DESIGN OF EXPERIMENT FOR EARTH ROTATION AND BASELINE PARAMETER DETERMINATION FROM VERY LONG BASELINE INTERFEROMETRY

by

Athanasios Dermanis

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The Ohio State University
Research Foundation
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PREFACE

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# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Chapter</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Preface</td>
<td>ii</td>
</tr>
<tr>
<td>1. Introduction</td>
<td>1</td>
</tr>
<tr>
<td>2. VLBI—Basic Technique and Mathematical Model</td>
<td>3</td>
</tr>
<tr>
<td>3. Earth Rotation Parametrization</td>
<td>6</td>
</tr>
<tr>
<td>4. Linearization of Observation Equations</td>
<td>11</td>
</tr>
<tr>
<td>5. Coordinate System Definition and Inner Constraints</td>
<td>16</td>
</tr>
<tr>
<td>6. Remarks Related to Design of Experiment Concepts</td>
<td>19</td>
</tr>
<tr>
<td>7. Simulation and Adjustment Philosophy</td>
<td>25</td>
</tr>
<tr>
<td>8. Arrangement of Experiments</td>
<td>28</td>
</tr>
<tr>
<td>9. Results and Conclusions</td>
<td>35</td>
</tr>
<tr>
<td>References</td>
<td>67</td>
</tr>
<tr>
<td>Appendix A: Inner Constraints and Their Relation to the &quot;Geometry&quot;</td>
<td>68</td>
</tr>
<tr>
<td>of the Pseudoinverse of an Operator</td>
<td></td>
</tr>
<tr>
<td>Appendix B: Algorithm for First-Order Partitioned Linear Regression</td>
<td>72</td>
</tr>
<tr>
<td>Including Inner Constraints</td>
<td></td>
</tr>
<tr>
<td>Appendix C: Computer Programs</td>
<td>76</td>
</tr>
</tbody>
</table>
LIST OF FIGURES

2.1 The geometry of VLBI observations ................................ 4
6.1 Geometric loci of parameter sensitivity vectors ................... 23
9.1 Geographic locations of stations and baselines in Experiments 1 through 10 ........................................ 34
9.2 Recovery of earth rotation parameters, Experiments 1 - 10 ........ 39
9.3 Recovery of baseline lengths and angles, Experiments 1 - 10 ....... 50
9.4 Variation of earth rotation parameter recovery with stepsize and the total time interval of observations (Experiment 2) ............. 61
9.5 Variation of baseline length and angle recovery with stepsize and the total time interval of observations (Experiment 2) ............. 62
9.6 Variation of earth rotation parameter recovery with earth step ....... 63
9.7 Variation of baseline length and angle recovery with earth step ....... 65
A.1 The geometry of the pseudoinverse operator ..................... 70

LIST OF TABLES

6.1 Sensitivity of Baseline - Radio Source Extreme Configurations with Respect to Various Parameters ......................... 21
8.1 Present and Prospective VLBI Station Locations and Their Approximate Coordinates ..................................... 29
8.2 Fictitious Stations Used in Experiments 6 and 9 .................... 30
8.3 Experiments and Participating Baselines ............................ 31
1. Introduction

Very Long Baseline Interferometry (VLBI) is one of the new techniques which will probably dominate geodesy and geophysics in the near future. Its main advantage lies in the fact that it brings the accuracy of direction measurements to a level previously possible only for range measurements. This closes the gap between powerful range determination techniques such as laser ranging and the much less accurate determination of directions through photographic tracking of artificial earth satellites.

The technique is geometric in the sense that the observations are independent of the gravity field of the earth. However, the "orbits" of the observed extragalactic radio sources with respect to an earth-fixed system are dominated and perturbed by the rotation of the earth with respect to inertial frame. This allows the determination of polar motion, precession-nutation and length-of-the-day variations, and the technique becomes also "dynamic" in this respect.

The capability of determining the geometry of a network of stations within a short time interval and with a centimeter level accuracy also allows the study of the variation of network geometry with time caused by earth tides and other periodic or secular station drifts.

