COMPUTER PROGRAM FOR DETERMINING MASS PROPERTIES OF A RIGID STRUCTURE

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SUMMARY

A computer program (ND0702) has been developed for the rapid computation of the mass properties of complex structural systems. The program uses rigid body analyses and permits differences in structural material throughout the total system. It is based on the premise that complex systems can be adequately described by a combination of basic elemental shapes. The following thirteen widely used structural shapes were selected for inclusion in the program:

1. Discrete mass
2. Cylinder
3. Truncated cone
4. Torus
5. Beam (arbitrary cross section)
6. Circular rod (arbitrary cross section)
7. Spherical segment
8. Sphere
9. Hemisphere
10. Parallelepiped
11. Swept Trapezoidal Panel
12. Symmetric Trapezoidal Panels
13. Curved Rectangular Panel

Simple geometric data describing size and location of each element and the respective material density or weight of each element are the only required input data. From this minimum input, the program yields system weight, center of gravity, moments of inertia and products of inertia with respect to mutually perpendicular axes through the system center of gravity. The program also yields mass properties of the individual shapes relative to component axes.

Permanent configuration records and the use of iterative calculations to investigate design systems or to determine optimums contribute to the cost-effectiveness of the program's use.

INTRODUCTION

Determining the mass properties of any rigid structure is a problem that at times becomes complex, but one which can easily be dealt with utilizing computer solutions.
For rigid structures the solution of the mass properties requires transformation to an axis parallel to the system axis and becomes laborious almost to the point of being impractical.

Any complex structure must be broken down into elements in order to exact a solution. The approach selected for the program presented in this paper was to automate the input to the point where an element's shape, geometry, density or weight, and three grid points are the only requirements. This approach was influenced by the simplicity of computing the direction cosines (Euler angle relationship) from the given three grid points. The program as outlined in this paper performs essentially the same process as calculations "by hand" and is extremely useful for rigid structures skewed in space. This program also provides improved accuracy, time savings, and complete permanent records for a mass properties analysis. (This TMX is a verified expansion of the LaRC working paper "Computer Program for Determining Mass Properties of a Composite Body", by Phillip J. Klich and John L. Gilbert dated Oct. 22, 1968.)

SYMBOLS

$I$ moments and products of inertia
$r$ displacement vector for differential mass
$T$ kinetic energy
$v$ linear velocity
$x', y', z'$ rectangular coordinate component displacement vectors
$x, y, z$ rectangular coordinate system displacement vectors
$\omega$ angular velocity
$'x'x$ cosine of angle between $x'$ and $x$ axes
$'y'x$ cosine of angle between $y'$ and $x$ axes
$'z'x$ cosine of angle between $z'$ and $x$ axes
$'x'y$ cosine of angle between $x'$ and $y$ axes
\( l_{y'y} \) \hspace{1cm} \text{cosine of angle between \( y' \) and \( y \) axes}

\( l_{z'y} \) \hspace{1cm} \text{cosine of angle between \( z' \) and \( y \) axes}

\( l_{x'z} \) \hspace{1cm} \text{cosine of angle between \( x' \) and \( z \) axes}

\( l_{y'z} \) \hspace{1cm} \text{cosine of angle between \( y' \) and \( z \) axes}

\( l_{z'z} \) \hspace{1cm} \text{cosine of angle between \( z' \) and \( z \) axes}

\textbf{Subscripts}

\( co \) \hspace{1cm} \text{system coordinates to component center of mass}

\( xxco \) \hspace{1cm} \text{refer to component axis to which moments of inertia are calculated}

\( yyco \) \hspace{1cm} \text{refer to component axis to which moments of inertia are calculated}

\( zzco \) \hspace{1cm} \text{refer to system axis to which the moments of inertia are rotated parallel to the system coordinates}

\textbf{Superscripts}

\( prime \) \hspace{1cm} \text{denotes component coordinate system}

\textbf{INERTIA EQUATIONS}

The mass properties of shapes such as a cylinder, sphere, etc., are easily calculated and therefore were selected as the basic component shapes for handling a system such as a spacecraft structure. Since the component shape mass properties are measured with respect to their respective center of mass, these properties have to be transferred to the system center of mass. The transformation can be made in two steps: first, the component properties are transferred to a system parallel to the system axis, and then transferred by the usual parallel axis theorem. The rotational transformation is derived by using the principle of kinetic energy. An introductory derivation of the moment of inertia, and product of inertia expressions are derived first and then transformation from the component to system coordinates is presented.

\textbf{Derivation of Inertia Equations}

Using the expressions of kinetic energy of a rigid body, the equations of moments of inertia and products of inertia are derived. Consider a component spinning with an angular velocity \( \omega \) as shown next.
The angular kinetic energy can be written as

\[ T = \frac{1}{2} \int \mathbf{V} \cdot \mathbf{V} \, dm \]

where the velocity is expressed as

\[ \mathbf{V} = \mathbf{\omega} \times \mathbf{r} \]

Substituting in the kinetic energy expression gives

\[ T = \frac{1}{2} \int (\mathbf{\omega} \times \mathbf{r}) \cdot (\mathbf{\omega} \times \mathbf{r}) \, dm \]

where

\[ \mathbf{\omega} = i\omega_x' + j\omega_y' + k\omega_z' \]

and

\[ \mathbf{r} = ix' + iy' + iz' \]

Taking the cross product

\[
\begin{vmatrix}
1 & j & k \\
\omega_x' & \omega_y' & \omega_z' \\
x' & y' & z'
\end{vmatrix} = i(\omega_y'z' - \omega_z'y') + j(\omega_z'x' - \omega_x'z') + k(\omega_x'y' - \omega_y'x')
\]
and performing the dot product results in

\[(\omega'z')^2 - 2(\omega'y')(\omega'y') + (\omega'y')^2 + (\omega'y')^2 - 2(\omega'y')(\omega'y') + (\omega'y')^2 \]

\[+ (\omega'x')^2 + (\omega'y')^2 - 2(\omega'y')(\omega'y') + (\omega'y')^2 \]

Which upon substituting into kinetic energy equation gives

\[T = \frac{1}{2} \int \left[ \omega_x^2(y'^2 + z'^2) + \omega_y^2(x'^2 + z'^2) + \omega_z^2(x'^2 + y'^2) \right. \]

\[\left. - 2\omega'\omega'x'y'z' - 2\omega'\omega'y'z' - 2\omega'\omega'x'y' \right] \, dm \]

This represents the rotational kinetic energy of one component of a system. Recognizing the definitions of moments and products of inertia and selecting the component coordinate system as the principal axes, we can write

\[T_{\text{comp}} = \frac{1}{2} \left( I_x\omega_x^2 + I_y\omega_y^2 + I_z\omega_z^2 \right) \]

This particular selection of coordinates does not affect the final answers because kinetic energy is constant with regards to the coordinate orientation; however, it does simplify the input data and also reduces computer time.

Rotating From Component Coordinates to System Coordinates

At this point we have the kinetic energy about the component axis system. In the computer program it is at this level that mass properties are computed for the preselected shapes such as the cylinder, sphere, etc.

It is necessary to resolve the component mass properties to an axis system parallel to the system coordinates. Once parallel to this system axis then we can translate to the system center of gravity by the usual parallel axis theorem. The derivation is similar to that presented in reference 5.

In deriving the rotational transformation from the component to the system coordinates the expression for kinetic energy is again used. Given a body rotating with an angular velocity \(\omega\), we know that its kinetic energy is invariant with regard to the coordinate orientation.
The kinetic energy in the system coordinates is

\[ T_{\text{sys}} = \frac{1}{2} \int (\vec{\omega} \times \vec{R}) \cdot (\vec{\omega} \times \vec{R}) \, dm \]

Defining the products of inertia as

\[ I_{xz} = \int yz \, dm, \quad I_{yz} = \int yz \, dm, \quad I_{xy} = \int xy \, dm \]

the energy equation becomes

\[ T_{\text{sys}} = \frac{1}{2} \left[ I_{xx} \dot{x}^2 + I_{yy} \dot{y}^2 + I_{zz} \dot{z}^2 + 2I_{xz} \dot{x} \dot{z} + 2I_{yz} \dot{y} \dot{z} + 2I_{xy} \dot{x} \dot{y} \right] \]

The products of inertia will not necessarily be zero in the system coordinates due to the coordinate rotations and, therefore, the products must be included.
It is necessary to write the angular velocity of the component system coordinates in terms of the system coordinates. For an arbitrary vector it can be written

\[ \vec{\omega} = \vec{\omega}'_{\text{comp}} = \vec{\omega}_{\text{system}} \]

\[ = i'\omega_x' + j'\omega_y' + k'\omega_z' = i\omega_x + j\omega_y + k\omega_z \]

Performing the dot product gives

\[ \omega_x' = i' \cdot i\omega + i' \cdot j\omega + i' \cdot k\omega \]

\[ \omega_y' = j' \cdot i\omega + j' \cdot j\omega + j' \cdot k\omega \]

\[ \omega_z' = k' \cdot i\omega + k' \cdot j\omega + k' \cdot k\omega \]

