DESIGN OF EXPERIMENT FOR EARTH ROTATION AND BASELINE PARAMETER DETERMINATION FROM VERY LONG BASELINE INTERFEROMETRY

by

Athanasios Dermanis

Prepared for the
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The Ohio State University
Research Foundation
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PREFACE

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1. Introduction

Very Long Baseline Interferometry (VLBI) is one of the new techniques which will probably dominate geodesy and geophysics in the near future. Its main advantage lies in the fact that it brings the accuracy of direction measurements to a level previously possible only for range measurements. This closes the gap between powerful range determination techniques such as laser ranging and the much less accurate determination of directions through photographic tracking of artificial earth satellites.

The technique is geometric in the sense that the observations are independent of the gravity field of the earth. However, the "orbits" of the observed extragalactic radio sources with respect to an earth-fixed system are dominated and perturbed by the rotation of the earth with respect to inertial frame. This allows the determination of polar motion, precession-nutation and length-of-the-day variations, and the technique becomes also "dynamic" in this respect.

The capability of determining the geometry of a network of stations within a short time interval and with a centimeter level accuracy also allows the study of the variation of network geometry with time caused by earth tides and other periodic or secular station drifts.

The obvious importance of VLBI for both geodetic and geophysical applications is only limited by the effect of atmospheric refraction and unwanted noise associated with instrumentation [Shapiro and Knight, 1970, Section 3]. At present the development of the technique is at the stage of establishing its capabilities by direct comparison with classical surveys in relatively short baselines where geophysical effects are negligible, and of making the system more transportable.

In the present work the effect of nonwhite instrumentation noise, atmospheric refraction and earth tides are ignored, and the emphasis is on the possibility of recovering earth rotation and network geometry (baseline) parameters. It is assumed that nonwhite instrumentation noise is either eliminated or separately determined through calibration except for a linear effect which cannot be
distinguished from clock errors included in the adjustment parameters. Errors in the determination of atmospheric refraction are assumed to be either absent or uncorrelated in which case their effect is incorporated in the variance of the considered instrumentation white noise. The effect of earth tides on the variation of station coordinates is assumed to be calculated from separate information and subtracted from the actual observations.

The numerical simulated experiments performed here are set up in an environment where station coordinates vary with respect to inertial space according to a simulated earth rotation model similar to the actual but unknown rotation of the earth.

In Chapter 2 the basic technique of VLBI and its mathematical model are presented. The parametrization of earth rotation chosen is described in Chapter 3, and the resulting model is linearized in Chapter 4. Cartesian station coordinates are not estimable quantities in the analysis of VLBI observations. It is possible to use a model where only estimable quantities are considered (see, for example, [Arnold, 1974]). However, the choice of Cartesian coordinates leads to a simpler model, and the problem of coordinate system definition is resolved in Chapter 5 with the use of inner constraints. A simple analysis of the geometry of the observations in Chapter 6 leads to some useful hints on achieving maximum sensitivity of the observations with respect to the parameters considered. The basic philosophy for the simulation of data and their analysis through standard least squares adjustment techniques is presented in Chapter 7.

The main objective of the present work is the exploration of the capabilities of VLBI for the recovery of earth rotation and baseline parameters. For this purpose, a number of characteristic network designs based on present and candidate station locations is chosen in Chapter 8. The results of the simulations for each design are presented in Chapter 9 together with a summary of the conclusions.
2. VLBI—Basic Technique and Mathematical Model

Very Long Baseline Interferometry (VLBI) is a technique for observing time intervals and/or their derivatives at two widely separated antennas between the two instants of reception of the same wavefront emitted from extragalactic radio sources. The fundamental difference with respect to classical radio (short baseline) interferometry is in the fact that the antennas are not directly connected by cables for the comparison of the received signals. Sufficiently stable atomic frequency standards at each site make it possible to record the signals as functions of time. The recorded signals are then crosscorrelated afterwards in a computer. The fact that maximum correlation occurs when the two recordings are shifted in time by an amount equal to the wave travel time between the two instances of reception, makes it possible in principle to determine this time interval as well as its time derivative. Extragalactic radio sources are sufficiently far away for the wavefront to be considered as a plane and thus from the known velocity of the wave (velocity of light) the geometric distance corresponding to the observed time delay can be calculated.

The crosscorrelation procedure is described in detail in a series of articles by Thomas (1972a, 1972b and 1973), and we shall be concerned here only with the geometry of the observations. In Fig. 1, XYZ is a geocentric Cartesian reference frame fixed with respect to the radio sources (assumed to be an inertial frame). A certain wavefront arrives at station i at epoch $t_1$, when the position vector of the station is $X_i(t_1)$, and at the station $j$ at a slightly different epoch $t_2$ when the position vector of this station is $X_j(t_2)$. The true time delay $t_2 - t_1$ is the travel time corresponding to the projection $D_{ij}$ of the retarded baseline $X_j(t_2) - X_i(t_1)$ on the direction of the radio source. If $e$ is the unit vector in the radio source direction and $c$ the velocity of light,

$$D_{ij} = c (t_2 - t_1) = [X_j(t_2) - X_i(t_1)] \cdot e$$

\[(2-1)\]
Fig. 2.1  The geometry of VLBI observations.
Since \((t_2 - t_1)\) is a very short time interval, the time variation of the position vector \(X_j(t)\) may be assumed to be solely due to the earth's rotation. Therefore

\[
X_j(t_2) = X_j(t_1) + \int_{t_1}^{t_2} \Omega(t) \times X_j(t) \, dt
\]  

(2-2)

where \(\Omega(t)\) is the rotational velocity vector of the earth. \(\Omega(t)\) is practically constant over the short time interval \((t_1 - t_2)\), and thus, by setting \(\Omega(t) = \Omega(t_1)\)

\[
X_j(t_2) = X_j(t_1) + (t_2 - t_1) [\Omega(t_1) \times X_j(t_1)]
\]

(2-3)

Setting \(B_{ij}(t) = X_j(t) - X_i(t)\) and \(V_j(t) = \Omega(t) \times X_j(t)\), one obtains

\[
D_{ij} = B_{ij}(t_1) \cdot e - \frac{1}{c} D_{ij} V_j(t_1) \cdot e
\]

(2-4)

If approximate values of \(\Omega(t_1), X_j(t_1)\) and \(e\) are known with sufficient accuracy such that the corresponding uncertainty in the retardation effect

\[
\delta D_{ij} = \frac{1}{c} D_{ij} V_j \cdot e
\]

(2-5)

is negligible, one obtains, by setting \(d_{ij} = D_{ij} + \delta D_{ij}\) and switching to matrix (column vector) notation, for an observation from the \(ij\) baseline to the radio source \(p\) at epoch \(t_k\), the much simpler model

\[
d_{ijpk} = B_{ij}(t_k) e_p
\]

(2-6)

Assume that \(d_{ijpk}\) has been corrected for effects giving rise to discrepancies between modeled and true distance, e.g., atmospheric errors, relativistic effects, aberration, except for clock errors. Assuming that the two local oscillators at stations \(i\) and \(j\) differ only by a constant offset \(C_{otj}\) (at some epoch \(t_0\)) and a linear drift \(C_{ij}\), the model becomes

\[
d_{ijpk} = B_{ij}(t_k) e_p + c[C_{otj} + C_{ij}(t_k - t_0)]
\]

(2-7)

Since the inner product of two vectors is invariant with respect to the choice of reference frame, the model may be rewritten

\[
d_{ijpk} = b_{ij}^* e_p(t_k) + c[C_{otj} + C_{ij}(t_k - t_0)]
\]

(2-8)
with

\[ b_{i,j}^T = [x_j - x_i, \ y_j - y_i, \ z_j - z_i] \quad (2-9) \]

where \( b_{i,j} \) and \( e_p^* \) are the baseline and the radio source direction unit vector, respectively, with respect to an earth-fixed reference frame \( x, y, z \). One also has

\[ e_p^*(t_k) = M(t_k) \ e_p \quad (2-10) \]

where \( M(t) \) is the earth rotation matrix of transformation from the inertial to the earth-fixed frame. The model with respect to the earth-fixed reference frame becomes

\[ d_{i,j} = b_{i,j} \ M(t_k) \ e_p^* + c[C_{ij} + C_{i1} (t_k - t_o)] \quad (2-11) \]

Further development of the model depends on the particular parametrization chosen for the rotation of the earth.

3. Earth Rotation Parametrization

The most straightforward way of parametrizing earth rotation is that of expressing the inertial to earth-fixed system transformation matrix \( M(t) \) in terms of three rotation angles at every epoch \( t \). The most logical choice seems to be that of the Eulerian angles \( \phi, \ \theta, \ \psi \), defined as

\[ M(t) = R_{3} \ [\phi(t)] \ R_{2} \ [\theta(t)] \ R_{1} \ [\psi(t)] \quad (3-1) \]

where \( R_1, R_2, R_3 \) denote conventional rotation matrices about the \( x, y, z \) axes, respectively. The rotating earth can be viewed as a dynamical system whose state evolves in time according to a second-order differential equation of the form

\[ \frac{d^2 E}{dt^2} = f(E, t) \quad (3-2) \]

where \( E = [\phi, \ \theta, \ \psi]^T \). If such equations of motion could be exactly known and solved, their solution

\[ E(t) = F(E(t_0), \dot{E}(t_0), \ t) \quad (\dot{E} = dE/dt) \quad (3-3) \]
would provide a parametrization of the transformation matrix

\[
M(t) = M(\varphi_0, \theta_0, \psi_o, \dot{\varphi}_0, \dot{\theta}_0, \dot{\psi}_0, t_0, t)
\]  

(3-4)

in terms of the six initial values (integration constants):

\[
\begin{align*}
\varphi_0 &= \varphi(t_0) \\
\theta_0 &= \theta(t_0) \\
\psi_0 &= \psi(t_0) \\
\dot{\varphi}_0 &= \frac{d\varphi}{dt} \mid_{t_0} \\
\dot{\theta}_0 &= \frac{d\theta}{dt} \mid_{t_0} \\
\dot{\psi}_0 &= \frac{d\psi}{dt} \mid_{t_0}
\end{align*}
\]

However, such an approach is not practical because of uncertainties surrounding
the current knowledge of the earth's rotation. The alternative is the representation
of the functions \(\varphi(t), \theta(t), \psi(t)\) in terms of a finite number of parameters to be
estimated from the observations. Such representations as polynomials, trigonometric
series, etc. can sufficiently approximate a wide class of functions, provided that
appropriate number of terms is included. The efficiency of the approximation can
be judged only when the function to be approximated is known over some time interval,
while in our case an unknown function is to be approximated. The unknown approxima-
tion error (representation error, modeling error) may well be significant compared
to the observational errors, thus affecting the validity of the final parameter error
estimates based upon real data analysis.

Among various finite parameter representations, the simplest perhaps is
that of representing a continuous bounded function over an interval by a step function,
which has a constant value over each subinterval, into which the original time
interval has been partitioned.

To formally represent a step function, we introduce the characteristic or
indicator function \(\chi_A\) of a set \(A\), defined as follows:

\[
\chi_A(x) = \begin{cases} 
1 & \text{if } x \in A \\
0 & \text{if } x \notin A 
\end{cases}
\]  

(3-5)

Let \([a, b]\) be the original interval and

\[
a = t_0 < t_1 < t_2 < \ldots < t_{a-1} < t_a = b
\]
If \( \chi_i \) denotes the characteristic function of the set \([t_{i-1}, t_i]\) (\(i = 1, 2, \ldots, n-1\)) and \( \chi_n \) that of \([t_{n-1}, t_n]\), a step function with respect to the above partition of \([a,b]\) can be written in the form

\[
y(t) = \sum_{i=1}^{n} \alpha_i \chi_i(t)
\]

A continuous bounded function \( f(t) \) can be approximated arbitrarily well by a step function \( y(t) \) with an appropriate choice and increase in the number of subintervals. In the case of this investigation, the functions to be approximated are the Eulerian angles \( \phi(t), \theta(t), \psi(t) \), and the effectiveness of step function approximation may be increased if the known parts of \( \phi, \theta, \psi \) are filtered out. Let \( \phi^0(t), \theta^0(t), \psi^0(t) \) be the "reference" Eulerian angles as computable from classical astronomy. Then the earth rotation transformation matrix may be represented as

\[
M(t) = R_3[\phi^0 + \delta \phi] \ R_1[\theta^0 + \delta \theta] \ R_3[\psi^0 + \delta \psi]
\]

where \( \delta \phi(t), \delta \theta(t) \) and \( \delta \psi(t) \) are step functions. An alternative parametrization may be of the form

\[
M(t) = M^0(t) \ \delta M(t) = R_3(\phi^0) \ R_1(\theta^0) \ R_3(\psi^0) \ R_3(\delta \phi) \ R_3(\delta \theta) \ R_3(\delta \psi)
\]

Traditionally, earth rotation representations are separated into three parts with the help of the instantaneous rotation vector \( \omega(t) \) related to the Eulerian angles through Euler's geometric equations

\[
\begin{align*}
\omega_x &= \sin \theta \sin \phi \ \ddot{\psi} + \cos \phi \ \dot{\theta} \\
\omega_y &= \sin \theta \cos \phi \ \ddot{\psi} + \sin \phi \ \dot{\theta} \\
\omega_z &= \cos \theta \ \ddot{\psi} + \dot{\phi}
\end{align*}
\]

where \( \omega_x, \omega_y, \omega_z \) are the rotation vector components with respect to the earth-fixed system. The variation in the direction of the vector \( \omega \) with respect to the inertial system constitutes precession and nutation, while polar motion is the corresponding variation with respect to the earth-fixed system. The third part of earth rotation is the variation in the length of the vector \( \omega \), i.e., the variation of the rotational velocity (length of the day) \( \Omega = \| \omega \| \).
To obtain a parametrization of earth rotation where precession-nutation, polar motion and rotational velocity are separated, we introduce an intermediate moving geocentric Cartesian reference frame $x', y', z'$. The $z'$-axis direction coincides at any epoch with the direction of the instantaneous rotation axis, while the $x'$-axis direction is arbitrary. The transformation from the $X, Y, Z$ (inertial) to the $x', y', z'$ frame may be expressed in terms of three rotations:

$$R_3 (\Theta_2) \ R_2 (\Xi) \ R_1 (H)$$

and the transformation from $x', y', z'$ to the $x, y, z$ (earth-fixed) frame can be similarly expressed as

$$R_1 (\eta) \ R_2 (\xi) \ R_3 (\Theta)$$

Setting $\Theta = \Theta_1 + \Theta_2$, the total transformation from the inertial to the earth-fixed frame becomes

$$M = R_1 (\eta) \ R_2 (\xi) \ R_3 (\Theta) R_3 (\Xi) R_1 (H)$$

where $\eta, \xi, \Theta, \Xi, H$ are all functions of time. (Note that the $\Theta_1$ and $\Theta_2$ components of $\Theta$ cannot be separated, and this justifies the arbitrariness of the $x'$-axis direction.)

In order to introduce the instantaneous rotational velocity $\Omega$ in the model, noting that the $R_3 (\Theta)$ rotation is about the instantaneous rotation axis (diurnal rotation), one may set

$$\Theta (t) = \Theta_0 + \int_{t_0}^{t} \Omega (\tau) \ d\tau \quad \Theta_0 = \Theta (t_0)$$

The three angles $\Xi, H, \Theta_0$ represent the traditional precession-nutation, and the angles $\eta$ and $\xi$ the polar motion. The angle $(\Theta - \Theta_0)$ is similar to GAST (Greenwich Apparent Sidereal Time). The functions $\eta, \xi, \Omega, \Xi, H$ can now be approximated by step functions. To improve the approximation, we shall include in the transformation $M$ the traditional precession-nutation based on current knowledge, as follows:

$$M = R_1 (\eta)\ R_2 (\xi)\ R_3 (\Theta)\ R_3 (\Xi)\ R_1 (H)\ R^C$$

where the known part of precession and nutation may be computed from [Mueller, 1969, Ch. 4].
\[
R^0 = R_2(\epsilon - \Delta \epsilon) R_3(\Delta \psi) R_1(\epsilon) R_2(-z) R_3(\theta) R_3(-\zeta_0)
\]  

(3-13)

In the above expression, \(z, \theta, \zeta_0\) are the Newcomb components of precession, and \(\Delta \psi\) and \(\Delta \epsilon\) the nutations in longitude and obliquity by Woolard. The transformation \(R^0\) defines the traditional "true" frame of reference, an intermediate, noninertial, moving reference frame \(X', Y', Z'\) of known orientation, such that the \(Z'\)-axis is at any epoch near the instantaneous rotation axis. The angles \(\Xi, H, \Theta_0\) are thus small corrections to the reference precession-nutation parameters used in (3-13), and as such they may be considered as the transformation parameters between the "traditional (Newcomb/Woolard) true" frame and the "actual true" frame.

There are two alternatives in the step function representation of the angle \(\Theta\). One may represent \(\Omega\) as a step function with constant value \(\Omega_1\) over the \(i\)th time interval (step), in which case

\[
\Theta(t) = \Theta_0 + \sum_{k=1}^{i-1} \Omega_k (t_k - t_{k-1}) + \Omega_1 (t - t_{i-1}) \quad t_{i-1} < t < t_i
\]  

(3-14)

A second choice is to approximate \(\Theta\) over each step by a function of the form

\[
\Theta_i(t) = \Theta_{oi} + \Omega_i (t - t_{i-1}) \quad t_{i-1} \leq t < t_i
\]  

(3-15)

This is a weaker representation from the adjustment point of view because of the larger number of parameters \(\Theta_{oi}, i = 1, 2, \ldots, N\ vs. \Theta_0\), but it leads to estimates of variances of the estimates of the step function constants \(\eta, \xi, \Theta_{oi}, \Omega_i, \Xi, H\), which are uniform (equal) over all steps \((t_{i-1}, t_i)\). On the contrary, the first representation leads to variances which are smaller for steps in the middle and larger for steps at the ends of the original time span of the available observations.

Over each step the model for the VLBI observations becomes

\[
d_{i,j;k} = b_{i,j}^* R_1(-\eta) R_2(-\xi) R_3 [\Theta_0 + \Omega (t_k - t_0)] R_2(\Xi) R_1(H) R^0 e_{ij} + \\
+ c [C_{oi1} + C_{ij}(t_k - t_0)]
\]  

(3-16)

where \(\eta, \xi, \Theta_0, \Omega, \Xi, H\) are constants over the considered step (the subscripts identifying the steps have been dropped for simplicity), \(t_k\) is the epoch of observation, and \(t_0\) is the beginning of the step containing \(t_k\).
4. Linearization of Observation Equations

For the time span of available observations considered in this study (maximum of one month), the effect of precession-nutation on the experimental design (relative radio sources - stations configuration) is negligible. One can therefore drop the known transformation matrix \( R^0 \) from the model without any significant change on the simulation results. This simplifies somewhat the equations following, but it must be understood that appropriate modifications must be made when analyzing real data by including the effect of \( R^0 \).