The obvious importance of VLBI for both geodetic and geophysical applications is only limited by the effect of atmospheric refraction and unwanted noise associated with instrumentation [Shapiro and Knight, 1970, Section 3]. At present the development of the technique is at the stage of establishing its capabilities by direct comparison with classical surveys in relatively short baselines where geophysical effects are negligible, and of making the system more transportable.

In the present work the effect of nonwhite instrumentation noise, atmospheric refraction and earth tides are ignored, and the emphasis is on the possibility of recovering earth rotation and network geometry (baseline) parameters. It is assumed that nonwhite instrumentation noise is either eliminated or separately determined through calibration except for a linear effect which cannot be
distinguished from clock errors included in the adjustment parameters. Errors in the determination of atmospheric refraction are assumed to be either absent or uncorrelated in which case their effect is incorporated in the variance of the considered instrumentation white noise. The effect of earth tides on the variation of station coordinates is assumed to be calculated from separate information and subtracted from the actual observations.

The numerical simulated experiments performed here are set up in an environment where station coordinates vary with respect to inertial space according to a simulated earth rotation model similar to the actual but unknown rotation of the earth.

In Chapter 2 the basic technique of VLBI and its mathematical model are presented. The parametrization of earth rotation chosen is described in Chapter 3, and the resulting model is linearized in Chapter 4. Cartesian station coordinates are not estimable quantities in the analysis of VLBI observations. It is possible to use a model where only estimable quantities are considered (see, for example, [Arnold, 1974]). However, the choice of Cartesian coordinates leads to a simpler model, and the problem of coordinate system definition is resolved in Chapter 5 with the use of inner constraints. A simple analysis of the geometry of the observations in Chapter 6 leads to some useful hints on achieving maximum sensitivity of the observations with respect to the parameters considered. The basic philosophy for the simulation of data and their analysis through standard least squares adjustment techniques is presented in Chapter 7.

The main objective of the present work is the exploration of the capabilities of VLBI for the recovery of earth rotation and baseline parameters. For this purpose, a number of characteristic network designs based on present and candidate station locations is chosen in Chapter 8. The results of the simulations for each design are presented in Chapter 9 together with a summary of the conclusions.
2. **VLBI—Basic Technique and Mathematical Model**

Very Long Baseline Interferometry (VLBI) is a technique for observing time intervals and/or their derivatives at two widely separated antennas between the two instants of reception of the same wavefront emitted from extragalactic radio sources. The fundamental difference with respect to classical radio (short baseline) interferometry is in the fact that the antennas are not directly connected by cables for the comparison of the received signals. Sufficiently stable atomic frequency standards at each site make it possible to record the signals as functions of time. The recorded signals are then crosscorrelated afterwards in a computer. The fact that maximum correlation occurs when the two recordings are shifted in time by an amount equal to the wave travel time between the two instances of reception, makes it possible in principle to determine this time interval as well as its time derivative. Extragalactic radio sources are sufficiently far away for the wavefront to be considered as a plane and thus from the known velocity of the wave (velocity of light) the geometric distance corresponding to the observed time delay can be calculated.

The crosscorrelation procedure is described in detail in a series of articles by Thomas (1972a, 1972b and 1973), and we shall be concerned here only with the geometry of the observations. In Fig. 1, XYZ is a geocentric Cartesian reference frame fixed with respect to the radio sources (assumed to be an inertial frame). A certain wavefront arrives at station i at epoch $t_i$, when the position vector of the station is $X_i(t_i)$, and at the station j at a slightly different epoch $t_j$ when the position vector of this station is $X_j(t_j)$. The true time delay $t_j - t_i$ is the travel time corresponding to the projection $D_{ij}$ of the retarded baseline $X_j(t_j) - X_i(t_i)$ on the direction of the radio source. If $e$ is the unit vector in the radio source direction and $c$ the velocity of light,

$$D_{ij} = c (t_j - t_i) = [X_j(t_j) - X_i(t_i)] \cdot e \quad (2-1)$$
Fig. 2.1 The geometry of VLBI observations.
Since \((t_2 - t_1)\) is a very short time interval, the time variation of the position vector \(X_j(t)\) may be assumed to be solely due to the earth's rotation. Therefore

\[
X_j(t_2) = X_j(t_1) + \int_{t_1}^{t_2} \Omega(t) \times X_j(t) \, dt
\]  

