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Simple Torsion Test for Shear Moduli Determination of Orthotropic Composites

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With the increasing concern over the environmental degradation of composites, and the concomitant need to develop accelerated test methods for obtaining moisture-induced property changes in these materials, we have re-examined the torsion test. The torsion test would appear to have definite potential as a measure of property degradation caused by environmental effects, because, in the torsional analysis, material properties at and near the surface (where moisture degradation must initially occur) are given more weight than material properties in the bulk specimen. Also, for interfacial bonding between fibers and matrix of composites and for matrix failure itself, as in compressive or off-axis loads, shear properties can be critical. Degradation of stiffness or in-plane shear modulus of graphite-epoxy composites under repeated stress applications has previously been shown to be sensitive to environmental effects (1).

A good shear test method is one that is easy to run, requires no complex specimen preparation, gives reproducible results, and provides all shear properties. Chiao et al. have evaluated a number of selected shear test methods (2). They have recommended that the 45° angle-ply laminate tensile shear test be used for the measurement of the shear properties of fiber composites. However, this method, like all other test methods for determining shear properties, does not provide the out-of-plane shear modulus (3). In actual applications, out-of-plane shear stresses are, of course, going to exist, and out-of-plane material properties, if available, would be of value to designers. A method has been suggested by one of the authors for the determination of two shear moduli of a homogeneous, orthotropic material in the shape of a rectangular parallelepiped (4). From torsion tests performed on specimens of the same material having a minimum of two different cross sections (flat sheet of different widths), the effective in-plane (G13) and out-of-plane (G23) moduli can be determined. The objective of our overall composite program is to determine environmental effects on mechanical properties. The objective of this phase of the program is to evaluate the torsion test of flat specimens of rectangular cross section as a useful means of examining environment-induced mechanical property changes. Shear moduli

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G_{13} and G_{23} have been experimentally determined for two different composites (AS/3501 and T300/5209) of uniaxial and cross-ply fiber orientations. Presented here are the results of that investigation.

Experimental Procedures

The two different composite materials used in this investigation were Hercules AS fiber in 3501 resin containing 63 ± 2% fiber volume, and Union Carbide T300 fiber in Narmco 5209 resin containing 65 ± 2% fiber volume. Material layups studied were (0)_16 for both composite systems plus (0°/±45°/0°)_2 for AS/3501 and (+45°/-45°/+45°)_4 for the T300/5209. Prepreg properties of the AS/3501, as given by the producer, were UTS of 3.1 GPa, modulus of 222 GPa, and density of 1.803 g/cc, and the prepreg properties of the T300/5209 were UTS of 2.5 GPa, modulus of 228 GPa, and density of 1.589 g/cc. Void content was negligible (0.5%) for both materials. The specimen stock was prepared by Lockheed Missiles and Space Co., Sunnyvale, California, in sheets of nominal 2-mm thickness. The T300/5209 uniaxial specimens were 2.25 mm ± 0.8%, the cross-ply 2.35 mm ± 0.8%. The AS/3501 material showed greater thickness variation; namely, 2.00 mm ± 7.3% for uniaxial specimens and 2.45 ± 3.4% for cross-ply specimens. The method of preparation is described elsewhere (5). The as-received sheets were cut using a diamond saw into specimens 100 mm in length and widths of 6.3, 9.6, 12.5, and 15.8 mm.

Torsion tests were run under controlled deflection (angle of twist) using an electro-hydraulic servo-controlled test system. In this system, a torsional force is applied at one end of the specimen, and the torque developed is measured by the output of a strain-gaged torque cell at the other end. The output of both the torque cell and the potentiometer measuring the angle of twist was fed to an XY recorder, and the torque versus torsion or twist graph was automatically obtained. The gage length between the grips was fixed and for all tests was 53.1 mm.

The modulus of rigidity of rectangular bars of isotropic materials can be calculated from a formula derived in the theory of elasticity (6). When applying this formula for torsion of a bar or plate, one should measure the deformation of the specimen in the center portion to avoid effects of load concentrations at the ends (6). Therefore, in addition to the potentiometer to measure the angle of twist, small mirrors (strips of a microscope glass slide approximately 2 x 4 mm, coated with vapor-deposited aluminum) were glued to the specimens at one-third intervals of the length between the specimen grips. Using a low-power laser as a high-intensity light beam easily visible in the undarkened laboratory, we determined the angle of twist by measuring the tangent of the angle θ between the incident ray and the beam reflected from the mirror strip on the surface of the specimen as shown schematically in Figure 1. The laser was positioned on the vertical post of a cathetometer.

