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Technical Report No. 1092-1

DEVELOPMENT OF AUTOMATIC AND MANUAL FLIGHT DIRECTOR LANDING SYSTEMS FOR THE XV-15 TILT ROTOR AIRCRAFT IN HELICOPTER MODE

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Development of Automatic and Manual Flight Director Landing Systems for the XV-15 Tilt Rotor Aircraft in Helicopter Mode


Systems Technology, Inc.
Hawthorne, California

National Aeronautics and Space Administration
Ames Research Center
Moffett Field, California

The objective of this effort is to determine IFR approach path and touchdown dispersions for manual and automatic XV-15 tilt rotor landings, and to develop missed approach criteria. Only helicopter mode XV-15 operation is considered. The results will be used to support the development of certification criteria for the XV-15. The analysis and design sections develop the automatic and flight director guidance equations for decelerating curved and straight-in approaches into a typical VTOL landing site equipped with an MLS navigation aid. These system designs satisfy all known pilot-centered, guidance and control requirements for this flying task. Performance data, obtained from nonstationary covariance propagation dispersion analysis for the system, are used to develop the approach monitoring criteria. The autoland and flight director guidance equations are programmed for the VSTOLAND 1819B digital computer. The system design dispersion data developed through analysis and the 1819B digital computer program are verified and refined using the fixed-base, man-in-the-loop XV-15 VSTOLAND simulation at NASA Ames.

Abstract

Key Words
Automatic Landing
Flight Director
Touchdown Dispersion
Decelerating Approach
Missed Approach Criteria

VTOL
VSTOLAND
Autoland

Distribution Statement
Unclassified

Unclassified

Unclassified
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<td>ALT</td>
<td>Altitude hold mode</td>
</tr>
<tr>
<td>AS</td>
<td>Airspeed hold mode</td>
</tr>
<tr>
<td>CTOL</td>
<td>Conventional takeoff and landing</td>
</tr>
<tr>
<td>DEC, DECL, DECEL</td>
<td>Constant attitude deceleration mode</td>
</tr>
<tr>
<td>DME</td>
<td>Distance measuring equipment</td>
</tr>
<tr>
<td>GPIP</td>
<td>Glide path initial point</td>
</tr>
<tr>
<td>GS</td>
<td>Glide slope track mode</td>
</tr>
<tr>
<td>HH</td>
<td>Heading hold mode</td>
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<td>HOV</td>
<td>Longitudinal hover mode</td>
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<td>IAS</td>
<td>Indicated airspeed</td>
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<td>IMC</td>
<td>Instrument meteorological conditions</td>
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<td>LOC A</td>
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<td>Bank-to-translate localizer mode</td>
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<td>MLS</td>
<td>Microwave landing system</td>
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<td>RCAH</td>
<td>Rate command, attitude hold</td>
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<td>SCAS</td>
<td>Stability and control augmentation system</td>
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<td>TRC</td>
<td>Translational rate command</td>
</tr>
<tr>
<td>VD</td>
<td>Vertical descent mode</td>
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<td>VTOL</td>
<td>Vertical takeoff and landing</td>
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<tr>
<td>ALTH, H, h</td>
<td>Altitude (AGL)</td>
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<td>AX, ax</td>
<td>Longitudinal acceleration of e.g.</td>
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<td>AYP, a_y</td>
<td>Lateral body-axis acceleration at pilot's location</td>
</tr>
<tr>
<td>AZP, a_z</td>
<td>Normal acceleration at pilot's location</td>
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<td>d</td>
<td>Actual glide slope deviation, positive for aircraft above glide path, linear units</td>
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<td>DC, δ_c</td>
<td>Longitudinal cyclic stick deflection</td>
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<tr>
<td>DCL, δ_CL</td>
<td>Total collective pitch deflection in lever units</td>
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<td>DE, δ_e</td>
<td>Indicated glide slope deviation in linear units</td>
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<td>DEH, δ_e</td>
<td>Input to beam rate command limiter during glide slope tracking, altitude command during vertical descent</td>
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<td>DLAT, δ_LAT</td>
<td>Total lateral cyclic deflection</td>
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<td>DLN, δ_LN</td>
<td>Total longitudinal cyclic pitch deflection in stick units</td>
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<td>DLNS, δ_LNS</td>
<td>Longitudinal cyclic series servo deflection</td>
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<td>DPDS, δ_PDS</td>
<td>Rudder series servo deflection</td>
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<td>DS, δ_s</td>
<td>Lateral cyclic stick deflection</td>
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<td>e</td>
<td>Base of natural logarithm, 2.718 ...</td>
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<td>e_x</td>
<td>Switching criterion signal for point hover mode initiation</td>
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<td>e_θ</td>
<td>Switching criterion signal (or error signal) for constant attitude deceleration initiation (or control law)</td>
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<td>ENU, ν</td>
<td>MLS localizer structure (noise) in angular units</td>
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<td>ETA, η</td>
<td>MLS glide slope structure (noise) in angular units</td>
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<td>f</td>
<td>Input to flight director shaping network</td>
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<td>FDC, FD_C</td>
<td>Flight director longitudinal cyclic command bar deflection</td>
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<tr>
<td>FDCL, FD_CL</td>
<td>Flight director collective pitch command bar deflection</td>
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FDL, FDL  Flight director lateral cyclic command bar deflection
FDP, FDP  Flight director rudder pedal command bar

\( g \)  Gravity constant

\( G_{FFS} \)  Force feel system transfer function for stick deflection in, parallel servo deflection out

\( h^* \)  Total altitude of aircraft center of gravity above GPIP on runway

\( h_{baro} \)  Pressure altitude

\( \dot{h}_{TCSI} \)  Instantaneous vertical speed indication

\( h_0 \)  Altitude at output of synchronizer

\( h_{ocg} \)  Aircraft center of gravity altitude at touchdown

\( h_{radar} \)  Radar altitude

\( h_c \)  Asymptotic flare altitude

\( HD, \dot{h} \)  Rate of climb

\( I(\cdot) \)  Moment of inertia about ( ) axis

\( k \)  Complementary filtering gain

\( K(\cdot) \)  Control system gain specified by (·)

\( K_{BUG} \)  Calibration parameter, ideally equal to unity

\( K(W) \)  Wind profile shaping gain

\( K_{\theta V} \)  Trim pitch attitude gradient with airspeed, \( \theta_V/\dot{V}_A \)

\( L_{xa, z_a} \)  Pilot's location with respect to aircraft center of gravity

\( L \)  Rolling moment

\( L(\cdot) \)  Dimensional rolling moment derivative, \( (1/I_x)[\dot{\alpha}/\dot{\gamma}(\cdot)] \)

\( L(\cdot) \)  Turbulence or MIB noise characteristic wave lengths specified by (·)

\( LS, W \)  Longitudinal control system mode indicator

\( \dot{m} \)  Mass

\( M \)  Pitching moment

\( M(\cdot) \)  Dimensional pitching moment derivative, \( (1/I_y)[\dot{\alpha}/\dot{\gamma}(\cdot)] \)
N Yawing moment

\[ N(\cdot) \]
Dimensional yawing moment derivative, \((1/I_2)[\Theta N/\Theta(\cdot)]\)

\[ N[\cdot] \]
Transfer function numerator between [\cdot] input and (\cdot) output

\[ P, p \]
Angular rate about x-axis

\[ PG, p_g \]
Effective rolling gust component

\[ PHDC, \Phi_{DC} \]
Input to bank angle command limiter

\[ PHI, \phi \]
Euler roll angle

\[ P_{MA} \]
Probability of having to execute a missed approach on any given IMC approach because of flight technical error

\[ PSI, \psi \]
Euler yaw angle with respect to runway heading

\[ Q, q \]
Angular rate about y-axis

\[ R, r \]
Angular rate about z-axis

\[ R \]
Slant range between center of gravity of the approaching aircraft and MLS elevation antenna

\[ R_A \]
Slant range between aircraft and MLS azimuth transmitting antenna

\[ R_G \]
Distance between MLS elevation and azimuth transmitting antennas

s Laplace transform variable

\[ S(\cdot) \]
Control system switch variable specified by (\cdot). Permissible values are 0. and 1.

\[ SA, S_A \]
Lateral-direction control system mode indicator and switch value

\[ SR \]
Input to sink rate command limiter

\[ TIME, t \]
Time

\[ T(\cdot) \]
Control system time constant specified by (\cdot)

\[ THCD, \Theta_{cd} \]
Input to pitch attitude command limiter

\[ THEE, \Theta_e \]
Pitch attitude deviation from commanded value

\[ THET, \theta \]
Euler pitch angle

\[ u \]
Perturbation x-velocity

\[ u_A \]
Body-fixed x-axis component of headwind and crosswind
\( U \)  
Total x-velocity

\( \dot{u} \)  
Longitudinal, body-axis kinematic acceleration, \( (a_{xog} - g \sin \theta) \)

\( U_G, u_G \)  
x-axis gust component (body-fixed reference frame)

\( U_W, u_W \)  
Headwind component with respect to runway direction

\( v \)  
Perturbation y-velocity

\( v_A \)  
Body-fixed y-axis component of headwind and crosswind

\( V \)  
Total y-velocity

\( V_A, V_A \)  
Airspeed

\( V_{AE}, V_{AE} \)  
Smoothed compensatedairspeed error

\( V_{AX} \)  
Airspeed below whichairspeed crossfeedto collective pitchis enabled

\( V_e \)  
Equivalentairspeed

\( V_G, V_G \)  
y-axis gust component (body-fixed reference frame)

\( V_{HW} \)  
Mean headwind component in runway direction at problem initial attitude

\( V_{IAS} \)  
Indicatedairspeed

\( W, v_w \)  
Crosswind component with respect to runway direction, positive for wind blowing from right to left

\( w \)  
Perturbation z-velocity

\( w_A \)  
Body-fixed z-axis component of headwind and crosswind

\( W \)  
Total z-velocity

\( W_G, w_G \)  
z-axis gust component (body-fixed reference frame)

\( X \)  
Force along x body axis

\( X \)  
Ground range from intended touchdown point, positive in the down-runway direction

\( X_D, X \)  
Velocity in down-runway direction

\( X( ) \)  
Dimensional x-force derivative, \((1/m)(\Delta X/\Delta( ))\)

\( X_C, X_C \)  
DME noise
$X_f$  
Ground range for first occurrence of $e_x = 0$

$X_{IND}, X_i$  
Indicated DME ground range

$X_o$  
Ground range for first occurrence of $e_\theta = 0$

$X_{PED}$  
Rudder pedal deflection

$\dot{y}$  
Velocity in the cross-runway direction, positive when moving to right of runway

$Y$  
Force along $y$ body-axis

$Y, y$  
Lateral deviation in cross-runway direction, positive for aircraft to right of runway centerline

$Y(\ )$  
Dimensional $y$-force derivative, $(1/m)\dot{y}/\theta(\ )$

$YD, \dot{y}$  
Lateral deviation rate

$YE, \gamma_e$  
Indicated localizer deviation, linear units

$Y_F(\ )$  
Transfer function representing pilot's control action in response to flight director specified by $(-)$

$Z$  
Force along $z$ body-axis

$Z(\ )$  
Dimensional $z$-force derivative, $(1/m)\dot{z}/\gamma(\ )$

$\alpha$  
Angle of attack

$\beta$  
Sideslip angle

$\gamma$  
Flight path angle

$\gamma_A$  
Aerodynamic flight path angle

$\delta$  
Control deflection

$\delta_{PED}$  
Total rudder deflection in pedal units

$\Delta$  
Transfer function denominator

$\Delta\delta_{CL}$  
Collective pitch component resulting from airspeed crossfeed

$\Delta\theta_{DEC\_CL}$  
Computed pitch attitude reference change for deceleration mode

$\epsilon_d$  
Switching criterion signal for glide slope capture initiation

$\epsilon_y$  
Switching criterion signal for localizer capture initiation

$\zeta$  
Damping ratio
\( \theta_c \) Pitch attitude hold reference

\( \theta_d \) Computed pitch attitude reference for constant attitude deceleration

\( \theta_h \) Trim pitch attitude at hover in zero wind

\( \theta_{\text{INC}} \) Constant attitude increment controlling rapidity of deceleration

\( \theta_{\text{O}(\cdot)} \) Attitude at output of synchronizer (\( \cdot \))

\( \theta_p \) Trim pitch attitude as function of airspeed

\( \phi \) Glide slope alignment angle (\( = \gamma \))

\( \rho \) Probability density function, or correlation coefficient if subscripted, also air density at altitude

\( \rho_0 \) Standard sea level air density

\( \sigma \) Denotes one standard deviation in general. May be particularized by subscript

\( \psi_c \) Yaw attitude hold reference

\( \psi \) Localizer alignment angle

\( \psi_{\text{REF}} \) Reference heading for heading hold control system mode

\( \omega \) Angular frequency

\( \omega_{\text{\cdot}(\cdot)} \) Control system break frequency specified by (\( \cdot \))

**MATRIX AND VECTOR SYMBOLS**

A System matrix

b Input vector

c Covariance matrix for \( x \)

D Covariance matrix for \( y \)

h Input vector

H State-to-output distribution matrix

n Discrete interval index

Q Power spectral density matrix for \( w \)
\( w \)  \hspace{1cm} \text{Process noise vector}

\( x \)  \hspace{1cm} \text{State vector}

\( y \)  \hspace{1cm} \text{Alternative state vector}

\( z \)  \hspace{1cm} \text{Output vector}

\( \Gamma \)  \hspace{1cm} \text{Forcing matrix for covariance propagation equation}

\( \Theta, \Theta \)  \hspace{1cm} \text{Forcing matrix or vector for state transition equation}

\( \Phi, \Phi \)  \hspace{1cm} \text{State transition matrices}

\textbf{PREFIXES AND SUFFIXES}

\( S(\cdot) \)  \hspace{1cm} \text{Standard deviation of (\( \cdot \))}

\( (\cdot)D \)  \hspace{1cm} \text{Derivative with respect of time of (\( \cdot \))}

\textbf{SPECIAL NOTATION}

\( E[\cdot] \)  \hspace{1cm} \text{Expected value of [\( \cdot \)]}

\( (\cdot)_{TD} \)  \hspace{1cm} \text{Touchdown-related value of (\( \cdot \))}

\( (\cdot) \)  \hspace{1cm} \text{Denotes mean or expected value of (\( \cdot \))}

\( (\cdot) \)  \hspace{1cm} \text{Derivative with respect to time of (\( \cdot \))}

\( (\cdot)' \)  \hspace{1cm} \text{Transpose of matrix (\( \cdot \))}

\( |(\cdot)| \)  \hspace{1cm} \text{Absolute value of scalar quantity (\( \cdot \)), or determinant of square matrix (\( \cdot \))}

\( (\cdot)_{\text{max}} \)  \hspace{1cm} \text{Maximum allowable value for (\( \cdot \))}

\( (\cdot)_{\text{min}} \)  \hspace{1cm} \text{Minimum allowable value for (\( \cdot \))}

\( (\cdot)_{\text{DP}} \)  \hspace{1cm} \text{Value of (\( \cdot \)) just prior to decision point}

\( (\cdot)_{\text{DP}}^{+} \)  \hspace{1cm} \text{Value of (\( \cdot \)) just subsequent to decision point}

\( (\cdot)_{o} \)  \hspace{1cm} \text{Initial value, or value at trim}

\( (\cdot)^{*} \)  \hspace{1cm} \text{Value in presence of a steady wind}

\( (\cdot)_{c} \)  \hspace{1cm} \text{Commanded or reference value}
SECTION I

INTRODUCTION

This report summarizes the results of a combined analytical and experimental study to determine IFR approach path dispersions under manual and automatic control. These dispersions, in turn, are the basis for missed approach criteria. The basic motivation for this research stems from recognition that the future of V/STOL aircraft as a viable transportation medium depends upon implementation of appropriate terminal control techniques and safety philosophies. Accordingly, the emphasis is directed at the generation of a dual (automatic and manually controlled) system for the guidance and control of the XV-15 aircraft on approach to evaluate performance in this critical terminal flight regime.

A. SCOPE

The purpose of this program is to generate performance data for the XV-15 Tilt Rotor aircraft and control system operating in the terminal area with a digital avionics system. This data is required to assess effects of the system elements (XV-15, flight paths, navigation aids, guidance and flight director laws, etc.), command inputs and disturbances in terms of the precision of control, pilot workload and available margins of safety. This data, in turn, provides insight for design of appropriate automatic or manual approach monitoring criteria for the XV-15.

This research assumes operation within the practical constraints and operating limits of the XV-15 aircraft. XV-15 performance margins (e.g., thrust/power and rate of sink), vibration, and rotor structural fatigue limits as defined in Refs. 1 and 2 preclude descending on the steep glide slopes (e.g., -6 deg (-0.10 rad), -3 deg (-0.17 rad) and -10 deg (-0.17 rad) while decelerating in airspeed in other than the helicopter mode of configuration (i.e., pylon at 90 deg (1.6 rad)). Therefore, neither the automatic nor the manual systems require consideration of a configuration management command mode (e.g., modulation of pylon and flaps). Furthermore, only the conventional displays and control associated with the XV-15 aircraft are considered. Thus, the many and varied
considerations associated with abort procedures, safety margins, engine-out conditions which are of concern before and during the transition to the helicopter mode of flight are non-operative. Therefore, the pilot need only concern himself with the final stages of the approach — deceleration to a hover on a descending, perhaps curved, flight path while in the terminal area.

B. APPROACH

The research consists of three major elements. The first of these is the development of a compatible set of guidance, autoland, and flight director laws. The word "compatible" refers specifically to the requirement that manual (using the flight director) and automatic (with the autoland system) control laws be compatible not only with the approach requirements but also with each other. Autoland systems capable of decelerating flight to a hover have been built and flown (Refs. 3 and 4). Flight director configurations with similar capabilities have also been flown (Ref. 4). Designs to date are not without pilot reservations — primarily because the flight director was apparently treated as an adjunct to the autoland system rather than a separate system with its own (pilot-centered) requirements in addition to the fundamental guidance requirements common to both manual and automatic systems.

The second element is the assessment of this system by means of exercise of a digital simulation of its characteristics. A statistical approach to the evaluation of the system performance is used. While the necessary statistical performance data could be developed using a Monte Carlo approach, the more effective and efficient covariance propagation technique is used. This provides statistical descriptions of performance directly in terms of dispersions. Further application of this technique is used to develop approach monitoring criteria.

The third key element is verification of system suitability and performance by means of man-in-the-loop simulation (fixed base) at Ames Research Center.
C. ORGANIZATION OF THE REPORT

The remainder of this report is organized into seven sections and three appendices. The next three sections detail development of the guidance, auto-land and flight director control laws. The first of these three sections summarizes the qualitative design requirements in consideration of the tasks, XV-15 characteristics, pilot-centered considerations, operating environment, etc. Quantitative development of the longitudinal and lateral-directional designs is contained in Sections III and IV, respectively.

A statistical analysis of the approach and touchdown dispersions for these longitudinal and lateral-directional systems is reported in Section V. Section VI recommends missed approach criteria based on the statistical analysis.

Results of the fixed-base, man-in-the-loop simulation are reported in Section VII, while the main conclusions from this research program are summarized in Section VIII.

Two appendices cover basic XV-15 aircraft data, equations of motion and parameter values used in the system performance analysis, and the equations suitable for programming the guidance, auto-land and flight director control laws and logic for the VSTOLAND 181-B Research Computer.
SECTION II

SYSTEM REQUIREMENTS

System requirements are established on the basis of mission phase or task definitions; fixed characteristics of aircraft, SCAS and disturbance environment; feasible competing systems capable of meeting mission or task objectives; and assessment of the favorable and unfavorable points for each competing system. Establishing requirements is a qualitative process for narrowing the field of potential system solutions to be analyzed in quantitative detail in Sections III and IV.

A. TASK DEFINITION

The mission phases of concern in this study are limited to terminal area vectoring, initial and final approach, and landing. The XV-15 is assumed to be operating in the helicopter mode over its complete speed range in that mode. Vertical landing, curved initial approach path and variable glide slope capability must be provided. All navigation and guidance measurements required for these tasks are obtained from the VSTOAND navigation system.

B. FIXED CHARACTERISTICS

1. XV-15 Aircraft

The unaugmented XV-15 aircraft in the helicopter mode has the usual helicopter handling deficiencies plus a unique lateral-directional roll control reversal problem in the 40 kt (21 m/s) regime and significant lack of directional stability below 40 kt (.1 m/s). These latter points, in turn, place special requirements on the lateral-directional SCAS. All basic handling deficiencies are summarized in Table 1. (Table 1 is based upon the stability derivative data in Tables A-4 and A-6 and the transfer function data in Table 7.)
TABLE I
BASIC XV-15 HANDLING DEFICIENCIES

**LONGITUDINAL**

Low heave damping below 40 kt*

Long-term attitude response to stick input is near zero

Unacceptable short-term attitude to stick responses at low speed

**LATERAL-DIRECTIONAL**

Very low dutch roll frequency at all speeds (low \( \omega_d \))

Negative or low dutch roll damping at all speeds (low \( \zeta_d \))

Unstable spiral mode

Large shift in instantaneous center of rotation \( (-X_{Spd}/N_{Spd}) \) between 60 and 80 kt

Roll reversal at 40 kt

Poor yaw rate to pedal characteristics

Large adverse yaw at speeds below about 70 kt

*1 kt = 5.144 x 10\(^{-1}\) m/s.

2. Disturbance Environment

The disturbance environment considered includes longitudinal, normal, side and effective rolling gusts, steady winds and wind shears, MLS azimuth, elevation and DME noise. Math models for each of these disturbance effects are given
in Appendix A. Effects of curved path commands and of pilot abuse of system operating procedures are also considered.

3. Stability and Control Augmentation System (SCAS)

The installed SCAS is a rate-command, attitude retention system. Installed series actuators have very limited [± in. (±0.025 m)] authority because the augmentation is single-thread. The original intent of this project was to design the guidance, flight director and autoland control laws to work with the SCAS as installed. However, the fact that the electrical inputs to the series servos shown in Fig. 16 of Ref. 5 are not in actuality available, and that the attitude hold (more properly, retention) feature is only introduced when there are no pilot inputs, caused modification of the original ground rules. Modification is necessary for manual flight director operation because the attitude feedbacks are cancelled whenever the control stick is moved from the detent. This deficiency might be corrected by using the attitude SCAS feedback full-time (an available option), but then an attitude command system results. Attitude command systems demand series servo deflections which substantially exceed the installed series servo authority. That this is so is illustrated in Fig. 1. Parts a and b compare the series servo deflection required for attitude command and rate command pitch SCAS systems in order to cope with a reasonably large wind shear. It is evident in part a that much of the series servo deflection is devoted to cancelling the parallel servo output for the attitude command system.

An alternative solution might be to include the required attitude feedback signal on the flight directors, but this is sure to result in an unacceptable level of flight director activity from the pilot’s viewpoint.

For the above reasons, the ground rules were modified to permit design of a new, rate-command, attitude-hold SCAS operating within the authority limits for the installed series servos.

Provisions for interfacing the VSTOLAND system with the XV-15 is through the force feel/autopilot actuators in the longitudinal, lateral, and directional control systems. The cockpit control motions are also introduced into the
Figure 1a. Response to Large Wind Shear — Attitude Command SCAS; \( V = 60 \text{ kt} \ (31 \text{ m/s}) \)
Figure 1b. Response to Large Wind Shear — Rate Command SCAS; \( V = 60 \) kt (31 m/s)
aircraft primary controls through the force feel actuators. Input to the power management system is through the power lever actuator.

C. **REQUIREMENTS AND COMPETING SYSTEM ALTERNATIVES**

Definition of competing candidate systems proceeds directly from formulation of system requirements. Requirements for both the manual (using flight director) and automatic (autoland plus autopilot) landing systems are addressed simultaneously. This is possible because of a NASA stated requirement for similarity in behavior and performance for the two systems. This dictates systems which are functionally similar, that is, have similar loop structures.

However, there are a few significant fundamental differences between the manual and automatic systems which are appropriately identified at the outset, e.g.:

- **Control authority and limits** — The automatic system has less because the pilot has access to additional controls (the flight director controls pilot inputs, not outputs); and it allows for pilot takeover (e.g., the pilot can override the FFS actuator because of its limited force capability).

- **Integration** — An automatic system will often have integrators to drive the steady-state errors to zero while the flight director system cannot include such functions because of excessive attentional demands.

- **Monitoring criteria** — The monitoring and associated takeover criteria for the two systems may differ for a number of reasons. Basically, the pilot has only conventional instruments while the automatic system potentially has access to more variables and may operate on different computed combinations of variables.

With these qualifications, we can proceed to establishing requirements and then to definition of the candidate competing systems.

Requirements can be grouped as follows:

- **Guidance and control requirements** — Fundamental and independent of whether the controller is an automatic or human pilot.

- **Pilot-centered requirements** — Relate to the fact that the controller is a man (pertains to the flight director design).
A summary of the requirements central to design of these systems is given in Table 2. The satisfaction of these requirements from basic considerations leads to the selection, sensing, shaping, and relative weighting of appropriate feedbacks (and feedforwards) in a way which is best for manual control using the flight director and (with adjustments) to autoland. The pilot-centered

TABLE 2. PILOT/VEHICLE SYSTEM REQUIREMENTS

<table>
<thead>
<tr>
<th>Guidance and Control</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Command Following</td>
</tr>
<tr>
<td>• Disturbance Regulation</td>
</tr>
<tr>
<td>• Stability and Damping</td>
</tr>
</tbody>
</table>

Pilot-Centered (Refs. 6 and 7)

• Minimum Pilot Compensation
  — Feedback paths
  — Equalization
• Response Quality
• Frequency Separation of Controls (Table 3)
• Non-Interaction of Controls
• Insensitivity to Pilot Response Variations
• Remnant Suppression

requirement for frequency separation of controls is very important for keeping pilot workload within acceptable bounds. This requirement can be met by two alternative choices for axes to be controlled by manual flight director or automatic systems. These two choices are Option A and Option B.

Option A
- All height control is manual.
- FDLAT goes automatic in hover to be consistent with FDLN and to minimize workload.
- Would require series actuator on collective.

Option B
- Lateral and longitudinal cyclic are primarily manual controls for entire approach.
- Heading is fully automatic.
- Display collective trim shifts on FDCL, e.g., step commands for vertical descent
The specific use of manual flight director or automatic control is detailed by control axis and approach mode in Table 3 for both options. The requirements of Table 2 are related to key longitudinal and lateral-directional feedbacks in Table 4.

Once the key alternative longitudinal and lateral-directional feedbacks for meeting the requirements are identified, additional pilot preferences concerning VTOL approach technique are imposed. These preferences are summarized in Table 5.

TABLE 3
MANUAL/AUTOMATIC CONTROL AXIS ALLOCATION OPTIONS FOR THE "FREQUENCY SEPARATION OF CONTROLS" REQUIREMENT

<table>
<thead>
<tr>
<th>OPTION A</th>
<th>MLS</th>
<th>FDCL</th>
<th>FDLAT</th>
<th>Automatic</th>
<th>Automatic</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(crab or wing low)</td>
<td></td>
</tr>
<tr>
<td>Deceleration</td>
<td>FDCL</td>
<td>FDLAT</td>
<td>Automatic</td>
<td>Automatic</td>
<td></td>
</tr>
<tr>
<td>Hover</td>
<td>FDCL</td>
<td>Automatic</td>
<td>Automatic</td>
<td>Automatic</td>
<td></td>
</tr>
<tr>
<td>Vertical Descent</td>
<td>FDCL</td>
<td>Automatic</td>
<td>Automatic</td>
<td>Automatic</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>OPTION B</th>
<th>MLS</th>
<th>Automatic + trim</th>
<th>FDLAT</th>
<th>FDLN</th>
<th>Automatic</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Automatic + trim</td>
<td>FDLAT</td>
<td>FDLN</td>
<td>Automatic</td>
</tr>
<tr>
<td>Deceleration</td>
<td>Automatic + trim</td>
<td>FDLAT</td>
<td>FDLN</td>
<td>Automatic</td>
<td></td>
</tr>
<tr>
<td>Hover</td>
<td>Automatic + trim</td>
<td>FDLAT</td>
<td>FDLN</td>
<td>Automatic</td>
<td></td>
</tr>
<tr>
<td>Vertical Descent</td>
<td>Automatic + trim</td>
<td>FDLAT</td>
<td>FDLN</td>
<td>Automatic</td>
<td></td>
</tr>
<tr>
<td>DIRECT REQUIREMENTS</td>
<td>CRITICAL DISTURBANCES AND COMMANDS</td>
<td>IMPLIED REQUIREMENTS</td>
<td>COMPETING SYSTEMS</td>
<td>COMMENTS</td>
<td></td>
</tr>
<tr>
<td>----------------------</td>
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<td>-----------------</td>
<td>----------</td>
<td></td>
</tr>
<tr>
<td>Altitude hold</td>
<td>w gusts</td>
<td>Good ( h/\delta_{IN} )</td>
<td>( h, h \rightarrow \delta_{IN} ) (IAS hold off) ( \theta, \dot{\theta} \rightarrow \delta_{IN} ) SCAS</td>
<td>Pilot option to use on ( \delta_{IN} ) or automatic system</td>
<td></td>
</tr>
<tr>
<td>Airspeed Hold</td>
<td>u gusts</td>
<td>Good low-frequency attitude control ( (1/T_g \gg 0) )</td>
<td>IAS ( \rightarrow \delta_{FD} )</td>
<td>( \theta, \dot{\theta} \rightarrow \delta_{IN} ) SCAS</td>
<td>Pilot option ( FD ) or automatic system</td>
</tr>
<tr>
<td>Vertical Speed Hold</td>
<td>Wind shear</td>
<td>Good ( h/\delta_{IN} ) or ( h/\delta_{CL} )</td>
<td>( \delta_{CL} \rightarrow \delta_{FD} )</td>
<td>( \theta, \dot{\theta} \rightarrow \delta_{IN} ) SCAS</td>
<td>Pilot option ( FD ) or automatic system ( \delta_{FD} ) will not work on backside</td>
</tr>
<tr>
<td>Pitch Attitude Control</td>
<td>u gusts</td>
<td>( t_p, a_b, 1/T_g ) in region of good handling ( K_0 ) ( \delta_{IN} \rightarrow \phi ) optimized ( \delta_{IN} ) ( \rightarrow \phi )</td>
<td>( \dot{x} \rightarrow c ) ( \delta_{IN} \rightarrow \phi ) FFS ( \dot{x} \rightarrow c ) Same as IAS ( \rightarrow c )</td>
<td>( \theta, \dot{\theta} \rightarrow \delta_{IN} ) SCAS ( \dot{x} \rightarrow c ) ( \delta_{IN} \rightarrow \phi ) ( \dot{x} \rightarrow c ) ( \delta_{IN} \rightarrow \phi ) ( \theta, \phi \rightarrow \delta_{LA} ) ( \delta_{LA} \rightarrow \phi )</td>
<td>Basic choice is ( \dot{c}_0 ) or ( \phi_0 ). Should be ( \dot{c}_0 ) for hover, e.g., ( \dot{x} \rightarrow c ) ( \dot{x} \rightarrow c ) Always on</td>
</tr>
<tr>
<td>Bank Angle Control</td>
<td>( v ) gusts</td>
<td>( \phi_{c} ) during approach ( \dot{\phi}<em>{c} ) ( \rightarrow \dot{c} ) ( \phi</em>{LAT} \rightarrow \phi_{c} )</td>
<td>( \dot{x} \rightarrow c ) ( \delta_{LAT} \rightarrow \phi_{c} )</td>
<td>( \phi \rightarrow \delta_{LA} ) ( \delta_{LA} \rightarrow \phi )</td>
<td>Basic choice is ( \dot{c}<em>{0} ) or ( \dot{\phi}</em>{c} ). Should be ( \dot{c}<em>{0} ) in hover but ( \phi</em>{c} ) is very undesirable on approach ( \dot{x} \rightarrow c ) Always on</td>
</tr>
<tr>
<td>Heading Control</td>
<td>( \delta_{LAT} ) inputs ( \delta_{PED} ) inputs Outer loop requirements ( v ) gusts</td>
<td>( \mu ), ( I_{h}/N_{a} ) in favorable region when ( \delta_{LAT} ) is primary ( \psi ) control ( \delta_{PED} \rightarrow \dot{c} ) ( \psi, \dot{\psi} \rightarrow \delta_{PED} ) SCAS</td>
<td>Latter stages of approach ( \delta_{PED} \rightarrow \dot{c} ) Early in approach ( \psi \rightarrow \dot{c} ) ( \delta_{PED} \rightarrow \dot{c} ) SCAS</td>
<td>Basic philosophy is to use ( \delta_{LAT} ) for heading control until some point on localizer ( \psi \rightarrow \dot{c} ) ( \delta_{PED} \rightarrow \dot{c} ) SCAS</td>
<td>During latter stages of approach use ( \delta_{LAT} ) for lateral corrections with basically constant heading automatically controlled through pedal FFS</td>
</tr>
<tr>
<td>DIRECT REQUIREMENTS</td>
<td>CRITICAL DISTURBANCES AND COMMAS</td>
<td>IMplied REQUIREMENTS</td>
<td>COMPETING SYSTEMS</td>
<td>COMMENTS</td>
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<tr>
<td>Localiser Capture</td>
<td>Crosswind from side of capture</td>
<td>Smooth transition from heading hold to localiser track</td>
<td>Switch when $\psi = 0$ SCAS</td>
<td>$\phi \rightarrow \phi_{LAT}$ SCAS</td>
<td></td>
</tr>
<tr>
<td>Curved Localiser Capture and Track</td>
<td>Tailwind at capture Tailwind at straight course intercept Low values of $\beta$ for given approach speed, e.g., $V_{AS}/\beta_{DF}$</td>
<td>Switch $\psi$ sec before before curved course intercept $\psi_{LIN} = 10^\circ$ Path damping all with derived and complementary filtered $\psi$</td>
<td>$\phi = V_{AS}/\beta_{DF}$ Feedforward $\psi_{LAT}$ IAS $\rightarrow \phi_{LAT}$</td>
<td>$\phi = \phi_{LAT}$ SCAS</td>
<td></td>
</tr>
<tr>
<td>Localiser Track (Before Final Deceleration)</td>
<td>Crosswind shear Crosswinds</td>
<td>All inner loops must be washed out to avoid standoffs (except $\phi$)</td>
<td>$\phi = \phi_{LAT}$ (attitude command) or $\phi = \phi_{LAT}$ SCAS (rate command) or $\phi = \phi_{LAT}$ SCAS $\psi = \psi_{LOC}$ SCAS $V_{AS} = V_{LOC}$ SCAS $\psi_{LAT}$</td>
<td>$\phi = \phi_{LAT}$ SCAS $\psi = \psi_{LAT}$ SCAS $V_{AS} = V_{LOC}$ SCAS</td>
<td></td>
</tr>
<tr>
<td>Localiser Track (During Final Deceleration)</td>
<td>Crosswind shear Crosswinds</td>
<td>All inner loops must be washed out to avoid standoffs Minimize shear offsets Disallow large heading rate excursions when approaching hover in a crosswind ($\psi$ or $\phi$ limit)</td>
<td>$\phi = \phi_{LAT}$ SCAS with $\psi$ limit reduced to $10^\circ$ $\psi_{LAT} = 15$ deg/sec $\psi = \psi_{LAT}$ SCAS (rate command) or $\phi = \phi_{LAT}$ SCAS (attitude command)</td>
<td>$\phi = \phi_{LAT}$ SCAS $\psi_{LAT} = 100^\circ$ $\phi = \phi_{LAT}$ SCAS $\psi = \psi_{LAT}$ SCAS</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Need high quality servo for $\phi$ Washed-out director is less complex to mechanize but will not follow very small radius paths due to initial overshoot Airspeed hold shall be automatic Active controls will be $\beta_{DF}$ and $\phi_{LAT}$ Cannot have $\phi_{LOC}$ SAS if System B is selected</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>System B is generally higher risk Can track LOC with wing low or conventional. Wing low popular but crosswind limited. Pilots selected wing low at Longley and Calypso.</td>
<td></td>
</tr>
<tr>
<td>DIRECT REQUIREMENTS</td>
<td>CRITICAL DISTURBANCES AND COMMANDS</td>
<td>IMPLIED REQUIREMENTS</td>
<td>COMPETING SYSTEMS</td>
<td>COMMENTS</td>
<td></td>
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<tr>
<td>---------------------</td>
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<td></td>
</tr>
<tr>
<td>Glide Slope Capture</td>
<td>Vertical gusts, Tailwind</td>
<td>Smooth transition from altitude hold to glide slope track</td>
<td><strong>SYSTEM A</strong></td>
<td>Airspeed control is automatic except for trim changes which exceed SCAS authority. System B was used successfully (see Ref. 6). Tentatively use B.</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$K_d + K_d = 0$ is switch point</td>
<td>PARALLEL</td>
<td>SERIES</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Step $b_{CL} \rightarrow F_{CCL}$ (set up trim collective)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\theta, \phi \rightarrow b_{LM}$ SCAS</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Glide Slope Track</td>
<td>Decreasing headwind shear</td>
<td>Good $\frac{\dot{h}}{b_{CL}}$ bandwidth, e.g., $\frac{1}{T_{z2}}$ large</td>
<td><strong>SYSTEM B</strong></td>
<td>Use backside control technique at all speeds.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Decreasing tailwind shear</td>
<td>$\frac{b_{CL}}{\ddot{h}} \frac{\dot{h}}{\dot{b}<em>{CL}}$ $\frac{2\dot{b}</em>{CL}}{b_{CL} \ddot{b}<em>{CL} + \frac{1}{T</em>{z2}}}$</td>
<td>PARALLEL</td>
<td>SERIES</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Good margins from $\gamma_{max}$ and $\gamma_{min}$. $\gamma_{min}$ is expected to be limiting factor on FD A due to low acceleration capability. No requirement to convert control technique. Airspeed command to allow deceleration if glide slope.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\theta, \phi \rightarrow b_{LM}$ SCAS</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$I_{AS} \rightarrow b_{LM}$ FFS</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\tilde{b}<em>{CL} \rightarrow b</em>{CL}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$I_{AS} \rightarrow b_{LM}$ FFS</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Deceleration to Hover</td>
<td>Decreasing headwind shear</td>
<td>Same as above plus following</td>
<td><strong>SYSTEM A</strong></td>
<td>Use constant $\dot{b}$ deceleration profile with $\dot{X} + \dot{X}_M = 0$ at the end.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Decreasing tailwind shear</td>
<td>Account for large trim shift in $b_{CL}$</td>
<td>PARALLEL</td>
<td>SERIES</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Deceleration profile acceptable to pilots</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\dot{b}<em>{CL} \rightarrow F</em>{CCL}$ for trim shift</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\theta, \phi \rightarrow b_{LM}$ SCAS</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\tilde{b}<em>{CL} \rightarrow b</em>{CL}$ FFS</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\dot{X} + \dot{X}<em>M \rightarrow b</em>{LM}$ FFS at very end</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hover</td>
<td>Horizontal winds</td>
<td>Need $\dot{b}<em>{C}$ and $\dot{b}</em>{C}$ attitude systems Good $\frac{\dot{h}}{b_{CL}}$</td>
<td><strong>SYSTEM B</strong></td>
<td>System A - $\dot{b}<em>{LM}$ and $\dot{b}</em>{LM}$ are used to command translational rates. System B - $\dot{b}<em>{LM}$ and $\dot{b}</em>{LM}$ are conventional $\dot{b}<em>{C}$ and $\dot{b}</em>{C}$, see langley, Calaparan.</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$x \rightarrow F_{CL}$</td>
<td>PARALLEL</td>
<td>SERIES</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$y \rightarrow F_{LAT}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\dot{b}<em>{TRIM} \rightarrow F</em>{CCL}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\dot{h} \rightarrow b_{CL}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\tilde{x}, \dot{y}, \phi \rightarrow F_{CCL}$ SCAS</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\tilde{y}, \dot{y}, \phi \rightarrow b_{LM}$ FFS</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vertical Descent</td>
<td>Horizontal winds</td>
<td>Flare low</td>
<td><strong>SYSTEM A</strong></td>
<td>Use exponential flare. Altitude automatic with pilot &quot;tweaker&quot; on $b_{CL}$. Pilot controls $X, y$.</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\dot{x}, \dot{y}, \phi \rightarrow b_{LM}$</td>
<td>PARALLEL</td>
<td>SERIES</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\tilde{h} \rightarrow b_{LM}$ FFS</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
TABLE 5

