General Disclaimer

One or more of the Following Statements may affect this Document

- This document has been reproduced from the best copy furnished by the organizational source. It is being released in the interest of making available as much information as possible.

- This document may contain data, which exceeds the sheet parameters. It was furnished in this condition by the organizational source and is the best copy available.

- This document may contain tone-on-tone or color graphs, charts and/or pictures, which have been reproduced in black and white.

- This document is paginated as submitted by the original source.

- Portions of this document are not fully legible due to the historical nature of some of the material. However, it is the best reproduction available from the original submission.

Produced by the NASA Center for Aerospace Information (CASI)
Models for Interrupted Monitoring of a Stochastic Process

Everett Palmer

October 1977

NASA
National Aeronautics and Space Administration
Ames Research Center
Moffett Field, California 94035
MODELS FOR INTERRUPTED MONITORING OF A STOCHASTIC PROCESS

by Everett Palmer

Ames Research Center, NASA, Moffett Field, CA 94035

ABSTRACT

As computers are added to the cockpit, the pilot's job is changing from one of manually flying the aircraft, to one of supervising computers which are doing navigation, guidance and energy management calculations as well as automatically flying the aircraft. In this supervisory role the pilot must divide his attention between monitoring the aircraft's performance and giving commands to the computer. In this paper, normative strategies are developed for tasks where the pilot must interrupt his monitoring of a stochastic process in order to attend to other duties. Results are given as to how characteristics of the stochastic process and the other tasks affect the optimal strategies.

INTRODUCTION

"New York control, this is NASA 1 arriving on CARMEL 2 with an expected arrival time at MERGE waypoint of 14:31:00." "NASA 1, you are cleared to arrive on CARMEL 1, with a merge time of 14:32:10." This exchange between pilot and controller occurred in a recent Ames simulation study of 4D RNAV in the terminal area(1,2). The pilot was cleared for a different PNAV approach route and arrival time. The pilot next entered this data into
his onboard navigation and guidance computer. In doing this he had to divide his attention between monitoring the autopilot's performance with his flight instruments and entering data into the computer through his multifunction display and keyboard. Observations of how pilots divided their attention between monitoring and data entry tasks in this simulation study were the motivation for the modeling of attention sharing presented in the present paper.

The environment in which the pilot interacts with his onboard computer is quite different from other jobs where a person interacts with a computer. In a management information system, teleoperator control, or in most human interaction with a computer, the computer is, or can easily be halted to allow the person time to think and plan his next input. Here the person and the computer work sequentially. When an aircraft is being controlled in real time by a computer it can not be stopped while the pilot leisurely inputs his commands. In this environment both computer and man must work in parallel. The pilot must interrupt his monitoring to interact with the computer. He must also interrupt the discrete tasks to monitor. Other characteristics of discrete tasks and monitoring in the cockpit are the following. The discrete tasks are presented at random. They should be accomplished by a certain time but usually plenty of time is available to do the tasks. Attention must be diverted from monitoring for fairly long blocks of time (seconds) to do the discrete tasks. The displays the pilot must monitor show the error between his vehicle's state and the desired state. When the aircraft is con-
trolled by an autopilot, these signals are relatively low bandwidth signals that should be monitored for out of tolerance readings.

The objective of this research is to determine how design parameters of both displays and the computer interface affect monitoring and data entry performance. In this paper, a task is developed which has many of the above characteristics but which is simple enough so that the attention allocation problem has an optimal solution. Three models based on the internal model concept are developed for this task. In the first two, the pilot is rewarded for diverting his attention from monitoring to do discrete tasks. The third model treats the discrete tasks as a constraint and then uses a dynamic programming formulation to maximize monitoring performance subject to the constraint of finishing all of the discrete tasks on time.

**SPECIFIC PROBLEM**

**Process Dynamics:** The subject is to monitor the output of a first order filter driven by white gaussian noise. The display (fig.1) is quantized in both time and position. The display is updated every 2 seconds and is quantized into 11 cells, \( \sigma \) wide. The display is defined as being out of tolerance if it is in the outermost 2 cells (\(|x| > 1.75 \sigma \)). The process bandwidth determines how predictable the signal is. The ratio of the tolerance to the output variance determines how frequently the signal will be out of tolerance.

**Monitoring Task:** Whenever the subject observes the process as be-
iring out of tolerance he gets a reward of one unit.

