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INVESTIGATION OF THE BURNING CONFIGURATION OF A COAXIAL INJECTOR IN A COMBUSTION CHAMBER

by

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prepared for

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NASA Lewis Research Center
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Richard J. Priem, Project Manager
An analytical investigation has been made into the stability of the burning configuration of a single coaxial injector surrounded by similar injectors. The stability criteria is based on an average pressure difference along the boundaries of the adjacent stream tubes as calculated using Spaulding's numerical method.

The results indicate qualitatively that there is a tendency for the injectors to have different burning configurations. It is believed that the configuration achieved is random; however once the burning configuration is established, it is believed to persist. These results are consistent with previous experimental observations.
FORWARD

The research reported herein was performed by the Civil & Mechanical Engineering Department of Texas A&I University under NASA Grant NSG-3112 from September 1976 through February 1978. The project manager was Dr. Richard J. Priem, Chemical Rockets Division, NASA Lewis Research Center.
ABSTRACT

An analytical investigation has been made into the stability of the burning configuration of a single coaxial injector surrounded by similar injectors. The stability criteria is based on an average pressure difference along the boundaries of the adjacent stream tubes as calculated using Spaulding's numerical method.

The results indicate qualitatively that there is a tendency for the injectors to have different burning configurations. It is believed that the configuration achieved is random; however once the burning configuration is established, it is believed to persist. These results are consistent with previous experimental observations.
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LIST OF SYMBOLS

**Latin Characters**

- $C_p$: Pressure Coefficient
- $f$: Darcy friction factor
- $L/D$: Length to diameter ratio
- $N$: Number of surrounding injectors
- $P$: Static pressure
- $P_r$: Reference static pressure, $r = 0$, $z = 36.88$ cm
- $r$: Radial coordinate
- $R$: Radius of injector stream tube
- $V$: Injector velocity of propellants
- $z$: Longitudinal coordinate

**Greek Characters**

- $\phi$: Pressure function
- $\rho$: Density

**Subscripts**

- $b$: Longitudinal boundary
- $c$: Central injector
- $f$: Injector face boundary
- $fu$: Fuel (gaseous Hydrogen)
- $inj$: Injector
- $s$: Stream tube larger than nominal
- $o$: Nominal operating conditions
- $ox$: Oxidizer (gaseous Oxygen)
- $s$: Stream tube smaller than nominal
I. INTRODUCTION

Over the past several years a series of experiments have been conducted (1, 2, 3, 4,) for the purpose of determining the turbulent mixing parameters in a liquid rocket engine. A tracer gas diffusion method with helium gas injected at a point along the combustion chamber centerline was used. (See figure 1) Downstream of the helium injector, gas samples were withdrawn at a series of points along a major diameter. Later these samples were analyzed to determine the helium concentration in each sample. The turbulent mixing parameters were calculated using the theories of turbulent diffusion and the measured helium concentration values.

The same small rocket engine was used for all the tests. This engine had a cylindrical combustion chamber 5.94 cm in diameter and approximately 37 cm long with a 2.54 cm throat. The initial series of runs were made using liquid oxygen and heptane as propellants and a like-on-like impingement injector. During these runs the sampling and measuring techniques were developed, and consistent and repeatable data were obtained.

A series of runs were then conducted using gaseous oxygen and gaseous hydrogen and later liquid oxygen and gaseous hydrogen as propellants. For both of these propellant combinations, a seven element coaxial injector was used as shown in figure 1.

Throughout both series of hydrogen-oxygen runs, a large amount of scatter was observed in the helium concentration measurement both within a run and between runs. Six gas samples could be taken per run, and it became customary to make four runs at each test configuration thus having twenty-four data points to define a helium concentration curve.
FIGURE 1 - EXPERIMENTAL CONFIGURATION
The six ports in the sample probe were distributed along the entire diameter and the probe could be moved along the diameter in small fixed increments between runs. Thus subsequent runs would provide data points to fill in the spaces between points from the previous runs. Figure 2 shows the data from a typical set of four runs. The large amount of scatter is readily apparent.

Repeated efforts were made to determine the cause of these inconsistencies. All measuring and metering equipment was recalibrated and/or replaced. The entire system was inspected for leaks many times. An error analysis which indicated the accuracy of the helium concentration measurements should have been within about ±5% was made; however, the actual scatter was many times this value.