The angles \( \xi, \eta, \Xi, H \) are very small since both the \( z \) and \( Z \) axes (actually the noninertial \( Z' \)-axis on account of ignoring \( R^0 \)) are or can be chosen to be near the rotation axis. The usual approximations, \( \cos \alpha = 1, \sin \alpha = \alpha, \alpha^2 = 0 \), for a small angle \( \alpha \) may be used, leading to

\[
R_1(-\eta) \quad R_2(-\xi) = \begin{pmatrix}
1 & 0 & \xi \\
0 & 1 & -\eta \\
-\xi & \eta & 1
\end{pmatrix}
\]

and

\[
R_2(\Xi) \quad R_1(H) = \begin{pmatrix}
1 & 0 & -\Xi \\
0 & 1 & H \\
\Xi & -H & 1
\end{pmatrix}
\]

The model now becomes

\[
d_{1j,k} = (r_j - r_i)^T S^T(\xi, \eta) \quad R_3(\Theta_0 + \Omega \tau_k) \quad S(\Xi, H) \quad e_p + \\
+ c [C_{0} + C_{1} \tau_k]
\]

where \( \tau_k = t_k - t_0 \); \( e_p = [\cos \delta_p \cos \alpha_p, \cos \delta_p \sin \alpha_p, \sin \delta_p]^T \), \( \alpha_p \) and \( \delta_p \) are the right-ascension and declination of the \( p \)th radio source with respect to the \( X, Y, Z \) inertial frame; \( r_1 = [x_1, y_1, z_1]^T \), \( x_1, y_1, z_1 \) are the coordinates of the \( i \)th station with respect to the \( x, y, z \) earth-fixed frame; and

\[
S(a, b) = \begin{pmatrix}
1 & 0 & -a \\
0 & 1 & b \\
a & -b & 1
\end{pmatrix}
\]
The available observations $d_{ij,k}$ form a vector of observations $L_b$, corresponding to a vector of observables $L_a = F(X_a)$, where $X_a$ is a vector of unknown parameters. The unknowns are the station coordinates $x_i, y_i, z_i, i = 1, 2, \ldots, N_s$, the earth rotation parameters $\xi, \eta, \Theta_0, \Omega, \Xi, H$ (one set of six per each step), radio source coordinates $\alpha_p, \delta_p, p = 1, 2, \ldots, N_{rs}$, and clock synchronization parameters $C_{0ij}, C_{1ij}, i,j = 1, 2, \ldots, N_s$.

The $d_{ij,k}$ components of the vector $L_a$ are linear with respect to most of the parameters with the exception of $\Theta_0$, $\Omega$, $\alpha_p$, and $\delta_p$. A completely linearized model can be obtained by means of approximate values $X^0$ of the parameters $X_a$. Expansion in Taylor series and neglect of second- and higher-order terms leads to

$$L_a = F(X^0) + \frac{\partial F(X_a)}{\partial X_a} X^0 (X_a - X^0) \quad (4-3)$$

The actual observations $L_b$ differ from the observables $L_a$ because of the presence of unknown observations errors $V$, according to $L_a = L_b + V$. Setting $A = (\partial F/\partial X_a)|X^0$, $L^0 = F(X^0)$, $X = X_a - X^0$, and $L = L^0 - L_b$, the linearized observation equations are obtained:

$$V = AX + L \quad (4-4)$$

$A$ and $L$ are known, while $X$ and $V$ are unknown. The vector $V$ is assumed to be an outcome of a random vector with $E[V] = 0$ and known covariance matrix $E[V V^T] = S$.

Application of standard least squares techniques leads to a solution $X$, minimizing the quadratic form $V^T P V$ where $P = S^2$, which satisfies the "normal equations,"

$$(A^T PA) X = -A^T P L \quad \text{or} \quad N X = U \quad (4-5)$$

To compute the elements of the design matrix $A$, the partial derivatives of $d_{ij,k}$ with respect to the parameters are required. Introducing the notation, $\psi_k = \Theta_0 + \Omega \tau_k$, $X_k = \psi_k - \alpha_p$, $\dot{x}_{ij} = x_j - x_i$, $y_{ij} = y_j - y_i$, $z_{ij} = z_j - z_i$, and dropping subscripts where confusion is not likely to arise, the partial derivatives are as follows:
\[
\frac{\partial d}{\partial x_i} = -\frac{\partial d}{\partial x_i} = \cos \delta \cos \chi + \sin \delta (-\Xi \cos \psi + H \sin \psi + \xi)
\]

\[
\frac{\partial d}{\partial y_i} = -\frac{\partial d}{\partial y_i} = -\cos \delta \sin \chi + \sin \delta (\Xi \sin \psi + H \cos \psi - \eta)
\]

\[
\frac{\partial d}{\partial z_i} = -\frac{\partial d}{\partial z_i} = \sin \delta - \cos \delta (\xi \cos \chi + \eta \sin \chi - \Xi \cos \alpha + H \sin \alpha)
\]

\[
\frac{\partial d}{\partial \xi} = x_{11} [\sin \delta + \cos \delta (\Xi \cos \alpha - H \sin \alpha - \xi \cos \chi)] +
+ y_{11} [\eta \cos \delta \cos \chi] +
+ z_{11} [-\cos \delta \cos \chi - \sin \delta (-\Xi \cos \psi + H \sin \psi + \xi)]
\]

\[
\frac{\partial d}{\partial \eta} = y_{11} [-\sin \delta + \cos \delta (\xi \cos \chi + \eta \sin \chi - \Xi \cos \alpha + H \sin \alpha)] +
+ z_{11} [-\cos \delta \sin \chi + \sin \delta (\Xi \sin \psi + H \cos \psi - \eta)]
\]

\[
\frac{\partial d}{\partial \Theta_0} = x_{11} [-\cos \delta \sin \chi + \sin \delta (\Xi \sin \psi + H \cos \psi)] +
+ y_{11} [-\cos \delta \cos \chi + \sin \delta (\Xi \cos \psi - H \sin \psi)] +
+ z_{11} [\cos \delta (\xi \sin \chi - \eta \cos \chi)]
\]

\[
\frac{\partial d}{\partial \Omega} = \tau_k \frac{\partial d}{\partial \Theta_0}, \quad \frac{\partial d}{\partial C_{ij}} = c, \quad \frac{\partial d}{\partial C_{ij}} = c \tau_k
\]

\[
\frac{\partial d}{\partial \Xi} = x_{11} \left\{ -\cos \psi \cos \delta + \cos \delta [\xi \cos \alpha + \cos \psi (-\Xi \cos \alpha + H \sin \alpha)] \right\} +
\]
\[
+ y_{11} \left\{ \sin \psi \sin \delta + \cos \delta [-\eta \cos \alpha + \sin \psi (\Xi \cos \alpha - H \sin \alpha)] \right\} +
\]
\[
+ z_{11} \left[ \cos \delta \cos \alpha + \sin \delta (\xi \cos \psi + \eta \sin \psi - \Xi) \right]
\]

\[
\frac{\partial d}{\partial \Psi} = x_{11} [\sin \delta \sin \psi + \cos \delta \sin \alpha (\Xi \cos \psi - H \sin \psi - \xi)] +
\]
\[
+ y_{11} [\sin \delta \cos \psi + \cos \delta \sin \alpha (-\Xi \sin \psi - H \cos \psi + \eta)] +
\]
\[
+ z_{11} [-\cos \delta \sin \alpha + \sin \delta (-\xi \sin \psi + \eta \cos \psi - H)]
\]

\[
\frac{\partial d}{\partial \alpha} = x_{11} [\cos \delta \sin \chi] + y_{11} [\cos \delta \cos \chi] +
\]
\[
+ z_{11} \cos \delta (-\xi \sin \chi + \eta \cos \chi - \Xi \sin \alpha - H \cos \alpha)
\]
\[
\frac{\partial d}{\partial \delta} = x_{ij} \left[ -\sin \delta \cos \chi + \cos \delta \left( -\Xi \cos \psi + H \sin \psi + \xi \right) \right] + \\
+ y_{ij} \left[ \sin \delta \sin \chi + \cos \delta \left( \Xi \sin \psi + H \cos \psi - \eta \right) \right] + \\
+ z_{ij} \left[ \cos \delta + \sin \delta \left( \xi \cos \chi + \eta \sin \chi - \Xi \cos \alpha + H \sin \alpha \right) \right]
\]

(4-6)

A second type of possible observations is the time derivative of \(d_{ijjk}\)

\[
f_{ijkp} = \frac{\partial d_{ijjk}}{\partial t} = \Omega (r_1 - r_4)^T S^T(\xi, \eta) \quad P_3 \quad R_3(\Theta_0 + \Omega \tau_k) \quad S(\Xi, H) \quad e_p + c C_{ijj}
\]

where

\[
P_3 = \begin{pmatrix}
0 & 1 & 0 \\
-1 & 0 & 0 \\
0 & 0 & 0
\end{pmatrix}
\]

(4-7)

The related partial derivatives of \(f_{ijkp}\) with respect to the parameters are as follows:

\[
\frac{1}{\Omega} \frac{\partial f}{\partial \xi} = x_{ij} \left[ \xi \cos \delta \sin \chi \right] - y_{ij} \left[ \eta \cos \delta \sin \chi \right] \\
+ z_{ij} \left[ \cos \delta \sin \chi - \sin \delta \left( \Xi \sin \psi + H \cos \psi \right) \right]
\]

\[
\frac{1}{\Omega} \frac{\partial f}{\partial \eta} = y_{ij} \left[ \cos \delta \left( -\xi \sin \chi + \eta \cos \chi \right) \right] \\
+ z_{ij} \left[ -\cos \delta \cos \chi + \sin \delta \left( \Xi \cos \psi - H \sin \psi \right) \right]
\]

\[
\frac{1}{\Omega} \frac{\partial f}{\partial \Xi} = x_{ij} \left[ \sin \delta \sin \psi - \cos \delta \sin \psi \left( -\Xi \cos \alpha + H \sin \alpha \right) \right] \\
+ y_{ij} \left[ \sin \delta \cos \psi - \cos \delta \cos \psi \left( -\Xi \cos \alpha + H \sin \alpha \right) \right] \\
+ z_{ij} \left[ \sin \delta \left( -\xi \sin \psi + \eta \cos \psi \right) \right]
\]

\[
\frac{1}{\Omega} \frac{\partial f}{\partial H} = x_{ij} \left[ \cos \psi \sin \delta - \cos \delta \sin \alpha \left( \Xi \sin \psi + H \cos \psi \right) \right] \\
+ y_{ij} \left[ -\sin \psi \sin \delta + \cos \delta \sin \alpha \left( -\Xi \cos \psi + H \sin \psi \right) \right] \\
+ z_{ij} \left[ \sin \delta \left( -\xi \cos \psi - \eta \sin \psi \right) \right]
\]
\[
\frac{1}{\Omega} \frac{\partial f}{\partial \Theta_\omega} = x_{1j} \left[ \cos \delta \cos \chi + \sin \delta \left( \Xi \cos \psi - H \sin \psi \right) \right] \\
+ y_{1j} \left[ \cos \delta \sin \chi - \sin \delta \left( -\Xi \sin \psi - H \cos \psi \right) \right] \\
+ z_{1j} \left[ \cos \delta \left( \xi \cos \chi + \eta \sin \chi \right) \right]
\]

\[
\frac{1}{\Omega} \frac{\partial f}{\partial x_j} = - \frac{1}{\Omega} \frac{\partial f}{\partial x_j} = - \cos \delta \sin \chi + \sin \delta \left( \Xi \sin \psi + H \cos \psi \right)
\]

\[
\frac{1}{\Omega} \frac{\partial f}{\partial y_j} = - \frac{1}{\Omega} \frac{\partial f}{\partial y_j} = - \cos \delta \cos \chi + \sin \delta \left( \Xi \cos \psi - H \sin \psi \right)
\]

\[
\frac{1}{\Omega} \frac{\partial f}{\partial z_j} = - \frac{1}{\Omega} \frac{\partial f}{\partial z_j} = \cos \delta \left( \xi \sin \chi - \eta \cos \chi \right)
\]

\[
\frac{1}{\Omega} \frac{\partial f}{\partial \alpha} = x_{1j} \left[ \cos \delta \cos \chi \right] + y_{1j} \left[ - \cos \delta \sin \chi \right] \\
+ z_{1j} \left[ \cos \delta \left( -\xi \cos \chi - \eta \sin \chi \right) \right]
\]

\[
\frac{1}{\Omega} \frac{\partial f}{\partial \delta} = x_{1j} \left[ \sin \delta \sin \chi + \cos \delta \left( \Xi \sin \psi + H \cos \psi \right) \right] \\
+ y_{1j} \left[ \sin \delta \cos \chi + \cos \delta \left( \Xi \cos \psi - H \sin \psi \right) \right] \\
+ z_{1j} \left[ \sin \delta \left( -\xi \sin \chi + \eta \cos \chi \right) \right]
\]

\[
\frac{\partial f}{\partial \Omega} = \tau_k \frac{\partial f}{\partial \Theta_\omega}, \quad \frac{\partial f}{\partial C_{ij}} = 0, \quad \frac{\partial f}{\partial \Theta_{ij}} = c \quad (4-9)
\]
5. Coordinate System Definition and Inner Constraints

The combination of all observations leads to the linearized observation equations

\[ nV_1 = nA_u X_1 + nL_1 \]  \hspace{1cm} (5-1)

Minimization of \( V^TPV \) leads to a solution satisfying the normal equations [Uotila, 1967]

\[ NX = U \]  \hspace{1cm} (5-2)

where

\[ N = A^TPA \] and \[ U = -A^TPL. \]

Because of the lack of coordinate system definition, \( N \) is singular with rank \( N = r < u \), where \( u \) is the number of parameters and \( s = u - r \) is the rank deficiency of \( N \).

Originally \( s = 9 \), corresponding to the six degrees of freedom in the definition of the earth-fixed coordinate system (three for origin position and three for orientation) plus the three degrees of freedom in the inertial system definition (orientation only).

Since a radio source catalogue of a certain accuracy is assumed to be available, radio source coordinates are treated as observations (weighted unknowns) rather than as unknown parameters. The inertial frame is therefore defined through the catalogued radio source coordinates and the remaining rank deficiency of \( N \) becomes \( s = 6 \). A unique solution to the normal equations may be obtained if in addition a set of minimal linear constraints are imposed on the parameters, of the form

\[ CTX = 0 \] \hspace{1cm} (5-3)

where \( C \) is a \( u \times s \) matrix and rank \( (C) = s \).

Among the various possible solutions to the normal equations, the unique one given by \( X = N^+ U \), where \( N^+ \) is the pseudoinverse of \( N \), has the following properties [Blaha, 1971]: \( X^TX = \min \) and \( \text{trace } N^+ = \min \). In view of the interpretation of \( N^+ \) as the variance-covariance matrix of the parameters, the second property makes the solution optimal in the sense that smaller variances of the parameters provide a better representation of parameter related estimable quantities in terms of the
nonestimable parameters. To avoid the use of pseudoinverse computation algorithm, one may resort to a particular set of minimal constraints called inner constraints [Blaha, 1971],

\[ E' X = 0 \]  

which leads to the same solution for \( X \) as the one obtained with the use of the pseudoinverse \( N^+ \). It can be shown that such a set of inner constraints can be obtained with the help of a \( u \times s \) matrix \( E \) (rank \( E \) = \( s \)) satisfying

\[ NE = 0 \]  

An algebraic type of proof can be found in [Pope, 1971] which settles the truth of the matter but throws little light on the interrelation between inner constraints and pseudoinverse. A different type of proof is given in Appendix A of this work, based on the geometry of the operator represented by the matrix \( N \).

In view of the fact that \( AE = 0 \Rightarrow NE = A^T P A E = 0 \), a set of inner constraints has been analytically constructed using six independent solutions to the set of equations

\[ A^j y = 0 \quad j = 1, 2, \ldots, u \]  

where \( A^j \) denotes the \( j^{th} \) row of \( A \). We assume that the approximate values of \( \eta, \xi, \Xi, \gamma \) are zero in the computation of partials of observations with respect to the unknowns, thus obtaining simpler analytical expressions for the elements of \( A \).

Letting the order of the unknowns in \( X \) to be the following

\[ X' = [d\eta, d\xi, d\Theta_0, d\Omega, dx_1, dy_1, d\xi_1, \ldots, dx_N, dy_N, dz_N] \]

one may set

\[ y' = [w_1, w_2, w_3, w_4, \alpha_1, \beta_1, \gamma_1, \ldots, \alpha_N, \beta_N, \gamma_N] \]

For a row of \( A \) corresponding to a \( d_{1k} \) observation, one has

\[ w_1 \frac{\partial d}{\partial \eta} + w_2 \frac{\partial d}{\partial \xi} + w_3 \frac{\partial d}{\partial \Theta_0} + w_4 \frac{\partial d}{\partial \Omega} + \alpha_1 \frac{\partial d}{\partial x_1} + \beta_1 \frac{\partial d}{\partial y_1} + \gamma_1 \frac{\partial d}{\partial z_1} + \]

\[ + \alpha_j \frac{\partial d}{\partial x_j} + \beta_j \frac{\partial d}{\partial y_j} + \gamma_j \frac{\partial d}{\partial z_j} = 0 \]  

(5-7)
With the analytical expressions of the partials and after a considerable algebraic effort, one arrives at six independent solutions $y_1, i = 1, 2, \ldots, 6$, and the inner constraint matrix $E$ is formulated by setting

$$E = [y_1 \ y_2 \ \ldots \ y_6]$$

The final result is

$$
\begin{bmatrix}
\frac{\partial \eta}{\partial x_1} & \frac{\partial \eta}{\partial y_1} & \frac{\partial \eta}{\partial z_1} & \ldots & \frac{\partial \eta}{\partial x_N} & \frac{\partial \eta}{\partial y_N} & \frac{\partial \eta}{\partial z_N} \\
0 & 0 & 0 & 1 & 0 & 0 & \ldots & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & \ldots & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & \ldots & 0 & 0 & 1 \\
1 & 0 & 0 & 0 & 0 & -z_1^0 & y_1^0 & \ldots & 0 & -z_N^0 & y_N^0 \\
0 & 1 & 0 & 0 & z_1^0 & 0 & -x_1^0 & \ldots & z_N^0 & 0 & -x_N^0 \\
0 & 0 & -1 & 0 & -y_1^0 & x_1^0 & 0 & \ldots & y_N^0 & x_N^0 & 0 \\
\end{bmatrix}
$$

where $x_i^0, y_i^0, z_i^0$ are the approximate values of station coordinates used in the computation of the design matrix $A$. More explicitly there are two sets of constraints. The first one

$$\sum_i \begin{pmatrix} dx_1 \\ dy_1 \\ dz_1 \end{pmatrix} = 0 \quad (5-9)$$

"defines" the origin of the earth-fixed system, while the second one

$$\begin{pmatrix} \frac{-d\eta}{d\xi} \\ \frac{-d\xi}{d\Theta_0} \end{pmatrix} = \sum_i \begin{pmatrix} 0 & -z_i^0 & y_i^0 \\ z_i^0 & 0 & -x_i^0 \\ -y_i^0 & x_i^0 & 0 \end{pmatrix} \begin{pmatrix} dx_1 \\ dy_1 \\ dz_1 \end{pmatrix} \quad (5-10)$$

"defines" its orientation.

In the discussion above, the problem of epoch and time scale definition for the set of station clocks has been ignored. The parameters $C_{ij}$ and $C_{ij}$ refer only to relative offsets and drifts between the clocks at stations $i$ and $j$. 

\[ \text{18} \]
Usually epoch and time scale are provided by the readings of one of the
clocks (master clock), and they are transformed to other station clocks through the
parameters $C_{\text{opt}}$, $C_m$ where the master clock is at the $m$th station.