**Discrete Task:** At each time at which the display is observed, the subject decides to either monitor next time or to divert his attention to the discrete tasks for one or more units of time. Two methods of rewarding the subject for doing discrete tasks were investigated. In the first method, the subject is given a reward of \( R \) units for every discrete task done. If \( R \) is zero the subject would always monitor and if \( R \) is greater than the steady state probability that the signal is out of tolerance the subject would always do discrete tasks. The objective was to maximize the total reward from monitoring and discrete tasks. In the second method, the subject was constrained to do \( m \) discrete tasks in the next \( n \) time units. The objective was to maximize the reward for monitoring subject to the constraint of finishing all of the discrete tasks. The constraint formulation seems to be more accurate description of the real situation. In addition, it has the large advantage of not requiring the experimenter to specify the relative worth of time spent on monitoring and discrete tasks. Unfortunately this formulation is computationally more difficult.

**THEORY**

A review of the literature in the fields of manual control, human factors and psychology found a number of empirical studies which required the operator to interrupt monitoring tasks to do discrete tasks. Models have also been developed for either instrument monitoring or discrete tasks. No papers were found
which addressed the problem of what strategies operators use or should use, to time share their attention between monitoring and discrete tasks. However, Smallwood's paper (3) on human instrument monitoring proposes an approach which can be applied to the present problem. This approach makes the reasonable assumption that the operator has an internal model of the process he is monitoring and of the environmental factors that affect the process. This internal model can be used to predict the future behavior of the process. Smallwood makes the following assumptions that describe how the operator reacts to environmental inputs.

Assumption 1: The human operator bases his state of information about his environment upon an internal model of this environment; the model is formed as a result of past perceptions of his environment.

Assumption 2: The human operator behaves optimally with respect to his task and his correct state of information within his psycho-physical limitations.

The structure of this model is shown in figure 2. The key problems in using this approach are to discover the form of the operators internal model and the optimal response. If the operators model of the process is exact and he has no psycho-physical limitations the resulting model is normative. Introducing errors in the internal model and psycho-physical limitations such as observation noise and errors in the operators internal model convert the original normative model into a descriptive model of human behavior.

In the following models, it is assumed that the operators
internal model of the process and environmental disturbances is exact. He knows the parameters of the process and can use this knowledge to predict the probability of being in a particular state given he was in a known state \( n \) seconds ago and has not observed the process since that time. For a first order process with bandwidth \( w \), the distribution of the position of the display after last observing the display \( t \) seconds ago at position \( x_0 \) is a gaussian distribution with

\[
\text{mean} \quad m(t) = x_0 e^{-w t}
\]

\[
\text{variance} \quad v(t) = \sigma^2 (1-e^{-2w t})
\]

Figure 3 plots the mean and variance of this distribution and the probability that the signal will be out of tolerance in the future for various values of \( x_0 \).

**Myopic Model:** In this model a decision is made at each stage to either monitor or do a discrete task next time depending on which activity maximizes the immediate expected reward. In other words find

\[
z = \max_{x=0,1} \left[ (1-x) \sum_{i \text{ out}} P_{ij}(k) + xR \right]
\]

if

- \( x=0 \) then monitor next time.
- \( x=1 \) then do a discrete task next time.

where

\[P_{ij}(k)\] is the probability that the process will be in state \( i \) next stage, given the process was in state \( i, k \)
stages ago.

\[ P(\text{out} | i, k) = \sum_j P_{ij}(k) \text{ the probability that the process } \]
\[ \text{j out will be out of tolerance next stage given the process was in state } \]
\[ i, k \text{ stages ago.} \]
\[ R= \text{ the reward for doing the discrete task for one stage.} \]

Another way to think of this strategy is to find the maximum value of \( k \) such that \( P(\text{out} | i, k) < R \) after each observation of the state \( i \) and then direct attention to discrete tasks for \( k \) stages.

As a specific example, consider the case where \( \sigma_0 = 1.4 \) and \( \gamma = 0.2 \). Table 1 gives values of \( P(\text{out} | i, k) \). If the above decision rule is followed at each stage the strategy in Table 2 will be observed for different values of reward \( R \). For example if \( R=0.049 \), the subject should continue to monitor whenever the process is observed in states 1 to 4 and divert attention to the other duties for three stages whenever the process is observed in state 5 or 6.