(2-2)

where \(\Omega(t)\) is the rotational velocity vector of the earth. \(\Omega(t)\) is practically constant over the short time interval \((t_1 - t_2)\), and thus, by setting \(\Omega(t) = \Omega(t_1)\)

\[
X_j(t_2) = X_j(t_1) + (t_2 - t_1) [\Omega(t_1) \times X_j(t_1)]
\]

(2-3)

Setting \(B_{ij}(t) = X_j(t) - X_i(t)\) and \(V_j(t) = \Omega(t) \times X_j(t)\), one obtains

\[
D_{ij} = B_{ij}(t_1) \cdot e - \frac{1}{c} D_{ij} V_j(t_1) \cdot e
\]

(2-4)

If approximate values of \(\Omega(t_1), X_j(t_1)\) and \(e\) are known with sufficient accuracy such that the corresponding uncertainty in the retardation effect

\[
\delta D_{ij} = \frac{1}{c} D_{ij} V_j \cdot e
\]

(2-5)

is negligible, one obtains, by setting \(d_{ij} = D_{ij} + \delta D_{ij}\) and switching to matrix (column vector) notation, for an observation from the \(ij\) baseline to the radio source \(p\) at epoch \(t_k\), the much simpler model

\[
d_{ijpk} = B_{ij}^p(t_k) e_p
\]

(2-6)

Assume that \(d_{ijpk}\) has been corrected for effects giving rise to discrepancies between modeled and true distance, e.g., atmospheric errors, relativistic effects, aberration, except for clock errors. Assuming that the two local oscillators at stations \(i\) and \(j\) differ only by a constant offset \(C_{oij}\) (at some epoch \(t_0\)) and a linear drift \(C_{ij}\), the model becomes

\[
d_{ijpk} = B_{ij}^p(t_k) e_p + c [C_{oij} + C_{ij}(t_k - t_0)]
\]

(2-7)

Since the inner product of two vectors is invariant with respect to the choice of reference frame, the model may be rewritten

\[
d_{ijpk} = b_{ij}^p e_p^*(t_k) + c [C_{oij} + C_{ij}(t_k - t_0)]
\]

(2-8)
with

\[ b_{ij}^* = [x_j - x_i, y_j - y_i, z_j - z_i] \]  \hspace{1cm} (2-9)

where \( b_{ij} \) and \( e^*_p \) are the baseline and the radio source direction unit vector, respectively, with respect to an earth-fixed reference frame \( x, y, z \). One also has

\[ e^*_p(t_k) = M(t_k) e_p \]  \hspace{1cm} (2-10)

where \( M(t) \) is the earth rotation matrix of transformation from the inertial to the earth-fixed frame. The model with respect to the earth-fixed reference frame becomes

\[ d_{ij}^{tk} = b_{ij}^* M(t_k) e_p + c[C_{ij} + C_{ij}(t_k - t_0)] \]  \hspace{1cm} (2-11)

Further development of the model depends on the particular parametrization chosen for the rotation of the earth.

### 3. Earth Rotation Parametrization

The most straightforward way of parametrizing earth rotation is that of expressing the inertial to earth-fixed system transformation matrix \( M(t) \) in terms of three rotation angles at every epoch \( t \). The most logical choice seems to be that of the Eulerian angles \( \varphi, \theta, \psi \), defined as

\[ M(t) = R_3[\varphi(t)] R_2[\theta(t)] R_1[\psi(t)] \]  \hspace{1cm} (3-1)

where \( R_1, R_2, R_3 \) denote conventional rotation matrices about the \( x, y, z \) axes, respectively. The rotating earth can be viewed as a dynamical system whose state evolves in time according to a second-order differential equation of the form

\[ \frac{d^2 E}{dt^2} = f(E, t) \]  \hspace{1cm} (3-2)

where \( E = [\varphi, \theta, \psi]^T \). If such equations of motion could be exactly known and solved, their solution

\[ E(t) = F(E(t_0), \dot{E}(t_0), t) \hspace{1cm} (\dot{E} = dE/dt) \]  \hspace{1cm} (3-3)
would provide a parametrization of the transformation matrix

