Digital Computer Processing
of LANDSAT Data for North Alabama

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The rationale for the division into chapters in this report was to describe the processing of Landsat data from various viewpoints. Chapter I is a description of the Landsat system, and the origin, type, and handling of its generated data. Chapter II is a verbal description of the analysis procedures; Chapter III is a mathematical description of the analysis techniques. Chapter IV is a presentation of the results achieved for Landsat coverage of North Alabama, while Chapter V is documentation of the major computer programs used in the analysis.
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I. THE LANDSAT SYSTEM

1-1. INTRODUCTION

Landsat-1 (formerly ERTS-1) is an experimental satellite whose purpose is to investigate the feasibility of remote sensing from space as a practical approach to efficient management of the earth's resources. The principal disciplines involved are agriculture, forestry, geology, geography, hydrology, and oceanography.

For this purpose, the satellite acquires repetitive multispectral images of the earth's surface and transmits this raw data through ground stations to a data processing center at the NASA Goddard Space Flight Center, for conversion into black and white or color photographs and computer tapes to fulfill the varied requirements of investigators and user agencies. Thus, such a remote sensing vehicle, due to its ability to cover large areas and generate large amounts of data, becomes a major element in an earth survey system, such as that illustrated in Figure 1. Several elements of this system will be discussed in the present report.

1-2. DESCRIPTION OF THE LANDSAT SYSTEM

Landsat-1 was launched in July, 1972 and traverses a circular, sun synchronous, near-polar orbit at an altitude of 915 km. (570 miles). This orbit provides repetitive earth coverage under nearly constant observation conditions, i.e. solar times. The satellite circles the earth every 103 minutes, completing 14 orbits per day, and views the entire earth every 18 days. Orbit specifications require that the satellite ground trace repeat its earth coverage at the same local time every 18 day period within 37 km. (23 miles). A typical one-day ground coverage trace is shown in Figure 2 for the daylight portion of each orbital revolution.

The overall Landsat system is illustrated in Figure 3. The satellite carries a payload of imaging multispectral sensors, wideband video tape recorders, and the spaceborne portion of a Data Collection System. The video data is received at Fairbanks, Alaska, Goldstone, California, and the Network Test and Training Facility (NTTF) at Goddard Space Flight Center (GSFC). Video data stored on magnetic tapes is received by the NASA Data Processing Facility (NDPF). The NDPF then performs the video-to-film conversion and correction, producing black and white images from individual spectral bands and color composites from several spectral bands. The NDPF includes a storage and retrieval system for delivery of data products and services to the investigators and other data users.

The satellite system consists of the earth resources payload subsystem and the various support subsystems comprising the spacecraft vehicle. The configuration is shown in Figure 4. Control of observatory attitude to the local
Figure 1. Basic Elements of an Earth Survey Information System.
Figure 2. Landsat Ground Coverage.
Figure 3. Landsat Support System
Figure 4. Landsat Satellite Configuration
vertical and orbit velocity vectors within 0.7 degree of each axis is achieved by a three axis active Attitude Control Subsystem. An independent passive Attitude Measurement Sensor provides pitch and roll attitude data accurate to within 0.07 degree to aid in image location. Orbit adjustment capability is provided by a system of one-pound thrusters. Payload video data are transmitted to ground stations over two wideband S-Band data links. Electrical power is generated by two solar arrays, with storage provided by batteries for spacecraft eclipse periods.

The earth resources payloads are the return beam vidicon (RBV) camera system and the multispectral scanner (MSS). The return beam vidicon camera operates by shuttering three independent cameras simultaneously, each sensing a different spectral band in the range 0.48 to 0.83 micrometers. The viewed ground scene, 185 x 185 km. (115 x 115 miles) in size, is stored on the photosensitive surface of the camera tube and, after shuttering, the image is scanned by an electron beam to produce a video signal output.

The multispectral scanner is a scanning device which uses an oscillating mirror to scan over lines perpendicular to the spacecraft ground track as shown in Figure 5. The surface of the earth is imaged in four spectral bands through the same optical system, so that optical energy is sensed simultaneously in the four bands. The bands lie in the solar reflected spectral region, and their wavelength limits are as follows:

- Band 1: 0.5 to 0.6 micrometers
- Band 2: 0.6 to 0.7 micrometers
- Band 3: 0.7 to 0.8 micrometers
- Band 4: 0.8 to 1.1 micrometers

Bands 1 through 3 use photomultiplier tubes as detectors; Band 4 uses silicon photodiodes. The cross track ground coverage of 185 km. (115 miles) is obtained as the flat mirror oscillates ± 2.89 degrees at a rate of 13.62 Hz. As the image is thus swept across an array of optical fibers, light impinging on each glass fiber is conducted to an individual detector through an optical filter, unique to the appropriate spectral band. The detector outputs are sampled, digitized, and formatted into a continuous data stream of 15 megabits per second. The along-track scan is produced by the orbital velocity of the satellite, which causes an along-track motion of the subsatellite point of 6.47 km/sec. (4.0 miles/sec). The mirror oscillation frequency is such that the subsatellite point traverses 474 meters during the 73.42 millisecond scan and retrace cycle. During each mirror cycle, six lines of 79 meters width are scanned, and hence the line scanned by the first detector in a cycle is adjacent to the sixth line of the previous cycle. This scan pattern is shown in Figure 6. The instantaneous field of view of 79 meters (86.4 yards) square on the ground is delineated by the square input end of each optical fiber. The area sampled to form the reflectance data for each picture element (pixel) is 6241 square meters (1.54 acres). Along
Figure 5. MSS Scanning Arrangement

Figure 6. Ground Scan Pattern of the MSS.
a scan line the sampling rate is approximately 100,000 Hz, which results in overlap of the samples along the scan lines such that the effective area covered per sample is 1.1 acres.

1-3. ORIGIN AND TYPE OF DATA

The physical origin of the electromagnetic energy that is detected quantitatively in the Landsat sensors is reflection of sunlight from the target scenes on the earth's surface. The ability to classify the measurements according to their origin from various objects arises from variations in the reflection as a function of wavelength. The physiological equivalent of this process is the recognizing of an object by its color only.

The quantitative definition of spectral composition is often called the spectral 'signature', and this represents the distribution of intensity of radiation as a function of wavelength. Each category, species or material will in general have a unique distribution, and it is this distribution, or signature, that is used for identifying the species. Over the visible range of the spectrum, the spectral signature may be thought of as the color distribution of the species in question.

Spectral signatures may be illustrated by plots of radiance intensity versus wavelength, ideally as is shown in Figure 7. In order to apply data analysis algorithms, it is necessary to represent these curves in numerical form, but a numerical point by point plot is not used in order to avoid data proliferation. Rather, a set of wavebands is selected over which the variation of spectral radiance is sufficiently large to permit discrimination between curves. Each spectral signature can then be represented as a set of numerical values that provide measures of the predominant spectral components present. In many multispectral sensor systems, the chosen separation of the wavebands is determined by the spectral resolution of the detector arrays or interference filter.

For analysis purposes, it is convenient to consider the set of numerical values derived from a single signature spectrum as a vector, and to represent this vector in an orthogonal vector-space whose axes are identified with the spectral components of the signature. This form of representation is illustrated in Figure 8.

The vector space interpretation of spectral signatures is particularly convenient for automatic data analysis since the algorithms of pattern recognition and feature classification can be applied directly. Each signature may be considered as characteristic of a certain class, species, or category. The vector components of any signature may be considered as characteristic features that enable the associated class to be distinguished from other, perhaps related, classes. As long as known characteristic signatures are available for comparison, feature vector measurements derived from remote sensor data may be sorted or classified by automatic decision logic according to the species from which they originated.
Figure 7. Relative Spectral Radiance Signatures of Agriculture Scenes

Figure 8. Spectral Signatures Interpreted as Feature Vectors
The format of the data output from the multi-channel detector array is particularly convenient for computer data processing. The signal output of each detector is recorded on a magnetic tape, and the variation of the recorded signal along the length of the magnetic tape then bears a very close correlation to the physical content of the ground scene, and each channel recorded on the tape may be regarded as the record of the ground scene viewed in a different color band, as depicted in Figure 9.

![Diagram](image)

**Figure 9. Correlation of Tape Record of Detector Outputs with Physical Features of Ground Scene**

The four spectral channels employed in the Landsat multispectral scanner lie approximately in the green, red, near infrared, and infrared ranges. Figure 10 shows data from the four bands in a 500×500 pixel area including Huntsville, Alabama, acquired on November 4, 1972.

1-4. DATA HANDLING AND DATA PRODUCTS

The Landsat data in digital format is segmented into four computer compatible tapes (CCTs). Each tape contains identification and annotation records,
Figure 10. Landsat Data in Four Bands Covering Huntsville, Alabama.
followed by the data in eight-bit bytes. The annotation record contains information regarding the conditions of exposure, such as date and time, sun elevation, coordinates of the image center, locations of lines of latitude and longitude intersecting the image. Data values in the four channels from pairs of adjacent pixels are interleaved according to the following order of channel numbers:

1 1 2 2 3 3 4 4.

Thus, the first step in data handling is the identification of the data segment required, based on latitude and longitude, possibly reordering to place data values for each pixel in channel order, and conversion to integer or decimal type numbers.

It is then necessary to use some means of examining the data visually in terms of the density levels, that is, reconstruct the image of the ground scene. This is to verify that the scene of interest has indeed been selected, and to locate specific locations and land use types for input to certain processing steps.

One method of displaying digital data is by plotting on the computer line printer with certain data values being represented by specific characters. The darkness of the printing is increased by overprinting several characters at the same location. This method has the advantages of showing the values of the data and of allowing exact determination of the coordinates of each data value. Disadvantages are poor gray level rendition and the inability to display large regions. An example of a printer plot is given in Figure 11, showing the Landsat data of the Huntsville Jetport.

The Landsat scanner data is more easily interpreted when displayed on a cathode ray tube (CRT), which may be either a storage type or a TV monitor. In the present work, the former type, a Dicomex was used. This device is capable of displaying 64 gray levels, and the screen size is 2048 by 2048 pixels.

In order to obtain photographic prints of scanner data and land use maps, a film writer is used. This device reproduces scan lines read from magnetic tape on film, with the film density being proportional to the numbers read from tape. An Optronics model Photowrite was used, and the film writer, tape drive, and film scanner-digitizer are shown in Figure 12. Prints in this report, such as Figure 10, were produced by this method.

The line printer plots are a very useful output product since they can be obtained in the same computer run which has processed the data. A 500 by 500 pixel area, such as that shown in Figure 10, can be plotted in the width of the printout if every fourth pixel in each direction is plotted. This allows, for example, immediate examination of a land use map during the process of computer classification.
Figure 11. Printer Plot of Landsat Data Coverage of Huntsville Jetport
Figure 12. Film Writing and Scanning Equipment with Magnetic Tape Unit
1-5. APPLICATION TO TARCOG AREA

TARCOG (Top of Alabama Regional Council of Governments) is a coalition of the five counties (Limestone, Madison, Jackson, Marshall, DeKalb) located in the extreme northeast corner of Alabama, which was formed to better evaluate and respond to socio-economic problems of the area. One such problem is the land usage of the area, which in cooperation with NASA/Marshall Space Flight Center had been surveyed using low altitude (3000-6000 ft.) aerial photography. In addition, some RB-57 aircraft coverage (60,000 ft.) had been obtained and one frame of three band photography analyzed by digital computer. The present study was initiated in order to evaluate the feasibility of Landsat data analysis of land usage in the TARCOG area, and make comparisons with the low altitude and high altitude aircraft coverage. Figure 13 shows the TARCOG area, and the RB-57 and satellite frame sizes.
Figure 13. TARCOG Area Showing RB-57 and Satellite Frame Sizes.
II. ANALYSIS PROCEDURES

2-1. PRELIMINARY DATA HANDLING

Multispectral scanner (MSS) data is transmitted to ground-based receiving sites and hence to the NASA Data Processing Facility (NDPF). The NDPF corrects, calibrates and formats the raw MSS data and converts it to a usable binary form on computer compatible tape (CCT). The annotated and corrected 185-km² square ground scene on the CCT is a final product of the MSS. A scene is made up of 2340 parallel scan lines, each containing approximately 3240 data samples.

The NDPF transmits completed ground scenes to data users on four separate CCTs, each containing image data for one 46.25 x 185 km strip. The images are registered with respect to spectral bands, and are calibrated using a calibration wedge which is introduced into the data during every other scan retrace interval. The CCTs also contain, as part of the annotation record, the geographic coordinates of the image center and the locations of the tick-mark reference system. The tick-marks are located at the intersections of the scene edges with latitudes and longitudes at intervals of one-half degree.

With this degree of geographic coordinate information available, it is possible to construct a computer program which reads the CCT and determines the image coordinates of an area specified by latitude and longitude bounds.

The pixel coordinates are determined using the following five steps:

(i) The pixel coordinates of the format center are found.
(ii) The latitude and longitude of the format center are found.
(iii) The latitudes or longitudes of the tick-marks and their locations relative to the format center are determined.
(iv) Scale factors needed to convert projected latitudes and longitude differences to pixel differences are computed.
(v) Pixel coordinates of the four corners of the area to be extracted are calculated.

The segment of data thus defined may then be transferred to additional magnetic tape to be used in subsequent operations. This output may contain the reflectance measurements from each band in four adjacent storage locations (feature vector format) for each pixel, or may contain all measurements from a spectral band in separate tape files. The feature vector format is most useful when the four spectral measurements are to be considered simultaneously, as in a multispectral classification into land uses.
Computer Classification

Classification algorithms may be defined as sequences of mathematical operations which determine from a set of measurements the class or type of object which is being measured. When the determination is done by computer, the process is termed automatic feature classification. Generally, measurements of more than one characteristic or feature of the objects in question are made simultaneously. In the present case, the set of measurements is the intensity of sunlight reflected in specified wavelength bands of the visible and near infrared regions of the spectrum. Each set of \( n \) measurements is said to define an \( n \)-dimensional feature vector, \( \{x_1, x_2, x_3, \ldots, x_n\} \), which thus contains all the information obtained by the sensor.

The mathematical criteria which are employed in classifying the feature vectors are called decision functions or discriminant functions. The particular method being used determines the form of these functions. The unknown parameters in these functions are determined in a preliminary process called learning or training. A small part of the data, called the training set, is used by the learning algorithm.

When the classification of the training samples is unknown, the determination of the decision function is said to be unsupervised. The algorithm attempts to find trends in the data and separate the given unknown samples into distinct groups.

Supervised algorithms may be employed when one is supplied with a set of training sample patterns of known classification. These samples are used to develop decision functions, which may then be used to classify unknown samples. The classification will be reasonably accurate if the training samples are truly representative of the classes and an appropriate type of decision function is computed.

Thus, a crucial aspect of the classification problem is the selection of data to be used as training samples. This is generally accomplished by visual inspection of the imagery, coupled with additional sources of information such as topographic maps and personal knowledge of the area. During this process it may be desirable to display the imagery at various levels of magnification, to enhance the imagery by adjusting density levels, and to indicate on the imagery the sites from which data is to be extracted.

In the present study, training data was selected from a region which was a small fraction of the TARCOG area, but which included the city of Huntsville, MSFC, the Huntsville-Madison County Jetport, Monte Sano, and a portion of the Tennessee River. Thus a wide range of land use categories could be defined within a relatively small area. Land usage in this area had been previously studied from aircraft photography, using manual and computer classification.
techniques. Thus, it was possible to identify in this section of imagery training sites which represent several land use categories.

The line and sample coordinates of the selected regions may be determined from a CRT display of the imagery, or from a computer line printer display, such as that in Figure 11. The latter has the advantage that individual pixels are readily identifiable.

Using the initial training site regions, a classification map may be prepared. The shape and extent of the various land use areas will be more easily seen on the classification map than on the input data. Hence if the training site boundaries are indicated on the classification map, it may be possible to adjust the defining coordinates so that the training data is extracted from areas whose land use corresponds to the desired classes. In addition, misclassifications may indicate that the training data was not sufficiently representative for all areas in the scene. In this case, training data may be extracted from additional regions of the scene.

Locations of the training sites for seven land use classes are shown in Figure 14. The classes and locations are as follows:

- **Urban**: City of Huntsville and Jetport terminal area.
- **Agriculture**: South of Jetport and south of Tennessee River near Highway 231.
- **Forest**: Mountains on east and south of MSFC.
- **Wetland**: Marshes southwest of MSFC.
- **Pasture**: Jones Valley farm and flanking Rideout Road.
- **Water**: Tennessee River
- **Barren**: Quarries northwest of Huntsville.

In separating one class of objects from one or more other classes, it is desirable to de-emphasize the characteristic features that the classes may have in common, and to emphasize where possible the features that are unique to the class of interest. The most obvious first approach is to say that the distinctive character of an object or class of objects is the sum total of its features, some features being more distinctive than others in certain environments. The Linear Classifier concept depends upon this assumption, and aims at developing a single measure of a class's composite features. This measure, the discriminant, is formed by adding the value of each feature (reflectance value or brightness in the case of multiband imagery), after each feature has been weighted according to its usefulness in separating the class of interest from the other classes.
Figure 14. Locations of Training Data for Seven Land Use Classes
Nonparametric methods are so termed because parameters (such as mean values and covariances) of the distribution functions of the data are not used. The training algorithm determines the values of the weighting factors \( w \) to be used in a discriminant function of the form

\[
G = w_0 + w_1x_1 + w_2x_2 + \ldots + w_nx_n
\]

A set of weights is determined for each class of data, the value of a weight reflecting the significance of its associated feature in separating the class from its companion class. Thus for each unknown feature vector, a value of \( G \) is obtained for each class.

There are two approaches possible in the application of linear classifiers. In the first, the discriminant functions are designed such that one class may be separated from each of the other classes, pairwise. Then, in determining the class to which a particular feature vector (the reflectance values from one pixel) should be assigned, the value of \( G \) is calculated by substituting the values of the feature vector in the discriminant function for each of the classes. The class for which the value of \( G \) is largest is the class to which the feature vector is assigned.

In the second approach, the one employed at NASA/MSFC, the discriminant functions are designed such that one class may be separated from all of the other classes considered collectively as one class. Unlike the first approach in which all discriminants are calculated concurrently, here the discriminants are calculated sequentially. Referring to Figure 15, the straight line corresponds to the discriminant function that will separate Class 4 from Classes 1, 2, and 3 taken together. If a given feature vector lies to the right of this line, the discriminant has a positive value and the vector is assigned to Class 4. If it lies to the left of the line, the discriminant has a negative value, and the vector is not assigned to Class 4. Class 4 may then be removed from consideration, and a further test is applied using the discriminant function for Class 3, say. These tests are repeated until the feature vector is assigned to a particular class, at which time testing ceases, and a new unknown feature vector is called in. The sequential nature of testing results in a speed advantage over the parallel procedure employed in the first approach.

The linear classification scheme described here is combined with a feature selection algorithm that determines which of the features of any class are of greatest significance in separating that class from the others. The method of feature selection is based on the concept that the classification is more accurate if

data values from different classes are widely separated (interclass distance is large), and
data values within each class are closely grouped (intraclass distance is small).

These effects are illustrated in Figure 16.

The interclass and intraclass distances are computed for each feature by calculating the totals of the separations between all pairs of points in different classes (interclass) and within each class (intraclass). The optimum is obtained when the interclass distance is maximized and the intraclass distance is minimized.

After calculating the criterion for best features (based on separations between training data of the various classes), the feature selection values are combined to yield a value which determines the most easily separable class (Class 4 in Figure 15), for which the discriminant function coefficients (w's) are then computed.

The analysis process in the training phase is illustrated in Figure 17. After the training samples have been selected, they are processed by the feature selection algorithm EFFECT. This determines which class is the most easily separable from all others, and the optimum subset of features (spectral bands) for separating that class. This latter option may be bypassed if not many (three of four for example) spectral bands of data are available, but it is very useful if many bands of multispectral scanner data have been acquired. The discriminant weights for the most easily separate class are then calculated, using the algorithm SNOPAL.

The values of the weights are determined by an iterative procedure. In each iteration, the value of w is changed slightly from its previous value to produce an improved set of weights. Several options are available in the algorithm for terminating the iteration. Once the weights for the most easily separable class have been determined, the training samples for that class are removed from the data set, and EFFECT then determines the next most easily separable class and its optimum feature subset. Then SNOPAL computes the required discriminant function coefficients. This process of identifying an easily separable class and its discriminant, supressing its data and moving on to the next easily separable class, is repeated until a discriminant function has been calculated for all of the classes in the training data set.

The training phase is completed by performing a test classification of all training samples. Ideally, the classifier should assign the training samples to the class from which they were selected by photointerpretation. If the classifier assigns more than a few samples from Class 4 to Class 1, for example, this will suggest an unsatisfactory choice of training samples, and that some of Class 4's training samples were inadvertently selected for Class 1. The choice of training samples must then be revised, and the entire training phase repeated.
Figure 15. Decision Function for Assigning Samples to Class 4.

Figure 16. Interclass and Intraclass Distance.
Figure 17. Discriminant Training Phase of Sequential Linear Classifier.
In the classification process for an unknown feature vector, shown in Figure 18, the values "G" of the discriminant functions are computed in the same order as the functions were defined, and the assignment is made to that class for which G first becomes a positive number.

Figure 18. Classification Phase of Linear Classifier.
The Landsat-1 system, described in Chapter I, provided the data used in this investigation. Data samples are gathered along scan lines normal to the direction of spacecraft travel, and a ground track of width 185 km. is imaged. However, the resultant image does not correspond to conventional map-making standards, i.e., equally spaced latitude and longitude lines with longitude vertical and latitude horizontal. In the present situation, there are three principal contributions to the geometric distortion. They are the non North-South heading of the satellite, non-uniform data sampling rates along track and across track, and the rotation of the earth from west to east beneath the satellite. Because for interpretative purposes it is mandatory to associate each Landsat resolution element with a precisely known geographic location on Earth’s surface in order to correlate the imagery with aerial photography and existing maps, the distortions in raw imagery must be corrected and the data established in a consistent geographic framework. Then the correlation of sequentially, seasonally or annually observed scenes is greatly simplified, and interpretation errors due to differences of image scale or orientation are minimized. Also standard reference data, for example political boundaries, can be overlaid on the CCT data as a routine processing procedure. Further, the correlation with airborne remote sensors, cameras and line scanners, is simplified by establishing a unified geographic datum. The Universal Transverse Mercator (UTM) projection is used in the present work.

In this system, the surface of the Earth is divided into sixty transverse (i.e. north-south) zones. In international usage these zones are numbered 1 to 60. The center of each zone is called the central meridian. The relation of the UTM zones to the Earth’s surface is shown in Figure 19 and the shape of a zone is shown in Figure 20.

Each UTM zone has superimposed on it a rectangular grid of vertical and horizontal lines. The vertical lines lie parallel to the meridian that runs down the center of each zone, and the horizontal lines run parallel to the equator. The basic grid lines are drawn 100,000 meters, or about 62 miles, apart. These grid lines are shown in Figure 21. The squares formed by the intersection of the 100,000 meter lines are usually subdivided by 10,000 meter lines, 1,000 meter lines, or 100 meter lines, depending on the scale and purpose of the map.

The 100,000 meter grid lines are referenced by their "northing" and "easting" values. The northing value is the distance of the line from the equator. Vertical lines are counted from the central meridian which is the 500,000 meter line, those on the left of it having an easting value of less than 500,000 meters and those on the right having a value above that. This is shown in Figure 21.
Figure 19. Universal transverse Mercator zones.

Figure 20. Shape of UTM zone.

Figure 21. A UTM zone with 100,000-meter grid superimposed.
Two approaches may be taken to the geographic referencing problem. The cartographic approach consists of correcting (or adjusting) the image data to match an Earth coordinate system. This is convenient for the preparation of displays or maps. The other approach systematically warps Earth coordinates to match row and column coordinates in the imagery. In either case, it is necessary to find the equations of transformation between points in the image and locations on the ground.

There are two methods that can be considered for determining the transformations: theoretically, by calculating the effects of all the processes involved in producing the image, and empirically, by comparing the image with a model (e.g., a map) of the terrain. The first requires detailed knowledge of the flight path or orbit, attitude and motions of the sensor-carrying vehicle, characteristics of the sensor, and important error sources. This approach is impractical in all but the simplest cases. This leaves the empirical method. If it is possible to determine the geographic coordinates of every point in the image, an expression for the transformation can be found exactly (assuming the requirements of the sampling theorem are satisfied). In any case, the geographic referencing problem is solved for that image. However in a practical case it is difficult to do more than locate a relatively small number of landmarks in the image, and often virtually impossible to find their exact row and column coordinates. In MSS data, for instance, this problem is accentuated because the instantaneous field of view is large and the resolution is relatively coarse. This suggests using a regression technique to fit a model of the transformation to the landmarks or control points, expressed in both coordinate systems. Very accurate maps are available for the United States and many other parts of the world, from which to obtain the geographic coordinates.

Landmarks or ground control points (GCP) ideally should possess well defined characteristics of shape, should exhibit high contrast against their background in one or more spectral bands, and should not change materially in contrast because of seasonal or climatic influences. Prominent land/water interfaces in the infrared, and man-made constructions such as major highway intersections or airport runways in the green, are well suited as GCP's. To pinpoint the geographic coordinates of a GCP within a Landsat scene, the CCT data is presented for inspection on a digital image display, candidate GCP regions are identified, their data arrays are extracted, magnified by a scaling algorithm to the point that individual resolution elements can be easily seen, and redisplayed. The data address (sample number and scan line number) of a single resolution element (pixel) within the GCP that can be associated with a unique geographical location on a map is then determined. Repeating this procedure for a number of GCP's provides the data required to establish the transformation between the image (pixel) and the Earth coordinate frameworks. If a digital image display is unavailable, this procedure can be followed, although with greater difficulty, using a computer line printer pseudo grey-level plot of the GCP region for pinpointing pixel coordinates.
After identifying the control points in an image and determining their geographic coordinates, it is necessary to find the amount of geometric correction required. In outline the method is as follows:

The coordinates of the control points in both systems (satellite data and UTM coordinates) are found. With the geographic UTM coordinate taken as the independent variable, the equations of transformation give expected values for the coordinates in the image reference frame. In general, it is desirable to use as large a number as available of control points to compensate for errors, if any, between the given image and the standard reference map. Also, it is desirable to perform geometric manipulations with a small number of parameters. This results in a large number of equations to solve for a small number of parameters. More often than not, exact solutions for parameters do not exist in such cases. Therefore the parameters should be determined such that the error between the estimates of the control points' coordinates using the parameters and their exact coordinates is minimized in some sense. The method chosen for solving for the fit parameters was classical Gaussian least squares, modified to work with vector observations. The basic generalization to vector observations consists of replacing the sum (over observations) of the squares of the deviations between model and observations with the sum of the squares of the Euclidean norms of the difference vectors. Then vector components are treated the same as scalar observations in the simpler case.

The procedure followed was as follows:

(i) The geographic coordinates \((x, y)\) of control points are taken as the independent variable, and the picture coordinates \((u, v)\) as the dependent variable (observations),

(ii) The image was displayed on the Dicommed display screen. With the aid of a map, several control points were identified and their approximate \(u-v\) coordinate locations were found.

(iii) Previously existing image processing software was used to extract small regions surrounding these approximate locations, magnify them several times by repeating pixels and lines, and format the enlarged regions into a multiple-frame output display.

(iv) The result was viewed on the Dicommed display, and the \(u-v\) coordinates of the control points were estimated as accurately as possible. The \(x-y\) coordinates were taken from a USGS map.

(v) These coordinates were used in the least squares program to obtain the parameters of the transformation.
2-4. GEOMETRIC CORRECTION

Geometric manipulation of images is needed in handling remotely sensed data in order to match the data obtained by various sensors and/or at several times with respect to a single standard frame of reference. The geometric transformations that need to be implemented may be simple rotations and scaling as in the case of aerial photographs of small regions or combinations of several more complex transformations as in the case of multispectral (linear or conical) scanner output from satellites of large areas on earth wherein the rotation and curvature of earth need be compensated for. The main problem involved in applying the transformations using a digital computer is the bulk of data one has to handle. For instance, in the MSS images there are over 7.5 X 10^6 bytes of data per frame in each of the four spectral bands. Data is generally supplied on magnetic tapes and the output is required to be on tapes. In contrast with point operations on image densities, geometric manipulation of images generally requires more than one input data record to generate one output record. Also, in many cases it may not be possible to contain all the input records needed to generate one record of output within the main memory of a computer and hence segmentation of input data and reassembly of output records may be required. Further, the sample locations in the geometrically transformed image do not necessarily correspond to integral sample locations in the input image. This requires that some type of interpolation be used for assigning the image intensity values at the output sample locations.

Any geometric distortion of a two dimensional image in a continuous domain may be expressed in the form

\[ x' = f(x, y) \]
\[ y' = g(x, y) \]

where \((x', y')\) is the location to which the point \((x, y)\) in the image should be moved. Thus a geometric distortion consists in finding \((x', y')\) for every \((x, y)\) in an image and setting the density of the new image at \((x', y')\) to that of the given image at \((x, y)\). Equivalently, when the inverse transformation

\[ x = \varphi (x', y') \]
\[ y = \psi (x', y') \]

exists one could compute \((x, y)\) for every \((x', y')\) and set the density at \((x', y')\) to that at \((x, y)\) in the given image.

In the case of a digitized image, it is possible that the sample point \((x_{i}, y_{i})\) in the new image does not map into any point \((x_{K}, y_{L})\) on the sampling grid of the original image. Therefore it is necessary to define the image density at \((x'_{i}, y'_{j})\) in some manner. If the continuous image function is band limited and
the sampling fine enough, the sampling theorem can be used to obtain the exact density at \((x^i_1, y^i_1)\). However, since this is a slow process and there is no guarantee that the sampling frequency is sufficiently large, some simple techniques of interpolation are used instead. Some common approaches are the nearest neighbor rule (causing some geometric uncertainty particularly at boundaries between different types of ground cover), and bilinear or cubic interpolation (leading to radiometric distortion).

Of a variety of models \(f(x, y)\), and \(g(x, y)\) appropriate for the characterization of Landsat image distortions, it is found that a linear transformation between original and corrected image coordinates compensates the predominant distortion components. Using 23 GCP's, for example, the root mean square compensation error is less than the dimension of the Landsat resolution cell.
2-5. SUPERPOSITION OF BOUNDARIES

In the study of remotely sensed images for land use analysis and planning, it is generally of interest to determine the distribution of land use classes within politically delineated regions such as states, counties or cities. Therefore, it is necessary to first associate the boundary information of the desired type with the remotely sensed images and then extract the region in the interior of a certain political entity as required for further evaluation. In this section we shall describe the steps involved in superposing boundaries on images and separating the image data into individual political entities.

The steps involved depend upon the type of equipment available to digitize the boundary data. The method described below was designed for a system employing a microdensitometer capable of digitizing transparencies. Some of the steps would be obviated if a draftsman's table with a digitizing plotter/tracer attachment were employed.

The steps required when a microdensitometer is used for digitizing are:

(i) Drafting
(ii) Photographic reduction
(iii) Digitization
(iv) Thinning and conversion to "scan line intersection code"
(v) Smoothing to assure continuity
(vi) Finding control point coordinates in pixels and UTM system
(vii) Determination of the required geometric transformation to assure that the image and boundary data are in the same coordinate system
(viii) Application of the geometric transformation
(ix) Thickening of boundary data (if desired) and superposition on image to obtain a combined picture for visual inspection
(x) Identification of separate regions and extraction of data corresponding to each region from the remotely sensed image

A general description of the above steps follows.

(i) Drafting

A standard map of a convenient size is used to obtain the desired boundary lines. The lines are traced in black on a translucent paper. The tracing should be as accurate as possible in order to assure geometric fidelity when matched with the remotely sensed image.
(ii) **Photographic Reduction**

The tracing is reduced photographically to a transparency of size convenient for digitization on the microdensitometer. (It is preferable to do this rather than try to get a tracing of the size the microdensitometer can handle, since the effects of drafting errors would be more pronounced in a small size tracing.)

(iii) **Digitization**

The image on the transparency is digitized at a resolution close to or finer than the final anticipated resolution (in km/pixel). This should be done in preparation for the geometric correction step. If the digitization is too coarse, most of the points after correction will have to be generated by interpolation and the resulting image of the boundary will be inaccurate and will show jaggedness, depending on the type of interpolation used.

(iv) **Thinning and Conversion to Scan Line Intersection Code**

When digitized with a microdensitometer, the data generated are the density values at all pixel locations within a rectangular region on film. Thus, a 25 x 25 mm$^2$ region scanned at a resolution of 12.5 µ generates 2000x2000 = 4 x 10$^6$ density values. But, when the image under consideration is a boundary image where most locations are blank and only the positions of a few lines constitute the relevant information, it is more efficient to store and expeditious to handle the boundary points' coordinates. Typically, the boundary lines in the above example may be represented by the coordinates of 10,000 to 20,000 points.

Several methods of boundary encoding are available (see [1], for example). The most convenient method for our purposes is the "scan line intersection code" (SLIC). With this code we represent the digitized boundary image by giving the sample numbers corresponding to the boundary locations in each row. For instance, while storing the boundary information on a tape, each record can be used to represent one scan line, the record consisting of the number of intersections of the scan line with the boundary lines followed by the sample numbers of those intersections arranged in ascending order. The information can be handled in a computer memory by using two arrays, the first array consisting of all the column coordinates corresponding to the boundary intersections and the second array providing a means of finding the bounds on the addresses in the first array of the coordinates corresponding to a given row (scan line). As an example, consider a simple boundary image shown in Figure 22. A digital version of it is shown in Figure 23. Each grid intersection in 23 is a sample location, and those marked with a dot correspond to the boundary pixels. Now, if we were to represent the boundary image by the densities at all sample locations as generated by a microdensitometer (for example), then the array would consist of 169 values (say, 0 for non-boundary points and N for boundary points). The same data can be represented as 13 records shown below, requiring 63 values.
Also, as two arrays in core, the same data can be represented as follows:

Index array: 1, 1, 1, 11, 15, 19, 23, 27, 30, 33, 36, 39, 51, 51

Data array: 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 3, 7, 10, 13, 2, 7, 10, 13, 2, 7, 12, 13, 2, 6, 13, 2, 5, 13, 2, 5, 13, 2, 5, 13, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13

Now, we shall consider the problem of converting the digitized data to the SLIC. To do this, we first need to detect the locations of boundary points. An adequate criterion for this is a threshold on the density values. However, the boundary lines thus detected turn out, in general, to be more than one pixel in thickness. Since in most problems involving boundaries it is desirable to have as thin a boundary as possible, we reduce the thickness of the lines using a thinning algorithm. Referring to Figure 24, the purpose of the thinning algorithm is to generate an approximation to the dashed line given the "thick" lines in digital form. After a thin boundary line is obtained, the coordinate information is converted to the SLIC.

(v) **Smoothing**

Discontinuities might occur in the thinned boundary data due to drafting and photographic defects or thresholding and thinning. For interior extraction or political entity separation, it is important that the boundary be continuous. Therefore, the thinned data are examined at every point for continuity and patches are generated between locations of discontinuity and the nearest boundary point (if any, within a pre-specified maximum distance).
Figure 22. A Simple Boundary Image

Figure 23. Digital Version of the Boundary Image

Figure 24. An Exaggerated View of Thick Boundaries
(vi) Finding Control Points

To find the corrections in scale and orientation required for matching the boundary data with a standard coordinate system, it is necessary to establish the coordinates of some known locations called control points. A convenient set of points while handling boundaries are intersections of boundary lines. Figure 25 shows a boundary map of the five-county area (TARCOG, in North Alabama) on which the work was performed. The control points are shown on the map. The ground coordinates of these points can be determined in the UTM system by reference to standard maps. The pixel coordinates of the same points can be determined by obtaining binary line printer plots of small sections of the boundary data including the control points and manually counting the pixel numbers.

(vii) Determination of the Geometric Transformation

The transformation needed to convert from the pixel numbers as obtained from the microdensitometer to the standard coordinate system (at a specified sampling interval) can be found from a knowledge of the control point coordinates. A parametric model is assumed depending on the types of correction required. A linear transformation with six parameters is sufficient to account for translation, rotation, and scale change. The parameters are then determined by minimizing the mean squared error between the observed UTM coordinates and those obtained by converting the pixel coordinates using the assumed transformation.

(viii) Application of the Geometric Transformation

The boundary data are converted to the UTM coordinate system by using the transformation determined as mentioned above. A resampling problem enters into the picture at this point. The boundary points which have integer coordinates in the original system do not necessarily transform into integer sample numbers in the UTM system. Therefore, the transformed coordinates are approximated by rounding them off to the nearest integers. Also, when there is a scale change, the number of boundary points in the output image is not necessarily the same as that in the input. The points that were contiguous in the input image might transform into non-contiguous points. Thus, to preserve continuity, it is sometimes necessary to interpolate and generate extra boundary points. A simple approach to this is to transform all the input boundary points via the given transformation, join the output points corresponding to contiguous input points by straight lines and obtain integer coordinate values by rounding off.

(ix) Thickening and Superposition

Whereas, to extract a region within a given boundary, it is necessary to have as thin a line as possible, for visual presentation of boundaries on remotely sensed images, it is desirable to thicken the boundary lines. The data in the SLIC format can be conveniently used to generate thickened boundary data in the same format by producing new boundary points at locations surrounding each old
Figure 25. Boundary Map of Five TARCOG counties showing Control Points.
boundary point up to a given thickness. The boundary lines are superposed on
the given image (which is of the same scale and orientation) by assigning a unique
density to all points in the image corresponding to the boundary point coordinates.

(x) Identifying and Extracting Individual Regions

Each political entity can be extracted separately using the boundary data
for the entire region in SLIC format after geometric correction. The first step
in doing this is to identify connected regions separated by the boundary lines and
generate a unique label for each of the regions. For example, the digital boundary
image shown in Figure 23 leads to a "region identification map" (RIM) shown below.

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Here, 0 is used for boundary points, 1 for the exterior and 2, 3, and 4 identify
the interior of the three separate regions. Such a map is easy to generate from
the boundary data in SLIC format using tests for connectivity.

A RIM can be used to extract data corresponding to any given region from
a remotely sensed image. For example, all points in region 2 can be highlighted
by reading the RIM and the given image record by record and setting all densities
to 0 except where the RIM values are 2.
III. MATHEMATICAL TECHNIQUES

3-1. PRELIMINARY DATA HANDLING

This section describes the extraction of a geographic location from a computer compatible tape (CCT), given the latitudes and longitudes bounding the area. Since the scanning direction is not parallel to latitudes or longitudes, the smallest rectangular region containing the desired part of the image is determined using the identification and annotation data read from the CCT.

The first step is very simple. The number of records in the CCT is a constant equal to 2340. The number \( n \) of pixels per record (per band) is given by "MSS adjusted line length" contained in the 39th and 40th characters \([2]\). Thus the pixel coordinates of the format center are given by \((1170.5, (n+1)/2)\).

In order to find the bounds on the region to be extracted in terms of pixel coordinates, it is sufficient to determine the pixel coordinates of the four corners of the rectangle bounded by the given latitudes and longitudes. Therefore, we shall describe the method for determining the pixel coordinates of a given point where its latitude and longitude are given.

Let \( E-N \) and \( R-P \) represent the geographic and pixel coordinates of a given point. Let \((e_0, n_0)\) and \((r_0, p_0)\) be the corresponding coordinates of a reference point, say the format center. The satellite heading is given by the angle \( \theta \) between the \( N \)-axis and the \( R \)-axis (see Figure 26).

From the equations for rotation of a Cartesian coordinate system about the origin, it follows that

\[
\begin{align*}
 r - r_0 &= (n - n_0) \cos \theta + (e - e_0) \sin \theta \\
 p - p_0 &= -(n - n_0) \sin \theta + (e - e_0) \cos \theta
\end{align*}
\]

if the units of measurement are the same for the two coordinate systems. (An approximation is made here in that the longitudes are assumed parallel to one another - a reasonable assumption if the region to be extracted is sufficiently small and sufficiently far from the poles. Hence, the use of plane geometry instead of spherical trigonometry.)

The scale factors required to convert the distances \((r-r_0)\) and \((p-p_0)\) are computed as follows. The tick-marks indicate the intersections of known longitudes or latitudes with edges parallel to the pixel coordinate axes. Also the number of pixels (records) between two tick-marks along a top or bottom (left or right) edge can be determined. Therefore, the number of pixels (records) per degree of change in longitude (latitude) in the horizontal (vertical) direction
Figure 26. Geographic and Pixel Coordinate Systems
can be determined from the tick-mark data. These then are the scale factors to
be used to convert \((r-r_0)\) and \((p-p_0)\) into increments in record and pixel numbers.

The procedure is to first read the ID record, find \(r_0\) and \(p_0\), then read
the annotation record to find \(n_0\) and \(e_0\), and the tick-mark information and con-
vert them to floating point numbers, compute the scale factors and find the
heading \(\theta\) and use the formulas above and the scale factors to find the record and
pixel numbers corresponding to each of the four corners of the rectangular region
to be extracted. Finally, these coordinates are converted into integers, the
smaller of the bounds being truncated and the larger being rounded to the next
higher integers.
3-2. COMPUTER CLASSIFICATION

The linear discriminant functions presently being used to separate classes of data divide a set of data into two regions, arising from the positive and negative results obtained when individual data samples are substituted into the discriminant function. Thus, as a starting position, it is necessary to define that class of data which can be separated from the remainder of the data by the appropriate decision function. Using the set of training data, the separation between classes is determined according to the following conditions:

1. the two clusters of class data values are widely separated (interclass distance ($S_1$) is large), and
2. data values within each class exhibit low dispersion, i.e., are closely grouped (intraclass distance ($S_2$) is small).

In the ideal case, the intraclass distance is negligible compared to the interclass distance, i.e., $S_2/S_1 \to 0$. If the data samples of different classes fall in the same region, $S_2/S_1 \to 1$. In the extremely poor case, where data values of class 2 all lie within the extreme data values of class 1, the intraclass distance ($S_2$) for the more widely dispersed class 1 is greater than the interclass distance ($S_1$) to the data values of class 2, and $S_2/S_1 > 1$. A normalized figure of merit for assessing the discriminatory effectiveness of a given feature is defined as

$$F = \exp \left( -\frac{S_2}{S_1} \right)$$

an index that is suitably bounded between 0 and 1.

The generalized distances $S_1$ and $S_2$ are based on the average distances between all pairs of data samples in the class or classes involved. As a first step in the computation, an array $D$ is defined whose elements are the interclass and intraclass distances along each feature. Assuming that an $N$-dimensional feature vector $X = \{X_i\}, i = 1 \ldots N$, defines the sample measurements over $M$ classes, the dimensions of the array are $M \times M \times N$. For classes $p$ and $q$, containing $n_p$ and $n_q$ samples, respectively, and considering feature $f$, the array element corresponding to the distance between classes $p$ and $q$ along feature axis $f$ is

$$D_{p,q,f} = \sum_{i=1}^{n_p} \sum_{j=1}^{n_q} \left| X_{p,f,i} - X_{q,f,j} \right|. $$

This distance element comprises a total of $n_p n_q$ terms. The diagonal elements of the distance array are the intraclass distances and are defined as
Each such element comprises \( np(n_p-1)/2 \) nonzero terms. The various elements of the array \( D \) are combined to form the total inter- and intraclass distances among the classes under consideration.

In the present scheme as already outlined, a linear discriminant is to be identified that will separate any one class from all of the remaining classes taken together, and the process is repeated with the number of classes under consideration being reduced by one each time. Thus, the problem of separating \( M \) classes is reduced to \((M-1)\) two-class problems in which each discriminant hyperplane successively partitions the sample space. If class \( p \) is under consideration, the second class \((q)\) consists of all the remaining classes \((q_1, q_2, q_3, \ldots)\) considered together. These original data classes are now, in effect, subclasses of the class \( q \).

The total interclass distance for feature \( f \) is then the sum of the distances between class \( p \) and each of the subclasses and is defined by

\[
\Sigma_{1,p,f} = D_{p,q_1,f} + D_{p,q_2,f} + D_{p,q_3,f} + \ldots
\]

The total number of individual distance terms is \( n_p(n_q_1 + n_q_2 + n_q_3 + \ldots) = n_p n_q \) where \( n_q = n_q_1 + n_q_2 + n_q_3 + \ldots \) and hence the average interclass distance from class \( p \) to all other classes along feature axis \( f \) is given by

\[
S_{1,p,f} = \Sigma_{1,p,f}/n_p n_q
\]

The interclass distance for class \( p \) itself is simply the array element \( D_{p,p,f} \) which is the sum of \( n_p(n_p-1)/2 \) terms.

