THE UNIVERSITY OF TEXAS AT AUSTIN

THE ANALYSIS OF A NONSIMILAR LAMINAR BOUNDARY LAYER

Dennis D. Stalmach and John J. Bertin

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Department of Aerospace Engineering and Engineering Mechanics
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A computer code is described which yields accurate solutions for a broad range of laminar, nonsimilar boundary layers, providing the inviscid flow field is known. The boundary layer may be subject to mass injection for perfect-gas, nonreacting flows. If no mass injection is present, the code can be used with either perfect-gas or real-gas thermodynamic models. Solutions, ranging from two-dimensional similarity solutions to solutions for the boundary layer on the Space Shuttle Orbiter during reentry conditions, have been obtained with the code. Comparisons of these solutions, and others, with solutions presented in the literature; and with solutions obtained from other codes, demonstrate the accuracy of the present code.
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INTRODUCTION

Calculation of the convective heat-transfer-rate distribution for a given configuration requires information about the character of the viscous boundary layer. The present work discusses a numerical procedure which provides solutions of a thin, laminar boundary layer bounded by a specified inviscid flow field. The inviscid flow field is required as input to the present code. This code can be used as a tool for analysis of data from wind tunnels and for the extrapolation of the correlations to flight conditions. Furthermore, it will serve as an instructional aid for courses in boundary layer theory and convective heat transfer at the University of Texas at Austin.

For compressible flow problems, where the wall temperature is constant and the static pressure at the edge of the boundary layer is constant, the convective heat-transfer rate can be calculated using the Eckert reference temperature relations [1]. The heating rates calculated using this method, which does not require the complete solution of the viscous equations, compare favorably with experimental measurements even for flows with moderate, favorable pressure gradients [2]. However, this technique cannot be applied for numerous problems of interest, e.g., those involving gas injection at the surface, nonuniform surface temperatures, real gas effects, etc.
A more rigorous calculation of the heat transfer rates could be obtained from a solution of the viscous equations, assuming the similarity conditions are satisfied [3]. As noted by Hayes and Proebstein [4], these conditions are satisfied in the following cases:

(a) for flow with a uniform free stream (i.e., constant pressure solutions), and a constant surface temperature,

(b) near a stagnation point, if the fluid properties in the stagnation region are assumed approximately constant, and

(c) for the limiting case of locally hypersonic flow, \( \frac{u_2}{u_e} + 2 \).

For other flow problems, approximate solutions can be obtained, if the flow is assumed to be locally similar. Special cases involving mass injection can also be treated assuming similarity. One such example would be:

\[ (\rho v)_{\text{inj}} = \frac{\text{const}}{\sqrt{x}} \]

where the injectant is the same gas as the free stream.

The procedures described above are still too limited to be of use in analyzing heat transfer data from many experimental programs. With these requirements in mind, Bertin and Byrd [5] developed a finite difference scheme (NONSIMBL) to obtain solutions for a laminar boundary layer for a flow which is:
(a) either compressible or incompressible,
(b) possibly subject to a pressure gradient in the external flow,
(c) either two-dimensional or axisymmetric,
(d) of arbitrary free-stream gas,
(e) possibly subjected to an arbitrary injectant, injection rate, and injection-rate distribution, and
(f) bounded by an arbitrary wall temperature distribution.

The original code was limited to a mixture of perfect-gas, nonreacting flows having a constant stagnation temperature. Their solution technique closely follows that developed by Marvin and Sheaffer [6].

The NONSIMBL code was later modified to handle real-gas flows as well as perfect-gas flows for either variable entropy or isentropic flow conditions at the edge of the boundary layer. It has been used extensively with good success but the computed solutions exhibited problems in certain applications, specifically accelerating flow past a highly cooled body [7]. The problems which occurred in these cases were due to the relation used to evaluate the transformed \( y \)-coordinate,

\[
q = 1 - e^{-an}
\]
where \( \eta \) is the transformed \( y \)-coordinate using the standard Lees-Dorodnitsyn transformation and \( \alpha \) is a scale factor which was assumed constant. This transformation allowed numerical integrations to be carried out over a fixed interval (zero to one) rather than the usual interval in the \( \eta \)-coordinate system (zero to infinity). Therefore, the need for an iteration to define the boundary layer edge was eliminated. However, the coding procedure also eliminated the iterative procedure which required that zero velocity and temperature gradients exist at the edge of the boundary layer. Note that if the scale factor \( \alpha \) is assumed a constant for every station, as it was in [5], there is a unique relation between \( \eta \) and \( n \). As a result, \( \eta_{\text{edge}} \) is forced to be a constant. However, the Faulkner-Skan solution shows that, for a boundary layer in the presence of a given pressure gradient, \( \eta_{\text{edge}} \) is not a constant but depends on the value of the pressure gradient parameter \( \beta \) [8]. For small streamwise variations of \( \beta \), a constant value of the scale factor is not a bad assumption, but for accelerating flows past highly cooled walls, such as in [7], it is unrealistic.

For flow conditions such as those discussed in [7], the solutions generated by the NONSIMBL code had relatively large velocity and temperature gradients at the boundary layer edge due to the constant \( \alpha \) assumption. Although these gradients had only a small effect on the heat transfer and skin friction (which were of primary importance at that time) they had a significant effect.
on the temperature and the velocity distributions. Subsequent applications have required that the code be capable of providing accurate calculations of displacement and momentum thicknesses which are very sensitive to the velocity and temperature profiles. The present code includes an iterative procedure in which $\alpha$ is varied so that the velocity gradient at the edge of the boundary layer is essentially zero. There is still no restriction on the temperature gradient at the edge of the boundary layer, but for most cases computed to date it is small.

Another refinement added to the code removes the need to input velocity and temperature profiles at the initial station. The code now calculates the initial profile using a Faulkner-Skan solution for a perfect gas with $Pr = 0.7$. The initial guesses for $f''(0)$ and $g'(0)$ needed to obtain the Faulkner-Skan solution are calculated using a correlation of these parameters as functions of $g(0)$ and $\beta$. Thus, a large range of flow conditions can be solved with a minimum of external calculations.

The modified NONSIMBL code which is called NSBL, is coded in FORTRAN IV for the CDC 6600 at the University of Texas at Austin and is described herein along with a discussion of its uses and limitations.
NOMENCLATURE

$A, B, C, D$  matrices, equation 31

$C$  Chapman-Rubesin factor $\frac{\rho u}{\rho u e}$

$C_f$  local skin friction coefficient

$C_i$  mass fraction for species $i$

$C_{pi}$  specific heat of species $i$

$\overline{C_p}$  specific heat, $\overline{\sum_i C_i C_{pi}}$

$D^*$  displacement thickness including mass injection, equation 42

$D_i$  diffusion coefficient for species $i$

$F$  dimensionless streamwise component of the local velocity $\frac{u}{u_e}$

$f$  stream function, equation 7

$G$  matrix, equation 35

$H$  matrix, equation 34

$H$  total enthalpy

$i$  mesh point coordinate normal to the wall

$k$  thermal conductivity

$L$  length of the Space Shuttle Orbiter
Mach number

\( \dot{\mu} \)

molecular weight of species \( i \)

\( m \)

mesh point coordinate in streamwise direction

\( N \)

number of mesh points normal to the wall

\( n \)

transformed \( y \)-coordinate, equation 11

\( p \)

pressure

\( Pr \)

Prandtl number, \( \frac{\mu c_p}{k} \)

\( q \)

local convective heat-transfer rate

\( R \)

universal gas constant

\( r \)

distance from surface of body to axis of symmetry, measured normal to the axis of symmetry

\( s \)

transformed \( x \)-coordinate, equation 6

\( Sc \)

Schmidt number, \( \frac{\mu}{\rho D_2} \)

\( St \)

Stanton number, \( \frac{q}{\rho u C_p(T_e - T_w)} \)

\( T \)

temperature

\( u \)

velocity in streamwise direction

\( v \)

velocity normal to the wall

\( x \)

physical streamwise wetted distance from the stagnation point

\( x_a \)

axial distance from the nose of the Space Shuttle Orbiter

\( y \)

physical distance normal to the wall
\( \alpha \)  
- scale factor, equation 11

\( \beta \)  
- pressure gradient parameter, \( \frac{2s}{u_e} \frac{du_e}{ds} \)

\( \delta^* \)  
- displacement thickness

\( \eta \)  
- transformed y-coordinate, equation 6

\( \eta_{\text{edge}} \)  
- that point in the boundary layer where \( u = 0.99u_e \)

\( \Theta \)  
- non-dimensional temperature, \( \frac{T}{T_\text{te}} \)

\( \theta \)  
- momentum thickness, equation 48

\( \mu \)  
- viscosity

\( \rho \)  
- density

\( \omega \)  
- column matrix representing the unknowns, equation 31

\( \tau \)  
- local skin friction

\( \psi \)  
- stream function, equation 7

**Subscripts**

\( a, b, c, d, e, f \)  
- mesh points, sketch 1

\( e \)  
- edge value

\( i \)  
- species \( i \)

\( \text{inj} \)  
- properties of the injected species

\( N \)  
- evaluated at boundary layer edge when used with matrices only

\( r \)  
- denotes recovery temperature

\( t \)  
- stagnation value

\( w \)  
- wall value
properties of stream species evaluated at the wall (which is the first node) when used with matrices only

injectant species

Superscripts

k

body geometry factor, \( k=0 \), for two-dimensional flow; and \( k=1 \) for axisymmetric flow
THEORETICAL ANALYSIS

Governing Equations

Solutions are sought for the laminar boundary layer for an axisymmetric or a two-dimensional configuration with possible ablation or transpiration cooling. To model this flow, the present analysis is not restricted to the requirements for a similar, laminar boundary layer with mass injection. The injected gas may differ from the stream species and a general injection-rate distribution may be specified. The body may be either axisymmetric or two-dimensional providing the radius of curvature is large in comparison to the boundary layer thickness, i.e., centrifugal forces are neglected. Approximate solutions for a three-dimensional boundary layer with small cross flow can be obtained using the axisymmetric analog [9] in which an effective radius of curvature is used to describe the streamline divergence. For flow with no mass injection at the wall, the thermodynamic properties of the free-stream gas may be modeled with the ideal gas relations [10] or with real gas properties using the thermodynamic subroutine, "MOLIER." For flows with mass injection, the thermodynamic properties of the mixture of injectant and stream gases are approximated with the ideal gas relations. Chemical reactions between
the species are not considered. The governing equations applicable to the flow model are as follows:

continuity: \[
\frac{\partial (\rho u r_k)}{\partial x} + \frac{\partial (\rho v r_k)}{\partial y} = 0
\] (1)

where \( k=1 \) for axisymmetric flows and \( k=0 \) for two-dimensional flow.

species:

\[
\rho u \frac{\partial c_i}{\partial x} + \rho v \frac{\partial c_i}{\partial y} = \frac{\partial}{\partial y} \left( \rho D_i \frac{\partial c_i}{\partial y} \right)
\] (2)

\( x \)-component of momentum:

\[
\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial y} \left( u \frac{\partial u}{\partial y} \right)
\] (3)

\( y \)-component of momentum:

\[
\frac{\partial p}{\partial y} = 0
\] (4)

which represents the standard boundary-layer assumption regarding the pressure gradients normal to the wall.

energy:

\[
\rho u \frac{\partial h}{\partial x} + \rho v \frac{\partial h}{\partial y} = u \frac{\partial p}{\partial x} + \frac{\partial}{\partial y} \left( k \frac{\partial T}{\partial y} + \rho D_i \sum h_i \frac{\partial c_i}{\partial y} \right) + u \left( \frac{\partial u}{\partial y} \right)^2
\] (5)

