A Design Handbook for
Phase Change Thermal Control
and Energy Storage Devices

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A DESIGN HANDBOOK FOR PHASE CHANGE THERMAL CONTROL AND ENERGY STORAGE DEVICES

I. INTRODUCTION

A. Definition

Phase change thermal control devices have been discussed extensively in the literature. These articles often refer to a device of this type by different names such as thermal capacitor, thermal flywheel, heat of fusion device, latent heat device, and fusible mass device. However, all these terms refer to a component which is used to either thermally control a medium or store energy by utilizing a material which undergoes a change of phase.

There are a number of phase change transformation classes given by Lorsch [1] as

- Solid-liquid transformations (melting/freezing) (e.g., melting or freezing of ice or water)
- Liquid-gas transformations (vaporization) (e.g., boiling of water)
- Solid-gas transformations (sublimation) (e.g., sublimation of solid carbon dioxide at atmospheric pressure, “dry ice”)
- Solid-solid transformations (e.g., transition of the rhombic form of sulfur to the monoclinic form)
- Liquid-liquid transformations (e.g., the transformation occurring when two liquids are immisible such as water and phenol).

Very little energy is released or absorbed by liquid-liquid transformations, and it is questionable whether this is a true class of phase change. The liquid-liquid transformation is neglected in the remainder of this work. Liquid-gas and solid-gas transformation classes have large volume changes associated with them, a feature which requires heavy and complicated pressure vessels or special design features, such as internal bellows. For this reason, they also are not considered herein. Solid-liquid transformations are of great importance because most classes of materials undergo this type of transformation without exhibiting large volume changes while releasing or absorbing relatively large quantities of energy. The fourth transformation is also of interest because the energy exchange can be significant and a number of materials display this phenomenon in a temperature range near their melting point. Consequently, for the purpose of this document, phase change processes are limited to solid-liquid and solid-solid transformations.
B. Applications

There are a number of uses to which phase change devices have been applied. Some of these are:

- Thermal damping of oscillatory outputs (e.g., Skylab space radiator fluid outlet)
- Inhibition of thermal excursions (e.g., Lunar Roving Vehicle drive control electronics)
- Maintaining constant temperature (e.g., Pegasus III coating experiment-thermocouple reference)
- Energy storage (e.g., solar heating and cooling energy storage such as illustrated by the MIT house application)
- Control mechanisms (e.g., Skylab 40°F vernatherm valve)
- Temperature indication (e.g., mushroom thaw warning indicator).

Brief descriptions of most of these applications through 1973 are given by Humphries [2]. Figure 1 depicts thermal responses for the first three applications.

Typically, a phase change device is passive with no moving parts, consisting of an external housing and the phase change material (PCM), with or without interspersed filler material (Fig. 2). Usually this device is in intimate thermal contact with the medium which it is thermally controlling. Although shown in a simplified form, this device can be applied in an almost infinite number of geometrical configurations with a myriad of different filler configurations.

C. Phase Change Material

Studies have shown that to qualify as a good PCM the material should possess the following characteristics:

- High heat of fusion per unit mass and volume
- Proper melting point (or range).

Other property requisites, but not requirements, for a good PCM are:

- High thermal diffusivity
- High coefficient of thermal conductivity
Figure 1. Responses of a phase change device.
Figure 2. A phase change device.

- Noncorrosive
- Low coefficient of expansion
- High flash point
- Good wetting characteristics
- Minimum of void spaces (in solid-solid state)
- Stable
- Congruent phase change
- Little or no supercooling
- Relatively pure
- Nontoxic.

Economic factors of importance are:

- Low cost
- Present and future availability.
There are a number of classes of materials which have been investigated for use in phase change devices. Some of the more important are:

- Inorganic salt hydrates (e.g., glauber's salt \( \text{Na}_2\text{SO}_4 \cdot 10\text{H}_2\text{O} \))
- Organic compounds (e.g., paraffins \( \text{C}_n\text{H}_{2n+2} \))
- Eutectics of the above (e.g., 88 mole percent acetic acid + 12 mole percent benzoic acid)
- Natural elements (e.g., sulphur, \( \text{S} \))
- Water.

Of these materials, paraffins have been the most widely used, primarily in the space program.

D. Scope of Study

This section presents an overview of phase change devices; that is, it gives definitions, typical applications, and general limitations of this study. Section II presents PCM property data. Since most of the past devices have used paraffin as the PCM, Section II is primarily devoted to this class of materials. However, a large volume of nonparaffin data excerpted from other studies is given in the appendices. These data should allow the designer to use this document without resorting to time consuming literature searches for property data. Section III details the derivation and use of a nondimensional computer program. Section IV presents results of parametric studies performed using this nondimensional model. Using typical paraffin property data and thermal boundary conditions commensurate with earlier applications, optimum thermal design of the containment device is given parametrically. Examples for typical designs are given to illustrate use of these data. The intent of this study is to present sufficient data to allow the thermal designer to “short cut” time consuming analyses by either using the given numerical model and altering it to suit his application or, where possible, by using the parametric data directly. At a minimum, these data should allow the designer to get starting point estimates for his design. Section V outlines related topics. These topics include data which compare analytical and experimental results, discussing some of the discrepancies between the two. Section V also includes schemes for estimating convective effects. Other containment cases not covered by the examples used in Section IV are also given in Section V.

The reader should keep in mind that the treatment of phase change devices given in this document is limited to thermal aspects. As such, other topics of importance such as structural and material designs and selection are not addressed. Also, specialized subject matter such as the kinetics of the phase change process and chemical
phenomenon (e.g., supercooling, incongruent melting, and dendrite formation) are not discussed. Although convection analysis is discussed, details of convection stimuli such as buoyancy and surface tension forces are ignored. The reader is referred to survey reports by Grodzka [3,4] and a text by Chalmers [5] for information in these areas. Due to the large number of possible design configurations, this report addresses only typical and simplified schemes.

II. PROPERTY DATA

In several space-related applications of phase change devices, a paraffin has served as the PCM. Major emphasis in this section is given to those properties which are typically needed in a thermal design of selected paraffins. These include the melting temperature, transition temperature, heat of fusion, heat of transition, specific heat, density, thermal conductivity, viscosity, and coefficient of volume expansion.

Since hydrocarbon properties are dependent on purity, differences in reported property values may be directly related to differences in purity, a quantity often not specified.

Although this section is devoted to properties of paraffins, some of the properties of certain nonparaffins are presented in Appendix A. These have been extracted from several different references, and, in some cases, some paraffins and paraffin mixtures are included there also.

It is understood that the term paraffin generally denotes any of the saturated aliphatic hydrocarbons of the methane series C\textsubscript{n}H\textsubscript{2n+2}. Property data herein are principally limited, however, to those members of the family lying between n-Undecane (C\textsubscript{11}H\textsubscript{24}) and n-Triacontane (C\textsubscript{30}H\textsubscript{62}) because the corresponding range on melting temperature has included that of previously considered applications.

A. Sources, Availability, and Estimated Cost of Selected Paraffins

Table 1, which is based on a reported survey [6], shows a limited selection of companies and laboratories capable of supplying several selected paraffins in quantities adequate to be used in phase change devices. Also, the availability of Sunoco P-116 paraffin wax from the Sun Oil Co., C\textsubscript{14}-C\textsubscript{16} paraffin from Conoco, and a C\textsubscript{15}-C\textsubscript{16} paraffin mixture from Enjay is noted in Reference 1.

Information relative to type, grade, and physical property data availability from the suppliers listed in Table 1 is outlined in Table 2 based on a review [6] of their products.
The cost of paraffins is influenced significantly by purity, and it also varies considerably with source. The ultrahigh purity standard reference materials available from the American Petroleum Institute (API) cost $95 for a 5 ml unit [6]. Some price estimates for two grades of several paraffins are given in Table 3. All but the last three entries in Table 3 correspond to prices for relatively small quantities of reasonably high purity paraffins, probably made available for laboratory rather than large-scale commercial purposes. The last three entries denote some paraffin mixtures which can be obtained in large quantities.

### TABLE 1. POTENTIAL SUPPLIERS OF SELECTED PARAFFINS

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<td>American Petroleum Institute</td>
<td>Carnegie-Mellon University Schenley Park</td>
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<tr>
<td>Standard Reference Materials</td>
<td>Pittsburgh, Pennsylvania 15213</td>
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<td></td>
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<tr>
<td>Aldrich Chemical Co., Inc.</td>
<td>940 West Saint Paul Avenue</td>
</tr>
<tr>
<td></td>
<td>Milwaukee, Wisconsin 53233</td>
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<tr>
<td></td>
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<tr>
<td>Analabs Inc.</td>
<td>80 Republic Drive</td>
</tr>
<tr>
<td></td>
<td>North Haven, Connecticut 06473</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Chemical Sample Co.</td>
<td>4692 Kenny Road</td>
</tr>
<tr>
<td></td>
<td>Columbus, Ohio 43221</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Eastman (Kodak) Organic Chemicals</td>
<td>2400 Mt. Read Boulevard</td>
</tr>
<tr>
<td></td>
<td>Rochester, New York 14650</td>
</tr>
<tr>
<td></td>
<td></td>
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<tr>
<td>Gallard-Schlesinger Chemical Manufacturing Corp.</td>
<td>584 Mineola Avenue</td>
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<tr>
<td></td>
<td>Carle Place, New York 11514</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>The Humphrey Chemical Co.</td>
<td>Devine Street</td>
</tr>
<tr>
<td></td>
<td>North Haven, Connecticut 06473</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Lachat Chemicals Inc.</td>
<td>20200 Ashland Avenue</td>
</tr>
<tr>
<td></td>
<td>Chicago Heights, Illinois 60411</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Phillips Petroleum Co.</td>
<td>Bartlesville, Oklahoma 74004</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Polysciences Inc.</td>
<td>Paul Valley Industrial Park</td>
</tr>
<tr>
<td></td>
<td>Warrington, Pennsylvania 18976</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a. From Table 9 of Reference 6.
<table>
<thead>
<tr>
<th>Company</th>
<th>Ability to Supply Paraffin Hydrocarbons</th>
<th>Grades of Paraffin Hydrocarbon Available</th>
</tr>
</thead>
<tbody>
<tr>
<td>American Petroleum Institute</td>
<td>Can be supplied</td>
<td>Ultrahigh purity</td>
</tr>
<tr>
<td>Aldrich Chemical Co., Inc.</td>
<td>Can be supplied</td>
<td>Only one grade available, unspecified purity</td>
</tr>
<tr>
<td>Analabs Inc.</td>
<td>Can be supplied</td>
<td>Only one grade available, purity specified greater than 99 percent</td>
</tr>
<tr>
<td>Chemical Sample Co.</td>
<td>Can be supplied</td>
<td>Several grades available</td>
</tr>
<tr>
<td>Eastman Organic Chemicals</td>
<td>Can be supplied</td>
<td>Only one grade available, unspecified purity</td>
</tr>
<tr>
<td>Gallard-Schlenzinger Chemical Manufacturing</td>
<td>Does not manufacture paraffin hydrocarbons but can obtain them on request from other source</td>
<td>Only one grade available, unspecified purity</td>
</tr>
<tr>
<td>The Humphrey Chemical Co.</td>
<td>Can be supplied</td>
<td>Only one grade available, specified purity 99 percent</td>
</tr>
<tr>
<td>Lachat Chemical Inc.</td>
<td>Can be supplied</td>
<td>Only one grade available, specified purity 99 percent</td>
</tr>
<tr>
<td>Phillips Petroleum Co.</td>
<td>Can be supplied</td>
<td>Several grades available</td>
</tr>
<tr>
<td>Polysciences Inc.</td>
<td>Can be supplied as specialized items</td>
<td>Can provide materials at any purity level</td>
</tr>
</tbody>
</table>

a. From Table 10 of Reference 6.
<table>
<thead>
<tr>
<th>Material</th>
<th>Formula</th>
<th>Purity (percent)</th>
<th>Approximate Unit Cost $/kg ($/lb)</th>
<th>Quantity kg (lb)</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>n-Nonane</td>
<td>C₉H₂₀</td>
<td>99</td>
<td>26.46 (12.00)</td>
<td>2.72 (6)</td>
<td>6</td>
</tr>
<tr>
<td>n-Nonane</td>
<td>C₉H₂₀</td>
<td>95</td>
<td>14.29 (6.48)</td>
<td>2.72 (6)</td>
<td>6</td>
</tr>
<tr>
<td>n-Decane</td>
<td>C₁₀H₂₂</td>
<td>99</td>
<td>16.26 (7.38)</td>
<td>2.77 (6.1)</td>
<td>6</td>
</tr>
<tr>
<td>n-Decane</td>
<td>C₁₀H₂₂</td>
<td>95</td>
<td>12.87 (5.84)</td>
<td>2.77 (6.1)</td>
<td>6</td>
</tr>
<tr>
<td>n-Undecane</td>
<td>C₁₁H₂₄</td>
<td>99</td>
<td>23.47 (10.65)</td>
<td>2.81 (6.2)</td>
<td>6</td>
</tr>
<tr>
<td>n-Undecane</td>
<td>C₁₁H₂₄</td>
<td>95</td>
<td>14.21 (6.44)</td>
<td>2.81 (6.2)</td>
<td>6</td>
</tr>
<tr>
<td>n-Dodecane</td>
<td>C₁₂H₂₆</td>
<td>99</td>
<td>16.45 (7.46)</td>
<td>2.86 (6.3)</td>
<td>6</td>
</tr>
<tr>
<td>n-Dodecane</td>
<td>C₁₂H₂₆</td>
<td>95</td>
<td>12.65 (5.74)</td>
<td>2.86 (6.3)</td>
<td>6</td>
</tr>
<tr>
<td>n-Tridecane</td>
<td>C₁₃H₂₈</td>
<td>99</td>
<td>34.99 (15.87)</td>
<td>2.86 (6.3)</td>
<td>6</td>
</tr>
<tr>
<td>n-Tridecane</td>
<td>C₁₃H₂₈</td>
<td>95</td>
<td>22.29 (10.11)</td>
<td>2.86 (6.3)</td>
<td>6</td>
</tr>
<tr>
<td>n-Tetradecane</td>
<td>C₁₄H₃₀</td>
<td>99</td>
<td>18.95 (8.59)</td>
<td>2.90 (6.4)</td>
<td>6</td>
</tr>
<tr>
<td>n-Tetradecane</td>
<td>C₁₄H₃₀</td>
<td>95</td>
<td>15.62 (7.09)</td>
<td>2.90 (6.4)</td>
<td>6</td>
</tr>
<tr>
<td>n-Pentadecane</td>
<td>C₁₅H₃₂</td>
<td>99</td>
<td>41.34 (18.75)</td>
<td>2.90 (6.4)</td>
<td>6</td>
</tr>
<tr>
<td>n-Pentadecane</td>
<td>C₁₅H₃₂</td>
<td>95</td>
<td>30.13 (13.67)</td>
<td>0.68 (1.5)</td>
<td>6</td>
</tr>
<tr>
<td>n-Hexadecane</td>
<td>C₁₆H₃₄</td>
<td>99</td>
<td>35.27 (16.00)</td>
<td>0.68 (1.5)</td>
<td>6</td>
</tr>
<tr>
<td>n-Hexadecane</td>
<td>C₁₆H₃₄</td>
<td>95</td>
<td>20.08 (9.11)</td>
<td>2.90 (6.4)</td>
<td>6</td>
</tr>
<tr>
<td>n-Heptadecane</td>
<td>C₁₇H₃₆</td>
<td>99</td>
<td>37.89 (17.19)</td>
<td>2.90 (6.4)</td>
<td>6</td>
</tr>
<tr>
<td>n-Heptadecane</td>
<td>C₁₇H₃₆</td>
<td>95</td>
<td>30.13 (13.67)</td>
<td>0.68 (1.5)</td>
<td>6</td>
</tr>
<tr>
<td>n-Octadecane</td>
<td>C₁₈H₃₈</td>
<td>99</td>
<td>18.65 (8.46)</td>
<td>2.95 (6.5)</td>
<td>6</td>
</tr>
<tr>
<td>n-Octadecane</td>
<td>C₁₈H₃₈</td>
<td>95</td>
<td>30.13 (13.67)</td>
<td>0.68 (1.5)</td>
<td>6</td>
</tr>
<tr>
<td>n-Nonodecane</td>
<td>C₁₉H₄₀</td>
<td>99</td>
<td>40.70 (18.46)</td>
<td>2.95 (6.5)</td>
<td>6</td>
</tr>
<tr>
<td>n-Nonodecane</td>
<td>C₁₉H₄₀</td>
<td>95</td>
<td>27.93 (12.67)</td>
<td>0.68 (1.5)</td>
<td>6</td>
</tr>
<tr>
<td>n-Eicosane</td>
<td>C₂₀H₄₂</td>
<td>99</td>
<td>26.05 (11.82)</td>
<td>2.99 (6.6)</td>
<td>6</td>
</tr>
<tr>
<td>n-Eicosane</td>
<td>C₂₀H₄₂</td>
<td>95</td>
<td>41.15 (18.67)</td>
<td>0.68 (1.5)</td>
<td>6</td>
</tr>
<tr>
<td>P-116 Paraffin</td>
<td></td>
<td></td>
<td>0.14 (0.065)</td>
<td>Not specified</td>
<td>1</td>
</tr>
<tr>
<td>C₁₅⁻C₁₆ Paraffin mixture</td>
<td></td>
<td></td>
<td>0.11 (0.05)</td>
<td>4536 (10,000)</td>
<td>1</td>
</tr>
<tr>
<td>C₁₄⁻C₁₆ Paraffin</td>
<td></td>
<td></td>
<td>0.11 (0.05)</td>
<td>Not specified</td>
<td>1</td>
</tr>
</tbody>
</table>

a. Prices shown are approximate based on 1974 publications. All but the last three entries shown in the table correspond to prices quoted for the small quantities indicated which, more than likely, are for laboratory applications and are not totally indicative for large commercial orders. Also, it should be noted that there is also variation in prices with supplier.
B. Melting and Transition Temperatures

Table 4 contains published [7,8] values of the melting and transition temperatures for the listed paraffins.

For commercial grades of paraffins, some variations in the phase change temperatures are to be expected. Impurities even tend to cause some difference in the melting and freezing temperatures. Some reported [6] experimental results which show the effect of purity on the fusion temperature for several grades of certain paraffins supplied by two manufacturers are given in the lower two segments of Table 5 together with the corresponding fusion temperature for the high purity paraffins available from API. Similar results and API data for solid-solid phase transition temperatures are given in Table 6.

For the seven paraffins listed in Tables 5 and 6, correlations of phase change temperatures corresponding to API data with the number of carbon atoms in the chain are shown in Figure 3.

C. Heat of Fusion and Heat of Transition

The energy values associated with solid-liquid and solid-solid transitions as published by API [7] or either by Timmermans [8] are also listed in Table 4. The effects of purity on these energy values based on comparisons between measurements [6] on two commercial grades (lower two segments of Tables 7 and 8) and values reported by the API for high purity paraffins are listed in Tables 7 and 8, respectively, for several paraffins.

For the seven paraffins listed in Tables 7 and 8, correlations of the heat of fusion and the heat of transition with the number of carbon atoms in the chain are shown in Figures 4 and 5, respectively, for the data available from API.

It is interesting to note also the effect of the mixing of two different paraffins on the heats of fusion and transition. As an example, the results reported [6] for a hexadecane-octadecane binary system are shown in Figure 6.

D. Specific Heat

The specific heat is temperature dependent in each phase with an increase in specific heat occurring with an increase in temperature. There is a change in specific heat between two phases. Data for all the paraffins listed in Table 4 have not been obtained. For some of the listed paraffins, specific heat data over a range of temperature are given in Reference 8. These have been plotted in Figures 7, 8, and 9. There are obvious gaps in the data in the vicinity of phase transitions. Data given in Reference 9 for \( \text{C}_{14}\text{H}_{30}, \text{C}_{16}\text{H}_{34}, \text{C}_{18}\text{H}_{38}, \text{and C}_{20}\text{H}_{42} \) are shown in Figures 10 through 13, respectively.
<table>
<thead>
<tr>
<th>Name</th>
<th>Chemical Formula</th>
<th>Molecular Weight</th>
<th>Melting Temperature °C (°F)</th>
<th>Transition Temperature °C (°F)</th>
<th>Heat of Fusion kJ/kg (Btu/lb)</th>
<th>Heat of Transition kJ/kg (Btu/lb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>n-Undecane</td>
<td>C_{11}H_{22}</td>
<td>156.314</td>
<td>-25.6 (-14.1)</td>
<td>-36.6 (-33.8)</td>
<td>141.3 (60.8)</td>
<td>43.9 (18.9)</td>
</tr>
<tr>
<td>n-Dodecane</td>
<td>C_{12}H_{26}</td>
<td>170.341</td>
<td>-9.6 (14.7)</td>
<td>None</td>
<td>210.5 (90.6)</td>
<td>None</td>
</tr>
<tr>
<td>n-Tridecane</td>
<td>C_{13}H_{28}</td>
<td>184.368</td>
<td>-5.4 (22.3)</td>
<td>-18.2 (-0.7)</td>
<td>154.8 (66.6)</td>
<td>41.6 (17.9)</td>
</tr>
<tr>
<td>n-Tetradecane</td>
<td>C_{14}H_{30}</td>
<td>198.395</td>
<td>5.9 (42.5)</td>
<td>None</td>
<td>229.9 (98.9)</td>
<td>None</td>
</tr>
<tr>
<td>n-Pentadecane</td>
<td>C_{15}H_{32}</td>
<td>212.422</td>
<td>10.0 (49.8)</td>
<td>-2.3 (27.9)</td>
<td>163.7 (70.4)</td>
<td>43.2 (18.6)</td>
</tr>
<tr>
<td>n-Hexadecane</td>
<td>C_{16}H_{34}</td>
<td>226.449</td>
<td>18.2 (64.7)</td>
<td>None</td>
<td>228.9 (98.5)</td>
<td>None</td>
</tr>
<tr>
<td>n-Heptadecane</td>
<td>C_{17}H_{36}</td>
<td>240.476</td>
<td>22.0 (71.5)</td>
<td>11.1 (51.9)</td>
<td>168.3 (72.4)</td>
<td>45.5 (19.6)</td>
</tr>
<tr>
<td>n-Octadecane</td>
<td>C_{18}H_{38}</td>
<td>254.504</td>
<td>28.2 (82.7)</td>
<td>None</td>
<td>243.5 (104.7)</td>
<td>None</td>
</tr>
<tr>
<td>n-Nonadecane</td>
<td>C_{19}H_{40}</td>
<td>268.531</td>
<td>31.9 (89.4)</td>
<td>22.8 (72.9)</td>
<td>170.6 (73.4)</td>
<td>51.4 (22.1)</td>
</tr>
<tr>
<td>n-Eicosane</td>
<td>C_{20}H_{42}</td>
<td>282.558</td>
<td>36.4 (97.6)</td>
<td>36.2 (97.2)</td>
<td>247.3 (106.4)</td>
<td>None</td>
</tr>
<tr>
<td>n-Henicosane</td>
<td>C_{21}H_{44}</td>
<td>296.585</td>
<td>40.2 (104.3)</td>
<td>32.5 (90.5)</td>
<td>160.8 (69.2)</td>
<td>52.2 (22.5)</td>
</tr>
<tr>
<td>n-Docosane</td>
<td>C_{22}H_{46}</td>
<td>310.612</td>
<td>44.0 (111.2)</td>
<td>43.0 (109.4)</td>
<td>157.6 (67.8)</td>
<td>90.8 (39.1)</td>
</tr>
<tr>
<td>n-Tricosane</td>
<td>C_{23}H_{48}</td>
<td>324.639</td>
<td>47.5 (117.5)</td>
<td>40.5 (104.9)</td>
<td>166.3 (71.5)</td>
<td>67.0 (28.8)</td>
</tr>
<tr>
<td>n-Tetracosane</td>
<td>C_{24}H_{50}</td>
<td>338.666</td>
<td>50.6 (123.1)</td>
<td>48.1 (118.6)</td>
<td>162.1 (69.7)</td>
<td>92.4 (39.8)</td>
</tr>
<tr>
<td>n-Pentacosane</td>
<td>C_{25}H_{52}</td>
<td>352.693</td>
<td>53.5 (128.3)</td>
<td>47.0 (116.6)</td>
<td>178.4 (76.7)</td>
<td>73.9 (31.8)</td>
</tr>
<tr>
<td>n-Hexacosane</td>
<td>C_{26}H_{54}</td>
<td>366.720</td>
<td>56.3 (133.3)</td>
<td>53.3 (127.9)</td>
<td>162.2 (69.8)</td>
<td>87.9 (37.8)</td>
</tr>
<tr>
<td>n-Heptacosane</td>
<td>C_{27}H_{56}</td>
<td>380.747</td>
<td>58.8 (137.8)</td>
<td>53.0 (127.4)</td>
<td>158.7 (68.3)</td>
<td>76.0 (32.7)</td>
</tr>
<tr>
<td>n-Octacosane</td>
<td>C_{28}H_{58}</td>
<td>394.774</td>
<td>61.4 (142.5)</td>
<td>58.0 (136.4)</td>
<td>163.7 (70.4)</td>
<td>89.8 (38.6)</td>
</tr>
<tr>
<td>n-Nonacosane</td>
<td>C_{29}H_{60}</td>
<td>408.802</td>
<td>63.4 (146.1)</td>
<td>58.2 (136.7)</td>
<td>161.7 (69.6)</td>
<td>72.7 (31.3)</td>
</tr>
<tr>
<td>n-Triacontane</td>
<td>C_{30}H_{62}</td>
<td>422.829</td>
<td>65.4 (149.7)</td>
<td>62.0 (143.6)</td>
<td>251.1 (108.0)</td>
<td>None</td>
</tr>
</tbody>
</table>
### TABLE 5. FUSION TEMPERATURE FOR SELECTED PARAFFINS

<table>
<thead>
<tr>
<th>Material</th>
<th>Formula</th>
<th>Heating (Solid → Liquid) °C (°F)</th>
<th>Cooling (Liquid → Solid) °C (°F)</th>
<th>Purity (Mole Percent) or Grade</th>
</tr>
</thead>
<tbody>
<tr>
<td>n-Undecane</td>
<td>C_{11}H_{24}</td>
<td>-25.74 (-14.05)</td>
<td>-25.74 (-14.05)</td>
<td>99.96 ± 0.03</td>
</tr>
<tr>
<td>n-Dodecane</td>
<td>C_{12}H_{26}</td>
<td>-9.74 (14.74)</td>
<td>-9.74 (14.74)</td>
<td>99.969 ± 0.025</td>
</tr>
<tr>
<td>n-Tridecane</td>
<td>C_{13}H_{28}</td>
<td>-5.54 (22.30)</td>
<td>-5.54 (22.30)</td>
<td>99.91 ± 0.06</td>
</tr>
<tr>
<td>n-Hexadecane</td>
<td>C_{16}H_{34}</td>
<td>18.02 (64.70)</td>
<td>18.02 (64.70)</td>
<td>99.90 ± 0.06</td>
</tr>
<tr>
<td>n-Nonadecane</td>
<td>C_{19}H_{40}</td>
<td>31.95 (89.80)</td>
<td>21.95 (89.80)</td>
<td>99.90 ± 0.08</td>
</tr>
<tr>
<td>n-Eicosane</td>
<td>C_{20}H_{42}</td>
<td>36.65 (98.20)</td>
<td>36.65 (98.20)</td>
<td>99.90 ± 0.08</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Material</th>
<th>Formula</th>
<th>Heating (Solid → Liquid) °C (°F)</th>
<th>Cooling (Liquid → Solid) °C (°F)</th>
<th>Purity (Mole Percent) or Grade</th>
</tr>
</thead>
<tbody>
<tr>
<td>n-Undecane</td>
<td>C_{11}H_{24}</td>
<td>-26.7 (-15.7)</td>
<td>-26.2 (-14.8)</td>
<td>99</td>
</tr>
<tr>
<td>n-Dodecane</td>
<td>C_{12}H_{26}</td>
<td>-11.7 (11.3)</td>
<td>-10.7 (13.1)</td>
<td>99</td>
</tr>
<tr>
<td>n-Tridecane</td>
<td>C_{13}H_{28}</td>
<td>-6.2 (21.2)</td>
<td>-6.0 (21.6)</td>
<td>99</td>
</tr>
<tr>
<td>n-Hexadecane</td>
<td>C_{16}H_{34}</td>
<td>18.9 (66.2)</td>
<td>18.4 (65.3)</td>
<td>99</td>
</tr>
<tr>
<td>n-Nonadecane</td>
<td>C_{19}H_{40}</td>
<td>27.0 (80.8)</td>
<td>27.1 (81.0)</td>
<td>99</td>
</tr>
<tr>
<td>n-Eicosane</td>
<td>C_{20}H_{42}</td>
<td>29.4 (85.1)</td>
<td>29.4 (85.2)</td>
<td>99</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Material</th>
<th>Formula</th>
<th>Heating (Solid → Liquid) °C (°F)</th>
<th>Cooling (Liquid → Solid) °C (°F)</th>
<th>Purity (Mole Percent) or Grade</th>
</tr>
</thead>
<tbody>
<tr>
<td>n-Undecane</td>
<td>C_{11}H_{24}</td>
<td>-28.55 (-19.2)</td>
<td>-26.2 (-14.8)</td>
<td>99.75</td>
</tr>
<tr>
<td>n-Dodecane</td>
<td>C_{12}H_{26}</td>
<td>-11.35 (11.8)</td>
<td>-11.2 (12.2)</td>
<td>99</td>
</tr>
<tr>
<td>n-Tridecane</td>
<td>C_{13}H_{28}</td>
<td>-7.35 (19.0)</td>
<td>-7.2 (19.4)</td>
<td>99</td>
</tr>
<tr>
<td>n-Hexadecane</td>
<td>C_{16}H_{34}</td>
<td>18.9 (66.2)</td>
<td>18.6 (65.7)</td>
<td>99</td>
</tr>
<tr>
<td>n-Nonadecane</td>
<td>C_{19}H_{40}</td>
<td>31.4 (88.7)</td>
<td>32.4 (90.5)</td>
<td>95</td>
</tr>
<tr>
<td>n-Eicosane</td>
<td>C_{20}H_{42}</td>
<td>34.9 (95.0)</td>
<td>34.9 (95.0)</td>
<td>95</td>
</tr>
</tbody>
</table>

---

a. Based on Tables 1 and 2 of Reference 6.

* Supplier A

** Supplier B

*** Supplier C
TABLE 6. TRANSITION TEMPERATURES FOR CERTAIN PARAFFINS\textsuperscript{a}

<table>
<thead>
<tr>
<th>Material</th>
<th>Heating (Solid → Solid) °C (°F)</th>
<th>Cooling (Solid → Solid) °C (°F)</th>
<th>Purity (Mole Percent) or Grade</th>
</tr>
</thead>
<tbody>
<tr>
<td>n-Undecane</td>
<td>-36.6 (-33.5)</td>
<td>-36.6 (-33.5)</td>
<td>99.96 ± 0.03 *</td>
</tr>
<tr>
<td>n-Tridecane</td>
<td>-18.2 (-0.4)</td>
<td>-18.2 (-0.4)</td>
<td>99.91 ± 0.06 *</td>
</tr>
<tr>
<td>n-Nonadecane</td>
<td>22.8 (73.2)</td>
<td>22.8 (73.2)</td>
<td>99.90 ± 0.08 *</td>
</tr>
<tr>
<td>n-Undecane</td>
<td>-36.2 (-32.8)</td>
<td>-36.6 (-33.5)</td>
<td>99 *</td>
</tr>
<tr>
<td>n-Tridecane</td>
<td>-19.1 (2.0)</td>
<td>-18.2 (-0.4)</td>
<td>99 **</td>
</tr>
<tr>
<td>n-Nonadecane</td>
<td>21.7 (71.2)</td>
<td>20.9 (69.8)</td>
<td>99</td>
</tr>
<tr>
<td>n-Undecane</td>
<td>-37.0 (-34.3)</td>
<td>-36.2 (-32.8)</td>
<td>99.75 ***</td>
</tr>
<tr>
<td>n-Tridecane</td>
<td>-21.1 (-5.65)</td>
<td>-19.2 (-2.2)</td>
<td>99</td>
</tr>
<tr>
<td>n-Nonadecane</td>
<td>20.5 (69.0)</td>
<td>20.9 (69.8)</td>
<td>95</td>
</tr>
</tbody>
</table>

\textsuperscript{a} Based on Tables 1 and 3 of Reference 6.

\* Supplier A

\*\* Supplier B

\*\*\* Supplier C
Figure 3. Correlation of phase change temperature with number of carbon atoms in the chain (from Fig. 1 of Reference 6).
<table>
<thead>
<tr>
<th>Material</th>
<th>Heating (Solid → Liquid) kJ/kg (Btu/lb)</th>
<th>Cooling (Liquid → Solid) kJ/kg (Btu/lb)</th>
<th>Purity (Mole %) or Grade</th>
</tr>
</thead>
<tbody>
<tr>
<td>n-Undecane</td>
<td>142 (61.2)</td>
<td>142 (61.2)</td>
<td>99.96 ± 0.03</td>
</tr>
<tr>
<td>n-Dodecane</td>
<td>216 (93.2)</td>
<td>216 (93.2)</td>
<td>99.969 ± 0.025</td>
</tr>
<tr>
<td>n-Tridecane</td>
<td>154 (66.5)</td>
<td>154 (66.5)</td>
<td>99.91 ± 0.06</td>
</tr>
<tr>
<td>n-Hexadecane</td>
<td>235 (101.5)</td>
<td>235 (101.5)</td>
<td>99.90 ± 0.06</td>
</tr>
<tr>
<td>n-Octadecane</td>
<td>244 (105.0)</td>
<td>244 (105.0)</td>
<td>99.90 ± 0.08</td>
</tr>
<tr>
<td>n-Nonadecane</td>
<td>187 (80.6)</td>
<td>187 (80.6)</td>
<td>99.90 ± 0.08</td>
</tr>
<tr>
<td>n-Eicosane</td>
<td>248 (107.0)</td>
<td>248 (107.0)</td>
<td>99.90 ± 0.08</td>
</tr>
<tr>
<td>n-Undecane</td>
<td>140 (60.4)</td>
<td>143 (61.7)</td>
<td>99</td>
</tr>
<tr>
<td>n-Dodecane</td>
<td>218 (94.0)</td>
<td>220 (94.9)</td>
<td>99</td>
</tr>
<tr>
<td>n-Tridecane</td>
<td>145 (62.5)</td>
<td>159 (68.5)</td>
<td>99</td>
</tr>
<tr>
<td>n-Hexadecane</td>
<td>237 (102.0)</td>
<td>235 (101.5)</td>
<td>99</td>
</tr>
<tr>
<td>n-Octadecane</td>
<td>248 (107.0)</td>
<td>245 (105.6)</td>
<td>99</td>
</tr>
<tr>
<td>n-Nonadecane</td>
<td>180 (77.5)</td>
<td>180 (77.5)</td>
<td>99</td>
</tr>
<tr>
<td>n-Eicosane</td>
<td>249 (107.5)</td>
<td>245 (105.5)</td>
<td>99</td>
</tr>
<tr>
<td>n-Undecane</td>
<td>142 (61.4)</td>
<td>141 (61.0)</td>
<td>99.75</td>
</tr>
<tr>
<td>n-Dodecane</td>
<td>218 (94.0)</td>
<td>212 (91.5)</td>
<td>99</td>
</tr>
<tr>
<td>n-Tridecane</td>
<td>154 (66.2)</td>
<td>150 (64.8)</td>
<td>99</td>
</tr>
<tr>
<td>n-Hexadecane</td>
<td>239 (103.0)</td>
<td>235 (101.5)</td>
<td>99</td>
</tr>
<tr>
<td>n-Hexadecane</td>
<td>244 (105.0)</td>
<td>237 (102.0)</td>
<td>95</td>
</tr>
<tr>
<td>n-Octadecane</td>
<td>243 (104.6)</td>
<td>242 (104.5)</td>
<td>95</td>
</tr>
<tr>
<td>n-Nonadecane</td>
<td>182 (78.3)</td>
<td>179 (77.2)</td>
<td>95</td>
</tr>
<tr>
<td>n-Eicosane</td>
<td>255 (110.0)</td>
<td>243 (104.8)</td>
<td>95</td>
</tr>
<tr>
<td>n-Eicosane</td>
<td>241 (104.0)</td>
<td>234 (101.0)</td>
<td>90</td>
</tr>
</tbody>
</table>

* Supplier A  
** Supplier B  
*** Supplier C
TABLE 8. COMPARISON OF HEAT OF TRANSITION FOR SEVERAL GRADES OF SELECTED PARAFFINS

<table>
<thead>
<tr>
<th>Material</th>
<th>Heating (Solid → Solid) kJ/kg (Btu/lb)</th>
<th>Cooling (Solid → Solid) kJ/kg (Btu/lb)</th>
<th>Purity (Mole %) or Grade</th>
</tr>
</thead>
<tbody>
<tr>
<td>n-Undecane</td>
<td>439 (18.9)</td>
<td>439 (18.9)</td>
<td>99.96 ± 0.03</td>
</tr>
<tr>
<td>n-Tridecane</td>
<td>416 (17.9)</td>
<td>416 (17.9)</td>
<td>99.91 ± 0.06</td>
</tr>
<tr>
<td>n-Nonadecane</td>
<td>514 (22.2)</td>
<td>514 (22.2)</td>
<td>99.90 ± 0.08</td>
</tr>
<tr>
<td>n-Undecane</td>
<td>409 (17.6)</td>
<td>405 (17.4)</td>
<td>99</td>
</tr>
<tr>
<td>n-Tridecane</td>
<td>433 (18.7)</td>
<td>470 (20.3)</td>
<td>99</td>
</tr>
<tr>
<td>n-Nonadecane</td>
<td>513 (22.1)</td>
<td>516 (22.3)</td>
<td>99</td>
</tr>
<tr>
<td>n-Undecane</td>
<td>403 (17.4)</td>
<td>422 (18.2)</td>
<td>99.75</td>
</tr>
<tr>
<td>n-Tridecane</td>
<td>403 (17.4)</td>
<td>406 (17.5)</td>
<td>99</td>
</tr>
<tr>
<td>n-Nonadecane</td>
<td>495 (21.4)</td>
<td>499 (21.5)</td>
<td>95</td>
</tr>
</tbody>
</table>

* Supplier A
** Supplier B
*** Supplier C

Figure 4. Correlation of heat of fusion with number of carbon atoms in the chain [from Fig. 5(a) of Reference 6].
Figure 5. Correlation of heat of transition with number of carbon atoms in the chain [from Fig. 5(b) of Reference 6].