Recognizing the direction cosines results in

\[
\begin{pmatrix}
\omega_x' \\
\omega_y' \\
\omega_z'
\end{pmatrix} =
\begin{pmatrix}
x'x & x'y & x'z \\
y'x & y'y & y'z \\
z'x & z'y & z'z
\end{pmatrix}
\begin{pmatrix}
\omega_x \\
\omega_y \\
\omega_z
\end{pmatrix}
\]

Writing the component kinetic energy in matrix algebra

\[ T_{\text{comp}} = \frac{1}{2} \{\omega'\}^T [I'] [\omega'] = \frac{1}{2} \{[\text{DC}] \{\omega}\}^T [I'] \{[\text{DC}] \{\omega}\} \]

where \([\text{DC}]\) is the direction cosine matrix shown above. Equating the system and component energy results in

\[ T_{\text{system}} = T_{\text{comp}} \]

\[ \frac{1}{2} \{\omega\}^T [I] \{\omega\} = \frac{1}{2} \{\omega\}^T [\text{DC}]^T [I'] [\text{DC}] \{\omega\} \]

and the resulting system inertial matrix is found to be
Expanding the matrix equation we get

\[
\begin{bmatrix}
I_{xxco} & I_{xyco} & I_{xzco} \\
I_{yxco} & I_{yyco} & I_{yzco} \\
I_{zxco} & I_{zyco} & I_{zzco}
\end{bmatrix} = 
\begin{bmatrix}
l'_{xx} & l'_{xy} & l'_{xz} \\
l'_{yx} & l'_{yy} & l'_{yz} \\
l'_{zx} & l'_{zy} & l'_{zz}
\end{bmatrix}
\begin{bmatrix}
l_x & 0 & 0 \\
0 & l_y & 0 \\
0 & 0 & l_z
\end{bmatrix}
\begin{bmatrix}
l'_{xx} & l'_{xy} & l'_{xz} \\
l'_{yx} & l'_{yy} & l'_{yz} \\
l'_{zx} & l'_{zy} & l'_{zz}
\end{bmatrix}
\]

The direction cosines are determined by the method presented in appendix A.

Transferring by Parallel Axis Theorem

Now that the moments of inertia are in a system parallel to the system coordinates, we now translate by the parallel axis theorem

\[
I_{xx} = I_{xxco} + m[(Y_{co} - \bar{Y})^2 + (Z_{co} - \bar{Z})^2]
\]

\[
I_{yy} = I_{yyco} + m[(X_{co} - \bar{X})^2 + (Z_{co} - \bar{Z})^2]
\]

\[
I_{zz} = I_{zzco} + m[(X_{co} - \bar{X})^2 + (Y_{co} - \bar{Y})^2]
\]

\[
I_{xy} = I_{xyco} + m[(X_{co} - \bar{X})(Y_{co} - \bar{Y})]
\]

\[
I_{xz} = I_{xzco} + m[(X_{co} - \bar{X})(Z_{co} - \bar{Z})]
\]

\[
I_{yz} = I_{yzco} + m[(Y_{co} - \bar{Y})(Z_{co} - \bar{Z})]
\]
Where $I_{XXCO}$, $I_{YYCO}$, and $I_{ZZCO}$ are the component moments of inertia rotated parallel to the system coordinates, and $X$, $Y$, $Z$ are the system center of mass coordinates and $X_{CO}$, $Y_{CO}$, $Z_{CO}$ are the coordinates of the component center of mass.

**COMPUTER PROGRAM**

This computer program is written in Fortran IV computer language. All names and descriptions are assigned in the first part of the program. Thirteen sections have been written using 13 common shapes usually found in spacecraft. The program is directed by the input data which singles out the section or shape factor desired to be used through the "go to" statement. The operation of the program is illustrated in figure 1 with a computer flow diagram.

The input data for each item is listed on two data cards. The basic input for each item will vary depending on the shape factor used. Each shape factor with the necessary data is discussed in the input data instructions.

After the data cards are supplied to the program the following operations are performed. The component mass properties are first printed with the moments of inertia about the component axis rotated parallel to the system coordinates. These mass properties are transferred to the system center of gravity and the following are computed: System weight, inertias about the system center of gravity, inertias about the origin, center of gravity of the system and products of inertias of the system. Based on this generated information, inertias about the system principal axes and their location is subsequently computed.

A listing of the computer program is found in appendix B.

**Selection of Coordinate Points**

The selection of points "i" and "j" determines the length of the member as well as the first three direction cosines. Point "k" is required to calculate the other six direction cosines. Shown in figure 2 are the two coordinate systems used in this program. Point "i" locates the system coordinates ($X_i$, $Y_i$, $Z_i$) for the origin of the component axes and point "j" determines the direction of the "x" axis of the component coordinates.

In order to determine the directions of the "y" and "z" axes, point "k" is required. This point can be anywhere in the x-y plane. If it is omitted, then the program automatically positions the "y" axis parallel to the X-Y plane. For a body of revolution point "k" is not required. The following figures (2(a) and 2(b)) describe points i, j, and k.

The main deck is referred to as the computer program without the necessary data. It is always necessary to have a 789 card following the main deck and a 789 card, then a 6789 card following the data. The 6789 card separates one program from another. A description of these cards and their formats follows figure 2.
LOCATION OF POINTS "i" AND 'j"
Figure 2(a)

LOCATION OF POINTS "k"
Figure 2(b)
1st Data Card

Item No.  I3 format
           Columns 1 through 3
Description  2A9 format
               Columns 4 through 21
Shape        I2 format
               Columns 22 and 23
Weight or density  F9.4 format
                     Columns 24 through 32
A             F8.3 format
               Columns 33 through 40
B             F8.3 format
               Columns 41 through 48
C             F8.3 format
               Columns 49 through 56
D             F8.3 format
               Columns 57 through 64
F             F8.3 format
               Columns 65 through 72

It is to be noted that the input data variables A, B, C, D, and F can be geometric dimensions, cross-sectional areas, area moments of inertia, and mass moment of inertia.

2nd Data Card

XI             F8.3 format
               Columns 1 through 8
YI             F8.3 format
               Columns 9 through 16
ZI             F8.3 format
               Columns 17 through 24
XJ             F8.3 format
               Columns 25 through 32
YJ             F8.3 format
               Columns 33 through 40
ZJ             F8.3 format
               Columns 41 through 48
XK             F8.3 format
               Columns 49 through 56
YK             F8.3 format
               Columns 57 through 64
ZK             F8.3 format
               Columns 65 through 72
Component shapes in the existing programs are:

Shape 1  Discrete Mass
Shape 2  Cylinder
Shape 3  Truncated Cone
Shape 4  Torus
Shape 5  Beam (Arbitrary Cross Section)
Shape 6  Circular Road (Arbitrary Cross Section)
Shape 7  Spherical Segment
Shape 8  Sphere
Shape 9  Hemisphere
Shape 10  Parallelepiped
Shape 11  Swept Trapezoidal Panel
Shape 12  Symmetric Swept Trapezoidal Panels
Shape 13  Curved Rectangular Panel

This program can easily be modified to include additional shapes.

Frequently where precise weights of components are known it is more convenient to input this weight rather than an average density which would have to be calculated. The value of .4 has been chosen as the limiting value for inputing \( \rho \) as density (lb/cu in). A value greater than .4 is used as total weight (lbs). In some cases referred to as "thin wall" the total weight of the component must be input. The degree of flexibility for inputing various shapes can be determined from the "shape data input instructions".

The program can be used to determine the mass properties of a component with hollows or voids. The component is treated as a standard shape with the hollows or voids included as solid material. The hollows or voids are then input as standard shapes having negative values for weight or density. The program will compute the actual weight, center of gravity, and moments and products of inertia of the component.

In the following programs for a variety of shapes the inertias about the \( x \), \( y \) and \( z \) component axes are represented respectively by computer symbols \( I_{XXCG} \), \( I_{YYCG} \) and \( I_{ZZCG} \). For some of the shapes expressions for the moments of inertia are given; however, for the more complex shapes they are omitted but can be found by referring to the computer program in Appendix B.

Note that component axes have been located so the \( x \) axis is a principal axis and the \( y \) and \( z \) axes either coincide with, or are parallel to principal axes. Any other than the aforementioned \( x \), \( y \), and \( z \) axes locations will incur an error in the main computer program with shape no. 11 being the sole exception. In this instance, the \( x \), \( y \), and \( z \) axes locations were chosen for convenience in accordance with the "shape data input instruction" and the \( x \) axis is rotated to a principal axis by the program.
SHAPE DATA INPUT INSTRUCTION

SHAPE 1
DISCRETE MASS

Center of gravity

\[ \begin{align*}
\text{IXXCG} &= A \\
\text{IYXCG} &= B \\
\text{IZZCG} &= C
\end{align*} \]

Input Data: Item, Description, Shape, RHO (wt.)
\[ \begin{align*}
A(\text{Ixx}), B(\text{Iyy}), C(\text{Izz}) \\
\text{XI, Yi, Zi, Xj, YJ, ZJ, Xk, Yk, ZK}
\end{align*} \]

OPTIONS AVAILABLE
No. 1 Input data just as indicated above.
No. 2 If negligible, inertias \( A(\text{Ixx}), B(\text{Iyy}) \) and \( C(\text{Izz}) \) may be omitted, in which case points \( j \) and \( k \) are not required.