CONSIDERATIONS AFFECTING CHOICE OF LATERAL PATH
CONTROL TECHNIQUE (STRAIGHT LOCALIZER)
(From Refs. 8 and 9)

<table>
<thead>
<tr>
<th>WING LOW</th>
<th>CRAB</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Pro:</strong> Flight test showed pilots controlled localizer with greater precision using wing low. Consistently selected wing low shortly after intercepting localizer (pages 11 and 12 of Ref. 8.)</td>
<td><strong>Pro:</strong> Crab can handle large cross-winds. Only way to descend vertically and land in significant wind is to point into the wind if $Y_v$ is significant.</td>
</tr>
<tr>
<td><strong>Con:</strong> Wing low limited to ±5 deg* of bank for pilot acceptance (about 10-15 kt† crosswind). Pilots show some concern for steady lateral acceleration. Concerned about lack of excess control capacity when at 4 or 5 deg bank.</td>
<td><strong>Con:</strong> Precise heading control is very difficult at low speed. Small deviations in $\varphi$ result in large $\psi$ excursions since $r = g\varphi/V$.</td>
</tr>
</tbody>
</table>

**Conclusion:** Best solution is to use crabbed approach for higher speeds wherein precision of heading control is not a problem and wing low at lower speeds. At lower speed $Y_v$ is very small for the XV-15. This permits the wing low technique to be used without requiring large bank angles, and avoids the heading control problems associated with crabbed approaches at low speeds.

* $\deg = 1.745 \times 10^{-2}$ rad.
† $1 \text{kt} = 0.5144 \times 10^{-1} \text{m/s}$.

D. SUMMARY

The longitudinal SCAS is a rate-command/attitude-hold system for all phases of the approach. Analysis of each phase of the approach has shown that attitude command is actually closer to optimum in terms of meeting the pilot-centered and guidance and control requirements. In addition, it minimizes system complexity. The decision to use rate-command/attitude-hold is based entirely upon requirements imposed by the limited authority series servo. A 2 kt/sec (1 m/s) wind
shear for 15 sec requires a series actuator displacement of 2 in. (0.05 m) for an attitude command system, which is double the existing authority. The rate command system requires only 1/2 in. (0.013 m) series actuator displacement for the same input. The basic deficiency of the rate command system is that it adds an additional integration to the effective aircraft dynamics, requires more complex equalization, and produces, in some cases, less desirable flight director and aircraft response characteristics. Points favoring the rate command system include the elimination of trim requirements (with a corresponding decrease in pilot workload) and considerable pilot acceptance in recent years in test aircraft.

The lateral SCAS has been configured as a rate-command/attitude-hold system for all flight conditions from cruise to hover, vertical descent and touchdown. It is a well-established fact that attitude command is superior to rate command during hover. However, lateral attitude command at significant forward speed is less desirable than rate command because of the need to hold a constant lateral stick input for the wing-low mode or for the curved path tracking mode. Hence, it would be necessary to use rate command during the approach and phase into attitude command at some point approaching hover. To be successful such blending would require a series of manned simulator experiments to establish its pilot acceptability. These experiments are beyond the scope of this effort.

In the final analysis, the low authority series serves [±1 in. (±0.025 m)] of travel] effectively eliminates all but one manual control system (rate command, attitude hold). It is therefore not possible to exercise the full range of usual considerations of guidance and control and pilot-centered requirements in the competing systems tradeoff analyses. In fact, there is considerable evidence indicating that rate command is among the least desirable SCAS configurations for precision IFR hovering. This results in the difficult position of having to optimize a system within given constraints which are known to lead to a fundamentally deficient manual control system. Based on currently available data Cooper-Harper ratings on the order of 4 to 6 are expected for hovering in turbulence under instrument meteorological conditions (IMC). An advance in the current state of the art would require a tradeoff between an attitude command and a translational rate command (TRC) SCAS for hover. Issues such as consonance
between pitch attitude and stick position with a TRC SCAS in a helicopter would represent the key tradeoffs if actuator authority were not the limiting factor. A good TRC SCAS would be expected to yield pilot ratings on the order of 2 to 3 for hovering in IMC in turbulence.

The system requirements developed in this section lead to the selection of the "best" longitudinal and lateral-directional systems which are analyzed in detail in Sections III and IV, respectively.
SECTION III
LONGITUDINAL SYSTEM DESIGN

A. SYSTEM SUMMARY

A summary of the longitudinal system is given in the block diagram in Fig. 2. The feedback path equalization and switching logic shown in Fig. 2 includes provision for the following modes:

- Altitude hold
- Constant speed approach on MLS flight path
- Deceleration on MLS flight path
- Hover on MLS flight path
- Hover at constant altitude
- Vertical descent to touchdown

A summary of the gains and time constants for the longitudinal system is given in Table 6.

The basic design philosophy is to treat the flight director design as primary. The automatic system design is then obtained by a simple replacement of the pilot block in Fig. 2 with an automatic system gain. The gain values labeled \( K_c \) in Table 6 reflect the value to be inserted for the \( Y_{PC} \) block in the automatic mode. The collective axis is fully automatic throughout the approach.

Plots of open-loop flight director frequency response to longitudinal cyclic are shown for each mode throughout this section to illustrate the degree to which the desired K/s frequency response characteristic has been achieved. The airplane plus flight director sensitivity (K) was not optimized at the time these plots were made. Reference 10 has established that the optimum K is 0.4 inches/sec of flight director bar deflection per inch of stick displacement. The values of \( K_{FDc} \) in Table 6 reflect the final adjustment necessary to achieve a 0.4/s flight director. The autopilot gain \( K_c \) is simply the pilot gain used for the closures in this section adjusted for the final \( K_{FDc} \).
Figure 2. Longitudinal System Block Diagram
| Table 6. System Gains and Time Constants |

**1. Rate Command/Attitude Hold SCAS (All Modes)**
- $K_\theta = 12$ in./rad
- $K_\delta = 8$ in./(rad/sec)
- $\alpha = 1$ (1/sec)
- $K_{\delta c} = 2.09$ (1/sec)

**2. Constant Speed Glide Slope Tracking, $X < X_0$**
- $Tv_\theta = 5$ sec = $1/\alpha_\theta$
- $Tv_\alpha = 0.67$ sec = $1/\alpha_\alpha$
- $Tv_\delta = 10$ sec
- $T_{FD1} = 0.222$ sec = $1/\alpha_{FD1}$
- $T_{FD2} = 0.222$ sec = $1/\alpha_{FD2}$
- $k = 7.5$
- $K_{\delta c} = c_{FD}(0)K_c = 3.2$
- $T_d = 0.25$ sec = $1/\alpha_{T_1}$
- $K_d = 0.01$ in./ft
- $K_{\delta d} = \frac{0.01}{\sqrt{V_{L_2}^2 + 0.0329V_{L_2}}} \text{ in./}(\text{ft/sec})$
- $K_{\delta d} = 1.0$ sec
- $K_{FDc} = 1.0$ in./rad
- $K_{CLO} = 5.0$
- $K_{ICL} = 1.0$ (1/sec)
- $K_{d} = -0.005 \text{ rad/}(\text{ft/sec})$

**3. Deceleration to Hover, $X_0 < X < X_f$**
- $T_{CLO} = 1$ sec = $1/\alpha_\theta$
- $T_{FD1} = 0.222$ sec = $1/\alpha_{FD1}$
- $T_{FD2} = 0.222$ sec = $1/\alpha_{FD2}$
- $K_{\delta d} = 0.5$ sec
- $K_{FDc} = 2.52$ in./rad
- $K_d \{ \text{ Same as for } X < X_0 \}$
- $K_{CLO}$
- $T_d = 0.25$ sec

**4. Hover, $X \geq X_f$**
- $Tv_\theta = 4.0$ sec = $1/\alpha_\theta$
- $T_{FD1} = 0.5$ sec = $1/\alpha_{FD1}$
- $T_{FD2} = 0.222$ sec = $1/\alpha_{FD2}$
- $K_{\delta d} = 1.33$ sec
- $K_x = 16 \text{ sec}^2$
- $K_x = 8 \text{ sec}$
- $K_x = -0.005 \text{ rad/ft}$
- $K_{FDc} = 1.8$ in./rad
- $K_{FDc} = 0_{FD}(0)K_c = 3.2$
- $K_d = K_{\delta d} = 0.01$ in./ft
- $K_d = K_{\delta d} = 0.053 \text{ in./}(\text{ft/sec})$
- $K_{CLO} = 5.0$
- $K_{ICL} = 1.0$ (1/sec)

**5. Vertical Descent**
- $Tv_\theta$, $T_{FD1}$, $T_{FD2}$
- $K_{\delta d}, K_x, K_x, K_y$
- $K_{FDc}, K_{FDc}, \text{ and } K_c$
- $K_d$ and $K_{\delta d}$
- $K_{CLO}$
- $K_{ICL} = 1.0$ (1/sec)
- $K_{CLO}$
- $K_d = 7 \text{ ft} (16 \text{ ft in performance evaluation})$
The remainder of this section contains a description of each of the modes of the longitudinal system, and includes a brief account of the rationale used to select equalization feedbacks and time constants.

A basic design concept for this system has been to limit the number of active flight directors to one in the longitudinal axis (and one in the lateral axis). This design concept follows directly from the pilot-centered requirement for frequency separation of controls (discussed in more detail in Ref. 10). Furthermore, a single control should remain primary throughout the approach. That is, the same control should be primary from glide slope intercept to touchdown. Longitudinal and lateral cyclic have been designated the primary controls for this design. This turns out to be a fortunate choice since the possibility of adding a series servo for the collective axis is currently unlikely. However, a strategy for including the collective control as an inherent part of the manual mode has been included to provide a backup position. In the event that a series actuator should become available for the collective axis, the manual flight director signal for this axis is available directly in the current system design.

B. LONGITUDINAL SCAS

The longitudinal SCAS is a rate-command/attitude-hold system for all phases of the approach.

The stick shaping equalization shown in Fig. 2 was designed to produce a pure rate command in the presence of a mechanical parallel path. The forward loop integrator, \((s + a)/s\), is required to get good mid- and low-frequency attitude responses. The mid- and low-frequency attitude response of the basic XV-15 is poor in the extreme. The feedback gains \(K_g\) and \(K_j\) are set to achieve an attitude bandwidth of between 1.5 and 2 rad/sec. This corresponds to a \(K_g\) of 0.2 in. (0.005 m) of series servo travel per degree of attitude. Since the maximum series servo authority is 1 in. (0.025 m), SCAS limiting will occur at 5 deg (0.09 rad) of pitch attitude deviation from trim. This is expected to be marginal but acceptable. Lower values of pitch attitude feedback gain would minimize the limiting problem at the expense of unacceptably low attitude bandwidth. The frequency response characteristics in attitude with and without
the SCAS are shown in Fig. 3 for 60, 40, 20 and 0 kt (31, 21, 10 and 0 m/s). It is not necessary to change the gain as a function of flight condition to achieve these results. The rate command feature of the SCAS is obtained by setting $K_{c}$ in Fig. 2 equal to 2.09. This results in 10 deg/sec (0.17 rad/sec) of pitch attitude per inch (0.025 m) of longitudinal cyclic stick deflection. This control sensitivity was found to be optimum in the study done in Ref. 10.

C. CONSTANT SPEED GLIDE SLOPE TRACKING

The feedbacks which are active (switched in) for constant-speed glide slope tracking are listed below:

- **Washed-out pitch attitude** — Pitch attitude is required for flight director equalization and is washed out to avoid standoffs between the indicated airspeed and trim attitude.

- **Pitch attitude rate** — This feedback is also required for flight director equalization. Roll stabilized pitch rate must be used (that is, $\dot{\theta}$ in distinction to $\dot{q}$) to avoid large pitch-down commands during turns.

- **Airspeed feedback** — The airspeed feedback has been lagged by $1/T_u$ to keep high-frequency gusts from exciting the system. This in turn results in unacceptably low phase margin in the flight director loop. To overcome this, longitudinal acceleration-independent-of-pitch is pseudo-integrated and complemented with lagged airspeed. For a perfect complementary filter, the constant $k$ is set equal to unity (see Fig. 2 at right-hand side). If $k$ is greater than 1, a lead-lag on the speed feedback results; and if $k$ is less than 1 a lag-lead results. Flight director equalization requirements dictate a lead-lag corresponding to $k = 7.5$.

The frequency response characteristics of the longitudinal flight director to longitudinal cyclic input are shown for 60 and 40 kt (31 and 21 m/s) in Fig. 4.* This figure indicates that the desired $K/s$ response is achieved up to about 2 rad/sec at 60 kt (31 m/s) and 3 rad/sec at 40 kt (21 m/s). While it would be desirable to have the $K/s$ region extend to a higher frequency, espe-

*Figure 4 has root locus information overplotted on the frequency response, $G(j\omega)$. This root locus information plots the modulus for each closed-loop root (read on the frequency axis) as a function of the inverse of the open loop gain (read on the dB magnitude axis). The interested reader can find a more detailed exposition on pages 153-155 of Ref. 14.
Figure 3. Frequency Response Characteristics of the Attitude Hold Feature of the Rate Command/Attitude Hold SCAS
Figure 3b. Open-Loop Frequency Response Characteristics for Pitch Attitude in Response to Longitudinal Cyclic Pitch
Figure 4. Open-Loop Response Characteristics for Longitudinal Cyclic Flight Director
cially at 60 kt (31 m/s), it is felt that the current system will be adequate. Extending the K/s region to higher frequency would require increased SCAS bandwidth which is currently limited by the series servo authority. Therefore bandwidth extension is not possible with the current system.

The requirements imposed on the longitudinal flight director to achieve regulation of the glide path with collective while pitch attitude is constrained by means of the longitudinal cyclic control are best illustrated by the following equation (from Ref. 7):

$$\frac{\dot{d}}{\delta_{CL}} = \frac{N^{\delta}_{IN5CL}}{N^{\delta}_{CL}} = \frac{A_d\delta \left(s + \frac{1}{T_{d\delta}}\right)}{(s + \frac{1}{T_{\delta_1}})(s + \frac{1}{T_{\delta_2}})}$$

(1)

An examination of the transfer functions in Table 7 reveals the following facts concerning the above equation:

- $\frac{1}{T_{\delta_1}}$ cancels $\frac{1}{T_{d\delta}}$ at all flight conditions so that the form of $\frac{\dot{d}}{\delta_{CL}}$ is

$$\frac{\dot{d}}{\delta_{CL}} = \frac{A_d\delta}{(s + \frac{1}{T_{\delta_2}})}$$

(2)

- The bandwidth of $\frac{\dot{d}}{\delta_{CL}}$ is defined by $\frac{1}{T_{\delta_2}}$ and is unacceptably low at 0 and 20 kt (0 and 10 m/s) and is marginal at 40 and 60 kt (21 and 31 m/s).

- Augmentation of $\frac{1}{T_{\delta_2}}$ will be achieved by feedback of beam rate (approximately vertical velocity) to the collective control. This indicates that the fully automatic collective control will act simultaneously as a SCAS to augment $\frac{1}{T_{\delta_0}}$, and as a path control function to regulate the glide slope errors to zero by virtue of beam deviation feedback to the collective control.

The transfer functions for flight path angles of -6 and -10 deg (-0.10 and -0.17) at 40 and 60 kt (21 and 31 m/s) (see Table 7) are essentially identical, indicating that the effect of flight path angle on vehicle dynamics is small and can be neglected.
## Table 7. XV-15 Transfer Functions*

(HELICOPTER MODE ONLY)

<table>
<thead>
<tr>
<th>F.C.</th>
<th>57-0</th>
<th>58-6</th>
<th>59-6</th>
<th>60-6</th>
<th>61-6</th>
<th>59-10</th>
<th>61-10</th>
</tr>
</thead>
<tbody>
<tr>
<td>V (at)</td>
<td>0</td>
<td>20</td>
<td>40</td>
<td>60</td>
<td>80</td>
<td>80</td>
<td>80</td>
</tr>
<tr>
<td>γ (deg)</td>
<td>0</td>
<td>-6</td>
<td>-6</td>
<td>-6</td>
<td>-10</td>
<td>-10</td>
<td>-10</td>
</tr>
<tr>
<td>αLCN</td>
<td>(-4) (-5) [-29, -28]</td>
<td>(-11, -13) [-03, -33]</td>
<td>(-11.19) [-61, 147]</td>
<td>(-10) (2.1) [-18.19]</td>
<td>(-0.9) (-0.9) [-0.9, 1.51]</td>
<td>[-0.9, 0.8] [-7, 1.21]</td>
<td>[-0.8, -0.5] [-39, 1.14]</td>
</tr>
<tr>
<td>αULN</td>
<td>-28 (-0.06) (2)</td>
<td>-31 (-01) (1.5)</td>
<td>-36 (-0.06) (32)</td>
<td>-41 (-0.06) (33)</td>
<td>-46 (-0.05) (39)</td>
<td>-40 (-0.04) (35)</td>
<td>-45 (-0.04) (35)</td>
</tr>
<tr>
<td>αRTL</td>
<td>1.2 (-2) [-08, 2.71]</td>
<td>1.2 (-2) [-31, 3.02]</td>
<td>1.06 (-46) [-47, 3.44]</td>
<td>0.95 (-50) [-38, 3.5]</td>
<td>0.79 (-76) [-09, 4.1]</td>
<td>1.23 (-47) [-43, 3.08]</td>
<td>0.71 (-69) [-14, 4.5]</td>
</tr>
<tr>
<td>α2COL</td>
<td>4.06 (-57) [-20, 28]</td>
<td>4.01 (-07) [-47, 1.04]</td>
<td>4.00 (-09) [-34, 1.87]</td>
<td>4.51 (-56) [-55, 8.09]</td>
<td>4.82 (-18) [-31, 3.35]</td>
<td>4.01 (-08) [-61, 1.02]</td>
<td>1.81 (-15) [-27, 25]</td>
</tr>
<tr>
<td>α2LNL</td>
<td>0.06 (-2.22) [-41, 1.44]</td>
<td>-0.17 (-0.91) [-09, 3.4]</td>
<td>-0.18 (-20) [-11, 4.63]</td>
<td>-0.19 (-0.06) [-18, 4.71]</td>
<td>-0.00 (-0.06) [-17, 5.67]</td>
<td>-0.67 (-0.30) [-11, 4.2]</td>
<td>-1.30 (-0.30) [-13, 5.4]</td>
</tr>
<tr>
<td>α2LNL3</td>
<td>-21 (-13) [-06]</td>
<td>-12 (-0.104)</td>
<td>-1.47 (0.024)</td>
<td>-2.3 (0.04)</td>
<td>-1.37 (0.06)</td>
<td>-1.37 (0.06)</td>
<td>-2.3 (0.06)</td>
</tr>
<tr>
<td>βLAT</td>
<td>-1.10 (-0.5) [-13, 81]</td>
<td>-1.19 (-0.6) [-08, 61]</td>
<td>-2.1 (1.14) [-73, 26]</td>
<td>-0.05 (1.42) [-05, 47]</td>
<td>-0.05 (1.29) [-17, 60]</td>
<td>-0.08 (1.15) [-10, 25]</td>
<td>-0.05 (1.19) [-05, 75]</td>
</tr>
<tr>
<td>βLNL</td>
<td>-30 (-21) [0.5]</td>
<td>-29 (-14, 51)</td>
<td>-3.3 (3.6) (-0.8)</td>
<td>-35 (-25) [-25]</td>
<td>-56 (-14, 68)</td>
<td>-29 (-27) [-35]</td>
<td>-2.1 (21) [-60]</td>
</tr>
<tr>
<td>βRTL</td>
<td>-0.02 (-3.7) [-17, 87]</td>
<td>-0.02 (-2.6) [-31, 1.2]</td>
<td>-0.02 (-1.0) [-30, 75]</td>
<td>-0.02 (-0.32) [-0.56, 56]</td>
<td>-0.016 (-7.4) [-06, 37]</td>
<td>-0.0006 (-07) [3.3]</td>
<td>-0.0005 (-0.15) [-47, 25]</td>
</tr>
<tr>
<td>β2COL</td>
<td>0.0660 [-0.07] [-37.38]</td>
<td>-0.003 (-1.2) [-19.7], 0.007 (-4.02) [-26.9]</td>
<td>-0.0007 (-1.1) [2.6] (-89)</td>
<td>-0.0007 (-0.062) [3.1] (-191)</td>
<td>-0.0004 (-0.39) [-5.5] (16.8)</td>
<td>x (-0.65)</td>
<td>-0.0007 (-0.13) [-0.3] x (-200)</td>
</tr>
<tr>
<td>β2LNL</td>
<td>0.21 (62) [-19, 1.93]</td>
<td>0.005 [-0.39] [1.1] (-27.4)</td>
<td>0.0060 [-0.16] [1.3] (-234)</td>
<td>0.001 (0.42) [1.55] (-161)</td>
<td>0.0006 (0.61) [24.2] (0.62)</td>
<td>0.0007 (-0.13) [-1.36] x (-200)</td>
<td>-0.0009 (-0.012) [-32] x (145)</td>
</tr>
<tr>
<td>β2LNL3</td>
<td>-1.12 (85) [-26, 45]</td>
<td>-1.13 (0.94) [-16, 30]</td>
<td>-1.15 (1.35) [-16, 37]</td>
<td>-1.0 (1.35) [-0.6, 37]</td>
<td>-0.00 (1.12) [-15, 38]</td>
<td>-0.14 (1.12) [-13, 37]</td>
<td>-0.13 (1.11) [-0.8, 34]</td>
</tr>
<tr>
<td>β2LNL2</td>
<td>-0.21 (-21) [-89] [6, 1.2]</td>
<td>-0.17 (-20) [1.9] [0.04, 1.25]</td>
<td>-0.14 (-21) [1.21] [-0.25, 93]</td>
<td>-0.04 (-1.59)</td>
<td>-0.32 (0.01) [1.2] (-1.8)</td>
<td>0.08 (-12) [1.24]</td>
<td>-0.11 (0.03) [-1.56]</td>
</tr>
<tr>
<td>β2LNL3</td>
<td>-0.25 [-16, 0.63]</td>
<td>-0.27 (11.2) [-21.3]</td>
<td>-1.19 (3.5) (48)</td>
<td>-0.16 (15.24)</td>
<td>0.05 (-17.6)</td>
<td>0.13 (26) [-34]</td>
<td>0.86 (-13.59)</td>
</tr>
</tbody>
</table>

*The shorthand transfer function notation used is $s + \frac{1}{s} \Rightarrow (1, 0)$ and $s^2 + 2K\omega_n + \omega_n^2 \Rightarrow [1, 0]$. 
The above discussion indicates that the beam-rate-to-collective transfer function can be approximated by a first-order lag located at \(1/T_{\theta 2}\). The collective flight director response to a collective control input can be approximated as follows:

\[
\frac{FD_{\text{CL}}}{\dot{\text{CL}}} = - \frac{Z_{\text{CL}}K_{\text{CL}}K_{\text{d}}T_{\text{d}}}{\text{M}_{\text{CL}}N_{\text{s}}} \left[ s^2 + \frac{1}{T_{\theta 2}} s + \frac{K_{\text{d}}}{K_{\text{d}}} \frac{1}{T_{\text{d}}} \right] \tag{3}
\]

From Refs. 6 and 9 a value of \(K = 0.01\) in. (0.00025 m) of flight director per foot (meter) of altitude error appears to give the appropriate sensitivity throughout the approach. The glide slope noise filter time constant \(T_{\text{d}}\) was set to 0.25 sec for noise rejection. \(K_{\text{d}}\) was set so that Eq. 3 could be factored as follows:

\[
\left[ s^2 + \frac{1}{T_{\text{d}}} s + \frac{K_{\text{d}}}{K_{\text{d}}} \frac{1}{T_{\text{d}}} \right] = \left( s + \frac{1}{T_{\theta 2}} \right) \left( s + \frac{1}{T_{\text{d}}} \right) \tag{4}
\]

thus providing the required zero at \(1/T_{\theta 2}\). The resulting values of \(K_{\text{d}}/K_{\text{d}}\) were plotted versus speed and fitted to obtain the following expression for \(K_{\text{d}}\):

\[
K_{\text{d}} = \frac{K_{\text{d}}}{0.19 + 0.005V_{\text{IAS}}} \quad V_{\text{IAS}} \text{ in kt} \tag{5}
\]

The above values for \(K_{\text{d}}, K_{\text{d}},\) and \(T_{\text{d}}\) result in a \(K/s\) open-loop collective flight director response at all frequencies. This is shown in Fig. 5 for 80 and 40 kt (41 and 21 m/s). While the collective axis is fully automatic in the current system, the basic approach of first designing the flight director was used to improve monitoring characteristics for this control axis and to allow for possible future low-frequency manual control of this axis.

Referring to Fig. 2, the block normally allocated to the pilot (just downstream of \(T\)) is shown as a constant \(K_{\text{CL}}\) and a parallel integration \(K_{\text{ICL}}\). The parallel integration is included in the forward loop to eliminate the
Figure 5. Open-Loop Response Characteristics of Collective Flight Director
possibility of standoffs. These automatic control paths provide control action which is equivalent to human pilot manual control action. That is, a pilot would not allow a constant flight director error to exist.

The speed control characteristics of the closed-loop longitudinal cyclic flight director system are checked by means of the speed-error-to-speed-command and speed-error-to-horizontal-gust frequency responses. A pilot model with a crossover frequency of 1.5 rad/sec and an effective delay of 0.17 sec is assumed for longitudinal cyclic control.

\[ G_{FPS}(0)Y_{pc} = 0.8e^{-0.17j\omega} \] (6)

The frequency response characteristics are shown in Fig. 6. These responses reveal the following facts:

- Speed responses to horizontal gusts and speed commands exhibit zero steady-state errors. (\( T_{uc} \) was set to zero to check this aspect of closed-loop system performance.)

- A bandwidth of about 0.12 rad/sec is achieved for speed command inputs. This bandwidth results from a tradeoff between conflicting objectives for tight speed control and minimum pitch attitude excursions. Aircraft with an additional control for controlling airspeed generally have higher speed control system bandwidth; however, a bandwidth of 0.12 rad/sec is well within the acceptable range for an aircraft using pitch attitude to control airspeed.

- Maximum gust sensitivity exists between 0.12 and 2.0 rad/sec. This reflects the selection of the airspeed feedback time constant of 0.67 sec at the upper end and the basic airspeed bandwidth of 0.12 rad/sec at the lower end.

The frequency response characteristics of the glide path control system are shown in Fig. 7 and indicate a bandwidth of about 0.85 rad/sec. This is a fairly tight glide slope system.

The final performance metric utilized to evaluate the system design is a critical wind shear disturbance taken from Ref. 11. This wind shear is 2 kt/sec (1 m/s) for 15 sec, followed by a steady 30 kt (15 m/s) wind.
Figure 6. Performance Characteristics of Speed Control at 60 kt (31 m/s)
Figure 7. Frequency Response Characteristics of Glide Path Control at 60 kt (31 m/s)

\[ G_{PPS}(s) = \frac{8e^{-17s}}{s}, \quad Y_C = \frac{5(s+1)}{s} \]
To put this in perspective, Ref. 11 indicates that this shear resulted in marginal glide path control for a DHC-6 Twin Otter Aircraft. The time histories of the system response to this wind shear are shown in Fig. 8 which is subject to the following interpretation:

- The maximum speed error was only 9.5 ft/sec (2.9 m/s) out of a total 51 ft/sec (155 m/s) disturbance.
- The peak pitch attitude excursion was about \(-0.1\) rad or \(-5.7\) deg.
- The maximum series servo excursion was 0.5 in. (0.013 m) (half its total travel).
- The peak glide path excursion was only 1 ft (1.2 m).
- The collective control moved to maximum of 1.9 in. (0.05 m) or less than half its total travel.

D. DECELERATION ON GLIDE SLOPE

Final deceleration to hover is a constant attitude maneuver in order to minimize pilot workload. Inasmuch as the attitude response to a step longitudinal cyclic input is a pitch rate, equalizing the flight director to a K/s response is a simple matter. Attitude and attitude rate feedbacks are used to extend the K/s region as far as possible beyond the closed-loop short-period frequency of the rate-command/attitude-hold SCAS. Open-loop flight director responses to longitudinal cyclic inputs for 20, 40 and 60 kt (10, 11 and 31 m/s) are shown in Fig. 9. The method used to compute the incremental pitch attitude command (\(\Delta \phi_{\text{decel}}\) in Fig. 2) is discussed later in this section. Inasmuch as the pitch attitude is the state variable being commanded during this phase of the approach, it is inappropriate to wash out the pitch attitude feedback to the flight director. Therefore, it was necessary to include the switching logic shown in Fig. 2 to remove the washout equalization on pitch attitude during the deceleration phase. The rationale for the switching logic shown in Fig. 2 is discussed as follows:
Figure 9. Open-Loop Flight Director Frequency Response, Deceleration to Hover.
1. \( X < X_o \) (constant-speed approach phase)
   - Attitude is washed out via \( G_{\theta_1} \).
   - Attitude is fed to both sides of the pitch attitude limiter to account for washout in the feedback.

