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CALCULATION OF THE MAGNETIC FIELD IN THE ACTIVE ZONE OF A HYSTERESIS CLUTCH

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Translation of "K raschetu magnitnogo polya v aktivnoi zone gisteresisnoy mufy", Elektrochestvo, No. 4, April, 1972, pp. 61-64.
The problem of calculating the initial distribution of magnetic induction in the armature stationary relative to the polar system of a hysteresis clutch is considered. Using several assumptions, the problem is reduced to calculating the static magnetic field in the ferromagnetic plate with finite and continuous magnetic permeability placed in the air gap between two identical, parallel semiconductors with rack fixed relative to the "tooth" or "slot" position. 13 ref. in Russian.

Translation of "К расчету магнитного поля в активной зоне гистерезисного муфты", Электрошедство, № 4, April, 1975, pp. 61-64.
Hysteresis clutches are a widely distributed type of asynchronous-synchronous clutch with magnetic coupling. Structurally, a hysteresis clutch is a dual rotation device. On one of the rotating parts - usually the driving part - there is a pole system of the inductor. On the other rotating part there is an armature in the form of a cylindrical or disk-type layer of "hysteresis" material (i.e., solid magnetic material with higher-than-usual remagnetization losses). Transmission of torque from the hysteresis clutch is the result of the interaction between the magnetic field of the inductor pole system and the field due to the residual magnetism of the hysteresis layer. If the hysteresis layer consists of electrically insulated rings made of solid magnetic material, then the transmission of torque from the clutch is practically independent of slippage [1-3]. Hysteresis clutches (HC) which have permanent magnets are called magneto-hysteresis clutches (MHC) in contrast to clutches with electromagnetic excitation (EHC).

The static and dynamic characteristics of EHC are analogous to the corresponding characteristics of electromagnetic particle clutches (EPC). With respect to over-all mass and energy indices, the ECH are inferior to the EPC; however, they have higher reliability indices and stable characteristics. The ability of the EHC to transmit torque proportional to the excitation current permits its use in high-speed servomechanisms with an actuating device operating on the controlled electromagnetic clutches of a transmission [4]. From the point of view of high-speed action and control capacity, the most efficient design for an EHC includes a driven rotor in the form of a thin walled sleeve and a "bomb-type" system of poles with a two-way, "checkerboard" distribution of...
the toothed poles (relative to the walls of the sleeve armature) [5].

Figure 1 contains a schematic drawing of an EHC of this type with a stationary field winding [6]. 1 is the drive shaft; 2 is the driven shaft, 3 is the stationary magnetic circuit; 4 is the excitation coil, 5 is the external magnetic circuit, 6 is the external pole ring, 7 is the junction element made of nonferromagnetic material, 8 is the driven rotor (armature), 9 is the internal pole ring, the dotted line shows the path of the basic magnetic flux.

Figure 1.

The procedure for calculating the maximum torque delivered by an EHC for a given control current when the drive shaft and the slave shaft are rotating in synchrony is based on the so-called energy approach to the analysis of the operation of hysteresis electrical machines [2]. In accordance with this approach, there is an "initial" distribution of magnetic induction into the hysteresis layer, i.e., after the field winding is first plugged into the constant voltage and the current is established in the winding, then the stationary magnetic field is distributed in the armature, which is stationary relative to the polar system, and applied to the EHC. Under the conditions stated, the magnetic state of the armature is characterized by the points on the initial magnetization curve of the solid magnetic material employed. On the basis of the initial induction distribution, the parameters of the remagnetization cycle are found for each elementary ring layer of the solid magnetic material. These determinations are made when the polar system is rotating at a very low speed relative to the braked armature - a speed which is insufficient to cause eddy currents to appear in the armature. In conformity with the energy balance, the electromagnetic moment is directly proportional to the energy losses during a single remagnetization of all elements of the armature ring layers.
An approximate solution of the problem concerning the initial distribution of induction in an EHC of the type under consideration is presented in [7] and [8]. In this case, a technique for simplifying certain boundary value problems in magnetostatics is used which is analogous to that in [9]. However, such important questions as the influence of the extent of overlap of the toothed poles, and the effect of the size of the air gap between the armature and the system of poles has on the field pattern in the armature have not been investigated.

