 28), and is $\sim 15-20\%$ with currents rectified by the diode to several milliamperes.

Thus, variable capacitance of the barrier in a diode with a Schottky barrier has a substantial effect on the output noise temperature of the frequency converter, especially with $\omega/\omega_c$ on the order of several hundredths in a converter at the 1st harmonic of the heterodyne.

The effect of variable capacitance on the noise characteristics of harmonic frequency converters is insignificant in a wide-band mode for an image frequency and a short circuit of the remaining combination frequencies.

The intrinsic noise temperature of a frequency converter for the two cases of capacitance proves roughly equal with $\omega/\omega_c > 0.1$. With optimization for the transmission coefficient, an appreciable decline of $T_{I\omega r\omega}$ is observed with low $\omega/\omega_c$ and currents rectified by the diode $< 2$ mA. Input of the direct bias voltage leads to the disappearance of the effect of variability of the barrier capacitance of the diode with a Schottky barrier.

With cooling of the frequency converter, twofold reduction of the intrinsic noise temperature is possible.
3. Matters of Matching with Transmission Lines

During construction of frequency converters, calculated data on the transmission and impedance characteristics prove insufficient, and it is necessary to take other elements of the design into account. With the inclusion of non-linear elements into the signal circuit, the inductivity of the contact needle and the parasitic capacitance of the needle, which shunts the diode (\( C_p \)), are added together. The values of the magnitudes of the parasitic elements in the millimeter wave range are such that their strong influence is observed on the characteristics of the frequency converter. An equivalent schematic of the frequency converter is given in figure 29, where \( R_{in} \) is the input resistor, \( Y_{in} \) is the parallel-connected reactive portion of the input resistor, and \( Z_0 \) is the impedance of the wave guide.

In order to match the frequency converter, the height of the wave guide at the input is usually selected so that the entire power radiated by the signal generator is absorbed in \( R_{in} \). The determination of the maximum wave guide height at which matching is possible is given, for example, in study [3] for a frequency converter of the 80-120 gigahertz range. For a diode structure with a ratio \( (1/\omega C_p)/Z_0 \sim 0.5-1 \), matching is possible with \( \omega L/Z_0 \sim 0.9-1.2 \), which corresponds to a wave guide height of about 0.25 of the standard height. The calculations are carried out for the case when the crystal is inserted slightly into the wave guide.

The analysis of matters of matching carried out in studies [3,15] does not take all of the points into account. It is considered that \( |Y_{in}| \) is determined as \( 1/\omega C_p \), and the capacitance \( C_p \) is not examined. However, this is wrong. The measurements we carried out show that \( C_p \sim 0.005-0.01 \) picofarads, and proves comparable with the capacitance of the barrier of a diode
with a Schottky barrier. In addition, \(|\chi_{in}|\) differs substantially from \(1/\omega C_p\). Given in figure 30 are the results of calculation of \(\chi_{in}\) for a frequency converter with \(\omega/\omega_c = 0.15\) with a direct bias voltage of 0 V and 0.6 V. It is evident that an increase in \(|\chi_{in}|\) occurs in proportion to the increase in rectified current and also the direct bias voltage, whereas \(1/\omega C_p\) diminishes with a bias input because of the increase in capacitance. As a result, it can turn out that \(|\chi_{in}|\) becomes greater than \(1/\omega C_p\), and it is necessary to take the effect of the capacitance \(C_p\) into account during the solution of matching problems.

In order to find the conditions of matching, we will represent the equivalent schematic of the frequency converter (see fig. 29) in the following form (fig. 31a). After transformations, we obtain

\[
x_1 = \frac{x_{in} R_{in}^2 (1 - \omega C_p x_{in})}{R_{in}^2 (1 - \omega C_p x_{in})^2 + x_{in}^2}, \quad (34)
\]

\[
R_1 = \frac{x_{in}^2 R_{in}}{R_{in}^2 (1 - \omega C_p x_{in})^2 + x_{in}^2}. \quad (35)
\]

Then, we switch to an equivalent schematic (fig. 31b) and find the values of \(R_2\) and \(x_2\).

\[
R_2 = R_1 + \left(\frac{x_1 + \omega L}{R_1}\right)^2, \quad (36)
\]

\[
x_2 = \frac{R_1^2 + (x_1 + \omega L)^2}{x_1 + \omega L}. \quad (37)
\]

In order for matching to take place, it is necessary to
fulfill the following equalities

\[ R_1 + \frac{R_1}{R_1^2 + (x_1 + \omega L)^2} = Z_0, \quad (38) \]

\[ \frac{R_1^2 + (x_1 + \omega L)^2}{x_1 + \omega L} = Z_0 \cdot \tan \frac{2\pi \ell'}{\lambda_w}, \quad (39) \]

where \( \ell' \) is the distance from the plane of inclusion of the non-linear elements to the short-circuiting device (see fig. 29).