---

Fig. 1 - Experimental setup for measuring torsion angle θ (schematic).
and was moved up and down to impinge first upon one mirror strip and then upon
the other. The angle of twist over the 17.7-mm center portion of the specimen
was the difference between the angle $\theta$ as measured with the light beam
reflected from the mirror strips at the two different positions. The shear
modulus calculated using the applied torque and this torsion angle over the
center part of the specimen is subsequently identified as $G_c$. The shear
modulus calculated using the torsion angle measured by the potentiometer over
the 53.1-mm gage length is identified as $G_g$. For each specimen, three read-
ings were taken of the measured angle of twist $\theta$ for the three torsion angles
(4.5°, 9.0°, and 13.5°) applied at the grip ends of the specimen. A repre-
sentative torque versus torsion graph is shown in Figure 2. The torsional
deformation resulting from the largest applied twist was well below the elastic
limit in all specimens. From these three readings, the average torque per unit
of twist per unit gage length ($TL/\theta$) was determined. From this parameter and
the specimens' dimensions, shear moduli were calculated as described in the
following paragraphs.

The equation relating the applied torque to the angle of twist for torsion
of an isotropic rectangular bar (Figure 3), in the linear elastic range is (6)

$$T = G\theta wt^3/6L \quad \therefore \quad G = TL/3\theta wt^3$$

where

$$\beta = \frac{1}{3} \left( 1 - \frac{192}{5} \frac{t}{w} \sum_{n=1,3,5\ldots}^{\infty} \frac{1}{n} \tanh \frac{n\pi w}{2t} \right)$$

and

$T$ torsional moment
$\theta$ angle of twist
$L$ gage length
$w$ width of specimen cross section
$t$ specimen thickness
$G$ shear modulus

For small values of $t/w$, $\tanh (n\pi w/2t) = 1$, and we have the approximation

$$T = \frac{1}{3} \frac{G\theta t^3}{L} \left( 1 - 0.63 \frac{t}{w} \right)$$

![Graph](image.png)

**Fig. 2** — Torque versus torsion angle for specimen
AS/3501 (0°) 13.00 x 2.11 mm.
Several investigators have used Equation (3) for estimating the shear modulus of unidirectional composite materials \((7, 8)\). In using Equation (3), it is tacitly assumed that these materials are transversely isotropic. We have suggested that, in the case of torsion of unidirectional laminated rectangular specimens, it is more appropriate to use the formulae for torsion of an orthotropic rectangular bar \((4)\). For a linearly elastic orthotropic rectangular parallelepiped, the relationship between the applied torque and the angle of twist is \((9)\):

\[
T = G_{13}^\beta(c)\tau^3 \theta / L \quad \text{or} \quad G_{13}^\beta = TL/\theta \beta(c)\tau^3
\]

where

\[
\beta(c) = \sum_{n = 1, 3, 5, \ldots}^{\infty} \frac{1}{n^2} \left( 1 - \frac{2c}{n^2} \tanh \frac{n\pi}{2c} \right)
\]

\[
c = \frac{w}{t} \sqrt{\frac{G_{23}}{G_{13}}}
\]

Fig. 3 — Test specimen (schematic).

Inasmuch as \(c\) depends on the two ratios \(w/t\) and \(G_{13}/G_{23}\), Equation (4) involves both shear moduli \(G_{13}\) and \(G_{23}\).

In order to determine the two shear moduli, experimental data on at least two different widths are used, say \(w_1\) and \(w_2 = 2w_1\). Equation (4) then yields two experimentally measurable quantities \(p_1 = G_{13}^\beta(c_1)\) and \(p_2 = G_{13}^\beta(c_2) = G_{13}^\beta(2c_1)\). The equation \(\delta(2c_1)/\delta(c_1) = p_2/p_1\) can now be solved for \(c_1\). The relation \(p_1 = G_{13}^\beta(c_1)\) yields \(G_{13}\), and Equation (6) gives \(G_{23}\). In view of the variability in properties of fiber-reinforced composite materials, it is preferable to obtain experimental data on three or more different widths and use a least squares fit of the data to Equation (4) written in the form

\[
\frac{TL}{\theta \tau^3} = G_{13}^\beta \left( \frac{w}{t} \sqrt{\frac{G_{23}}{G_{13}}} \right)
\]

The infinite series in Equation (5) converges rapidly for the parameters used in our calculations, and it is necessary to retain only a few terms in order to calculate the value of \(\delta(c)\) accurately. A simple computer program has
been written for this analysis, and effective shear moduli $G_{13}$ and $G_{23}$ calculated for the four test materials.