2. \( X \geq X_o \) (deceleration phase)
   - The input to \( G_{\theta_1} \) goes to zero rapidly. This will excite the washout circuit (looks like an input of the opposite sign) resulting in a transient output from \( G_{\theta_1} \). Therefore, a switch is included downstream of the washout equalization to avoid this transient.
   - The switch at the input of \( G_{\theta_2} \) moves to the decel position. The synchronizer at the input of \( G_{\theta_2} \) eliminates any transient at the switch point.
   - The initial input to \( G_{\theta_2} \) is zero and becomes \( \theta - \theta_{o_2} \) \((\theta_{o_2} = \theta \text{ at } x = X_o)\) for \( x > X_o \).
   - The commanded value \((\Delta \theta_{\text{decel}})\) must be referenced to the output of the synchronizer \( \theta_{o_2} \). Since \( \theta_{o_2} \) is the last value of pitch attitude before switching from constant speed flight, \( \theta_{o_2} \) represents the nominal pitch attitude for zero acceleration along the glide path at the trim approach speed. The constant attitude for deceleration \((\Delta \theta_{\text{decel}})\) is blended in via the \( M \) function shown in Fig. 2. This is done primarily to avoid sudden pitch transients when the system is in the automatic mode. Note that as the \( M \) function approaches unity, the speed feedback approaches zero due to the \((1 - M)\) function in that feedback path.

3. \( X \geq X_f \) (hover phase)
   - The washout circuit \( G_{\theta_1} \) is switched back in and the nominal attitude reference is \( \theta_{o_1} \). \( \theta_{o_1} \) is the nominal total pitch attitude used during the deceleration phase and is also defined as \( \theta_f \).
   - \( G_{\theta_2}, \Delta \theta_{\text{decel}} \) and \( K_u \) are switched out.
   - The longitudinal position feedback for hover control is blended in via the \( N \) function.

The switching ranges, \( X_o \) and \( X_f \), are specified later in subsection H.2 of this section.

Power required to maintain a constant glide slope increases drastically during the deceleration to hover as the airspeed falls below 60 kt (31 m/s) (see Fig. 10). This fact results in initial excursions below the glide path.
Figure 10. $\gamma$-V Plot for XV-15 Aircraft
(in the absence of anticipatory control action) which are ultimately eliminated by the closed-loop action of collective system in Fig. 2. Experience has shown that the initial excursions can be unacceptably large (see Ref. 6). Therefore, an airspeed crossfeed signal ($\Delta^c_{CL}$ in Fig. 2) has been designed to add collective in accordance with the power-required characteristics of the XV-15. This is akin to the pilot's usual precognitive action to increase collective in a helicopter approaching hover. Review of the XV-15 performance data indicated that the $\Delta^c_{CL}$ required is essentially the same for glide path angles -6 to -10 deg (-0.10 to -0.17 rad). The power increase required is significantly greater for the heavier weights and varies directly with the density ratio $\rho_o/\rho$.

Straight-line approximations to the power-required curves yield the following:

$$\Delta^c_{CL} = \frac{\rho_o}{\rho} [K(W)](101.4 - V_{IAS})$$  \hspace{1cm} (7)

$$K(W) = 0.021 \frac{\text{in}}{\text{ft/sec}} \left( \frac{0.13 \text{ cm}}{\text{m/sec}} \right); \hspace{1cm} W = 10,000 \text{ lb (4536 kg)}$$

$$K(W) = 0.033 \frac{\text{in}}{\text{ft/sec}} \left( \frac{0.13 \text{ cm}}{\text{m/sec}} \right); \hspace{1cm} W = 13,000 \text{ lb (5871 kg)}$$

$$K(W) = 0.053 \frac{\text{in}}{\text{ft/sec}} \left( \frac{0.13 \text{ cm}}{\text{m/sec}} \right); \hspace{1cm} W = 15,000 \text{ lb (6804 kg)}$$ \hspace{1cm} (8)

$\Delta^c_{CL}$ in in. (cm) of collective; $V_{IAS}$ in ft/sec (m/sec) \hspace{1cm} (9)

The straight-line approximation is least accurate at the 10,000 lb (4536 kg) weight where the peak error is about 70 percent at 50 kt (15 m/s). This is of little practical consequence as the closed-loop glide-slope-to-collective system tends to suppress small errors due to improper trim power. Wind and wind shear will also require small trim power increments which must be developed via closed-loop regulation.
E. HOVER

There are numerous references (e.g., Refs. 12 and 13) which illustrate that hover control is best accomplished with an attitude command system. However, as stated earlier, the limited authority XV-15 series actuator characteristics preclude using an attitude command system. The active feedbacks during hover control are washed-out pitch attitude, pitch attitude rate, range rate, range and body-fixed longitudinal acceleration. These feedbacks are weighted to produce a K/s frequency response characteristic for the longitudinal cyclic flight director as shown in Fig. 11. The lead-lag network included at the input to the flight director eliminates phase lag between the region where pitch attitude leaves off and longitudinal acceleration picks up to produce a K/s response. (Neither shaping nor body-fixed longitudinal accelerometer feedback is required to produce an ideal K/s response in the design of the system with the attitude hold SCAS.)

The longitudinal position holding characteristics in hover are shown by the frequency responses of position error to position command and position error to horizontal gusts in Fig. 12. The pilot model used is:

\[ G_{TF}(s)Y_{PC} = 1.0e^{-17\omega} \]  

(10)

The position control bandwidth is about 0.35 rad/sec. This is well within the acceptable range. The position error response to horizontal gust characteristics indicates effective regulation against horizontal gusts at all frequencies and no tendency toward low-frequency standoff.

An indication of the attitude-to-position harmony characteristics of hover can be obtained from a physical interpretation of the hover position feedback gain \( K_x \) (see Fig. 2). This gain is indicative of the attitude commanded per unit position error. The system commands about 0.3 deg (0.005 rad) of pitch attitude per foot (0.30 m) position error. Thinking of it another way, 10 ft (3 m) of position error will result in a 3 deg (0.052 rad) pitch attitude command. Hence, the bandwidth of 0.35 rad/sec shown in Fig. 11 is not obtained at the expense of excessive pitch attitude excursions during hover.
Figure 11. Open-Loop Frequency Response of Longitudinal Cyclic Flight Director for Hover
Figure 12. Longitudinal Position Holding Performance Characteristics of Hover System
F. ALTIMETER HOLD IN HOVER

When the radar altimeter reads 50 ft (15 m), the collective system reverts to altitude hold. The beam deviation and deviation rate gains used for glide slope tracking during the terminal phases of the approach are equally appropriate for the altitude hold mode; hence, $K_h = K_d$ and $K_h = K_d$ in the altitude hold mode at 50 ft (15 m). The same forward loop gain and parallel integrator time constant are also used [$K_{OL} = 5 \text{ in./in. (}0.13 \text{ m/m)}$] and $K_{IOL} = 1.0 (1/\text{sec})$.

Precision altitude control in the hover mode is required to maintain pilot confidence in proximity to the ground. The characteristics of the altitude hold system are verified by the frequency response in altitude to vertical gusts and the altitude time response to a 15 ft/sec (4.6 m/s) vertical gust. These are shown in Fig. 13. The time responses in Fig. 13a indicate a peak altitude error of only 2.5 ft (0.76 m) and peak collective displacement of 1.0 in. (0.025 m). Figure 13b shows that the sensitivity to vertical gusts is low with no tendency for low-frequency standoff.

G. VERTICAL DESCENT

Once established in altitude hold over the hover point, the pilot manually initiates vertical descent. Inasmuch as the collective axis is fully automatic, the pilot simply continues to hold his longitudinal and lateral position with cyclic stick and monitors aircraft sink rate. The vertical descent mode is an exponential flare accomplished using collective control. The descent system uses a sink rate command which is proportional to altitude via the gain $K_h$ ($K_h$ is set to unity). At initiation of vertical descent an altitude command ($h_0 + H_c$ in Fig. 2) is introduced through the lag network $1/(s + 1)$. This lag prevents an abrupt down command at initiation of vertical descent which would be disconcerting to the crew. The shape of the descent profile depends on the altitude command $H_c$ and the exponential time constant $K_h$. $H_c$ is set equal to a few feet below the ground to insure a positive but not hard touchdown ($h$ between $-1.5$ and $-3 \text{ ft/sec (} -0.45 \text{ and } -0.92 \text{ m/s)}$). $K_h$ is set to achieve a reasonable descent profile. A maximum descent rate of 500 ft/min (152 m/min) was set somewhat arbitrarily. Also, a descent time of between 10 and 15 sec was chosen to
Figure 13a. Altitude Time Response to a 15 ft/sec (4.6 m/s) Vertical Gust, Altitude Hold During Hover

\[ Y_{cl} = \frac{5(s+1)}{s} \]

Figure 13b. Altitude Error Frequency Response for Vertical Gust Inputs During Hover
avoid prolonging the maneuver. Because of the relatively short duration of the maneuver, the parallel integrator in the forward loop of the collective system is not required. It is removed to eliminate the phase lag penalties introduced by this integrator. Removal is accomplished by fading P to zero at vertical descent initiation.

Time responses indicating performance of the vertical descent system in the absence of disturbances are given in Fig. 14. This descent profile results in a 14 sec descent time with a peak sink rate of 6 ft/sec (1.8 m/s) [360 ft/min (110 m/min)] and a nominal touchdown sink rate of 1.8 ft/sec (0.55 m/s). Deviations in sink rate (—d) are eliminated well before touchdown.

H. APPROACH LOGIC

The switching logic in Fig. 2 divides the approach into five basic segments: altitude hold, glide slope track, constant attitude deceleration, hover and vertical descent to touchdown.

1. Glide Slope Tracking

The altitude hold/glide slope track switching occurs when \( \epsilon_d \leq 0^* \) and \( |d_{\epsilon}| \leq 100 \text{ ft} (30 \text{ m}) \). This logic causes the system to switch from altitude hold to glide slope track without transients. A blending function \( T \) insures that undesirable transients do not occur at the switch point. Specifically, the glide slope intercept logic is as follows:

- **Altitude hold (AH)** when \( |d_{\epsilon}| \geq 100 \text{ ft} (30 \text{ m}) \) or when \( |d_{\epsilon}| \leq 100 \text{ ft} (30 \text{ m}) \) and \( \epsilon_d > 0 \).

- **Glide slope track (GS)** when \( 60 \leq V_A \leq 80 \text{ kt} (31 \text{ to } 41 \text{ m/s}) \) , \( |d_{\epsilon}| \leq 100 \text{ ft} (30 \text{ m}) \) and \( \epsilon_d > 0 \). Initiated by start of blender function, \( T \).

The glide slope tracking proceeds at constant airspeed until the point \( X_0 \) at which time a constant attitude deceleration is commanded.

\[ *\epsilon_d = K_d d + K_d d'. \]
Figure 14. Vertical Descent Profile in Calm Air
2. Constant Attitude Deceleration

The deceleration capability of the XV-15 can be obtained from the \( \gamma-V \) curves in Fig. 10 and the following interpretations. First, note that the specific force acting along the flight path may be written as:

\[
\alpha_x = \dot{V}_I + \beta \gamma = \dot{V}_I + \beta \gamma_a
\]  

where \( \dot{V}_I \) is inertial acceleration along the inertial flight path, \( \gamma \) is the inertial flight path angle, and \( \gamma_a \) is the aerodynamic flight path angle (angle between airspeed vector and horizon). Physical interpretation of the aircraft specific force capability in the down direction is made easier if \( \alpha_x \) is expressed as "minimum achievable trim flight path angle," e.g., \( \alpha_{x\text{min}} = g \gamma_{a\text{min}} \).

The available deceleration capability may therefore be read directly from Fig. 10 as \( \dot{V}_{I\text{min}}/g = \gamma_{a\text{min}} - \gamma_a \). For deceleration, the 20 percent power line defines the "minimum achievable trim" flight path angle (in the down direction).

An excellent approximation for deceleration as a function of pitch attitude is:

\[
\dot{V}_I = -g(\theta - \theta_T)
\]  

where \( \theta_T \) is the trim pitch attitude for the flight path angle and speed being flown. Since the lines of constant attitude are nearly vertical between \( \gamma = 0 \text{ deg} \) (0 rad) and \( \gamma = -10 \text{ deg} \) (0.17 rad), a unique relationship between trim pitch attitude and airspeed can be derived using Fig. 10. Furthermore, it turns out that this function may be approximated accurately a linear function:

\[
\theta_T = \theta_H + \frac{d\theta_T}{dV_a} V_a
\]
where $\theta_H = 3$ deg (0.052 rad) is the no-wind hover trim attitude, and

$$K_{\theta VA} = \frac{d\theta_T}{dV_a} = -0.117 \text{ deg} \text{ kt}^{-1} = -0.069 \text{ deg} \text{ ft/sec}^{-1} = -0.0004 \text{ rad} \text{ m/sec}^{-1} \quad (14)$$

The deceleration is therefore approximated as:

$$\dot{V}_I = -\theta_D (\theta_D - \theta_H - \frac{d\theta_T}{dV_a} V_a) \quad (15)$$

where $\theta_D$ is the constant attitude used for deceleration. If $\theta_D < \theta_H$, the aircraft will stop decelerating at some positive airspeed. Thereafter, the aircraft will proceed at (trim airspeed for $\theta_D +$ wind speed). This slow closure on the hover reference point is undesirable. Therefore, a deceleration attitude of one degree greater than the no-wind hover attitude is selected [$\theta_D = 4$ deg (0.070 rad)] to provide more rapid closure on the hover reference point. This results in the deceleration profile shown in Fig. 15.

![Figure 15. Nominal Deceleration Profile](image)

1 kt = 0.514 m/sec
1 deg = 0.0175 rad
For a constant wind situation \( \dot{X} = V_a + V_w \) the approach trajectory is defined as:

\[
\ddot{X} + K_2 \dot{X} = K_1
\]  

(16)

where

\[
K_1 = -g(\theta_D - \theta_H) - g \frac{d \theta}{dV_a} V_w
\]

\[
K_2 = -g \frac{d \theta}{dV_a}
\]

Solving for \( \dot{X} = f(X) \) yields:

\[
X = -\frac{1}{K_2} \left[ \dot{X} + \frac{K_1}{K_2} \ln \left( 1 - \frac{K_2 \dot{X}}{K_1} \right) \right]
\]

(17)*

Hence, the switch from constant speed glide slope tracking to constant attitude deceleration should occur when:

\[
\epsilon_\theta = -X + \frac{57.3}{g} \left\{ \dot{X} + \left[ \frac{(\theta_D - \theta_H)}{d \theta / dV_a} + V_w \right] \ln \left[ 1 - \frac{\dot{X}}{\left( \frac{\theta_D - \theta_H}{d \theta / dV_a} + V_w \right)} \right] \right\} \leq 0
\]

(18)

The first value of \( X \) satisfying Eq. 18 is defined as \( X_0 \). Note that the effect of a steady headwind in Eq. 16 or 18 is equivalent to a change in the deceleration attitude (1 deg (0.0175 rad) of \( \theta_D \) is the same as 8.5 kt (4.3 m/s) of headwind). The effect of steady winds (or equivalent \( \theta_D \) value) is shown in Fig. 16. An approximation to the hover control law \( \dot{X} = 0 \) is also plotted in Fig. 16 to indicate where the guidance strategy would switch from the constant attitude deceleration mode to the hover mode \( (X = X_f) \). For the large headwind cases this occurs at a large closing rate [50 ft/sec (15 m/s) for a 20 kt (10 m/s) headwind]. Since by its definition the hover control law commands deceleration proportional to closing rate \( \dot{X} + 8X = 0 \), the commanded

---

*This solution was developed by W. A. Johnson.
pitch attitude at $X_f$ could become excessive. To avoid this, the value of $\theta_D$ is adjusted to account for steady winds in order to follow more closely the no-wind trajectory in Fig. 16. This is accomplished by holding $K_1$ in Eq. 16 constant, e.g.

$$K_1 = -\frac{32.2}{57.3} \theta_{\text{INC}} = -32.2 \frac{(\theta_D - \theta_H)}{57.3} - \frac{32.2}{57.3} K_{\text{VA}} V_w$$

Hence

$$\theta_D = \theta_H + \theta_{\text{INC}} - K_{\text{VA}} V_w$$

$$\theta_H = 3 \text{ deg (0.052 rad)}, \text{ } K_{\text{VA}} = -0.06 \frac{\text{deg}}{\text{ft/sec}} \text{ } \left( -0.25 \frac{\text{deg}}{\text{m/sec}} \right)$$

where $V_w$ is in ft/sec (m/sec) and $\theta_D$ and $\theta_H$ are in degrees (radians). The value of $V_w$ can be estimated by computing $(\dot{x} - V_A)$; closure rate minus indicated airspeed.
The effect of wind shear during the constant attitude deceleration will be as follows:

- Decreasing headwind shear. The initial \( \dot{X} \) will be low due to the headwind resulting in initiation of deceleration at shorter range. \( \theta_p \) will be less than \( \frac{1}{2} \) deg (0.070 rad). As the headwind shears away, groundspeed will increase, resulting in intercept of the hover mode switch line at larger than normal closure rates. This might result in relatively large attitude requirements at \( X_f \). For example, if \( \dot{X} = 50 \text{ ft/sec} \) (15 m/s) at \( X_f \); the commanded attitude will be 11 deg (0.19 rad) prior to the attitude limiter.

- Decreasing tailwind shear. The initial \( \dot{X} \) will be high, resulting in initiation of constant attitude deceleration at increased range. \( \theta_p \) will be greater than \( \frac{1}{2} \) deg (0.070 rad). As the tailwind shears away groundspeed will decrease, resulting in a tendency to come to hover short of the target and the \( X + 8\dot{X} = 0 \) switch line. The hover control law will be initiated any time the closure rate \( (\dot{X}) \) decreases below 5.92 [10.0 ft/sec (3.05 m/s)] to avoid this problem.

The switching boundaries which define \( X_0 \) and \( X_f \) are shown in Fig. 17.

**Figure 17. Definitions of \( X_f \) and \( X_0 \)**
The equations to be mechanized are summarized as follows:

\[ X = X_0 \]

when

\[ X = \frac{57.3}{52.2 K_{\text{VA}}} \left[ \dot{X} + \frac{\theta_{\text{INC}}}{K_{\text{VA}}} \ln \left( 1 - \frac{K_{\text{VA}} \dot{X}}{\theta_{\text{INC}}} \right) \right] \]  \hspace{1cm} (21)

\[ X = X_f \]

when

\[ X + 8 \dot{X} + 16 \ddot{X} = 0 \]  \hspace{1cm} (22a)

(The 16\dddot{X} term is added to the switching criterion to eliminate a pitching transient at Hover mode initiation.)

or

\[ \dot{X} = 10 \text{ ft/sec} \left[ 5.92 \text{ kt (5.05 m/s)} \right] \]  \hspace{1cm} (22b)

All switching is irreversible so that \( X_0 \) and \( X_f \) can be defined only once during a single approach.

\[ \Delta \theta_{\text{decel}} = K_{\text{BUG}} (\theta_D - (\theta_F)_{X=X_0}) \]

\[ = K_{\text{BUG}} \left\{ \theta_H + \theta_{\text{INC}} - K_{\text{VA}} (\dot{X} - V_a)_{X=X_0} - [\theta_H + K_{\text{VA}} (V_a)_{X=X_0}] \right\} / 57.3 \]

\[ = K_{\text{BUG}} \left\{ \theta_{\text{INC}} - K_{\text{VA}} (\dot{X})_{X=X_0} \right\} / 57.3 \]  \hspace{1cm} (23)

\( (K_{\text{BUG}} = 1.0 \text{ nominal value}) \)

where \( \Delta \theta_{\text{decel}} \) is in radians and \( \dot{X} \) is in ft/sec. \( K_{\text{BUG}} \) is a parameter introduced to adjust for the fact that \( \theta_D \) is not a completely linear function of airspeed for the actual aircraft model.

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3. **Altitude Hold at 50 ft (15 m/s)**

The longitudinal hover position is defined as the point where the MLS beam passes through 50 ft (15 m/s) above ground level. Hence the horizontal and vertical velocities will be nominally zero as the XV-15 reaches 50 ft (15 m/s). The vertical descent is made using altimeter data for altitude information. The conversion from MLS to radar altitude vertical guidance is initiated when radar altitude equals 50 ft (15 m/s). This is accomplished via the function T in Fig. 2. Recall that T ramps to unity at glide slope intercept. At 50 ft (15 m/s), \( T = [1 - 0.3(t - t_{50})] \); where \( t_{50} \) is the time at which the aircraft passes through 50 ft (15 m/s) and the minimum value of T is zero. This blends the MLS beam signals out and the altitude and attitude rate signals in over a time of 3.3 sec.

4. **Vertical Descent**

Vertical descent is initiated manually by the pilot at any time following \( T = 0 \) in the altitude hold mode. It is terminated following a positive indication for weight-on-wheels.

I. **COMMAND LIMITING**

The three limiters shown in Fig. 2 are included in the design to prevent the flight director or automatic flight control system from commanding excessive pitch attitudes, rates of climb, or sink rates which could lead to unfavorable pilot opinion.

The attitude limiter (shown in the upper part of Fig. 2) must account for the output of the synchronizer, which is effectively an attitude command inserted downstream of the limiter. This is accomplished as follows:
\[ L_3a = \theta_{LA} - \theta_o \]
\[ L_3b = \theta_{LB} - \theta_o \]  

The attitude limits are tentatively set to:

\[ \theta_{LA} = +10 \text{ deg} (+0.17 \text{ rad}) \]
\[ \theta_{LB} = -10 \text{ deg} (-0.17 \text{ rad}) \]  

The beam rate limiter is included to insure that large glide slope errors do not result in excessive rates of sink or climb. The limits are set so the maximum commanded rate of climb is 0 and the maximum commanded rate of descent is 1000 ft/min (305 m/min). This limiter would be removed, of course, if a missed approach mode were added to the design. The upper and lower limit values are defined by the following equations:

\[ L_{2a} = K_d \dot{\theta}_{LA} \]
\[ L_{2b} = K_d \dot{\theta}_{LB} \]  

where

\[ \dot{\theta}_{LA} = -\dot{\theta}_{LB} = 8.33 \text{ ft/sec} (2.53 \text{ m/sec}) \]
The altitude rate limiter shown in the upper right part of Fig. 2 is set to limit sink rate or rate of climb commands to 500 ft/min (152 m/min). This limiter is in effect in the altitude hold and in the vertical descent modes. Its values are defined by the following equations:

\[ L2a = \frac{K_a}{h} h_{\text{max}} \]

\[ L2b = \frac{K_b}{h} h_{\text{min}} \]  \hspace{1cm} (27)

\[ h_{\text{max}} = -h_{\text{min}} = 8.33 \text{ ft/soc (2.53 m/s)} \]  \hspace{1cm} (28)
SECTION IV
LATERAL-DIRECTIONAL SYSTEM DESIGN

A. SUMMARY

A block diagram which summarizes the feedback selection, shaping and switching for the lateral stability and command augmentation system (SCAS) and the lateral flight director system is shown in Fig. 18. The numerical values of the time constants and gains in Fig. 18 are given in Table 8. The fully automatic approach mode is achieved by simply replacing the pilot in Fig. 18 with a gain element. Since low frequency standoffs are eliminated by washing out the inner-loop feedbacks to the flight director, it is not necessary to add forward loop integrations upon changing from the flight director mode to the fully automatic approach mode.

The lateral SCAS has been configured as a rate-command/attitude-hold system for all flight conditions from cruise to hover, vertical descent and touchdown.

Referring to Fig. 18, it can be seen that the switching involves three basic modes, e.g., Localizer A (LOC A), Localizer B (LOC B) and heading hold (HH). A brief description of each of these modes is given as follows.

- **Heading hold (HH).** This is a conventional heading hold mode and is based on coordinated turns to pilot-selected headings (ref in Fig. 18).
- **LOC A.** Straight or curved localizer tracking via coordinated turns to correct for lateral errors.
- **LOC B.** Straight localizer tracking using bank angle regulation at constant heading to correct for localizer errors. A constant pilot-selected heading is obtained via pilot input to the pedals.

The XV-15 aerodynamic data indicate that the side force characteristics 
\(Y_y\) are very low. This means large magnitude crosswinds can be handled with small bank angles in the LOC B mode; hence, a complex system to cause the
### TABLE 8. SUMMARY OF LATERAL DIRECTIONAL SCAS, FLIGHT DIRECTOR, AND AUTOPILOT GAINS AND TIME CONSTANTS

**Turn Following SCAS (used for LOC A and HH)**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_p$</td>
<td>5.6 in$/\text{rad/sec}$</td>
</tr>
<tr>
<td>$K_{\Phi}$</td>
<td>11.2 in$/\text{rad}$</td>
</tr>
<tr>
<td>$K_{\beta}$</td>
<td>10.0 in$/\text{rad/sec}$</td>
</tr>
<tr>
<td>$K_{\nu_H}$</td>
<td>1.0</td>
</tr>
<tr>
<td>$K_{\nu_C}$</td>
<td>2.93 1/sec</td>
</tr>
<tr>
<td>$K_{\Phi_C}$</td>
<td>3.14</td>
</tr>
<tr>
<td>$K_{\psi_c}$</td>
<td>0.087 (rad/sec)/in.</td>
</tr>
</tbody>
</table>

**Wing Low SCAS (used for LOC B)**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_p$</td>
<td>8 in$/\text{rad/sec}$</td>
</tr>
<tr>
<td>$K_{\Phi}$</td>
<td>12 in$/\text{rad}$</td>
</tr>
<tr>
<td>$K_{\beta}$</td>
<td>0</td>
</tr>
<tr>
<td>$K_{\psi}$</td>
<td>13 in$/\text{rad}$</td>
</tr>
<tr>
<td>$K_{\nu_H}$</td>
<td>1.06</td>
</tr>
<tr>
<td>$K_{\nu_C}$</td>
<td>3.14</td>
</tr>
<tr>
<td>$K_{\psi_c}$</td>
<td>0.087 (rad/sec)/in.</td>
</tr>
</tbody>
</table>

**LOC A and LOC B Flight Directors**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_y$</td>
<td>0.002 rad/ft$^*$</td>
</tr>
<tr>
<td>$K_{\psi}$</td>
<td>0.017 rad/(ft/sec)</td>
</tr>
<tr>
<td>$K_{DP}$</td>
<td>0.425 sec</td>
</tr>
<tr>
<td>$K_{D\Phi}$</td>
<td>1.06</td>
</tr>
</tbody>
</table>

**Autopilot**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_{\Phi}$</td>
<td>$G_{FFS}(0)Y_{\Phi}$</td>
</tr>
<tr>
<td>$K_{\Phi}$</td>
<td>5.62 (LOC A)</td>
</tr>
<tr>
<td>$K_{\Phi}$</td>
<td>2.5 (LOC B)</td>
</tr>
</tbody>
</table>

---

$^*$1 in. $= 2.54 \times 10^{-2} \text{ m.}$

$^+1 \text{ ft} = 3.048 \times 10^{-1} \text{ m.}$
vehicle to be pointed into the wind in hover is not warranted. LOC B will be used for localizer tracking at low speeds (below about 60 kt (31 m/s) including hover.

B. STABILITY AND COMMAND AUGMENTATION SYSTEM (SCAS)

A review of the vehicle transfer functions at speeds from 80 kt (41 m/s) down to hover (see Table 7) indicates the following basic airplane deficiencies for lateral-directional control:

- Very low dutch roll frequency at all speeds (low \( \alpha_d \))
- Negative or low dutch roll damping at all speeds (low \( \zeta_d \))
- Unstable spiral mode
- Large shift in instantaneous center of rotation (\(-V_\text{ped}/N_\text{ped}\)) between 60 and 80 kt (31 and 41 m/s). This characteristic makes it impractical to use a lateral acceleration-to-pedal feedback to improve the low dutch roll damping.
- Roll reversal at 40 kt (21 m/s). The bank angle to lateral cyclic numerator consists of two real zeros, one of which is in the right half plane, indicating that the aircraft will ultimately roll left to a right lateral cyclic input. This unusual characteristic only occurs at speeds near 40 kt (21 m/s) and is attributed to rotor wash characteristics on the horizontal tail at this speed.
- Poor yaw rate-to-pedal response characteristics resulting in marginal improvements in dutch roll damping with a conventional yaw damper feedback.
- Large adverse yaw at speeds below about 70 kt (36 m/s) at \( \gamma = -10 \text{ deg} (-0.17 \text{ rad}) \).

Two separate stability and command augmentation systems have been developed to resolve the above deficiencies. At higher speeds heading changes will be made in the conventional way, that is, utilizing bank angle to develop a turn rate. At low speeds, the heading response to bank angle changes becomes too sensitive for effective closed-loop path control (\( \dot{\psi} = \omega_p/V \)). Experience has shown that this characteristic becomes unacceptable at speeds below 60 kt (31 m/s); hence, the system is designed to be switched from the turn-following SCAS to the
wing-low SCAS as the aircraft is slowed to speeds below 60 kt (31 m/s). The wing-low SCAS is a constant heading mode where lateral position changes are made by varying the bank angle. It is intended that this mode be used during straight localizer tracking on final approach when speed reductions below 60 kt (31 m/s) will occur. Nominally, the pilot will switch manually from the turn-following SCAS to the wing-low SCAS once he is established on the straight-in localizer approach course. However, if the speed decreases to 50 kt (26 m/s), this switch will be accomplished automatically to avoid getting into a region of unfavorable dynamics with the turn-following SCAS. If the system automatically switches (due to inadvertent low-speed excursions), the pilot must switch back manually to turn-following mode if he so wishes. However, if the aircraft is in the wing-low SCAS mode, and the speed exceeds 70 kt (36 m/s), the system will automatically switch back to the turn-following mode. This is done to avoid getting into a region of unfavorable dynamics with the wing-low SCAS.

The following paragraphs describe the turn-following SCAS and the wing-low SCAS.

1. Turn-Following SCAS [V ≥ 50 kt (26 m/s)]

Bank angle (ψ) and body-fixed roll rate (p) are fed to the lateral cyclic series servo to stabilize the spiral mode and achieve a bank angle command SCAS. The ratio of $K_p/K_p$ was set to the desired bandwidth of the bank angle command system, or 1.5 rad/sec. The fact that the ratio of $K_p/K_p$ is approximately equal to the bandwidth of the closed-loop system can be seen from the following approximation:

$$\lim_{K_p \to \infty} \left( \frac{\psi}{\psi_e} \right) = \lim_{K_p \to \infty} \frac{K_p \psi_p}{s + K_p \left( s \cdot \frac{K_p}{K_p} \right) N_\psi}$$

$$= \frac{K_p}{s + K_p}$$

\[ (29) \]
Based on the flight test results of Ref. 9, a bandwidth of 1.5 rad/sec achieves acceptable bank angle regulation characteristics. Larger bandwidth tends to result in poor ride qualities due to jerky responses to stick inputs. The effect of feeding bank angle and roll rate to the lateral cyclic series servo on the vehicle lateral characteristic equation is shown in Fig. 19 for the 60 kt (31 m/s) flight condition. Figure 19 indicates that additional augmentation is required to increase the damping and frequency of the closed-loop dutch roll mode. The possible alternatives that were considered to achieve these objectives are listed below. (See Ref. 14.)

- Utilize feedback of yaw rate and lateral acceleration to the pedal series servo. This is a classic combination utilized to increase the dutch roll frequency via the lateral acceleration feedback and to improve the dutch roll damping via the yaw rate to pedal feedback. In order to be effective, the lateral accelerometer must be at or near the instantaneous center of rotation, which is located forward of the center of gravity for aircraft with aft-mounted vertical tails. In the case of the XV-15, the vertical tail effectively moves from a forward location to a rearward location as directional control is shifted from differential cyclic to conventional rudders. This occurs as the speed is increased from 60 to 80 kt (31 to 41 m/s). This large shift is in the instantaneous center of rotation [approximately 0.65 ft (0.2 m) behind the c.g. at 60 kt (31 m/s) to 2.58 ft (0.8 m) forward of the c.g. at 80 kt (41 m/s)] makes the use of a lateral acceleration feedback impractical for this airplane. Additionally, the location of the zeros of the yaw rate-to-pedal-accumulator make the feedback of yaw rate to pedals ineffective in terms of increasing the dutch roll damping.

- Another conventional way of increasing the dutch roll damping and frequency is to use lateral acceleration-to-pedal feedback (with a lateral accelerometer located at the instantaneous center of rotation) with a lead/lag network. However, because of the above discussed movement of the instantaneous center of rotation at speeds below 60 and 80 kt (31 and 41 m/s), this scheme is also impractical for the XV-15.

- The feedback of sideslip angle to the pedal series servo with a lead/lag network is very effective for increasing both the damping and frequency of the dutch roll mode. Recognizing that the measurement of sideslip angle in a rotor craft is highly impractical, an attractive alternative is to use other signals, which when combined, have the same characteristics as sideslip angle with a lead/lag network.
Figure 19. Effect of Roll Loop Closure on Lateral Characteristic Equation;
\[ V = 60 \text{ kt (31 m/s)}; \text{ Turn-Following SCAS (LOC A)} \]
The turn-following SCAS design for the XV-15 is based on the third of the above alternatives and uses the following approximation for (inertial) sideslip angle rate:

\[ \dot{\beta} = -(r - \frac{\delta_{\phi}}{V}) \]  

(30)

This approximation assumes that \( Y_\phi \) and \( Y_\gamma \) are small. A review of the derivatives presented in Table A-6 indicates that this is a very good approximation. Inasmuch as \( r - \frac{\delta_{\phi}}{V} \) is being fed back to simulate \( \dot{\beta} \), the feedback gain was labeled \( K_\beta \). The effect of \( K_\beta \) on the characteristics equation as modified by the \( \varphi \) and \( \psi \) feedbacks is shown in Fig. 20. The root locus plot in Fig. 20 clearly illustrates that the feedback \( r - \frac{\delta_{\phi}}{V} \) has very little effect on the \( \omega_{sr} \) mode (the combined spiral and roll subsidence mode obtained from the bank angle and roll rate feedback), but does drive the undesirable low-frequency dutch roll mode to the real axis to form two real roots. The highest frequency real root becomes the dominant response to rudder or gust inputs. The dominant response to lateral cyclic inputs is \( \omega_{sr} \). \( \omega_{sr} \) is nearly equal to \( \omega_{sr} \), that is, the \( r - \frac{\delta_{\phi}}{V} \) feedback does little to change the bandwidth of the bank angle loop. The feedback gain \( K_\beta \) is set to 10 in. (0.25 m) of series serve motion per rad/sec of \( \dot{\beta} \) so that the real dominant dutch roll root would be slightly greater than 1 rad/sec. Consideration of pedal series serve limiting reveals that a value of \( K_\beta = 10 \) implies that 5.7 deg/sec of yaw rate will result in saturation. This is felt to be marginal but not unreasonable.

