USE OF LEANING VANES IN A TWO-STAGE PAN
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USE OF LEANING VANES
IN A TWO STAGE FAN

by

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SUMMARY

The use of leaning vanes for tone noise reduction is examined in terms of their application in a typical two-stage high pressure ratio fan. In particular for stages designed with outlet guide vanes and zero swirl between stages, leaning the vanes of the first stage stator is examined, since increasing the number of vanes and the gap between stages do not provide the desired advantage. It is shown that noise reduction at higher harmonics of blade passing frequency can be obtained by leaning the vanes. The work reported here is carried out under Contract No. NAS 2-8680 with the Large Scale Aerodynamics Branch of NASA Ames Research Center, Moffett Field, California.
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<tr>
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<td>vane section drag coefficient</td>
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<td>$R$</td>
<td>distance of field point from the rotor center</td>
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<td>$U_a$</td>
<td>axial velocity</td>
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<td>$U_r$</td>
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<td>inclination of wake center line to radial direction</td>
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Subscripts

\( a \)  blade loading harmonic index  \\
\( h \)  at the hub  \\
\( n \)  blade passing frequency harmonic index  \\
\( t \)  at the tip

Superscripts

\( ^\)  amplitude  \\
\( * \)  complex conjugate
I. INTRODUCTION

Noise reduction at blade passing frequency in single stage fans by the proper choices of vane-blade number ratio and stator-rotor gap has been well documented in the literature. For a single stage, an OGV (outlet guide vanes) fan is seen to be less noisy than an IGV (inlet guide vanes) fan due to the fact that the fluctuative forces on the vane elements in the former are generally of lower magnitude than those on the blade elements in latter type. For certain V/STOL airplane applications, such as an augmentor wing configuration, the fan pressure requirements dictate a two-stage fan. One can utilize, to some extent, the techniques developed in acoustic design of single stage fans for the two stage fans. However, the configurations are obviously not identical and considerably less experience has been obtained on them.

A representative two-stage fan designed for high pressure rise is described in reference (1). Each stage is of an OGV type, with energy input equally distributed between the stages, and with the flow leaving the first stage axially. The wake-vane interaction effects in each stage can be alleviated by choosing the proper blade-vane number ratio and increasing the rotor-stator gap as mentioned above. Due to the swirl in the flow downstream of the rotor, the wakes from the blades become non-radial as they approach the outlet guide vanes. The resulting obliquity of wake-vane intersections can be increased by increasing the rotor-stator gap. The velocity
deficit in the wake region also becomes reduced by increasing the gap, but the obliquity of wake intersections plays a more important role in noise reduction arising from the wake-vane interactions.

The impingement of wakes from the first stage stator on the following rotor is similar to the situation in an IGV fan, and can be a major source of noise in a two-stage fan. The flow being axial between the stages, increasing the gap in between does not impart the advantageous non-radial orientation to the wakes impinging on the second rotor. Consequently, the investigations in this report deal with only the interaction of the first stage stator wakes with the second stage rotor blades.

For reasons mentioned above, the only means of providing oblique wake interactions at the second rotor is by leaning the outlet guide vanes of the first stage, and thereby reduce the noise generated by wake impingement on the second stage rotor blades. The influence of such leaning vane configurations was examined in reference (2) by placing a stator of zero-turning vanes ahead of a 12 inch diameter low speed rotor with 15 blades. The analyses of the oblique wake-blade interactions is extended in this report to the conditions occurring over the second stage rotor of the representative two-stage high pressure ratio fan. For the sake of comparison, the interaction of the rotor blades with the turbulent velocity fluctuations typically occurring in the region are also examined here.

In the following section are described flow conditions and other pertinent parameters that would be typical for a second stage rotor. The next two sections deal with the methods employed in estimating the noise generated by the fluctuations in blade loading of the second stage rotor. The results of the acoustic computations and their discussion are given in the fifth section. In the final section are given conclusions based on our analytical results.
In the present study we concern ourselves only with the noise generated by the second stage rotor, in a typical two-stage fan, and the relative effects of the number of rotor blades and the number of upstream stator vanes. A tip radius of 2 ft (0.601 m) and a hub-tip ratio of 0.4 are considered as typical dimensions for the rotor. A forty bladed rotor with the blade solidity linearly varying from 2.25 at the hub to 1.27 at the tip is taken as a reference configuration. Blade stagger angle varying from 19.5° at the hub up to 56.5° at the tip is assumed in the estimation of loading on the blade elements. At a design tip speed of 1220 ft/sec (371.8 m/sec) corresponding to 97 rps, the mean axial flow through the rotor is taken as 750 ft/sec (228.6 m/sec). The rotor parameters, flow conditions, and their radial variations are listed in reference (1).