The governing equations which describe the nonsimilar, possibly compressible, flow in physical coordinates are nonlinear, partial differential equations. Therefore, a transformation is sought to
simplify the solution procedures. Using the standard Lee-
Dorodnitsyn coordinate transformation [11]:

\[
 s = \int_{0}^{x} \rho_{e} u_{e} e_{e} r^{2k} dx
\]

\[
 \eta = \frac{\rho_{e} u_{e} r^{k}}{\sqrt{2s}} \int_{0}^{y} \frac{\rho_{e}}{\rho_{e}} dy
\]

where the stream function is defined as:

\[
 \psi = (2s)^{0.5} f
\]

one obtains the following equations:

**species:**

\[
 \left( \frac{C_{1}}{Sc} c_{1} \right) + f c_{1} = 2s \left( c_{1} f' - f^{s} c_{1}' \right)
\]

\[
 \left( \frac{C_{1}e}{Sc} \right) + f \left( \frac{\rho_{e}}{\rho} \right) \left( c_{1} f' - f^{s} c_{1}' \right) = 2s \left( f' (f')^{s} - f^{s} f'' \right)
\]

**energy:**

\[
 f \theta' + \frac{C_{e}}{\tau} \left( c_{p1} - c_{p2} \right) \theta c_{1}' + \frac{Cu_{e}^{2}}{c_{p} T e} \left( f'' \right)^{2} + \frac{1}{c_{p}} \left( \frac{C_{Pr}}{c_{p}} \right) \theta'
\]

\[
 - \frac{\rho_{e} u_{e}^{2}}{\rho C_{p} T e} = 2s \left( f' \theta'^{s} - \theta' f^{s} \right)
\]
where superscript prime denotes differentiation with respect to \( \eta \) and superscript \( s \), with respect to \( s \). Having introduced the stream function, which automatically satisfies the overall continuity equation, the new set of governing equations contains one less dependent variable. Except for a limited number of injection-rate distributions, it is not possible to reduce the governing equations to a form analogous for similar solutions, as discussed in the Introduction. An additional coordinate transformation can be made, as suggested in [12] and as mentioned in the Introduction:

\[
n = 1 - e^{-\alpha n}
\]

(11)

This transformation is for numerical purposes. Numerical integrations can now be carried out over a fixed interval (zero to one) rather than the usual interval in the \( \eta \)-coordinate system (zero to infinity). This coordinate system eliminates the need for an iteration to define the boundary layer edge. Note that in the present approach it is assumed that the edge of the viscous boundary layer, the edge of the thermal boundary layer, and the edge of the species concentration layer, all occur at the same \( \eta \).

Also, the transformation affects nodal point spacing in the physical-coordinate plane. Points which are evenly spaced with respect to the \( n \)-coordinate are not evenly spaced in physical space. Spacing of the \( y \)-coordinates of the nodal locations varies with position, such that \( \Delta y \) increases with distance from the wall.
This results in more nodal points in the region near the wall, where gradients are large, and fewer points in the region away from the wall, where gradients are smaller. The scale factor $\alpha$ will be treated as a constant for any two adjacent streamwise stations.

The resultant governing equations in the transformed system with $F = \frac{u}{u_e}$ replacing $f'$, are as follows.

Species:

$$a^2(1-n)^2 \left( \frac{C}{Sc} \right)_n C_{ln} + a^2(1-n) \left( (1-n) C_{n} - C_{ln} \right) \frac{C}{Sc} + \alpha(1-n) f C_{ln}$$

$$= 2s \left( C_{ls} F - \alpha(1-n) F_s C_{ln} \right)$$

(12)

Momentum:

$$ff_n a(1-n) + a^2(1-n)^2 C_n F_n + Ca^2(1-n) \left( (1-n) F_{nn} - F_n \right)$$

$$+ \beta \frac{\rho e}{\rho} F^2 = 2s \left( FF_s - f_s F_{nn} \alpha(1-n) \right)$$

(13)

Energy:

$$\frac{1}{C_p} a^2(1-n)^2 \left( \frac{C_C}{Fr} \right)_n \theta_n + \left( \frac{C}{Fr} \right) a^2(1-n) \left( (1-n) \theta_{nn} - \theta_n \right) + \alpha(1-n) f \theta_n$$

$$+ \frac{C_{le}}{C_p} \frac{C}{Sc} \left( C_{p_1} - C_{p_2} \right) a^2(1-n)^2 \theta_n C_{ln} + \frac{Cu_e^2}{C_p T_e} F_n a^2(1-n)^2$$

$$- \frac{u_e^2}{C_p T_e} \beta \frac{\rho e}{\rho} F = 2s \left( F \theta_s - \alpha(1-n) \theta_{ns} \right)$$

(14)
where the subscripts \( n \) and \( s \) denote differentiation with respect to \( n \) and \( s \), respectively.

**Boundary Conditions**

In the previous section, the governing equations were written in terms of three dimensionless, dependent variables, \( F \), \( C_1 \), and \( \theta \). Since the surface temperature and the inviscid flow field are known a priori, values for \( F \) and \( \theta \) are immediately determined at both boundaries.

At the wall, i.e., \( n = 0 \)

\[
F = 0, \quad \theta = T_w / T_{\text{te}}
\]  
(15)

At the boundary layer edge, i.e., \( n = 1 \)

\[
F = 1, \quad \theta = T_e / T_{\text{te}}
\]  
(16)

Also, it is possible to readily define the species concentrations at \( n = 1 \)

\[
C_1 = 1, \quad C_2 = 0
\]  
(17)

The values of the concentration fractions at the wall, however, are not explicitly known. The appropriate wall boundary condition is obtained from the conservation of species equation:

\[
(\rho v)_w C_{1w} = \rho_w D_{12} \left( \frac{\partial C_1}{\partial y} \right)_w
\]  
(18)
Physically, this boundary condition specifies the balance between the convective flux away from the wall and the diffusive flux toward the surface for the free-stream species. Written for the \( n,s \) coordinate system in terms of nondimensional parameters, this boundary condition becomes

\[
C_{1nW} = \frac{(\rho v)_W C_{1W} Sc_W \sqrt{2s}}{\alpha \rho_w u_w u_e r^k}
\] (19)

In addition to the dependent variables \( C_1, \theta, \) and \( F, \) the governing equations also contain the stream function, \( f. \) By definition:

\[
f' \equiv F
\]

\[
f = \frac{F}{\alpha(1-n)} \, dn + f(0)
\] (20)

From the continuity equation it follows that

\[
f(0) = -\frac{1}{\sqrt{2s}} \int_0^r (\rho v)_W r^k \, dx
\] (21)

In summary, the boundary conditions for \( C_1 \) and \( f(0) \) at \( n = 0 \) are determined by substituting the specified injection-rate distribution into the appropriate equations. The other boundary conditions are determined from the usual physical constraints. A complete listing of the boundary conditions is as follows:
At the wall, \( n = 0 \):

\[
F = 0
\]

\[
\theta = \frac{T_w}{T_{te}}
\]

\[
C_{ln} = \frac{(\rho v)_w C_{lw} S_{cw} \sqrt{2s}}{\alpha \rho_w \mu_w u_e r^k}
\]

\[
f(0) = -\frac{1}{\sqrt{2s}} \int_{0}^{x} (\rho v)_w r^k \, dx
\]

At the boundary layer edge, \( n = 1 \):

\[
F = 1
\]

\[
\theta = \frac{T_e}{T_{te}}
\]

\[
C_1 = 1
\]

\[
C_2 = 0
\]

Formulation of the Solution Algorithm

The governing partial differential equations, and the appropriate boundary conditions have been developed. However, to obtain a solution it will be necessary to use the finite difference form of the equations. These finite difference approximations are computed using the nodal scheme shown in Sketch 1.
The finite difference approximations for the partial derivatives of the dependent variables are functions of their values at the streamwise coordinate, \( m + 1 \), where the solutions are to be calculated; and at the station, \( m \), where the flow field is already known.

\[ G \] represents a particular dependent variable

\[ G = G_e \]

\[ G_s = \frac{1}{\Delta s} (G_e - G_b) \]  
(22)

\[ G_n = \frac{1}{4\Delta n} (G_c - G_a + G_f - G_d) \]

\[ G_{nn} = \frac{1}{2(\Delta n)^2} (G_c - 2G_b + G_a + G_f - 2G_e + G_d) \]

The approximations for the thermodynamic properties are evaluated only in terms of their values at streamwise coordinate, \( m \):

\[ R \] represents a thermodynamic property

\[ R_n = \frac{1}{2\Delta n} (R_c - R_a) \]  
(23)

\[ R_{nn} = \frac{1}{(\Delta n)^2} (R_c - 2R_b + R_a) \]
Difference approximations for partial derivatives involving products are defined so that the dependent variable at the \( m + 1 \) stream-wise coordinate appear linearly in the resulting equation.

\[
G_M = \frac{1}{\Delta s} G_b \left( M_e - M_b \right)
\]

\[
G_n = \frac{1}{8(\Delta n)^2} \left[ (G_c - G_a)(M_f - M_d) + (G_f - G_d)(M_c - M_a) \right]
\]

\[
G^2 = G_b G_e
\]  

(24)

\[
G_M = G_b \frac{1}{4\Delta n} \left( M_c - M_a + M_f - M_d \right)
\]

\[
G^2_n = \frac{1}{4(\Delta n)^2} \left( G_c - G_a \right) \left( G_f - G_d \right)
\]

Substitution of the appropriate approximations into the momentum equation, using \( \bar{a} = a(1-n) \), results in the following:

\[
\bar{a}^2 C_n \cdot \frac{1}{4\Delta n} \left( F_c - F_a + F_f - F_d \right) + \bar{a} f \frac{1}{4\Delta n} \left( F_c - F_a + F_f - F_d \right)
\]

\[+ C \bar{a}^2 \frac{1}{2(\Delta n)^2} \left( F_c - 2F_b + F_a + F_f - 2F_e + F_d \right)
\]

\[- C \alpha \bar{a} \frac{1}{4\Delta n} \left( F_c - F_a + F_f - F_d \right)
\]

\[+ \beta \left( \frac{\rho_e}{\rho} - F_b e \right) = 2s \left( F_e - F_b \right) \frac{\Delta s}{\Delta s}
\]

\[- \bar{a} \left( 2s \right) F_s \frac{1}{4\Delta n} \left( F_c - F_a + F_f - F_d \right)
\]
In the above equation, \( f_s \) is evaluated using the relations for the thermodynamic properties. Although the derivatives of the properties are not represented in finite difference forms, it is understood that the values are to be obtained in this manner. The finite difference form of the momentum equation takes on the form

\[
F_f \left( \frac{\tilde{a} f}{\Delta n} + \frac{\tilde{a}^2 C_n}{4 \Delta n} + \frac{\tilde{a}^2 C}{2(\Delta n)^2} - \frac{\tilde{a} \alpha C}{4 \Delta n} + \frac{2s \tilde{a} f_s}{4 \Delta n} \right) \\
+ F_e \left( - \frac{2 \tilde{a}^2 C}{2(\Delta n)^2} - \beta \frac{F_b}{\Delta s} - \frac{2s F_b}{\Delta s} \right) \\
+ F_d \left( - \frac{\tilde{a} f (F_c - F_a)}{\Delta n} - \frac{\tilde{a}^2 C_n (F_c - F_a)}{4 \Delta n} - \frac{\tilde{a}^2 C (F_c - 2F_b + F_a)}{2(\Delta n)^2} \right)
\]

\[= \frac{\tilde{a} f (F_c - F_a)}{\Delta n} - \frac{\tilde{a}^2 C_n (F_c - F_a)}{4 \Delta n} - \frac{\tilde{a}^2 C (F_c - 2F_b + F_a)}{2(\Delta n)^2} \]

\[+ \frac{\tilde{a} \alpha C (F_c - F_a)}{4 \Delta n} - \beta \frac{F_b}{\Delta s} - \frac{2s F_b^2}{\Delta s} - \frac{2s \tilde{a} f_s (F_c - F_a)}{4 \Delta n} \]