For class \( q \), the intraclass distance is the sum of all the distances involving the subclasses \( q_1, q_2, q_3, \ldots \), namely

\[
D_{q,q,f} = D_{q_1,q_1,f} + D_{q_1,q_2,f} + D_{q_1,q_3,f} + \ldots
+ D_{q_2,q_2,f} + D_{q_2,q_3,f} + \ldots
+ D_{q_3,q_3,f} + \ldots
+ \ldots
\]

This expression is the sum of \( n_q(n_q-1)/2 \) terms.
The total average intraclass distance for the two classes, therefore, is

\[ S_{2,p,f} = \frac{D_{p,p,f}}{n_p(n_p - 1)} + \frac{D_{q,q,f}}{n_q(n_q - 1)} \]

and the figure of merit for determining the effectiveness of feature \( f \) in separating class \( p \) from all other classes is

\[ F_{p,f} = \text{Exp} \left( -\frac{S_{2,p,f}}{S_{1,p,f}} \right). \]

An \( M \times N \) array is computed using this expression giving a figure-of-merit matrix that exhibits the effectiveness of all features in separating each of the classes present.

In parallel with this merit figure evaluation, a further computation determines the figures of merit between class \( p \) and each of the subclasses \( q_1, q_2, q_3 \ldots \). The purpose of this calculation is to determine whether any of the subclasses are poorly separable from class \( p \), even though the figure of merit of separating class \( p \) from class \( q \) may have a high value. This can occur when several of the subclasses are very well separated from class \( p \) and, hence, heavily weight the value of \( F_{p,f} \) while at the same time one or more of the subclasses \( q_i \) are poorly separated from \( p \) and, hence, the classification ambiguity between class \( p \) and these particular subclasses \( q_i \) would be considerable. The smallest of these individual figures of merit is multiplied by the overall figure of merit defined above. Thus, the final figure of merit contains two factors:

1. The separability of class \( p \) from the remaining classes considered together as the second class,
2. the separability of class \( p \) from the nearest neighboring class.

In order to determine the order of separability of the training classes, the figures of merit for individual features of a class are combined to form a single figure of merit for that class. By ordering these values according to magnitude, the most easily separable class is identified.

The determination of the linear classifier discriminant functions is discussed in the following section.

The problem of designing a pattern classifier may in general be expressed as one of determining the discriminant functions \( G_i(x), i = 1, \ldots, M \), such that for any pattern sample vector \( X_j \), the inequality

\[ G_i(X_j) \geq 0 \]

implies that \( X_j \) belongs to pattern class \( C_i \).
The structure of the classifier is dependent upon the functional forms of the discriminants and also upon the availability of a sufficient quantity of a priori data that adequately characterize representative samples of the patterns to be classified. As long as the assumption may be justified that the pattern classes can be separated by a linear hyperplane, a linear discriminant function leads to the simplest structure of classifier. In this case, a variety of techniques exists for determining the actual structural properties of the classifier. [3]

In multiclass problems, a linear classifier may be applied in either a parallel or a sequential mode. In the parallel mode, the discriminant functions for a given sample vector are computed simultaneously, the largest resulting value identified, and sample assignment is made to the class corresponding to the largest discriminant. The resulting classifier is cumbersome, since the discriminant for any one class must be capable of separating that class from each other class taken individually, and requires \( M(M-1)/2 \) linear segments when \( M \) classes are present. In the sequential mode, the classifier structure is simpler since sample classification into \( M \) categories is performed by a sequence of \( (M-1) \) dichotomies, and each discriminant is required only to separate its corresponding class from all other classes taken together.

It is well known that a dichotomous linear classifier or Threshold Logic Unit (TLU) defined by the discriminant function

\[
G(x) = w_0 + \sum_{i=1}^{N} w_i x_i
\]

exhibits the following properties: [4]
1. The classifier separates patterns by a hyperplane decision surface in measurement space.

2. The hyperplane has an orientation given by the weight values $w_1, w_2, \ldots w_n$.

3. The hyperplane has a position proportional to $w_0$.

4. The distance from the hyperplane to an arbitrary pattern vector $X_j$ is proportional to the value $G(X_j)$.

Given two distinct classes of patterns, therefore, classifier structural design reduces to the problem of determining (a) the orientation and (b) the position of the separating hyperplane. In general, these quantities must be derived iteratively from information contained in the distances of misclassified samples from a trial hyperplane.

In the system reported here, the discriminant functions are determined by employing a gradient procedure, the Ho-Kashyap algorithm, [5, 6] that iteratively minimizes the least-squared classification error over the representative sample classes or training classes.

In the case in which two pattern classes $p$ and $q$ are present, containing respectively $n_p$ and $n_q$ $N$-dimensional pattern vectors $X$, the discriminant function is

$$G(X_{pj}) = w_0 + w^T X_{pj} = d_{pj}, \quad j=1 \rightarrow n_p$$

and

$$G(X_{qj}) = w_0 + w^T X_{qj} = d_{qj}, \quad j=1 \rightarrow n_q$$

where

$$w^T = \text{transpose of the hyperplane weight vector} \ (w_1, w_2, \ldots w_N).$$

The values $d_{pj}$ and $d_{qj}$ are measures of the perpendicular distances of the respective sample patterns from the separating hyperplane. The above $n_p + n_q$ equations may be expressed as
where the matrix \( A \) of dimensions \((n_p + n_q) \times (N + 1)\) defines the entire set of patterns, the \((N + 1)\) vector \( \alpha \) defines the separating hyperplane, and the \((n_p + n_q)\) vector \( d \) defines the pattern-hyperplane separations, to within a normalization factor. The feature vectors of class \( q \) are negated to ensure \( A \alpha > 0 \).

The components of \( d \) are positive or zero in the ideal case of totally separable patterns although in practice this condition is unattainable because of misclassification due to imperfectly separate pattern clusters. The optimum hyperplane, however, will minimize the number of misclassifications, i.e., will minimize the number of elements of \( d \) having incorrect sign, and will therefore minimize a classification error vector \( e = (\beta - d) \), where \( \beta \) is a \((n_p + n_q)\) vector of positive constants. The minimization criterion, of course, is arbitrary, but a quadratic criterion is advantageous since a steepest-descent minimization procedure results in a linear recursion relationship. Therefore, let

\[
J = \frac{1}{2} || \beta - d ||^2 = \frac{1}{2} || A \alpha - \beta ||^2
\]

The condition for minimum \( J \) is given by

\[
\partial J / \partial \alpha = A^T [A \alpha - \beta] = 0, \quad \beta > 0
\]
and for a given $\beta$, the corresponding hyperplane is determined by

$$\alpha = [A^TA]^{-1} A^T \beta$$

Since $\beta$ is initially an unknown positive vector, it must be determined iteratively from the relation

$$\beta(k + 1) = \beta(k) + \delta \beta; \quad \beta(0) \text{ arbitrary, } k = \text{iteration index.}$$

Logically, to minimize $J$, the iteration increments $\delta \beta$ should be proportional to the gradient $\partial J / \partial \beta$. Since

$$\frac{\partial J}{\partial \beta} |_{\beta(k)} = \beta(k) - A \alpha(k)$$

several possibilities arise due to the constraint $\beta > 0$.

1. $\frac{\partial J}{\partial \beta} |_{\beta(k)} = 0$, then $\beta(k) - A \alpha(k) = 0$, the ideal solution

2. $\frac{\partial J}{\partial \beta} |_{\beta(k)} > 0$, i.e., $\beta(k) - A \alpha(k) > 0$, hence an increment $\delta \beta$ will tend to increase the classification error vector, and preferably $\delta \beta = 0$.

3. $\frac{\partial J}{\partial \beta} |_{\beta(k)} < 0$, i.e., $\beta(k) - A \alpha(k) < 0$, hence a positive increment $\delta \beta$ proportional to the gradient may be made.

The rationale for incrementing $\beta$ therefore is

$$\delta \beta = \rho \left\{ A \alpha(k) - \beta(k) + \left| A \alpha(k) - \beta(k) \right| \right\} = 0, \frac{\partial J}{\partial \beta} > 0$$

$$= 2\rho, \frac{\partial J}{\partial \beta} < 0$$

where $\rho$ is a positive constant vector.

The Ho-Kashyap training algorithm thus may be summarized as follows

1. $\alpha(0) = [A^TA]^{-1} A^T \beta(0); \beta(0) > 0$, otherwise arbitrary
2. \[ \beta(k+1) = \beta(k) + \rho \left\{ A \alpha(k) - \beta(k) + | A \alpha(k) - \beta(k) | \right\} \]

3. \[ \alpha(k+1) = [A^T A]^{-1} A^T \beta(k+1) \]
\[ = \alpha(k) + \rho [A^T A]^{-1} A^T \left\{ A \alpha(k) = \beta(k) + | A \alpha(k) - \beta(k) | \right\} \]

The convergence properties and other details of the algorithm have been discussed elsewhere. [6]
Aside from simple rotation and scaling, there are three categories of geometric distortions that may be present in remotely sensed image data. First, there are effects due to geometry. Primarily, these are the result of projection of features from the curved surface of the earth into the image plane. This may also include the map projection involved; the map coordinate system of primary interest in this document is the Universal Transverse Mercator (UTM) projection. The point of view incorporated in the mathematics to be developed may be illustrated by assuming that a set of geographic grid lines are painted on the ground and transformed into image coordinates by the sensor (and by the equations to be developed). Other distortions are due to dynamics - the motion of the satellite, and rotation of the earth. Then, there may be distortions introduced by the instrumentation. For example, in the case of a scanning imager the relationship between position in the projective image plane along a scan line and the data stream itself may not be linear. (In fact, the ground trace of a scan line may not be a straight line.) Another possible instrumentation effect is a direction-dependent scale factor.

It will be seen that the distortions produced by some of these causes are considerable, while others are (more or less) negligible. Fortunately, the big distortions will also turn out to be the easiest to solve for. It will be seen that a simple mathematical model of the coordinate transformation provides accuracy high enough for many uses.

It will be assumed henceforth that geographic coordinates of points on the earth's surface are in the UTM system. Many projections are used in an effort to display the curved surface of the earth on a flat map, all necessarily involving some distortion. Mercator projections have several useful properties. For one, they are conformal. So, taking any small area, the shape of the regions is the same as on the globe. (The shapes of large areas are distorted because the scale is position-dependent.) Also, standard Mercator projections are the only ones in which lines of constant compass heading (rhumb lines or loxodromes) appear as straight lines. This makes them useful in navigation. A standard Mercator projection is related to a projection from the earth's surface onto a cylinder tangent at the equator. Parallels are horizontal and meridians are vertical. Meridians are equally spaced, while the spacing between parallels varies as the secant of the latitude.

The transverse Mercator projection turns the projection system (or the earth) 90°. It is related to a horizontal cylinder tangent along a meridian. A standard meridian great circle replaces the equator, and the zone on either side of that meridian is fairly well represented. The UTM system is a collection of transverse Mercator projections. In the UTM system, the earth is divided into 60 zones bounded by meridians whose longitudes are multiples of 6° west or
east of Greenwich. The zones are numbered sequentially, beginning with 1 for
the zone from 180° W. to 174° W., and proceeding eastward. The origin of
coordinates for each zone is at the intersection of the central meridian of the
zone and the equator. Distances in UTM coordinates ("easting" and "northing")
are measured in meters. The central meridian is given a "false easting" of
500,000 meters so all easting coordinates are positive. There is no false northing
in the Northern Hemisphere; in the Southern Hemisphere a false northing of
10,000,000 meters is assumed. The latitude limits are 80° N. and S. (A different
projection must be used in polar regions.) UTM coordinates of a point on the
earth's surface consist of the zone number and the easting and northing coordinates.

The equations sought to account for the geometrical effect of projection
would relate picture coordinates to UTM coordinates. However, the mathematics
involved is quite intractable, and it has not been found possible to obtain such
equations except in a form whose complexity conceals their content. It is some-
what easier to write a sequence of equations describing the situation. The
following equations are taken from Reference 7. (Although they refer to satellite
observations, only a change in terminology is needed to apply them to aircraft.)
They relate latitude ϕ and longitude λ to x and y, Cartesian image coordinates
with origin (corresponding to the satellite subpoint) at the center of the image.
The subscript SP will be used to refer to the subpoint (picture center), and P will
be used to designate the coordinates of an arbitrary point. Figure 27 illustrates
the situation being described. A spherical earth is assumed. Also, the line of
sight of the sensor is assumed to be straight downward.

The plane of the Landsat orbit is inclined at an angle of 8.906° (0.1554
radian) from a polar orbit. With reference to Figure 28, the equatorial incli-
nation i is 81.094°. Because of this inclination, the satellite crosses
meridians of longitude with increasing frequency and at increasing angles at the
higher latitudes. The heading of the satellite relative to the local longitude line
(azimuth) is obtained by applying the law of sines to the shaded spherical triangle
in Figure 28. Noting that the two sides which are also longitude lines have arc
lengths related to angles of 90° and 90° - ϕSP, we have

\[
\frac{\sin(180 - H)}{\sin 90} = \frac{\sin \epsilon}{\sin(90 - \phi_{SP})}
\]

or

\[
\sin H = \frac{\sin \epsilon}{\cos \phi_{SP}} = \frac{0.1548}{\cos \phi_{SP}}
\]

The following sequence of equations relate the subpoint latitude and longi-
tude (ϕ, λ)SP to Cartesian coordinates in the image plane, (x, y).
Figure 27. Orientation of picture along Subpoint Track or Heading Line. [7]

Figure 28. Azimuth of the Heading Line. [7]
Applying the law of cosines to the spherical triangle in Figure 29, we obtain

\[
\cos \delta = \cos(90 - \phi_{sp}) \cos(90 - \phi_p) + \sin(90 - \phi_{sp}) \sin(90 - \phi_p) \cos \Delta \lambda
\]

\[
= \sin \phi_{sp} \sin \phi_p + \cos \phi_{sp} \cos \phi_p \cos (\lambda_p - \lambda_{sp})
\]

and by applying the law of sines, \( \sin \alpha = \frac{\cos \phi_{sp} \sin (\lambda_p - \lambda_{sp})}{\sin \delta} \).

The transformation from \((\delta, \alpha)\) to the nadir angle \(\eta\) subtended at the satellite by \(\delta\) and the image plane azimuth \(\psi\) measured from the heading line is illustrated in Figure 30. The transformation is

\[
\tan \eta = \frac{R \sin \delta}{R(1 - \cos \delta) + H}
\]

\[
\psi - \alpha = \alpha^*
\]

As illustrated in Figure 31, coordinates \((\eta, \psi)\) transform to Cartesian coordinates in the image plane according to

\[
x = (f \tan \eta) \sin \psi
\]

\[
y = (f \tan \eta) \cos \psi
\]

In these equations \(f\) is a scale factor related to the field of view of the imaging device. The \(y\) axis is along the heading line and, as mentioned above, the origin of coordinates is at the image center (the image of the subpoint).

The equations that connect the UTM system with latitude and longitude are

\[
E = \alpha \sin^{-1} \left[ \sin (\lambda - cm) \cos \phi \right]
\]

\[
N = \alpha \sin^{-1} \left[ \frac{\sin \phi}{\cos (E/\alpha)} \right]
\]

where \(E\) is the easting coordinate, \(N\) is the northing coordinate, \(\alpha = 0.9996R\), and \(cm\) is the longitude of the central meridian of the UTM zone. These equations are also specialized to a spherical earth, and do not show the false easting (and false northing in the Southern Hemisphere) that must be added.

A large effect on the geometry of the satellite image is the rotation of the earth. The earth's rotation causes the heading of the ground track to
Figure 29. Transformation from Latitude-Longitude to Great Circle-Azimuth. [7]

Figure 30. Transformation from Great Circle-Azimuth to Image Nadir-Azimuth. [7]
deviate, and produces skew in the image. The earth’s rotation causes the plane viewed to move eastward, so successive points along the ground track are farther and farther to the west of where they would be in the absence of earth rotation. During the scan period of 33 msec., a point at the equator is displaced due to earth rotation by approximately 15 meters. During the 25 seconds required to scan a frame, the shift at the equator is 13,500 meters, and decreases as the distance from the point on the earth’s surface to the earth’s axis, which is proportional to the cosine of the latitude. Referring to Figure 32 [8] the dotted meridian of longitude through S rotates to point S₀ which is a subsatellite point at position given by latitude φ, longitude λₘ. Thus the satellite views the point S on the earth’s surface, located a distance Δ from the point S₀ which would have been viewed in the absence of earth rotation. The effective satellite track is that through S, which has heading H greater than the original heading H₀. The distance Δ varies as the angular velocity Vₑ of the earth and the cosine of the latitude, and the distance Δx covered by the satellite during the same period of time varies as the velocity Vₛ of the satellite. Hence

\[
\frac{\Delta}{\Delta x} = \frac{V_e}{V_s} \cos \phi
\]

where \( V_e/V_s = 0.0717 \) for Landsat -1.

In the MSS image (Figure 33), point S on the earth’s surface appears at point S₀ due to west-to-east movement of the earth’s surface by a distance Δ while the satellite covers a distance Δx, and hence is scanning along the line through S₀. Lines of latitude are rotated by the local heading angle, H₀, plus an additional amount dH due to the earth’s rotation. The change in position Δ has components dx (along satellite motion) and dy (along scan directions). In the practical case, for small heading angle, dx is small and the effect is that successive scan lines cover a portion of the earth farther and farther to the west, and skew is introduced into the resulting image.

The apparent change in heading dH can be determined by writing

\[
\tan dH = \frac{\Delta}{OS} = \frac{\Delta}{\Delta x} \cos H₀ \left( 1 + \frac{\Delta}{\Delta x} \sin H₀ \right)^{-1},
\]

using \( OS = \frac{\Delta x + dx}{\cos H₀} \) and \( dx = \Delta \sin H₀ \).

Substituting for \( \Delta/\Delta x \) in terms of earth and spacecraft velocities,
Figure 31. Transformation from Image Nadir-Azimuth to Cartesian Coordinates.

Figure 32. Effect of Earth Rotation on Satellite Track.
Figure 33. Orientation of UTM Axes in MSS Scene After Rotation and Skew.
\[
\tan dH = \frac{\frac{VE}{Vs} \cos \phi \cos H_O}{1 + \frac{VE}{Vs} \cos \phi \sin H_O}
\]

\[
\sim \frac{VE}{Vs} \cos \phi \cos H_O.
\]

For the latitude (\(\phi = 34.75^\circ\)) of Huntsville, Alabama, the heading \(H_O\) with respect to the local meridian is given by

\[
H_O = \sin^{-1} \left[ \frac{\sin i}{\cos \phi} \right] = 10.86^\circ
\]

using 8.906° as the polar inclination of the orbit [9]. The skew angle \(dH\) is computed to be 3.27°.

The locations of the scan line and pixel axes in the UTM system on the ground are shown for rotation and skew in Figure 34. The correction for rotation by the heading angle \(H_O\) is

\[
x' = -E \sin H_O - N \cos H_O
\]

\[
y' = E \cos H_O - N \sin H_O.
\]

The correction for skew, assuming scan lines in an east-west direction, is

\[
x'' = x' / \cos dH
\]

\[
y'' = x' \tan dH + y' = x' \frac{\sin dH}{\cos dH} + y'
\]

Substituting for \(x'\) and \(y'\), we obtain

\[
x'' = \frac{-E \sin H_O}{\cos dH} - \frac{N \cos H_O}{\cos dH}
\]

\[
y'' = -E \left[ \frac{\sin H_O \sin dH - \cos H_O}{\cos dH} \right] - N \left[ \frac{\cos H_O \sin dH + \sin H_O}{\cos dH} \right].
\]
Figure 34. MSS Axes in Terms of UTM Axes for Rotation and Skew.
The transformation from the ground into the image pixel coordinates becomes

\[
\begin{bmatrix}
  x \\
y
\end{bmatrix} = \frac{1}{\cos dH} \begin{bmatrix}
  -\sin H_0 & -\cos H_0 \\
  \cos (H_0 + dH) & -\sin (H_0 + dH)
\end{bmatrix}
\begin{bmatrix}
  E \\
  N
\end{bmatrix} + \begin{bmatrix}
  x_0 \\
y_0
\end{bmatrix}
\]

where \(x\) is the line count coordinate, \(y\) is the scan line point count coordinate, and, from right to left: \(x_0\) and \(y_0\) are the components of an origin shift vector (so the first point in the first line can have the coordinates \((x, y) = (1, 1)\)), \(E\) and \(N\) are the easting and northing UTM coordinates, \(H_0\) is the nominal heading angle, and \(dH\) is the skew caused by the earth's rotation. Both the \(E-N\) and \(x-y\) coordinate systems are right-hand systems; in the image, the line count \(x\) increases downward, while the point count coordinate \(y\) increases to the right. Although \(dH\) does depend on \(H_0\), it is not completely determined by \(H_0\); it also depends on the satellite's angular velocity, the earth's rotational rate, and the latitude, according to the previous expression for \(\tan dH\).

The significant thing about the transformation matrix that has been obtained, from the point of view of this discussion, is that for one image the elements of the matrix are (almost exactly) constants. (\(H_0\) and \(dH\) change due to their latitude dependence, which changes by \(1-2/3^\circ\) across a scene.) Further, an arbitrary \(2 \times 2\) matrix with constant elements can be assumed to be of the form given in the transformation.

Implicit in the transformation is the assumption that equal distances along a scan line correspond to equal distances on the ground, anywhere along the scan line. This may not actually be so. The multispectral scanner carried on Landsat will be chosen as an example. That scanner is an electromechanical device with scanning performed by a rotating mirror, swinging back and forth (with no imaging performed during the 'back' part of the motion). It is clear that, if the angular rate of the mirror is constant, the velocity of the intercept of the line of sight with the ground is not constant. (The combination of this with the forward motion of the spacecraft causes the ground sweep to be slightly S-shaped.) Since the maximum angle of sweep away from the nadir is small, this effect is quite small. In fact, the angular rate is not exactly constant during the sweep. The velocity profile is slightly sinusoidal. The effect of the latter is somewhat greater than that of the former in the case of the Landsat scanner. Other small effects, such as the angular bend due to the change in \(dH\) across a scene, are discussed in Reference [10]. Effects such as these can be accounted for by making the matrix elements functions of position. However, the constant-matrix formulation is at least an excellent approximation, so its use in the geographic referencing problem will now be described.
The transformation

\[
\begin{bmatrix}
  x \\
  y
\end{bmatrix} =
\begin{bmatrix}
  a_{11} & a_{12} \\
  a_{21} & a_{22}
\end{bmatrix}
\begin{bmatrix}
  E \\
  N
\end{bmatrix} +
\begin{bmatrix}
  x_0 \\
  y_0
\end{bmatrix}
\]

is completely determined if the elements of \((a_{ij})\) and \(x_0, y_0\) are known. One approach to geographic referencing could be to compute the transformation from orbit parameters and similar information. Then inversion of the transformation, as long as \((a_{ij})\) is not singular, would give the geographic coordinates corresponding to every point in the picture.

However, this approach has serious weaknesses. Because satellite velocities are in the range 5-10 km/sec, a rather small ephemeris error could cause a significant location error. Also, the approach assumes that there are no attitude, pointing, or motion errors. Those, as well as the nonlinearities discussed in the preceding section, could cause location errors or have the result that values \(a_{ij}\) different from those calculated actually characterize the transformation.

There is an approach circumventing these difficulties. The transformation contains six unknown parameters: \(a_{11}, a_{12}, a_{21}, a_{22}, x_0, y_0\). If both sets of coordinates \((x, y)\) and \((E, N)\) are known for at least three points (six components), the unknown parameters can be solved for. If three points are used, the solution is algebraic. If there are more than three points, the resulting set of equations is overdetermined. In this case, a method such as least squares can be used to solve for the six unknowns. The latter approach is preferable; the influence of modeling and observational errors is minimized when a sufficient number of judiciously located "control points" (known points) is used.

Following is an outline of the well-known classical generalized least squares method. Suppose \(N\) observations are made of some "observable" \(y\), and \(y\) is assumed to have the form

\[
y(x) = \sum_{k=1}^{n} a_k f_k(x), \quad n < N
\]

The least-squares assumption states that the "best" estimate \(^\wedge y\) of \(y\) minimizes the function

\[
Q = \sum_{i=1}^{N} w_i (y_i - ^\wedge y_i)^2
\]
where \( y_i \) is the \( i \)th observed value of \( y \), \( \hat{y}_i \) is the value of (1) at \( x=x_i \) with some set of values assigned to \( \{a_k\} \), and \( \{w_i\} \) is a set of weights. The resulting equation for the set of best estimates of the coefficients \( a_k \) is, in matrix notation,

\[
\hat{a} = [F' W F]^{-1} F' W \mathbf{y}
\]

(3)

Here \( \mathbf{y} \) is the (column) vector whose components are \( y_i \), \( W \) is the matrix whose diagonal elements are \( w_i \) and whose off-diagonal elements are zero, \( F \) is the matrix whose rows are \( f' \), the transposes (row vectors) of the set of vectors whose elements are \( f_k(x_i) \) (one vector for each \( x_i \)), and \( F' \) is the transpose of \( F \). If \( y(x) \) is not a linear function of the fit parameters \( \{a_k\} \), the formulation can still be applied. The expression for \( y(x) \) is linearized by expanding in a Taylor series and retaining only the leading terms, and then proceeding similarly. Because of the approximation made, the solution is iterative. An equation similar to Equation (3) gives each successive estimate of \( \hat{a} \), where the right-hand side depends on the result of the previous iteration.

The transformation is linear in the six parameters to be adjusted by the fit, so an iterative formulation is not necessary. However, both the dependent and independent variables are vectors, whereas Equations (1) - (3) only considered scalars. Fortunately, the generalization from one to several dimensions is straightforward. There is essentially no change for the independent variable, which only appears in the equations indirectly, as a summation index labeling points at which observations are made. The dependent variable causes no more trouble. It is perfectly logical to minimize a function of the vector \( \vec{\varepsilon}_i \) (the deviation between the observed quantity and its estimate at the \( i \)th observation), such as

\[
Q = \sum_{i=1}^{N} w_i \vec{\varepsilon}_i \cdot \vec{\varepsilon}_i
\]

(4)

Performing the steps of the analysis shows that Equation (4) treats components of vector observations, for all observations, in the same way that Equation (2) treats scalar observations. Equation (4) is a straightforward generalization of (2) to several dimensions. Another simple change puts the treatment of all components on an equal footing, whether they are the components of the same or different error vectors. It will be noted that, in Equation (4), all components of one vector are given the same weight. This is an unnecessary restriction, and it actually simplifies the mechanization of the equations to remove it and allow each component to have a different weight. Suppose the vectors have \( M \) components, labeled by \( j \) \((j=1, 2, \ldots, M)\), and define \( m=(i-1)M+j \) \((i \) labels observations, \( i=1, 2, \ldots, N)\). Then Equation (4) can be generalized to
Q = \sum_{m=1}^{NM} w_m \varepsilon_m^2 \tag{5}

where the first \( M \) terms apply to the first observation, the second \( M \) to the second observation, etc. Then a computer program implementing Equation (3) can be used to perform the solution of a multi-dimensional regression problem, with only some changes in indexing.

In order to solve for the parameters of the transformation by the method just described, it is necessary to know the coordinates of several points, called "control points," as accurately as possible in both the UTM and the scan line-point count systems (the latter is called the pixel coordinate system). Control points can be any features that can be readily identified, such as highway intersections, projecting tips of islands or peninsulas, ends of bridges, distinctive buildings, etc. The UTM coordinates of control points can be found quite accurately by reference to standard maps.

Accurate determination of pixel coordinates is more of a problem. For the case of the scanner carried aboard Landsat, for example, an error of a few pixels corresponds to several hundred meters on the ground. Although the least squares fitting may absorb some error, it is unwise to rely on this. (In particular, systematic errors in the same direction will not be removed by the least squares process.) It is highly desirable to locate control points to the nearest pixel. Unless special equipment capable of making highly accurate measurements on small-scale imagery (small enough so that the eye blends pixels together) is available, this requires enlargement sufficient to permit the discrimination of individual pixels. As is discussed below, it is useful to employ some image enhancement techniques in addition to simple enlargement.

The procedures to be described are illustrated in Figures 35-37. Figure 35 shows a portion of (one band of) Landsat frame covering part of North Alabama. The squares marked on it indicate small regions containing distinctive features to be used as control points. The next two figures concentrate on one of those regions (the area around Guntersville, Alabama). Figure 36 shows the result of enlarging the image of the small region by repeating each pixel 15 times in both dimensions. (Approximately the same effect would be produced by photographic enlargement.) It is seen that the blockiness of the image - which makes it easy to count pixels - tends to interfere with the recognition of features. When one is sufficiently far from the picture, the eye smooths out the blockiness; however, then it is impossible to count pixels.

It would appear that smoothing the image would help. Since the effect in general of smoothing is to assign different values to adjacent pixels, where each pixel in the enlarged image corresponds to a fraction of a pixel in
ERTS 1104-15552-6, (1,741) TO (1500,3240) TARCOG REGION

Figure 35. Locations of Ground Control Points for the TARCOG Region.
Figure 36. Enlargement of One Region by Pixel Repetition
the original, smoothing is equivalent to an increase in resolution. The "right" way of achieving this is to use the sampling theorem for interpolation to obtain the intermediate values. The implementation of the appropriate formula is slow computationally, so it is desirable to use approximations requiring less computer time. Figure 37 shows the result of using a bicubic (two-dimensional cubic) interpolation formula approximating the "right" expression to perform the enlargement. In addition, linear density stretching has been applied to effect contrast enhancement.

Working with enlarged imagery, a small measurement error corresponds to only a fraction of a pixel in the original. So it is feasible to locate control points in pixel coordinates to the nearest pixel. Then, with control point locations in both coordinate systems, the least squares solution for the transformation parameters can be performed. It may then be found that the residuals at some control points are excessively large. This situation may persist at a few points even after all errors that can be accounted for have been corrected, due perhaps to errors in the maps used. Such points should be discarded and the solution repeated. (Possibly, subsequent analysis will explain the discrepancy.) The solution is relatively insensitive to the number of points used, unless that number is close to the minimum.

It is appropriate to mention here that these methods may only need to be used the first time geographic referencing is applied to a scene. For other observations of the same scene - for example, subsequent Landsat passes - it may be possible to avoid repeating all of the same steps. Instead, small regions surrounding the control points (whose locations are known) from the first image can be used as templates to search for the locations (pixel coordinates) of the control points in the other observations. Fast sequential methods for doing so exist.\[11, 12\]
Figure 37. Enlargement of One Region Using Bicubic Interpolation, Enhanced by Linear Density Stretching
3-4. GEOMETRIC CORRECTION

The transformation from ground or UTM coordinates to image pixel coordinates may be used to determine the pixels required in constructing an image in conformance to a UTM map projection. For every UTM grid position (typically at 50 meter spacing) the corresponding pixel coordinates are calculated, and the density at that point becomes the output pixel density. In general, the calculated pixel coordinates are not integers, i.e. the location is between image pixels. Hence, the density must be interpolated from the neighboring pixels. Three interpolation methods will be presented.

For nearest neighbor resampling, the pixel value closest to the position of the correct image pixel is chosen for the result of the interpolation operation. In other words, the coordinates \((x', y')\) of the desired pixel are computed by rounding off the computed coordinates \((x, y)\) to the nearest integer, using

\[
\begin{align*}
x' &= x + 0.5 \\
y' &= y + 0.5.
\end{align*}
\]

This leads to a position error in the nearest neighbor resampled image as large as ± 0.5 pixel spacing. However, an advantage is that the magnitudes of the samples are retained exactly.

The bilinear interpolation method scales the output value linearly between the density values of two neighboring pixels. If the neighboring pixels have densities \(A\) and \(B\), then the scaled density at a distance \(\Delta x\) from \(A\) is

\[
Q_1 = A + \Delta x (B - A).
\]

In two dimensions, the input values are the four corners of the square containing the calculated pixel location. If \(A\) and \(B\) are the densities of the top two corners and \(C\), \(D\) of the bottom, the interpolated output along the bottom line is

\[
Q_2 = C + \Delta x (D - C).
\]

The values \(Q_1\) and \(Q_2\) are then interpolated in the orthogonal direction to give the final result:

\[
Q_1 + \Delta y (Q_2 - Q_1).
\]

For spatial frequency band-limited data, the ideal interpolation function is \(\frac{\sin (x)}{x}\). A continuous signal can be sampled at discrete intervals and then the \(\frac{\sin (x)}{x}\) filter can be applied to the discrete data to completely reconstruct the continuous signal. This can be done provided the sampling frequency meets the Nyquist criterion, i.e. it is at least twice the highest spatial frequency.
However, this function has significant magnitude until very high x, requiring an impractically large number of terms (>1000) for each interpolated value. In addition, Landsat MSS data is not band-limited, but in fact contains aliasing errors, which are not removable after resampling without severe resolution degradation. Thus a limited extent approximation is made to this function.

The cubic convolution function is an approximation to the \( \sin(x)/x \) function, maintaining the main positive lobe and the first negative lobe on either side. No term beyond \( x=2 \) exists. The functions are shown in Figure 38. In these graphs the x axis can be taken as distance from the resample location to the discrete data locations and the y axis is the response value. The equations of the cubic function for the two lobes may be expressed as

\[
\begin{align*}
\text{f}_1(x) &= a_1 |x|^3 + b_1 x^2 + c_1 |x| + d_1 & 0 \leq |x| \leq 1 \\
\text{f}_2(x) &= a_2 |x|^3 + b_2 x^2 + c_2 |x| + d_2 & 1 \leq |x| \leq 2.
\end{align*}
\]

The eight coefficients may be determined by applying the following eight conditions:

\[
\begin{align*}
\text{f}_1(0) &= 1 \\
\text{f}_1(1) &= 0 \\
\text{f}_2(1) &= 0 \\
\text{f}_2(2) &= 0 \\
\text{f}_1'(0) &= 0 \\
\text{f}_1'(1) &= \text{f}_2'(1) \\
\text{f}_1''(0) &= 0 \\
\text{f}_2'(1) &= 0.
\end{align*}
\]

The cubic convolution polynomials then become \([13]\)

\[
\begin{align*}
\text{f}_1(x) &= |x|^3 - 2 x^2 + 1 & 0 \leq |x| \leq 1 \\
\text{f}_2(x) &= -|x|^3 + 5 x^2 - 8 |x| + 4 & 1 \leq |x| \leq 2.
\end{align*}
\]

Cubic convolution is accomplished using a 4 x 4 pixel subimage about the resample location. First, a vertical axis is passed through the resample location. Next, four horizontal axes are made through the four rows of pixels. At the intersection of the vertical axis and each of the four horizontal axes an interpolation value is computed. Finally, these four interpolated values are reinterpolated along the vertical axis to produce a value at the resample location. The interpolation formula above is used to do each of the five interpolations.
Figure 38. Interpolation Functions for Resampling.
3-5. SUPERPOSITION OF BOUNDARIES

The mathematical details of some of the steps described in Section 2-5 are presented below.

(i). Thinning and Conversion to SLIC

Let \( P_{ij} \) be the density at the point \((i, j)\) in the digitized boundary image produced by the microdensitometer. Then, the values \( P_{ij} \) are generally available as a sequential file consisting of several records, the \( i \)th record consisting of

\[ \{P_{ij} \mid j=1, \ldots, J \} \quad \text{for} \quad i = 1, \ldots, I. \]

Now, let a threshold \( t \) be selected such that all points in the image satisfying \( P_{ij} \leq t \) can be decided to be boundary points. The boundary data may then be compressed by setting single bits to "1" at the boundary positions. The boundary lines were thinned by a peeling algorithm which remains outer layers of thick lines while ensuring that connectivities are preserved.

To decide whether a particular boundary point should be deleted (i.e. the bit corresponding to it changed to 0), we examine a 3 x 3 neighborhood centered around the point. Consider the array

\[
\begin{array}{ccc}
  a & b & c \\
  d & e & f \\
  g & h & i \\
\end{array}
\]

where each letter represents a binary pixel. It is to be decided whether \( e \), which is presently equal to 1 should he changed to 0. The conditions for a 'top peel' will be derived below and those for peeling from the other directions follow by symmetry.

First of all, \( e \) should be a top boundary point. That is, there should be no boundary point directly above \( e \) and there should be a boundary point below \( e \). Therefore \( b = 0 \) and \( h = 1 \) are necessary conditions. Suppose \( b h = 1 \). (Here, \( \bar{b} \) denotes the complement of \( b \)). Then, we need only check whether \( e \) is a nonessential boundary point, that is, whether two 0's in the 3 x 3 array which are disconnected will stay disconnected where \( e \) is made 0. Connectivity, in this context, is defined as the existence of a path not including 1's and consisting only of horizontal and vertical segments.

Now, it is easy to see that \( e \) is essential if and only if \( \bar{a} \bar{d} = 1 \) or \( \bar{c} \bar{f} = 1 \). Therefore, the condition for a top peel is that

\[ \bar{b} h (\bar{a} + d) (\bar{c} + f) = 1. \]
Equivalently, to perform a top peel we set

\[ e = e \cdot (b + h + a \bar{d} + cf). \]

It is convenient to implement the above equation by employing bit manipulation routines operating on pairs of 32 bit words, thereby performing the top-peel operation in parallel on 32 pixels. This is done by using the "current" array in place of \( e \), the "previous" array for \( b \), the "next" array in place of \( h \). Also, the previous, current, and next arrays are right (left) shifted by one bit and used for \( a, d \) and \( g \) (\( c, f \) and \( i \)) respectively in the peeling formulas.

The program minimizes the movement of data in core by using circular buffers for storing the "previous, current and next" arrays. An array \( J \) dimensioned 3 is used to store the indices pointing to these arrays (\( J(1) \to \) previous, \( J(2) \to \) current, \( J(3) \to \) next) and after finishing each record, only the array \( J \) is updated.

Also, top, left, bottom and right peels are performed one after the other by just one pass through the data (thus minimizing I/O) by storing the intermediate results in core and operating with a phase lag.

(ii). Smoothing

For the boundary data to be useful for extracting interior of regions, it is necessary that the boundary represented be "contiguous" at all points. Continuity and connectivity in the digital domain can be defined as follows [15].

The points \((x_1, y_1) \) and \((x_2, y_2) \) are said to be 4-adjacent if

\[ |x_1 - x_2| + |y_1 - y_2| = 1. \]

The points \((x_1, y_1) \) and \((x_2, y_2) \) are said to be 8-adjacent if

\[ \text{Max}\ (|x_1 - x_2|, |y_1 - y_2|) = 1 \]

A curve is said to be continuous at a point \((x, y) \) on it, if there exists at least one point on the curve which is 8-adjacent to \((x, y) \).

The contiguity count for a point \( P \) on a curve is defined as the number of points on the curve that are 8-connected to \( P \).

Two points \( P \) and \( Q \) on a curve (in a region) are said to be connected if there exists a sequence of points \( P_0, P_1, P_2, \ldots, P_n \) such that
A region is said to be connected if all points in it are connected to each other.

Now, it is easy to see that closed curves are necessary to separate a given region into several connected subregions. Also, if the contiguity count for every point on a curve is greater than or equal to 2, then the curve is closed.

If closed curves in the continuous domain without retracing (or "burrs") were digitized, then the digital curves would be closed according to the above definition. However, when the boundaries are digitized using a microdensitometer and undergo a thinning process, it is impossible to produce closed boundaries as defined above. But an approximation to closed boundaries can be produced wherein there are closed components which contain the major connected regions of interest and a few "burrs" are retained. Smoothing is the process which converts thinned boundaries into (approximately) closed boundaries.

The smoothing algorithm proceeds as follows. At each point, the contiguity count is determined. This is done by testing the row containing the point and the two adjacent rows to see whether there are any 8-adjacent boundary points. The search is quite simple, if it is remembered that the column coordinates in the SLIC format are arranged in ascending order. Therefore, when looking for boundary points adjacent to (i, j) we need only check the (i-1)st and (i+1)st rows until the column coordinates exceed j+1. Also, in the ith row we need only check the column coordinates previous and next to j (assuming no repetitions).

Now, if the contiguity count of a point is less than 2 a neighborhood of the point is examined. The size of the neighborhood determines how large a discontinuity will be patched and should be pre-specified. (A square neighborhood with sides of the order of the thickness of the original, i.e., unthinned, boundaries is generally satisfactory.) Two nearest points (if any) which are not connected either to each other or to the given point are determined. Digital approximations to straight lines joining the given point to these two points are generated and stored as row and column coordinates.

After producing the patches at all points as required, the new boundary points are sorted, merged with the input and arranged in the SLIC format.
The problem of applying a general geometric transformation on a given boundary image can be stated as follows.

Let $B = \{(i_1, j_1), (i_2, j_2), \ldots, (i_n, j_n)\}$ be a set of points obtained by digitizing a curve using a unit grid in the $x$-$y$ plane. Let

\[
x' = f(x, y) \\
y' = g(x, y)
\]

be a coordinate transformation. Then, the problem is to find a set of integer coordinates

\[
B' = \{(k_1, t_1), (k_2, t_2), \ldots, (k_m, t_m)\}
\]

which represent the digitization of the same curve using a unit grid in the $x'$-$y'$ plane.

This is a resampling problem. It can be solved "exactly" if the original curve in the continuous domain has a bandlimited spectrum and the sampling in the $x$-$y$ plane is fine enough. In that case, one could reconstruct the curve in the continuous domain using sampling theorem and resample in the $x'$-$y'$ plane. Since this is a time-consuming process, we use an approximation as follows.

First, generate the set of points $\{(x'_r, y'_r) \mid r = 1, \ldots, n\}$

where

\[
x'_r = f(i'_r, j'_r) \\
y'_r = g(i'_r, j'_r)
\]

Now, $x'_r$ and $y'_r$ are, in general, nonintegral. Therefore, we choose the nearest integers to $x'_r, y'_r$ and let them represent the boundary points. Further, to assure that connectivity is preserved after the transformation, we join $(x'_r, y'_r)$ and $(x'_s, y'_s)$ by a straight line whenever $(i'_r, j'_r)$ and $(i'_s, j'_s)$ are 8-adjacent, and generate a digital approximation to the straight line.

This method can be conveniently implemented with the data in SLIC format (a more convenient format for this particular operation is the "chain code" [1]). The only tricky part of the algorithm is to handle the storage and rearrangement of the coordinates of the new boundary points generated when large images are handled. If the boundary coordinates produced for the entire geometrically transformed image can be held in the main memory at a time, it can be written
out on a sequential file in SLIC format by array sorting in core. Otherwise, it is necessary to dump the coordinate data on a direct access device whenever the core capacity is exceeded and then sort the data from the direct access device.

(iv). Thickening

Boundary lines can be thickened by "growing" each boundary point around itself by a given amount. This is, if \((i, j)\) is a boundary point, \((k, l)\) is also treated as a boundary point for all \((k, l)\) such that \(|i - k| \leq h\) and \(|j - l| \leq h\). Thickening boundaries in two dimensions starting from the data in SLIC format is accomplished as follows. If \(j_1, j_2, \ldots, j_n\) are the column coordinates corresponding to the \(i\)th row in the given boundary image, then the set

\[ \theta_i = \{ j \mid |j - j_r| \leq h \text{ for some } r \in [1, n] \} \]

is formed. This represents the horizontally thickened \(i\)th row. Now, to thicken in the vertical direction, we simply set the output column coordinate set \(T_i\) for the \(i\)th row to be

\[ T_i = \bigcup_{r=-h}^{h} \theta_{i+r} \]

When \(T_i\) is generated, it is arranged in ascending order, repetitions, if any, are eliminated, and the coordinate set is written out as a record on a sequential file.

(v). Generation of Region Identification Maps (RIM)

Starting from the basic definition of connectivity for regions given in Section (ii), we can develop an algorithm to identify separate connected regions given the boundary data. An image consisting of a unique number assigned to each connected region is called a region identification map (RIM). We shall adopt the convention that 0 be used for boundary points and 1 for "exterior" points (i.e., points connected to points in the region outside the rectangle containing the given boundary points). The algorithm to generate a RIM proceeds as follows.