Two performance metrics are used to evaluate the turn-following SCAS. The attitude hold feature of the roll rate-command/attitude-hold system was evaluated by consideration of the \( \varphi/\varphi_c \) frequency response where \( \varphi_c \) is the output of the lateral stick shaping in Fig. 18. The feedback of \( r - \frac{\delta_{\phi}}{V} \) inherently tends to minimize adverse yaw. The time response of yaw rate to a step \( \varphi_c \) was used to evaluate the adverse yaw characteristics of the turn-following SCAS.
Figure 20. Effect of \( r - \frac{g_p}{V} \) Feedback to Pedals on Lateral Characteristic Equation; \( V = 50 \text{ kt (31 m/s)} \); Turn-Following SCAS (LOC A)
The frequency response characteristics of $\varphi/\varphi_c$ are shown in Fig. 21. It can be seen that the system bandwidth is 1.5 rad/sec at 60 kt (31 m/s) and 1.75 rad/sec at 80 kt (41 m/s). Notice that this is consistent with the Eq. 29 approximation which was the basis for setting $K_p/\varphi_c = 1.5$. The stick shaping network in Fig. 18 when combined with the proportional-plus-integral ($1 + b/s$) in the series actuator path and the parallel actuator path results in an integration between stick and the effective $\varphi_c$. The value of $K_p\varphi_c$ was set to achieve a sensitivity of 15 deg/sec of roll rate per inch (0.025 m) of lateral cyclic on the basis of the results achieved in Ref. 6. The time histories of bank angle and yaw rate to a step $\varphi_c$ input are shown in Fig. 22. Here it is seen that there is essentially no adverse yaw at 80 kt (41 m/s) and a small amount of adverse yaw at 60 kt (31 m/s), that is, there is an effective delay between developing the proper sign of yaw rate to $\varphi_c$ of about 0.6 sec. This is felt to be negligible. The steady-state turn rate at 60 kt (41 m/s) is slightly less than that at 60 kt (31 m/s), indicating the presence of a small steady sideslip angle during turns at 80 kt (41 m/s). This effect is not felt to be important enough to warrant additional SCAS feedbacks. Because the turns are automatically coordinated, very little pedal usage is expected in LOC A or HN modes. Therefore, no pedal shaping has been included and pedal inputs are transmitted via the parallel servo only (see Fig. 18).

It is felt that the stability and command augmentation system in the turn-following mode will receive reasonably good pilot ratings because of its snappy but not oversensitive roll response and lack of any appreciable aileron/rudder coordination requirements arising from adverse yaw. The primary system limitation is expected to be possible saturation of the pedal and lateral cyclic series servos.

2. Wing-Low Stability and Command Augmentation System

\[ V \leq 50 \text{ kt (26 m/s)} \]

The nominal wing-low SCAS was designed at the 40 kt (21 m/s) condition. This was done assuming that a SCAS designed to yield acceptable flying qualities with the extremely poor basic vehicle dynamics at 40 kt (21 m/s) should also work well at other flight conditions. This in fact turned out to be the case.
Figure 21. Frequency Response Characteristics of Roll Rate to Lateral Stick; Turn Following SCAS (LOC A)
Figure 22. Bank Angle and Yaw Rate Responses to $\omega_c = 1$ in. (0.025 m) at Output of Stick Shaping Network.
Since, by definition, heading is constant in this mode it is possible to use heading feedback to the pedal series servo to provide the necessary increase in the dutch roll frequency. Yaw rate to pedal feedback was also utilized to provide the necessary damping. Proportional-plus-integral (1 + c/s) is required in the forward loop pedal series servo path to insure that heading error is zero at low frequency. Important tradeoff considerations in this loop closure require maximum c for the best heading error suppression and minimum c for stability. The best compromise is reached by decreasing the ratio of $K_r/K_p$ to make up for damping lost because of the parallel integrator. The ultimate outcome is a small decrease in the closed-loop dutch roll frequency, $\xi_d'$. The effect on the lateral characteristic equation of heading and yaw rate feedback to pedals in the presence of this parallel integrator is shown in Fig. 23. Notice that for a given $K_r$ the total damping, $\xi_d''$, is independent of $K_p/K_r$ but that $\xi_d'$ increases with decreasing $K_p/K_r$. $K_p$ and $K_r$ are set equal to 15 ($K_p/K_r = 1.0$) as a best compromise between maximizing $\xi_d'$ and minimizing $K_p$ to avoid an unacceptable degree of series servo limiting. $K_p = 13$ in. (0.33 m) per radian of heading results in pedal series servo limiting when heading excursions exceed 4.1 deg (0.076 rad). This is felt to be marginal but probably acceptable. Notice also the kinematic roots at the origin are driven into the low frequency zero ($\omega_c$) (see Fig. 23). This lightly damped, low-frequency closed-loop mode has been labeled $\omega_{CAS}$.

The pedal shaping network in Fig. 18 when combined with the parallel integrator (1 + c/s) results in an integration between pedals and $\psi_c$. This results in a rate command attitude hold SCAS in heading. $K_p$ was set to 1.13 so that 1 in. (0.025 m) of pedal commands 5 deg/sec (0.087 m/s) of heading rate.

It is shown in Fig. 24 that feedback of bank angle and roll rate to the lateral cyclic series servo results in $\alpha_{CAS}''$ being driven to the approximate location of $\omega_2''$ while $\omega_1''$ is driven to higher values of frequency and damping (resulting in $\omega_1'$). A parallel integrator (1 + b/s) was required in the roll loop for good mid- and low-frequency regulation. A value of $K_p = 8.0$ in./(rad/sec) [0.2 m/(rad/sec)] was picked to maximize the bandwidth of the closed-loop bank angle-to-lateral cyclic system (e.g., to maximize $\omega_1''$ and $\xi_d''$).
Figure 23. Effect of $\psi$ and $\eta$ Feedback to Pedals on Lateral Characteristic Equation; $V = 40$ at (21 m/s); Wing Low SCAS (LOC B)
Figure 24. Effect of $\phi$ and $p$ Feedback to Lateral Stick on the Lateral-Directional Characteristic Equation; $V = 40$ kt (22 m/s); Wing Low SCAS (LOC B)
while keeping $K_p$ low enough to minimize the possibility of lateral cyclic series servo saturation. For a series servo limit of 1 in. (0.025 m) and $K_p/K_p = 1.5$, this implies limiting for a 4.8 deg (0.084 rad) roll excursion from the commanded value (output of the lateral stick shaping network).

Several performance metrics were utilized to evaluate the wing-low SCAS system before proceeding with the flight director and automatic flight control system design. These consisted of frequency response of bank angle to bank angle command, heading to pedal command and heading to lateral cyclic input.

The $\dot{\phi}/\dot{\phi}_c$ frequency response bandwidth is primarily set by $\omega_n$ and is reasonably flat out to about 1.5 rad/sec for the 0, 20, 40 and 60 kt (0, 10, 21 and 31 m/s) flight conditions. These frequency responses are shown in Fig. 25 and indicate that the bandwidth of the bank angle response is 2 rad/sec at 60 and 40 kt (21 and 21 m/s) and improves to 2.4 rad/sec at 20 (10 m/s) and hover.

An attempt was made to relax the roll gain, $K_p$, from 8 to 6 and thereby increase the magnitude of roll excursion required to saturate the lateral cyclic series servo. This gain change would allow an increase in bank angle error from 4.8 to 6.4 deg (0.08 to 0.11 rad) before saturation occurs. Time histories of the resulting $\dot{\phi}/\dot{\phi}_c$ time responses indicate undesirable transient characteristics at the lower gain (see Fig. 26). This verifies that the design is tightly constrained by servo saturation on the one hand and unacceptable transient response characteristics on the other.

The directional SCAS is also a rate-command/attitude-hold system. Like the roll SCAS, the rate command feature is obtained via shaping of the pedal input. The frequency response characteristics of heading to pedal are shown in Fig. 27 for 0, 20 and 40 kt (0, 10 and 21 m/s). The responses appear to be rate-like out to about 1.4 to 1.5 rad/sec at which point $\omega_n$ cuts off the rate-like response. Recall that $\omega_n$ is set by $K_\psi$ and $K_p$ which are both set to 13. Any further increases in these feedback gains would require an increase in the pedal series servo authority. It is felt that the bandwidth of the heading-to-pedal SCAS loop is adequate. The possibility of saturating the series servo is moderate in that only 4.4 deg or deg/sec of heading or yaw rate will result in 1 in. (0.025 m) of servo travel.
Figure 25. Frequency Response of Roll Rate to Lateral Cyclic Inputs; Wing Low SCAS (LOC H)
Figure 26. Bank Angle Response to $\phi_c = 1$ in. (0.025 m) Step at Output of Stick Shaping network; $V = 40$ kt (21 m/s);
Wind Low SCAS (LOC B)
REFRIGERATION OF THE ORIGIN. PAGE IS POOR.
Another metric chosen to evaluate the system was the heading response to a lateral cyclic input. For a wing-low approach we would like to minimize the heading response to lateral cyclic inputs at all frequencies. The heading to lateral cyclic frequency responses at 60, 40 and 20 kt (31, 21 and 10 m/s) and hover are shown in Fig. 28. These results indicate that moderate to large stick deflections will produce measurable heading excursions. For example, lateral cyclic excursions of 1 in. (0.025 m) at a frequency of 1.5 rad/sec will produce heading excursions which vary from a minimum of 1.1 deg (0.019 rad) at 60 kt (31 m/s) to a maximum of 3 deg (0.052 rad) at 0 kt (21 m/s). Recalling that the pedal series servo saturation occurs at heading excursions of 4.4 deg (0.076 rad), we can see again that the series servo authority could be marginal.

C. FLIGHT DIRECTOR FOR LOC A

LOC A is a conventional beam tracking mode where bank angle is used to develop a turn rate which results in heading changes. This mode is nominally used at speeds at and above 60 kt (31 m/s) for straight and curved localizer tracking. The analysis techniques used to select and shape the feedbacks were taken directly from Ref. 10. From Fig. 18 it can be seen that localizer error and derived error rate are fed back to the flight director to provide path following and path damping, respectively. Inner-loop stabilization is achieved via a bank angle feedback to the flight director. This feedback is washed out to avoid requiring a trajectory-dependent feedforward. Washout use allows tracking of arbitrary curved path without external inputs. Complementary filtering schemes using $a_y + g(\phi - \phi_c)$ to replace high frequency beam rate cannot satisfy the desired "no external inputs" design requirement. This is because the desired design must be capable of following any beam shape (within system limits) without prior knowledge of the beam geometry. Notice that $\phi_c = \tan^{-1}(V_{gs}^2/gR)$ depends on beam geometry (R is the turn radius). Furthermore, the noise characteristics of the beam must be of low enough level and/or broad enough bandwidth to allow a sufficiently small value of $T_y$ (beam rate filter time constant) so that unacceptable lags in following curved paths will not result. Normal values of $T_y$ are about 2 sec in an ILS system, whereas a value of 0.25 sec or less are possible with an Mls system. This is so because
Figure 3.3. Frequency Response of Heading to Lateral Cyclic Inputs: Wing Low SCAS (103 II)
Figure 3B (Concluded)
the noise characteristics of microwave landing systems (MLS) are generally of much lower level and of broader bandwidth than for ILS.

1. Localizer Capture

The transition from heading hold (HH) to LOC A occurs automatically when
\[ e_y = 0 \quad (e_y = K_y y + K_p \dot{y}) \]. Transition back to heading hold must be manually selected by the pilot. Transition to LOC B may be selected manually by the pilot or automatically based upon airspeed.

2. Parameter Adjustments

The feedback selection and shaping are as shown in Fig. 18 with the switches set to the LOC A position. The values of the time constants and gains were adjusted by the methods of Ref. 10 to obtain a K/s frequency response for the open-loop lateral flight director to lateral cyclic input in the region of the unit gain crossover frequency for manual flight director control. As shown in Fig. 29, the desired K/s-like characteristic is obtained with reasonable success in that the lateral flight director to lateral cyclic input is K/s in a frequency region from about 0.5 to 5 rad/sec.

The performance metrics used to evaluate the closed-loop system regulatory characteristics in the LOC A mode are the frequency responses of lateral beam error to lateral beam command (\( y_c / y_c \)) and lateral beam offset to side gust inputs (\( y / y_g \)) and the time response to an initial condition offset. These closed-loop performance measures were obtained using an assumed unit gain crossover frequency under pilot control of 1.5 rad/sec and a neuromuscular lag of 0.17 sec. The resulting pilot model is:

\[ G_{FFS}(0)Y_{PP} = 3.7e^{-17} \text{sec} \]

The ability of the closed-loop pilot/vehicle system to regulate against side gust disturbances is shown in terms of the lateral beam deviation to side gust frequency response in Fig. 30. The system is seen to regulate the errors to zero at frequencies below 0.3 rad/sec. This performance is considered to be acceptable.
Figure 20. Open-loop lateral flight director to lateral cyclic input
  Frequency Response: Turn following 8033 (100 A)
Figure 20. Closed-Loop Frequency Response in Lateral Body Deviation to Side Gust Inputs; Turn-Following SCAS (LOC A)
The ability of the closed-loop pilot/vehicle system to follow external beam commands is shown in Fig. 51 in terms of the $y_c/y_e$ frequency response. The closed-loop bandwidth for error regulation is seen to be 0.25 rad/sec at 60 and 80 kt (31 and 41 m/s). This is consistent with the performance achieved with the flight director developed in Ref. 10. That director received very good pilot ratings.

A time response of the closed-loop pilot/airplane system to a 100 ft (30 m) initial condition offset is shown in Fig. 52. A similar time response obtained from the Ref. 10 system is shown superimposed on the XV-15 response and indicates that the two responses are nearly identical. The pilot ratings for the Ref. 10 flight director were very good (order of 3) indicating that the long tail (due to the washout of the bank angle feedback) is of little or no consequence in practice.

5. Flight Director for LOC B

The analysis techniques developed in Ref. 10 to select and shape the flight director feedbacks are for a turn-following SCAC. However, they also apply for a wing-low SCAC if the relationship between lateral deviation and bank angle can be shown to be the same for the two systems. Comparing the lateral deviation to bank angle characteristics of each system reveals the following:

- Turn-following
  \[ y' = \frac{\alpha}{\alpha_0} \]

- Wing-low
  \[ y' = \frac{\alpha V}{\lambda - \alpha V} \]

Since $-\alpha V$ is orders of magnitude below the path mode response frequency, the $y'$ response is essentially identical for wing-low and turn-following stability and command augmentation systems. Therefore the Ref. 10 methods apply directly for synthesis of both the LOC B and LOC A males. The resultant feedback selection and shaping are as shown in Fig. 18 with the switches in the LOC B position. The gains and time constants in Table 3 yield the open-loop lateral flight director to lateral stick frequency responses shown in Fig. 52 for 18, 30, 42 kt (31, 47 and 84 m/s), and lower. As in the case of the turn-following system, the wing-low system (LOC B) exhibits an
Figure 3.1. Closed-Loop Frequency Response in Beam Error to Beam Command; Turn-Following SCAS (100 A)
Flight Director B of Ref. No. 4

Response to Initial Condition Offset
\[ y_p = 5.62 e^{-0.17t} \]
\[ V = 60 \text{ kts} \]

Figure 4.4. Comparison Between Initial Condition Responses of XV-15 and Flight Director B of Ref. 10
Then-Following Scale (100 A)
Figure 33. Open-loop Frequency Response of Lateral Flight Director to Lateral Cyclic Inputs; King low SCAS (100 ft) Rate
Figure 5a. (Concluded)
open-loop lateral flight director-to-lateral cyclic input characteristic which is essentially \( \frac{5}{\pi} \) between 0.2 and 5 rad/sec. However, at 30 kt (10 m/s) and hover the frequency response exhibits a bulge at about 2 rad/sec. This is a direct result of a decreased closed-loop dutch roll damping ratio \( (\zeta_d) \) at lower airspeeds in the wing-low SCAS. The origin of this may be better understood by examining an equivalent system transfer function which is a close approximation for the wing-low SCAS system:

\[
\frac{\varphi}{\delta_s} = \frac{.2\zeta_d\omega_d}{s(s^2 + .2\zeta_d\omega_d^2 + \omega_d^2)} \quad (32)
\]

where \( \zeta_d \) and \( \omega_d \) vary with speed as follows:

<table>
<thead>
<tr>
<th>Airspeed (kt)*</th>
<th>( \zeta_d )</th>
<th>( \omega_d ) (rad/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>.50</td>
<td>2.02</td>
</tr>
<tr>
<td>20</td>
<td>.53</td>
<td>1.97</td>
</tr>
<tr>
<td>40</td>
<td>.66</td>
<td>1.95</td>
</tr>
<tr>
<td>60</td>
<td>.79</td>
<td>1.91</td>
</tr>
</tbody>
</table>

As shown in Fig. 3h, an increase in \( \zeta_d \) would require increased feedback gains which is not possible because of the SCAS feedback gain limitations imposed by the limited authority actuator servo.

It is also evident in Fig. 3h that the SAS mode has an even lower damping ratio than the dutch roll mode. However, the SAS mode poles are nearly canceled by the \( \omega_d \) zeros with the result that there is virtually no net effect upon the open-loop flight director frequency response.

*1 kt = 1.144 x 10^{-1} m/s.
The \( K/s \) shape of the lateral flight director to lateral cyclic input transfer function is obtained by adjusting the first-order zeros arising from \( 1/T_1 \) and \( K_D/K_Dp \) to have the magnitude of \( a_1^\prime \). Hence, the residues for the \( a_1^\prime \) poles increase with decreasing \( \xi_1 \) from 1.0. When the residues are large because of low damping of the closed-loop dutch roll mode, the result is a bulge in the open-loop amplitude ratio even though the transfer function asymptotes are \( K/s \)-like. This effect is significant at damping ratios as high as 0.7, and becomes very pronounced at damping ratios less than 0.5.

It is felt that the average slope of the amplitude ratio is close enough to \( K/s \) so that the pilot opinion will not suffer excessively at 20 kt (10 m/s) and hover. However, it must be recognized as a marginal situation which should receive attention during the piloted simulator evaluations. It will, of course, have little or no effect on the automatic system operation.

The performance of the closed-loop pilot/vehicle system was evaluated assuming a unit gain crossover frequency under pilot control of 1.5 rad/sec in the lateral flight director loop. This resulting pilot model is:

\[
G_{PPs}(0)Y_{Dp} = 3.1e^{-1.7j\omega}
\]  

(53)

The same performance metrics are used as for the LOC A system. These are the frequency responses of lateral beam offset to lateral gusts \( (y/v_{g}) \), lateral beam error to lateral beam command \( (y_{c}/v_{c}) \), and time histories of the path response to lateral initial condition offsets.

Figure 34 indicates that the lateral gust sensitivity of the closed-loop pilot/vehicle system varies widely with flight condition. The 60 and 20 kt (31 and 10 m/s) flight conditions exhibit the highest overall gust sensitivity, followed by the hover flight condition. The 40 kt (21 m/s) case exhibited the best regulation against side gusts. The gust sensitivity turns out to be more a function of basic aerodynamic characteristics than of variation in closed-loop regulation characteristics with speed. This can be seen from the following table of \( Y_y \) as a function of speed.
Figure 5.1: Closed-loop Frequency Response of Integral Formulation to
Side-Gap Drift; Wind-Driven (10WP)
Figure 34 (Concluded)
A plot of $D_e v$ at the end that there are \( \text{flight} \) achieved. Table 1 comparison with Fig. 32 shows that the bandwidth for reducing errors to zero is about 0.25 rad/sec for all flight conditions. This is well within the acceptable range.

The time responses to a lateral initial condition offset of 100 ft (30 m) are shown in Fig. 36 for the 40 kt (21 m/s) and hover flight conditions. Comparison with Fig. 32 shows the responses are essentially identical to the 60 kt (31 m/s) LOC A flight condition, which in turn was nearly the same as the Ref. 10 response. Hence, the objective of augmenting the airplane so that its performance is nearly invariant with flight condition has been achieved. Furthermore, the system performance is consistent with a lateral flight director system which is known to have pilot acceptable performance.

D. INTERAXIS COUPLING

A preliminary investigation of the XV-15 aerodynamic crosscoupling revealed that the pitching moment due to sideslip $M(\beta)$ can be quite large at sideslip angles greater than 1 deg (0.025 rad). This did not show up initially because the perturbation derivatives supplied to STI were obtained for very small sideslip angles. At small sideslip angle, $M|\beta|$ is very small. A plot of pitching moment vs. side velocity at the 40 kt (21 m/s) flight

<table>
<thead>
<tr>
<th>Airspeed (kt)</th>
<th>$Y_v$ (sec$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-0.007</td>
</tr>
<tr>
<td>20</td>
<td>-0.038</td>
</tr>
<tr>
<td>40</td>
<td>-0.0054</td>
</tr>
<tr>
<td>60</td>
<td>-0.046</td>
</tr>
</tbody>
</table>

The values of $Y_v$ correlate directly with the closed-loop $y/v_g$ responses in Fig. 34. The lateral beam deviations at all flight conditions are seen to be decreasing to zero at frequencies less than 0.1 rad/sec.

\[ \text{Airspeed (kt)} \]

\[ \text{Y}_v \text{ (sec}^{-1}) \]

\[ \begin{array}{cc}
0 & -0.007 \\
20 & -0.038 \\
40 & -0.0054 \\
60 & -0.046 \\
\end{array} \]

\( ^*1 \text{ kt} = 5.144 \times 10^{-1} \text{ m/s}. \)
Figure 35. Closed-Loop Frequency Response in Beam Error to Beam Command
Figure 56. Lateral Beam Deviation Time Response for a 100 ft (30 m) Initial Condition Offset; Wing-Low SCAS (LOC B)
condition is given in Fig. 37a. The longitudinal cyclic required at 40 kt (21 m/s) to trim out the pitching moment due to steady sideslip (as occurs on a wing-low approach) is plotted in Fig. 37b. It is expected that these already large trim values will be still larger at hover.

The dynamic effects of $M|\beta|$ were investigated briefly by examining the open-loop pitch attitude response to lateral cyclic inputs, and by examining the pitch flight director responses to lateral cyclic inputs with the pitch flight director loop closed by the pilot at 1.5 rad/sec. These results are shown in Figs. 38 and 39 respectively. Looking first at $\theta/\delta_8$, it should be noted that with a rate command SCAS, the low frequency lateral cyclic activity will be extremely small. In fact we would expect that the majority of lateral cyclic activity will be concentrated in the region of crossover, say between 1 and 2 rad/sec. At $\omega = 1$ rad/sec the ratio of pitch to lateral cyclic is $-32$ dB or $1.4$ deg (0.024 rad) of $\omega$ per inch (0.025 m) $\delta_8$. Put another way, a roll rate command of $15$ deg/sec (0.26 rad/s) will result in a $1.4$ deg (0.024 rad) pitch attitude excursion in LOC B.

Assuming that the pilot is actively closing the longitudinal and lateral flight director loops at 1.5 rad/sec results in the $\Phi_{dc}/\delta_8$ response shown in Fig. 39. This plot indicates that at 1 rad/sec the ratio of longitudinal cyclic flight director to lateral cyclic inputs is $-14$ dB, or 0.2 in. (0.005 m) of longitudinal cyclic flight director per inch (0.025 m) of lateral cyclic. This indicates that in spite of the pitch loop closure at 1.5 rad/sec the pilot will observe $\Phi_{dc}$ excursions which are 20 percent of full scale per inch (0.025 m) of lateral cyclic stick. The acceptability of these excursions can only be established in the simulator. Inasmuch as the pitch SCAS gains are already at a maximum, the only possible further refinement appears to be a crossover from lateral to longitudinal cyclic.

E. LIMITERS

The bank angle limit will be set to $\pm 50$ deg ($\pm 0.87$ rad) until LOC B is selected. At that time it will be reduced to $\pm 10$ deg ($\pm 0.17$ rad). Note that this involves setting the limits to $|\Phi_{lim}|/\Phi_d$. 

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Figure 37a. Pitch Acceleration Due to Side Velocity at \( V = 40 \text{ kt} \) (21 m/s)

Figure 37b. Variation in Trim Longitudinal Cyclic Position with Side Velocity at \( V = 40 \text{ kt} \) (21 m/s)

\[ M_{\delta_{LN}} = 0.36 \frac{\text{rad/sec}^2}{\text{in.}} \]
Figure 38. Frequency Response of Pitch Attitude to Lateral Cyclic Inputs; $V_e = 40$ kt ($21$ m/s)
Figure 39. Closed-Loop Frequency Response of Longitudinal Cyclic Flight Director to Lateral Cyclic Inputs; \( V = 40 \) kt \((21 \text{ m/s})\); LOC B Mode
The course rate limiter is set to preclude the possibility of commanding large bank angles rapidly. This would occur if the aircraft were significantly offset from course due to a combination of winds and pilot inattention. The limit level is set as a function of ground speed to achieve a 20 deg (0.35 rad) reintercept angle by the following method:

\[
|\gamma_{lim}| = \frac{K_y}{K_y} \dot{\gamma} = \frac{K_y}{K_y} v_{GS} \sin 20^\circ \quad \text{for } |\gamma_{lim}| \geq 24 \text{ ft} \ (\geq 7.2 \text{ m})
\]

otherwise

\[
|\gamma_{lim}| = 24 \text{ ft} \ (= 7.2 \text{ m})
\]

Note that initial course intercepts are made in the heading hold mode so that the \( \dot{\gamma} \) limiter will have no effect.
SECTION V
SYSTEM PERFORMANCE EVALUATION

The purpose of this section is to evaluate performance of guidance, auto-
land and flight director control laws described in Sections III and IV. These
control laws are designed in a stationary (time invariant) system context to
be suitable at several fixed speeds. The designs are also based upon perturba-
tion equations in distinction to equations using the total quantities (per-
turbation + operating point) actually measured by sensors. Both approximations
are removed in the system model used for performance evaluation. The remaining
operative approximations then involve only assumptions of linearity and of
separability of the longitudinal and lateral-directional performance evaluation
problems.

Evaluation of the fully automatic and of the manually controlled flight
director systems can be accomplished using the same math model. This is pos-
sible because the automatic mode control laws have been designed to automate
the pilot's control function and use the same guidance and control computations
as are used for the flight director. This means that the only difference
between fully automatic and manual flight director operation is whether the
gain constant relating flight director computer output to the force feel sys-
tem input is supplied by the automatic system or by the pilot. Since this
difference will not result in different performance for the fully automatic
and manual flight director systems (assuming full pilot attention to the
task), no distinction between these cases is necessary in evaluation. Care
has been taken in generating performance data to include the variables for
all flight director and status cockpit instruments indications and stick and
lever positions required for complete performance evaluation.

Performance evaluation is based upon the control system block diagrams in
Figs. 2 and 18 and the aircraft and disturbance models given in Appendix A.
The equations and parameter values actually used are summarized in Appendix B.
A. APPROACH TO PERFORMANCE EVALUATION

The overall system model is the basis for performance evaluation. It includes parts representing
- Steady wind and wind shear
- Atmospheric turbulence
- MLS guidance geometry and structure
inputs, a dynamic model of aircraft response to the above atmospheric inputs and to control inputs obtained from dynamic models of
- Automatic system response to MLS inputs and aircraft motions or alternatively the pilot's manual control response to the flight directors
- Flight control system SCAS response to aircraft motions and inputs from the automatic system or pilot.

The overall model is such that it makes the mean value and the variance of every input and response variable available as a function of time. The model has two sections, namely:
- A deterministic section which produces the mean value of every input and response variable
- A stochastic section which produces the covariance matrix for the input and response variables. (The diagonal elements of the covariance matrix are the variances or mean square or $\sigma^2$ values of the input and response variables. $\sigma$ is the standard deviation.)

Next consider these two sections of the complete model.

1. Deterministic Section of the Model

The deterministic section for the overall longitudinal system is described by the block diagram in Fig. 40. The block diagram for lateral system is similar. The mean values of variables are denoted by the bars over the variables in this figure. The block diagram indicates that the mean values of the aircraft, flight control system SCAS and coupler response are obtained as the result of forcing the model with the mean wind, $\bar{u}_w$, and the mean glide path,
Figure 40. Block Diagram of the Deterministic Section of the Complete Longitudinal Model

Note: \( \bar{\cdot} \) indicates the mean or average value of \( \cdot \)
\( \overline{\mathbf{u}}_d \). The level of the mean wind, \( \overline{\mathbf{u}}_d \), is the average headwind magnitude with respect to active runway landing direction.

The models in the blocks of Fig. 40 are the dynamic equations describing the particular subsystem. For example, the longitudinal aircraft equations of motion are the aircraft dynamic model, and so on, for the approach coupler, pilot's manual control and flight control system SCAS dynamic models. The complete details of the models actually used (for the MLS wind, wind shear and turbulence environment; the aircraft; approach couplers and flight control systems) are given in Appendices A and B.

The model shown in Fig. 40 will not be linear in general. However, between capture completion and touchdown an approximate linearized model of the complete system has been shown to be accurate (Ref. 15).

2. Stochastic Section of the Model

The stochastic section of the model for the overall longitudinal system is described by the block diagram in Fig. 41. The block diagram for lateral system is similar. Here the variances of the variables are denoted by \( \sigma^2 \) with the particular variable designated by the subscript. The dynamic models of the aircraft, flight control system, SCAS and approach coupler in Fig. 41 are different from, but are closely related to the corresponding blocks of Fig. 40.

3. Mathematical Basis for System Performance Evaluation

Between capture completion and touchdown the dynamic models in the blocks of Fig. 40 can be described by linear differential equations. It can be shown that the time histories for the atmospheric and MLS inputs can also be described by linear differential equations (operating upon white noise). When this is the case, the entire system model can be written in the form of a first-order vector differential (state) equation and a vector algebraic equation. The specific equations are given in Appendix B for the longitudinal and lateral-directional systems. These are of the form
Figure 4.1. Block Diagram of the Stochastic Section of the Complete Longitudinal Model
\[
\dot{x} = A(t)x + b(t) + w(t) , \quad x(0) = x_0 \tag{35}
\]
\[
y = H(t)x + g(t) \tag{36}
\]

where \( b(t) \) and \( g(t) \) are deterministic input vectors, \( w(t) \) is a vector of independent white noise processes with zero means. If we let \( E[\cdot] \) denote the expected value of \([\cdot]\), then define the mean or expected value for \( x \) as \( \bar{x} \), the differential and algebraic equations for the mean values are

\[
\dot{\bar{x}} = A(t)\bar{x} + b(t) , \quad \bar{x}(0) = \bar{x}_0 \tag{37}^*
\]
\[
\bar{y} = H(t)\bar{x} + g(t) \tag{38}
\]

given that \( E[w] = 0 \). The covariance matrix for \( x \), \( E[x(t)x'(t)] \), is \( C \). The differential equations for the covariance matrix are (e.g., Ref. 16):

\[
\dot{C} = A(t)C + CA'(t) + Q(t) , \quad C(0) = C_0 \tag{39}^*
\]

where \( E[w(t)w'(t + \tau)] = Q(t)\delta(\tau) \). The covariance for the output, \( E[y(t)y'(t)] \), is \( D \).

\[
D = H(t)C'H'(t) \tag{40}
\]

Now the importance of Eqs. 37 through 40 derives from the fact that \( \bar{x}(t) \) and \( C(t) \) completely determine the joint probability density function for \( x(t) \) as a function of time. Namely:

---

*Discretized versions of Eqs. 37 and 39 are used in actual computation. The propagation interval used for the discretized equations is 2 sec.*
\[ \rho(x_1, \ldots x_n, t) = \frac{\exp[-(1/2)(x - \bar{x})^T C^{-1}(x - \bar{x})]}{(2\pi)^{n/2} \sqrt{|C|}} \]  

(41)

where \( \rho(x_1, \ldots x_n, t) \) denotes the n-dimensional joint Gaussian probability density function for \( x(t) \). \( \bar{y}(t) \) and \( D(t) \) similarly define the joint probability density function for \( y(t) \). And, of course, \( x_1, x_2, \ldots \) and \( y_1, y_2, \ldots \) can be used to represent all system variables in the problem of interest. The above equation for \( \dot{C} \) (Eq. 39) and the last equation for \( D \) (Eq. 40) constitute the stochastic section for the complete model shown in Fig. 41. This model is "closely related" to the one in Fig. 40 in that the same parameter matrices (which represent aircraft stability derivatives, flight control system, SCAS and approach coupler gains, etc.) \( A(t) \), \( b(t) \), \( g(t) \) and \( H(t) \) characterize the equations for \( \dot{x} \) and \( y \) as well as the equations for \( \dot{C} \) and \( D \).

Since the probability density function in Eq. 41 is Gaussian, the longitudinal touchdown dispersion \( \sigma_{X_{TD}} \) is:

\[ \sigma_{X_{TD}} = \sqrt{\sigma_X^2 - \rho_{XH}^2} \]  

(42)

\( \rho_{XH} \) is the correlation coefficient for ground range, \( X \), and altitude, \( H \), and \( \sigma_X \) is the standard deviation for \( X \).

4. Results

Performance evaluation results and interpretations are presented in this subsection. Table 9 is a guide to these results. The system mode designations are given in terms of shorthand and code designations in Table 10. Results are in terms of time histories for the approach ensemble means and standard deviations of key variables. The noise and disturbance environment used to produce these results is summarized in Fig. 42. The disturbance environment includes a mean headwind profile (UW) which is a function of altitude to simulate wind shear. Also included are variable headwind (SUW) and crosswind (SVW) components.
### TABLE 9
GUIDE TO PERFORMANCE EVALUATION RESULTS

<table>
<thead>
<tr>
<th>CASE</th>
<th>CONDITIONS</th>
<th>FIGURE NUMBER</th>
</tr>
</thead>
<tbody>
<tr>
<td>Disturbance Environment</td>
<td>Mean and Stochastic and Disturbances</td>
<td>42</td>
</tr>
<tr>
<td>Longitudinal</td>
<td>Mean ( \gamma_o = -6 \text{ deg}^* )</td>
<td>43</td>
</tr>
<tr>
<td>Longitudinal</td>
<td>Standard Deviation ( \gamma_o = -6 \text{ deg} )</td>
<td>44</td>
</tr>
<tr>
<td>Longitudinal</td>
<td>Mean ( \gamma_o = -10 \text{ deg} )</td>
<td>45</td>
</tr>
<tr>
<td>Longitudinal, Vertical Descent</td>
<td>Expanded Scale Mean and Standard Deviation ( \gamma_o = -90 \text{ deg} )</td>
<td>46</td>
</tr>
<tr>
<td>Lateral-Directional</td>
<td>Standard Deviation ( \text{VTOL pad} )</td>
<td>47</td>
</tr>
<tr>
<td>Lateral-Directional</td>
<td>Standard Deviation ( \text{CTOL runway} )</td>
<td>48</td>
</tr>
</tbody>
</table>

\*1 deg = 1.745 \times 10^{-2} \text{ rad.}

### TABLE 10. CONTROL SYSTEM MODE CODE

<table>
<thead>
<tr>
<th>LONGITUDINAL SYSTEM MODE</th>
<th>ISW CODE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Airspeed hold, Glide slope track (AS, GS)</td>
<td>1, 2</td>
</tr>
<tr>
<td>Deceleration initialization, Glide slope track (DECL I, GS)</td>
<td>3</td>
</tr>
<tr>
<td>Deceleration, Glide slope track (DECL, GS)</td>
<td>4</td>
</tr>
<tr>
<td>Point hover, Glide slope track (HOV, GS)</td>
<td>5</td>
</tr>
<tr>
<td>Point hover, Altitude hold (HOV, ALT)</td>
<td>6</td>
</tr>
<tr>
<td>Point hover, Vertical descent (HOV, VD)</td>
<td>7</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>LATERAL-DIRECTIONAL SYSTEM MODE</th>
<th>SA CODE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Localizer track, turn following (LOC A)</td>
<td>1</td>
</tr>
<tr>
<td>Localizer track, wing low (LOC B)</td>
<td>0</td>
</tr>
</tbody>
</table>
Legend: "S" prefix denotes standard deviation for stochastic variable

Figure 42. Noise and Disturbance Environment Characteristics ($\gamma_0 = -6$ deg)
b) Lateral-Directional Components

Figure 42. (Concluded)
profiles. These components are also functions of altitude to simulate wind shear and have levels which are random variables from one approach to the next, but are constant during any one approach. Fifty percent-probability-of-exceedence turbulence is used. Selection of the 50 percent level was arbitrary, even though representing a typical level in operation. (Larger levels might result in significant series servo limiting.) Longitudinal (SUG), normal (SWG), side (SVG) and effective rolling (SPG) components are included. The levels and integral scale lengths are functions of altitude and airspeed in the manner of the Dryden turbulence model. MLS noise is modeled on the basis of the so-called path-following error budget described in Ref. 17. MLS glide slope (SEFA), azimuth (SENU) and DME (SXC) components are included.