Figure 6. Example illustrating effect of composition of heats of phase change for hexadecane-octadecane binary system (from Fig. 29 of Reference 6).
Figure 7. Specific heat versus temperature for $C_{11}H_{24}$, $C_{12}H_{26}$, and $C_{13}H_{28}$ (based on data given in Reference 8).
Figure 8. Specific heat versus temperature for C\textsubscript{14}H\textsubscript{30}, C\textsubscript{15}H\textsubscript{32}, and C\textsubscript{16}H\textsubscript{34} (based on data given in Reference 8).
Measurements of specific heat for several paraffins using samples from two manufacturers are reported by Reference 6. These are tabulated in Table 9 together with values for the same materials from API.

Comparative examination of data shown in Figures 7 through 13 and listed in Table 9 indicates that the specific heat of paraffins is sensitive to impurities.

E. Density

The density of the liquid phase decreases with temperature and data for most of the considered paraffins are shown in Figure 14. The values agree with those reported in Reference 6. For \( C_{14}H_{29}, \ C_{16}H_{34}, \ C_{18}H_{38}, \) and \( C_{20}H_{42}, \) density data are also plotted in Figures 15 through 18, respectively, which are from Reference 9 and which include some solid phase density measurements. For these paraffins, density changes across the solid-liquid phase transition ranged from 5 to 8 percent.

F. Thermal Conductivity

There is a limited amount of thermal conductivity data available for paraffins. Most reported values are only for the liquid phase. For selected paraffins Table 10 lists a limited amount of liquid thermal conductivity data, which were reported in Reference 6.
Figure 10. Specific heat of tetradecane \((C_{14}H_{30})\) (from Fig. 8 of Reference 9).
Figure 11. Specific heat of hexadecane (C_{16}H_{34}) (from Fig. 9 of Reference 9).
Figure 12. Specific heat of octadecane (C<sub>18</sub>H<sub>38</sub>) (from Fig. 10 of Reference 9).
Figure 13. Specific heat of eicosane (C₂₀H₄₂) (from Fig. 11 of Reference 9).
<table>
<thead>
<tr>
<th>Material</th>
<th>Formula</th>
<th>Purity (Mole %)</th>
<th>Liquid Specific Heat</th>
<th>Solid Specific Heat</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Units</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>kJ/kg-°C (Btu/lb-°F)</td>
<td>Temperature °C (°F)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Units</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>kJ/kg-°C (Btu/lb-°F)</td>
</tr>
<tr>
<td>n-Undecane</td>
<td>C_{11}H_{24}</td>
<td>99.96 ± 0.03</td>
<td>1.53 (0.366)</td>
<td>0 (32)</td>
</tr>
<tr>
<td>n-Dodecane</td>
<td>C_{12}H_{26}</td>
<td>99.969 ± 0.025</td>
<td>1.53 (0.366)</td>
<td>0 (32)</td>
</tr>
<tr>
<td>n-Tridecane</td>
<td>C_{13}H_{28}</td>
<td>99.91 ± 0.06</td>
<td>1.53 (0.366)</td>
<td>0 (32)</td>
</tr>
<tr>
<td>n-Hexadecane</td>
<td>C_{16}H_{34}</td>
<td>99.90 ± 0.06</td>
<td>1.70 (0.406)</td>
<td>38 (100)</td>
</tr>
<tr>
<td>n-Octadecane</td>
<td>C_{18}H_{38}</td>
<td>99.90 ± 0.08</td>
<td>1.70 (0.406)</td>
<td>38 (100)</td>
</tr>
<tr>
<td>n-Nonadecane</td>
<td>C_{19}H_{40}</td>
<td>99.90 ± 0.08</td>
<td>1.69 (0.405)</td>
<td>38 (100)</td>
</tr>
<tr>
<td>n-Eicosane</td>
<td>C_{20}H_{42}</td>
<td>99.90 ± 0.08</td>
<td>1.94 (0.464)</td>
<td>93 (200)</td>
</tr>
<tr>
<td>n-Undecane</td>
<td>C_{11}H_{24}</td>
<td>99</td>
<td>1.89 (0.451)</td>
<td>-3 (26.6)</td>
</tr>
<tr>
<td>n-Dodecane</td>
<td>C_{12}H_{26}</td>
<td>99</td>
<td>2.15 (0.513)</td>
<td>17 (62.6)</td>
</tr>
<tr>
<td>n-Tridecane</td>
<td>C_{13}H_{28}</td>
<td>99</td>
<td>2.16 (0.516)</td>
<td>27 (80.2)</td>
</tr>
<tr>
<td>n-Hexadecane</td>
<td>C_{16}H_{34}</td>
<td>99</td>
<td>2.31 (0.551)</td>
<td>57 (134.5)</td>
</tr>
<tr>
<td>n-Octadecane</td>
<td>C_{18}H_{38}</td>
<td>99</td>
<td>2.33 (0.556)</td>
<td>57 (134.5)</td>
</tr>
<tr>
<td>n-Nonadecane</td>
<td>C_{19}H_{40}</td>
<td>99</td>
<td>2.30 (0.549)</td>
<td>67 (153.5)</td>
</tr>
<tr>
<td>n-Eicosane</td>
<td>C_{20}H_{42}</td>
<td>99</td>
<td>2.46 (0.587)</td>
<td>77 (171.0)</td>
</tr>
<tr>
<td>n-Undecane</td>
<td>C_{11}H_{24}</td>
<td>99.75</td>
<td>2.13 (0.509)</td>
<td>-3 (26.6)</td>
</tr>
<tr>
<td>n-Dodecane</td>
<td>C_{12}H_{26}</td>
<td>99</td>
<td>2.26 (0.540)</td>
<td>17 (62.6)</td>
</tr>
<tr>
<td>n-Tridecane</td>
<td>C_{13}H_{28}</td>
<td>99</td>
<td>2.20 (0.526)</td>
<td>27 (80.6)</td>
</tr>
<tr>
<td>n-Hexadecane</td>
<td>C_{16}H_{34}</td>
<td>99</td>
<td>2.31 (0.552)</td>
<td>57 (134.5)</td>
</tr>
<tr>
<td>n-Octadecane</td>
<td>C_{18}H_{38}</td>
<td>95</td>
<td>2.42 (0.578)</td>
<td>57 (134.5)</td>
</tr>
<tr>
<td>n-Nonadecane</td>
<td>C_{19}H_{40}</td>
<td>95</td>
<td>2.24 (0.536)</td>
<td>67 (153.5)</td>
</tr>
<tr>
<td>n-Eicosane</td>
<td>C_{20}H_{42}</td>
<td>95</td>
<td>2.24 (0.536)</td>
<td>67 (153.5)</td>
</tr>
<tr>
<td>n-Eicosane</td>
<td>C_{20}H_{42}</td>
<td>90</td>
<td>2.34 (0.559)</td>
<td>67 (153.5)</td>
</tr>
</tbody>
</table>

a. Data based on Reference 6. The first segment corresponds to API. The last two segments correspond to samples acquired from two different companies.
Figure 14. Liquid phase density of several paraffins.
Figure 15. Density of tetradecane (C_{14}H_{30}) (from Fig. 4 of Reference 9).

Figure 16. Density of hexadecane (C_{16}H_{34}) (from Fig. 5 of Reference 9).
Figure 17. Density of octadecane (C\textsubscript{18}H\textsubscript{38}) (from Fig. 6 of Reference 9).

Figure 18. Density of eicosane (C\textsubscript{20}H\textsubscript{42}) (from Fig. 7 of Reference 9).
TABLE 10. THERMAL CONDUCTIVITY OF SELECTED PARAFFINS (EVALUATED AT PHASE CHANGE TEMPERATURE)\(^a\)

<table>
<thead>
<tr>
<th>Material</th>
<th>W/m·°C</th>
<th>Btu/h-ft·°F</th>
</tr>
</thead>
<tbody>
<tr>
<td>n-Undecane</td>
<td>0.1496</td>
<td>0.0865</td>
</tr>
<tr>
<td>n-Dodecane</td>
<td>0.1488</td>
<td>0.0860</td>
</tr>
<tr>
<td>n-Tridecane</td>
<td>0.1496</td>
<td>0.0865</td>
</tr>
<tr>
<td>n-Hexadecane</td>
<td>0.1505</td>
<td>0.0870</td>
</tr>
<tr>
<td>n-Octadecane</td>
<td>0.1505</td>
<td>0.0870</td>
</tr>
<tr>
<td>n-Nonadecane</td>
<td>0.1505</td>
<td>0.0870</td>
</tr>
<tr>
<td>n-Eicosane</td>
<td>0.1505</td>
<td>0.0870</td>
</tr>
</tbody>
</table>

\(^a\) From Table 8 of Reference 6.

as being available from the API. Thermal conductivity data which are presented in Reference 9 for the liquid phase of \(C_{14}H_{30}\), \(C_{16}H_{34}\), \(C_{18}H_{38}\), and \(C_{20}H_{42}\), are shown in Figures 19 through 23. Figure 23 shows a correlation which was extrapolated to obtain thermal conductivities for \(C_{20}H_{42}\).

The lack of thermal conductivity data for the solid phase poses some analytical uncertainties when attempting to analyze the phase change process. This concern and related considerations are treated in more detail in Section V.

G. Viscosity

Absolute viscosity data for liquid paraffins \(C_{11}H_{24}\) through \(C_{20}H_{42}\), which are given in Reference 7, are shown plotted as a function of temperature in Figure 24.

H. Surface Tension

Values of the surface tension for liquid paraffins \(C_{11}H_{24}\) through \(C_{20}H_{42}\), which are tabulated in Reference 7, are plotted versus temperature in Figure 25. The data varies linearly with temperature with an increase of temperature resulting in a decrease of surface tension.

I. Coefficient of Expansion

Single values of the coefficient of expansion for four of the considered paraffins are shown in Table 11 which were reported in Reference 6 as being from API.
Figure 19. Thermal conductivity of tetradecane (C_{14}H_{30})
(from Fig 12 of Reference 9).
Figure 20. Thermal conductivity of hexadecane ($C_{16}H_{34}$) (from Fig. 13 of Reference 9).
Figure 21. Thermal conductivity of octadecane (C_{18}H_{36})
(from Fig. 14 of Reference 9).
Figure 22. Thermal conductivity of eicosane (C_{20}H_{42}) extrapolated from data for lower carbon number paraffins (from Fig. 15 of Reference 9).
Figure 23. Correlation of thermal conductivity data for normal paraffins \( \text{C}_n\text{H}_{2n+2} \) (from Fig. 16 of Reference 9).
Figure 24. Absolute viscosity of paraffins $C_{11}H_{24}$ through $C_{20}H_{42}$ versus temperature (based on data given in Reference 7).
Figure 25. Surface tension for C_{11}H_{24} through C_{20}H_{42} (based on data given in Reference 7).
<table>
<thead>
<tr>
<th>Material</th>
<th>1/K</th>
<th>1/°F</th>
</tr>
</thead>
<tbody>
<tr>
<td>n-Undecane</td>
<td>10.1 × 10⁻⁴</td>
<td>5.6 × 10⁻⁴</td>
</tr>
<tr>
<td>n-Dodecane</td>
<td>9.9 × 10⁻⁴</td>
<td>5.5 × 10⁻⁴</td>
</tr>
<tr>
<td>n-Tridecane</td>
<td>9.4 × 10⁻⁴</td>
<td>5.2 × 10⁻⁴</td>
</tr>
<tr>
<td>n-Eicosane</td>
<td>8.5 × 10⁻⁴</td>
<td>4.7 × 10⁻⁴</td>
</tr>
</tbody>
</table>

a. From Table 18 of Reference 6.

III. TWO-DIMENSIONAL ANALYSIS OF A PHASE CHANGE DEVICE

A. Introduction

The analysis of two-dimensional conductive heat transfer within a phase change device is outlined in this section. Since convection may be present in certain applications, the design of a phase change device based solely on conduction will in most cases be conservative (i.e., heat transfer rate to and from material will be lower than in a case where convection is present). Additional attention is given in Section V to the effect of convection and how it can be included in the analysis.

The two-dimensional transient heat conduction equation for an isotropic, homogeneous medium in the absence of sources, sinks, or phase change is

\[ \alpha \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) = \frac{\partial T}{\partial t} \]  (1)

In general, solutions of equation (1) for typical applications can best be obtained via numerical techniques, other techniques were outlined by Muehlbauer and Sunderland [10]. For numerical solutions of equation (1), the partial derivatives are approximated by finite difference analogs. A forward difference analog for the time derivatives can be used. With regard to the latter, there are two possibilities for a transient problem [11]. If all temperatures appearing in the spatial analogs are evaluated at the old time level, the formulation is explicit, and the resulting equation for a node involves only one unknown at the new time level. The solution for a system of nodes by this method is fairly simple.
and straightforward. If all temperatures appearing in the spatial analogs are evaluated at the new time level, the formulation is implicit. The resulting equation for a node involves five unknown temperatures at the new time level. Solution for a system of nodes involves solving a set of simultaneous equations equal to the number of nodes.

Figure 26 represents a general node (I, J), its surrounding nodes, and the notational scheme for interconnecting thermal conductances.

![Figure 26. General designations for nodes and connecting conductances.](image)

Physical arguments can be used to formulate the difference analogs previously discussed. The summation of heat transfer to node (I, J) from all surrounding nodes should equal the product of the nodal capacitance and the difference in its old and new temperature. With reference to Figure 26, the explicit formulation based on this physical approach is

\[
GH(I,J)[T(I-1,J) - T(I,J)] + GH(I+1,J)[T(I+1,J) - T(I,J)] + GV(I,J)[T(I,J-1) - T(I,J)] + GV(I,J+1)[T(I,J+1) - T(I,J)] = \frac{C(I,J)[\tilde{T}(I,J) - T(I,J)]}{\Delta t}
\]  

(2)
The implicit formulation is

\[ \begin{align*}
\text{GH}(I,J) \left[ \tilde{T}(I-1,J) - \tilde{T}(I,J) \right] + \text{GH}(I+1,J) \left[ \tilde{T}(I+1,J) - \tilde{T}(I,J) \right] + \text{GV}(I,J) \left[ \tilde{T}(I,J-1) - \tilde{T}(I,J) \right]
- \tilde{T}(I,J) + \text{GV}(I,J+1) \left[ \tilde{T}(I,J+1) - \tilde{T}(I,J) \right] &= \frac{C(I,J) \left[ \tilde{T}(I,J) - T(I,J) \right]}{\Delta t} \\
\text{(3)}
\end{align*} \]

The tildes denote temperatures at the new time level.

Numerical solution of the explicit formulation given by equation (2) is subject to a restriction on the allowable time step given by

\[ \Delta t \leq \frac{C(I,J)}{\text{GH}(I,J) + \text{GH}(I+1,J) + \text{GV}(I,J) + \text{GV}(I,J+1)} \]

For a nodal arrangement, the node having the smallest time step given by equation (4) governs the entire solution.

The implicit formulation given by equation (3) is not restricted by a stability requirement, but the solution is not as simple as one using the explicit formula. In general, it requires a matrix inversion routine and may be impractical for a large number of nodes.

A technique for handling the phase change when using the explicit formulation involves keeping a record of the stored energy at each PCM node. The stored energy at any time is compared to that associated with the initiation and termination of the phase change. When the accumulated energy at a node lies between these two values, the temperature prediction is overridden and the temperature is forced to be equal to the phase change temperature.

The technique for handling the phase change when using the implicit formulation involves comparison of the predicted temperature with the phase change temperature. When the predicted temperature first exceeds the latter value, the predicted value is overridden and the nodal temperature is specified to be the phase change temperature. The energy associated with the excess of the predicted value over the phase change temperature is calculated and allowed to accumulate. Once this accumulated energy reaches a value sufficient to accomplish the phase change, the predicted temperature is allowed to rise in accordance with the governing conduction equations.
B. Physical Model

The physical model selected for study corresponds to a phase change device application which is depicted in Figure 27. Basically, the phase change device consists of a metallic housing in the shape of a parallelepiped which is filled with PCM and straight metallic fins which are included to improve the heat transfer mechanism between the housing and the PCM.

![Phase change device application selected for study.](image)

When the phase change device is operating as an energy sink (i.e., it is storing energy), either a hot fluid or a hot surface is considered to be in contact with one surface of the housing resulting in heat transfer into the unit. Hereafter, this surface which is in contact with the hot medium is referred to as the base. The PCM is used to absorb the energy primarily via the phase change process. The stored energy may be later rejected by means of exposing the base or the cover or both to a cooler medium, which in some applications may be the same medium from which the energy was absorbed earlier.

A typical single cell of the device is shown in Figure 28. It is assumed that planes parallel to the fins and passing through their centers represent planes of symmetry as do planes parallel to the fins and located midway between two adjacent fins. Consequently, the analysis is reduced to that of a half-cell represented by the schematic given in Figure 29.
The following thermal analysis pertains to the symmetrical half-cell shown in Figure 29 with the base exposed to a constant heat flux density. The resulting computer program is also modified for the case of a constant base temperature.

The limitations of the analysis are given by the following assumptions:

- **Conduction** is the only heat transfer mechanism within the two-dimensional half-cell; the overall phase change device is considered to be composed of many such cells. (The effects of convection are discussed in Section V).

- The problem is two-dimensional; the cell, therefore, is sufficiently long that end effects are small.

- The cover is perfectly insulated.
The physical properties, density, specific heat, and thermal conductivity of the metallic fin, cell base, and cell cover are considered to be identical and to have constant values. Likewise, the density, specific heat, and thermal conductivity of the solid and liquid phases of the PCM are considered to be identical and to have constant values. (The model may be easily modified to input temperature variant properties.)

To cast the problem in a nondimensional formulation, appropriate dimensionless parameters are sought. As a means of obtaining the pertinent dimensionless parameters, differential equations for the temperature in the PCM, the fin, the base, and the cover are presented and nondimensionalized. The differential analysis is followed by a numerical analysis which is used for the solutions, the results of which are presented in Section IV.
C. Differential Formulation

Since the principal objective of the differential formulation is to find the appropriate dimensionless parameters for the physical model previously described, the reader interested only in using the results can go directly to the next section on the numerical analysis.

The differential equations pertinent to each region of the model are presented below.

1. **PCM Region.** The energy differential equation is the usual two-dimensional conduction equation

\[
\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = \frac{1}{\alpha} \frac{\partial T}{\partial t}
\]  

(5)

and the two-dimensional interfacial boundary condition according to Rathjen and Jiji [12] is

\[
k \left[ \left( \frac{\partial T}{\partial y} \right)_{y=\delta} - \left( \frac{\partial T}{\partial y} \right) \right] \left[ 1 + \left( \frac{\partial \delta(x,t)}{\partial x} \right)^2 \right] = \rho L_f \frac{\partial \delta}{\partial t} \bigg|_{y=\delta}
\]

(6)

where \( \delta(x,t) \) is the interfacial location or fusion front. Choosing the width \( W \) of the PCM region shown in the symmetrical half-cell in Figure 29 as the characteristic x-dimension, the PCM height \( H \) as the characteristic y-dimension, the PCM fusion temperature \( T_f \) as the characteristic temperature, and denoting a characteristic time by \( t_c \) result in the following dimensionless quantities:

\[
x' \equiv x/W
\]

(7)

\[
y' \equiv y/H
\]

(8)

\[
\theta \equiv T/T_f
\]

(9)

\[
t' \equiv t/t_c
\]

(10)
In terms of these, equation (5) can be written as

\[
\left(\frac{H}{W}\right)^2 \frac{\partial^2 \theta}{\partial x'^2} + \frac{\partial^2 \theta}{\partial y'^2} = \frac{H^2}{\alpha t_c} \frac{\partial \theta}{\partial t'}
\]  

and equation (6) becomes

\[
\left[\left(\frac{\partial \theta}{\partial y'}\right)_s - \left(\frac{\partial \theta}{\partial y'}\right)_L\right]\left[1 + \left(\frac{H}{W}\right)^2 \left(\frac{\partial \delta'}{\partial x'}\right)^2\right] = \frac{h_f H^2}{\alpha c_p T_f t_c} \left(\frac{\partial \delta'}{\partial t'}\right)
\]

2. Fin. Treating the temperature distribution in the fin as one-dimensional, i.e., \(T_m = T_m(y,t)\), with conductive heat transfer from the fin to the PCM, which is analogous to the treatment of convective heat transfer in the conventional one-dimensional fin problem, results in

\[
\frac{\partial^2 T_m}{\partial y'^2} + \frac{2 k_m}{\rho_m c_p m} \left(\frac{1}{S_2}\right) \left(\frac{\partial T}{\partial x'}\right)_{0,y} = \frac{\rho_m c_p m}{k_m} \frac{\partial T_m}{\partial t}
\]

where the coordinate system is given in Figure 30. The second term on the left in equation (13) results from considering conduction in the PCM. The temperature in the fin at \(x = 0\), however, is the same as that in the PCM at \(x = 0\).

![Figure 30. Coordinate system and dimensions for fin.](image)
Nondimensionalization of equation (13) using equations (7) through (10) results in

\[ \frac{\partial^2 \theta_m}{\partial y^2} + 2 \frac{k_m}{k_m (S_2 W)} \left( \frac{\partial \theta}{\partial x} \right)_{0,y} = \left( \frac{H^2}{\alpha} \right) \left( \frac{\alpha}{\alpha_m} \right) \frac{\partial \theta_m}{\partial t} \]  

(14)

3. Cell Base. Treating the cell base as a lumped system, i.e., \( T_b = T_b(t) \), and assuming the base and fin properties to be the same result in the following energy equation:

\[ q'' \left( W + \frac{S_2}{2} \right) + k_m \left( \frac{\partial \theta_m}{\partial y} \right)_{0,0} + kW \left( \frac{\partial \theta}{\partial y} \right)_{x,0} = \rho_m c_p m S_1 \left( W + \frac{S_2}{2} \right) \frac{dT_b}{dt} \]

(15)

where it has been assumed that the base is exposed to a constant heat flux density, \( q'' \). The coordinate system and physical dimensions for the cell base are given in Figure 31. The subscripts \( x,0 \) on the third term on the left side of equation (15) denote evaluation at \( 0.5 \leq x \leq W \) and \( y = 0 \). In terms of the previously defined dimensionless variables, equation (15) becomes

\[ \frac{q''H}{k T_f} \left( 1 + \frac{S_2}{2W} \right) + \frac{k_m}{k} \frac{S_2}{2W} \left( \frac{\partial \theta_m}{\partial y} \right)_{0,0} + \frac{\partial \theta}{\partial y'}_{x',0} = \frac{S_1}{H} \left( 1 + \frac{S_2}{2W} \right) \left( \frac{H^2}{\alpha} \right) \left( \frac{k_m}{k} \right) \left( \frac{\alpha}{\alpha_m} \right) \frac{d\theta_b}{dt} \]

(16)

In equations (15) and (16) the overbar on the third term on the left side denotes a value spatially averaged over the \( x \)-length \( W \).

Figure 31. Coordinate system and dimensions for base.
4. **Cell Cover.** Since the treatment of the cell cover is analogous to that of the cell base except for the assumption that the cover is insulated, no new dimensionless parameters should appear; therefore, the appropriate equation is omitted for brevity.

To this point the dimensionless equations contain a undefined reference time, \( t_c \). By selecting this characteristic time to be given by

\[
t_c = \frac{H^2}{\alpha}
\]

the ratio \( \frac{H^2}{\alpha t_c} \) becomes 1.0 in equations (11), (12), (14), and (16). Examination of the reduced set of equations reveals seven salient dimensionless ratios for this class of problem. Summarizing these ratios by category, with pure constants omitted, yields

**Geometric:**

\[
R_1 = \frac{H}{W}
\]

\[
R_2 = \frac{H}{S_1}
\]

\[
R_3 = \frac{W}{S_2}
\]

**Material Properties:**

\[
R_4 = \frac{k_m}{k}
\]

\[
R_5 = \frac{\alpha_m}{\alpha}
\]

\[
R_6 = \frac{h_f}{c_p T_f}
\]

**Thermal Loading:**

\[
R_7 = \frac{\dot{q}'' H}{k T_f}
\]

These ratios developed with the differential equation formulation of the problem are also appropriate for the finite difference formulation. Hence, the finite difference equations are cast in dimensionless form with the seven aforementioned dimensionless parameters appearing explicitly and serving as focal points for a parametric numerical study.
D. Finite Difference Formulation

The finite difference formulation begins with a subdivision of the region of interest into a network of nodal points. For the two-dimensional, symmetrical half-cell shown in Figure 29, the nodal network and nodal point designation scheme are shown in Figure 32. The approach provides flexibility for increasing or decreasing the number of PCM nodes in accordance with the dictates of size and shape, there being $M$ by $N$ PCM nodes. Also note that all base nodes correspond to $J = 1$ and all fin nodes correspond to $I = 1$. The number of base or fin nodes is, of course, directly related to the PCM subdivision. Figure 26 depicts a general node $(I,J)$, its four surrounding nodes, and the notational scheme for interconnecting thermal conductances.

Figure 32. Nodal grid designation.

Physical arguments can be used to formulate the temperature finite difference equations. With reference to Figure 26 the net heat transfer to node $(I,J)$ from its surrounding nodes (during time $\Delta t$) should equal the product of the nodal thermal capacitance and the temperature difference occurring during the time increment. The explicit formulation based on this approach in nondimensional form is
\[
GH'(I,J) [\theta(I-1,J) - \theta(I,J)] + GH'(I+1,J) [\theta(I+1,J) - \theta(I,J)] + GV'(I,J) [\theta(I,J) - \theta(I,J-1)] + GV'(I,J+1) [\theta(I,J+1) - \theta(I,J)] = C'(I,J) [\theta(I,J) - \theta(I,J)]
\]

\[
\frac{C'(I,J) [\theta(I,J) - \theta(I,J)]}{R_1^2 M^2 \Delta t'}
\]  

(18)

The implicit formulation is

\[
GH'(I,J) [\tilde{\theta}(I-1,J) - \tilde{\theta}(I,J)] + GH'(I+1,J) [\tilde{\theta}(I+1,J) - \tilde{\theta}(I,J)] + GV'(I,J) [\tilde{\theta}(I,J) - \tilde{\theta}(I,J-1)] + GV'(I,J+1) [\tilde{\theta}(I,J+1) - \tilde{\theta}(I,J)] = C'(I,J) [\tilde{\theta}(I,J) - \tilde{\theta}(I,J)]
\]

\[
\frac{C'(I,J) [\tilde{\theta}(I,J) - \tilde{\theta}(I,J)]}{R_1^2 M^2 \Delta t'}
\]  

(19)

In the nondimensionalization of all thermal conductances, the nondimensional value is obtained by dividing the respective conductance by \(k_B\) which is the conductance between two general PCM nodes. Similarly, nondimensional thermal capacitance values are obtained by dividing the respective dimensional value by \(\rho S^2 B_c p\) which is the thermal capacitance of a general PCM node. It should be noted that the nondimensional conductances and capacitances that are entirely within the PCM have the numerical value of 1.0. All other dimensionless conductances and capacitances have values other than unity. As an example, the dimensionless horizontal conductance between the fin and the lower left hand corner PCM node is

\[
GH'(2,2) = \frac{2}{M} \frac{1}{1 + \frac{2}{R_3 R_4}}
\]  

(20)

A dimensionless base nodal capacitance, for example, is

\[
C'(2,1) = \frac{M R_1 R_4}{R_2 R_5}
\]  

(21)

1. Computer Programs. Numerical solutions of the problem described herein via computers can be affected by use of either the explicit formulation given by equation (18) or the implicit formulation given by equation (19). The two general approaches together with the inherent advantages and disadvantages related to each are discussed in
the general heat transfer literature; see, for example, Chapman [11] or Özişik [13]. It is well established that solution of the explicitly formulated set of equations is subject to a stability restriction which dictates the maximum allowable time increment. This restriction is given by equation (4). For a small grid size, this requirement leads to excessive computer time. While solutions of the implicitly formulated set of equations is not restricted by a stability requirement, the method involves the solution of a set of simultaneous equations equal in number to the number of nodes. For a small grid size, large computer storage is needed.

The problem under consideration involves thin metallic fins; most attention, consequently, has been given to the implicit approach. The stability restriction dictated by a node located in the fin resulted in extremely small time increments for the explicit approach.

A computer program based on the implicit approach, equation (19), for the problem described in this section is given in Table 12. A modification of the program to accommodate a sudden change in the base temperature rather than imposition of a constant heat flux density is given in Table 13. In both programs, a band algorithm technique given by Funderlic [14] is used to solve the equations. A dimensional computer program based on the explicit approach is presented and briefly discussed in Appendix D.

2. Instructions for Using Computer Programs. To use the programs, the following steps are suggested:

a. Establish the grid size (see Fig. 32) by specifying M and N. M represents the number of vertical columns and N represents the number of horizontal rows within the PCM. The ratio of N to M must equal \( R_1 \). The accuracy of the transient results may be affected by the choice of M and N, particularly in the early part of the transient. The user should conduct a sensitivity study by altering M and N along with DT (Step f) in a systematic manner until the results appear to be unaltered to within an acceptable level by further changes. The influence of the choice of M and N is discussed and illustrated in Section IV along with discussion of the parametric study.

b. Determine appropriate values for the dimensionless ratios \( R_1 \) through \( R_7 \) as defined in the analysis. Note that \( R_7 \) is denoted as \( R_8 \) in the program. Also, for the constant temperature case \( R_7 \) is not needed, but a temperature difference (DELT) is needed. This difference is defined by

\[
\text{DELT} = \frac{T_B}{T_F} - 1
\]

c. The dimension card must be set correctly. The dimensions depend on M and N as indicated below.

\[
\begin{align*}
A(LL,L1) \\
T1(M1,N2)
\end{align*}
\]
T2(LL)
GH(M2,N2)
GV(M1,N2)
C(M1,N2)
F(LL)
TEXC(LL)
D(LL)
MELT(LL)

where

\[ LL = (N2) \times (M1) \]
\[ L1 = (2) \times (M1 + 1) \]
\[ M1 = M + 1 \]
\[ M2 = M + 2 \]
\[ N2 = N + 2 \]

\[ d. \] Input required parameters in accordance with Formats 998 and 999. The required inputs are shown by the Read statements. NR is reserved for a reference run number.

\[ e. \] The program is written with the initial PCM temperature at \( T_f \). If it is desired to run for other values of initial temperature, set TIN by

\[ TIN = T_{\text{initial}} / T_f \]

\[ f. \] Select a nondimensional time set by specifying DT. It is suggested that the magnitude of this time increment be explored for its influence on the accuracy of the results. Note the discussion given in Step a also.

\[ g. \] Set ITT. This quantity controls the printout of results. The program is written to yield output at the end of the first time increment and then on intervals given by

\[ (ITT - 1) \times (DT) \]
The output includes

- **TIME** - Nondimensional time measured from start of melting
- **TRISE** - \((T_{\text{Base}} - T_f)/T_f\) (use absolute temperatures)
- **UCOEF** - Nondimensional overall heat transfer coefficient defined by
  \[
  UCOEF = \frac{R_7}{T_{\text{Base}} - T_f}
  \]
- **FMELT** - Fraction of PCM melted at given time (or solidified for freeze case)
- **SQF** - Energy stored in fin
- **SQB** - Energy stored in base
- **SQT** - Energy stored in top
- **SQS** - Energy stored in solid
- **SQL** - Energy stored in liquid
- **QLAT** - Energy used to change phase
- **QTOT** - Summation of SQF, SQB, SQT, SQS, SQL and QLAT
- **EIN** - Total energy transferred into base which is dependent on \(q''\) and time
- **RATIO** - \(QTOT/EIN\) - check on energy balance.

All the energy values which were listed are nondimensional and evaluated with respect to initial values of zero. The energy value used in the nondimensionalization is the latent energy for a single PCM node.