Let \((b_{ij} \mid i = 1, \ldots, I, j = 1, \ldots, J)\) be the set of column coordinates of the boundary points (stored in SLIC format). Choose \(N \geq B - 1 + 1 \) where
\[ \bar{b} = \max_{i,j} b_{ij} \quad \text{and} \quad \underline{b} = \min_{i,j} b_{ij} \]

Let \( p \) and \( q \) be two \( N \)-vectors. These are the vectors in which the region identification numbers (RIN) will be generated. The vector \( p \) will be used to store the RIN for the previous row and \( q \) will be used to store those for the current row as they are generated. Also a scalar \( \lambda \) is used to count the number of regions found.

Initially, all points in the "previous" row are in the exterior. Therefore, the vector \( p \) is initialized with all components equal to unity. Also, \( \lambda \) is set to unity. Now, consider the \( i \)th row. The boundary data

\[ \{ b_{ij} \mid j = 1, 2, \ldots, J_i \} \]

are read from the sequential file. Since in the SLIC format \( b_{ij} \) are in ascending order, the points before \( b_{i1} \) and after \( b_{iJ_i} \) are exterior points. Therefore,

\[ q_k = 1 \quad \text{for} \quad 1 \leq k \leq b_{i1} - \underline{b} + 1 \quad \text{and} \quad b_{iJ_i} - \underline{b} + 1 \leq k \leq N. \]

Also,

\[ q_k = 0 \quad \text{for} \quad k = b_{ij} - \underline{b} + 1, \quad j = 1, 2, \ldots, J_i. \]

Now, \( q_k \) must be found only for values of \( k \) in intervals

\[ A_j = \{ k \mid b_{ij} - \underline{b} + 1 < k < b_{i,j+1} - \underline{b} + 1 \} \quad \text{for} \quad j = 1, 2, \ldots, J_i - 1. \]

For every such interval, there are two possibilities.

Case 1: There is a \( k_0 \in A_j \) such that \( p_{k_0} \neq 0 \). In this case, all points in the interval in the current row are in the region \( p_{k_0} \). Therefore \( q_k = p_{k_0} \) for all \( k \in A_j \).

Case 2: \( p_k = 0 \) for all \( k \in A_j \). In this case, it is decided that a new region might be beginning. The region count \( \lambda \) is changed to \( \lambda + 1 \). Also, \( q_k = \lambda \) for all \( k \in A_j \).

Now, \( q \) contains the RIN for the current row. The array \( q \) can be written out and also moved into array \( p \), to make it the "previous" row for handling the \((i+1)\)st row.

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This procedure, as described so far, produces RIN's assuring that no two unconnected regions will have the same RIN. However, in many cases, the same region may have more than one RIN. This happens since connectivity in the \((i+1)\)st through \(i\)th rows is not known when the \(i\)th row of the RIM is being generated. Therefore, it is necessary to update the region numbers after connectivity between differently numbered regions is discovered. To do this, a "Region Identity Matrix (RIMX)" \(D\) is used to store the information about the connectivity between differently numbered regions. The matrix \(D\) is a binary matrix with \(d_{ij} = 1\) if regions numbered \(i\) and \(j\) are connected and 0 otherwise. Initially, \(D\) is set equal to a null matrix. When a new region number \(\lambda\) is started, \(d_{\lambda\lambda}\) is set to 1. Also, after the vector \(q\) is found for the \(i\)th row, \(D\) is modified by letting

\[
d_{k\ell} = d_{kq} = 1 \quad \text{for all} \quad (k, \ell) \text{ such that } k = p_j \neq 0, \ell = q_j \neq 0, j = 1, \ldots, N.
\]

Now, at any stage, the matrix \(D\) indicates which region numbers determined thus far represent the same region. This is analogous to the connectivity matrix used widely in graph theory [16].

Connectivity matrices have some interesting properties which are very useful in this application. These will be introduced briefly here. Let \(\lambda\) and \(\mu\) be two region numbers. Suppose there exists a sequence of region numbers \(\lambda_0, \lambda_1, \lambda_2, \ldots, \lambda_n\) such that \(\lambda = \lambda_0, \mu = \lambda_n\) and \(d_{\lambda_i\lambda_{i+1}} = 1\) for \(i = 0, 1, \ldots, n-1\). Then the regions \(\lambda\) and \(\mu\) are said to be connected by a path of length \(n\). Now, if \(D_n\) is evaluated using ordinary matrix multiplication, then the \((\lambda, \mu)\)th element will be equal to the number of paths of length \(n\) between \(\lambda\) and \(\mu\). Instead, if a logical matrix product is used (using \(1+1 = 1, 1+0 = 1, 1 \times 1 = 1\) and \(1 \times 0 = 0\)) find \(D^n\), then the \((\lambda, \mu)\)th element of \(D\) will indicate whether region \(\mu\) can be reached from \(\lambda\) via a path of length \(n\). If the matrix \(R\) is defined as

\[
R = D + D^2 + D^3 + \ldots + D^n
\]

where \(n\) is chosen such that \(D^{n+1} = D^n\), then \(R_\lambda = 1\) if there exists a path between \(\lambda\) and \(\mu\) of any length and \(R_\lambda = 0\) otherwise.

An efficient method to obtain \(R\) is to generate a sequence of matrices recursively:

\[
R^0 = D
\]

\[
R^i = R^{(i-1)} + (R^i \times R^i) \quad \text{for } i = 1, 2, 3, \ldots.
\]

The computations are stopped when \(R^n = R^{(n-1)}\). (The matrix \(R^i\) then indicates paths of length less than or equal to \(2^i\).)
Now, the matrix $R$ can be used to find the smallest RIN to be assigned to each connected region to which several RIN's have been given. The records of the RIM computed can then be updated using table lookup.

When handling large images, it might become necessary to perform several such updates, depending on the memory assigned to the computation of $D$ and $R$. If the size assigned to $D$ is exceeded during the computation of the $i^{th}$ row, the $(i-1)$ rows before that are updated using the corresponding matrix $R$, the updated $(i-1)^{st}$ row is stored in $p$, the value of $\lambda$ is set to the largest region number in the updated $(i-1)$ rows of the RIM, $D$ is set equal to an identity matrix and the computation for the $i^{th}$ row is restarted.

Several steps are involved in superposing political boundaries on remotely sensed images. The complexity of handling this problem depends on the facilities available for digitizing the boundary information. The steps described in this memorandum have been designed to handle data digitized using a microdensitometer. The process is considerably simplified if a digitizing plotter/tracer is used so that the boundaries can be digitized by manually tracing the curves from standard maps. In that case, each region can be digitized separately as indicated in [16]. Converting the data corresponding to each region after geometric correction into the so-called "Tightly Closed Boundary" (TCB) format, wherein the extrema and inflections of the boundary are repeated, we would then have a very simple method for extracting individual regions or generating an RIM.
IV. RESULTS

4-1. PRELIMINARY DATA HANDLING

Landsat coverage of the TARCOG region was extracted from the computer compatible tape of scene 1104-15552, obtained on November 4, 1972. The region extracted was lines 1 to 741 and samples 1500 to 3240. (Sample 3240 is the last sample in the scene, due to the fact that the TARCOG region extends out of the Landsat scene slightly.)
Training samples were selected from a region of size 500 x 500 pixels centered on the city of Huntsville. Two sets of training samples were selected. One set was chosen to be representative of four major land use classes (urban, agriculture, forest, and water); and the other of seven Level I land use classes (urban and built-up, agriculture, forest, wetland, pasture, water, and barren). The training areas were shown in Figure 14.

Linear decision functions were then computed using these sets of training data. The coefficients of the decision functions, in the order in which testing for a positive result is performed, are given in Tables 1 and 2.

The decision functions are then tested by using them to classify the training data. This procedure gives a measure of the accuracy of the decision functions in classifying the training data, but is no guarantee of the results when applied to unknown data from other parts of the scene. This is because there may be present data corresponding to a certain class, but differing sufficiently from the training data of that class that it is classified incorrectly. This situation arises when the training data is not representative of all data corresponding to each class type. However, the separation of the training sample data by the discriminant functions is accurate to approximately 95 percent. The classification assignments of the training data are given in Tables 3 and 4. Using an IBM 360/65 computer, the computer time required to calculate the four class discriminants was 39 seconds. For the seven class discriminants the CPU time was 75 seconds. In each case, 100 training samples for each class were used in the calculations.

The discriminant functions were then tested on the 500 x 500 pixel Huntsville scene, since the land usage of this relatively small area was well known. The class occupancy of this area by number of samples and percentages is given in Tables 5 and 6.

A classification map showing seven classes in the Huntsville region is given in Figure 39.

The classification into four classes required 21 minutes, 18 seconds of computer time. The rate of classification is 2933 pixels per second or 0.3409 millisec. per pixel.

For seven classes, the corresponding values are 1869 pixels per second or 0.5350 millisec. per pixel.

For comparison purposes, Figures 40 and 41 show land use maps of the Jetport region obtained by computer analysis of high altitude (60,000 ft.) three
band photography and by manual analysis of low altitude (6,000–12,000 ft.) four band photography. It is apparent that the areas of significant sizes are classified into the same land usage in each case.
Table 1. Four Class Linear Discriminant Coefficients
(In order of testing; discriminant function is
\[ G = w_1x_1 + w_2x_2 + w_3x_3 + w_4x_4 + w_5 \])

<table>
<thead>
<tr>
<th>Land Use Class</th>
<th>Coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( w_1 )</td>
</tr>
<tr>
<td>Water</td>
<td>0.1100</td>
</tr>
<tr>
<td>Forest</td>
<td>-0.2492</td>
</tr>
<tr>
<td>Agriculture</td>
<td>-0.4470</td>
</tr>
<tr>
<td>Urban</td>
<td>0.4470</td>
</tr>
</tbody>
</table>

Table 2. Seven Class Linear Discriminant Coefficients

<table>
<thead>
<tr>
<th>Land Use Class</th>
<th>Coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( w_1 )</td>
</tr>
<tr>
<td>Barren</td>
<td>0.07296</td>
</tr>
<tr>
<td>Pasture</td>
<td>0.06251</td>
</tr>
<tr>
<td>Water</td>
<td>0.2855</td>
</tr>
<tr>
<td>Wetland</td>
<td>0.01383</td>
</tr>
<tr>
<td>Urban</td>
<td>0.5952</td>
</tr>
<tr>
<td>Forest</td>
<td>-0.4142</td>
</tr>
<tr>
<td>Agriculture</td>
<td>0.4142</td>
</tr>
</tbody>
</table>
Table 3. Classification of 4 Class Training Samples

<table>
<thead>
<tr>
<th>Land Use Class</th>
<th>Percent Correct</th>
<th>Number of Samples Classified as:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Urban</td>
<td>Agriculture</td>
</tr>
<tr>
<td>Urban</td>
<td>95</td>
<td>95</td>
</tr>
<tr>
<td>Agriculture</td>
<td>97</td>
<td>2</td>
</tr>
<tr>
<td>Forest</td>
<td>98</td>
<td>0</td>
</tr>
<tr>
<td>Water</td>
<td>100</td>
<td>0</td>
</tr>
</tbody>
</table>

Average Accuracy = 97.5%

Table 4. Classification of 7 Class Training Samples

<table>
<thead>
<tr>
<th>Land Use Class</th>
<th>Percent Correct</th>
<th>Number of Samples Classified as:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Urban</td>
<td>Agriculture</td>
</tr>
<tr>
<td>Urban</td>
<td>82</td>
<td>82</td>
</tr>
<tr>
<td>Agriculture</td>
<td>99</td>
<td>0</td>
</tr>
<tr>
<td>Forest</td>
<td>99</td>
<td>0</td>
</tr>
<tr>
<td>Wetland</td>
<td>97</td>
<td>0</td>
</tr>
<tr>
<td>Pasture</td>
<td>99</td>
<td>0</td>
</tr>
<tr>
<td>Water</td>
<td>100</td>
<td>0</td>
</tr>
<tr>
<td>Barren</td>
<td>87</td>
<td>13</td>
</tr>
</tbody>
</table>

Average Accuracy = 94.7%
Table 5. Four Class TARCOG Land Use

<table>
<thead>
<tr>
<th>Class</th>
<th>Number of Samples</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Urban</td>
<td>241,445</td>
<td>6.44</td>
</tr>
<tr>
<td>Agriculture</td>
<td>1,378,609</td>
<td>36.76</td>
</tr>
<tr>
<td>Forest</td>
<td>2,003,588</td>
<td>53.43</td>
</tr>
<tr>
<td>Water</td>
<td>126,358</td>
<td>3.37</td>
</tr>
<tr>
<td>TOTAL</td>
<td>3,750,000</td>
<td></td>
</tr>
</tbody>
</table>

Table 6. Seven Class TARCOG Land Use

<table>
<thead>
<tr>
<th>Class</th>
<th>Number of Samples</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Urban</td>
<td>303,545</td>
<td>8.09</td>
</tr>
<tr>
<td>Agriculture</td>
<td>969,926</td>
<td>25.86</td>
</tr>
<tr>
<td>Forest</td>
<td>2,021,475</td>
<td>53.91</td>
</tr>
<tr>
<td>Wetland</td>
<td>38,745</td>
<td>1.03</td>
</tr>
<tr>
<td>Pasture</td>
<td>321,830</td>
<td>8.58</td>
</tr>
<tr>
<td>Water</td>
<td>92,104</td>
<td>2.46</td>
</tr>
<tr>
<td>Barren</td>
<td>2,375</td>
<td>0.06</td>
</tr>
<tr>
<td>TOTAL</td>
<td>3,750,000</td>
<td></td>
</tr>
</tbody>
</table>
Figure 39. Seven Land use Classes in the Huntsville Region using Landsat Data.

Urban - Red
Agriculture - Yellow
Forest - Green
Wetland - Cyan
Pasture - Magenta
Barren - Black
Water - Blue
Figure 40. Four Class Map of the Jetport Region, obtained from RB-57 Photography.
Figure 41. Manually Determined Land use Map of the Jetport Region, obtained from Low Altitude Photography
The accuracy of the classification has been studied in detail. In one procedure, a ground truth study was conducted in 101 randomly selected study areas located in Madison County. Each study area consisted of a five pixel by five pixel matrix, centered on the random location. Thus the 101 study areas resulted in 2525 pixels, the locations of which were visited in the field, classified and compared with the computer designations. It was determined that 67.4 percent of the study pixels were correctly identified. Since agriculture and pasture are the same in Level I classification, these two groups can be combined. When this is done, the percentage of correctly identified pixels rises to 76.3 percent. Table 7 gives the classifications of the 2525 pixels whose actual land use was determined. From Table 8 it is seen that for a pixel classified as urban there is a 0.675 probability that it is actually urban, a 0.134 probability that it is agriculture/pasture, a 0.025 probability that it is water, and a 0.162 probability that it is actually water. The Bayesian probability of a pixel being classified correctly is the probability of correct classification divided by the sum of the probabilities of other actual classes being so classified. The Bayesian probabilities of correct classification in each land use category are:

- urban: 0.677
- agriculture/pasture: 0.465
- forest: 0.618
- water: 0.992
- wetland: 0.519

A second accuracy analysis was performed by examining low altitude photographs of rural areas on Sand Mountain, since it is known that bare soil in agricultural areas is easily confused with urban areas. This is illustrated in Figure 42, in which the outlined areas appear as urban, since they are bright in the green band image, but are in fact are agricultural land usage, as determined from the low altitude photography. In the classification map, light areas are urban, gray agriculture, and dark are forest.
Table 7. Classification of Actual Land Use Classes [17]

<table>
<thead>
<tr>
<th>Actual Land Use Class</th>
<th>Number of Pixels</th>
<th>Percent Classified Correctly</th>
<th>Urban</th>
<th>Agriculture &amp; Pasture</th>
<th>Forest</th>
<th>Water</th>
<th>Wetland</th>
<th>Barren</th>
</tr>
</thead>
<tbody>
<tr>
<td>Urban</td>
<td>126</td>
<td>67.5</td>
<td>85</td>
<td>35</td>
<td>4</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Agriculture &amp; Pasture</td>
<td>1444</td>
<td>72.8</td>
<td>193</td>
<td>1051</td>
<td>194</td>
<td>5</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Forest</td>
<td>906</td>
<td>85.2</td>
<td>23</td>
<td>108</td>
<td>772</td>
<td>0</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>Water</td>
<td>37</td>
<td>40.5</td>
<td>6</td>
<td>4</td>
<td>1</td>
<td>15</td>
<td>11</td>
<td>0</td>
</tr>
<tr>
<td>Wetland</td>
<td>12</td>
<td>33.3</td>
<td>0</td>
<td>4</td>
<td>4</td>
<td>0</td>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>Barren</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
Table 8. Classification Probabilities of Actual Land Use Pixels

<table>
<thead>
<tr>
<th>Actual Land Use Class</th>
<th>Number of Pixels</th>
<th>Probability of Actual Land Use Pixels Classified as:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Urban</td>
</tr>
<tr>
<td>Urban</td>
<td>126</td>
<td>.675</td>
</tr>
<tr>
<td>Agriculture &amp; Pasture</td>
<td>1444</td>
<td>.134</td>
</tr>
<tr>
<td>Forest</td>
<td>906</td>
<td>.025</td>
</tr>
<tr>
<td>Water</td>
<td>37</td>
<td>.162</td>
</tr>
<tr>
<td>Wetland</td>
<td>12</td>
<td>.333</td>
</tr>
</tbody>
</table>
Figure 42. Example of Agriculture Misclassification on Sand Mountain.
The area assigned to the urban and built-up class for cities in the TARCOG region should bear a constant ratio to their populations, insofar as the type of housing and the proportion of commercial and industrial development remain constant. The built-up areas of eleven cities in the TARCOG region were determined by counting the pixels assigned to the urban category within a rectangular region encompassing each city. The populations used were the published values for the 1970 census. The populations, areas, and population densities are given in Table 9. The pixel counts given were obtained from a geometrically corrected image in which each pixel represented an area of 57 m x 57 m. The populations vs. area are shown plotted in Figure 43. The solid line in the figure is a least squares fit of a linear function. The equation of fit is

\[ p = 1312.8 A - 974.4 \]

where \( p \) is the population and \( A \) is the area in square kilometers.

A previous study [14] of forty urban areas in the Tennessee River Valley using aerial photography yielded the following fit equations for the years 1953 and 1963, respectively:

\[ p = 1778.3 A - 549.4 \]
\[ p = 1118.7 A - 2928.4 \]

The slope of the fit curve reported in this study falls between these two values. Thus it appears that reasonably consistent results are obtained using computer classified satellite imagery and manually interpreted aerial photography. The average ratio obtained from the three fits is 1403 persons per square kilometer.
### Table 9. Population Data For TARCOG Cities

<table>
<thead>
<tr>
<th>City</th>
<th>Population</th>
<th>Area pixels (57 m²)</th>
<th>km²</th>
<th>PoI Det PoI</th>
</tr>
</thead>
<tbody>
<tr>
<td>Huntsville</td>
<td>137802</td>
<td>31267</td>
<td>101.59</td>
<td>135</td>
</tr>
<tr>
<td>Decatur</td>
<td>41800</td>
<td>13273</td>
<td>43.12</td>
<td>96</td>
</tr>
<tr>
<td>Athens</td>
<td>14360</td>
<td>3583</td>
<td>11.64</td>
<td>123</td>
</tr>
<tr>
<td>Cullman</td>
<td>12900</td>
<td>2457</td>
<td>7.98</td>
<td>161</td>
</tr>
<tr>
<td>Albertville</td>
<td>9963</td>
<td>2485</td>
<td>8.07</td>
<td>123</td>
</tr>
<tr>
<td>Scottsboro</td>
<td>9324</td>
<td>1849</td>
<td>6.01</td>
<td>155</td>
</tr>
<tr>
<td>Hartselle</td>
<td>7355</td>
<td>2192</td>
<td>7.12</td>
<td>103</td>
</tr>
<tr>
<td>Guntersville</td>
<td>6491</td>
<td>902</td>
<td>2.93</td>
<td>221</td>
</tr>
<tr>
<td>Boaz</td>
<td>5621</td>
<td>1551</td>
<td>5.04</td>
<td>111</td>
</tr>
<tr>
<td>Arab</td>
<td>4399</td>
<td>1261</td>
<td>4.10</td>
<td>107</td>
</tr>
<tr>
<td>Madison</td>
<td>3086</td>
<td>1032</td>
<td>3.35</td>
<td>92</td>
</tr>
</tbody>
</table>
Figure 43. Population Density of TARCOG Cities
4-3. GEOGRAPHIC REFERENCING

Twenty-three control points from the 11 areas marked in Figure 35 were used in the geographic referencing solution. Figure 44 shows a UTM grid superimposed on the region to illustrate the solution. The lines nearly parallel to the sides of the picture run north-south, while the other grid lines run east-west. The spacing between grid lines is 10 km in both directions. The heading, skew, and scale factor distortions are clearly apparent in the figure, as grid squares appear as parallelograms.

The theoretical transformation matrix, considering heading and skew effects, was given previously as

\[
\begin{bmatrix}
\frac{1}{\cos dH} & -\sin H_0 & -\cos H_0 \\
\cos (H_0 + dH) & -\sin (H_0 + dH)
\end{bmatrix}
\]

where \( H_0 \) is the heading angle and \( dH \) is the angle of skew.

Evaluating at the center of the scene, latitude 34.5°, the matrix becomes (using \( H_0 = 10.83^\circ \) and \( dH = 3.29^\circ \))

\[
\begin{bmatrix}
-0.18816 & -0.98381 \\
0.97141 & -0.24423
\end{bmatrix}
\]

The scale change between the pixel axes and the UTM axes must be taken into account. The scale in the line count (x) direction is

\[
\frac{1}{.079} = 12.66 \text{ pixels/km.}
\]

and in the pixel count (y) direction is

\[
\frac{1}{.057} = 17.54 \text{ pixels/km.}
\]

applying these factors, the matrix becomes

\[
\begin{bmatrix}
-2.382 & -12.453 \\
17.042 & -4.285
\end{bmatrix}
\]

The empirical matrix from the least squares fit is
\[
\begin{bmatrix}
-2.363 & -12.271 \\
16.784 & -4.231
\end{bmatrix}
\]

The percentage differences of each element in the theoretical matrix are

\[
\begin{bmatrix}
0.80\% & 1.49\% \\
-1.54\% & 1.27\%
\end{bmatrix}
\]

due to the approximations in the theoretical matrix.
4-4. GEOMETRIC CORRECTION

The transformation determined by least squares minimization was applied to the Huntsville area data and the classification maps. The red spectral band image of the Huntsville area after geometric correction is shown in Figure 45. A segment of data sized 80 lines by 100 samples containing the Huntsville Madison County Jetport is shown in Figure 46. Cubic convolution was used, and the scale was chosen to obtain magnification of the image. The axis labels are kilometers in the UTM system. The geometrically corrected four class map with UTM grid superimposed is shown in Figure 47. A land use map of urban and built-up areas as shown in Figure 48 reveals the locations of cities and major roads and airports. The results may also be tabulated in terms of UTM cells of various sizes, as is illustrated in Figure 49.
Figure 45. Red Band Coverage of Huntsville Region, Geometrically Corrected.
Figure 46. Coverage of Huntsville-Madison County Jetport, Geometrically Corrected and Magnified by Cubic Interpolation.
Figure 47. Four Class Map of TARCOG Region Geometrically Corrected with UTM Grid Superimposed.
Figure 48. Urban Land use Areas in TARCOG.
**LAND USE CLASSIFICATION SUMMARY**

**CONTENTS OF 10 KM BY 10 KM UTM CELL WITH SOUTHWEST CORNER (560.4, 3760.1)**

<table>
<thead>
<tr>
<th>Land Use</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>URBAN</td>
<td>4.114 %</td>
</tr>
<tr>
<td>AGRICULTURE</td>
<td>23.315 %</td>
</tr>
<tr>
<td>FOREST</td>
<td>71.917 %</td>
</tr>
<tr>
<td>WATER</td>
<td>0.653 %</td>
</tr>
</tbody>
</table>

**STATISTICS BASED ON ANALYSIS OF 7656 SAMPLES**

**BREAKDOWN BY 1 KM SQUARE CELLS**

<table>
<thead>
<tr>
<th>Land Use</th>
<th>Percentage Occupancy</th>
</tr>
</thead>
<tbody>
<tr>
<td>URBAN</td>
<td></td>
</tr>
<tr>
<td>AGRICULTURE</td>
<td></td>
</tr>
<tr>
<td>FOREST</td>
<td></td>
</tr>
<tr>
<td>WATER</td>
<td></td>
</tr>
</tbody>
</table>

**Figure 49. Classification summary of a 10 km. by 10 km. UTM cell.**

101
4-4-1 EFFECT OF RESAMPLING ON CLASSIFICATION

The uncorrected ERTS MSS data, while good for visual identification of gross ground features, require geometric corrections for comparison with standard maps and images from other sensors. Exact resampling can be performed using sinc functions (under the assumption of band-limitedness of the data), but practical considerations require approximate interpolation to produce the radiometric values in the geometrically corrected images. Two approaches can be used to study the radiometric fidelity of such images. The errors relative to \( \sin(x)/x \) function interpolation are studied or the effects of interpolation on the performance of classifier are experimentally evaluated. It is seen that the overall class occupancy statistics change only slightly, but the point-by-point differences between the classifications of corrected and uncorrected data are noticeable.

In order to study the effect of resampling for geometric correction on classification accuracy, it is necessary to compare the classifications before and after geometric correction with the ground truth. With supervised classification, however, the actual classifications depend on the choice of training samples. Therefore, to have a uniform basis for comparing the classification performance, it is desirable to use the same set of training samples for the "before" and "after" classifications.

Several types of before-and-after comparisons can be made. The ground truth is difficult to gather and convert into machine-readable format for areas large enough to be statistically significant. Therefore, the classification map of the uncorrected data based on training samples chosen as accurately as possible is chosen as a standard. When this classification map is geometrically corrected using the nearest neighbor rule for resampling, the resulting map can be used for point-by-point comparison with the classification of geometrically corrected data. This map can be compared with the classification maps obtained when geometric correction is made using linear or cubic interpolation and the training is performed using samples from the uncorrected image or the corresponding locations in the corrected image. While such comparisons do not show which type of classification is the most accurate, they do indicate whether the effect of geometric correction is significant.

One such study was made on a 500 x 500 pixel four-band Landsat image of a region containing Huntsville, Alabama. A map showing four land use classes (urban, agriculture, forest, water) was obtained using a sequential linear classifier whose discriminant hyperplanes were obtained by training on the raw data samples from each of these classes. The classification map was geometrically corrected for the heading angle and earth rotation effects using nearest neighbor values after resampling. Also, the four bands were individually corrected using the same correction transformation, but using linear and cubic interpolation rules. Four classification maps were produced, two with the original training
samples and two more with training samples taken from the geometrically corrected images. The following abbreviations will be introduced to facilitate reference to the experimental results.

\[ C = \text{Classification map of the uncorrected data}. \]
\[ \text{NN} = \text{Result of geometrically correcting } C \text{ using nearest neighbor values}. \]
\[ L/U = \text{Classification map of the geometrically corrected data using linear interpolation/training samples being from the uncorrected image}. \]
\[ C/U = \text{Classification map of the geometrically corrected data using cubic interpolation/training samples being from the uncorrected image}. \]
\[ L/L = \text{Same as } L/U, \text{ except that the training samples are from the corrected image}. \]
\[ C/C = \text{Same as } C/U, \text{ except that the training samples are from the corrected image}. \]

The four classes are denoted by
\[ 1 = \text{Urban} \]
\[ 2 = \text{Agriculture} \]
\[ 3 = \text{Forest} \]
\[ 4 = \text{Water} \]

When images are geometrically corrected, in general, they become non-rectangular with edges not parallel and perpendicular to the scan lines. For convenience, they are stored in a circumscribing rectangle. Therefore, there are several points in the corrected images files which do not belong in the images. These points are indicated by the class number 0.

Tables 10 through 15 indicate the number of occurrences of each of the classes 0 through 4 in the various classification maps. It can be seen that there is no significant change in the percentage occupancy of each of the classes (1 through 4).

The point-by-point differences between the classification maps can be summarized in various ways. Let \( D(X,Y) \) denote the 5x5 matrix whose \( ij^{th} \) element consists of the number of occurrences of the \( i^{th} \) class in image \( X \) and \( j^{th} \) class in image \( Y \) at the same location. Then, the matrices of the NN corrected classification vs. classifications of corrected data are shown in Tables 16-19.
Table 10. Class Occupancy in C

<table>
<thead>
<tr>
<th>Class</th>
<th>Number of Samples</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Urban</td>
<td>28475</td>
<td>11.39</td>
</tr>
<tr>
<td>Agriculture</td>
<td>111196</td>
<td>44.48</td>
</tr>
<tr>
<td>Forest</td>
<td>104978</td>
<td>41.99</td>
</tr>
<tr>
<td>Water</td>
<td>5351</td>
<td>2.14</td>
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</tbody>
</table>

Table 11. Class Occupancy in NN

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<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Urban</td>
<td>40559</td>
<td>11.43</td>
</tr>
<tr>
<td>Agriculture</td>
<td>157644</td>
<td>44.42</td>
</tr>
<tr>
<td>Forest</td>
<td>149089</td>
<td>42.01</td>
</tr>
<tr>
<td>Water</td>
<td>7609</td>
<td>2.14</td>
</tr>
</tbody>
</table>

Table 12. Class Occupancy in L/U

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<th>Class</th>
<th>Number of Samples</th>
<th>Percentage</th>
</tr>
</thead>
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<td>Urban</td>
<td>37781</td>
<td>10.65</td>
</tr>
<tr>
<td>Agriculture</td>
<td>165926</td>
<td>46.75</td>
</tr>
<tr>
<td>Forest</td>
<td>144186</td>
<td>40.63</td>
</tr>
<tr>
<td>Water</td>
<td>7008</td>
<td>1.97</td>
</tr>
</tbody>
</table>
Table 13. Class Occupancy in L/L

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<thead>
<tr>
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<th>Number of Samples</th>
<th>Percentage</th>
</tr>
</thead>
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<td>Urban</td>
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<td>10.09</td>
</tr>
<tr>
<td>Agriculture</td>
<td>166760</td>
<td>46.99</td>
</tr>
<tr>
<td>Forest</td>
<td>144043</td>
<td>40.59</td>
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<tr>
<td>Water</td>
<td>8273</td>
<td>2.33</td>
</tr>
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Table 14. Class Occupancy in C/U

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<th>Number of Samples</th>
<th>Percentage</th>
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<td>44.79</td>
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<td>Forest</td>
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<td>Water</td>
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Table 15. Class Occupancy of C/C

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<th>Percentage</th>
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<td>Water</td>
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Table 17. Matrix D(NN, L/C)

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<th>[4]</th>
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</table>
### Table 18. Matrix D(NN, C/U)

<table>
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<th>1</th>
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<th>3</th>
<th>4</th>
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<td>5884</td>
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### Table 19. Matrix D(NN, C/C)

<table>
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<th>Class in NN</th>
<th>0</th>
<th>1</th>
<th>2</th>
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<th>4</th>
</tr>
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</table>
Note that in these matrices the off-diagonal elements are generally much smaller than the corresponding diagonal elements. Also, the $ij^{th}$ and $ji^{th}$ elements are of the same order for all $i$ and $j$. This accounts for the smallness in the percentage differences in class occupancies between the classification maps.

The differences between NN and L/U or C/U are caused solely by the interpolation process since the training samples used are identical and hence the discriminant hyperplanes are also identical. The dependence of the classifications in L/U or C/U on interpolation can be illustrated as follows. The feature vector at any point $A$ in the geometrically corrected image is obtained by interpolation from 4 (or 16) feature vectors in the uncorrected image at the points on a $2 \times 2$ (or $4 \times 4$) array surrounding the point corresponding to $A$. The feature vectors participating in interpolation may not all be in one class. The classes that do enter into interpolation can be found by applying the geometric correction to the classification map and, instead of using any type of interpolation, generating a unique number indicative of the class combinations in the $2 \times 2$ (or $4 \times 4$) array. A matrix of the type shown in Tables 16 through 19 can be obtained for each subset of points in the image having a given class combination. Such matrices are shown for all class combinations possible showing differences between NN and L/U in Table 20. In this table the class combination $(n_1, n_2, n_3, n_4)$ indicates that $n_i$ feature vectors from class $i$ entered into interpolation. The "nearest neighbor" is the value in NN. The table, then, consists of the number of points with interpolation class combination $(n_1, n_2, n_3, n_4)$ and NN value $i$ that got classified as $j$ in L/U. Thus, it can be seen that there were a total of 30,391 points at which the class combination $(0, 2, 2, 0)$ occurred (i.e., interpolation was between two samples each of agriculture and forest) and 2483 of them were classified into the forest class, even though the nearest neighbors were in the agriculture class. Some general observations can be made from this table.

(i) When only one class enters into interpolation, all but a negligible percentage of points in L/U fall into that class.

(ii) When two classes enter into interpolation, a significant portion of points in L/U might belong to classes other than the two classes involved (e.g., $(2, 0, 2, 0)$, $(0, 2, 0, 2)$ and $(0, 0, 2, 2)$).

(iii) When more than one class enters into interpolation, the nearest neighbors tend to dominate the classifications in L/U.

These empirical conclusions are easily justified from theoretical considerations. For, a feature vector $q$ in the geometrically corrected image using linear interpolation is obtained by

$$q = (\alpha p_1 + (1 - \alpha) p_2) \beta + (\alpha p_3 + (1 - \alpha) p_4) (1 - \beta)$$
Table 20. Class Occupancy vs. Combinations for Interpolation in L/U

<table>
<thead>
<tr>
<th>CLASS COMBINATION</th>
<th>NEAREST NEIGHBOR</th>
<th>CLASS 0</th>
<th>CLASS 1</th>
<th>CLASS 2</th>
<th>CLASS 3</th>
<th>CLASS 4</th>
</tr>
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<tbody>
<tr>
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<td></td>
</tr>
</tbody>
</table>

*The number of occurrences* refer to the occurrence of classes 0, 1, 2, 3, 4 in the classification map of the geometrically corrected data at locations having the corresponding class combinations and nearest neighbors as determined using the classification map of the uncorrected data.
Table 20. (Continued)

<table>
<thead>
<tr>
<th>CLASS COMBINATION</th>
<th>Nearest Neighbor</th>
<th>CLASS 0</th>
<th>CLASS 1</th>
<th>CLASS 2</th>
<th>CLASS 3</th>
<th>CLASS 4</th>
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where $p_1, p_2, p_3, p_4$ are feature vectors in the uncorrected image and $\alpha, \beta$ are constants between 0 and 1. The vectors $p_1, p_2, p_3, p_4$ are classified using the rule

"Assign $p$ to class $k$ iff $(d^\lambda p + d_0^\lambda) < 0$ for $\lambda < k$ and $(d_k^\lambda p + d_0^k) > 0"$

where $d_{\lambda}$ is a vector and $d_{0\lambda}$ is a scalar defining discriminants for each $\lambda$. Now if $p_1$ and $p_2$ are assigned to classes $i$ and $j$, it is easy to see that $\alpha p_1 + (1 - \alpha)p_2$ cannot be assigned to any class number less than $\text{Min}(i, j)$. Also, if $p_1$ and $p_2$ are assigned to a class $i$, it is found that the discriminant conditions for class $i$ are satisfied by $\alpha p_1 + (1 - \alpha)p_2$ also.

In producing C, NN and L/U, the order of testing discriminant functions was water, forest, agriculture and urban. Remembering this and examining Table 20, the above theoretical conclusions are confirmed (except for a few anomalies caused, possibly, by round-off errors).

In conclusion, the overall statistics of class occupancy are only negligibly affected by geometric correction. But the effects on a pixel-by-pixel level are noticeable and the differences between the classifications of corrected and uncorrected data tend to compensate such that the overall class occupancies stay approximately the same. Some peculiarities may be introduced by interpolation such as obtaining an urban pixel from samples which were classified as forest and water in the uncorrected image. The correct classifications in those cases can only be found by comparison with the ground truth for those locations. Almost all the differences occur at locations where more than one class is involved in interpolation. It is precisely at these points that the raw data from the spacecraft would consist of mixtures of reflectances from different classes. Therefore, it might well be that if the radiometric values at the resampled coordinate locations are estimated accurately (as with a sinc function or cubic convolution) then the resulting classifications would be more accurate than those obtained in NN. Further tests along these lines (other than ground truth surveys) could be made using mixture proportion estimation methods [18, 19].
4-5. SUPERPOSITION OF BOUNDARIES

The final step in the generation of the TARCOG land use map was the superposition of the county boundaries, as shown in Figure 25, after geometric correction to UTM coordinates. The complete seven class land use map is shown in Figure 50.

Using these county boundaries, it was determined, for example, that the forest area in Madison County covers 590 square kilometers or 146 thousand acres.
Figure 50. Seven class TARCOG land use map with UTM grids and county boundaries superimposed.
V. COMPUTER PROGRAM DOCUMENTATION

5-1. PRELIMINARY DATA HANDLING

1. NAME

ERTXTM

2. PURPOSE

Extraction and reformatting of a desired rectangular region from four files (corresponding to the four strips) of Landsat data on CCT's. The desired region is specified in terms of latitude and longitude or pixel coordinates.

3. CALLING SEQUENCE

This is a main program. It is currently on a partitioned data set as an executable module.

4. INPUT-OUTPUT

4.1 Input

The following input parameters should be supplied in data cards according to the formats and read statements indicated below.

```
READ  500, JFLG, KFLG
READ  500, NBDS, (IBDS(I), I=1, NBDS)
READ  820, FVECT, BANDS, HIST
IF(JFLG, EQ, 0) READ  500, IRI, IRF, ICI, ICF
IF(JFLG, GT, 0) READ  600, RLATI, RLATF, RLNGI, RLNGF
```

where

JFLG, KFLG are "task-indicator" flags.

JFLG=0 indicates that the region is specified by pixel coordinates and should be extracted.

JFLG=1 indicates that the region is specified by geographic coordinates, the corresponding pixel coordinates are to be found and printed, but the region should not be extracted.

JFLG=2 indicates that the region is specified by geographic coordinates, the corresponding pixel coordinates are to be found and printed, and the region should be extracted.

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KFLG=0(1) indicates that while extracting, the "synthetic" pixels (which are extra pixels added to adjust the line length) should not (should) be suppressed.

NBDS = number of bands (≤ 4) to be extracted.

IBDS = band numbers (the order specified will dictate the order in which the components in the feature vectors and the individual band files are arranged).

FVECT, BANDS, HIST are logical variables which should be .TRUE. to indicate that a feature vector file, individual band files, and histograms of individual bands (respectively) are desired. Histograms are produced only when BANDS.AND. HIST = .TRUE..

IRI, IRF, ICI, ICF are initial and final rows and initial and final columns respectively of the region to be extracted.

RLATI, RLATF, RLANGI, RLANGF are initial and final latitude and initial and final longitude of the region of interest. They should be supplied in degrees (not degrees and minutes, but decimal degrees). Northern latitudes and Eastern longitudes are considered positive.

The data from the Landsat CCT's should be on four separate data sets which can be "OPEN" at the same time. The DDNAMES for these data sets should be TAPEiFO1 with i = 1, 2, 3, 4 in order to be compatible with the non-FORTRAN read routine READNL. If the four strips of data are on the same tape, each strip should be copied to a separate tape (or disk file) before using this program.

4.2 Output

The output of this program will consist of printout of the coordinates requested (as illustrated in the attached example) and, depending on FVECT and BANDS, a file of feature vectors and NBDS files of individual band images. These files will be written as unformatted FORTRAN records. The number of records = IRF - IRI + 1 in the feature vector file and IRF - IRI + 2 in the individual band files. The first record in the individual band files consists of NREC, NEL where NREC = IRF - IRI + 1 and NEL = ICF - ICI + 1 where IRI, IRF, ICI, ICF are the values supplied when JFLG=0 or computed from RLATI, RLATF, RLANGI, RLANGF when JFLG=2. When KFLG=1, NEL is the number of pixels in the shortest record after synthetic pixel removal.
The data in the output files are in bytes. Thus, each record of the feature vector file consists of \( NEL \times NBDS \) bytes, with \( NBDS \) bytes per pixel. Each record of the individual band file consists of \( NEL \) bytes.

4.3 File Storage

This program requires a direct access work space of \( 9360 \times 3240 \) bytes (on logical unit 90) to be able to handle the four-band separation for a full (2340 by 3240) frame. This can, however, be reduced when smaller regions are to be extracted. A convenient way to avoid excessive demand on direct access space is to have the DEFINE FILE statement

\[
\text{DEFINE FILE 90 (9360,3240, L, IAV)}
\]

provide for the maximum space, but use a DD card for unit 90 with the SPACE parameter

\[
\text{SPACE = (3240, (585,585))}
\]

5. EXITS

Not applicable.

6. USAGE

The program is in FORTRAN IV and implemented on the IBM 360 using the H compiler. The program, in its executable form, is in the users' library.
7. **EXTERNAL INTERFACES**

This program calls several routines. The linkage is indicated in the following table.

<table>
<thead>
<tr>
<th>Calling Program</th>
<th>Programs Called</th>
</tr>
</thead>
<tbody>
<tr>
<td>ERTXTM</td>
<td>STRTMR, PET, READNL, SVSCI, TIKBIN, SCLREL, GEOPIX, ERTXT5, ERTXT6, ERTXT7, PRTHST</td>
</tr>
<tr>
<td>TIKBIN</td>
<td>IDIGIT</td>
</tr>
<tr>
<td>SCLREL</td>
<td>ENDS, CROSS</td>
</tr>
<tr>
<td>ERTXT5</td>
<td>READNL, ERTXT3, ERTXT4, DAWN, SAWN</td>
</tr>
<tr>
<td>ERTXT6</td>
<td>SVSCI, LRHSTG</td>
</tr>
<tr>
<td>ERTXT7</td>
<td>READNL, LINFIX, ERTXT8, SAWN, ERTXT9</td>
</tr>
</tbody>
</table>
8. PERFORMANCE SPECIFICATIONS

8.1 Storage

The program is 34844 bytes long, but including external references required and the buffers, this program needs 128K bytes of storage.

8.2 Execution Time

The execution time depends on the size of the image to be extracted. With FVECT = BANDS = HIST = .TRUE., a 1200 x 1200 region can be processed by this program in approximately 4-1/2 minutes.

8.3 I/O Load

None except as specified by Section 4.

8.4 Restrictions

None

9. METHOD

The program reads the ID and annotation records, finds the number of pixels per record (called the adjusted line length) and prints the exposure date and time and the scene ID. If JFLG=0, the pixel coordinates are read from a card. Otherwise, the routines CTRBIN, TIKBIN, and SCLREL are used to determine a transformation matrix A and the skew angle (due to earth's rotation) using the information about center latitude and longitude and the tick mark locations from the annotation record. The geographic coordinates bounding the region of interest are read from a data card and converted pixel coordinate bounds using GEOPIX. If the pixel coordinates exceed the limits (i.e., if IRI < 1, IRF > 2340, ICI < 1 or ICF > adjusted line length), they are changed to the nearest limits. If the synthetic pixel removal flag KFLG is 0, then the region bounded by IRI, IRF, ICI, ICF is extracted using the routine ERTXT5. If KFLG=1, the region is extracted using ERTXT7 which also removes synthetic pixels. When ERTXT5 is used the number of pixels per line of output is ICF - ICI + 1. When ERTXT7 is used there will be fewer pixels per line, the number being computed and printed by the program.

Other than this, there are no external differences between ERTXT5 and ERTXT7. If FVECT = .TRUE., both routines write a feature vector file on unit NTPFV (=13). If BANDS = .TRUE., the individual bands are separated and written on the direct access unit 90. The order in which
the bands (and components of the feature vectors) are written is dictated by IBDS. If BANDS = .TRUE., the routine ERTXT6 is used to read the individual bands from the direct access file and write as separate files on unit 13. If, in addition, HIST = .TRUE., the histograms of the individual bands are found by ERTXT6 and printed by the routine PRTHST.

10. COMMENTS

The details of the subroutines are omitted here. The methods used in most of the routines are quite straightforward and are apparent from the listings. The routine SCLREL uses some simple concepts from elementary geometry. A useful modification to this program (ERTXTM) would be to include specification of geographic coordinates of the vertices of a more general polygon instead of RLATI, RLATF, RLNGI, RLNGF. As it is, the program extracts a rectangle with sides parallel and perpendicular to the scan lines containing the given rectangle and, in some cases, may yield too large a region. The only routine to be changed to include this generalization is GEOPIX.