$$\mathbf{M}(t) = \mathbf{M}(\varphi_0, \theta_0, \psi_0, \dot{\varphi}_0, \dot{\theta}_0, \dot{\psi}_0, t_0, t)$$ (3-4)

in terms of the six initial values (integration constants):

$$\varphi_0 = \varphi(t_0) \quad \theta_0 = \theta(t_0) \quad \psi_0 = \psi(t_0)$$

$$\dot{\varphi}_0 = \frac{d\varphi}{dt} \bigg|_{t_0} \quad \dot{\theta}_0 = \frac{d\theta}{dt} \bigg|_{t_0} \quad \dot{\psi}_0 = \frac{d\psi}{dt} \bigg|_{t_0}$$

However, such an approach is not practical because of uncertainties surrounding the current knowledge of the earth's rotation. The alternative is the representation of the functions $\varphi(t)$, $\theta(t)$, $\psi(t)$ in terms of a finite number of parameters to be estimated from the observations. Such representations as polynomials, trigonometric series, etc. can sufficiently approximate a wide class of functions, provided that appropriate number of terms is included. The efficiency of the approximation can be judged only when the function to be approximated is known over some time interval, while in our case an unknown function is to be approximated. The unknown approximation error (representation error, modeling error) may well be significant compared to the observational errors, thus affecting the validity of the final parameter error estimates based upon real data analysis.

Among various finite parameter representations, the simplest perhaps is that of representing a continuous bounded function over an interval by a step function, which has a constant value over each subinterval, into which the original time interval has been partitioned.

To formally represent a step function, we introduce the characteristic or indicator function $\chi_A$ of a set $A$, defined as follows:

$$\chi_A(x) = \begin{cases} 1 & \text{if } x \in A \\ 0 & \text{if } x \notin A \end{cases}$$ (3-5)

Let $[a, b]$ be the original interval and

$$a = t_0 < t_1 < t_2 < \ldots < t_{n-1} < t_n = b$$
If \( X_i \) denotes the characteristic function of the set \([t_{i-1}, t_i]\) (i = 1, 2, ..., n-1) and \( X_n \) that of \([t_{n-1}, t_n]\), a step function with respect to the above partition of \([a, b]\) can be written in the form

\[
y(t) = \sum_{i=1}^{n} \alpha_i X_i(t)
\]

(3-6)

A continuous bounded function \( f(t) \) can be approximated arbitrarily well by a step function \( y(t) \) with an appropriate choice and increase in the number of subintervals. In the case of this investigation, the functions to be approximated are the Eulerian angles \( \phi(t) \), \( \theta(t) \), \( \psi(t) \), and the effectiveness of step function approximation may be increased if the known parts of \( \phi \), \( \theta \), \( \psi \) are filtered out. Let \( \phi^0(t) \), \( \theta^0(t) \), \( \psi^0(t) \) be the "reference" Eulerian angles as computable from classical astronomy. Then the earth rotation transformation matrix may be represented as

\[
M(t) = R_3[\phi^0 + \delta \phi] R_2[\theta^0 + \delta \theta] R_3[\psi^0 + \delta \psi]
\]

(3-7)

where \( \delta \phi(t) \), \( \delta \theta(t) \) and \( \delta \psi(t) \) are step functions. An alternative parametrization may be of the form

\[
M(t) = M^0(t) \delta M(t) = R_3(\phi^0) R_2(\theta^0) R_3(\psi^0) R_3(\delta \phi) R_3(\delta \theta) R_3(\delta \psi)
\]

(3-8)

Traditionally, earth rotation representations are separated into three parts with the help of the instantaneous rotation vector \( \omega(t) \) related to the Eulerian angles through Euler's geometric equations

\[
\begin{align*}
\omega_x &= \sin \theta \sin \phi \dot{\psi} + \cos \phi \dot{\theta} \\
\omega_y &= \sin \theta \cos \phi \dot{\psi} + \sin \phi \dot{\theta} \\
\omega_z &= \cos \theta \dot{\psi} + \phi
\end{align*}
\]