The programs given in Tables 12 and 13 can be altered slightly to accommodate freezing for the case of a constant heat flux density withdrawal at the base and a step reduction in the base temperature, respectively. The changes needed to accomplish this are:

a. In the parameters at the beginning, replace \(QLATM\) by \(-QLATM\).

b. In the third, second, and first lines above statement 150, change the statements to
TABLE 12. COMPUTER PROGRAM FOR BASE EXPOSED TO A CONSTANT HEAT FLUX DENSITY

C *** CONSTANT HEAT RATE PROGRAM
DIMENSION A(126,7), T1(3,42), T2(126), Q(4,2), G(3,42), F(126), TEXC(126), D(126), TELT(126)
IMPLICIT REAL*8(A-H,O-Z)
C *** R1 = H/W / R2 = 4/S1 / R3 = W/S2 / R4 = K/W / ***********
C *** R5 = (K/DEN*CP)/(K/DENW*CPW) / R6 = H*MLT/CPW*TMELT *
C *** R7 = H*MLT/HTR / R8 = Q*H/KW*TMELT ************************
C *****
READ(5,998) R1,R2,R3,R4,R5,R6,R8
READ(5,999) NR,M,N
C *** COMPUTATIONAL PARAMETERS ***
TIN=1.0
DT=0.0005
ITT=41
CW = 1.0
TMELT = 1.0
ITTT = 1
TPHASE = R6
AN = N
AM = M
QLATM = AM*AN
AM1 = AM + 1.0
M1 = M + 1
M2 = M1 + 1
N1 = N + 1
N2=N+2
LL=N2*M1
L=2*M1 + 1
L = M2
CD = (R1**2)*(AM**2)
C *** INITIAL VALUES
SQS = 0.0
SQR = 0.0
SQB = 0.0
SQT=0.0
SQL = 0.0
QLAT = 0.0
EBOT=0.0
TCB=0.0
MM = 0
TBOT = 0.0
TIME = 0.0
DO 10 I = 1,LL
F(I) = 1.0
MELT(I)=0
10 TEXC(I) = 0.0
DO 20 J=1,N2
DO 20 I = 1,M1
20 T1(I,J) = TIN

52
C **HORIZONTAL CONDUCTANCES, VERTICAL CONDUCTANCES**
C **CAPACITANCES**
C **** ARE CALCULATED ****

\[ GV(1,1) = 0.0 \]
\[ DO 30 J=2*N1 \]
\[ DO 30 I=2*M1 \]
\[ GH(I,J) = 1.0 \]
\[ C(I,J) = 1.0 \]

\[ 30 \]
\[ GV(I,J) = 1.0 \]
\[ DO 40 J = 2*N1 \]
\[ C(I,J) = (R4/R1+R4)/(2*R5*R3) \]
\[ GV(I,J) = (AM*R4)/(2*R5*R3) \]
\[ GH(I,J) = 2.0/(1.0 + AM/(2.0*R3*R4)) \]
\[ GH(M2,J) = 0.0 \]

\[ 40 \]
\[ GH(I,J) = 0.0 \]
\[ DO 50 I = 2*M1 \]
\[ C(I,J) = (AM*R1+R4)/(R2*R5) \]
\[ C(I,J) = C(I,J) \]
\[ GV(I,J) = (AM*R4)/(R2*R5) \]
\[ GV(I,J) = GV(I,J) \]
\[ GH(I,J) = 0.0 \]
\[ GH(I,J) = R4*R4*R1/R2 \]

\[ 50 \]
\[ GH(I,J) = GH(I,J) \]
\[ GH(I,J) = 0.0 \]
\[ GH(M2,J) = 0.0 \]
\[ GH(M2,J) = 0.0 \]
\[ GH(2,J) = 2.0*R4/(R2/R1)*(1.0/(2.0*R3) + 1.0/AM) \]
\[ GH(2,J) = GH(2,J) \]
\[ GV(I,J) = R4/(R3*(1.0/AM + R1/R2)) \]
\[ GV(I,J) = GV(I,J) \]
\[ C(I,J) = (R1*R4*(AM+2))/2*R2*R3*R4 \]
\[ C(I,J) = C(I,J) \]
\[ DO 55 I=2, M1 \]

\[ 55 \]
\[ TTCB = TTCB + C(I,J) \]

C **** TEMPERATURES AT EACH NODE CALCULATED ****
\[ WRITE(6,7500) R1, R2, R3, R4, R5, R6, R8, T, N, NR \]
\[ 60 \]
\[ TIME = TIME + DT \]
\[ DO 70 I = 1, LL \]
\[ DO 70 J = 1, L1 \]

\[ 70 \]
\[ A(I,J) = 0.0 \]
\[ DO 80 J = 2, N1 \]
\[ DO 80 I = 1, M1 \]
\[ K = (J=1)*M1 + I \]
\[ A(K,L+M1) = GV(I,J) \]
\[ A(K,L+M1) = A(K,L+M1) \]
\[ A(K,L) = GH(I+1,J) = GV(I,J) = GV(I,J) = (C(I,J)/DT \]
\[ 1*CD)) = GH(I,J) \]
\[ A(K,L+1) = GH(I,J) \]
\[ A(K,L+1) = GH(I,J) \]

\[ 80 \]
\[ U(K) = C(I,J)*T1(I,J)/(CD*DT) \]

53
TABLE 12. (Continued)

DO 90 I=1,M1
K=N1*M1+I
A(I,L) = -GH(I+1,1) = GH(I,1) = GV(I,2) - C(I,1)/(DT*CD)
A(I,L+1) = GH(I+1,1)
A(I,L-1) = GH(I,1)
A(I,L+M1) = GV(I,2)
A(K,L) = -GH(I,N2) = GH(I+1,N2) = GV(I,N2) = C(I,N2)/
1(DT*CD)
A(K,L+1) = GH(I,N2)
A(K,L-1) = GH(I+1,N2)
90 A(K,L=M1) = GV(I,N2)
DO 110 I=1,M1
K=N1*M1+I
D(I) = -C(I,1)*T(I,1)/(DT*CD) = R8/(R1*AM)
110 D(K) = -C(I,N2)*T(I,N2)/(DT*CD)
D(I) = -C(I,1)*T(I,1)/(CD*DT) = R8/(2*O*R3*R1)
CALL BANSOL(A,LL,L,JI,LL)
DO 120 I = 1,LL
120 T2(I) = DI1
C *** QUALITY CALCULATED ***
DO 150 J=2,N1
DO 150 I=2,M1
K=(J-1)*M1+I
IF(MELT(K)*EQ,1) 30 TO 150
IF(T2(K)+GT*MELT)TEXC(K) = TEC(K) + (T2(K) = MELT)
IF(T2(K)+GT*MELT)F(K) = 1.0 = TEC(K)/(R6*TAMEL) T2
1(K)=MELT
IF(TEXC(K)+GE,TPHASE)T2(K)=MELT+TEXC(K)+T3PHASE+F
1(K)=F.0+MELT(K)=1
150 CONTINUE
DO 160 J = 1,N2
DO 160 I = 1,M1
K=(J-1)*M1+I
160 T1(I,J) = T2(K)
QLAT = 0.0
DO 170 J = 2,N1
DO 170 I = 2,M1
K = (J-1)*M1+I
170 QLAT = QLAT + (1.0) = F(K)
IF(ABS(QLATM = QLAT)+LT*U+U001) GO TO 180
IF(TIME*EQ*DT)ITTT = ITTT + 1
IF(TIME*EQ*DT)GO TO 180
IF(ITTNE*ITTT)ITTT = ITTT + 1
IF(ITTNE*ITTT)GO TO 240
ITTT = 1
180 DO 175 I=1,M1
175 EBOT=EBOT+I(C(I,1)*T1(I,1)=TIN)
TBOT=TIN+(EBOT/TTC)
EBOT=0.0
TRISE = TBOT = T3MELT

54
TABLE 12. (Continued)

\[
UCOEF = \frac{R8}{(TBOT - TMELT)}
\]

WRITE(6$B000) TIME, TRISE, UCOEF

C *** ENERGY BALANCE ***

DO 200 J = 2, N1

200 SQF = SQF + (C1(J)*T1(J,J) - TIN)/R6

DO 210 I = 1, M1

210 SQT = SQT + (C1(N2)*T1(I,N2) - TIN)/R6

DO 220 J = 2, N1

DO 220 I = 2, M1

K = (J-1)*M1 + I

SQB = SQB + (C(I,N2)*T1(J,N2) - TIN)/R6

DO 220 J = 2, N1

DO 220 I = 2, M1

K = (J-1)*M1 + I

SQL = SQL + (C(I,J)*T1(J,I) - TIN)/R6

QTOT = SQF + SQB + SQL + QLAT + SQT

EIN = (AM*2*R1*R8/R6)*(1.0 + (1.0/(2.0*R3)))*TIME

RATIO = QTOT/EIN

FMELT = QLAT/QLAT

WRITE(6$1500) SQF, SQB, SQT, SQL, QLAT, QTOT, EIN, RATIO,

IF (ABS(QLATM - QLAT) * 1000) GO TO 60

999 FORMAT(6F10.5)

200 FORMAT(6F10.5)

300 FORMAT(6F10.5)

400 FORMAT(6F10.5)

500 FORMAT(6F10.5)

600 FORMAT(6F10.5)

700 FORMAT(6F10.5)

750 FORMAT(6F10.5)

800 FORMAT(4H, 'TIME = ', F7.5, 5X, 'TRISE = ', F7.5, 5X, 'UCOEF = ', F10.5)

CALL EXIT

END

SUBROUTINE BANSG(L(C, M, V, IO)

DIMENSION C(IO, M), V(IO)

IMPLICIT REAL(A-H, O-Z)

L = (M+1)/2

L1 = L + 1


```
DO 2 IR = 1, L1
LR = L - IR
DO 2 I = 1, LR
DO 1 J = 2, M
1 C(IR,J-1) = C(IR,J)
NP1 = N + 1 = IR
MP1 = M + 1 = I
C(NP1,MP1) = 0.0
DO 0 IR = MP1
2 C(IR,M) = C(NP1,MP1)
NI = N + 1
DO 9 I = 1, NI
IPIV = I
IRE = I + 1
DO 3 IR = IRE - L
IF(ABS(C(IR+1)) * E* ABS(C(IPIV+1))) GO TO 3
IPIV = IR
3 CONTINUE
IF(IPIV EQ I) GO TO 5
T = V(I)
V(I) = V(IPIV)
V(IPIV) = T
DO 4 J = 1, M
T = C(I,J)
C(I,J) = C(IPIV,J)
4 C(IPIV,J) = T
5 V(I) = V(I)/C(I1)
DO 6 J = 2, M
C(I,J) = C(I,J)/C(I,1)
DO 8 IR = IRE - L
T = C(IR1)
V(IR) = V(IR) - T*V(I)
DO 7 J = 2, M
7 C(IR,J-1) = C(IR,J) = T*C(I,J)
8 C(IR,M) = 0.0
IF(L EQ N) GO TO 9
L = L + 1
9 CONTINUE
V(N) = V(N)/C(N1)
JM = 2
DO 11 ICE = 1, V1
IR = N - ICE
DO 10 J = 2, JM
IRM1 = IR - 1 + J
10 V(IR) = V(IR) - C(IR,J)*V(IRM1)
IF(JM EQ M) GO TO 11
JM = JM + 1
11 CONTINUE
RETURN
END
```
TABLE 13. COMPUTER PROGRAM FOR BASE EXPOSED TO A STEP INCREASE IN TEMPERATURE

C *** CONSTANT BASE TEMPERATURE PROGRAM
DIMENSION A(243,7), T1(3,82), T2(243), GH(4,82), GV(3,82), C(3,82), F(243), TEXC(243), D(243), MELT(243)
IMPLICIT REAL*8(A,H,Z)
C *** R1 = H/W ; R2 = H/S1 ; R3 = H/S2 ; R4 = </W ; ********
C *************
C *** R5 = (K/DEN*CP)/(<K/DEN*CP) ; R6 = HMELT/CP*TMELT *
C *************
C *** R7 = HMELT/HTR ; R8 = Q*H/K*TMELT *********************
C *************
READ(5,998) R1,R2,R3,R4,R5,R6,DELT
READ(5,999) NR,M,N
C *** COMPUTATIONAL PARAMETERS ***
TIN=1*0
DT = 0.00050
ITT=6
CW = 1.0
TMELT = 1.0
ITTT = 1
TPHASE = R6
AN = N
AM = M
QLATM = AN*AN
M1 = M + 1
M2 = M1 + 1
N1 = N + 1
N2=N+2
LL=M1*M1
L1 = 2*M1 + 1
L = M2
CD = (R1**2)+(AM**2)
C *** INITIAL VALUES
SQS = 0.0
SQF = 0.0
SUB = 0.0
SQT=0.0
SQL = 0.0
QLAT = 0.0
EBOT=0.0
TTCB=0.0
TBOT = 0.0
TIME = 0.0
QTOT1 = 0.0
DO 10 I = 1,LL
  F(I) = 1.0
  MELT(I)=0
10 TEXC(I) = 0.0
DO 20 I=1,M1
  T1(I+1)=TIN+DELT
DO 20 J=2,N2
20 T1(I+J) = TIN

57
TABLE 13. (Continued)

C  ***** HORIZONTAL CONDUCTANCES, VERTICAL CONDUCTANCES
C  *** CAPACITANCES **
C  ***** ARE CALCULATED  ******

GV(I,J) = 0.0
DO 30 J=2,N1
DO 30 I=2,M1
GH(I,J) = 1.0
CI(I,J) = 1.0
30   GV(I,J) = 1.0

DO 40 J = 2,N1
CI(1,J) = (R4*A4)/(2.0*R5*R3)
GV(1,J) = (AM*R4)/(2.0*R3)
GH(2,J) = 2.0/(1.0 + AM/(2.0*R3*R4))
GH(M2,J) = 0.0
40   GH(I,J) = 0.0

DO 50 I = 2,M1
CI(I,1) = (AM*R1*R4)/(R2*R5)
CI(I,N2) = CI(I,1)
GV(I,2) = 2.0/(1.0 + (R1*AM/(R2*R4)))
GV(I,N2) = GV(I,2)
GV(I,1) = 0.0
GH(I,1) = R4*AM*R1/R2

50   GH(I,N2) = GH(I,1)
GH(I,1) = 0.0
GH(I,N2) = 0.0
GH(M2,1) = 0.0
GH(M2,N2) = 0.0
GH(2,1) = 2.0*R4/(R2/R1)*(1.0/(2.0*R3) + 1.0/AM)
GH(2,N2) = GH(2,1)
GV(I,2) = R4/(R3*(1.0/AM + R1/R2))
GV(I,N2) = GV(I,2)
CI(I,1) = (R1*R4*(AM**2))/(2.0*R2*R3*R5)
CI(I,N2) = CI(I,1)
DO 55 I = 1,M1

55 TTCB = TTCB + CI(I,1)

C  ***** TEMPERATURES AT EACH NODE CALCULATED  ***
WRITE (6,7250) NR
WRITE(6,7500) R1, R2, R3, R4, R5, R6, DELTA, T, N

60   TIME = TIME + DT
DO 70 I = 1,LL
DO 70 J = 1,LL

70   A(I,J) = 0.0
DO 80 J = 2,N1
DO 80 I = 1,M1
K = (J-2)*M + I
IF (J.NE.2) GO TO 75
D(K) = -C(I,J)*T1(I,J)/(CD*DT) + GV(I,J)*T1(I,J-1)
GO TO 79
75   D(K) = -C(I,J)*T1(I,J)/(CD*DT)
A(K+L+M1) = GV(I,J)
79   A(K+L+M1) = GV(I,J+1)
TABLE 13. (Continued)

\[ A(K,L) = \text{GHI}(I+1,J) - \text{GVI}(I,J) = \text{GVI}(I,J+1) = (C(I,J)) / (DT) \]

1(CU) = \text{GHI}(I,J)

A(K,L+1) = \text{GHI}(I,J)

DO 90 I=1, M1
K = N*M1+I
A(K,L) = \text{GHI}(I,J) - \text{GII}(I,J) - \text{GII}(I,J+1) - \text{CII}(I,J)
1(DT*CU)
A(K,L+1) = \text{GHI}(I,J)
A(K,L+1) = \text{GHI}(I,J+1)
90 A(K,L=M1) = \text{GVI}(I,J)
DO 110 I=1, M1
K = N*M1+I
110 D(K) = \text{CI}(I,J)*T11(I,J) / (DT*CU)
CALL HANSOL (A, L, L1, D, LL)
DO 120 I = 1, LL
120 T2(I) = D(I)

C *** QUALITY CALCULATED ***
DO 150 J=2, M1
DO 150 I=2, M1
K = (J+2)*M1 + I
IF(TMELT(K) EQ 1) GO TO 150
IF(T2(K) GT TMELT) TMEC(K) = TMEC(K) + (T2(K) - TMELT)
IF((T2(K) GT TMELT) I F(K) = 1*0 = TMEC(K) / (R*TMELT) T2
1(K) = TMELT
1IF(TMEC(K) GT TPHASE) T2(K) = TMELT + TMEC(K) - TPHASE; F
1(K) = 0.0; MELT(K) = 1
150 CONTINUE
DO 160 J = 2, M1
DO 160 I = 1, M1
K = (J+2)*M1 + I
160 T1(I,J) = T2(K)
QLAT = 0.0
DO 170 J = 2, M1
DO 170 I = 2, M1
K = (J+2)*M1 + I
170 QLAT = QLAT + (1*0 = F(K))
TBUT = 0.0
DO 175 I=1, M1
175 EBUT = EBUT + (C(I,1) * (T1(I,J) - TIN))
TBUT = TBUT + (EBUT / TT3)
EBUT = 0.0
TRISE = TBUT = TMELT

C *** ENERGY BALANCED ***
SQF = 0.0
SQB = 0.0
SQT = 0.0
SQS = 0.0
SQL = 0.0
DO 200 J = 2, M1
200 SQF = SQF + (C(I,J) * (T1(I,J) - TIN)) / R6)
TABLE 13. (Continued)

DO 210 I = 1,M1
SQT=SQT+(C(I,N2)*(T1(I,N2)=TIN)/R6)
210 SQB = SQB + (C(I,1)*(T1(I,1) = TIN)/R6)
DO 220 J = 2,N1
DO 220 I = 2,M1
K = (J=2)*M1 + 1
SQS=SQS+(C(W*(T2(<>))=TIN)*F(K)/R6)
220 SQL=SQL+(C(W*(T2(<>))=TIN)*Q2=F(K))/R6
QTOT2 = SQF + SQB + SQS + SQL + QLAT + SQT
QRATE = (QTOT2-QTOT1)/dT
UCOEF = (QRATE*R6)/((TBOT=TMELT)*(M**2)*R1* (1+2*R3)/2)
QTOT1=QTOT2
F M E L T = Q L A T / (A M A N)
IF (ABS(QLATM = QLAT) .LT.0.0001) GO TO 180
IF (TIME.EQ.DT) ITTT = ITTT + 1
IF (TIME.EQ.DT) GO TO 180
IF (ITT+NE+ITTT) ITTT = ITTT + 1
IF (ITT+NE+ITTT) GO TO 240
ITYT = 1
180 WRITE (6,8000) TIME,TRISE,UCOEF
WRITE (6,1500) SQF,SQB,SQT,SQS,SQL,QLAT,QTOT2,F M E L T
240 IF (ABS(QLATM = QLAT) .GT.0.0001) GO TO 60
1500 FORMAT(1H1,10X,'SQF = ',F10.5,'5X,'SQB = ',F10.5,'5X,'SQT
1 = ',F10.5,'5X,'SQS = ',F10.5,'5X,'SQL = ',F10.5,'4X
2*QLAT = ',F10.5,'5X,'QTOT = ',F10.5,'4X,'FMELT = '
3,F10.5)
998 FORMAT(7F10.5)
999 FORMAT(3Z3)
2000 FORMAT(1H1)
7250 FORMAT (1H1,4X,'RUN NO. ',I4)
7500 FORMAT(1H0,9X,'R1 = ',F7.3,'3X,'R2 = ',F6.1,'3X,'R3 = '
1,F5.1,'3X,'R4 = ',F7.1,'5X,'R5 = ',F5.1,'5X,'R6 = ',F5.3
2*4X,'DEL = ',F6.4,'3X,'M = ',I2,5X,'N = ',I4)
8000 FORMAT(1H0,9X,'TIME = ',F7.5,'5X,'TRISE = ',F7.5,'3X
1,'UCOEF = ',F10.5)
WRITE (6,2000)
CALL EXIT
END

SUBROUTINE BANSOL(C,NNEW,MNEW,V,ID)
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION C(ID,M),V(ID)
L = (MNEW+1)/2
L1 = L = 1
DO 2 IR = 1,L1
LR = L = IR
DO 2 I = 1,LR
DO 1 J = 2,MNEW
1 C(IR,J-1) = C(IR,J)
NPI = NNEW + 1 = IR
MPI = MNEW + 1 = I

60
TABLE 13. (Concluded)

\[ \begin{align*}
C(NP1, MP1) & = 0.0 \\
C(IR, MNEW) & = C(NP1, MP1) \\
N1 & = NNEW = 1 \\
\text{DO } 9 & I = 1, N1 \\
IPIV & = I \\
IRE & = I + 1 \\
\text{DO } 3 & IR = IRE, L \\
\text{IF}(ABS(C(IR, 1)) \cdot L \cdot ABS(C(IPIV, 1))) & \text{ GO TO 3} \\
IPIV & = IR \\
\text{CONTINUE} \\
\text{IF}(IPIV = EQ, I) & \text{ GO TO 5} \\
T & = V(I) \\
V(I) & = V(IPIV) \\
V(IPIV) & = T \\
\text{DO } 7 & J = 1, MNEW \\
T & = C(I, J) \\
C(I, J) & = C(IPIV, J) \\
\text{CONTINUE} \\
V(I) & = V(I)/C(I, 1) \\
\text{DO } 6 & J = 2, MNEW \\
C(I, J) & = C(I, J)/C(I, 1) \\
\text{DO } 10 & IR = IRE, L \\
T & = C(IR, 1) \\
V(IR) & = V(IR) = T \cdot V(I) \\
\text{DO } 9 & J = 2, MNEW \\
C(IR, J-1) & = C(IR, J) = T \cdot C(I, J) \\
\text{CONTINUE} \\
V(NNEW) & = V(NNEW)/C(NNEW, 1) \\
\text{JM} & = 2 \\
\text{DO } 11 & ICE = 1, N1 \\
IR & = NNEW = ICE \\
\text{DO } 10 & J = 2, JM \\
IRM1 & = IR = 1 + J \\
V(IR) & = V(IR) = C(IR, J) \cdot V(IRM1) \\
\text{IF}(JM \cdot EQ \cdot MNEW) & \text{ GO TO 11} \\
JM & = JM + 1 \\
\text{CONTINUE} \\
\text{RETURN} \\
\text{END}
\end{align*} \]
IF(T2(K).LT.TMELT)TEXC(K)=TEXC(K)-(T2(K)-TMELT)

IF(T2(K).LT.TMELT)F(K)=1.0-TEXC(K)/(R6*TMELT); T2(K)=TMELT

IF(TEXC(K).GE.TPHASE)T2(K)=TMELT-TEXC(K)+TPHASE;F(K)=0.0;MELT(K) =1.0.

c. In statement 170, change sign to give QLAT = QLAT - (1.0 - F(K)).

d. In the line above statement 220 and statement 220, make changes to give

\[ SQS = SQS + (CW \times (T_2(K) - T_{IN}) \times (1.0 - F(K)/R6) \]

\[ 220 \quad SQL = SQL + (CW \times (T_2(K) - T_{IN}) \times F(K)/R6) \]

An extensive parametric study for the case of melting with the initial PCM temperature at the fusion temperature was conducted using the computer program given in Table 12. This parametric study is discussed in more detail and the results are presented in Section IV.

### IV. GENERAL PARAMETRIC STUDY

#### A. Introduction

Presented in this section are some details and typical results of a parametric study for a particular phase change device application. Since there are many phase change device applications, each having its own unique constraints, it would be unrealistic to propose that the results obtained for a particular set of conditions and constraints could be quantitatively extrapolated to other conditions and constraints. However, results obtained for a particular case such as those presented herein may afford valuable insight, qualitatively, into such considerations as trends and acceptable operating ranges.

The parametric study consisted of a thermal analysis of the symmetrical half-cell shown in Figure 29 with the base exposed to a constant heat flux density. The model and assumptions are described in Section III.

#### B. Scope and Definitions

Numerical results have been obtained using the implicit formulation described in Section III.
A band algorithm [14] was used to reduce the required computer storage in comparison to that which would have been required using a general matrix inversion routine.

Of the seven dimensionless variables previously given, some of the physical parameters were considered to be fixed for the computations. Independent variation of all seven parameters would render the problem untractable.

By selection of a particular PCM and metal for the housing and fins, the three dimensionless parameters denoted by \( R_4 \), \( R_5 \), and \( R_6 \) are all fixed. Since aluminum was typically used in most of the previous phase change device applications and paraffin was commonly used as the PCM, properties corresponding to these materials were used in the computations. For the results presented herein, the representative properties used in the computations for these materials are given in Table 14. With the property values given in Table 14, the values for the fixed dimensionless ratios are

\[
R_4 = 1069 \quad , \quad R_5 = 753 \quad , \quad R_6 = 0.381
\]

Values of the geometrical variables and the heat flux density chosen for the numerical predictions are based on a survey of past applications. The values are listed in Table 15. For the range of variables outlined in Table 15, there are 300 possible combinations which indicates the need for an equal number of computer runs to obtain results encompassing these conditions. Because of the large number of computer runs involved, it was not practical to conduct a sensitivity study for each run on the choice of \( M \) and \( N \) and \( DT \), a suggestion outlined in Section III under the instructions for using the computer programs. The choice of \( M \) and \( N \) was made such that the node size \( S \) for the range of geometric variables given in Table 15 never exceeded an approximate value of 0.254 cm (0.1 in.). This value, for some cases, was not sufficiently small to avoid there being an effect on the character of the transient results. This is discussed later in this section.

Specifically, for the results shown hereafter in this section, the parametric computations were performed for the case of melting only. The entire cell was considered to be initially at the fusion temperature of the PCM and in the solid phase.

If one assumes that the phase change is reversible, the results can also be applied to freezing. There is evidence, however, that the rate of freezing is typically underpredicted by the model. However, in most designs, these data will indicate conservatively slower freeze rates than actually occur. Additional discussion of the freezing case is presented in Section V.
The problem is transient in character. The computer program provides a printout of a number of computed quantities at time intervals which can be chosen as any multiple of the time increment used in the numerical expressions. The quantities computed at each time included the temperature difference between the base and the melt temperature ($\Delta T_b$), an overall heat transfer coefficient ($U$), the latent energy which represents the amount of PCM which has melted ($Q_{LAT}$), and the sensible energy stored in the fin ($S_{QS}$), the base ($S_{QB}$), the cover ($S_{QT}$), the solid PCM ($S_{QS}$), and the liquid PCM ($S_{QL}$). Computations automatically terminate when all the PCM has melted.

For the geometry considered here, the latent energy associated with the PCM per unit area of the base is expressed by
\[ E''_q = \frac{2 \rho h_f H}{2 + S_2/W} \]  

This parameter is defined because it is useful in expressing the latent energy storage capability of the device. In using equation (22) to determine the latent energy storage capability, the appropriate area to use with this equation is the inside plan area of the device. This leads to a negligible error since equation (22) is based on a symmetrical cell and the two outermost end cells of the device are unsymmetrical. The latent energy is linearly related to \( H \) for fixed values of \( S_2 \) and \( W \).

C. Results

For the range of \( H, W, S_2 \), and the physical properties considered in this study, equation (22) is represented graphically in Figures 33 through 35. Also shown is the case for infinite \( W \) which corresponds to the case for which the entire housing is filled with PCM without fins. The effect of increasing fin thickness is readily apparent by comparing the curves shown in Figures 33 through 35.

An important consideration, especially in temperature control design, is the temperature rise of the base. Ratios of the base temperature rise at the termination of melting to the absolute fusion temperature of the PCM for the range of conditions considered herein are shown plotted versus \( W \) in Figures 36 through 50. Over the examined range, the base temperature rise increases with an increase in \( W \). For a specified set of parameters, the data indicate that the temperature rise approaches an asymptotic value which corresponds to expected behavior since the problem should reduce to that of a one-dimensional case as \( W \) becomes large. It is also expected that the temperature rise would increase at very small values of \( W \) as the problem approaches that for the heating of a metal slab. Computations were not performed for these very low values of \( W \). It should also be noted that some of the values for the base temperature rise which are shown are too large to be of much practical significance; some of these are included, however, to indicate trends and to facilitate approximate interpolation. For some of the larger heat-flux-density cases, the curves are terminated at lower values of \( W \) than they are for the lower heat-flux-density cases because the resultant temperature rises are much too large for practical considerations.

In Figures 36 through 50 as well as in all subsequent figures showing results of the parametric study, computed data are represented by solid circles. Curves have been faired through the points; it should be recognized, however, that the value for points on the curve between two computed values are only approximate.

The data shown in Figures 36 through 50 are cross plotted versus fin height in Figures 51 through 62. For some geometries within the considered range, the curves indicate the presence of a relative maximum and a relative minimum. This feature is attributable to the two-dimensional character of the problem. This characteristic is discussed in more detail in Appendix B.
Figure 33. Latent energy storage per unit base area versus tin height for tin thickness of 0.0127 cm (0.005 in.).
Figure 34. Latent energy storage per unit base area versus fin height for fin thickness of 0.0508 cm (0.02 in.).
Figure 35. Latent energy storage per unit base area versus fin height for fin thickness of 0.127 cm (0.05 in.).
Figure 36. Base temperature rise at the termination of melting versus $W$ for $H = 1.27$ cm (0.5 in.) and $S_2 = 0.127$ cm (0.05 in.).
Figure 37. Base temperature rise of the termination at melting versus \( W \) for \( H = 2.54 \text{ cm (1.0 in.)} \) and \( S_2 = 0.127 \text{ cm (0.05 in.)} \).
Figure 38. Base temperature rise at the termination of melting versus W for
H = 5.08 cm (2.0 in.) and S₂ = 0.127 cm (0.05 in.).
Figure 39. Base temperature rise at the termination of melting versus W for 
$H = 10.16 \text{ cm (4.0 in.)}$ and $S_2 = 0.127 \text{ cm (0.05 in.)}$. 

$H = 10.16 \text{ cm (4.0 in.)}$
$S_2 = 0.127 \text{ cm (0.05 in.)}$
Figure 40. Base temperature rise at the termination of melting versus $W$ for $H = 20.32$ cm (8.0 in.) and $S_2 = 0.127$ cm (0.05 in.).
Figure 41. Base temperature rise at the termination of melting versus $W$ for
$H = 1.27$ cm (0.5 in.) and $S_2 = 0.0508$ cm (0.02 in.).
Figure 42. Base temperature rise at termination of melting versus $W$ for $H = 2.54$ cm (1.0 in.) and $S_2 = 0.0508$ cm (0.02 in.).
Figure 43. Base temperature rise at termination of melting versus \( W \) for \( H = 5.08 \text{ cm} \ (2.0 \text{ in.}) \) and \( S_2 = 0.0508 \text{ cm} \ (0.02 \text{ in.}) \).
Figure 44. Base temperature rise at termination of melting versus W for 
H = 10.16 cm (4.0 in.) and S₂ = 0.0508 cm (0.02 in.).
Figure 45. Base temperature rise at termination of melting versus $W$ for $H = 20.32$ cm (8.0 in.) and $S_2 = 0.0508$ cm (0.02 in.).
Figure 46. Base temperature rise at termination of melting versus W for $H = 1.27$ cm (0.5 in.) and $S_2 = 0.0127$ cm (0.005 in.).
Figure 47. Base temperature rise at termination of melting versus $W$ for $H = 2.54 \text{ cm (1.0 in.)}$ and $S_2 = 0.0127 \text{ cm (0.005 in.)}$. 

$\Delta T = 0.0R = \frac{1}{W}$
Figure 48. Base temperature rise at termination of melting versus W for H = 5.08 cm (2.0 in.) and S = 0.0127 cm (0.005 in.).

\[
\frac{q}{J/m^2} = \frac{1076.4}{(882.9)} \text{ (341.4) Btu/h-ft}^2
\]

\[
H = 5.08 \text{ cm (2.0 in.)}
\]

\[
S = 0.0127 \text{ cm (0.005 in.)}
\]
Figure 49. Base temperature rise at termination of melting versus W for
\( H = 10.16 \text{ cm (4.0 in.)} \) and \( S_2 = 0.0127 \text{ cm (0.005 in.)} \).
Figure 50. Base temperature rise at termination of melting versus W for $H = 20.32$ cm (8.0 in.) and $S_2 = 0.0127$ cm (0.005 in.).
\[ q'' = 1076.4 \text{ J/s-m}^2 \ (341.4 \text{ Btu/h-ft}^2) \]
\[ S_2 = 0.0127 \text{ cm (0.005 in.)} \]

**Figure 51.** Base temperature rise at the termination of melting versus H for \( q'' = 1076.4 \text{ J/s-m}^2 \) (341.4 Btu/h-ft\(^2\)) and \( S_2 = 0.0127 \text{ cm (0.005 in.)} \).
Figure 52. Base temperature rise at the termination of melting versus H for \( q'' = 1076.4 \text{ J/s-m}^2 (341.4 \text{ Btu/h-ft}^2) \) and \( S_2 = 0.0508 \text{ cm (0.020 in.)} \).
\[ \dot{q}'' = 1076.4 \text{ J/s-m}^2 (341.4 \text{ Btu/h-ft}^2) \]
\[ S_2 = 0.127 \text{ cm (0.050 in.)} \]

Figure 53. Base temperature rise at the termination of melting versus \( H \) for \( \dot{q}'' = 1076.4 \text{ J/s-m}^2 (341.4 \text{ Btu/h-ft}^2) \) and \( S_2 = 0.127 \text{ cm (0.050 in.)} \).
\[
q'' = 2152.8 \text{ J/s-m}^2 (682.9 \text{ Btu/h-ft}^2)
\]
\[
S_2 = 0.0127 \text{ cm (0.005 in.)}
\]

Figure 54. Base temperature rise at the termination of melting versus \(H\) for \(q'' = 2152.8 \text{ J/s-m}^2 (682.9 \text{ Btu/h-ft}^2)\) and \(S_2 = 0.0127 \text{ cm (0.005 in.)}\).
Figure 55. Base temperature rise at the termination of melting versus H for $\dot{q}'' = 2152.8$ J/s-m$^2$ (682.9 Btu/h-ft$^2$) and $S_2 = 0.0508$ cm (0.020 in.).
\( \dot{q}'' = 2152.8 \text{ J/s-m}^2 (682.9 \text{ Btu/h-ft}^2) \)
\( S_2 = 0.127 \text{ cm (0.050 in.)} \)

Figure 56. Base temperature rise at the termination of melting versus \( H \) for \( \dot{q}'' = 2152.8 \text{ J/s-m}^2 (682.9 \text{ Btu/h-ft}^2) \) and \( S_2 = 0.127 \text{ cm (0.05 in.)} \).
Figure 57. Base temperature rise at the termination of melting versus H for $q'' = 4305.6 \text{ J/s-m}^2 (1365.7 \text{ Btu/h-ft}^2)$ and $S_2 = 0.0127 \text{ cm (0.005 in.)}$. 
Figure 58. Base temperature rise at the termination of melting versus H for $\dot{q}'' = 4305.6$ J/s-m$^2$ (1365.7 Btu/h-ft$^2$) and $S_2 = 0.0508$ cm (0.020 in.).
Figure 59. Base temperature rise at the termination of melting versus H for $\dot{q}'' = 4305.6$ J/s-m$^2$ (1365.7 Btu/h-ft$^2$) and $S_2 = 0.127$ cm (0.050 in.).
Figure 60. Base temperature rise at the termination of melting versus H for \( q'' = 6458.3 \text{ J/s-m}^2 \) (2048.6 Btu/h-ft\(^2\)) and \( S_2 = 0.0127 \text{ cm (0.005 in.)}. \)
\( \dot{q}' = 6458.3 \text{ J/s-m}^2 \) (2048.6 Btu/h-ft\(^2\))

\( S_2 = 0.0508 \text{ cm (0.020 in.)} \)

Figure 61. Base temperature rise at the termination of melting versus \( H \) for \( \dot{q}' = 6458.3 \text{ J/s-m}^2 \) (2048.6 Btu/h-ft\(^2\)) and \( S_2 = 0.0508 \text{ cm (0.020 in.)} \).
Figure 62. Base temperature rise at the termination of melting versus H for $q'' = 6458.3$ J/s-m$^2$ (2048.6 Btu/h-ft$^2$) and $S_2 = 0.127$ cm (0.050 in.).
The base temperature rises shown in Figures 36 through 62 correspond to the termination of melting. If the entire amount of PCM is not melted, the temperature rise of the base may be considerably less than that reflected in Figures 36 through 62. As an indication of the transient behavior of the base temperature, certain transient data are plotted in Figures 63 through 77. For some cases, the base temperature rise remains relatively low over a large percentage of the phase change process and rises rapidly near the end of the process. In other cases, the rise occurs nearly uniformly throughout the phase change process. In either case, designs based on the data shown in Figures 36 through 62 should be conservative, especially if the design does not require complete melting of the PCM. However, as indicated by these data, in cases where base plate temperature control is the object, it may be more desirable to limit phase change in order to avert unreasonable fin spacing and/or heights.