11. LISTINGS

The listings of the program are attached at the end.

12. TESTS

The program has been tested, used for the extraction of various Landsat data sets with all the options available, and found to operate satisfactorily.
MAIN PROGRAM .... ERTXTM ....

DIMENSION RLUC(6,4),RLTLLNG(6,4),IFLG(6,4),LTLNG(6,4)
JFLG=1: COMPUTE REGION TO BE EXTRACTED. DO NOT EXTRACT.
JFLG=0: NO COMPUTATION OF REGION TO BE EXTRACTED. EXTRACT GIVEN REGION.
JFLG=2: COMPUTE REGION TO BE EXTRACTED AND EXTRACT.
WHEN JFLG=C OR 2, KFLG=0 OR 1 CORRESPOND TO NO OR YES FOR SYNTHETIC PIXEL REMOVAL.
IF(BANDS) EXTRACT INDIVIDUAL BANDS ON NTPFV.
IF(FVECT) EXTRACT FEATURE VECTORS ON NTPFV.

IF(BANDS.AND.HIST) FIND AND PRINT HISTOGRAMS OF INDIVIDUAL BANDS IN ADDITION TO EXTRACTING THE BANDS.
NB=NUMBER OF BANDS TO BE SEPARATED AND/OR FORMED INTO A F. V. COMPONENTS.
IBDS IS THE ARRAY SPECIFYING THE ORDER IN WHICH F. V. COMPONENTS OCCUR AND/OR THE FILES OF BANDS SHD. BE ARRANGED ON NTAP0.

NOTE*** THE FOLLOWING DIMENSIONS SHD BE CHECKED W. R. T. THE IMAGE SIZE TO BE EXTRACTED.
DIMENSION IBDS(4)
DIMENSION NSAMPS(4)
LOGICAL=1 IX(13200),IY(13200)
DIMENSION IH(256,4)
INTEGER=2 IZ(20)
DIMENSION A(2,2)
LOGICAL=1 HIST
LOGICAL=1 BANDS,FVECT
COMM/GEOLIM/RLATI,RLATF,RLNGI,RLNGF
COMM/CE/CTLAT,CTLONG,CNTRY,CNTRY
COMM/COORD/IRI,IRF,ICI,ICF
COMM/DSK90/NRW90,NBYT90
DATA NTAP0,NTPFV/13,13/
DEFINE FILE 90(9360,3240,1,IAV90)
NRW90=9360
NBYT90=3240
READ 500,JFLG,KFLG
WRITE(6,800)JFLG,KFLG
READ 500,NBDS,(IBDS(1),I=1,NBDS)
WRITE(6,810)NBDS,(IBDS(1),I=1,NBDS)
READ 820,FVECT,BANDS,HIST
WRITE(6,830)FVECT,BANDS,HIST

FIND NUMBER OF PIXELS IPIXT PER RECORD
CALL STRTMR
CALL PET(0)
CALL READNL(I2,1000,LRECL,1)
LRECL2=LRECL/2
CNTRY=117C.5

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NPIXS=IPIXT/4
CNTRY=FLOAT(IPIXT)/2. + 5

CALL READNL(IX, IEND, LRECl, 1)
WRITE(6, 400)
WRITE(6, 1100)(IX(I), I=1, 17)
WRITE(6, 1200)(IX(I), I=107, 111)
WRITE(6, 1300)(IZ(I), I=1, 6)
IF(JFLG.NE.0)GO TO 30
READ 500, IRx, IREF, ICI, ICF
WRITE(6, 5000)IRx, IREF, ICI, ICF
GO TO 40

30 CONTINUE

C READ ANNOTATION RECORD

C FIND LATITUDE AND LONGITUDE OF FORMAT CENTER
CALL CTRB(IN(IY(II), CILAT, CTILO
C FIND TICK MARK COORDINATES
J=1
K=1
CALL SVSCI(RLOC, 24, C.
CALL SVSCI(RLTLNG, 24, 0.
CALL SVSCI(IFLLG, 24, 0.
CALL SVSCI(RLTLNG, 24, 0)
DO 20 I=385, 615, 10
CALL TIKBIN(IY(I), RLOC(J, K), RLTLNG(J, K), IFLG(J, K), LTLNG(J, K))
J=1+1
IF(J.LT.6)GO TO 20
J=1
K=K+1
20 CONTINUE
WRITE(6, 1000)CNTRX, CNTRY
WRITE(6, 2000)CILAT, CTILO
WRITE(6, 1004)
WRITE(6, 1005)
DO 35 I=1, 6
35 WRITE(6, 3000)(RLOC(I, J), RLTLNG(I, J), RLTLLNG(I, J), J=1, 4)
C FIND SATELLITE HEADING (CHARACTERS 70, 71, 72)
C HEADING=IDIGIT(IY(701, 3)
C COMPUTE TRANSFORMATION MATRIX A AND TANGENT T OF SKEW ANGLE CR E
C TO EARTH'S ROTATION.
CALL SCLREL(RLOC, RLTLNG, LTLNG, HEADING, A, T)
C DETERMINE PIXEL COORDINATES OF THE FOUR CORNERS OF THE RECTANGULAR
C AREA SPECIFIED IN TERMS OF LATITUDES AND LONGITUDES.
C HENCE FIND LIMITS OF AREA TO BE EXTRACTED (ICI, ICF, IRx, IREF)
C

READ 600, R LATI,RLATF,RANGI,RLNGF
WRITE (6,600) LATI,RLATF,RANGI,RLNGF
CALL GEOPIX(A,T,2340,FLOAT(IPIXT))

40 CONTINUE
WRITE (6,700) IRI,IRF,ICI,ICF

C
IF(I .LE. IRI.AND.IRF.LE.2340.AND.I .LE.ICI.AND.ICF.LE.IP IXT)GO TO 61
IF( IRI.GT.2340 OR.IRF.LT.1 OR. ICI.GT.IP IXT OR.ICF.LT.1) STOP
IF( IRI.LT.1) IRI=1
IF( IRF.GT.2340) IRF=2340
IF (ICI.LT.1) ICI=1
IF (ICF.GT.IP IXT) ICF=IP IXT
WRITE (6,701)
WRITE (6,700) IRI,IRF,ICI,ICF
CONTINUE
CALL PET(1)
IF(JFLG.EQ.1) STOP

C
EXTRACT THE AREA FROM APPROPRIATE STRIPS OF FR YTS DATA TAPES.
C
NEL=ICF-ICI+1
IF(KFLG.EQ.0) CALL FRTXT5(NPIXS,IX,YY,NBDS,NEL,IBDS,NSAMPS,NSTRP,
BANDS,FVECT,NPEV)
IF(KFLG.EQ.1) CALL FRTXT7(NPIXS,IX,YY,NBDS,IBDS,NELMIN,BANDS,FVECT,
NPEV)
IF(KFLG.EQ.1) NEL=NELMIN
CALL PET(1)
IF(NSTRP.EQ.0) STOP
IF(FVECT) END FILE NPEV
IF(.NOT.BANDS) STOP
CALL FRTXT6(IX,NBDS,NEL,NPAPD,HIST,TH,IRF-TRI+1)
CALL PET(1)
IF(.NOT.HIST) STOP
DO 60 I=1,NBDS
WRITE (6,1100) IBDS(I)
60 CALL PRTHST(IH(1,1),256)
CALL PET(1)
STOP
100 FORMAT(1X,29HPIXEL COORDINATES OF CENTER=(,F7.1,1H,,F7.1,1H)/
110 FORMAT(' EXPOSURE DATE='2A1,1X3A1,1X2A1)
120 FORMAT(' TIME='2A1,'-2A1,'-1A1,'0')
130 FORMAT(' SCENE/FRAME ID='6A2)
200 FORMAT(4X,26H L AT. AND LONG. OF CENTER=(,F8.2,1H,,F8.2,1H)/
300 FORMAT(4F18.3,1X,12,F9.2))
400 FORMAT(1H1)
500 FORMAT(12I6)
600 FORMAT(6F12.3)
700 FORMAT(/,5H IRI=,15.5H IRF=,15.5H ICF=,15)
701 FORMAT(9H COMPUTED REGION EXCEEDS THAT AVAILABLE ON THE SUPPLIED
FRAME. THE PART EXTRACTED IS GIVEN BY)
800 FORMAT(' JFLG=12,'0: EXTRACT SPECIFIED REGION; 1: FIND PIXEL ADD
RESSES OF REGION(GIVEN LAT AND LONG.); 2: FIND ADDRESSES AND EXTRA
CT'/' KFLG=12,'0: DO NOT REMOVE SYNTHETIC PIXELS; 1: REMOVE SYNT
HEtic pixels.')
810 FORMAT(' NO. OF BANDS TO BE EXTRACTED='12,' BANDS REQUIRED='412)
820 FORMAT('12L6')
830 FORMAT(' FEATURE VECTORS REQUIRED? 'L3/
        ' INDIVIDUAL BAND FILES REQUIRED? 'L6/
        ' HISTOGRAMS OF BANDS REQUIRED? 'L3)
1004 FORMAT ('57X,19HM5S TICK MARK DATA/57X,18(1H*)///')
1005 FORMAT ('15X,8HTOP EDGE,23X,9HLEFT EDGE,21X,10HRIGHT EDGE,21X,11HBD
        TTON EDGE/15X,8(1H*),23X,9(1H*),21X,10(1H*),21X,11(1H*)///')
1100 FORMAT('1H1,10X,18HHISTOGRAM FOR BAND,12)
END
SUBROUTINE PET(I)
IF(I.NE.0)GO TO 10
CALL TIMER(ITIME1)
ITIME = 0.
WRITE(6,200)
200 FORMAT(10X,'BEGINNING TIMING** TIME NOW IS 0')
RETURN
10 CALL TIMER(ITIME2)
ITIME = (ITIME2-ITIME1)/100.
ITIME = ITIME + TIME
ITIME1 = ITIME2
WRITE(6,100)TIME,ITIME
100 FORMAT(10X,'TIME ELAPSED SINCE LAST PRINTING OF TIME='E12.3','SEC., TOTAL TIME ELAPSED='E12.3','SEC.')
RETURN
END
READNL START

SUBROUTINE READNL(INBUF, MEND, LRECL, NTAPI)

BC 15, 12(15)
DC '7'
DC CL7'READNL'
STM 14, 12, 12(13)
BALR 2, 0
USING *, 2
L 3, 0(1) LOAD ADDRESS OF BUFFER
USING INAREA, 3
L 5, 4(1) LOAD ADDRESS OF MEND
L 6, 8(1) LOAD ADDRESS OF LRECL
L 7, 12(1) LOAD ADDRESS OF NTAPI
LR 10, 13 1 SET UP
LA 13, TSAVE 1 LINKAGE FOR
ST 13, 8(0, 10) 1 CALLING OTHER
ST 10, 4(0, 13) 1 ROUTINES
L 12, 0(0, 7) R12 = NTAPI
SH 12, =H'1' R12 = NTAPI - 1
SLL 12 R12 = (NTAPI - 1) * 4
A 12 = ALGPNTAB1. R12 = ADDRESS OF GPNTAB + NTAPI - 1 * 4
L 12, 0(0, 12) R12 = ADDRESS OF NTAPI * TH INDCB
OPEN 4(12), (INPUT)
LA 9, EDFEXIT
ST 9, EDFADD
MVC 33(3, 12), EDFADD + 1
GET (12), INBUF
LH 8, 82(12)

KFRN L 13, TSAVE + 4
LM 2, 12, 28(13) RESTORE REGISTERS
L 14, 12(13)

MVI 12(13), X'EE' SIGNAL COMPLETION OF SUBROUTINE

BCR 15, 14 RETURN

EDFEXIT L 3, =F'1'
ST 8, 0(5) SFT MEND = 1
CLOSE ((12), LEAVE)

B KFRN

DPNTAB DC A(LINDC1, LINDC2, LINDC3, LINDC4, LINDC5, LINDC6)
DC A(LINDC7, LINDC8, LINDC9)
DS 4E

LINDC1 DCB DDNAME = TAPE1FO1, DEVD = TA, DSORG = PS, BUFNO = 2, MACRF = (GM), EROPT = ACC

LINDC2 DCB DDNAME = TAPE2FO1, DEVD = TA, DSORG = PS, BUFNO = 2, MACRF = (GM), EROPT = ACC

LINDC3 DCB DDNAME = TAPE3FO1, DEVD = TA, DSORG = PS, BUFNO = 2, MACRF = (GM), EROPT = ACC

LINDC4 DCB DDNAME = TAPE4FO1, DEVD = TA, DSORG = PS, BUFNO = 2, MACRF = (GM), EROPT = ACC

LINDC5 DCB DDNAME = TAPE5FO1, DEVD = TA, DSORG = PS, BUFNO = 2, MACRF = (GM), EROPT = ACC

LINDC6 DCB DDNAME = TAPE6FO1, DEVD = TA, DSORG = PS, BUFNO = 2, MACRF = (GM), EROPT = ACC

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INDCB7 DC8 DDNAME=TAPE7F01,DEVD=TA,DSORG=PS,BUFNO=2,MACRF=(GM), EROPT=ACC

INDCB8 DCB DDNAME=TAPE8F01,DEVD=TA,DSORG=PS,BUFNO=2,MACRF=(GM), EROPT=ACC

INDCB9 DCB DDNAME=TAPE9F01,DEVD=TA,DSORG=PS,BUFNO=2,MACRF=(GM), EROPT=ACC

COUNT DC CL38'02030405060708091011121314151617181920'

TSAVE DS 9D

EDFADD DS 1F

INAREA DSECT

INBUF DS 900F THIS SIZE CAN BE CHANGED TO DESIRED VALUE
SUBROUTINE CRTBN(IY,RLAT,RLONG)
C
C TO CONVERT COORDINATES OF FORMAT CENTER(LAT. AND LONG.) OF FRTS
C CCT IMAGE TO BINARY.
C
LOGICAL*1 IX(14)
LOGICAL*1 IS,'S', IW,'W'
RLAT=IDIGIT(IX(2),2)+IDIGIT(IX(5),2)/60.
RLONG=IDIGIT(IX(9),3)+IDIGIT(IX(13),2)/60.
IF(IX(1).EQ.IS)RLAT=-RLAT
IF(IX(8).EQ.IW)RLONG=-RLONG
RETURN
END
SUBROUTINE TIKBIN(IX, RLOC, RITNG, IFLG, IITNG)
C C TO CONVERT LATITUDE OR LONGITUDE OF TICK MARKS TO FLOATING POINT.
C BINARY.
C C
LOGICAL*1 IX(10)
LOGICAL*1 IN, IS, IE, IW, ITIK1, ITIK2
INTEGER*2 II
LOGICAL*1 LW(2)
EQUIVALENCE (II, LW(1))
DATA I1/I0/
DATA IN, IS, IE, IW, ITIK1, ITIK2/"N", "S", "E", "W", "/
DATA I2/I0/I5/I8/I0000/
C C FIRST FIND LOCATION OF TICK MARK IF ANY.
C C
IFLAG=0
IF (IX(3).EQ.ITIK1 .OR. IX(3).EQ.ITIK2) IFLAG=4
IF (IX(10).EQ.ITIK1 .OR. IX(10).EQ.ITIK2) IFLAG=3
IF (IFLAG.EQ.0) RETURN
C C IF(IFLG.EQ.0) THERE IS NO TICK MARK CORR. TO VECTOR IX SUPPLIED.
C C
LW(1)=IX(1)
LW(2)=IX(2)
RLOC=II/FLOAT(I2/I0/I5)
IF (IX(IFLAG).EQ.IN) IITNG=1
IF (IX(IFLAG).EQ.IS) IITNG=-1
IF (IX(IFLAG).EQ.IE) IITNG=+2
IF (IX(IFLAG).EQ.IW) IITNG=-2
ISTRNG=IDIGIT(IX(IFLAG+1).3)+IDIGIT(IX(IFLAG+5).2)/60.
IF (ISTRNG.LT.0) RITNG=-RITNG
ITNG=ABS(ISTRNG)
RETURN
END
SUBROUTINE SCLREL(RLOC, RLTNLG, LTLNLG, HEADNG, A, T)
REAL ITBL
DIMENSION RLOC(6,4), RLTNLG(6,4), LTLNLG(6,4), A(2,2), ITBL(4)
COMMON/CNTRE/CLAT, CLNG, CNTRX, CNTRY

C TO FIND THE TRANSFORMATION MATRIX A AND T, THE TANGENT OF THE SKE:
C ANGLE IN THE FILM IMAGE.
C (LATITUDE ) (X) (CLAT)
C ( ) = A ( ) + ( )
C (LONGITUDE) (Y) (CLNG)
C RLOC, RLTNLG ARE ARRAYS CONTAINING FILM COORDINATES, GEOGRAPHIC
C COORDINATES AND LATITUDE/LONGITUDE INDICATORS CORR. TO TICK MARKS.
C LTNLG(i, j) = 1 IF (i, j) TH TICKMARK IS A LATITUDE AND 2 IF IT IS A
C LONGITUDE. J=1,2,3,4 FOR TOP, LEFT, RIGHT, AND BOTTOM EDGES OF T:
C IMAGE ON FILM.

CALL ENDS(LTLNLG(1,1), 6, 2, IT1, IT2)
CALL ENDS(LTLNLG(1,4), 6, 2, IT1, IT2)
DTIK=RLOC(IT2, 1)-RLOC(IT1, 1)+RLOC(ITB2, 4)-RLOC(IT1, 4)
DLNLG=RLTNLG(IT2, 1)-RLTNLG(IT1, 1)+RLTNLG(IB2, 4)-RLTNLG(IB1, 4)
A(1, 1)=DLNLG/DTIK

CALL ENDS(LTLNLG(1,2), 6, 2, IT1, IT2)
CALL ENDS(LTLNLG(1,3), 6, 2, IT1, IT2)
DLNLG=RLTNLG(IL2, 1)-RLTNLG(IL1, 1)+RLTNLG(IR2, 3)-RLTNLG(IR1, 3)
DTIK=RLOC(IL2, 1)-RLOC(IL1, 1)+RLOC(IR2, 3)-RLOC(IR1, 3)
A(2, 1)=DLNLG/DTIK
A(2, 2)=0.
N=0
ITBL(1)=-1./(.5+5.5/180.)
ITBL(4)=1./(.5+3.25/180.)
DO 10 I=1, 6
DO 10 J=1, 5, 3
IF(LTNLG(I, J).NE.2) GO TO 10
N=N+1
A(1, 2)=A(1, 2)+PDFNLG(I, J)*RLOC(I, J)*ITBL(J)
10 CONTINUE
A(1, 2)=A(1, 2)/N

C FIND X COORDINATES OF LEFT AND RIGHT EDGES FOR TICK MARKS.

XL=0.
N=0
DO 20 I=1, 6
IF(LTNLG(I, 2).NE.2) GO TO 20
XL=XL+(RLTNLG(I, 2)-CLNG-A(1, 2)*RLOC(I, 2))
N=N+1
20 CONTINUE
IF(N.NE.0)XL=XL/N/A(1, 1)
IF(N.EQ.0)XL=.55
XR=0.

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N=0
DO 30 I=1,6
    IF(LT(LN(I,3),NE.2)GO TO 30
    XR=XR+(RT(LN(I,3)-CLNG-A(I,2)*RLC(I,3))
N=N+1
    CONTINUE
30
    IF(N.EQ.0)XR=XR/N/A(I,1)
    IF(N.EQ.0)XR=-.55
    IT(2)=1./XL
    IT(3)=1./XR
N=0
    A(I,1)=0.
    DO 40 I=1,6
    DO 40 J=2,3
        IF(LT(LN(I,J),NE.1)GO TO 40
        N=N+1
        A(I,1)=A(I,1)+(RLT(I,J)-CLG-I(I,J)*RLOC(I,J))*IT(J)
        CONTINUE
40
        A(I,1)=A(I,1)/N
        DEGRAD=ATAN(1./)/45.
        H=(HEADNG-180.)*DEGRAD
        T=18./251.*COS(CLATION*DEGRAD)
        T=T*COS(H)/(1.-T*SIN(H))
        SKEW=ATAN(T)*180./3.1415926
    PRINT 100,((A(I,J),J=1,2),I=1,2),SKEW
    FORMAT('/* TRANSFORMATION FROM FILM COORDINATES TO GEOGRAPHIC COU
DINATES*/2(2E15.6)'/ 'SKEW ANGLE ON FILM= 'F8.3,' DEGREES')
    PRINT 400
    400
    FORMAT('/* RESIDUALS AT TICK MARKS WHEN TRANSFORMATION IS USED*/')
    PRINT 500
    500
    FORMAT('/* TOP AND BOTTOM EDGES*/')
    DO 60 J=1,4,3
    Y=1./IT(J)
    DO 60 I=1,6
        IF(LT(LN(I,J),NE.2)GO TO 70
C
C Y AND LONGITUDE ARE GIVEN.
C
    CALL CROSS(A(I,2),A(I,1),A(2,2),A(2,1),Y,RLT(I,J)-CLG,I,PHI)
    GO TO 75
70
    IF(LT(LN(I,J),NE.1)GO TO 50
C
C Y AND LATITUDE ARE GIVEN.
C
    CALL CROSS(A(I,2),A(I,1),A(2,2),A(2,1),Y,RLT(I,J)-CLG,I,PHI)
    DX=RLOC(I,J)-X
    PRINT 600,X,Y,RLT(I,J),PHI,RLOC(I,J),DX
    FORMAT('6F15.5')
50
    CONTINUE
60
    CONTINUE
    PRINT 700
    700
    FORMAT('/* LEFT AND RIGHT EDGES*/')
    DO 80 J=2,3
    X=1./IT(J)
    DO 80 I=1,6
    END
IF(LTLNG(I,J).NE.1) GO TO 90

C X AND LATITUDE ARE GIVEN.
C CALL CROSS(A(2,1),A(2,2),A(1,1),A(1,2),X,RLTLNG(I,J)-CLAT,Y,PHI)
GO TO 95

90 IF(LTLNG(I,J).NE.2) GO TO 80

C X AND LONGITUDE ARE GIVEN
C CALL CROSS(A(1,1),A(1,2),A(2,1),A(2,2),X,RLTLNG(I,J)-CLNG,Y,PHI)

95 DY=RLOC(I,J)-Y
PRINT 600,X,Y,RLTLNG(I,J),PHI,RLOC(I,J),DY
CONTINUE
RETURN

200 FORMAT(20X4110)
300 FORMAT(10X3F15.5)
END
SUBROUTINE ENDS(IX,N.I,11,12)
DIMENSION IX(N)

C FIND I1, I2, THE SMALLEST AND LARGEST INDICES I BETWEEN 1 AND N SUCH THAT
IX(I) = J.
C I1 = 0 IF NO SUCH I EXISTS.

I1 = 0
DO 10 I = 1, N
IF(IX(I).NE.J)GO TO 10
I1 = I
GO TO 20
10 CONTINUE
RETURN

20 I2 = I1
I1N = I1 + N
DO 30 I = I1, N
K = I1N - I
IF(IX(K).NE.J)GO TO 30
I2 = K
RETURN
30 CONTINUE
RETURN
END

SUBROUTINE CROSS(A,B,C,D,Y,U,V)
C SOLVE FOR Y, V GIVEN X, U.
C U = A*X + B*Y; V = C*X + D*Y
C
Y = (U - A*X) / B
V = C*X + D*Y
RETURN
END
SUBROUTINE GEOPIX(A,T,XUP,YUP)
COMMON/CNTRE/CLAT,CLNG,CNTRX,CNTRY
COMMON/GEOL/IM/RLAT1,RLATF,RLNGI,RLNGF
COMMON/COORD/IRF,ICI,ICF
DIMENSION A(2,2)
DIMENSION B(2,2)
DIMENSION CORN(2,4)

C FIND INVERSE OF MATRIX A.

DET=A(1,1)*A(2,2)-A(1,2)*A(2,1)
B(1,1)=A(2,2)/DET
B(1,2)=-A(1,2)/DET
B(2,1)=-A(2,1)/DET
B(2,2)=A(1,1)/DET

C FIND AND PRINT FILM COORDINATES AND PIXEL INCREMENTS (FROM CENTER) OF THE 4 CORNERS OF THE RECTANGLE TO BE EXTRACTED.

CORN1=1.E10
CORN2=1.E10
CORNX1=-CORN1
CORNX2=-CORN1
PRINT 100
DO 10 I=1,4
RLAT=RLAT1
IF(I.GT.2)RLAT=RLATF
RLAT=RLAT-CLAT
RLNG=RLNGI
IF(MOD(I,2).EQ.0)RLNG=RLNGF
RLNG=RLNG-CLNG
CORN(1,I)=B(1,1)*RLNG+B(1,2)*RLAT
CORN(2,I)=B(2,1)*RLNG+B(2,2)*RLAT
CORN1=CORN(2,I)*XUP-85
CORN2=(-CORN(1,I)+CORN(2,I)+T1)*(YUP-1)
CORN1=AMIN1(CORN1,CORN1)
CORN2=AMIN1(CORN2,CORN2)
CORNX1=AMAX1(CORNX1,CORN1)
CORNX2=AMAX1(CORNX2,CORN2)
10 PRINT 200,CORN(1,I),CORN(2,I),CORN1,CORN2,RLAT,RLNG
IRI=CORN1+CNTRX
ICI=CORN2+CNTRY
IRF=CORNX1+CNTRX+1
ICF=CORNX2+CNTRY+1
RETURN

100 FORMAT(//' FILM COORDINATES AND PIXEL INCREMENTS (FROM CENTER) OF THE FOUR CORNERS OF THE RECTANGLE SPECIFIED,')
200 FORMAT(1X4F10.3,10X2F10.3)
END
SUBROUTINE ERXT5(NPIXS,LY,NB,NEL,IBDS,NSAMPS,NSTRP,BANDS,
              FVEC,NTAPO)
          LOGICAL*BANDS,FVEC
          DIMENSION IBDS(NB),ISTRPS(4),NSAMPS(4),ISAMPI(4),ISAMPF(4)
          DIMENSION IPRTYI(4),IPRTYF(4),IBYTEI(4),IBYTEF(4)
          LOGICAL*1 IX(4000),IY(NEL,NB)
          IF(BANDS .AND. FVEC) DIM IX(MAX(4000,NEL*NB))
          DEFINE FILE 90(IRE=IRI+1)*NB,NEL,L,IAV)
          DEFINE FILE 90(9460,3240,L,IAV) WORKS FOR ALL ERTS CTS WITH LE
          THAN OR EQUAL TO 2340*3240 PIXELS.
          DEFINE FILE 90(NRW90,NBYT90,L,IAV)
          IF DEFINED FILE SPACE IS INSUFFICIENT, PROGRAM WILL RETURN NSTRP
          COMMON/COORD/IRI,IRE,ICI,ICF
          COMMON/DSK90/NRW90,NBYT90
          NSTRP=0
          DO 10 I=1,4
          IF(NPIXS(I-1)+1.GT.ICE.OR.NPIXS(I-1).LT.ICI)GO TO 10
          NSTRP=NSTRP+1
          ISTRP(1NSTRP)=1
          10 CONTINUE
          CONTINUE
          PRINT 1200, (ISTRP(I),I=1,NSTRP)
          1200 FORMAT(//' CTS STRIPS CONTAINING DESIRED DATA ARE'\6)
          COMMON/NECR/NRW90,NBYT90
          FIND IF ALLOCATED DISK SPACE IS SUFFICIENT, IF NOT PRINT ERROR
          MESSAGE, SET NSTRP=0 AND RETURN.
          IF(IREC*.EQ.*NB)
          IF(IREC.LE.NRW90.AND.NEL.LE.NBYT90)GO TO 48
          PRINT 1000, JREC,NEL,NRW90,NBYT90
          1000 FORMAT(//' INSUFFICIENT FILE ALLOCATION FOR ERXT5'\)
          ' REQUIRED NRW90= '15/\)
          ' REQUIRED NBYT90= '15/\)
          ' SUPPLIED NRW90= '15/\)
          ' SUPPLIED NBYT90= '15/\)
          NSTRP=0
          RETURN
          48 CONTINUE
          SKIP IRI+1 RECORDS ON THE DESIRED STRIPS.
          DO 30 I=1,NSTRP
          IRI1=IRI+1
          IF(ISTRP(I)+1)IRI1=IRI-1
          IF(IRI1.LE.0)GO TO 30
          DO 40 IREC=1,IRI1
          40 CALL READNL(IX,IEND,LRECL,ISTRP(I))
          30 CONTINUE
          FIND INITIAL AND FINAL SAMPLES TO BE EXTRACTED ON ISTRP(I) FOR
C     I=1,NSTRP.
      DO 20 I=1,NSTRP
      ISAMPF(I)=MAX0(I,ICT-(ISTRPS(I)-1)*NPIXS)
      IPRTYF(I)=MOD(ISAMPF(I),2)
      TBYTEF(I)=(ISAMPF(I)-1)/2*N+2-IPRTYF(I)
      ISAMPF(I)=MIN0(NPIXS,ICT-(ISTRPS(I)-1)*NPIXS)
      IPRTYF(I)=MOD(ISAMPF(I),2)
      TBYTEF(I)=(ISAMPF(I)-1)/2*N+2-IPRTYF(I)+8
      NSAMPS(I)=ISAMPF(I)-ISAMPI(I)+1
      IF(I.GT.1)NSAMPS(I)=NSAMPS(I)+NSAMPS(I-1)
      20 CONTINUE
C     EXTRACT AND COPY DATA ON DISK.
C     JREC=1
      DO 70 JREC=1,IRF
C     READ DATA FROM EACH STRIP AND MOVE INTO ARRAY IY
      DO 80 I=1,NSTRP
      CALL READN(LX,IXN,ISTRPS(I))
      IF(BANDS)CALL FRTXT2(NB,NEL,IBYTES,IBYTEF,IPRTYF,IPRTYF,LDOS,NSAMPS,
                        IX,IXY)
      IF(.NOT.BANDS)CALL FRTXT4(NB,NEL,IBYTES,IBYTEF,IPRTYF,IPRTYF,LDOS,NSAMPS,
                        IX,IXY)
      80 CONTINUE
C     ONE RECORD HAS BEEN FORMED IN ALL THE BANDS, WRITE IT AS NB RECORDS
C     ON DISK.
      IF(.NOT.BANDS)GO TO 130
      DO 120 J=1,NB
      CALL DAWX(0,JREC,IX(Y(IJ+1),IJB)
120     JREC=JREC+1
      IF(.NOT.FVECT)GO TO 70
      JELL=0
      DO 50 J=1,NB
      JELL=JELL+1
      50 CONTINUE
      CALL SAWN(NTBQ,IX,JELL,NB)
      GO TO 70
130    CALL SAWN(NTBQ,IX,NB)
      70 CONTINUE
C     END OF LOOP ON RECORDS
C     RETURN
      END
SUBROUTINE FRTY3(INB, NEL, IBYTE1, IBYTEF, IPRTY1, IBDS, NSAMPS, IX, IY
DIMENSION IBYTE1(NB), IBYTEF(NB), IPRTY1(NB), IBDS(NB), NSAMPS(NB)
LOGICAL IX(1), IY(NEL, NB)
DO 90 IB=1, NB
IBYTE1=IBYTE1(IBDS(IB)-1)*2
IBYTE2=IBYTEF(IB)
IF(1.EQ.1) JEL=1
IF(1.GT.1) JEL=NSAMPS(IB-1)+1
IF(IPRTY1(IB).EQ.1) GO TO 100
IY(JEL, IB)=IX(IBYTE1)
JEL=JEL+1
IBYTE1=IBYTE1+7
100 CONTINUE
DO 110 IEL=IBYTE1, IBYTE2, 8
IY(JEL, IB)=IX(IYEL)
JFL=JEL+1
IY(JEL, IB)=IX(JEL+1)
110 JFL=JFL+1
90 CONTINUE
RETURN
END
SUBROUTINE FRTXT4(NR,NFL,IBYTE1,IBYTEF,IPRTYI,IBDS,NSAMPS,I,IX,IY)
DIMENSION IBYTE1(NB),IBYTEF(NB),IPRTYI(NB),IBDS(NB),NSAMPS(NB)
LOGICAL I IX(1),IY(NB,NEL)
DO 90 IB=1,NB
   IBYTE1=IBYTE1(IB)+(IBDS(IB)-1)*2
   IBYTE2=IBYTEF(IB)
   IF(I.EQ.1)JEL=1
   IF(I.GT.1)JEL=NSAMPS(I-1)+1
   IF(IPRTYI(IB).EQ.1)GO TO 100
   IY(IB,JEL)=IX(IBYTE1)
   JEL=JEL+1
90 CONTINUE
   IBYTE1=IBYTE1+7
100 CONTINUE
   DO 110 IEL=IBYTE1,IBYTE2,8
      IY(IB,JEL)=IX(IEL)
      JEL=JEL+1
110 JEL=JEL+1
90 CONTINUE
RETURN
END
SUBROUTINE ERTXTA(IX,NB,NEL,NTapo,HIST,IH,NREC)
LOGICAL*1 IX,NEL,HIST
DIMENSION IH(256,NB)

C FORM INDIVIDUAL BAND IMAGES BY READING DATA FROM DISK WRITTEN BY
C ERTXT5. NB FILES OF OUTPUT ARE WRITTEN ON NTapo. IF HIST, GENERATE
C HISTOGRAMS OF EACH OF THE BANDS IN IH.

C IF(HIST)CALL SWSC1(IH,256,NB,0)
DO 10 IB=1,NB
WRITE(NTapo,NREC,NEL
JREC=IB
DO 20 JREC=1,NREC
READ(90,JREC)IX
JREC=JREC+NB
IF(HIST)CALL LRHSTG(IX,NEL,1H(1,IB))
20 IF(NTapo.GT.0)WRITE(NTapo,IX
10 IF(NTapo.GT.0)END FILE NTapo
RETURN
END
SUBROUTINE IRHSTG(IY,N,IH)
DIMENSION IH(256)
LOGICAL*1 LX(N),LW(4)
equivalence(IW,LW(1))
data IW/0/.
do 10 I=1,N
lw(4)=lx(I)
10 ih(IW+1)=ih(IW+1)+1
return
end

SUBROUTINE DARN(IDEV,IREC,X,N)
LOGICAL*1 X(N)
read(idev,irec)X
return
entry daw(i,idev,irec,x,n)
write(idev,irec)X
return
entry sar(n,ntapi,x,n)
read(ntapi)X
return
entry sar(n,ntapi,x,n)
write(ntapi)X
return
end
SUBROUTINE ERTXTB(NPIXS,IX,IY,NBDS,IBDS,NELMIN,BANDS,FVECT,NTPFY)
COMMON/TNDRO/IRI,IRF,ICI,ICF
LOGICAL*1 IX(1),IY(4,1),BANDS,FVECT,SCRMBL
DIMENSION IBDS(4)
INTEGER*2 LLC
INTEGER*2 LLX
SCRMBL=NBDS.NE.4
DO 5 I=1,NBDS
5 SCRMBL=SCRMBL.OR.IBDS(I).NE.1
C
C DIMENSION IX, IY (4*3300)
LLA=NPIXS*4
NELMIN=ICF-ICI+1
C
C SKIP TO IRI1TH RECORD OF DATA (NOTE: FIRST 2 RECORDS ARE ID AND
C ANNOTATION; THEY ARE ASSUMED TO HAVE BEEN SKIPPED ON UNIT 1)
C
C DO 10 I=1,4
IRI1=IRI1+1
IF(I.EQ.1)IRI1=IRI1-1
IF(IRI1.EQ.0)GO TO 10
DO 20 IR=1,IRI1
20 CALL READNL(IX,IEND,LRECL,I)
10 CONTINUE
C
C EXTRACT AND REARRANGE DATA FOR THE REGION OF INTEREST.
C
C DO 30 IREC=IRI1,IRF
C
C MERGE DOUBLE BAND INTERLEAVED DATA.
C
C IAD=1
DO 80 I=1,4
CALL READNL(IX(IAD),IEND,LRECL,I)
IF(I.EQ.1)CALL VMOV1(IX(LRECL-1),2,LLC)
IAD=IAD+LRECL-56
80
C
C FIND IDEL=NO. OF GENUINE PIXELS BETWEEN SYNTHETIC PIXELS.
C
C IDEL=LLC/(LLA-LLC-6)
CALL LINFIX(IX,IY,LLA,1,IDEL)
C
C NOW IY HAS LLC FEATURE VECTORS. FIND COLUMN ADDRESSES CORRESPONDING
C TO ICI,ICF (IT IS ASSUMED THAT ICI,ICF REFER TO THE FRAME INCLUDING
C SYNTHETIC PIXELS)
C
C ICI=ICI-(ICI-1)/(IDEL+1)
ICF=ICF-((ICF-1)/(IDEL+1)
NEL=ICF-ICI+1
NELMIN=MIN(NELMIN,NEL)
C
C IF(BANDS)CALL ERTXTB TO REARRANGE IY INTO BANDS AND WRITE ON DISK.
IF(.NOT.SCRMBL AND FVECT) WRITE FV'S ON NTPFV.
IF(SRMBL AND FVECT) CALL ERXT9 TO SCRAMBLE THE FV'S AND WRITE ON NTPFV.

IF(BANDS) CALL ERXT8(IY(1,ICIO),IX,NBDS,IBDS,NELO,IREC-IRI*1)
IF(.NOT.SRMBL AND FVECT) CALL SAWN(NTPFV,IY(1,ICIO),NELO*4)
IF(SRMBL AND FVECT) CALL ERXT9(IY(1,ICIO),IX,NBDS,IBDS,NELO)
IF(SRMBL AND FVECT) CALL SAWN(NTPFV,IX,NELO*NBDS)
CONTINUE
PRINT 100,NELMIN
FORMAT('NUMBER OF PIXELS PER LINE AFTER REMOVAL OF SYNTHETIC PIXELS
LS='15)
RETURN
END
**LINFIX CSECT**
* THIS ROUTINE REARRANGES THE EIGHT BYTE ERTS PIXEL PAIRS TO
* SEPARATE THE ADJACENT PIXELS. THE MAPPING IS
* 12345678 BECOMES 13572468

```plaintext
USING *.12
SAVE (14,12),
LR 12,15
LR 11,13
LA 13,SAVE
ST 11,SAVE+4
ST 13,8(11)
```

* LOAD PARAMETER LIST

```plaintext
LM 2,6,0(1) FETCH PARAMETER LIST
ST 2,REC1 SAVE ADR OF INPUT ARRAY
ST 3,LPIX SAVE ADR OF OUTPUT ARRAY
SR 0,0
LH 0,0(4) FETCH VALUE OF NPIXLN
ST 0,NPIXLN SAVE VALUE OF NPIXLN
L 0,0(5) LOAD LLC SWITCH VALUE
ST 0,LLC SAVE LLC 0=FALSE 1=TRUE
L 0,0(6) LOAD IDLE PIXEL REPEAT SPACING
ST 0,IDEL
```

* SET UP INDICES FOR INNER LOOP

```plaintext
L 2,REC1 ADR OF INPUT ARRAY
L 3,LPIX ADR OF OUTPUT ARRAY
SR 4,4 ZERO OUT INDEX REGISTER
SR 5,5 ZERO OUT TRANSITION REGISTER
L 6,F*4 LOAD INCREMENT REGISTER
L 7,NPIXLN SET UP COMPARAND
SLA 7,2 * MULTIPLY BY 4
S 7,F1* NOW COMPARAND = NPIXLN*4 -1
SR 8,8 COUNTER - UP TO REPT. PIXEL
L 10,F1* INCREMENT FOR BYX
L 11,IDEF COMPARAND FOR BYX - SPACING
L 9,LLC LOAD LINE LENGTH CORRECTION SWITCH
LTR 9,9 TEST LINE LENGTH CORRECT SWITCH
BNZ LADJ IF .TRUE. GO TO LINE ADJ CODING
```

* INNER LOOP - MAP BYTES INTO NEW POSITIONS

```plaintext
TOP IC 5,0(4,2) BAND 4 PIXEL LEFT
STC 5,0(4,3)
IC 5,1(4,2) 4 RIGHT
STC 5,4(4,3)
IC 5,2(4,2) 5 LEFT
STC 5,1(4,3)
IC 5,3(4,2) 6 RIGHT
STC 5,5(4,3)
```

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<table>
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B     END

**LADJ** BXH 8,10, FIX1  COUNT PIXELS UP TO IDEL - REPEAT INTERVAL
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**PART2** BXH 8,10, FIX2
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</table>

**BOTTOM** BXLE 4,6, LADJ
B     END

**FIX1** SP 3,8
| S    | 3, =F14' | ADJUST OUTPUT POINTER FOR DELETED PIXEL |
| B    | PART2    | PROCESS RIGHT PIXEL OF THIS PAIR |

**FIX2** SR 8,8
| S    | 3, =F14' | RESET OUTPUT POINTER FOR DELETED PIXEL |
| B    | BOTTOM   | END OF GROUP |

*     END OR ROUTINE

L     13, SAVE +4
RETURN (14,12), T, RC=0
SAVE DS 1F
RFC1 DS 1F
LPiX DS 1F
NPiXLN DS 1F
LLC DS 1F
IDEL DS 1F
END
SUBROUTINE ERTX8(IX, IY, NBDS, IBDS, NEL, IRC)

    LOGICAL *1 IX(4, NEL), IY(NEL)

    DIMENSION IBDS(NBDS)

    IREC = (IREC - 1) * NBDS

    DO 10 I = 1, NBDS
    DO 20 J = 1, NEL

    20 IY(J) = IX(IBDS(I), J)
    IREC = IREC + 1

    10 WRITE(90, IREC) IY

    RETURN

    END

SUBROUTINE ERTX9(IX, IY, NBDS, IBDS, NEL)

    LOGICAL *1 IX(4, NEL), IY(NBDS, NEL)

    DIMENSION IBDS(NBDS)

    DO 10 I = 1, NBDS
    DO 10 J = 1, NEL

    10 IY(I, J) = IX(IBDS(I), J)

    RETURN

    END

SUBROUTINE PTHST(IH, N)

    DIMENSION IH(N)

    DO 10 I = 1, N

    IF (IH(I) .EQ. 0) GO TO 10

    J = I - 1

    10 WRITE(6, 100) J, IH(I)

    100 FORMAT(10X, 14, 18X, I7)

    CONTINUE

    RETURN

    END
5-2. COMPUTER CLASSIFICATION

1. **NAME**

   EFFECT – Effective Figure of Merit Feature Selection Criterion

2. **PURPOSE**

   This subroutine is used to implement a nonparametric feature selection criterion. The separability of classes of data from the remaining classes is required in the design of a sequential linear classifier.

3. **CALLING SEQUENCE**

   CALL EFFECT (X, NS, CLASS, MOC, DE, NW1, NN, MM)

   X          - the array of data samples, with subscripts corresponding to feature number, class number, and sample number
   NS         - array containing number of data samples per class
   CLASS      - array containing class names (8 characters)
   MOC        - array containing class numbers in separability order
   DE         - array of interclass and intraclass distances for each feature
   NW1        - class counter
   NN          - number of features
   MM         - number of classes

4. **INPUT/OUTPUT**

   4.1 Input

   All input is via the arguments of the calling statement.

   4.2 Output

   Printed output consists of the values of the normalized figure of merit for each feature, for each class remaining under consideration. This is followed by a list of the effective figure of merit for each class, listed in descending order of merit.

5. **EXITS**

   There are no nonstandard exits.
6. **USAGE**

Computer: IBM 360/65
Language: FORTRAN IV

7. **EXTERNAL INTERFACES**

7.1 System Subroutines

The subroutine SORTLS is called to arrange the effective figures of merit in descending order.

8. **PERFORMANCE SPECIFICATIONS**

8.1 Storage

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<td></td>
</tr>
</tbody>
</table>

8.2 Execution Time

In a typical case of 100 training samples for each of six classes of four band data, the interclass and intraclass distances (all elements of array DE) are computed in 9 seconds.