**Results and Discussion**

Results of the investigation are summarized in Table I. The first column identifies the material system and the fiber orientation of the test specimens. The second column lists the number of specimens tested for each width/thickness ratio shown in the third column. The first double column, $G_c$, gives the apparent shear modulus calculated from the "isotropic" formula, Equation (3), by using the applied torque and the torsion angle $\theta_c$ measured at the center of the gage section with the laser beam. The arithmetic mean or average value for the number of specimens tested is given and also the coefficient of variation (c.v.) or percent standard deviation. The next double column, $G$, gives corresponding data for the apparent torsion shear modulus calculated from Equation (3) by using applied torque and the angle of twist $\theta$ over the length of the gage section between the grips measured by the potentiometer read-out. The next column gives the torsional shear modulus calculated by using a least squares fit to Equation (1), which is valid for transversely isotropic materials. The last two columns give the in-plane and out-of-plane shear moduli obtained by using a least squares fit to Equation (7), which is valid for orthotropic materials. As noted previously, these latter calculations were made by using average values of TL/6 for specimens of the same nominal widths.

<table>
<thead>
<tr>
<th>Composite Fiber Orientation</th>
<th>Number of Specimens</th>
<th>Ratio w/t</th>
<th>$G_c$ Avg. c.v.</th>
<th>$G_0$ Avg. c.v. (Isotropic)</th>
<th>$G_0$ Eq. 1 (Orthotropic)</th>
<th>$G_{13}$ Eq. 7 Eq. 7</th>
</tr>
</thead>
<tbody>
<tr>
<td>AS/3501 (0°) 16</td>
<td>7</td>
<td>2.93</td>
<td>6.2 6.4</td>
<td>6.3 6.3</td>
<td>2.9</td>
<td></td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>4.44</td>
<td>6.4 3.5</td>
<td>6.4 5.6</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>6.00</td>
<td>6.3 5.3</td>
<td>6.6 4.3</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>7.95</td>
<td>6.5 5.0</td>
<td>6.2 3.2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Avg. 4 sets</td>
<td>Avg. 4.33</td>
<td>6.4 2.0</td>
<td>6.4 2.7</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(0°/±45°/0°) 2s</td>
<td>4</td>
<td>2.56</td>
<td>11.7 3.5</td>
<td>11.8 4.3</td>
<td>6.4</td>
<td>6.6 4.2</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>3.89</td>
<td>12.5 6.4</td>
<td>12.2 6.6</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>5.45</td>
<td>14.9 6.3</td>
<td>13.8 8.3</td>
<td>13.7</td>
<td>17.4 3.4</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>6.59</td>
<td>15.2 4.6</td>
<td>14.6 4.0</td>
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<td></td>
</tr>
<tr>
<td>Avg. 4 sets</td>
<td>Avg. 4.62</td>
<td>13.6</td>
<td>13.1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>T300/5209 (0°) 16</td>
<td>3</td>
<td>3.15</td>
<td>6.2 1.9</td>
<td>6.1 4.8</td>
<td>6.3</td>
<td>6.7 3.7</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>4.27</td>
<td>6.6 4.0</td>
<td>6.7 4.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>5.72</td>
<td>6.3 3.0</td>
<td>6.4 3.2</td>
<td>28.7</td>
<td>47.3 3.7</td>
</tr>
<tr>
<td>Avg. 3 sets</td>
<td>Avg. 4.39</td>
<td>6.4 3.3</td>
<td>6.4 4.7</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(+45°/-45°/+45°) 4</td>
<td>4</td>
<td>2.90</td>
<td>20.0 6.2</td>
<td>16.4 5.1</td>
<td>28.7</td>
<td>47.3 3.7</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>4.15</td>
<td>25.2 8.8</td>
<td>20.1 7.1</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>5.55</td>
<td>31.6 9.4</td>
<td>24.3 2.3</td>
<td>28.7</td>
<td>47.3 3.7</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>6.85</td>
<td>34.3 5.4</td>
<td>26.8 4.3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Avg. 4 sets</td>
<td>Avg. 4.86</td>
<td>27.8</td>
<td>21.9</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
From the $G_c$ and $G_g$ columns, it can be seen that, for uniaxial materials, there is little variation in the apparent shear moduli versus width/thickness ratios. For both AS/3501 and T300/5209, the coefficient of variation of $G_c$ and $G_g$ for specimens of each of the four different nominal widths is less than the maximum c.v. for specimens of any one width. It can be seen that the method of measuring torsion angle does not affect calculated torsional shear modulus inasmuch as $G_c$ and $G_g$ for both materials are 6.4 GPa. Modulus $G_{13}$ calculated from the orthotropic equation, Equation (7), is approximately the same as that calculated from the isotropic formula, Equation (1). From this, it appears that longitudinal, in-plane torsional shear properties of uniaxial fiber composites can be determined to a close approximation by using the simpler Equation (3) and the torsion angle $\theta_g$ measured by the test machine readout.