The configuration of the stator upstream of the rotor, referred above, is of prime importance since the rotor blade elements suffer fluctuations in loading due to their crossing the wakes from the upstream vanes. Since the flow upstream of the rotor is assumed to contain zero swirl, the important parameters are the number of vanes, chord length, drag coefficient, and rotor-stator gap. It is suggested in reference (3) that choosing blade number \( B \) and vane number \( V \) satisfying the relation

\[
V = 2B + 6
\]  

(1)
would cut off the propagation of the blade passing frequency tone arising from wake-blade interaction. The tone levels at higher harmonics depend upon the number of stator vanes, rotor blades and the rotor speed. Consequently, we included blade numbers 30, 40, and 50 in our study. The blade-vane number combinations studied are listed below.

| Blade number B | 30 | 40 | 50 |
| Vane number V  | 66 | 86 | 106 |
| Blade passing frequency at design speed - KHz | 2.91 | 3.88 | 4.85 |

The solidity of the vanes is considered to vary linearly from 2.25 at the hub to 1.55 at the tip. As the vane number is changed, their chord length is proportionately changed to keep the vane solidity the same. Defining the stator-rotor gap as the axial distance between the vane trailing edge and blade leading edge, the ratio of gap to vane chord is held at unity in each of the above configurations. Furthermore, we examined the effect of leaning the vanes in each configuration. The vane lean angles are limited to $\psi_t = 15^\circ$ to avoid local blockage of flow in the regions where the vanes meet the hub.

The flow conditions met by the rotor can be described in terms of the steady mean flow through the rotor annulus and turbulent velocity fluctuations. The mean flow consists of axial flow $U_\alpha$ with deficit velocity profiles present in the regions of the wakes shed from the upstream stator vanes. Based on the two-stage fan design discussed in reference (1) typical radial variation of $U_\alpha$ met at the second rotor at the design speed is assumed in our present studies. Apart from
the blade number and the vane lean angle, the rotor speed itself has a strong influence in our acoustic computations. Consequently, we considered rotor speeds of 60% and 75% of the design value. At higher rotor speeds, the relative velocity of the blades becomes supersonic and the approximations employed in our study become invalid. The radial variation of \( U_a \) and relative velocity \( U_r \) at 60% design speed are shown in figure (1). The blade stagger angles at different radii are also shown in the figure.

The flow conditions within the wakes are approximated in the following manner using results given in reference (4) for isolated airfoils. The deficit velocity profile in the wake is assumed as

\[
\frac{U_c}{U_a} = \cos^2 \left( \frac{\pi}{2} \frac{Y}{y} \right) \text{ with } |y| \leq Y
\]

The velocity deficit \( U_c \), at wake center, and the wake half-width \( Y \) are given by

\[
\frac{U_c}{U_a} = -2.42 C_D^{1/2} \{0.6 + \frac{2\xi}{C_V}\}^{-1}
\]

\[
Y = 0.68 C_D^{1/2} \{0.15 + \frac{\xi}{C_V}\}^{1/2} C_V
\]

where \( C_D \) = vane section drag coefficient and

\( \xi \) = axial distance from the vane trailing edge

\( U_a \) = mean flow over the stator vanes, considered here to be same as that at the rotor.
In evaluating loading on rotor blade element, we consider the conditions at a cross sectional plane passing through the rotor center. However, the vane chord $c_v$ and the distance $\xi$ depend upon the number $V$ of the vanes in the upstream stator. Also the mean axial velocity $U_a$ depends upon the radial location. For example, at the pitch radial location in forty bladed rotor with 86 vanes, we obtained $u_c = 34.5$ ft/sec ($10.51$ m/sec) and $\gamma = 0.027$ ft ($0.00828$ m) by using vane section drag coefficient $C_D = 0.02$. The variation of these parameters are included in our acoustic computations.