The quantities enclosed in brackets depend only upon properties and dependent variables evaluated at the streamwise coordinate \( m \), where the solution is already known. The equation can be rewritten:
Conservation of momentum:

\[ A_{11} F_d + B_{11} F_e + C_{11} F_f = D_1 \]  \hspace{1cm} (27)

Conservation of energy:

\[ A_{21} F_d + B_{21} F_e + C_{21} F_f + A_{22} \theta_d + B_{22} \theta_e + C_{22} \theta_f + A_{23} C_{1d} + C_{23} C_{1f} = D_2 \]  \hspace{1cm} (28)

Conservation of species:

\[ A_{33} C_{1d} + B_{33} C_{1e} + C_{33} C_{1f} = D_3 \]  \hspace{1cm} (29)

The following matrix equation can be used to represent these three equations, where the A, B, and C matrix elements are defined in Appendix A.

\[
\begin{align*}
\begin{bmatrix}
A_{11} & 0 & 0 \\
A_{21} & A_{22} & A_{23} \\
0 & 0 & A_{33}
\end{bmatrix}
\begin{bmatrix}
F_d \\
\theta_d \\
C_{1d}
\end{bmatrix}
+ 
\begin{bmatrix}
B_{11} & 0 & 0 \\
B_{21} & B_{22} & 0 \\
0 & 0 & B_{33}
\end{bmatrix}
\begin{bmatrix}
F_e \\
\theta_e \\
C_{1e}
\end{bmatrix}
+ 
\begin{bmatrix}
C_{11} & 0 & 0 \\
C_{21} & C_{22} & C_{23} \\
0 & 0 & C_{33}
\end{bmatrix}
\begin{bmatrix}
F_f \\
\theta_f \\
C_{1f}
\end{bmatrix}
= 
\begin{bmatrix}
D_1 \\
D_2 \\
D_3
\end{bmatrix}
\end{align*}
\]  \hspace{1cm} (30)
Replace

\[
\begin{pmatrix}
F_d \\
θ_d \\
C_d
\end{pmatrix} = \begin{pmatrix}
A_{11} & 0 & 0 \\
A_{21} & A_{22} & A_{23} \\
0 & 0 & A_{33}
\end{pmatrix}
\]

Noting that \( d \) corresponds to \( i-1 \), \( e \) to \( i \), and \( f \) to \( i+1 \), the matrix equation can be rewritten in more compact notation:

\[
A_{i,m} \omega_{i-1,m+1} + B_{i,m} \omega_{i,m+1} + C_{i,m} \omega_{i+1,m+1} = D_{i,m}
\]

(31)

The subscripts \( i,m \) refer to the nodal point at which the matrix is evaluated. The terms in the \( A, B, C \) and \( D \) matrices depend only upon quantities at station \( m \). Since these quantities are known, the \( A, B, C \) and \( D \) matrices can be evaluated. As the value of \( i \) varies from 1 at the wall to \( N \) at the boundary-layer edge, \( N-2 \) matrix equations of the form shown above are produced. The equation is not valid at \( i=1 \) or \( i=N \) because the values of the elements in the \( A, B, C \) and \( D \) matrices for any point \( n,m \) also depend upon properties at points \( i+1,m \) and \( i-1,m \). At the wall and at the boundary layer edge, the boundary conditions will be used to evaluate the flow variables. In general, the set of matrix equations for all \( N \) points can be represented using still another matrix representation:
To solve this set of linear matrix equations, the Gauss-Jordan method of elimination is used as described in [13]. Briefly,

\[
\begin{bmatrix}
B_1 & C_1 & 0 & 0 & \cdots & 0 \\
A_2 & B_2 & C_2 & 0 & \cdots & 0 \\
& & & \ddots & \ddots & \vdots \\
0 & - & A_1 & B_1 & C_1 & 0 \\
& & & & - & \cdots \\
0 & - & - & A_{N-1} & B_{N-1} & C_{N-1} \\
0 & - & - & - & A_N & B_N
\end{bmatrix}
\begin{bmatrix}
\omega_1 \\
\omega_2 \\
\vdots \\
\omega_{N-1} \\
\omega_N
\end{bmatrix}
=
\begin{bmatrix}
D_1 \\
D_2 \\
\vdots \\
D_{N-1} \\
D_N
\end{bmatrix}
\tag{32}
\]

The values \( \omega_1 \) are obtained using the boundary conditions at the wall.

Recall that, at the wall:

\[
\begin{align*}
\omega = 0 \\
\theta = \theta_w
\end{align*}
\]
Following the method outlined in [6], one can write

\[ C_{1}(i=2) = C_{1}(i=1) + \Delta n[C_{1}(i=1)]_{n} + (\Delta n)^{2}[C_{1}(i-1)]_{nn} \]

where \( C_{1}(i=2) \) is the value of \( C_{1} \) at the \( i=2 \) node in the direction normal to the surface. The value of \( [C_{1}(i+1)]_{n} \) can be determined using equation (19). The value of \( [C_{1}(i+1)]_{nn} \) can be obtained, after some manipulation, from the species equation for \( i=1 \). The above equation results in the relation:

\[ C_{1}(i=1) = \frac{C_{1}(i=2)}{1 + ENS} \quad (36) \]

where

\[ ENS = \frac{\sqrt{2} \omega}{\alpha(pu)_{w} u_{e} r_{k}^{2}} \left\{ \Delta n + \frac{(\Delta n)^{2}}{2} \left[ 1 - \frac{Sc_{w}}{C_{w}^{2}} \left( \frac{C_{w}}{Sc_{w}} \right)_{n} \right] \right\} \]

Using this expression for the boundary condition, the appropriate values for \( h_{1} \) and \( C_{1} \) are:
The solution for the ω-vector at each of the N-stations of the boundary layer for a given streamwise coordinate, \( m+1 \), begins at the outer edge of the boundary layer. Using the known boundary conditions:

\[ F = 1, \ \theta = \theta_e, \ \text{and} \ \omega_1 = 1 \]

one finds that:

\[ \omega_N = \begin{bmatrix} 1 \\ \theta_e \\ 1 \end{bmatrix} \]  

\[ H_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{1}{1 + ENS} \end{pmatrix} \]  

\[ G_1 = \begin{pmatrix} 0 \\ \theta_w \\ 0 \end{pmatrix} \]  

The solution proceeds inward, i.e., toward the wall, applying equation (33) sequentially to determine the velocity, the temperature, and the species profiles. Having obtained numerical values for these parameters, a convergence check is made. The convergence criteria is satisfied if consecutive solutions of the boundary layer have the slope of the velocity function at the wall within
0.05% of each other. For reference, this term is titled "CONVG" in the Fortran listing of the program. Should the convergence criteria not be satisfied, the elements of the A, B, C and D matrices are reevaluated using the average of the newly calculated value of F (or $\theta$ or $C_1$) and its value at the previous streamwise station. The iteration process continues until consecutive values of the velocity gradient agree. Once the convergence criteria is satisfied, the y-gradient of velocity at the edge of the boundary layer is checked to see if it is within a specified tolerance. Only if the value of the edge shear, which is defined as

\[
\frac{F_n - F_{n-1}}{\Delta n},
\]

is greater than zero and less than 0.2, is the solution considered to be the desired one. Otherwise, $\alpha$ is changed by 5% and a linear extrapolation method is used to recalculate the boundary layer until the edge shear criteria is met. In addition to the viscous profiles, numerical values are obtained for the heat-transfer rate, the skin-friction, the displacement thickness, etc. The procedure is then repeated to obtain the boundary layer profiles at the next streamwise station, until solutions are obtained at all stations for which the inviscid flow has been specified.

The solution of the initial station is obtained using a Faulkner-Skan formulation for a compressible, laminar boundary layer for a perfect gas with $Pr = 0.7$. This initial profile is calculated.
in the NSBL code by the subroutine entitled PIGYBAK. A linear double interpolation routine is used to calculate reasonable first guesses for \( f''(0) \) and \( g'(0) \) based on the values of \( g(0) \) and \( \beta \), as determined from the boundary conditions.

The numerical routine described herein computes the viscous-layer profile of the velocity function \( F \), the temperature function \( \theta \), and the stream species concentration \( C_1 \). Values are also obtained for the following parameters:

**CF**: the local skin-friction coefficient

\[
C_f = \frac{\tau}{0.5\rho_e u_e^2} \tag{40}
\]

**DELST**: the displacement thickness, feet

\[
\delta^* = \int_0^Y \left( 1 - \frac{\rho u}{\rho e u_e} \right) dy = \frac{\sqrt{2s}}{\rho e u_e r_k} \int_0^m \left( \frac{\rho e}{\rho} - \frac{u}{u_e} \right) \frac{dn}{a(1-n)} \tag{41}
\]

**DSTAR**: a thickness which represents the free-stream mass flow entrained in the boundary layer with injection

\[
D^* = \delta^* + \frac{1}{r_k} \int_0^x \frac{(\rho v)_w r^k}{\rho_e u_e} dx \tag{42}
\]

This equation corresponds to the definition of Hayasi [14].
HCOEF: the local heat-transfer coefficient, Btu/ft² sec

\[ h = \frac{q}{T_{te} - T_w} \]  
(43)

HRATIO: the ratio of the local heat-transfer coefficient to the reference heat-transfer coefficient

\[ HRATIO = \frac{h}{h_{ref}} \]  
(44)

QDOT: the local convective heat-transfer rate, Btu/ft² sec

\[ q = \left( k \frac{\partial T}{\partial y} \right)_w = \frac{k_w \rho_e u_e T_{te} r^{k} \alpha(1-n)}{\sqrt{2s}} \left( \frac{\partial \theta}{\partial n} \right)_w \]  
(45)

STNO: the Stanton number

\[ St = \frac{q}{\rho_e u_e C_p (T_r - T_w)} \]  
(46)

TAU: the local skin friction, lbs/ft²

\[ \tau = \left( \mu \frac{\partial u}{\partial y} \right)_w = \frac{\rho_w u_w u_e^{2} r^{k} \alpha(1-n)}{\sqrt{2s}} \left( \frac{\partial \psi}{\partial n} \right)_w \]  
(47)

THMOM: the momentum thickness, feet

\[ \theta = \int_0^y \frac{\rho u}{\rho_e u_e} \left( 1 - \frac{u}{u_e} \right) dy = \frac{\sqrt{2s}}{\rho_e u_e r^{k}} \int_0^n \frac{F(1-F)}{\alpha(1-n)} \, dn \]  
(48)

TREC: the recovery temperature, °R

\[ T_r = r(T_{te} - T_e) + T_e \]  
(49)
DISCUSSION OF RESULTS

The NSBL code has been used to generate numerical solutions for a variety of problems. Presented herein are the solutions for some representative flows. Specifically, there are two flows where the similarity conditions are satisfied and two flows where they are not. The two flows where the similarity conditions are satisfied are: (1) incompressible flow over a flat plate (the Blasius problem); and (2) supersonic flow past a sharp cone. The two flows which do not satisfy the similarity conditions are reentry flight conditions of the Space Shuttle Orbiter at (1) a relatively low Mach number ($M = 9.49$); and (2) a very high Mach number ($M = 29.86$).