9. **METHOD**

The interclass and intraclass distances are labeled SUM 1 and SUM 2, respectively. They are computed for the I-th feature by the formulas:

\[
\text{SUM 1 (I)} = \sum_{I3=1}^{MM} \sum_{I4=1}^{I3-1} \sum_{LK1=1}^{LKK1} \sum_{LK2=1}^{LKK2} \left[ X(I, I3, LK1) - X(I, I4, LK2) \right]
\]

\[
\text{SUM 2 (I)} = \sum_{I3=1}^{MM} \sum_{LK1=1}^{LKK1} \sum_{LK2=1}^{LKK2} \left[ X(I, I3, LK1) - X(I, I3, LK2) \right]
\]

where MM is the number of classes and LKK1, LKK2 are the number of samples of the classes I3 and I4, respectively. The sums over sample numbers are the elements of array DE.
The normalized figure of merit is given by:

\[ F(I) = \frac{\text{SUM } 1(I)}{\text{SUM } 2(I)} \]

The figures of merit for each feature are then combined to give the figure of merit for the class.

10. **COMMENTS**

The elements of array DE are computed when the routine is called the first time (NW1 = 1). On succeeding calls, this calculation is bypassed.
11. LISTING

SUBROUTINE EFFECT (X, NS, CLASS, MOC, DF, NW1, NN, MM)

C EFFECTIVE FIGURE OF MERIT FEATURE SELECTION CRITERION
C
DIMENSION X(NN,MM,1), NS(MM), MOC(MM), DF(MM,MM,NN), FC(20),
.CFM(20)
DOUBLE PRECISION CLASS(MM)

142 FORMAT ('1'/'2OX, 'EFFECTIVE FIGURES OF MERIT'/20X,26('*')/17X,16I1)

143 FORMAT (/2OX,'COMBINED FIGURES OF MERIT'/20X,25('*')/)

150 FORMAT (/2OX, 'COMBINED FIGURES OF MERIT'/20X,25('*')/)

151 FORMAT (I12,1A10,F14.5)

C COMPUTES INTER-CLASS AND INTRA-CLASS DISTANCES

WRITE (6,142) (I, I=1,NN)

IF (NW1.NE.1) GO TO 2000

DO 1005 NF1=1,NN

1005 CONTINUE

1111 MOC(NC) = NC

C COMPUTES THE NORMALIZED FIGURE OF MERIT OF ALL REMAINING PATTERN
C CLASSES ALONG EACH OF THE FEATURE DIRECTIONS

C
2000 CONTINUE

DO 1100 J3=NW1,MM

CMF(J3) = 1.0

I3 = MOC(J3)

AI3 = NS(I3)

DO 3000 I=1,NN

FCMIN = 1.0 / 50

NST = 0

SUM1 = 0.0

SUM2 = 0.0

C COMPUTE SUM1 - TOTAL OF INTERCLASS DISTANCES FROM CLASS I3 TO ALL
C REMAINING CLASSES

C

DO 6 J4=NW1,MM

IF (J4.EQ.J3) GO TO 6

I4 = MOC(J4)

AI4 = NS(I4)

NST = NST + NS(I4)

SUM1 = SUM1 + DE(I3,I4,1)

C COMPUTE SUM2 - TOTAL OF DISTANCES AMONG ALL REMAINING CLASSES
C EQUIVALENT TO INTRACLASS DISTANCE OF ALL REMAINING CLASSES CONSIDERED
C AS ONE CLASS

C

180
DO 7 J5=N+1, J4
 IF (J5.EQ.J3) GO TO 7
 I5 = MOC(J5)
 SUM2 = SUM2 + DE(I4, I5, I)
 7 CONTINUE

C COMPUTE MINIMUM FIGURE OF MERIT FOR INDIVIDUAL CLASSES I3 AND I4

C
S1 = DE(I3, I4, I) / (A13*A14)
S2 = DE(I3, I3, I) / (A13*(A13-1.0)) * DE(I4, I4, I) / (A14*(A14-1.0))
F = S1 / S2
IF (F.LT.FCMIN) FCMIN = F
 6 CONTINUE

C
ANST = NST
SUM1 = SUM1 / (NS(I3)*NST)
SUM2 = DE(I3, I3, I) / (A13*(A13-1.0)) * SUM2 / (ANST*(ANST-1.0))
FC(I) = FCMIN * SUM1 / SUM2
FC(I) = EXP(-1.0/FC(I))

C
THE MOST SEPARABLE CLASS

C
3000 CFM(J3) = CFM(J3) * FC(I)
 CFM(J3) = CFM(J3) ** (1.0/NF1)
 WRITE (6,143) I3, CLASS(I3), (FC(NF), NF=1,NN)
 1100 CONTINUE
 CALL SORTL (CFM, MOC, NW1, MM)

C
PRINT 150
DO 1251 NC1=N+1, MM
 1251 PRINT 151, MOC(NC1), CLASS(MOC(NC1)), CFM(NC1)
 CALL SORTLS (MOC, MOC, NW1+1, MM)
RETJR
END
12. **TEST RESULTS**

<table>
<thead>
<tr>
<th>Category</th>
<th>EFFECTIVE FIGURES OF MERIT</th>
<th>COMBINED FIGURES OF MERIT</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 URBAN 11</td>
<td>0.5182 0.5195 0.6611 0.6613</td>
<td>0.58904</td>
</tr>
<tr>
<td>2 TRANS 15</td>
<td>0.4766 0.4914 0.5934 0.5568</td>
<td>0.59543</td>
</tr>
<tr>
<td>3 AGRIC 21</td>
<td>0.4895 0.4442 0.7357 0.7526</td>
<td>0.52743</td>
</tr>
<tr>
<td>4 DECID 31</td>
<td>0.5884 0.4832 0.4919 0.4551</td>
<td>0.50228</td>
</tr>
<tr>
<td>5 EVGRN 32</td>
<td>0.7841 0.6479 0.5169 0.4727</td>
<td></td>
</tr>
<tr>
<td>6 WATER 61</td>
<td>0.4613 0.5555 0.9130 0.9263</td>
<td></td>
</tr>
</tbody>
</table>
1. **NAME**

SNOPAL - Supervised Nonparametric Learning

2. **PURPOSE**

This subroutine is used to derive the coefficients of the linear functions used in a sequential linear classifier.

3. **CALLING SEQUENCE**

CALL SNOPAL (X, NS, CLASS, W, MOC, S, Y, B, NW1, NN, MM, NN1, MM1)

- **X** - the array of training data, labeled by feature number, class number, sample number
- **NS** - array containing number of data samples per class
- **CLASS** - array containing class names (8 characters)
- **W** - array of coefficients of the linear discriminant functions
- **MOC** - array containing class numbers in order of testing
- **S** - double precision array of dimension NN1 x NN1
- **Y** - work array of length total number of training samples
- **B** - same as Y
- **NW1** - class counter
- **NN** - number of features
- **MM** - number of classes
- **NN1** - NN + 1
- **MM1** - MM - 1
4. INPUT/OUTPUT

4.1 Input

All input is via the calling arguments.

4.2 Output

The coefficients are output by the argument "W". Printed output is the coefficients and the errors for each iteration of the algorithm.

5. EXITS

There are no nonstandard exits.

6. USAGE

Computer: IBM 360/65

Language: FORTRAN IV

7. EXTERNAL INTERFACES

The subroutine GASINV is called to invert a matrix.

8. PERFORMANCE SPECIFICATIONS

8.1 Storage

<table>
<thead>
<tr>
<th>Subroutine</th>
<th>Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>SNOPAL</td>
<td>4172</td>
</tr>
<tr>
<td>GASINV</td>
<td>1838</td>
</tr>
<tr>
<td>Total</td>
<td>6010 bytes</td>
</tr>
</tbody>
</table>

8.2 Execution Time

The execution time for each call in constructing a sequential linear classifier varies because of variations in the number of iterations required and the number of training samples. In a typical four-band, six-class problem, the time spent by this routine approximately 1 minute.
9. **METHOD**

The method consists of maximizing the total distance of the training samples from the discriminant hyperplane, as described by J. C. Ho and R. L. Rasyap, "A Class of Iterative Procedures for Linear Inequalities," *J. Siam on Control*, 1966.

10. **COMMENTS**

The algorithm performs a maximum of 100 iterations. Otherwise, for four-band data, iterations cease when all coefficients change by less than 1 percent.
11. LISTING

SHAROUTINE SNORPAL (X,NS,CLASS,W,MOC,S,Y,A,NW1,NN,MM,NN1,MM1)

SUPERVISED NON-PARAMETRIC LEARNING

DIMENSION X(NN,MM,1), NS(MM), W(MM1,NN1), MOC(MM), Y(1), B(1)
DOUBLE PRECISION CLASS(MM), S(NN1,NN1), DET
LOGICAL TEST
DATA NI /1CO/
100 FORMAT (18,111,18,4X,1P7E14.3/(31X,1P7E14.3))
101 FORMAT (/113,110,10X,1P7E14.3/(33X,1P7E14.3))
102 FORMAT (/22X,22(*1)/22X,22(*1)/) /* ITE
RATION NO. 5X, 'ERRORS', 10X, 'LINEAR DISCRIMINANT COEFFICIENTS'/*
   A22, 'OTHER'/)
216 FORMAT (*1/10X, 'ORDERED CLASSES', 20X, 'ELEMENTS OF THE DISCRIMINAN

T VECTOR'/10X, 15(*1), 20X, 35(*1)/)
220 FORMAT (*7X, 'TOTAL ERRORS', 15)

C

C INITIALIZE W, Y, AND B ARRAYS

NW = MOC(NW1)
NSW = NS(NW)
NW2 = NW1 + 1
DELTA = NN/400.0
DO 1112 NFA=1,NN1
   NST2 = 0
   DO 1110 NC1=1,NW1,MM
      NC = MOC(NC1)
      NSC = NS(NC)
      NST2 = NST2 + 1
      Y(NST2) = -1.0
   1110 B(NST2) = 1.0
   1112
   C

C COMPUTE INVERSE OF A(TRANSPOSE) A WHERE 'A' IS AUGMENTED MATRIX OF SAMPLE

DO 130 I=1,NN
   DO 130 J=I,NN1
      S(I,J) = 0.0
   130 S(I,J) = S(I,J) + A1
   DO 131 NC1=1,NW1,MM
      NC = MOC(NC1)
      NSC = NS(NC)
      DO 131 NS1=1,NSC
         A1 = X(I,NC,NS1)
         IF (.LE.NF-NN1) A1 = A1*X(I,NC,NS1)
      131 S(I,J) = S(I,J) + A1
   130 S(I,J) = S(I,J)
   S(NN1,NN1) = NST2
   CALL GASINV (S, NN1, DET)
   C

C DO NI ITERATIONS OF THE HD-KASHYAP ALGORITHM, UNLESS ALL COEFFICIENTS
C CHANGE BY LESS THAN DELTA = NN/4 PERCENT

PRINT 102, NW, CLASS(NW), CLASS(NW)
DO 1040 INDEX=1,NI
   TEST = .TRUE.
   DO 1101 I=1,NN1
      WO = W(NW1,I)
   1101
   1040

C 1101 CONTINUE
C COMPUTE NEW DISCRIMINANT VALUES AND CLASSIFICATION ERRORS
C
DO 1034 I = 1, NSN
Y(I) = w(NW1, I)
IF (Y(I).GT.0.0) B(I) = 6(I) + 2.O*Y(I)
1034 CONTINUE
C
GOTO 1040
C
IF (TEST) GOTO 1010
1010 NERR = NERK1 + NERK2
PRINT 220, NERR
C
IF (NW1.NE. NNA) RETURN
WRITE (6, 216)
DO 1220 N = 1, N41
NC = MC(NW1)
1220 PRINT 101, NC, CLASS(NC), w(NW1, NFA), NFA=1, NNA
PRINT 101, MC(NW1), CLASS(MC(NW1))
RETURN
END
### TEST OUTPUT

<table>
<thead>
<tr>
<th>ITERATION NO.</th>
<th>ERRORS</th>
<th>LINEAR DISCRIMINANT COEFFICIENTS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>DECID 31 OTHER</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>11</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
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<td>2</td>
<td>4</td>
</tr>
<tr>
<td>24</td>
<td>2</td>
<td>4</td>
</tr>
</tbody>
</table>

**TOTAL ERRORS 6**
1. **NAME**

NOPACA - Nonparametric Classification Algorithm

2. **PURPOSE**

This subroutine has as its purpose the implementation of this method of classification.

3. **CALLING SEQUENCE**

CALL NOPACA (X, NW, W, MOC, NSS, NN, NN1, MM1)

X - array of feature vectors to be classified
NW - array of class numbers assigned to input feature vectors
W - array of coefficients of the linear discriminant functions
MOC - array containing class numbers in order of testing
NSS - number of feature vectors to be classified
NN - number of spectral bands
NN1 - NN + 1
MM1 - number of classes less one

4. **INPUT/OUTPUT**

4.1 Input

All input is via the items in the calling statement.

4.2 Output

The output is described under CALLING SEQUENCE.

5. **EXITS**

There are no nonstandard exits.
6. **USAGE**

   Computer: IBM 360/65  
   Language: FORTRAN IV

7. **EXTERNAL INTERFACES**

   None.

8. **PERFORMANCE SPECIFICATIONS**

   8.1 **Storage**

      792 bytes

   8.2 **Execution Times**

      The time required to classify a four-band feature vector is 0.06 millisecond per class present.

9. **METHOD**

      The unknown feature vector is used to evaluate the discriminant functions in the order of class separability determined by subroutine EFFECT. The unknown feature vector is assigned to that class for which the evaluation is positive.

10. **COMMENTS**

     None.
11. LISTING

SUBROUTINE NOPACA (X, NW, W, MOC, NSS, NN, Nw1, MM1)
C
C NON-PARAMETRIC CLASSIFICATION OF A STRING OF NSS FEATURE VECTORS
C USING PRE-LEARNED LINEAR DISCRIMINANT FUNCTIONS
C
DIMENSION X(NN,NSS), NW(NSS), W(MM1,1), MOC(1)
C
DO 20 NS1=1,NSS
DO 1 NW1=1,MM1
G = w(NW1,NN1)
DO 2 NF1=1,NN
2 G = G + W(NW1,NF1) * X(NF1,NS1)
IF (G.GT.0.0) GO TO 3
1 CONTINUE
NW(NS1) = MOC(MM1+1)
GO TO 20
3 NW(NS1) = MOC(NW1)
20 CONTINUE
RETURN
END
5-3. GEOGRAPHIC REFERENCING

1. NAME

GEOGREF

2. PURPOSE

Find a geometric transformation from one coordinate system to another such that the mean squared error at a given set of control points is minimized. The transformation is linear, accounting for rotation, scale change, skew, and translation.

3. CALLING SEQUENCE

This is a main program. It is currently on a partitioned data set as an executable module.

4. INPUT-OUTPUT

4.1 Input

The following input parameters should be supplied in data cards according to the formats and read statements indicated below.

1     READ 100,NCP,ICI,ISP,NTRY
     IF(NCP.LE.0) STOP
     ..
     DO 10 N=1,NCP
     N2=N*2
     READ 102,X(N2-1),X(N2),Y(N2-1),Y(N2),TITL
     ..
     10    CONTINUE
     ..
     GO TO 1
100   FORMAT (4I6)
102   FORMAT (4D12.0,32A1)

where

NCP=Number of control points (if NCP<0, the program stops)
ICI,ISP are special parameters to be used when handling Landsat
images, with the ground control points' coordinates measured relative to the image without the synthetic pixels removed and the transformation required being from the image with no synthetic pixels. ICI is then equal to the initial pixel number on the Landsat frame starting from which the region of interest was extracted and ISP is the number of real pixels between synthetic pixels. If ISP is specified as zero, this special case is ignored and no corrections are applied to the coordinates \(Y(N2)\). It is generally more convenient to remove synthetic pixels in advance and supply ISP=0. NTRY=Number of fits to be found for the current set of control points using the successive elimination procedure (See Section 9).

\[X(N2-1), X(N2): \text{coordinates of the } N\text{'th control point in the reference image (e.g. UTM coordinates)}\]

\[Y(N2-1), Y(N2): \text{coordinates of } N\text{'th control point in the observed image (e.g. Landsat pixel coordinates).}\]

The transformation is found from the X's to the Y's. Note that the loop starting at statement number 1 indicates that the program finds transformations for several sets of control points, until terminated by, say, a blank card while reading NCP.

TITL: Arbitrary 32 character title.

4.2 Output

The output of this program is a printout of the control point coordinates, the fit parameters found and a table of errors at all the control points. A typical output is attached at the end.

4.3 File Storage

None.

5. EXITS

Not applicable.

6. USAGE

The program is in FORTRAN IV and implemented on the IBM360 using the H compiler. It is in the user's library in its executable form.
7. EXTERNAL INTERFACES

The linkage with the subroutines required by this program is shown in the following table.

<table>
<thead>
<tr>
<th>Calling Program</th>
<th>Programs Called</th>
</tr>
</thead>
<tbody>
<tr>
<td>GEOGREF</td>
<td>RSVP</td>
</tr>
<tr>
<td></td>
<td>EIVFIT</td>
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</tbody>
</table>

8. PERFORMANCE SPECIFICATIONS

8.1 Storage

The program is 7688 bytes long, but including external references required and the buffers, this program requires 62K bytes of storage.
8.2 Execution Time

Depends on the number of cases to be considered and NCP,NTRY in each case. With NCP=36 and NTRY=6, this program takes approximately four seconds per case.

8.3 I/O Load

None

8.4 Restrictions

None

9. METHOD

In the special case of Landsat data (described in Section 4.1) the routine RSVP is first used to modify the coordinates Y(N\times2) for N=1, \ldots,NCP. The means of both X and Y coordinates over N=1,\ldots,NCP are found and subtracted in order avoid possible inversion of a matrix with large numbers during the determination of the transformation.

The routine EHVFIT is used to determine the transformation. This routine considers a given subset of the control points supplied and, using the least squares fit program LNLLS [20], finds the geometric transformation parameters. The transformation so formed is used to compute the error at all the control points used for the fit. The mean and variance of the error are found. The points with error greater than or equal to the (mean + variance) are included in the set of points to be eliminated for the next trial.

The main program GEOGREF calls EHVFIT NTRY times. During the first call the set of points to be eliminated is null. During the subsequent calls this set is appended with points causing high error so that with each attempt the RMS error at the fit points is reduced. After each call to EHVFIT, the set of points used for finding the transformation, the RMS error at the fit points, the values of the parameters defining the transformation, a table of errors at all the control points, the overall RMS error and the set of points to be omitted next are printed.
10. COMMENTS

The details of the subroutines are omitted here. The least squares fit routine LNLLS is described elsewhere [20].

11. LISTINGS

The listings of the program, the associated subroutines are attached at the end.

12. TESTS

The program has been tested for several sets of control points.
TO FIND GEOMETRIC TRANSFORMATION NEEDED FOR GEOGRAPHIC REFERENCING

DIMENSION W(100), ICOMB(50), ICOMBO(50)
DOUBLE PRECISION NORM, X(100), Y(100), XP(100), YP(100), XOR(100),
YOR(100), W(12), GP1(6), TB(6), DX, DY
LOGICAL*1 TITL(32)

D*N W(2*NCP), ICOMB(NCP), ICOMBO(NCP)
D*P* X(2*NCP), Y(2*NCP), XP(2*NCP), YP(2*NCP), XOR(2*NCP), YOR(2*NCP)
WHERE NCP IS THE MAXIMUM NUMBER OF G. C. P.'S EXPECTED FOR ONE FIT.
COMMON /MORN/ NORM /PRINV/ INV
INV=0
CONTINUE
READ 100,NCP,ICI,ISP,NTRY
READ F.N ROW, COL.
READ 102,X(N2-1),X(N2),Y(N2-1),Y(N2),TITL
NEWY=Y(N2)
CALL RSVP(NEWY,ISP,ICI,NEWY)
Y(N2)=NEWY
XM1=XM1+X(N2-1)
XM2=XM2+X(N2)
YM1=YM1+Y(N2-1)
YM2=YM2+Y(N2)
PRINT 103,N,X(N2-1),X(N2),Y(N2-1),Y(N2),TITL
PRINT 105,XM1,XM2,YM1,YM2
XM1=XM1/NCP
XM2=XM2/NCP
YM1=YM1/NCP
YM2=YM2/NCP
PRINT 104,XM1,XM2,YM1,YM2
PRINT 101
DO 15 N=1,NCP
N2=N*2
X(N2-1)=X(N2-1)-XM1
X(N2)=X(N2)-XM2
Y(N2-1)=Y(N2-1)-YM1
Y(N2) = Y(N2) - YM2
15 PRINT 103, N, X(N2-1), Y(N2), X(N2-1), Y(N2)
C
W(1) = -1
MCP = NCP
K = 0
NERR = 0
DO 10 KK = 1, NTRY
NCPK = NCP - K
CALL EHVEIT(K, ICOMB, NCP, Y, YP, XP, YP, W, GP1, NERR, ERMIN, ERMEAN, ERVAR, ICOMBO)
IF(NERR .EQ. 1) GO TO 20
C
ERROR CONDITION -- NO LEAST SQUARES FIT
C
PRINT 106
STOP
C
20 CONTINUE
IF(ERMIN .GT. 0.) ERMIN = SORT(ERMIN/NCPK)
DX = GP1(5) + YM1 - GP1(1) * XH1 - GP1(2) * XM2
DY = GP1(6) + YM2 - GP1(3) * XH1 - GP1(4) * XM2
WRITE(6,121) NERR
WRITE(6,124) NCPK
WRITE(6,122) (ICOMBO(J), J = 1, NCPK)
WRITE(6,123) ERMIN, ERMEAN, ERVAR
WRITE(6,108) NRM
WRITE(6,107) (GP1(J), J = 1, 4), DX, DY
C
WRITE(6,111)
RMS = 0.
G0 25 N = 1, NCP
N2 = 2 * N
C
CALL SUBAT (N, X, GP1, W1, TB)
C
DU = Y(N2-1) - TB(1)
DV = Y(N2) - TB(2)
SQ = DU**2 + DV**2
RMS = RMS + SQ
XMAG = SQRT(SQ)
YDIR = RADDEG * ATAN2(DV, DU)
PRINT 109, N, Y(N2-1), Y(N2), TB(1), TB(2), XMAG, YDIR, DU, DV
C
CONTINUE
RMS = SQRT(RMS/FLOAT(NCP1))
WRITE(6,114) RMS
WRITE(6,119) K
IF(K .NE. 0) WRITE(6,120) (ICOMBO(J), J = 1, K)
IF(NCPK .LE. 2) GO TO 1
1000 CONTINUE
G0 1
C
FORMAT STATEMENTS.
C
100 FORMAT(1216)
SUBROUTINE CHVFIT(K, ICOMB, N, X, Y, XP, YP, W, GP, NERR, ERR, ERMEAN, ERVAR, ICOMBO)

C PERFORM FITS BY ELIMINATING POINTS WITH HIGH VARIANCE.
C FIRST, FIND THE FIT AND ERRORS AT THE POINTS SUPPLIED.
C NEXT, IDENTIFY LOCATIONS OF HIGH ERROR AND STORE THEIR INDICES IN
C ICOMB IN PREPARATION FOR THE NEXT CALL OF CHVFIT.
C
C DIMENSION ICOMB(N), W(2, N), ICOMBO(N)
REAL*8 X(2, N), Y(2, N), XP(2, N), YP(2, N), GP(6), T(6), W1(12)
NK=N-K
CALL DPMMV(X, XP, 2, N, ICOMB, K)
CALL DPMMV(Y, YP, 2, N, ICOMB, K)
CALL LNLLS(W, XP, YP, GP, NERR, 6, NK, 2, 0, 0, 6HLIN, Y)
IF (NERR .NE. 1) RETURN

C FIND MEAN AND VARIANCE OF ERROR.

C ERMEAN=0
ERVAR=0
DO 50 J=1,NK
CALL SUBRT(J, XP, GP, W1, T)
DU=YP(1, J)-T(1)
DV=YP(2, J)-T(2)
ERR=DU**2+DV**2
XP(1, J)=SQRT(ERR)
ERVAR=ERVAR+ERR
50 ERMEAN=ERMEAN*XP(1, J)
ERMEAN=ERMEAN/NK
ERR=ERVAR
ERVAR=ERVAR/NK-ERMEAN**2
IF (ERVAR .GT. 0) ERVAR=SQRT(ERVAR)
ERTHKR=ERMEAN+ERVAR

C FIND THE SET OF POINTS TO BE ELIMINATED.
C FIRST, SET ICOMB=(1, 2, ... N)-ICOMB
C
I=1
L=0
DO 10 J=1,N
IF (J .GT. K) GO TO 40
IF (J .EQ. ICOMB(1)) GO TO 20
10 L=L+1
ICOMBO(L)=J
GO TO 10
20 I=I+1
10 CONTINUE

C NOW, ICOMBO(L) IS THE INDEX IN THE X SET CORRESPONDING TO L IN THE
C XP SET.
L = K
GO TO J = 1, NR
IF (X(I, J) .LT. ETA) GO TO 30
L = L + 1
ICOMB(L) = ICOMB(J)
CONTINUE
R = L
CALL SORT (ICOMB, 1, K, R, L, P, PP)
RETURN
END
SUBROUTINE SORT(A, IT, JJ, MM, NN, T, IT)

DIMENSION A(MM, NN), T(NN), TT(NN), IU(16), IL(16)
INTEGER A, T, IT
M=1
I=1
J=1
IF(I .GE. J) GO TO 70
K=1
I=[(I+J)/2]
CALL MVMMMR(A, MM, NN, T, 1, I, J)
IF(A(I, I), LE, T(I)) GO TO 20
CALL MVMMMR(A, MM, NN, A, MM, I, J)
CALL MVMMMR(T, 1, NN, A, MM, 1, I)
CALL MVMMMR(A, MM, NN, T, 1, I, J)
L=J
IF(A(J, J), GT, T(I)) GO TO 40
CALL MVMMMR(A, MM, NN, A, MM, J, I)
CALL MVMMMR(T, 1, NN, A, MM, 1, J)
CALL MVMMMR(A, MM, NN, T, 1, I, J)
GO TO 20
CALL MVMMMR(A, MM, NN, A, MM, K, L)
CALL MVMMMR(TT, 1, NN, A, MM, 1, K)
L=L-1
IF(A(L, 1), GT, T(1)) GO TO 40
CALL MVMMMR(A, MM, NN, TT, 1, L, 1)
K=K+1
IF(A(K, 1), LT, T(1)) GO TO 50
IF(K, LE, L) GO TO 30
IF(L, LE, J-K) GO TO 60
IL(M)=T
IU(M)=L
I=K
M=M+1
GO TO 80
IL(M)=K
IU(M)=J
J= L
M=M+1
GO TO 80
IF(J, EQ, 0) RETURN
I=IL(M)
J=IU(M)
IF(J-1, GE, 11) GO TO 10
IF(T, EQ, 11) GO TO 5
I=1
90 I=I+1
IF(I.EQ.J)GO TO 70
CALL MVMRMR(A,MM,NN,T,1,I+1,1)
IF(A(I,1).LE.T(1))GO TO 90
K=I
100 CALL MVMRMR(A,MM,NN,A,MM,K,K+1)
K=K-1
IF(T(1).LT.A(K,1))GO TO 100
CALL MVMRMR(T,1,NN,A,MM,1,K+1)
GO TO 90
END

SUBROUTINE MVMRMR(A,MA,N,B,MB,IA,IB)
DIMENSION A(MA,N),B(MB,N)
DO 10 J=1,N
10 B(IB,J)=A(IA,J)
RETURN
END
SUBROUTINE DPMMV%X,Y,M,N,ICOMB,K<

TO MOVE PART OF A DOUBLE PRECISION MATRIX SPECIFIED BY INDICES
BETWEEN 1 AND N OTHER THAN ICOMB%1<...ICOMB%K< INTO D. P.
MATRIX Y.

DIMENSION ICOMB%K<
DOUBLE PRECISION X%M,N<,Y%M,N<

I#1
L#1
DO 10 J#1,N
IF%I.GT.K<GO TO 40
IF%J.EQ.ICOMB%I<GO TO 20
40 DO 30 JJ#1,M
30 Y%JJ,.L<#X%JJ. J<
L#LE1
GO TO 10
20 I#IE1
10 CONTINUE
RETURN
END
SUBROUTINE LNLLS (W, X, Y, GP1, NERR, HP, NDP, NDIM, EPS, NIT, LINNO, TBI)

C PROGRAM TO CALL GAUSS AND MANAGE ITERATIVE CALCULATION
FOR NONLINEAR PROBLEMS

C DOUBLE PRECISION X(I), Y(I), GP1(I), TBI(NP), XX(30,30)
DIMENSION W(I)
DATA LINEAR*4MLIN. /
COMMON /PRINV/ INV

IF (W(I) .GT. 0.0) GO TO 10
SET ALL WEIGHTS TO UNITY IF NONE WERE SUPPLIED
ILP = NDP * NDIM
DC 5 I=1, IUP
5 W(I) = 1.0
CONTINUE

HANDLE LINEAR AND NONLINEAR PROBLEMS DIFFERENTLY
IF (LINNO .EQ. LINEAR) GO TO 60
C NONLINEAR PROBLEM --
C SET IMODE = 2, AND START ITERATIVE PROCEDURE
IMODE = 2
DC 30 N=1, NIT
CALL GAUSS (W, X, Y, GP1, NERR, HP, NDP, NDIM, XX, IMODE, TBI)

IF (NERR .EQ. 2) GO TO 70
IF (IMODE .EQ. 1) GO TO 35
C TEST FOR CONVERGENCE
DC 20 I=1, NP
IF (GP1(I) .LT. 16.0) 16, 17, 16
16 CONV = DABS (TBI(I) / GP1(I))
DC TO 18
17 CONV = DABS (TBI(I))
18 IF (CONV .LT. EPS) GO TO 25
CONTINUE
GO TO 35
C CONVERGENCE NOT YET ATTAINED
C UPDATE GP1, CONTINUE ITERATING
DC 30 I=1, NP
30 GP1(I) = GP1(I) + TBI(I)
C CONVERGENCE WAS NOT ATTAINED WITHIN SPECIFIED NUMBER OF
C ITERATIONS
NERR = 3
WRITE (6,100) NIT, EPS
100 FORMAT (20H00DID NOT CONVERGE IN,14,
28H ITERATIONS WITH CRITERION,E15.6)
GO TO 45
C CONVERGENCE WAS ACHIEVED FOR PROBLEM IS LINEAR
C MAKE FINAL UPDATES OF GPI

30 DO 40 I=1,NP
40 GPI(I) = GPI(I) + T(I)*I

C PRINT INVERSE MATRIX

45 IF (INV.LE. 0) GO TO 55

101 FORMAT (/,'INVERSE MATRIX')

50 WRITE (6,102) (XX(I,I),I=1,NP)

102 FORMAT (1HO,1P014+6).

55 RETURN

C SET UP FOR LINEAR PROBLEM

60 IMODE = 1

65 DO 65 I=1,NP

65 GPI(I) = 0.

C NORMAL MATRIX WAS SINGULAR

70 WRITE (6,133)

133 FORMAT (16H0SINGULAR MATRIX)

RETURN

END
SUBROUTINE GAUSS (W, X, Y, GP1, NERR, NP, NDP, NDIM, XX, XI, XI1, XI2, X1, Y1, Z1, N1, T1, IMODE, TB)

VECTOR LEAST SQUARES SUBROUTINE

DOUBLE PRECISION X(1), Y(1), GP1(1), T8(1)
DOUBLE PRECISION XX(30,30), TB(1)
DOUBLE PRECISION A(30,30), B(30), DERIV(60), NORM, FI(24), Y(12)
DIMENSION W(11)

C THE FIRST NDIM ELEMENTS OF W AND Y ARE FOR THE FIRST OBSERVATION.
C THE NEXT NDIM ARE FOR THE SECOND, ETC. IF THE INDEPENDENT variable IS A VECTOR, A SIMILAR CONVENTION IS OBSERVED FOR X, Y, ETC.
C CURRENT DIMENSIONS ARE FOR NDIM = 24 FOR OBSERVATIONS WITH
C HIGHER DIMENSIONALITY, THE DIMENSIONS OF FI, Y, A, AND (PERHAPS) NORM MUST BE INCREASED.
C COMMON /MORB/ NORM, A(NDIM), B(NDIM), T8(NDIM)

C ASSUME NO ERROR INITIALLY
NERR = 1

C INITIALIZE A AND B MATRICES
DC 110 L=1, NP
DC 110 M=1, NP

110 AT(L,M) = C.

C FOR EACH OBSERVATION DETERMINE (OLD) ESTIMATES AND DERIVATIVES.
C FORM SCMs TO GIVE MATRIX ELEMENTS.
C
DC 111 J=1, NDP

CALL SUBR (V, X, Y, GP1, DERIV, YC)

C THE FIRST NDIM ELEMENTS OF DERIV ARE PARTIAL DERIVATIVES OF
C successive vector components with respect to the first.
C THE SECOND NDIM ARE PARTIALS WITH RESPECT TO THE SECOND,
C ETC.

GO TO (1111, 1113), IMODE

C DIRECT CALCULATION OF PARAMETERS
1111 DO 1112 I=1, NDI

RETURN

END

STOP
1112 F1(I) = Y(I,SUB)
       GO TO 1119
C
C ITERATIVE CALCULATION OF PARAMETERS
C
1112 DO 1114 I=1,NDIM
   1114 F1(I) = Y(I,SUB) = YC(I)
1115 CONTINUE
KSUB = 0
DO 111 K=1,NP
   111 L=1,NDIM
   111 CONTINUE
KSUB = KSUB + 1
LSUB = LSUB + 1
BST = BST + DERIV(KSUB) * W(LSUB) * F1(L)
MSUB = KSUB - NDIM
DO 111 M=K,NP
   111 M=K,NP
MSUB = MSUB + NDIM
111 A(I,K) = A(I,K) + DERIV(KSUB) * W(LSUB) * DERIV(MSUB)
C
C COMPLETE NORMAL MATRIX (LOWER LEFT TRIANGLE)
C
DO 112 M=2,NP
   K = M - 1
   DO 112 I=1,K
   112 A(M,I) = A(I,M)
C
C INVERT A
C
CALL BIPHTR (A, XX, NP)
C
C CHECK NORM AND SET NEKR ACCORDINGLY.
C
IF (NDIM .LT. 1.0D-08) GO TO 114
   NEKR = 2
C
RETURN
C
C COMPUTE ESTIMATE OF (CORRECTIONS TO) FIT PARAMETERS, IF MATRIX INVERSION WAS SUCCESSFUL.
C
114 DO 115 I=1,NP
   ID(I) = 0
   DO 115 J=1,NP
   115 ID(I) = XX(I,J) * B(J) + ID(I)
C
C FOR ITERATIVE SOLUTION, THIS IS AN INCREMENT TO BE ADDED TO C THE PREVIOUS ESTIMATE.
C
RETURN
C
END
SUBROUTINE SUBRT (J, X: GP1, DERIV, YC)

C PROGRAM TO GENERATE TRANSFORMED VECTORS AND THEIR PARTIAL
C DERIVATIVES, AS FUNCTIONS OF THE (PIT) PARAMETERS OF
C THE TRANSFORMATION
C TRANSFORMATION INCLUDES ROTATION AND SKEW
C (SCALE FACTORS ARE IMPLICIT IN THE COEFFICIENTS)
C
C DOUBLE PRECISION X(J), GP1(1), DERIV(J), YC(1)

I1 = 2 * (J - 1) + 1
I2 = I1 + 1
DERIV(I1) = X(I1)
DERIV(2) = 0.
DERIV(3) = X(I2)
DERIV(4) = 0.
DERIV(5) = 0.
DERIV(6) = X(I1)
DERIV(7) = 0.
DERIV(8) = X(I2)
DERIV(9) = 1.000
DERIV(10) = 0.
DERIV(11) = 0.
DERIV(12) = 1.000
YC(1) = GP1(1) * X(I1) + GP1(2) * X(I2) + GP1(3)
YC(2) = GP1(3) * X(I1) + GP1(4) * X(I2) + GP1(6)
RETURN
C
END
SUBROUTINE BIORTH (A, B, N)

WHERE - (1) A = AN N-BY-N MATRIX
    (2) B = THE LEFT INVERSE OF THE MATRIX A
    (3) N = ROW- AND COLUMN-DIMENSION OF THE MATRICES

DOUBLE PRECISION A(30,30), B(30,30), DOT, NORM, EPS, CJK,
    NSA

EXTERNAL DOT

DATA EPS /1.D-9/.

C CCMNC / NRM/ NRM
C
C NIT = 0
DC 100 I=1,N
DC 100 J=1,N
B(I,J) = A(I,J)
100 CONTINUE

C 200 CONTINUE

C NIT = NIT + 1
DC 300 K=1,N
CALL 5CMNC(1.D0, DOT, DOT, B(I,K), A(I,K), NIT, B(I), B(I,K), N)
DO 300 J=1,N
IF (B(I,J).LE.EPS) GO TO 500
C JK = DOT (B(I,J), A(I,K), N)
DO 301 I=1,N
B(I,J) = B(I,J) - CJK * B(I,K)
301 CONTINUE

C 300 CONTINUE

C NORM = EPS
DO 400 I=1,N
DO 400 J=1,N
NORM = NORM + (DOT(B(1,I), A(I,J), N) - AMNOS(I/J, J/I))**2
400 CONTINUE

C NORM = DSQRT(NORM)
IF (NORM .GT. (2.D0*NIT)*EPS) GO TO 200

C DC 500 I=1,N
DO 500 J=1,N
NSA = B(I,J)
B(I,J) = NSA
500 CONTINUE

C RETURN.
SUBROUTINE SCLR (Z, X, Y, N)
DOUBLE PRECISION X, Y, Z
DIMENSION X(1), Y(1)
DO 700 K = 1, N
Y(K) = Z * X(K)
700 CONTINUE
RETURN
END

DOUBLE PRECISION FUNCTION DOT (X, Y, N)
DOUBLE PRECISION X, Y
DIMENSION X(1), Y(1)
DOT = 0.0
DO 600 K = 1, N
DOT = DOT + X(K) * Y(K)
600 CONTINUE
RETURN
END
5-4. GEOMETRIC CORRECTION

1. NAME

GEOCOR

2. PURPOSE

Apply geometric corrections to large data sets. This program implements the affine transformation.

3. CALLING SEQUENCE

This is a main program. It is currently on a partitioned data set as an executable module.

4. INPUT-OUTPUT

4.1 Input

The following input parameters should be supplied on data cards according to the formats and read statements indicated below.

READ 100, A, XOP, YOP
READ 200, NREC, NEL, INTPL, INVFLG, SCALEX, SCALEY
100 FORMAT(6E12.3)
200 FORMAT(416,2F6.1)

where

A is a 2x2 matrix defining the geometric transformation.
XOP, YOP are the shifts defining the geometric transformation.
NREC = Number of records in the input image.
NEL = Number of pixels per record in the input image.
INTPL is a flag indicating the type of interpolation to be used in producing the output records. (0: nearest neighbor, 1: Bilinear, 2: Bicubic).
INVFLG is a flag indicating whether the given transformation should be inverted or should be used as supplied (1 for inversion).
SCALEX = Number of pixels per unit distance in the X-direction.
SCALEY = Number of pixels per unit distance in the Y-direction.
Note that SCALEX and SCALEY are floating point variables.

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The input image data should be on Unit 8 with one record per scan line, NEL pixels per record and one word (4 bytes) per pixel.

4.2 Output

The output of this program will consist of a printout of the coordinates of the extremities of the output image, the input and output image sizes, the desired transformation matrix and shift vector, the implemented transformation matrix and shift vector (which in some cases, could be different; e.g. when a 70° rotation is desired, a -20° rotation will be performed instead and the result will have to be rotated by 90° using a transposition program). Also, the inverse matrix and some implementation details are printed. The output image will appear on Unit 10 in the same format as input. Since the image after geometric transformation is not necessarily rectangular with the edges parallel and perpendicular to the scan lines, the "exterior" is filled with zeros and the output is stored as a rectangular image.

4.3 File Storage

This program requires space on two direct access units 90 and 91 depending on the core size supplied (MAXC, the dimension of the array IX), the input image size and the transformation desired. Currently 1500X1500 word direct access files are provided for. If these are not sufficient, the program prints an error message, specifies the number of records and words per record required for work areas of input (90) and output (91) images. The user should then change the DEFINE FILE statements and the values of NRW90, NRW91, MAXR90, MAXR91 in the source program, recompile and run the job. Here, NRW and MAXR refer to the number of records and number of words per record, respectively.

5. EXITS

No abnormal exits except as described in Section 4.3.
6. **USAGE**

The program is in FORTRAN IV and implemented on the IBM 360 using the H compiler. The program, in its executable form, is in the user's library.

7. **EXTERNAL REFERENCES**

As indicated below.

<table>
<thead>
<tr>
<th>Calling Program</th>
<th>Programs Called</th>
</tr>
</thead>
<tbody>
<tr>
<td>GEOCOR</td>
<td>ROTATM</td>
</tr>
<tr>
<td>ROTATM</td>
<td>ROTAT1, ROTAT2, ROTAT4, ROTAT3, ROTAT5, ROTAT6, ROTAT7</td>
</tr>
<tr>
<td>ROTAT1</td>
<td>VMOV</td>
</tr>
<tr>
<td>ROTAT4</td>
<td>ROTAT3</td>
</tr>
<tr>
<td>ROTAT5</td>
<td>READER, RITER, MOVVMR, SVSCI</td>
</tr>
<tr>
<td>ROTAT6</td>
<td>DOT</td>
</tr>
<tr>
<td>ROTAT7</td>
<td>DAWN, ROTAT3, DARN</td>
</tr>
<tr>
<td>READER</td>
<td>IRVCON, RIVCON</td>
</tr>
</tbody>
</table>

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8. PERFORMANCE SPECIFICATIONS

8.1 Storage

This program is 120840 bytes long (mainly due to array IX dimensioned 30000 words). Including external references and buffers it needs 184K bytes.

8.2 Execution Time

The time is largely dependent on the output image size (which is a function of the input image size and the transformation) and the type of interpolation used. Typical times for various output image sizes are shown below.

<table>
<thead>
<tr>
<th>INTPL</th>
<th>OUTPUT IMAGE SIZE</th>
<th>EXECUTION TIME (MINUTES)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>600X600</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>1600X2400</td>
<td>10</td>
</tr>
</tbody>
</table>

8.3 I/O Load

None except as specified by Section 4.

8.4 Restrictions

None.

9. METHOD

A detailed description of the method used for handling the geometric correction of large images is given elsewhere [21]. Here, we shall confine ourselves to the discussion of the scale factors SCALEX and SCALEY.

Suppose a matrix A and XOP, YOP are supplied to the routine ROTATM. Then the transformation applied is

\[
\begin{bmatrix}
XP \\
YP
\end{bmatrix} = A \begin{bmatrix}
X \\
Y
\end{bmatrix} + \begin{bmatrix}
XOP \\
YOP
\end{bmatrix}
\]
where \( X_P, Y_P \) are the coordinates in the transformed system and \( X, Y \) are those in the original (input) system.

Now, if the output image should be enlarged by factors \( \text{SCALEX} \) and \( \text{SCALEY} \) in the \( X \) and \( Y \) directions, we should modify the above equations to

\[
\begin{bmatrix}
X_P \\
Y_P
\end{bmatrix}
= \begin{bmatrix}
\text{SCALEX} & 0 \\
0 & \text{SCALEY}
\end{bmatrix}
\begin{bmatrix}
X \\
Y
\end{bmatrix}
+ \begin{bmatrix}
X_O \\
Y_O
\end{bmatrix}.
\]

As an example, consider the case where the matrix \( A \) and \( X_O, Y_O \) supplied to this program are the six parameters determined by GEOGREF [22]. Then the transformation yields the Landsat pixel coordinates in terms of UTM coordinates. If the UTM coordinates are supplied in kilometers, the matrix \( A \) will be in Landsat pixels per kilometer. Now, if the data are to be corrected to UTM coordinates, we should supply \( \text{INVFLG}=1 \). Also, the resulting transformation will yield one pixel per kilometer. If it is desired to have, say 20 pixels per kilometer (a convenient scale close to Landsat resolution), we should supply \( \text{SCALEX}=\text{SCALEY}=20.0 \). Also, sometimes it is desirable to have line printer plots on which the physical scale in the \( X \) and \( Y \) direction are the same. But line printers generally print at 10 characters per inch and 6 lines per inch. Thus, to get the same number of kilometers per inch of plot we can choose \( (\text{SCALEX},\text{SCALEY})= (10,6) \) or \( (20,12) \) etc.