The turbulent velocity fluctuations referred above can be described in terms of their spectral density distribution and spatial coherent distances. We note that there exist two distinct types of turbulence: one due to random fluctuation in the inflow to the two stage fan, and the other due to the viscous interactions in the wakes shed from the upstream vanes. The presence of the wakes from the blades of the first rotor will be ignored here, since they are not "chopped" by the second rotor. For the turbulence in the inflow, we assumed a root mean square value of 3% of the axial velocity with spectral density distribution as shown in figure (2). These are based on typical engine inlet measurements taken during static tests at Ames Research Center. In the absence of spatial correlation measurements, we used $l_r = l_\theta = 3$ inches in evaluating noise from the inflow turbulence effect. It is assumed that the upstream stator vanes, whether leaning or not, do not alter the above postulated turbulence intensity and its spectral distribution.
Whereas the inflow turbulence is homogenous over the entire annulus area, the wake turbulence exists only in the regions covered by the wakes. Since there are not sufficient measurements available regarding conditions within the wakes, we employed "frozen-convected-eddy" hypothesis to characterize the wake turbulence. An eddy size equal to half-width of wake convected with velocity $U_a$ leads to a spectral density distribution shown in figure(3), typically computed at the pitch radial location in forty-bladed rotor. Regarding the wake turbulence intensity, we assumed an r.m.s. value of 20% of the velocity deficit at wake center.

In describing the wakes from leaning vanes, it is assumed that only the locations of the wake regions depend upon the leaning angle, but the flow conditions within the wake remain the same for all lean angles.
III. TONE NOISE GENERATED BY ROTOR BLADES INTERACTING WITH WAKE VELOCITY DEFICIT:

Radiated noise from the sources on the blades of the second stage rotor depends upon the nature of the acoustic sources, as well as on the sound transmission characteristics of the fan duct and the adjacent blade and vane rows. Since the latter is not definitely known at the present stage of preliminary investigations, it is assumed that computations based on direct radiation from the sources serve as close approximation.

The interaction of the steady-state non-uniform flow, due to the presence of \( V \) number of upstream vanes, with the \( B \) number of rotor blades rotating at \( N \) rps gives rise to discrete tones at blade passing frequency \( BN \) and higher harmonics. In describing the non-radial wakes from the upstream leaning vanes, we note that the wake orientation is the same as that of the vanes due to zero swirl in the region. Let \( \psi_\tau \) denote the circumferential location of the vane at tip radius measured, positive in the direction of rotation, from the meridional plane passing through its hub location. Consequently, the location of the wake at any radius \( r \) at the rotor plane, as sketched in figure(4), is given by

\[
\psi_w = \frac{r - r_h}{r_t - r_h} \psi_\tau
\]
The angle $\beta_w$ at which the wake center line intersects the radial blade is given by

$$\tan\beta_w = r \frac{d\psi_w}{dr} \quad (6)$$

The values of $\beta_w$ resulting from the above two equations for various vane lean angles up to $\psi_t = 15^\circ$ are shown in figure(5).

Let us consider a blade element of chord $c_r$ and span length $dr$ at radius $r$. In travelling around the circumference $2\pi r$ and crossing $V$ number of wakes, the blade element experiences periodic fluctuations of incidence angle, which gives rise to harmonic loading on the blade element. The wake thickness and the mean velocity profile within the wake of a vane can be evaluated using equations (2), (3), and (4) given in the preceding section.

For the evaluation of the incidence angle, the $V$ number of wakes are assumed to be identical and equally spaced around the circumference. Replacing the cosine profile given in equation (2) by Gaussian error curve, and following the method described in reference(5), we obtain the following expression for the fluctuations of incidence angle on a blade element.

$$\delta\alpha = \sum_{a=1}^{\infty} (\delta\hat{a}) \cdot \exp\{-2\pi a VN\left(t - \frac{r}{U}\right)\} \quad (7)$$
with the amplitude of the $a$th harmonic given by

$$(\delta \alpha)_{\alpha} = \frac{u_c}{U_r} \sin \lambda_r \frac{2\sqrt{\pi}}{\kappa} \exp\{-\frac{\pi a}{k}\}$$

where $\kappa = \frac{2\pi r}{V} \frac{\sqrt{\pi}}{\sqrt{Y}}$

$x_r = \text{distance along blade chord measured from midchord}$

and $\lambda_r = \text{rotor blade stagger angle}.$

In the above equation, we note that the component normal to the relative velocity $U_r$ is approximated by $u_c \sin \lambda_r$ taken normal to blade chord. Consistent with the above notation of $x$, the value of $\xi$ in equation (2), (3) and (4) is the distance to rotor blade midchord.