Longitudinal and lateral-directional response plots have been grouped by area of interest for Figs. 43-45, 47 and 48. These areas of interest are

Part a: Trajectory variables
--- Longitudinal: HD, ALTH, X, XD
--- Lateral-Directional: PHI, PSI, YD, Y

Part b: Cockpit indications
--- Longitudinal: LSW, VA, XD, XIND, ALTH, DE, HD, THET, DC, FDC, DCL, FDCL
--- Lateral-Directional: SA, YE, PSI, R, PHI, AYP, DS, FDL, FDP

Part c: Pilot acceptability variables
--- Longitudinal: Q, THEE, AZP, AX, VAE
--- Lateral-Directional: P, PHI, R, AYP

Part d: Limited variables
--- Longitudinal: THCD, FDC, DLN, DLM, SR, DEH, FDCL, DCL
--- Lateral-Directional: PHDC, FDL, DLAT, DLTS, DPDS

The criteria for pilot acceptability are limits upon variability about the mean responses. These limits are listed in Table 11. These limits are generally accepted in the industry. Some have been stated in FAA Advisory Circulars or in ICAO Annex 10. In part c of the standard deviation response figures, these limits are shown by J-shaped brackets.

Variables which are limited by device constraints or by actual limiter functions have their limiting values listed in Table 12. These limiting values
### Table 11. Pilot Acceptance Limits for Final Approach

<table>
<thead>
<tr>
<th>Attitude Deviation</th>
<th>$\sigma_\theta, \sigma_\phi \leq 2 \text{ deg}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Attitude Rates</td>
<td>$\sigma_p, \sigma_q, \sigma_r \leq 2 \text{ deg/sec}$</td>
</tr>
<tr>
<td>Linear Acceleration Deviation</td>
<td>$\sigma_{nz} \leq 0.1 \text{ g}$</td>
</tr>
<tr>
<td></td>
<td>$\sigma_{nx}, \sigma_{ny} \leq 0.05 \text{ g}$</td>
</tr>
<tr>
<td>Airspeed Deviation (during airspeed hold)</td>
<td>$\sigma_{w_{AS}} \leq 5 \text{ kt}$</td>
</tr>
</tbody>
</table>

### Table 12. Summary of System Limit Levels

<table>
<thead>
<tr>
<th>VARIABLE</th>
<th>LIMIT LEVEL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pitch Attitude Command Limit</td>
<td>$\pm 10 \text{ deg}$</td>
</tr>
<tr>
<td>Flight Director Longitudinal Cyclic Command Bar</td>
<td>$\pm 1 \text{ in.}$</td>
</tr>
<tr>
<td>Longitudinal Cyclic Pitch Deflection (Stick Units)</td>
<td>$\pm 4.8 \text{ in.}$</td>
</tr>
<tr>
<td>Longitudinal Cyclic Series Servo Deflection</td>
<td>$\pm 1 \text{ in.}$</td>
</tr>
<tr>
<td>Sink Rate Command</td>
<td>$\pm 0.33 \text{ ft/sec}$</td>
</tr>
<tr>
<td>Beam Rate Command</td>
<td>$\pm 44.1 \text{ ft/sec}$</td>
</tr>
<tr>
<td>Flight Director Collective Command Bar</td>
<td>$\pm 1 \text{ in.}$</td>
</tr>
<tr>
<td>Collective Pitch Deflection (Lever Units)</td>
<td>$\pm 1.78, \pm 6.0 \text{ in.}$</td>
</tr>
<tr>
<td>Bank Angle Command</td>
<td>$\pm 30 \text{ deg (LOC A)}$</td>
</tr>
<tr>
<td>Flight Director Lateral Cyclic Command Bar</td>
<td>$\pm 1 \text{ in.}$</td>
</tr>
<tr>
<td>Lateral Cyclic Pitch Deflection (Stick Units)</td>
<td>$\pm 4.8 \text{ in.}$</td>
</tr>
<tr>
<td>Lateral Cyclic Series Servo Deflection</td>
<td>$\pm 1 \text{ in.}$</td>
</tr>
<tr>
<td>Rudder Series Servo Deflection</td>
<td>$\pm 1 \text{ in.}$</td>
</tr>
</tbody>
</table>

*1 deg = $1.745 \times 10^{-2} \text{ rad}$*

*1 in. = $2.54 \times 10^{-2} \text{ m}$

*1 ft = $3.048 \times 10^{-1} \text{ m}$
are represented by C-shaped brackets on the part d mean response plots and by F-shaped brackets on the part d standard deviation response plots. The upper horizontal stroke of the "F" represents the limiting value; the middle stroke of the "F" represents the limiting value less the absolute value of the mean response. The interpretation to be made is as follows: If the standard deviation for a variable is less than one-half the distance between the base of the "F" and the middle horizontal stroke, then the probability of encountering the operative limit is less than 5 percent; if less than one-third the distance; less than 0.26 percent; if less than the distance itself, less than 32 percent; etc. In the case of the lateral-directional variables, the middle and upper horizontal strokes of the "F" coincide because the mean response for all lateral-directional variables is zero.

5. Interpretation of Results

a. Response Means for Longitudinal Variables

The response means may be interpreted either as the mean responses in the stochastic disturbance environment or as the deterministic responses in the absences of all disturbances except the mean headwind/shear (W).

These responses (Figs. 43 and 45) show well-controlled glide slope tracking during the airspeed hold, constant attitude deceleration and point hover phases of the final approach for approach path angles of -6 and -10 deg (-0.10 and -0.17 rad). The aircraft continues to be well-controlled in the point hover mode through the transition from glide slope track to altitude hold at 50 ft (15 m) and during the exponential flare, vertical descent to touchdown. Figure 46 gives the key responses on expanded scales for the vertical descent maneuver.

Transient responses at initiation of the constant attitude deceleration and point hover phase are only very slightly different for the -6 and -10 deg (-0.10 and -0.17 rad) approach paths. This lack of sensitivity to approach path angle is desirable. It is the result of the particular switching logic used.

Initiation of the constant attitude deceleration maneuver results in modest "ballooning" above the glide path (refer to DR trace) because there
is no crossfeed of the pitch-up deceleration command to the cyclic pitch control. A crossfeed is deemed unnecessary because the peak glide slope deviation is only 5 ft (1.5 m) at a ground range (X) of 2000 ft (610 m) from the hover point.

Longitudinal (AX) and normal (AZP) accelerations encountered during deceleration and vertical descent phases are moderate. The longitudinal cyclic stick (DC) and collective pitch lever (DCL) deflections required are well within the available limits. The longitudinal series servo deflection (DLNS) required is well within the available authority.

b. Response Standard Deviations for Longitudinal Variables

Standard deviation responses are shown in Fig. 44 and on expanded scales for vertical descent in Fig. 46. The plots are for a -6 deg (-0.10 rad) approach path. Plots for other approach path angles are indistinguishable from the -6 deg (-0.10 rad) plots except for a slight stretching or shrinking of the time axis. There is virtually no dependence of the vertical descent results on approach path angle.

The standard deviation responses show low variability in all trajectory variables and cockpit indications with the exceptions discussed below. Variability in altitude (SALT) and ground range (SX) grow large during the constant airspeed and constant attitude deceleration phases of the approach. This is the (random walk) effect of the headwind (SW) variability in producing along-path variations in aircraft position for a given time into the approach. This results because slant range is uncontrolled by this system. This along-path or slant-range component of variability is reduced to zero at t = 54.5 sec by a mathematical procedure. It must be emphasized that this procedure is actually part of the system model; it is not an arbitrary feature. Its purpose is to avoid introducing a fictitious slant-range dependency into the hover approach phase performance statistics. Airspeed (SVA) variability becomes large following the airspeed hold phase of the approach. This occurs because speed reduction ceases during the constant attitude deceleration phase and because ground speed is regulated during the subsequent hover phase. Pitch attitude (STHET) variability tends to increase transiently during initiation of the constant attitude deceleration and hover phases. The generally larger pitch attitude
\[ \gamma_0 = -6 \text{ deg} \]

Control System Mode

Rate of Climb (Instantaneous)

Altitude (AGL)

Ground Range to TD Point

Ground Speed

a) Longitudinal Trajectory

Figure 43. Mean Responses for Longitudinal System on \(-6^\circ\) Approach from 340' Through Touchdown
\[ \gamma_0 = -6 \text{ deg} \]

Control System Mode

Airspeed

Groundspeed

Ground Range (DME)

Altitude (AGL)

b) Longitudinal Cockpit Indications (1)

Figure 43. (Continued)
\( \gamma_0 = -6 \text{deg} \)

Control System Mode

Glideslope Deviation

Instantaneous Vertical Speed

Pitch Attitude (Absolute)

Altitude (AGL)

b) Longitudinal Cockpit Indications (ii)

Figure 43. (Continued)
\( \gamma_0 = -6 \text{ deg} \)

Control System Mode

Longitudinal Cyclic Stick Deflection

Flight Director Longitudinal Cyclic Command Bar

Collective Pitch Deflection (lever units)

Flight Director Collective Command Bar

b) Longitudinal Cockpit Indications (iii)

Figure 43. (Continued)
c) Longitudinal Pilot Acceptance Variables

Figure 43. (Continued)
\( \gamma_0 = -6 \text{deg} \)

Control System Mode

Input to Pitch Attitude Command Limiter

Flight Director Longitudinal Cyclic Command Bar

Longitudinal Cyclic Pitch Deflection (Stick Units)

Longitudinal Cyclic Series Servo Deflection (Stick Units)

d) Longitudinal Limits (i)

Figure 43. (Continued)
\[ \gamma_0 = -6 \text{ deg} \]

Control System Mode

Input to Sink Rate Command Limiter

Input to Beam Rate Command Limiter (to A)
Vertical Descent Altitude Command (after A)

Flight Director Collective Command Bar
Collective Pitch Deflection (Lever Units)

*Effective scale change at A
**Effective scale change at C

**Figure 43. (Concluded)
\( \gamma_0 = -6 \text{ deg} \)

**Control System Mode**

**Rate of Climb (Instantaneous)**

**Altitude (AGL)**

**Ground Range to TD Point**

**Groundspeed**

Reinitialization to eliminate the along-path (time) component of variability arising from headwind variation

---

*a) Longitudinal Trajectory*

**Figure 44. Standard Deviation Responses for Longitudinal System on -6° Approach from 340° Through Touchdown**
b) Longitudinal Cockpit Indications (i)

Figure 44. (Continued)
$\gamma_0 = -6 \text{deg}$

Control System Mode

Glideslope Deviation

Instantaneous Vertical Speed

Pitch Attitude (Absolute)

Reinitialization to eliminate the along-path (time) component of variability arising from headwind variation

*Pilot Acceptance Variable (after B)*

b) Longitudinal Cockpit Indications (II)

Figure (Continued)
c) Longitudinal Pilot Acceptance Variables

Figure 44. (Continued)
$\gamma_0 = -6 \text{ deg}$

Control System Mode

Input to Sink Rate Command Limiter

Input to Beam Rate Command Limiter (to A)
Vertical Descent Altitude Command (After A)

Flight Director Collective Command Bar

Collective Pitch Deflection (Lever Units)

*Effective Scale Change at C

d) Longitudinal Limits (ii)

Figure b' (Concluded)
Figure 45. Mean Responses for Longitudinal System on -10° Approach from 500' through Touchdown
\[ \gamma_0 = -10 \text{ deg} \]

Control System Mode

Airspeed

Groundspeed

Ground Range (DME)

Altitude (AGL)

---

b) Longitudinal Cockpit Indications (1)

Figure 15. (Continued)
\( \gamma_0 = -10 \text{ deg} \)

Control System Mode

Glideslope Deviation

Instantaneous Vertical Speed

Pitch Attitude (Absolute)

Altitude (AGL)

b) Longitudinal Cockpit Indications (ii)

Figure 45. (Continued)
$\gamma_0 = -10$ deg

Control System Mode

Longitudinal Cyclic Stick Deflection

Flight Director Longitudinal Cyclic Command Bar

Collective Pitch Deflection (Lever Units)

Flight Director Collective Command Bar

b) Longitudinal Cockpit Indications (iii)

Figure b- (Continued)
\[ \gamma_0 = -10 \text{deg} \]

Pitch Rate

Pitch Attitude Deviation from Commanded Value

Normal Acceleration at Pilot Location

Longitudinal Acceleration

Airspeed Deviation from Approach Reference

c) Longitudinal Pilot Acceptance Variables

Figure 45. (Continued)
\[ \gamma_0 = -10 \text{ deg} \]

Control System Mode

Input to Pitch Attitude Command Limiter

Flight Director Longitudinal Cyclic Command Bar

Longitudinal Cyclic Pitch (Stick Units)

Longitudinal Cyclic Series Servo Deflection (Stick Units)

\textbf{d) Longitudinal Limits (1)}

Figure 45. (Continued)
\[ \gamma_0 = -10 \text{ deg} \]

Control System Mode

Input to Sink Rate Command Limiter

Input to Beam Rate Command Limiter (Up to A) Vertical Descent Altitude Command (After A)

Flight Director Collective Command Bar

Collective Pitch Deflection (Lever Units)

*Effective scale change at A
**Effective scale change at C

d) Longitudinal Limits (ii)

Figure 45. (Concluded)
Figure 46a. Expanded Scale Responses for Longitudinal System
for Vertical Descent - from 50' to Touchdown
Figure 45b. Expanded Scale Responses for Longitudinal System for Vertical Descent from 50' to Touchdown
a) Lateral-Directional Trajectory

Figure 47. Standard Deviation Responses for Lateral-Directional System on -6° Approach from 340' through T зна on VTOL Pad
b) Lateral-Directional Cockpit Indications (1)

Figure 47. (Continued)
A diagram showing various control parameters for a flight system. Parameters include:

- \( R_0 = 1000 \text{ft} \)
- Lateral Directional Control System Mode
- Lateral Cyclic Stick Deflection
- Flight Director Lateral Cyclic Command Bar
- Flight Director Rudder Pedal Command Bar
- Longitudinal Control System Mode

The diagram includes a timeline from 0.00 to 10.00, with specific values and points indicating control actions.

Figure 47. (Continued)
c) Lateral-Directional Pilot Acceptance Variables

Figure 47. (Continued)
d) Lateral-Directional Limits

Figure 3-7. (Concluded)
Figure 48. Standard Deviation Responses for Lateral-Directional System on -6° Approach from 340' through Touchdown on CTOL Runway
b) Lateral-Directional Cockpit Indications (ii)

Figure 10. (Continued)
c) Lateral-Directional Pilot Acceptance Variables, Trajectory and Cockpit Indications (ii)

Figure 48. (Continued)
R₀ = 10,000 ft

Input to Bank Angle Command Limiter

Flight Director Lateral Cyclic Command Bar

Lateral Cyclic Pitch Deflection (Stick Units)

Lateral Cyclic Series Servo Deflection (Stick Units)

Rudder Series Servo Deflection (Pedal Units)

Figure 48. (Concluded)
variability during these two phases is basically the effect of the pitch attitude required to trim in the face of various headwind components.

Variability in rate of climb (SHD), the flight director command bars (SFDC and SFDCL) and the controls (SDC and SDCL) is very low throughout approach and landing. Figure 46 shows that ground speed (SXD) and longitudinal position (SX) variability is small throughout vertical descent.

Variability in the pilot acceptance variables is satisfactory, but the following interpretations are required. Pitch attitude deviations from the commanded value (STHEE) show low variability before the hover phase. During the hover phase the high additional variability encountered is the result of DME noise. The actual pitch attitude (STHET) variability during hover is in the acceptable range (±2 deg). Variability in normal acceleration (SAZP) is larger than the acceptability level only for a very brief interval during initiation of the constant attitude deceleration phase. The acceptability level for airspeed error variability (SVAE) applies only during the airspeed hold phase of the approach.

Inputs to all limiting functions except for pitch attitude command (STHCD) are such that the probability of limiting is less than 5 percent. The probability of pitch attitude limiting exceeds 5 percent only during the short hover initiation transient. This is nevertheless expected to result in acceptable performance.

c. Response Standard Deviations for Lateral-Directional Variables

Recall that the mean response for all lateral-directional variables is zero. Response standard deviations are included for two landing site types, a VTOL pad in Fig. 47, and a CTOL runway in Fig. 48. The two sites have different separations between the azimuth and glide slope antennas. This separation is 1000 ft (305 m) for the VTOL pad and 10,000 ft (3,048 m) for the CTOL runway. The level of MLS noise disturbing the lateral-directional system is directly proportional to range from the azimuth antenna.

The standard deviation responses show low variability in all trajectory variables and cockpit indications except for bank angle (SPHI) in the CTOL runway case. The large bank angle variability is the result of coupling too
tight to the localizer at large ranges from the azimuth antenna in the Section IV design. Reduced $K_x$ and $K_y$ gains or range-scheduled values for these gains will be required in the final system. In particular, lateral deviation (SY) and lateral deviation rate (SVD) standard deviations are small throughout the approach. The standard deviation in heading deviation (referenced to runway centerline) (SFSI) is almost entirely due to Crab angle resulting from crosswind variability, one approach to the next.

All standard deviations for pilot acceptance variables are satisfactory for the VTOL pad case. For the CTOL runway case, the standard deviations for roll rate (SP) and bank angle (SPHI) exceed the acceptable level by a significant amount. This is for the reason explained in the preceding paragraph. This problem will be eliminated by the same fix in the final system.

Inputs to the crosstrack rate (SY) and bank angle command (SPHDC) limiters, and to lateral cyclic pitch (SDIAT) and stick (SDS), flight director lateral cyclic command bar (SDFP) are all less than one-fourth of the limit level for the VTOL pad case. The rudder series servo deflection (SDFDS) is such the probability of limiting is 16 percent during very low speed flight. The effective reduction in yaw damping resulting from this may be tolerable in the final system. If it is not, low frequency inputs to the rudder series servo will have to be shifted to the rudder force feel system actuator. Early in the approach record the standard deviation of the lateral cyclic series servo deflection (SDLTS) approaches the limit level. This again is the result of overly tight coupling to the localizer.

The inputs to the limiting functions have similar characteristics for the CTOL runway case, but the limiting is more severe in the case of bank angle command (SPHCD) and lateral cyclic series servo deflection (SDLTS). Less tight coupling to the localizer in the final system will reduce these limiter inputs to acceptable levels.

d. Touchdown Statistics

Criteria for a successful touchdown are compared with the corresponding values resulting from performance evaluation in Table 13. It turns out that these performance evaluation values are unchanged to two significant figures
TABLE 13. COMPARISON OF KEY VARIABLES AT TOUCHDOWN WITH ACCEPTABLE LIMITS

**PERFORMANCE**

<table>
<thead>
<tr>
<th>VTOL Pad</th>
<th>CTOL Runway</th>
<th>REQUIREMENT</th>
<th>SOURCE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.26 ft</td>
<td>7.35 ft*</td>
<td>$\sigma_{y_{TD}} \leq 0.1$ W (W = 150 ft, typical)</td>
<td>Touchdown point location</td>
</tr>
<tr>
<td>6.36 ft</td>
<td>$\sigma_{y_{TD}} \leq 0.1$ W (not critical for CTOL runway)</td>
<td>Touchdown point location</td>
<td></td>
</tr>
<tr>
<td>1.58 ft/sec</td>
<td>$\sigma_{y_{TD}} \leq 5$ ft/sec (not critical for CTOL runway)</td>
<td>Roll off pad</td>
<td></td>
</tr>
<tr>
<td>-2.28 ft/sec</td>
<td>$\overline{h_{TD}} + 3\sigma_{h_{TD}} &lt; 0$ ft/sec</td>
<td>Positiveness of touchdown</td>
<td></td>
</tr>
<tr>
<td>-2.98 ft/sec</td>
<td>$\overline{h_{TD}} - 3\sigma_{h_{TD}} \geq -12$ ft/sec</td>
<td>Landing gear strength</td>
<td></td>
</tr>
<tr>
<td>5.56 deg</td>
<td>$\overline{\varphi_{TD}} + 3\sigma_{\varphi_{TD}} \leq 12.75$ deg</td>
<td>Airframe ground clearance</td>
<td></td>
</tr>
<tr>
<td>-2.65 deg</td>
<td>$\overline{\varphi_{TD}} - 3\sigma_{\varphi_{TD}} \geq -5$ deg</td>
<td>Nose gear-first touchdown limit</td>
<td></td>
</tr>
<tr>
<td>0.12 ft</td>
<td>0.14 deg/sec</td>
<td>$\sigma_{y_{TD}} \leq 2.0$ deg/sec</td>
<td>Nose gear side load</td>
</tr>
<tr>
<td>0.23 ft/sec</td>
<td>$\sigma_{y_{TD}} \leq 2.0$ ft/sec</td>
<td>Landing side load</td>
<td></td>
</tr>
<tr>
<td>1.87 ft</td>
<td>2.25 deg</td>
<td>$3\sigma_{\varphi_{TD}} \leq 15$ deg</td>
<td>Airframe ground clearance</td>
</tr>
</tbody>
</table>

---

when the missed approach decision rule is operative. In every case, the performance achieved is far better than the criterion specified in the "Requirement" column. This may be interpreted as permitting safe system operation in a more severe disturbance environment or as permitting operation with systems of lesser effectiveness than that developed herein.

The basic limitations upon touchdown accuracy for the current system arise from the significant noise levels on the DME and azimuth (CTOL runway only) guidance signals.
SECTION VI
APPROACH MONITORING CRITERIA

Development of approach monitoring criteria for both manually and automatically controlled approaches proceeds interactively with performance evaluation. These criteria are then implemented in the missed approach decision rule. In this section the important considerations involved are discussed first, then the technical results are presented.

The actual decision to break off an approach may of course be made at any point in the approach prior to the decision point. However, concepts based upon a single decision point are used since they are sufficient to protect against a landing accident arising from an out-of-tolerance approach. The development concepts are the same for both automatically and manually controlled approaches, and regardless of whether criterion evaluation is a computer or pilot task. However, it is possible that distinctly different criteria are appropriate for computer and pilot evaluation. This is so because of the pilot-centered requirements when conventional aircraft instruments (ADI, HSI, etc.) are used. Requirements are for acceptable workload level when aircraft is manually controlled via the flight director, ability to discern the criterion levels accurately using the given instrument scales, etc. These pilot-centered requirements are not operative when criterion evaluation is implemented in the computer.

A. BASIC CONCEPTS

Approach monitoring criteria tend to derive from three basic considerations:

- Airframe operational performance, e.g., control power envelope limits defining the safe operation corridor
- Limits upon dynamic excursions for reasons of pilot acceptance.
- Limits upon dynamic excursions for reasons of achieving landing on the pad/runway with precision adequate for safety
Operational characteristics tend to establish the "location" of the latest point in the approach at which a missed approach may be elected, that is, it establishes a selected altitude and/or distance as a "decision point." The XV-15 decision point location is selected to occur during the stabilized point hover while in the altitude hold mode, just prior to committing to vertical descent. This decision point is at the latest possible time in the approach because missed approach execution from the vertical-descent maneuver appears unwise in the extreme. This is the case because later execution of a missed approach would require arrest of the established sink rate using the modest amount of remaining collective pitch control power. On the other hand, an earlier decision point is undesirable because the missed approach decision rule is then less effective in eliminating unsafe touchdown conditions. This is the case because of increased exposure to subsequent disturbances which can produce unsafe conditions. These are the key operational performance considerations.

Limits imposed by virtue of pilot acceptance of dynamic excursions in attitude, speed, etc., throughout the approach have already been evaluated in the previous section. Dynamic excursions in attitude, attitude rate, linear acceleration, and speed are found to remain generally within limits which inspire pilot confidence in the integrity of system performance throughout the approach.

The class of limits which arise from landing precision requirements is more difficult to determine. It requires that limits on acceptable touchdown dispersion and sink rate (as well as the other quantities listed in Table 13) be projected back up the approach path to establish limits upon the available approach status data at the decision point. The limits on the approach status data at the decision point must assure a high probability of achieving touchdown conditions which are within the "safe landing" limits without being unduly conservative (i.e., causing an excessive missed approach rate). By this description it is clear that this aspect of determining approach monitoring criteria can have an iterative interaction with the landing performance evaluation. Fortunately, selection of the covariance propagation method of performance evaluation provides a key relationship between dispersion at the decision point and dispersion at touchdown. This relationship enables identification
of the essential variables i.e. approach monitoring and critical decision levels for those variables. These variables and the corresponding decision levels define an approach "window." If an approach trajectory passes through this window, there is a high probability that the resulting landing will be acceptable and that approach is continued to touchdown. All other approaches are converted to missed approaches as go-arounds are executed.

The method for developing approach monitoring criteria arising from touchdown dispersion must accommodate two facts:

- Acceptable landing must essentially be predicted on the basis of airborne sensor measurements existing at the time the decision point is reached.
- Acceptable values of the key variables characterizing touchdown conditions must be inferred from a different, only somewhat related, set of variables which are the airborne sensor measurements.

As a first step, define the key variables and limits characterizing acceptable touchdown. These are listed in the "Requirement" column of Table 13. A list of the available airborne measurements is in Table 14. (The fact that only some of these measurements are available to the crew via conventional instrumentation must be taken into account when approach monitoring is accomplished by the crew.)

The key step is to model the interaction between approach performance just prior to the decision point, just after the decision point, the missed approach rate, and touchdown dispersion. Define the following variables:

\[ \bar{X} \] Expected value (mean) of state vector

\[ \bar{Y} \] Expected value (mean) of an alternate state vector including the key variables determining touchdown conditions (Table 13) and airborne measurements (Table 14), \[ \bar{Y} = H\bar{X} \]

\[ D \] Covariance matrix for an alternative state vector which includes the key variables determining touchdown conditions and airborne measurements

\[ (\cdot)^{DP}- \] (\cdot) evaluated just before the decision point

\[ (\cdot)^{DP}+ \] (\cdot) evaluated just after the decision point

\[ (\cdot)^{TD} \] (\cdot) evaluated at touchdown
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d_e$</td>
<td>Indicated MLS glide slope deviation*</td>
</tr>
<tr>
<td>$v_e$</td>
<td>Indicated MLS localizer deviation*</td>
</tr>
<tr>
<td>$\dot{d}$</td>
<td>MLS derived glide slope deviation rate</td>
</tr>
<tr>
<td>$h$</td>
<td>Instantaneous vertical speed*</td>
</tr>
<tr>
<td>$h$</td>
<td>Barometric and radar altitude*</td>
</tr>
<tr>
<td>$x_1$</td>
<td>DME (distance)*</td>
</tr>
<tr>
<td>$\theta, \phi, \psi$</td>
<td>Pitch, roll, heading*</td>
</tr>
<tr>
<td>$V_A$</td>
<td>Airspeed*</td>
</tr>
<tr>
<td>$\dot{x}$</td>
<td>Ground speed (via DME)*</td>
</tr>
<tr>
<td>$q, p, r$</td>
<td>Pitch, roll, yaw rate gyros</td>
</tr>
<tr>
<td>$a_x, a_y, a_y'$</td>
<td>Normal, longitudinal, and lateral* acceleration</td>
</tr>
<tr>
<td>Instrument flag signals*</td>
<td></td>
</tr>
<tr>
<td>Rotor speed*</td>
<td></td>
</tr>
<tr>
<td>Rotor cross-shaft torque*</td>
<td></td>
</tr>
<tr>
<td>Differential collective pitch</td>
<td></td>
</tr>
<tr>
<td>Flap position*</td>
<td></td>
</tr>
<tr>
<td>Nacelle (pylon) angle*</td>
<td></td>
</tr>
</tbody>
</table>

*Cockpit indications available.
State transition matrix from the decision point to touchdown

Covariance matrix component at touchdown arising from stochastic disturbance inputs acting during the interval from the decision point to touchdown

Mean state vector component at touchdown arising from mean inputs during the interval from the decision point to touchdown

The following relationships (based on linear system theory) hold between quantities just after the decision point and quantities at touchdown:

\[ \bar{Y}_{TD} = \Phi \bar{Y}_{DP} + \Theta \]

\[ D_{TD} = \Phi \bar{D}_{DP} + \Gamma \]

(These equations may be thought of as the result of integrating Eqs. 37 and 39 between the decision point and touchdown, followed by application of Eqs. 38 and 40.)

It turns out that the imposition of a missed approach decision rule based on approach monitoring criteria has virtually no effect upon \( \bar{Y}_{DP}^{+} \) and \( \bar{Y}_{TD} \).

Those characteristics are determined mainly by the selection of the nominal approach path, performance characteristics of the aircraft and the basic guidance and control law structure. \( D_{TD} \), the dispersion in key touchdown variables, can be very sensitive to the approach monitoring criteria because \( D_{DP}^{+} \) is very sensitive to the approach monitoring criteria. (Recall that \( D_{DP}^{-} \) is converted to \( D_{DP}^{+} \) by imposing the missed approach decision rule based on the approach monitoring criteria.) The next step consists of identifying those measurement variables (or combinations of variables) in \( D_{DP}^{+} \) which affect the key touchdown variables in \( D_{TD} \) in a sensitive way. The covariance propagation method of performance evaluation makes numerical evaluations of \( \Gamma \) and \( \Phi \) available so that this step is really less complex than it might appear. In the final step, the procedure described immediately below is used to convert quantities just prior to the decision point, \( \bar{Y}_{DP}^{+} \), to quantities just after the decision point, \( \bar{Y}_{TD} \), for a trial set of approach monitoring criteria, and Eq. 44 is used to evaluate dispersion in key touchdown variables. This process is
continued iteratively until approach monitoring criteria satisfying all performance objectives are found. In this latter step, effects of electing missed approach execution or approach continuation on landing performance are evaluated and the missed approach rate is determined. The upper limit placed on the missed approach rate, $R_{MA}$, is $R_{MA} \leq 0.05$ in this iterative procedure.

Since out-of-tolerance approaches are converted to missed approaches at the decision height in our model, there must also be a correction of the joint probability density function (refer to Eq. 41) for all problem variables at the decision height so that only those approaches which are continued to touchdown are represented. For illustrative purposes only, consider example approach monitoring criteria for indicated distance (DME), $X_i$, and MLS glide slope deviation, $d_e$. Let the decision levels for these variables be $\pm 60$ ft ($\pm 18$ m) and $\pm 6$ ft ($\pm 1.8$ m), respectively. Just prior to the decision point the joint probability density function is:

$$[p(X_i, d_e, x_3, \ldots x_n, t)]_{H=50 \text{ ft (15 m)}}$$

(45)

Just after the decision point it is

$$(1-R_{MA})^{-1} [p(X_i, d_e, x_3, \ldots x_n, t)]_{H=50 \text{ ft (15 m)}}$$

(46)

for $-60 \leq X_i \leq 60$ and $-6 \leq d_e \leq 6$, and is zero elsewhere. $(1 - R_{MA})$ is the "volume" under the joint probability density function surface (Eq. 41) between the above limits on $X_i$ and $d_e$. The joint probability density function just after the decision height is obviously non-Gaussian. This non-Gaussian joint probability density function for $X_i$ and $d_e$ is approximated by a Gaussian one having the same first and second moments. The influence of these first and second moments (i.e., the means and variances) upon the mean and covariance for other states is modeled in simulation as the expected outcome of a single discrete Kalman measurement update. The result of this update provides the initial conditions for continuing the solution of Eqs. 37 through 40 from the decision height to touchdown for IMC operations with the missed approach decision rule operative.
B. RESULTS

Recall that the standard deviations of the elements in the $Y_{TD}$ vector are square roots of the respective diagonal elements of $D_{TD}$. Referring to the right hand side of Eq. 44, two terms are apparent. The first term represents that component of $D_{TD}$ which arises because variability in $Y_{TD}$ just after the decision point. This component may be modified by changing the approach monitoring criteria. The second term represents that component of $D_{TD}$ which arises because of the stochastic components of turbulence and MLS noise acting between the decision point and touchdown. This component is not affected by the approach monitoring criteria. This component sets the lower bound on the reduction in standard deviations of $Y_{TD}$ obtainable via the approach monitoring criteria and the attendant missed approach decision rule.

Tables 15 through 17 summarize the standard deviation for $Y_{TD}$ vector in the absence of approach monitoring criteria, $\sqrt{(D_{TD})_{ii}}$, and the lower bound achievable with approach monitoring criteria, $\sqrt{(I)_{ii}}$. These tables together with Table 13 lead to the following conclusions.

- Dispersion in key variables at touchdown is well within acceptable safe limits in the absence of an approach monitoring criterion and attendant missed approach decision rule (Table 13).
- Potential for reduction in key variable dispersions at touchdown with use of an approach monitoring criterion and attendant missed approach decision rule is slight (Tables 15 through 17).

Regardless of the negative indication of necessity given above, approach monitoring criteria are imposed based on estimates of likely conditions resulting from pilot abuse of the approach and landing system operating procedures. These are given in Table 18. The resulting probability of missed approach assuming normal system use is $P_{MA} = 0.0082$ or a rate of about one in 122 IMC approaches. The significant contributions to $P_{MA}$ are from the second and third items in Table 18 in approximately equal amounts.