NOTES: Never locate point \( j \) at the system origin and if input, \( A(\text{Ixx}), B(\text{Iyy}) \) and \( C(\text{Izz}) \) must be inertias about principal axes in slug-ft.

SHAPE 2
CYLINDER

\[ \begin{align*}
\text{Identical segments}
\end{align*} \]

Input Data: Item, Description, Shape, RHO (density)
\[ \begin{align*}
A, B, C \text{ (option No. 2 only)} \\
\text{XI, Yi, Zi, Xj, YJ, ZJ}
\end{align*} \]

OPTIONS AVAILABLE
No. 1 Input data just as indicated above.
No. 2 Cylinder may be segmented requiring a \( C \) value be input.
No. 3 Total weight may be input for RHO in both previous options but only if it is input more than .4 pounds.
No. 4 Input \( A \) and \( B \) equal and total weight for RHO and program treats shape as a thin-wall cylinder.

NOTES: Point \( k \) is not required with any option and density as such must never be input more than .4 pounds per cu. in.

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SHAPE DATA INPUT INSTRUCTION

SHAPE 3
TRUNCATED CONE

Input Data: Item, Description, Shape, RHO (density), A, B, C, D, XI, YI, ZI, XJ, YJ, ZJ

OPTIONS AVAILABLE
No.1 Input data just as indicated above
No.2 A total weight may be input for RHO but only if it is more than .4 pounds.

NOTES: The values of A minus C and or B minus D must never equal zero.
Point i is always at the cones larger end and point k is not required.
Density as such must never be input greater than .4 lbs./cu.in.

SHAPE 3 (SPECIAL CASE)
THIN-WALL TRUNCATED CONE

Input Data: Item, Description, Shape
RHO (Density), A, B, C, D, F
XI, YI, ZI, XJ, YJ, ZJ

NOTES: In this case program assumes all mass is concentrated midway between inner and outer surfaces.
Point i is always at the cone's larger end and point x is not required.
SHAPE DATA INPUT INSTRUCTION

SHAPE 4
TORUS

\[ \begin{align*}
J_{xxCG} &= Y_{II} - Y_{I2} \\
J_{yyCG} &= X_{II} - X_{I2} \\
J_{zzCG} &= J_{yyCG}
\end{align*} \]

Input Data: Item, Description, Shape, RHO (density), A, D, XI, YI, ZI, XJ, YJ, ZJ

NOTE: Point "k" is not required.

SHAPE 5
BEAM
(ARBITRARY CROSS SECTION)

\[ \begin{align*}
J_{xxCG} &= \rho (\text{Length})(B + C) \\
J_{yyCG} &= \rho ((B)(\text{Length}) + 0.0133 (A)(\text{Length})^3) \\
J_{zzCG} &= \rho ((C)(\text{Length}) + 0.0133 (A)(\text{Length})^3)
\end{align*} \]

Input Data: Item, Description, Shape, RHO (density), A (area), B(I_{yy}), C(I_{zz}) (Area moment of inertia in inches^4), XI, YI, ZI, XJ, YJ, ZJ, XK, YK, ZK

NOTES: If beam is a body of revolution about the x (centroidal) axis, point k is not required. I_{yy} and I_{zz} are area moments of inertia about principal axes of the beam cross section taken in a plane normal to x axis.

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SHAPE DATA INPUT INSTRUCTION

SHAPE 6
CIRCULAR ROD
(ARBITRARY CROSS SECTION)

NOTES:
Point "k" is not required and weight instead of density is the required input for RHO.
The solution for this shape is approximate in that it may be as much as one % less than correct if rod dimensions t and w are as much as 25% of dimension A. (Error incurred tends to increase as this % increases)

SHAPE 7
SOLID SEGMENT

OPTIONS AVAILABLE
No.1 Input data just as indicated above
No.2 Total weight may be input for RHO if more than .4 lb.

NOTES:
Point "k" is not required. When computing as a solid spherical segment, B dim. will become equal the distance between points "i" and "j". Density as such must never be input greater than .4 lbs./cu. in.
SHAPE DATA INPUT INSTRUCTION

SHAPE 8
SPHERE

\[ \text{IxxCG} = \text{XII} - \text{XI}^2 \]
\[ \text{IyyCG} = \text{IXXCG} \]
\[ \text{IzzCG} = \text{IXXCG} \]

Input Data: Item, Description, Shape, RHO (density), A, B
XI, YI, ZI

NOTE: Points "j" and "k" are not required. If B = A the program selects thin-wall equations; therefore, weight instead of density should be input for RHO.

SHAPE 9
HEMISPHERE

\[ \text{IxxCG} = \text{XII} - \text{XI}^2 \]
\[ \text{IyyCG} = \text{XI}^3 - \text{XI}^4 \]
\[ \text{IzzCG} = \text{IyyCG} \]

Input Data: Item, Description, Shape, RHO (density), A
XI, YI, ZI, XJ, YJ, ZJ

NOTE: Point "k" is not required. If A = Lgth (i,j) the program selects thin-wall equations; therefore, weight instead of density should be input for RHO.
SHAPE DATA INPUT INSTRUCTION

SHAPE 10
PARALLELEPIPED

Point k
(Can lie anywhere in the xy plane)

Input Data:
Item, Description, Shape, RHO (density), A, B, C, D, F
XI, YI, ZI, XI, YJ, ZJ, XK, YK, ZK

OPTIONS AVAILABLE
No. 1 Input data just as indicated above.
No. 2 A total weight may be input for RHO but only if it is more than .4 pounds.
No. 3 Input D equal A and the program selects thin-wall equations therefore total weight must be input for RHO.

NOTE: Density as such must never be input greater than .4 lbs./cu.in.
SHAPE DATA INPUT INSTRUCTION

SHAPE 11
SWEPT TRAPEZOIDAL PANEL
(THICK WALL)

Input Data:
Item, Description, Shape, RHO (density), A, B, C, F
XI, YI, ZI, XJ, YJ, ZJ, XK, YK, ZK
(Assign 1 to C according to sweep of panel)

Formulas designed to facilitate input data determination:

\[ a = \frac{C(B)}{B-F} \]
\[ b = \frac{C(B+2F)}{3(B+F)} \]
\[ c = \frac{\text{Length}(B+2F)}{3(B+F)} \]

OPTIONS AVAILABLE
No. 1 Input data just as indicated above.
No. 2 Total weight may be input for RHO if more than .4 lbs.

SHAPE 11 (SPECIAL CASE)
UNSWEPT TRAPEZOIDAL PANEL
(THICK WALL)

Input Data:
Item, Description, Shape, RHO (density), A, B, F
XI, YI, ZI, XJ, YJ, ZJ, XK, YK, ZK

OPTIONS AVAILABLE
No. 1 Input data just as indicated above.
No. 2 Total weight may be input for RHO if more than .4 lbs.

Notes:
Dimension F must never be more than 98% of B.
Density as such must never be input greater than .4 lbs./cu.in.

Point k (Can lie anywhere on the y axis except at point i)

Notes:
Dimension F must never be more than 98% of B.
Density as such must never be input greater than .4 lbs./cu.in.
SHAPE DATA INPUT INSTRUCTION

SHAPE 12
SYMMETRIC SWEPT TRAPEZIODAL PANELS (THICK WALL)

Point k
(Can lie anywhere
in the xy plane)

Input Data:
Item, Description, Shape, RHO (density), A, B, C, D, F
XI, YI, ZI, XJ, YJ, ZJ, XK, YK, ZK
(Assign 2 to C according to sweep of panel)

OPTIONS AVAILABLE
No. 1 Input data just as indicated above.
No. 2 Total weight may be input for RHO
if it is more than 0.4 lb.

NOTES:
Dimension F must be no more than 0.8% of B.
Density as such must never be input greater
than 0.4 lb./cu.in.

SHAPE 13
CURVED RECTANGULAR PANEL
(THIN WALL)

Point k
(Can lie anywhere
in the xy plane)

Input Data:
Item, Description, Shape
RHO (density), A, B, C
XI, YI, ZI, XJ, YJ, ZJ, XK, YK, ZK

NOTE: Plane xy is one about which symmetrical exists.
EXAMPLE PROBLEMS

Two example problems are presented in order to show required input data. These problems were selected due to the simple calculations involved and thus could be checked by hand calculations.

Example Problem 1

Problem 1 was taken from reference 3. It is a cylinder skewed in the y-z plane. The moments of inertia about the system origins are given in this reference and are used for comparison in this paper. It should be noted that "g" (acceleration of gravity) was taken to be 32.0 ft/sec² in this reference rather than 32.2 ft/sec² (386 in./sec²). Depending on the value of "g" selected by the program user, the term "cons" has to be changed accordingly.

The axis through the center of the component must always be the x-axis for the computer program. Therefore, the y₃ axis of this problem corresponds with the x-axis of the computer program; the y₁ axis corresponds with the y-axis and the y₂ axis corresponds with the z-axis.