In (38), \( L \) and \( Z_0 \) are functions of the wave guide height; therefore, one can find those values of the wave guide height \( (h) \) at which equality (38) occurs.

The impedance of the wave guide is

\[ Z_0 = 2^{40} \pi \frac{k}{a} \frac{1}{\sqrt{1-(\lambda/2\alpha)^2}} = P \cdot h. \quad (40) \]

The value of the inductivity of the contact needle, located at the center of a wave guide of reduced cross-section, is [15]

\[ L = 2 \cdot 10^{-7} \cdot h \cdot \ln \left( \frac{2a}{\pi \alpha} \right). \]

We will also represent the value of \( \omega L \):

\[ \omega L = Q \cdot h, \]

then condition (38) has the form

\[ R_1 + \frac{(x_1 + Q \cdot h)^2}{R_1} = P \cdot h. \quad (41) \]

After simple transformations
The solution of this equation is
\[ h = \frac{PR_{r}-2x_{r}Q}{2Q^{2}} + \sqrt{(\frac{PR_{r}-2x_{r}Q}{2Q^{2}})^{2} - \frac{R_{i}^{2}+x_{i}^{2}}{Q^{2}}}. \] (42)\]

The solution exists if the radicand is greater than or equal to zero
\[ \frac{1}{Q^{2}} \left( \frac{PR_{r}-2x_{r}Q}{2Q} \right)^{2} - \frac{R_{i}^{2}+x_{i}^{2}}{Q^{2}} \geq 0. \] (44)\]

By transforming this expression, we obtain the inequality
\[ R_{i} \left[ \left( \frac{P}{2Q} \right)^{2} - 1 \right] \geq \frac{PX_{i}}{Q}, \] (45)\]
from which one can find the permissible changes if \( R_{i} \):

a) if \( 2Q>P \), then
\[ R_{i} \leq \frac{PX_{i}}{(\frac{P}{2Q})^{2} - 1}; \] (46)\]
in this case, a solution is possible if \( X_{1}<0 \);

b) \( 2Q<P \),
\[ R_{i} \geq \frac{PX_{i}}{(P/2Q)^{2} - 1}; \] (47)\]
and \( X_{1} \) can be both a negative and a positive value.

For the majority of cases, \( 2Q>P \) (see above), that is, condition (46) is fulfilled.

We will first find the height of the wave guide for the
boundary value $R_{1,\text{N}}$ (which corresponds to the sign of the equality in condition (46)), at which matching is possible, and the maximum case, for which $\chi_{1,\text{N}} = \infty$. Then, $\chi_1$ and $R_1$ are expressed by the following formulas

$$\chi_1^2 = -\frac{R_{1,\text{N}}^2 (\omega C_0)}{1 + R_{1,\text{N}}^2 (\omega C_0)^2}, \quad (48)$$

$$R_1 = \frac{R_{1,\text{N}}}{1 + R_{1,\text{N}}^2 (\omega C_0)^2} \quad (49)$$

We will determine the maximum value of $R_1^{\text{max}}$. For this purpose, we will express $\chi_1$ in (46) by $R_1$. Comparing (48) and (49), we obtain

$$\chi_1 = -R_1 R_{1,\text{N}} \omega C_0. \quad (50)$$

From (49), we find $R_{1,\text{N}}$ and substitute it into (50).

$$R_{1,\text{N}}^2 - \frac{R_{1,\text{N}}}{R_1 (\omega C_0)^2} + \frac{1}{(\omega C_0)^2} = 0. \quad (51)$$

Solving this equation, we obtain

$$R_{1,\text{N}} = \frac{1}{2 R_1 (\omega C_0)^2} + \frac{1}{\omega C_0} \sqrt{\frac{1}{4 R_1^2 (\omega C_0)^2} - 1}. \quad (52)$$

For $\chi_1$:

$$\chi_1 = -R_1 \omega C_0 \left[ \frac{1}{2 R_1 (\omega C_0)^2} + \frac{1}{\omega C_0} \sqrt{\frac{1}{4 R_1^2 (\omega C_0)^2} - 1} \right]. \quad (53)$$

Substituting (53) into (46), we find the $R_1^{\text{max}}$ at which matching is possible

$$R_{1,\text{max}}^{\text{max}} = -\frac{\psi}{\omega C_0 (\psi^2 + 1)}, \quad (54)$$

where

$$\psi = \frac{Q}{P} \left[ \left( \frac{P}{2Q} \right)^2 - 1 \right].$$
In order to give numerical evaluations, we will find the constants $Q$ and $P$, which were introduced in (40) and (41). For a wave guide of the 3 mm range with a 1.2 mm x 2.4 mm cross-section and a signal frequency of 100 gigahertz, $P=4 \cdot 10^5$ ohms/m. The value of $Q$ for 100$\mu$, 50$\mu$, and 25$\mu$ contact needle wire diameters is equal to $4.3 \cdot 10^5$ ohms/m, $5.2 \cdot 10^5$ ohms/m, and $6.05 \cdot 10^5$ ohms/m, respectively.