In the earlier stages of the final approach missed approach decision levels given in Table 19 are recommended. The resulting probability of missed
TABLE 15

COMPONENTS OF LONGITUDINAL TOUCHDOWN DISPERSION REDUCIBLE BY MISSED APPROACH DECISION RULE AT 50 FT

<table>
<thead>
<tr>
<th>VARIABLE</th>
<th>UNITS</th>
<th>KEY AT TOUCHDOWN</th>
<th>STANDARD DEVIATION</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Total, $\sqrt{\text{D}<em>{\text{TD}}}</em>{\text{ii}}$</td>
</tr>
<tr>
<td>$x$</td>
<td>ft/sec</td>
<td>$\checkmark$</td>
<td>1.58</td>
</tr>
<tr>
<td>$\dot{x}$</td>
<td>ft/sec</td>
<td>$\checkmark$</td>
<td>0.116</td>
</tr>
<tr>
<td>$q$</td>
<td>deg/sec</td>
<td>$\checkmark$</td>
<td>1.49</td>
</tr>
<tr>
<td>$\theta$</td>
<td>deg</td>
<td>$\checkmark$</td>
<td>1.37</td>
</tr>
<tr>
<td>$H$</td>
<td>ft</td>
<td></td>
<td>1.14</td>
</tr>
<tr>
<td>$x$</td>
<td>ft</td>
<td>$\checkmark$</td>
<td>6.35</td>
</tr>
<tr>
<td>$x_7$</td>
<td>in²</td>
<td></td>
<td>0.315</td>
</tr>
<tr>
<td>$\delta_{\text{CL}}$</td>
<td>in.</td>
<td></td>
<td>0.214</td>
</tr>
<tr>
<td>$x_{9}$</td>
<td>ft/sec²</td>
<td></td>
<td>0.663</td>
</tr>
<tr>
<td>$x_{10}$</td>
<td>in.</td>
<td></td>
<td>0.004</td>
</tr>
<tr>
<td>$\delta_e$</td>
<td>ft</td>
<td>$\checkmark$</td>
<td>0.00</td>
</tr>
<tr>
<td>$\delta_s$</td>
<td>in.</td>
<td></td>
<td>0.320</td>
</tr>
<tr>
<td>$\theta_c/K_\theta$</td>
<td>deg</td>
<td></td>
<td>2.45</td>
</tr>
<tr>
<td>$x_{14}$</td>
<td>in.</td>
<td></td>
<td>0.105</td>
</tr>
<tr>
<td>$\beta_{\text{DC}}$</td>
<td>in.</td>
<td></td>
<td>0.115</td>
</tr>
<tr>
<td>$\theta_{\text{cd}}$</td>
<td>deg</td>
<td></td>
<td>5.39</td>
</tr>
<tr>
<td>$x_{17}$</td>
<td>ft/sec</td>
<td></td>
<td>11.0</td>
</tr>
<tr>
<td>$\delta_e$</td>
<td>ft</td>
<td></td>
<td>1.33</td>
</tr>
<tr>
<td>$K$</td>
<td>ft</td>
<td></td>
<td>20.9</td>
</tr>
</tbody>
</table>

*1 ft = 3.048 x 10⁻¹ m.

†1 deg = 1.745 x 10⁻² rad.

‡1 in. = 2.54 x 10⁻² m.
### TABLE 16

**Components of Lateral Touchdown Dispersion Reducible by Missed Approach Decision Rule at 50 ft**
(*VTOL Pad Case*)

<table>
<thead>
<tr>
<th>VARIABLE</th>
<th>UNITS</th>
<th>KEY AT TOUCHDOWN</th>
<th>STAND. DEVIATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\dot{y}$</td>
<td>ft/sec</td>
<td>✓</td>
<td>0.227</td>
</tr>
<tr>
<td>$\cdot r$</td>
<td>deg/sec</td>
<td>✓</td>
<td>0.130</td>
</tr>
<tr>
<td>$r$</td>
<td>deg/sec</td>
<td>✓</td>
<td>0.116</td>
</tr>
<tr>
<td>$\varphi$</td>
<td>deg</td>
<td>✓</td>
<td>0.625</td>
</tr>
<tr>
<td>$\psi$</td>
<td>deg</td>
<td></td>
<td>10.9</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>ft</td>
<td>✓</td>
<td>1.26</td>
</tr>
<tr>
<td>$\delta_{FD}$</td>
<td>in.</td>
<td></td>
<td>0.606</td>
</tr>
<tr>
<td>$\psi - \psi_s$</td>
<td>deg</td>
<td></td>
<td>0.239</td>
</tr>
<tr>
<td>$\delta_{LSD}$</td>
<td>in.</td>
<td></td>
<td>0.235</td>
</tr>
<tr>
<td>$\varphi_c/K_p$</td>
<td>deg</td>
<td></td>
<td>0.618</td>
</tr>
<tr>
<td>$x_{11}$</td>
<td>in.</td>
<td></td>
<td>0.007</td>
</tr>
<tr>
<td>$FD_L$</td>
<td>in.</td>
<td></td>
<td>0.014</td>
</tr>
<tr>
<td>$\varepsilon_y$</td>
<td>deg</td>
<td></td>
<td>0.297</td>
</tr>
<tr>
<td>$\Phi_{DC}$</td>
<td>deg</td>
<td></td>
<td>0.611</td>
</tr>
<tr>
<td>$y_a$</td>
<td>ft</td>
<td></td>
<td>0.740</td>
</tr>
</tbody>
</table>

---

*1 ft = 3.048 x $10^{-1}$ m.

*1 deg = 1.745 x $10^{-2}$ rad.

*1 in. = 2.54 x $10^{-2}$ m.
TABLE 17
COMPONENTS OF LATERAL TOUCHDOWN DISPERSION REDUCIBLE
BY MISSED APPROACH DECISION RULE AT 50 FT*
(CTOL RUNWAY CASE)

<table>
<thead>
<tr>
<th>VARIABLE</th>
<th>UNITS</th>
<th>KEY AT TOUCHDOWN</th>
<th>STANDARD DEVIATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\dot{y})</td>
<td>ft/sec</td>
<td>✓</td>
<td>(0.435)</td>
</tr>
<tr>
<td>(p)</td>
<td>deg/sec</td>
<td>✓</td>
<td>(0.594)</td>
</tr>
<tr>
<td>(r)</td>
<td>deg/sec</td>
<td>✓</td>
<td>(0.129)</td>
</tr>
<tr>
<td>(\varphi)</td>
<td>deg</td>
<td>✓</td>
<td>(0.751)</td>
</tr>
<tr>
<td>(\psi)</td>
<td>deg</td>
<td></td>
<td>(10.8)</td>
</tr>
<tr>
<td>(y)</td>
<td>ft</td>
<td>✓</td>
<td>(7.35)</td>
</tr>
<tr>
<td>(\delta_{PDS})</td>
<td>in(^\dagger)</td>
<td></td>
<td>(0.606)</td>
</tr>
<tr>
<td>(\psi - \psi_s)</td>
<td>deg</td>
<td></td>
<td>(0.244)</td>
</tr>
<tr>
<td>(\delta_{LTS})</td>
<td>in.</td>
<td></td>
<td>(0.338)</td>
</tr>
<tr>
<td>(\varphi_0/K_\varphi)</td>
<td>deg</td>
<td></td>
<td>(1.47)</td>
</tr>
<tr>
<td>(x_{11})</td>
<td>in.</td>
<td></td>
<td>(0.033)</td>
</tr>
<tr>
<td>(FDL)</td>
<td>in.</td>
<td></td>
<td>(0.100)</td>
</tr>
<tr>
<td>(\epsilon_y)</td>
<td>deg</td>
<td></td>
<td>(1.07)</td>
</tr>
<tr>
<td>(\varphi_{DC})</td>
<td>deg</td>
<td></td>
<td>(1.19)</td>
</tr>
<tr>
<td>(y_e)</td>
<td>ft</td>
<td></td>
<td>(1.15)</td>
</tr>
</tbody>
</table>

\*1 ft = 3.048 x 10\(^{-1}\) m.
\^1 deg = 1.745 x 10\(^{-2}\) rad.
\*1 in. = 2.54 x 10\(^{-2}\) m.
### TABLE 18
MISSED APPROACH DECISION LEVELS
(JUST PRIOR TO VERTICAL DESCENT COMMITMENT)

<table>
<thead>
<tr>
<th>COCKPIT INDICATION</th>
<th>DECISION LEVEL</th>
<th>MOTIVATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>H</td>
<td>±6 ft*</td>
<td>Strong effect on touchdown sink rate</td>
</tr>
<tr>
<td></td>
<td>of hover</td>
<td></td>
</tr>
<tr>
<td></td>
<td>reference</td>
<td></td>
</tr>
<tr>
<td></td>
<td>altitude</td>
<td></td>
</tr>
<tr>
<td>θ</td>
<td>±5.5,</td>
<td>Indicative of excessive turbulence and/or</td>
</tr>
<tr>
<td></td>
<td>-2.5 deg+</td>
<td>wind effects upon airframe ground clearance</td>
</tr>
<tr>
<td></td>
<td></td>
<td>at touchdown</td>
</tr>
<tr>
<td>x_i</td>
<td>±50 ft</td>
<td>Indicative of gross system error</td>
</tr>
<tr>
<td>y_e</td>
<td>±15 ft</td>
<td>Indicative of gross system error</td>
</tr>
</tbody>
</table>

### TABLE 19
MISSED APPROACH DECISION LEVELS
(FINAL APPROACH)

<table>
<thead>
<tr>
<th>COCKPIT INDICATION</th>
<th>DECISION LEVEL</th>
<th>MOTIVATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>d_e</td>
<td>±12 ft</td>
<td>Indicative of gross system error</td>
</tr>
<tr>
<td>y_e</td>
<td>±24 ft</td>
<td>Indicative of gross system error</td>
</tr>
<tr>
<td>x_i</td>
<td>&gt; 20 kt</td>
<td>At hover mode engage</td>
</tr>
<tr>
<td></td>
<td>(33.8 ft/sec)</td>
<td></td>
</tr>
</tbody>
</table>

*1 ft = 3.048 x 10^-1 m.
†1 deg = 1.745 x 10^-2 rad.
approach assuming normal system use is zero during the earlier stages of the final approach.

The data presented above lead to the conclusion that no intricately computed approach monitoring criteria would be beneficial for significantly reducing either the probability of missed approach or touchdown dispersion. Therefore, computer evaluated approach monitoring criteria selected are the same as the pilot evaluated criteria given in Tables 18 and 19.
SECTION VII
PILOTED FIXED BASE SIMULATION RESULTS

An abbreviated fixed base simulation was conducted using the VSTOLAND 1819B airborne computers for the system equations and an EAI 8×00 for the equations of motion. The objectives of the simulation were as follows:

- Exercise system in an actual pilot-vehicle environment.
- Identify deficiencies not obvious from analysis and make required system modifications.
- Obtain preliminary evaluation of final system.
- Generate computer program tape of final system to allow direct use in XV-15 version of VSTOLAND.

All of the above objectives were achieved. One qualification is that the complementary filtered position and derived rate information from the VSTOLAND could not be used. This was due to the fact that the derivative relationships between the estimated positions and estimated velocities are not preserved when the values of the states (X, Y and Z) are small. This defect manifested itself as a loss of phase lead in the estimated velocities, and granularity in the estimated positions at low amplitudes. This in turn resulted in continuous low to moderate amplitude oscillations in path. In order to complete the program, the aircraft positions and velocities were taken directly from the equations of motion calculations (bypassing the VSTOLAND navigation equations). For this reason curved paths could not be flown.

Because of time and cost limitations, only the STI project pilot flew the simulation. Certain modifications were made to the system described in Section III. These modifications are discussed in the following subsection. The results of an abbreviated evaluation of the final system are given in Subsection B.
A. SYSTEM MODIFICATIONS

Manned simulation identified necessary refinements for the system described in Section III. These are discussed below.

1. Constant Speed Column Director

Unacceptably large airspeed errors (order of 7 percent) tended to persist even though the pitch flight director bar was kept reasonably centered. The problem turned out to be inadequate resolution on the display [e.g., 4 kt (2 m/s) was only 0.05 in. (0.0013 m) of flight director error]. Increasing the flight director gain (K_{FDc}) by a factor of 2 resolved the airspeed error problem but resulted in a "twitchy" flight director. This further confirmed the basic requirement that the overall open-loop sensitivity for a K/s pitch flight director should be about 0.4 in./sec (0.0102 m/s) of flight director per inch of stick. (See Section III.) Hence the problem reduces to a fundamental tradeoff between SCAS sensitivity (K_{SC}) and allowable airspeed errors. Based on limited simulation, it appeared that K_{SC} could be reduced by a factor of 2 in order to allow doubling of K_{FDc}.

In the final configuration, K_{4} was reduced from 10 deg/sec/in. to 5 deg/sec/in., and K_{FDc} was increased to 2.66 in./rad (0.073 m/rad).

2. Revised Deceleration Strategy

The constant attitude open-loop deceleration was found to be somewhat unsatisfactory because of flight director display resolution problems and sensitivity to pilot abuses (delay in initiating deceleration, etc.). Fig. 49 shows the effect of holding the flight director bar high or low on the center dot. Both cases result in undesirable transients at transition to the hover mode.

Based on these results it was decided to reinterpret the equation for constant attitude deceleration (derived in Subsection III-N)
Figure 49. Piloted Simulation Runs Showing Effect of Flight Director Resolution on Deceleration Trajectory — Open Loop Deceleration
where \( \theta_{\text{INC}} = \theta_D - \theta_H \) and \( \frac{d\theta_T}{dV_A} \) is the change in trim pitch attitude with air-speed (assumed to be a constant, \( K\theta_VA \)). When \( e_\theta = 0 \), the aircraft is on a trajectory which corresponds to deceleration at a constant attitude, \( \theta_D \). Hence, \( e_\theta \) is a logical choice for a feedback variable.

A relatively low feedback gain on \( e_\theta \) (\( K_{e_\theta} = -0.0275 \text{ rad/ft} \)) (-0.09 rad/m) yields sufficiently good regulation of \( e_\theta \) and does not alter the K/s characteristics of the flight director.

An attractive consequence of using \( e_\theta \) feedback to the pitch flight director arises from the fact that pitch attitude feedback becomes an inner loop. As an inner loop, pitch attitude must be washed out to avoid standoffs. This washout requirement, in turn, results in the constant attitude deceleration mode pitch feedback being the same as for the other modes (airspeed hold and hover). The end result is considerable simplification of the original complex switching and synchronizing used to remove the washout during constant attitude deceleration. (See Fig. 2.)

The effect on the deceleration trajectory of varying the value of \( \theta_{\text{INC}} \) in Eq. 47 is shown in Fig. 50. The trends are according to the analysis in Subsection III-H: increasing \( \theta_{\text{INC}} \) (\( \theta_{\text{INC}} = \theta_D - \theta_H \)) delays the initiation of deceleration. The resulting trajectories are characterized by higher levels of deceleration and shorter deceleration times. An attractive feature of this guidance scheme could be provision of a pilot-selected deceleration level. For example, the pilot may select low \( \theta_{\text{INC}} \)'s for IFR and larger \( \theta_{\text{INC}} \)'s for VFR.

The degree to which constant attitude deceleration is achieved decreases as \( \theta_{\text{INC}} \) increases as shown in the simulation time histories in Fig. 51. This is simply a result of the fact that delays in the initial pitch-up become more critical when the deceleration magnitude is greater and initiated later. Values of \( \theta_{\text{INC}} \) between 1.0 and 1.5 were found to be desirable for IFR decelerations.
Figure 50. Effect of Deceleration Attitude on Trajectory
(Closed-Loop Deceleration)
The use of $e_\theta$ feedback retains the basic philosophy of the open-loop, constant attitude deceleration scheme derived in Subsection III-H. That is, once a value of $\theta_{INC}$ is chosen, the deceleration trajectory is invariant with steady winds. (In the case of $e_\theta$ feedback the deceleration reference pitch attitude is automatically adjusted in the face of changing winds, whereas in the open-loop case it is calculated only once at initiation of deceleration.) Figure 52 demonstrates that similar trajectories are achieved for winds varying from a 10 kt (5 m/s) tailwind to a 40 kt (21 m/s) headwind.

A primary weakness of the original open-loop deceleration scheme was its inability to cope with wind shear. (See discussion on page 48.) As shown in Fig. 53, the $e_\theta$ feedback system does a reasonable job of regulating against a large [2 kt/sec (1 m/s)] decreasing headwind shear.
Figure 52. Effect of Steady Winds on Closed-Loop Deceleration Trajectories
Figure 53. Effect of a Large Decreasing Headwind Shear (Closed-Loop Deceleration)
The value of $d\theta_T/dV_A$ (also called $K_{0VA}$) used in Figs. 50 through 53 was $-0.0427 \, \text{deg/}(\text{ft/sec})$ which is somewhat lower than the actual value measured from the XV-15 trim characteristics. To further complicate matters, the trim attitude vs. airspeed obtained from the perturbation simulation differs somewhat from those obtained from the full nonlinear FSAA simulation. These trim characteristics are summarized for a $-6 \, \text{deg} (-0.10 \, \text{rad})$ flight path angle in Fig. 54 below.

![Diagram of trim characteristics](image)

**Figure 54. Comparison of Trim Characteristics**

From Fig. 54 it can be seen that a value of $-0.0427$ for $K_{0VA}$ is somewhat less than the actual slope $(d\theta_T/dV_A)$ in the deceleration region [between 60 and 20 kt (31 and 10 m/s)]. However, increasing $K_{0VA}$ tended to decrease the range at which deceleration was initiated. This has the same deleterious effect as increasing $\theta_{INC}$, e.g., attitude is less constant during deceleration. It is believed that this effect is primarily due to delays between commanded and actual pitchup at deceleration initiation. Fortunately, the constant attitude feature of the deceleration guidance scheme is preserved as long as $\theta_{INC}$ is not made too large [less than 2 deg (0.035 rad)].
3. \( \dot{x} \) Limiter

A limiter was added to prevent large velocity commands when the aircraft is displaced a large distance from the commanded hover position. The limiter is mechanized as shown in Fig. 55 below. The limiter is set so a maximum ground speed of \( \pm 20 \text{ft/sec} \) \((6 \text{m/s})\) could be commanded e.g., \( \pm 20 K_x = X_{\text{LIM}} = 160 \text{ft} \) \((48.8 \text{m})\).

![Diagram of \( \dot{x} \) Limiter](image)

Figure 55. Addition of \( \dot{x} \) Limiter

4. Vertical Descent Logic

The "P" blend in Fig. 2 was replaced with a simple switching function. This was necessitated by an undesirable transient output of the integrator \((K_{\text{ICL}}/s)\) during the initial phase of vertical descent.

5. Summary Block Diagram

A summary of the status of the longitudinal system at the completion of the piloted simulation program is given in Fig. 56.

B. LIMITED EVALUATION

Some evaluation runs were made by the STI project pilot (Roger Hoh) to assess the performance of the final system. The pilot ratings given must be tempered by the fact that the project pilot played a major role in the system design.
Figure 56. Final Longitudinal System at Conclusion of Piloted Simulation
1. Longitudinal System

The longitudinal system performance was found to be excellent by any standard in that the tracking errors were negligible for the following inputs.

- Steady winds varying from a 10 kt (5 m/s) tailwind to a 40 kt (21 m/s) headwind.
- 30 kt (15 m/s) wind shears at a rate of 2 kt/sec (0 m/s/s) (both decreasing and increasing headwind shears). These shears were induced at each phase of the approach.
- 4.5 ft/sec (1.4 m/s) rms turbulence.*
- Large pilot abuses including a 5 sec delay from commanded deceleration to initiation of pitchup.

The measured performance was essentially identical for manual (flight director) and fully automatic approaches on glide slopes varying from 6 deg to 10 deg (0.10 rad to 0.17 rad).

As expected, the ratings and commentary for manual approaches reflect a very high workload situation. For a nominal approach with no disturbances, the Cooper Harper pilot ratings for deceleration and hover were 4-1/2. Addition of the major disturbances listed above had little effect on the ratings. The pilot's major complaints centered on the poor attitude precision during the large attitude maneuver required to decelerate, and the constant attention required on the director during hover. Neither of these deficiencies was unexpected (see Section III) and both are attributable to the design constraint imposed by the very limited authority of the series servos on the XV-15. Marginal attitude precision is attributable to the low attitude bandwidth (see Fig. 3) which must be used to prevent series servo saturation. Use of a rate command SCAS for hover was dictated solely by series

*Subsequent to completion of the manned simulation evaluation, the turbulence representation in the XV-15 simulator math model was found to be invalid. The exact nature of this discrepancy is not known to the authors.
servo saturation. It is well known that attitude command is significantly superior to rate command for hover. (For example, see Ref. 12.) Hence the ratings of 4-1/2 for hover are indicative of the best that can be done with a rate command system when hovering in instrument meteorological conditions (IMC). It is interesting that the ratings vary very little (maximum of 5) with the addition of very large disturbances.

2. Lateral System

The lateral system worked well in a no-disturbance environment. The LOC B mode was especially effective at low speeds [below 60 kt (31 m/s)]. Unfortunately, a problem in the XV-15 simulation math model did not allow evaluations in steady crosswinds. All the problems noted in hover for the longitudinal system also apply for the lateral system.

Time histories showing localizer recapture from a 50 ft (15 m) offset for LOC A and LOC B are shown in Fig. 57.
Figure 57. Response of Manual System to 50 ft Lateral Offset

(a) LOC A Mode $V_A = 60$ ft

(b) LOC B Mode $V_A = 20$ ft
SECTION VIII
CONCLUSIONS

A. BASIC OBSERVATIONS

XV-15 flight path angle on final approach in the helicopter mode is limited in the downward direction by $-10$ deg ($-0.17$ rad). Limitation arises from the minimum flight idle power setting which is the usual case for VTOL aircraft. The $-10$ deg ($-0.17$ rad) limit is much more severe than the $-30$ deg ($-0.52$ rad) limit originally planned for however.

Significant force-feel system damping ratio increases with decreasing airspeed are undesirable.

The installed "rate command, attitude retention" SCAS is only approximately rate command. "Attitude retention" should not be confused with "attitude hold." The former is a part-time attitude feedback system, while the latter has full-time attitude feedback.

B. DESIGN CONSIDERATIONS

Pilot workload considerations in the flight director mode of operation dictate a maximum of two active director commands. Therefore the collective pitch and pedal control axes are made fully automatic for approach and landing.

Structuring the automatic modes of the approach and landing system in the manner of the manual flight director system is an effective design approach. This consists of replacing the pilot's control function by an automatic system gain coupling each flight director signal into the corresponding force-feel system actuator. All guidance and control and pilot-centered requirements are satisfied by both systems when this approach is used. Furthermore, automatic system operation is similar to the pilot's manual operation of the system. Still further, pilot monitoring of automatic system operation is facilitated by virtue of both systems satisfying the pilot-centered requirements.
Deceleration by means of a constant pitch attitude maneuver during final approach is an effective deceleration strategy which tends to have low workload contribution and acceptable duration (e.g., 45 sec). Required pitch attitudes (and longitudinal acceleration levels) are moderated by adjusting the range at which the constant attitude deceleration maneuver is initiated. Range is adjusted according to estimated wind speed and airspeed. The initiation point and reference pitch attitude are computed such that the range vs. range rate profile is not a function of steady wind speed. The latter two points are essential to viability of this scheme.

Combined lack of directional stability and an aerodynamic sideslip measurement for the XV-15 in the low speed regime requires minor loop heading regulation via the pedal series servo.

Pilot workload considerations indicate "rate command, attitude hold" as the minimum acceptable level of augmentation.

Limited authority of series actuation precludes use of higher levels of augmentation such as "attitude command" or "translational rate command."

Airspeed-to-collective crossfeeding is essential to precise glide path control below the speed for minimum power.

High turn rate sensitivity in a bank-to-turn lateral control mode at low speeds recommends transition to a bank-to-translate mode in this speed regime.

C. PERFORMANCE EVALUATION

Evaluation of landing system performance throughout a decelerating approach to touchdown can be accomplished economically and effectively using non-stationary covariance propagation techniques.

Approach monitoring criteria are required to protect against pilot abuse of normal system operating procedures (blunders). The precision of control attainable with the XV-15 + SCAS + MLS is such that reasonable disturbance levels do not result in violation of criteria for safe touchdown or for pilot acceptance in the case of normal system operation even in the absence of a missed approach decision rule. Therefore approach monitoring criteria are not
required for the usual primary purpose of protecting against unsafe levels of touchdown dispersion. Their usual purpose is to convert out-of-tolerance approaches to missed approaches by means of a missed approach decision rule.

High precision approach path and touchdown point control requires tight MLS guidance coupling. This in turn requires gain scheduling in coupling to the MLS guidance signals because of the range dependency of the guidance noise when expressed in linear units.

D. SIMULATOR EVALUATION

The system was able to regulate against very large disturbances with negligible errors in the longitudinal axis. The lateral axis worked very well but could not be tested against disturbances because of problems with the XV-15 simulation math model. Pilot ratings of 4-1/2 were obtained for hover and deceleration to hover with and without large disturbances. It is felt that these ratings would be improved with a higher bandwidth rate-command, attitude-hold SCAS during deceleration which would undergo conversion to an attitude or translational rate command SCAS in hover. Both of these options were disallowed in the present study due to a very limited series servo authority.
REFERENCES


APPENDIX A

MODELS USED FOR MLS GUIDANCE; WIND, WIND SHEAR AND TURBULENCE; AND AIRCRAFT

The equations and numerical parameter values actually used in the overall system performance model are summarized in this appendix.

GLIDE SLOPE BEAM ALIGNMENT AND NOISE MODEL

This subsection documents the model of the MLS Glide Slope signal used in the system performance analysis. The model represents the received signal in the aircraft (in distinction to representing the MLS signal in space). Consequently, only the deterministic portion of the received signal model is a function of the receiving antenna location for a given range.

The model of the received signal consists of three components:

- The selected ideal straight path line at angle, \( \Theta \), with respect to horizontal in the vertical plane containing the runway centerline
- The deviation of the mean alignment by angle, \( \Delta \Theta \), for the actual beam from the ideal above.
- The angular deviation, \( \eta \), arising from actual beam structure with respect to the mean alignment of the beam.

The first of the above components is deterministic.

The forms for the second and third components of the model are based upon the actual MLS Glide Slope error budget (Table 2 of Ref. 18).

LOCALIZER ALIGNMENT AND NOISE MODEL

The model of the MLS localizer signal to be used in the system performance analysis represents the received signal in the aircraft. Consequently, only the deterministic portion of the received signal model is a function of the receiving antenna location for a given range.
The model of the received signal consists of three components:

- The selected ideal straight course line at angle $\theta$ with respect to the extended runway centerline as measured in a horizontal plane.

- The deviation of the mean alignment, $\Delta \psi$, of the MLS localizer guidance reference in a horizontal plane with respect to the selected course line in azimuth.

- The angular deviation, $\psi$, arising from irregularities in the localizer course structure with respect to the beam mean alignment.

The first of the above components is deterministic. The forms for the second and third components of the model are based upon the actual MLS Localizer error budget (Table 2 of Ref. 18).

Alignment Errors

The alignment errors represent biases or mean deviations which vary from facility-to-facility over a relatively narrow range. These errors can, in principle, be reduced to zero by adjusting the elevation or azimuth transmitting antenna pattern. However, from one approach and landing to the next the value of the mean deviation changes in the simulation model in order to represent a population of MLS facilities. The models for these components are given in Table A-1. Alignment error effects are negligible, therefore alignment errors are not included in the system performance analyses.

Noise Errors

The noise errors represent stochastic disturbances arising from in-beam multipath effects. It represents the variability about the mean deviation which causes a noisy deviation signal to be received. This component is represented in the model by a power spectral density consistent with the actual MLS error budget for multipath effects falling within the aircraft path response bandwidth. The models for these noise components are given in Table A-2.
TABLE A-1

MIS GUIDANCE ERROR MODEL FOR ALIGNMENT (BIAS)

Alignment (Bias) Error (2σ)

<table>
<thead>
<tr>
<th></th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elevation</td>
<td>0.06 deg* [1.4 ft † MGA at 1145 ft]</td>
</tr>
<tr>
<td>Azimuth</td>
<td>0.044 deg [9.0 ft MGA at 13000 ft]</td>
</tr>
</tbody>
</table>

Alignment Error Math Models

Elevation

State equation and mean initial value
\[ \Delta \theta = 0 , \; \Delta \theta(0) = 0 \]
Initial value for variance of \( \Delta \theta \)
\[ \sigma^2_{\Delta \theta} = (0.022)^2 = 4.84 \times 10^{-4} \text{ deg}^2 \]

Azimuth

State equation and mean initial value
\[ \Delta \psi = 0 , \; \Delta \psi(0) = 0 \]
Initial value for variance of \( \Delta \psi \)
\[ \sigma^2_{\Delta \psi} = (0.03)^2 = 9.0 \times 10^{-4} \text{ deg}^2 \]

---

\*1 ft = 3.048 \times 10^{-1} \text{ m.}

†1 deg = 1.745 \times 10^{-2} \text{ rad.}
"Path Following" (Low Frequency) Fluctuations

Passband: (Assumed constant power spectral density in interval cited, zero elsewhere)
Elevation Passband
0 to 1.0 rad/sec
Azimuth Passband
0 to 0.5 rad/sec
Assumed approach inertial speed is 200 ft/sec

Level (2σ):
Elevation
0.07 deg [1.4 ft MGA at 114.5 ft]
Azimuth
0.04 deg [9.0 ft MGA at 13000 ft]

Fluctuation Math Models

Elevation
State equation and mean initial value
\[ \dot{\eta} = -\frac{|V_T^x|}{L_\eta} \eta + \sigma_\eta \sqrt{2|V_T^x|/L_\eta} \omega_1, \quad \eta(0) = 0 \]
Inertial speed is \( V_T^{x} \)
Fluctuation scale length is \( L_\eta = 200 \) ft
Standard deviation is \( \sigma_\eta = 0.07 \) deg
Independent, zero mean, unit white noise is \( \omega_1 \)

Azimuth
State equation and mean initial value
\[ \dot{\nu} = -\frac{|V_T^x|}{L_\nu} \nu + \sigma_\nu \sqrt{2|V_T^x|/L_\nu} \omega_5, \quad \nu(0) = 0 \]
Inertial speed is \( V_T^{x} \)
Fluctuation scale length is \( L_\nu = 400 \) ft
Standard deviation is \( \sigma_\nu = 0.04 \) deg
Independent, zero mean, unit white noise is \( \omega_5 \)

\( 1 \) ft = 3.048 x 10\(^{-1}\) m.
\( 1 \) deg = 1.745 x 10\(^{-2}\) rad.

A.4
Distance Measuring Equipment (DME) Errors

The DME output is assumed to contain zero bias (Ref. 19). Fluctuations are budgeted to have a 2σ error of 40 ft (12 m) (Ref. 19) over a zero to 2 rad/sec rectangular passband (Ref. 17). The DME error, \( X_c \), is modeled by the following differential equation and parameter values

\[
\dot{X}_c = -\omega_c X_c + \sigma_{X_c} \sqrt{2 \omega_c} \, w_4, \quad X_c(0) = 0 \quad (A-1)
\]

where \( w_4 \) is an independent, zero mean, unit white noise. The standard deviation is \( \sigma_{X_c} = 20 \) ft (6 m). The half-power frequency is \( \omega_c = 2 \) rad/sec.

WIND, WIND SHEAR, AND TURBULENCE MODELS

This subsection documents models for the atmospheric disturbance environment which forms part of the overall system performance model.

The atmospheric disturbance environment model represents disturbances of three types. These are the mean wind, wind shear and stochastic turbulence. All three types are characterized by parameters which are a function of altitude, and which themselves are possibly random variables.

The mean wind and wind shear are deterministic disturbances for any one approach and landing operation. However, from one approach and landing to the next, the level of the mean wind and wind shear is a random selection from a Gaussian distribution having a particular mean and standard deviation. These disturbances are therefore properly applied to the stochastic portion of the system performance model. The turbulence is a stochastic disturbance. The turbulence is therefore applied to the stochastic portion of the system performance model.

Mean Wind and Wind Shear (Ref. 15 and 20)

Headwind Component

The headwind component, \( w_H \), of the steady headwind profile of Ref. 20 is used. This results in a profile whose magnitude is determined by a random
selection from Gaussian distribution. Thus, for any given approach and landing, the profile is fixed, but from one approach to the next the profile changes. A sample profile is shown in Fig. A-1. To obtain any other profile, it is only necessary to scale up (or scale down) the wind magnitude. Conveniently, any particular profile can be completely determined by specifying the magnitude at a given reference altitude. For the purpose of discussion, a wind reference altitude of 10 ft (3 m) will be selected. This corresponds to the approximate altitude of the center of gravity for a typical aircraft at the instant of touchdown. At this altitude the wind magnitude varies from a 10 kt (5 m/s) tailwind to a 26 kt (13 m/s) headwind (-30°) and has mean value of 8 kt (4 m/s). These values are consistent with the design values specified by the FAA in Ref. 21.

The probability density function for the mean wind, \( \bar{u}_w \), is a Gaussian distribution with mean and standard deviation given by:

\[
\bar{u}_w = F_w e^{-h^*/H_w}\left(D_w \log_{10} h^* + E_w\right)/(D_w + E_w) \quad (A-2)
\]

\[
\sigma_{u_w} = 0.75 \bar{u}_w \quad (A-3)
\]

where

\[
F_w = 13.5 \text{ ft/sec (8 kt) (4.1 m/s)} \quad (A-4)
\]

\[
D_w = 0.43
\]

\[
E_w = 0.35
\]

\[
H_w = 10,000 \text{ ft (3048 m)}
\]

and

\[
h^* = H + h_{ocg}
\]

\( H \) is the altitude of the main landing gear wheels and \( h_{ocg} \) is the aircraft c.g. altitude at touchdown. The mean wind \( \bar{u}_w \) has the following shear characteristics (Ref. 20).

A-6
Figure A-1. Wind Profile Associated with 10 kt (5 m/s) Wind at 10 ft (3 m) Altitude
(From Ref. 16)
These characteristics also tend to be consistent with 1/3 of the 8 kt/100 ft (30 m) specified by the FAA in Ref. 21 at an altitude of 100 ft (30 m). However, the increasing shear with decreasing altitude of the present model poses a more severe but perhaps more realistic environment than does the Ref. 21 model.

The state equation for modeling the reference value of the steady headwind and its mean initial condition are

\[
\dot{\bar{u}_W} = 0; \quad \bar{u}_W = V_{HN_0} \text{ ft/sec} \tag{A-5}
\]

where the initial value of the mean headwind \( V_{HN_0} \) is:

\[
V_{HN_0} = \frac{F_w[D_w \log_{10} (H_o + h_{ocg}) + E_w]e^{-(H_o + h_{ocg})/H_w}}{D_w + E_w} \tag{A-6}
\]

\( H_o \) is the initial altitude of the main landing gear wheels. The initial condition for the covariance equation is

\[
E[\bar{u}_{WO}^2] = \sigma_{\bar{u}_{WO}}^2 \tag{A-7}
\]

where

\[
\sigma_{\bar{u}_{WO}} = \frac{0.75F_w[D_w \log_{10} (H_o + h_{ocg}) + E_w]e^{-(H_o + h_{ocg})/H_w}}{D_w + E_w} \tag{A-8}
\]
The headwind profile is then given by

\[ u_w = K_w u_{wo} \]

where

\[ K_w = K_w(H) = \frac{(D_w \log_{10} h^* + E_w)e^{-h^*/H_w}}{(D_w \log_{10} [H_o + h_{ocg}] + E_w)e^{-(H_o + h_{ocg})/H_w}} \]  

(A-9)

Crosswind Component

The probability density function for the crosswind, \( v_w \), is a Gaussian probability density function with a zero mean and standard deviation:

\[ \bar{v}_w = 0 \]  

(A-10)

\[ \sigma_{v_w} = J_w e^{-h^*/H_w}(D_w \log_{10} h^* + E_w)/(D_w + E_w) \]  

(A-11)

\[ J_w = 8.455 \text{ ft/sec (5 kt) (2.574 m/s)} \]  

(A-12)

This model results in 3\( \sigma \) crosswind components of 15 kt (7.7 m/s) at an altitude of 10 ft (3 m).