For the case of non-radial wakes interacting with radial blades, the analysis given by Filotas in reference (6) can be employed and one can represent the $a$th component of equation (7) by the following expression

$$ (\delta \alpha)_{\alpha} = (\delta \alpha)_{\alpha} \exp\{-i2\pi a V N(t - \frac{x_r}{U_r} - \frac{Y}{U_r \tan \beta}) \} \quad (8) $$

where the angle $\beta$ is related to the orientation of the wakes. The spanwise velocity of the above sinusoidal gust must be the same as the velocity with which the wake intersection travels along
the blade radius. Consequently the relation

\[ U_r \tan \beta = \frac{2\pi N}{\tan \beta_w} \tag{9} \]

determines the value of \( \beta \) to be used in the preceding equation (8).

The \( \alpha \)th harmonic of the fluctuating lift-force on the blade element due to the action of incidence angle fluctuation given by equation (8) is derived in reference (7) as

\[ (\Delta L)_\alpha = \frac{1}{2} \rho U_r^2 \cdot c_r \cdot dr \cdot (\delta \alpha)_\alpha \cdot 2\pi \cdot T_\alpha \tag{10} \]

where \[ T_\alpha = \exp\left\{-i \phi_\alpha \right\} \left\{1 + \pi K \left(1 + \sin^2 \beta + K \cos \beta\right)\right\}^{1/2} \]

with \[ \phi_\alpha = K \left(\sin \beta - \frac{\pi}{1 + 2\pi K (1 + \frac{1}{2} \cos \beta)} \right) \]

\[ K = \frac{2\pi N}{\sin \beta} \cdot \frac{c_r}{2U_r} \]

and \( \beta = \) the angle defined in equation (9).

In the above evaluation the lift on the blade element is represented as a concentrated force at its mid-chord, ignoring the nature of its chord-wise distribution. Also, the compressibility effect and the interference of the neighboring blades are ignored in the above representation. However, for the present preliminary computations such approximations are considered valid.
The fluctuating forces on the blade elements radiate noise as acoustic dipoles. Integrating the radiation from all the elements on the B number of blades rotating at circular frequency \( \Omega = 2\pi N \), and summing over all the blade loading harmonics, we find that farfield sound is present only at blade passing frequency \( BN \) and its higher harmonics.

It is shown in reference (7) that the acoustic pressure at the farfield point \((R, \psi)\) is given by

\[
\frac{p_n}{4\pi R} = \exp\{-ika_0 (t-R/a_0)\} \int_{r_h}^{r_t} \sum_{m=-\infty}^{\infty} (-i)^{m+1} \left\{\frac{f_m}{2} J_m(kr \sin \psi) \times \Delta L \right. \\
\left. \times \exp\{i(aV \psi - \phi)\}\right\} \, dr
\]

where

\[
f_m = \sin \lambda_r \cos \psi \frac{m a_0}{\sqrt{r}} \cos \lambda_r
\]

\[
k = nB_0 / a_0
\]

\[
n = \text{harmonic index}
\]

and \( m = nB \cdot aV \).

The pressure wave at the blade passing frequency is obtained by setting \( n = 1 \) in the above equation. For the first and higher harmonics of the blade passing frequency, we get \( n = 2 \) and respective higher integer values. The subscript \( a \) refers to the blade loading harmonic whose frequency is \( aV \).
Integrating the right hand side of equation (11) from \( r = r_h \) to \( r_t \) and then taking the root mean square of the sound pressure, one obtains the contribution to the \( n \)th harmonic of the fan noise from the wake-blade interaction and the associated periodic dipole sources. The tone noise thus computed can exhibit strong directivity, depending upon the vane-blade number ratio. Consequently, we directed our attention to the sound pressure radiated in the forward direction by evaluating

\[
(Sound \; power)_{nBN} = \int_{0}^{\pi/2} \left\{ \frac{p_n}{\rho_0 c_0} \right\} \pi R^2 \sin \varphi \; d\varphi
\]  

(12)

where \( p_n^* \) is the complex conjugate of \( p_n \) obtained from equation (11) at field point \((R, \varphi)\), and the subscript \( nBN \) on the left hand side indicates the frequency at which the sound power is evaluated.