It is easy to generalize the obtained expressions to the case when $X_{in}$ has a finite value. If $X_{in}$ has a capacitance nature ($-\frac{1}{\omega C_{in}}$) (where $C_{in}$ is the equivalent capacitor parallel-connected to $R_{in}$), which evidently takes place in the majority of cases, then the capacitance should be substituted for the sum $(C_p + C_{in})$ in the given expressions. Then, we will conduct the analysis, bearing in mind the impedance of the equivalent total capacitance, but we will limit the interval of values of the capacitance to the magnitudes of $C_{eff} \gg C_p$, since the minimum capacitance is equal to the parasitic capacitance of the contact needle.

Given in figure 32 are the calculated dependences of the wave guide height at which matching takes place for the minimum possible $R_{in}$, as a function of the impedance of the parallel-connected capacitor for contact needle wire diameters of 25, 50, and 100 $\mu$. Given in this very same figure are the magnitudes of the resistance of the input of the frequency converter (broken line) which correspond to the given wave guide height. The wave guide height is standardized to a standard height $h_5 = 0.5a \left( \xi = \lambda / h_5 \right)$. As has already been noted, $2Q > P$ for the examined cases, and inequality (46) is correct, and if $R_{in, min} > 1/\omega C_{eff}$, then matching is also possible with $R_{in} >$ greater than those given in figure 32.

The calculation, given in § 1, of the impedances of the frequency converter shows that the approximated equality $R_{in} \approx$
\( R_{\text{out}}/2 \) take place, and \( R_{\text{in}} \) should be 25-50 ohms for matching with an input line of \( \sim 50-100 \) ohms. Therefore, it is of interest to find the dependences of the wave guide height, necessary for matching, on the input resistance with a fixed reactive component of the frequency converter impedance. Given in figures 33 and 34 are the graphs for two parameters of the contact needle wire and the values of the impedances \(-j100 \) ohms and \(-j300 \) ohms (solid lines). For contact needle diameters of 50 \( \mu \) and 100 \( \mu \), as well as for \( P=4 \cdot 10^5 \) ohms/m, the ratio \( Q/P=1.3 \) and 1.07 (curves 1 and 2, respectively). The dependences of the wave guide height, necessary for matching, on the input resistance, with an impedance of \(-j300 \) ohms or \(-j100 \) ohms, for a boundary value with a change in the ratio \( Q/P \), are given by the broken line.

The ratio \( Q/P \) is proportional to the inductivity of the needle per unit of length. With a decrease in this ratio, matching proves possible for lesser \( R_{\text{in}} \), with the wave guide height proving maximum for boundary values of \( R_{\text{in}} \) on the order of \( 1/2\omega C_{\text{eff}} \). A decrease in the value of \( Q/P \) makes it possible to accomplish matching with two wave guide heights (if \( R_{\text{in}} > R_{\text{in}} \text{ boundary} \) for the given constants). It turns out that one of the possible values of the wave guide height, with a decrease in \( Q/P \), is greater than the value of the height, with a greater \( Q/P \), which is necessary for matching with the same input resistance.

For matching with a wave guide impedance of 50-100 ohms and values of \( Q/P>1 \), it is desirable that \( 1/\omega C_{\text{eff}} \sim R_{\text{in}} \). With \( Q/P<0.8 \), however, it is desirable that \( 1/\omega C_{\text{eff}} \) have a greater value, which makes it possible to increase \( \hat{\lambda} \) slightly.