Find moments of inertia I'₁₁, I'₂₂, I'₃₃ which corresponds to Iₓₓₒ, Iᵧᵧₒ, Iｚｚₒ in our coordinate system.

Given: W = 1 slug = 32 lb
R = 24 inches
L = 36 inches
θ = \cos^{-1} \frac{3}{5} = 53° 8'

Solution: (a) Direction cosines for the transformation are

\[
\begin{bmatrix}
1 & 0 & 0 \\
0 & 2 & \frac{4}{5} \\
0 & \frac{4}{5} & 3 \\
\end{bmatrix}
\]

And numerical values of the Iᵢⱼ are

\[
\begin{bmatrix}
3 & 0 & 0 \\
0 & 4 & 0 \\
0 & 0 & 2 \\
\end{bmatrix}
\]

Component Moment of Inertia
The results of problem 1 as computed in reference 3 are now given. Hence, from the transformation equations

System Moments of Inertia

\[ I'_{11} = a_{11}I_{11} + a_{12}^2I_{22} + a_{13}a_{13}I_{33} + 2a_{12}a_{13}I_{23} + 2a_{11}a_{13}I_{13} + 2a_{11}a_{12}I_{12} \]

\[ I'_{xx} = (1)^2 4 + 0 + 0 + 0 + 0 = 4 = \text{slug ft}^2 \quad (I'_{xx} = I'_{11}) \]

\[ I'_{22} = a_{21}a_{21}I_{11} + a_{22}^2I_{22} + a_{23}a_{23}I_{33} + 2a_{22}a_{23}I_{23} + 2a_{21}a_{23}I_{13} + 2a_{21}a_{22}I_{12} \]

\[ I'_{yy} = 0 + \left( \frac{3}{5} \right)^2 4 + \left( \frac{4}{5} \right)^2 2 + 0 + 0 + 0 = \frac{68}{25} = 2.72 \text{ slug ft}^2 \quad (I'_{yy} = I'_{22}) \]

\[ I'_{zz} = a_{31}a_{31}I_{11} + a_{32}a_{32}I_{22} + a_{33}a_{33}I_{33} + 2a_{32}a_{33}I_{23} + 2a_{31}a_{33}I_{13} + 2a_{31}a_{32}I_{12} \]

\[ I'_{zz} = 0 + \left( \frac{4}{5} \right)^2 4 + \left( \frac{3}{5} \right)^2 2 + 0 + 0 + 0 = \frac{52}{25} = 2.08 \text{ slug ft}^2 \]

This problem is now computed using Computer Program ND0702. Shown below are the coordinates of points 1, 1', and k for problem 1.

Point "1"
\[ \begin{align*}
  x_1 &= 0 \\
  y_1 &= 0 \\
  z_1 &= 0
\end{align*} \]

Point "1′" (\[ \begin{align*}
  x_1' &= 0 \\
  y_1' &= 0 \\
  z_1' &= 0
\end{align*} \]

Point "q" (\[ \begin{align*}
  y_q &= 30 \sin 55^\circ \quad 30 \text{ ft} = 20.3 \text{ in} \\
  z_q &= 30 \cos 55^\circ \quad 30 \text{ ft} = 21.6 \text{ in}
\end{align*} \]

Point "k" Unit since not required for cylinder

Reference is now made to the necessary data cards to compute this problem.
<table>
<thead>
<tr>
<th>ITEM NO.</th>
<th>SHAPE</th>
<th>DESCRIPTION</th>
<th>WT. OR DENSITY</th>
<th>A</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td></td>
<td>CYLINDER-TEST</td>
<td>32.0</td>
<td>24.0</td>
</tr>
</tbody>
</table>

**FORTRAN STATEMENT**

1ST DATA CARD

2ND DATA CARD

<table>
<thead>
<tr>
<th>YJ</th>
<th>ZJ</th>
</tr>
</thead>
<tbody>
<tr>
<td>28.8</td>
<td>21.8</td>
</tr>
</tbody>
</table>

**FORTRAN STATEMENT**

2ND DATA CARD
We now have the results of problem 1 using Computer Program
(Note that the gravitational constant used here was 32.166... rather than 32.0 used in ref.)

<table>
<thead>
<tr>
<th>ITEM</th>
<th>DESCRIPTION</th>
<th>SHAPE</th>
<th>POS</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>CYLINDER, TUBE</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

COMPONENT DATA LISTED BELOW

<table>
<thead>
<tr>
<th>ITEM</th>
<th>DESCRIPTION</th>
<th>IXX</th>
<th>IXY</th>
<th>IYZ</th>
<th>IYY</th>
<th>IZX</th>
<th>IZY</th>
<th>IZZ</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>CYLINDER, TUBE</td>
<td>32.0000</td>
<td>1.7484E3</td>
<td>1.7484E3</td>
<td>4.608E3</td>
<td>1.7484E3</td>
<td>1.7484E3</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

SYSTEM DATA LISTED BELOW (INTEGRAL, INERTIA ELEMENTS FT SQUARED, C.G., MOMENT, SECOND MOMENT, ELEMENT FT SQUARED)

<table>
<thead>
<tr>
<th>ITEM</th>
<th>IXX</th>
<th>IXY</th>
<th>IYY</th>
<th>IZX</th>
<th>IZY</th>
<th>IZZ</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.07E8</td>
<td>3.07E8</td>
<td>4.608E3</td>
<td>3.07E8</td>
<td>3.07E8</td>
<td>9.0000</td>
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<tr>
<td>2</td>
<td>1.74E7</td>
<td>1.74E7</td>
<td>0.0000</td>
<td>1.74E7</td>
<td>1.74E7</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

INERTIAS (EIGENVALUES): ABOUT SYSTEM PRINCIPAL AXES - 1ST AXIS DIRECTION COEFFICIENTS (EIGENVECTORS) RELATING THE PRINCIPLE AXES TO THE X, Y, AND Z SYSTEM AXES IN THAT ORDER

EIGENVALUE(1) = 1.74E6
EIGENVECTOR(1)

EIGENVALUE(2) = 1.74E6
EIGENVECTOR(2)

EIGENVALUE(3) = 9.000
EIGENVECTOR(3)

Example Problem 2

Problem 2 was taken from reference 2. In this space structure, the weight has been 'lumped' or concentrated at the joints. In the program being presented, this is not required but is used here only for illustration. To compare moment of inertia, the numbers given in reference 2 should be converted to slug-ft².

\[
\begin{align*}
IXX &= IYY - \frac{22038.464}{4.608E3} = 4.783 \text{ slug-ft}^2 \\
IZZ &= 9000.00 - \frac{4.0608E3}{4.0608E3} = 1.953 \text{ slug-ft}^2
\end{align*}
\]
Example Problem 2

INPUT
MEMBER AREAS - 0.01 in²
WEIGHT AT EACH JOINT - 150 lb
MODULUS OF ELASTICITY - 10¹⁴ psi
SEE THE FIRST TWO PAGES OF THE SAMPLE
PROBLEM FOR DETAILS OF THE INPUT

<table>
<thead>
<tr>
<th>JOINT</th>
<th>X1</th>
<th>X2</th>
<th>X3</th>
<th>X4</th>
<th>X5</th>
<th>X6</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
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<td>0</td>
<td>100</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>0</td>
<td>100</td>
<td>0</td>
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<td>0</td>
</tr>
<tr>
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<td>0</td>
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<td>0</td>
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<tr>
<td>6</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>100</td>
</tr>
</tbody>
</table>

The results of problem 2 as computed in reference 6 are now given.

<table>
<thead>
<tr>
<th>TOTAL WEIGHT</th>
<th>105,000</th>
<th>105,000</th>
<th>105,000</th>
<th>9</th>
<th>8</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>CENTER OF GRAVITY</td>
<td>X = 1500, Y = 5000, Z = 11,730</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CENTER OF INERTIA ABOUT CENTER OF GRAVITY</td>
<td>X = 1215 in⁴, Y = 1215 in⁴, Z = 9000 in⁴</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

NOTE: Inertias are given in pound inches squared and must be converted to compare with results program yields.

This problem is now computed using Computer Program ND0702
<table>
<thead>
<tr>
<th>ITEM</th>
<th>DESCRIPTION</th>
<th>VT</th>
<th>Y1</th>
<th>Y2</th>
<th>Y3</th>
<th>Y4</th>
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<th>Y6</th>
<th>Y7</th>
<th>Y8</th>
<th>Y9</th>
<th>Y10</th>
<th>Y11</th>
<th>Y12</th>
<th>Y13</th>
<th>Y14</th>
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<tbody>
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<tr>
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<tr>
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<tr>
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<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

**SYSTEM DATA LISTED BELOW (TYPES, INERTIAL VALUES PT EQUATED, C.G., WIND, EROTIC ++ INERTIALS PT EQUATED)**

<table>
<thead>
<tr>
<th>918</th>
<th>190</th>
<th>11,475</th>
<th>12</th>
<th>5,989</th>
<th>5,970</th>
</tr>
</thead>
<tbody>
<tr>
<td>122</td>
<td>130</td>
<td>11,975</td>
<td>6,723</td>
<td>1,799</td>
<td>1,799</td>
</tr>
</tbody>
</table>

Note: No principal axes definition is included since they will lie on one of symmetry.
EXAMPLE PROBLEM 3
A COMPOSITE STRUCTURE
(\frac{1}{8} SCALE)

NOTES:
Points i and j shown on the structure are those assumed when data was input to the inertia program. Also, some salient dimensions are included on the components to assist in relating component data to the computer input. Except for items No. 1 and No. 7 density is input for RHO.