Incompressible, Laminar Boundary Layer on a Flat Plate

A solution of the incompressible, laminar boundary layer was obtained with the NSBL code for flow past a flat plate at zero angle of attack. The free-stream velocity was 167 ft/sec; the stagnation pressure was 1.0 atmosphere. Both the stagnation temperature and the wall temperature were 540°F. The skin-friction coefficient, the displacement thickness, and the momentum thickness as calculated using the NSBL code are compared in Table I with the results reported by Schlichting [15], and the values calculated using the BLAST code [3]. The values obtained using the NSBL code are in close agreement.
with the values from the other solutions. The velocity profile computed using the NSBL code is presented in Figure 1. Included for comparison are the velocity profile presented in [15] and that obtained using the BLAST code. The NSBL profile is that calculated for $x = 1.0$ ft, i.e., for a distance of 1.0 ft from the leading edge. It should be noted that the $y$-transformation used in [15] differs by a factor of $\sqrt{2}$ from that used in the two codes developed at the University of Texas and in the earlier reference of Blasius [16]. However, the solution given in [15] has been converted to the present definition for $n$. The velocity profiles calculated by the three methods are in excellent agreement.

Laminar boundary Layer for Supersonic Flow
Past a Sharp Cone

The NSBL code was also used to obtain a solution for the laminar boundary layer on a sharp cone which had a half angle of 12°. The cone was at zero angle of attack in a supersonic stream. The free-stream Mach number was such that the flow downstream of the shock was supersonic and the flow properties were constant along the surface of the cone. At the edge of the boundary layer the static pressure was 0.05 atmosphere, the static temperature was 1000°R, and the Mach number was 4.84. The wall temperature of the cone was 540°R. The velocity profile at a distance of 1.0 ft from the apex of the cone as calculated with the NSBL code is compared in Figure 2
with that computed using the BLAST code. The temperature profiles obtained from the two codes are compared in Figure 3. Note that the maximum temperature in the boundary layer is greater than either the wall temperature or the static temperature at the edge of the boundary layer. This region of elevated temperature within the boundary layer is due to the effect of viscous dissipation. Once again, the agreement between the different calculation procedures is excellent. The convective heat-transfer-rate distributions as calculated by the two codes are presented in Figure 4. Included for comparison is the distribution calculated using Eckert's reference temperature technique (as discussed briefly in the Introduction). The heat-transfer rates were nondimensionalized with the theoretical heat-transfer rate to the stagnation point of a 1.0 ft radius sphere under the same flow conditions as calculated using the theory of Detra, Kemp, and Riddell [17]. The agreement between these three solutions is very good.

Compressible, Laminar Boundary Layers for Reentry Flow Conditions

As was mentioned in the Introduction, it was desired to write a code that could accurately solve for the boundary layer on a highly cooled wall in an accelerating, high-speed flow. The NSBL code was used to solve for the boundary layer on the Space
Shuttle Orbiter for two flight conditions from the reentry trajectory. These conditions included a relatively low Mach number ($M = 9.49$) and a very high Mach number ($M = 29.86$). The inviscid flow fields for these two flight conditions, which were provided by Dr. W. D. Goodrich of NASA Johnson Space Flight Center [18], are presented in Table 2. Additional points were added to those in [18] to improve the accuracy of the NSBL solution. Velocity and temperature profiles, displacement-thickness distributions, heat-transfer-rate distributions, and skin-friction coefficient distributions for these two flight cases, as calculated by the NSBL code, are presented in Figures 5 through 14. Also shown for comparison purposes are the solutions obtained using the BLIMP code [19], as provided by JSC.

For the first flight condition, the free-stream Mach number was 9.49 and the angle of attack was 30.8 degrees. The conditions at the stagnation point and the streamwise distributions for the static pressure, the wall temperature, the entropy at the edge of the boundary layer, and the radius of the equivalent body of revolution are presented in Table 2(a). These conditions were used as the input boundary conditions for the NSBL code and for the BLIMP code. Solutions using the NSBL code were obtained for two thermodynamic flow models: (1) for air that behaves as a thermally perfect ($p = \rho RT$) and a calorically perfect ($C_p = $ constant) gas; and (2) for air that behaves as a real gas. For the perfect-gas
model the viscosity was computed using Sutherland's equation,

\[
\mu = 2.270 \frac{T^{1.5}}{T + 198.6} \times 10^{-8} \text{ lbf sec ft}^{-2}
\]

Furthermore, the Prandtl number was assumed a constant, \( Pr = 0.70 \). The thermal conductivity was calculated using the definition of the Prandtl number, \( k = \frac{\mu C_p}{Pr} \). For the real-gas model, thermodynamic properties were calculated using the tabulated values of [20]. The viscosity, thermal conductivity, and Prandtl number were calculated using linear, double-interpolation of the properties tabulated by Hansen [21]. The specific heat was calculated using the definition of the Prandtl number. In the BLIMP solutions the real-gas model for air was used both for the thermodynamic properties and for the transport properties.

The velocity profiles as calculated by the NSBL code are compared to the BLIMP calculations in Figure 5. Figure 5(a) presents the velocity profiles at \( x_a = 0.10L \) and Figure 5(b) presents the profiles at \( x_a = 0.498L \). At both streamwise locations, the agreement between the NSBL real-gas calculations and the BLIMP calculations is very good. However, the solutions obtained for the perfect-gas model differ substantially from those obtained for the real-gas model. The temperature profiles at these same two streamwise locations are compared in Figure 6. Again, the agreement between the NSBL real-gas and the BLIMP calculations is very good.
The NSBL perfect-gas calculations differ significantly, especially at the downstream location. Note that, at \( x_a = 0.498L \), \( T = 0.385T_e \) at the wall for the real-gas solution, whereas the corresponding temperature is \( 0.685T_e \) for the perfect-gas solution. Since the wall temperature is specified as 1500\(^\circ\)F for both gas models (see Table 2a), the difference in the two dimensionless values of \( T_w/T_e \) indicates a difference between the values of \( T_e \). The same inviscid flow field, as defined by the pressure ratio \( (p_e/p_{t2}) \), the local entropy \( (s_e/R) \), and the streamline divergence metric \( (R_{DS}) \), was used for both gas property models. Therefore, it is clear that the thermodynamic model used in the calculations can have a significant effect on the resultant solutions.

The displacement-thickness distributions are compared in Figure 7. The BLIMP and the NC\,3L real-gas distributions are in good agreement. The displacement-thickness distribution calculated using the NSBL perfect-gas model compares reasonably well with the other two.

The heat-transfer-rate distributions are compared in Figure 8. The local heat-transfer rates were divided by the theoretical heat-transfer rate to the stagnation point of a 1.0 ft radius sphere as calculated using the theory in [17]. The NSBL real-gas distribution compares very well with the BLIMP results. However, the values for the NSBL perfect-gas solution are substantially lower.
than those for the other two models. The skin-friction-coefficient distributions are compared in Figure 9. The agreement between the NSBL real-gas solution and the BLIMP solution is very good. Although the values from the NSBL perfect-gas solution are below those of the other two models, the differences are not as great as they are for the heat-transfer-rate distributions. In summary, the NSBL real-gas calculations and the BLIMP calculations compare very favorably for this flight case. The NSBL perfect-gas calculations, however, differ significantly from those of the other two models and demonstrate the error that could be introduced by using perfect-gas assumptions for a real-gas situation.

For the second flight condition, the free-stream Mach number was 9.86 and the angle of attack was 41.4°. The input boundary conditions for the NSBL code and the BLIMP code are presented in Table 2(b). The NSBL code was used to obtain a solution for the real-gas model only. The velocity profiles at \( x_a = 0.107L \) and at \( x_a = 0.444L \) as calculated by the NSBL code are compared to the BLIMP solutions in Figures 10(a) and 10(b), respectively. The agreement between the profiles calculated by the two codes is excellent at both streamwise locations. The temperature profiles at the same two streamwise locations are compared in Figures 11(a) and 11(b). Again, the agreement between the solutions provided by the two codes is excellent. It is interesting to note that both codes indicate an inflection point in the temperature profiles at the two
streamwise locations. In both cases the physical temperature in the boundary layer in the region of the inflection point was on the order of 4500 to 5000°F. In this temperature range, the oxygen molecules begin to dissociate at the static pressure of this flow (which is approximately 0.01 atm). In this temperature range, the thermal conductivity changes rapidly with temperature. Furthermore, the value of the thermal conductivity at a specific temperature also depends on the static pressure.

The displacement-thickness, the heat-transfer-rate (non-dimensionalized as before), and the skin-friction-coefficient distributions as calculated by the two codes are compared in Figures 12, 13 and 14, respectively. Once again, the real-gas calculations of the NSBL code compare very well with those of the BLIMP code.
CONCLUDING REMARKS

The agreement between the sample calculations made using the NSBL code and the results available in the literature, along with the calculations of other codes, demonstrates the validity of the solution algorithm of the NSBL code. It should be noted, however, that the equations employed in the code incorporate the assumptions typically applied to thin boundary layers. Furthermore, it is assumed that the viscous boundary layer, the thermal boundary layer, and the species concentration all have the same thickness. Thus, because the iterative solution procedure requires that only the shear be negligible at the edge of the boundary layer, the code should be applied with caution to flows where the Prandtl number and the Schmidt number are significantly different than unity. It should also be noted that the NSBL code is applicable only to laminar boundary layers. Therefore, comparisons between experimental results and the solutions obtained with the code are valid only when the physical boundary layer actually is laminar.
Table 1. - The laminar boundary layer for incompressible flow over a flat plate.

<table>
<thead>
<tr>
<th></th>
<th>NSBL</th>
<th>BLAST (Ref. 3)</th>
<th>Schlichting (Ref. 15)</th>
</tr>
</thead>
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<tr>
<td>$C_f$</td>
<td>$\frac{0.6816}{\sqrt{Re_x}}$</td>
<td>$\frac{0.6644}{\sqrt{Re_x}}$</td>
<td>$\frac{0.6641}{\sqrt{Re_x}}$</td>
</tr>
<tr>
<td>$\frac{\delta^*}{x}$</td>
<td>$\frac{1.7206}{\sqrt{Re_x}}$</td>
<td>$\frac{1.7963}{\sqrt{Re_x}}$</td>
<td>$\frac{1.7208}{\sqrt{Re_x}}$</td>
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<tr>
<td>$\frac{\theta}{x}$</td>
<td>$\frac{0.6541}{\sqrt{Re_x}}$</td>
<td>$\frac{0.6633}{\sqrt{Re_x}}$</td>
<td>$\frac{0.664}{\sqrt{Re_x}}$</td>
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Table 2. - The input boundary conditions for Space Shuttle Orbiter Flight Design Trajectory.

(a) $M = 9.49$; altitude = 162,000 ft; angle of attack = 30.83°; $P_{t2} = 215.8$ lbf/ft$^2$; $T_{t2} = 5508^\circ R$.

<table>
<thead>
<tr>
<th>$M$</th>
<th>$x$ (ft)</th>
<th>$p_e/p_{t2}$</th>
<th>$T_w$ ($^\circ R$)</th>
<th>$S_e/R$</th>
<th>RDS (ft)</th>
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<tbody>
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Table 2. - (a) Continued.

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<th>$T_w(°R)$</th>
<th>$S_e/R$</th>
<th>RDS(ft)</th>
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Table 2. - Continued

(b) \( M_{w} = 29.86; \) altitude = 246,000 ft; angle of attack = 41.4°; 
\( p_{t2} = 46.26 \text{ lbf/ft}^2; \) \( T_{t2} = 10,798\text{°R}. \)

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Table 2. - (b) Conclusion.