Some of the curves shown in Figures 63 through 77 exhibit fluctuations in the transient base temperature. For those curves where a sharp rise occurs in the computed base temperature (e.g., the upper curve in Figure 63), the accuracy with which the predictions describe the actual physical variation is questionable, particularly during the early part of the transient. The computing model attempts to predict the behavior of a continuous system with use of a limited number of finite size nodes. Since the phase change is handled in the model by forcing an entire node to remain at the phase change temperature until sufficient energy has accumulated to account for the phase change, one would expect some stepping in the results unless the nodes are made very small. This, however, is related to the choice of M and N, which in turn dictates computed requirements. Since the problem examined in the parametric study involved a step input of a constant heat flux at the base, the initial level to which the computer temperature rises is dependent on the node size. The influence of N on the computed results is illustrated in Figure 63 for three runs, in Figure 64 for one run, and in Figure 65 for one run. For these selected runs, computations are shown for N values of 5 and 10. When 5 is used, a horizontal row of nodes represents 20 percent of the PCM. Obviously, the precise detail of the transient variation is affected in each case, but the principal difference is at the beginning; it appears that increasing N results in a smoothing out of the fluctuations and shortens the initial plateau. The general trend of the results remains consistent and there appears to be only a slight effect in the latter stages of a run. For reference purposes the values of N used in the computations are given for the respective curve in Figures 63 through 77.

The time required for the PCM to melt is depicted graphically in Figures 78 through 89. These times exceed those which would be required by purely latent considerations because of the effect of sensible heating.

Since there is a certain amount of sensible energy storage, ratios of the total stored energy at the termination of melting to the latent energy required to change phase of the PCM are shown plotted versus heat flux density at the base in Figures 90 through 104.

A few additional computer runs were made for selected values of the parameters to examine the influence of the fusion temperature on the predicted temperature rise of the base at the termination of melting. The results of these computations are shown in Figures 105 through 107.
Figure 63. Transient base temperature rise versus fraction of PCM melted for $H = 1.27$ cm (0.5 in.) and $S_2 = 0.0127$ cm (0.005 in.).
\[
\dot{q}'' = 1076.4 \text{ J/s-m}^2 (341.4 \text{ Btu/h-ft}^2)
\]
\[
S_2 = 0.0508 \text{ cm (0.020 in.)}
\]
\[
H = 1.27 \text{ cm (0.5 in.)}
\]

Figure 64. Transient base temperature rise versus fraction of PCM melted for \(H = 1.27 \text{ cm (0.5 in.)}\) and \(S_2 = 0.0508 \text{ cm (0.020 in.)}\).
\( \dot{q} = 1076.4 \text{ J/s-m}^2 \) (341.4 Btu/h-ft\(^2\))

\( S_2 = 0.127 \text{ cm (0.050 in.)} \)

\( H = 1.27 \text{ cm (0.5 in.)} \)

Figure 65. Transient base temperature rise versus fraction of PCM melted for \( H = 1.27 \text{ cm (0.5 in.)} \) and \( S_2 = 0.127 \text{ cm (0.05 in.)} \).
Figure 66. Transient base temperature rise versus fraction of PCM melted for $H = 2.54$ cm (1 in.) and $S_2 = 0.0127$ cm (0.005 in.).
\[
q'' = 1076.4 \text{ J/s-m}^2 \quad (341.4 \text{ Btu/h-ft}^2)
\]
\[
S_2 = 0.0508 \text{ cm (0.020 in.)}
\]
\[
H = 2.54 \text{ cm (1 in.)}
\]

Figure 67. Transient base temperature rise versus fraction of PCM melted for \(H = 2.54 \text{ cm (1 in.)}\) and \(S_2 = 0.0508 \text{ cm (0.020 in.)}\).
\[
\dot{q}'' = 1076.4 \text{ J/s-m}^2 (341.4 \text{ Btu/h-ft}^2) \\
S_2 = 0.127 \text{ cm (0.050 in.)} \\
H = 2.54 \text{ cm (1.0 in.)}
\]

Figure 68. Transient base temperature rise versus fraction of PCM melted for \(H = 2.54 \text{ cm (1 in.) and } S_2 = 0.127 \text{ cm (0.05 in.)}.\)
Figure 69. Transient base temperature rise versus fraction of PCM melted for $H = 5.08 \text{ cm (2 in.)}$ and $S_2 = 0.0127 \text{ cm (0.005 in.)}$. 

$q'' = 1076.4 \text{ J/s-m}^2$ (341.4 Btu/h-ft$^2$) 

$S_2 = 0.0127 \text{ cm (0.005 in.)}$ 

$H = 5.08 \text{ cm (2.0 in.)}$
Figure 70. Transient base temperature rise versus fraction of PCM melted for $H = 5.08 \text{ cm (2 in.)}$ and $S_2 = 0.0508 \text{ cm (0.02 in.)}$.
\( q'' = 1076.4 \text{ J/m}^2 \) (341.4 Btu/h-ft\( ^2 \))
\( S_2 = 0.127 \text{ cm} \) (0.050 in.)
\( H = 5.08 \text{ cm} \) (2.0 in.)

Figure 71. Transient base temperature rise versus fraction of PCM melted for \( H = 5.08 \text{ cm} \) (2 in.) and \( S_2 = 0.127 \text{ cm} \) (0.05 in.).
Figure 72. Transient base temperature rise versus fraction of PCM melted for $H = 10.16$ cm (4 in.) and $S_2 = 0.0127$ cm (0.005 in.).
\( \dot{q}^" = 1076.4 \text{ J/s-m}^2 \) (341.4 Btu/h-ft²)

\( S_2 = 0.0508 \text{ cm (0.020 in.)} \)

\( H = 10.16 \text{ cm (4 in.)} \)

Figure 73. Transient base temperature rise versus fraction of PCM melted for \( H = 10.16 \text{ cm (4 in.) and } S_2 = 0.0508 \text{ cm (0.02 in.)} \).
Figure 74. Transient base temperature rise versus fraction of PCM melted for $H = 10.16 \text{ cm (4 in.)}$ and $S_2 = 0.127 \text{ cm (0.05 in.)}$
Figure 75. Transient base temperature rise versus fraction of PCM melted for $H = 20.32 \text{ cm (8 in.)}$ and $S_2 = 0.0127 \text{ cm (0.005 in.)}$. 
\( \dot{q}'' = 1076.4 \text{ J/s-m}^2 \) (341.4 Btu/h-ft²)

\( S_2 = 0.0508 \text{ cm (0.02 in.)} \)

\( H = 20.32 \text{ cm (8 in.)} \)

Figure 76. Transient base temperature rise versus fraction of PCM melted for \( H = 20.32 \text{ cm (8 in.)} \) and \( S_2 = 0.0508 \text{ cm (0.02 in.)} \).
$\dot{q}'' = 1076.4 \text{ J/s-m}^2 (341.4 \text{ Btu/h-ft}^2)$

$S_2 = 0.127 \text{ cm (0.050 in.)}$

$H = 20.32 \text{ cm (8 in.)}$

Figure 77. Transient base temperature rise versus fraction of PCM melted for $H = 20.32 \text{ cm (8 in.)}$ and $S_2 = 0.127 \text{ cm (0.05 in.)}$. 
Figure 78. Time required for PCM to melt for $q'' = 1076.4 \, \text{J/s-m}^2$ (341.4 Btu/h-ft$^2$) and $S_2 = 0.0127 \, \text{cm} \, (0.005 \, \text{in.})$. 
Figure 79. Time required for PCM to melt for $q'' = 1076.4 \text{ J/s-m}^2$ (341.4 Btu/h-ft$^2$) and $S_2 = 0.0508 \text{ cm (0.02 in.)}$. 
Figure 80. Time required for PCM to melt for $q'' = 1076.4 \text{ J/s-m}^2\;\text{(341.4 Btu/h-ft}^2\) and $S_2 = 0.127 \text{ cm (0.05 in.)}$.
Figure 81. Time required for PCM to melt for $q'' = 2152.8 \text{ J/s-m}^2$ (682.9 Btu/h-ft$^2$) and $S_2 = 0.0127 \text{ cm (0.005 in.)}$.
Figure 82. Time required for PCM to melt for $\dot{q}'' = 2152.8 \text{ J/m}^2 \text{ (682.9 Btu/h-ft}^2\text{)}$ and $S_2 = 0.0508 \text{ cm (0.02 in.).}$
\[ \dot{q}'' = 2152.8 \text{ J/s-m}^2 (682.9 \text{ Btu/h-ft}^2) \]
\[ S_2 = 0.127 \text{ cm} (0.05 \text{ in.}) \]
Figure 84. Time required for PCM to melt for $q'' = 4305.6 \text{ J/s-m}^2$ (1365.7 Btu/h-ft$^2$) and $S_2 = 0.0127 \text{ cm (0.005 in.)}$. 

![Graph showing time required for PCM to melt with different values of H cm and H in.](image)
Figure 85. Time required for PCM to melt for $\dot{q}'' = 4305.6 \text{ J/s-m}^2$ ($341.4 \text{ Btu/h-ft}^2$) and $S_2 = 0.0508 \text{ cm (0.02 in.)}$. 
Figure 86. Time required for PCM to melt and $\dot{q}'' = 4305.6 \text{ J/s-m}^2$ (1365.7 Btu/h-ft$^2$) and $S_2 = 0.127 \text{ cm (0.05 in.)}$.
Figure 87. Time required for PCM to melt for $q'' = 6458.3 \text{ J/s-m}^2$ (2048.6 Btu/h-ft$^2$) and $S_2 = 0.0127 \text{ cm (0.005 in.)}$.
\[ \dot{q}'' = 6458.3 \text{ J/s-m}^2 \text{ (2048.6 Btu/h-ft}^2) \]
\[ S_2 = 0.0508 \text{ cm (0.02 in.)} \]

Figure 88. Time required for PCM to melt for \( \dot{q}'' = 6458.3 \text{ J/s-m}^2 \)
\( \text{ (2048.6 Btu/h-ft}^2) \) and \( S_2 = 0.0508 \text{ cm (0.02 in.)} \).
Figure 89. Time required for PCM to melt for $\dot{q}' = 6458.3 \text{ J/s-m}^2$ (2048.6 Btu/h-ft$^2$) and $S_2 = 0.127 \text{ cm (0.05 in.)}$. 

$\dot{q}' = 6458.3 \text{ J/s-m}^2$ (2048.6 Btu/h-ft$^2$) and $S_2 = 0.127 \text{ cm (0.05 in.)}$.
Figure 90. Energy ratio versus heat flux density for $W = 0.127$ cm (0.05 in.) and $S_2 = 0.0127$ cm (0.005 in.).
Figure 91. Energy ratio versus heat flux density for $W = 0.127$ cm (0.05 in.) and $S_2 = 0.0508$ cm (0.02 in.).
Figure 92. Energy ratio versus heat flux density for $W = 0.127$ cm (0.05 in.) and $S_2 = 0.127$ cm (0.05 in.).
Figure 93. Energy ratio versus heat flux density for $W = 0.254$ cm (0.1 in.) and $S_2 = 0.0127$ cm (0.005 in.).
Figure 94. Energy ratio versus heat flux density for $W = 0.254$ cm (0.1 in.) and $S_2 = 0.0508$ cm (0.02 in.).
Figure 95. Energy ratio versus heat flux density for $W = 0.254$ cm (0.1 in.) and $S_2 = 0.127$ cm (0.05 in.).
Figure 96. Energy ratio versus heat flux density for $W = 0.508$ cm (0.2 in.) and $S_2 = 0.0127$ cm (0.005 in.).
Figure 97. Energy ratio versus heat flux density for $W = 0.508$ cm (0.2 in.) and $S_2 = 0.0508$ cm (0.02 in.).
Figure 98. Energy ratio versus heat flux density for $W = 0.508$ cm (0.2 in.) and $S_2 = 0.127$ cm (0.05 in.).
Figure 99. Energy ratio versus heat flux density for $W = 1.016$ cm (0.4 in.) and $S_2 = 0.0127$ cm (0.005 in.).
Figure 100. Energy ratio versus heat flux density for $W = 1.016$ cm (0.4 in.) and $S_2 = 0.0508$ cm (0.02 in.).
W = 1.016 cm (0.4 in.)
S₂ = 0.127 cm (0.05 in.)

Figure 101. Energy ratio versus heat flux density for W = 1.016 cm (0.4 in.) and S₂ = 0.127 cm (0.05 in.).
Figure 102. Energy ratio versus heat flux density for $W = 2.032$ cm (0.8 in.) and $S_2 = 0.0127$ cm (0.005 in.).
Figure 103. Energy ratio versus heat flux density for $W = 2.032$ cm (0.8 in.) and $S_2 = 0.0508$ cm (0.02 in.).
Figure 104. Energy ratio versus heat flux density for $W = 2.032$ cm (0.8 in.) and $S_2 = 0.127$ cm (0.05 in.).
Figure 105. Base temperature rise versus $W$ for three different fusion temperatures and $H = 1.27$ cm (0.5 in.) and $S_2 = 0.127$ cm (0.05 in.).
Figure 106. Base temperature rise versus W for three different fusion temperatures and H = 2.54 cm (1.0 in.) and S2 = 0.127 cm (0.05 in.).
Figure 107. Base temperature rise versus W for three different fusion temperatures and \( H = 5.08 \text{ cm (2.0 in.)} \) and \( S_2 = 0.127 \text{ cm (0.05 in.)} \).
Some representative examples illustrating how the results can be used by a designer are given in the following section. For conditions significantly different from those used in the computations (Tables 14 and 15), it is suggested that the program described in Section III be used to determine the required design information.

D. Examples

Some hypothetical examples which a thermal designer may encounter illustrating use of the results are outlined as follows. All examples assume the designer has selected a housing design and materials as discussed earlier.

Example 1

Given: The heat flux density is specified to be 1076.4 J/s-m$^2$ (341.4 Btu/h-ft$^2$) and the designer wishes to limit the temperature increase to 5.56 K (10°R). The PCM melt temperature is 291.67 K (525°R), using these inputs the maximum permissible base temperature rise ratio is computed to be

$$\Delta T_b/T_f = \frac{5.56}{291.67} = 19.05 \times 10^{-3}$$

Case a -- The designer wishes to use a fin thickness of 0.0127 cm (0.005 in.).

- Find an acceptable geometry to satisfy the given conditions. From Figure 51, a height of 2.54 cm (1 in.) can be used with a half-fin spacing of 0.127 cm (0.05 in.) to achieve the desired conditions.

- Determine the energy stored. From equation (22) or from Figure 33, the latent energy storage is 4506 kJ/m$^2$ (397 Btu/ft$^2$). The results shown in Figure 90 show that the total energy stored at the termination of melting for these conditions is approximately 5 percent higher than the latent energy stored, the difference being sensible energy storage.

Case b -- The designer wishes to use a fin thickness of 0.127 cm (0.05 in.).

- Find an acceptable geometry. From Figure 53, it is seen that a fin height of 10.16 cm (4 in.) with a half-fin spacing of 0.254 cm (0.1 in.) can be used.

- Determine the energy stored. From equation (22) or from Figure 35, the corresponding latent energy storage is 15 128 kJ/m$^2$ (1333 Btu/ft$^2$). At the termination of melting, the total stored energy exceeds the latent value by approximately 5 percent as shown in Figure 95.
Example 2

Given: The designer is limited by space to a plan area for the phase change device of 0.0697 m² (0.75 ft²); the heat flux density is specified to be 2152.8 J/s-m² (682.9 Btu/h-ft²); the duty cycle of the controlled medium requires latent energy 948.9 kJ (900 Btu) of storage. For these conditions, latent energy storage per unit area is

\[ E''_L = \frac{948.9}{0.0697} = 1.3614 \times 10^4 \text{ kJ/m}^2 (1200 \text{ Btu/ft}^2) \]

Case a – The designer wishes to use a fin thickness of 0.0127 cm (0.005 in.).

- Find the geometry. From Figure 33, it is noted that the required height is approximately 7.62 cm (3 in.) regardless of fin spacing over the range shown.

- Determine the base temperature rise. Assume a half-fin spacing of 0.254 cm (0.1 in.) and a height of 7.62 cm (3 in.). From Figure 54, the corresponding base temperature rise at the termination of melting is

\[ \Delta T_b = 0.132 \times 291.67 \approx 38.5 \text{ K (69°R)} \]

Case b – The designer selects a fin thickness of 0.127 cm (0.05 in.).

- Find the geometry. From Figure 35, it can be seen that there is a wider range of fin height-spacing combinations which will satisfy the latent energy storage requirements.

- Determine the base temperature rise. First, assume a fin height of 10.92 cm (4.3 in.) and a half-fin spacing of 0.127 cm (0.05 in.). From Figure 56, the corresponding temperature rise of the base at the termination of melting is approximately

\[ \Delta T_b = 0.027 \times 291.67 \approx 7.88 \text{ K (14°R)} \]

Second, assume a fin height of 8.38 cm (3.3 in.) and a half-fin spacing of 0.508 cm (0.2 in.). From Figure 56, the corresponding temperature rise of the base at the termination of melting is approximately

\[ \Delta T_b = 0.06 \times 291.67 \approx 17.5 \text{ K (32°R)} \]
Example 3

Given: In a phase change device the designer wishes to utilize 0.0127 cm (0.005 in.) thick fins and he wants to limit the base temperature rise to 13.89 K (25°R). Consequently, the base temperature rise ratio is not to exceed

\[
\frac{\Delta T_b}{T_f} = \frac{13.89}{291.67} = 48 \times 10^{-3}
\]

Case a -- A heat flux density of 1076.4 J/s-m² (341.4 Btu/h-ft²) is required.

- Determine acceptable geometry and the corresponding latent energy storage capacity. From Figure 46, a height of 1.27 cm (0.5 in.) can be used with a half-fin spacing up to approximately 0.48 cm (0.19 in.). The corresponding latent energy storage capacity, from equation (22), is 2338 kJ/m² (206 Btu/ft²).

- Determine acceptable geometry and the corresponding latent energy storage capacity. From Figure 51, heights up to 3.81 cm (1.5 in.) for a half-fin spacing of 0.254 cm (0.1 in.) and up to 9.65 cm (3.8 in.) can be used for a half-fin spacing 0.127 cm (0.05 in.). The corresponding upper limits on the latent energy storage capacity, from either equation (22) or Figure 33, are 6923 kJ/m² (610 Btu/ft²) and 17 114 kJ/m² (1508 Btu/ft²), respectively.

Case b - Assume an imposed heat flux density of 4305.6 J/s-m² (1365.7 Btu/h-ft²).

- Determine an acceptable geometry and the corresponding latent energy storage capacity. From either Figure 46 or 57, the fin height cannot exceed 1.27 cm (0.5 in.) for a half-fin spacing 0.127 cm (0.05 in.). From equation (22) or Figure 33, the corresponding latent energy storage capacity is 2247 kJ/m² (198 Btu/ft²).

Example 4

Given: A phase change device is required to have a fin height of no more than 5.08 cm (2.0 in.) and its base is to be exposed to a heat flux density of 2152.8 J/s-m² (682.9 Btu/h-ft²).

Case a – The designer wishes to use a fin thickness of 0.0127 cm (0.005 in.) to store as much energy as possible.

- Determine the base temperature rise if the designer selects a half-fin spacing of 0.3175 cm (0.125 in). From Figure 48, the maximum base temperature rise is
\[ \Delta T_b \approx 0.21 \times 291.67 \approx 61.3 \text{ K (110.3}^\circ \text{R)} \]

which is too large for most practical considerations.

- Determine the base temperature rise for a half-fin spacing of 0.127 cm (0.05 in.). From Figure 48, the maximum base temperature rise is

\[ \Delta T_b \approx 0.095 \times 291.67 \approx 27.7 \text{ K (49.9}^\circ \text{R)} \]

Case b – Assume the designer selects a fin thickness of 0.0508 cm (0.02 in.)

- Determine the base temperature rise for a half-fin spacing of 0.3175 cm (0.125 in.). From Figure 43, the maximum base temperature rise is

\[ \Delta T_b \approx 0.088 \times 291.67 \approx 25.7 \text{ K (46.2}^\circ \text{R)} \]

- Determine the base temperature rise for a half-fin spacing of 0.127 cm (0.05 in.). From Figure 43, the maximum base temperature rise is

\[ \Delta T_b \approx 0.043 \times 291.67 \approx 12.5 \text{ K (22.6}^\circ \text{R)} \]

Case c – Assume a fin thickness of 0.127 cm (0.05 in) is selected.

- Determine the base temperature rise for a half-fin spacing of 0.3175 cm (0.125 in.). From Figure 38, the maximum base temperature rise is

\[ \Delta T_b \approx 0.060 \times 291.67 \approx 17.5 \text{ K (31.5}^\circ \text{R)} \]

- Determine the base temperature rise for a half-fin spacing of 0.127 cm (0.05 in.). From Figure 38, the maximum base temperature rise is

\[ \Delta T_b \approx 0.030 \times 291.67 \approx 8.8 \text{ K (15.8}^\circ \text{R)} \]

Example 5

Given: A phase change device is to be designed to have its base exposed to a heat flux density of 2152.8 J/s-m² (682.9 Btu/h-ft²). Space constraints restrict the plan area
to 0.0557 m² (0.6 ft²) and the maximum base temperature at 5400 s (1.5 h) is not to exceed the PCM fusion temperature by more than 8.33 K (15°R). Consequently,

$$\Delta T_b \approx \frac{8.33}{291.67} \approx 28.6 \times 10^{-3}$$

An estimate of the required latent energy storage capacity is

$$E'_r = 2152.8 \times \frac{5400}{1000} = 11,625 \text{ kJ/m}^2 (1024 \text{ Btu/ft}^2)$$

Determine the acceptable geometry to satisfy these conditions. An examination of Figures 33 through 35 reveals that the maximum acceptable height is 6.35 cm (2.5 in.) and that the required height must be greater than this as the fin spacing is decreased and as the fin thickness is increased.

Case a – Consider a fin thickness of 0.0127 cm (0.005 in.). The specified conditions cannot be met for this case (at least considering geometries used herein) as can be seen in Figure 54.

Case b – Consider a fin thickness of 0.0508 cm (0.02 in.). Figure 55 shows that a fin height of 7.62 cm (3.0 in.) and a half-fin spacing of 0.127 cm (0.05 in.) will facilitate maintenance of the base temperature rise constraint. An examination of Figure 34 indicates that the latent energy storage capacity is also satisfied. Figure 67 shows that the time required for the PCM to melt is 5760 s (1.6 h.).

V. RELATED TOPICS

In the preceding section, attention has been given to certain properties of several paraffins and to an extensive numerical study of a particular phase change device. The results of the parametric study should be helpful to the designer of phase change devices for applications compatible with the model, but the user needs an awareness of features that may require more specialized treatment in a critical design. Some of these important features which are discussed in this section include: (1) filler, (2) convection, (3) solid-phase thermal conductivity, and (4) consideration of an application with a nonuniform thermal boundary condition.

A. Filler

The principal reason for using a filler is to improve the flow of energy to or from the PCM. Most nonmetallic PCM's, particularly the paraffins, have very low thermal
conductivities; consequently, large thermal gradients may be required to effect the desired heat transfer between the PCM and the heated or cooled surface of the phase change device. Incorporation of the metallic filler within the PCM region yields a composite medium with a higher effective thermal conductivity. If the application imposes a size limitation on the phase change device, utilization of a filler reduces the volume available for the PCM. Consequently, when considering the energy storage capacity, the large heat of fusion of the displaced PCM is replaced by the specific heat-temperature rise product for the filler.

Fillers are discussed in References 2, 15, and 16. Some types which have been considered are:

- Aluminum powder
- Aluminum foam
- Aluminum wool
- Aluminum honeycomb
- Copper foam
- Alumina (Al₂O₃) foam
- Alumina (Al₂O₃) powder
- Aluminum fins (straight).

Based on tests involving aluminum wool, aluminum foam, copper foam, and aluminum honeycomb, it is reported [15] that honeycomb offers distinct advantages over the other three types. The results of a number of performance tests on honeycomb are given.

Straight fins offer some advantage over honeycomb in certain cases since better thermal contact between the filler and the housing of the phase-change device may be possible. Fins can be welded to the housing or they may be provided integrally with the housing via milling or casting.

The numerical study treated in Sections III and IV applies specifically to straight-finned arrangements. Likewise the experimental work presented in Reference 2 pertains to applications with straight fins.

The designer should evaluate the advantages and penalties of using a filler. Fabrication, thermal contact resistance, added weight, reduced energy storage capacity, and improved effective thermal conductivity are all interrelated factors. The effects of straight-fin spacing, height, and width on the thermal performance of the model treated in Sections III and IV are reflected in the results. Some comparative insight into the effect of the use of honeycomb filler may be obtained from the results of tests presented in Reference 15.
B. Convection

The physical model and the parametric numerical study which are presented in Sections III and IV are based on the conductive mode of heat transfer only within the PCM. If motion occurs within the liquid phase of the PCM, the heat transfer is affected. This behavior may require special consideration for applications where the liquid is subjected to surface or body forces of sufficient magnitude to initiate and sustain the motion. For the considerations given herein, there appears to be no previous or contemporary treatment of the formidable problem of an exact analysis of convective motion in enclosed cells of liquid. Some experimental correlations exist for certain cases, and these have been applied with some success to phase change problems. A brief background review and some considerations related to PCM studies are presented in the following paragraphs to aid the user in his study of convective effects.

1. Brief Review of Convective Considerations. If one neglects body force fields other than gravity and surface forces other than surface tension, convective motion within the liquid of a PCM could still be due to one or more of the following effects:

- Buoyancy forces
- Surface tension forces
- Density change as the PCM undergoes a phase change.

In early studies of buoyancy driven convection, Lord Rayleigh [17] related the Nusselt numbers (Nu) to the product of the Grashoff number (Gr) and the Prandtl number (Pr). The product of the latter two is known as the Rayleigh number (Ra).

Numerous experimenters have conducted studies that were related to hydrodynamic instability caused by buoyancy as it effects the heat transfer process, including the studies cited in References 18 through 29. O'Toole and Silveston [30] verified the Nusselt number versus Rayleigh number correlation for a fluid confined between two parallel plates (Fig. 108). A number of authors, including Edwards and Catton [31] later showed the effect of L/d ratios for closed cells (Fig. 109), where L is the cell height and d represents fin cell spacings.

The critical Rayleigh number is defined as the value at which convection begins. In the region below the critical value, the heat transfer is substantially by conduction only (i.e., Nu ≈ 1). The critical value, which depends on the boundary conditions, is 1708 for a fluid confined between two infinite horizontal, isothermal, conducting walls; it is 720 for the case of a fluid confined within a nonconducting walled container [29]. As shown for a bounded cell (Fig. 109a), this critical value tends to increase as the cell sides approach one another (i.e., as L/d increases), and it also tends to increase as the walls become more conducting (Fig. 109b).

Although scientists have been aware of surface tension driven convection for some time, quantitative studies of this phenomena appear to be scarce. The flow patterns created in Bénard's classic experiment [32], which produced cellular circulation patterns
Figure 108. Silveston's experimental results in the neighborhood of instability in various liquids (Fig. 11 of Reference 2).

Figure 109. Region of influence of L/d on heat transfer (Fig. 12 of Reference 2).
in a very shallow liquid, were initially attributed to buoyancy effects; however, Block [33], Pearson [34], Scivvy [35], and Sternling [36] later proved that this phenomenon was due to surface tension driven convection. The initial discovery of this phenomenon is attributed to Marangoni [37], and the term "Marangoni Flow" is commonly associated with this phenomenon.

The nature of the Marangoni flow, as discussed by Young, Goldstein and Block [38], is that temperature variations across a free gas/liquid interface produced variations in the shear force along the surface. This is due to the dependency of surface tension on temperature, which is estimated by Gambrill [39] to be linear:

\[ \sigma = \sigma_0 + bT \]  \hspace{1cm} (23)

Since the coefficient "b" is negative, an increase in temperature at the surface is accompanied by a subsequent decrease in the surface tension. Hershey [40] has shown qualitatively that a depression occurs in the surface at a local hot spot, causing the liquid to flow away from the hot zone and toward the cold zone (Fig. 110). McGrew and Larkin [41] photographed this effect for a number of configurations, producing dramatic verification of the phenomenon.

![Diagram of surface tension convection patterns](image)

Figure 110. Surface tension convection patterns (Fig. 13 of Reference 2).

The level of surface tension driven convection is correlated by using the Marangoni number, \( Ma \), which is given by

\[
Ma = \frac{-d\sigma}{dT} \frac{dT}{dy} \frac{L^2}{\rho \nu \alpha}
\]  \hspace{1cm} (24)
The critical Marangoni number for a fluid fixed between a rigid and free surface is given as 80.

Nields [42] has stated that the onset of convection in a standard one-g field might be better determined by correlating the Nusselt number with a normalized parameter, $R$, given by

$$R = \frac{Ra}{Ra_{cr}} + \frac{Ma}{Ma_{cr}}$$

(25)

This assumes that buoyancy and Marangoni driven convection are additive. Grodzka [43], however, noted that experimental data on buoyancy and Marangoni surface effects indicated that such an additive relation does not hold. Regardless, the convective currents caused by Marangoni flow in a standard one-g gravitational field are usually small compared to those caused by buoyancy. For most fluids at normal temperatures, Pearson [34] has shown that a liquid thickness of 1 cm or less must be attained before Marangoni effects overshadow buoyancy effects.

For systems with small characteristic dimensions, the nondimensional Bond number, $Bo$, is given by

$$Bo = \frac{\text{Gravity Forces}}{\frac{\rho g L^2}{\sigma}}$$

(26)

This number is sometimes used to evaluate the relative importance of Marangoni effects as compared to buoyancy effects. From equation (26), it follows that a low Bond number indicates a high degree of surface tension effects.

Although surface tension effects exist at all unlike interfaces, only the liquid/gas interface (as opposed to liquid/liquid and liquid/solid interfaces) is expected to produce appreciable resulting flows. However, no proof of this conjecture was noted in the literature surveyed.

Volume change driven convection can be caused by the phase change process. During freezing, the new layer of frozen material at the interface tends to contract, since the solid density is usually greater than the liquid density. Consequently, the liquid near the front will flow toward the interface to fill the volume shrinkage caused by solidification. During melting, the liquid at the interface tends to flow away to allow for the volume created by melting. Tien and Koump [44] have stated that this effect will cause both the freezing and melting process to be retarded. This retardation is due to the ingress of warm fluid during freezing and the egress of warm fluid during melting (Fig. 111)
Figure 111. Volume change driven flow patterns (Fig. 14 of Reference 2).

If liquid circulation is created in the melting processes, warm fluid could be drawn into the interface, thereby, augmenting the melting process.

Tien and Koump [44] have also shown, in a computational exercise for a fictitious system where the solid density is 25 percent greater than the liquid density, that only a 10 percent reduction in the freezing rate occurs due to volume change effects. Since paraffins experience only a 5 to 10 percent volume increase on melting, it can be inferred that an even smaller effect can be expected in paraffins for similar conditions.

In summary, the principal modes of heat transfer that are involved in a typical one-g static phase change process are conduction and convection. Convection is caused only by buoyancy driven currents for reasonable container sizes. Surface tension effects are negligible, except for very thin films, and volume change effects may be ignored.

2. Zero Gravity. Since a number of applications of phase change devices [2] have been in space flight situations, some attention is devoted here to zero-g performance.

When comparing environs of near Earth orbital space with that of the Earth, several differences are noted in the heat transfer process. In Earth orbit, the reduced gravitational force is nearly balanced by the centripetal orbital force, creating an effective zero-g environment. Effects caused by reduced pressure, radiation field, meteoroid bombardment, and three-dimensional spacecraft maneuvers are also possible.

Restricting the hypothetical phase change device under consideration to be a hermetically sealed container that is isolated from exterior thermal effects by insulation and antipenetration shields and to be aboard a nonmaneuvering vehicle, then reduced gravity remains as the only important alien effect.

The primary effect of reduced gravity on the heat transfer mechanism is in the lessening or elimination of buoyancy convection. Typical measurements of the net
gravitational acceleration force on a spacecraft indicates acceleration levels of the order $1 \times 10^{-7}$ g. Using nonadecane paraffin properties, a typical Rayleigh number of this low level of gravitational acceleration is

$$Ra = 0.01 L^3 \Delta T$$  

(27)

Equation (27) indicates that, for reasonable container sizes (i.e., less than 15 cm (6 in.) cell depths), the temperature difference across the liquidus portion of the cell must be $\geq 444^\circ C$ ($830^\circ F$) to produce buoyancy driven convection. Since this temperature difference is well in excess of normal operating values, buoyancy stimulated convection may be considered negligible.

At one-g, Marangoni flow or surface tension-driven convection is normally unimportant; however, at zero-g this is not necessarily true. As discussed earlier, liquid flow caused by surface tension at a liquid/vapor interface may occur.

A group of experiments, performed during the mission of NASA’s Apollo-14, revealed that the surface tension driven phenomena in zero-g is a reality and can produce significant convection [45,46]. Using data from these experiments, Grodzka [46] has plotted the relation existing between the Marangoni number and the ratio of effective thermal conductivity to actual conductivity (Fig. 112). These data show a rapid increase in the convective level at a Marangoni number slightly greater than 300. The temperature difference in the Krytox test liquid was only $2.5^\circ C$ ($4.1^\circ F$) in this instance.

Close examination of these data stimulates some questions. Applying these data to nonadecane paraffin contained in a cell with a 15.24 cm (6 in.) characteristic dimension at a Marangoni number of 300, a $K_{eff}/K$ of 12 is predicted in Figure 112. This is a very high convective level. The temperature difference across a parallel plate system required to reach this convective level is only $0.0011^\circ C$ ($0.002^\circ F$); however, for this same system, a Rayleigh number of $4.32 \times 10^4$ is predicted at one-g. From this, a $K_{eff}/K$ of only 2.5 is indicated, which is a convective level well below that predicted by the tentative surface tension data. This conflicts with earlier investigators, who reported that for this condition at one-g, Marangoni driven convection is unimportant. This could be explained, at least partially, by the fact that the Apollo 14 data were taken for a free surface, whereas the Figure 108 correlations were taken for top and bottom bounded cells.

Using nonadecane paraffin properties, evaluated at the phase change temperature, the Marangoni number is given by

$$Ma = 2374 \Delta TL$$  

(28)

This indicates that for reasonably sized cells, a temperature difference of only 0.003 K ($0.006^\circ F$) is necessary for the onset of Marangoni driven convection in a paraffin filled
device at zero-g. This fact, along with the high levels indicated in Figure 112, suggests that Marangoni convection can be appreciable at zero-g.