Another way to provide epoch and time scale is the use of inner constraints
defining a fictitious master clock. We may shift to a new set of parameters by
setting

$$
C_{\text{opt}} = C_{\text{opt}} - C_{\text{opt}}, \quad C_J = C_{\text{opt}} - C_{\text{opt}}
$$

(5-11)

The new parameters are offsets $C_{\text{opt}}$ and drifts $C_{\text{opt}}$ with respect to the fictitious
master clock, and the additional inner constraints are

$$
\sum_i dC_{\text{opt}} = 0 \quad \text{and} \quad \sum_i dC_{\text{opt}} = 0
$$

(5-12)

6. Remarks Related to Design of Experiment Concepts

Since the usual objective of a geodetic experiment is the estimation of
certain parameters, an experimental design may be considered optimal when the
errors in parameter estimates are minimized in some certain sense. Within the
linear least squares model, the a posteriori variance-covariance matrix of parameter
estimates is given in general by

$$
S_i = \sigma_0^2 (A^T P A)^+ = \sigma_0^2 Q^+
$$

(6-1)

where $Q^+$ denotes the pseudoinverse of a square matrix $Q$, $P$ is the weight matrix
and $\sigma_0^2$ the a posteriori estimate of the variance of unit weight. The "errors" in
parameter estimates therefore depend on the observational accuracy and on the
design matrix $A$. The design matrix $A = A(X^0)$ itself depends on the approximate
values of parameters $X^0$. The determination of optimal $X^0$ values minimizing some
risk function (e.g., trace $S_i$) is usually referred to as the "configuration problem,"
or the "first-order optimal design" [Grafarend, 1974]. On the assumption that the
risk function chosen is insensitive to small changes of $X^0$, the approximate values
of parameters may be identified with the true ones. In a similar way the small
angles \( \eta, \xi, \Xi, H \) in our model may be considered to be zero. Of the remaining parameters \( \Theta_0 \) depends on the choice of reference frames, \( \Omega \) is approximated by the fixed average rotational velocity of the earth, while in the case of transcontinental baselines the choice of radio source coordinates \( \alpha_p, \delta_p \) is strongly restricted by observability conditions. Therefore, the first-order optimal design is practically limited to the determination of the optimal configuration of the network of observing stations.

The order of magnitude of elements in a row of the design matrix \( A \) determines the sensitivity of the corresponding observation with respect to each of the parameters. Next a short geometric interpretation of these partials is given to provide possible "hints" towards the determination of the optimal design.

The partial derivatives with respect to \( \eta, \xi, \Theta_0, \Omega, \Xi, H \) (earth rotation parameters) and \( \alpha_p, \delta_p \) are proportional to the baseline length. Indeed, they all contain linear terms in \( x_{ij} = r_{ij} \hat{x}_{ij}, y_{ij} = r_{ij} \hat{y}_{ij}, \) and \( z_{ij} = r_{ij} \hat{z}_{ij}, \) where \( r_{ij} \) is the length of the \( ij \) baseline and \( x_{ij}, y_{ij}, z_{ij} \) are the components of the unit vector \( \hat{r}_{ij} \) in the direction of the baseline. It is therefore possible to define "sensitivity per unit of baseline length" by means of \( \frac{1}{r_{ij}} \frac{\partial d}{\partial \beta} \) where \( \beta \) stands for any of the earth rotation or radio source coordinates parameters. It can be shown that

\[
\frac{1}{r_{ij}} \frac{\partial d}{\partial \beta} = (r_{ij})^T S_\beta \tag{6-2}
\]

i.e., that the sensitivity per unit of baseline length is the projection of some vector \( S_\beta \) (called the parameter \( \beta \) sensitivity vector) on the direction of the baseline. It can be easily verified that the parameter sensitivity vectors are given by:

\[
S_\eta = -R_1 (\pi/2) P_{yz} R_3 (X) e_6 \\
S_\xi = -R_2 (\pi/2) P_{xz} R_3 (X) e_5 \\
S_{\Theta_0} = \tau_x^3 S_{\Omega} = P_{xy} R_3 (X + \pi/2) e_6 \\
S_{\Xi} = R_3 (\psi) R_2 (\pi/2) P_{xz} R_3 (-\alpha) e_6 \\
S_{H} = R_3 (\psi) R_1 (\pi/2) P_{yz} R_3 (-\alpha) e_5 \\
S_\alpha = -R_3 (X + \pi/2) P_{xy} e_6 \\
S_\delta = R_3 (X) R_2 (\pi/2) e_6 \tag{6-3}
\]
where
\[ e_6 = [\cos \delta, 0, \sin \delta]^T, \quad e_p = R_3(-\alpha) e_6, \] and
\[ P_{xy} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad P_{yz} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad P_{xz} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \]
are projection matrices on the xy, yz and xz planes, respectively. The position of the sensitivity vectors varies with time, and their loci, all contained within the unit sphere, are depicted in Fig. 6.1. Maximum sensitivity occurs when the baseline is parallel to the corresponding parameter sensitivity vector, while an observation is insensitive to a parameter when the baseline and parameter sensitivity vector are perpendicular. With this in mind and with the help of Fig. 6.1, Table 6.1 is constructed, summarizing the sensitivity to various parameters for extreme cases of baseline and radio source directions.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Baseline Parallel to Equator ($z_{11} = 0$)</th>
<th>Baseline Parallel to Rotation Axis ($x_{11} = y_{11} = 0$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\xi, \eta$</td>
<td>no</td>
<td>yes</td>
</tr>
<tr>
<td>$\Xi, \varnothing$</td>
<td>no</td>
<td>yes</td>
</tr>
<tr>
<td>$\Theta, \Omega, \alpha$</td>
<td>yes</td>
<td>no</td>
</tr>
<tr>
<td>$\delta$</td>
<td>no</td>
<td>yes</td>
</tr>
</tbody>
</table>

Table 6.1
Sensitivity of Baseline - Radio Source Extreme Configurations with Respect to Various Parameters
Note that observations in the fourth column are impossible due to observability conditions (a baseline parallel to the rotation axis must be located near the equator and observe equatorial radio sources only).

The sensitivity with respect to $\Omega$ as well as to $C_{ij}$ is proportional to the time span $\tau_k$ of the available observations while that to $C_{ij}$ and $C_{oc}$ is independent of the baseline - radio source configuration.

Sensitivity with respect to station coordinates is independent of baseline length. If $e_x, e_y, e_z$ denote unit vectors in the directions of the $x, y, z$ axes, respectively, the relevant partial derivatives under the same approximations are

$$\frac{\partial d}{\partial x_j} = e^T_x S_c, \quad \frac{\partial d}{\partial y_j} = e^T_y S_c, \quad \frac{\partial d}{\partial z_j} = e^T_z S_c$$

where $S_c = R_{\chi} (-\chi) e_6$. The sensitivity of observations with respect to station coordinates is determined by the projection of the coordinate sensitivity vector $S_c$ on the coordinate axes. The locus of $S_c$ is depicted in Fig. 6.1. It can readily be seen that observations to equatorial radio sources are insensitive to the $z$ station coordinates, while observations to polar radio sources are insensitive to $x$ and $y$ station coordinates.

All the above remarks are of a rather general nature and simply provide "hints" towards the design of an optimal station configuration for the VLBI experiment, which also depends on which of the parameters in particular is to be determined.
Fig. 6.1 Geometric loci of parameter sensitivity vectors.
Fig. 6.1 (cont'd)
7. Simulation and Adjustment Philosophy

The objective of this simulation is the determination of standard deviations of parameter estimates relevant to VLBI observations for a number of characteristic network designs. In addition to this design (relative configuration of station - radio sources and arrangement of observations in time), the a posteriori standard deviations of parameters also depend on a number of other factors. These include the accuracy of observations, their total time span, and the modeling of earth rotation and relative station motions in the adjustment. Since step function models are used here, the time interval over which earth rotation parameters are considered to be constant ("earth step") and the time interval over which station coordinates are considered constants ("station step") become also variables. It has also been assumed that a radio source coordinate catalogue is available, and its accuracy is one additional variable affecting final results. Summarizing, the problem is to examine the variation of standard deviations of the following parameters:

i. station coordinates

ii. earth rotation parameters

with respect to the following independent variables:

a. observational noise

b. time span of observations

c. earth step

d. station step

e. accuracy of radio source catalogue

f. station - radio source configuration, arrangement of observations in time.

With respect to the last item above, three different variables appear in the same group. This is justified in view of the fact that once a transcontinental network design has been decided upon, observability conditions impose strong limitations on both the choice of radio sources to be observed and the arrangement of observations in time.
The fact that observations are not instantaneous, but averages over some time interval (integration time of 7-10 minutes), imposes also limitations on the maximum density of observations in time.

The choice of earth and station step in real data analysis depends on the frequency content of the corresponding time functions. In view of the aliasing effect [Koopmans, 1974], the smaller the step size, the higher the spectral resolution of the corresponding functions. On the other hand, larger step size leads to a smaller number of parameters to be estimated and a "stronger" adjustment with more degrees of freedom. The choice of optimal step size in the analysis of real data constitutes a very interesting problem worthy of a separate study.

In the present simulation study, step size has been considered as a variable, while the corresponding functions generated in the simulation of observations are continuous rather than step functions. However, precautions have been taken so that the total variation $V_{f,I}$ of the function $f(t)$ in question over each step time interval $I$ is insignificant compared to observational noise. In fact,

$$V_{f,I} = \max_{t \in I} [f(t)] - \min_{t \in I} [f(t)] \leq \sigma_b / N \quad i = 1, 2, \ldots, n$$

where $\sigma_b$ is the standard deviation of the observations, and $N$ is a large-integer (specifically here $N = 10$).

In view of a 7-10-minute integration time, and allowing for antenna motion, observations (time delays and their time derivatives) are uniformly spaced at 15-minute intervals over the total time span considered. Their corresponding standard deviations $\sigma_b$ and $\sigma_b'$ are taken to be of a constant ratio $\sigma_b / \sigma_b' = 1/2$, where $\sigma_b$ is in nsec and $\sigma_b'$ in nsec/hr. This value is typical, but the ratio may be somewhat different depending on antenna and observed signal characteristics. The choice of a fixed ratio reduces the number of considered parameters by one with a corresponding significant reduction in computational effort.

Once a certain observational pattern has been decided upon, a simulation program (VLBI SIMULATOR) produces perfect observations using the simplified model (Eq. 4-1). This program requires as input station and radio source true coordinates and the pattern of the observations (baseline - radio source - epoch of
observation). The station coordinates are taken to be constants, while the values of earth rotation parameters at the epoch of observation are provided by a subroutine (SUBROUTINE EARTH). \( \Theta_0 \) is fixed to a realistic value for the beginning of the observations \( t_0 \). The effect of traditional precession-nutation is absent from the simulation as well as from the adjustment model for computational simplicity. The inertial frame thus is taken to be the true equatorial system of the epoch \( t_0 \) so that its Z-axis is near the rotational axis. For the same reason, the earth-fixed frame is taken to be the CIO/BIH zero-meridian terrestrial system.

The perfect observations are processed through a program which adds "Observational errors" drawn from a Gaussian population with zero mean and desired variance.

For the adjustment of the observations, two separate programs are used. The first one (PROG01) processes observations of total time span equal to both earth and station step (one day). An option is available for processing any desired number of different radio source catalogue accuracies in the same run.

The second program (PROG02) processes observations of time span equal to the station step only, which in turn is taken to be an integer multiple of the earth step. It is, however, possible to process observations using unequal earth steps within the total span of available observations. Within each earth step the linearized observations equations are of the form

\[
V_i = \dot{A}_i \dot{X} + \ddot{A}_i \ddot{X}_i + L_i
\]

where \( \dot{X} \) refers to parameters common to all earth steps (station and radio source coordinates) and \( \ddot{X}_i \) refers to parameters particular to each earth step (earth rotation parameters).

The total set of such individual observation equations is combined in a unified solution through a "second-order partitioned linear regression" scheme, extended to incorporate inner constraints. The detailed algorithm is presented in Appendix B. The related programs and subroutines are in Appendix C.
8. Arrangement of Experiments

To obtain more realistic results, station locations considered here are not completely arbitrary, but limited to present and prospective locations of VLBI stations. Such candidate station locations are presented in Table 8.1. In the same table the stations actually used in this study are indicated by asterisks. Two criteria have been considered in the selection. First, stations close to plate boundaries, i.e., in geophysically active areas, have been excluded to avoid possible comparatively large station drifts deviating from those considered in our simulations. Secondly, from groups of stations close to each other, only one station has been included on the assumption that final results would be similar for a nearby station. It is recognized that these criteria probably exclude the most active stations currently in operation, but this should not influence the general conclusions sought.

An experiment consists of a group of stations simultaneously participating in the observations. In Table 8.3 the station groups for the experiments considered in this study are presented. After a certain station group has been selected, the arrangement of observations is one observations every 15 minutes, which interval and choice of the radio sources are limited by observability conditions. The observability condition naturally imposed is that the observed radio source must be above the horizon at both ends of the observing baseline. To avoid large atmospheric refraction errors, the zenith distance of the radio source is limited, in general, to be less than $z_{\text{max}} = 45^\circ$, with some exceptional cases where $z_{\text{max}} = 60^\circ$.

Such observability conditions limit the observable part of the sky to within the intersection of two cones with the local station verticals as axes and $z_{\text{max}}$ as the vertex angle. For transcontinental baselines, the region of observability is small and the choice is between observing a certain quasar or a neighboring one without significant effect on the experimental design.

In setting out a certain sequence of observations, the choice between possible alternatives has been guided by the hints obtained in Chapter 6, with emphasis on sensitivity with respect to earth motion and station coordinate parameters.
Table 8.1
Present and Prospective VLBI Station Locations and
Their Approximate Coordinates

<table>
<thead>
<tr>
<th>No.</th>
<th>Location</th>
<th>Longitude</th>
<th>Latitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Canberra, Australia</td>
<td>149</td>
<td>-35</td>
</tr>
<tr>
<td>2</td>
<td>Kauai, Hawaii</td>
<td>200</td>
<td>22</td>
</tr>
<tr>
<td>3</td>
<td>Fairbanks, Alaska</td>
<td>212</td>
<td>65</td>
</tr>
<tr>
<td></td>
<td>Goldstone, California</td>
<td>243</td>
<td>35</td>
</tr>
<tr>
<td>4</td>
<td>Algonquin, Canada</td>
<td>277</td>
<td>51</td>
</tr>
<tr>
<td></td>
<td>Greenbank, Virginia</td>
<td>282</td>
<td>38</td>
</tr>
<tr>
<td>5</td>
<td>Haystack, Massachusetts</td>
<td>288</td>
<td>41</td>
</tr>
<tr>
<td></td>
<td>Santiago, Chile</td>
<td>289</td>
<td>-34</td>
</tr>
<tr>
<td>6</td>
<td>Sao Paulo, Brazil</td>
<td>313</td>
<td>-24</td>
</tr>
<tr>
<td></td>
<td>Madrid, Spain</td>
<td>356</td>
<td>40</td>
</tr>
<tr>
<td>7</td>
<td>Bonn, Germany</td>
<td>7</td>
<td>51</td>
</tr>
<tr>
<td>8</td>
<td>Onsala, Sweden</td>
<td>17</td>
<td>59</td>
</tr>
<tr>
<td>8</td>
<td>Johannesburg, So. Africa</td>
<td>28</td>
<td>-26</td>
</tr>
<tr>
<td></td>
<td>Crimea, USSR</td>
<td>34</td>
<td>45</td>
</tr>
</tbody>
</table>

* indicates stations considered in this study
Table 8.2

Fictitious Stations Used in Experiments 6 and 9

<table>
<thead>
<tr>
<th>No.</th>
<th>Location</th>
<th>Longitude</th>
<th>Latitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>equatorial</td>
<td>0°</td>
<td>0°</td>
</tr>
<tr>
<td>10</td>
<td>&quot;</td>
<td>60</td>
<td>0</td>
</tr>
<tr>
<td>11</td>
<td>&quot;</td>
<td>120</td>
<td>0</td>
</tr>
<tr>
<td>12</td>
<td>&quot;</td>
<td>180</td>
<td>0</td>
</tr>
<tr>
<td>13</td>
<td>&quot;</td>
<td>240</td>
<td>0</td>
</tr>
<tr>
<td>14</td>
<td>&quot;</td>
<td>300</td>
<td>0</td>
</tr>
<tr>
<td>15</td>
<td>meridian</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>16</td>
<td>&quot;</td>
<td>0</td>
<td>60</td>
</tr>
<tr>
<td>17</td>
<td>&quot;</td>
<td>180</td>
<td>60</td>
</tr>
<tr>
<td>18</td>
<td>&quot;</td>
<td>180</td>
<td>0</td>
</tr>
<tr>
<td>19</td>
<td>&quot;</td>
<td>180</td>
<td>-60</td>
</tr>
<tr>
<td>20</td>
<td>&quot;</td>
<td>0</td>
<td>-60</td>
</tr>
</tbody>
</table>
Table 8.3
Experiments and Participating Baselines
(Station Nos. Refer to Numbering in Table 8.1 or 8.2)

<table>
<thead>
<tr>
<th>Experiment No.</th>
<th>Participating Baselines</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2-3, 2-4, 3-4</td>
</tr>
<tr>
<td>2</td>
<td>3-4, 3-7, 4-7</td>
</tr>
<tr>
<td>3</td>
<td>4-5, 4-6, 5-6</td>
</tr>
<tr>
<td>4</td>
<td>5-6, 5-8, 6-8</td>
</tr>
<tr>
<td>5</td>
<td>1-2, 2-3, 2-4, 3-4, 3-7, 4-5, 4-6, 4-7, 5-6, 5-8, 6-7, 6-8</td>
</tr>
<tr>
<td>6</td>
<td>9-10, 10-11, 11-12, 13-14, 14-9</td>
</tr>
<tr>
<td>7</td>
<td>2-4, 4-5</td>
</tr>
<tr>
<td>8</td>
<td>2-4, 4-6</td>
</tr>
<tr>
<td>9</td>
<td>15-16, 16-17, 17-18, 18-19, 19-20, 20-15</td>
</tr>
<tr>
<td>10</td>
<td>3-4, 4-7</td>
</tr>
</tbody>
</table>
An effort has also been made to observe radio sources of a variety of declinations to avoid the critical configuration appearing when, for example, all radio sources are of the same declination. From a larger set of available radio sources, only a rather uniformly distributed set has been considered. The criterion for inclusion has been the use of the radio source in previous experiments so that its appropriateness for VLBI observations is assured.

Since a radio source catalogue of a certain accuracy is assumed to be available, no effort has been made to optimize radio source coordinate recovery. As a result, this study is of no "astrometric" value, and no significant improvement of radio source coordinate accuracy can be found in the results of the adjustment performed.

One sidereal day can be taken as the "unit" for an observational pattern since a sequence of observations performed over one day can be repeated over the next days. For observations of a total time span of a number of days, the same daily pattern is repeated in the simulation. Various experimental designs are compared to each other on the basis of one day of observations where earth rotation and baseline parameters are considered as constants over the whole day.

The choice of experiments performed has been primarily directed towards the possibility of recovering earth rotation parameters from a minimum number of observing stations. The minimum number is the three stations necessary for the definition of an earth-fixed system through minimal or inner constraints. Experiments 1, 2, 3 and 4 are cases of such minimal "triangle networks" where all the sides of the triangle are observing baselines.