For cross-ply materials, columns $G_c$ and $G_g$ show that there is a definite and consistent increase of apparent modulus with increasing $w/t$ ratio for both methods of determining the angle of twist. The apparent in-plane shear moduli for the cross-ply composites obtained from Equation (7) are significantly greater than those for the uniaxial composites. For AS/3501 ($0^\circ/\pm45^\circ/0^\circ$)$_{2S}$, the ratio is 2:1; for T300/5209 ($45^\circ/-45^\circ+/45^\circ$)$_4$, the ratio is approximately 4:1. For cross-ply composites, the method of measuring the angle of twist affects the apparent shear modulus, as does the method of calculation. For the former material, shear moduli $G_c$, $G_g$, and $G_{13}$ (Equation (1)), and $G_{13}$ (Equation (7)) are 13.6, 13.1, 13.7, and 17.4 GPa respectively. For the latter material, the corresponding moduli are 27.8, 21.9, 28.7, and 47.3 GPa.

Data from other sources give 5.6 GPa for in-plane shear of uniaxial AS/3501 (10), and for T300/5209 shear values of 5.2 GPa for uniaxial and 15.2 GPa for ($0^\circ/\pm45^\circ$) oriented material (11). These values for the uniaxial composites are approximately 15% lower than the comparable values shown in Table 1. This is in agreement with Johnson's analysis which shows that tests of constrained specimens will give higher shear values than unconstrained ones (12). The in-plane shear modulus of 15.2 GPa noted above for ($0^\circ/\pm45^\circ$) T300/5209 is one-half to one-third of the comparable shear modulus of cross-ply T300/5209 in Table 1 (28.7 and 47.3 GPa for $G_{13}$ Equation (1) and Equation (7), respectively). However, with the fibers of the latter consisting of 100% $\pm45^\circ$ orientation and the former 67%, such a difference would be expected.

The out-of-plane shear modulus ($G_{23}$), which is generally considered to be the same as the in-plane shear modulus ($G_{13}$) for uniaxial material (7,8), is in fact appreciably lower: 4.2 GPa compared with 6.6 GPa or 37% lower for AS/3501; 3.7 compared with 6.7 GPa for 44% lower for T300/5209. A difference of this magnitude is sufficient to be considered in some design applications. For the cross-ply material the difference between out-of-plane and in-plane shear moduli is much greater. For AS/3501, shear modulus $G_{23}$ is 3.4 GPa compared with 17.4 for $G_{13}$ calculated with Equation (7) or 80% lower; compared with the isotropic equation value of 13.7 GPa, the out-of-plane modulus $G_{23}$ is 75% lower. For T300/5209, comparison of $G_{23}$ of 3.7 GPa with corresponding $G_{13}$ values of 47.3 and 28.7 GPa shows $G_{23}$ shear moduli to be 92% and 87% lower, respectively.