The computer output relative to example problem 3 follows and includes, the data input, the computer calculated component data, the summed data and lastly, the inertias about the system principal axes and the individual axis locations.
### INPUT DATA LISTED BELOW

<table>
<thead>
<tr>
<th>Item</th>
<th>Description</th>
<th>Shape</th>
<th>Rho</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Antenna</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Dielectric</td>
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<td></td>
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<td></td>
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</tr>
<tr>
<td>3</td>
<td>Rod Cyl</td>
<td></td>
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<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>4</td>
<td>Pins</td>
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</tr>
<tr>
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</tr>
<tr>
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<tr>
<td>7</td>
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</tbody>
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<table>
<thead>
<tr>
<th>Component Data Listed Below</th>
</tr>
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<table>
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<tr>
<th>Item</th>
<th>Description</th>
<th>m</th>
<th>IYECO</th>
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<td>5</td>
<td>Flared Adapter</td>
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<td>.05261</td>
<td>.05261</td>
<td>14.71107</td>
<td>0.00000</td>
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<td>-0.0130</td>
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<td>-0.00470</td>
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<tr>
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<td>.03779</td>
<td>.00012</td>
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**SYSTEM DATA LISTED BELOW** (INLV=SLM, INERTIAS=SLUGS FT SQUARED, C.G.=INS SECOND MOMENT=SLUGS FT SQUARED)

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<th>IYECO</th>
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<td>7</td>
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<td>0.00000</td>
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</table>

**INERTIAS (EIGENVALUES) ABOUT SYSTEM PRINCIPAL AXES WITH AXIS DIRECTION COSINES (EIGENVECTORS) RELATING THE PRINCIPAL AXES TO THE X, Y, AND Z SYSTEM AXES IN THAT SEQUENCE**

Eigenvalue 1 = .20760E+00

Eigenvector 1:

- .999916E+00 .100000E+01 0.

Eigenvalue 2 = .23740E+01

Eigenvector 2:

- .100000E+00 .999916E+00 0.

Eigenvalue 3 = .12400E+01

Eigenvector 3:

- .999916E+00 .100000E+01 0.

Eigenvalue 4 = .10000E+01

Eigenvector 4:

- 0. 0. 1.00000E+01

30
### CONVERSION TO INTERNATIONAL SYSTEM OF UNITS

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<th>TO</th>
<th>MULTIPLY BY</th>
</tr>
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<td>METERS</td>
<td>0.025 400</td>
</tr>
<tr>
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<td>METERS$^2$</td>
<td>0.000 645 160</td>
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<tr>
<td>FEET</td>
<td>METERS</td>
<td>0.304 800</td>
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<td>METERS$^2$</td>
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<td>POUNDS</td>
<td>KILOGRAMS</td>
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<td>POUNDS/FEET$^3$</td>
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<td>16.018 463</td>
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<td>1.355 8179</td>
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CONCLUSIONS

A computer program for determining mass properties of a rigid structure is presented. The structure is broken down into preselected shapes with known properties, and input data are supplied to completely describe each shape. For complicated structures skewed in space, this program offers a practical solution to a tedious and time-consuming task. It is also practical to use this program for problems that involve repetitious or lengthy calculations.
REFERENCES

APPENDIX A

DIRECTION COSINE DERIVATION

The direction cosines are used to transform the properties from the component coordinate system \( x, y, z \) to system coordinates \( \Xi, \Sigma, \Zeta \). Three points \( (i, j, k) \) shown in the figure below, define the vectors \( V_1 \) and \( V_2 \).

Vector \( V_1 \) is arbitrarily selected to be coincident with the "x" axis. A unit vector on this axis can be written

\[
\overrightarrow{1_x} = \frac{\overrightarrow{V_1}}{V_1}
\]

Taking the vector cross product of \( \overrightarrow{1_x} \) with \( \overrightarrow{V_2} \) and dividing by the resulting magnitude gives a unit vector on the z axis

\[
\overrightarrow{1_z} = \frac{\overrightarrow{1_x} \times \overrightarrow{V_2}}{V_z}
\]

Similarly, a unit vector on the "y" axis is found from

\[
\overrightarrow{1_y} = \overrightarrow{1_z} \times \overrightarrow{1_x}
\]

The direction cosines are the \( \Xi, \Sigma, \Zeta \), components of the unit vectors on the x,y,z axes.
The direction cosine for the x axis is written as

\[ \begin{align*}
LX &= (XJ - XI) / LGTH \\
MX &= (YJ - YI) / LGTH \\
NX &= (ZJ - ZI) / LGTH
\end{align*} \]

where the length is

\[ LGTH = \sqrt{((XJ - XI)^2 + (YJ - YI)^2 + (ZJ - ZI)^2)} \]

The vector \( \mathbf{V}_2 \) can be written

\[ \begin{align*}
T1 &= XK - XI \\
T2 &= YK - YI \\
T3 &= ZK - ZI
\end{align*} \]

A vector on the z axis is found by taking the vector cross product of \( \mathbf{i}_x \) and \( \mathbf{V}_2 \)

\[ \begin{align*}
LZ &= MX*NX - NX*MX \\
MZ &= NX*T1 - T3*MX \\
NZ &= T2*NX - NX*T1
\end{align*} \]

The length is

\[ T4 = \sqrt{(LZ^2 + MZ^2 + NZ^2)} \]

Normalizing to get a unit vector

\[ \begin{align*}
LZ &= LZ/T4; MZ = MZ/T4; NZ = NZ/T4
\end{align*} \]

The unit vector on the y axis has a magnitude of one and is determined by the vector cross product \( \mathbf{i}_z \) and \( \mathbf{i}_x \)

\[ \begin{align*}
LY &= NZ*MX - MX*NZ \\
MY &= NZ*NX - NX*LY \\
NY &= MX*NZ - NZ*MX
\end{align*} \]

Writing the nine terms in matrix form, we get

\[ \begin{bmatrix}
LX & MX & NX \\
LY & MY & NY \\
LZ & MZ & NZ
\end{bmatrix} \]
APPENDIX B

PROGRAM NAME: INPUT, OUTPUT, TAPE=INPUT, TAPE6=OUTPUT

C LATEST DIRCOS IN MAIN DECK AND 13 SHAPES 12/20/72

DIMENSION ITEM(200), SHAPE(200), RH0(200), A(200), R(200), C(200),
C 1 X1(200), Y1(200), Z1(200), XJ(200), YJ(200), Z(J(2)
C 200), XK(200), YK(200), ZK(200), IXXC(200), IYYC(200), IZZC(200), IXYC
C 3(200), IYYC(200), IXXC(200), IXYC(200), YC(200), YC(200), YL(200), YL
C 4(200), ZL(200), IXXC(200), IYYC(200), IZZC(200), IXYC(200), IYYC(200),
C IXCN(200), IXXCN(200), IYYCN(200), IZZCN(200), IXYCN(200), IYYCN(200),
C IXCN(200)
C 5), IYYCN(200), DES(3, 200), W(200), D(200), P(200), ARH(3, 3), F(3), CRH(3)
C REAL IXX, IYY, IZZ, IXNC, IYYC, IZZC, IXCN, IYYCN, IZZCN,
C IYYCN, IXCN, IXCN, IXCN, IYYCN, IYYCN, IYYCN, IYYCN, IYYCN, IYYCN, IXXCN,
C IYYCN, IXYC, IXXCN, IYYCN, IYYCN, IYYCN, IYYCN, IYYCN, IYYCN, IYYCN,
C IXXCN, IYYCN, IYYCN, IYYCN, IYYCN, IYYCN, IYYCN, IYYCN, IYYCN, IYYCN,
C INTEGER SHAPE
C 1 = 0

1010 I = I + 1
105 FORMAT(15),
READ(5, 101) ITEM(I), DES(I, 1), DES(I, 2), SHAPE(I), RH0(I), A(I), R(I),
C 1 C(I), R(I), F(I), X1(I), Y1(I), Z1(I), XJ(I), YJ(I), Z(J(I), YJ(I),
C 2 YK(I), ZK(I)
101 FORMAT (13, 2A9, 17, F3, 4, 5E9, 2/4, 3)
1009 INDX=I-1

WRITE(4, 271)
271 FORMAT (19, 20, INPUT DATA LISTIN RELAY) //
WRITE(4, 103) ITEM(I), DES(I, 1), DES(I, 2), SHAPE(I), RH0(I), A(I), R(I),
C 3 C(I), R(I), F(I), X1(I), Y1(I), Z1(I), XJ(I), YJ(I), Z(J(I), YJ(I),
C 4 YK(I), ZK(I), I = I + 1, INDX
103 FORMAT (12X, ITEM), DESCRITION SHAPE
C 1 1
C 2 = 3
C 3 4
C 4 5
WRITE(6, 261)
261 FORMAT (19, 20, ITEM, DESCRITION SHAPE) //
WRITE(6, 250)
250 FORMAT (14, ITEM), DESCRITION SHAPE
C 1 IXYC
C 2 IXCN, IYYC, IXXC, IYYCN, IXXCN, IYYCN, IYYCN
C 3 IXCN, IYYCN, IYYCN, IXXCN, IYYCN
C 4 IXCN, IYYCN, IYYCN, IYYCN, IYYCN