A weaker design (Experiments 7, 8, 10) is a three station configuration where only two of the triangle sides are observing baselines. In particular, Experiment 10 is a counterpart of Experiment 2, such that the effect of removing one baseline (other design parameters remaining the same) can be studied. The opposite to such minimal three-station designs is provided in Experiment 5 where all eight stations participate in the observations.
In addition to these "realistic" station locations, two "fictitious" designs are also considered, their common characteristic being the closure of a network of stations on a great circle around the earth. Experiment 6 is such a network of uniformly spaced stations along the equator, while Experiment 9 is of a similar design along a meridian. The coordinates of stations involved in these experiments are listed in Table 8.2.

Fig. 9.1 depicts the geographic locations of the networks in the above-mentioned experiments.
Fig. 9.1  Geographic locations of stations and baselines in Experiments 1 through 10.
9. Results and Conclusions

The results of the simulations are presented in three parts. The first part (Figs. 9.2 - 9.3) deals with the comparison of the ten designs described in the previous chapter, on the basis of one day of observations. Fig. 9.2 depicts the recovery (standard deviations) of earth rotation parameters \( (\xi, \eta, \Xi, \Theta, \Omega) \), while Fig. 9.3 shows the recovery of network geometry in terms of baseline lengths and angles (estimable quantities).

Standard deviations \( \sigma \) for each parameter are given as a function of the standard deviation \( \sigma_Q \) of radio source coordinates and for various values of observational precision \( \sigma_\theta \). Values used in the simulation are \( \sigma_\theta, \sigma_Q = 1, 5, 10, 20, 50 \) cm. To achieve uniformity, \( \sigma_Q \) is expressed as arc length on a reference sphere with a radius of 6371 km (1° ≈ 30 m).

The second part (Figs. 9.4 - 9.5) is a study of the improvement of the standard deviations of the parameters with the total time interval \( T \) of available observations. Station coordinates are assumed to be constant over \( T \) while earth rotation parameters are constant only over the subintervals of duration \( \Delta t \) (earth step). The ratio \( \rho = \sigma/\sigma_\theta \) is given for each parameter as a function of \( T \) for various values of \( \Delta t \). In the simulations, the following values have been used: \( T = 1, 2, 3, 4, 5, 10, 30 \) days; \( \Delta t = 6, 12, 24 \) hours. Experiment 2 has been chosen for this study. Radio source coordinate standard deviations are assumed to be \( \sigma_Q = \sigma_\theta \) for all cases. However, the variation of the results with \( \sigma_Q \) is insignificant for \( T \geq 5 \) days.

The third part (Figs. 9.6 and 9.7) is a study of the variation of \( \rho = \sigma/\sigma_\theta \) with \( \Delta t \) (earth step) based on one day of observations (\( T = 1 \) day) for some of the other designs (Experiments 1, 3, 4, 6, 9, 10) is shown.

Since the results of the simulations are fully presented here, one may draw his own conclusions. However, the following general remarks are made in an attempt to summarize the results:

(1) Earth Rotation Parameters from One-Day Observations

For one day of observations and the recovery of earth rotation parameters, the following are noted:

(a) Three-baseline "triangle" designs (Experiments 1, 2, 3, 4 in Fig. 9.2) are capable of recovering polar motion \( (\xi, \eta) \) with a precision of the same order of magnitude as that of the observations, even for low radio source accuracies \( (\sigma_Q = 50 \) cm).
For two-baseline designs (Experiments 7, 8, 10 in Fig. 9.2), polar motion recovery is of the same quality as for the "triangle" station configurations. However, the removal of one baseline in Experiment 10 results in an increase of polar motion standard deviations by a factor of 1.5 compared to Experiment 2. The more complex design of Experiment 5 offers no dramatic improvement in polar motion recovery. An essential improvement can be seen in the equatorial design of Experiment 6 and even more in Experiment 9 which is the "best," but admittedly not very practical design for polar motion recovery.

(b) The standard deviations of the (precession-nutation) parameters \( \Xi \) and \( \Upsilon \) are practically the same for all two- or three-baseline designs (Experiments 1, 2, 3, 4, 7, 8, 10). In particular, the removal of one baseline in Experiment 10 with respect to Experiment 2 has no effect on the recovery of \( \Xi \) and \( \Upsilon \). The best recovery of \( \Xi \), \( \Upsilon \) can be seen in Experiments 5 and 9. Although the station configuration in Experiment 6 is of the same shape as in Experiment 9, the change in their position (equatorial vs. meridian) has a great effect on \( \Xi \), \( \Upsilon \) standard deviations. Experiment 6 has the weakest recovery, while Experiment 9 provides the strongest one.

(c) The standard deviation of \( \Theta_0 \) varies with the station configurations. It is highest for Experiments 4, 8, 7; medium for Experiments 10, 3, 5; and smallest for Experiments 1, 2, 9, and especially 6 (best).

(d) Recovery of \( \Omega \) varies strongly even for designs of the same type. It is generally weak for two-baseline designs (Experiments 7, 8, 10) and somewhat better for three-baseline designs with the exception of the extremely weak recovery in Experiment 4. Many-station designs (Experiments 5, 6, 9) improve the recovery especially in the equatorial configuration of Experiment 6.

(e) \( \Xi \) and \( \Theta_0 \) are the only parameters favored by the artificial equatorial design of Experiment 6 vs. the meridian one of Experiment 9. Recovery of the two components \( \xi \) and \( \eta \) of polar motion is not equal but depends on the position of the stations in each experiment with the exception of Experiment 6. On the contrary, both the \( \Xi \) and \( \Upsilon \) parameters of precession-nutation are recovered with practically the same precision for every design.
The recovery of earth rotation parameters varies with the accuracy $(\sigma_0)$ of radio source coordinates. However, this dependence is much stronger for $\Xi, H$; milder for the rest of the parameters; and even disappearing for $\Omega, \xi, \eta$ in the strongest designs of Experiments 6 and 9.

(f) The results of the idealized designs of Experiments 6 and 9 can be used to verify some of the results of Chapter 6. Recovery of $\xi, \eta, \Xi, H$ is better for Experiment 9 than 6, while recovery of $\Theta_0, \Omega$ is better for Experiment 6. This is explained by the fact that Experiment 9 involves baselines parallel to the equator (16-17, 19-20) observing polar radio sources, a design sensitive to $\xi, \eta, \Xi, H$. Experiment 6 involves equatorial baselines observing equatorial radio sources, the design most sensitive to $\Theta_0, \Omega$ (see Table 6.1).

(g) The recovery of baselines varies with station configuration. However, larger number of baselines does not in general improve the individual recovery as in the case of earth rotation parameters. For example, recovery in the three-baseline design of Experiment 2 is much better than in any of the many-station designs (Experiments 5, 6, 9). The relative recovery of angles in the same experiment is inversely proportional to the recovery of the opposite baselines. Dependence on radio source accuracy varies and disappears in the case of the equatorial configuration of Experiment 6, especially in comparison to the meridian configuration of Experiment 9.

(2) **Earth Rotation and Baseline Parameters As a Function of the Length of Observations and Earth Step**

For the variation of the recovery of earth rotation and baseline parameters with respect to the total time interval of observations $T$, it is noted that the standard deviations $\sigma$ of the parameters are directly proportional to the standard deviations $\sigma_b$ of the observations. (Note that $\sigma_0 = \sigma_b$.) For this reason only the ratio $\rho = \sigma/\sigma_b$ is depicted in Figs. 9.4 and 9.5.

The dependence of $\rho$ on the earth step $\Delta t$ is very strong for earth rotation parameters, especially for $\Omega$ (see Fig. 9.4), while not significant for baseline
parameters (see Fig. 9.5). In general, the recovery improves as \( \Delta t \) increases since the corresponding number of unknown parameters in the adjustment decreases. For earth rotation parameters (Fig. 9.4), standard deviation decreases strongly as \( T \) increases from 1 to 5 days. An additional increase of \( T \) from 5 to 10 days offers a small improvement, while no significant additional improvement is gained from the extension of \( T \) from 10 to 30 days. The same is true for baseline parameters (lengths and angles, see Fig. 9.5) with the exception of an additional small improvement with the increase of \( T \) from 10 to 30 days.

(3) Earth Rotation and Baseline Parameters As a Function of Earth Step

As mentioned, the recovery of earth rotation and network geometry parameters improves with larger earth step \( \Delta t \) (Figs. 9.6 and 9.7). The only exceptions are with respect to the \( \xi, \eta \) parameters in Experiments 1, 2, 4, 10 and with respect to \( \Theta_0 \) in Experiment 4 where \( \Delta t = 24 \) hours results in worse recovery than \( \Delta t = 12 \) hours or even \( \Delta t = 6 \) hours in some cases. This "inversion" is correlated to the weak recovery of polar motion in Experiments 1, 2, 4, 10 (see Fig. 9.2) and to the weak recovery of \( \Theta_0 \) in Experiment 4. On the contrary, for Experiments 6 and 9, where \( \xi, \eta \) and \( \Theta_0 \) recovery is better, variation of \( \rho \) with respect to \( \Delta t \) is "normal." Such variation is negligible for \( \Theta_0 \) in Experiment 6 and also for network geometry parameters in Experiments 6 and 9. Naturally an increase in the length of the earth step is a disadvantage when short periodic variations in the earth rotation parameters are sought.
Fig. 9.2  Recovery of earth rotation parameters, Experiments 1 - 10.

Total time span of observations: \( T = 1 \text{ day} \)

Earth step: \( \Delta t = 1 \text{ day} \)

\( \sigma \) = a posteriori standard deviation of earth rotation parameters
\( \sigma_\theta \) = a priori standard deviation of radio source coordinates
\( \sigma_b \) = standard deviation of observational errors

Note: Angular quantities when expressed in length units are arc lengths on a sphere of radius \( R = 6371000 \text{ m} \) (1" \( \approx \) 30 m).
Fig. 9.2 Recovery of earth rotation parameters.
EXPERIMENT 2

Fig. 9.2 (cont'd)
Fig. 9.2 (cont'd)

EXPERIMENT 3
EXPERIMENT 4

Fig. 9.2 (cont'd)
EXPERIMENT 5

Fig. 9.2 (cont'd)
EXPERIMENT 6

Fig. 9.2 (cont'd)
EXPERIMENT 7

Fig. 9.2 (cont'd)
EXPERIMENT 8

Fig. 9.2 (cont’d)
EXPERIMENT 9

Fig. 9.2 (cont'd)
Fig. 9.2 (cont'd)
Fig. 9.3  Recovery of baseline lengths and angles, Experiments 1 - 10

Total time span of observations: $T = 1$ day
Earth step: $\Delta t = 1$ day

For explanation of other symbols, see Fig. 9.2, p. 40.
Fig. 9.3 Recovery of baseline lengths and angles.

EXPERIMENT 1
EXPERIMENT 2

Fig. 9.3 (cont'd)

52
EXPERIMENT 3

Fig. 9.3 (cont'd)
EXPERIMENT 4

Fig. 9.3 (cont'd)
EXPERIMENT 5

Fig. 9.3 (cont'd)
Fig. 9.3 (cont'd)
EXPERIMENT 7

Fig. 9.3 (cont'd)
Fig. 9.3 (cont’d)
EXPERIMENT 9

Fig. 9.3 (cont'd)
EXPERIMENT 10

Fig. 9.3 (cont'd)
Fig. 9.4 Variation of earth rotation parameter recovery with stepsize and the total time interval of observations (Experiment 2).
Fig. 9.5 Variation of baseline length and angle recovery with stepsize and the total time interval of observations (Experiment 2).
Fig. 9.6 Variation of earth rotation parameter recovery with earth step.
Fig. 9.6 (cont'd)
Fig. 9.7 Variation of baseline length and angle recovery with earth step.
Fig. 9.7 (cont'd)
References


Uotila, U.A. (1967). "Introduction to Adjustment Computations with Matrices." Lecture Notes, Department of Geodetic Science, The Ohio State University, Columbus.
Inner Constraints and Their Relation to the "Geometry" of the
Pseudoinverse of an Operator

Suppose that a system of linear equations
\[
\begin{align*}
N & \times x = y \\
u \times u & \quad u \times 1 & \quad u \times 1
\end{align*}
\]
is given \((N, y)\) known; \(x\) unknown) which is consistent (i.e., it has at least one solution), and furthermore it has more than one solution. The objective is to show that there exists a set of linear constraints
\[
E^T x = 0
\]
called inner constraints, such that the solution to the simultaneous system of equations \(N x = y\) and \(E^T x = 0\) is unique and identical to the solution \(x_0 = N^+ y\), where \(N^+\) is the pseudoinverse of the matrix \(N\).

The matrix \(N\) is a representation of an operator \(N: X \rightarrow Y\), where \(X\) and \(Y\) are both \(u\)-dimensional inner product spaces with elements \(u \times 1\) vectors and the usual Euclidean inner product
\[
<f, g> = g^T f \quad f, g \in X \quad \text{or} \quad f, g \in Y
\]
Two subspaces can be introduced with respect to the operator \(N\), the range \(R(N) = R\) of \(N\) defined as
\[
R = \{ y; y \in Y \quad \text{and} \quad y = Nx \text{ for some } x \in X \}, \quad R \subseteq Y
\]
and the kernel or null space \(K(N) = K \subseteq X\) of \(N\),
\[
K = \{ x; x \in X \quad \text{and} \quad Nx = 0 \}
\]
The consistency of the linear equations \(N x = y\) refers to the fact that \(y \in R\), while the non-uniqueness of solution to the fact that \(K \cap \{0\}^c\) is non-empty, i.e., \(K\) has other elements in addition to \(0\). If \(x'\) is a solution to \(N x = y\), then \(x = x' + \bar{x}\) is also a solution where \(\bar{x}\) is any element of \(K\). We can therefore identify the solution.
space \( S_y \subset X \), defined as

\[
S_y = \{ x ; \ x \in X \text{ and } \ N x = y, \ y \in R \text{ fixed} \}
\]

with a linear variety

\[
S_y = \{ x ; \ x = x' + \bar{x}, \ \bar{x} \in K, \ x' \text{ fixed with} \ N x' = y \}
\]

If \( K^\perp \) is the orthogonal complement of \( K \) with respect to \( X \), the projection \( x_0 = \mathcal{P}_{K^\perp}(x) \) of any element \( x \in S_y \) on \( K^\perp \) is unique, and furthermore [Dermanis, 1977, §3.5]

\[
\| x_0 \| = \min_{x \in S_y} \| x \|
\]

We can now define the pseudoinverse \( N^+ \) of the operator \( N \) as an operator

\[
N^+ : R \to X
\]

where

\[
N^+ y = x_0 = \mathcal{P}_{K^\perp}(x)
\]

for any \( y \in R \) and \( x \in S_y \)

\( N^+ \) is therefore the ordinary inverse of the restriction of the operator \( N \) to \( K^\perp \) (see [Desoer and Whalen, 1963, p. 444] for a more rigorous definition). The domain of definition of \( N^+ \) can be extended to the whole space \( Y \) (\( N^+ : Y \to X \)) by setting

\[
N^+(y) = N^+(y')
\]

where \( y' = \mathcal{P}_R(y) \) is the projection of \( y \in Y \) on \( R \).

The range of \( N^+ \) is \( K^\perp \), while its kernel (null) space is \( R^\perp \). For \( y \in R \) we have \( x_0 = N^+ y \) and \( x_0 \) can be alternatively defined as the unique element in the intersection \( S_y \cap K^\perp \). This means that \( x_0 \) is uniquely defined by its two properties:

\[
x_0 \in S_y \quad \text{and} \quad x_0 \in K^\perp
\]

The first property is expressed by the original equations \( N x = y \), while a similar expression must be established for the second property. Let \( r = \text{rank } (N) < u \) and \( s = u - r \) be the rank deficiency of \( N \). The dimension of the linear subspace \( K \subset X \) is then \( s \), and suppose that \( \{ e_i \}, \ i = 1, 2, \ldots, s \) is a basis in \( K \). Since \( x_0 \in K^\perp \), we have \( x_0 \perp e_i \) for \( i = 1, 2, \ldots, s \), i.e.,

\[
< x_0, e_i > = e_i^T x_0 = 0 \quad \text{for } i = 1, 2, \ldots, s
\]
Fig. A.1 The geometry of the pseudoinverse operator.
Setting

\[ E = [e_1, e_2, \ldots, e_s] \]
\[ u \times s \]

the above equations can be written in the compact matrix form

\[ E^T x_0 = 0 \]

Therefore, \( x_0 \) is uniquely determined as the vector satisfying both equations

\[ N x = y \quad \text{and} \quad E^T x = 0 \]

where the columns of \( E \) constitute a basis in \( K \), the kernel (null space) of \( N \). The constraints \( E^T x = 0 \) are referred to as the "inner" constraints.

The problem has now been reduced to that of finding a basis in \( K \), i.e., of finding a set of \( s \) linearly independent \( u \times 1 \) vectors \( \{e_i\} \) satisfying

\[ N e_i = 0 \quad \text{for} \quad i = 1, 2, \ldots, s \]

The equations above can be written again in matrix form

\[ N E = 0 \quad \text{where} \quad E = [e_1, e_2, \ldots, e_s] \]

We therefore only need to find a \( u \times s \) matrix \( E \) with rank \( (E) = s = u - \text{rank} \ (N) \) (to secure linear independence of its columns \( e_i \)) satisfying \( NE = 0 \).—Summarizing, we have the following theorem

Given the system of linear equations

\[ \begin{align*}
N x &= y \\
u \times u \times 1 x &= u \times 1
\end{align*} \]

where \( \text{rank} \ (N) = r < u \), \( s = u - r \), and a \( u \times s \) matrix \( E \) with rank \( (E) = s \) satisfying \( NE = 0 \), the vector \( x \) satisfying simultaneously \( N x = y \) and \( E^T x = 0 \) is unique and identical to \( x = N^+ y \), where \( N^+ \) is the pseudoinverse of \( N \).