Figure 4 is a plot of two measures of a constant sampling strategy. They are the fraction of time spent doing other tasks or not monitoring the display, \( f(\text{tasks}) \), and the fraction of observed out of tolerance signals to the total out of tolerance signals, \( f(\text{hit}) = p(\text{hit}) / p(\text{out}) \), for various values of discrete task reward and strategy. These values are calculated as follows:

\[ f(\text{tasks}) = \frac{\sum_i d_i \pi_i}{\sum_i (d_i + 1) \pi_i} \]
\[ P(\text{hits}) = \sum_{i = \text{out}} \pi_i (1 - f(\text{tasks})) \]

where

\[ d_1 = \text{the number of stages devoted to discrete tasks after the display is observed in state } i. \]

\[ \pi_i = \text{the steady state probability of the process being observed in state } i \text{ when the fixed time sharing strategy specified by } d_1 \text{ is followed. The elements of the observed "single" step transition matrix are } P_{ij}(d_1). \]

The expected reward for following this fixed strategy is,

\[ E(R) = P(\text{hit}) + R \cdot f(\text{tasks}). \]

Figure 4 shows that for a first order display with a bandwidth of .2, 85% of the out of tolerance signals will be observed even if only 50% of the time is spent monitoring and this myopic strategy is followed. Figure 4 also shows the monitoring performance that would be expected if the pilot could make perfect predictions and his expected performance if he could make no predictions. As the bandwidth of the process decreases and the signal becomes more predictable performance approaches that possible with perfect information.

This model has the advantage of being very simple and it can be easily extended to continuous state and continuous time processes. Unfortunately this model does not appear to maximize the long term expected reward for both monitoring and other...
tasks. This model neglects the future value of knowing what state the process is in. The next three models use a dynamic programming formulation to explicitly account for these future values.

**Dynamic Programming models with Rewards.** The following dynamic programming model maximizes the sum of the expected immediate rewards and the future rewards.

Define

\[ f_n(i,k) = \text{the maximum expected return when the process was observed in state } i, \text{ } k \text{ stages ago and there are } n \text{ stages left to go.} \]

\[ j_n(i,k) = x = 0 \text{ then monitor next time.} \]

\[ x = 1 \text{ then discrete task next time.} \]

\[ R = \text{discrete task reward.} \]

then

\[ f_n(i,k) = \max_{x=0,1} \left[ (1-x) \left( \sum_{l} P_{il}(k) + D \sum_{j} P_{ij}(k) f_{n-1}(i,l) \right) + x \left( R + D \sum_{j} P_{ij}(k) f_{n-1}(j,k+1) \right) \right] \]

\[ f_0(i,k) = 0 \]

The terms premultiplied by D are the future values. If D=1, we have the optimum dynamic programming solution. If D=0 this model reduces to the myopic model.

In the next formulation a decision is made after each monitoring observation of how many discrete tasks to do next. The
decision may be to do no discrete tasks in which case the operator continues to monitor. To make a decision involves using the "internal model" of the process, the $P_{ij}(k)$'s, to predict the probability of being in each state in the future. This formulation is equivalent to the above formulation but because it uses one less state it is computationally more attractive.

Define

$$f_n(i) = \text{the maximum expected return when the process is observed in state } i \text{ with } n \text{ stages to go.}$$

$$d_n(i) = q = \text{the number of discrete tasks done before the next monitoring observation when the process is observed in state } i \text{ with } n \text{ stages to go.}$$

$$R = \text{reward for doing one discrete task. then}$$

$$f_n(i) = \max_{q=0,1,...,n} \left[ R q + \sum_{j \in \text{out}} P_{ij}(q) + \sum_{i} P_{ij}(q) f_{n-q-1}(i) \right]$$

$$f_0(i) = 0$$

Table 3 shows the steady state solution to this normative model for the same parameters used in the myopic model, $\tau = .2, \sigma = 1.9, T = 1.75$, for various values of $R$. Note that for a discrete task reward of 0.05 the steady state decision for state 6, the center state, is to look away for two stages where as the myopic strategy is to look away for 4 stages. This difference in strategies is because the value of knowing the process is out of tolerance is greater than the immediate reward for observing the process out of tolerance. For a given discrete task reward, $R$, the steady state decisions of the myopic model and the dynamic
programming model become more divergent as the process bandwidth decreases and the signal becomes more predictable. These steady state solutions were obtained by using value iteration and assuming that the steady state had been reached by stage 40. They can also be solved by a modification of Howard's policy iteration technique (4) that allows for looking ahead g+1 stages.