10. COMMENTS

It is possible that this program will exceed the estimated time. However, to get a better estimate of time needed for a subsequent run, the printed output provides an indication of how many output records have been processed. The "number of partitions" multiplied by the number of records in the output image is the total number of records to be processed. "Partitions" and "column groups" are used synonymously. A message is printed after processing every 500 records in each partition.

11. LISTINGS

This listing of the program, the associated subroutines are attached at the end.
12. TESTS

The program has been tested thoroughly for several sizes of images and various geometric transformations. A sample output is attached.
C MAIN PROGRAM TO APPLY GEOMETRIC CORRECTION.
C
C DATA MAXC/30000/
DIMENSION IX(30000),A(2,2)
LOGICAL I,LX(4,1500),LY(1500)
EQUIVALENCE(IX(1),LX(1))(IX(1501),LY(1))
DEFINE FILE 90(1500,1500),U,IAV90
DEFINE FILE 91(1500,1500),U,IAV91
COMMON AVE/C(2,2),B(2,2),XPP,YPP,INTPL,IA(2,2),ILD,1HI,JLO,JHI
COMMON WRIXDSK/WR90,MAXR90,MAXR91
NRX90=1500
NRX91=1500
MAXR90=1500
MAXR91=1500
C READ TRANSFORMATION TO BE USED
C MATRIX A AND XUP,YDP DEFINE THE TRANSFORMATION
C
C READ 100 A,XUP,YDP
100 FORMAT(6E12.3)
C READ IN IMAGE SIZE AND THE TYPE OF INTERPOLATION TO BE USED
C
C INTPL=0,1,2 FOR NEAREST NEIGHBOR, BILINEAR AND BICUBIC
C READ 200 NREC,NEL,INTPL,INVFLG,SCALEX,SCALEY
200 FORMAT(4I6,2F6.1)
IF(INVFLG.EQ.1)GO TO 40
A(1,1)=A(1,1)*SCALEX
A(1,2)=A(1,2)*SCALEX
A(2,1)=A(2,1)*SCALEY
A(2,2)=A(2,2)*SCALEY
XPP=XUP*SCALEX
YPP=YUP*SCALEY
Cn Tu 50
40 CONTINUE
C INVERT THE TRANSFORMATION.
C
C DET=A(1,1)*A(2,2)-A(1,2)*A(2,1)
C(1,1)=A(2,2)/DET
C(2,1)=-A(1,2)/DET
C(1,2)=-A(1,1)/DET
C(2,2)=A(1,1)/DET
A(1,1)=C(1,1)*SCALEX
A(1,2)=C(1,2)*SCALEX
A(2,1)=C(2,1)*SCALEY
A(2,2)=C(2,2)*SCALEY
XPP=A(1,1)*XUP+A(1,2)*YPP
YPP=A(2,1)*XUP+A(2,2)*YPP
YPP=-YPP
50 CONTINUE
C PERFORM CORRECTION.
C
CALL ROTATM(IX,MAXC,NREC,NEL,A,XOP,YDP,8,10,NRECO,NEL0)
STOP
END
SUBROUTINE ROTATEIIX, MAXC, NREC, NEL, AMAT, XPO, YPO, NTAPI, NTAPU, NRECO, NELO)

C DIMENSION Ix(MAXC), AMAT(2,2)
COMMON/FINE/ A(2,2), b(2,2), XOP, YOP, INTPL, IA(2,2), ILO, IHI, JLO, JHI
COMMON/ROTS/ IRINIT
C C DEFINE FILE 90(NRWO, MAXR90, NRW91, MAXR91)
C C DEFINE FILE 91(NRWO, MAXR90, NRW91, MAXR91)
C C COMPUTE OUTPUT IMAGE ARRAY SIZE.
C WRITE (6,1000)
CALL ROTATE(NREC, NEL, AMAT, XPO, YPO, NRECO, NELO)
WRITE (6,1010) NREC, NEL
WRITE (6,1100) NRECO, NELO
WRITE (6,1200) (AMAT (I,J), J = 1, 2, I = 1, 2), XPO, YPO
WRITE (6,1210) (AI (I,J), J = 1, 2, I = 1, 2), XOP, YPO
WRITE (6,1220) (BI (I,J), J = 1, 2, I = 1, 2)

1000 FORMAT (1H1)
1010 FORMAT (2H INPUT PICTURE SIZE=(,I5,1H,,I5,1H))
1100 FORMAT (2H OUTPUT PICTURE SIZE=(,I5,1H,,I5,1H))
1200 FORMAT (1H DESIRED TRANSFORMATION MATRIX,2(/2E15.4),//,
1210 FORMAT (1H IMPLEMENTED SHIFT VECTOR,2(/E15.4))
1220 FORMAT (1H IMPLEMENTED SHIFT VECTOR,2(/E15.4))
1230 FORMAT (1H INVERSE MATRIX,2(/2E15.4))

C COMPUTE DIMENSIONS OF INPUT ARRAY WHICH CAN BE HELD IN CORE AT A
C TIME AND NUMBER OF PARTITIONS REQUIRED.
C MAXCP=MAXC-MAXO(NEL,NELO)
MAXCP1=MAXCP+1
CALL ROTATE2(MAXCP, NEL, NR, NC, NCGP)
WRITE (6,1230) MAXC, MAXCP, MAXCP1
IF (INTPL.EQ.0) WRITE (6,1240)
IF (INTPL.EQ.1) WRITE (6,1241)
IF (INTPL.EQ.2) WRITE (6,1242)
WRITE (6,1300) NR, NC
WRITE (6,1400) NCGP
1230 FORMAT (2H MAX. CORE SUPPLIED=,I6,/, 1240 FORMAT (1H3H MAX. CORE AVAILABLE FOR INPUT ARRAY=,I6)
1241 FORMAT (1H RESAMPLING METHOD--- NEAREST NEIGHBOR)
1242 FORMAT (1H RESAMPLING METHOD--- BILINEAR INTERPOLATION)
1300 FORMAT (2H TEMP. 2D CORE ARRAY SIZE=(,I5,1H,,I5,1H))
1400 FORMAT (1H NO. OF PARTITIONS=,I3)
C C FIND NUMBER OF WORK RECORDS AND LONGEST RECORD LENGTH TO BE
C WRITTEN ON AUXILIARY DISK MODULE FOR INTERMEDIATE OUTPUT.
CALL ROTATE (NCGP, NC, NREC, NEL, MAXR, NROUT)

C FIND AND PRINT SIZE OF DIRECT ACCESS WORKFILES REQUIRED FOR INPUT/OUTPUT.

NELDI = NEL - IFIX (B(2,2) + FLOAT (NC-4))
IF (NCGP .LE. 1) GO TO 50
PRINT 1500, NREC, NELDI, NROUT, MAXR
IF (NREC .LE. NRW90, .AND. NELDI .LE. MAXR)
AND. NROUT .LE. NRW91, .AND. MAXR .LE. MAXR91) GO TO 50
NRECO = 0
NEL0 = 0
PRINT 1600
1500 FORMAT (' NO. OF RECORDS IN INPUT WORK FILE='15/
* NO. OF WORDS PER RECORD IN INPUT WORK FILE='15/
* NO. OF RECORDS IN OUTPUT WORK FILE='15/
* NO. OF WORDS PER RECORD IN OUTPUT WORK FILE='15/
1600 FORMAT ('// ERROR CONDITION IN ROTATE. SUPPLIED WORKFILE SPACE INSUFFICIENT. THEREFORE THE PROGRAM RETURNED WITH NRECO=NEL0=0:
RETURN

50 CONTINUE
DO 10 ICGP = 1, NCGP
121 = 0
YLOC = (ICGP - 1) * IFIX (B(2,2) + FLOAT (NC-4)) + 1
IRINIT = 1
NC1 = NC
IF (ICGP .EQ. NCGP) NC1 = NEL - (NCGP - 1) * IFIX (B(2,2) + FLOAT (NC-4))
DO 20 IREC = 1, NRECO
CALL ROTATE (ICGP, NCGP, NC, IREC, NEL, JP1, JP2, I2, 1)

C UPDATE ARRAY IX IF NECESSARY. IX IS TREATED AS IF IT WERE DIMENSIONED (NR, NC)
C I2=LAST INPUT ROW NUMBER PRESENTLY IN CORE. I2=LAST INPUT ROW NUMBER NEEDED FOR COMPUTING IREC'TH ROW OF OUTPUT.
C I21=ROW NUMBER THE INPUT TAPE IS READY TO READ.
C CALL ROTATE (I20, I21, I2, IX, IX(MAXCP1), NREC, NEL, NC, ICNP, NCGP, NTAPI)

C GENERATE IY(JP1) THRU IY(JP2) USING IX.
C CALL ROTATE (IX, IX, IX(MAXCP1), NR, NC1, IREC, JP1, JP2, YLOC, NREC, NEL)
C WRITE IY(JP1) THRU IY(JP2) ON DISC OR TAPE DEPENDING ON ICGP VALUE
C CALL ROTATE (ICGP, NCGP, IREC, NEL0, NC, JP1, JP2, IX(MAXCP1), NTAPI)
IF (MOD (IREC, 500) .EQ. 0) PRINT 1700, IREC, ICGP
20 CONTINUE
10 CONTINUE
RETURN
1700 FORMAT (' FINISHED PROCESSING '15,' RECORDS IN COLUMN GROUP '13)
END
SUBROUTINE ROTATI(NREC,NEL,AMAT,XPU,YPU,NREC0,NELU)
C
GIVEN MATRIX A AND XOP, YOP FIND B-INV(A). DECIDE IF A SHD
BE MODIFIED FOR CONVENIENCE OF IMPLEMENTATION AND FIND THE TRUE
INITIAL AND FINAL COORDINATES OF THE OUTPUT IMAGE GENERATED.
TRANSFORMATION TO BE APPLIED IS
XPR=a*x+x0, WHERE A IS A 2X2 MATRIX AND XP IS A 2-VECTOR.
C
COMMON/AFINE/A(2,2),B(2,2),XUP,YPU,INTPL,IA(2,2),L0,IHI,JL0,JHI
DIMENSION AMAT(2,2)
C
CALL_VM0V(AMAT,4,A)
C
XOP=XPO
C
YOP=YP0
C
DET=A(1,1)*A(2,2)-A(1,2)*A(2,1)
B(1,1)=A(2,2)/DET
B(2,2)=A(1,1)/DET
B(1,2)=-A(1,2)/DET
B(2,1)=-A(2,1)/DET
IA(1,1)=1
IA(1,2)=0
IA(2,1)=0
IA(2,2)=1
P=NREC
C
Q=NEL
C
IF DET.LE.1.E-8 PRINT MESSAGE.
C
IF(DET.LE.1.E-8)WRITE(6,100)
IF(DET.LE.1.E-8)PRINT 100
C
FORMAT(1X,5I1)HCAUTION*** REQUESTED TRANSFORMATION MAY BE SINGULAR
RAT1=ABS(B(1,1)/B(2,1))
RAT2=ABS(B(1,2)/B(2,2))
IF(ABS(B(2,1)).LE.1.E-8)RAT1=1.E20
IF(ABS(B(2,2)).LE.1.E-8)RAT2=1.E20
IFLG=0
IF(RAT1.GE.RAT2) GO TO 10
IFLG=1
IA(1,1)=0
IA(1,2)=1
IA(2,1)=0
IA(2,2)=1
W=B(1,1)
B(1,1)=B(1,2)
B(1,2)=W
W=B(2,1)
B(2,1)=B(2,2)
B(2,2)=W
C
10 IF(B(1,1).GE.0.)GO TO 20
IFLG=1
IA(1,1)=-IA(1,1)
IA(2,1)=-IA(2,1)
C
191
B(1,1) = B(1,1)
B(2,1) = B(2,1)

CONTINUE

IF (B(T,?),GE.0.160 TO 30
IFLG=1
IA(T,2) = IA(T,2)
IF (T,?) = IA(T,2)

B(T,2) = B(T,2)
B(2,2) = B(2,2)

CONTINUE
IPASS=1
WRITE(6,151)

FORMAT(/)
WRITE(6,150)

FORMAT(/,72H COORDINATES OF IMAGE EXTREMITIES IF DESIRED TRANS-
MATION IS PERFORMED)
C
C COMPUTE COORDINATES OF TOP, BOTTOM, LEFT, RIGHT CORNERS OF THE
C IMAGE IN THE TRANSFORMED COORDINATE SYSTEM.
C
CONTINUE

W1 = A(1,2) + XDP
W2 = W1 + A(1,1) * P
W1 = A(1,1) + W1
W3 = A(1,2) * Q + XDP
W4 = W3 * A(1,1) * P
W3 = A(1,1) + W3
KLC = AMAX(W1, W2, W3, W4)

IF (KLC > GT, AND, RL, NE. FLOAT(IK0), IKU = IKU + 1)
RHI = AMAX1(W1, W2, W3, W4)

IF (RHI.GT.0., AND, RHI, NE. FLOAT(IHI), IHI = IHI - 1
WRITE(6,200) RLO, IKU, RHI, IHI

FORMAT(/,8H TOP ROW, F10.2,3H OR, I6,11H BOTTOM ROW, F10.2,3H OR, 1
W1 = A(2,21 + YCP
W2 = W1 + A(2,1) * P
W1 = A(2,1) + W1
W3 = A(2,21 * Q + YCP
W4 = W3 * A(2,1) * P
W3 = A(2,1) + W3
KLC = AMAXI(W1, W2, W3, W4)

JLC = RLO

IF (LJ, GT, 0., AND, RL, NE. FLOAT(JL0), JL0 = JL0 + 1)
RHI = AMAX1(W1, W2, W3, W4)

JE = RHT

IF (RHT .LT. 0., AND, RHT, NE. FLOAT(JHI), JHI = JHI - 1
WRITE(6,300) RL0, JL0, RHT, JHI

FORMAT(12H LEFT COLUMN, F10.2,3H OR, I6,13H RIGHT COLUMN, F10.2, 3H OR)

IF (IFLG .EQ. 0. OR. IPASS .EQ. 2) GO TO 40
IPASS = 2
WRITE(6,151)

WRITE (6,150)

FORMAT(13H COORDINATES OF EXTREMITIES OF IMAGE ACTUALLY PRODUCED,$
\begin{verbatim}

IDET = IA(1,1) * IA(2,2) - IA(1,2) * IA(2,1).
IW = IA(2,2) / IDET
IA(2,1) = IW
IA(1,2) = IA(1,2) / IDET
IA(2,1) = IW
IA(1,2) = IW
IA(1,1) = IW
A(1,1) = W1
A(1,2) = W2
A(2,1) = W3
A(2,2) = W4
XOP = W1
YOP = W2
<ll

CONTINUE
RETURN
END
\end{verbatim}
SUBROUTINE ROTATE(MAXC,NEL,NR,NC,NCGP)

C FIND MAX NO. OF RECORDS THAT MUST BE HELD IN CORE TO GENERATE
C LONGEST POSSIBLE OUTPUT RECORD GIVEN CORE SIZE LIMIT. ALSO, FI
C NO. OF COLUMN GROUPS INTO WHICH INPUT DATA SHOULDBE SECTIONED.

C

COMMNL/AFINE/A(2,2),B(2,2),XOP,YOP,INTPL,TA(2,2),ULO,IHI,JLU,JH

TT=ABS(B(1,2)/B(2,2))

J=MAXC/3

IF(TT.LE.1.E-10)GO TO 20

C=MAXC

T12=2.*TT

NC1=(TT-5.*SQR((TT-5.)*2+4.*TT*(C+T12-5.1))/T12)

NC2=(TT-4.*SQR((TT-4.)*2+4.*TT*(C+T12-4.1))/T12+1.

DO 10 I=NC1,NC2

J=NC1+NC2-I

IT=TT*FLDAT(J=2)

IF((I1+5)*(J+1).LE.MAXC)GO TO 20

10 CONTINUE

20 NC=MIN0(NEL,J+1)

NR=MAXC/NC

C FIND NCGP, THE NUMBER OF COLUMN GROUPS.

NCGP=1

NCP=IFIX(B(2,2)+NC-4

IF(NC.LE.NEL/NCGP=(NEL-NC)/NCP+2

RETURN

END
SUBROUTINE ROTAT3 (ICGP,NCGP,NC,IREC,NELO,JPI,JP2,J2,IFLG)

FOR GIVEN OUTPUT RECORD NUMBER IREC AND PARTITION NUMBER ICGP, TO
FIND OUTPUT WORD NUMBERS THAT CAN BE COMPUTED AND INPUT RECORD
NUMBERS REQUIRED.

CUMMCNZ/AFINE#A(2,2),B(2,2),XOP,YOP,INTPL,TA(2,2),TCL,THI,JCLO,JHI
Y1=1.+FLOAT((ICGP-1)×(NC-4+IFIX(B(2,2)))))
Y2=Y1+FLOAT(NC-1)
XP=IREC+ILD-1
W=YOP-B(2,1)×XP=XOP)/B(2,2)
YP1=(Y1+1.)/B(2,2)+W
YP2=(Y2-1.)/B(2,2)+W
JPI=YP1
IF(YPI.GT.0.5 AND. YPI.NE.FLOAT(JPI)) JPI=JPI+1
JP2=YP2
IF(YP2.LT.0.5 OR. YP2.EQ.FLOAT(JPI)) JPI=JPI-1
KP1=MINC(NELO,MAXO(1,JP1-JLO+1))
KP2=MINC(NELO,MAXO(1,JP2-JLO+1))
JPI=KP1
JP2=KP2
IF(ICGP.EQ.1) JPI=1
I=TPFIP.EQ.1 JCP=JP2=NELO
IF(IFLG.EQ.0) RETURN

YP2=KP2+JLO-1
IF(TP1,1,2...LT.0) YP2=KPI+JLO-1
X2=B(1,1)×(XP-XOP)+B(1,2)×(YP2-YOP)
J2=X2×2.
RETURN.
END
SUBROUTINE ROTATG(NCGP, NC, NRECO, NELU, MAXR, NKOUT)

IF (NCGP .EQ. 1) RETURN

MAXR = 1
CALL ROTAT3 (1, NCGP, NC, 1, NELU, JP1, JP2, I2, 0)
MAXR = MAXR (MAXR, JP2 = JP1)
CALL ROTAT3 (1, NCGP, NC, NRECO, NELU, JP1, JP2, I2, 0)
MAXR = MAXR (MAXR, JP2 = JP1)
CALL ROTAT3 (NCGP, NCGP, NC, NRECO, NELU, JP1, JP2, I2, 0)
MAXR = MAXR (MAXR, JP2 = JP1)
MAXR = MAXR + 1
NKOUT = NRECO * (NCGP + 1)
RETURN
END
SUBROUTINE ROTAT5(I20, I21, I2, IX, IY, NREC, NEL, NR, NC, ICGP, NCGP, NTAPT)
DIMENSION IX(NR, NC), IY(NEL)
C
C     COMM/AFINE/A(2,2), B(2,2), XLP, YLP, INTPL, IA(2,2), ILU, IHI, JLU, JHI
C
C     FIND NUMBER OF RECORDS TO BE READ.
C
IF(I2T.GT.NREC) RETURN
NRWS=I2-I20
IF(NRWS.LE.0) RETURN
C
IF ICGP=1 READ DATA FROM INPUT TAPE, ELSE READ FROM DISK.
IF IMCP.EQ.1 AND ICGP=1 WRITE THE LAST NEL-NC+4 WORDS OF INPUT ON DISK UNIT 90.
C
IF(ICGP.EQ.1) DEH=NTAPI
IF(ICGP.EQ.1) IDHV=0
JDEV=1
IF(ICGP.EQ.1) JDEV=-1
NC1=I2X6(2,2)+FTET(NC-3)
NEL2=NEL-NC+1
IEL=1
IF(ICGP.EQ.1) NEL1=(ICGP-2)*(NC1-1)+1
IYPE=0
IF(INTPL.NE.2) IYPE=1
C
CALL CSCL(IY, H, NC, L, IY(NEL1), MOD(IKINT+I-2, NR)+1)
IF(I12.LT.NREC) GO TO 10
JDEV=-1
CALL SVSCI(IY, NEL, 0)
GO TO 20
CONTINUE

10
IRINIT=MOD(IRINIT+I-1, NR)+1
C
C     NEW IRINIT IS THE ROW NUMBER IN IX CONTAINING THE EARLIEST RECORD
C     OF THE INPUT IMAGE.

RETURN
END
SUBROUTINE ROTATE (IX, RX, IY, NR, NC, IREC, JP1, JP2, YLOC, NREC, NEL)
   DIMENSION IX(NR, NC), RX(NR, NC), IY(1)
   COMMON/AFINE/ANM1(2,2), BM(2,2), XOP, YOP, INTPL, IA(2,2), ILO, IHI, JLO
   IRND(A)=MAXO(0,MINO(63,IFIX(A+.5)))

C
C GENERATE IY(JP1) THRU IY(JP2) USING IX.
C
C INTPL=0, 1, 2 RESULT IN NEAREST, BILINEAR, BICUBIC INTERPOLATION
C RESAMPLING
C
IF(INTPL.GT.0)GO TO 11
P=NREC
Q=NEL
YLOC1=YLOC-1.5
XP=IREC+IL0-1
X=XP-XOP
YOP1=FLOAT(JLO-1)-YOP
DO 10 JP=JP1,JP2
YP=FLOAT(JP)+YOP1
   X=BM(1,1)*XP+BM(1,2)*YP
   IF(X.LT.1. OR. X.GT.P)GO TO 20
   Y=BM(2,1)*XP+BM(2,2)*YP
   IF(Y.LT.1. OR. Y.GT.Q)GO TO 20
   I=INT(X+.5)
   I=MLD(I-1, NR)+1
   J=YP+YLOC1
   IY(JP)=IX(I,J)
   GO TO 10
20 CONTINUE
10 RETURN

C
11 IF(INTPL.GT.1)GO TO 12
P=NREC
Q=NEL
YLOC1=YLOC-1.5
XP=IREC+IL0-1
X=XP-XOP
YOP1=FLOAT(JLO-1)-YOP
DO 101 JP=JP1,JP2
YP=FLOAT(JP)+YOP1
   X=BM(1,1)*XP+BM(1,2)*YP
   IF(X.LT.1. OR. X.GT.P)GO TO 201
   Y=BM(2,1)*XP+BM(2,2)*YP
   IF(Y.LT.1. OR. Y.GT.Q)GO TO 201
   B=Y-YLOC1
   I=X
   J=B
   A=X-FLOAT(I)
   A1=1.-A
   B=B-FLOAT(J)
   I=MLD(I-1, NR)+1

   101 CONTINUE
10 RETURN
I1 = Mod(I, NR) + 1
J1 = J + 1
GO TO 101
201 IY(JP) = 0
101 CONTINUE
C
12 CONTINUE
DIMENSION H(4), W(4)

P = NREC - 1
P1 = NREC
Q = NEL - 1
Q1 = NEL
YLOC1 = YLOC - 1.
XP = IREC + ILO - 1
YP = X + XCP
YP1 = FLOAT(JLO - 1) - YP
DO 102 JP = JP1, JP2
YP = FLOAT(JP + YP1
X = BM(1, 1) * XP + BM(1, 2) * YP
IF (XTLT.2..CR. X GT. P) GO TO 202
Y = BM(2, 1) * XP + BM(2, 2) * YP
IF (YT.2..CR. Y GT. Q) GO TO 502
I = X
B = Y - YLOC1
J = B
A = X - FLOAT(I)
A1 = 1. - A
B = B - FLOAT(J)
B1 = 1. - B
A11 = A * A1
AA11 = AA1 + 1.
H11 = - A1 * AA1
H(2) = A1 * AA11
H(31 = A * AA11
H(4) = A * AA1
I = Mod(I - 2, NR) + 1
I1 = Mod(I, NR) + 1
I2 = Mod(I1, NR) + 1
J = J - 2
DO 302 JJ = 1, 4
JK = J + JJ
BBI = 8 * B1
BB11 = 8B1 + 1.
H11 = B1 * BB1
H(21 = B1 * BB11
H(31 = 9 * BB11
H(4) = 8 * BB1
IY(JP) = IRAND(DOT(W, 4))
GO TO 102
199
202 IF(X .LT. 1.0 OR X .GT. P1) GO TO 402
    Y = 6M(2,1) * XP + 6M(2,2) * YP
502 IF(Y .LT. 1.0 OR Y .GT. Q1) GO TO 402
    B = Y = YLOC1
    I = X
    J = B
    H(2) = X = FLOAT(1)
    H(1) = 1. = H(2)
    I = MED(I - 1, NR) + 1
    I1 = MOD(I, NR) + 1
    J1 = J + 1
    W(11) = H(1) * RX(I, J1) + H(2) * RX(I1, J)
    W(2) = H(1) * RX(I, J1) + H(2) * RX(I, J1)
    H(2) = B = FLOAT(1)
    H(1) = 1. = H(2)
    TY(JP) = DOT(W, H, T2) + .5
    GO TO 1C2
402 TY(JP) = 1
102 CONTINUE
    RETURN
    END
SUBROUTINE ROTAT7 (ICGP, NCGP, IREC, NELD, NC, JP1, JP2, IY, NTAP0)
DIMENSION IY(NELD)

C IF NCGP=1, WRITE IY ON NTAP0
C IF ICGP .NE. NCGP WRITE IY ON DISK UNIT 91. IF ICGP=NCGP READ PARTS
C OF IY CORRESPONDING TO JCGP.LT.NCGP FROM DISK AND WRITE ASSEMBLED
C IY ON NTAP0.
C IF (NCGP.EQ.1) GO TO 10
C
NCGP1=NCGP-1
NDAREC=NCGP1*(IREC-1)
IF (ICGP.EQ.NCGP) GO TO 20
C
CALL DAVN (91, NDAREC+ICGP, IY(JP1), 4*(JP2-JP1+1))
RETURN
C 20 DC 3C JCGP-1, NCGP1
CALL ROTAT3 (JCGP, NCGP, NC, IREC, NELD, KP1, KP2, C; 0)
CALL DAVN (91, NDAREC+JCGP, IY(KP1), 4*(KP2-KP1+1))
CONTINUE
C 10 WRITE (NTAP0) IY
RETURN
END
SUBROUTINE VMOV(IX,N,ITY)
C TO MOVE VECTOR IX INTO VECTOR IY.
C
DIMENSION IX(N),IY(N)
IF(N.EQ.0)RETURN
DO 10 I=1,N
10 IY(I)=IX(I)
RETURN
END

SUBROUTINE READER(IDEV,IX,X,NEL,IREC,ITYPI,ITYPO)
DIMENSION IX(NEL),X(NEL)
C
THIS ROUTINE RETURNS IF IDEV.LT.0. IT READS NEXT RECORD ON SEQUE
C TIAL UNIT IF IDEV.GE.2. IT READS IREC*TH RECORD ON DIRECT ACCESS
C UNIT IF IDEV.LT.2 AND IDEV.GE.0.
C TYPE CONVERSION IS AN OPTION. ITYPI AND ITYPO REPRESENT INPUT AND
C OUTPUT DATA TYPES RESPECTIVELY. (0-- INTEGER, 1-- REAL).
C
MLST EQ'CE(IX,X)
C
IF(IDEV.LT.0)RETURN
IF(IDEV.LT.2)GO TO 20
READ(IDEV)IX
GO TO 30
20 IDEV90=IDEV+90
READ(IDEV90+IREC)X
30 IF(ITYPI.EQ.ITYPO)RETURN
IF(ITYPI.EQ.0.AND.ITYPO.EQ.1)CALL IRVCON(IX,X,NEL)
IF(ITYPI.EQ.1.AND.ITYPO.EQ.0)CALL RIVCON(IX,X,NEL)
RETURN
END

202
SUBROUTINE RVCON(I X, X, N)
DIMENSION I X(N), X(N)
DO 10 I = 1, N
10 X(I) = I X(I)
RETURN
ENTRY RVCON(I X, X, N)
DO 20 I = 1, N
20 I X(I) = X(I)
RETURN
END

SUBROUTINE RITER(I DEV, X, I REC, NEL)
C
TO WRITE NEL WORDS OF X ON DEVICE I DEV. IF I DEV.LT.0 RETURN.
C
IF(I DEV.GT.1) (IX(I EL), I EL = 1, NEL) IS WRITTEN AS ONE RECORD ON TAP
C
IF I DEV IS 0 OR 1 THE RECORD IS WRITTEN ON I DEV+90, A DISC UNIT.
C
DIMENSION X(N EL)
IF(I DEV.LT.0) RETURN
IF(I DEV.LT.2) GO TO 20
WRITE(I DEV) X
RETURN
20 I DEV90 = I DEV+90
WRITE(I DEV90+I REC) X
RETURN
END
SUBROUTINE F07VR(A,M,N,X,I)
C
MOVE A VECTOR X TO I'TH ROW OF M X N MATRIX A.
C
DIMENSION A(M,N),X(N)
DO 10 J=1,N
10 XI,J=X(J)
RETURN
C
ENTRY MVMARK(A,M,N,X,I)
C
MOVE I'TH ROW OF A TO X.
C
DO 20 J=1,N
20 XI,J=J(J,J)
RETURN
END

SUBROUTINE SVSCI(I,X,N,IS)
DIMENSION IX(N)
DO 10 I=1,N
10 IX(I)=IS
RETURN
END
FUNCTION DATA(X,Y,N)
DIMENSION X(N),Y(N)
DATA 0,
DC 10 I=1,N
 10 DET=DET*X(I)*Y(I)
RETURN
END

SUBROUTINE DARN(IDEV,IREC,X,N)
LOGICAL=X(N)
READ(IDEV,IREC)X
RETURN
ENTRY DAWN(IDEV,IREC,X,N)
WRITE(IDEV,IREC)X
RETURN
ENTRY SARN(NTAPI,X,N)
READ(NTAPI)X
RETURN
ENTRY SAWN(NTAPO,X,N)
WRITE(NTAPO)X
RETURN
END
COORDINATES OF IMAGE EXTREMITIES IF DESIRED TRANSFORMATION IS PERFORMED

TOP ROW 2.11 OR 3 BOTTOM ROW 1350.07 OR 1350

LEFT COLUMN -271.44 OR -271 RIGHT COLUMN 1015.10 OR 1015

INPUT PICTURE SIZE=( 625, 755)
OUTPUT PICTURE SIZE=( 1348, 1287)

DESIRED TRANSFORMATION MATRIX

\[
\begin{bmatrix}
0.1859E+01 & 0.2504E+00 \\
-0.4364E+00 & 0.1345E+01 \\
\end{bmatrix}
\]

DESIRED SHIFT VECTOR

\[
\begin{bmatrix}
0.0 \\
0.0 \\
\end{bmatrix}
\]

IMPLEMENTED TRANSFORMATION MATRIX

\[
\begin{bmatrix}
0.1858E+01 & 0.2504E+00 \\
-0.6264E+00 & 0.1345E+01 \\
\end{bmatrix}
\]

IMPLEMENTED SHIFT VECTOR

\[
\begin{bmatrix}
0.0 \\
0.0 \\
\end{bmatrix}
\]

INVERSE MATRIX

\[
\begin{bmatrix}
0.5158E+00 & -0.9603E-01 \\
0.1674E+00 & 0.7123E+00 \\
\end{bmatrix}
\]

MAX. CORE SUPPLIED= 50000
MAX. CORE AVAILABLE FOR INPUT ARRAY= 48713

RESAMPLING METHOD--- NEAREST NEIGHBOR

YEAR 2D CORE ARRAY SIZE=( 93, 586)

NO. OF PARTITIONS= 2

NO. OF RECORDS IN INPUT WORK FILE= 625

NO. OF WORDS PER RECORD IN INPUT WORK FILE= 173

NO. OF RECORDS IN OUTPUT WORK FILE= 1348

NO. OF WORDS PER RECORD IN OUTPUT WORK FILE= 1092

FINISHED PROCESSING 500 RECORDS IN COLUMN GROUP 1
FINISHED PROCESSING 1000 RECORDS IN COLUMN GROUP 1
FINISHED PROCESSING 500 RECORDS IN COLUMN GROUP 2
FINISHED PROCESSING 1000 RECORDS IN COLUMN GROUP 2

ELAPSED TIME -- MINUTES - 3 SECONDS - 29.20

206
GEOMETRIC TRANSFORMATION OF CURVES - I

1 NAME: GETC1

2 PURPOSE: To apply a given geometric transformation to a curve given in SLIC (scan line intersection code) format. This routine generates the row and column coordinates of all points on the curve in the transformed coordinate system and writes them on a direct access device. It can handle cases where the number of row and column coordinates produced exceeds the core capacity.

3 CALLING SEQUENCE:
   CALL GETC1(NTAPI, IY1, MAXC, IDUM, IY2, A, XPO, YPO, NTOT, IRC, IWR, IWC, MINR, MAXR, NREC)
   where
   IY1, IY2, IRC, IWR, IWC are work arrays to be dimensioned as indicated in the attached listings.
   NTAPI = Logical unit number of the input sequential data set (input).
   MAXC = Core capacity available for row (or column) coordinates produced (input, should be as large as possible).
   IDUM = Number of dumps on to the direct access device (output).
   A = Matrix defining the geometric transformation (rotation, skew and scale change) (input).
   (XPO, YPO) = Vector defining the geometric transformation (translation) (input).
   NTOT = Number of row (or column) coordinates in the final dump on the direct access device (output).
   MINR, MAXR = Vectors containing the minima and maxima of the row coordinates in each of the dumps in the output (output).
   NREC = Number of records in the input data (input).
INPUT-OUTPUT:

The input, consisting of NREC records, is in the SLIC format on a sequential file (e.g., output of SMOD).

The output of this routine consists of 2^IDUM records on a direct access file (logical unit 90). Every (2*I-1)^st record consists of a set of row coordinates arranged in ascending order. Every (2*I)^th record consists of the corresponding set of column coordinates. The first 2*(IDUM-1) records have MAXC words each and each of the last two records has NTOT words.

EXITS: No nonstandard exits.

USAGE: The program is in FORTRAN IV and is implemented on the IBM 360/65 system. An IBM 7094 version is also available.

EXTERNAL INTERFACES:

7.1 System Subroutines: IBCOM#

7.2 Other Routines Called: SORT, JOIN1, VMOV.

7.3 External Storage: None

PERFORMANCE SPECIFICATIONS:

8.1 Storage: 8B2 Hexadecimal locations.

8.2 Execution Time: Depends on the image size, MAXC and the transformation to be implemented. The timing for a test case for both GETC1 and the next step, GETC2, is shown.

8.3 I/O Load: None

8.4 Restrictions: None
METHOD:
Initially NTOT = 1 and IDUM = 0. For each record of input which has a non-zero number of boundary points, the following computations are performed. The column coordinates in the record are sorted in ascending order (this is not necessary if the input records contain data already in ascending order). The column coordinates IY(J) in the Ith record are examined one by one. If \( \text{IY}(J+1) - \text{IY}(J) \leq 1 \), then the routine JOIN1 is called to generate a straight line in the transformed coordinate system between (I, IY(J)) and (I, IY(J+1)). Also, if there are points in the (I+1)st record connected to the point (I, IY(J)), then the routine JOIN1 is used to generate straight lines joining them to it in the transformed system.

While handling any record, the current record is held in IY1 and the next record is held in IY2. After finishing each record, IY2 is moved to IY1 and a new record is read into IY2.

The routine JOIN1 works as follows. Given two points \((X_1, Y_1)\) and \((X_2, Y_2)\), the transformed coordinates \((X_{P1}, Y_{P1})\) and \((X_{P2}, Y_{P2})\) are computed using

\[
\begin{bmatrix}
X_{P1} \\
Y_{P1}
\end{bmatrix} = A \begin{bmatrix}
X_1 \\
Y_1
\end{bmatrix} + \begin{bmatrix}
X_{P0} \\
Y_{P0}
\end{bmatrix}
\]

Next, a digital approximation to the straight line joining \((X_{P1}, Y_{P1})\) to \((X_{P2}, Y_{P2})\) is found using a routine "JOIN". The row and column coordinates of the points on this line are stored in arrays IWR and IWC. The number of such points after one call to JOIN is given by K. The total number of points computed and held in the array IRC is given by NTOT-1. Now, if there are any points \((\text{IWR}(I), \text{IWC}(I))\) which are identical to points in IRC, then they are eliminated and K is corrected accordingly. If NTOT+K < MAXC+1, then IWR, IWC are moved into
IRC and NTOT is set to NTOT+K. Otherwise, the parts of IWR, IWC corresponding to the first MAXC+1 - NTOT points are moved into IRC and the array IRC is dumped on the direct access device 90 as two records of length MAXC each. Now, the remainder of IWR, IWC is moved into IRC and NTOT is changed to K - (MAXC - NTOT). Also IDUM is set to IDUM+1.

After all the records have been processed, NTOT is changed to (NTOT-1), the data in IRC are dumped on Unit 90 as two records of length NTOT each and IDUM is set to IDUM+1.

10 COMMENTS: None

11 LISTINGS: The listings for GETC1 follow along with GETC2 and the subroutines required.
GEOMETRIC TRANSFORMATION OF CURVES - II

NAME: GETC2

PURPOSE: To rearrange the row and column coordinates on the direct access file (produced by a routine such as GETC1) in SLIC format and write on a sequential file.

CALLING SEQUENCE:

CALL GETC2(IDUM, IA, IDA, ISEQ, ISKIP, MAXC, MINR, MAXR, NWDS, IRC, NTAP0, IRMN, IRMX, ICMN, ICMX)

where

IDUM = Number of sets of row and column coordinates to be read from the direct access file (Unit 90).
   = 1/2 (Number of records on the file) (input)

IA is an array dimensioned (IDA, 5) with MINR, MAXR, ISEQ, NWDS and ISKIP equivalenced to IA(1, 1), ..., IA(1, 5), respectively.

IDA is a number greater than or equal to IDUM. (input)

ISEQ, ISKIP and IRC are work arrays.

MAXC = Maximum core capacity available for reading the row and column data. IRC is dimensioned (MAXC, 2). (input)

MINR, MAXR = Arrays containing the minima and maxima of row coordinates in each of the "row records" on the input file (input).

NWDS = Array containing the number of words to be read from each of the "row (or column) records" on the input file (input).

NTAPO = Logical unit number of the output sequential data set (input).

IRMN, IRMX = Minimum and maximum row coordinates for the entire image (output).

ICMN, ICMX = Minimum and maximum column coordinates for the entire image (output).
4 INPUT-OUTPUT

The input data for this routine is on a direct access file (Unit 90) which has 2*IDUM records. The input value of NWDS(I) indicates the number of words of relevant data in the (2*I-1)st and the (2*I)th records. Each odd numbered record contains the row coordinates followed, in the next record, by the corresponding column coordinates. The row coordinates must be in ascending order.

The output consists of IRMX-IRMN+1 records written in SLIC format on unit NTAPO, the first record corresponding to the IRMN'th row of the image.

5 EXITS: No nonstandard exits. However, there is an error exit in the case where the supplied MAXC is not sufficient to handle the data. In this case, an error message is printed and IRMX is set to IRMN-1.

6 USAGE: The program is in FORTRAN IV and is implemented on IBM 360/65. An IBM 7094 version is also available.

7 EXTERNAL INTERFACES:

7.1 System Subroutines: IBCOM#
7.2 Other Routines Called: VMINI4, VMAXI4, SORT, VMOV
7.3 External Storage: None

8 PERFORMANCE SPECIFICATIONS:

8.1 Storage: C3C Hexadecimal bytes

8.2 Execution Time: Depends on the size and content of the input file. A test for the geometric correction of TARCOG county boundary data consisting of 1553 records with approximately 12000 boundary points took about four minutes with MAXC=7993.
8.3 I/O Load: None

8.4 Restrictions: None

9 METHOD:

A flow chart describing the steps in the routine is shown. Briefly, the algorithm consists in (i) reading as much of the data as possible into core, (ii) sorting them in ascending order of row coordinates, (iii) finding the largest row coordinate $R$ in core that has no unread parts on the input file, (iv) reformatting and writing the column coordinates corresponding to the data in core whose row coordinates are less than or equal to $R$ and (v) repeating steps (i) through (iv) until all the data are processed.

10 COMMENTS: None

11 LISTINGS: The listings for GETC1, GETC2 and the subroutines required are attached at the end of this section.

12 TESTS: The routines GETC1 and GETC2, in combination, have been tested using test patterns and on the TARCOG county boundary map. Visual inspection of the picture of the corrected TARCOG county boundaries superposed on a land use classification scanner data indicates accurate performance of the programs.
FOR I = 1, IDUM
ISEQ(I) = I, ISKIP(I) = 0

IRMN = Min (MINR(I), I = 1, IDUM)
IRMX = Max (MAXR(I), I = 1, IDUM)
N = MAXC; IPASS = 1

SORT MAXR, ISEQ, NWDS, ISKIP
WITH MINR AS KEY
(MINR TO BE ARRANGED IN ASCENDING ORDER)

I = 0
J = 1

I = I + 1
K = Min (NWDS(I), N - J)

READ K ROW AND COLUMN
COORDINATES INTO IRC FROM THE ISEQ(I)'TH
PAIR OF INPUT RECORDS, SKIPPING
ISKIP(I) COORDINATES. READ
(K + 1)'ST ROW COORDINATE INTO NEXTR.

IF K = NWDS(I)
MINR(I) = MINR(IDUM) + 1000000
ELSE
MINR(I) = NEXTR

NWDS(I) = NWDS(I) - K
ISKIP(I) = ISKIP(I) + K
J = J + K

IF J ≤ N & I < IDUM

FLOW CHART FOR GETC2
NW = J − 1 = NUMBER OF (ROW, COLUMN) COORDINATES READ

JJ = NW − N  
= NUMBER OF OLD COORDINATES STILL IN CORE

NW = JJ + J − 1  
= NUMBER OF COORDINATES IN CORE FOR THE PRESENT PASS

MORE COORDINATES WERE PROCESSED AND WRITTEN OUT ON TAPE DURING THE LAST PASS THAN WERE READ IN DURING THE CURRENT PASS

LEFT-SHIFT JJ OLD COORDINATES (STARTING FROM IRC (N + 1, *)) TO LOCATIONS STARTING FROM IRC (J, *).