The results of computations using above relations for various blade numbers in second stage rotor and various number of leaning vanes in the preceding stator are given in Section 5.
Turbulent velocity fluctuations in the flow upstream of the rotor give rise to random lift forces on the blades related to the random fluctuations in the velocity component normal to the blades. If these random fluctuations are mostly at low frequency, there can exist certain amount of correlation in the fluctuating forces experienced by several adjacent blades of the rotor. Such is the case for turbulence in the inflow to the fan, and the consequent discrete tone noise caused by the random blade loading is examined in reference(8). On the other hand, most of the energy of the random velocity fluctuations can be in the high frequency range as in the case of turbulence in the wakes from upstream stator vanes. The loading on any one blade will then be uncorrelated with that on the other blades, and the resulting broad band noise is examined in reference(9). It is to be noted that the turbulent conditions in the inflow occur over the entire fan annulus, whereas the wake turbulence is limited to regions covered by the wakes. The pertinent relations for evaluating the rotor noise from the above described sources are given in the following subsections.

(1) Interaction of Rotor Blades with Inflow Turbulence

The blade stagger angles in a high-speed rotor are close to 90°, and the fluctuations in the velocity component normal to the blade chord, which govern the loading on blade elements, can be approximated by those of the axial velocity component. Such an approximation eliminates the need for measurements with hot-wire probe direction varying with its radial location in
the inlet annulus. As discussed in reference (10) it is found appropriate to employ the measured spectral intensity of turbulent velocity fluctuations in our acoustic computations instead of "frozen-convected eddy" concept. In the absence of detailed spatial correlation measurements one can employ reasonable estimates for $\lambda_\theta$ and $\lambda_r$, the velocity correlation lengths along circumferential and radial directions respectively. Considering the turbulent velocity fluctuations to be of the same character in the entire rotor annulus, a method is developed in reference (8) for evaluating the random blade loading and the following expression is derived for the spectral intensity of the far field sound pressure at radian frequency $\nu$.

$$\Phi(\nu) = \frac{1}{2} \int_{r_h}^{r_t} \left( \pi \rho_0 \frac{\bar{U} c_r^2}{2} \cdot \frac{v}{4 \pi R_0} \cdot \nu^2 \cdot \frac{2 \pi r}{\nu} \right) \cdot \frac{2 \pi r}{\nu_0}$$

$$\cdot \sum_m \sum_g A_g^2 \cdot \hat{f}_\lambda^2 \cdot \hat{f}_r^2 \cdot J_m \left( \frac{\nu}{a_0} \sin \psi \right) \cdot \hat{\phi}_u(\omega) \cdot |S(\gamma^{'})|^2 \cdot dr$$

(13)

where $\hat{\phi}_u(\omega)$ is the spectral intensity of velocity fluctuations normal to blade chord, assumed here to be same as that in the axial direction

$$\hat{f}_\lambda = \sin \lambda_r \cos \psi - (m a_0 / \nu r) \cos \lambda_r$$

$$S(\gamma^{'}) = \text{Sear's lift response transfer function evaluated at reduced circular frequency } \gamma^{'}$$

and

$$A_g^2 = \begin{cases} \frac{1}{g \pi} \sin(g \theta_0 / 2r) \frac{1}{2} \cos \frac{g \theta_0 / 2r}{2r} & \text{for } |g \theta_0 / 2r| < \pi / 2, \\
\{(g \theta_0 / 2r)(1 + g \theta_0 / 2r)^2\}^{-\frac{1}{2}} & \text{otherwise} \end{cases}$$
The circular frequencies $v, \omega$, and $\gamma'$ are related to the rotor circular frequency $\Omega = 2\pi N$ by