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Figure 1. - Velocity profile for incompressible, laminar flow past a flat plate; angle of attack = 0°.
Figure 2. - Velocity profile for supersonic, laminar flow past a 12° cone; $M_e = 4.84$; angle of attack = 0°; $x = 1.0$ ft.
Figure 3. - Temperature profile for supersonic, laminar flow past a 12° cone, $M_e = 4.84$; angle of attack = 0°. $x = 1.0$ ft.
Figure 4. - Heat-transfer-rate distribution for supersonic flow past a 12° cone, $M_e = 4.84$; angle of attack = 0°.
Figure 5. - Velocity distribution across a laminar boundary layer, Orbiter Flight Design Trajectory; $M = 9.49$; angle of attack $= 30.83^\circ$. 

(a) $x_a/L = 0.10$
Figure 5. - Concluded.
Figure 6. - Temperature distribution across a laminar boundary layer, Orbiter Flight Design Trajectory; $M = 9.49$; angle of attack $= 30.83^\circ$. 

(a) $x_a/L = 0.10$
Figure 6. - Concluded.
Figure 7. - Displacement-thickness distribution, Orbiter Flight

Design Trajectory; $M = 9.49$; angle of attack $= 30.83^\circ$. 
Figure 8. Heat-transfer-rate distribution, Orbiter Flight Design Trajectory; $M = 9.49$; angle of attack $= 30.83^\circ$. 
Figure 9. - Skin-friction-coefficient distribution, Orbiter Flight Design Trajectory; $M = 9.49$; angle of attack = 30.83°.
Figure 10. - Velocity distribution across a laminar boundary layer, Orbiter Flight Design Trajectory; $M = 29.86$; angle of attack $= 41.4^\circ$. 

(a) $x_a/L = 0.107$
Figure 10. - Concluded.
Figure 11. - Temperature distribution across a laminar boundary layer, Orbiter Flight Design Trajectory; $M = 29.86$; angle of attack $= 41.4^\circ$. 

(a) $x_d/L = 0.107$
Figure 11. - Concluded.
Figure 12. - Displacement-thickness distribution, Orbiter Flight Design Trajectory; $M = 29.86$; angle of attack $= 41.4^\circ$. 

\[
\frac{x_a}{L}
\]
Figure 13. - Heat-transfer-rate distribution, Orbiter Flight Design Trajectory; $M = 29.86$; angle of attack = $41.4^\circ$. 

\[ \frac{\dot{q}}{q_{t,\text{ref}}} \] 

\[ \frac{x_m}{L} \]
Figure 14. - Skin-friction-coefficient distribution; Orbiter Flight Design Trajectory; $M = 29.86$; angle of attack = $41.4^\circ$. 

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APPENDIX A

DEFINITION OF MATRIX ELEMENTS

The individual elements of the coefficient matrices which appear in equations (28), (29) and (30) are as follows:

\[ A_{11} = \frac{\ddot{\alpha} f}{4\Delta n} + \frac{\ddot{\alpha}^2 C_n}{4\Delta n} - \frac{\ddot{\alpha}^2 C}{2(\Delta n)^2} - \frac{\ddot{\alpha} C}{4\Delta n} + \frac{2s \ddot{\alpha} f_s}{4\Delta n} \]

\[ A_{21} = \frac{C u^2}{2\Delta n C_p} \left( \frac{\Gamma_c - \Gamma_a}{2\Delta n} \right) \]

\[ A_{22} = \frac{\ddot{\alpha}^2}{4\Delta n C_p} \left( \frac{C_{p1} - C_p}{Pr} \right)_n - \frac{\ddot{\alpha}^2}{2(\Delta n)^2} \left( \frac{C}{Pr} \right) - \frac{\ddot{\alpha} C}{4\Delta n Pr} + \frac{\ddot{\alpha} f}{4\Delta n} \]

\[ + \frac{C_{le}}{C_p} \frac{C_{p1} - C_p}{4\Delta n} \frac{\ddot{\alpha}^2}{2(\Delta n)} \left( \frac{C_{1c} - C_{1a}}{2\Delta n} \right) + \frac{2s \ddot{\alpha} f_s}{4\Delta n} \]

\[ A_{23} = \frac{C_{le}}{C_p} \frac{C_{p1} - C_p}{Sc} \frac{\ddot{\alpha}^2}{4\Delta n} \left( \frac{\theta_c - \theta_a}{2\Delta n} \right) \]

\[ A_{33} = \frac{\ddot{\alpha}^2}{4\Delta n} \left( \frac{C}{Sc} \right)_n - \frac{\ddot{\alpha}^2}{2(\Delta n)^2} \frac{C}{Sc} - \frac{\ddot{\alpha} C}{4\Delta n Sc} + \frac{\ddot{\alpha} f}{4\Delta n} + \frac{2s \ddot{\alpha} f_s}{4\Delta n} \]

\[ B_{11} = \frac{\ddot{\alpha} C}{(\Delta n)^2} + \beta r_p + \frac{2s r_b}{\Delta s} \]

\[ B_{21} = \frac{3u e}{C_p T} \frac{\rho e}{\rho} \]
\[ B_{22} = \frac{\tilde{a}^2}{(\Delta n)^2} \frac{C}{Pr} + \frac{2s \Gamma_b}{\Delta s} \]

\[ B_{33} = \frac{\tilde{a}^2}{(\Delta n)^2} \frac{C}{Sc} + \frac{2s \Gamma_b}{\Delta s} \]

\[ C_{11} = \frac{\tilde{a} \alpha}{4 \Delta n} - \frac{\tilde{a} \xi}{4 \Delta n} + \frac{\tilde{a}^2 C}{4 \Delta n} - \frac{\tilde{a}^2 C}{2 (\Delta n)^2} - \frac{2 s \tilde{a}^2}{\Delta n} \]

\[ C_{21} = - \frac{C_{\varepsilon}}{2} \frac{\tilde{a}^2}{\Delta n} \left( \frac{F_c - F_a}{2 \Delta n} \right) \]

\[ C_{22} = - \frac{\tilde{a}^2}{4 \Delta n} \left( \frac{C \varepsilon_p}{\varepsilon_p} \right) - \frac{\tilde{a}^2}{2 (\Delta n)^2} \frac{C}{Pr} + \frac{\tilde{a} \alpha}{4 \Delta n} \frac{C}{Pr} - \frac{\tilde{a} f}{4 \Delta n} \]

\[ - \frac{C_{\varepsilon}}{C_p} \frac{C}{Sc} \left( \frac{C_p^1 - C_p^2}{4 \Delta n} \right) \left( \frac{C_{1c} - C_{1a}}{2 \Delta n} \right) - \frac{2 s \tilde{a}^2}{4 \Delta n} \]

\[ C_{23} = - \frac{C_{\varepsilon}}{C_p} \frac{C}{Sc} \left( \frac{C_p^1 - C_p^2}{4 \Delta n} \right) \left( \frac{\theta_c - \theta_a}{2 \Delta n} \right) \]

\[ C_{33} = \frac{\tilde{a} \alpha}{4 \Delta n} \frac{C}{Sc} - \frac{\tilde{a}^2}{4 \Delta n} \left( \frac{C}{Sc} \right) - \frac{\tilde{a}^2}{2 (\Delta n)^2} \frac{C}{Sc} - \frac{\tilde{a} f}{4 \Delta n} - \frac{2 s \tilde{a}^2}{4 \Delta n} \]

*ORIGINAL PAGE IS OF POOR QUALITY*
\[ D_1 = \frac{\tilde{\alpha}f(F_c - F_a)}{\Delta n} + \frac{\tilde{\alpha}^2 C_c (F_c - F_a)}{4 \Delta n} + \frac{\tilde{\alpha}^2 C_c (F_c - 2F_b + F_a)}{2 (\Delta n)^2} - \frac{\alpha C_c (F_c - F_a)}{4 \Delta n} \]

\[ + \frac{\rho e}{\Delta s} + \frac{2sF_b^2}{\Delta s} + \frac{2s \tilde{\alpha} f_s (F_c - F_a)}{4 \Delta n} \]

\[ D_2 = \frac{\tilde{\alpha}^2}{C_p} \left( \frac{C_T}{Pr} \right) \left( \frac{\theta_c - \theta_a}{4 \Delta n} \right) + \frac{\tilde{\alpha}^2}{2 (\Delta n)^2} \]

\[ - \frac{u \tilde{\alpha} (\theta_c - \theta_a)}{4 \Delta n} \frac{C}{Pr} + \frac{\tilde{\alpha} f_s (\theta_c - \theta_a)}{4 \Delta n} + \frac{2sF_b \theta_b}{\Delta s} \]

\[ + \frac{2s \tilde{\alpha} f_s (\theta_c - \theta_a)}{4 \Delta n} \]

\[ D_3 = \frac{\tilde{\alpha}^2 (C_{lc} - C_{la})}{4 \Delta n} \left( \frac{C}{Sc} \right) + \frac{\tilde{\alpha}^2 (C_{lc} - 2C_{lb} + C_{la})}{2 (\Delta n)^2} \]

\[- \frac{u \tilde{\alpha} (C_{lc} - C_{la})}{4 \Delta n} \frac{C}{Sc} + \frac{\tilde{\alpha} f_s (C_{lc} - C_{la})}{4 \Delta n} + \frac{2sF_b C_{lb}}{\Delta s} \]

\[ + \frac{2s \tilde{\alpha} f_s (C_{lc} - C_{la})}{4 \Delta n} \]
APPENDIX B

NSBL USERS GUIDE

Program Contents

The NSBL code is made up of the following blocks:

Program NSBL: calculates conditions at the edge of the boundary layer as well as $R_{e_x}, s, \beta, \frac{f_{inj}}{f_w}$.

Subroutine PIGYBAK: called by NSBL - calculates the initial boundary layer profile using a Faulkner-Skan solution technique.


Subroutine DERIV: called by NUMIN - calculates derivatives

Subroutine FGES: called by PIGYBAK - calculates first guesses for $f'(0)$ and $g'(0)$.

Function TWOVAR: called by FGES and EJOYCE - does linear double interpolation.

Subroutine EJOYCE: called by NSBL - calculates the downstream boundary layer profiles using finite difference methods.
Subroutine MOLIER: called by EJOYCE - calculates thermodynamic properties for air as a real gas.

Subroutine DINT2: called by MOLIER - does double interpolation.

Subroutine DINT1: called by DINT2 - does double interpolation.

Description of Input

The definitions of the input parameters are given below.

Figure B-1 presents a flowchart of the input sequence with the formats used.