In-plane shear moduli $G_{13}$ of the cross-ply materials are much larger than those for the uniaxial orientations. For AS/3501, the comparison is 13.7 with 6.4 GPa or 17.4 with 6.6 GPa, ratios of 2.1 or 2.6, depending upon whether the approximate or exact equation (Equation (3) or Equation (7)) is used. For T300/5209 material, the corresponding comparisons are 28.7 with 6.3 and 47.3 with 6.7 GPa, or ratios of 4.6 and 7.0. Inasmuch as the T300/5209 has 100% of the fibers oriented at $\pm45^\circ$, at which the shear modulus is maximized, and the AS/3501 has only 50% of the fibers at this orientation, shear modulus ratios of 4 and 2, respectively, would have been expected.
If one considers that the degree of anisotropy can be represented by the ratio of in-plane to out-of-plane shear moduli, $G_{13}/G_{23}$, then both uniaxial materials show anisotropy of slightly less than 2, while the anisotropy of the AS/3501 (0°/±45°/0°)$_2$ is 5 and that of the T300/5209 (-45°/+45°/-45°)$_4$ is greater than 12. As the difference between the effective values of $G_{13}$ and $G_{23}$ increases, the inaccuracy involved in using Equation (1) or Equation (3) (i.e., assuming transverse isotropy) for the calculation of the in-plane shear modulus can be expected to increase. Table I shows that this is indeed the case; namely, the difference between the calculated values of the in-plane shear modulus using the orthotropic formula, Equation (7), and the isotropic formula, Equation (1), increases as the ratio $G_{13}/G_{23}$ increases.

Figures 4 and 5 show plots of $\frac{TL}{\theta wt^2}$ versus $w/t$ for the two unidirectional materials. The experimental data are shown together with the least squares best fit curves to the orthotropic formula, Equation (7), and to the isotropic formula, Equation (1). The orthotropic model appears to give a much better fit to the experimental data, showing that, as conjectured, for the unidirectional laminated composite, the out-of-plane shear modulus differs from the in-plane shear modulus (4). Figures 6 and 7 show the corresponding plots for the two cross-ply materials studied. Here too it is clear that the orthotropic formula gives a much better fit to the experimental data than the isotropic formula. From these comparisons, it is evident that the unidirectional as well as the angle-ply materials are not transversely isotropic, and, therefore, that the resulting difference between in-plane and out-of-plane shear moduli needs to be considered in design applications.
Fig. 5 TL/\theta w t^3 versus w/t for T300/5209 uniaxial graphite-epoxy composite.
Fig. 6 $\frac{TL}{\theta wt^3}$ versus $w/t$ for AS/3501 (0°/±45°/0°)$_{2S}$ graphite-epoxy composite.
Fig. 7 TL/dwt$^3$ versus w/t for T300/5209 (+45°/-45°/+45°)$_4$ graphite-epoxy composite.

Conclusions

(1) Torsional shear moduli can be determined by simple torsion tests of flat specimens of rectangular cross section.

(2) Using the solution for torsion of an orthotropic rectangular bar, the out-of-plane as well as the in-plane shear moduli can be determined.

(3) Neither the uniaxial nor angle-ply composite materials studied are transversely isotropic. The degree of anisotropy for the angle-ply materials was several times greater than that of the uniaxial composites.

(4) For specimens of uniaxial fiber orientation, the in-plane shear moduli can be calculated to a good approximation by using the isotropic formula and test machine deflection data.

References


By means of torsion tests performed on test specimens of the same material having a minimum of two different cross sections (flat sheet of different widths), the effective in-plane \((G_{13})\) and out-of-plane \((G_{23})\) shear moduli were determined for two composite materials (AS/3501 and T300/5209) of uniaxial and angleply fiber orientations. Test specimens were 16 plies (nominal 2 mm) thick, 100 mm in length, and in widths of 6.3, 9.5, 12.5, and 15.8 mm. Torsion tests were run under controlled deflection (constant angle of twist) using an electro-hydraulic servo-controlled test system. In-plane and out-of-plane shear moduli were calculated from an equation derived in the theory of elasticity which relates applied torque, the torsional angle of twist, the specimen width/thickness ratio and the ratio of the two shear moduli \(G_{13}/G_{23}\). Results demonstrate that torsional shear moduli, \(G_{23}\) as well as \(G_{13}\), can be determined by simple torsion tests of flat specimens of rectangular cross section. Neither the uniaxial nor angleply composite material was transversely isotropic. The degree of anisotropy of the angleply materials was several times greater than that of the uniaxial composites.