36
1 \[ \begin{align*}
  & \text{PI} = 3.141593 \\
  & \text{CON}\text{\#} = 4632 \\
  & \text{DOS} = \text{JX} & \text{AX} \\
  & \text{CQNS} = 4632 \\
  & \text{SFL} = \text{JX} & \text{AX} \\
  & \text{STL} = \text{JX} & \text{AX} \\
  & \text{ITM} = \text{JX} & \text{AX} \\
  & \text{12} * (\text{LGTH} + \text{C(I)}) \\
  & \text{?} = \text{JX} & \text{AX} \\
  & \text{G} = \text{JX} & \text{AX} \\
  \end{align*} \]

**DISCRETE MASS**

1 \[ \begin{align*}
  & \text{POINT J MUST BE SELECTED ANYWHERE ON THE X AXIS FOR DIR. COSINES} \\
  & \text{I} = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13 \text{, ITM}
  \end{align*} \]

2 \[ \begin{align*}
  & \text{IF}(\text{A(I)}, \text{E} = \text{A(I)}) \text{GO TO 33} \\
  & \text{IF}(\text{R(I)}, \text{L} = \text{R(I)}) \text{GO TO 33} \\
  & \text{IF}(\text{C(I)}, \text{L} = \text{C(I)}) \text{GO TO 33} \\
  & \text{IF}(\text{VOLCYL} \neq \text{A(I)} + \text{R(I)} + \text{C(I)} \text{GO TO 33}) \\
  & \text{IF}(\text{A(I)}, \text{R} = \text{A(I)}) \text{GO TO 33} \\
  & \text{IF}(\text{R(I)}, \text{A} = \text{R(I)}) \text{GO TO 33} \\
  & \text{IF}(\text{C(I)}, \text{A} = \text{C(I)}) \text{GO TO 33} \\
  & \text{IF}(\text{VOLCYL} = \text{A(I)} + \text{R(I)} + \text{C(I)} \text{GO TO 33}) \\
  & \text{IF}(\text{A(I)}, \text{R} = \text{C(I)} \text{GO TO 33}) \\
  & \text{IF}(\text{R(I)}, \text{A} = \text{C(I)} \text{GO TO 33}) \\
  & \text{IF}(\text{C(I)}, \text{A} = \text{C(I)} \text{GO TO 33}) \\
  \end{align*} \]
33 *(I)*RHO(I)
XL(I)=LGMH/2
IXYC(I)=RHO(I)*A(I)**2
IYYCG(I)=5*RHO(I)*A(I)**2+RHO(I)*LGMH**3/C(I)**3)/
2*(LGMH-C(I))*12.0
IZZCG(I)=IYYCG(I)
GO TO 60

TRUNCATED CONE *SHAPE 3* (REF->R. HULL)

3 IF(B(I),EQ,A(I)) GO TO 30
VOL =
$1.0472*LGTH*(A(I)**2+A(I)*C(I)+C(I)**2)*B(I)**2*B(I)*D(I)**2)
/R(I) = RHO(I) / VOL
IF(RHO(I),LT,.4) *(I)=RHO(I)
RHO(I) = *(I) / VOL
XL(I) = (LGTH**3)*(A(I)**2+2*A(I)*C(I)+C(I)**2+R(I)**2+B(I)**2)
/R(I) = 3*(I)*RHO(I) *2.0 *26.18/VOL
< I = .053.141593*RHO(I)*(C(I)**5+A(I)**5)*LGTH/(C(I)*A(I)) =
$0/(I)**5 = B(I)**5)*LGTH/(D(I)**5)
< I = .053.141593*RHO(I)*LGTH**3/(A(I)**2+B(I)**2)/12.0 = .5*(A(I) =
/G(I) = 2)*RHO(I)*XL(I)**2
IXYC(I)=2.0*(X(I)
I= C(I)=X(I)*2=I
IYYCG(I)=IYYCG(I)
GO TO 60

TRUNCATED CONE (THIN WALL) *SHAPE 3*

30 VOL=(I)*SORT((A(I)-C(I)**2+LGMH**2)**3.141593*(A(I)+C(I))
/ R(I) = VOL/RHO(I)
XL(I)=LGMH/3.0*(P+C(I)+A(I))/((C(I)+A(I))
IYYCG(I)=RHO(I)**4*(A(I)**2+C(I)**2)*RHO(I)*LGMH**2/18.0*(1.0+A(I)
1)**C(I)**(A(I)+C(I)**2))
IZZCG(I)=IYYCG(I)
IXYC(I)=(RHO(I)**2*A(I)**2+2*A(I)**2*C(I)**2))
GO TO 60

TUBS *SHAPE 4*

A(I)=A(I)+LGMH
IF (B(I),LT,0.0) D(I)=0.
VOL=2.*PI**2*LGMH**2*A(I)
VOL=2.*PI**2*B(I)**2*A(I)
ACvol=VOL/VOL
/1*/ = RHO(I) * ACvol
V=1* RHO(I) * VOL(1)

38
**BEAM (ARBITRARY CROSS SECTION)**

*SHAPE 5*

5

\[ X_{L(I)} = L_{GH}/2 \]

\[ I_{XXCG(I)} = (\rho_0(I) * L_{GH} * (B(I) + C(I))) \]

\[ I_{YYCG(I)} = (\rho_0(I) * (B(I) * L_{GH} + 0.833 * A(I) * L_{GH}^2)) \]

\[ I_{ZZCG(I)} = (\rho_0(I) * (C(I) * L_{GH} + 0.833 * A(I) * L_{GH}^2)) \]

\[ V_{OL} = A(I) * L_{GH} \]

\[ \rho_0(I) * V_{OL} \]

GO TO 60

**CIRCULAR ROD (ARBITRARY CROSS SECTION)**

*SHAPE 6*

6

\[ \rho_0(I) = \rho_0(I) \]

\[ R_{SCG} = A(I) * \pi * C(I) + B(I) / (A(I) * C(I)) \]

\[ I_{XXCG(I)} = R_{SCG} * \pi * (I) \]

\[ I_{YYCG(I)} = 0.5 * R_{SCG} * \pi * (I) \]

\[ I_{ZZCG(I)} = I_{YYCG(I)} \]

GO TO 60

**SPHERICAL SEGMENT**

*SHAPE 7*

7

\[ F(I) = C(I) \]

\[ P(I) = R(I) \]

\[ V_{OL} = L_{GH} \]

\[ G = L_{GH} - R(I) \]

\[ V_{OL} = V_{OL} / V_{OL} \]

\[ B = B(I) \]

\[ \rho_0(I) = \rho_0(I) \]

\[ x = 0.75 * (2 * C(I) + L_{GH} * P(I) + C(I) * L_{GH}^2) \]

\[ x = 0.75 * (P(I) * F(I) + R(I) * F(I) + C(I) * L_{GH}) \]

\[ x = x^2 + V_{OL} \]

\[ x = x^2 + V_{OL} \]

\[ x = x^2 + V_{OL} \]

\[ x = x^2 + V_{OL} \]

39

*ORIGINAL PAGE IS OF POOR QUALITY*
XI2*{(2.9G +x2/(3.*F(I)+G)))*(F(I)**2*75*F(I)+G +15*G
1*21)
IXXCG(I)=X(I)*X2
TEMP1=.052363RHO(I)*(15.*C(I)**4*C(I)+10.*C(I)**4*C(I)**3+C(I)
1**5)+.2094e*RHO(I)*(5.*C(I)**2*C(I)**3+C(I)**5)
TEMP2=.052363RHO(I)*(15.*C(I)**4*C(I)+10.*C(I)**2*C(I)**3+C(I)
1**5)+.2094e*RHO(I)*(5.*C(I)**2*C(I)**3+C(I)**5)
TEMP3=.052363RHO(I)*(15.*F(I)**4*C(I)**F(I)**3*C(I)
1**5)+.2094e*RHO(I)*(5.*F(I)**2*C(I)**3+C(I)**5)
ACTEMP=((TEMP1=TEMP2)=((TEMP3=TEMP4))=(TEMP3*LYT)**2)
IXYCG(I)=ACTEMP
IZZCG(I)=IXYCG(I)
GO TO 40

SPHEREF #SHAPE 8#

9 LGTH=4(I)
XJ(I)=X(I)+LGTH
YJ(I)=Y(I)
ZJ(I)=Z(I)
.:I(I) = X(I)**2.
IF(III,EO,A(I))GO TO 34
VOL1=-.188791*A(I)**3
VOL2=-.188791*A(I)**3
ACVOL=VOL1*VOL2
X1=(R+0(I)-ACVOL)
Y2=(R+0(I)-VOL1)
Y2=(R+0(I)-VOL2)
X1=4.*X1**1*LGTH**2
Y1=4.*Y1**2*R(I)**2
IXXCG(I)=X(I)*X1=X(I)
IXYCG(I)=IXXCG(I)
IZZCG(I)=IXXCG(I)
GO TO AC