References


APPENDIX B

Algorithm for First-Order Partitioned Linear Regression
including Inner Constraints

The problem in question is the construction of an algorithm for the least squares adjustment of a group of linearized observation equations of the form

$$ V_i = \tilde{A}_i \tilde{X}_i + \hat{A}_i \hat{X} + L_i \quad i = 1, 2, \ldots, m $$

(A 1)

with $E \{V_i\} = 0$ and $E \{V_i V_i^T\} = P_i$. $\hat{X}$ refers to parameters common to all sets of observations, while $\tilde{X}_i$ refers to those appearing only in the $i$th set of observations. Each set of observation equations gives rise to a contribution to the normal equations of the form

$$
\begin{bmatrix}
\bar{N}_i & \bar{N}_i \\
\bar{N}_i^T & \bar{N}_i 
\end{bmatrix}
\begin{bmatrix}
\bar{X}_i \\
\bar{X}
\end{bmatrix}
+ 
\begin{bmatrix}
\bar{U}_i \\
\bar{U}
\end{bmatrix}
= 0
$$

(A 2)

where

$$
\bar{N}_i = \tilde{A}_i^T P_i \tilde{A}_i \quad \bar{U}_i = \tilde{A}_i^T P_i L_i$

$$
\bar{N}_i = \tilde{A}_i^T P_i \tilde{A}_i \quad \bar{U}_i = \tilde{A}_i^T P_i L_i
$$

and

$$
\bar{N}_i = \tilde{A}_i^T P_i A_i
$$

The combination of such individual sets of normal equations leads to the final set of normal equations incorporating all observations

$$
\begin{bmatrix}
\bar{N} & \bar{N} \\
\bar{N}^T & \bar{N}
\end{bmatrix}
\begin{bmatrix}
\bar{X} \\
\bar{X}
\end{bmatrix}
+ 
\begin{bmatrix}
\bar{U} \\
\bar{U}
\end{bmatrix}
= 0
$$

(A 3)

where

$$
\begin{bmatrix}
\bar{N}_1 & 0 & 0 \\
0 & \bar{N}_2 & 0 \\
0 & \cdots & \bar{N}_n
\end{bmatrix}, \quad \begin{bmatrix}
\bar{N}_1 \\
\bar{N}_2 \\
\cdots \\
\bar{N}_n
\end{bmatrix}, \quad \begin{bmatrix}
\bar{U}_1 \\
\bar{U}_2 \\
\cdots \\
\bar{U}_n
\end{bmatrix}, \quad \begin{bmatrix}
\bar{X}_1 \\
\bar{X}_2 \\
\cdots \\
\bar{X}_n
\end{bmatrix}
$$

72
\[ \dot{N} = \sum \dot{N}_i, \quad \dot{U} = \sum \dot{U}_i \]

In case \( N = \begin{bmatrix} \tilde{N} & \tilde{N} \\ N' & \tilde{N} \end{bmatrix} \) is nonsingular (\( \det N \neq 0 \)), the normal equations above can be solved through a First-Order Partitioned Linear Regression scheme as explained in [Brown and Trotter, 1969; and Uotila, 1973]. We give here an extension of this scheme for the case when \( N \) is singular and a unique solution is ascertained only after the introduction of a set of inner constraints

\[ \tilde{E}'X + \tilde{E}'X = 0 \quad \text{(A.4)} \]

with

\[ \tilde{E}' = \begin{bmatrix} \tilde{E}_1 & \tilde{E}_2 & \ldots & \tilde{E}_l \end{bmatrix} = \begin{bmatrix} \tilde{E}_0 & \tilde{E}_0 & \ldots & \tilde{E}_0 \end{bmatrix} \]

The normal equations can now be augmented as follows [Uotila, 1967]:

\[
\begin{bmatrix}
\tilde{N} & \tilde{N} & \tilde{E} \\
\tilde{N}' & \tilde{N} & \tilde{E} \\
\tilde{E}' & \tilde{E}' & 0
\end{bmatrix}
\begin{bmatrix}
\dot{X} \\
\dot{X} \\
K_l
\end{bmatrix}
+ 
\begin{bmatrix}
\dot{U} \\
\dot{U} \\
0
\end{bmatrix} = 0
\quad \text{(A.5)}
\]

The new augmented coefficient matrix is nonsingular and inversion with the introduction of some new notation leads to

\[
\begin{bmatrix}
\tilde{N} & \tilde{N} & \tilde{E} \\
\tilde{N}' & \tilde{N} & \tilde{E} \\
\tilde{E}' & \tilde{E}' & 0
\end{bmatrix}^{-1}
\begin{bmatrix}
\tilde{M} & \tilde{M} \\
\tilde{M}' & \tilde{M}
\end{bmatrix}^{-1}
\begin{bmatrix}
\tilde{Q} & \tilde{Q} \\
\tilde{Q}' & \tilde{Q}
\end{bmatrix}
= 
\begin{bmatrix}
\tilde{G} & \tilde{G} & \tilde{F} \\
\tilde{G}' & \tilde{G} & \tilde{F}'
\end{bmatrix}
\quad \text{(A.6)}
\]

With the help of this inverse, the solution of the normal equations becomes

\[
\begin{aligned}
\dot{X} &= \tilde{G} \tilde{U} + \tilde{G} \dot{U} \\
\dot{X} &= \tilde{G}' \tilde{U} + \tilde{G} \dot{U}
\end{aligned}
\quad \text{(A.7)}
\]

If \( \tilde{G}_i, \tilde{G}_{ij} \) denote submatrices of \( \tilde{G} \) and \( \tilde{G} \), respectively, with dimensions equal to those of \( \tilde{N}_i \) and \( \tilde{N}_{ij} \), respectively, we have

\[
\begin{aligned}
\dot{X} &= \sum \tilde{G}_j \tilde{U}_j + \tilde{G} \dot{U} \\
\dot{X}_i &= \sum \tilde{G}_{ij} \tilde{U}_j + \tilde{G}_i \dot{U}
\end{aligned}
\quad \text{(A.8)}
\]
The inversion of Eq. (A 6) leads to

\[
\dot{\mathbf{Q}} = [\mathbf{M} - \mathbf{M}^T \mathbf{M}^{-1} \mathbf{M}]^3
\]

\[
\mathbf{Q} = -\mathbf{M}^3 \mathbf{M} \dot{\mathbf{Q}}
\]

\[
\ddot{\mathbf{Q}} = \mathbf{M}^3 + \mathbf{M}^3 \mathbf{M} \dot{\mathbf{Q}} \mathbf{M}^T \mathbf{M}^3
\]  

Replacing \( \tilde{\mathbf{M}} \), \( \dot{\mathbf{M}} \), \( \ddot{\mathbf{M}} \) from (A 6) in terms of submatrices of \( \tilde{\mathbf{N}} \), \( \dot{\mathbf{N}} \), \( \ddot{\mathbf{N}} \), and introducing the following notation:

\[
\tilde{H}_1 = \tilde{N}_i^3 \tilde{N}_i \quad \quad R_1 = N_i^7 H_1
\]

\[
S_1 = N_1 - R_1 \quad \quad S = \Sigma S_1
\]

\[
H = \Sigma H_1 \quad \quad K = \Sigma \tilde{N}_i^3
\]

\[
\tilde{L}_1 = \tilde{H}_1 \tilde{U}_1 \quad \quad L = \Sigma \tilde{L}_1
\]

\[
T_1 = \tilde{N}_i^3 \tilde{U}_1 \quad \quad T = \Sigma T_1
\]

we finally obtain after some algebraic manipulations

\[
\dot{\mathbf{Q}}^3 = \begin{bmatrix}
S & (\mathbf{E} - H^T \mathbf{E}_0) \\
(E^T - \mathbf{E}_0^T H) & (-\mathbf{E}_0^T K \mathbf{E}_0)
\end{bmatrix}
\]  

\[
\dot{\mathbf{X}} = \mathbf{G} \mathbf{L} + \mathbf{F} \mathbf{E}_0^T T - \mathbf{G} \dot{\mathbf{U}}
\]  

\[
\ddot{\mathbf{X}}_1 = -T_1 - \dot{\mathbf{N}}_i^3 [\mathbf{E}_0 \mathbf{F}^T (-\dot{\mathbf{U}} + \mathbf{L}) + (\mathbf{E}_0 \mathbf{F} \mathbf{E}_0) T] - \mathbf{H}_1 \dot{\mathbf{X}}
\]

\[
\ddot{G}_{11} = \delta_{1j} \tilde{N}_i^3 + H_1 \dot{G} \mathbf{H}_j^T + H_1 (F \mathbf{E}_0) \tilde{N}_i^3 + 

+ \tilde{N}_i^3 (\mathbf{E}_0 \mathbf{F}^T) \mathbf{H}_j^T + \dot{\mathbf{N}}_i^3 (\mathbf{E}_0 \mathbf{F} \mathbf{E}_0) \tilde{N}_i^3
\]
The algorithm can now be summarized in the following steps:

1. Compute $\tilde{A}_1, \tilde{A}_4, L_4; \tilde{E}, \tilde{E}_0$
2. Compute $\tilde{N}_1, \tilde{N}_4, \tilde{N}_4, \tilde{U}_1, \tilde{U}_4$
3. Compute $H_1, R_1, S_1, T_1, L_4$
4. Compute $H_1, S, \tilde{N}, \tilde{U}, K, L, T$
5. Obtain $\hat{G}, \hat{F}, F$ by inversion
   \[
   \begin{bmatrix}
   S & \tilde{E} - H^T \tilde{E}_0 \\
   (\tilde{E} - H^T \tilde{E}_0)^T & -\tilde{E}_0^T K \tilde{E}_0
   \end{bmatrix} \tilde{A} =
   \begin{bmatrix}
   \hat{G} \\
   \hat{F}
   \end{bmatrix}
   \]
6. Compute $\dot{X}$ from (A11), $\tilde{X}_1$ from (A12), and $\tilde{G}_{ij}$ from (A13)

References


The simulation of observations and their adjustment are carried out with the help of three programs, VLBI SIMULATOR, PROG$1$, and PROG$2$ as explained in Chapter 7. These programs are presented here in detail together with their supporting subroutines.

1. Name of Program: VLBI SIMULATOR

   **Function:** Simulates distance and distance rate VLBI observations corresponding to time delays and their derivatives, according to a previously decided observational pattern.

   **Input:** Number of participating stations and radio sources, radio source and station coordinates, initial and final epochs of observations, time interval between successive observations, identifying numbers of stations and radio source participating in each observation.

   **Output:** Identification numbers for participating stations and radio source, observed (perfect) distance and distance rate, epoch of observation.

Subroutines required:

- GRESID: Calculates Greenwich sidereal time for a given epoch
- JULIA: Converts Universal Time to Julian date
- EARTH: Provides simulated earth rotation parameters for a given epoch
- REDPI: Converts an angle in radians to the \((-\pi, \pi)\) interval.

The following is a listing of VLBI SIMULATOR program and supporting subroutines.
**CONVERT RA AND DEC TO RADIANS**

```fortran
RA = RA(I) * PI / 180.0
DEC = DEC(I) * PI / 180.0
(:,:,)
C = COS(DEC)
SD = SIN(DEC)
C = COS(RA)
S = SIN(RA)
F1(I) = C * CA
F3(I) = S * CA
```

**READ STATION COORDINATES**

```fortran
DO 6 I = 1, IN
RERAD(5, 701) IDUM, IL1, IL2, LONG(I), PHI(I), R(I)
6 FORMAT(I2, 1X, A4, A2, 3FI5.1)
WRITE(6, 701) I, IL1, IL2, LONG(I), PHI(I), R(I)
701 FORMAT(3FI5.1)
```

**CONVERT PHI AND LONG TO RADIANS**

```fortran
PHIR = PHI(I) * PI / 180.0
LOGR = LONG(I) * PI / 180.0
F = COS(PHI)
S = SIN(PHI)
CL = COS(LONG)
SL = SIN(LONG)
X(I) = F * CL
Y(I) = S * CL
Z(I) = SF
```

**READ INITIAL EPOCH IN UT**

```fortran
READ(5, 201) IYEAR, IMO, IDAY, IHOUR, IMIN, SEC
WRITE(6, 201) IYEAR, IMO, IDAY, IHOUR, IMIN, SEC
201 FORMAT(1X, 14, 4FI5.1)
```

**FIND SRF-EWITCH SIDERIAL TIME AT EPOCH TO CALL GRES**

```fortran
CALL GRES(IY, IMO, IDAY, IHOUR, IMIN, THO)
```

**READ FINAL EPOCH TF AND TIME INTERVAL IN HOURS**

```fortran
READ(5, 201) JYEAR, JMO, JDAY, JHOUR, JMIN, SEC
WRITE(6, 201) JYEAR, JMO, JDAY, JHOUR, JMIN, SEC
201 FORMAT(I2, 6FI5.1)
```