Since the magnitude of volume change effects discussed earlier were low for freezing (where buoyancy driven convection is negligible) as well as for melting, it may be implied that the volumetric effects are negligible for pure conduction as well as convective processes. Consequently, the volumetric effects on the heat transfer process are negligible for zero-g operation. Heat transfer modes occurring in a phase change device at zero-g are then reduced to Marangoni-driven convection and pure conduction.

![Figure 112. Heat transfer characteristics of Bénard cells (Fig. 15 of Reference 2).](image)
A secondary effect of reduced gravity which can significantly alter the heat transfer process is the ullage gas position. In a typical rectangular cell containing ullage volume, the ullage gas may configure itself in any of a number of possible modes. A number of possible cell ullage locations are given by Reference 2, those of more importance are shown in Figure 113. When heating/cooling from the bottom, only configurations d and e (Fig. 113) would alter the normal process; however, when heating/cooling from the top or sides, configurations a, b, d, and e would all reduce the rate of heat transfer due to the insulating effect of the ullage gas. Small bubbles occurring in the liquid could induce convective currents. In a zero-g field, this motion could be caused by the Marangoni flow phenomenon causing the bubbles to migrate toward warm zones. Also, bubbles could be entrapped in the freezing solid (configuration d of Fig. 113), thereby decreasing the apparent thermal conductivity of the PCM. Fortunately, paraffins have the property of being good surface wetters, which in zero-g tends to force the ullage to form in the center of the cell (configuration c of Fig. 113); however, insufficient quantitative data are available on these phenomena to determine the effect on the heat transfer process under given conditions.

![Diagram of various ullage configurations](image)

**Figure 113.** Zero-g ullage configurations (Fig. 16 of Reference 2).
Grodzka [4] has examined effects of the space environment on the microscopic processes. She concluded that complex coupling effects between phase change kinetics and various possible complex coupling effects between phase change kinetics and various possible modes of convective motion cannot be predicted accurately without actual flight data. She also concluded that the magnitude of magnetic and electric fields likely to be encountered in the Earth’s orbit are not expected to alter phase change behavior significantly from that observed on Earth. Finally, she stated that radiation fields encountered in Earth orbit are expected to have little effect, except perhaps in the case of an organic PCM where long time exposures will result in the buildup of impurity. Although the later statement applies to a paraffin, no definitive information on this effect could be found in the literature surveyed.

Although insufficient reduced gravity data are available to corroborate these findings, some observations have been made as to the mechanisms of heat transfer in a phase change device. When a free surface is present and thermal conditions are proper, Marangoni convection may be present. Insufficient data are currently available to determine quantitatively the influence of this phenomenon. In the absence of surface tension effects, conduction is the primary process governing heat transfer. The conduction process may, however, be altered by the ullage gas location. Furthermore, secondary effects of property degradation due to radiation may occur if the PCM is organic.

3. PCM Related Considerations. Regarding buoyancy-driven convective currents in the liquid during the melting of a PCM, a major factor is the positioning of the heated surface. If the heated surface is located at the bottom of a PCM cell during melting, convection may significantly augment the heat transfer. Conversely, the role of convection is minor, vanishing for one-dimensional cases, when the heated surface is at the top of the cell. The contribution of convection in the liquid to the heat transfer during solidification depends on positioning of the cooled surface, but it also depends on the amount by which the initial liquid temperature exceeds the fusion temperature of the PCM.

Prensner and Hsu [47] reported on studies of natural convection in air within closed cells. Their work revealed the possibility of a number of different geometry-dependent convective patterns, even under steady state conditions. Such studies attest to the formidable task of analytically predicting the transport mechanism occurring during natural convection within a bounded fluid. When considering convection within the liquid during melting of a PCM, the problem is further complicated by the fact that one of the boundaries is moving (i.e., the interface). Even if accurate predictions of the convective mechanism were possible, another unpredictable feature occurring in finned PCM devices is the tendency for the PCM to melt up the fin with the solid PCM core, subsequently, falling to the heated surface.

While the aforementioned difficulties appear pessimistic, some success has been reported [2, 48] on approximately accounting for convection during the melting of a paraffin. The approach made use of the correlations given by O'Toole and Silveston [30] which are:
Although the correlations given by equation (29) were determined from data on steady state heat transfer through horizontal layers of fluid, they were used as a means of estimating the convective effect within the liquid of finned PCM cells similar to the one modeled in Section III. The scheme involved continuous replacement of the thermal conductivity of the liquid by an effective thermal conductivity, which was calculated by

\[ K_{\text{eff}} = \text{Nu} \times k \]  

(30)

It was recognized that this scheme would not yield true temperatures in the liquid because the convective mechanism therein was being artificially replaced by conduction in a hypothetical medium with a larger thermal conductivity than that of the liquid PCM.

Figure 114 shows some experimental data for the interface location during melting of a paraffin inside a cell similar to the one modeled in Section III. Also shown are numerical predictions for conduction only and for inclusion of convection in the manner previously described. The numerical predictions shown in Figure 114 were achieved with an explicit formulation which was subject to a stability requirement. For nodes located in the fin, this requirement demanded excessive computer time; consequently, temperatures along the fin were approximated with the aid of measurements from thermocouples attached to the fins. With the approximate inclusion of convection, the predictions were promising. The predicted interfacial positions followed the same trend as the experimental data. Although magnitudes did not exactly agree, the difference appeared to be due to a shift in time and might have been associated with starting conditions. Use of this approach for including convection is discussed more extensively in Reference 2. Also presented is a more detailed discussion of the computer program and several comparisons between predictions and experimental data. The computer program is also listed and briefly discussed in Appendix D.

4. Solid Phase Thermal Conductivity. Nearly all published thermal conductivity values for paraffins apply to the liquid phase. Since the mechanism of heat conduction is typically different in solids than in liquids, one should expect some difference between the thermal conductivity value for the solid phase and that for the liquid phase of a PCM.

In studies reported in References 2 and 49, attempts to analytically predict the transient interfacial location during the phase change of a paraffin were more successful...
Figure 114. Comparison between predicted and measured fusion front location for an example melt run.
for the case of melting than for the case of solidification. Two plausible explanations for
the lack of success with solidification existed. One explanation was simply that the true
value of the thermal conductivity of the solid phase may not have been used in the
analysis. Knowledge of the thermal conductivity of the solid is more critical in analyses
of solidification than in those of melting. Another possible explanation was related to the
character of the solid-liquid interface during the phase transition. During melting, the
interface appeared smooth. During solidification, however, it was covered with numerous
projections called dendrites. Their presence may have affected the interfacial heat transfer
rate. The discrepancy may also have been due to a combination of both effects.

Thomas and Westwater [50] studied the nature of the interface during freezing
and melting of n-octadecane. Measured interfacial velocities exceeded analytical
predictions by as much as 100 percent; the difference was attributed to irregularities at
the interface.

Bailey and Davila [51] monitored the position of the solid-liquid interface during
solidification of hexadecane and octadecane. Bailey and Liao [52] found that effective
solid thermal conductivities, which yielded agreement between predictions and
experimental measurements, exceeded published values by factors ranging from 2.5 to 3.

Dyer and Griggs [49] measured interfacial positions during solidification of
nonadecane and hexadecane; they also determined the values of solid thermal
conductivity which forced agreement between the measurements and numerical
predictions. The work was extended to include measurement of the thermal
conductivities of solid nonadecane and hexadecane. The measured values did not agree
with the values required to force agreement between experimental and analytical results
(Table 16).

As previously mentioned, values of thermal conductivity for the solid phase of
paraffins are less prominent in the literature than are those for the liquid phase. Some of
the available values are summarized in Table 16 for reference.

In summary, analytical predictions of the solidification of paraffins suffer
accuracy due to the aforementioned effects. The designer needs to exercise some
judgement in the selection of an appropriate thermal conductivity value to use for the
solid phase. Uncertainty associated with the effect of interfacial irregularities (i.e.,
dendrites) renders the problem less amenable to an exact treatment.

C. Example Application with a Nonuniform Thermal
   Boundary Condition

The parametric study, which is presented in Section IV, pertains to a numerical
analysis of a symmetrical half of a single PCM cell. The results can be extended to
multicell phase change devices by appropriate scaling provided that the thermal boundary
condition imposed on the full-scale device is uniform and agrees with the type analyzed.
TABLE 16. SOME REPORTED THERMAL CONDUCTIVITY VALUES FOR THE SOLID PHASE OF CERTAIN PARAFFINS IN W/m·°C (Btu/h-ft·°F)

<table>
<thead>
<tr>
<th>Source</th>
<th>Paraffin Wax</th>
<th>Nonadecane</th>
<th>Hexadecane</th>
<th>Octadecane</th>
</tr>
</thead>
<tbody>
<tr>
<td>McAdams [53]</td>
<td>0.24 (0.14)</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Bailey and Liao [52]a</td>
<td>-</td>
<td>-</td>
<td>0.49 (0.28)</td>
<td>0.41 (0.24)</td>
</tr>
<tr>
<td>Dyer and Griggs [49]a</td>
<td>-</td>
<td>0.35 (0.20)</td>
<td>0.52 (0.30)</td>
<td>-</td>
</tr>
<tr>
<td>Dyer and Griggs [49]</td>
<td>-</td>
<td>0.26 (0.15)</td>
<td>0.35 (0.20)</td>
<td>-</td>
</tr>
<tr>
<td>Powell and Challar [54]</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>≈0.19 (≈0.11)</td>
</tr>
</tbody>
</table>

a. Value used to force agreement between analytical and experimental results.

One application of phase change devices involves heat transfer between the device and a flowing fluid. In such applications, the thermal condition existing at the separating surface between the PCM and the fluid is not necessarily uniform. An analysis of the heat transfer between the fluid and the PCM is not a simple problem. The entire apparatus including PCM, housing, and flow passage could be subdivided into a thermal network of nodes and interconnecting thermal resistances, and an appropriate numerical study could be performed. Such an approach, however, could involve a large number of nodes, particularly if much internal detail is needed. For a specific case, this approach may be the most desirable as well as productive; however, it certainly does not appear feasible to attempt to generalize it into an extensive parametric study, not only because of the vast nature of the task but also because of the large number of parameters which would be involved. The unwieldiness of the problem is evident from the integral analysis presented by Bailey and Liao [55] even where some simplifying assumptions were incorporated. An approximate approach which utilizes input from the parametric study is outlined in the following paragraphs.

The model under consideration is depicted in Figure 115. Fluid flows through the flow passage and makes intimate thermal contact with a surface separating it from a device filled with a PCM. For this model, it is assumed that the lower side of the flow passage is insulated or that a symmetrical PCM device is also located on the other side.
An energy balance on a differential length of the flow passage yields the following differential equation:

\[
\rho A_p c_p \frac{\partial T_m}{\partial t} + m c_p \frac{\partial T_m}{\partial x} = k A_p \frac{\partial^2 T_m}{\partial x^2} - k B (T_m - T_s)
\]  

Equation (31)

Neglecting any effect due to the surface separating the PCM and fluid, the instantaneous heat transfer between the surface and the PCM can be equated to the instantaneous heat transfer between the flowing fluid and the surface. This equality can be expressed by

\[
h(T_m - T_s) = U_0 (T_s - T_f)
\]  

Equation (32)

The right-hand side of equation (32) represents the instantaneous heat transfer per unit area between the surface and the PCM. The two-dimensional character of the problem has been masked by use of an overall heat transfer coefficient based on the difference between the surface temperature and the PCM fusion temperature. Equation (32) can be rearranged to yield the surface temperature as
\[
T_s = \frac{T_m}{1 + (U_0/h)} + \frac{T_f}{1 + h/U_0} \tag{33}
\]

Equation (33) implies that \(T_s \approx T_m\) when \(h\) greatly exceeds \(U_0\) and \(T_s \approx T_f\) when \(U_0\) greatly exceeds \(h\).

The elimination of \(T_s\) in equation (31) with the aid of equation (33) and the neglection of heat conduction in the flowing fluid give

\[
\rho c_p A \frac{\partial T_m}{\partial t} + \dot{m} c_p \frac{\partial T_m}{\partial x} = hB \left( \frac{T_f}{1 + h/U_0} - \frac{T_m}{1 + h/U_0} \right) \tag{34}
\]

Equation (34) can be solved numerically for both spatial and temporal variations of \(T_m\); the solution, however, requires input of the convective heat transfer coefficient \(h\) for the flowing fluid and the overall coefficient \(U_0\) for heat transfer with the PCM. The computer programs (listed in Section IV) provided a value of \(U_0\) as part of the output. This value is determined by dividing the instantaneous heat transfer per unit area between the surface and the PCM by the difference in the surface temperature and the fusion temperature of the PCM. Although the computations apply to a symmetrical cell, use of the \(U_0\) values may be acceptable since lateral heat conduction between adjacent PCM cells is probably much less significant that heat transfer between the surface and the PCM. If this hypothesis is valid, the \(U_0\) values may serve as a convenient aid in analyzing the variation of \(T_m\). The solutions discussed in the following paragraphs are restricted to those cases for which the aforementioned assumptions are meaningful.

The time derivative can be approximated by

\[
\frac{\partial T_m}{\partial t} \approx \frac{T_{m_{i+1,n}} - T_{m_{i,n}}}{\Delta t} \tag{35}
\]

The space derivative can be approximated by

\[
\frac{\partial T_m}{\partial x} \approx \frac{T_{m_{i+1,n}} - T_{m_{i+1,n-1}}}{\Delta x} \tag{36}
\]

The first subscript denotes time while the second subscript denotes space.
Insertion of the difference expressions given in equations (35) and (36) into equation (34) and some rearrangement result in

\[
T_{m_{i+1,n}} = \frac{1}{\rho c_p A \frac{\dot{m} c_p}{(1 + h/\rho \omega_0)} + \frac{hB}{\Delta t} + \frac{\dot{m} c_p}{\Delta x} + \frac{h}{\rho \omega_0}} \left[ h B T_f \left( \frac{\dot{m} c_p}{(1 + h/\rho \omega_0)} \right) = \frac{\dot{m} c_p}{\Delta t} T_{m_{i+1,n-1}} + \frac{\rho c_p A}{\Delta t} T_{m_{i,n}} \right]
\]

Equation (37) facilitates computation of the temperature of a node in the fluid at time \( t + \Delta t \) from the previous temperature of that node at time \( t \) and the temperature of the adjacent node at \( x - \Delta x \) and at time \( t + \Delta t \). The nodal pattern is illustrated in Figure 116. The computations require input of the initial temperature distribution in the fluid (i.e., temperatures along the horizontal axis of Figure 116) and the transient variation of the fluid temperature at the inlet to the flow passage (i.e., temperatures along the vertical axis of Fig. 116). The internal convective heat transfer coefficient for the flowing fluid must be estimated from knowledge of the character of the flow (e.g., laminar, fully-developed, turbulent, etc.). The value of \( \omega_0 \) is dependent on the amount of PCM which has undergone phase change; consequently, the computational scheme should incorporate sequential alteration of \( \omega_0 \). One method employed in the following computations involved representing \( \omega_0 \) as a function of the fraction of PCM which has undergone phase change. The fraction is determined at each location and used in the functional representation to determine \( \omega_0 \) for use in the computation of \( T_m \).

Figure 116. Nodal pattern for calculating bulk temperature of flowing fluid.
A computer program which facilitates computations of the type previously outlined is listed in Appendix C. Exploratory calculations have been made, but the predictions have not been corroborated by any experimental data. The program was developed and intended as a tool for investigating the effect of the phase change process, flow rate, and internal convective heat transfer coefficient on the temperature at the exit of a flow passage which exchanges energy with the phase change device. Without some history of comparison with experimental data, the integrity of the results can only be conjectured at this stage.

The following parameters were used in two computer runs to examine the influence of the internal convective heat transfer coefficient inside the flow passage. The results were generated for the case of a sinusoidal variation of the fluid temperature at the inlet of the flow passage. Initially, the flow was stagnant with the fluid in the passage and the entire phase change device considered to be at the fusion temperature of the PCM.

- Length of phase change device = 31.7 cm (1.04 ft)
- Width of phase change device = 7.6 cm (3 in.)
- PCM thickness = 3.8 cm (1.5 in.)
- Flowing fluid = Water
- Flow rate = 1.89 gm/s (15 lb/h)
- PCM fusion temperature = 32.2°C (90°F)
- \( U_0 \) variation with fraction of PCM melted, \( f \) → \( U_0 = 86.02 - 76.45 f \) Btu/h-ft\(^2\)-°F or \( U_0 = 488.36 - 434.03 f \) J/s-m\(^2\)-°C
- Temperature variation of fluid at passage inlet → \( T_{inlet} = 110 + 10 \sin(6.283 t) \) °F with \( t \) (h) or \( T_{inlet} = 43.33 + 5.56 \sin(2.26 \times 10^4 t) \) °C with \( t \) (s)
- Convective heat transfer coefficient inside flow passage → 1st Run: \( h = 283.86 \) J/s-m\(^2\)-°C (50 Btu/h-ft\(^2\)-°F); 2nd Run: \( h = 28.39 \) J/s-m\(^2\)-°C (5 Btu/h-ft\(^2\)-°F).

The results for the fluid temperature at the exit of the flow passage are shown in comparison to the inlet temperature variation in Figure 117. For both heat transfer coefficients, the temperature drop across the passage decreases as the PCM melts (as would be expected). The relative influence of the heat transfer coefficient on the temperature drop across the passage is evident.
Figure 117. Example exit temperature variations for a sinusoidal temperature at inlet.
APPENDIX A
PROPERTY APPENDIX

Tabulations of certain properties for a number of phase change materials are given in this appendix. They have been included principally to serve as a convenient reference for the user; they are not intended to be exhaustive. For more detail regarding the advantages and disadvantages of the listed materials, the referenced literature should be consulted.

Following is an index for the tabulations:

Table A-1. Fusible Materials with a Heat of Fusion Greater than 185.96 kJ/kg (80 Btu/lb) Listed in Order of Increasing Melt Temperature from 4.4 to 65°C (40 to 150°F). (Table 3-1 of Reference 15).

Table A-2. Comparisons of Fusion Properties of Selected PCM's Reported in Reference 56 (From Table 3 of Reference 56)

Table A-3. Properties of Six PCM’s Studied in Reference 57 with a Fusion Temperature Between 273 and 373 K (From Table 2 of Reference 57)

Table A-4. Melt Temperatures of Fifteen Low Temperature PCM Candidates (From Table 6 of Reference 57)

Table A-5. Four Low Temperature PCM’s Recommended in Reference 57 (From Table 7 of Reference 57)

Table A-6. PCM’s Used in Tests Reported in Reference 58 (From Table 1 of Reference 58)

Table A-7. Comparative Data on Waxes and Inorganic Hydrates (From Reference 59)

Table A-8. Melting Temperature of Some Mixtures Investigated for Air Conditioning Thermal Energy Storage (From Table 3-1 of Reference 1)

Table A-9. Melting Temperature and Heat of Fusion of Inorganic Salt Hydrates (From Table 3-3 of Reference 1)

Table A-10. Melting Point, Heat of Fusion, and Latent Heat Density of Some Inorganic Hydrates (From Table 3-4 of Reference 1)

Table A-11. Melting Temperature and Heat of Fusion of Some Inorganic Hydrate Eutectics (From Table 3-5 of Reference 1)
Table A-12. Calorimetric Data for Some Organic Thermal Energy Storage Materials for Solar Heating (From Table 3-6 of Reference 1)

Table A-13. Melting Temperature and Heat of Fusion of Organic Materials for Air Conditioning-Hydrocarbons and Halogenated Hydrocarbons (From Table 3-7 of Reference 1)

Table A-14. Melting Point and Heat of Fusion for Some Organic Eutectics (From Table 3-7 of Reference 1)

Table A-15. Melting Temperature and Heat of Fusion for Some Acetamide-Organic Eutectics and Compounds (From Table 3-7 of Reference 1)

Table A-16. PCM Data for Some Alcohols, Phenols, Aldehydes, Ketones, and Ethers (From Table 3-7 of Reference 1)

Table A-17. PCM Data for Organic Acids and Miscellaneous Organics (From Table 3-7 of Reference 1)

Table A-18. Calorimetric Data for Air Conditioning Organic Materials (From Table 3-8 of Reference 1)

Table A-19. Melting Point and Heat of Fusion for Clathrates and Semi-Clathrates Melting Congruently Above 0°C (32°F) (From Table 3-9 of Reference 1)

Table A-20. Melting Point for Organic-Inorganic Eutectics (From Table 3-10 of Reference 1).
TABLE A-1. FUSIBLE MATERIALS WITH A HEAT OF FUSION GREATER THAN 185.96 kJ/kg (80 Btu/lb) LISTED IN ORDER OF INCREASING MELT TEMPERATURE FROM 4.4 TO 65°C (40 TO 150°F). (FROM TABLE 3-1 OF REFERENCE 15).

<table>
<thead>
<tr>
<th>Material</th>
<th>Melting Point °C (°F)</th>
<th>Heat of Fusion kJ/kg (Btu/lb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tetradecane C\textsubscript{14}H\textsubscript{30}</td>
<td>5.6 (42)</td>
<td>227.6 (98)</td>
</tr>
<tr>
<td>Formic Acid HCOOH</td>
<td>7.8 (46)</td>
<td>246.1 (106)</td>
</tr>
<tr>
<td>Pentadecane C\textsubscript{15}H\textsubscript{32}</td>
<td>10 (50)</td>
<td>206.7 (89)</td>
</tr>
<tr>
<td>Myristic Acid Ethyl Ester (\text{CH}_3(\text{CH}<em>2)</em>{12}\text{COOC}_2\text{H}_5)</td>
<td>11 (51)</td>
<td>185.8 (80)</td>
</tr>
<tr>
<td>Acetic Acid CH\textsubscript{3}CO\textsubscript{2}H</td>
<td>17 (62)</td>
<td>185.8 (80)</td>
</tr>
<tr>
<td>Hexadecane C\textsubscript{16}H\textsubscript{34}</td>
<td>18 (64)</td>
<td>236.8 (102)</td>
</tr>
<tr>
<td>Lithium Chloride Ethanolate LiCl·4C\textsubscript{2}H\textsubscript{6}O</td>
<td>21 (69)</td>
<td>185.8 (80)</td>
</tr>
<tr>
<td>n-Heptadecane C\textsubscript{17}H\textsubscript{36}</td>
<td>22 (71)</td>
<td>213.6 (92)</td>
</tr>
<tr>
<td>d-Lactic Acid CH\textsubscript{3}CHOHCOOH</td>
<td>26 (79)</td>
<td>185.8 (80)</td>
</tr>
<tr>
<td>Octadecane C\textsubscript{18}H\textsubscript{38}</td>
<td>28 (82)</td>
<td>243.8 (105)</td>
</tr>
<tr>
<td>13-Methyl Pentacosane C\textsubscript{26}H\textsubscript{54}</td>
<td>29 (84)</td>
<td>195.1 (84)</td>
</tr>
<tr>
<td>Methyl Palmitate C\textsubscript{27}H\textsubscript{34}O\textsubscript{2}</td>
<td>29 (84)</td>
<td>104.3 (88)</td>
</tr>
<tr>
<td>Nonadecane C\textsubscript{19}H\textsubscript{40}</td>
<td>32 (90)</td>
<td>220.6 (95)</td>
</tr>
<tr>
<td>Trimyristin (C\textsubscript{13}H\textsubscript{27}COO)\textsubscript{3}C\textsubscript{3}H\textsubscript{3}</td>
<td>33 (91)</td>
<td>202.0 (87)</td>
</tr>
<tr>
<td>2-Dimethyl-n-docosane C\textsubscript{24}H\textsubscript{50}</td>
<td>35 (95)</td>
<td>197.4 (85)</td>
</tr>
<tr>
<td>Eicosane C\textsubscript{20}H\textsubscript{42}</td>
<td>37 (98)</td>
<td>246.1 (106)</td>
</tr>
<tr>
<td>1-Tetradecanol CH\textsubscript{3}(CH\textsubscript{2})\textsubscript{12}CH\textsubscript{2}OH</td>
<td>38 (100)</td>
<td>229.9 (99)</td>
</tr>
<tr>
<td>Camphenilone C\textsubscript{9}H\textsubscript{14}O</td>
<td>39 (102)</td>
<td>204.3 (88)</td>
</tr>
<tr>
<td>Caprylone (CH\textsubscript{3}(CH\textsubscript{2})\textsubscript{6})\textsubscript{2}CO</td>
<td>40 (104)</td>
<td>257.7 (111)</td>
</tr>
<tr>
<td>Docosyl Bromide C\textsubscript{22}H\textsubscript{45}BR</td>
<td>40 (104)</td>
<td>202.0 (87)</td>
</tr>
<tr>
<td>Heneicosane C\textsubscript{21}H\textsubscript{44}</td>
<td>41 (105)</td>
<td>213.6 (92)</td>
</tr>
<tr>
<td>7-Heptadecamone C\textsubscript{17}H\textsubscript{34}O</td>
<td>41 (105)</td>
<td>199.7 (86)</td>
</tr>
<tr>
<td>1-Cyclohexyloctadecane C\textsubscript{24}H\textsubscript{28}</td>
<td>41 (106)</td>
<td>218.3 (94)</td>
</tr>
<tr>
<td>4-Heptadecanone C\textsubscript{17}H\textsubscript{34}O</td>
<td>41 (106)</td>
<td>197.4 (85)</td>
</tr>
<tr>
<td>Material</td>
<td>Melting Point °C (°F)</td>
<td>Heat of Fusion kJ/kg (Btu/lb)</td>
</tr>
<tr>
<td>----------</td>
<td>------------------------</td>
<td>-------------------------------</td>
</tr>
<tr>
<td>8-Heptadecanone C_{17}H_{34}O</td>
<td>42 (107)</td>
<td>202.0 (87)</td>
</tr>
<tr>
<td>Cyanamide CH_{2}N_{2}</td>
<td>44 (111)</td>
<td>209.0 (90)</td>
</tr>
<tr>
<td>Docosane C_{22}H_{46}</td>
<td>44 (112)</td>
<td>248.5 (107)</td>
</tr>
<tr>
<td>Methyl Eicosanate C_{21}H_{42}O_{2}</td>
<td>45 (113)</td>
<td>227.6 (98)</td>
</tr>
<tr>
<td>Tricosane C_{23}H_{48}</td>
<td>47 (117)</td>
<td>232.2 (100)</td>
</tr>
<tr>
<td>3-Heptadecanone C_{17}H_{34}O</td>
<td>48 (118)</td>
<td>216.0 (93)</td>
</tr>
<tr>
<td>2-Eptadecanone C_{17}H_{34}O</td>
<td>48 (119)</td>
<td>216.0 (93)</td>
</tr>
<tr>
<td>Camphene C_{10}H_{16}</td>
<td>50 (122)</td>
<td>239.2 (103)</td>
</tr>
<tr>
<td>9-Heptadecanone C_{17}H_{34}O</td>
<td>51 (123)</td>
<td>211.3 (91)</td>
</tr>
<tr>
<td>Tetracosane C_{24}H_{50}</td>
<td>51 (124)</td>
<td>253.1 (109)</td>
</tr>
<tr>
<td>Elaidic Acid C_{18}H_{34}O_{2}</td>
<td>51 (124)</td>
<td>218.3 (94)</td>
</tr>
<tr>
<td>Methyl Behenate C_{24}H_{46}O_{2}</td>
<td>52 (126)</td>
<td>232.2 (100)</td>
</tr>
<tr>
<td>Pentacosane C_{25}H_{52}</td>
<td>54 (129)</td>
<td>236.8 (102)</td>
</tr>
<tr>
<td>Ethyl Lignocerate C_{26}H_{52}O_{2}</td>
<td>54 (129)</td>
<td>216.0 (93)</td>
</tr>
<tr>
<td>Hypo Phosphoric Acid H_{4}P_{2}O_{6}</td>
<td>55 (131)</td>
<td>213.6 (92)</td>
</tr>
<tr>
<td>n-Hexacosane C_{26}H_{54}</td>
<td>56 (133)</td>
<td>255.4 (110)</td>
</tr>
<tr>
<td>Myristic Acid C_{13}H_{27}COOH</td>
<td>57 (135)</td>
<td>199.7 (86)</td>
</tr>
<tr>
<td>Heptacosane C_{27}H_{56}</td>
<td>59 (138)</td>
<td>234.5 (101)</td>
</tr>
<tr>
<td>Ethyl Cerotate C_{28}H_{56}O_{2}</td>
<td>60 (140)</td>
<td>222.9 (96)</td>
</tr>
<tr>
<td>Octacosane C_{28}H_{58}</td>
<td>61 (142)</td>
<td>253.1 (109)</td>
</tr>
<tr>
<td>Nonacosane C_{29}H_{60}</td>
<td>64 (147)</td>
<td>239.2 (103)</td>
</tr>
<tr>
<td>Stearic Acid C_{17}H_{35}CO_{2}H</td>
<td>64 (148)</td>
<td>199.7 (86)</td>
</tr>
<tr>
<td>Triacontane C_{30}H_{62}</td>
<td>65 (150)</td>
<td>250.8 (108)</td>
</tr>
<tr>
<td></td>
<td>Formic Acid</td>
<td>Tetradecane</td>
</tr>
<tr>
<td>--------------------------------</td>
<td>-------------</td>
<td>-------------</td>
</tr>
<tr>
<td>Heat of Fusion</td>
<td></td>
<td></td>
</tr>
<tr>
<td>kJ/kg (Btu/lb)</td>
<td>246.1 (106)</td>
<td>227.6 (98)</td>
</tr>
<tr>
<td>From Literature</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Melt Temperature</td>
<td></td>
<td></td>
</tr>
<tr>
<td>°C (°F)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>From Literature</td>
<td>7.8 (46)</td>
<td>5.6 (42)</td>
</tr>
<tr>
<td>Supplier&lt;sup&gt;a&lt;/sup&gt;</td>
<td>—</td>
<td>1.7-3.9</td>
</tr>
<tr>
<td>Tests&lt;sup&gt;a&lt;/sup&gt;</td>
<td>8.3 (47)</td>
<td>5 (41)</td>
</tr>
<tr>
<td>Supercooling to</td>
<td>0.6-1.7</td>
<td>None</td>
</tr>
<tr>
<td></td>
<td>(33-35)</td>
<td>Observed</td>
</tr>
</tbody>
</table>

<sup>a</sup> As reported in Reference 56.
<table>
<thead>
<tr>
<th>PCM</th>
<th>Formula</th>
<th>Melt Temperature K (°R)</th>
<th>Heat of Fusion kJ (Btu) kg⁻¹ (lb⁻¹)</th>
<th>Phase</th>
<th>Density kg m⁻³ (lbm ft⁻³)</th>
<th>Thermal Conductivity W m⁻²°C⁻¹ (Btu h⁻¹°F⁻¹)</th>
<th>Constant Pressure Specific Heat kJ kg⁻¹ (Btu lbm⁻¹°F⁻¹)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Octadecane</td>
<td>C₁₈H₃₈</td>
<td>301 (541.8)</td>
<td>242.2 (104.4)</td>
<td>Solid</td>
<td>810 (50.6)</td>
<td>0.15 (0.087)</td>
<td>2.18 (0.52)</td>
</tr>
<tr>
<td>LiNO₃·3H₂O</td>
<td></td>
<td>303 (545.4)</td>
<td>296.5 (127.8)</td>
<td>Solid</td>
<td>1550 (96.8)</td>
<td>0.50 (0.29)</td>
<td>2.09 (0.50)</td>
</tr>
<tr>
<td>Sodium Hydrogen Phosphate Dodecahydrate</td>
<td>Na₂HPO₄·12H₂O</td>
<td>309 (556.2)</td>
<td>279.8 (120.6)</td>
<td>Solid</td>
<td>1520 (94.9)</td>
<td>0.51 (0.30)</td>
<td>1.67 (0.40)</td>
</tr>
<tr>
<td>Myristic Acid</td>
<td>CH₃(CH₂)₁₂COOH</td>
<td>331 (595.8)</td>
<td>200.5 (86.4)</td>
<td>Solid</td>
<td>860 (53.7)</td>
<td>0.10 (0.058)</td>
<td>1.59 (0.38)</td>
</tr>
<tr>
<td>Cerrobend</td>
<td>Alloy</td>
<td>343 (617.4)</td>
<td>33.4 (14.4)</td>
<td>Solid</td>
<td>9400 (586.8)</td>
<td>18.8 (10.89)</td>
<td>0.17 (0.04)</td>
</tr>
<tr>
<td>Acetamide</td>
<td>CH₃CONH₂</td>
<td>354 (637.2)</td>
<td>242.2 (104.4)</td>
<td>Solid</td>
<td>1160 (72.4)</td>
<td>0.25 (0.15)</td>
<td>2.80 (0.67)</td>
</tr>
</tbody>
</table>
TABLE A-4. MELT TEMPERATURES OF FIFTEEN LOW TEMPERATURE PCM CANDIDATES (FROM TABLE 6 OF REFERENCE 57)