Figure 5 plots the expected steady state reward per stage for both monitoring and discrete tasks as a function of the discrete task reward, R, for 4 values of process bandwidth. When R=0, the total reward is just 0.08, the steady state probability that the process is out of tolerance. This is the expected reward per stage of always monitoring. The diagonal line shows the reward received for always doing discrete tasks. The upper diagonal line is the maximum reward with perfect knowledge of what the signal will be in the future. The graph shows that an optimal time sharing strategy results in a gain above the two lower bounds and below the upper bound. As the process bandwidth decreases the gain becomes closer to that possible with perfect information. Note that the maximum advantage of an optimal strategy over a nonsampling strategy occurs for R=0.08 - the steady state probability that the process will be out of tolerance.

One disadvantage of this dynamic programming model is that it requires R, the reward for doing discrete tasks, to be specified. In any real task it would be very difficult to determine an actual numerical value for R. Even in a laboratory task in which the experimenter tells the subject the value of R and the subject uses a time sharing strategy similar to the model, it is doubt-
ful that the subject's implicit value for doing discrete tasks would agree with the explicitly specified value of \( R \). One solution to the problem of rewards is to use the reward only to generate the sampling strategies and then pick the strategy that devotes the appropriate amount of time to discrete tasks. Figure 6 plots the fraction of hits, \( f(\text{hits}) = p(\text{hit})/p(\text{out}) \), vs. the fraction of time devoted to discrete tasks, \( f(\text{tasks}) \), for both the dynamic programming strategies and the myopic strategies. As can be seen, the curves are essentially identical. The rewards for a given strategy and the strategies are different but when monitoring performance is plotted against \( f(\text{tasks}) \) instead of reward, the graphs are essentially identical. This very is promising for modeling monitoring of multiple higher order processes because the myopic strategy is only based on the probability that the signal will be out of tolerance in the future, not the probability of which specific state the process will be in as required in the dynamic programming formulations.

Figure 7 shows how the fraction of hits changes when an optimal sampling strategy is followed for various process bandwidths. As the bandwidth decreases the performance approaches that possible with perfect information.

In many discrete tasks there is the equivalent of a set-up cost each time the task is started or restarted after being interrupted. For example, in entering data into a keyboard, some time is lost while the pilot shifts his attention to the keyboard and positions his hands. This type of set-up cost can be included by introducing nonlinearities into the reward per discrete task.
function. In figure 9, C is the set up cost. When C=0 we have
the normal case considered above.

Figure 9 shows that as set up cost increases monitoring perfor-
manace rapidly decreases to that possible with no predictive
information of the processes future state. When a set up cost is
involved the formulas derived above for f(tasks) must be modified
as follows.

\[ f(tasks) = \frac{\sum d_i^* \, \pi_i}{\sum (d_i + 1) \, \pi_i} \]

where

\[ C = \text{the number of stages wasted due to the set up cost,} \]

\[ d_i^* = \begin{cases} d_i - C & \text{if } d_i \geq C \\ 0 & \text{if } d_i < C \end{cases} \]

\[ f(tasks) \text{ is now the productive fraction of time spent on discrete} \]

\[ \text{tasks. The fraction of time wasted because of the set up cost is} \]

\[ f(\text{waste}) = \frac{\sum (d_i - d_i^*)}{\sum (d_i + 1) \, \pi_i} \]

\[ \text{The fraction of time spent monitoring is just;} \]

\[ f(\text{mon}) = 1 - f(tasks) - f(\text{waste}) \]

**Dynamic Programming model with Constraint.** Like the last for-
mulation, a decision is made after each monitoring observation
of how many stages to devote to discrete tasks. However instead
of rewarding the subject for doing discrete tasks, we will con-
strain him to do exactly \( m \) discrete tasks in the next \( n \) stages \((m<n)\).