(3-9)

where \( \omega_x \), \( \omega_y \), \( \omega_z \) are the rotation vector components with respect to the earth-fixed system. The variation in the direction of the vector \( \omega \) with respect to the inertial system constitutes precession and nutation, while polar motion is the corresponding variation with respect to the earth-fixed system. The third part of earth rotation is the variation in the length of the vector \( \omega \), i.e., the variation of the rotational velocity (length of the day) \( \Omega = \| \omega \| \).
To obtain a parametrization of earth rotation where precession-nutation, polar motion and rotational velocity are separated, we introduce an intermediate moving geocentric Cartesian reference frame $x', y', z'$. The $z'$-axis direction coincides at any epoch with the direction of the instantaneous rotation axis, while the $x'$-axis direction is arbitrary. The transformation from the $X, Y, Z$ (inertial) to the $x', y', z'$ frame may be expressed in terms of three rotations:

$$R_3 (\Theta_3) R_2 (\Xi) R_1 (H)$$

and the transformation from $x', y', z'$ to the $x, y, z$ (earth-fixed) frame can be similarly expressed as

$$R_1 (-\eta) R_3 (-\xi) R_3 (\Theta_1)$$

Setting $\Theta = \Theta_1 + \Theta_2$, the total transformation from the inertial to the earth-fixed frame becomes

$$M = R_1 (-\eta) R_3 (-\xi) R_3 (\Theta) R_3 (\Xi) R_1 (H)$$

(3-10)

where $\eta$, $\xi$, $\Theta$, $\Xi$, $H$ are all functions of time. (Note that the $\Theta_1$ and $\Theta_2$ components of $\Theta$ cannot be separated, and this justifies the arbitrariness of the $x'$-axis direction.)

In order to introduce the instantaneous rotational velocity $\Omega$ in the model, noting that the $R_3 (\Theta)$ rotation is about the instantaneous rotation axis (diurnal rotation), one may set

$$\Theta (t) = \Theta_0 + \int_{t_0}^{t} \Omega (\tau) d\tau$$

$$\Theta_0 = \Theta (t_0)$$

(3-11)

The three angles $\Xi$, $H$, $\Theta_0$ represent the traditional precession-nutation, and the angles $\eta$ and $\xi$ the polar motion. The angle $(\Theta - \Theta_0)$ is similar to GAST (Greenwich Apparent Sidereal Time). The functions $\eta$, $\xi$, $\Omega$, $\Xi$, $H$ can now be approximated by step functions. To improve the approximation, we shall include in the transformation $M$ the traditional precession-nutation based on current knowledge, as follows:

$$M = R_1 (-\eta) R_3 (-\xi) R_3 (\Theta) R_3 (\Xi) R_1 (H) R^C$$

(3-12)

where the known part of precession and nutation may be computed from [Mueller, 1969, Ch. 4].
\[ R^0 = R_z(-\epsilon - \Delta \epsilon) \ R_3(-\Delta \psi) \ R_z(\epsilon) \ R_3(-\Omega) \ R_3(-\zeta_0) \] (3.13)

In the above expression, \( \zeta, \theta, \zeta_0 \) are the Newcomb components of precession, and \( \Delta \psi \) and \( \Delta \epsilon \) the nutations in longitude and obliquity by Woolard. The transformation \( R^0 \) defines the traditional "true" frame of reference, an intermediate, noninertial, moving reference frame \( X', Y', Z' \) of known orientation, such that the \( Z' \)-axis is at any epoch near the instantaneous rotation axis. The angles \( \Xi, H, \Theta_0 \) are thus small corrections to the reference precession-nutation parameters used in (3.13), and as such they may be considered as the transformation parameters between the "traditional (Newcomb/Woolard) true" frame and the "actual true" frame.