In the present article, we present a more rigorous solution of the problem concerning the initial distribution of induction in the active zone of an EHC. This makes it possible, in particular, to evaluate the effect produced by the degree of overlap of the toothed poles and by the size of the operating gap on the diagrams of the vector components of the magnetic induction in the armature.

This solution was obtained under the following assumptions: 1) the depth of the operating gap between the toothed surfaces of the polar rings is small in comparison to the length of the active zone in the direction of the rotation axis of the clutch; therefore the field in the active zone is plane-parallel; 2) the toothed spacings of the polar rings are small compared with the diameters of the operating annular gap; hence, the cylindrical surfaces of the armature and the crowns of the teeth can be replaced by parallel planes (of infinite extent); 3) the teeth along both sides of the armature are rectangular in shape, and have identical dimensions and spacings, 4) in the elements of the active zone there are no eddy or displacement currents (since the state of the magnetic system is under consideration, when the armature is stationary relative to the toothed poles and the excitation current is constant after the winding is first plugged into the constant voltage); 5) the magnetic permeability of the solid magnetic armature material does not depend on the field intensity and has a finite value which is determined from a linear approximation of the magnetization curve; 6) the magnetically soft material of the teeth is unsaturated and thus its magnetic permeability is infinite.

With the aid of the above assumptions, the problem is reduced to calculating the magnetic field in an idealized model of the active zone; i.e., calculating the magnetic field in a magnetic system of two
identical parallel semiconductors having the profile of a toothed rack and fixed in a "tooth opposite gap" relative position; in the gap between these two semiconductors there is placed a plate of ferromagnetic material with constant, finite magnetic permeability.

Figure 2 shows a partial cross-section of the model of the active zone. The width is equal to that of the toothed spacing. The following notation is used for the geometrical measurements: \( t_z \) denotes the toothed spacing, \( a \), the width of the slot, \( h \), the depth of the slot (height of a tooth); \( A \), the thickness of the ferromagnetic plate, representing the armature; \( \delta \), the size of the unidirectional gap.

The plane-parallel magnetic field in the region under consideration satisfies the Laplace equation for the scalar magnetic potential, \( u(x, y) \), when no current flows in the region:

\[
\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0. \tag{1}
\]

The equations

\[
B_x = -\mu_0 \mu_r \frac{\partial u}{\partial x}; \\
B_y = -\mu_0 \mu_r \frac{\partial u}{\partial y}, \tag{2}
\]

express the relationships between the projections of the magnetic induction vector and the potential function. \( \mu_0 \) is the magnetic constant, \( \mu_r \) is the relative magnetic permeability of the region.

Figure 2.
A) Region 3  B) Region 2  C) Region 1  D) Region 4  E) Region 5
Analysis of the probable paths of the magnetic flux in the model of the active zone (Figure 2) indicates the presence of periodicity and symmetry properties in the potential function. The period in the longitudinal direction equals the toothed spacing and is identical for the upper and lower semiconductor, hence

\[ a(x, y) = a(x + kT, y) \text{ for } \frac{\lambda}{2} + \delta > y > -\left(\frac{\lambda}{2} + \delta\right), \]  

(3)

The pattern of the lines of force of the field under study is characterized by plane symmetry with respect to the trace of the plane \( x = 0 \), and by central symmetry relative to the points \( A(t_Z/4, 0) \) and \( Q(-t_Z/4, 0) \). Plane symmetry is due to the fact that the semiconductors are fixed in the "tooth opposite gap" relative position, and central symmetry results from the fact that the profiles of the toothed surfaces of the semiconductors are identical. Mathematically the property of plane symmetry is expressed by

\[ B_x(x, y) = -B_x(-x, y); \]
\[ B_y(x, y) = B_y(-x, y) \]

or

\[ a(x, y) = a(-x, y). \]  

(5)

The central symmetry property is expressed by

\[ B(x, y) = B\left(\frac{t_Z}{2} - x, -y\right) \]

or

\[ a(x, y) = -a\left(\frac{t_Z}{2} - x, -y\right) \text{ for } u\left(\frac{t_Z}{2}, 0\right). \]  

(6)