Thus, the wave guide height, necessary for matching with the input resistance of the frequency converter, is determined by \( R_{\text{in}} \), the magnitude of the inductivity of the contact needle,
and the reactive component of the input resistor. For practical purposes, it is convenient to select $R_{in} \approx 25-100$ ohms, which makes it possible to simplify the problem of wide-band matching of the output of the frequency converter with the normal input resistance of the intermediate-frequency amplifier. Matching at the input would present no difficulties if there were no difficulty in making up millimeter wave guides of reduced cross-section. For example, it is evident from figure 34 that the height of the wave guide is $\sim 1/7$ of standard for matching with $R_{in}=100$ ohms ($j\chi_{in} = -j100$ ohms) and a contact needle wire diameter of 50-100 μ. One can avoid a considerable reduction in the wave guide height by decreasing the inductivity of the contact needle. For example, with $Q/P=0.6$, the standardized wave guide height for that same case is $\sim 0.4$, which proves acceptable up to frequencies of $\sim 300$ gigahertz. The parallel-connected reactive component of the input resistance also has an appreciable effect, which can differ in each case. With $Q/P=1.3$ and $j\chi_{in} = j300$ ohms, matching for $R_{in}=100$ ohms is impossible (standing wave ratio $\sim 3$), while with $j\chi_{in} = -j100$ ohms, matching takes place. If $Q/P$ is reduced to 0.6, then, with $j\chi_{in} = -j300$ ohms, matching is possible with a wave guide height 1.5-2 times greater than that for $j\chi_{in} = -j100$ ohms. As was noted above, $\chi_{in}$ is determined by the operating conditions of the frequency converter (heterodyne voltage, bias voltage, barrier capacitance), as well as by the parasitic capacitance of the contact needle, and can be changed within broad limits. In each concrete case, in order to solve the problem, the values of the parasitic parameters of the diode with a Schottky barrier, the heterodyne voltage, and the bias voltage are selected in accordance with what was set forth in § 1-2.

Conclusion

The conducted study of the transmission, impedance, and noise characteristics of millimeter wave frequency converters
diodes with a Schottky barrier in the range $\omega/\omega_c = 0.04 - 0.2$ revealed the appreciable effect of the barrier's variable capacitance on the characteristics of the converters.

With low values of the ratio of the working frequency to the maximum frequency of the non-linear elements (on the order of several hundredths), the conversion losses of the frequency converter at the 1st harmonic, for an image frequency in the wide band mode, can become less than 3 dB, which leads, however, to an increase in the output noise temperature, and, as a result, the intrinsic noise temperature of the frequency converter does not change substantially (it increases considerably only with small rectified currents). In addition, an increase in the optimal impedance of the signal generator, which is difficult to bring about in practice, occurs in modes with small conversion losses. With $\omega/\omega_c > 0.1$, the phenomena associated with the variable capacitance of the barrier layer become insignificant. As a result, the variable capacitance of the barrier layer can not lead to substantial improvement in the characteristics of the frequency converter at the 1st harmonic, although in this case, the intrinsic noise temperature has values which are smaller by 10-15%.

With conversion at the harmonics of the heterodyne, worsening of the conversion losses is $\sim 1$ dB with an increase in the harmonic number by one. A considerable increase in the conversion losses of a harmonic frequency converter is observed with an increase in the diode's saturation current. The variable capacitance of the barrier practically does not affect the characteristics of the harmonic frequency converter if $\omega/\omega_c > 0.1$.

The input of direct bias voltage, for the purpose of decreasing the power input of the heterodyne, leads to a reduction
in the optimal resistance of the signal generator and the output resistor, which vary within broad limits. In this case, an increase takes place in the conversion losses, which is especially noticeable in harmonic frequency converters (with the input of a direct bias of 0.5 V for a gallium arsenide diode and $\omega/\omega_c = 0.15$, the increase in conversion losses is $\sim 3$ dB in a frequency converter at the 2nd harmonic, whereas the increase is $\sim 1$ dB for a frequency converter at the 1st harmonic).

By cooling millimeter wave frequency converters for the short wave section, it is possible to bring about roughly a twofold reduction in the intrinsic noise temperature of the converter at the 1st harmonic. The intrinsic noise temperature of the frequency converter is equal to 140 K with single band receiving and $\omega/\omega_c = 0.15$.

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REFERENCES

Fig. 4

Fig. 5

Fig. 6

\( R_{out}, \text{Ohms} \)

\( \omega/\omega_c = 0.15 \)

\( \omega/\omega_c = 0.15 \)

\( L_{min}, \text{dB} \)

\( \omega/\omega_c = 0.15 \)

\( \omega/\omega_c = 0.15 \)

\( \omega/\omega_c = 0.15 \)
Fig. 25

1 HARM.
\[ V = 0.3V \]
\[ R_3 = 10 \text{ ohms} \]

Fig. 26

2 HARM.
\[ V = 0.6V \]
\[ R_3 = 10 \text{ ohms} \]

Fig. 27

1 HARM.
\[ V = 0.017V \]
\[ V_0 = 2.012V \]
\[ T_0 = T' = 9 \text{ K} \]
\[ C = \text{const.} \]
\[ C = \frac{1}{T} \]
\[ R_3 = 10 \text{ ohms} \]
\[ U_B = 0V \]
Fig. 28

Fig. 29

Fig. 30
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