Define

\[ f_n(m,i) = \text{the maximum expected return when the process is observed in state } i \text{ with } n \text{ stages to go and } m \text{ discrete tasks remain to be done.} \]

\[ d_n(m,i)=q= \text{the number of stages devoted to discrete tasks before the next monitoring observation when the process is observed in state } i \text{ with } n \text{ stages to go.} \]

\[ C= \text{set up cost, the number of stages wasted when attention is shifted to discrete tasks.} \]

\[ q^* = q-C \text{ if } q-C \geq 0 \]

\[ = 0 \text{ if } q-C < 0 \]

then

\[ f_n(m,i) = \max_{0 \leq q \leq m} \left[ \sum_{j} P_{ij}(q) + \sum_{j} P_{ij}(q) f_{n-q-1}(m-q^*,i) \right] \]

\[ f_n(m,i) = 0 \text{ if } n=m \]

In this formulation, the fraction of the remaining time which must be spent on discrete tasks is just, \( f(\text{tasks})=m/n \) and the fraction of hits is \( f_n(m,i)/n \) at state \((m,i)\) and stage \( n \).

Figure 10 plots the fraction of hits vs the fraction of time spent on discrete tasks for three different values of \( n \), the number of stages to go. Note that in this formulation the monitoring performance is slightly less than the other two formulations and performance degrades further as the number of stages, \( n \),
allowed to do the tasks is further reduced. This is as expected because the subject is constrained to spend exactly \( t \) out of the next \( n \) stages on discrete tasks whereas in the earlier model the subject had no limit as to how long he could postpone the discrete tasks.

Figures 11 and 12 show the effect of a discrete task set up cost on sampling strategy and monitoring performance. As the set up cost increases, the best strategy is to look away for longer and longer periods of time when the display is observed near the center. With a set up cost of 2, if the display is observed in the center the best policy is to complete all of the discrete tasks with no interruption. This is why the monitoring performance shown in figure 12 for a set up cost of 2 is so close to the performance that is possible when no predictions are made.

Figure 13 shows the sensitivity of monitoring performance to discrete task chunk size - the minimum number of stages which must be spent on discrete tasks. Note that when the minimum chunk size is 5 that the decrement in performance is only large when less than about 60\% of the time must be spent on discrete tasks. This is because above 60\% the optimum strategy is to look away for more than 5 stages so that chunk size is less of a constraint on performance. Finally figure 14 shows the sensitivity of monitoring performance to display tolerance.

CONCLUDING REMARKS

In this paper the general problem of time sharing attention
between monitoring and other duties has been described and one myopic and three dynamic programming models have been presented. Model performance was presented in terms of the fraction of out of tolerance signals seen as a function of the amount of time spent on non-monitoring duties. This way of viewing performance eliminates the difficult problem of specifying relative rewards for monitoring and other duties. It allows an appropriate strategy to be chosen based on the fraction of time that must be devoted to other duties. The effect of such parameters as process bandwidth and tolerance and discrete task set up cost and chunk size on monitoring performance and normative time sharing strategies was shown. Future work will extend these models to multiple second order processes and incorporate human limitations such as observation noise and internal model errors.
REFERENCES


Table 1. Values of \( P(\text{outli},k) \) the probability the process will be out of tolerance next stage given the process was in state \( i, k \) stages ago for a first order process with \( \sigma =1.0, \tau =.2 \text{ rad/stage} \) and \( T=1.75 \).

<table>
<thead>
<tr>
<th>( k )</th>
<th>( i=1 )</th>
<th>( i=2 )</th>
<th>( i=3 )</th>
<th>( i=4 )</th>
<th>( i=5 )</th>
<th>( i=6 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>.697</td>
<td>.422</td>
<td>.192</td>
<td>.057</td>
<td>.018</td>
<td>.002</td>
</tr>
<tr>
<td>1</td>
<td>.469</td>
<td>.291</td>
<td>.159</td>
<td>.073</td>
<td>.031</td>
<td>.019</td>
</tr>
<tr>
<td>2</td>
<td>.326</td>
<td>.219</td>
<td>.135</td>
<td>.070</td>
<td>.047</td>
<td>.036</td>
</tr>
<tr>
<td>3</td>
<td>.242</td>
<td>.172</td>
<td>.118</td>
<td>.050</td>
<td>.057</td>
<td>.054</td>
</tr>
<tr>
<td>4</td>
<td>.185</td>
<td>.142</td>
<td>.105</td>
<td>.080</td>
<td>.046</td>
<td>.060</td>
</tr>
<tr>
<td>5</td>
<td>.152</td>
<td>.121</td>
<td>.097</td>
<td>.090</td>
<td>.070</td>
<td>.067</td>
</tr>
<tr>
<td>6</td>
<td>.125</td>
<td>.108</td>
<td>.092</td>
<td>.089</td>
<td>.073</td>
<td>.071</td>
</tr>
</tbody>
</table>