MM number of x-points to be calculated
N number of y-points in the boundary layer
KK equals zero for two-dimensional flow; equals one for axisymmetric flow
ISEN equals zero for variable entropy flow; equals one for isentropic flow
IPERF equals zero for real-gas properties; equals one for perfect-gas properties
NTEMP number of elements in the transport property temperature vector
NPRES number of elements in the transport property pressure vector
DDSTR displacement thickness due to injection at the first station
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>FLCW</td>
<td>value of the stream function at the wall at the first station</td>
</tr>
<tr>
<td>FINJ</td>
<td>value of the stream function of the injectant species at the first station</td>
</tr>
<tr>
<td>WMI</td>
<td>molecular weight of the injectant species</td>
</tr>
<tr>
<td>WMS</td>
<td>molecular weight of the stream species</td>
</tr>
<tr>
<td>TEMP(M)</td>
<td>Mth temperature of the transport property temperature vector, °R (temperatures must be in increasing order)</td>
</tr>
<tr>
<td>PRES(M)</td>
<td>Mth pressure of the transport property pressure vector, log_{10}(lbf/ft^2), (pressures must be in increasing order)</td>
</tr>
<tr>
<td>VISS(I,J)</td>
<td>viscosity of the stream species at TEMP(I) and PRES(J), lbf sec/ft^2</td>
</tr>
<tr>
<td>TCS(I,J)</td>
<td>thermal conductivity of the stream species at TEMP(I) and PRES(J), Btu/ft sec °R</td>
</tr>
<tr>
<td>CPS(I,J)</td>
<td>specific heat of the stream species at TEMP(I) and PRES(J), Btu/slug °R</td>
</tr>
<tr>
<td>VISI(I,J)</td>
<td>viscosity of the injectant species at TEMP(I) and PRES(J), lbf sec/ft^2</td>
</tr>
<tr>
<td>TCI(I,J)</td>
<td>thermal conductivity of the injectant species of TEMP(I) and PRES(J), Btu/ft sec °R</td>
</tr>
<tr>
<td>CPI(I,J)</td>
<td>specific heat of the injectant species at TEMP(I) and PRES(J), Btu/slug °R</td>
</tr>
<tr>
<td>E</td>
<td>convergence criterion on a boundary condition</td>
</tr>
<tr>
<td></td>
<td>(typically 0.0005)</td>
</tr>
</tbody>
</table>
**DELT** step size in the y-direction (typically .05)

**EPS** convergence criterion when a variable step size is used

**PTE(1)** total pressure behind the shock, lbf/ft²

**TTE(1)** total temperature behind the shock, °R

**HTREF** reference value of the heat transfer coefficient, Btu/ft² sec °R

**RGAS** gas constant of the stream species, ft lbf/lbm °R

**GK** ratio of the specific heats of the stream species

**PRATIO** ratio of the local edge pressure to PTE(1)

**TW** wall temperature, °R

**ROVIN** density of the injectant species, slug/ft³

**RDS** radius of the equivalent body of revolution, ft

**X** streamwise wetted distance from the stagnation point, ft

**SEL** local entropy at the edge of the boundary layer divided by
the stream species gas constant, \( S_e / RGAS \)

**SLE** total entropy behind the shock minus the local entropy at
the edge of the boundary layer, Btu/lbm °R

**SELT** total entropy behind the shock, Btu/lbm °R

**IINIT** equals zero if the initial profile is to be calculated
in the code; equals one if the initial profile is to
be input

**ALF1** initial value of the scale factor, α, at the x-location
of the input profile

---

*ORIGINAL PAGE IS OF POOR QUALITY*
REXINT(1) integrated value of the Reynolds number at the x-location of the input profile

S(1) transformed x-coordinate at the x-location of the input profile

BETA(1) pressure gradient parameter at the x-location of the input profile

F(I) nondimensional velocity \( \frac{u}{u_e} \) at the \( I^{th} \) point in the boundary layer

THETA(I) nondimensional temperature \( \frac{T}{T_e} \) at the \( I^{th} \) point in the boundary layer

CS(I) mass fraction of the stream species at the \( I^{th} \) point in the boundary layer

Description of Output

The output for the NSBL code includes the input parameters, the free-stream parameters, the boundary layer profiles, and the boundary layer parameters. The output for the free-stream flow includes:

M station number in the x-direction

PE(M) static pressure at the edge of the boundary layer, lbf/ft\(^2\)

PTE(M) total pressure at the edge of the boundary layer, lbf/ft\(^2\)
THETA\(E(M)\) temperature ratio \(T_e/T_{\infty}\) at the edge of the boundary layer

\(T_{\infty}(M)\) total temperature at the edge of the boundary layer, \(^\circ\)R

\(EMACH(M)\) Mach number at the edge of the boundary layer

\(UE(M)\) velocity at the edge of the boundary layer, ft/sec

\(URENOE(M)\) unit Reynolds number at the edge of the boundary layer, 1/ft

\(REXINT(M)\) Reynolds number at the edge of the boundary layer integrated over the wetted distance from the stagnation point

\(SEL(M)\) local entropy at the edge of the boundary layer divided by the stream species gas constant

\(S(M)\) transformed \(x\)-coordinate

\(ROVIN(M)\) density of the injectant species, slug/ft\(^3\)

\(BETA(M)\) pressure gradient parameter, \(\beta\)

\(FLCW(M)\) value of the stream function at the wall

The output for the boundary layer profiles and parameters for \(M\)-1 \(x\)-stations includes:

\(M\) station number in the \(x\)-direction

\(X(M)\) physical wetted distance from the stagnation point, ft

\(I\) station number in the \(y\)-direction

\(YN(I)\) transformed \(\eta\)-coordinate, \(\eta\), at the \(I^{th}\) point in the boundary layer
Y(I) physical y-coordinate at the \textsuperscript{I}th point in the boundary layer, ft

ETA(I) transformed y-coordinate, \(n\), at the \textsuperscript{I}th point in the boundary layer

F(I) nondimensional velocity at the \textsuperscript{I}th point in the boundary layer, \(u(I)/u_e(M)\)

THETA(I) nondimensional temperature at the \textsuperscript{I}th point in the boundary layer, \(T(I)/T_w(M)\)

CS(I) mass fraction of the stream species at the \textsuperscript{I}th point in the boundary layer

TAU local skin friction, lbf/ft\textsuperscript{2}

CF local skin-friction coefficient

HCOEF local heat-transfer coefficient

TREC recovery temperature, \(^{\circ}\)R

QDOT local convective heat-transfer rate, Btu/ft\textsuperscript{2} sec

STNO local Stanton number

HRATIO HCOEF/HTREF

DELST local displacement thickness, ft

THMOM local momentum thickness, ft

DSTAR local displacement thickness with mass injection, ft
Figure B-1. Flowchart of the input sequence and FORMATS for the NSBL code.
Figure B-1. - Continued
Figure B-1. - Continued

Format (2E12.5)  
RGAS, GK

Format (5E12.5)  
(PRATIO(M), TW(M), ROVIN(M), RDS(M), X(M), M=1, MM)

ISEN=1?  

IPREP=1?  

Format (6E12.5)  
(SLE(M), M=1, MM)

Format (6E12.5)  
(SEL(M), M=1, MM)

Format (E12.5)  
SELT

Figure B-1. - Continued

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Figure B-1. - Conclusion.
APPENDIX C

THERMODYNAMIC AND TRANSPORT PROPERTIES

Calculation Procedures for a Perfect Gas

Before one can solve equations (12), (13), and (14) for the independent variables F, θ, and C, it is necessary to evaluate the thermodynamic properties which appear in the coefficients. The thermodynamic properties of the individual species are readily computed as a function of pressure and temperature. For cases of mass injection or when the free stream is assumed to be a perfect gas, the properties of the mixture are approximated with the ideal gas relations [10] once the relative concentrations of the stream species and of the injectant species are known.

DENSITY: The density of the mixture is calculated using the relation

\[ \rho = \frac{P}{RT} \]

which can be written for a mixture of two gases as:

\[ \rho = \frac{P}{RT \frac{M_1}{M_1 + C_1(M_2 - M_1)}} \]

where, to have consistent units, the value of the universal gas constant, R, is

\[ 1545 \ \text{ft lbf} \]

\[ 1 \text{lb mole} \ \text{°K} \ . \]
The transport properties (specific heat, viscosity, and thermal conductivity) for a given component are computed by linearly interpolating the input tables as a function of temperature (the transport properties are independent of pressure for a perfect gas).

**SPECIFIC HEAT:** The specific heat of the mixture is computed using

\[
\tilde{C}_p = C_{p2} \cdot (C_{p1} - C_{p2})
\]

**VISCOSITY:** The viscosity of the mixture is computed using the relation [11]:

\[
\mu = \frac{\mu_1 x_2}{1 + G_{12} \frac{x_2}{x_1}} + \frac{\mu_2 x_1}{1 + G_{21} \frac{x_1}{x_2}}
\]

where

\[
x_1 = \frac{M_2}{M_1 + C_1(M_2 - M_1)}
\]

\[
x_2 = 1 - x_1
\]

\[
G_{12} = \frac{\left[1 + \left(\frac{\mu_1}{\mu_2}\right)^{0.5} \left(\frac{M_2}{M_1}\right) \cdot 0.25\right]^2}{2.828 \left(1 + \frac{M_1}{M_2}\right)^{0.5}}
\]

and

\[
G_{21} = \frac{\left[1 + \left(\frac{\mu_2}{\mu_1}\right)^{0.5} \left(\frac{M_1}{M_2}\right) \cdot 0.25\right]^2}{2.828 \left(1 + \frac{M_2}{M_1}\right)^{0.5}}
\]
THERMAL CONDUCTIVITY: The thermal conductivity of the mixture is computed using:

\[
k = \frac{k_1}{1 + 1.055 G_{12} \frac{X_2}{X_1}} + \frac{k_2}{1 + 1.065 G_{21} \frac{X_1}{X_2}}
\]  

(C-4)

where \( G_{12}, G_{21}, X_1, \) and \( X_2 \) have been defined above.

DIFFUSION COEFFICIENT: The binary diffusion coefficient is computed using the procedure outlined in [22] for diffusion in a multicomponent mixture:

\[
D_{ij} = \frac{\bar{D}}{F_i F_j}
\]  

(C-5a)

Where \( \bar{D} \) is a reference coefficient which can be approximated by:

\[
\bar{D} = \frac{cT^{1.5}}{p}
\]  

(C-5c)

Further,

\[
F_i = \left( \frac{M_i}{26} \right)^{0.461}
\]  

(C-5c)

and

\[
F_j = \left( \frac{M_j}{26} \right)^{0.461}
\]  

(C-5d)

The constant of equation (C-5b) was evaluated by fitting the equation to the tabulated values for binary diffusion coefficients. Specifically, the diffusion coefficients for mixtures of \( \text{N}_2: \text{CH}_4 \).
H₂:CO₂, N₂:H₂, Air:CO₂, N₂:N₂, for temperatures from 520°R to 12,000°R, as tabulated in [22]. Using a value of \( c = 2.64 \times 10^{-6} \), the calculated values were within 20% of the tabulated values over the entire range of pressure and temperature considered. Thus, the equation used in the numerical code is:

\[
D_{ij} = \frac{2.64 \times 10^{-6} T^{1.5}}{p(F_i/F_j)}
\]

DIMENSIONLESS PARAMETERS: Once the dimensional thermodynamics of the mixture have been solved, it is a simple matter to compute the Chapman-PRubesin factor, \( \frac{\rho u}{\rho c^* u_e} \), the Prandtl number, \( \frac{\mu c^*}{k} \), and the Schmidt number, \( \frac{\rho D_{ij} c^*}{\mu} \).