IPASS = IPASS + 1

SORT THE NW COORDINATES IN CORE ARRANGING ROW COORDINATES IN ASCENDING ORDER
MROW = Min \( M \{ \text{MINR}(i), i = 1, \text{IDUM} \} \)
- Minimum row coordinate not read into core

\[ N = \text{Max} \{ M \mid 1 \leq M \leq \text{NW AND IRC}(M,1) < \text{MROW} \} \]

N FOUNDED NO

PRINT ERROR MESSAGE

YES

REFORMAT AND WRITE THE FIRST N COLUMN COORDINATES ON OUTPUT FILE AS IRC(N,1) - IRC(1,1) + 1 VARIABLE LENGTH RECORDS

IRMN = IRMX + 1

RETURN

ICMN, ICMX = MIN, MAX of column coordinates written out

FIND I SUCH THAT
\[ 1 \leq I \leq \text{IDUM AND NWDS}(I) \neq 0 \]

YES

I FOUND

RETURN

NO
ISN 0002   LOGICAL I Z(200)
ISN 0003   DIMENSION I A(20,5), I SEQ(20), I SKIP(20), NWDS(20)
ISN 0004   DIMENSION I Y(11,25), I Z(125), A(2,2), I RC(500,2), I WR(20), I WC(20),
            HMIN(20), MAXK(20)
ISN 0005   EQUIVALENCE (I A(I 6), NWDS(I 1)), (I A(I 6), I SKIP(I 1))
ISN 0006   DEFINE CTTE-E=90.20, S0G,U, I NWD
ISN 0007   DATA NTAPI, MAXK, NREC/10, 500, 60/
ISN 0008   DATA : , XPD, YPO, 707, -707, 707, 0, 0,
ISN 0009   DATA NTAPW/12/
ISN 0010   DATA I DAt, NTAPD/25, 8/
ISN 0011   CALL GETC1(NTAPI, I Y, MAXK, IDUM, I Y, A, XPD, YPO, NTQT, I RC,
            HMIN, MAXK, MAXK, NREC)
ISN 0012   CALL SVSCI(NWDS, IDUM, MAXK)
ISN 0013   NWDS(IDUM)=NTOT
ISN 0014   CALL GETC2(IDUM, IA, IDA, I SEQ, I SKIP, MAXK, MINR, MAXK, NWDS, I RC,
            NTAP, I RMX, I CMN, I CMN)
ISN 0015   REWIND NTAPD
ISN 0016   PRINT I GC, =IRMN, I RMX, I CMN, I CMN
ISN 0017   100  FORMAT(1 MIN, ROW NO., =15/,
            MAX, ROW NO., =15/
            MIN, CUL, NO., =15/
            MAX, CUL, NO., =15/
            I RMX, =IRMN)
ISN 0018   NREC=IRMX=IRMN+1
ISN 0019   IF(NREC.EQ.0) STOP
ISN 0020   NELD=ICMN=ICMN+1
ISN 0021   BU=10, I =1, NREDC
ISN 0022   READ(NTAPD) N(I), I Y(J), I Z, I N
ISN 0023   CALL SETFI(I Y, I , I Z, NELD, ICN, I M)
ISN 0024   10  WH I NTAPW/11, I Z, I J, NELD)
ISN 0025   10  WHI NTAPW/11, I Z, I J, NELD)
ISN 0026   REWIND NTAPW
ISN 0027   NPAGE=NELO=1/125+1
ISN 0028   D U , IP=1, NPAG
ISN 0029   JI=IP-1,125+1
ISN 0030   IF(IP.EQ.1) STOP
ISN 0031   IF(I P, EP, NPAG = AND, NEL0 GT, 124) J1 = NELQ-124
ISN 0032   J2=MIN(J1+124, NELQ)
ISN 0033   K1=J1+ICMN=1
ISN 0034   K2=J2+ICMN=1
ISN 0035   PRINT 200, =K1, K2
ISN 0036   200  FORMAT(1 GEOMETRICALLY CORRECTED BOUNDARY IMAGE- COLUMNS=15,
            THROUGH=15, **)
ISN 0037   DO 3 0 C =1, NREC
ISN 0038   READ(NTAPW/11, I Z, J-I, NELO)
ISN 0039   RE WIND NTAPW
ISN 0040   100  FORMAT(1 I X(1254), J1, J2)
ISN 0041   120  RE WIND NTAPW
ISN 0042   310  END
TO APPLY A GEOMETRIC TRANSFORMATION TO A GIVEN SET OF CURVES

WHERE COORDINATE DATA ARE STORED UN-NTAPI AND GENERATE NEW COORD IN NOOT OF ISOLATED POINTS WILL BE SUPPRESSED.

DATA. THE OUTPUT WILL BE ON DISK. CONNECTIVITY WILL BE PRESERVED.

D* N 41 AND 42 MAX. NO. OF WORDS EXPECTED IN ANY RECORD OF INPUT.

D* N INR AND INC AS IN JOINER.

D*N MINK AND MAXX NUMBER UP EXPECTED SUBDIVISIONS OF DATA NEEDED TO COMPUTE ALL BOUNDARY DATA, I.E., EXPECTED VALUE OF IDUM.

DIMENSION 41(1), 42(1, J=1, NRC2), 43(1, J=1, NRC2)

C IF NC1.EQ.0 NO COMPUTATIONS ARE NEEDED. GO TO END OF LOOP AND UP.

C DATE THE ARRAYS.

C COMPUTE LINES CORRESPONDING TO CONNECTED POINTS IN THE ARRAY 41.

X1=1

DO 30 J=1, NRC1

IF (J, J=1, NRC1) GO TO 40

DO 29 J=1, J1

IF (J1, J=1, NRC1) GO TO 50

X2=X1

Y1=IY1(J1)

Y2=IY1(J1)

CALL JOIN1(X1, Y1, X2, Y2, AXPB, YPC, MAXC1, MAXC, NTCF, INRC1, IDUN, IRC, INRC, MINK, MAXX)

IF NC2.EQ.0 OR 1, EW NREC GO TO 30

CALL JOIN1(X1, Y1, X2, Y2, AXPB, YPC, MAXC1, MAXC, NTCF, INRC1, IDUN, IRC, INRC, MINK, MAXX)

C COMPUTE LINES CORRESPONDING TO CONNECTIONS BETWEEN 11-11'S AND 11'S REG.

C 11 TH REG.
SUBROUTINE GETC2(IDUM,IA,IDA,ISEQ,ISKIP,MINR,MAXR,NWDS,IRC)
  NTAP,IRRM,IRNX,ICHN,ICHX
  DIMENSION IA(IDA+5),TIS,TT(5)
  DIMENSION ISEQ(IDA),ISKIP(IDA),MINR(IDA),MAXR(IDA),NWDS(IDA),
  IRC(MINR+2)

  DEFINE FILE 90 AS IN GETC1.
  MUST EQCE CLHS 1,2,3,4,5 OF IA TO MINR,MAXR,ISEQ,NWDS,ISKIP.
  INITIALIZE ISEQ, ISKIP AND N.

  DO 10 I=1,IDUM
  ISEQ(I)=I
  ISKIP(I)=0
  N=MAXC
  IPASS=1
  IRMN=10**6
  IRNX=IRMN
  IRC=IRC
  CALL VMINS(MINR,IDUM,IRRM)
  CALL VMAXI4(MAXR,IDUM,IRNX)

  ARRANGE MINR IN ASCENDING ORDER AND MAXR,ISEQ,NWDS,ISKIP ACCLY.

  NREC=0
  10  IF(ISKIP.LT.N)
  PRINT 100, IPASS,NREC,(MINR(I),MAXR(I),ISEQ(I),NWDS(I),ISKIP(I)),
  1,1,IDUM)

  100  FUXMAT('PASS', 'PASST', 'N'. NUMEBK OF RECORDS DUMPED OUT='15'/
  (* MINR=*16, MAXR=*16, ISEQ=*13, NWDS=*16, ISKIP=*16)*/

  READ N WORDS INTO ARRAYS IDUM AND ICLM FROM DISK WITH
  ISEQ(I)ST SET OF DATA. NWDS ISKIP(J)+1 THRU ISKIP(J)+NWDS(J)
  AKE READ FROM ISEQ(J)TH SET OF DATA.

  NREC=0
  20  IF(ISKIP.EQ.0)
  PRINT 200, IPASS,NREC, (MINR(I),MAXR(I),ISEQ(I),NWDS(I),ISKIP(I)),
  1,1,IDUM)

  200  FUXMAT('PASS', 'PASST', 'N'. NUMEBK OF RECORDS DUMPED OUT='15'/
  (* MINR=*16, MAXR=*16, ISEQ=*13, NWDS=*16, ISKIP=*16)*/

  READ N WORDS INTO ARRAYS ICLM AND ICLM FRUM DISK WITH
  ISEQ(I)ST SET OF DATA. NWDS ISKIP(J)+1 THRU ISKIP(J)+NWDS(J)
  AKE READ FROM ISEQ(J)TH SET OF DATA.

  IF(ISKIP.LE.0)
  READ(90*I,ILEC) (IRC inclusive),NEXTR
  (I, IEL=J,JK1),NEXTR
  IF(ISKIP.LE.0)
  READ(90*I,IMN) (I, IEL=J,JK1),NEXTR
  IF(ISKIP.LE.0)
  READ(90*I,IMN) (I, IEL=J,JK1),NEXTR
  IF(ISKIP.LE.0)
  READ(90*I,IMN) (I, IEL=J,JK1),NEXTR
READ(90,1WREC+1)(DUM,IEL=1,ISKIP1),(IRC(I,EL2),IEL=1,JKI)

100 0039  N=DS(I+1)=N=DS(I+1)-K
100 0040  ISKIP(I)=ISKIP(I)+K
100 0041  J=J+K
100 0042  IF(J=JE,N=I,L.DIM(I)) GO TO 30

C (J-1) SNS HAVE NOW BEEN READ FROM DISK INTO CORE
C N=NUMBER OF SNS IN CORE ON THE PREVIOUS PASS OUT OF WHICH N HAVE
C BEEN TRANSFERRED TO OUTPUT TAPE. THUS (NW-N) IS THE NO. OF OLD
C SNS STILL IN CORE.

100 0044  IF(IPASS,GT,1) GO TO 40
100 0046  NW=J-1
100 0047  GO TO 45
100 0048  40 JJ=NW-N
100 0049  NW=JJ+J-1

C NW=NUMBER OF WORDS IN CORE FOR THE PRESENT PASS

100 0050  IF(J,GT,N) GO TO 45
100 0052  N1=N+1
100 0053  CALL VMOV(I,N(1),JJ,IRC(J,1))
100 0054  CALL VMOV(I,N(2),JJ,IRC(J,2))
100 0055  45 CONTINUE
100 0056  IPASS=IPASS+1

C SORT ROW AND COLUMN

100 0057  CALL SORT(I,RNC,1,NW,MAXC,2,TR,TI)

C FIND PART OF DATA IN CORE TO BE REFORMATTED AND WRITTEN ON TAPE.
C THAT IS, FIND MAX, N SUCH THAT IROWN(N).LT.MROW WHERE MROW=MINIMUM
C ROW NUMBER CORRESPONDING TO DATA ON DISK THAT HAVE NOT BEEN TAKEN
C INTO CORE YET.

100 0058  MROW=MINR(1)
100 0059  CALL VMIN14(MINR,1,NUM,HRW)
100 0060  NW=NW+1
100 0061  DO 60 J=1,NW

C DO 60 J=1,NW
100 0062  N=NW-J
100 0063  IF(IR(N(1)).LT.MROW) GO TO 50
100 0064  60 CONTINUE

C NO PART OF DATA IN CORE CAN BE WRITTEN ON TAPE BECAUSE THE RECORDS
C ARE INCORRECT. PRINT ERROR MESSAGES AND STOP.

100 0065  PRINT 400,NREC
100 0066  IKMN=IKMX+1
100 0067  RETURN
100 0068  400 FOKMATI ERROR IN GETC2, TRY LARGER MAXC, NREC='16'
100 0069  50 CONTINUE

C WRITE N WORDS OF COLUMN DATA ON TAPE AFTER REFORMATTING.

100 0070  J=0
100 0071  70 J=J+1
100 0072  IF(J,LE.N) GO TO 80
100 0073  IF(IR(J).EQ.I,1) GO TO 70
100 0074  IF(IR(J).EQ.I,1) GO TO 70
100 0075  JJ=J-J+1
100 0076  NREC=NREC+1
100 0077  WRITE(NTAP0,1JJ,1IRC(I,2),L=J,1,J)
100 0078  CALL VMIN14(I,IRC(J,1),J,IN)
100 0079  CALL VMAX14(I,IRC(J,2),JJ,ICMX)
100 0080  80 IF(J=1) GO TO 70
100 0081  IF(IR(J).EQ.I,1) GO TO 70

220
SUBROUTINE ELAPTN(IWR, IWC, IRNW, ICLM, IK, N)

DIMENSION IWR(I), IWC(I), IRNW(N), ICLM(N)

C TO ELIMINATE COORDINATES IN (IWR, IWC) WHICH ARE EQUAL TO ANY OF
C THE COORDINATES IN (IRNW, ICLM).
C 'N' IN IWR(K), IWC(K), K IS BOTH INPUT AND OUTPUT (NUMBER OF COORDI-
C NATES).

ISN 0004 30 I = 0
ISN 0005 10 I = I + 1
ISN 0006 10 IF (I .LT. K) RETURN
ISN 0006 10 DO 10 J = 1, N
ISN 0009 10 IF (I .EQ. IRNW(J), IWC(J), ICLM(J)) GO TO 10
ISN 0110 10 GO TO 20
ISN 0120 10 CONTINUE
ISN 0130 10 GO TO 30
ISN 0140 10 IF (I .EQ. K) GO TO 40
ISN 0150 CALL VNUV(IWR(I+1), K-I, IWR(I))
ISN 0170 CALL VNUV(IWC(I+1), K-I, IWC(I))
ISN 0180 10 I = I - 1
ISN 0190 10 K = K - 1
ISN 0200 GO TO 30
ISN 0210 40 K = K - 1
ISN 0220 RETURN
ISN 0230 END
5-5 SUPERPOSITION OF BOUNDARIES

5-5-1 THINNING OF BOUNDARY IMAGES

1 NAME

PEELS

2 PURPOSE

Starting with the output of a microdensitometer digitizing a boundary image, to apply a given threshold of density and reduce the thickness of the boundary lines by "peeling" their outer layers while preserving the distinctness of regions separated by them.

3 CALLING SEQUENCE

CALL PEELS (NTAPI, NTAP0, NREC, NEL, IT, MPASS, MDEV, NDEV, LX, LY, IBDY)

where

NTAPI, NTAP0 are the logical unit numbers of the input and output sequential data sets;

NREC, NEL are the number of records and the number of pixels (bytes) per record in the input image;

IT is a threshold on density; if IT is positive (negative) all points with densities ≥ IT (≤ IT) will be regarded as boundary points;

MPASS is the maximum number of iterations permitted (see Section 9, Method);

MDEV, NDEV are logical unit numbers of two direct access scratch data sets defined as indicated in the listing of PEELS;

LX, LY, IBDY are scratch arrays with LX, LY dimensioned as indicated in the listing and IBDY dimensioned NEL.

4 INPUT-OUTPUT

4.1 Input

The input image should be on a sequential data set with unit number NTAPI and consist of NREC records and NEL bytes per record, each record corresponding to a line of the digitized image and each byte, to a pixel. All other inputs are as indicated in the calling sequence.
4.2 Output

The output of this program will be on unit NTAPO as a sequential data set with NREC records. The records will be in SLIC (scan line intersection code) format. That is, the first word of the I'th record indicates the number of words that follow and each subsequent word is a column coordinate of the intersection of the I'th scan line with the boundary image.

4.3 File Storage

This program requires two direct access scratch data sets to handle the intermediate iterations of the boundary data. The sizes of these data sets are indicated in the listings attached.

5 EXITS

No nonstandard exits.

6 USAGE

The program is in FORTRAN IV and implemented on the IBM 360 with the II compiler. The program is in the user's library as a load module.

7 EXTERNAL INTERFACES

This subroutine calls several subroutines and the linkage is shown in the following table.

8 PERFORMANCE SPECIFICATIONS

8.1 Storage

The subroutine PEELS is 1458 bytes long. However, including a driver (whose size depends largely on the dimensions of LX, LY, IBDY which are functions of NEL), the required subroutines and the buffers the program needs approximately 70K for handling NEL = 2100.

8.2 Execution Time

The execution time is highly dependent on the size and complexity of the boundary image, the thickness of the boundary lines and the maximum number of passes (MPASS) requested. In the case of the Mobile Bay GTM (a 4000 × 2100 level II map with boundaries 3 and 4 pixels thick) the initial thresholding and reformatting took about 10 minutes and the subsequent iterations about 6 minutes each, with a final reformatting and copying step taking about 7 minutes. Thus, with MPASS = 4, it takes about 40 minutes of CPU time to process the image.
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<tr>
<th>Calling Program</th>
<th>Program Called</th>
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<td></td>
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<td>BRSFTV</td>
<td>ILOAD⁺</td>
</tr>
<tr>
<td></td>
<td>ISTORE⁺</td>
</tr>
</tbody>
</table>

* Entry under DARN
⁺ Logical function available in the user's library under a main member name LOGFUNC
• Entry under BLSFTV

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8.3 Restrictions

None

9 METHOD

The program has three major steps:

(i) Thresholding, compressing and writing on a direct access unit.
(ii) Iterating to "peel" boundaries.
(iii) Changing to SLIC format and writing on output sequential data set.

9.1 Thresholding and Compressing

The routine SARN reads each record (of NEL bytes) of the input data set into the array LX. The routine VLTHR thresholds each of the NEL bytes in LX. A logical vector LY is defined as follows:

IF (IT.GE.0) LY(I) = LX(I).GE.IT
IT (IT.LT.0) LX(I) = LX(I).LE.IABS(IT)
for I = 1, NEL.

The routine CMPRES is then used to pack the information in LY into the first NEL bits of the array LX. The I'th bit of LX is "set" if and only if LY (I) is .TRUE..

The compressed boundary information is then written on the direct access unit MDEV using the routine DAWN.

9.2 Iterating to Peel

The main peeling routine is called PEELER. The input to this routine is from MDEV whenever IPASS, the iteration number, is odd and the output then will be written on NDEV. When IPASS is even, the input and output designations are interchanged. One call to PEELER removes one "layer" of the thick boundaries from top, left, bottom and right.

To decide whether a particular boundary point should be deleted (i.e. the bit corresponding to it changed to 0), we examine a 3x3 neighborhood centered around the point. Consider the array

```
 a b c 
 d e f 
 g h i
```
where each letter represents a binary pixel. It is to be decided whether \( e \), which is presently equal to 1 should be changed to 0. The conditions for a 'top peel' will be derived below and those for peeling from the other directions follow by symmetry.

First of all, \( e \) should be a top boundary point. That is, there should be no boundary point directly above \( e \) and there should be a boundary point below \( e \). Therefore \( b = 0 \) and \( h = 1 \) are necessary conditions. Suppose \( \bar{b} \ h = 1 \). (Here, \( \bar{b} \) denotes the complement of \( b \)). Then, we need only check whether \( e \) is a nonessential boundary point, that is, whether two 0's in the 3x3 array which are disconnected will stay disconnected where \( e \) is made 0. Connectivity, in this context, is defined as the existence of a path not including 1's and consisting only of horizontal and vertical segments.

Now, it is easy to see that \( e \) is essential if and only if \( \bar{a} \ d = 1 \) or \( \bar{c} \ f = 1 \). Therefore, the condition for a top peel is that

\[
\bar{b} \ h \ (\bar{a} + d) \ (\bar{c} + f) = 1.
\]

Equivalently, to perform a top peel we set

\[
e = e \ (\bar{b} + \bar{h} + \bar{a} \ d + \bar{c} \ f).
\]

It is convenient to implement the above equation by employing bit manipulation routines operating on pairs of 32 bit words, thereby performing the top-peel operation in parallel on 32 pixels. This is done by using the "current" array in place of \( e \), the "previous" array for \( b \), the "next" array in place of \( h \). Also, the previous, current, and next arrays are right (left) shifted by one bit and used for \( a \), \( d \) and \( g \) (\( c \), \( f \) and \( i \)) respectively in the peeling formulas.

The routine PEELER minimizes the movement of data in core by using circular buffers for storing the "previous, current and next" arrays. An array \( J \) dimensioned 3 is used to store the indices pointing to these arrays (\( J(1) \rightarrow \) previous, \( J(2) \rightarrow \) current, \( J(3) \rightarrow \) next) and after finishing each record, only the array \( J \) is updated.

Also, top, left, bottom and right peels are performed one after the other by just one pass through the data (thus minimizing I/O) by storing the intermediate results in core and operating with a phase lag.

When the \( I \)th record \( LX \) is read from the input data set (see PEELR1), BLSFTV and BRSFTV are used to generate arrays \( LXL \) and \( LXR \) with the bits in \( LX \) shifted by one bit to the left and right, respectively. Next, the \((I-1)\)th record is peeled from the top. The top-peeled output of the \((I-2)\)nd record is peeled from the left. The top-and left-peeled output of the \((I-3)\)rd record is peeled from the bottom. The top-, left- and bottom- peeled output of the \((I-4)\)th record is right-peeled and written on the output data set. Also, whenever any peeling is done other
than from the right the output is shifted to the left and right by one bit and the results are stored in the appropriate core locations pointed by J(3), K + 1.

The routine PEELRO with the appropriate ISIDE will perform the peeling of one record. The above operations performed for I = 1, NREC + 4 will complete one iteration of peeling, constituting one call to PEELER. The number, NP, of words of input that were changed is counted during each call to PEELER. If NP = 0 or the number of calls to PEELER has been MPASS, the iterations are stopped.

9.3 Converting to SLIC

Each record is read from the last scratch unit on which the output image was created. The routine EXPBDY is used to sense each bit in the record. The bit number of each 1-bit is stored in IBDY. The total number, N, of 1-bits followed by N words of the array IBDY are written on unit NTAPO.

10 COMMENTS

On large images this program takes a long time to execute. To avoid loss of data on long runs it is suggested that the direct access data sets be saved so that, with slight modifications, the routine PEELS can continue where the last run stopped due to insufficient CPU time.

11 LISTINGS

The listings of PEELS and most of the associated routines are attached at the end of this section. The routines not included are: PET, a routine used for printing time elapsed between sections of a program; SVSCI, a routine which sets all elements of an array to a given constant; DARN and the associated entry points for array read/write and the logical functions under member name LOGFUNC.

12 TESTS

The program was tested on a small portion of a boundary image, the image printed before and after peeling and was found to work satisfactorily.
SUBROUTINE PEEL...(INTI,NTG,NREC,NEL,IT,MPASS,MDEV,NDEV,LX,LY,IBDY).

DIMENSION LX(I),LY(I),IBDY(I)
DIMENSION LX(36*(NEL-1)/32+1),LY((NEL-1)/4+1)
DEFINE FILE MDEV/NREC,(NEL-1)/32+1,U,IAV1)
DEFINE FILE NDEV(NREC,(NEL-1)/32+1,U,IAV2)

N=(NEL-1)/2+1
CALL PET(2)
DO 10 I=1,NREC
CALL SARN(INTI,LX,NEL)
CALL VLTHR(LX,NEL,IT,LY)
LX(N)=0
CALL CMPSR(NEL,LX)
DO 10 CALL DAWN(NDEV,1,LX,N=4)
CALL PET(2)
DO 20 IPASS=1,HPASS
IF(LXPOD IPASS,Z1. EQ.1)CALL PEELER (W),NDEV,NREC,CrN,LY(N),LX(N)=0
IF(LX IPOD IPASS.2, .EO.O)CALL PEELER(NDEV,~DEV,NREC,N,LX,LX=12*N+4)
PRINT 100,IPASS,NP
IF(INP.EQ.O)GO TO 30
CONTINUE
DO 20 1-l ,N
IF(LXI GE.ITLY(I)=T.
DO 20 I=1,NREC
CALL DARN(LX,NEL,IT,LY)
CALL EXCPB(LX,N,LX,IBDY,J)
WRITE(NTO)J,LIBDY(L),L=1,J)
RETURN
END

SUBROUTINE VLTHR(LX,N,IT,LY)
LOGICAL*1 LX,N,IT,LY
THRESHOLD A VECTOR LX OF 8 BIT INTEGERS TO GET A T-F VECTOR.
IF IT GE.0,LX(I)=GE.IT IMPLIES LY(I)=T.
IF ITCO LXII,LE.IABS(1)
IMPLIES LY(I))=.T.

INT=1ABS(1)
CALL ITL.LT.01GO TO 10
DO 20 I=1,N
LY(I)=F
DO 20 I=1,N
IF(LXI GE.1TLY(I)=T
RETURN
DO 20 I=1,N
LY(I)=E.
IF(LXI LE.IITLY(I)=T
RETURN
END

230
SUBROUTINE CMPRES(LX,NEL,LY)
LOGICAL*1 LX(NEL)
DIMENSION LY(I)
JWRD=1
JBIT=33
DO 10 I=1,NEL
JBIT=JBIT+1
IF(JBIT.NE.0)GO TO 20
JBIT=32
JWRD=JWRD+1
IX=LX(I)
LY(JWRD)=ISTORE(IX,LY(JWRD),JBIT,1)
CONTINUE
RETURN
END

SUBROUTINE PEELER(NDEV,NREC,N,LX,LXR,LXL,LY,NP)
DIMENSION LX(N,3,4),LXR(N,3,4),LXL(N,3,4),LY(N),J(3)
NREC=NREC+1
NREC2=NREC*2
NREC3=NREC+3
NREC4=NREC*4
J(1)=1
J(2)=2
J(3)=3
CALL SVEL0(LX,N+1,2,0)
CALL SVEL0(LXR,N,2,0)
CALL SVEL0(LXL,N,2,0)
NP=0
DO 10 I =1,NREC4
DO 20 K=1,4
IF1 I.LE.NREC+KGO TO 20
CALL SVEL0(LX,J(3)+K,N,0)
CALL SVEL0(LXR,J(3)+K,N,0)
CALL SVEL0(LXL,J(3)+K,N,0)
20 CONTINUE
IF1 I.LE.NREC1
IF1 I.GT.1.AND.I.LE.NREC2)
CALL SVEL0(LX,J(3)+3,N,0)
CALL SVEL0(LXR,J(3)+3,N,0)
CALL SVEL0(LXL,J(3)+3,N,0)
IF1 I.GT.3.AND.I.LE.NREC3)
CALL SVEL0(LX,J(3)+3,N,0)
CALL SVEL0(LXR,J(3)+3,N,0)
CALL SVEL0(LXL,J(3)+3,N,0)
IF1 I.GT.4)
CALL SVEL0(LX,J(3)+4,N,0)
CALL SVEL0(LXR,J(3)+4,N,0)
CALL SVEL0(LXL,J(3)+4,N,0)
NP=0
DO 30 K=1,3
J(K)=MOD(J(K),3)+1
30 CONTINUE
RETURN
END

231
SUBROUTINE PEELROLX(LX, LXR, LXL, J, h, SIDE, LY, LYM, LYN)
DIMENSION LX(N,3), LXR(N,3), LXL(N,3), LY(N), J(3), I(3), LYM(N), LYN(N)
DO 60 I = 1, N
CALL DARN(NDEV, I, LX, J, LXR, LXL, J, h, SIDE)
CALL BLSFTV(LX, N, LYM)
CALL BRSTV(LY, N, LYN)
RETURN
END

SUBROUTINE BLSFTV(LX, N, IY)
DIMENSION IX(N), IY(N)
N1 = N - 1
DO 10 I = 1, N1
IY(I) = ILOAD(IX(I+1), J, 32, 1)
10  IY(I) = ISTORE(IX(I), IY(I), J, 32, 1)
IY(N) = 0
RETURN
END

ENTRY BRSTV(LX, N, IY)
IY(1) = ILOAD(IX(1), 32, 31)
DO 20 I = 2, N
IY(I) = ILOAD(IX(I), 32, 31)
20  IY(I) = ISTORE(IX(I-1), IY(I), 32, 31)
RETURN
END

SUBROUTINE PEELROLX(NDEV, I, LX, J, LXR, LXL)
DIMENSION LX(N,3), LXR(N,3), LXL(N,3), J(3)
CALL DARN(NDEV, I, LX, J, LXR, LXL, J, h, SIDE)
CALL BLSFTV(LX(I), J(3), J, LXR, LXL, J(3), N)
CALL BRSTV(LX(I), J(3), N, LXR, LXL, J(3))
RETURN
END
SUBROUTINE EXPDY(LX,N,NEL,IBDY,J)
DIMENSION LX(N),IBDY(J)
LOGICAL ILOAD
JHRD=1
JBIT=33
J=0
DO 10 I=1,NEL
JBIT=JBIT-1
IF(JBIT,NE,0)GO TO 20
JBIT=32
JHRD=JHRD+1
20 IF(.NOT.ILOAD(LX(JHRD),JBIT,J))GO TO 10
J=J+1
IBDY(J)=1
10 CONTINUE
RETURN
END
5-5-2 FINDING DISCONTINUITIES IN BOUNDARY DATA

1 NAME

BOUDIM

2 PURPOSE

To find the discontinuities in digital curves stored in SLIC format.

3 CALLING SEQUENCE

CALL BOUDIM (IBDY, NTAPI, NREC, NEL, IRC, ND, NDIS)

where

IBDY is a scratch array to be dimensioned NEL*3 where NEL is the maximum number of boundary points in a given line;
NTAPI is the logical unit number on which the input boundary data are stored;
NREC is the number of lines (records) in the input data set;
IRC is the output array of coordinates of the discontinuities;
ND is the maximum number of discontinuities expected [IRC is dimensioned (ND, 2)];
NDIS is the output integer giving the actual number of discontinuities found.

NTAPI, NREC, NEL, ND are inputs to this routine IRC, NDIS are outputs.

4 INPUT-OUTPUT

4.1 Input

The input data should be on logical unit NTAPI as a sequential data set consisting of NREC records. Each record should consist of the coordinates of the intersection of the corresponding scan line with the boundary image written as

\[ J, (X(I), I = 1, J) \]

where \( J \) is the number of such intersections and \( X(I) \) are the coordinates.
4.2 Output

The output of this program is only through the calling sequence.

4.3 File Storage

None

5 EXITS

No nonstandard exits.

6 USAGE

This program is written in FORTRAN IV and is implemented on the IBM 360 with the H compiler. It is available on the users' library as a load module.

7 EXTERNAL INTERFACES

The linkage with other subroutines needed with this routine is indicated in the following table.

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<thead>
<tr>
<th>Calling Program</th>
<th>Program(s) Called</th>
</tr>
</thead>
<tbody>
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<td>BOUDIM</td>
<td>BOUDIS</td>
</tr>
<tr>
<td>BOUDIS</td>
<td>JCOINT</td>
</tr>
</tbody>
</table>

8 PERFORMANCE SPECIFICATIONS

8.1 Storage

This program is 834 bytes long. Including the external references listed above, the storage needed will be 2578 bytes (excluding the calling program which should provide storage for the arrays IRC and IRDY).

8.2 Execution Time

TBD

8.3 I/O Load

None
8.4 Restrictions

None

9 METHOD

The routine BOUDIM simply handles the I/O needed for finding the discontinuities. Connectivity, in this context, is defined in terms of the eight nearest neighbors of the point under consideration. Therefore, while examining the ith record of data, it is necessary to have the (i-1)st and (i+1)st records in core. The movement of data in core is avoided by using a circular buffer IBDY dimensioned (NEL, 3) and indexed by the pointer array IND dimensioned 3. Initially, IND is set to (1, 2, 3). Always, IND (2) points to the current row. The numbers of boundary points in the three rows stored in core are in (NB(IND(J)), J=1, 3). The routine BOUDIM starts by reading the first record into IBDY (*, 2). Then, for I=1, NREC the (I+1)st record is read into IBDY (*, IND(3)). The (NREC+1)th record is undefined. Therefore, in that case, NB (IND(3)) is simply set to 0. The routine BOUDIS is called to determine the coordinates of the discontinuities on the Ith record. Then the pointer array IND is updated.

The functioning of BOUDIS is as follows. Each of the boundary points in the current record is treated as the point e in the following array.

a b c
d e f
g h i

The number of boundary points in this array excepting e is called the connectivity count of e. The connectivity count is calculated by examining the arrays IBDY (*, IND(2)), IBDY (*, IND(1)) and IBDY (*, IND(3)), stopping the calculations when the count equals 2. If the count is smaller than 2, then the point e is a discontinuity. The row and column coordinates of e and the continuity count are then stored in (IRC(NDIS,I<) , I< = 1, 3).

10 COMMENTS

None

11 LISTINGS

The listings of this routine, with BOUDIS and JCOUNT are attached at the end.
This program has been tested in conjunction with SMOB2, a smoothing routine documented in the next section.
SUBROUTINE BOUNDARY(IBDY,NREC,NEL,IRC,ND,NDIS)
C
C FIND DISCONTINUITIES ON BOUNDARIES GIVEN THE INFO. ON NTAPI IN SLIC
C
DIMENSION IBDY(NREC,3),IND(3),NB(3)

DO 10 I=1,NREC
  IND(I)=1
  NB(I)=0
  READ(INTAPI)NB2,(IBDY(I,J),J=1,NB2)
  NB(2)=NB2
  NDIS=0
  DO 20 J=1,NREC
    IF(IND(J).GT.1.AND.IBDY(J,J-1,IND(J-1).GT.J,IND(J)).GT.1.NB(J).GT.0) THEN
      IBDY(J,IND(J-1),J-1,IND(J-1)).GT.1.NB(J).GT.0
      CALL BOUNDARY(IBDY,IND,NB,NEL,IRC,ND)
    ELSE
      CALL BOUNDARY(IBDY,IND,NB,NEL,IRC,ND)
      CALL BOUNDARY(IBDY,IND,NB,NEL,IRC,ND)
    END IF
    IND(J)=IND(J)+1
    IRC(NDIS,3)=IRC(NDIS,3)+1
  END DO
10    CONTINUE
RETURN
END
SMOOTHING BOUNDARY DATA

1 NAME

SMOB2

2 PURPOSE

To patch discontinuities in a digital curve.

3 CALLING SEQUENCE

CALL SMOB2(IRC, MDIS, IDIS, NDIS, NDEV, IBDY, IW1, IW2, NREC, K)

where

IRC is an input array dimensioned (MDIS, 3) with IRC(I, 1), IRC(I, 2)) giving the row and column coordinates of the I'th discontinuity and IRC(I, 3) giving its connectivity count for I = 1 through NDIS;

IDIS is the discontinuity number at which the patching should be started (only the discontinuities corresponding to I = IDIS through NDIS will be patched);

NDEV is the logical unit number of a direct access device on which the input boundary data set is located; the output after smoothing is written back on NDEV.

IBDY, IW1, IW2 are work arrays to be dimensioned as indicated in the listing attached;

NREC is the number of records in the boundary image;

K is maximum coordinate difference over which the nearest boundary points are checked for patching a discontinuity. (See 9, Method).

All parameters except the work arrays are inputs.

4 INPUT-OUTPUT

4.1 Input

The input data should be on the direct access unit NDEV, consisting of NREC records, the I'th record readable by

READ(NDEV'1)N, (IBDY(J, 1)), J = 1, N).
4.2 Output

The output data will be on NDEV in the same format as the input.

4.3 File Storage

None.

5 EXITS

No nonstandard exits.

6 USAGE

The program is in FORTRAN IV and is presently implemented on IBM 360 using the H compiler. It is available on the user's library in the form of a load module.

7 EXTERNAL REFERENCES

The linkage is indicated in the following table:

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<th>Programs Called</th>
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<td>PATCH3</td>
</tr>
<tr>
<td>PATCH3</td>
<td>SVSCI PATCH1</td>
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<tr>
<td></td>
<td>SORT ELIRPT</td>
</tr>
<tr>
<td>PATCH1</td>
<td>CONTEL PATCH4</td>
</tr>
<tr>
<td></td>
<td>PATCH2 PRTVEC</td>
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<td>SORT</td>
<td>MVMRMR</td>
</tr>
<tr>
<td>ELIRPT</td>
<td>VMOV</td>
</tr>
</tbody>
</table>
8 PERFORMANCE SPECIFICATIONS

8.1 Storage

The size of SMOB2 is 1068 bytes. Including a main program to supply the arrays required to handle a maximum of 2100 boundary points per record with K = 20 and the buffers, this program needs approximately 114K bytes for execution.

8.2 Execution Time

Highly dependent on the image size, complexity and the number of discontinuities to be patched. In the case of the Mobile Bay, Alabama level II GTM which had 4000 records with 728 discontinuities of which about 530 required patches to be generated, the execution time on IBM 360/65 was about 9 minutes. Since there is a considerable amount of I/O involved on the direct access unit NDEV, a significant improvement in execution time can be achieved by using the array read/write routines DARN and DAWN wherever implied DO loops have been used in the subroutine PATCH3.

8.3 I/O Load

None

8.4 Restrictions

None

9 METHOD

The routine SMOB2 simply consists of a DO loop which calls PATCH3 to generate the patch points needed for the L'th discontinuity and prints the details of the patches produced, with L ranging from IDIS through NDIS.

Consider the routine PATCH3. Suppose (I,J) is the address of the discontinuity at which a patch is to be produced. Then, the records I-K through I+K (bounded, of course, by 1 and NREC) are read from NDEV. While each record is read one row of a 2K+1 by 2K+1 binary matrix IW1 is defined. The elements of the row are initially set to 0 and whenever the (J-K+L)'th column in the present row of the input image has a boundary point, the (L+1)st element is set to 1.

After defining IW1, the routine PATCH1 is used to check the array IW1, eliminate the 1's contiguous with the (K+1,K+1)th element, find the nearest 1 among the remaining and join it to that element by a straight line and store the row and column coordinates of the points so produced in an array IW2. Further, if the contiguity count of the point of interest is 0, then the 1's contiguous with
the point joined to the point of interest are also eliminated and a straight line patch is produced to the nearest remaining 1.

The addresses in IW2 are then merged with the data on the input direct access data set by reading the corresponding records of input, sorting the column coordinates in each record using SORT, eliminating repetitions of column coordinates using ELIRPT and writing back on NDEV.

10 COMMENTS

The routine SMOB2 can be used in conjunction with BOUDIM or independently. If used independently, the coordinates of discontinuities may be supplied by reading a sequential data set produced by a separate run of BOUDIM. If the program terminates due to lack of time, the execution can be continued by a subsequent run with an updated value of IDIS provided the output data set on NDEV is kept.

11 LISTINGS

Listings of SMOB2 and the important routines called by it are shown at the end of this section.

12 TESTS

This routine has been tested by using the coordinates of the discontinuities produced by BOUDIM on the Mobile Bay GTM. The first 40 discontinuities were examined in detail by printing the arrays IW1. The performance of the routine was found to be satisfactory.
SUBROUTINE SMU2(IW1,IW2,M,N,I,J,ICOUNT)
COMMON/PATCHAD/II,J1,J2,NP
DATA IPASS/0/
IPASS=IPASS+1
CALL PRVECA(IW1,M,N,1)
C GENERATE PATCH POINTS IN IW1 STARTING FROM (I,J) TO THE NEAREST
C (2-ICOUNT) NONCONTIGUOUS NEIGHBORS.
C SEE CONTEL FOR DIMENSIONING INFO FOR IW2.
DIMENSION IW1(M,N),IW2(2,1)
IW2(1)=I
IW2(1)=J
C ELIMINATE POINTS CONTIGUOUS WITH (I,J).
CALL CONTEL(IW1,IW2,M,N)
C FIND NEAREST NEIGHBOR OF (I,J1) WHICH IS SET.
I2=0
CALL PATCHA(IW1,M,N,1,I1,J1)
NOW (I1,J1) IS THE NEAREST NEIGHBOR.
IF(ICOUNT.NE.0)GO TO 10
C ELIMINATE POINTS CONTIGUOUS WITH (I1,J1)
IW2(1)=I1
IW2(2)=J1
CALL CONTEL(IW1,IW2,M,N)
CALL PATCHA(IW1,M,N,1,12,J2)
NOW (I2,J2) IS THE NEXT NEAREST NEIGHBOR.
CONTINUE
MP=0
NP=0
IF(I1.EQ.0)RETURN
C PRODUCE PATCH ADDRESSES IN IW2.
CALL PATCH2(IW2,MP,NP,I1,J1)
IF(I2.EQ.0)CALL PATCH2(IW2,NP+1,MP,I2,J2)
MP=NP+1
IF(IPASS.LT.60)CALL PRVECA(IW2,2,MP+2)
RETURN
END
SUBROUTINE PATCH2(1W*,NP*,11,J1,J2,J2)
DIMENSION 1W(2,1)

C TO GENERATE COORDINATES OF LINE JOINING (11,J1) AND (12,J2).
C DIMENSION 1W(2,NP) WHERE NP= MAX. NO. OF POINTS EXPECTED TO BE
C PRODUCED. NP= NO. OF POINTS ACTUALLY PRODUCED BY THIS ROUTINE.

ISN 0004 IMA=MINO(11,J1)+1
ISN 0005 IMX=MAXO(11,J1)-1
ISN 0006 JHN=MINO(J1,J2)+1
ISN 0007 JMX=MAXO(J1,J2)-1
ISN 0008 I12=11-I1
ISN 0009 J12=J1-J2
ISN 0010 RI12=I12
ISN 0011 RJ12=J12
ISN 0012 NP=0
ISN 0013 IF(IMX=IMN.GT.JMY-JMN)GO TO 10
ISN 0014 IF(JHN.GT.JMX)RETURN
ISN 0015 DO 20 J=JMN,JMX
ISN 0016 I=I1+(J-J1)*I12/RJ12+.5
ISN 0017 NP=NP+1
ISN 0018 1W(1,NP)=I
ISN 0019 1W(2,NP)=J
ISN 0020 RETURN
ISN 0021 CONTINUE

SUBROUTINE CONTENT(IW1,IW2,M,N)
DIMENSION 1W1(M,N),1W2(2,1)

C THIS PROGRAM ELIMINATES ALL 1'S IN IW1 CONNECTED TO THE 1 AT
C (IW1(11,J1),IW2(2,1)). IW2 SHOuld BE DIMENSIONED (2,K) WHERE K IS TWICE
C THE NUMBER OF NODES IN THE PIECE OF DIGITAL CURVE CONNECTED TO THE
C POINT OF INTEREST.
C
K=1
L=1
40 DO 20 KK=1,K
ISN 0006 I=IW2(1,KK)
ISN 0007 J=IW2(2,KK)
ISN 0008 IW1(I,J)=0
ISN 0009 I1=MAXO(I-1,1)
ISN 0010 J1=MAXO(J-1,1)
ISN 0011 I2=MNO(I+1,M)
ISN 0012 J2=MNO(J+1,N)
ISN 0013 DO 10 II=11,J1
ISN 0014 DO 10 JJ=J2,J1
ISN 0015 IF(II.IJ,II.EQ.0)GO TO 10
ISN 0016 IW2(IJ,II)=II
ISN 0017 IW1(IJ,II)=0
ISN 0018 LI=LI+1
ISN 0019 IW2(IJ,II)=JJ
ISN 0020 IW1(IJ,II)=I
ISN 0021 IW1(IJ,II)=0
ISN 0022 CONTINUE
ISN 0023 CONTINUE
ISN 0024 IF(L.EQ.K1)RETURN
K1=K1+1
ISN 0025 LL=0
ISN 0026 DO 30 KK=K1,1
ISN 0027 LL=LL+1
ISN 0028 30 LL=LL+1
ISN 0029 30 IW2(IJ,II)=IW2(IJ,II)
ISN 0030 K=LL
ISN 0031 LL=LL
ISN 0032 GO TO 40
ISN 0033 END
SUBROUTINE PATCH3(IBDY1,1,ICOUNT,K,1W1,1W2,NREX,NDEV)

COMMON/PTCHAD/I1,J1,J2,J3,NP

DIMENSION 1W1(1,K),1W2(2,NP),WHERE NP=MAX(NO. OF PATCH POINTS

C EXPECTED TO BE GENERATED BY PATCH2, DIMENSION REQUIRED BY CONTEL)

C PATCH DISCONTINUITY AT I+N

DIMENSION 1W1(1),1W2(2,1),IBDY(1)

K21=K2+1

K1=MAXO(I-K,1)

K2=K2+1

KK21=K2-K1+1

CALL SWSC1(1W1,1K21*1K21+1

ICLMMK=1-J-K

ICLMPK=1+K

DO 10 KK=K1,K2

READ(NDEV,ΚΚΚΚΚΚΙΚΜΚΚΜΚΚ1W1(1),L=1,N)

IF(IN.EQ.0)GO TO 10

DO 20 L=1,N

IF(1BDY(1,LT.ICLMMK)GO TO 20

IF(1BDY(1,GT.ICLMPK)GO TO 10

C TREAT 1W AS A 2-D ARRAY DIMENSIONED K21*K21 WITH THE GIVEN POINT

10 AT (I-K1+1,K+1)TH LOCATION.

1W1(IBDY(I1-ICLMMK)*KK21+KK-K1+1)=1

CONTINUE

CONTINUE

DO 10 KK=K1,K2

IF(V.ET.0)RETURN

IP=1

IP=IP+1

30 IP=IP+1

IF(IP.GT.NP)GO TO 40

IF(1W2(1,IP).EQ.1W2(1,IP-1))GO TO 30

IP2=IP-1

KK=1W2(1,IP2)+K1-1

READ(NDEV,ΚΚΚΚΚΚΙΚΜΚΚΜΚΚ1W1(1),L=1,N)

DO 50 JP=1IP1,IP2

IF(JP.NE.1)RETURN

50 IBDY(N)=1W2(2,JP)+ICLMMK-1

CALL SORT(IP,IBDY1,K1+1,K1+1,N1,N1)

CALL ELKPTN(IP,IBDY1)

WRITE(NDEV,ΚΚΚΚΚΚΙΚΜΚΚΜΚΚ1W1(1),L=1,N)

IP=IP+1

IF(IP.LE.NP)GO TO 30

RETURN

END
SUBROUTINE PATCH(IW1,H,N,J1,JO,JO)
DIMENSION IW1(H,N)
ID=0
IF(I.EQ.0)RETURN
IDMIN=H**2+N**2+1
DO 10 II=1,H
IF(W1(I,II).EQ.0)GO TO 10
10 CONTINUE
ID=II-1**2+(JJ-J)**2
IF(ID.GE.IDMIN)GO TO 10
ID=II
JJ=JJ
END

SUBROUTINE PRTVEC(IX,H,IFMT)
DIMENSION IX(H)
IF(IFMT.EQ.1)PRINT 100,IX
IF(IFMT.EQ.2)PRINT 200,IX
RETURN
100 FORMAT(10X4111)
200 FORMAT(1X4013)
END
IDENTIFICATION OF CONNECTED REGIONS

1 NAME

REGIONS

2 PURPOSE

To identify all distinct connected regions in an image given the boundary data in SLIC format and produce a map with a number at each point showing the region to which it belongs. The region numbers will be in descending order of area.