It was finally concluded that the concentration measurements must be representative of the conditions in the engine. The possibility that the problem was caused by some type of combustion instability in the usual sense was ruled out. Even low-frequency combustion instability will be in the order of several hundred hertz. The sampling time for each of these runs was approximately 1.5 seconds. Certainly variations caused by combustion instability would be averaged in 1.5 seconds. If it were some type of very low frequency "chugging," the existence of the condition should be apparent on either the chamber pressure or thrust measurements; however, it was not.

It is the purpose of the work herein to investigate analytically the possibility that the engine had more than one stable mode of burning.
FIGURE 2  TYPICAL HELIUM CONCENTRATION DATA
To analyze such a complex flow situation involving a bi-propellant system with turbulent combustion is very difficult even under steady state conditions, and to include the time variable would be out of the question. The approach chosen was the Spalding steady state computer analysis given by Gosman et al. (5) This method was used to calculate the pressure along the boundary of a single element stream tube under different conditions of stream tube diameter and propellant flow rate. The results of these calculations would then be analyzed.

It was also decided that due to the nature of the problem and the large amount of computer time required for each case considered a very simplistic approach would be taken. Only gross trends would be sought rather than numerical bounds.
II. CALCULATION OF PRESSURE COEFFICIENTS

A. The Spaulding Numerical Method

The experimental combustion chamber had seven coaxial propellant injectors, thus the flow was three-dimensional. Mathematically this is a very difficult problem. To simplify the problem, it was assumed that each injector formed an axisymmetric jet with turbulent mixing and chemical reaction. The axisymmetric jet was assumed to be confined in a constant diameter cylindrical stream tube. This boundary condition is believed to closely approximate the case of a single injector surrounded by similar injectors.

The Spaulding numerical method is applicable to this situation and was used to calculate the pressure distribution along the boundary. The uniqueness of this method, which is restricted to two-dimensional or axisymmetric problems, lies in the transformation of each of the conservation equations into a standard form which is a nonlinear partial differential equation of the elliptic type. The standard equation is solved by a finite difference procedure using the Gauss-Seidel iterative technique.

A very important consideration is the method of modeling the turbulent flow. In the present calculations, the turbulent flow was modeled by an equation of the effective viscosity. The equation was that proposed by Gosman et al.\(^{(5)}\)

The details of this calculating procedure are given by Gosman et al including a complete listing of the program. Tou\(^{(6)}\) presents details of the calculating procedure as applicable to the configuration considered herein.
B. Configuration considered

The configuration considered was a single injector element with a circular stream tube of constant radius. The same injector dimensions were maintained throughout and are those shown by figure 1. The nominal or reference case was assumed to be that in which the cross sectional area of the stream tube is one seventh of the chamber area. Under this assumption, the stream tube radius is 1.123 cm. A free longitudinal boundary was assumed, i.e., the gradient of the longitudinal velocity with respect to the radius equals zero. The nominal or reference injector velocities for oxygen and hydrogen were 233.5 m/sec. and 780 m/sec. respectively.

Computer runs were made for five cases as listed in TABLE I. These were: nominal, radius increased 10%, radius decreased 10%, radius increased and injector velocities decreased by 10%, and radius decreased and injector velocities increased by 10%.

The longitudinal boundary and injector face pressure coefficients as calculated by the Spaulding computer program are plotted on figure 3 and 4, respectively. The pressure coefficient used is referenced to the center line pressure, \( P_r \), far down stream \((z=36.88 \text{ cm})\) and defined as

\[
C_p = \frac{P}{P_r} - 1 \tag{1}
\]
FIGURE 3  PRESSURE COEFFICIENT ALONG LONGITUDINAL BOUNDARY
(a) Absolute Values

<table>
<thead>
<tr>
<th>SYM</th>
<th>$\Delta R/R_0$</th>
<th>$\Delta V/V_0$</th>
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<tr>
<td></td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>-0.1</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>+0.1</td>
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</tr>
<tr>
<td></td>
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<td>+0.1</td>
</tr>
<tr>
<td></td>
<td>+0.1</td>
<td>-0.1</td>
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</table>

(b) Change From Nominal

FIGURE 4 PRESSURE COEFFICIENT ALONG INJECTOR FACE
### TABLE I-Listing of Computer Cases