**NOTICE THAT IF TF TO TF ARE CLOSE BUT IN DIFFERENT MONTHS SECOND DATE TF MUST BE EXPRESSED AS A DAY OF PREVIOUS MONTH, E.G. AUG 33**

```fortran
TF = FLOAT(JDAY-IDAY)*24.0 + FLOAT(JHOUR-IMOUR) + FLOAT(JMIN-IMIN)```

TRANSFORM QUASAR UNIT VECTOR TO EARTH FIXED SYSTEM AT EPOCH T

\[ \text{ANGLE} = 0.95(T - T_0) + THO \]

\[ CA = \cos(\text{ANGLE}) \]

\[ SA = \sin(\text{ANGLE}) \]

\[ Q1 = CA \times E1 + SA \times E2 \]

\[ Q2 = -SA \times E1 + CA \times E2 \]

\[ Q3 = E3 \]

DO 8 I = 1, IM

DO 8 J = 1, IN

ZNT = ARCOS(X(J) \times Q1 + Y(J) \times Q2 + Z(J) \times Q3)

ARGU = X(J) \times Q1 + Y(J) \times Q2 + Z(J) \times Q3

IF(ARGU .LT. -1.0D0) ARGU = -1.0D0

IF(ARGU .GT. 1.0D0) ARGU = 1.0D0

ZNT = DARCOS(ARGU)

ZNT = DABS(ZNT) * 180.0D0 / PI

ZNTMAX = 60.0D0

IF(ZNT .GT. ZNTMAX) ZNT = 10.0D0 * 10

INDEX(I,J) = ZNT

CONTINUE

WRITE(6, 207) T

207 FORMAT(*1/10X,*EPOCH = *,F15.5/*20X,*1*,2X,*2*,2X,*3*,2X,)

\[ \ldots \]

\[ 71X,*13*,1X,*14*,1X,*15*,1X,*16*,1X,*17*,1X,*18*,1X,*19*,1X,*20*, \]

\[ 71X,*21*,1X,*22*,1X,*23*,1X,*24*,1X,*25*,1X,*26*,1X,*27*,1X,*28*, \]

\[ 71X,*29*,1X,*30*,//) \]

DO 9 I = 1, IM

WRITE(6, 105) I, RA(I), DEC(I), {INDEX(I,J), J = 1, IN}

105 FORMAT(1X,I2,2F7.0,1X,30(1X,I2))

CONTINUE

INM = IN - 1

DO 555 J = 1, INM

IF(INDEX(I,J) .GT. 1000) GO TO 555

JP = J + 1

555 CONTINUE

DO 955 K = JP, IN
0084 IF(INDEX(I,K).GT.1000) GO TO 555
0085 WRITE(7,705) J,K,I,T
0086 705 FORMAT(3I5,F15.2)
0087 555 CONTINUE

C
C
0088 99 T=T+DT
0089 GO TO 33
0090 88 WRITE(*,500)
0091 500 FORMAT(1X///10X,'WRONG INITIAL AND FINAL DATE'/
?10X,'PROGRAM INTERRUPTED THROUGH STATEMENT 88')
0092 40 STOP
0093 END
C***** RA AND DEC ARE IN DEGREES

DO 5 I=1,IM
READ(5,321) IDM,IL1,IL2,IL3,RA(I),DEC(I)
WRITE(6,321) I,IL1,IL2,IL3,RA(I),DEC(I)
321 FORMAT(1X,I3,7X,3A4,F17.7,F20.9)

C***** CONVERT RA AND DEC TO RADIANS

RAR=RA(I)*PI/180.DO
DECR=DEC(I)*PI/180.DO
CD=DCOS(DECR)
SD=DSIN(DECR)
CA=DCOS(RAR)
SA=DSIN(RAR)
E1(I)=CD*CA
E2(I)=CD*SA
E3(I)=SD
5 CONTINUE

C

C***** READ STATION COORDINATES

C***** PHI(LATITUDE) AND LONG ARE IN DEGREES, R IS IN METERS

DO 6 I=1,IN
READ(5,701) IDM,IL1,IL2,LONG(I),PHI(I),R(I)
WRITE(6,701) I,IL1,IL2,LONG(I),PHI(IR(I)
701 FORMAT(1X,12,12X,A4,A2,3F15.1)

C***** CONVERT PHI AND LONG TO RADIANS

PHIR=PHI(I)*PI/180.DO
LONGR=LONG(I)*PI/180.DO
CF=DCOS(PHIR)
SF=DSIN(PHIR)
CL=DCOS(LONGR)
SL=DSIN(LONGR)
X(I)=CF*CL
Y(I)=CF*SL
Z(I)=SF
6 CONTINUE

C

C***** READ CORRECTION TO GREENWITCH SIDERIAL TIME DDTH IN ARC SEC

READ(5,101) DDTH
WRITE(6,101) DDTH
101 FORMAT(3F15.5)

C***** CONVERT TO RADIANS

DDTH=DDTH*PI/(180.DO*3600.DO)

C

C***** READ INITIAL EPOCH IN UT

READ(5,201) IYEAR,IMO,IDAY,IHOUR,IMIN,SEC
WRITE(6,201) IYEAR,IMO,IDAY,IHOUR,IMIN,SEC
201 FORMAT(5S1,F15.5)

C

C***** FIND GREENWITCH SIDERIAL TIME AT EPOCH TO

CALL GRESID(IYEAR,IMO,IDAY,IHOUR,IMIN,SEC,TDO)
COPY(1)=THO
THD=THO+DDTH*PI
WRITE(6,136) THO
136 FORMAT(///10X'APPR. THO= ',F25.9///)
WRITE(7,101) THO
C**************************************************************************
C************ VLBI NETWORK, OBSERVATION SIMULATOR
C************
C**************************************************************************
C************ CONTINUOUS VERSION: EARTH MOTION PARAMETERS
C************ PROVIDED BY SUBROUTINE EARTH
C************
C**************************************************************************

C************ THE MODEL USED IS:
C************
C************ T
C************ D(I,J,P,K)=(XJ-XI)*S(XP,HP)*
C************ T
C************ *R3(OMG*TK+THO)*S (XX,HH)*QP
C************
C************
C************ S(A,B) =
C************ ( 1 0 A )
C************ ( -A B 1 )
C************
C************ T
C************ QP = (COS(DECP)*COS(RAP),
C************ COS(DECP)*SIN(RAP), SIN(DECP) )
C************
C************ XJ, XI: POSITION VECTORS OF J AND I
C************ STATIONS IN EARTH FIXED SYSTEM
C************ DECP, RAP: DECLINATION AND RIGHT ASCENSION
C************ OF P QUASAR IN QUASAR FIXED SYSTEM
C************ QUASAR FIXED SYSTEM
C************ OMG: ROTATIONAL VELOCITY OF EARTH
C************ XP, HP: POLAR MOTION SMALL ANGLES
C************ XX, HH: PRECESSION NUTATION SMALL ANGLES
C************ D(I,J,P,K): OBSERVED DISTANCE INVOLVING IJ
C************ AFTER INITIAL REFERENCE EPOCH TO
C************ THO: GREENWICH SIDERIAL TIME AT EPOCH TO
C************ THETA: UT1 ANGLE
C************
C**************************************************************************

IMPLICIT REAL*8(A-H,L-Z)
DIMENSION X(4),Y(4),Z(4),EI(18),E2(18),E3(18),RA(18),DEC(18)
2,PHI(4),LONG(4),R(4),COPY(3)
C X(IN),Y(IN),Z(IN),EI(IN),E2(IN),E3(IN),INDEX(IM,IN),RA(IM),DEC(IM),
C PHI(IN),LONG(IN),R(IN)
C IN=NO OF STATIONS, IM=NO OF QUASARS, IN.LT.35
C
PI=4.DO*DATAN(1.D0)
REART=6371000.DO
C
READ(5,100) IN,IM
WRITE(6,100) IN,IM
100 FORMAT(215)
C
C************ READ QUASAR COORDINATES
C****** READ FINAL EPOCH TF AND TIME INTERVAL IN HOURS
0050 READ(5,201) JYEAR,JMO,JDAY,JHOUR,JMIN,SECJ
0051 WRITE(6,201) JYEAR,JMO,JDAY,JHOUR,JMIN,SECJ
0052 READ(5,101) DT
0053 WRITE(6,101) DT

C
C
0054 IF(JYEAR.NE.IYEAR) GO TO 88
0055 IF(JMO.NE.I4Q) GO TO 88

C****** NOTICE THAT IF TF, TF ARE CLOSE BUT IN DIFFERENT MONTHS SECOND
C****** DATE TF MUST BE EXPRESSED AS A DAY OF PREVIOUS MONTH, E.G. AUG 33
0056 TF=DFLOAT(JDAY-IDAY)*24.000+DFLOAT(JHOUR-IHOUR)+DFLOAT(JMIN-IMIN)
    \  2  30.00+(SECJ-SECJ)/3600.00
    \  4  

C
0057 T0=0.00
0058 T=0.00
0059 33 IF(T.GT.TF) GO TO 40

C
0060 555 READ(5,107) I,J,K
0061 IF(I.EQ.0) GO TO 99

C****** I=0 IS A CODE TO INDICATE END OF OBSERVATIONS FOR THIS EPOCH
C
0062 CALL EARTH(T,XP,HP,OMG,THETA,XX,HH)

C
0063 MODIFY EARTH PARAMETERS AND WRITE / PUNCH
0064 XP=XP*REAXT
0065 HP=HP*REAxT
0066 XX=XX*REAXT
0067 HHD=HH*REAXT
0068 OMG=OMG*(1.0E0/PI)
0069 THETA=THETA
0070 THET=THET*(180.00/PI)
0071 TH1=THETA
0072 TH2=(THET-DFLOAT(I1TH1))*60.00
0073 TH2=TH1
0074 WRITE(6,801) T,XP,HP,XX,HH,ITH1,ITH2,TH1,TH2,THET,DMGD
0075 801 FORMAT(2X,F7.2,4F6.4,2X,2F7.2,4F6.4,2X,2F7.2,4F6.4,2X,2F7.2,4F6.4,2X,2F7.2,4F6.4)
0076 COPY(1)= E1(K)-XX*E3(K)
0077 COPY(2)= + E2(K)+HH*E3(K)
0078 COPY(3)=XX*E1(K)-HH*E2(K)+ E3(K)

C
0079 ANGLE=THETA+THO
0080 CA=DCOS(ANGLE)
0081 SA=DSIN(ANGLE)

0082 Q1= CA*COPY(1)+SA*COPY(2)
0083 Q2=SA*COPY(1)-CA*COPY(2)
0084 Q3= COPY(3)

0085 QF1=SA*COPY(1)+CA*COPY(2)
0086 QF2=CA*COPY(1)-SA*COPY(2)
0087 QF3=XP*QF1+HP*QF2
C
COPY(1)= X*Q3
COPY(2)= Y*Q3
COPY(3)= Z*Q3

C
DS=(X(I)+R(I)-X(J)+R(J))*COPY(1)+(Y(I)*R(I)-Y(J)*R(J))*COPY(2)
+Z(I)*R(I)-Z(J)*R(J))*COPY(3)

C
FRNG=(X(I)+R(I)-X(J)+R(J))*QF1+(Y(I)*R(I)-Y(J)*R(J))*QF2
+Z(I)*R(I)-Z(J)*R(J))*QF3

C FRNG= DERIVED OF DELAY DISTANCE DS (METERS/HOUR)

C
WRITE(6,107) I,J,K,DS,FRNG,T
WRITF(7,107) I,J,K,DS,FRNG,T

C
FORMAT(315,3F15.5)
GO TO 33

C
T=T+DT
GO TO 33

C
WRITE(6,500)
FORMAT(*I10X,'WRONG INITIAL AND FINAL DATE'/
?10X,'PROGRAM INTERRUPTED THROUGH STATEMENT 88*)
STOP
END
CROUTINE GRSID(IYEAR, IMD, IDAY, IHR, IMIN, SEC, THO)

CALCULATES THE GREENWICH SIDERIAL TIME
THO, FOR EPOCH WITH DATE IYEAR IMD IDAY

IMPLICIT REAL*8(A-H,L-Z)

F1=4.10&DATAN(1.0)
I1=0
I2=0
A=0.0
CALL JULIA(IYEAR, IMD, IDAY, I1, I2, A, MJD)
T=(MJD-2415020.0)/36525.0
T2=T*T
THC=99.69019E3+F 0.007689D0*T+0.00038708D0*T2
THO=THO*PI/180.0
CALL REDPI(THO)
T=DFLOAT(IHR)*60.0+DFLOAT(IMIN)+SEC/60.0
DTT=0.0003752695D0/(10.0**3)
T0=TH0+DT*DTT
CALL REDPI(T0)
END
SUBROUTINE JULIA(YFAR, IM, IDAY, IMM, S, MJD)

C** Subroutine to convert Universal Time to Julian Date
C**
C******************************************************************************
C**
C******************************************************************************
C**
IMPLICIT REAL*8 (A-H,L-Z)
DIMENSION IMONTH(12)
DATA IMONTH /0,31,59,90,120,151,181,212,243,273,304,334/
H=DFLOAT(IM)
P=DFLOAT(_IMM)
I=DFLOAT(IY)
M=365.0*(IY/4)+IY-(IY/4)
IF(IY.EQ.1900) ICO=0
IF(IY.EQ.1900) ICO=0
IF(IY.EQ.1900) ICO=0
IF(IY.EQ.1900) ICO=0
IF(IY.EQ.1900) ICO=0
X=2415020+(IY-1900)*365.0D0+IDIS+IMONTH(IM)+ICOD+0.5D0+IDAY
X=X/24.0D0+H/1440.0D0+S/86400.0D0
RETURN
END
SUBROUTINE EARTH(T, XP, HP, OMG, THETA, XX, HH)

T : TIME INTERVAL AFTER INITIAL EPOCH (HOURS)
XP, HP: POLAR MOTION COORDINATES (RADIANS)
THETA: UT ANGLE IN RADIANS
OMG : ROTATIONAL VELOCITY OF EARTH (RADIANS PER HOUR)
XX, HH: PRELESSION-NUTATION SMALL ANGLES (RADIANS)

IMPLICIT REAL*8(A-H,L-Z)
DIMENSION P(10), A(10), B(10), PS(10), F(10)

DATA P/8760.00, 380.00, 2190.00, 720.00, 360.00, 2160.00, 24.00, 12.00, 1.00, 0.50,
360.00, 0.10, 0.05, 0.00, 0.00, 0.00, 0.00, 0.00,
500.00, 0.50, 0.00, 0.00, 0.00, 0.00, 0.00,
600.00, 0.04, 0.00, 0.00, 0.00, 0.00, 0.00,
7PS/3.1500, 5.2800, 1.3200, 0.1500, 0.00, 0.9000, 2.0000,
86.1400, 7.7500, 4.1200,
9F/1.1300, 2.8500, 6.1700, 4.3500, 0.0500, 5.1800, 3.7500, 1.2400,
70.3500, 1.6700/

PI = 3.1416 * ATAN(1.0)
PI2 = 2.00 * PI

REART = 637100.00
M10 = 6.00
M20 = 0.00

PC = 429.00 * 24.00
C = COS(PI2 * T / PC)
S = SIN(PI2 * T / PC)
M1 = C * M10 + S * M20
M2 = S * M10 + C * M20

CONTINUE

DO 5 I = 1, 10

ARG = (PI2 * T) / F(I)

M1 = M1 + A(I) * DCOS(ARG + PS(I))
M2 = M2 + P(I) * DCOS(ARG + F(I))

CONTINUE

XP = M2 / REART
HP = M1 / REART

XX = 0.0600 + 0.1800 * DCOS(2.3400 + T/24.00) - 0.0030 * DCOS(0.1000 + T/12.00)

HH = 0.0080 + 0.2000 * DCOS(3.1100 + T/24.00) - 0.0050 * DCOS(0.0700 + T/12.00)

THETA = 0.0017 * T / 24.00 + 25.1000 * DCOS(ARG1) - 9.2000 * DSIN(ARG1)

FACTOR = PI2 / 66400000.00

THETA = PI2 / 24.00 * THETA * FACTOR

OMG = 0.001500 / 24.00 + (25.1000 * DCOS(ARG1) - 18.4000 * DCOS(ARG2))

RETURN

END
SUPROUTINE REDPI(A)
C******************************************************************************
C**** REDUCES AN ANGLE A IN RADIANS TO THE (-PI, +PI) INTERVAL
C******************************************************************************
C******************************************************************************
IMPLICIT REAL*8(A-H,L-Z)
PI=4.DO*DATAN(1.DO)
PI2=PI*2.DO
IA=COPY
IF(A.GE.0.DO) A=A-CFLOAT(IA)*PI2
IF(A.LT.0.DO) A=A+DFLOAT(IA)*PI2
5 IF(A.GT.-PI.AND.A.LT.PI) GO TO 8
IF(A.LT.0.DO) A=A+PI2
IF(A.GT.0.DO) A=A-PI2
GO TO 5
8 RETURN
END
2. Name of Program: PROG01

Function:

Adjustment of a set of observations with total time-span equal to both earth and station step. Earth rotation parameters and station coordinates are treated as constants over the total time span of observations. A number of weighting options for radio source coordinates are available.

Input:

Number of weighting options for radio source coordinates, standard deviations of observations and radio source coordinates, approximate values of parameters, identification numbers of participating stations and radio source, distance and distance rate observations.

Output:

Standard deviations of adjusted parameters.

Subroutines required:

REDPI.

(Following is a listing of PROG01.)
VLBI ADJUSTMENT PROGRAM

THIS VERSION PERFORMS AN ADJUSTMENT OF ONE DAY OF OBSERVATIONS TREATING ALL PARAMETERS AS CONSTANTS OVER THE ONE DAY INTERVAL.

AN OPTION IS AVAILABLE FOR APPLYING A NUMBER OF DIFFERENT WEIGHTS ON RADIO SOURCE COORDINATES.

OBSERVATIONS: DELAY DISTANCES AND THEIR DERIVATIVES.

UNKNOWNs: EARTH MOTION PARAMETERS, STATION & QUASAR COORDINATES, CLOCK OFFSETS AND DRIFTS.

SYSTEMS DEFINITION:
QUASAR FIXED: WEIGHTED QUASAR COORDINATES
EARTH FIXED: BY MEANS OF INFR CONSTRAINTS

SUBROUTINES REQUIRED: REDPI

THE ORDER OF THE PARAMETERS IS:
XP, HP POLAR MOTION ANGLES KSI AND ETA
XH, HN PRECESSION-NUTATION ANGLES (CAPITAL) KSI & ETA
TH THETA ZFRO ANGLE (GAST AT INITIAL EPOCH)
OMG ANGULAR VELOCITY OF EARTH ROTATION
XI(J), Y(J), Z(J) COORDINATES OF STATION I (I=1,2,...,IN)
RA(J), D(J) RIGHT ASCENSION AND DECLINATION OF RADIO SOURCE J (J=1,2,...,IM)
CLOCK OFFSET AND DRIFT AT STATION I (I=1,2,...,IN)

IMPLICIT REAL*8(A-H,L-Z)
DIMENSION A(27),N(45,45),X(3),Y(3),Z(3),RA(8),D(8),U(37)
X(2),XX(27)+L(I(45)+L(45)+CC(8)+37)+D(37)+D(I(37)+NKEEP(45,45)
2,SMOQA(10),PRATI(10),F(I(37)+2)
DIMENSION A(KK),N(K,K)+8,RA(N),D(N),RA(KK),XX(KK)
C 2,L1(KK)+L2(KK),CC(8,8),DD(N),DD(N)
C 3,B(KK),NKEEP(K,K8),SDQUA(IO),PRATIO(IO)
C WHERE: N=NO OF STATIONS, M=NO OF QUASARS, K=6+3*N+2*M, KK=K+2*N, K8=KK+8
C C READ EXPERIMENT IDENTIFICATION NO
0003 READ(5,100) IEXP
0004 WRITE(6,977) IEXP
0005 977 FORMAT(1x,15x,'EXPERIMENT NO =',I5,1x)
C C READ NUMBER OF WEIGHTING OPTIONS
0006 READ(5,100) IOPT
0007 WRITE(6,978) IOPT
0008 978 FORMAT(1x,15x,'REQUESTED NO OF WEIGHTING OPTIONS =',I5,1x)
C C READ A PRIORI STANDARD DEVIATIONS:
C SDRD : OF DELAY DISTANCE IN CM
C SDBF : OF DELAY DISTANCE DERIVATIVE IN M/HOUR
C SDQUA: OF QUASAR COORDINATES IN CM
0009 READ(5,101) SDRD,SDBF
0010 101 FORMAT(5F15.5)
C C DO 181 I=1,IOPT
0012 READ(5,101) SDQUA(I)
0013 PRATIO(I)=SDBF**2/SDQUA(I)**2
0614 181 CONTINUE
C C R=E37106.00
C CLIGHT=3.00*(10.00**R)
C PI=4.00*DATAN(1.00)
C RIO0=R*10.00
C RH404=(112.00*(PI)**9)/(10.00**9)
C RMS2=(180.00/(PI)**2600000.00
C C PF=(SDRD/RIO0)/(SDBF/R)
0022 PF=PF**2
0023 2EAD="NO OF STATIONS & IM= NO OF QUASARS"
C C READ IN= NO OF STATIONS & IM= NO OF QUASARS
0024 READ(5,100) IN, IM
0025 100 FORMAT(5I5)
0026 WRITE(6,920) IN,IM
0027 920 FORMAT(5x,15x,'NO OF STATIONS =',I5,1x,15x,'NO OF QUASARS =',I5)
0028 K=6+2*IN+2*IM
0029 KK=K+2*IN
0030 KB=KK+8
C C INITIALIZE POLAR MOTION (XP,HP) AND PRECESSION-NUTATION (XN,HN)
C PAKAMETFRS (APPROXIMATE VALUES) TO ZERO
0031 XP=0.00
0032 HP=0.00
0033 XN=0.00
0034 HN=0.00
0035 C DO 51 I=1,IN
DO I=1,IN
READ(5,701) IDM,IL1,IL2,ALONG,APHI,AR
WRITE(6,701) I,IL1,IL2,ALONG,APHI,AR
701 FORMAT(IXI2,1X,6A6,A2,3F15.1)
APHI=APHI*PI/180.DO
ALONG=ALONG*PI/180.DO
CONVERT TO CARTESIAN
X(I)=AR*VCOS(APHI)*DCOS(ALONG)
Y(I)=AR*r'CSO(APHI)*DSIN(ALONG)
Z(I)=AR*DSIN(APHI)
CONV=RT TO EARTH RADII UNITS
X(I)=X(I)/R
Y(I)=Y(I)/R
Z(I)=Z(I)/R
CONTINUE
WRITE(6,451)
451 FORMAT(//1X,'RADIA SOURCE',10X,'RA',17X,'DEC',//)
DO J=1,IM
READ(5,321) JDM,IL1,IL2,IL3,RA(J),D(J)
WRITE(6,321) J,IL1,IL2,IL3,RA(J),D(J)
321 FORMAT(IXI2,1X,6A6,A2,3F15.1)
RA(J)=RA(J)*PI/180.DO
D(J) = D(J)*PI/180.DO
CONTINUE
DO 7 I=1,KK
XX(I)=0.DO
U(I)=0.DO
7 CONTINUE
CC(J,I)=0.DO
CONTINUE

DO 16 I=1,K8
DO 16 J=1,K8
NII(J)=0.DO
NKEEP(I,J)=0.DO
CONTINUE

FORMULATE CONSTRAINT MATRIX CC

CC(5,1)=1.DO
CC(4,2)=1.DO
CC(4,5)=-1.DO

DO 3 I=1,IN
IB=4+3*T
!82=I81+1
1113=Iez+I
CC(I,IB)=1.DO
CC(2,I)=1.DO
CC(3,13=1.DO
CC(4,152)=-Z(I)
CC(4,IP3)=Y(I)
CC(5,I81)=Z(I)
CC(iIF3)=-X(I)
CC(AI81)=-Y(I)
CC(6,I82)=X(I)
ICI=KI
IC2=ICI+IN
CL(7,IC1)=1.DO
CC(EIC2)=I.DO
CONTINUE

READ OBSERVATIONS AND IDENTIFIERS:

DS= DISTANCE IN METERS OF IJ BASELINE OBSERVATION TO QUASAR IP
AT TIME TK (IN HOURS) AFTER SOME INITIAL EPOCH TO

ICOUNT=0

READ(5,104) I,J,IP,DS,FRNG,T,K,ICHEK
FORMAT(315.3F15.5,215)
ICHEK IS A CODE INDICATING END OF DATA FOR ICHEK=1
IF(ICHEK.EQ.1) GO TO 66
ICOUNT=ICOUNT+1
WRITE(I,100) I,J,IP
100 ICOUNT=0
DS=DS/R
FRNG=FRNG/R
DX=X(J)-X(I)
DY=Y(J)-Y(I)
DZ=Z(J)-Z(I)
0111  CA=DCOS(RA(IP))
0112  SA=DSIN(RA(IP))
0113  CD=DCOS(D(IP))
0114  SD=DSIN(D(IP))
0115  Y1=OMG*TX+TH
0116  Y2=Y1-RA(IP)
0117  CK=DCOS(Y1)
0118  SK=DSIN(Y1)
0119  CKF=FCOS(Y2)
0120  SKP=DSIN(Y2)

FORMULATE PARTIALS

0121  F1=-CD*DZ*CKP+DX*SD
0122  F1=F1+(-DX*XP*CY*HP)*CD*CKP-DZ*XP*SD
0123  F1=F1+DZ*CD*(XN*CA-HN*SA)+DZ*SD*(XN*CK-HN*SK)
0124  F2=-CD*DZ*CKP-DY*SD
0125  F2=F2+DZ*CD*(XN*CA-HN*SA)+DZ*SD*(XN*CK-HN*SK)
0126  F3=DX*(CD*CKP*(-XN*CA+HN*SA)+DZ*SD*(XN*CK+HN*SK)

FORMULATE PARTIALS

0127  2 +DY*(-CD*SKP*(-XN*CA+HN*SA)+DZ*SKP-HP*CD*CA)
0128  3 +DZ*(CD*SA*(-XN*SK-HN*CK)+HP*CKP+SKP)
0129  F5=(DX+SPK-DY*CKP)*CD
0130  F5=F5+DZ*CD*(XN*SK+HN*CK)*CKP
0131  F5=F5+DZ*CD*(-XN*SK-HN*CK)+DZ*SD*(XN*CK-HN*SK)
0132  F6=TK*F5
0133  F7=CD*CKP
0134  F7=F7+XP*SD
0135  F7=F7+SN*(-XN*CK+HN*SK)
0136  F8=-CD*SPK
0137  F8=F8+HP*SD
0138  F8=F8+SD*(XN*SK+HN*CK)
0139  F9=SD
0140  F9=FR+SD*(XN*SK+HN*CK)
0141  F9=FR+SD*(XN*CA-HN*SA)
0142  F10=CD*(IX*SPK-DY*CKP)
0143  F10=F10+DX*CD*(-XN*SPK+HP*CKP)
0144  F10=F10+DX*CD*(-XN*CA-HN*CA)
0145  F11=-DX*CD*(-XN*CKP+HP*SKP)
0146  F11=F11+DX*CD*(-XN*CK+HN*SK)+DZ*SD*(XN*SPK+HP*CKP)
0147  F11=F11+DX*CD*(-XN*CK+HN*SK)+DZ*CD*(XN*SK+HN*CK)

FORMULATE PARTIALS

0148  F12=1.0
0149  F12=1.0
0150  F12=1.0
0151  F12=1.0
0152  F12=1.0
0153  F12=1.0
0154  F12=1.0
0155  F12=1.0
0156  F12=1.0
0157  F12=1.0
0158  F12=1.0
0159  F12=1.0
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0160 G3=G3+DZ*SD*(-XP*SK+HP*CK)
0161 G3=G3+DZ*G3
0162 G4=DX*CK+SD-DY*SK*SD
0163 G4=G4+DZ*SD*(-XP*CK-HP*SK)
0164 G4=G4+DZ*SA*(-XP*SK-HN*CK)+DY*CD*SA*(-XN*CK+HN*SK)
0165 G4=OMG*G4
0166 G5=DX*CK*SD-DY*SK*SD
0167 G5=G5+DZ*CD*(XP*SK+HP*CK)
0168 G5=G5+DX*SD*(XN*CK-HN*SK)-DY*SD*(XN*SK+HN*CK)
0169 G5=OMG*G5
0170 G6=DK*G5
0171 G7=CD*G5
0172 G7=G7+SD*(XN*SK+HN*CK)
0173 G7=G7+OMG*G7
0174 G8=CD*G7
0175 G6=G6+SD*(XN*CK-HN*SK)
0176 G8=OMG*G8
0177 G9=CD*G8
0178 G9=G9+CD*(XP*SK-HP*CK)
0179 G9=G9+OMG*G9
0180 G10=DX*CD*CKP-DY*CD*SKP
0181 G10=G10+DZ*CD*(XP*CKP+HP*SKP)
0182 G10=OMG*G10
0183 G11=DX*CD*CKP-DY*CD*SKP
0184 G11=G11+DZ*CD*(XP*CKP+HP*SKP)
0185 G11=G11+DX*CD*(XN*CK-HN*CK)+DY*CD*(XN*SK+HN*CK)
0186 G11=OMG*G11
0187 G12=0.00
0188 G13=0.00

C A, B ARE ROWS OF THE DESIGN MATRIX CORRESPONDING TO DELAY
C AND DELAY DERIVATIVE OBSERVATIONS RESPECTIVELY
C INITIATE A AND B TO ZERO
C
DO 8 II=1, KK
A(II)=0.00
B(II)=0.00
8 CONTINUE

C SET PARTIALS IN APPROPRIATE LOCATIONS OF A AND B ROWS
C
0193 A(1)=F1
0194 A(2)=F2
0195 A(3)=F3
0196 A(4)=F4
0197 A(5)=F5
0198 A(6)=F6

B(1)=G1
B(2)=G2
B(3)=G3
B(4)=G4
B(5)=G5
B(6)=G6

C I1=I1+4
0205 I2=I1+1
C
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0207 I3=I1+2
0208 A(I1)=-F7
0209 A(I2)=-F6
0210 A(I3)=-F5

0211 B(I1)=G7
0212 B(I2)=G8
0213 B(I3)=G9
0214 IC1=K+I
0215 IC2=IC1+I
0216 A(IC1)=-F12
0217 A(IC2)=-F13

0218 C B(IC1)=-G12
0219 C B(IC2)=-G13

0220 J1=3*J+4
0221 J2=J1+1
0222 J3=J1+2
0223 A(J1)= F7
0224 A(J2)= F8
0225 A(J3)= F9

0226 B(J1)=G7
0227 B(J2)=G8
0228 B(J3)=G9

0229 JC1=K+J
0230 JC2=JC1+I
0231 A(JC1)=-F12
0232 A(JC2)=-F13

0233 C B(JC1)=G12
0234 C B(JC2)=G13

0235 IP1=3*JN+2*IP+5
0236 IP2=IP1+1
0237 A(IP1)=F10
0238 A(IP2)=F11

0239 B(IP1)=G10
0240 B(IP2)=G11

C FORMULATION OF L VECTOR ENTRY

0241 *****
0242 L=DSO-DS

C STORE PARTIALS / NON-ZERO ELEMENTS OF A 1 8 ROWS ON DISK 8 FOR
C LATER USE IN COMPUTATION OF RESIDUALS AND VTPV
C ADD CONTRIBUTIONS OF CURRENT OBSERVATIONS TO NORMAL EQUATIONS
C

C
WRITE(8,351) F1,F2,F3,F4,F5,F6,F7,F8,F9,F10,F11,F12,F13,L
0245    351 FORMAT(10X,F40.15)

DO 12 II=1,KK
0246     U(II)=U(II)-A(II)*L
0248 DO 12 JJ=1,KK
0249     NKEEP(II,JJ)=NKEEP(II,JJ)+A(II)*A(JJ)
0250 12 CONTINUE

DO 65 II=1,KK
0251     U(II)=U(II)-A(II)*L
0253 DO 65 JJ=1,KK
0254     NKEEP(II,JJ)=NKEEP(II,JJ)+A(II)*A(JJ)
0255 65 CONTINUE

WRITE(8,351) G1,G2,G3,G4,G5,G6,G7,G8,G9,G10,G11,G12,G13,L
0256    351 FORMAT(IOX,F40.I5)

DO 12 JJ=IKK
0257     NKEEP(II,JJ)=NKEEP(II,JJ)+A(I)*A(JJ)
0258 12 CONTINUE

FRNGO=P*X*G7+DY*G8+DZ*G9
FRNGO=FRNGO+(DD(J)-DD(I))/24.DO

WRITE(B,351) G12,G3,G4,G5,G6,G7,G8,G9,G10,G11,G12,G13,L
0260    351 FORMAT(IOX,F40.I5)

DO 100 65 II=1,KK
0261     U(II)=U(II)-B(XI)*L*PF
0263 65 CONTINUE

DO 182 I=1,IOPT
0264     N(I,J)=NKEEP(I,J)
0266 182 CONTINUE

INI=7+IN*3
DO 2 INI=1,K
0267     N(I,I)=N(I,I)+PRATIO(I)*T
0269 2 CONTINUE

CALLDMINV(NKR,DET,L1,L2)
DO 187 I=1,KK
0271     XX(I)=0.DO
0273 187 CONTINUE
DO 14 I=1,IKK
XX(I)=XX(I)+N(I,J)*U(J)
CONTINUE

C WRITE XX

DO 9 I=1,5
CALL REDPI(XX(I))
CONTINUE
C ISTART=7+3*IN
DO 26 I=ISTART,K
CALL REDPI(XX(I))
CONTINUE

C READ A AND B (ROWS OF DESIGN MATRIX) FROM DISK 8,
FORMULATE RESIDUALS AND COMPUTE VARIANCE COVARIANCE MATRIX

REWIND 8
VTPV=0.00D0
VABS1=0.00
VABS2=0.00
DO 22 II=1,ICOUNT
READ(8,100) I,J,IP
DO 27 IDUM=1,2
READ(351) (F(JJ),JJ=1,13),L
V=0.0D0
DO 23 JJ=1,6
V=V+F(JJ)*XX(JJ)
CONTINUE
I1=2*I+4
I2=I1+1
I3=I1+2
J1=3*J+4
J2=J1+1
J3=J1+2
V=V+F(7)*(XX(J1)-XX(I1))+F(8)*(XX(J2)-XX(I2))
2+F(9)*(XX(J3)-XX(I3))
IP1=3*IN+2*IP+5
IP2=IP1+1
V=V+F(10)*XX(IP1)+F(11)*XX(IP2)
IC1=K+I
IC2=IC1+IN
JC1=K+J
0317 JC2=JC1+IN
0318 V=V+F(12)*(XX(JC1)-XX(IC1))+F(13)*(XX(JC2)-XX(IC2))
0319 C
0320 Y=V+L
0321 C
0322 IF(IDUM.EQ.2) GO TO 28
0323 VARS1=VARS1+DABS(V)
0324 VTPV=VTPV+V**2
0325 27 CONTINUE
0326 C
0327 22 CONTINUE
0328 C
0329 VABS2=VABS2+DABS(V)
0330 VTPV=VTPV+PF*V**2
0331 27 CONTINUE
0332 C
0333 ALTERNATIVE FOR S0 COMPUTATION
0334 C
0335 DO 172 I=IN1,K
0336 VTPV=VTPV+PRATIO*XX(I)**2
0337 CONTINUE
0338 C
0339 DO 24 I=1,KK
0340 DO 24 J=I,KK
0341 N(I,J)=N(I,J)*S0
0342 CONTINUE
0343 C
0344 DO 41 J=1,KK
0345 N(1,J)=RHO*N(1,J)
0346 N(2,J)=RHO*N(2,J)
0347 N(3,J)=RHO*N(3,J)
0348 N(4,J)=RHO*N(4,J)
0349 N(5,J)=RHO*N(5,J)
0350 C
0351 DO 42 I=7,KIN
0352 N(I,J)=N(I,J)*RHO
0353 CONTINUE
0354 C
0355 DO 43 I=IN1,K
0356 N(I,J)=N(I,J)*RHO
0357 CONTINUE
0358 C
0359 DO 47 I=1,KIN
0360 N(I,J)=N(I,J)/CLIGHT*1000*DO
DO 48 I=KIN1,KK
   N(I,J)=(N(I,J)/CLIGHT)*1000.DO
   CONTINUE

DO 44 I=1,KK
   N(I,1)=R100*N(I,1)
   N(I,2)=R100*N(I,2)
   N(I,3)=R100*N(I,3)
   N(I,4)=R100*N(I,4)
   N(I,5)=RASM3*N(I,5)
   N(I,6)=RHROM9*N(I,6)
   CONTINUE

DO 45 J=7,IIN
   N(I,J)=N(I,J)*R100
   CONTINUE

DO 46 J=IIN1,K
   N(I,J)=N(I,J)*RASM3
   CONTINUE

DO 49 J=K1,KIN
   N(I,J)=(N(I,J)/CLIGHT)*1000.DO
   CONTINUE

DO 52 J=KIN,KK
   N(I,J)=(N(I,J)/CLIGHT)*100.DO
   CONTINUE

FORMAT(IOX,3F20.4)
FORMAT(IOX,2F22.6)
FORMAT(SXF40.9)

WRITE(6,696) ICOUNT,ICNT2,KK,ICNTZ,ICNT2-IDF
FORMAT('NO OF OBSERVATIONS =',I5,' DEGREES OF FREEDOM =',I5)

WRITE(6,850) EXP,SDBD,SDBF,SCQUA(IW)
FORMAT('EXPERIMENT NO =',I5,' A PRIORI STANDARD DEVIATIONS',F15.5,' DEVIATIONS',F15.5)

WRITE(6,356) (N(I,I),I=1,6)
FORMAT('STANDARD DEVIATIONS',F15.5)

WRITE(6,356) (7X,F15.5,3X,CM',10X,F15.5,3X,CM',/26X,F15.5,3X,CM')
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MAIN

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5. *PRECESSION-NUTATION = */F14.5,3X,*CM*/,/26X,F14.5,3X,*CM*/,/5X
4. *THETA ZERO = */F14.5,3X,*ARCSEC/1000*/,/5X
3. *OMG = */16X,F14.5,3X,*REV PER DAY/1000*/,/10X
2. *STATION COORDINATES (X,Y,Z) IN CM*/

397 WRITE(6,357) (N(I,I),I=I,I,N)
357 FORMAT(IOX,3F20.5)
0399 WRITE(6,359)
0400 358 FORMAT(//IOX,'QUASAR COORDINATES (RA,DEC) IN ARCSEC/1000')
0401 WRITE(6,359) (N(I,I),I=I,I,N1,K)
0402 359 FORMAT(10X,2F20.5)
C
0403 WRITE(6,361)
0404 361 FORMAT(//10X,'STATION CLOCK OFFSETS IN MSEC')
0405 WRITE(6,362) (I(I,I),I=I,K,K)
0406 362 FORMAT(2X,F20.9)
0407 WRITE(6,363)
0408 363 FORMAT(//10X,'STATION CLOCK DRIFTS IN MSEC PER DAY')
0409 WRITE(6,362) (N(I,I),I=K,K,1,KK)
C
C
0410 CONTINUE
C
0411 STOP
0412 END
3. Name of Program: PROG02

Function:

Adjustment of observations with a time span equal to the station step. Station coordinates are considered constant over the whole set of observations, while earth rotation parameters are treated as constant but different over a prearranged set of subintervals.

Input:

Standard deviations of observations and radio source coordinates, approximate values of parameters, station and radio source identification numbers, distance and distance rate observations.

Output:

Standard deviations of adjusted parameters.

Subroutines required:

MADD, MATPRO, TRAPRO, PROTRA for matrix operations and REDPI.

(Following is a listing of PROG02 and supporting subroutines, except for REDPI already included in VLBI SIMULATOR.)
PROGRAM VLBI ADJUSTMENT PROGRAM

** THIS VERSION PERFORMS THE ADJUSTMENT OF **
** A SET OF OBSERVATIONS TREATING STATION **
** COORDINATES AS CONSTANTS. EARTH ROTATION **
** PARAMETERS ARE CONSTANT BUT DIFFERENT **
** OVER SUBINTERVALS OF THE ORIGINAL TOTAL **
** TIME INTERVAL OF THE AVAILABLE OBSERVATIONS. **

** OBSERVATIONS: DELAY DISTANCES AND **
** THEIR DERIVATIVES **

** UNKNOWNS: EARTH MOTION PARAMETERS **
** STATION & QUASAR COORDINATES **
** CLOCK OFFSETS AND DRIFTS **

** SYSTEMS DEFINITION: **
** QUASAR FIXED: WEIGHTED QUASAR COORDINATES **
** EARTH FIXED: BY MEANS OF INNER CONSTRAINTS **

** SUBROUTINES REQUIRED: **

REDPI
MADD
MATPRO
TRAPRO
PROTRA

** WHERE: K=5*N+IN, IN=NO OF STATIONS, IN=NO OF QUASARS **

EQUIVALENCE (HTKD, F8), (UD1, L1K, GB1), (RL1, L2K, FET), (R1, GB)
READ EXPERIMENT IDENTIFICATION NO
RFAD(5,100) IEXP
WRITE(6,977) IEXP
FORMAT(*11///10X,*EXPERIMENT NO =',I5/10X
?**********(**********11///)
C
C
READ A PRIORI STANDARD DEVIATIONS:
SDBD : OF DELAY DISTANCE IN CM
SCBF : OF DELAY DISTANCE DERIVATIVE IN M/HOUR
SOQUA: OF QUASAR COORDINATES IN CM
C
READ(5,101) SDBD,SCBF
FORMAT(*15.5)
C
WRITE(6,240) SDBD,SCBF
FORMAT(*10X,*OBSERVATIONAL NOISE*/15X,*DISTANCES
*10.5/15X,*DISTANCE RATES */,F10.5//)
C
READ(5,101) SOQUA
FORMAT(*20*
PRATIO=SCED**2/SDQUA**2
C
READ EARTH STEP INTERVAL IN HOURS
C
READ(5,101) ERSTEP
WRITE(6,243) ERSTEP
FORMAT(*10X,*EARTH STEP INTERVAL (HRS) =',F10.2//)
C
EH=6371000.DO
CLIGHT=3.DO*(10.DO**8)
CLIGHT=CLIGHT/R
PI=4.DO*ATAN(1.DO)
RI00=R*10.DO
RHM9=(12.DO/PI)*(10.DO**9)
RASH3=(180.DO/PI)*3600000.DO
C
PF=(SDOD/RIOC)/(SD5F/R)
PF=PF**2
PF IS NOW IN (EARTH RADII/HOUR)**-2, PRATIO IN (EARTH RADII)**-2
C
READ IN= NO OF STATIONS & IM= NO OF QUASARS
C**************************************************************************************
RFAD(5,100) IN, IM
FORMAT(*15)
WRITE(6,920) IN,IM
FORMAT(*5X,*NO OF STATIONS =',I5/5X,*NO OF QUASARS =',I5)
K=5*IN+2*IM
K=K+6
KP=K+8
C
CONTINUE
C
DO 51 I=1,IN
DM(I)=0.DO
D0(1)=0.DO
CONTINUE
C
READ APPROXIMATE VALUES OF PARAMETERS THETA ZERO, OMG,
C X(I), Y(I), Z(I), (I=1,IN), RA(J), D(J), (J=1,IM)
C ANGLES ARE IN DEGREES, DISTANCES IN METERS AND OMG IN RADIANS PER HOUR
C
0626 READ(5,101) TH
0037 WRITE(6,100) TH
0038 660 FORMAT(/5X,'THETA ZERO = *,F20.9/)
0039 THE0=TY*PI/180.DO
0040 OMG=PI/12.DO
C
0041 WRITE(6,450)
0042 450 FORMAT(/,5X,*STATION & RADIO SOURCES APPROXIMATE COORDINATES*
C **/2X,*STATIONS,10X,*LONG*
C **/2X,*LAT*,11X,*R*,//)
C
0043 DO 9 I=1,IN
0044 READ(5,701) IDM,I11,I12,ALONG,APHI,AR
0045 WRITE(6,701) I,I11,I12,APHI,AR
0046 701 FORMAT(1X,I2,1X,5F15.1)
0047 APHI=APHI*PI/180.DO
0048 ALONG=ALONG*PI/180.DO
C CONVERT TO CARTESIAN
C
0049 X(I)=AR*DCOS(APHI)*DCOS(ALONG)
0050 Y(I)=AR*DCOS(APHI)*DSIN(ALONG)
0051 Z(I)=AR*DSINC(APHI)
C CONVERT TO EARTH RADII UNITS
C
0052 X(T)=X(I)/R
0053 Y(T)=Y(I)/R
0054 Z(T)=Z(I)/R
0055 9 CONTINUE
C
0056 WRITE(6,451)
0057 451 FORMAT(/,5X,*RADIO SOURCE*,10X,*RA*,17X,*DEC*,//)
C
0058 6 J=1,IM
0059 READ(5,321) IDM,I11,I12,IL3,RA(J),D(J)
0060 WRITE(6,321) J,I11,I12,IL3,RA(J),D(J)
0061 321 FORMAT(1X,13,7X,3A4,F17.7,F20.9)
C CONVERT TO RADIAN UNITS
C
0062 RA(J)=RA(J)*PI/180.DO
0063 D(J)=D(J)*PI/180.DO
0064 6 CONTINUE
C
C FORMULATE CONSTRAINT MATRICES ED, EDO
C
0065 DO 11 J=1,6
0066 DO 9 I=1,6
0067 EDO(I,J)=0.DO
0068 9 CONTINUE
0069 DO 10 I=1,K
0070 EDO(I,J)=0.DO
0071 10 CONTINUE
C
0072 EDO(1,5)=1.DO
0073 EDO(2,4)=1.DO
0074 EDO(5,6)=1.DO
C
0075 DO 3 I=1,IN
0076 IE1=I+I-2
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IE2=IE1+1
IE3=IE2+1
ED(IP1,1)=1.0D0
ED(IP7+2)=1.0D0
ED(IP3,3)=1.0D0
ED(IP2,4)=-Z(I)
ED(IP3,4)=Y(I)
ED(IP1,5)=Z(I)
ED(IP3,5)=X(I)
ED(IP1,6)=Y(I)
ED(IP2,6)=X(I)
IC1=3*I+2+IN
IC2=IC1+IN
ED(IC1,7)=1.0D0
ED(IC2,8)=1.0D0
CONTINUE
ICCONT=0
ICNT=0

CONTINUE

IC = INITIATE H, RK, RL, RT, UD, OB TO ZERO

CONTINUE

DO 72 I=1,K
VL(I)=0.0D0
UD(I)=0.0D0
J=1,6
W(J,I)=0.0D0
CONTINUE

DO 73 I=1,K
RT(I)=0.0D0
J=1,6
SK(I,J)=0.0D0
CONTINUE

DO 73 I=1,K
RT(I)=0.0D0
J=1,6
NKEEP(I,J)=0.0D0
CONTINUE

TH=THET+(ERSTEP/12.00)*PI*FLOAT(IDCNT)

READ OBSERVATIONS AND IDENTIFIERS:

DISTANCE IN METERS OF IU BASELINE OBSERVATION TO QUASAR IP
AT TIME TK (IN HOURS) AFTER SOME INITIAL EPOCH TO
READ(5,104) I,J,IP,DS,FRNG,TK,ICHEK
FORMAT(315,3F15.5,2T5)

ICHEK IS A CODE INDICATING END OF DATA FOR ICHEK=1

IF(ICHEK.EQ.1) GO TO 66
IF(ICHEK.EQ.2) GO TO 66
ICDNT=ICDNT+1

TK=TK-FRSTEP*DFLOAT(IDCNT)

DS=DS/R
FRNG=FRNG/R
DX=K(J)-X(I)
DY=Y(J)-Y(I)
DZ=Z(J)-Z(I)
CA=COS(RA(IP))
SA=SCSIN(RA(IP))
CD=COS(D(IP))
SD=SCSIN(D(IP))
Y1=OMG+TK+TH
Y2=RA(IP)
CK=COS(Y1)
SK=SCSIN(Y1)

FORMULATE PARTIALS

F1=-CD*Z*CKP+DX*SD
F2=-CD*DZ*CKP-GY+SD
F3=-DX*SD*CK+DY*SK*SD+DZ*CD*CA
F4=DX*SK*SD+DY*CK*SD-DZ*CD*SA
F5=-(DX*SKP+DY*CKP)*CD
F6=TK*F5
F10=CD*(DX*SKP+DY*CKP)
F11=(-DX*SKP+DY*CKP)*SD+DZ*CD
F12=1.0
F13=TK/24.0

G1=DZ*CD+SKP
G1=OMG*G1
G2=-DZ*CD+SKP
G2=OMG*G2
G3=(DX*SK+DY*CK)*SD
G3=OMG*G3
G4=DX*CK+SD-DY*SK+SD
G4=OMG*G4
G5=(-DX*SKP+DY*CKP)*CD
G6=TG*G6
G7=CD*SKP
G7=OMG*G7
G8=CD*SKP
G8=OMG*G8
G9=0.0
G10 = DX*CD*CKP - DY*CD*SKP
G11 = DX*SD*SKP + DY*SD*CKP
G12 = 0.0
G13 = 1.0D0/24.0D0

C **INITIATE A AND B TO ZERO**
C*********************************************************************************
C
DD B II = 1, KK
A(II) = 0.0
R(II) = 0.0
CONTINUE

C **SET PARTIALS IN APPROPRIATE LOCATIONS OF A ROW**
C*********************************************************************************
C
A(1) = F1
A(2) = F2
A(3) = F3
A(4) = F4
A(5) = F5
A(6) = F6
B(1) = G1
B(2) = G2
B(3) = G3
B(4) = G4
C(5) = G5
C(6) = G6
I1 = 3*I + 4
I2 = I1 + 1
I3 = I1 + 2
A(I1) = -F7
A(I2) = -F8
A(I3) = -F9
R(I1) = -G7
B(I2) = -G8
B(I3) = -G9
J1 = 3*J + 4
J2 = J1 + 1
J3 = J1 + 2
A(J1) = F7
A(J2) = F8
A(J3) = F9
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0209 B(J1)=G7
0210 B(J2)=G8
0211 B(J3)=G9

C
0212 JC1=6+3*IN+2*IM+J
0213 JC2=JC1+IN
0214 A(JC1)=F12
0215 A(JC2)=F13

C
0216 B(JC1)=G12
0217 B(JC2)=G13

C
0218 IP1=3*IN+2*IP+5
0219 IP2=IP1+1
0220 A(IP1)=F10
0221 A(IP2)=F11

C
0222 B(IP1)=G10
0223 B(IP2)=G11

C FORMULATION OF L VECTOR ENTRY
C
0224 50 DS0=DX*F7+DY*F8+DZ*F9
0225 DS0=DS0+DO(J)-DO(J)-DD(D(J)-DD(I))]*(TK/24.DO)
0226 L=DS0-DS
0227 LW1=L*W100

C
C*************************************************************************
C STORE A, B, L AND OBSERVATION IDENTIFIERS ON DISK 8 FOR LATER
C USE IN COMPUTATION OF RESIDUALS AND COVARIANCES
C*************************************************************************
C
0227 WRITE(8,100) ICHEK,I,J,IP
0228 WRITE(8,351) F1,F2,F3,F4,F5,F6,F7,F8,F9,F10,F11,F12,F13,L
0229 351 FORMAT(10X,F40.15)

0230 DO 12 IT=1,IK
0231 U(II)=U(II)+U(A(II))*L
0232 DD 12 JJ=1,KK
0233 NKEEP11,JJ)=NKEEP(II,JJ)+B(II)*B(JJ)*PF
0234 12 CONTINUE

C
0235 FRNGO=DX*G7+DY*G8+DZ*G9
0236 FRNGO=FRNGO*(DD(J)-DD(I))/24.DO
0237 L=FRNGO-FRNG
0238 WRITE(6,350) LW1,LW2
0239 350 FORMAT(6OX,2F19.9)

0238 WRITE(8,351) G1,G2,G3,G4,G5,G6,G7,G8,G9,G10,G11,G12,G13,L'
C
0245
C
0246
C
WRITE(6,100) ICHEK
C
DO 20 I=1,6
0247
UDD(I)=U(I)
0248
DO 20 J=1,6
0250
NDD(I,J)=NKEEP(I,J)
0251
CONTINUE
0252
DO 21 I=1,6
0253
DO 21 J=1,K
0254
J6=J+6
0255
NB(I,J)=NKEEP(I,J6)
0256
CONTINUE
C
DO 22 I=1,K
0257
I6=I+6
0258
UDD(I)=U(I6)
0259
CONTINUE
C
CALL DMINV(NDD,6,DFT,L16,L26)
0260
CALL MATPRO(NDD,6,NB,HI,6,6,K)
0261
CALL TRAPRO(NB,HI,RI,K,6,6,K)
0262
DO 23 I=1,K
0263
DO 23 J=1,K
0264
I6=I+6
0265
J6=J+6
0266
QB(I,J)=QB(I,J)+NKEEP(I6,J6)-RI(XI)
0267
CONTINUE
C
CALL MATPRO(NDD,UDDI,TI,6,6,1)
0268
CALL TRAPRO(NB,TI,RLI,K,6,1)
0269
CALL MADD(HI,HI,6,K)
0270
CALL MADD(RK,NDD,6,6)
0271
CALL MADD(RL,RLI,K,1)
0272
CALL MADD(AR,TI,6,1)
0273
CALL MADD(UDD,UDI,K,1)
C
C*****************************************************************************
C STORE UDDI,NDD,HI,TI ON DISK 9
C*****************************************************************************
C
DO 25 I=1,6
0274
DO 25 J=1,6
0275
WRITE(9,351) NDD(I,J)
0276
CONTINUE
C
DO 26 I=1,6
0277
DO 26 J=1,K
0278
WRITE(9,351) HI(I,J)
0279
CONTINUE
C
DO 27 I=1,6
0280
CONTINUE
C
C*****************************************************************************
C*****************************************************************************
0286          WRITF(9,351) TI1(I)
0287          CONTINUE
0288          IDCNT=IDCNT+1
0289          IF(ICHK.EQ.2) GO TO 67
0290          GO TO 55
0291          CALL TRAPRO(H,EDO,HTEDO,K,6,8)
0292          DO 28 I=1,K
0293          DO 28 J=1,8
0294          JK=J+K
0295          QB(I,JK)=ED(I,J)-HTEDO(I,J)
0296          Qn(JK,I)=Qn(I,JK)
0297          CONTINUE
0298          CALL MATPRO(RK,EDO,RKEDO,EKE,8,6,8)
0300          DO 29 I=1,8
0301          IK=I+K
0302          DO 29 J=1,8
0303          JK=J+K
0304          QB(IK,JK)=-EKE(I,J)
0305          CONTINUE
0306          IF(3*IN+1)
0307          IPL=3*IN+2*IN
0308          DO 34 J=IPF,IPF
0309          QB(I,J)=CS(I,J)+PRATIO
0310          CONTINUE
0311          CALL DMINS(GB,K8,DET,LIK,L2K)
0312          DO 3G I=1,K
0313          DO 3G J=1,8
0314          GP(I,J)=CS(I,J)
0315          CONTINUE
0316          DO 3T I=1,K
0317          DO 3T J=1,8
0318          JK=J+K
0319          FB(I,J)=GB(I,JK)
0320          CONTINUE
0321          DO 32 I=1,8
0322          IK=I+K
0323          DO 32 J=1,8
0324          JK=J+K
0325          FF(I,J)=QB(IK,JK)
0326          CONTINUE
0327          CALL MATPRO(GB,RL,GBL,K,K,1)
0328          CALL TRAPRO(EDO,RT,ETOT,E6,1)
0329          CALL MATPRO(FB,ETOT,ETOT,K,8,1)
CALL MATPRO(GB, UD, GU, K, K+1)

DO 41 I=1, K
X8(I)=GBL(I)+FET(I)-GU(I)
CONTINUE

CALL MATPRO(FET, FET, FBT, 8, 8, 1)
CALL TRAFO(FB, RL, FBT, 8, K, 1)
CALL MANO(FET, FBT, 8, 1)
CALL TRAFO(FB, RD, FBT, 8, K, 1)

DO 44 I=1, 6
FET(I)=FET(I)-FBT(I)
CONTINUE

CALL MATPRO(EDO, FET, SOL, 6, 8, 1)
CALL PROTRA(FB, EDO, FBEOT, K, 8, 6)
CALL MATPRO(EDO, FF, EDF, 6, 8, 8)
CALL PROTRA(FEOT, EDO, EDF, 6, 8, 8)

REWOIND 9
DO 60 IDAY=1, INCNT

DO 35 I=1, 6
DO 35 J=1, 6
READ(9, 351) NDD(I, J)
CONTINUE

DO 36 I=1, 6
DO 36 J=1, K
READ(9, 351) HIC(J)
CONTINUE

DO 37 I=1, 6
DO 37 J=1, K
GDH(I, J)=NDD(I, J)
CONTINUE

CALL PROTRA(HI, GBHT, K, 6)
CALL MATPRO(HI, GBHT, HGT, 6, K, 6)
CALL MANO(GBHT, HGT, 6, 6)
CALL MATPRO(FBEOT, NDD, FEN, K, 6, 6)
CALL MATPRO(HI, FEN, HFEN, 6, 6)

DO 39 I=1, 6
DO 39 J=1, 6
GDD(I, J)=GDD(I, J)+HFEN(J, I)
CONTINUE

CALL MATPRO(FET, NDD, EFEN, 6, 6, 6)
CALL MATPRO(NDD, EFEN, NFFEN, 6, 6, 6)
CALL MANO(GDD, NFFEN, 6, 6)
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```
C C
0274 DD 48 I=1,6
0275 IF=6#IDAY-6+I
0276 STD(IF)=GDD(I,I)
0277 48 CONTINUE
C C
0278 CALL MATPRO(MDD,SOL,NSOL,6+6,I)
0279 CALL MATPRO(HI,XB,HIXB,6,K,1)
0280 DO 40 I=1,6
0281 XB(I,IDAY)=TI(I)-NSOL(I)-HIXB(I)
0282 40 CONTINUE
0283 CONTINUE
C C
0284 DO 49 I=1,K
0285 IPL=6#IPCNT+I
0286 STD(IPL)=GB(I,I)
0287 49 CONTINUE
C C
0288 READ(8,100) ICHEK,IJ,IP
0289 IF(ICHEK.EQ.1) GO TO 88
0290 IF(ICHEK.EQ.2) GO TO 89
0291 IS1=3*I-2
0292 IS2=IS1+1
0293 JS1=3*J-2
0294 JS2=JS1+1
0295 ID1=3*IN+2*IM+I
0296 JD1=3*IN+2*IM+J
0297 ID2=ID1+IN
0298 JD2=JD1+IN
0299 DD 45 JJ=1,2
0300 READ(8,511) (FI,I)=I(3L)
0301 VL=L
0302 DO 42 II=1,6
0303 VL=VL*FI(I)*XB(I,IDAY)
0304 42 CONTINUE
C C
0305 V=V+F(7)*(XB(IS1)-XB(IS2))+F(8)*(XB(JS1)-XB(JS2))
0306 2+F(9)*(XB(JS2)-XB(JS1))
0307 V=V+F(10)*XB(IP)+F(11)*XB(IP2)
0308 V=V+F(12)*(XB(JD1)-XB(ID1))+F(13)*(XB(JD2)-XB(ID2))
0309 IF(JJ.EQ.2) GO TO 43
C C
V=W=VR100
```
C WRITE(*,453) VW
VTPV = VTPV + V**2
GO TO 45