3 CALLING SEQUENCE

This is a main program. In its present version the image size is supplied through DATA statements.

4 INPUT-OUTPUT

4.1 Input

The input to this program is a sequential data set on logical unit 8, having NREC records stored as N, (IX(J), J=1, N) in unformatted FORTRAN mode.

4.2 Output

The output of this program will be a sequential data set on logical unit 12, having NREC records with NEL pixels each, with one half-word (2 bytes) per pixel.

4.3 File Storage

This program requires a direct access data set with NREC records and NEL half-words per record.

5 EXITS

Not applicable

6 USAGE

This program is in FORTRAN IV and is implemented on IBM 360 with the H compiler. The associated subroutines are available as load modules on the user's library. The deck for the main program is available with the authors and needs only slight modifications in the DEFINE FILE and DATA statements for use on any data set.
This program uses several subroutines as indicated by the linkage table below:

<table>
<thead>
<tr>
<th>Calling Program</th>
<th>Programs Called</th>
</tr>
</thead>
<tbody>
<tr>
<td>REGIONS</td>
<td>PET VMAXI4 VMINI4 RIDER SVSCI DARN SEQLS SAWN</td>
</tr>
<tr>
<td>RIDER</td>
<td>SVSCI2 SVSCL1 SORT RIDER1 RIDER4 DAWN VMOV2 RIDER2 PRTVE2 DARN VMAXI4</td>
</tr>
<tr>
<td>SEQLS</td>
<td>SORT FLIPV</td>
</tr>
<tr>
<td>SORT</td>
<td>MVMRMR</td>
</tr>
<tr>
<td>RIDER1</td>
<td>SVSCI2</td>
</tr>
<tr>
<td>RIDER4</td>
<td>RIDER5 SVSCI2 PRTVE2 RIDER7 RIDER6 SVSCL1</td>
</tr>
</tbody>
</table>
8 PERFORMANCE SPECIFICATIONS

8.1 Storage

The present version of the main program is 134,436 bytes long. The external references required and the buffers increase this to 192K bytes. However, the size is dependent on the data set to be handled and the dimension statements should be changed to satisfy specific requirements.

<table>
<thead>
<tr>
<th>Calling Program</th>
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</tr>
</thead>
<tbody>
<tr>
<td>RIDER2</td>
<td>PRTVE2</td>
</tr>
<tr>
<td>RIDER7</td>
<td>VMAXI2</td>
</tr>
<tr>
<td></td>
<td>VMINI2</td>
</tr>
</tbody>
</table>

where

\[
\begin{align*}
\text{NR} & = \text{Maximum number of regions expected;} \\
\text{N} & = \text{Maximum number of boundary points expected in a record;} \\
\text{MSEG} & = \text{Maximum number of "segments" required to handle the image (see 9, Method);} \\
\text{MR} & = \text{Maximum number of region identifiers permissible in a segment;} \\
\text{NEL} & = \text{Number of pixels per line in the output map.}
\end{align*}
\]

8.2 Execution Time

The time is highly dependent on the size and complexity of the image. The Mobile Bay GTM (level II) resulting in a region identification map with 400x2100 pixels and consisting of 1742 regions had to be handled in 15 sections and took 19.5 minutes of CPU time on IBM 360/65.

8.3 Restrictions

None
9 METHOD

This program has five major sections.

(i) Determination of the bounds on the column coordinates of boundaries on the input data set;
(ii) Finding a preliminary set of region identifiers;
(iii) Finding the areas of each of the regions;
(iv) Generating a mapping such that the region numbers are used in the order of decreasing areas;
(v) Modifying the region numbers by table look-up.

9.1 Determination of Bounds

The maximum and minimum values of the column coordinates of the boundary points are determined. If the minimum is greater than 1, it is set to 1. If the maximum is less than the value of NEL supplied, it is set to NEL. The value of NEL is then changed to Max-Min+1. The output image size will then be NREC by NEL.

9.2 Finding Preliminary Region Identifiers

This is the most important step in the program. The subroutine RIDER is used for this purpose. Its function is similar to the routine with the same name described in [23]. The routine in [23] was designed to print an error message and return with NR=0 when the number of distinct regions exceeded MR. But the present version can handle up to MR*MSEG distinct regions while still using a "region identity matrix" of size MR by MR (rather than MR*MSEG by MR*MSEG).

This routine uses the arrays IW1 and IW2 as the previous and current records of region identifiers. By convention, region numbers 1 and 0 indicate the "exterior" of the image and boundary points. The MR by MR array IDENT is used to store information about identity of regions, IDENT(I,J) = .TRUE., meaning that region numbers I and J refer to the same connected region.

Initially, the array IW1 is set to all 1's and IDENT is set to all .FALSE.. Each of the input records is read and the following operations are performed.

The boundary coordinates in the input record are arranged in ascending order. The routine RIDER1 is used to generate, in IW2, the region identification numbers corresponding to the present row. First, all the elements of IW2 corresponding to the boundary coordinates are set to zero. Each interval between
the zeros is compared with the corresponding segment of IW1. If there is no non-
zero element in that segment of IW1, a new region number is started and assigned
to the interval in IW2. If there is a nonzero element, that number is filled into
all elements in the interval. Finally, IDENT(IW1(I), IW2(I)) is set to .TRUE. for
I = 1, NEL wherever IW1(I) ≠ 0 and IW2(I) ≠ 0, indicating that IW1(I) and IW2(I)
refer to the same region. Also, when new region identifiers are to be used, the
routine RIDER1 verifies whether the number of identifiers exceeds MR. If so,
the value of NR, the total number identifiers, is set to -NRP, the total number
up to the previous record and the control goes back to the routine RIDER.

Now, if RIDER1 returns a positive NR, the array IW2 is written as the
I'th record on the direct access data set (unit number IDEVO in RIDER, same as
90 in the main program) and IW2 is moved into IW1 (so that it becomes the
"previous" record while handling the next record).

If RIDER1 returns a negative NR, then NR is changed to -NR and the routine
RIDER4 is called. The set of records handled between any two calls of RIDER4
will be referred to as a segment. Associated with each segment, a table is defined
which gives a mapping from the set of region identifiers obtained in that segment
to a new set reflecting the connectivities discovered up to the most recent segment
handled. Also, the initial record number for each of the segments is stored in an
array. The functions of the routine RIDER4 are to:

(i) Reduce the matrix IDENT (using RIDER5) examining all of the
available connectivity information in it and obtain a look-up table
for the current segment;

(ii) Modify the tables for the previous segments to reflect the newly
found connectivities, if any;

(iii) Find all the distinct region numbers occurring in the last record
IW1 of the current segment and change the numbers there which
are greater than 1 to consecutive numbers starting with 2; Let
NR be the largest number in IW1;

(iv) Set up an array IS consisting of the distinct region numbers in
IW1 and then change IS(I) to ITABL(IS(I), ISEG) where ITABL
is the look-up table for the current segment;

(v) Set all elements of IDENT TO .FALSE. except when IS(I) = IS(J)
for I, J in the range 1 through NR.

After each call to RIDER4, the segment count ISEG is incremented and
the initial record number for the next segment (which is really the record number
at which RIDER4 had to be called) is stored in IRES(ISEG). If MSEG is exceeded
by ISEG or if NR > MR (which means there are more than MR distinct regions in
the last record) the routine RIDER prints an error message, sets NR=0 and exits.
Otherwise, RIDER1 is called again, IW2 is found and written on IDEVO and the program proceeds normally to the next input record.

After the NREC input records have been processed the routine RIDER4 is called to get the look-up table for the final segment. A call to RIDER2 changes the look-up tables for all the segments such that consecutive region numbers are used.

Finally, each record from IDEVO is read, the appropriate look-up table is used to modify it and the record is written back on IDEVO. Also, NR is set to the maximum region number used after table look-up.

9.3 Finding Areas

A histogram of the region identification maps is found, giving the total number of occurrences of each of the region identifiers 0 through NR. These numbers indicate the areas of the regions.

9.4 Finding the Final Look-up Table

A sequence of natural numbers is used as a secondary array with the histogram as the primary array in a descending sort operation (routine SEQLS). The resulting secondary array then gives the sequence of original region identifiers corresponding to decreasing areas. An inverse mapping \( \{ \text{IX}(J) \} J=1, N \) is defined as \( \{ \text{IY}(J) \} J=1, N \) if \( \text{IY(IX(J))}=J \). This sequence gives the final look-up table. The actual coding follows these principles but is slightly different in detail to preserve the identities of regions 0 and 1 which have special significance.

9.5 Deriving the Final Region Identification Map

The look-up table generated above is used to modify the region identifiers on IDEVO, record by record, and write out the final sequential data set on unit 12.

10 COMMENTS

An approach suggested in [24] can be used instead of the one described above. With that method, the processing would be identical, except that the matrix IDENT is not defined. Instead, a table is updated every time a new connectivity is discovered. While this saves storage, it appears to take more execution time than the present method.

11 LISTINGS

The listings of the main program and the associated routines are attached at the end of this section.
This program has been tested on the Mobile Bay GTM both before and after smoothing and found to work satisfactorily. Also, the results have been found to be identical (on a smaller data set) with those obtained by the earlier version of this program.
For I = 1, NEL: IW1 (I) = 1
For I = 1, MR AND J = 1, MR:
IDENT (I, J) = FALSE
NR = 1

IREC = 0

IREC = IREC + 1

IF IREC > NREC

Call Rider 2 to produce look-up table ITABL

Modify NREC output records according to ITABL

RETURN

Call Rider 2 to produce look-up table ITABL

Read one record from NTAPB into IBDY

Arrange IBDY in ascending order

Call Rider 1 to generate IW2 using IW1 and IBDY; increment NR appropriately (see flowchart for Rider 1)

For I = 1, NEL:
IW1 (I) = IW2 (I)

NR > 0

Yes

Write IRECth record of output (IW2) on IDEVO.

NR = NR

No

IREC = 1

Yes

No

NR = NR

Call Rider 4 to change (IREC-1) records of output on IDEVO by table look-up

Call Rider 4 to change (IREC-1) records of output on IDEVO by table look-up

WRITE IRECth RECORD OF OUTPUT (IW2) ON IDEVO.

NR = NR

Yes

No

PRINT ERROR MESSAGE

RETURN
ISN 0002  DIMENSION IX(4000),IRES(21)
ISN 0003  INTEGER*I IX(2100),I(2100),I(4000),15(400)
ISN 0004  INTEGER*I IX(2100)
ISN 0005  LOGICAL*1 IDENT(300,300)
ISN 0006  DEFINE_FILE 9C(4000,4200,L,IA)

C
C N IX(MAXZ2NR+2,N) WHERE NR=MAX NO. OF REGIONS EXPECTED AND
C N MAX NO. OF BOUNDARY POINTS EXPECTED IN ANY RECORD.

C
ISN 0007  DATA NREC,MR,MSG/4000,300,20/
ISN 0008  DATA NSEG/2100/
ISN 0009  MAXX=1000000
ISN 0010  MINX=1000000
ISN 0011  CALL STATM
ISN 0012  CALL PET100
ISN 0013  DU 10 I=1,NREC
ISN 0014  READ(I,N(1IX(J),J=1,N)
ISN 0015  IF(N.EQ.0)GO TO 10
ISN 0016  CALL VMAXI4(IX,N,MAXX)
ISN 0017  CALL VMINI4(IX,N,MINX)
ISN 0018  CONTINUE
ISN 0019  PRINT I,IX
ISN 0020  REWIND 8

C IDENTIFY CONNECTED REGIONS.
ISN 0021  PRINT 690,MINX,MAXX,NEL
ISN 0022  MINX=MNIX(MINX,1)
ISN 0023  PRINT 600,MINX,MAXX,NEL
ISN 0024  MAXX=MXX(MAXX,NEL)
ISN 0025  PRINT 600,MINX,MAXX,NEL
ISN 0026  NEL=MAX-MINX+1
ISN 0027  PRINT 600,MINX,MAXX,NEL
ISN 0028  6CO FORMAT(\#MINX,NAXX,NAXX,NAXX)*118)
ISN 0029  PRINT 1000,NREC,NEL
ISN 0030  100 FORMAT(//\# IMAGE SIZE=(15,15,15)*)
ISN 0031  NDUH=1
ISN 0032  PRINT 1000,NUH
ISN 0033  1000 FORMAT(\# NDUH=15)
ISN 0034  CALL PET2
ISN 0035  CALL RIDER(8,NREC,NEL,9C,MINX,IX,IX,1,2,1TA,B,IDENT,1R ,NR,1LW)
ISN 0036  CALL PET12

C FIND AND PRINT HISTOGRAM OF REGION IDENTIFICATION MAP.
ISN 0037  PRINT 200
ISN 0038  200 FORMAT(//10X\#REGION NO.\#REX NO. OF PIXELS\#)
ISN 0039  CALL SYSCLI(IX,NR+1,0)
ISN 0040  DO 30 I=1,NREC
ISN 0041  CALL DARN(90,1,1,1,1,NEL+2)
ISN 0042  DO 30 I=1,NEL
ISN 0043  J=1W111J+1
ISN 0044  30 IX(J)=IX(J)+1
ISN 0045  NR=NR+1
ISN 0046  DU 40 I=1,NR1
ISN 0047  J=1-1
ISN 0048  IF(I>IX(J),N.EQ.0)PRINT 300,J,IX(J)
ISN 0049  40 CONTINUE
ISN 0050  300 FORMAT(11X16,16X19)
ISN 0051  CALL PET2

C REARRANGE NUMBERS IN DESCENDING ORDER OF POPULATIONS.
C LEAVE 0 AND 1 UNCHANGED SINCE THEY CORRESPOND TO EXTERIOR AND
C BOUNDARY POINTS RESPECTIVELY.
ISN 0053  CALL SEULS(IX(3),IX(NR+1),NR-1,NR-1)
ISN 0054  PRINT 400
ISN 0055  400 FORMAT(10 REGIONS AFTER REASSIGNMENTS:11)
ISN 0056  PRINT 200
ISN 0057  DO 50 I=1,NR1
ISN 0058  J=1-1
ISN 0059  IF(I.LE.2)PRINT 300,J,IX(J)
ISN 0060  50 CONTINUE
ISN 0061  IF(I.GT.2)PRINT 350,J,IX(J)+IX(I+NR-1)
ISN 0062  PRINT 350 FORMAT(11X16,16X19,17X16)
SUBROUTINE RIDER(INTABP,NREC,NEL,IDEV0,ICMN,IBDY,IM2,ITABL,IDENT,MR,NL,LW,MSG,E,IRES,I)
C TO IDENTIFY ALL DISTINCT CONNECTED REGIONS IN A PICTURE SEPARATED
C BY BOUNDARY LINES. THE BOUNDARY DATA ARE GIVEN AS NREC RECORDS ON
C SEQUENTIAL FILE NTABP, EACH RECORD BEING WRITTEN AS
C N,IBDY(1:1:1:N)
C THE OUTPUT OF THE PROGRAM IS AN NREC+NEL DIRECT ACCESS FILE ON
C IDEV0 CONSISTING OF D'S FOR BOUNDARY POINTS AND DISTINCT REGION
C NUMBERS FOR EACH OF THE CONNECTED REGIONS. ICMN=MINIMUM COLUMN
C NUMBER WHICH ON THE OUTPUT FILE WILL CORRESPOND TO THE FIRST
C COLUMN. IT IS NECESSARY THAT
C ICMN+LEMIN+1,IE,.MAX(IBDY)+1,IE.ICMN+NEL+1.
C DEFINE FILE IDEV0(NREC,NEL+2,L,IAV)
C
C DIMENSION IBDY(I)
C LOGICAL*1 IDENT(MR,NL)
C LOGICAL*1 LW(MR,NL)
C INTEGER*2 ITABL(NL,NL,ITABL(MR,MSG,E),IRES(I))
C
C INITIALIZE A WORK ARRAY IM2 WITH 1'S AND IDENT WITH .FALSE.
C
CALL SYSC2(IW1,NEL,1)
CALL SYSC1(IDENT,MR,NL,.FALSE.,1)
ISEG=1
IRES(11)=1
NR=1
C
C LOOP ON RECORDS
C
DO 10 IREC=1,NREC
C
C READ ONE RECORD OF BOUNDARY INFO.
C
READ(INTABP),(IBDY(I),I=1,N)
IF(N=.NE.0)CALL SORT1(IBDY(I),N,N,1,1,1)
C
C USE IM2 AND IBDY TO SET ARRAY IM2 AND MATRIX IDENT
C
30 CONTINUE
CALL RIDER2(IW1,IBDY,ICMN,NL,NI2,IDENT,MR,NR)
IF(MR.GT.0)GO TO 29
PRINT 200, IREC,NR
20 FORMAT(2I12) (IREC,NR)=4216)
IF(IRES(NR).GT.0)GO TO 40
NR=NR
CALL RIDER4(IDENT,MR,NL,ITABL,ISEG,IM2,NL,IS,.FALSE.,1)
ISEG=ISEG+1
IF(ISEG.GT.MSG,E)GO TO 40
IRES(ISEG)=IREC
29 CONTINUE
IF (NR.LE. MR) GO TO 30
40 PRINT 100, IREC
KR=0
100 FORMAT(*RULE-CONDITION IN-REC2...SUPPLIED-MA-OR-MSEG-WAS-EXCEEDED
AT RECORD NUMBER'16, 0'. RETURNING WITH NR=0' )
RETURN
20 CALL DANK(IDEVO,IREC,IN2,NEL*2)
CALL VMHEV2(IN2,NEL,IN1)
10 CONTINUE
CALL RIDE1(IDENT,LT,MR,MRTABL,ISEG,IN1,NEL,15, TRUE)
CALL RIDER2(ITABL,MK*ISEG)
JRES=ISEG+11=NR+1
PRINT 300
300 FORMAT(*FINAL TABLES FOR MODIFYING REGION NUMBERS' )
DO 60 JSEG=1, ISEG
PRINT 400, JSEG
60 FORMAT(*SEGMENT NUMBER'13)
CALL PRV2E1(ITABL,JSEG,MK)
JSEG=1
DO 70 IREC=1,NREC
IF (IREC.LT. IRES1) JSEG=11=ISEG
JSEG=JSEG+1
70 CONTINUE
CALL DANK(IDEVO,IREC,IN1,NEL*2)
DO 80 IEL=1,NEL
I=IN1(IEL)
IF (I.NE.0) IN1(IEL)=ITABL(I,JSEG)
80 CONTINUE
70 CALL DANK(IDEVO,IREC,IN1,NEL*2)
NR=0
CALL VMAXI2(ITABL,MK*ISEG,NR)
RETURN
END

SUBROUTINE PET(I)
IF (I.NE.0) GO TO 10
CALL TIMER(ITIME1)
ITIME=0.
WRITE(6,200)
200 FORMAT(*BEGINNING TIMING*** TIME NOW IS 0' )
RETURN
10 CALL TIMER(ITIME2)
TIME=(ITIME2-ITIME1)/100.
ITIME=ITIME+TIME
WRITE(6,100)ITIME, TIME
100 FORMAT(*TIME ELAPSED SINCE LAST PRINTING OF TIME='E12.3,
'SEC.', 'TOTAL TIME-ELAPSED='E12.3'y'SEC.' )
RETURN
END
SUBROUTINE RIDE1(IY, IBDY, ICHN, NEL, IY, IDENT, MR, NR)
GIVEN CURRENT SET OF BOUNDARY ADDRESSES (IBDY(I), I=1,N) AND THE
LAST ARRAY IX, FIND CURRENT ARRAY IY CONTAINING REGION IDENTIFICATIONS. 
ALSO, IF THE NONBOUNDARY ELEMENTS IN CURRENT ROW ARE CONTIGUOUS WITH ANY NONBOUNDARY POINTS OF THE LAST ROW, SET THE CORRESPONDING ELEMENTS IN IDENT-ARRAY.

DIMENSION IBDY(N), IY(NEL), IDENT(MR, NR)
LOGICAL IDENT(MR, NR)
IF (N, NEQ. 0) GO TO 10
CALL SVSCI2(IY, NEL, I)
GO TO 20
CONTINUE
NR = NR
ALL POINTS TO THE LEFT OF IBDY(1) ARE 'EXTERIOR' POINTS (REGION 1)
I = IBDY(1) - ICHN
IF (I, GT. 0) CALL SVSCI2(IY, I, I)

ALL POINTS TO THE RIGHT OF IBDY(N) ARE 'EXTERIOR' POINTS.
I = IBDY(N) - ICHN + 2
IF (NEL, GE. 1) CALL SVSCI2(IY(I), NEL - 1, I)

DESIGNATE ALL BOUNDARY POINTS AS 'REGION 0'.
DO 30 I = 1, N
J = IBDY(I) - ICHN + 1
30 IY(J) = 0
FOR I = 1, N - 1 EXAMINE IX(J) FOR IBDY(I), LT. J, LT. IBDY(I + 1)
AND SET IY ACCORDINGLY. A NEW REGION NUMBER IS STARTED WHEN IX(J)
IS 0 FOR ALL J IN THE ABOVE RANGE.
IF (N, NEQ. 1) GO TO 20
N = N - 1
DO 40 I = 1, N
L1 = IBDY(I) - ICHN + 2
L2 = IBDY(I) + 1 - ICHN
IF (I, LT. L2) GO TO 40
DO 50 L = L1, L2
IF (IX(L), EQ. 0) GO TO 50
LL = L
GO TO 40
CONTINUE
NR = NR + 1
IF (NR, LE. MR) GO TO 70
NR = NR
RETURN
CALL SVSCI2(IY(L1), L2 - L1 + 1, NR)
IDENT(NR, NR) = .TRUE.
GO TO 40
CONTINUE
LL = X1(L1)
CALL SVSCI2(IY(L1), L2 - L1 + 1, LL)
CONTINUE
SET IDENT MATRIX TO INDICATE REGION NUMBERS CORRESPONDING TO IDENTICAL REGIONS.
DO 80 IEL = 1, NEL
I = IX(IEL)
IF (I, EQ. 0) GO TO 80
J = IY(IEL)
IF (J, EQ. 0) GO TO 80
IDENT(I, J) = .TRUE.
CONTINUE
RETURN
END

260
SUBROUTINE XIDORT2(ITABL, NR)

CHANGE REGION NUMBERS SUCH THAT CONSECUTIVE NUMBERS ARE USED.

INTEGER*2 ITABL(NR, 21)

FIND THE SET OF NUMBERS IN ITABL(*)+1.

DO 5 I=1, NR
5 ITABL(I, 2)=0
DO 10 J=1, NR
10 J=ITABL(I, 1)
IF(J .NE. 0) ITABL(J, 2)=ITABL(J, 2)+1
PRINT 10
CALL PRVEZ2(ITABL(1:2), NR)

CHANGE ITABL(*)+21 TO GET A LOOKUP TABLE FOR ITABL(*)+1.

J = 0
DO 20 I=1, NR
20 IF(ITABL(I, 2)>EQ.0) GO TO 20
J=J+1
ITABL(J, 2)=J
CONTINUE
PRINT 200
CALL PRVEZ2(ITABL(1:2), NR)

CHANGE ITABL(*)+1.

DO 30 I=1, NR
30 ITABL(I, 1)=ITABL(I, 1)+ITABL(I, 1)
RETURN

FORMAT('NUMBERS OF OCCURRENCES OF REGION NUMBERS IN THE ABOVE TABLE', $1$)

FORMAT('LOOK-UP TABLE TO CHANGE NUMBERS IN THE ABOVE TABLE($1$) END

SUBROUTINE VMAX(IX14, N, MAXI4)

DIMENSION IX14(*)

DUP 1C(T=1, N)

MAXI4=MAXD(IX14(1), MAXI4)
RETURN

ENTRY VMIN(IX14, N, MINI4)

UV =C(I=1, N)

MINI4=MIND(IX14(1), MINI4)
RETURN

END

SUBROUTINE VSCLE(IX, N, L)

LOGICAL .T. IX(N), L

DO 10 I=1, N
10 IX(I)=L
RETURN

END
SUBROUTINE RIDER4(IDENT,LW,MR,NR,ITABL,ISEG,II1,NEL,L5,LAST)
   LOGICAL LAST
   LOGICAL LO IDENT(MT,MT)
   INTEGER 2 IDENT(MR,MR),ITABL(NR,1),NW(NR),ISII,II1(NEL)

C D'N ITABL(MR,ISEG),IS(MCR) WHERE MSEG IS THE MAXIMUM
C NUMBER OF SEGMENTS EXPECTED FOR HANDLING THE GIVEN BOUNDARY IMAGE
C NCR=MAX NUMBER OF REGION NUMBERS EXPECTED TO OCCUR IN ANY RECORD.
C
C THIS ROUTINE IS CALLED FROM RIDER WHEN ALL RECORDS ARE PROCESSED
C (LAST=.TRUE.) OR WHEN THE NUMBER OF REGION NUMBERS FOUND WHILE TEST-
C ING (IREC+1)TH RECORD EXCEEDS NCR. (LAST=.FALSE.).
C THEN,
C 1. THE REGION CONNECTIVITY MATRIX IDENT IS REDUCED TO GET A LOOK-
   UP TABLE FOR THE CURRENT SEGMENT.
C 2. THE LOOK-UP TABLES CORRESPONDING TO EARLIER SEGMENTS ARE
   MODIFIED-BASED ON NEWLY FOUND CONNECTIVITIES, IF ANY.
C 3. THE DISTINCT REGION NUMBERS OCCURRING IN THE IREC'TH RECORD
   ARE FOUND: A CORRESPONDENCE ARRAY IS BETWEEN CURRENT AND NEXT SEGMENT
   SET UP. THE LAST RECORD(IWI) IS MODIFIED TO MATCH THE NUMBERING
   IN THE NEXT SEGMENT.
C 4. THE CONNECTIVITY MATRIX IS MODIFIED TO PRESERVE THE INFORMATION ON
   THE CONNECTIONS BETWEEN REGIONS IN IREC'TH RECORD.
C
C SECTION 1.
C
   DO 50 I=1,NR
   DO 50 J=1,NR
   50 IDENT(I,J)=IDENT(I,J).OR. IDENT(J,1)
   CALL RID4S2(IDENT,MR,NR,ITABL,ISEG,LW)
   ISEG=ISEG+I
   DO 10 I=1,NR
   10 IF(ITABL(I,ISEG).GT.1) ITABL(I,ISEG)=ITABL(I,ISEG)+ISEG
   IF(MR.GT.NR) CALL SVSCI2(ITABL(MR+1,ISEG1,MR-NR,0)
   PRINT 10,1SEG
   CALL PRTVE3(I=ITABL(MR,ISEG),MR)
C
C SECTION 2.
C
   IF(ISEG.EQ.1) GO TO 60
   ISEG=ISEG+1
   CALL RID4S2(I=ITABL(MR,ISEG),IS,NCR,ITABL,MR+ISEG1)
   DO 40 ISEG=ISEG-ISEG1
   KSEG=ISEG-JSEG
   PRINT 10,KSEG
   CALL PRTVE3(I=ITABL(MR,KSEG),MR)
   40   CALL PRTVE3(I=ITABL(MR,KSEG),MR)
C
C SECTION 3.
C
   IF(LAST)RETURN
   CALL RIDER3(I=11,W,NEL,NR)
   NCR=NR
   PRINT 999,NR
   CALL PRTVE3(I=11,NR)
   DO 70 I=1,NR
   70 ISII=ITABL(ISII,ISEG)
   PRINT 499,
   CALL PRTVE3(I=ISII,NR)
CALL SYSCL1(IDENT,HR*HR,FALSE)
DO 20 I=1,HR
   IDENT(I,I)=.TRUE.
   IF(I.EQ.NR)GO TO 20
   I=I+1
   DO 30 J=1,HR
50 IDENT(I,J)=IS(I,I).EQ.IS(J)
20 CONTINUE
RETURN
100 FORMAT(7/1--THE FOLLOWING IS A PRELIMINARY TABLE FOR MODIFYING-SEGMENT NUMBER '13')
200 FORMAT(7/THE FOLLOWING IS AN UPDATED TABLE FOR MODIFYING-SEGMENT '13')
300 FORMAT(7/THE DISTINCT REGION NUMBERS PRESENT IN THE LAST RECORD.
   THE CURRENT SEGMENT TO BE REASSIGNED NOS. 1 THROUGH '14' IN THE NEXT SEGMENT '14'.)
400 FORMAT(7/ASSIGNMENTS FOR THE ABOVE REGIONS FROM THE PRELIMINARY LOCATION)
END

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SLERGUTINE RIVERS(IDENT,HR*N,IT(N))
C TO GENERATE A TABLE: MAPPING J=1,...,N TO 1=IT(J) = SMALLEST K
C SUCH THAT THERE EXISTS A SEQUENCE (K(1),ID=1,...,L) WITH K(1)=1,
C K(I)-J AND IDENT(EK(1),K(1)+1)=.TRUE.
C INTEGER IN(N),M(I)
C LOGICAL IDENT(N),IT(N)
DO 100 I=1,N
100 IT(I)=1
   J=0
   IF(I.LE.N)GO TO 20
   RETURN
20 IF(IT(I).LT.I)GO TO 10
   J=0
   K=0
   L=0
   IF(J.LE.N)GO TO 40
   L=L+1
   C IF(L.GT.K+1)GO TO 10
   J=0
   IF(J.GT.N)GO TO 50
   IF(J.EQ.1)GO TO 70
   IF(J.EQ.1+1)GO TO 75
   IF(J.EQ.1)GO TO 70
   IT(J)=1
   K=1
   M(K)=J
   GO TO 70
40 IF(J.EQ.1)GO TO 70
   IT(J)=1
   K=1
   M(K)=J
   GO TO 70
END

263
SUBROUTINE RIDE? (IX, IY, IS, N)
C
C FIND & SET IS OF DISTINCT NONZERO ELEMENTS IN IX, THE NUMBER OF
C SUCH ELEMENTS IS N.
C
INTEGER*2 IX(M), IY(N)

IS = 2
DO 10 I = 1, N
IF (IX(I).EQ.0) GO TO 10
IF (N.EQ.0) GO TO 20
DO 10 J = 2, N
10 IX(I) = I
IX(I) = N
GO TO 16
16 CONTINUE
RETURN
END

SUBROUTINE RIDE? (IX, IY, IS, N)
INTEGER*2 IX(M), IY(N)

C MODIFY RELEVANT ENTRIES IN IY ACCORDING TO CONNECTIVITIES FOUND
C
C IX = X.
C IF (N.EQ.1) RETURN
M = 1
K = 1
CALL VMAX12 (IX(2), M, K)
CALL VMIN12 (IX(2), M, K)
DO 20 J = 2, N
IF (IY(J).LT.KIN .OR. IY(J).GT.MAX) GO TO 20
20 M = M + 1
IF (IY(J).EQ.IS(I)) GO TO 20
GO TO 16
20 CONTINUE
16 CONTINUE
RETURN
END

SUBROUTINE VHUVZ (IX, IY, IS)

IS = 2
IY = IX
GO TO 11

10 CONTINUE
RETURN
END

SUBROUTINE VHUVZ (IX, IY, IS)

IS = 2
IY = IX
GO TO 11

10 CONTINUE
RETURN
END
SUBROUTINE PRTVE2(I,X,N)
INTEGER I2(I,N)
PRINT 100100
100 FORMAT(1X)
RETURN
END

SUBROUTINE SEQLS(X,ISEQ,N,ND)
DIMENSION X(ND),ISEQ(N),T(2),TT(2)
C
C MUST EQUIVALENCE (X(1,2),ISEQ(1))
C
C IFLAG=0
C)
ENTRY SEQLS(X,ISEQ,N,ND)
IFLAG=1
10 DO 20 I=1,N
20 ISEQ(I)=I
CALL SORT(X,1,N,ND,Z,T,TT)
IF (IFLAG.EQ.0) RETURN
CALL FLIPV(X,N,X)
CALL FLIPV(ISEQ,N,ISEQ)
RETURN
END

SUBROUTINE FLIPV(X,N,Y)
DIMENSION X(N),Y(N)
C
C N2=N/2
N+1
DO 10 I=1,N2
W=X(I)
I=N-I
Y(I)=X(I)
Y(11)=W
10 CONTINUE
RETURN
END

SUBROUTINE VSCE1(I,X,N,IS)
SUBROUTINE VSCE2(I,X,N,MAX)
SUBROUTINE VSCE4(I,X,N,MAX)
SUBROUTINE VSCE5(I,X,N,MAX)
SUBROUTINE VSCE7(I,X,N,MAX)
SUBROUTINE VSCE8(I,X,N,MAX)
SUBROUTINE VSCE9(I,X,N,MAX)
SUBROUTINE VSCE10(I,X,N,MAX)
SUBROUTINE VSCE12(I,X,N,MAX)
SUBROUTINE VSCE13(I,X,N,MAX)

265
DELETION OF BOUNDARY POINTS

1 NAME

DBOUND

2 PURPOSE

To modify each of the "0" pixels in an image to the most frequently occurring number in its 3 by 3 neighborhood. (This is useful, for example, in generating a level I GTM from a level II map and/or suppressing all the boundary points in a GTM and replacing them with reasonable class labels).

3 CALLING SEQUENCE

CALL DBOUND (NREC, NEL, NEL2, NTAPI, NTAP0, IX, IY)

where

NREC = Number of records in the input image;
NEL = Number of pixels per record;
NEL2 = NEL+2;
NTAPI, NTAP0 are the logical unit numbers of input and output sequential data sets;
IX, IY are work arrays to be dimensioned as indicated in the listings.

All the calling arguments except IX and IY are inputs.

4 INPUT-OUTPUT

Both the input and output sequential data sets have the same format. The number of records is NREC. The number of pixels per record is NEL and the number of bytes per pixel is 4. The records are in unformatted FORTRAN.

5 EXITS

No nonstandard exits

6 USAGE

The program is in FORTRAN IV and implemented on the IBM 360 using the H compiler. The program is in the users' library as a load module.
7 EXTERNAL INTERFACES

The subprograms required by this routine are:

SARN, a sequential access array read routine;
VMOV, a routine to move a vector in core;
MAJOR, a function giving the most frequently occurring number in a 3 by 3 neighborhood.

8 PERFORMANCE SPECIFICATIONS

8.1 Storage

This subroutine is 1036 bytes long. With the main program needed to call it for an image with NEL = 866, the external references and buffers, the storage required is 40K bytes.

8.2 Execution Time

Depends on image size. For a test case of 1624 by 866 pixels it took approximately 100 seconds.

8.3 I/O Load

None

8.4 Restrictions

None

9 METHOD

This program uses a circular buffer IX with pointers I1, I2, I3 indicating the previous, present and next records under consideration. Initially, I1, I2, I3 are set at 1, 2 and 3 respectively. After each record is processed, the pointers I1, I2, I3 are "rolled" upward. The processing of each record consists of checking the eight neighbors of each pixel whose value is zero. The function subprogram MAJOR is employed to determine the most frequent number occurring in the set of eight (If such a number is not unique, the first encountered number is taken).

Records 0 and NREC+1 are defined to be identical to records 1 and NREC respectively. Also, pixels 0 and NEL+1 in any record are defined to be the same as pixels 1 and NEL in the same record.
10 COMMENTS

None

11 LISTINGS

The listings of DBOUND and MAJOR are attached at the end of this section.

12 TESTS

This program was used in removing the extraneous boundary points after conversion of the level II GTM of the Mobile Bay region to a level I map. Line-printer plots of the maps before and after the application of DBOUND indicate satisfactory operation of this program.
PROGRAM TO MODIFY POINTS IN THE IMAGE ON NTAPI
WITH NUMBER 0 TO THE NUMBER OCCURRING MOST FREQUENTLY IN A 3 X 3 NEIGHBORHOOD.

CALL Service

CALL S4RV~NT~I-X, IFL4I

CALL VMOV(Ix(1,11), Ix(2,12))

DO 10 1=1,NFL

IF(1.LT.NREC) READ (1+1)ST RECORD INTO Ix(*,13).

CALL SARM(Ix(1,11),NFL)

CALL VMOV(Ix(1,11),NFL, Ix(2,12))

DO 20 J=1, NFL

IY(J)=IY(J+1,12)

20 IFIY(J,J+1)=MAJ0R(Ix,NFL,11,17,19, J+1)

WRITE(NTAPI,LY)

MODIFY 11,12,13 IN PREPARATION FOR THE NEXT RECORD.

FUNCTION MAJOR(Ix,NFL,11,12,13, J)

C

C FIND THE MOST FREQUENTLY OCCURRING NUMBER AMONG THE EIGHT NEIGHBORS
C OF Ix(J,J). NOTE THAT 1.LT.J,LT.NFL.

LABEL(1)=Ix(J-1,11)

NUMBER(1)=1

N=1

DO 30 I=2,8

20 IF(I.EQ.1)=1

30 IF(I.EQ.2)=2

IF(I.EQ.3)=3

KM=1

INC=1

IF(I.EQ.2) INC=2

DO 30 K=KM,3, INC

30 MAX=0

GO TO 40

LABEL(N)=Ix(J2+K,11)

NUMBER(N)=1

GO TO 10

GO TO 50

MAX=NUMBER(N)

GO TO 50

RETURN
THICKENING OF DIGITALLY DEFINED CURVES

1 NAME: THICK2

2 PURPOSE: To modify curve information in scan line intersection code so as to represent two-dimensionally thickened curves.

3 CALLING SEQUENCE:
CALL THICK2(NTAPI, NTAPO, IX, IY, IW, NREC, K)

4 INPUT-OUTPUT

The input to this program is a curve stored in SLIC format on unit NTAPI. NREC records are stored as J, (IX(L), L = 1, J) in FORTRAN binary format where J = number of coordinates in the record and IX is the array of coordinates.

The output of this program will consist of NREC records on unit NTAPO in the same format as of the input.

5 EXITS: No non-standard exits.
USAGE: The program exists in both IBM-7094 and IBM-360 versions and is written in FORTRAN IV. The decks are available with the author.

EXTERNAL INTERFACES:

System Routines: IBCOM#

Other Programs Called: SVSCI, SORT, ELIRPT, THICK1, VMOV

External Storage: None.

PERFORMANCE SPECIFICATIONS:

Storage: 518 hexadecimal bytes. Including the routines named in Section 7.2, the storage required is 14C2 hexadecimal bytes. (This does not include the storage needed for the work arrays which is data dependent.)

Execution Time: Depends largely on the number of points on the curve to be thickened and K. A test run on a file with 1753 records and approximately 12000 points on the curve took 6.8 minutes with K=2 on the IBM 360/65 system.

Restrictions: None.

METHOD:

The routine essentially consists of thickening in the horizontal direction by calls to THICK1 and taking unions of 2K+1 successive records to achieve thickening in the vertical direction.

The array IW is used to store the number of coordinates after thickening in the horizontal direction. IW(1) through IW(2K+1) are the numbers of coordinates in the (2K+1) successive records which are combined to form the current record of output.
Initially, the components of $IW$ are all set to 0. Next $(K+1)$ records of input are read, thickened in the horizontal direction and all the resulting boundary coordinates are stored in array $IX$. After the $I$'th record is thickened, the number of coordinates corresponding to it is stored in $IW(I+K)$.

Now, the following operations are performed for $I+1$ through $NREC$. The coordinates in $IX$ are moved into $IX$. $IX$ is sorted, repetitions are eliminated and an output record is written. Next, the array $IX$ is left-shifted by $IW(1)$ words since the first $IW(1)$ words (which correspond to the earliest horizontally thickened input record) are no longer required. Next, $IW$ is left-shifted by one word. Now, if there are any more input records left, the next input record is read into $IX$ and thickened. The thickened coordinate values are always loaded into the right end of $IX$ and the number of coordinates is stored in $IW(2K+1)$.

The routine $THICK1$ operates as follows. It assumes that the input array $IX$ is in ascending order. First, it sets $IX(1)$ through $IX(2K+1)$ to $IX(1)$-$K$ through $IX(1)+K$. This corresponds to thickening the first point in $IX$. After the $I$'th point is thickened, suppose $n$ values have been produced in $IX$. Then, the values corresponding to thickening the $(I+1)^{st}$ point are $\text{Max}(IX(I+1)-K, IX(n)+1)$ through $IX(I+1)+K$. When an $M$-vector $IX$ is thickened by $K$ elements using $THICK1$, the number $N$ of components in the output array $IX$ is bounded by

$$2K + M < N < (2K + 1) M.$$
TESTS: This program has been tested on synthetic data and on the boundary data for the TARCOG counties in North Alabama and found to work satisfactorily.

```fortran
SUBROUTINE THICKEN(IX,N,IY,N,K)
C TO THICKEN VECTOR IX BY K ELEMENTS AND PUT IN IY.

C DIMENSION IX(1:M),IY(1)
C ON IY(EXPECTED NUMBER OF VALUES AFTER THICKENING)
N=0
IF(M.EQ.0)RETURN
N=1
IY(1)=IX(1)-K
I=1
50 IF(IN=IX(I)+K
20 IF(IX(N).EQ.IFIN)GO TO 10
N=N+1
IY(N)=IY(N-1)+1
GO TO 20
10 IF(M.EQ.1)RETURN
40 I=I+1
IF(N+LT.IGEN(I)+K.OR.I.EQ.N)GO TO 30
GO TO 40
30 N=N+1
IY(N)=MAXO(IX(I)-K,I(Y(N-1)+1)
GO TO 50
END
```
SUBROUTINE THICK2(NTAPI,NTAPO,IX,IX,JY,IV,NREC,K)
DIMENSION IX(I),IV(I),IW(I)

C TO THICKEN BOUNDARY INFO ON TAPE NTAPI IN TWO DIMENSIONS BY K
C ELEMENTS ON EACH SIDE OF TEN BOUNDARY POINTS AND WRITE ON NTAPO.

K2=K+2
K21=K2+1
K1=K+1
NRECK1=NREC-K1
K1=K+1
CALL VSCL(WI,K21,0)

C READ K+1 RECORDS OF INPUT, THICKEN IN ONE DIMENSION AND INITIALIZE
C IV.

N=1
DO 10 I=1,K1
IR=I+K
READ(NTAPIJ,J,(IX(L),L=1,J)
IF(I,J,EQ.0)GO TO 19
CALL SORT(IX,J,J,Y,J,1,T,TT)
CALL ELIRPT(J,IX)
15 CONTINUE
CALL THICK2(IX,J,JY,J,IV(IR),IW(IR),K)
10 N=N+1W(IK)
DU-20 I=1,NREC
NI=N-1
IF(NI.EQ.0)GO TO 30
CALL VMOV(IV,N1,IX)
CALL SORT(IX,J,NI,J,M1,T,TT)
CALL ELIRPT(N1,IX)
30 WRITE(NTAPO1,J,(IX(L),L=1,M1)

C UPDATE ARRAY IY BY READING NEW RECORD OF INPUT.

IW=IW(I)+1
NI=N-IV
IF(IW11.NE.O.AND.NI.NE.O)CALL VMOVW(IY(IW11,NI,JY)
N=N-IV(I)
CALL VMOVW(M2),K2,IV

READ(NTAPIJ,J,(IX(L),L=1,J)
IF(I,J,EQ.0)GO TO 25
CALL SORT(IX,J,J,Y,J+1,T,TT)
CALL ELIRPT(J,IX)
25 CONTINUE
CALL THICK2(IX,J,JY,J,IV(K21),K)
N=N+TW(K21)
20 CONTINUE
97 FORMAT(IX,215,2514)
RETURN
END
REFERENCES


REFERENCES (Continued)


23. H. K. Ramapriyan, "Programs for Digital Manipulation of Curves such as Political Boundaries", CSC Memorandum to File, 5E3030-1-2-3, January 24, 1975.

Computer processing procedures and programs applied to Multispectral Scanner (MSS) data from Landsat are described. The output product produced is a Level I land use map in conformance with a Universal Transverse Mercator (UTM) projection. The region studied was a five-county area in North Alabama.