Table 2. The number of discrete tasks that will be done given the process is observed in state \( i \) for various ranges of reward \( R \) if the myopic strategy is followed for a first order process. (\( \sigma =1.0, \tau =0.2 \text{ rad/stage} \), \( T=1.75 \))

<table>
<thead>
<tr>
<th>( R ) gain/stage</th>
<th>( i=1 ) to ( 3 )</th>
<th>( 4 )</th>
<th>( 5 )</th>
<th>( 6 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \leq .02 )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( \leq .04 )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>( \leq .06 )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>( \leq .08 )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>( \leq .10 )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>( \leq .12 )</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>( \leq .14 )</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>( \leq .16 )</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>( \leq .18 )</td>
<td>0</td>
<td>0</td>
<td>29</td>
<td>4</td>
</tr>
</tbody>
</table>

Table 3. Steady state solution to the dynamic programming model with rewards. (\( \sigma =1.0, \tau =0.2 \text{ rad/stage} \), \( T=1.75 \), \( P(\text{outli})=0.99 \))

-18-
Figure 1. Drawing of the quantized monitoring display. A new line was added every 2 seconds. The display was quantized into 11 cells \(-0.50\) wide. The display was out of tolerance if it was in the outermost 2 cells indicated with the + signs. At stages 32 and 27 this subject decided to look away from the display to do discrete tasks for 2 and 1 stages respectively.

Figure 2. A block diagram of the human monitor (from Smallwood (3)).
Figure 3. The state of information of a perfect monitor after looking away from the output of a first order filter with bandwidth 0.2 rad/stage driven by white noise.

\[ m(t) = x_0 e^{-\omega t} \]
\[ \omega = 0.2 \text{ RAD/STAGE} \]

\[ a(t) = a_0 (1 - e^{-2\omega t})^{1/4} \]
\[ a_0 = 1.0 \]

OUT OF BOUNDS = |X| > 1.75 \( \sigma \)

1 = INITIAL STATE, \( i \)
Figure 4. The fraction of observed out of tolerance signals vs. the fraction of time spent doing discrete tasks for the myopic sampling strategy. 
($\omega = 0.2 \text{ rad/stage}, \sigma = 1.0, T = 1.75, P(\text{out}) = 0.081$)

<table>
<thead>
<tr>
<th>POINT ON GRAPH</th>
<th>REWARD R</th>
<th>STRATEGY IN STATE i, d,</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>0.000</td>
<td>0 0 0</td>
</tr>
<tr>
<td>b</td>
<td>0.002</td>
<td>0 0 1</td>
</tr>
<tr>
<td>c</td>
<td>0.018</td>
<td>0 0 1</td>
</tr>
<tr>
<td>d</td>
<td>0.031</td>
<td>0 0 2</td>
</tr>
<tr>
<td>e</td>
<td>0.036</td>
<td>0 0 2</td>
</tr>
<tr>
<td>f</td>
<td>0.047</td>
<td>0 0 3</td>
</tr>
<tr>
<td>g</td>
<td>0.050</td>
<td>0 0 3</td>
</tr>
<tr>
<td>h</td>
<td>0.052</td>
<td>0 0 4</td>
</tr>
<tr>
<td>i</td>
<td>0.060</td>
<td>0 1 4</td>
</tr>
<tr>
<td>j</td>
<td>0.065</td>
<td>0 1 4</td>
</tr>
<tr>
<td>k</td>
<td>0.074</td>
<td>0 2 7</td>
</tr>
</tbody>
</table>

Figure 5. The expected steady state reward per stage for both monitoring and discrete tasks when an optimal sampling policy is followed for a first order system. 
($\omega = 0.2 \text{ rad/stage}, \sigma = 1.0, T = 1.75, P(\text{out}) = 0.081$)
Figure 6. Comparison of the monitoring performance of the myopic and dynamic programming sampling strategies when performance is plotted against the fraction of time devoted to discrete tasks.