<table>
<thead>
<tr>
<th>PCM</th>
<th>Melt Temperature K (°R)</th>
</tr>
</thead>
<tbody>
<tr>
<td>n-Dodecane</td>
<td>263.4 (474.1)</td>
</tr>
<tr>
<td>Transit-Heat</td>
<td>257.2 (463.0)</td>
</tr>
<tr>
<td>Formic Acid/Formamide (75.5/24.5)</td>
<td>256.4 (461.5)</td>
</tr>
<tr>
<td>Transit-Heat</td>
<td>252.2 (414.0)</td>
</tr>
<tr>
<td>Acetic Acid/Formic Acid (53.1/46.9)</td>
<td>250.7 (451.3)</td>
</tr>
<tr>
<td>Acetic Acid/Water (58%)</td>
<td>246.8 (444.2)</td>
</tr>
<tr>
<td>n-Decane</td>
<td>243.3 (437.9)</td>
</tr>
<tr>
<td>Formic Acid/Potassium Formate (75/25)</td>
<td>241.7 (435.1)</td>
</tr>
<tr>
<td>Heptanone-4</td>
<td>240.2 (432.4)</td>
</tr>
<tr>
<td>Hydrazine Hydrate (N₂H₄ · H₂O)</td>
<td>233.2 (419.8)</td>
</tr>
<tr>
<td>Monomethylhydrazine</td>
<td>220.8 (397.4)</td>
</tr>
<tr>
<td>Tricaprolin</td>
<td>213 (383.4)</td>
</tr>
<tr>
<td>Tributyrin</td>
<td>198 (356.4)</td>
</tr>
<tr>
<td>Ammonia</td>
<td>195 (351.0)</td>
</tr>
<tr>
<td>Ammonia Hydrate (NH₃ · H₂O)</td>
<td>194 (349.2)</td>
</tr>
<tr>
<td>PCM</td>
<td>Melt Temperature K (°F)</td>
</tr>
<tr>
<td>--------------</td>
<td>-------------------------</td>
</tr>
<tr>
<td>n-Dodecane</td>
<td>263 (473.4)</td>
</tr>
<tr>
<td>Heptanone-4</td>
<td>240 (432)</td>
</tr>
<tr>
<td>Tricaprin</td>
<td>213 (383.4)</td>
</tr>
<tr>
<td>Ammonia</td>
<td>195 (351)</td>
</tr>
</tbody>
</table>

a. Represents estimated data.
<table>
<thead>
<tr>
<th>Pairings</th>
<th>Name</th>
<th>Formula</th>
<th>Melting Point °C (°F)</th>
<th>Density $\frac{\text{kg}}{\text{m}^3}$ (lb/ft$^3$)</th>
<th>Surface Tension $\frac{\text{N}}{\text{m}}$ (lb/ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>n-Octadecane</td>
<td>$C_{18}H_{38}$</td>
<td>28.0 (82.4)</td>
<td>774 (48.3)</td>
<td>$2.745 \times 10^{-3}$ ($1.880 \times 10^{-6}$)</td>
</tr>
<tr>
<td></td>
<td>Tripalmitin</td>
<td>$C_{51}H_{98}O_6$</td>
<td>65.1 (149.2)</td>
<td>866 (54.1)</td>
<td>$2.878 \times 10^{-3}$ ($1.971 \times 10^{-6}$)</td>
</tr>
<tr>
<td></td>
<td>m-Phenylenediamine</td>
<td>$C_6H_8O_2$</td>
<td>62.8 (145.1)</td>
<td>1107 (49.1)</td>
<td>$4.22 \times 10^{-3}$ ($2.890 \times 10^{-6}$)</td>
</tr>
<tr>
<td>2</td>
<td>Trilavrin</td>
<td>$C_{39}H_{74}O_6$</td>
<td>46.5 (115.7)</td>
<td>891 (55.6)</td>
<td>$2.00 \times 10^{-3}$ ($20.55 \times 10^{-6}$)</td>
</tr>
<tr>
<td></td>
<td>O-Iodonitro</td>
<td>$C_6H_4INO_2$</td>
<td>49.4 (120.9)</td>
<td>1810 (:13.0)</td>
<td>$4.31 \times 10^{-3}$ ($29.52 \times 10^{-6}$)</td>
</tr>
<tr>
<td>3</td>
<td>n-Tetracosane</td>
<td>$C_{24}H_{50}$</td>
<td>54.0 (129.2)</td>
<td>779 (48.6)</td>
<td>$3.00 \times 10^{-3}$ ($20.55 \times 10^{-6}$)</td>
</tr>
<tr>
<td></td>
<td>Tristearin</td>
<td>$C_{57}H_{110}O_6$</td>
<td>54.5 (130.1)</td>
<td>862 (53.8)</td>
<td>--</td>
</tr>
<tr>
<td>4</td>
<td>n-Octacosane</td>
<td>$C_{28}H_{58}$</td>
<td>65.0 (149.0)</td>
<td>779 (48.6)</td>
<td>$2.878 \times 10^{-3}$ ($1.971 \times 10^{-6}$)</td>
</tr>
<tr>
<td></td>
<td>Tripalmitin</td>
<td>$C_{51}H_{98}O_6$</td>
<td>65.1 (149.2)</td>
<td>866 (54.1)</td>
<td>--</td>
</tr>
<tr>
<td>Materials Compatibility</td>
<td>Waxes (Paraffins and Microcrystalline)</td>
<td>Inorganic Hydrates</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-------------------------</td>
<td>---------------------------------------</td>
<td>-------------------</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Compatible with most metals and plastics, waxes generally nonreactive and stable</td>
<td></td>
<td>Unknown, material composition proprietary</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Thermal Properties</td>
<td>Heat of Fusion: $\sim 1.67 \times 10^5 \text{J/kg} (\sim 71.8 \text{ Btu/lb})$</td>
<td>Heat of Fusion: $\sim 2.6 \times 10^5 \text{J/kg} (\sim 111.9 \text{ Btu/lb})$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Specific Heat:</td>
<td>Specific Heat:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Solid -- 1886-2019 J/kg°C (0.45-0.48 Btu/lb°F)</td>
<td>Solid -- 1464-2092 J/kg°C (0.35-0.50 Btu/lbm°F)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Liquid -- 2019-2837 J/kg°C (0.48-0.68 Btu/lb°F)</td>
<td>Liquid -- 2092-3347 J/kg°C (0.50-0.80 Btu/lb°F)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Thermal Conductivity: $\sim 0.146 \text{ J/m-s°C}$ ($\sim 0.085 \text{ Btu/h-ft°F}$)</td>
<td>Thermal Conductivity: Unknown</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Expansion on Liquification: 15-20%</td>
<td>Expansion on Liquification: Unknown</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Maximum Allowable Temperature</td>
<td>200°C (400°F) (Higher with oxidation inhibitor)</td>
<td>104°C (220°F) (Limited by decomposition/Vapor generation)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Phase Change Reversibility</td>
<td>High melting temperature paraffins should exhibit minimal supercooling, based on tests of low melting paraffins for many cycles. Microcrystallines should have similar behavior</td>
<td>Unknown, limited test conducted, data insufficient</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fabricability</td>
<td>Small sample fabricated to examine PCM behavior, no special problems</td>
<td>Unknown, proprietary to Royal Industries</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cost/Availability</td>
<td>Cheap, readily available</td>
<td>Uncertain</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Composition</td>
<td>Mole Weight (gram)</td>
<td>Relative Weight (%)</td>
<td>Melting Point °C (°F)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-------------</td>
<td>--------------------</td>
<td>---------------------</td>
<td>----------------------</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Na₂SO₄</td>
<td>142.04</td>
<td>37.2</td>
<td>18 (64)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10 H₂O</td>
<td>180.15</td>
<td>47.4</td>
<td></td>
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</tr>
<tr>
<td>NaCl</td>
<td>58.44</td>
<td>15.4</td>
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</tr>
<tr>
<td>Total</td>
<td>380.63</td>
<td>100.0</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Na₂SO₄</td>
<td>142.04</td>
<td>37.8</td>
<td>10 (50)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10 H₂O</td>
<td>180.15</td>
<td>48.0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NH₄Cl</td>
<td>53.49</td>
<td>14.2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>375.68</td>
<td>100.0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Na₂SO₄</td>
<td>142.04</td>
<td>37.6</td>
<td>13 (55)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10 H₂O</td>
<td>180.15</td>
<td>47.6</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1/2 NaCl</td>
<td>29.22</td>
<td>7.7</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1/2 NH₄Cl</td>
<td>26.75</td>
<td>7.1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>378.16</td>
<td>100.0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Na₂SO₄</td>
<td>142.04</td>
<td>35.8</td>
<td>7 (45)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10 H₂O</td>
<td>180.15</td>
<td>45.4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>KCl</td>
<td>74.55</td>
<td>18.8</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>396.74</td>
<td>100.0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Compound</td>
<td>Heat of Fusion $\frac{\text{kJ}}{\text{kg}}$</td>
<td>Melting Point $^\circ\text{C}$ ($^\circ\text{F}$)</td>
<td>Remarks</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-----------------</td>
<td>---------------------------------------------</td>
<td>-----------------------------------------------</td>
<td>-------------------------------------------------------------------------</td>
<td></td>
<td></td>
</tr>
<tr>
<td>LiClO$_4$·3H$_2$O</td>
<td>253.1 (108.9)</td>
<td>8.1 (46.5)</td>
<td>Needs cold finger to prevent supercooling. Not available commercially. The least expensive lithium salt is the carbonate: 1971 carlot price $0.52-0.64$ per lb. Safety hazard. Containment difficult.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>H$_2$SO$_4$·H$_2$O</td>
<td>163.6 (70.4)</td>
<td></td>
<td>Strong tendency to form glasses. Cold finger needed to prevent supercooling.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ZnCl$_2$·3H$_2$O</td>
<td>108.8 (46.8)</td>
<td></td>
<td>Crystalization velocity at all temperatures very slow. Cold finger needed to prevent glass formation. Safety hazard, containment difficult.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>K$_2$HPO$_4$·5H$_2$O</td>
<td>129.0 (55.4)</td>
<td></td>
<td>Congruent only if melted quickly. Slow melting forms metastable $\alpha$CaCl$_2$·4H$_2$O incongruently.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>NaOH·7H$_2$O</td>
<td>192.5 (82.9)</td>
<td></td>
<td>Crystalization velocity at all temperatures very slow. Cold finger needed to prevent supercooling.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Na$_2$CrO$_4$·10H$_2$O</td>
<td>164.0 (70.6)</td>
<td></td>
<td>Congruent only if melted quickly. Slow melting forms metastable $\alpha$CaCl$_2$·4H$_2$O incongruently.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>CaCl$_2$·6H$_2$O</td>
<td>170.3 (76.3)</td>
<td></td>
<td>Requires cold finger to prevent supercooling.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>LiNO$_3$·3H$_2$O</td>
<td>295.8 (127.3)</td>
<td></td>
<td>Forms complex system with HCl. Crystallization velocities at all temperatures very slow.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Na$_2$SO$_4$·10H$_2$O</td>
<td>30.0 (86)</td>
<td></td>
<td>Reduction in apparent latent with freeze/melt cycling.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>FeCl$_3$·6H$_2$O</td>
<td>36.1 (97.3)</td>
<td></td>
<td>Requires cold finger to prevent supercooling.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Compound</td>
<td>Melting Point °C (°F)</td>
<td>Heat of Fusion (\frac{kJ}{kg}) ((\frac{Btu}{lb}))</td>
<td>Remarks</td>
<td></td>
<td></td>
</tr>
<tr>
<td>----------------------</td>
<td>-----------------------</td>
<td>-------------------------------------------------</td>
<td>------------------------------------------</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Zn((NO_3)_2) · 6H_2O</td>
<td>36.1 (97)</td>
<td>133.9 (57.6)</td>
<td>Nucleating agent available; expensive</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Na_2HPO_4 · 12H_2O</td>
<td>36.1 (97)</td>
<td>279.5 (129.3)</td>
<td>Supercools; nucleates unstable heptahydrate</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Na_2S_2O_3 · 5W</td>
<td>45 (113)</td>
<td>200.0 (86.1)</td>
<td>Supercools; unstable relative to sulfur and sodium sulfate</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ca((NO_3)_2) · 4H_2O</td>
<td>47.2 (117)</td>
<td>161.9 (69.7)</td>
<td>Supercools</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mg((NO_3)_2) · 6H_2O</td>
<td>90.0 (194)</td>
<td>159.0 (68.4)</td>
<td>Corrosive</td>
<td></td>
<td></td>
</tr>
<tr>
<td>MgCl_2 · 6H_2O</td>
<td>115.0 (239)</td>
<td>164.9 (71.0)</td>
<td>Corrosive</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
TABLE A-10. MELTING POINT, HEAT OF FUSION, AND LATENT HEAT DENSITY OF SOME INORGANIC HYDRATES (FROM TABLE 3-4 OF REFERENCE 1)

<table>
<thead>
<tr>
<th>Material</th>
<th>Melting Point °C (°F)</th>
<th>Heat of Fusion (Literature) kJ/kg (Btu/lb)</th>
<th>Latent Heat Density kJ/m³ (Btu/ft³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CaCl₂ • 6H₂O</td>
<td>28.9 (84)</td>
<td>170.2 (73.3)</td>
<td>2.86 × 10⁵ (7688)</td>
</tr>
<tr>
<td>MgCl₂ • 6H₄O</td>
<td>117.2 (243)</td>
<td>172.3 (74.2)</td>
<td>2.69 × 10⁵ (7226)</td>
</tr>
<tr>
<td>Ca(NO₃)₂ • 4H₂O</td>
<td>42.8 (109)</td>
<td>142.1 (61.2)</td>
<td>2.59 × 10⁵ (6954)</td>
</tr>
<tr>
<td>Mg(NO₃)₂ • 6H₂O</td>
<td>95.0 (203)</td>
<td>159.8 (68.8)</td>
<td>2.34 × 10⁵ (6271)</td>
</tr>
<tr>
<td>Zn(NO₃)₂ • 6H₂O</td>
<td>36.1 (97)</td>
<td>130.0 (56.0)</td>
<td>2.69 × 10⁵ (7219)</td>
</tr>
<tr>
<td>Al(NO₃)₃ • 9H₂O</td>
<td>70.1 (158)</td>
<td>155.6 (67)</td>
<td></td>
</tr>
</tbody>
</table>

a. Experimental value
<table>
<thead>
<tr>
<th>Composition</th>
<th>Wt.%</th>
<th>Melting Point °C (°F)</th>
<th>Heat of Fusion kJ (Btu/ (kg lb))</th>
</tr>
</thead>
<tbody>
<tr>
<td>CaCl₂</td>
<td>37</td>
<td>13 (56)</td>
<td></td>
</tr>
<tr>
<td>Ca(NO₃)₂ H₂O</td>
<td>23 approx.</td>
<td>25 (57)</td>
<td>174.5 (75.1) Experimental</td>
</tr>
<tr>
<td>CaCl₂ MgCl₂ H₂O</td>
<td>40</td>
<td>25 (57)</td>
<td></td>
</tr>
<tr>
<td>Ca(NO₃)₂ · 4H₂O</td>
<td>41</td>
<td>25 (57)</td>
<td></td>
</tr>
<tr>
<td>KCl</td>
<td>4.3</td>
<td>27 (81)</td>
<td></td>
</tr>
<tr>
<td>NaCl</td>
<td>0.4</td>
<td>47.3</td>
<td></td>
</tr>
<tr>
<td>Ca(NO₃)₂ · 4H₂O Mg(NO₃)₂ · 6H₂O</td>
<td>45</td>
<td>25 (77)</td>
<td>129.7 (55.8) Calculated</td>
</tr>
<tr>
<td>Mg(NO₃)₂ · 6H₂O</td>
<td>55</td>
<td>27 (81)</td>
<td></td>
</tr>
<tr>
<td>Ca(NO₃)₂ · 4H₂O Al(NO₃)₂ · 9H₂O</td>
<td>67</td>
<td>30 (86)</td>
<td>136.0 (58.5) Calculated</td>
</tr>
<tr>
<td>Mg(NO₃)₂ · 6H₂O Al(NO₃)₂ · 9H₂O</td>
<td>33</td>
<td>35 (95)</td>
<td>138.9 (59.8) Calculated</td>
</tr>
<tr>
<td>Mg(NO₃)₂ · 6H₂O 2n(NO₃)₂ · 6H₂O</td>
<td>72</td>
<td>35 (95)</td>
<td>148.1 (63.8) Calculated</td>
</tr>
<tr>
<td>Mg(NO₃)₂ · 6H₂O 2n(NO₃)₂ · 6H₂O</td>
<td>28</td>
<td>32 (90)</td>
<td>130.1 (56.0) Calculated</td>
</tr>
<tr>
<td>Material</td>
<td>Supplier</td>
<td>Measurement Temperature Range °C (°F)</td>
<td>Heat of Fusiona (kJ kg⁻¹, Btu lb⁻¹)</td>
</tr>
<tr>
<td>---------------------------------</td>
<td>--------------------</td>
<td>--------------------------------------</td>
<td>-------------------------------------</td>
</tr>
<tr>
<td>Artificial Spermaceti</td>
<td>Lipo Chemicals</td>
<td>35-47.8 (95-118)</td>
<td>195.0 (84)</td>
</tr>
<tr>
<td>Artificial Spermaceti</td>
<td>Lipo Chemicals</td>
<td>40-47.8 (104-118)</td>
<td>133.9 (58)</td>
</tr>
<tr>
<td>Paraffin Wax +25% Wt. CCl</td>
<td>Arthur H. Thomas</td>
<td>40-50 (104-122)</td>
<td>95.2 (41)</td>
</tr>
<tr>
<td>Paraffin Wax +33% Wt. CCl</td>
<td>Arthur H. Thomas</td>
<td>40-17.18 (104-118)</td>
<td>104.5 (45)</td>
</tr>
<tr>
<td>P116 Paraffin Wax</td>
<td>Sun Oil Company</td>
<td>45.6-46.7 (114-116)</td>
<td>209.0 (90)</td>
</tr>
</tbody>
</table>

a. Includes specific heat contribution over the indicated temperature range.
### TABLE A-13. MELTING TEMPERATURE AND HEAT OF FUSION OF ORGANIC MATERIALS FOR AIR CONDITIONING-HYDROCARBONS AND HALOGENATED HYDROCARBONS (FROM TABLE 3-7 OF REFERENCE 1)

<table>
<thead>
<tr>
<th>Compound</th>
<th>Melting Point °C (°F)</th>
<th>Heat of Fusion kJ/kg (Btu/lb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>n-Tetradecane</td>
<td>5.9 (42.6)</td>
<td>225.5 (97.1)</td>
</tr>
<tr>
<td>n-Pentadecane</td>
<td>9.0 (48.2)</td>
<td>162.9 (70.1)</td>
</tr>
<tr>
<td>n-Hexadecane</td>
<td>18.2 (64.8)</td>
<td>233.9 (100.7)</td>
</tr>
<tr>
<td>n-Hexadecylbromide</td>
<td>14 (57.2)</td>
<td></td>
</tr>
<tr>
<td>l-Heptadecane</td>
<td>11.2 (52.2)</td>
<td>131.1 (56.5)</td>
</tr>
<tr>
<td>l-Octadecane</td>
<td>17.9 (64.2)</td>
<td>129.1 (55.6)</td>
</tr>
<tr>
<td>l-Pentadecyne</td>
<td>10 (50)</td>
<td></td>
</tr>
<tr>
<td>l-Hexodecyne</td>
<td>15 (59)</td>
<td></td>
</tr>
<tr>
<td>p-Xylene</td>
<td>13.2 (55.8)</td>
<td>159.9 (68.9)</td>
</tr>
<tr>
<td>Tridecylbenzene</td>
<td>10 (50)</td>
<td>168.3 (72.5)</td>
</tr>
<tr>
<td>Tetradecylbenzene</td>
<td>16 (60.8)</td>
<td>173.3 (74.6)</td>
</tr>
<tr>
<td>2-Nonyl naphthaleno</td>
<td>12 (53.6)</td>
<td></td>
</tr>
<tr>
<td>3-Chlorobiophenyl</td>
<td>17 (62.6)</td>
<td></td>
</tr>
<tr>
<td>Compound</td>
<td>Mole %</td>
<td>Melting Point °C (°F)</td>
</tr>
<tr>
<td>----------------------------</td>
<td>--------</td>
<td>-----------------------</td>
</tr>
<tr>
<td>α Chloroacetic Acid</td>
<td>31.0</td>
<td>16.5 (61.7)</td>
</tr>
<tr>
<td>Phenol</td>
<td>69.0</td>
<td></td>
</tr>
<tr>
<td>β Chloroacetic Acid Phenol</td>
<td>35.0</td>
<td>11.8 (53.2)</td>
</tr>
<tr>
<td></td>
<td>65.0</td>
<td></td>
</tr>
<tr>
<td>α Chloroacetic Acid p Cresol</td>
<td>28.0</td>
<td>13.4 (56.1)</td>
</tr>
<tr>
<td></td>
<td>72.0</td>
<td></td>
</tr>
<tr>
<td>α Chloroacetic Acid o Cresol</td>
<td>30.9</td>
<td>15.8 (60.4)</td>
</tr>
<tr>
<td></td>
<td>69.1</td>
<td></td>
</tr>
<tr>
<td>Acetic Acid Benzoic Acid</td>
<td>88.0</td>
<td>9.5 (49.1)</td>
</tr>
<tr>
<td></td>
<td>12.0</td>
<td></td>
</tr>
<tr>
<td>Naphthalene Diphenyl Methane</td>
<td>25.0</td>
<td>14.5 (58.1)</td>
</tr>
<tr>
<td></td>
<td>75.0</td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>Mole % A</td>
<td>Wt. % A</td>
</tr>
<tr>
<td>------------------</td>
<td>----------</td>
<td>---------</td>
</tr>
<tr>
<td>Urea</td>
<td>37</td>
<td>37</td>
</tr>
<tr>
<td>o-Cresol</td>
<td>46</td>
<td>61</td>
</tr>
<tr>
<td>Phenol</td>
<td>42</td>
<td>53.5</td>
</tr>
<tr>
<td>Stearic Acid</td>
<td>50</td>
<td>83</td>
</tr>
<tr>
<td>Palmitic Acid</td>
<td>50</td>
<td>81</td>
</tr>
<tr>
<td>Palmitic Acid</td>
<td>64.8</td>
<td>89</td>
</tr>
<tr>
<td>Myristic Acid</td>
<td>69.0</td>
<td>89</td>
</tr>
<tr>
<td>Lauric Acid</td>
<td>75.8</td>
<td>91</td>
</tr>
<tr>
<td>Compound</td>
<td>Melting Point °C (°F)</td>
<td>Heat of Fusion kJ (Btu) kg \ lb</td>
</tr>
<tr>
<td>--------------------------------</td>
<td>------------------------</td>
<td>---------------------------------</td>
</tr>
<tr>
<td><strong>Alcohols and Phenols</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1-Decanol</td>
<td>6.1 (43.0)</td>
<td></td>
</tr>
<tr>
<td>1-Undecanol</td>
<td>15 (59.0)</td>
<td></td>
</tr>
<tr>
<td>Glycerol</td>
<td>18 (64.4)</td>
<td></td>
</tr>
<tr>
<td>m-Cresol</td>
<td>12 (53.6)</td>
<td></td>
</tr>
<tr>
<td>2-Chlorophenol</td>
<td>7 (44.6)</td>
<td></td>
</tr>
<tr>
<td>3,3 Dimethyl Cyclohexanol</td>
<td>11 (51.8)</td>
<td></td>
</tr>
<tr>
<td><strong>Aldehydes, Ketones, and Ethers</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1,4 Dioxane</td>
<td>11.8 (53.2)</td>
<td></td>
</tr>
<tr>
<td>2-Decanone</td>
<td>14 (57.2)</td>
<td>145.3 (62.6) kW lb</td>
</tr>
<tr>
<td>n-Tridecanol</td>
<td>15 (59.0)</td>
<td></td>
</tr>
<tr>
<td>Parahlehyde</td>
<td>12.6 (54.7)</td>
<td>104.4 (45.0) kW lb</td>
</tr>
<tr>
<td>2,5 Dichloroaceiophenone</td>
<td>14 (57.2)</td>
<td></td>
</tr>
<tr>
<td>Compound</td>
<td>Melting Point °C (°F)</td>
<td>Heat of Fusion kJ (kg lb / ( \text{Btu} ))</td>
</tr>
<tr>
<td>---------------------------</td>
<td>-----------------------</td>
<td>---------------------------------------------</td>
</tr>
<tr>
<td><strong>Organic Acids</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Formic Acid</td>
<td>8.4 (47.1)</td>
<td>276.7 (119.0)</td>
</tr>
<tr>
<td>Acetic Acid</td>
<td>16.6 (61.9)</td>
<td>201.3 (86.6)</td>
</tr>
<tr>
<td>Acrylic Acid</td>
<td>12.3 (54.1)</td>
<td>154.9 (66.6)</td>
</tr>
<tr>
<td>n-Octanoic Acid</td>
<td>16.3 (61.3)</td>
<td>148.2 (63.7)</td>
</tr>
<tr>
<td>n-Nonanoic Acid</td>
<td>12.3 (54.1)</td>
<td>128.1 (55.1)</td>
</tr>
<tr>
<td>Oleic Acid</td>
<td>14 (57.2)</td>
<td></td>
</tr>
<tr>
<td><strong>Other</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Diethyl Isophthalate</td>
<td>11.5 (52.7)</td>
<td></td>
</tr>
<tr>
<td>Dimethyl Adipate</td>
<td>8.5 (47.3)</td>
<td></td>
</tr>
<tr>
<td>1,2 Ethylene Diamine</td>
<td>8.5 (47.3)</td>
<td></td>
</tr>
<tr>
<td>m-Cresyl Acetate</td>
<td>12 (53.6)</td>
<td></td>
</tr>
<tr>
<td>2,5 Dimethyl Analine</td>
<td>14.2 (57.7)</td>
<td></td>
</tr>
<tr>
<td>Material</td>
<td>Supplier</td>
<td>Freezing Point °C (°F)</td>
</tr>
<tr>
<td>-------------------</td>
<td>----------</td>
<td>------------------------</td>
</tr>
<tr>
<td>C(<em>{14})-C(</em>{16}) Paraffin</td>
<td>Conoco</td>
<td>5.6-6.1 (42-43)</td>
</tr>
<tr>
<td>C(<em>{15})-C(</em>{16}) Paraffin</td>
<td>Enjay</td>
<td></td>
</tr>
<tr>
<td>Batch No. 1</td>
<td></td>
<td>8.3 (47)</td>
</tr>
<tr>
<td>Batch No. 2</td>
<td></td>
<td>8.3 (47)</td>
</tr>
<tr>
<td>I-Decanol</td>
<td>Eastman</td>
<td>5.6 (42)</td>
</tr>
<tr>
<td>C(<em>{15})-C(</em>{18}) Oleins</td>
<td>Chevron</td>
<td>3.3 (38)</td>
</tr>
<tr>
<td>C(_{14}) Paraffin</td>
<td>Humphrey</td>
<td>4.4 (40)</td>
</tr>
<tr>
<td>C(_{16}) Paraffin</td>
<td>Humphrey</td>
<td>17.8 (64)</td>
</tr>
</tbody>
</table>

\(^a\) Not including any specific heat contribution; the average specific heat of the paraffins between 1.7° C (35° F) and room temperature is 2.8 kJ/kg·°C (0.52 Btu/lb·°F).
TABLE A-19. MELTING POINT AND HEAT OF FUSION FOR CLATHRATES AND SEMI-CLATHRATES MELTING CONGRUENTLY ABOVE 0°C (32°F)  
(FROM TABLE 3-9 OF REFERENCE 1)

<table>
<thead>
<tr>
<th>Composition</th>
<th>Melting Point °C (°F)</th>
<th>Heat of Fusion kJ (Btu/lb)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Type I Clathrate Hydrates</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>H₂S · 6.1 H₂O</td>
<td>0.4 (33)</td>
<td></td>
</tr>
<tr>
<td>H₂S · 6.1 H₂O</td>
<td>29.5 (85)</td>
<td></td>
</tr>
<tr>
<td>CO₂ · 6.0 H₂O</td>
<td>9.9 (50)</td>
<td></td>
</tr>
<tr>
<td>Cl₂ · 7.3 H₂O</td>
<td>9.7 (50)</td>
<td></td>
</tr>
<tr>
<td>SO₂ · 6.0 H₂O</td>
<td>7.0 (45)</td>
<td></td>
</tr>
<tr>
<td>SO₂ · 6.1 H₂O</td>
<td>12.1 (54)</td>
<td></td>
</tr>
<tr>
<td>C₂H₄0 · 6.9 H₂O</td>
<td>11.1 (52)</td>
<td></td>
</tr>
<tr>
<td><strong>Type II Clathrate Hydrates</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C₄H₈0 · 17.2 H₂O</td>
<td>4.4 (40)</td>
<td>255.2 (109.0) Experimental</td>
</tr>
<tr>
<td>(Tetrahydrofuran)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C₄H₈0 · 2H₂S · 17 H₂O</td>
<td>21.3 (70)</td>
<td>276.1 (118.9) Experimental</td>
</tr>
<tr>
<td>(CH₃)₂COH · 2H₂S · 17H₂O</td>
<td>7.3 (45)</td>
<td></td>
</tr>
<tr>
<td><strong>Amine Semi-Clathrate Hydrate</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(CH₃)₃N · 10 1/4 H₂O</td>
<td>5.9 (43)</td>
<td>238.5 (102.7) Experimental</td>
</tr>
<tr>
<td><strong>Tetrabutylammonium Salt Semi-Clathrate Hydrates</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bu₄NF · 32H₂O</td>
<td>24.9 (77)</td>
<td></td>
</tr>
<tr>
<td>Bu₄NCl · 32H₂O</td>
<td>15.7 (60)</td>
<td></td>
</tr>
<tr>
<td>Bu₄NBr · 32H₂O</td>
<td>12.5 (54)</td>
<td></td>
</tr>
<tr>
<td>Bu₄NNO₃ · 32H₂O</td>
<td>5.8 (42)</td>
<td></td>
</tr>
<tr>
<td>Bu₄NOH · 32H₂O</td>
<td>30.2 (86)</td>
<td></td>
</tr>
<tr>
<td>Bu₄NHCO₃ · 32H₂O</td>
<td>17.8 (64)</td>
<td>184.1 (79.3) Experimental</td>
</tr>
<tr>
<td>(Bu₄N)₂HPO₄ · 64H₂O</td>
<td>17.2 (63)</td>
<td></td>
</tr>
<tr>
<td>(Bu₄N)₂C₂O₄ · 64H₂O</td>
<td>16.8 (62)</td>
<td></td>
</tr>
<tr>
<td>Bu₄NCHO₂ · 32H₂O</td>
<td>12.5 (54)</td>
<td>209.2 (90.1) Experimental</td>
</tr>
<tr>
<td>Bu₄NCH₃CO₂ · 32H₂O</td>
<td>15.1 (59)</td>
<td></td>
</tr>
<tr>
<td>Bu₄NC₆H₅CO₂ · 32H₂O</td>
<td>3.5 (38)</td>
<td></td>
</tr>
<tr>
<td><strong>Tetraisoamylammonium Salt Semi-Clathrate Hydrates</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>i-Am₄NF · 40 H₂O</td>
<td>31.2 (88)</td>
<td></td>
</tr>
<tr>
<td>i-Am₄NCl · 38 H₂O</td>
<td>29.8 (86)</td>
<td></td>
</tr>
<tr>
<td>i-Am₄NOH · 40 H₂O</td>
<td>31 (88)</td>
<td></td>
</tr>
<tr>
<td>i-Am₄NCHO₂ · 40 H₂O</td>
<td>15-20 (59-68)</td>
<td></td>
</tr>
</tbody>
</table>
### Table A-20. Melting Point for Organic-Inorganic Eutectics

*(From Table 3-10 of Reference 1)*

#### Urea Eutectics—Binary

<table>
<thead>
<tr>
<th>A</th>
<th>Mole % A</th>
<th>Wt. % A</th>
<th>Melting Point °C (°F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>NH₄Cl</td>
<td>17.5</td>
<td>15.9</td>
<td>101 (214)</td>
</tr>
<tr>
<td>NH₄Br</td>
<td>23.5</td>
<td>33.4</td>
<td>76 (169)</td>
</tr>
<tr>
<td>NH₄I</td>
<td>25.5</td>
<td>45.2</td>
<td>66 (151)</td>
</tr>
<tr>
<td>NH₄NO₃</td>
<td>47.5</td>
<td>54.7</td>
<td>46 (115)</td>
</tr>
<tr>
<td>NH₄SCN</td>
<td>39.5</td>
<td>48.4</td>
<td>32 (90)</td>
</tr>
<tr>
<td>LiNO₃</td>
<td>16</td>
<td>18</td>
<td>76 (169)</td>
</tr>
<tr>
<td>LiNO₃</td>
<td>45.5</td>
<td>48.1</td>
<td>104 (219)</td>
</tr>
<tr>
<td>NaN₃</td>
<td>23</td>
<td>29.5</td>
<td>84 (183)</td>
</tr>
<tr>
<td>KNO₃</td>
<td>16</td>
<td>24.3</td>
<td>110 (230)</td>
</tr>
<tr>
<td>Ca(NO₃)₂</td>
<td>9.4</td>
<td>22</td>
<td>92 (198)</td>
</tr>
<tr>
<td>Ca(NO₃)₂</td>
<td>25</td>
<td>48</td>
<td>140 (286)</td>
</tr>
<tr>
<td>Sr(NO₃)₂</td>
<td>15</td>
<td>38</td>
<td>67 (153)</td>
</tr>
<tr>
<td>Ba(NO₃)₂</td>
<td>10.5</td>
<td>33.8</td>
<td>96 (205)</td>
</tr>
<tr>
<td>K₂CO₃</td>
<td>7</td>
<td>14.8</td>
<td>102 (216)</td>
</tr>
</tbody>
</table>

#### Urea Eutectics—Ternary

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>Mole % A</th>
<th>Wt. % A</th>
<th>Mole % B</th>
<th>Wt. % B</th>
<th>Melting Point °C (°F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>NH₄Cl</td>
<td>NH₄NO₃</td>
<td>4</td>
<td>2.4</td>
<td>46</td>
<td>53</td>
<td>35 (95)</td>
</tr>
<tr>
<td>NH₄Cl</td>
<td>NH₄SCN</td>
<td>4</td>
<td>2.5</td>
<td>40</td>
<td>46</td>
<td>20 (68)</td>
</tr>
<tr>
<td>NH₄NO₃</td>
<td>LiNO₃</td>
<td>34</td>
<td>40</td>
<td>7</td>
<td>7.1</td>
<td>34 (93)</td>
</tr>
<tr>
<td>NH₄NO₃</td>
<td>LiNO₃</td>
<td>46</td>
<td>51</td>
<td>32</td>
<td>31</td>
<td>65 (149)</td>
</tr>
<tr>
<td>NH₄NO₃</td>
<td>NaNO₃</td>
<td>41</td>
<td>47</td>
<td>8</td>
<td>9.7</td>
<td>35 (95)</td>
</tr>
<tr>
<td>NH₄NO₃</td>
<td>KNO₃</td>
<td>44</td>
<td>49.9</td>
<td>4</td>
<td>5.7</td>
<td>43 (109)</td>
</tr>
<tr>
<td>LiNO₃</td>
<td>NaNO₃</td>
<td>24</td>
<td>26</td>
<td>10</td>
<td>13</td>
<td>56 (133)</td>
</tr>
<tr>
<td>LiNO₃</td>
<td>NaNO₃</td>
<td>45</td>
<td>49</td>
<td>3.5</td>
<td>4.6</td>
<td>100 (212)</td>
</tr>
<tr>
<td>LiNO₃</td>
<td>KNO₃</td>
<td>16</td>
<td>17</td>
<td>6</td>
<td>9.5</td>
<td>62 (144)</td>
</tr>
<tr>
<td>LiNO₃</td>
<td>KNO₃</td>
<td>45</td>
<td>43</td>
<td>19</td>
<td>27</td>
<td>96 (205)</td>
</tr>
<tr>
<td>LiNO₃</td>
<td>Ca(NO₃)₂</td>
<td>16.5</td>
<td>18.1</td>
<td>1.5</td>
<td>4</td>
<td>62 (144)</td>
</tr>
<tr>
<td>LiNO₃</td>
<td>Ca(NO₃)₂</td>
<td>41</td>
<td>44.5</td>
<td>4</td>
<td>9.2</td>
<td>82 (180)</td>
</tr>
<tr>
<td>LiNO₃</td>
<td>Sr(NO₃)₂</td>
<td>12</td>
<td>11</td>
<td>8</td>
<td>23</td>
<td>45 (113)</td>
</tr>
<tr>
<td>LiNO₃</td>
<td>Ba(NO₃)₂</td>
<td>16</td>
<td>17</td>
<td>1</td>
<td>4</td>
<td>68 (154)</td>
</tr>
<tr>
<td>LiNO₃</td>
<td>Ba(NO₃)₂</td>
<td>44</td>
<td>46</td>
<td>1</td>
<td>4</td>
<td>92 (198)</td>
</tr>
<tr>
<td>NaN₃</td>
<td>KNO₃</td>
<td>22.3</td>
<td>27.7</td>
<td>7</td>
<td>10.3</td>
<td>73 (163)</td>
</tr>
<tr>
<td>NaN₃</td>
<td>Ba(NO₃)₂</td>
<td>17.5</td>
<td>20.5</td>
<td>4</td>
<td>14.4</td>
<td>68 (154)</td>
</tr>
<tr>
<td>KNO₃</td>
<td>Ca(NO₃)₂</td>
<td>3</td>
<td>4</td>
<td>14</td>
<td>20</td>
<td>89 (192)</td>
</tr>
<tr>
<td>KNO₃</td>
<td>Ca(NO₃)₂</td>
<td>18</td>
<td>20</td>
<td>24</td>
<td>42.5</td>
<td>125 (257)</td>
</tr>
<tr>
<td>KNO₃</td>
<td>Ba(NO₃)₂</td>
<td>8.5</td>
<td>11.5</td>
<td>5.5</td>
<td>19.3</td>
<td>80 (176)</td>
</tr>
</tbody>
</table>
TABLE A-20. (Concluded)

Acetamide Eutectics

<table>
<thead>
<tr>
<th>A</th>
<th>Mole % A</th>
<th>Wt. % A</th>
<th>Melting Point °C (°F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>NH₄NO₃</td>
<td>32</td>
<td>39</td>
<td>38 (100)</td>
</tr>
<tr>
<td>LiNO₃</td>
<td>20</td>
<td>23</td>
<td>25 (77)</td>
</tr>
<tr>
<td>NaNO₃</td>
<td>15</td>
<td>20</td>
<td>59 (138)</td>
</tr>
<tr>
<td>KNO₃</td>
<td>6</td>
<td>9.8</td>
<td>72 (162)</td>
</tr>
</tbody>
</table>
Some of the results of the parametric study which are shown in Section V exhibit interesting trends. Specifically, some plots of the temperature rise at the termination of melting versus fin height demonstrate a relative maximum, a relative minimum, or both. Several curves in Figures 51 through 62 indicate this behavior. A plausible explanation of this behavior is presented in this appendix.