SPHEREF (TWIN =ALL) *SHAPE 8#

34 =I(I)**40(I)
IXXCG(I)=I(I)**2*H(I)**2*H(I)**2
IXYCG(I)=IXXCG(I)
IZZCG(I)=IXXCG(I)
XI(I)=0
GO TO 34

IF VISPHERE *SHAPE 9#

9 IF (LGTH**2,AT(I))GO TO 35
T(I,LT,0)A(I)=0.
VOL1=2.00003*LGTH**3
VOL2=2.00003*LGTH**3

40
ACVOL=VOL1=VOL2
* (I) = RHO(I) * ACVOL
XBAR1=.375*LGTH
XBAR2=.375*A(I)
XL(I) = (XBAR1*VOL1+XBAR2*VOL2)/ACVOL
XM1=RHO(I)*VOL1
XM2=RHO(I)*VOL2
XII=(.4*x1*1*LGTH**2)
XIZ=.4*x1*2*a(I)**2
IXXCG(I) = (XII=XI2)
IXI3=(.26*x1*1*LGTH**2)
IXI4=(.26*x1*2*a(I)**2)
IYVCG(I)=XI3*XI4
IZZCG(I)=IYVCG(I)
GO TO 60

C HEMISPHERE (THIN WALL) #SHAPE 9#

35 * (I) = RHO(I)
XL(I) = LGTH/2,
IXXCG(I) = .666*RHO(I)*LGTH**2
IYVCG(I) = .416*RHO(I)*LGTH**2
IZZCG(I) = IYVCG(I)
GO TO 60

C PARALLEL PIPED #SHAPE 1#

10 IF((I),EQ,4(I))GO TO 36
IF((I),LT,0,I0(I)=0.
VOL1=LGTH*F(I)*A(I)
VOL2=C(I)*F(I)*D(I)
ACVOL=VOL1=VOL2
(C(I)=H(W(I))*ACVOL
X(I)=H(W(I))*ACVOL
XL(I) = LGTH/2.
X1=VOL1*P=0(I)
XI2=VOL2*P=0(I)
XI1=.6333*x1*(R(I)**2+4(A(I)**2))
XI2=.6333*x2*(F(I)**2+4(A(I)**2))
IXXCG(I) = (XI1=+1)
IXI3=.6333*x1*(1*LGTH**2+4(A(I)**2))
IXI4=.6333*x2*(1*LGTH**2+4(A(I)**2))
IYVCG(I) = (XI3=+1)
IZZCG(I) = (XI5=+1)
GO TO 56

ORIGINAL PAGE IS OF POOR QUALITY
PARALLELEPIPED (THIN WALL) *SHAPE 10*

36 XL(I) = LGTH/2,
   *I(I) = RHO(I),
   TEMP1 = (LGTH*B(I)*A(I)),
   TEMP2 = (LGTH*B(I)*A(I) + LGTH*A(I) + RHO(I)/6,)
   IXXCG(I) = 0.83333*A(I) + RHO(I)*((B(I)**2 + A(I)**2) + (RHO(I)/6)) * (TEMP1
   (B(I)+A(I))/TEMP2),
   IYYCG(I) = 0.83333*A(I) + RHO(I)*((LGTH**2 + A(I)**2) + (RHO(I)/6)) * (TEMP1
   (B(I)+A(I))/TEMP2),
   IZZCG(I) = 0.83333*A(I) + RHO(I)*((LGTH**2 + B(I)**2) + (RHO(I)/6)) * (TEMP1
   (B(I)+A(I))/TEMP2)
GO TO 60

SQUINT TRAPEZIODAL PANEL (THICK WALL) *SHAPE 11* (REF = R, HULL)

11 (*I(I) = A(I)*RHO(I)*((LGTH*(H(I) + F(I)))/2,)
   IF (RHO(I) < 0.04) *I(I) = RHO(I)
   RHO(I) = (*I(I)) * ((B(I) + F(I))/2) + LGTH*A(I))
   XL(I) = LGTH*(B(I) + 2.0*F(I))/((B(I) + F(I)) + LGTH*A(I))
   X = F(I)*LGTH/(B(I) + F(I))
   TNTA := C(I)/LGTH
   FETA := (F(I)/2 + B(I)/2 + C(I))/LGTH
   AFTA := (C(I) - F(I))/2 + B(I)/2)/LGTH
   THRR := LGTH + X
   RC := (FETA + AFTA)/2*(THRR*XL(I))
   XPA = AFTA + FETA
   IXXCG(I) = 4*I(I) + (XPA**.5)**3*(THRR**2 + X + 2)/24.0 + XPA**.5*I(I)*
   2*(THRR**4 + XT**4)/48.0 + (THRR**2 + XT**2) + R**2*RC*(FETA + AFTA)*
   1*(THRR**3 + XT**3)/3.0 + (FETA + AFTA)**2*(THRR**4 + XT**4)/16.0
   IYYCG(I) = RHO(I)**(A(I)**3)**3 + A(I)**(THRR**4 + XT**4)/48.0 + XPA**.5*I(I)**3
   IZCG(I) = A(I)**3 + RHO(I)**(XPA**.5)**3 + (THRR**2 + XT**2) + R**2*(Z(I) + XPA**.5)
   IXXCG(I) = (THRR**2 + XT**2)/3.0 + (Z(I) + AFTA + FETA)**2*(THRR**4 - XT**4)/16.0)
   IF (RHO(I) <= 0.04) GO TO 60
   ARES = 1/2 + ARES(TNTA = XL(I))
   = ARES*CG**.5*(THRR**2 + X + 2.0 + V(I) = Y(I) = Z(I) + Z(I)**2 + (Z(I) = Z(I) + X(I) + 2.0))
   CG**.5 = ARES
   PR = THRR*LGTH
   SY = ARES/THRR
   PRXY = 5*F(I)*(2 + SY)/F(I)*X(I) + LGTH + PR**2/12.0
   IYY = 2 + SY/(L - R(I))**2)/24.0 + PRXY

42
IXYCGP = ABS(IYYCG(I)) * RHO(I)  = (I) * CGK * XL(I) * C(I)/ABS(C(I))

PAIRC = 0.5 * ABS(C(I))/C(I)

IF(ABS(IIXCG(I)) = IYVCG(I)) ➔.001 GO TO 20

PAIRC = ATAN((2.0 * IIXCG(I))/IYVCG(I)) / IIXCG(I)

IF(IIXCG(I) * GT.1) GO TO 1

PAIRC = 5.0 * ABS(2.0 * PAIRC) = 3.1416

1 PAIRC = ABS(PAIRC)

20  TPIXX = IIXCG(I) * COS(PAIRC) * 2 + IYYCG(I) * SIN(PAIRC) * 2

12.0 * IIXCG(I) * SIN(PAIRC) * COS(PAIRC)

IYVCG(I) = IIXCG(I) * SIN(PAIRC) * 2 + IYYCG(I) * COS(PAIRC) * 2

2.0 * IIXCG(I) * SIN(PAIRC) * COS(PAIRC)

IIXCG(I) = TPIXX

21.0 = 3.0

SHTLT = ABS(TNAP(PAIRC)) * XL(I)

XJ(I) = (XJ(I) * (LGT - XL(I)) + XJ(I) * XL(I))/LGT

YJ(I) = (YJ(I) * (LGT - XL(I)) + YJ(I) * XL(I))/LGT

ZJ(I) = (ZJ(I) * (LGT - XL(I)) + ZJ(I) * XL(I))/LGT

XI(I) = XI(I) * SHFTI + CGK

YI(I) = YI(I) * SHFTI + CGK

ZI(I) = ZI(I) * SHFTI + CGK

LGT = 3.0

XI(I) = XI(I) * XII(I) * XI(I) * XI(I) + XI(I) * XI(I) * XI(I) * XI(I)

GO TO 19

C TRAPEZOIDAL PANELS (THICK = ALL) SHAPE 12* (REF = R, HULL)

12.0 = (LGT = 2.0) * N(I)/2.0

VOL = (LGT * R(I) * F(I)) * A(I)

X(I) = X(I) / VOL

RHY(I) = (X(I) - R(I)) / ((R(I) + F(I)) * A(I)) * LGT * A(I)

X(I) = X(I) * (R(I) + F(I)) / (3.0 * R(I) + F(I))

IYVCG(I) = (LGT * 4.0) * (F(I) * 2.0 + F(I) * R(I) + R(I) * 2.0 + 2.0) * A(I) * RHO(I)

2.0 * F(I) * R(I) * 2.0 + 2.0 * A(I) * 2.0 + (D(I) * XL(I)) * 2.0

XT = F(I) * LGT / (R(I) + F(I))