(\( \omega = 0.2 \text{ rad/stage}, \ \sigma = 1.0, T = 1.75 \) and \( P(\text{out}) = 0.080 \))

<table>
<thead>
<tr>
<th>POINT ON GRAPH</th>
<th>REWARD R</th>
<th>STRATEGY IN STATE i, d,</th>
<th>POINT ON GRAPH</th>
<th>REWARD R</th>
<th>STRATEGY IN STATE i, d,</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>.000</td>
<td>0 0 0 0</td>
<td>1</td>
<td>.000</td>
<td>0 0 0 0</td>
</tr>
<tr>
<td>b</td>
<td>.002</td>
<td>0 0 0 1</td>
<td>2</td>
<td>.020</td>
<td>0 0 1 1</td>
</tr>
<tr>
<td>c</td>
<td>.018</td>
<td>0 0 1 1</td>
<td>3</td>
<td>.040</td>
<td>0 0 1 2</td>
</tr>
<tr>
<td>d</td>
<td>.031</td>
<td>0 0 1 2</td>
<td>4</td>
<td>.060</td>
<td>0 0 2 2</td>
</tr>
<tr>
<td>e</td>
<td>.036</td>
<td>0 0 2 2</td>
<td>5</td>
<td>.080</td>
<td>0 1 2 3</td>
</tr>
<tr>
<td>f</td>
<td>.047</td>
<td>0 0 2 3</td>
<td>6</td>
<td>.100</td>
<td>0 1 3 4</td>
</tr>
<tr>
<td>g</td>
<td>.050</td>
<td>0 0 3 3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>h</td>
<td>.052</td>
<td>0 0 3 4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>i</td>
<td>.060</td>
<td>0 1 4 4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>j</td>
<td>.065</td>
<td>0 1 4 5</td>
<td>7</td>
<td>.120</td>
<td>0 1 4 5</td>
</tr>
<tr>
<td>k</td>
<td>.074</td>
<td>0 2 7 7</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Figure 7. Fraction of hits vs fraction of time on discrete tasks for four values of process bandwidth.
(σ = 1.0, τ = 1.75, P(out) = 0.800)

Figure 8. Discrete task reward with a setup cost, C.
Figure 9. The effect of a discrete task set of cost on monitoring performance.
(\(\omega=0.2\) rad/stage, \(\sigma=1.0\), \(T=1.75\), \(P(\text{out})=1.00\))

Figure 10. The effect of the number of stages to go \((n)\) on monitoring performance for the dynamic programming model with discrete task constraint.
(\(\omega=0.2\) rad/stage, \(\sigma=1.0\), \(T=1.75\), \(P(\text{out})=1.75\))
Figure 11. The effect of a discrete task set up cost (C) on the optimal time sharing strategy for states 4, 5 and 6. In states 1, 2 and 3 the optimal decision is 0 until the fraction of time which must be devoted to discrete tasks is very high.

(n=20 stages, \( w = 0.2 \) rad/stage, \( T = 1.75 \), \( P(out) = 0.080 \))

Figure 12. The effect of a discrete task set up cost on monitoring performance for the dynamic programming model with a discrete task constraint.

(n=39 stages, \( w = 0.2 \) rad/stage, \( T = 1.75 \), \( \sigma = 1.0 \), \( P(out) = 0.080 \))
Figure 13. The effect of discrete task chunk size on monitoring performance.
\[ \tau = 0.2 \text{ rad/stage}, \ n = 40 \text{ stages}, \ \sigma = 1.0, \ T = 1.75, \ P(\text{out}) = 0.080 \]

Figure 14. The effect of display tolerance on monitoring performance.
\[ \tau = 0.2 \text{ rad/stage}, \ \sigma = 1.0, \ n = 40 \text{ stages} \]
As computers are added to the cockpit, the pilot's job is changing from one of manually flying the aircraft, to one of supervising computers which are doing navigation, guidance and energy management calculations as well as automatically flying the aircraft. In this supervisory role the pilot must divide his attention between monitoring the aircraft's performance and giving commands to the computer. In this paper, normative strategies are developed for tasks where the pilot must interrupt his monitoring of a stochastic process in order to attend to other duties. Results are given as to how characteristics of the stochastic process and the other tasks affect the optimal strategies.