Calculation Procedures for a Real Gas

For cases where there is no mass injection, the free-stream species may, at the user's option, be assumed to behave as a real gas. When this is the case, the density of the gas is calculated by the subroutine MOLITR. The transport properties are computed by linearly interpolating the input tables as a function of temperature and pressure.
REFERENCES


VISE(M)@MOVAR(WRES,M,T1M,PLUG,TEV(H,M),PRE5,TEMP,VISE)
H13E(H,M)@TEV(H,M)`1CPS(13)/32,174
EA(M)@SORT(GK,32,174)AAGAST(EA(H,M))
THEA(M)@DIM(M)/T1F1
UE(M)@SORT((K+TIE)@H(EA(M))+3MD2,0)
EMACH(H)@UEF(M)/EA(H,M)
PT(E(H,M))@RPE(M)+1)/(GK-1)+2/2/EMACH(M)+EMACH(M)*GK/(GK=1))
WHUE(M)@PHAT1(OPE(L)PDE11/TE(M))AGAS=32,174
UEREUE(M)+WHUE(M)*UE(M)+VISE(M)

15H CONTINUE
16H CONTINUE
PRINT 463
PRINT 464
PRINT 6(M), (M+1), PHAT1(M), TIE(H,M),WUS(H,M), X(M), NOVU(M)@1, MM)
PRINT 260
NSPACE(57+MM)/6
IF (NSPACE,LH) NSPACE=N
DO 170 I=1,NSPACE
PRINT 260
17H CONTINUE
PRINT 463
PRINT 470
PRINT 464, (M+1), Y(H,M), RPE(M), PDE(M), THEA(M), TIE(M), EMACH(M)@1, MM)
PRINT 260
NSPACE(57+MM)/6
IF (NSPACE,LH) NSPACE=N
HU 1NH @1, NSPACE
PRINT 260
18H CONTINUE
PRINT 463
PRINT 470
C
ME4K 554, 6 1 1 1 ALF1, REXINT(1), S(1), BETA(1)
C
IF (IINIT,FLU) GO TO 19H
REXINT(1)@UREUE(1)+4/2*(WBS(1)+4.1)/4.1*(1/(1+5.2))
S(1)@NWHUE(1)+VISE(1)+4/2*(WBS(1)+4.1)/4.1*(1/(1+5.2))
19H MHz
PRINT S(H,M), UE(H,M), UMANGE(M), NFXINT(M), SEL(M)
C
NU=STREAM STATIONS
C
NO 2NH @2, MM
DELM(M)=EM(M)+X(M)+1
WEINT(M)+WEINT(M)+1+HMEM(EM)+HMEM(M)+DELM(M)/2.1
PRINT S(H,M), UE(H,M), HMEM(M), WFXINT(M), SEL(M)
ONSH(M)=HMEM(M)+HMEM(M)+HMEM(M)+1+HMEM(M)+4/2*(WBS(1)+4.1)/4.1*(1/(1+5.2))
2 (M)+1
10 (M)=1
C
EVALUATION OF S
C
S(M)+EM(M)+HMEM(M)+HMEM(M)+HMEM(M)+4/2*(WBS(1)+4.1)/4.1*(1/(1+5.2))
1)+(WHUE(M)+UE(M)+VISE(M)+WBS(M)+4/2)+(WHUE(M)+UE(M)+VISE(M)+WBS(M)+4/2))
20H CONTINUE
OS(1)=5(S(2)=5(1)
IF (IINIT,FLU) GO TO 21H
REXINT(1)=4*(2+5.1)@UE(M)+UE(M)+4/2*(WBS(1)+4.1)/4.1*(1/(1+5.2))
21H RETURN(REXINT(1))
DO 22H @2, MM
OS(M)=5(M)+S(1)+2.1
HMEM(M)+2*(S(M)+S(M)+S(M)+1+IF(S(M)+S(M)+1+IF(S(M)+S(M)+1))
C
22H RETURN(REXINT(1))
CONTINUE

CALCULATION OF FLC AT THE WALL

DO 230 M=2,MM
   FLC(M)=FINJ(M)+ROV(M)*HDS(M)*HUV(M)*HDS(M)
   1
   FLCL(M)=FLCL(M)+SU0(F,MM(M))
230 CONTINUE

PRINT 28M
NSPACE(62*MM)/6
IF (NSPACE.LT.0) NSPACE=M
PRINT 28M
28M CONTINUE
PRINT 51M
PRINT 52M, M,N(M),S(M),HUV(M),ETA(M),FLC(M),MM1,MMAX

CALCULATING THE FLM PARAMETERS AT THE INITIAL STATION

IF (1.IN1.EQ.4) GO TO 25M
ALPHA1

HNO 50, (F,THETA,CS,I,N)

GO TO 28M

25M H=1
H=H(1)
THET(1)
THET(1)
THET(1)
THET(1)
CALL PIGMILK (MTF1)

26M CALL EJUICE

27M FORMAT (11111)
28M FORMAT (11111)
29M FORMAT (3)
30M FORMAT (6E12.5)
31M FORMAT (5E12.5)
32M FORMAT (5X,24((1HE,1),/5X,24(1HE,1),/11,11)
33M FORMAT (G8X,9THREE ARE,13,2HM STATIONS IN THE X DIRECTION,/,4AX,9
   INITIAL AM/13,2HM STATIONS IN THE Y DIRECTION,/)34M FORMAT (5X,13THET(1),11NHPFMM HAVE INPUP AN ERRONEOUS KK, KK MUST EQU
   ALL X FIN A TWO DIMENSIONAL CONFIGURATION, ON 1 FOR A BODY OF REVOLU
   TIONS)35M FORMAT (5X,4THET THE BODY IS A TWO DIMENSIONAL CONFIGURATION)
36M FORMAT (5X,4THET THE BODY IS A BODY OF REVOLUTION)
37M FORMAT (6X,3THET THE MOLFCIL NAT IOM OF THE INJECTANT IS,F6,2,113M
   1 LM/MM HANDLE,/,3AX,5THET THE MILFCIL NAT IOM OF THE SPECIES IS,F6,2
   2.15L MU/MM HANDLE,/)38M FORMAT (5X,21THET THE GAS IS A HEAL GAS,/)39M FORMAT (5X,21THET THE GAS IS A PERFECT GAS,/)40M FORMAT (G5X,9THET(1M),(1M),F12.5,10M,FLC(M),(1M),E12.5,14M,FINJ(M),E1
   4.5,1/)41M FORMAT (5X,23E12.5,7M, DELTA,E12.5,14M, EPS,E12.5,7M)
42M FORMAT (5F12.5)
43M FORMAT (6F12.5)
44M FORMAT (6F12.5)
45M FORMAT (5X,13E12.5)
46M FORMAT (5X,13E12.5,5X,1E12.5)

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<table>
<thead>
<tr>
<th>FORMAT</th>
<th>(SIZE, NAME, DATA)</th>
<th>A</th>
</tr>
</thead>
<tbody>
<tr>
<td>4TH FORMAT</td>
<td>(2x, IM, AX, 1X, 11X, 5HPE (M), 6X, 8HPE (M), 7X, 9HPE (M), 8X, 10HPE (M))</td>
<td>319</td>
</tr>
<tr>
<td>4TH FORMAT</td>
<td>(2x, IM, 1X, 11X, 5HPE (M), 6X, 8HPE (M), 7X, 9HPE (M), 8X, 10HPE (M))</td>
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<tr>
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<td>4TH FORMAT</td>
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<td>4TH FORMAT</td>
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<td>319</td>
</tr>
<tr>
<td>4TH FORMAT</td>
<td>(2x, IM, 1X, 11X, 5HPE (M), 6X, 8HPE (M), 7X, 9HPE (M), 8X, 10HPE (M))</td>
<td>320</td>
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<tr>
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<td>321</td>
</tr>
<tr>
<td>4TH FORMAT</td>
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<td>322</td>
</tr>
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<td>4TH FORMAT</td>
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<td>323</td>
</tr>
</tbody>
</table>
GA(1)

IF (I.EQ.2) GO TO 80
IF (I.EQ.3) GO TO 40
Y(1)=AA
Y(2)=BB
Y(3)=FPPSAV
Y(4)=DD
Y(5)=GO SAVE
FPPUSY(3)
GPPOSYY(5)
GO TO 40

80 Y(1)=AA
Y(2)=BB
Y(3)=1,05*FPPSAV
Y(4)=DD
Y(5)=SAVE
FPPUSY(3)
GPPOSYY(5)
GO TO 40

GO TO 40

100 UFIAPAFPA(1)=FPA(2)

GOA=GA(1)=GA(2)

UFPBFPA(1)=FPA(3)

UAMA=GA(1)=GA(3)

UFPBFPPB1=FPPP1

DGPA=GP1=GP3

Ab=UFIAP/AUFIAP

H10FPA/NGP0

C111=H10FPA(1)

D1=GA/GFPPK

E1=DGA1/DGPA

F1=1.6565*GA(1)

DEFIAP(C111=F1*10)/Ab*1=D1*10

UEGF=(Ab*If=F1*10)/C11110*1=110

FPRFPPB1=DEPIAP

UFPBGP(1)=EP

Y(1)=AA
Y(2)=BB
Y(3)=FPP
Y(4)=DD
Y(5)=GP
GO TO 50

100 CONTINUE

Y(1)=AA
Y(2)=BB
Y(3)=FPP
Y(4)=DD
Y(5)=GP

CALL NUMIN (N3,DLT,TP,TE,EP,YN,EP,PS,PSAVE,KEY,DP,IV,1)


GO TO IPRS

C

RANGE-KUTTA PROCEDURE

C

00 VVUV*DELAY2
00 VVUV*DELAY2

00 VVUV*DELAY2

7H CONTINUE
CALL DERIV (VV,DU,V,P,J,N)
IF (IERM.EQ.1) RETURN
DO 90 J=1,N

80 CONTINUE
CALL DERIV (VV,DU,TE,J,N)
IF (IERM.EQ.1) RETURN
DO 90 J=1,N

90 CONTINUE
CALL DERIV (T,DU,P,5,N)
IF (IERM.EQ.1) RETURN

10 CONTINUE
CALL DERIV (T,DU,TE,J,N)
IF (IERM.EQ.1) RETURN

11 CONTINUE
CONTINUE
CALL DERIV (T,DU,TE,J,N)
IF (IERM.EQ.1) RETURN

C

CHECK THE NUMBER OF INTEGRATION STEPS MADE BY N=K

12 N=4
13 N=3
14 N=2
15 N=1

16 IF (KINT.LT.3) GO TO 12V
ASSIGN 200 TO IPR5

17 N=4
18 N=3
19 N=2
20 N=1

21 IF (KINT.EQ.1) RETURN
IF (KINT.EQ.2) RETURN
KINC=0
DO 131 M=1,N

22 IF (E(J),LT,0.0) GO TO 154
IF (E(J),LE,0.0) GO TO 154

23 IF (KINC.EQ.1) GO TO 198
DO 144 J=1,N

24 CONTINUE
RETURN

25 CONTINUE
IF (KINC.EQ.1) GO TO 198

26 CONTINUE
IF (KINC.EQ.1) GO TO 198

27 CONTINUE
IF (KINC.EQ.1) GO TO 198

28 CONTINUE
IF (KINC.EQ.1) GO TO 198

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IF (KINC.EQ.1) GO TO 198

123 CONTINUE
IF (KINC.EQ.1) GO TO 198

ORIGINAL PAGE IS OF POOR QUALITY
SUBROUTINE HENRY (Ta,alph,LMC,n)
DIMENSION Y(5), X(5)
COMMON YXS, XTA, YS, XTA
COMMON /CVST/ A1, A2, A3, A4, Ex, YS