<table>
<thead>
<tr>
<th>Case</th>
<th>$\Delta R/R_0$</th>
<th>$\Delta V/V_0$</th>
<th>R(cm)</th>
<th>$V_{ox}$(m/sec)</th>
<th>$V_{fu}$(m/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1*</td>
<td>0</td>
<td>0</td>
<td>1.123</td>
<td>233.5</td>
<td>780.0</td>
</tr>
<tr>
<td>2</td>
<td>+0.1</td>
<td>0</td>
<td>1.235</td>
<td>233.5</td>
<td>780.0</td>
</tr>
<tr>
<td>3</td>
<td>-0.1</td>
<td>0</td>
<td>1.011</td>
<td>233.5</td>
<td>780.0</td>
</tr>
<tr>
<td>4</td>
<td>+0.1</td>
<td>-0.1</td>
<td>1.235</td>
<td>210.1</td>
<td>702.0</td>
</tr>
<tr>
<td>5</td>
<td>-0.1</td>
<td>+0.1</td>
<td>1.011</td>
<td>256.5</td>
<td>858.0</td>
</tr>
</tbody>
</table>

*Nominal*
III. RESULTS

A. Development of Pressure Function

As mentioned in the introduction, it is believed that the complexity of this problem and the limitations of the analytical tools at hand make only the simplest analysis justifiable. Thus, in the analysis which follows, gross approximations and simplifying assumptions are made with the understanding that possible trends only are to be implied from the results obtained.

Inspection of the curves of figure 3 shows that changes in stream tube radius and injector velocities have an effect on the longitudinal boundary pressure distribution in the region from about one to six centimeters downstream of the injector. Further downstream, the effect is very small. This is believed significant because if the stream tube radius is altered by a pressure difference in the upstream region, the change will persist in the downstream region. This would give some justification for assuming a constant diameter stream tube.

Figure 4 shows that changes in stream tube radius and injector velocities have a large effect on the injector face pressure coefficient. These injector face pressure variations are large enough to cause significant changes in the pressure across the injector and thus influence the propellant flow rates.

The pressure data have been referenced to the pressure far downstream because it is assumed that this pressure will be the controlling boundary condition when several injectors are adjacent to one another.

The changes from nominal in the pressure distribution caused by stream tube radius changes and injector velocity changes are also shown on figure 3 and 4. Average changes in the longitudinal boundary pressure coefficient, $\Delta C_p^b$, and average changes in injector face pressure coefficient, $\Delta C_p^f$, have been assumed as indicated by the straight lines on the graphs.
These average changes were approximated by equations as follows:

\[ \overline{\Delta C_p_{b}} = 0.12|\Delta R/R_0| - 0.2 (\Delta V/V_0) \]  \hspace{1cm} (2)

and

\[ \overline{\Delta C_p_{f}} = 0.7(\Delta R/R_0) + 0.4 (\Delta V/V_0) \]  \hspace{1cm} (3)

The accuracies of these equations are illustrated by figure 5.

The relationship between the pressure drop across the injector and the injection velocity can be approximated by the incompressible flow equation.

\[ \Delta P_{\text{inj}} = (fL/D + 1) \rho V^2/2 \]  \hspace{1cm} (4)

where \( \Delta P_{\text{inj}} = \Delta P_{\text{inj}} + \Delta(\Delta P_{\text{inj}}) \). Assuming that variations in flow rate do not significantly change the upstream injector manifold pressures, then \( \Delta(\Delta P_{\text{inj}}) = -P_r(\overline{\Delta C_p_{f}}) \). Differentiating equation 4 for small deviations from nominal conditions it follows that:

\[ \overline{\Delta C_p_{f}} = \frac{-1}{P_r} (fL/D - 1) \rho V_0^2 (\Delta V/V_0) \]  \hspace{1cm} (5)

Substituting numerical values for the experimental configuration, the relationship can be represented to within \( \pm 10\% \) for both oxygen and hydrogen streams by the equation

\[ \overline{\Delta C_p_{f}} = -1.1 (\Delta V/V_0) \]  \hspace{1cm} (6)

Combining equations 2, 3 and 6 gives an expression for the average boundary pressure coefficient in terms of the change in radius.

\[ \overline{\Delta C_p_{b}} = 0.12 |\Delta R/R_0| + 0.10 (\Delta R/R_0) \]  \hspace{1cm} (7)

Consider now a configuration where one central injector element is surrounded by \( N \) other injectors and when the stream tube of the central injector decreases in radius the stream tubes of the \( N \) surrounding injectors increase in radii, and visa versa. Assume that the cross section of all \( N + 1 \) stream tubes remains constant as it would in a small engine.
\[
\Delta C_{P_b} = 0.12|\Delta R/R_0| - 0.2(\Delta V/V_0)
\]

\[
\Delta C_{P_f} = 0.7(\Delta R/P_0) + 0.4(\Delta V/V_0)
\]

FIGURE 5  ACCURACY OF EMPERICAL EQUATIONS FOR VARIATION IN PRESSURE COEFFICIENT
Define a pressure parameter
\[ \phi = \Delta C_{p_b} \xi - \Delta C_{p_b} \eta \]  
(8)
to represent the pressure difference between the central and surrounding stream tubes. The subscripts \( \xi \) and \( \eta \) refer to large and small, respectively. A positive value of \( \phi \) indicates an unstable condition because the large stream tube will tend to become larger.