The physical model, which was treated in Section IV, is shown in Figure B-1. Heat flows directly from the bottom into the PCM. Heat also flows into the PCM through the fin and top. The relative thermal resistances of these paths is dependent on the geometry.

Figure B-1. Physical model.
Some insight into the thermal resistance behavior may be gained by means of a simple but nonexact viewpoint. Suppose the effect of the top is ignored and the PCM and fin are each represented by a single node. The resulting thermal network, including thermal resistances, for the heat flow between the bottom and the PCM is indicated in Figure B-2. The equivalent thermal resistance for this arrangement is related to the parameters by

$$K_{eq} \frac{K_w}{B} = \frac{1}{2W/H + 2 (K/K_w)HS/(2H^2 + WS(K/K_w))}$$  \hspace{1cm} (B-1)

The right-hand side of equation (B-1) was evaluated for the range of geometrical parameters covered in the parametric study. The thermal conductivity ratio \((K/K_w)\) of \(1069\) was also used. The results are plotted in Figures B-3 through B-5. The effect of geometry on the equivalent thermal resistance for the simplified network is shown by the curves.

![Figure B-2. Simplified thermal network.](image-url)
Figure B-3. Variation of equivalent thermal resistance with height for a fin thickness of 0.0127 cm (0.005 in.).
Figure B-4. Variation of equivalent thermal resistance with height for a fin thickness of 0.0508 cm (0.02 in.).
Figure B-5. Variation of equivalent thermal resistance with height for a fin thickness of 0.127 cm (0.05 in.).
APPENDIX C

COMPUTER PROGRAM FOR EXAMPLE DESCRIBED IN SECTION V

An example application of a phase change device with a nonuniform thermal boundary condition is discussed in Section V. The equation used to compute the temperature variation of the flowing fluid is equation (37). The computer program shown in Table C-1 was written to facilitate the calculation of the mean fluid temperature from equation (37). This program occupies approximately 7 K words of storage. The data shown in Table C-2 is required for each run. In addition, the user must supply routines in the function subroutines COEFF and TEMP. COEFF calculates \( U_0 \) and TEMP generates the inlet temperature of the fluid. Note that there are provisions for up to three different functions for \( U_0 \) and the inlet temperature. The function used during any run is chosen by the data entered in the data card (Table C-2). There are provisions for multiple runs within one computer job. The program reads nine cards, processes the run, and then returns to read nine more. If none are present, the program terminates. For each run, header information is printed, such as flow rates, and fluid and PCM properties. Two time steps are printed simultaneously, one on the left-hand side of the page, the other on the right-hand side of the page. A description of the output data is given in Table C-3. At the end of each run, the total energy stored, \( Q_{TOT} \), is printed.
TABLE C-1. COMPUTER PROGRAM USED TO COMPUTE FLUID TEMPERATURE VARIATION AS IT FLOWS THROUGH A PHASE CHANGE DEVICE

<table>
<thead>
<tr>
<th>REAL LENGTH, MLUMP, MDOT</th>
</tr>
</thead>
<tbody>
<tr>
<td>INTEGER TIME, TIMECCT</td>
</tr>
<tr>
<td>DIMENSION INLEN(2,500), QTAB(500),OLQ3(500),TLUMP(2)</td>
</tr>
<tr>
<td>1(\times)500)</td>
</tr>
<tr>
<td>COMMON IBUP,F,LINCT,OLDTME,NODECT</td>
</tr>
</tbody>
</table>

9998 READ (5,10U+2) TFREEZ

100 FORMAT (14U+2)

READ (5,11U) SPHEAT, MDOT, HTXFER, WIDTH, LENGTH, XSTEP

110 FORMAT (3F10.2/2F10.2, F10.5/F10.2, F10.5)

READ (5,15U) RHO, XSECT

150 FORMAT (2F10.2)

READ (5,19U) SPAC, VISC, PRAN, COND

190 FORMAT (4F10.2)

READ (5,16U) HEIGHT, HTFUS, SPHLM, CONLMP, RHOLMP

160 FORMAT (11U+5/4F10.2)

READ (5,17U) IWRIT, ISUPX

170 FORMAT (2I10)

READ (5,18U) ITMPZ, IUORTN

180 FORMAT (2I10)

WRITE (6,19Y)

199 FORMAT (' 1')

WRITE (6,20U)

200 FORMAT (3I49, '********************************************************************

1********************************************************************')

WRITE (6,22U) LENGTH, XSECT

220 FORMAT (2I38, 'CAPACITOR * LENGTH =', 'F7.3, ' FT', ' * T8(1, ' FLOW PASS AREA =', 'F7.4, ' SQ FT', ' *, T115, *, *)

WRITE (6,230) HEIGHT, SPAC

230 FORMAT (2I38, 'DIMENSIONS * PCM HEIGHT =', 'F7.3, '

1 FT', ' * T8U, ' FLOW PASS HEIGHT =', 'F7.4, ' FT', '*, T115, *, *)

WRITE (6,240) WIDTH

240 FORMAT (2I49, '* M=WIDTH =', 'F7.4, ' # T8, ' , T115, *, *)

WRITE (6,210)

210 FORMAT (2I49, '********************************************************************

1********************************************************************')

WRITE (6,22U) MDOT, VISC

250 FORMAT (2I38, 'FLUID * FLOW RATE =', 'F7.3, ' LB

1/HR', ' *, T8U, ' VISCOSITY =', 'F7.4, ' LB/FT', ' * HR', ' *, T115, *, *)

WRITE (6,250) SPHEAT, PRAN

260 FORMAT (2I38, 'PROPERTIES * SPECIFIC H =', 'F5.3, '

1 BTU/LB =', 'T80, ' PRAND NO =', 'F7.3, T115, *, *)

WRITE (6,270) RHO, COND

270 FORMAT (2I49, '* DENSITY =', 'F7.3, ' LB/CUB FT', ' *, T80

1, ' THERM COND =', 'F7.3, ' BTU/HR', ' * FT', ' , T115, *, *)

WRITE (6,210)

WRITE (6,240)

WRITE (6,28U) HTFUS, CONLMP

280 FORMAT (2I38, 'PCM * HEAT CF FUS =', 'F5.0, '

1 BTU/CUB FT', ' *, T80, ' THERM COND =', 'F7.3, ' BTU/HR', ' * FT', ' ,')
TABLE C-1. (Continued)

```
2,T115,'*') WRITE (6,290) SPHLM,TFREEZ
290 FORMAT (' ',138,'PROPERTIES * SPECIFIC HT = ',F5.3,
1 BTU/LB'°F', 'TBO, FREEZE TEMP = ',F5.3,' ',T115,'*') WRITE (6,300) RHOLMP
300 FORMAT (' ',149,'DENSITY = ',F7.3,' ',L3/CUB FT',T115,'*')
WRITE (6,210)
WRITE (6,240)
WRITE (6,310) XSTEP,ITMPIN
310 FORMAT (' ',138,'RUN * X=STEP = ',F4.4,' ',IT',T8C
1 TEMP=IN KTN = ',11,115,'*') WRITE (6,320) THESDP,TDORTN
320 FORMAT (' ',138,'PARAMETERS * TIME=STEP = ',F7.3,' ',HR8
1,T80, UQ=RUN = ',11,115) WRITE (6,330) TIMELN
330 FORMAT (' ',149,' TIME LENGTH = ',F7.3,' ',HR8,T115,'*')
WRITE (6,210)
C1=RHO*XSECT*SPHEAT/THESTP C2=MDOT*SPHEAT/XSTEP NNODECT=LENGTH/XSTEP+2 TIMECT=TIMELN/THESTP VOLUME=NI0*HT8C*HEIGHT MLUMP=RHOLMP*VOLUME UCONST=1/(HEIGHT/(2*CONLMP)) DIA=2*SPH
REYN=DIAMDOT/(RHO*XSECT*IVISC) IHSTW=1 IF (HTXFEREQ0.0) IHSTW=0 DO 1 J=1,NNODECT TMEAN(1,J)=TFREEZ TLUMP(1,J)=TFREEZ 1 QTAB(J)=U=
IRITSH=I=MRT=1 I8UFF=0 LINCT=25 OLDTM=YY9=0 DO 3 TIME=1,TIMELCT T=TIME*THESTP IF (IRITSH*EW=IRIT) IRITW=0 IRITW=IRITSh+1 TMEAN(2,1)=ITMPIN(ITMPIN,T)
DO 4 NODE=2,NNODECT X=(NODE-1)*XSTEP UO=COEF(I,QLXAD(NODE),VOLUME,HTFUB,FRACR4,TDORTN)
IF (FRACR<LT0.99) GO TO 3C TLUMP(2,NODE)=OLDQ(NODE)/(MLUMP*SPHLM)+ITLUMP(1,NODE)
TMPCH=TLUMP(2,NODE)
UQ=UCON+1 GO TO 4C 30 TMPCH=TFREEZ
```

TABLE C-1. (Continued)

40 IF (IHTSW*EQ*1) GO TO 20
  GRATZ=X/(DIA*REYNOPRAN)
  HTXFER=(COND/DIA)*HEAT(1/GRATZ)
20 C4=(HTXFER*WIDTH/UO)/(UO+HTXFER)
  CS=1/(C1+C2+C4)
  TMEAN(2, NODE) = C5*(C4*TMRPCM+C2*TMEAN(2, NODE=1)+C3
  1*TMEAN(1, NODE))
  QA=C2*(TMEAN(2, NODE) - TMEAN(2, NODE=1))
  QB=C1*(TMEAN(2, NODE) - TMEAN(1, NODE))
  Q = 1*(QA+QB)*THESTP*STEP
  QTAB(NODE) = QTAB(NODE) + Q
  OLDG(NODE) = Q
  IF (IRTBN*EQ*1) CALL PRNTIT(TMEAN(2, NODE), QTAB
  1(NODE), FRCTN, UO, T, TMEAN(2, 1), X, NODE, HTXFER, 1SUPX)
  4 CONTINUE
   DO 2 J = 1, NODECT
   TUMP(1, J) = TUMP(2, J)
  2 TMEAN(1, J) = TMEAN(2, J)
  3 CONTINUE
  QTOT = 0
  DO 10 J = 2, NODECT
  10 QTOT = QTOT + QTAB(J)
  WRITE (6,101) QTOT
  140 FORMAT (5X, 'QTOT = ', E10.2)
  GO TO 9999
9999 CALL EXI1
END
SUBROUTINE PRNTIT(TMEAN, QTAB, FRCTN, JC, T, TEMPIN, X, NODE
  1, HTXFR, 1SUPX)
DIMENSION TMEAN1(500), QTAB1(500), FRCTN1(500), UO1(500)
  1, HTXFR1(500)
COMMON 1BUFF, LINCT, OLDTME, NODECT
IF (ISUPX*EQ*0. AND. NODE=1*NODECT) RETURN
IF (FRCTN*GT*1.0) FRCTN = 1.0
IF (IBUFF*EQ*0) GO TO 10
IF (OLDTME*EQ*T) 30 TO 20
IBUFF = 0
OLDTME = 1
GO TO 30
20 TMEAN1(NODE) = TMEAN
  QTAB1(NODE) = QTAB
  FRCTN1(NODE) = FRCTN
  UO1(NODE) = UC
  HTXFR1(NODE) = HTXFR
RETURN
10 IF (OLDTME*EQ*T) GO TO 40
IBUFF = 1
OLDTME = T
PRNTTH = OLDTME
PRNTTP = TEMPIN
GO TO 20
TABLE C-1. (Continued)

<table>
<thead>
<tr>
<th>Line</th>
<th>Code</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>IF</td>
<td>( \text{LINCT} \geq 65 ) ( \text{GO TO} ) 50</td>
</tr>
<tr>
<td>100</td>
<td>WRITE</td>
<td>( \text{WRITE (6,140) PRNTTH,PRNTTP,TEMPIN} )</td>
</tr>
<tr>
<td>110</td>
<td>WRITE</td>
<td>( \text{WRITE (6,140) PRNTTH,PRNTTP,TEMPIN} )</td>
</tr>
<tr>
<td>140</td>
<td>WRITE</td>
<td>( \text{WRITE (6,140) PRNTTH,PRNTTP,TEMPIN} )</td>
</tr>
<tr>
<td>40</td>
<td>IF</td>
<td>( \text{IF (LINCT} \lt 73 ) ( \text{GO TO} ) 60</td>
</tr>
<tr>
<td>160</td>
<td>WRITE</td>
<td>( \text{WRITE (6,160) X,TMEAN1(NODE),QTAB1(NODE),FRACTN1(NODE)} )</td>
</tr>
<tr>
<td>150</td>
<td>FORMAT</td>
<td>( \text{FORMAT (6,6F10.2,11x,6F10.2)} )</td>
</tr>
</tbody>
</table>

FUNCTION COEFF(QTAB, VOLUME, HTFUS, FRACTN, IUORTN)

\( \text{VOLUMT} = \text{AB1 / VOLUME} \)
\( \text{IF (FRACTN} \leq 0.99 \) \( \text{RETURN} \)
\( \text{GO TO (12/3), IUORTN} \)

1 CONTINUE

**C**
**C** USER SUPPLIED UO VARIATION ROUTINE
**C** GOES BETWEEN STATEMENTS 162
**C** SELECTED WHEN IUORTN=1
**C** IF (FRACTN \( \geq 25 \) \( \text{GO TO} \) 10
**C** COEFF=146-12-242,56\*FRACTN
**C** RETURN

10 COEFF=54+32-35,37\*FRACTN
**C** RETURN

2 CONTINUE

**C** A SECOND UO VARIATION ROUTINE
**C** MAY GO BETWEEN STATEMENTS 253
**C** SELECTED WHEN IUORTN=2
**C** COEFF=50,0
**C** RETURN

3 CONTINUE

**C** A THIRD UO VARIATION ROUTINE CANNOT GO TO 30
**C** AFTER STATEMENT 3
**C** SELECTED WHEN IUORTN=3
**C** RETURN
TABLE C-1. (Concluded)

END

FUNCTION TEMPIN(ITMPIN,T)
GO TO (1,2,3), ITMPIN
1 CONTINUE
   *** USER SUPPLIED INLET TEMPERATURE
   *** ROUTINE GOES BETWEEN STATEMENTS
   *** 1 & 2 SELECTED WHEN ITMPIN=1
   IF (T*GT*1.5) GO TO 10
   TEMPIN=1.38*T
   RETURN
10 IF (T*GT*5.5) GO TO 20
    TEMPIN=4.64+12.927*T+9.826*T*T
    RETURN
20 IF (T*GT*1.4*5) GO TO 30
    TEMPIN=6B*O*8IN((3*1415/15)*T)
    RETURN
30 IF (T*GT*14.0) GO TO 40
    TEMPIN=1485.165+157.533*T+3.898*T*T
    RETURN
40 TEMPIN=614.7=40.98*T
    RETURN
2 CONTINUE
   *** A SECOND TEMPERATURE VARIATION
   *** ROUTINE MAY GO BETWEEN STATEMENTS
   *** 2 & 3 SELECTED WHEN ITMPIN=2
   TEMPIN=10.0*O+20.0*T
   RETURN
3 CONTINUE
   *** A THIRD INLET TEMPERATURE VARIATION
   *** ROUTINE MAY GO AFTER STATEMENT
   *** 3 SELECTED WHEN ITMPIN=3
   RETURN
END

FUNCTION HEAT(GRATZ)
IF (GRATZ*GT*01) GO TO 10
   EX=0.26
   HEAT=2.27*GRATZ**EX
   RETURN
10 IF (GRATZ*GT*03) GO TO 20
   EX=0.07
   HEAT=7.26*GRATZ**EX
   RETURN
20 HEAT=7.5*1
RETURN
END
<table>
<thead>
<tr>
<th>Card Number</th>
<th>Punch Position</th>
<th>Data Description</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1-10</td>
<td>Freezing temperature of the PCM</td>
<td>°F</td>
</tr>
<tr>
<td>2</td>
<td>1-10</td>
<td>Specific heat of working fluid</td>
<td>Btu/lbm·°F</td>
</tr>
<tr>
<td></td>
<td>11-20</td>
<td>Mass flow rate of working fluid</td>
<td>lbm/h</td>
</tr>
<tr>
<td></td>
<td>21-30</td>
<td>Heat transfer coefficient between working fluid and the flow passage wall Note:</td>
<td>Btu/h·ft²</td>
</tr>
<tr>
<td></td>
<td></td>
<td>If this piece of data is omitted, a heat transfer coefficient will be calculated</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>by the program for flow between parallel flat plates</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>1-10</td>
<td>Area of flow passage wall over which heat transfer takes place</td>
<td>ft²</td>
</tr>
<tr>
<td></td>
<td>11-20</td>
<td>Length of the capacitor</td>
<td>ft</td>
</tr>
<tr>
<td></td>
<td>21-30</td>
<td>Length of X increment used in finite difference formula</td>
<td>ft</td>
</tr>
<tr>
<td>4</td>
<td>1-10</td>
<td>Duration of the simulation run</td>
<td>h</td>
</tr>
<tr>
<td></td>
<td>11-20</td>
<td>Length of time increment used in finite difference formula</td>
<td>h</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>Density of the working fluid</td>
<td>lbm/ft³</td>
</tr>
<tr>
<td></td>
<td>11-20</td>
<td>Cross-sectional area of flow passage</td>
<td>ft²</td>
</tr>
<tr>
<td>6</td>
<td>1-10</td>
<td>Height of the flow passage</td>
<td>ft</td>
</tr>
<tr>
<td></td>
<td>11-20</td>
<td>Viscosity of the working fluid</td>
<td>lbm/ft·h</td>
</tr>
<tr>
<td></td>
<td>21-30</td>
<td>Prandtl number of the working fluid</td>
<td>N/A</td>
</tr>
<tr>
<td></td>
<td>31-40</td>
<td>Thermal conductivity of the working fluid</td>
<td>Btu/h·ft·°F</td>
</tr>
<tr>
<td>7</td>
<td>1-10</td>
<td>Height of the PCM</td>
<td>ft</td>
</tr>
<tr>
<td></td>
<td>11-20</td>
<td>Heat of fusion of the PCM</td>
<td>Btu/lbm·°F</td>
</tr>
<tr>
<td></td>
<td>21-30</td>
<td>Specific heat of the PCM</td>
<td>Btu/h·ft·°F</td>
</tr>
<tr>
<td></td>
<td>31-40</td>
<td>Thermal conductivity of the PCM</td>
<td>lbm/ft³</td>
</tr>
<tr>
<td></td>
<td>41-50</td>
<td>Density of the PCM</td>
<td>N/A</td>
</tr>
<tr>
<td>8</td>
<td>10</td>
<td>An integer number, N, that specifies the number of time increments between prints</td>
<td></td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>A blank here will cause only information about the last node to be printed.</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>An integer 1 will cause information about all nodes to be printed</td>
<td></td>
</tr>
<tr>
<td>Card Number</td>
<td>Punch Position</td>
<td>Data Description</td>
<td>Units</td>
</tr>
<tr>
<td>-------------</td>
<td>----------------</td>
<td>-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------</td>
<td>-------</td>
</tr>
<tr>
<td>9</td>
<td>10</td>
<td>An integer 1, 2, or 3. This number selects which inlet temperature routine will be used \</td>
<td>N/A</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>An integer 1, 2, or 3 that selects the $U_0$ generator routine that will be used \</td>
<td>N/A</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Note: These routines must be user supplied. They will go in the COEFF and the TEMPIN function subroutines</td>
<td></td>
</tr>
</tbody>
</table>
TABLE C-3. DESCRIPTION OF OUTPUT DATA FROM PROGRAM LISTED IN TABLE C-1

<table>
<thead>
<tr>
<th>Data Name</th>
<th>Description</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>TEMP IN</td>
<td>Fluid inlet temperature</td>
<td>°F</td>
</tr>
<tr>
<td>X</td>
<td>X location in the capacitor</td>
<td>ft</td>
</tr>
<tr>
<td>TEMP</td>
<td>Temperature at X</td>
<td>°F</td>
</tr>
<tr>
<td>Q</td>
<td>Accumulated heat transfer into the PCM at X</td>
<td>Btu</td>
</tr>
<tr>
<td>CHANGE</td>
<td>Fraction undergone change of phase</td>
<td></td>
</tr>
<tr>
<td>U0</td>
<td>$U_0$</td>
<td>$\frac{\text{Btu}}{\text{h-ft}^2\cdot\circ\text{F}}$</td>
</tr>
<tr>
<td>HTXFER</td>
<td>Convective heat transfer coefficient between fluid and flow passage wall</td>
<td>$\frac{\text{Btu}}{\text{h-ft}^2\cdot\circ\text{F}}$</td>
</tr>
</tbody>
</table>
APPENDIX D

COMPUTER PROGRAM USED TO FACILITATE THE EXPLICIT NUMERICAL STUDY

The computer program used to facilitate the explicit numerical study is outlined and described in this appendix. A copy of the program for a melting run with inclusion of convective effects is given in Section D.1; the corresponding notation is described in Section D.2; and the steps are discussed in Section D.3.

After listing this program, it was discovered that the program described in Section D.1 would not run for the case of $M = 1$. The statements causing this incompatibility are indicated in Section D.1 by an arrow placed at the left of the appropriate statement.

A copy of the computer program for a solidification case is given in Section D.4. A comparison of the appropriate statements with those shown in Section D.3 generally illustrates the modification to accommodate $M = 1$.

The statements which were included in the solidification program to allow for changing the effective interfacial area for nodes experiencing melting are underlined.
D.I. COMPUTER PROGRAM FOR MELTING

1  DIMENSION  RV(5,31), RH(5,31), C(5,31), Q5(5,31), T1(5,31)
2  DIMENSION  T2(5,31), F(5,31), Q(5,31), QRT(5,31), T3(1,31)
3  DIMENSION  TM1(9), TM2(9), TM3(9), TM4(9), T1M(9)
4  DIMENSION  QRA(5,31)

COMPUTATIONAL PARAMETERS

5  N=27
6  M=4
7  MM=M+1
8  NN=N+2
9  NJ=N+3
10  NI=N+4
11  ND=9
12  NDP=8
13  TAU=1.1
14  KCOUNT=1
15  MCOUNT=1
16  MFN=500
17  EPS=1.E-06
18  KCHK=1000
19  JOE=1
20  DT=1.0E-04
21  TAU2=(10.*DT)+(DT/3.)

PHYSICAL PROPERTIES

22  TAMB=80.
23  TIN=73.5
24  HTOP=1.E-08
25  HBOT=5.0
26  G=(32.2*3600.*3600.)
27  WAX
28  DEN=47.2
29  TK=0.087
30  CP=0.5
31  TTR=73.04
32  HTR=22.108
33  TMELT=89.8
34  HMELT=73.357
35  TREF=50.0
36  BETA=0.00045
37  VIS=14.3
38  CL=CP
39  TKL=TK
40  BOTTOM PLATE DENOTED BY 1
41  DEN1=171.0
42  TK1=93.0
43  CP1=0.22
44  FIN DENOTED BY 2
45  DEN2=171.0
46  TK2=93.0
47  CP2=0.22
48  TOP PLATE DENOTED BY 3
49  DEN3=72.5
50  TK3=0.09
51  CP3=0.33

207
GEOMETRY PARAMETERS
55
56  W=0.75/12.0
57  H=2.625/12.0
58  B=5.0/12.0
59  S=W/(2.0*AM)
60  S1=0.032/12.0
61  S2=0.008/12.0
62  S3=0.25/12.0
63  F2JM=1.0/(32.0*S*12.0)

VERTICAL RESISTANCES
64
65  RV(1,2)=(S1/(TK1*S2*B))+(1./(HBOT*S2*B))
66  DO 10 I=2,MM
67  10 RV(I,2)=(S1/(TK2*S2*B))+(S1/(TK1*S2*B))
68  DO 20 I=2,MM
69  20 RV(I,3)=(1./(2.*TK*B))+(S1/(2.*TK1*S*B))
70  DO 30 J=4,NN
71  30 RV(I,J)=(2./(HBOT*S2*B))
72  DO 40 J=4,NN
73  40 RV(I,J)=((S)/(TK*B))+(S2/(2.*TK3*S*B))
74  DO 50 I=2,MM
75  50 RV(I,3)=(S1/(2.*TK*S*B))+(I./(HBOT*S*B))
76  DO 60 I=2,MM
77  60 RV(I,N+3)=(S3/(2.*TK2*S*B))+(S1/(2.*TK1*S*B))
78  DO 70 I=3,MM
79  70 RV(I,N+3)=(S2/(2.*TK2*S*B))+(S1/(2.*TK1*S*B))
80  DO 80 J=4,NN
81  80 RV(I,J)=(2./(HBOT*S2*B))
82  DO 90 J=4,NN
83  90 RV(I,J)=((S)/(TK*B))+(S2/(2.*TK3*S*B))
84  DO 100 I=3,MM
85  100 RV(I,2)=(S1/(TK1*S*B))
86  RV(2,N+3)=RH(2,N+3)+RV(I, N+3)
87
88 HORIZONTAL RESISTANCES
89
90  RH(2,N+3)=((S2+S)/(2.*TK2*S3*B))
91  DO 70 I=3,MM
92  70 RH(I,N+3)=(S3/(2.*TK*S3*B))
93  DO 80 J=3,NN
94  80 RH(2,J)=(S2/(2.*TK2*S*B))+(I./(2.*TK*B))
95  DO 90 J=3,NN
96  90 RH(I,2)=(S3/(2.*TK3*S*B))+(I./(2.*TK*B))
97  DO 100 I=3,MM
98  100 RH(I,2)=(S/(TK1*S*B))
99  RH(2,N+3)=RH(2,N+3)+RV(1, N+3)
100
101 NODAL CAPACITANCES
102
103  C(I,2)=((DEN1*S1*S2*B*CP1)/2.)
104  DO 110 I=2,NN
105  110 C(I,2)=((DEN1*S1*S*B*CP1)/2.)
106  DO 120 J=3,NN
107  120 C(I,J)=((DEN2*S2*S*B*CP2)/2.)
108  C(I,N+3)=((DEN3*S3*S*B*CP3)/2.)
109  C(2,N+3)=(S3*B*(S+(S2/2.))*DEN3*CP3)
110  DO 130 I=3,MM
111  130 C(I,2)=((DEN1*S1*S2*B*CP1)/2.)
112  DO 140 J=3,NN
113  140 C(I,J)=((DEN1*S1*S*B*CP1)/2.)
114  Q1=(C(3,4)*(TTR-TREF))
115  Q2=Q1+((DEN*(S**2)*B*HTR))
Q3=Q2+((C(3,4))*(TMELT-TTR))
Q4=Q3+((DEN*(S**2)*B*HMELT))

INITIALIZATION OF PERTINENT QUANTITIES

TIME=0.0
QWAX=0.0
QBW=0.0
QSW=0.0
QTW=0.0
QTOP=0.0
QFTR=0.0
QTTR=0.0
V1=0.0
V2=0.0
DMO=0.0

DO 150 J=2,NJ
  DO 150 I=I,MM
    TI(I,J)=TIN
    IF(TIN.LT.TTR) QSIN=(C(3,4)*(TIN-TREF))
    IF(TIN.GT.TTR.AND.TIN.LT.TMELT) QSIN=Q2+((C(3,4))*(TIN-TTR))
    IF(TIN.GT.TMELT) QSIN=Q4+((C(3,4))*(TIN-TMELT))
  DO 160 I=I,MM
    T1(I,N+4)=TAMB
    DO 170 J=3,NN
      DO 170 I=2,MM
        QS(I,J)=QSIN
      DO 180 I=I,MM
        Q(I,2)=0.0
      DO 190 J=3,NN
        Q(I,N+3)=0.0
      DO 190 J=2,NN
        Q(I,J)=0.0
      DO 199 J=2,NN
        F(I,J)=0.0
      READ(5,11) (TM1(I), I=I,ND)
      READ(5,11) (TM2(I), I=I,ND)
      READ(5,11) (TM3(I), I=I,ND)
      READ(5,11) (TM4(I), I=I,ND)
      READ(5,11) (TIM(I), I=I,ND)
      WRITE(6,22) TIME,W,H,N,M
      FORMAT(1X,5HTIME=,E15.8,10X,2HN=,E15.8,10X,2HN=,E15.8,10X,2HN=,12,
      15X,2HM=,12)
      WRITE(6,33)Q1,Q2,Q3,Q4
      FORMAT(1X,3HQ1=,E15.8,10X,3HQ2=,E15.8,10X,3HQ3=,E15.8,10X,3HQ4=,E1
      5.8)
      WRITE(6,44)
      FORMAT(2X,4HI J,5X,19HVERTICAL RESISTANCE,6X,21HHORIZONTAL RESIST
      ANCE,6X,17HNODAL CAPACITANCE,6X,11HTEMPERATURE,5X,7HQS(I,J))
      DO 200 J=2,NI
      DO 200 I=1,MM
        IF(J.EQ.N+4) GO TO 1
        GO TO 2
      QS(I,J)=0.0
      RH(I,J)=1.E08
      C(I,J)=0.0
IF(I.EQ.1)RH(I,J)=1.E08
IF(J.EQ.2.OR.J.EQ.N+3)QS(I,J)=0.0
IF(I.EQ.1)QS(I,J)=0.0
WRITE(6,55)I,J,RV(I,J),RH(I,J),C(I,J),T1(I,J),QS(I,J)
55 FORMAT(IX,12,1X,12,5X,E15.8,10X,E15.8,10X,E15.8,6X,E15.8,4X,E15.8)
CONTINUE
DO 889 J=3,NN
889 T3(I,J)=T1(I,J)

3 TIME=TIME+DT
DO 210 J=3,NJ
3 TIME=TIME+DT

210 CONTINUE
DO 211 J=3,NJ
211 CONTINUE
DO 212 J=3,NN
212 DO 220 I=2,M
220 QS(I,J)=QS(I,J)+(QRAT(I,J)*DT)
220 CONTINUE
DO 213 J=3,NJ
213 DO 211 I=2,M
211 DO 250 J=3,NN
250 CONTINUE
DO 240 I=2,M
240 T2(I,N+3)=T1(I,N+3)+((QP_AT(I,N+3)*DT)/C(I,N+3))
240 DO 250 J=3,NN
250 DO 240 I=2,M
240 DO 255 J=3,NN
255 T2(I,J)=T3(I,J)
255 CONTINUE
DO 260 L=I,NDP
260 IF(TIME.GE.TIM(L).AND.TIME.LE.TIM(L+I)) GO TO 4
260 CONTINUE
4 FAC=(TIME-TIM(L))/(TIM(L+I)-TIM(L))
T2(I,2)=TM1(L)+((TM1(L+I)-TM1(L))*FAC)
T2(I,8)=TM2(L)+((TM2(L+I)-TM2(L))*FAC)
T2(I,15)=TM3(L)+((TM3(L+I)-TM3(L))*FAC)
T2(I,21)=TM4(L)+((TM4(L+I)-TM4(L))*FAC)
DO 270 I=1,MM
270 IF(QS(I,J).LT.Q1)T2(I,J)=TREF+(QS(I,J)/C(I,J))
270 IF(QS(I,J).GE.Q1.AND.QS(I,J).LE.Q2) T2(I,J)=TTR
270 IF(QS(I,J).GT.Q2.AND.QS(I,J).LT.Q3) T2(I,J)=TTR+((QS(I,J)-Q2)/C(I,
270 J))
270 IF(QS(I,J).GE.Q3.AND.QS(I,J).LE.Q4) T2(I,J)=TMELT
270 WHEN GOING FROM MELT TO FREEZE OR VICE-VERSA CHANGE THE FOLLOWING CARD
270 IF(QS(I,J).GT.Q3.AND.QS(I,J).LT.Q4) F(I,J)=(QS(I,J)-Q3)/(DEN*(S**2*
270 HMELT)
270 IF(QS(I,J).GT.Q4) T2(I,J)=TMELT+((QS(I,J)-Q4)/C(I,J))
270 WHEN GOING FROM MELT TO FREEZE OR VICE-VERSA CHANGE THE FOLLOWING CARD
270 IF(QS(I,J).GE.Q4)F(I,J)=1.0
270 IF(QS(I,J).LE.Q3) F(I,J)=0.0
270 CONTINUE
250 CONTINUE
250 SPECIFICATION AND/OR DETERMINATION OF FIN TEMPERATURES
250 THE FOLLOWING DO LOOP ASSUMES FIN TEMPERATURES FOR ITERATION
250 DO 255 J=3,NN
255 T2(I,J)=T3(I,J)
255 CONTINUE
250 CONTINUE
250 CONTINUE
270 CONTINUE
270 CONTINUE
270 CONTINUE
270 CONTINUE
270 CONTINUE
270 CONTINUE
270 CONTINUE
270 CONTINUE
270 CONTINUE
T2(I,2)=T2(I,2)

UNSPECIFIED FIN TEMPERATURES DETERMINED BY STEADY STATE EQUATIONS

MCOUNT=MCOUNT+I

DO 280 J=3,NN

T3(I,J)=T2(I,J)

IF(J.EQ.8.OR.J.EQ.15) GO TO 5

IF(J.EQ.21) GO TO 5

T2(I,J)=((T2(I,J-1)/RV(I,J))+(T2(J)/RH(2,J))+(T2(I,J+1)/RV(I,J+1)))

5 CONTINUE

DO 280 J=3,NN

IF(MCOUNT.GT.MFIN) GO TO 8

DO 281 J=3,NN

DIF=T2(I,J)-T3(I,J)

IF(ABS(DIF).GT.EPS) GO TO 256

281 CONTINUE

IF(JOE.EQ.2) GO TO 285

JQE=JQE+1

DO 888 I=T,MM

DO 888 J=3,NJ

888 QS(I,J)=QS(I,J)-(QRAT(I,J)*DT)

DO 282 J=3,NJ

QRA2(I,J)=((T2(I,J)-T2(I,J))/RH(2,J))+(T2(I,J-1)-T2(I,J))/RV(I,J)

QRA2(MM,J)=((T2(M,J)-T2(MM,J))/RH(MM,J))+(T2(MM,J-I)-T2(MM,J))/RV(MM,J)

282 CONTINUE

DO 283 J=3,NJ

QRA2(I,J)=((T2(I-I,J)-T2(I,J))/RN(I,J))+((T2(I+I,J)-T2(I,J))/RH(I,J)+((T2(I,J-I)-T2(I,J))/RV(I,J)+((T2(I,J+I)-T2(I,J))/RV(I,J+1)

283 CONTINUE

DO 284 J=3,NJ

QRAT(I,J)=(QRAT(I,J)+QRA2(I,J))/2.0

284 CONTINUE

MCOUNT=MCOUNT+1

DO 286 I=1,MM

DO 287 I=1,MM

QS(I,2)=QS(I,2)+(C(I,2)*(T2(I,2)-TI(I,2)))