$$v = \omega + nB\Omega \quad \gamma' = (\omega - g\Omega) (c_r / 2U_r).$$

In the summation on the right hand side, the integer indices $g, m$ and $n$ are related by

$$m = nB + g,$$

with $g$ and $m$ taking on all values $-l, 0, 1, ...$

After evaluating $\phi(v)$ from above equation (13) at various azimuthal angles $\Psi$, the acoustic power radiated in the forward direction can be computed as

$$\text{(Sound Power)}_v = \frac{\rho_0 a_0}{2\pi R^2} \cdot \sin \Psi \cdot d\Psi \quad (14),$$

where the subscript $v$ indicates the radian frequency at which the spectral density of the sound power is computed.

The computational results obtained by using above equations (13) and (14) with the parameters representative of the second stage rotor are given in a later section.
The load fluctuations on the second stage rotor blades and the noise generated thereof also depend upon the turbulence within the wakes shed from the upstream stator vanes. However, the length scale being small, the noise generated will be in broad band as discussed in reference(9).

The farfield noise radiated from the rotor blades in this case can be evaluated in the same manner as in the preceding subsection with the following modifications. The turbulent velocity fluctuations exist only in the wake regions. The root mean square value of the axial component of these fluctuations, considered constant over the wake width, is assumed to 20% of the deficit of mean velocity at wake center. Furthermore, the correlation lengths for this component along the axial, radial and circumferential directions are assumed to be all equal to $Y$, the wake half-width. As in the previous subsection, the turbulent fluctuations of the axial velocity component can be used to estimate the blade loading. However, for the spectral intensity $\Phi_u(\omega)$, it is convenient to use the "Frozen-convected" hypothesis and define

$$\Phi_u(\omega) = \frac{U_z}{\pi U_a} \frac{L_z}{1 + (L_z \omega / U_a)^2}^{-1} \quad \text{(45)}$$

with $\langle \bar{u}^2 \rangle^{1/2} = 0.2 U_c$

and $L_z = Y$. 

(2) Interaction of Rotor Blades with Wake Turbulence
The analysis of the effect of inflow turbulence presented in reference 8 shows that the origin of the term $2\pi r/\lambda_0$ on the right hand side of equation (13) is from the number of coherent regions of length $\lambda_0$ within the circumference $2\pi r$. In considering the turbulence in the wakes from V upstream vanes, the number of such coherent regions is $2V$, since the coherent length in the circumferential direction is assumed to be equal to wake half-width.

Incorporating the above described modifications into equation (13), the following expression is obtained for the spectral intensity of farfield sound pressure arising from interaction of rotor blades with wake turbulence.

$$\Phi(\nu) = \frac{1}{2} \int_{r_h}^{r_t} \left( \frac{\nu}{\nu_0} \right)^4 \cdot \frac{B^2}{4\pi R_0} \cdot \nu \cdot 2V \cdot \frac{1}{2} \sum_{m} \sum_{g} A_g^2 \cdot f_{\lambda}^2 \cdot J_m^2 \left( \frac{\nu}{\nu_0} r \sin \psi \right) \cdot S_{\nu}(\omega) \cdot S(\nu') \cdot dr$$

(16)

For $A_g^2$ in the above equation, we replace $\lambda_0$ by $Y$ in the definition previously given in equation (13). The wake half-width $Y$ and velocity deficit $u_c$ for use on the right hand side of equation (16) can be evaluated at the blade mid-chord at each radius $r$ using equations (2) and (3).

From equation (14) with the results of equation (16) substituted therein, the sound power radiated in the forward direction can be evaluated. The results of such computations for the conditions occurring at the second stage rotor are discussed in the next section.
Using the theoretical methods described in the preceding sections, we computed farfield sound pressure levels and sound power levels due to the noise sources on the second-stage rotor for the fan configuration listed in Section II. Considering the forty bladed rotor as a representative case, the loading on the blade elements and the noise radiated thereof are examined in detail for this configuration only.