C WRITE(6,454) VW
VW = VW + R
C FORTRAN 4.1
C C
C
0419 C CONTINUE
0420 GO TO 77
0421 86 IDAY = IDAY + 1
0422 GO TO 77
0423 09 IPF = 2 * IN + 1
0424 IPL = 3 * IN + 2 * IM
0425 DO 46 I = IPF, IPL
0426 VTPV = VTPV + P**V**2
VW = VW * R
WRITE(6,454) VW
FORMAT(*,F30.15)
CONTINUE
0427 46 CONTINUE
0428 C
C
IS2 = 2 * INCT
JS2 = 5 * IN + 6 * IDCT
IS3 = IS2 - JS2
IDC = (24.00/ERSTEP + O.IDO)
INC = IDCT / IDC
0432 IC = IDCT / IDC
0433 C WRITE(6,242) IDC, IS2, JS2, IS3
0434 242 FORMAT(*,10X,*DAYS OF OBSERVATION = ,15/10X,*NO OF OBSERVATIONS = *
2 * 15/10X,*NO OF UNKNOWNS = ,15/10X,*DEGREES OF FREEDOM = ,15/*)
0435 C
C
0436 C
C 1ST = 2 * INCT
C JS2 = 5 * IN + 6 * IDCT
C ID = IC = JS2 - JS2
C INC = IC = IDCT / IDC
0437 DO 47 I = 1, IPL
0438 IF(STD(I) .GT. 0.00) GO TO 999
0439 C
C
0440 999 STD(I) = DSQRT(STD(I) * S0)
C 347 CONTINUE
C
0442 C
C
0443 888 FORMAT(*, *STANDARD DEVIATIONS/*,10X,*STANDARD DEVIATIONS/*,10X,*STANDARD DEVIATIONS/*)
0444 888 FORMAT(*, **STANDARD DEVIATIONS/*,10X,**STANDARD DEVIATIONS/*,10X,**STANDARD DEVIATIONS/*)
0447 4 * *KSI (CM)*, 7X, *ETA (CM)*, 7X, *(REV/DAY)* 10**9/*)
C
0448 DO 85 IDAY = 1, IDCT
0449 IPL = 6 * IDAY
0450 IPF = IPL - 5
0451 DO 86 I = 1, 5
0452 JJ = IPF + I - 1
0453 CALL KDEPI(STD(JJ))
0454 STD(JJ) = STD(JJ) * RHO
0455 CONTINUE
0456 86 CONTINUE
0457 85 CONTINUE
0458 85 CONTINUE
0459 WRITE(6,889) IDAY, STD(I), J = IPF, IPL
0460 889 FORMAT(*, 1X, I4, 6F15.5)
0461 CONTINUE
2 * 2 * 15X, *Z* *X*, *(CM)*, 6X, *(CM)*, 6X, *(CM)*)
IPF=6*IDCNT+1
IPL=6*IDCNT+3*IN+2*IM
II=IPL-2*IM+1
DO 91 I=II,IPL
CALL REDPI(STD(I))
91 CONTINUE