The state equation for modeling the reference value of the steady crosswind and its mean initial condition are

\[ \dot{v}_{wo} = 0 \; ; \; \bar{v}_{wo} = 0 \]  

(A-13)

The initial condition for the covariance equation is

\[ E[v_{wo}^2] = \sigma_{v_{wo}}^2 \]  

(A-14)

where

\[ \sigma_{v_{wo}} = \frac{J_w(D_w \log_{10} [H_o + h_{ocg}] + E_w)e^{-(H_o + h_{ocg})/H_w}}{D_w + E_w} \]  

(A-15)
The wind profile is then given by

\[ v_w = K_w v_{w0} \]  \hspace{1cm} (A-16)

where

\[ K_w = K_w(n) = \frac{(D_w \log_{10} h^* + E_w)e^{-h^*/H_w}}{(D_w \log_{10} [H_0 + f_{ocg}] + E_w)e^{-[H_0 + f_{ocg}]/H_w}} \]  \hspace{1cm} (A-17)

Random Turbulence Model (Ref. 22)

Longitudinal Components

The model for random turbulence is a simplified version of that given in Ref. 22. Gradient effects associated with the normal turbulence component are neglected. For any one approach the random turbulence components have Gaussian probability density functions with zero means. The standard deviation \( \sigma_{u_g} \) should be chosen for each approach from a Rayleigh probability density function* having a characteristic speed of \( \sigma_{u_g} \) ft/sec. However, for the sake of simplicity, the mean value of \( \sigma_{u_g} \), which is \( \sigma_{u_g} \), is used for all approaches in the overall system performance model.

\[ \sigma_{u_g} = \sigma_{u_g} = 2.79 - 0.245 \log_{10} h^* \text{ ft/sec} \quad h^* > 100 \text{ ft (30 m)} \]

\[ = 2.3 \text{ ft/sec (0.7 m)} \quad h^* \leq 100 \text{ ft (30 m)} \]  \hspace{1cm} (A-18)

The standard deviation \( \sigma_{u_g} \) is a function of \( u_g \). The frequency content of the random turbulence and \( \sigma_{u_g} \) are functions of altitude.

*The Rayleigh probability density function is for \( u_g \) (rather than \( w_g \) as stated in Ref. 22. This reinterpretation is based on Ref. 23, paragraph 3.7.3, A, 1.

A-10
The power spectral densities for the longitudinal and normal random turbulence components at a given altitude are respectively:

\[
\Phi_{ug} = \frac{\sigma_{ug}^2 V_A / L_u}{\omega^2 + (V_A / L_u)^2}
\]  
(A-19)

\[
\Phi_{wg} = \frac{\sigma_{wg}^2 (1.594 V_A / L_w)}{\omega^2 + (1.594 V_A / L_w)^2}
\]  
(A-20)

where

\[
\sigma^2 = \frac{1}{2\pi} \int_{-\infty}^{\infty} \Phi dw
\]  
(A-21)

\(\Phi_{wg}\) is a lower order approximation to the power spectral density given in Ref. 23. The approximation is such that the mean-square level and half-power frequency are preserved.

The differential equations for unit-white-noise shaping filters producing output variables \(u_g\) and \(w_g\) having power spectral densities \(\Phi_{ug}\) and \(\Phi_{wg}\) respectively are:

\[
\dot{u}_g = -|V_A^*| / L_u u_g + \sigma_{ug} \sqrt{2} |V_A^*| / L_u w_2
\]  
(A-22)

\[
\dot{w}_g = -1.594 |V_A^*| / L_w w_g + \sigma_{wg} \sqrt{2} (1.594) |V_A^*| / L_w w_3
\]  
(A-23)

where \(w_2\) and \(w_3\) are independent, zero-mean, unit white noises. \(V_A^*\) is the trim approach airspeed.

The integral scale lengths \(L_u\) and \(L_w\) are given as functions of altitude \(h^*\) by

A-11
\[ L_u = 145[h^*]^{1/3} \quad 100 \leq h^* \leq 1750 \text{ ft (533 m)} \]

\[ = 145[100]^{1/3} = 673 \quad h^* \leq 100 \text{ ft (30 m)} \quad (A-24) \]

\[ L_w = h^* \quad h^* \leq 1750 \text{ ft (533 m)} \quad (A-25) \]

The standard deviation for the normal turbulence component \( \sigma_{w_g} \) is related to the standard deviation for the longitudinal turbulence component \( \sigma_{u_g} \) through the integral scale lengths.

\[ \sigma_{w_g} = \sqrt{L_w/L_u} \sigma_{u_g} \quad (A-26) \]

The random turbulence model is used in the stochastic portion of the system performance model throughout the approach and landing.

**Lateral Components**

The model for random turbulence is a simplified version that given in Ref. 22. Gradient effects associated with the lateral turbulence component are neglected. For any one approach the random turbulence components have normal probability density functions with zero means. The standard deviation \( \sigma_{v_g} \) is chosen from a Rayleigh probability density function having a characteristic speed of \( \sigma_{v_g} \) ft/sec. The frequency content of the random turbulence and \( \sigma_{v_g} \) are functions of altitude.

The power spectral density for the lateral random turbulence components at a given altitude is:

\[ \Phi_{v_g}(\omega) = \frac{\sigma_{v_g}^2}{\omega^2 + (1.594 \frac{V_A}{L_v})^2} \quad (A-27) \]
\( \Phi_v(\omega) \) is a lower order approximation to the power spectral density given in Ref. 22. The approximation preserves the half-power frequency and mean-square level.

The differential equation for a unit-white-noise shaping filter producing \( v_g \) having a power spectral density \( \Phi_v \) is

\[
\dot{v}_g = -1.594 \frac{|V_a^2/L_v v_g|}{L_v v_g} + \sigma_v \sqrt{2(1.594)|V_a^2/L_v|/L_v w_6} \tag{A-28}
\]

where \( w_6 \) is an independent unit-white-noise. \( V_a \) is the approach airspeed.

The integral scale length \( L_v = L_u \). The standard deviation \( \sigma_v \) is related to the standard deviation for the longitudinal turbulence component \( \sigma_u \) through the integral scale lengths, viz.,

\[
\sigma_v = \sqrt{L_v/L_u} \sigma_u \tag{A-29}
\]

which reduces to

\[
\sigma_v = \sigma_u \tag{A-30}
\]

The Rayleigh probability density function for \( \sigma_v \) is independent of that for \( \sigma_u \), but otherwise identical in formulation to that given by Eq. A-18.

This random turbulence model is used in the stochastic portion of the system performance model throughout the approach and landing.

Spanwise gradient effects associated with the normal turbulence component will be represented by the simplified model of rolling gust velocity in Ref. 22. The power spectral density for this rolling turbulence component at a given altitude is
\[
\Phi_p^t(\Omega) = \frac{\sigma^2_p}{L_w} \frac{0.8 \left(\frac{\pi L_w}{4b}\right)^{1/3}}{1 + \left(\frac{4b}{\pi} \Omega\right)^2}
\]

(A-31)

where

\[
\sigma^2 = \int_0^{\infty} \Phi_t(\Omega) d\Omega
\]

(A-32)

\[
\Omega = \frac{\omega}{V_A^*}
\]

(A-33)

\[
\Phi^t_p(\omega) = \frac{1}{V_A} \Phi^t_p \left(\frac{\omega}{V_A^*}\right)
\]

(A-34)

If

\[
\sigma^2 \Delta = \frac{1}{2\pi} \int_{-\infty}^{\infty} \Phi(\omega) d\omega
\]

(A-35)

then

\[
\Phi_p^t(\omega) = \pi \Phi_p^t(\omega)
\]

(A-36)

\[
= \sigma^2_p \frac{0.8 \left(\pi/4b\right)^{7/3} \left(V_A^*/L_w^{2/3}\right)}{\left[\omega^2 + \left(\pi V_A^*/4b\right)^2\right]}
\]

(A-37)
The differential equation for a corresponding unit-white-noise $p_g$-filter is

$$\dot{p}_g = -(\pi |V_A^*|/4b)p_g + \sigma_{p_g} \sqrt{0.8 \pi (\pi/4b)^{7/3} |V_A^*|/L_w^{2/3}} \ w_7$$

(A-37)

where $w_7$ is an independent unit-white-noise. $b$ is the reference wing span.

The effective value of $\sigma_{p_g}$ is

$$\sigma_{p_g} = \sigma_{w_g} \frac{\sqrt{0.4 \pi (\pi/4b)^{2/3}}}{L_w^{1/3}} \text{ (rad/sec)}$$

(A-38)

so that the differential equation for the $p_g$ shaping filter might be written in the alternative form

$$\dot{p}_g = -(\pi |V_A^*|/4b)p_g + \sigma_{p_g} \sqrt{2(\pi |V_A^*|/4b)} \ w_7$$

(A-39)

AIRCRAFT LONGITUDINAL MOTION MODEL

The method used for system performance analysis requires that equations of motion for the aircraft be in state vector form, include the pertinent kinematic equations, and that appropriate measures be taken to incorporate steady wind effects. All of these considerations force some minor changes upon the customary equations-of-motion model.

The next three subsections cover in turn the kinematic equations, incorporation of deterministic wind effects and the final set of state equations for the aircraft and kinematics plus auxiliary equations for sensor inputs which are not states.
Kinematic Equations

The scenario for the system performance model is shown in Fig. A-2. A perfectly level runway is assumed. Figure A-3 defines the perturbed coordinates for the aircraft body axes with respect to the unperturbed (or nominal) coordinates. Kinematic equations locating the aircraft center of gravity with respect to the apparent source of the MIS Glide Slope and with respect to the runway are

\[ \dot{X} = V_{T0}^* \cos \gamma_o + u \cos \theta_o^* + w \sin \theta_o^* - (U_o^* \sin \theta_o^* - W_o^* \cos \theta_o^*) \theta \]  

(A-40)

\[ H = - V_{T0}^* \sin \gamma_o + \theta(U_o^* \cos \theta_o^* + W_o^* \sin \theta_o^*) + u \sin \theta_o^* - w \cos \theta_o^* \]  

(A-41)

\[ U_o^* = V_{T0}^* \cos (\theta_o^* - \gamma_o) \]  

(A-42)

\[ W_o^* = V_{T0}^* \sin (\theta_o^* - \gamma_o) \]  

(A-43)

Additional kinematic relationships of interest are

\[ d = H \cos \gamma_o - X \sin \gamma_o - 50. \cos \gamma_o \]  

(A-44)

\[ R = \sqrt{H^2 + (X + 50/\tan \gamma_o)^2} \]  

(A-45)

\[ d_c = d_c - d = R \eta - d \]  

(A-46)

R is the distance between the aircraft GPIP in Fig. A-3.

The steady wind* acts in the horizontal direction only. For a given airspeed, \( V_{Ao} \), and flight path angle, \( \gamma_o \), and steady headwind, \( V_{HW} \), the

*The "steady wind" \( V_{HW} \) is here taken as the mean value of the wind \( u_w \) in the system performance model. See the second subsection of this appendix for a description of the mean wind.

A-16
Figure A-2. Scenario for Approach and Landing for System Performance Analysis
Figure A-3. Perturbed Coordinates Locating the Aircraft
corresponding steady inertial speed, \( V_{T_o}^* \), and aerodynamic flight path angle, \( \gamma_{A_o}^* \), are

\[
V_{T_o}^* = \sqrt{V_{A_o}^2 + V_{H_W}^2 \sin^2 \gamma_o} - V_{H_W} \cos \gamma_o \quad (A-47)
\]

\[
\gamma_{A_o}^* = \sin^{-1} \left( \frac{\left( \sqrt{V_{A_o}^2 + V_{H_W}^2 \sin^2 \gamma_o} - V_{H_W} \cos \gamma_o \right) \sin \gamma_o}{V_{A_o}} \right) \quad (A-48)
\]

When the steady headwind is zero, then \( V_{A_o} = V_{T_o}^* = V_{T_o}, \theta_o^* = \theta_o, \gamma_{A_o}^* = \gamma_o, \alpha_o^* = \alpha_o, U_o^* = U_c, \) and \( w_o^* = W_o \) where the unstarred qualities have the customary definitions.

**Deterministic Wind Effects**

The mean wind and wind shear components of the atmospheric disturbance environment act in a horizontal direction and therefore must be resolved into aircraft body-fixed axis coordinates for proper application via the aircraft equations of motion. Let the longitudinal and normal components (with respect to body-fixed axes) of the deterministic atmospheric disturbance environment be designated \( u_A \) and \( w_A \), respectively.

\[
u_A = -u_w \cos (\theta_o^* + \theta) = -u_w (\cos \theta_o^* - \theta \sin \theta_o^*) \quad (A-49)
\]

\[
w_A = -u_w \sin (\theta_o^* + \theta) = -u_w (\sin \theta_o^* + \theta \cos \theta_o^*) \quad (A-50)
\]

\( u_w \) represents the mean wind and wind shear component described in the second subsection of this appendix. The linearized approximate expressions for \( u_A \) and \( w_A \) are used in the system performance model. \( u_A \) and \( w_A \) enter the equations of motion in the manner of \( u_c \) and \( w_c \).
State and Output Equations for the Aircraft

Aircraft perturbation equations of motion are customarily expressed in terms of states u, w, q and θ. Assuming \( Z_w = 0 \) and neglecting normal gust gradient effects, the aircraft state equations are (Ref. 14)

\[
\dot{u} = X_uu + X_ww - W_o q + (V_{HW_o} K_w X_w - g \cos \theta_o) \theta
\]

\[
+ X_{CL}^\delta LN + X_{CL}^\delta CL - X_u u_g - X_w w_g
\]

\[
+ X_{w}^w u_w - X_{w}^w v_{HW_o} ; \quad u(0) = 0
\]  \hspace{1cm} (A-51)

\[
\dot{w} = Z_uu + Z_ww + U_o q + (V_{HW_o} Z_w - g \sin \theta_o) \theta
\]

\[
+ Z_{CL}^\delta LN + Z_{CL}^\delta CL - Z_u u_g - Z_w w_g
\]

\[
+ Z_{w}^w u_w - Z_{w}^w v_{HW_o} ; \quad w(0) = 0
\]  \hspace{1cm} (A-52)

\[
\dot{q} = (M_u + M_w Z_u) u + (M_w + M_w Z_w) w + (M_q + M_w U_o) q
\]

\[
+ V_{HW_o} K_w (M_u + M_w Z_u) \theta + (M_{CL}^\delta LN + M_w Z_{CL}^\delta LN) \theta
\]

\[
+ (M_{CL}^\delta CL + M_w Z_{CL}^\delta CL) - (M_u + M_w Z_u) u_g
\]

\[
- (M_w + M_w Z_w) v_g + (M_u + M_w Z_u) K_w u_w
\]

\[
- (M_u + M_w Z_u) v_{HW_o} ; \quad q(0) = 0
\]  \hspace{1cm} (A-53)

\[
\dot{\theta} = q
\]  \hspace{1cm} (A-54)
where

\[ X_u^* = X_u \cos \theta^*_o + X_w \sin \theta^*_o \]  
\[ Z_u^* = Z_u \cos \theta^*_o + Z_w \sin \theta^*_o \]  
\[ M_u^* = M_u \cos \theta^*_o + M_w \sin \theta^*_o \]

and

\[ X_w^* = X_w \cos \theta^*_o - X_u \sin \theta^*_o \]  
\[ Z_w^* = Z_w \cos \theta^*_o - Z_u \sin \theta^*_o \]  
\[ M_w^* = M_w \cos \theta^*_o - M_u \sin \theta^*_o \]

The airspeed equation is

\[ V_A = V_T^* \cos (\gamma^*_A - \gamma_A) + u \cos (\theta^*_o - \gamma^*_A) - u_g \cos (\theta^*_o - \gamma^*_A) \]

\[ + w \sin (\theta^*_o - \gamma^*_A) - w_g \sin (\theta^*_o - \gamma^*_A) - V_{HW} \sin \gamma_{A0} \]

\[ + u_{WO} \cos \gamma_{A0} \]  
\[ \text{(A-61)} \]

The output equation for \( \dot{H} \) is given above with the kinematic equations.

Numerical Data for Example Aircraft
and Kinematic Constants

Numerical stability and control data for the XV-15 aircraft (Fig. A-4) is given in Table A-4 for flight path angles of -6 and -10 deg (-0.10 and -0.17 rad) and airspeeds of 0, 20, 40, 60 and 80 kt (0, 10, 21, 31 and 41 m/s). Numerical trim map data is given in Table A-5. Linear interpolation: between data points are used to compute \( g_\delta(V_A^*, \gamma_{A0}^*) \) and \( 5_{DL0}(V_A^*, \gamma_{A0}^*) \) trim values.
Figure A-4. XV-15 Three-View
<table>
<thead>
<tr>
<th>UNITS</th>
<th>( V_{A0} )</th>
<th>1.0</th>
<th>33.8</th>
<th>67.6</th>
<th>101.4</th>
<th>135.2</th>
<th>33.8</th>
<th>67.6</th>
<th>101.4</th>
<th>135.2</th>
</tr>
</thead>
<tbody>
<tr>
<td>ft/sec</td>
<td>( V_{A0} )</td>
<td>0</td>
<td>20</td>
<td>40</td>
<td>60</td>
<td>80</td>
<td>40</td>
<td>60</td>
<td>80</td>
<td>40</td>
</tr>
<tr>
<td>deg</td>
<td>( \alpha_0 )</td>
<td>2.2</td>
<td>6.5</td>
<td>4.5</td>
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<td>10.7</td>
<td>-4.4</td>
<td>6.0</td>
<td>-1.8</td>
</tr>
<tr>
<td>deg</td>
<td>( \gamma_0 )</td>
<td>0</td>
<td>-6.0</td>
<td>-6.0</td>
<td>-6.0</td>
<td>-6.0</td>
<td>-10.0</td>
<td>-10.0</td>
<td>-10.0</td>
<td>-10.0</td>
</tr>
<tr>
<td>1/sec</td>
<td>( \chi_u )</td>
<td>-0.29</td>
<td>-0.4</td>
<td>-0.54</td>
<td>-0.59</td>
<td>-0.28</td>
<td>-0.4</td>
<td>-0.52</td>
<td>-0.60</td>
<td></td>
</tr>
<tr>
<td>1/sec</td>
<td>( \zeta_u )</td>
<td>-1.142</td>
<td>-2.11</td>
<td>-1.86</td>
<td>-1.52</td>
<td>-1.55</td>
<td>-2.27</td>
<td>-2.20</td>
<td>-1.165</td>
<td></td>
</tr>
<tr>
<td>1/sec</td>
<td>( \mu_u )</td>
<td>+0.0042</td>
<td>+0.0073</td>
<td>+0.0028</td>
<td>+0.0071</td>
<td>+0.0028</td>
<td>+0.0025</td>
<td>+0.0025</td>
<td>+0.014</td>
<td></td>
</tr>
<tr>
<td>1/sec</td>
<td>( \zeta_u )</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>1/sec</td>
<td>( \chi_v )</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>1/sec</td>
<td>( \zeta_v )</td>
<td>+0.028</td>
<td>+0.017</td>
<td>+0.029</td>
<td>+0.014</td>
<td>+0.035</td>
<td>-0.015</td>
<td>+0.035</td>
<td>-0.041</td>
<td>+0.0016</td>
</tr>
<tr>
<td>1/sec</td>
<td>( \chi_v )</td>
<td>+0.29</td>
<td>-3.0</td>
<td>-4.66</td>
<td>-5.84</td>
<td>-6.80</td>
<td>-2.60</td>
<td>-4.55</td>
<td>-5.80</td>
<td>-6.81</td>
</tr>
<tr>
<td>1/sec</td>
<td>( \zeta_v )</td>
<td>0.0</td>
<td>-0.29</td>
<td>-0.22</td>
<td>+0.0093</td>
<td>-0.0076</td>
<td>-0.0069</td>
<td>-0.013</td>
<td>-0.005</td>
<td>-0.0056</td>
</tr>
<tr>
<td>1/sec</td>
<td>( \chi_v )</td>
<td>-0.429</td>
<td>-0.879</td>
<td>-1.24</td>
<td>-1.64</td>
<td>-2.00</td>
<td>-0.850</td>
<td>-1.18</td>
<td>-1.26</td>
<td>-1.39</td>
</tr>
<tr>
<td>ft/sec^2/in.</td>
<td>( \chi_{BLN} )</td>
<td>+1.21</td>
<td>+1.20</td>
<td>+1.06</td>
<td>+0.97</td>
<td>+0.792</td>
<td>1.19</td>
<td>+1.23</td>
<td>-0.95</td>
<td>0.705</td>
</tr>
<tr>
<td>ft/sec^2/in.</td>
<td>( \zeta_{BLN} )</td>
<td>-0.22</td>
<td>+3.08</td>
<td>+5.62</td>
<td>+9.35</td>
<td>+1.29</td>
<td>-0.314</td>
<td>+0.570</td>
<td>-0.95</td>
<td>1.34</td>
</tr>
<tr>
<td>ft/sec^2/in.</td>
<td>( \mu_{BLN} )</td>
<td>-2.77</td>
<td>-3.08</td>
<td>-3.60</td>
<td>-4.09</td>
<td>-4.62</td>
<td>-3.04</td>
<td>-3.43</td>
<td>-3.77</td>
<td>-4.46</td>
</tr>
<tr>
<td>ft/sec^2/in.</td>
<td>( \chi_{BCL} )</td>
<td>+0.64</td>
<td>+0.36</td>
<td>-0.4</td>
<td>-1.19</td>
<td>+1.152</td>
<td>0.021</td>
<td>-0.071</td>
<td>-0.153</td>
<td>-0.475</td>
</tr>
<tr>
<td>ft/sec^2/in.</td>
<td>( \zeta_{BCL} )</td>
<td>+4.06</td>
<td>+4.03</td>
<td>+4.10</td>
<td>+4.51</td>
<td>+4.82</td>
<td>+4.00</td>
<td>+4.03</td>
<td>+4.41</td>
<td>+4.82</td>
</tr>
<tr>
<td>ft/sec^2/in.</td>
<td>( \mu_{BCL} )</td>
<td>+0.0028</td>
<td>+0.0094</td>
<td>+0.012</td>
<td>+0.0044</td>
<td>+0.0051</td>
<td>+0.0080</td>
<td>+0.0080</td>
<td>+0.0080</td>
<td>+0.0080</td>
</tr>
<tr>
<td>1/sec^2</td>
<td>( \mu_u )</td>
<td>0.0</td>
<td>-0.98</td>
<td>-1.49</td>
<td>+0.638</td>
<td>-1.02</td>
<td>-0.23</td>
<td>-0.879</td>
<td>+0.304</td>
<td>-0.487</td>
</tr>
</tbody>
</table>

*1 ft = 3.048 x 10^-1 m.
*1 kt = 5.144 x 10^-1 m/s.
**TABLE A-5**

**TRIM DATA (W = 13000 lb)**

<table>
<thead>
<tr>
<th>$V_{AO}$</th>
<th>$\theta_{AO}$</th>
<th>$\gamma_{AO}$</th>
<th>$\delta_{CLO}$</th>
<th>$\delta_{LNO}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>kt ${}^a$</td>
<td>ft/sec ${}^b$</td>
<td>deg</td>
<td>deg</td>
<td>%</td>
</tr>
<tr>
<td>2</td>
<td>3.38</td>
<td>2</td>
<td>0</td>
<td>80</td>
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<tr>
<td>28</td>
<td>47.32</td>
<td>0</td>
<td>62.5</td>
<td>4.08</td>
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<tr>
<td>42</td>
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<td>-2</td>
<td>51</td>
<td>3.45</td>
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<td>57</td>
<td>96.53</td>
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<td>45</td>
<td>3.13</td>
</tr>
<tr>
<td>68</td>
<td>114.92</td>
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<td>45</td>
<td>3.13</td>
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<td>79</td>
<td>133.51</td>
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<td>48</td>
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</tr>
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<td>1.75</td>
<td>2.96</td>
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<td>80</td>
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<tr>
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<td>43.94</td>
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<td>61</td>
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<td>67.60</td>
<td>-2</td>
<td>45</td>
<td>3.13</td>
</tr>
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<td>92.95</td>
<td>-4</td>
<td>33</td>
<td>2.48</td>
</tr>
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<td>68.5</td>
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<td>2.27</td>
</tr>
<tr>
<td>80.5</td>
<td>136.05</td>
<td>-8</td>
<td>29</td>
<td>2.27</td>
</tr>
<tr>
<td>96</td>
<td>162.24</td>
<td>-10</td>
<td>-6</td>
<td>33</td>
</tr>
<tr>
<td>1.5</td>
<td>2.54</td>
<td>2</td>
<td>-12</td>
<td>80</td>
</tr>
<tr>
<td>24</td>
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<td>0</td>
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<td>40</td>
<td>2.86</td>
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<td>92.95</td>
<td>-4</td>
<td>21</td>
<td>1.83</td>
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<td>71</td>
<td>119.99</td>
<td>-6</td>
<td>11.5</td>
<td>1.32</td>
</tr>
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<td>85</td>
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<td>9</td>
<td>1.19</td>
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<tr>
<td>101</td>
<td>170.69</td>
<td>-10</td>
<td>-12</td>
<td>9.5</td>
</tr>
</tbody>
</table>

---

* $^a$ kt = 5.144 x 10^{-1} m/s.
* $^b$ ft = 3.048 x 10^{-1} m.

---

A-24
The trim value for $\delta_{IN_0}$ was taken as zero for all conditions. The data for Table A-5 was obtained from Fig. 10 supplied by NASA.

The kinematic constants of interest are the initial unperturbed altitude of the wheels above the runway, $H_0$, and the pilot-selected Glide Slope angle, $\Theta$, which is also equal to the negative of trimmed flight path angle, $\gamma_0$. Values for these are

$$H_0 = 340 \text{ ft} (104 \text{ m}) \text{ for } \gamma_0 = -6 \text{ deg }, \quad 500 \text{ ft} (152 \text{ m}) \text{ for } \gamma_0 = -10 \text{ deg}$$

(A-62)

$$\gamma_0 = -6.0 \quad , 
-10.0 \text{ deg (} -0.10, -0.17 \text{ rad})$$

(A-63)

$$X_0 = (H_0 - 50)/\tan \gamma_0 \text{ ft}$$

(A-64)

$X_0$ is the initial location of the (unperturbed) aircraft, e.g., in the horizontal direction with respect to the touchdown point. The initial altitude of the aircraft e.g. above the touchdown point is $H_0 + h_{oeg}$.

**AIRCRAFT LATERAL-DIRECTIONAL MOTION MODEL**

Kinematic Equations

The scenario for the system performance model is shown in Fig. A-5. A perfectly level runway is assumed. Figure A-6 defines the perturbed coordinates for the aircraft body axes with respect to the unperturbed (or nominal) coordinates. The longitudinal kinematic location of the aircraft center of gravity with respect to the touchdown point is obtained as a tabular function as the result of exercising the longitudinal model. The following mean value data is collected at 2 sec intervals and stored in a file: $t, H, X, R, (V_{A_0} + u_0) = V_A, (\theta^{*}_0 + \Theta) = \theta^*$ and $(V^{*}_{T0} + u) = V^{*}_T$.

Kinematic equations locating the aircraft center of gravity laterally with respect to the extended runway centerline are
Figure A-5. Typical MLS Geometry for CTOL Runway
Figure A-6. Perturbed Coordinates Locating the Aircraft in a Horizontal Plane
\[ \dot{y} = v_A \hat{\beta} + \psi[U^* \cos \theta^* + \omega^* \sin \theta^*] - \omega^* \varphi \]  
(A-65)

\[ \dot{\psi} = r / \cos \theta^* \]  
(A-66)

\[ \dot{\varphi} = p + r \tan \theta^* \]  
(A-67)

\[ a_y = v_A \hat{\beta} - \omega^* \ddot{\phi} + U^* r - g \cos \theta^* \varphi \] 
(A-68)

\[ u^* = v_T \cos(\theta^* - \gamma_o) \]  
(A-69)

\[ w^* = v_T \sin(\theta^* - \gamma_o) \]  
(A-70)

The crosswind and crosswind shear act horizontally and in a direction perpendicular to the runway. The positive sense of the crosswind is directed away from the runway centerline toward the left wingtip of an unperturbed landing aircraft. The crosswind, \( v_w \), therefore must be resolved into aircraft body-fixed axis coordinates for proper application via the aircraft equations of motion. Let the longitudinal, side and normal components (with respect to body-fixed axes) of the deterministic crosswind disturbance be designated \( u_A \), \( v_A \) and \( w_A \), respectively.

\[ u_A = -v_w \sin \psi \cos (\theta_o^* + \theta) \] 
(A-71)

\[ v_A = -v_w \left[ \sin \psi \sin (\theta_o^* + \theta) \sin \varphi + \cos \psi \cos \varphi \right] \]  
(A-72)

\[ w_A = -v_w \left[ \sin \psi \sin (\theta_o^* + \theta) \cos \varphi - \cos \psi \sin \varphi \right] \]  
(A-73)
The linearized approximate expression for $v_A$ is used in the system performance model. $v_A$ enters the equations of motion in the manner of $v_g$.

The lateral displacement of the aircraft with respect to the MLS guidance reference is

$$\gamma_e = \gamma_c - \gamma \quad (A-74)$$

$\gamma_c$ is obtained from a statistical description of the MLS guidance signal in terms of the received signal $v_c$ (deg) as

$$\gamma_c = \frac{v_c}{57.3} R_A \quad (ft) \quad (A-75)$$

where

$$R_A = \sqrt{[R_g - (X + 50\tan \gamma_0)]^2 + H^2} \quad (ft) \quad (A-76)$$

State Equations for the Aircraft

The aircraft state variable equations are

$$\dot{\beta} = Y_\beta \beta + \frac{W^*}{V_A} p - \frac{U^*}{V_A} r + \frac{g \cos \theta^*}{V_A} \phi$$

$$+ \frac{Y_S}{V_A} \delta_{LT} + \frac{Y_{SPD}}{V_A} \delta_{PD} - \frac{Y_v}{V_A} v_g$$

$$+ \frac{Y_v}{V_A} K_w v_{wo} \quad (\beta \neq \frac{\dot{\gamma}}{V_A} ; v_A = \text{const.}) \quad (A-77')$$

$$\dot{p} = L_\beta \beta + L_p p + L_r r + L_{S_{LT}} \delta_{LT} + L_{S_{PD}} \delta_{PD}$$

$$- L_{p} \frac{\dot{\gamma}}{V_A} v_g - L_{p} p + L_{p} v_A K_w v_{wo} \quad (A-78')$$

A-29
\[
\dot{r} = N_{\beta}\beta + N_{\delta P}\delta + N_{\alpha}\alpha + N_{\phi LT}\phi + N_{\phi PD}\phi_P + N_{\phi}^{\prime} + N_{\phi}^{\prime}\phi_P + N_{\phi}^{\prime}/V_A \frac{v}{v_o} + N_{\phi}^{\prime}/V_A \frac{v}{v_o} \frac{k_w v}{v_o}
\]

The coefficients for these equations—and the kinematic equations given earlier for the lateral-directional model—are time-varying by virtue of their dependence upon kinematic variable values determined as a result of the longitudinal performance analysis calculations. The kinematic variables are \(t, H, X, R, V_A, \theta^*, \) and \(V_T^*\). Numerical values for the stability derivatives in Eqs. A-77 through A-79 are given in Table A-6 as functions of airspeed and aerodynamic flight path angle.
### TABLE A-6

**XV-15 LATERAL-DIRECTIONAL DERIVATIVES**

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<tr>
<th>UNITS</th>
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<th></th>
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</thead>
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<td>$V_{AO}$</td>
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<td>33.8</td>
<td>67.6</td>
<td>101.4</td>
</tr>
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<td>$V_{AO}$</td>
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<td>20</td>
<td>40</td>
<td>60</td>
</tr>
<tr>
<td>deg†</td>
<td>$\alpha_0$</td>
<td>$\theta_0 = 2.2$</td>
<td>6.5</td>
<td>4.5</td>
<td>1.6</td>
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<tr>
<td>deg</td>
<td>$\gamma_0$</td>
<td>—</td>
<td>-6</td>
<td>-6</td>
<td>-6</td>
</tr>
<tr>
<td>1/sec</td>
<td>$Y_v$</td>
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<td>-.038</td>
<td>-.0034</td>
<td>-.0460</td>
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<td>1/sec²</td>
<td>$L'_\beta$</td>
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<td>-.181</td>
<td>-.315</td>
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<tr>
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<td>$L'_T$</td>
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<td>.393</td>
<td>.444</td>
<td>.3980</td>
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<td>.276</td>
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<td>.187</td>
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<td>-.129</td>
<td>.0646</td>
<td>-.10678</td>
</tr>
<tr>
<td>ft/sec²/in.</td>
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<td>.0066</td>
<td>-.108</td>
<td>-.045</td>
<td>-.019</td>
</tr>
</tbody>
</table>

*1 ft = 3.048 x 10⁻¹ m.
†1 kt = 5.144 x 10⁻ⁱ m/s.
‡1 deg = 1.745 x 10⁻² rad.

1 in. = 2.54 x 10⁻² m.
TABLE A-6 (Concluded)

<table>
<thead>
<tr>
<th>UNITS</th>
<th>VARIABLE</th>
<th>1/ser²/in.</th>
<th>1/ser²/in.</th>
<th>ft/sec²/in.</th>
<th>1/ser²/in.</th>
<th>1/ser²/in.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>L&lt;sub&gt;0&lt;/sub&gt;&lt;sup&gt;LT&lt;/sup&gt;</td>
<td>.297</td>
<td>.290</td>
<td>.290</td>
<td>.3248</td>
<td>.362</td>
</tr>
<tr>
<td></td>
<td>N&lt;sub&gt;0&lt;/sub&gt;&lt;sup&gt;LT&lt;/sup&gt;</td>
<td>-.0196</td>
<td>-.0207</td>
<td>-.0417</td>
<td>-.02637</td>
<td>-.0161</td>
</tr>
<tr>
<td></td>
<td>Y&lt;sub&gt;0&lt;/sub&gt;&lt;sup&gt;PD&lt;/sup&gt;</td>
<td>.214</td>
<td>.166</td>
<td>.043</td>
<td>.10</td>
<td>-.324</td>
</tr>
<tr>
<td></td>
<td>L&lt;sub&gt;0&lt;/sub&gt;&lt;sup&gt;PD&lt;/sup&gt;</td>
<td>.0201</td>
<td>-.0258</td>
<td>-.0707</td>
<td>-.13354</td>
<td>-.131</td>
</tr>
<tr>
<td></td>
<td>N&lt;sub&gt;0&lt;/sub&gt;&lt;sup&gt;PD&lt;/sup&gt;</td>
<td>.121</td>
<td>.133</td>
<td>.143</td>
<td>.15492</td>
<td>.134</td>
</tr>
</tbody>
</table>

l<sub>x</sub><sub>a</sub> = 8.0 ft
l<sub>z</sub><sub>a</sub> = 0.0 ft
APPENDIX B
EQUATIONS AND DATA FOR SYSTEM PERFORMANCE EVALUATION

METHOD FOR USING PERTURBATION STABILITY DERIVATIVES

The XV-15 parametric data are in terms of small perturbation stability and control derivatives at specific static trim conditions in distinction to parametric data wherein the stability and control derivatives are expressed as functions of the trim variables. This and the fact that the performance evaluation required in this research program involves fairly rapid deceleration during final approach from 60-80 kt (31-41 m/s) to hover, in turn, has required development of a special procedure for using the available data. This has been necessary for three reasons:

- Validity of the (small) perturbation equations of motion requires that the perturbation variables indeed be kept reasonably small. This is particularly important for longitudinal perturbation velocity and airspeed which tend to grow large because of the wind shear and deceleration effects.