The deficit in the mean velocity within the wakes from the vanes as computed from equations (2), (3) and (4) give rise to periodic lift-forces on the rotor blades. The amplitudes and phase angles of the blade loading harmonics computed at various radii of the forty bladed rotor using equation (10) with various lean angles for the 86 upstream vanes are presented in figures (6) and (7). The fundamental, first and second loading harmonics are obtained by setting $a = 1, 2, \text{ and } 3$ respectively in equation (10). The phase angles shown in figure (7) are relative to their values at the hub since only the radial variation is of consequence in the use of equation (11). Substituting the blade loading harmonics into equation (11), we computed the farfield acoustic pressure and the sound power radiated at the blade passing frequency harmonics is calculated from equation (12).

Apart from the periodic blade loading discussed above, the turbulence in the flow through the rotor generates random forces on the blade elements. However, the nature of the
farfield noise spectrum depends upon the characteristics of the turbulent velocity fluctuations. The spectral density distribution shown in figure (2), for the turbulence in the inflow to the fan, upon substitution into equation (13), leads to prominent peaks in the farfield noise spectrum at the blade passing frequency harmonics. On the other hand, the wake turbulence characterized by spectral distribution shown in figure (3), when substituted into equation (16), leads to a broad band spectrum for the farfield noise. The spectra of the sound pressure from these two sources at farfield points on axis and on the side line are respectively shown in figures (8) and (9). We observe that the sound pressure levels of the discrete tones are considerably higher than the broad band. We also note that the noise generated by the rotor blades interacting with wake turbulence can be neglected when compared to that from interactions with inflow turbulence. The tone levels arising from interaction of blades with velocity deficit in the wakes are also indicated in these figure (9). Since the vane number is not an integer multiple of the blade number, tone noise levels from this source do not appear on figure (8). Furthermore, the vane-blade number ratio being 86 to 40, there is no noise at blade passing frequency along the side line. However, considerable noise appears at the higher harmonics as indicated on figure (9). The amounts of noise reduction one obtains on the side line by leaning the vanes are also shown in the figure. From the sound pressure levels at various azimuthal angles, we computed the sound power level and the results are shown in figure (10). We note that the velocity defect in the wakes is the dominant factor in estimating the sound power levels at the blade passing frequency harmonics.

Sound pressure levels on the side line by themselves are not indicative of sound power levels as can be seen from figures (9) and (10). Consequently, we examined the directivity of sound pressure
level at blade passing frequency harmonics along a 100 ft (30.48 m) arc computed separately from the various sources discussed above and the results are shown in figure(11). As mentioned earlier, only the noise due to interaction with turbulence remains at the fundamental blade passing frequency.

Since the directivity of sound pressure is significantly affected by the vane lean angle, as seen from figures (11.b) and (11.c) we considered the reductions in radiated sound power obtained at various lean angles of the upstream vanes. The results of the computations using equations (12) and (14) at the first and second harmonics of the blade passing frequency are shown in figure(12). The noise due to rotor interaction with turbulence is not affected by vane lean angle and hence is shown as a floor level in the figure. We note that it does not help to employ vane lean to reduce the noise to levels lower than that arising from turbulence interactions. The results of similar computations on the 30 bladed and 50 bladed rotors are shown in figures(13) and (14) respectively.

For the three rotors with the different blade numbers and corresponding vane numbers, we carried out the acoustic computations at 75% design speed also. The effect of vane lean angle at this rotor speed on the sound power radiated at blade passing frequency harmonics for each case considered is shown in figures (15), (16), and (17). It appears that rotor speed and the blade number have a strong influence on the advantage afforded by vane lean angle, which is also evident from the various factors in equation (11).
VI CONCLUSIONS

From the analyses and computational results presented in the preceding sections, the following conclusions can be made regarding the noise generated by the second rotor of a typical two-stage high pressure ratio fan operating at subsonic flow conditions.

1. Even though the farfield noise at the blade passing frequency can be cut off by using appropriate number of vanes in the upstream stator, considerable noise levels will be present at higher harmonics of the blade passing frequency.

2. For design condition of zero swirl downstream of first stage, increasing the gap between stages does not impart non-radial orientation to the wakes as they impinge on the rotor blades. Since oblique intersections with the wakes cause lower fluctuating loads on the rotor blades, leaning the vanes of the upstream stator will provide noise reduction at higher harmonics of blade passing frequency.