From the plane symmetry and periodicity properties, it follows that the axes of the teeth (slots) coincide with the magnetic induction lines. Therefore, \[ \left. \frac{\partial u}{\partial x} \right|_{y = \frac{\lambda}{2} + \delta} = 0 \text{ for } \frac{\lambda}{2} + \delta + h > y > -\left(\frac{\lambda}{2} + \delta\right); \]
\[ \left. \frac{\partial u}{\partial x} \right|_{y = -\frac{\lambda}{2} + \delta} = 0 \text{ for } \frac{\lambda}{2} + \delta > y > -\left(\frac{\lambda}{2} + \delta + h\right), \]  

(7)

and the least recurrent part of the active zone is included between the lines \( x = 0 \) and \( x = t_Z/2 \). This part consists of regions with different magnetic permeability, i.e., it is piecewise uniform. The analytic expression for the potential function in each of the regions can be found by realizing one of the methods for solving boundary value problems for the Laplace equation. In [10], an effective method is recommended for finding the potential function in an air region with orthogonal boundaries when the potential varies on the boundary of the
air gap and the ferromagnetic plate according to a known law. This method is based on representing the potential function in a trigonometric series in the slot and the air gap, and subsequently connecting the solutions at the separation boundary between the slot and the gap by means of an infinite system of equations. This method, which is conventionally designated as "sewing together", has been successfully applied by a number of investigators to the calculation of the magnetic field in the active zone of induction type electrical machines. Below, we present its realization as applied to the formulation of the above problem.

The region which models the EHC active zone is divided into five rectangular subregions with sides parallel to the coordinate axes: the ferromagnetic sheet, 1; the air gaps 2 and 4, the slots 3 and 5 (Figure 2). The external boundary conditions for calculating the field in this system of subregions are the conditions (7) and the equi-potentiality condition for the toothed surface of a semiconductor. The difference in the potential of the semiconductors is equal to the magnetizing force, $F_{\Delta\delta}$, expanded in conducting the magnetic flux across the operating air gaps and the ferromagnetic plate.

From central symmetry (6) it follows that when calculating the potentials relative to the point A (i.e., when $u_A = 0$), the potentials of the upper and lower semiconductors are equal modulo $0.5 F_{\Delta\delta}$, and are opposite in sign.

For the purpose of representing the results of the solution in criteria form, we introduce relative values of the coordinates, the dimensions, the potential and the induction; as basic values for the length, the potential and the induction we adopt

$$t_0 = t_1; \ u_0 = 0.5 F_{\Delta\delta}; \ \beta_0 = \mu_0 \frac{u_0}{l_0} = \frac{0.5 F_{\Delta\delta}}{l_0}.$$

The relative values are connected by the pertinent relationships

$$\hat{t}(\hat{x}, \hat{y}, \hat{z}, \hat{\alpha}, \hat{\beta}, \hat{\delta}) = \frac{t}{t_0};$$

$$\hat{u} = \frac{u}{0.5 F_{\Delta\delta}}; \ \hat{B} = B \frac{l_0}{\mu_0 0.5 F_{\Delta\delta}}.$$

Hereafter, the sign # will be omitted.
In accordance with the recommendations in [11], the potential function on the boundary of regions 2 and 3 is represented in the form

\[ u|_{\gamma=0.5, \beta=1} = -1 + \sum_{n=1}^{\infty} c_{n} \cos \left( \frac{(2n-1)\pi x}{a} \right) \]

for \(-\frac{a}{2} \leq x \leq \frac{a}{2}\).

for the realization of the "sewing together" method.

The trigonometric series in (8) has no even harmonics; however, it was decided not to take account of this fact in the numbering of the amplitudes of the harmonics, \(c_{n}\), since by so doing, the programming of the problem for a small digital computer is simplified. In particular, the formula for computing the coefficients of the system of linear algebraic equations obtained by "sewing together" subregions are described more compactly. The matrix representation of this system of equations is

\[ [a_{m, n}] [c_{n}] = [b_{m}] \]

where \(m\) is the number of rows; \(n\) the number of columns, \([a_{m, n}]\), the square coefficient matrix of the system, \([c_{n}]\) is the column matrix of unknowns; \([b_{m}]\) is the column matrix of the right-hand members of the system.