- Coefficients in the perturbation equations of motion for the aircraft are functions of trim airspeed, wind speed, pitch attitude, collective pitch, and longitudinal cyclic pitch, i.e., of the operating point.

- Trim data are for static, that is, unaccelerated conditions only. Valid application of the data requires that the operating point be a constant airspeed, wind speed, flight path angle and pitch attitude condition. Wind shear and deceleration to hover obviously make it impossible to select a single operating point for the landing approach which will cause all perturbation variables to be reasonably small.

The method developed for circumventing this difficulty without necessity for additional assumptions is given below.

Linear perturbation differential equations for the aircraft are in the form:

\[ \dot{x} = Ax + b, \quad x(0) = x_0 \]  \hspace{1cm} (B-1)
The actual response, \(X\), consists of the perturbation response plus the operating point, \(X_0\).

\[
X = x + X_0
\]  

(B-2)

The elements of \(A\) and \(b\) are functions of the operating point, \(X_0\).

Next consider how Eq. B-1 may be modified to allow for periodic adjustment of the operating point in the course of solution. This periodic adjustment is for two purposes:

- To select a new operating point for which the perturbation variables, \(x\), are once again small (ideally, zero).
- To revise the coefficients, \(A\) and \(b\), in the equations so their values correspond closely (ideally, exactly) to the coefficient values for the actual values of pitch attitude, wind speed, airspeed, flight path angle, collective and longitudinal cyclic pitch.

The ideal cannot be achieved for either of the above objectives if the system is not in static equilibrium (i.e., \(\dot{x} \neq 0\)). However, it is possible to approach the ideal closely for the decelerating conditions of interest. This is accomplished by using the actual values of airspeed, wind speed and flight path angle existing at the time of adjustment to partially define the revised operating point. The remaining parameters defining the revised operating point are determined by requiring static trim given those first several parameters. The difference between the static trim values for these latter operating point parameters and the actual values defines revised initial conditions for the perturbation variables. These are used in resuming solution of the differential equations. The specific procedures are given below.

Perturbation Equations

Let \(x_{0n}\) be the constant, incremental revision to the operating point for the \(n\)th interval. Then the operating point for the \(n\)th time interval, \(X_{0n}\), is given by
The perturbation differential equation for the nth time interval may be written:

\[ \dot{x} = (\dot{x} + \dot{x}_{on}) = A(x + x_{on}) + b - A\theta_{on} = Ax + b \]  
(B-4)

The initial conditions are obtained by requiring that \( (x + x_{on}) \) at the beginning of the nth time interval be equal to \( (x + x_{on-1}) \) at the end of the \((n-1)\)st time interval. The quantities \( \dot{x}_{on} \) and \( A\theta_{on} \) are zero by virtue of the definition of trim for small increments of the operating point, \( x_{on} \).

These differential equations must be discretized for use in the performance analysis program. Let the discretization interval be \( T \). Then the transition from the beginning to the end of the nth time interval is given by

\[ x_{n+1} = \varphi x_n + \theta b \]  
(B-5)

where \( \varphi = e^{AT} \) and \( \theta = \int_0^T e^{A(T-\tau)} d\tau \). The quantities used previously may be used to obtain the recursion equations for \( x \) and \( X \).

\[ X_{on} = x_{on} + x_{on-1} \]  
(B-6)

\[ x_n = x_n + x_{on} \quad \text{(value of } x \text{ at } t^+) \]  
(B-7a)

\[ x_{n+1} = \varphi x_n + \theta b \quad \text{(value of } x \text{ at } t_{n+1}^-) \]  
(B-7b)

\[ X_{n+1} = x_{n+1} + X_{on} \quad \text{(value of } X \text{ at } t_{n+1}^+) \]  
(B-7c)

Operating Point Revision

The operating point revision concept is to maintain the initial value of the airspeed perturbation variable zero for each time interval. Operating
point airspeed ($V_{Ao_n}$), flight path angle ($\gamma_o$), and wind speed ($V_{HW_n}$) are, in effect, given. Calculation of aerodynamic flight path angle ($\gamma_{Ao_n}$) enables us to calculate or look up values for all other trim quantities if equilibrium flight is assumed.

$$\gamma_{Ao_n} = \sin^{-1} \left( \frac{\sqrt{V_{Ao_n}^2 + V_{HW_n}^2 \sin^2 \gamma_o - V_{HW_n} \cos \gamma_o}}{V_{Ao_n}} \right) \sin \gamma_o$$

(B-8)

Trim pitch attitude, collective pitch, and longitudinal cyclic pitch are obtained from trim maps as functions of $V_{Ao_n}$ and $\gamma_{Ao_n}$. Trim values for inertial speed, longitudinal speed, normal speed are given by

$$V_{To_n} = \sqrt{V_{To_n}^2 + V_{HW_n}^2 \sin^2 \gamma_o - V_{HW_n} \cos \gamma_o}$$

(B-9)

$$U_{on} = V_{To_n} \cos (\theta_{on} - \gamma_o)$$

$$W_{on} = V_{To_n} \sin (\theta_{on} - \gamma_o)$$

The aircraft stability and control derivatives are functions of $V_{Ao_n}$ and $\gamma_{Ao_n}$.

Consideration of the above procedure for operating point revision reveals that assumption of static equilibrium results in different values for collective pitch, longitudinal cyclic pitch, and pitch attitude than exist in the actual, decelerating case. The differences tend to be small. The pitch attitude difference, in turn, affects the longitudinal and normal trim speeds, but also in a small way. These differences in each case are, of course, compensated by initial conditions on corresponding perturbation variables.
Equations and Data

Longitudinal equations used in performance evaluation are summarized in Table B-1. Numerical values for the control system parameters, switch settings and switching criteria for the longitudinal system modes are given in Tables B-2 through B-4. The lateral-directional equations used are summarized in Table B-5. Numerical values for the control system parameters are given in Table B-6. Switch settings and switching criteria for the lateral-directional control system modes are given in Table B-7.
TABLE B-1
LONGITUDINAL EQUATIONS FOR SYSTEM PERFORMANCE EVALUATION

STATE EQUATION: \[ \dot{x} = A_1 x + A_2 v + b_1 \]

1) \[ \dot{u} = X_u + X_w - u_0 u + (V_{1500} K_x X^2 - g \cos \theta_0) + X_{u_0} u_0 - X_{w_0} w \]
   \[ + X_{1500}^2 L + X_{BCL}^2 CL - X_{v_1} v_1 + X_{v_1} v_1 - X_{v_1} v_1 \]

2) \[ \dot{v} = Z_u + Z_w + u_0 u_0 + (V_{1500} K_x Z^2 - g \sin \theta_0) + Z_{u_0} u_0 - Z_{w_0} w \]
   \[ + Z_{v_1} v_1 + Z_{v_1} v_1 + Z_{v_1} v_1 \]

3) \[ \dot{q} = M_u + M_w + M_0 + X_{1500} K_x X^2 \]
   \[ + M_{1500}^2 L + M_{BCL}^2 CL - M_{v_1} v_1 + M_{v_1} v_1 - M_{v_1} v_1 \]

4) \[ \dot{\theta} = q \]

5) \[ \dot{u} = \sin \theta_0 u - \cos \theta_0 w + [u_0 \cos \theta_0 + w_0 \sin \theta_0] \theta \]

6) \[ \dot{x} = \cos \theta_0 u + \sin \theta_0 w + [-u_0 \sin \theta_0 + w_0 \cos \theta_0] \theta \]

7) \[ \dot{x_7} = -\omega x_7 \]

8) \[ \dot{x_8} = K_{15CL} K_{CLPD} \]

9) \[ \dot{x_9} = -(a_1 + a_2) x_9 - a_1 a_2 x_10 - a_1 a_2 (a_1 + a_2) (\nu/\theta_3.3) \eta + a_1 a_2 x_11 \cos \gamma \dot{H} - a_1 a_2 \sin \gamma \dot{x} + a_1 a_2 (a_1 + a_2) d \]

10) \[ \dot{x_10} = x_9 + a_1 a_2 (\nu/\theta_3.3) \eta \]

11) \[ \dot{x_11} = -\omega x_{11} \]

REPRODUCIBILITY OF THE ORIGINAL PAGE IS POOR
<table>
<thead>
<tr>
<th>Table B-1 (Continued)</th>
</tr>
</thead>
<tbody>
<tr>
<td>12) $\dot{x}_{12} = -aK\theta - aXg$</td>
</tr>
<tr>
<td>13) $\dot{x}_{13} = -$</td>
</tr>
<tr>
<td>14) $\dot{x}<em>{14} = -aX</em>{14}$</td>
</tr>
<tr>
<td>15) $\dot{x}<em>{15} = -mPD</em>{2}\theta_{15}$</td>
</tr>
<tr>
<td>16) $\dot{x}<em>{16} = m\theta</em>{9} - m\theta_{16}$</td>
</tr>
<tr>
<td>17) $\dot{x}<em>{17} = -kg - m</em>{x_{17}}$</td>
</tr>
<tr>
<td>18) $\dot{w}_{o} = 0$</td>
</tr>
<tr>
<td>19) $\dot{u}<em>{x} = -(\nu</em>{x_{o}}^{*}/L_{x})u_{g}$</td>
</tr>
<tr>
<td>20) $\dot{v}<em>{g} = -(1.5\nu^{*}</em>{x_{o}}/L_{u})u_{g}$</td>
</tr>
<tr>
<td>21) $\dot{\eta} = -(\nu_{x_{o}}^{*}/L_{\eta})\eta$</td>
</tr>
<tr>
<td>22) $\dot{x}<em>{c} = -ax</em>{c}x_{c}$</td>
</tr>
</tbody>
</table>

Reproducibility of the original page is poor.
### TABLE B-1 (Continued)

**AUXILIARY EQUATION:** \( v = A_2 + A_4 x + b_2 \)

1. \( \dot{x} = \cos \theta_x + \sin \theta_x \omega + \left[-U_x \sin \theta_x + W_x \cos \theta_x\right] \theta \) + \( v_x \cos \gamma_0 \) + \( v_x \sin \gamma_0 \)

2. \( \dot{\theta} = \sin \theta_x - \cos \theta_x \omega + \left[U_x \cos \theta_x + W_x \sin \theta_x\right] \theta \)

3. \( v_A = \cos(\theta^*_x - \gamma_0^*)u + \sin(\theta^*_x - \gamma_0^*)v - V_D u \) \( \theta_x + \left[1 - (1 - S_T)\right] \dot{\theta} - (1 - X_T) \theta_x S_A H \)

4. \( d = \cos \gamma_0 \dot{H} - \sin \gamma_0 \dot{x} \)

5. \( F_{DCL} = -\left[(1 - X_T) \theta_x + X_T \theta_4 \right] + \left[K_x \theta_x + (1 - S_T)\right] \dot{\theta} - (1 - X_T) \theta_x S_A H \)

6. \( \theta_{CL} = \theta_{CLDCL} + x_7 + x_8 \)

7. \( \Delta_{DEC} = -\left(\theta_y v_A K_B D / 57.3\right) \dot{x} \)

8. \( \theta_{LN} = K_9 \theta + K_q \dot{q} - x_{12} - x_{13} - x_{14} \)

9. \( a_x = x_{BCL} + x_{BCL} \dot{a}_x + x_{BCL} + x_{BCL} + (V_D u - V_D u) \theta + x_{BCL} \dot{x}_{10} - x_{BCL} - x_{BCL} \dot{x}_{10} - x_{BCL} - x_{BCL} \dot{x}_{10} - x_{BCL} + g \left(\sin \theta_x^* - \sin \theta_x^*\right) \)

10. \( \theta_{CD} = K_9 \dot{x}_x - K_9 \dot{x}_x + x_{17} + \left(1 - \theta_{DC}\right) x_{16} - B_u \theta_x \theta_x \)

11. \( f = \theta_{CD} \) - \( \theta - K_{\theta_0} \)

12. \( F_D = K_{FDC} f \) + \( K_{FDC} \cdot x_{15} \)

13. \( \theta_c = K_{PC} F_D \)

14. \( \theta_c = -\theta_c \) + \( x_{13} + x_{14} \)

15. \( \theta_b = \theta_c \) - \( K_9 \theta - K_9 \theta + x_{12} \)

**Reproducibility of the original page is poor.**
### TABLE B-1 (Continued)

**Alternate State Equation:**  \( y = A_5 v + A_6 x + B_3 \)

<table>
<thead>
<tr>
<th>( y )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) ( \dot{x} = v(1) )</td>
<td>12) ( \theta_s = v(15) )</td>
</tr>
<tr>
<td>2) ( \dot{H} = v(2) )</td>
<td>13) ( \theta_c/H = v(14)/H )</td>
</tr>
<tr>
<td>3) ( q = x(3) )</td>
<td>14) ( y_{14} = x(14) )</td>
</tr>
<tr>
<td>4) ( \Theta = x(4) + \theta_s^* )</td>
<td>5) ( FD_c = v(12) )</td>
</tr>
<tr>
<td>5) ( H = x(5) )</td>
<td>16) ( \theta_{CD} = v(10) )</td>
</tr>
<tr>
<td>6) ( X = x(6) )</td>
<td>17) ( y_{17} = x(17) )</td>
</tr>
<tr>
<td>7) ( \Delta \theta_{CL} = x(7) )</td>
<td>18) ( u_w = K_w u_{w_0} )</td>
</tr>
<tr>
<td>8) ( \delta_{CL} = v(6) )</td>
<td>19) ( u_g = x(19) )</td>
</tr>
<tr>
<td>9) ( Y_9 = x(11) )</td>
<td>20) ( v_g = x(20) )</td>
</tr>
<tr>
<td>10) ( Y_{10} = x(10) )</td>
<td>21) ( d_e = -\cos \gamma_0 \ H + \sin \gamma_0 \ X + Rn/57.3 + 50 \cos \gamma_0 )</td>
</tr>
<tr>
<td>11) ( Y_{11} = x(11) )</td>
<td>22) ( X_i = -X + X_c )</td>
</tr>
</tbody>
</table>
TABLE B-1 (Concluded)

OUTPUT EQUATION: \( z = A_7x + A_8v + b_4 \)

\( z \)

1) \( FD_{CL} = v(5) \)

2) \( A_2 = \dot{\omega} - \ell_x \dot{q} + (g \sin \theta_0^*) \theta \)

3) \( A_x = v(9) \)

4) \( \theta_e = \theta_{CD} - (1 - S_{DEC})(\theta + \theta_0^* - \theta_{oo}^*) \)

5) \( \varepsilon_{LN} = v(8) \)

6) \( e_{\theta} = \frac{\lambda_{78}}{K_{\theta VA}} \left\{ \cos \theta_0^* u + \sin \theta_0^* v - [v_0^* \sin \theta_0^* - W_0^* \cos \theta_0^*] \theta \right\} \)

\[ - x - \frac{\lambda_{78}}{K_{\theta VA}} \left\{ v_0^* \cos \theta_0^* + \frac{\theta_{NC}}{K_{\theta VA}} \ln \left[ 1 - \frac{K_{\theta VA} \dot{x}}{\theta_{INC}} \right] \right\} - (T_{D_{5}} + 2.0) \dot{x} \]

7) \( e_X = -K_2 \dot{x} - K_A x + gK_{ax} \theta - X + X_c + gK_{ax} (\theta_0^* - \theta_{oo}^*) - K_{ND_{5}} \dot{x} / 2.0 \)

\[ = x(22) - x(6) - [K_2 \dot{x} + K_{ND_{5}} / 2.0] y(1) - K_A z(3) + gK_{ax} x(4) + gK_{ax} (\theta_0^* - \theta_{oo}^*) \]

8) \( V_A = v(3) \)
# TABLE B-2

LONGITUDINAL CONTROL SYSTEM PARAMETERS USED IN PERFORMANCE EVALUATION

<table>
<thead>
<tr>
<th>ALL MODES</th>
<th>( K_\phi ) (in./rad)</th>
<th>( K_\dot\phi ) [in./(rad/sec)]</th>
<th>( K_{\phi C} ) (1/sec)</th>
<th>( \omega_a ) (rad/sec)</th>
<th>( K_d ) (in./ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>12.0</td>
<td>8.0</td>
<td>2.09</td>
<td>1.0</td>
<td>0.01</td>
</tr>
<tr>
<td>( K_{CL} )</td>
<td>(–)</td>
<td>( K_{ICL} ) (1/sec)</td>
<td>( V_{AX} ) (ft/sec)</td>
<td>( H_c ) (ft)</td>
<td>( T_{D3} ) (sec)</td>
</tr>
<tr>
<td></td>
<td>5.0</td>
<td>1.0</td>
<td>101.4</td>
<td>16.0</td>
<td>1.99</td>
</tr>
<tr>
<td>( \alpha_{d1} ) (rad/sec)</td>
<td>1.0</td>
<td>( \alpha_{d2} ) (rad/sec)</td>
<td>0.5</td>
<td>( \alpha_{CL} ) (rad/sec)</td>
<td>1.0</td>
</tr>
<tr>
<td>( K_{h} ) (rad/ft)</td>
<td>–0.005</td>
<td>( K_{h} ) (sec)</td>
<td>8.0</td>
<td>( K_{h} ) (sec)</td>
<td>16.0</td>
</tr>
<tr>
<td>( K_{p_c} ) (–)</td>
<td>3.2</td>
<td>( x_{MIN} ) (ft/sec)</td>
<td>10.0</td>
<td>( \omega_7 ) (rad/sec)</td>
<td>1.0</td>
</tr>
<tr>
<td>( \theta_{INC} ) (deg)</td>
<td>( K_{BUG} ) (–)</td>
<td>1.0</td>
<td>1.5</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

REPRODUCIBILITY OF THE ORIGINAL PAGE IS POOR
### TABLE E-2 (Concluded)

<table>
<thead>
<tr>
<th>Cyclic Axis</th>
<th>Collective Axis</th>
<th>LSW Code</th>
<th>$K_d$ (in./ft)</th>
<th>$K_h$ [in./(ft/sec)]</th>
<th>$\delta_D$ (sec)</th>
<th>$K_{FDc}$ (in./rad)</th>
<th>$\omega_{PD}$ (rad/sec)</th>
<th>$K_{WT}$ [in./(ft/sec)]</th>
<th>$\omega_{11}$ (rad/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>AS</td>
<td>GS</td>
<td>1, 2*</td>
<td>0.01</td>
<td>0.053</td>
<td>1.0</td>
<td>1.43</td>
<td>4.5</td>
<td>0</td>
<td>4.0</td>
</tr>
<tr>
<td>DECL</td>
<td>GS</td>
<td>3*</td>
<td>0.01</td>
<td>0.053</td>
<td>0.5</td>
<td>2.52</td>
<td>4.5</td>
<td>0</td>
<td>4.0</td>
</tr>
<tr>
<td>DECL</td>
<td>GS</td>
<td>4*</td>
<td>0.01</td>
<td>0.053</td>
<td>0.5</td>
<td>2.52</td>
<td>4.5</td>
<td>0.029</td>
<td>4.0</td>
</tr>
<tr>
<td>HOV</td>
<td>GS</td>
<td>5†</td>
<td>0.01</td>
<td>0.053</td>
<td>1.55</td>
<td>1.8</td>
<td>2.0</td>
<td>0.029</td>
<td>4.0</td>
</tr>
<tr>
<td>HOV</td>
<td>ALT</td>
<td>6†</td>
<td>0.01</td>
<td>0.053</td>
<td>1.55</td>
<td>1.8</td>
<td>2.0</td>
<td>0.029</td>
<td>1.0</td>
</tr>
<tr>
<td>HOV</td>
<td>VD</td>
<td>7†</td>
<td>0.0064</td>
<td>0.04</td>
<td>1.55</td>
<td>1.8</td>
<td>2.0</td>
<td>0.029</td>
<td>1.0</td>
</tr>
</tbody>
</table>

---

$K_d = \frac{0.01}{0.19 + 0.00296V_A}$ [in./(ft/sec)], $V_A$ in ft/sec units.

$K_{d} = 0.053$ [in./(ft/sec)].
### TABLE B-3. SWITCH SETTINGS FOR LONGITUDINAL PERFORMANCE EVALUATION

<table>
<thead>
<tr>
<th>CONTROL SYSTEM MODE</th>
<th>CYCLIC AXIS</th>
<th>COLLECTIVE AXIS</th>
<th>LSW CODE</th>
<th>(\omega_y) (rad/sec)</th>
<th>Su</th>
<th>SDEC</th>
<th>SDECL</th>
<th>SAH</th>
<th>ST</th>
<th>SVD</th>
</tr>
</thead>
<tbody>
<tr>
<td>AS</td>
<td>GS</td>
<td>1, 2</td>
<td>0.2</td>
<td>1.0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1.0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>DECL I</td>
<td>GS</td>
<td>3</td>
<td>4.0</td>
<td>0</td>
<td>1.0</td>
<td>0</td>
<td>0</td>
<td>1.0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>DECL</td>
<td>GS</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>1.0</td>
<td>0</td>
<td>0</td>
<td>1.0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>HOV</td>
<td>GS</td>
<td>5</td>
<td>0.25</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1.0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>HOV</td>
<td>ALT</td>
<td>6</td>
<td>0.25</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1.0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>HOV</td>
<td>VD</td>
<td>7</td>
<td>0.25</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1.0</td>
<td>0</td>
<td>0</td>
<td>1.0</td>
</tr>
</tbody>
</table>

**Legend for Control System Modes:**

- **ALT**: Altitude Hold  
- **AS**: Airspeed Hold  
- **DECL**: Constant Attitude Deceleration  
- **DECL I**: Fictitious mode introduced for performance evaluation to simulate pitch synchronizers without requiring additional model states  
- **GS**: Glide Slope Track  
- **HOV**: Longitudinal Point Hover  
- **VD**: Vertical Descent
### TABLE B-4

LONGITUDINAL CONTROL SYSTEM MODE SWITCHING CRITERIA

<table>
<thead>
<tr>
<th>CONTROL SYSTEM MODE</th>
<th>CYCLIC AXIS</th>
<th>COLLECTIVE AXIS</th>
<th>LSW CODE</th>
<th>CRITERION</th>
</tr>
</thead>
<tbody>
<tr>
<td>AS</td>
<td>GS</td>
<td>1</td>
<td></td>
<td>Problem Initialization: ( e_\vartheta &gt; 0 \text{ ft} ), ( 101.4 \leq V_A \leq 136.0 \text{ ft/sec} )</td>
</tr>
<tr>
<td>AS</td>
<td>GS</td>
<td>2</td>
<td></td>
<td>( e_\vartheta &gt; 0 \text{ ft} )</td>
</tr>
<tr>
<td>DECL I</td>
<td>GS</td>
<td>3</td>
<td></td>
<td>( e_\vartheta \leq 0 \text{ ft}, T_D^t &lt; 2.1 \text{ sec} )</td>
</tr>
<tr>
<td>DECL</td>
<td>GS</td>
<td>4</td>
<td></td>
<td>( T_D \geq 2.1 \text{ sec}, e_x \geq 0 \text{ ft}, \dot{x} &gt; \dot{x}_{MIN} \text{ ft/sec} )</td>
</tr>
<tr>
<td>HOV</td>
<td>GS</td>
<td>5</td>
<td></td>
<td>( e_x &lt; 0 \text{ ft or } \dot{x} \leq \dot{x}_{MIN} \text{ ft/sec and } H &gt; 51.0 \text{ ft} )</td>
</tr>
<tr>
<td>HOV</td>
<td>ALT</td>
<td>6</td>
<td></td>
<td>( H \leq 51.0 \text{ ft}, T_H^t &lt; 15.0 \text{ sec} )</td>
</tr>
<tr>
<td>HOV</td>
<td>VD</td>
<td>7</td>
<td></td>
<td>( T_H \geq 15.0 \text{ sec}, H \geq 0 \text{ ft} )</td>
</tr>
</tbody>
</table>

*1 ft = 3.048 x 10^{-1} m.

*Elapsed time from initiation of mode.
TABLE B-5

LATERAL-DIRECTIONAL EQUATIONS FOR SYSTEM PERFORMANCE EVALUATION

STATE EQUATION: \( \dot{x} = A_1 x + A_2 v + b_1 \)

1) \( \dot{\theta} = \dot{v} \theta + \frac{v}{v_A} p - \frac{v}{v_A} r + \frac{q}{v_A} \theta + \left( \frac{v}{v_A} \right) K_{uv} \theta \) \( - \frac{v}{v_A} V_{AV} \)

2) \( \dot{\psi} = L_{dp} \theta + L_{r} p + L_{h} r + \left( L_{dp} \frac{v}{v_A} \right) K_{uv} \theta \) \( - L_{dp} V_{AV} - L_{r} p \)

3) \( \dot{z} = N_{dp} \theta + N_{r} p + N_{h} r + \left( N_{dp} \frac{v}{v_A} \right) K_{uv} \theta \) \( - N_{dp} V_{AV} - N_{r} p \)

4) \( \dot{p} = \dot{v} - \tan \theta \dot{r} \)

5) \( \dot{v} = \frac{1}{\cos \theta} \dot{r} \)

6) \( \dot{x} = v_A \theta \) \( - v^2 \phi \) \( + \left( u^2 \cos \theta + w^2 \sin \theta \right) \psi \) \( - \omega_c \phi \)

7) \( \dot{x}_7 = \)

8) \( \dot{x}_8 = s_A \psi + s_A \dot{\psi} \)

9) \( \dot{x}_9 = - a_d \psi - K_{uv} \psi \) \( m_\psi \)

10) \( \dot{x}_{10} = \)

11) \( \dot{x}_{11} = - a_b \xi_{11} \)

REPRODUCIBILITY OF THE ORIGINAL PAGE IS POOR
TABLE B-5 (Continued)

12) \( \dot{x}_{12} = -\alpha_{FD2} x_{12} \)

13) \( \dot{x}_{13} = -\alpha_{y} x_{13} - \alpha_{y} y \)

14) \( \dot{x}_{14} = -\alpha_{\theta} x_{14} + \alpha_{dp} \phi \)

15) \( \dot{\psi}_{w} = \)

16) \( \dot{\psi} = -1.594 V_{A}/L_{v} \psi_{g} \)

17) \( \dot{\psi}_{g} = -x_{g} / (h_{p} g_{g}) \psi_{g} \)

18) \( \dot{v} = (-V_{E}^{2} / L_{v}) \psi \)

Output equation: \( z = A_{y} x + A_{v} v + b_{v} \)

\( z \)

1) \( A_{y} = (V_{A} y_{v} + \ell_{A} y_{P} ) S_{E} + \ell_{x} y_{P} P_{D} + \ell_{x} y_{P} P_{E} \) + \( (Y_{v} + \ell_{x} N_{F} / V_{A}) x_{w} V_{W0} - (Y_{v} + \ell_{x} N_{F} / V_{A}) y_{g} - \ell_{x} N_{F} v_{g} + (Y_{D} I_{D} + \ell_{x} N_{F} I_{D}) \delta_{LAT} + (Y_{D} I_{D} + \ell_{x} N_{F} I_{D}) \delta_{FD} \)

2) \( \delta_{LAT} = \)

3) \( \delta_{g} = \)

4) \( S_{A} = \)

5) \( -\dot{\gamma} = \frac{\partial L_{v}}{\partial \psi} \)
| 1) \( \epsilon_y = \) & \(- (K_y + \omega_y x_y) \nu - \omega_y x_{13} \) & \(+ [K_y + \omega_y x_y] R_B / 57.3 \nu \) \\
| 2) \( \Phi_{DC} = \) & \( \epsilon_y + K_{dp} x_{14} \) \\
| 3) \( q = \) & \( \Phi_{DC} \) & \(- k_{DP} - k_{Dp} \varphi \) \\
| 4) \( F_{DL} = (K_{FDL} \alpha_{FDL} / \alpha_{FD}) \nu \) & \( (K_{FDL} \alpha_{FDL} / \alpha_{FD}) x_{12} \) \\
| 5) \( \delta_s = K_{DP} F_{DL} \) & \(+ x_{10} + x_{11} \) \\
| 6) \( \varphi_c = - \delta_s \) \\
| 7) \( \xi_{LAT} = \) & \(- k_{DP} - k_{Dp} \) & \(+ x_9 + x_{10} + x_{11} \) \\
| 8) \( \tilde{\nu} = \) & \( K_{R} \nu + S_A K_{DP} (1 - S_A) \cdot + S_A K_B (\sigma / V_A) \varphi + K_{\nu} (1 - S_A) \varphi - K_{\nu} (1 - S_A) \psi \) & \(+ x_7 \) \\
| 9) \( \varphi_{FED} = \) & \( \tilde{\nu} \)
TABLE B-5 (Continued)

**ALTERNATE STATE EQUATION:** \( \dot{y} = A_2v + A_6x + b_3 \)

<p>| | | | | | | | | | | |</p>
<table>
<thead>
<tr>
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<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1)</td>
<td>( \dot{y} )</td>
<td>=</td>
<td>( v_{AB} )</td>
<td>( -w^*\phi )</td>
<td>( (u^* \cos \theta^* + w^* \sin \phi^*) )</td>
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<td></td>
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<tr>
<td>2)</td>
<td>( p )</td>
<td>=</td>
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<tr>
<td>3)</td>
<td>( r )</td>
<td>=</td>
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<td></td>
<td></td>
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<tr>
<td>4)</td>
<td>( \phi )</td>
<td>=</td>
<td></td>
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<td></td>
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<tr>
<td>5)</td>
<td>( \psi )</td>
<td>=</td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6)</td>
<td>( y )</td>
<td>=</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>7)</td>
<td>( \delta_{EDS} )</td>
<td>=</td>
<td>( \delta_{PED} )</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8)</td>
<td>( (\psi - \psi_b) )</td>
<td>=</td>
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<tr>
<td>9)</td>
<td>( \delta_{LTS} )</td>
<td>=</td>
<td>( \delta_{LAT} - \delta_b )</td>
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<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>10)</td>
<td>( \phi_c/K_\phi )</td>
<td>=</td>
<td>( 1/K_\phi \phi_c )</td>
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<td></td>
<td></td>
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<tr>
<td>11)</td>
<td>( y_{11} )</td>
<td>=</td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>
TABLE B-5 (Concluded)

12) $P_{DL} = P_{DL}$
13) $\epsilon_y = \epsilon_y$
14) $\Phi_{DC} = \Phi_{DC}$
15) $v_w = v_w$
16) $v_g = v_g$
17) $P_g = P_g$
18) $\gamma = \gamma + \left(\frac{B_0}{57.3}\right)\nu$

**EQUATIONS OF THE ORIGINAL PAGE IS WW**
<table>
<thead>
<tr>
<th>LOC A AND LOC B</th>
<th></th>
<th></th>
<th></th>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_f$</td>
<td>$K_r$</td>
<td>$K_\psi$</td>
<td>$K_{SYN}$</td>
<td>$K_y$</td>
<td></td>
</tr>
<tr>
<td>$[\text{in}^2/(\text{rad}/\text{sec})]$</td>
<td>$[\text{in}./(\text{rad}/\text{sec})]$</td>
<td>$(\text{in.}/\text{rad})$</td>
<td>$(\text{rad}/\text{sec})$</td>
<td>$(\text{rad}/\text{ft}^\dagger)$</td>
<td></td>
</tr>
<tr>
<td>10.0</td>
<td>13.0</td>
<td>13.0</td>
<td>10.0</td>
<td>0.002</td>
<td></td>
</tr>
<tr>
<td>$K_r$</td>
<td>$K_{DP}$</td>
<td>$\omega_{MP}$</td>
<td>$\omega_{FD1}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$[(\text{rad}/\text{ft}/\text{sec})]$</td>
<td>$(-)$</td>
<td>$(\text{sec})$</td>
<td>$(\text{rad}/\text{sec})$</td>
<td>$(\text{rad}/\text{sec})$</td>
<td></td>
</tr>
<tr>
<td>0.017</td>
<td>1.06</td>
<td>0.425</td>
<td>0.1</td>
<td>2.0</td>
<td></td>
</tr>
<tr>
<td>$\omega_{FD2}$</td>
<td>$\omega_y$</td>
<td>$R_G$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$(\text{rad}/\text{sec})$</td>
<td>$(\text{rad}/\text{sec})$</td>
<td>$(\text{ft})$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>14.9</td>
<td>4.0</td>
<td>1000$^\dagger$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**LOC A ($S_A = 1.0$)**

| $K_p$          | $K_\varphi$    | $K_\theta$      | $\omega_y$ | $\omega_c$ |
| $[\text{in}./(\text{rad}/\text{sec})]$ | $(\text{in.}/\text{rad})$ | $(1/\text{sec})$ | $(\text{rad}/\text{sec})$ | $(\text{rad}/\text{sec})$ |
| 5.6            | 11.2           | 2.93            | 0            | 0            |

| $K_{FDL}$      | $K_{DP}$       |
| $(\text{in.}/\text{rad})$ | $(-)$           |
| 1.0            | 5.62           |

---

*1 in. = 2.54 x 10^{-2} m.

†1 ft = 3.048 x 10^{-1} m.

*Number shown is for landing on VTOL pad. CTOL runway value is 10,000.
TABLE B-6 (Concluded)

LOC B ($S_A = 0$)

<table>
<thead>
<tr>
<th>$K_p$ [in./(rad/sec)]</th>
<th>$K_{\phi}$ (in./rad)</th>
<th>$K_{\phi_c}$ (1/sec)</th>
<th>$\omega_b$ (rad/sec)</th>
<th>$\omega_c$ (rad/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.0</td>
<td>12.0</td>
<td>3.14</td>
<td>1.0</td>
<td>0.5</td>
</tr>
</tbody>
</table>

$K_{FDL}$

(in./rad)

1.6

$K_{\phi}$

(\(-\))

2.5

REPRODUCIBILITY OF THE ORIGINAL PAGE IS POOR
TABLE B-7
LATERAL-DIRECTIONAL CONTROL SYSTEM MODE
SWITCHING CRITERIA AND SWITCH SETTINGS

<table>
<thead>
<tr>
<th>CONTROL SYSTEM MODE</th>
<th>SA CODE/ VALUE</th>
<th>CRITERION</th>
</tr>
</thead>
<tbody>
<tr>
<td>LOC A</td>
<td>1</td>
<td>( V_A &gt; 84.5 \text{ ft}^* /\text{sec} )</td>
</tr>
<tr>
<td>LOC B</td>
<td>0</td>
<td>( V_A \leq 84.5 \text{ ft} /\text{sec} )</td>
</tr>
</tbody>
</table>

*1 ft = 3.048 x 10^{-1} m.