C(IF(I),2) =C(IF(I),2)+(T2(I,2)-T2(I,3))/RV(I,3)+QBW

290 QTW=(T2(I,N+3)-T2(I,N+2))/RV(I,N+3)+QTW

QBT=QBW+((T2(I,2)-T2(I,3))/RV(1,3))

DO 300 J=3,NN

QSW=QSW+((T2(I,J)-T2(I,J))/RH(I,J))

QTOP=((T2(I,N+3)-TAMB)/RV(I,N+4))+QTOP

QFTR=QFTR+QRAT(I,J)

QTTR=QTTR+QRAT(I,NJ)

QWAX=(QBW+QTW+QSW)*DT+(QWAX)

290 QTW=(T2(I,N+3)-T2(I,N+2))/RV(I,N+3)+QTW

QBT=QBW+((T2(I,2)-T2(I,3))/RV(1,3))

DO 300 J=3,NN

QSW=QSW+((T2(I,J)-T2(I,J))/RH(I,J))

QTOP=((T2(I,N+3)-TAMB)/RV(I,N+4))+QTOP

QFTR=QFTR+QRAT(I,J)

QTTR=QTTR+QRAT(I,NJ)

QWAX=(QBW+QTW+QSW)*DT+(QWAX)

RATIO=QSW/QBW
ERROR=((QBT-(QBW+QSW+QTW+QTOP+QFTR+QTTR))*100.0)/QBT
DO 930 J=3,NN
DO 930 I=2,MM
V2=V2+(F(I,J)*(S**2)*B)
930 CONTINUE
DAVG=((2*V2)/(W*B))*12.0
QMELT=((V2-V1)*DEN*HMELT)/DT
V1=V2
V2=0.0
DELT=ABS(T2(1,2)-TMELT)
PR=(VIS*CL)/TKL
RA=((DEN**2)*G*CL*BETA*DELT*(DAVG**3))/(VIS*TKL*1728.0)
IF(T2(1,2).LE.TMELT) RA=0.0
IF(RA. GT. 1.0E05) TK=(TKL*0.104*(RA**0.305)*(PR**0.084))
IF(RA. GE. 3500.0 .AND. RA. LE. 1.0E05) TK=(TKL*0.229*(RA**0.252))
IF(RA. GE. 1700.0 .AND. RA. LT. 3500.0) TK=(TKL*0.00238*(RA**0.816))
TKR=TK/TKL
DO 945 I=2,MM
IF(F(I,3).LT.0.25) GO TO 945
RV(I,3)=(1./(2.*TK*B))+(S1/(2.*TK1*S*B))
945 CONTINUE
DO 946 J=4,NN
DO 946 I=2,MM
IF(F(I,J).LT.EPS) GO TO 946
RV(I,J)=(1./(TK*B))
946 CONTINUE
DO 947 I=2,MM
IF(F(I,NN).LT.0.75) GO TO 947
RV(I,NN+3)=(S3/(2.*TK3*S*B))+(I./(2.*TK*B))
947 CONTINUE
DO 950 J=3,NN
IF(F(2,J).LT.0.25) GO TO 949
RH(2,J)=(S2/(2.*TK2*S*B))+(I./(2.*TK*B))
949 CONTINUE
IF(KCOUNT.EQ.KCHK) GO TG 6
KCOUNT=KCOUNT+1
GO TO 7
WRITE(6,66) TIME, QWAX, ERROR
66 FORMAT(1X,5HTIME=,E15.8,10X,5HQWAX=,E15.8,10X,6HERROR=,E15.8)
DO 313 J=3,NN
AJ=J-3
BJ=J-2
IF(F(2,J).GT.F2JM) DFLO=(BJ*S*12.0)
DFIN=(AJ*S*12.0)
IF(F(2,J).LT.EPS) GO TO 314
313 CONTINUE
314 DO 315 J=3,NN
AJ=J-3
315 CONTINUE
IF(315) 313
312 IF(F(I,J).LT.EPS) GO TO 950
RH(I,J)=(1./(TK*B))
316 IF(KCOUNT.EQ.KCHK) GO TG 6
KCOUNT=KCOUNT+1
GO TO 7
WRITE(6,66) TIME, QWAX, ERROR
66 FORMAT(1X,5HTIME=,E15.8,10X,5HQWAX=,E15.8,10X,6HERROR=,E15.8)
DO 313 J=3,NN
AJ=J-3
BJ=J-2
IF(F(2,J).GT.F2JM) DFLO=(BJ*S*12.0)
DFIN=(AJ*S*12.0)
IF(F(2,J).LT.EPS) GO TO 314
313 CONTINUE
314 DO 315 J=3,NN
AJ=J-3
315 CONTINUE
IF(315) 313
312 IF(F(I,J).LT.EPS) GO TO 950
RH(I,J)=(1./(TK*B))
316 IF(KCOUNT.EQ.KCHK) GO TG 6
KCOUNT=KCOUNT+1
GO TO 7
WRITE(6,66) TIME, QWAX, ERROR
66 FORMAT(1X,5HTIME=,E15.8,10X,5HQWAX=,E15.8,10X,6HERROR=,E15.8)
DO 313 J=3,NN
AJ=J-3
BJ=J-2
IF(F(2,J).GT.F2JM) DFLO=(BJ*S*12.0)
DFIN=(AJ*S*12.0)
IF(F(2,J).LT.EPS) GO TO 314
334  67 FORMAT(1X,5HDFIN=,E15.8,10X,5HDFLO=,E15.8,10X,5HDAVG=,E15.8,10X,5H
335    DMID=,E15.8)
336    DDOT=(DMID-DMO)/DT
337    DMO=DMID
338    WRITE(6,68) RA, TKR, QMELT, DDOT
339  68 FORMAT(1X,3HRA=,E15.8,10X,4HTKR=,E15.8,10X,6HQMELT=,E15.8,10X,5HDD
340      DT=,E15.8)
341    WRITE(6,77) QBW, QSW, QTW, QBT, RATIO
342  77 FORMAT(I X,4HQBW=,E15.8,3X,4HQSW=,E15.8,3X,4HQTW=,E15.8,3X,4HQBT=,E
343      15.8,3X,6HRATIO=,E15.8)
344    WRITE(6,88)
345  88 FORMAT(2X,1HI,2X,1HJ,10X,11HTEMPERATURE,10X,15HFRACTION MELTED,10X
346      ,11HENERGY RATE,10X,13HENERGY STORED)
347    DO 320 J=2,NJ
348    DO 320 I=1,MM
349  99 FORMAT(I X,12,1X,12,8X,E15.8,8X,E15.8,8X,E15.8,8X,E15.8)
350  320 CONTINUE
351  7 QBW=0.0
352  8 QSW=0.0
353  9 QTW=0.0
354 10 QTOP=0.0
355 11 QTTR=0.0
356 12 QFTR=0.0
357 13 DO 330 J=2,NJ
358 14 DO 330 I=1,MM
359 15 T(I,J)=T2(I,J)
360 16 JOE=1
361 17 IF(TIME.LT.TAU) GO TO 3
362 18 WRITE(6,111) MCOUNT
363 19 111 FORMAT(IX,13)
364 20 STOP
365 21 END
D.2. DESCRIPTION OF PROGRAM NOTATION

AJ  J-3, used in computing interface location
AM  value of integer M converted to floating point
B   length of section, ft
BETA volume expansivity of wax, R⁻¹
BJ  J-2, used in computing interface location
C(I,J) thermal capacitance of node (I,J), Btu/°F
CL  constant pressure specific heat of wax, Btu/lbm°F
CP  constant pressure specific heat of wax, Btu/lbm°F
CP1 constant pressure specific heat of bottom plate, Btu/lbm°F
CP2 constant pressure specific heat of fin, Btu/lbm°F
CP3 constant pressure specific heat of top plate, Btu/lbm°F
DAVG average height of liquid based on amount melted, in.
DDOT interfacial velocity for nodes adjacent to centerline (I = MM), in./h
DELT absolute value of temperature difference between bottom plate and interface, °F
DEN  wax density, lbm/ft³
DEN1 bottom plate density, lbm/ft
DEN2 fin density, lbm/ft³
DEN3 top plate density, lbm/ft³
DFLO⁻¹ approximate interfacial location for nodes adjacent to fin (I = 2) based on amount melted being equal to F2JM, in.
DFIN interfacial position for nodes adjacent to fin (I = 2) based on any amount being melted, in.
DIF  temperature difference used in comparing new and old temperatures during iteration when solving steady state equations for unspecified fin temperatures, °F.

DMID interfacial position for node adjacent to centerline, in.

DMO DMID evaluated at previous time, in.

DT time increment, h.

EPS arbitrarily set small number used as a comparator

ERROR percent error in computed energy balance based on transfer rates, percent.

F(I,J) mass fraction of node (I,J) which has undergone phase change since start of process

FAC time ratio used in linearly interpolating specified fin temperatures at a particular time in terms of bracketed data values.

F2JM fraction of S which corresponds to 1/32 in. (arbitrary)

G acceleration of gravity, ft/h^2

H PCM section height (see Fig. 40), ft.

HBOT^2 heat transfer coefficient between external fluid and bottom plate, Btu/h-ft^2·°F

HMELT heat of fusion, Btu/lbm

HTOP^3 heat transfer coefficient between external fluid and top plate, Btu/h-ft^2·°F

HTR heat of transition, Btu/lbm.

I integer designation of vertical column in which a mode is located (see Fig. 40)

J integer designation of horizontal row in which a mode is located (see Fig. 40)

JOE counter used in refining the heat transfer computation before progressing in time

KCHK integer used to control printing of results at desired times (see definition of KCOUNT)
KCOUNT  integer counter used to print our results at times when KCOUNT = KCHK
M  number of wax nodes in a horizontal row
MCOUNT  counter used in determining unspecified fin temperatures
MFIN  maximum value of MCOUNT which when exceeded causes program to stop
MM  M+1 (see Fig. 40)
N  number of wax nodes in a vertical column
ND  number of data points for measured fin temperatures
NDP  ND-1
NI  N+4
NJ  N+3
NN  N+2
PR  Prandtl number
Q(I,J)  unnecessary variable — replaced where needed by QS(I,J)
QBT  instantaneous heat transfer rate through bottom, Btu/h
QBW  instantaneous heat transfer rate through bottom to wax only, Btu/h
QFTR  instantaneous rate of heat transfer to fin, Btu/h
QMELT  energy which accounts for amount of wax melted at any time, Btu
QRAT(I,J)  the instantaneous net rate of heat transfer to node I,J, Btu/h
QS(I,J)  the energy stored by node I,J above TREF for wax and above 0 for metal nodes, Btu
QSIN  the energy stored by node I,J above TREF corresponding to initial temperature throughout network, Btu
QSW  instantaneous heat transfer rate from fin to wax, Btu/h
TIME = instantaneous value of time, h.
TIN = initial temperature of all nodes, °F
TK = wax thermal conductivity (artificially allowed to vary in liquid to account for convection), Btu/h-ft-°F
TKL = thermal conductivity of liquid, Btu/h-ft-°F
TKR = ratio of effective thermal conductivity to thermal conductivity
TMELT = fusion temperature of wax, °F
TREF = arbitrary reference temperature (should be less than TIN), °F
TTR = transition temperature, °F
T1(I,J) = temperature of node I,J at time t, °F
T2(I,J) = temperature of node I,J at time t+Δt, °F
T3(I,J) = temperature of node I,J in fin at beginning of each iterative step used in finding steady state solution, °F
TAU2 = arbitrarily defined time value used in print-out control, h.
TK1 = thermal conductivity of bottom plate, Btu/h-ft-°F
TK2 = thermal conductivity of fin, Btu/h-ft-°F
TK3 = thermal conductivity of top plate, Btu/h-ft-°F
TIM(L) = time value corresponding to input data of measured fin temperatures, h
TM1(L) = measured bottom plate temperature (input data), °F
TM2(L) = first measured fin temperature (input data), °F
TM3(L) = second measured fin temperature (input data), °F
TM4(L) = third measured fin temperature (input data), °F
VIS = viscosity of liquid, lbm/h-ft
V1     volume of wax melted at time $t$, $ft^3$ 

V2     volume of wax melted at time $t+\Delta t$, $ft^3$ 

W     width of wax cell, ft 

1. This assumes that some finite thickness must have melted before it could be detectable on the film. The number DFIN is the height corresponding to a node with any amount melted. 

2. This was included to be general but has not been used to date as bottom plate temperatures were specified as input data. 

3. This has been included but set at a small value to essentially correspond to the top being insulated.
### D.3. DISCUSSION OF COMPUTER PROGRAM FOR MELTING

In the following discussion, references are made to line numbers corresponding to those designated on the copy of the program.

<table>
<thead>
<tr>
<th>LINES</th>
<th>DISCUSSION</th>
</tr>
</thead>
<tbody>
<tr>
<td>1–4</td>
<td>required dimension statements for subscripted variables; values should be (MM, NI) for all double subscripted variables except T3 for which they should be (1, NJ); values should be ND for single subscripted variables. NOTE: Q(MM,NI) is superfluous and can be omitted with lines 136-140.</td>
</tr>
<tr>
<td>5–23</td>
<td>specification of computational parameters</td>
</tr>
<tr>
<td>24–54</td>
<td>specification of physical properties</td>
</tr>
<tr>
<td>55–63</td>
<td>specification of geometry parameters</td>
</tr>
<tr>
<td>64–82</td>
<td>computation of all vertical thermal resistance values RV (I,J)</td>
</tr>
<tr>
<td>83–95</td>
<td>computation of all horizontal thermal resistance values RH(I,J); note that line 95 is a special definition which amounts to bypassing node (1,N+3) which was done to overcome stability criterion required by this small corner node</td>
</tr>
<tr>
<td>96–108</td>
<td>computation of all nodal capacitance values C(I,J)</td>
</tr>
<tr>
<td>109–112</td>
<td>computation of energy stored by a wax node relative to TREF for the start and end of phase transition and the start and end of fusion, respectively</td>
</tr>
<tr>
<td>113–144</td>
<td>initialization of pertinent quantities; the initial value of the stored energy depends on the relationship of the initial temperature to the reference temperature TREF; note that lines 136–140 are superfluous and can be omitted; some initialization of certain parameters is done in the DO loop between lines 160 and 172 which are set primarily to avoid random printout and are not essential to the computation done in the heart of the program</td>
</tr>
<tr>
<td>145–150</td>
<td>input data values for measured fin and bottom plate temperatures and corresponding time values</td>
</tr>
<tr>
<td>151–172</td>
<td>printout of initial values for checking purposes and printout of certain computed quantities for informational purposes</td>
</tr>
<tr>
<td>173–174</td>
<td>initially defines T3(I,J) for all fin nodes and sets these equal to the initial temperatures T1(I,J)</td>
</tr>
</tbody>
</table>
beginning of main computation scheme

time is stepped forward by $\Delta t$

loops which compute and store the net rate of heat transfer to nodes $(i,j)$ based on old temperatures $T_1(i,j)$; the rate of heat transfer to node $(i,j)$ is given by

$$q(i,j) = \frac{T(i-1,j)-T(i,j)}{RH(i,j)} + \frac{T(i+1,j)-T(i,j)}{RH(i,j)} + \frac{T(i,j-1)-T(i,j)}{RV(i,j)} + \frac{T(i,j+1)-T(i,j)}{RV(i,j+1)}$$

this expression must be modified accordingly for nodes near a boundary which are not surrounded by four neighbors

computation of total energy stored by node $(i,j)$ since the start which is given by

$$Q_{stored} = \sum_{time} q \Delta t$$

computation of new top plate temperatures $T_2(i,N+3)$ from the expression

$$T_2(i,j) = T_1(i,j) + \frac{q(i,j)\Delta t}{C(i,j)}$$

sets the corner plate node $(1,N+3)$ temperature equal to that of the second node $(2,N+3)$; omission of the corner node in the computation scheme was done to avoid stability problems due to its small size

computation of new wax node temperatures from the energy stored by the nodes and their capacity and/or phase change enthalpy values; when the stored energy lies between $Q_1$ and $Q_2$ the new temperature is forced to be the transition temperature and when it lies between $Q_3$ and $Q_4$ the new temperature; is forced to be the fusion temperature; also the fraction of the node which has undergone phase change is calculated from the relationship of the stored energy to $Q_3$ and $Q_4$; note that certain designated cards need to be changed when running the program for freezing as contrasted to melting

beginning of determination of fin temperatures
all new fin temperatures are set to $T_3(I,J)$ which simply represents an assumed value always corresponding to the previously computed value except at the very beginning at which time it is set as the initial temperature.

Interpolation scheme which assigns new temperatures to the three nodes on the fin and one on the bottom plate corresponding to positions where temperature measurements were made; the new temperatures are linearly interpolated from the input data.

Assigns all nodes along the bottom plate the same value of new temperature.

Beginning of iteration process to determine unspecified fin temperatures from steady state equation; fin nodes were not treated as transient cases due to their extremely small capacitances that would impose a severe stability criterion.

Iteration process used to determine unspecified fin temperatures; in each iteration, $T_2(I,J)$ is computed from steady state equations and then compared with $T_3(I,J)$ which corresponds to the calculated temperatures during the previous iterative step; the iteration is continued until the differences between computed fin temperatures and their corresponding values for the previous iterative step are all acceptably small; should the iteration exceed MFIN counts the program is directed to stop.

When counter JOE equals 2, the new temperatures at all nodes are considered to be the solution at the particular time and the program advances to line 259.

Increase of counter JOE to 2.

The energy stored at each node $(I,J)$ is reset back to its original value; this is to allow for an improved computation of the net heat transfer rate to each node to be made and then a recomputation of the stored energy and the corresponding new temperatures.

Computation of net heat transfer rate to each node using new temperatures $T_2$.

Calculation of net heat transfer rate to each node as the average of that based on old temperatures $T_1$ and new temperatures $T_2$.

Return to line 189 which consists of redirecting the computation through that of computing improved new temperatures, energy storage values, and fractional melted values by using the improved (averaged) heat transfer rate (Lines 255-257); this corrective technique is only employed once.
computation of various heat transfer quantities from the new temperatures obtained at time $t+\Delta t$

computation of volume of melted wax

computation of average liquid depth from the volume melted

computation of energy required to melt the wax which has melted

resetting of $V_1$ and $V_2$ for next time step

determination of absolute value of temperature difference between bottom plate and the fusion temperature

computation of the Prandtl number

computation of the Rayleigh number

determination of effective liquid conductivity due to convection by using correlations of O'Toole and Silveston

computation of ratio of effective liquid conductivity to actual value

recomputation of thermal resistances in the liquid by using the effective thermal conductivity rather than the actual value

counter check which controls printing out of desired results as well as computation of interfacial position

counter advance

by-pass of printing results except when line 316 is executed

write statement for printing results

computation of interface position for columns next to fin and next to centerline

write statement for printing results

calculation of interfacial velocity and renaming interfacial position to provide for determining its change at the next time step

write statements and corresponding formats for printing results
reinitialization of pertinent quantities for next time step

setting new temperatures for current time step to be old temperatures for
the next time step

reinitialization of counter

comparison of time to upper limit value which when exceeded results in
stopping the program

printing out of value of counter used in fin temperature iteration

STOP

END
Figure D-1. Skelton flow chart.
D.4. COMPUTER PROGRAM FOR SOLIDIFICATION

C TEST 230-15 FREEZE TEST 3/4 INCH CELL
DIMENSION RV(5,31), RH(5,31), C(5,31), QS(5,31), T1(5,1,31)
DIMENSION T2(5,31), F(5,31), QRA(5,31), T3(1,31), QRA2(5,31)
DIMENSION TM1(25), TM2(25), TM3(25), TM4(25), TIM(25)
DIMENSION VR(5,31), HR(5,31)

C COMPUTATIONAL PARAMETERS
N=27
M=4
MM=M
NN=N+2
NJ=N+3
NI=N+4
ND=23
NDP=22
TAU=1.5
KCOUNT=1
MCOUNT=1
MFIN=500
EPS=1.E-06
KCH=1000
JOE=1
DT=0.00005
TAU2=(10**DT)+(DT/3**)
AF=40.

C PHYSICAL PROPERTIES
TAMB=80*
TIN=100*
HCP=1.E-08
HOT=5*
G=(32.174*3600.*3600.)*

C WAX
DEN=47.2
TK=0.087
CP=0.5
TR=73.04
HTR=22.108
TMELT=89.8
HMELT=73.357
TREF=50.*
BETA=0.00045
VIS=1.43
CL=CP
tkl=TK

C BOTTOM PLATE DENOTED BY 1
DEN1=171*
TK1=93*
CP1=0.22
C FIN DENOTED BY 2
  DEN2=171
  TK2=93
  CP2=0.22
C TOP PLATE DENOTED BY 3
  DEN3=72.5
  TK3=0.09
  CP3=0.33
C GEOMETRY PARAMETERS
  w=0.75/12.0
  H=2.625/12.0
  B=5.0/12.0
  S=H/(2.*AM)
  S1=0.032/12.0
  S2=0.008/12.0
  S3=0.25/12.0
  F2JW=1.0/32.0*5.0/12.0
C VERTICAL RESISTANCES
  RV(I,2)=((S1/(TK1*S2*B))+(1/(HT1*S2*B))
  DO 10 J=2,MM
10 RV(I,2)=((S1/(2.*TK1*S2*B))+(1/(HT1*S2*B))
  RV(I,3)=((S3/(TK2*S2*B))+(S1/(TK1*S2*B))
  DO 20 J=2,MM
20 RV(I,3)=((S3/(2.*TK3*S2*B))+(S1/(2.*TK1*S2*B))
  DO 30 J=4,NN
30 RV(I,J)=(2.*S)/(TK2*S2*B))
  DO 40 J=4,NN
40 RV(I,J)=(1.0/(TK*B))
  RV(I,N+3)=((S3/(TK3*S2*B))+(S1/(TK2*S2*B))
  DO 50 J=2,MM
50 RV(I,N+3)=((S3/(2.*TK3*S2*B))+(1.0/(HT0*S*B))
  RV(I,N+4)=((S3/(TK3*S2*B))+(2.0/(HT0*S*B))
  RV(2.,N+4)=((S3/(2.*TK3*S2*B)+S2/(S2/2.*)))+(1.0/(HT0*B*(S+
1.S2/2.*))
C HORIZONTAL RESISTANCES
  DO 70 J=2,MM
70 RH(I,2)=(S/(TK1*S1*B))
  DO 80 J=3,NN
80 RH(I,J)=(1.0/(TK*B))
  DO 90 J=3,NN
90 RH(2,J)=(S2/(2.*TK2*S*B))+(1.0/(2.*TK*B))
  RH(2,J)=(S2+S)/(2.*TK1*S1*B))
  RH(2,N+3)=(S2+S)/(2.*TK3*S3*B))
  RH(2,N+3)=RH(2,N+3)+RV(I,N+3)
C NODAL CAPACITANCES
  C(1,2)=((DEN1*S1*S2*B*CP1)/2.)
  DO 110 I=2,MM
C(IN) = (DEN1*S1*S*BP1)
   DO 120 J = 3, NN
120 C(IN) = ((DEN2*S2*S*BP2)/2)
   C(IN+1) = ((DEN3*S3*S*BP3)/2)
   DO 130 I = 2, MM
130 C(IN+1) = (DEN3*S3*S*BP3)
   C(IN+1) = (S3*S*(S+(S2/2)))*DEN3*BP3
   DO 140 J = 3, NN
   DO 140 I = 2, MM
140 C(IN) = (DEN*(S**2)*BP)
   Q1 = (C(2,3)*(TTR=TREF))
   Q2 = Q1 + (DEN*(S**2)*BP*HTR)
   Q3 = Q2 + ((C(2,3))*(TMELT=TTR))
   Q4 = Q3 + (DEN*(S**2)*BP*HMLT)
C INITIALIZATION OF PERTINENT QUANTITIES
TIME = 0.0
QWAX = 0.0
GBX = 0.0
QSH = 0.0
QTH = 0.0
QTOP = 0.0
QTTR = 0.0
V1 = 0.0
V2 = 0.0
DMO = 0.0
   DO 150 J = 2, NJ
   DO 150 I = 1, MM
150 T(I,J) = TIN
   IF (TIN = T*TR) QSIN = (C(2,3)*(TIN=TREF))
   IF (TIN = T*TR AND TIN = TMELT) QSIN = Q2 + (C(2,3)*(TIN
1 = TTR))
   IF (TIN = T*TR AND TIN = TMELT) QSIN = Q4 + (C(2,3)*(TIN=TMELT))
   DO 160 I = 1, MM
160 T(I,N+1) = TAMB
   DO 170 J = 3, NN
   DO 170 I = 2, MM
170 QS(I,J) = QSIN
   DO 199 J = 2, NI
   DO 199 I = 1, MM
F(I,J) = 0.0
199 QRT(I,J) = 0.0
   READ(5,11) (TM1(I), I = 1, ND)
   READ(5,11) (TM2(I), I = 1, ND)
   READ(5,11) (TM3(I), I = 1, ND)
   READ(5,11) (TM4(I), I = 1, ND)
   READ(5,11) (TM5(I), I = 1, ND)
   WRITE(6,22) TIME,N,M,M
11 FORMAT(8F10.0)
910 KTC = 1, ND
227
```plaintext
22 FORMAT(1X,5HTIME=E15.8,10X,2H=,E15.8,10X,2H=,E15.8
1,10X,2H=,12,15X,2H=,12)
WRITE(6,33)G1,02,03,G4
33 FORMAT(1X,3HQ1=E15.8,10X,3HQ2=E15.8,10X,3HQ3=E15.8
1,10X,3HQ4=E15.8)
WRITE(6,44)
44 FORMAT(2X,4HI J,5X,19HVERTICAL RESISTANCE=6X
1,21HORIZONTAL RESISTANCE=6X,17HNODAL CAPACITANCE=6X
2,11HTEMPERATURE=5X,7HGS(I,J))
DO 200 J=2,NI
DO 200 I=1,MM
IF(J.EQ.N+4) GO TO 1
GO TO 2
1 QS(I,J)=0.0
RH(I,J)=1.0E08
C(I,J)=0.0
2 IF(I.EQ.1) RH(I,J)=1.0E08
IF(J.EQ.2.OR.J.EQ.N+3) QS(I,J)=0.0
IF(I.EQ.1) QS(I,J)=0.0
WRITE(6,55)I,J,RV(I,J),RH(I,J),C(I,J),T(I,J),QS(I,J)
55 FORMAT(1X,12,1X,12,5X,E15.8,10X,E15.8,10X,E15.8,6X
1,E15.8,4X,E15.8)
200 CONTINUE
DO 400 J=3,NI
DO 400 I=2,MM
VR(I,J)=RV(I,J)
400 HR(I,J)=RH(I,J)
DO 889 J=3,NN
889 T(I,J)=T(I,1)
C COMPUTATION SECTION
SECTION COMPUTATION
SECTION COMPUTATION
C
3 TIME=TIME+DT
DO 210 J=3,NI
QRA(T(I,J)=(T(I+1,J)-T(I,J))/RH(I,J))+(T(I,J-1)-T(I,J))/RH(I,J))
QRA(T(MM,J)=(T(MM,J)-T(1,MJ))/RH(MM,J))+(T(MM,J-1)-T(MM,J))/RH(MM,J))
DO 210 CONTINUE
IF(M.EQ.1) GO TO 212
DO 211 J=3,NI
DO 211 I=2,MM
QRA(I,J)=(T(I+1,J)-T(I,J))/RH(I,J)+(T(I,J-1)-T(I,J))/RH(I,J)
QRA(I,J)+QRA(I,J)+QRA(I,J)+QRA(I,J)+QRA(I,J)+QRA(I,J)+QRA(I,J)+QRA(I,J)
DO 211 CONTINUE
212 DO 220 J=3,NI
DO 220 I=1,MM
220 QS(I,J)=QS(I,J)+(QRA(I,J)*DT)
DO 240 I=2,MM
240 T(I,N+3)=T(I,N+3)+(QRA(I,N+3)*DT)/C(I,N+3)
```
T2(1,s+3)=T2(2,s+3)
DO 250 J=3,NN
DO 250 I=1,MM
IF(QS(I,J)<LT,Q1) T2(I,J)=TREF+(QS(I,J)/C(I,J))
IF(QS(I,J)>GE,Q1 AND QS(I,J)<LE,Q2) T2(I,J)=TTR
IF(QS(I,J)>GT,Q2 AND QS(I,J)<LT,Q3) T2(I,J)=TTR+(QS(I
1,J)>Q2)/C(I,J))
IF(QS(I,J)<GE,Q4 AND QS(I,J)<LE,Q4) T2(I,J)=TMELT
WHEN GOING FROM MELT TO FREEZE OR VICE-VERSA CHANGE THE
FOLLOWING CA
IF(QS(I,J)>GT,Q4 AND QS(I,J)<LT,Q4) F(I,J)=(Q4-QS(I
1,J))/DEN*(8*2)*3*HMELT)
IF(QS(I,J)<GE,Q4) F(I,J)=0.0
IF(QS(I,J)>GT,Q4) T2(I,J)=TMELT+(QS(I,J)-Q4)/C(I,J))
WHEN GOING FROM MELT TO FREEZE OR VICE-VERSA CHANGE THE
FOLLOWING CA
IF(QS(I,J)<LE,Q3) F(I,J)=1.0
250 CONTINUE
C SPECIFICATION AND/OR DETERMINATION OF FIN TEMPERATURES
THE FOLLOWING DO LOOP ASSUMES FIN TEMPERATURES
FOR ITERATION
DO 255 J=3,NN
255 T2(1,J)=T3(1,J)
DO 260 L=1,NDP
IF(TIME>GE,TIM(L) AND TIME<LE,TIM(L+1)) GO TO 4
260 CONTINUE
4 FAC=(TIME=TIM(L))/(TIM(L+1)-TIM(L))
T2(1,2)=TM1(1)+((TM1(L+1)=TM1(L))*FAC)
T2(1,8)=TM2(1)+((TM2(L+1)=TM2(L))*FAC)
T2(1,15)=TM3(1)+((TM3(L+1)=TM3(L))*FAC)
T2(1,21)=TM4(1)+((TM4(L+1)=TM4(L))*FAC)
DO 270 I=1,MM
270 T2(I,2)=T2(I,2)
C UNSPECIFIED FIN TEMPERATURES DETERMINED BY STEADY STATE
EQUATIONS
256 MCOUNT=MCOUNT+1
DO 280 J=3,NN
T3(1,J)=T2(1,J)
IF(J=EQ.8 OR J=EQ.15) GO TO 5
IF(J=EQ.21) GO TO 5
T2(1,J)=((T2(1,J-1)/RV(1,J))+(T2(2,J)/RH(2,J)))+(T2(1,J
1+1)/RV(1,J+1))/((1/RV(1,J))+(1/RH(2,J))+(1/RV(1,J
2+1))
5 CONTINUE
280 CONTINUE
IF(MCOUNT>GT*MFIN) GO TO 3
DO 281 J=3,NN
DIF=T2(1,J)-T3(1,J)
IF(Abs(DIF)>GT*EPS) GO TO 256
281 CONTINUE
IF(JOE=EQ.2) GO TO 285
312 \[\text{QTTR} = \text{QTTR} + \text{QRAT}(I, NJ)\]
\[\text{QWAX} = (\text{QBw} + \text{QTW} + \text{QSW}) \times \text{DT} + (\text{QWAX})\]
\[\text{RATIO} = \text{QSW} / \text{QBw}\]
\[\text{ERROR} = ((\text{QBT} - (\text{QBw} + \text{QSW} + \text{QTW} + \text{QTOP} + \text{JFTR} + \text{QTTR})) \times 100 \times 0) / \text{QBT}\]

DO 930 J = 3, NN
DO 930 I = 2, MM
V2 = V2 + (F(I, J) \times (S \times 2) \times B)

930 CONTINUE

DAVG = (2.0 \times V2) / (W \times B) * 10.0
QMELT = ((V2 = V1) \times \text{DEN} \times \text{HMELT}) / \text{DT}
QFREQ = QMELT
V1 = V2
V2 = 0.0
IF (COUNT \times EQ \times KCHK) GO TO 6
KCOUNT = KCOUNT + 1
GO TO 7

6 WRITE(6, 66) \text{TIME, QWAX, ERROR}
66 FORMAT(1X, 5HTIME, =E15.8, 10X, 5HGWAX, =E15.8, 1O\times, 6HERROUR= 1,E15.8)

DO 313 J = 3, NN
AJ = J = 3
BJ = J = 2
IF (F(2, J) \times GT \times F2JM) DFL0 = (BJ \times S \times 12 \times 0)
DFIN = (AJ = S \times 12 \times 0)
IF (F(2, J) \times LT \times EPS) GO TO 314

313 CONTINUE

314 DO 315 J = 3, NN
AJ = J = 3
IF (F(MM, J) \times LT \times 1 \times 0) DMD = ((AJ = S) \times (F(MM, J) \times S)) \times 12 \times 0
IF (F(MM, J) \times LT \times 1 \times 0) GO TO 316

315 CONTINUE

316 WRITE(6, 67) DFIN, DFO, DAVG, DMIN
67 FORMAT(1X, 5HDFIN = E15.8, 10X, 5HDFLO = E15.8, 10X, 5HDAVG = 1,E15.8)

DDOT = (DMIN \times DMD) / \text{DT}
DMD = DMIN
WRITE(6, 69) QFREZ, DDOT

69 FORMAT(1X, 5HDFREZ = E15.8, 10X, 5HD_DOT = E15.8)

WRITE(6, 77) GBs, GSW, QTW, QBT, RATIO
77 FORMAT(1X, 5HQBS = E15.8, 3X, 4HGUI = E15.8, 3X, 4HQTW = E15.8, 1,3X, 4HGBT = E15.8, 3X, 6HRATIO = E15.8)

WRITE(6, 88)

88 FORMAT(2X, 1HI, 2X, 1HJD, 10X, 11HTEMPcdc, 10X, 15HFRACTIO
1N MELTED, 10X, 11HERGNY RATE, 10X, 13ENERGY STORED)

DO 320 J = 2, NJ
DO 320 I = 1, MM
WRITE(6, 99) I, J, D(T(I, J), F(I, J), QRAT(I, J), QSW(I, J))
99 FORMAT(1X, 12, 1X, 12, 8X, E15.8, 8X, E15.8, 8X, E15.8, 8X, E15.8, 8X)

320 CONTINUE

KCOUNT = 1
7 QBK=0.0
QSWM=0.0
QTW=0.0
QTOP=0.0
QTTR=0.0
QFTR=0.0
DO 330 J=2,NJ
DO 330 I=1,MM
330 T1(I,J)=T2(I,J)
JOE=1
IF(TIME.LT.TAU) GO TO 3
8 WRITE(6,111) MCOUNT
111 FORMAT(1X,I3)
STOP
END
REFERENCES


REFERENCES (Continued)


REFERENCES (Concluded)


This document gives a comprehensive survey of the thermal aspects of phase change material devices. Fundamental mechanisms of heat transfer within the phase change device are discussed. Performance in zero-g and one-g fields are examined as it relates to such a device. Computer models for phase change materials, with metal fillers, undergoing conductive and convective processes are detailed. Using these models, extensive parametric data are presented for a hypothetical configuration with a rectangular phase change housing, using straight fins as the filler, and paraffin as the phase change material. These data are generated over a range of realistic sizes, material properties, and thermal boundary conditions. A number of illustrative examples are given to demonstrate use of the parametric data. Also a complete listing of phase change material property data are reproduced herein as an aid to the reader.