The scheme for realization of the "sewing together" method is described in sufficient detail in [12]. With the aid of an analogous scheme, the following formulas for computing the elements of the matrix were obtained:

\[ b_{m} = \frac{4 (-1)^{m}}{n (2n + \mu_{1}^{-1} \delta) (2m - 1)} \]

\[ a_{m, n} = \frac{8 (-1)^{m} (-1)^{n}}{n^{2} (2n + \mu_{1}^{-1} \delta) (2m - 1)} \times \sum_{k=1, 2, \ldots}^{\infty} k |c_{n} \times \]

\[ \times \mu_{1} [-(-1)^{k} + ch 2k \pi \Lambda] + sh 2k \pi \Lambda \times \th 2k \pi \delta \]

\[ \times \mu_{1} [-(-1)^{k} + ch 2k \pi \Lambda] + sh 2k \pi \Lambda \times \]

where
I_r = \begin{cases} \frac{\pi}{2a} (2m - 1) \cosh (\frac{2n - 1}{a} r) & \text{when } m = n \\ 0 & \text{when } m \neq n \end{cases} \quad (12)

\frac{(1 - 1)^{2n-1}}{2n - 1} \left( \frac{k_0}{2i - 1} \right) \left( \frac{1}{1 - 1} \right) \quad \text{when } \frac{2i-1}{2k} \neq n; \quad (13)

Also, in (13) \( i \) identifies the first subscript - i.e., during the calculation, substitute \( m \) or \( n \) for \( i \).

For the potential function in the region 1, the following expression is obtained:

\( u(x, y) = -\frac{1}{0,55 + \mu_0} \left[ 1 + 2\pi \sum_{n=1, 2, 3} \cdots \sum_{m=1}^{\infty} \frac{(-1)^n c_n}{2n - 1} y + \right. \)

\( + a \sum_{k=1}^{\infty} \sum_{n=1}^{\infty} f_{m, n} \left[ \frac{\sin 2kn(y + 0,5\delta)}{\left((-1)^k + \cosh 2kn\delta\right) \sin 2kn\delta} + \frac{\sin 2kn(y - 0,5\delta)}{\left((-1)^k + \cosh 2kn\delta\right) \sin 2kn\delta} \right] \cos 2knx. \quad (14) \)

On the basis of (2) and (14), the projections of the magnetic induction vector in region 1 are determined:

\( B_x = 2\pi \sum_{k=1}^{\infty} \sum_{n=1}^{\infty} c_{m, n} \chi \times \)

\( \times \frac{\sin 2kn(y + 0,5\delta) + (-1)^k \sin 2kn(y - 0,5\delta)}{\left((-1)^k + \cosh 2kn\delta\right) \sin 2kn\delta} \sin 2knx; \quad (15) \)

\( B_y = \frac{1}{\delta + 0,5\mu_1} \left[ 1 + 2\pi \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{(-1)^n c_n}{2n - 1} y + \right. \)

\( + 2\pi \sum_{k=1}^{\infty} \sum_{n=1}^{\infty} f_{m, n} \chi \times \)

\( \times \frac{\sin 2kn(y + 0,5\delta) + (-1)^k \sin 2kn(y - 0,5\delta)}{\left((-1)^k + \cosh 2kn\delta\right) \sin 2kn\delta} \cos 2knx. \quad (16) \)

The average value (constant component) of the component of the induction vector normal to the surface of the ferromagnetic plate is the same in all horizontal sections of the regions 1, 2, and 4:
This quantity is connected to the magnetic conductivity of the flux in a toothed division (on a unit length in the active zone, directed toward the axis of rotation of the clutch) by an equation which is analogous to a familiar equation in the theory of induction-type electrical machines [13]:

\[ B_2 = \int B_2 dx = \frac{1}{\delta + 0.5\mu_1^{-1}L} \left[ 1 + \frac{2\delta}{\mu_1} \sum_{n=1}^{\infty} \frac{(-1)^n c_n}{2n-1} \right]. \quad (17) \]

Also in (18) and (19), and in all succeeding equations we use absolute units of measurement. From (17)-(19), follows:

\[ \kappa_2 = \frac{B_2}{L_{15}}, \quad \text{wb/a.m} \]

where

\[ B_2 = \frac{\mu_0 B_0}{L_{15}}, \quad \kappa_2 = \frac{0.5L_{15}}{L_{15}}, \quad \text{t1}, \quad (19) \]

\[ \lambda_{21} = \mu_0 \kappa_2 = \lambda_2' \left[ 1 + \frac{2\delta}{\mu_1} \sum_{n=1}^{\infty} \frac{(-1)^n c_n}{2n-1} \right], \quad (20) \]

where

\[ \kappa_2 = 0.5B_2 = \frac{1}{2\delta + \mu_1^{-1}L} \left[ 1 + \frac{2\delta}{\mu_1} \sum_{n=1}^{\infty} \frac{(-1)^n c_n}{2n-1} \right]; \quad (21) \]