IF(R(I) = F(I)) XT = R(I) * LGT

TIRA = CT * LGT

FEFTA = (F(I) / 2.0 + C(I))/LGT

AFTA = C(I) + F(I) / (1.0 + 2.0 + 3.0) / LGT

IF(R(I) = F(I)) LGT = 0.0

IKM = SHTC - R(I) - 4.0 * FEFTA * 3.0 * FEFTA * 3.0 / 3.0

KK = (R(I) + 1.0) * (R(I) + 1.0) / (LGT * XT) + TIRA * (LGT * XT + XL(I))

IKM = SHTC

IKM = 0.0

IF(R(I) = F(I)) LGT = 1.0 + F(I) / R(I) + XT

LXT = LGT + 0.0

IYVCG(I) = ATT + (X(I) * LGT) / 4.0 / 4.0 / 2.0.0 * KIT * ((XT * LGT) * 2.0 + XT * 2.0)

ITV = ATT

12.0 = ATT

IYVCG(I) = IYVCG(I) + IYVCG(I)

XL(I) = LGT + 0.0

LGT = 3.0 + X(I)

GO TO 19

43
CURVED THIN WALL PANEL #SHAPE 13#

15 \( n(I) = 2, \ast \text{LGM} \ast a(I) \ast b(I) \ast c(I) \ast RHO(I) \)

IF(RHO(I), GT, 0, 4) \( n(I) = RHO(I) \)

RHO(I) = \( n(I)/(2, \ast \text{LGM} \ast a(I) \ast b(I) \ast c(I)) \)

KX = 2, \ast c(I) \ast a(I) \ast b(I) \ast \text{LGM} \ast RHO(I)

KY = \text{LGM} * 2/12,

XL(I) = \text{LGM} \ast 2,

IXC(I) = \text{LGM}

\( S = RHO(I) \ast a(I) \ast s(R(I) \ast (2, \ast c(I) \ast (2, \ast \sin(c(I)) \ast 2)/c(I)) \)

IYY(C(I)) = KI

\( S = (A(I) \ast 2 \ast c(I) \ast \sin(c(I)) \ast \cos(c(I)))/(2, \ast c(I)) \ast KI \)

IZZC(I) = KI

\( S = (A(I) \ast 2 \ast c(I) \ast \sin(c(I)) \ast \cos(c(I)) = 2, \ast \sin(c(I)) \ast 2/ \)

1C(I))/(2, \ast c(I) \ast KI)

GO TO 60

BEGIN DIRCOS

60 IF(ABS(X(I)), NE, 0, ), OR, (ABS(Y(I)), NE, 0, ), OR, (ABS(Z(I)), NE, 0, )

IF((ABS(X(I)), NE, 0, ), OR, (ABS(Y(I)), NE, 0, ), OR, (ABS(Z(I)), NE, 0, )) GO TO 90

\( V(I) = \text{LGM} + 1, I \)

90 \( LX = (X(I) \ast V(I))/\text{LGM} \)

\( LX = (Y(I) \ast V(I))/\text{LGM} \)

\( LX = (Z(I) \ast V(I))/\text{LGM} \)

\( T(I) = \text{LGM} \ast V(I) + 1, I \)

T2 = Y(I) \ast V(I)

T3 = Z(I) \ast V(I)

LZ = X(T) \ast T2 \ast N

4Z = X(T) \ast T3 \ast N

Z = T2 \ast LX \ast T1 \ast N

T4 = SQRT(LZ \ast 2 \ast 4Z \ast 2 \ast 4N \ast 2)

LZ = TX \ast T4 \ast 4Z \ast TX \ast 4N \ast TX

L4 = 4Z \ast X = 4X \ast X

4Z = X \ast LX \ast N \ast LX

4X = X \ast LX \ast LX \ast N

END DIRCOS
C ROTATE COMPONENT MOMENT OF INERTIA TO SYSTEM COORDINATES

IXXCO(I) = (1XXCG(I) * (LX)**2 + IYYCG(I) * (LY)**2 + IZZCG(I) * (LZ)**2) / CONS
IYYCO(I) = (1XXCG(I) * (MX)**2 + IYYCG(I) * (MY)**2 + IZZCG(I) * (MZ)**2) / CONS
IZZCO(I) = (1XXCG(I) * (NX)**2 + IYYCG(I) * (NY)**2 + IZZCG(I) * (NZ)**2) / CONS
IXYCO(I) = ((1XXCG(I) * (LX) * (MX) + IYYCG(I) * (LY) * (MY)) / CONS
IZZCO(I) = ((1XXCG(I) * (NX) * (MX) + IYYCG(I) * (NY) * (MY)) / CONS
IYYCO(I) = ((1XXCG(I) * (LX) * (NX) + IYYCG(I) * (LY) * (NY)) / CONS

C CALCULATE COMPONENT CENTER OF MASS COORDINATES AND WRITE OUT

XCG(I) = YI(I) * XL(I) / XL
YCG(I) = YI(I) * XL(I) / XL
ZCG(I) = ZI(I) * XL(I) / XL

WRITE(*,300) ITEM(I),DE(1,1),DE(2,1),DE(3,1),IXXCO(I),IYYCO(I),IZZCO(I),IXYCO(I),IZZCO(I),IYYCO(I)

300 FORMAT(15,5X,24G,6F11.5,4F9.5)

C CALCULATE SYSTEM WEIGHT, SECOND MOMENT AT ORIGIN AND C.G.

T = T(I)
IXXCO = IXXCO + (1) * (ZCG(I)**2 + YCG(I)**2)
IYYCO = IYYCO + (1) * (ZCG(I)**2 + XCG(I)**2)
IZZCO = IZZCO + (1) * (XCG(I)**2 + YCG(I)**2)

WRITE(*,500) ITEM(I),DE(I),DE(I),DE(I),DE(I),DE(I),DE(I)
500 FORMAT(15,5X,24G,6F11.5,4F9.5)

C COMPLETE SYSTEM C.G. COORDINATES

XBAR = XBAR + X(I)
YBAR = YBAR + Y(I)
ZBAR = ZBAR + Z(I)

WRITE(*,200) ITEM(1),C.G. SYSTEM DATA LISTED BELOW (*=lbs, INERTIAS=SLUGS FT SQ
1/2RAR= C.G. IN'S, SECOND MOMENT=SLUG FT SQUARED)///
200 FORMAT(6,10X,T,XCO,YCO,ZCO,XBAR,YBAR,ZBAR
199 FORMAT(11X,3F9.5)1X2=12X4H1XX014XW1Y4O14XH17Z01AXHXB14XK34HAR
1)4X=ZBAR/7*1RI,3)

ORIGINAL PAGE IS OF POOR QUALITY
TRANSFER MASS PROPERTIES TO SYSTEM C.G., SUM AND WRITE OUT

DO 70 IM1,IMAX
DELT X%XCG(I)=XBAR
DELT Y%YCG(I)=YBAR
DELT Z%ZCG(I)=ZBAR
IXX=IXX+IXXCO(I)++(I)*(DELY**2+DELZ**2)/CONS
IYY=IYY+IYCYCO(I)++(I)*(DELY**2+DELZ**2)/CONS
IZZ=IZZ+IZZCO(I)++(I)*(DELY**2+DELZ**2)/CONS
IXY=IXY+IXYCO(I)++(I)*DELY*DELY/CONS
IXZ=IXZ+IXZCO(I)++(I)*DELY*DELY/CONS

70 IVZ=IYZ+IVZCO(I)++(I)*DELY*DELY/CONS
WRITE(6,200)IXX,IYY,IZZ,IXY,IXZ,IZY
200 FORMAT(12X3H1X16X3H1Y15X3H1Z15X3H1X15X3H1Y15X3H1Z/6F18.5/)

COMPUTE INERTIAS (EIGENVALUES) ABOUT PRINCIPAL AXES AND EACH AXIS
DIRECTION COSINES (EIGENVECTORS) AND WRITE OUT

PRINT 340
340 FORMAT(1X,*INERTIAS (EIGENVALUES) ABOUT SYSTEM PRINCIPAL AXES WITH
1H AXIS DIRECTION COSINES (EIGENVECTORS) RELATING THE PRINCIPAL AXE
1S TO THE X, Y, AND Z SYSTEM AXES IN THAT ORDER//)
MAX=3

*BEGIN
ARR(1,1)=IXX
ARR(2,1)=IYY
ARR(3,1)=IZZ
ARR(1,2)=IYY
ARR(2,2)=IYY
ARR(3,2)=IZZ
CALL SYV7L(MAX,N,ARR,E,CRR,IERR)
IF(IERR .NE. 0) GO TO 332
N=356 J=1.3
PRINT 337,J,E(J)
337 FORMAT(1X,*EIGENVALF (*I*) = *612.3//)
PRINT 339
339 FORMAT(1X,*EIGENVECTORS (*I*)//)
PRINT 338,ARR(I,J),I=1,3
338 CONTINUE
GO TO 330
332 PRINT 333,IERF
333 FORMAT(1X,*ERRNO = IERF = *15)
334 STOP
END
A method is presented for determining the complete mass properties of a rigid structure.

Computer technology is utilized to exercise the greatest savings possible in solving for and printing out this information including properties relative to the total structure's principal axes.

**Key Words (Suggested by Author(s))**
- Mass Properties
- Computer
- Rigid Structure

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