Under the conditions outlined above, if the central stream tube is smaller than the surrounding stream tubes, i.e., \( \Delta R_c < 0 \)
\[ \phi = -0.12\left|\frac{\Delta R_c}{R_o}\right| (1 - 1/N) - 0.10 \left(\frac{\Delta R_c}{R_o}\right) (1 + 1/N) \]  
(9)
and if the central stream tube is larger, i.e., \( \Delta R_c > 0 \)
\[ \phi = 0.12\left|\frac{\Delta R_c}{R_o}\right| (1 - 1/N) + 0.10 \left(\frac{\Delta R_c}{R_o}\right) (1 + 1/N) \]  
(10)
where \( \Delta R_c \) is change in radius of the central stream tube.
B. Discussion of Results

The pressure function, $\phi$, is shown on figure 6 as a function of the fractional change in the radius, $\Delta R_c/R_0$, and the number of surrounding injectors, $N$. It must be remembered that the pressure function represents the pressure coefficient of the larger stream tube minus the pressure coefficient of the smaller no matter which happens to be in the center. Therefore, a positive value of pressure function, $\phi$, means that a larger stream tube will tend to get larger, i.e., there is a destabilizing effect.

The pressure for a two injector combination is represented by $N = 1$ and is seen to have a very large destabilizing tendency. According to this analysis, the two injector combination would always tend to burn in an asymmetric configuration.

The case of a single injector surrounded by six other injectors where the six surrounding injectors have identical burning patterns is represented by $N = 6$. For $\Delta R_c/R_0 < 0$, i.e., the center stream tube is smaller, the system is seen to be neutrally stable. On the other hand for $\Delta R_c/R_0 > 0$, i.e., the center stream tube is larger, the results indicate a large destabilizing influence.

While this analysis is admittedly very crude, it does give an indication of the type burning patterns which might tend to form. In particular, it indicates that the case in which all injectors have the same burning pattern is not very likely. Figure 7 shows some of the burning patterns which it is believed will tend to form. The particular pattern which actually forms may be a matter of chance depending on which injector ignites first.

Likewise, the crudeness of the analysis makes it impossible to make any predictions regarding the details of the final steady state burning configurations. Certainly, the steady state configuration would be decidedly three-dimensional and many non-linearities would be present.
**FIGURE 6** PRESSURE FUNCTION

**FIGURE 7** POSSIBLE BURNING CONFIGURATIONS
The destabilizing influence predicted above is for a particular coupling between the propellant manifold and the combustion chamber and for a constant manifold pressure. Both of these situations are believed to be representative of the typical hydrogen oxygen engine; and while the numerical values of the pressure function might change, the trends would be the same.

These results do not prove that the scatter in the experimental data discussed in the introduction was the result of the stable but dissimilar burning of the individual injectors. It does, however, indicate that such burning might be possible and shows the mechanism whereby it could occur.

The analysis indicates that the injector face pressures surrounding the individual injectors could differ by an easily measurable amount. If such measurements were made, they should indicate whether or not the dissimilar burning condition actually exists. Also, if such a condition were found, it would add significant creditability to the Spaulding method.
IV. CONCLUSIONS & RECOMMENDATIONS

The conclusions drawn from this work are:

(1) The Spaulding numerical method can be used to predict that the individual injectors of a multi-coaxial injector system will tend toward dissimilar burning configurations.

(2) There may be several stable steady-state burning configurations, and the particular configuration occurring may depend on the ignition process.

(3) The dissimilar burning condition is a result of coupling with the propellant manifolds.

(4) The dissimilar burning conditions could be the cause of the large scatter observed in previous experimental turbulent diffusion data.

It is recommended that:

(1) Further analysis should be conducted in order to define the steady-state burning configuration(s) of multi-coaxial injector systems.

(2) Experimental verification of the destabilizing tendency predicted herein should be sought both for the purpose of determining the burning characteristics and for the purpose of evaluating the Spaulding numerical method.