\( \lambda_{21}' \) is the magnetic conductivity when there are no slots (i.e., when the semiconductors are smooth):

\[ \lambda_{21}' = \mu_0 \frac{1}{2\delta + \mu_1^{-1}L}. \quad (22) \]

Analysis of the formulas for the coefficients of the system (9) shows that undue increase in the depth of the inductor slots is inadvisable in practice. In fact, it follows from (12), that when \( h > a \) the magnitude of \( \gamma_m, n \) changes very little as \( h \) increases (since \( \cosh (2m-1) \pi = 1 \), even when \( m = 1 \), i.e., the field pattern preserves its previous form.

From (10), (11), (15), and (16), it follows that when the conditions
are satisfied simultaneously, the ferromagnetic plate (armature) is equivalent to a plate with infinite magnetic permeability, and the field pattern in the air gap between the armature surface and the toothed surface of the semiconductor is analogous to the pattern of the excitation field in the operating gap of an inductor generator with constant flux [13]. Here, the initial field distribution in the mass of the plate (armature) is practically independent of the values of its magnetic permeability. When $\Delta > \delta$, it is sufficient to set $k = 1$ in (24).

The relations obtained were used in the formulation of the algorithm for computing the initial induction distribution in the EHC armature. On the basis of this algorithm, an operating program for the digital computer "Naira-2" was drawn up and calculations of the parameters $h$, $h^*$, $\Delta$, $\delta$, and $u_1$ were carried out. The results of the calculations were checked on physical electromagnetic models - in particular, on a large-scale mockup of the active zone of inductor generators with ripple flux [13].

The ferromagnetic plate which models the armature was made from mark EKHZ chrome steel. This steel is isotropic with respect to magnetic properties. The maximum value of the sine of the hysteresis angle for this steel is attained during remagnetization along the partial hysteresis loop with vertex at the point $H_{mY} = 6$ ka/m, $B_{mY} = 0.9$ t1 [2].

The relative magnetic permeability matching this point of the magnetization curve is $\mu_1 = 120$ relative units, also when $B_t < 1.1$ t1, a straight line through the origin is a good approximation to the magnetization curve.
Figure 3

Figure 4
The geometric dimensions of the active zone elements were obtained: \( t = 70 \text{ mm}, \ a = 35 \text{ mm}, \ h = 35 \text{ mm}, \ \Delta = 4 \text{ mm}, \ \delta = 2 \text{ mm} \). To these values there corresponds the following set of initial parameters for a calculation in relative units: \( \hat{a} = 0.5, \ \hat{h} = 0.5, \ \hat{\Delta} = 0.05714, \) and \( \hat{\delta} = 0.02857 \).

Figure 3 shows the curves of the initial distribution of the tangent to the surface of the hysteresis layer for the component of the vector of the magnetic induction in the layer mass when \( y = 0 \) (curve 1) and at its surface when \( y = 0.5\Delta \) (curve 2). These curves were constructed from the initial data introduced above. During the experiment, the magnetization force, \( F_{\Delta\delta} \), was fixed at 1115 A; also the basic value of the induction corresponding to 1 relative unit amounted to 0.01 Tl. With the help of milliweber meters, the normal component of the induction vector in the air gap was measured at the surface of the hysteresis layer.

Figure 4 shows the curves of the initial impact distribution normal to the surface of the hysteresis layer for the component of the magnetic induction vector in the layer mass when \( y = 0 \) (curve 1) and at its surface (curve 2). The curves were constructed using a digital computer; however, values of the induction which were measured are indicated by small circles. The high degree of linearity of the magnetization curve of the hysteresis material in the operating induction zone (up to the region of the bend) contributed to the close agreement between the computed and experimental data.

The analytic relations presented in the article show that quite a large number of parameters influence the induction distribution in the active zone of a clutch. A detailed investigation of the influence of each parameter can be conducted by introducing statistical methods in planning a numerical experiment. The results obtained can be used when designing EHC, and also as a first approximation when computing the field in the active zone, taking into account the saturation of the toothed poles and the nonlinearity of the magnetization curve for the hysteresis material.
REFERENCES