DO 75 I=IPF,IPL
STD(I)=STD(I)*100
CONTINUE

IPL=IPL-2*IM
WRITE(6,222) (STD(I),I=IPF,IPL)
222 FORMAT(IOX,3F10.2)

WRITE(6,223)
223 FORMAT(///IOX,'OUASAR COORDINATES',//16X,'RA*',8X,'DEC',//,
2X,'(CM)',6X,'(CM)'/)

IF=IPL+I
IPL=IPL+2*IM
WRITE(6,224) (STD(I),I=IPF,IPL)
224 FORMAT(IOX,2F10.2)

WRITE(6,225)
225 FORMAT(///IOX,'STATION CLOCK OFFSETS AND DRIFTS',//13X,
2X,'(MSEC)',6X,'(MSEC/DAY)'/)

DO 76 I=I,IN
IC1=IPL+I
IC2=IC1+IN
STC(IC1)=(STD(IC1)/CLIGHT)*1000.DO
STC(IC2)=(STD(IC2)/CLIGHT)*1000.DO
WRITE(6,226) STD(IC1),STD(IC2)
226 FORMAT(4X,2F16.12)
CONTINUE

DO 76 I=I,IN
IC1=IPL+I
IC2=IC1+IN
STC(IC1)=(STD(IC1)/CLIGHT)*1000.DO
STC(IC2)=(STD(IC2)/CLIGHT)*1000.DO
WRITE(6,226) STD(IC1),STD(IC2)
226 FORMAT(4X,2F16.12)
CONTINUE

STOP
DEBJG SUBCHK
END
SUBROUTINE MADD(A,B,L,M)

      A = A + B
      L*M  L*M  L*M

IMPLICIT REAL*8(A-H,O-Z)
DIMENSION A(L,M),B(L,M)

DO 5 I=1,L
   DO 5 J=1,M
      A(I,J)=A(I,J)+B(I,J)
   5 CONTINUE
RETURN
END
SUBROUTINE MATPRO(A,B,C,L,M,N)

C    C = A * B
C    L*N   L*M M*N

IMPLICIT RFAL*8(A-H,O-Z)
DIMENSION A(L,M),B(M,N),C(L,N)

DO 5 I=1,L
    DO 5 J=1,N
        C(I,J)=0.0
    DO 5 K=1,M
        C(I,J)=C(I,J)+A(I,K)*B(K,J)
    CONTINUE
5 CONTINUE
RETURN
END
SUBROUTINE TRAPRO(A, B, C, L, M, N)

      C = A * B
      L*N   L*M M*N

IMPLICIT REAL*8(A-H,O-Z)
DIMENSION A(M, L), B(M, N), C(L, N)

DO 5 I=1, L
   DO 5 J=1, N
   C(I, J) = 0.0
5 CONTINUE
   DO 9 K=1, M
   C(I, J) = C(I, J) + A(K, I) * B(K, J)
9 CONTINUE
RETURN
END
SUBROUTINE PROTRA(A,B,C,L,M,N)

C = A * B
L=M M=N

IMPLICIT REAL*8(A-H,O-Z)

DIMENSION A(L*M),B(N*M),C(L*N)

DO 5 I=1,L
DO 5 J=1,N
C(I,J)=0.00
DO 5 K=1,M
C(I,J)=C(I,J)+A(I,K)*B(J,K)
5 CONTINUE
RETURN
END