IF (Y(1) /= 1.0) THEN
  go to 10
K2 = (X(3)+X(4))/(X(1)+X(2)+X(3)+X(4)+X(5)+X(6))
K10 = (X(1)+X(2)+X(3)+X(4)+X(5)+X(6))
Y(5) = Y(5)*K10/(K10+K2)
Y(4) = Y(4)*K10/(K10+K2)
Y(3) = Y(3)*K10/(K10+K2)
Y(2) = Y(2)*K10/(K10+K2)
Y(1) = Y(1)*K10/(K10+K2)
RETURN
END
10 WRITE (6,*) 'Alpha=1.0'
SUMMANTINE FLOYER
C
C SUMMANTINE FLOYER USES FINITE DIFFERENCE METHODS TO CALCULATE THE DOWNSWEEP PHASES
C
C DIMENSION F(I5N), THETA(I5N), CS1(I5N), PS2(I5N), NFCS2(I5N)
C 150, 150, 150, 150,
C 200, 200, 200, 200,
C 200, 200, 200, 200,
C 200, 200, 200, 200,
C
C COMMON /CFL2/ (FLC(I5N), TET(5N), CS1(I5N), PS2(I5N), NFCS2(I5N), FCL2(5N),
C 150, 150, 150, 150,
C 200, 200, 200, 200,
C 200, 200, 200, 200,
C 200, 200, 200, 200,
C
C COMMON /CFL1/ (FLC(I5N), TET(5N), CS1(I5N), PS2(I5N), NFCS2(I5N), FCL1(5N),
C 150, 150, 150, 150,
C 200, 200, 200, 200,
C 200, 200, 200, 200,
C 200, 200, 200, 200,
C
C COMMON /CFL3/ (FLC(I5N), TET(5N), CS1(I5N), PS2(I5N), NFCS2(I5N), FCL3(5N),
C 150, 150, 150, 150,
C 200, 200, 200, 200,
C 200, 200, 200, 200,
C 200, 200, 200, 200,
C
C COMMON /CFL4/ (FLC(I5N), TET(5N), CS1(I5N), PS2(I5N), NFCS2(I5N), FCL4(5N),
C 150, 150, 150, 150,
C 200, 200, 200, 200,
C 200, 200, 200, 200,
C 200, 200, 200, 200,
C
C COMMON /CFL5/ (FLC(I5N), TET(5N), CS1(I5N), PS2(I5N), NFCS2(I5N), FCL5(5N),
C 150, 150, 150, 150,
C 200, 200, 200, 200,
C 200, 200, 200, 200,
C 200, 200, 200, 200,
C
C COMMON /CFL6/ (FLC(I5N), TET(5N), CS1(I5N), PS2(I5N), NFCS2(I5N), FCL6(5N),
C 150, 150, 150, 150,
C 200, 200, 200, 200,
C 200, 200, 200, 200,
C 200, 200, 200, 200,
C
C COMMON /CFL7/ (FLC(I5N), TET(5N), CS1(I5N), PS2(I5N), NFCS2(I5N), FCL7(5N),
C 150, 150, 150, 150,
C 200, 200, 200, 200,
C 200, 200, 200, 200,
C 200, 200, 200, 200,
C
C COMMON /CFL8/ (FLC(I5N), TET(5N), CS1(I5N), PS2(I5N), NFCS2(I5N), FCL8(5N),
C 150, 150, 150, 150,
C 200, 200, 200, 200,
C 200, 200, 200, 200,
C 200, 200, 200, 200,
C
C COMMON /CFL9/ (FLC(I5N), TET(5N), CS1(I5N), PS2(I5N), NFCS2(I5N), FCL9(5N),
C 150, 150, 150, 150,
C 200, 200, 200, 200,
C 200, 200, 200, 200,
C 200, 200, 200, 200,
C
C COMMON /CFL10/ (FLC(I5N), TET(5N), CS1(I5N), PS2(I5N), NFCS2(I5N), FCL10(5N),
C 150, 150, 150, 150,
C 200, 200, 200, 200,
C 200, 200, 200, 200,
C 200, 200, 200, 200,
C
C COMMON /CFL11/ (FLC(I5N), TET(5N), CS1(I5N), PS2(I5N), NFCS2(I5N), FCL11(5N),
C 150, 150, 150, 150,
C 200, 200, 200, 200,
C 200, 200, 200, 200,
C 200, 200, 200, 200,
C
C COMMON /CFL12/ (FLC(I5N), TET(5N), CS1(I5N), PS2(I5N), NFCS2(I5N), FCL12(5N),
C 150, 150, 150, 150,
C 200, 200, 200, 200,
C 200, 200, 200, 200,
C 200, 200, 200, 200,
M(I,J,K18)&iw
DO 320 LM1,3
H(I,J,K)=H(I,J,K)+DAMTV(I,J,L)*C(I,L,K)
320 CONTINUE
DO 330 JB1,3
AG(I,J,1)=H
DO 330 LM1,3
AG(I,J,1)=AG(I,J,1)+A(I,J,L)*G(I=L,1)
330 CONTINUE
DO 340 JB1,3
DAG(I,J,1)=0(I,J,1)=AG(I,J,1)
340 CONTINUE
DO 350 JB1,3
G(I,J,1)=H
DO 350 LM1,3
G(I,J,1)=G(I,J,1)*C(I,L,K)
350 CONTINUE
DO 360 JB1,3
DAG(I,J,1)=G(I,J,1)+DAMTV(I,J,L)*DAG(I=L,1)
360 CONTINUE

SOLUTION FOR THE X VECTOR
C
K(N,1,1)=N
K(N,2,1)=TMAE(M)
K(N,3,1)=N
F(N)=K(N,1,1)
THETA(N)=K(N,2,1)
CS(N)=K(N,3,1)
DO 370 JB1,NN
1B=II
DO 370 JB1,3
2H=I,1,1)
DO 370 LB1,3
3H=I,1,1)
DO 370 JB1,3
4H=I,1,1)
370 CONTINUE
F(I)=K(I,1,1)
THETA(I)=K(I,2,1)
CS(I)=K(I,3,1)
380 CONTINUE
CONVG=F(2)=F(1)=F(2)=F(1)/F(2)
PRINT 70,M,CONVG
IF (KLOOKP(LT,2)) G0=0 444
IF (ABS(CONVG)>LF=0.0005) G0=0 444
444 DO 410 IN1,N
F2(I)=F(I)
THETA2(I)=THETA(I)
CS2(I)=CS(I)
410 CONTINUE
DO 420 IN1,N
F(I)=F(I)+(F(1)*F(I))
THETA(I)=THETA(I)+THETA2(I)
T(I)=T(I)+THETA(I)
CS(I)=CS(I)+CS2(I)
420 CONTINUE

EVALUATION OF THE STREAM FUNCTION
C
DO 460 IN1,N
FF(I)=F(I)/(1.0=F(1)+FNLAT(I=1)*DELT))
460 CONTINUE

ORIGINAL PAGE IS OF POOR QUALITY
FUNCTION TMOVAR(ITVMX, JTVMX, VALX, VALY, X, Y, FN)
DIMENSION Y(JTVMX), X(ITVMX), FN(ITVMX, JTVMX)
DO 10 J=1, JTVMX
   IF (Y(J)·VALY) JH, 2N, 6N
10 CONTINUE
20 DO 30 I=1, ITVMX
   IF (X(I)·VALX) 30, 4N, 7N
30 CONTINUE
40 FNHNN=FNH(N, 1)
TMOVAR=FMNN
RETURN
50 IP=1
   I=1
   TMOVAR=FMN(I, J)=(FN(IP, J)+FN(I, J))*(VALX·X(I))/X(IP, X(I))
RETURN
60 J=1
   DO 70 I=1, ITVMX
      IF (X(I)·VALX) 70, 8N, 9N
70 CONTINUE
80 FNHNN=FNH(N, 1)
GO TO 1ND
90 IP=1
   I=1
   FNHNN=FNH(1, J)=(FN(IP, J)+FN(1, J))*(VALX·X(1))/X(IP, X(1))
100 J=1
   DO 110 I=1, ITVMX
      IF (X(I)·VALX) 110, 12N, 13N
110 CONTINUE
120 FNHNN=FN(1, J)
GO TO 1ND
130 IP=1
   I=1
   FNHNN=FN(1, J)=(FN(IP, J)+FN(1, J))*(VALX·X(1))/X(IP, X(1))
140 CONTINUE
TMOVAR=FNHNN*FNHNN*(Y(J)-VALY)/(Y(J)-Y(J))
RETURN
END
SUBROUTINE MILIEH (NP, NOPT, T, Z, S, HMO), GAMMA)

C NOPT*1 LOOK UP Props BASED ON P AND H
C NOPT*2 LOOK UP Props BASED ON P AND S
C NOPT*3 LOOK UP Props BASED ON h AND S

DIMENSION FLPR(33,2H), HZ(33,2H), TT(33,2H), ZT(33,2H), GAME(33,2H)
1, ENTROC(33,2H), FLPO(66B), ZPO(66B), TTO(66B), ZTO(66B), GAMEO(66B)
2, ENTROO(66B), FLPE(2H), MLTH(33), ENTHOV(2H,33), FLPV(2H,33)

EQUIVALENCE (FLPR,FLP), (HZ,MT), (TT,TT), (ZT,FT), (GAME,GAME

1. C (ENTROC,ENTRO)

DATA PO,CPOR,MO,SH,G,H,FLM,CP/111,13,3,4,15H,117,560,23,60,37,2
1,5,3,2,3NP55,33,755,23d666

IDEAL#1

Z#1

GAMMA=#1

IF (IZ,Eq,33) GO TO 3W
DO 14 K=1,20
LL=35*(K-1)
DO 10 L=1,35
FLPO(LL+L)*FLPZ(1)
IF (K,KT,1)) GO TO 1W
H2O(LL+L)*H2U(L)
1W CONTINUE

IZ=33
JZ=2W
DL 2H I=1,JZ
MLXW(I)=H2(1,1)
NO 2H J=1,JZ
ENTR0W(I,J)=ENTR0D(1,JZ=J+1)
FLPV(J,1)=FLP(1,JZ=J+1)
2W CONTINUE

3A IF (NOPT,Eq,5) Go TO 2W
PL=ALOG1((P/215)+1)*
IF (NOPT=1) G1,4W,21H
3A IF (H,LT,1W,10) Go TO 1W
5A CONTINUE

C ALL UNI3 (MH,LMZT,PL,FLPZ,IZ,JZ,T,T,S,MLTH,M,HMO), GAMMA,GAME)
IF (NOPT,Eq,6) Go TO 8W
IC ANS((SSS)=IP,2**4=1PLACE),LT,1,1) Go TO 9A
C IF (S=SS) A$ W,W,7W
6W PLLPL
PL=(PL*PL)/(2.6)
GO TO 5A
7W PLLPL
PL=(PL+PLL)/2.6
GO TO 5A
8A SSS

A% CONTINUE
9A IF (IZ,22222+3W) 1W,13,13,13
10A AMMUP(33,2+33,35+4CT)
GO TO 33A
11W IF (H,LT,2W) GO TO 13W
12A Sa(CP0H,ALG1N(M,HG)=AL0G1(P/P1))=ALF+SN
GO TO 33A
13A IFP=1
IDEAL#1
GO TO 33A

ORIGINAL PAGE IS OF POOR QUALITY
SUBROUTINE DINT1 (XX, XT, Y1, YT1, ZZ, ZT, ML, NL)

C DOUBLE INTERPOLATION SUBROUTINE.
C
DIMENSION ZT(NL), XT(NL, NL), YT1(NL, NL)
IF (ZZ=ZT(1)) 20, 10, 10
10 IF (XX=XT(1,1)) 20, 10, 30
20 Y1 = ZT ZT(1)
RETURN
20 GO TO 10
30 DO 10 J = 1, NL
   40 IF (ZZ=ZT(J)) 50, 60, 40
40 CONTINUE
50 GO TO 10
60 IF (XX>XT(J, J)) 70, 80, 70
70 CONTINUE
80 GO TO 10
90 Y1 = YT1(LM, J)
RETURN
100 IF (XX=XT(J, J)) 110, 120, 110
110 CONTINUE
120 GO TO 10
130 IF (XX>XT(JM, J)) 140, 150, 130
140 GO TO 10
150 IF (XX=XT(JM, J)) 160, 170, 150
160 CONTINUE
170 IF (XX>XT(JM, J)) 180, 190, 170
180 CONTINUE
190 IF (XX=XT(JM, J)) 200, 210, 190
200 CONTINUE
210 IF (XX>XT(JM, J)) 220, 230, 210
220 CONTINUE
230 RETURN
C
END