3. In addition to the vane-blade number ratio, the rotor blade number and its speed play an implicit role in determining the effect of vane lean angle. Consequently, leaning vanes may not lead to noise reduction at all values of parameters involved. It is necessary to carry out the acoustic computations in each case to determine the advantage of vane lean.
4. The effect of inflow turbulence on the rotor blades gives rise to discrete tones at blade passing frequency and its harmonics. When the use of leaning vanes reduces the effect of the velocity deficit in the wakes, the interactions of the blades with inflow turbulence will become important in evaluating noise from the rotor.

5. The turbulence within the wake regions interacting with the rotor blades generates broad band noise, which is considerably lower than that generated by the inflow turbulence.
REFERENCES


Fig. 1. Blade Stagger Angle and Flow Conditions.
(sixty percent speed)
Fig. 2. Spectrum of Turbulent Velocity Fluctuations Assumed for the Inflow to the Fan.
Fig. 3. Spectrum of Turbulent Velocity Fluctuations Assumed for the Wake Regions at the Rotor Plane.
(typically shown for pitch radial location with \( V = 86 \))
Fig. 4. Sketch of Wake-Blade Orientation.

(angles $\theta$ and $\psi$ measured positive in the direction of rotation).
Fig. 5. Effect of Vane Lean on Inclination of Wake to Blade Center Line.
Fig. 6. The Effect of Vane Lean on the Amplitude of Blade Loading in the Forty Bladed Rotor.
(sixty percent speed)
(b) first blade loading

Fig. 6. Continued.
Fig. 6. Continued

(c) second harmonic of blade loading

\[ \psi_t = 0 \]
(a) fundamental blade loading harmonic

Fig. 7. Effect of Vane Lean on the Phase Angle of the Blade Loading Harmonic in the Forty Bladed Rotor.
(sixty percent speed)
(b) First harmonic of blade loading.

Fig. 7. Continued.
(c) second harmonic of blade loading

Fig. 7. Continued.
Fig. 8. Spectral Density of Sound Pressure at 100 ft (30.48 m) Distance on Axis From the 40 Bladed Rotor. (sixty percent speed)
Fig. 9. Spectral Density of Sound Pressure at 100 ft (30.48 m) Distance on Side Line From the 40' Bladed Rotor (sixty percent speed)
Fig. 10. Spectral Density of Sound Power from the Forty Bladed Rotor.
(sixty percent speed)
interaction with inflow turbulence

interaction with wake turbulence

Fig. 11. Directivity of Sound Pressure Levels from the Forty Bladed Rotor at 100 ft (30.48 m) from Rotor.

(sixty percent speed)
interaction with wake velocity deficit ($\psi_0 = 0$)

interaction with wake velocity deficit ($\psi_t = 10$)

interaction with inflow turbulence

interaction with wake turbulence

(b) first harmonic of bpf

Fig. 11. Continued.
interaction with wake velocity deficit
\( (\psi_t = 0) \)

interaction with wake velocity deficit
\( (\psi_t = 10) \)

interaction with inflow turbulence

interaction with wake turbulence

SPL in dB ref 0.0002 \# bar

(c) second harmonic of bpf

Fig. 11. Continued.
Fig. 12. Effect of Vane Lean on Sound Power in the Forty Bladed Rotor (sixty percent speed)
Fig. 13. Effect of Vane Lean on Sound Power in the Thirty Bladed Rotor.

(sixty percent speed)
fig. 14. effect of vane lean on sound power in the fifty bladed rotor.
(sixty percent speed)
interaction with wake velocity deficit
interaction with inflow turbulence

(a) first harmonic of bpf
(b) second harmonic of bpf

Fig. 15. Effect of Vane Lean on Sound Power from the Forty Bladed Rotor.
(seventy-five percent speed)
interaction with wake velocity deficit

interaction with inflow turbulence

(a) first harmonic of bpf

(b) second harmonic of bpf

Fig. 16. Effect of Vane Lean on Sound Power from the Thirty Bladed Rotor. (seventy-five percent speed)
interaction with wake velocity deficit

interaction with inflow turbulence

(a) first harmonic of bpf

(b) second harmonic of bpf

Fig. 17. Effect of Vane Lean on Sound Power from the Fifty Bladed Rotor.

(seventy-five percent speed)