There are two alternatives in the step function representation of the angle \( \Theta \). One may represent \( \Theta \) as a step function with constant value \( \Theta_0 \) over the \( i \)th time interval (step), in which case

\[ \Theta(t) = \Theta_0 + \sum_{k=1}^{t-1} \Omega_k (t_k - t_{k-1}) + \Omega_1 (t - t_{i-1}) \quad \text{if} \quad t_{i-1} < t \leq t_i \] (3.14)

A second choice is to approximate \( \Theta \) over each step by a function of the form

\[ \Theta_i(t) = \Theta_{o_i} + \Omega_{r_i} (t - t_{i-1}) \quad \text{if} \quad t_{i-1} \leq t < t_i \] (3.15)

This is a weaker representation from the adjustment point of view because of the larger number of parameters \( (\Theta_{o_i}, i = 1, 2, \ldots, N) \) vs. \( \Theta_0 \), but it leads to estimates of variances of the estimates of the step function constants \( \eta_i, \xi_i, \Theta_{o_i}, \Omega_{r_i}, \Xi, H \), which are uniform (equal) over all steps \( (t_{i-1}, t_i) \). On the contrary, the first representation leads to variances which are smaller for steps in the middle and larger for steps at the ends of the original time span of the available observations.

Over each step the model for the VLBI observations becomes

\[ d_{ij,k} = b_{ij}^* \ R_z(-\eta) \ R_3(-\xi) \ R_3(\Theta_0 + \Omega (t_k - t_0)) \ R_3(\Xi) \ R_z(H) \ R^0 e_p + c [C_{oi} + C_{ij}(t_k - t_0)] \] (3.16)

where \( \eta, \xi, \Theta_0, \Omega, \Xi, H \) are constants over the considered step (the subscripts identifying the steps have been dropped for simplicity), \( t_k \) is the epoch of observation, and \( t_0 \) is the beginning of the step containing \( t_k \).
4. Linearization of Observation Equations

For the time span of available observations considered in this study (maximum of one month), the effect of precession-nutation on the experimental design (relative radio sources - stations configuration) is negligible. One can therefore drop the known transformation matrix $R^0$ from the model without any significant change on the simulation results. This simplifies somewhat the equations following, but it must be understood that appropriate modifications must be made when analyzing real data by including the effect of $R^0$.

The angles $\xi$, $\eta$, $\Xi$, $H$ are very small since both the z and Z axes (actually the noninertial Z'-axis on account of ignoring $R^0$) are or can be chosen to be near the rotation axis. The usual approximations, $\cos \alpha = 1$, $\sin \alpha = \alpha$, $\alpha^2 = 0$, for a small angle $\alpha$ may be used, leading to

$$
\begin{bmatrix}
1 & 0 & \xi \\
0 & 1 & -\eta \\
-\xi & \eta & 1
\end{bmatrix}
$$

and

$$
\begin{bmatrix}
1 & 0 & -\Xi \\
0 & 1 & H \\
\Xi & -H & 1
\end{bmatrix}
$$

The model now becomes

$$
d_{ijk} = (r_j - r_i)^T S^T (\xi, \eta) R_3 (\Theta_0 + \Omega \tau_k) S(\Xi, H) e_p + c [C_{01j} + C_{11} \tau_k] \quad (4-1)
$$

where $\tau_k = t_k - t_0$; $e_p = [\cos \delta_p \cos \alpha_p, \cos \delta_p \sin \alpha_p, \sin \delta_p]^T$, $\alpha_p$ and $\delta_p$ are the right-ascension and declination of the $p^{th}$ radio source with respect to the $X, Y, Z$ inertial frame; $r_i = [x_{i1}, y_{i1}, z_{i1}]^T$, $x_{i1}$, $y_{i1}$, $z_{i1}$ are the coordinates of the $i^{th}$ station with respect to the $x, y, z$ earth-fixed frame; and

$$
S(a, b) = \begin{bmatrix} 1 & 0 & -a \\ 0 & 1 & b \\ a & -b & 1 \end{bmatrix} \quad (4-2)
$$
The available observations \( d_{ij} \) form a vector of observations \( L_b \), corresponding to a vector of observables \( L_a = F(X_a) \), where \( X_a \) is a vector of unknown parameters. The unknowns are the station coordinates \( x_i, y_i, z_i, i = 1, 2, \ldots, N_s \), the earth rotation parameters \( \xi, \eta, \Theta_0, \Omega, \Xi, H \) (one set of six per each step), radio source coordinates \( \alpha_p, \delta_p, p = 1, 2, \ldots, N_{ra} \), and clock synchronization parameters \( C_{0ij}, C_{1ij}, i,j = 1, 2, \ldots, N_s \).

The \( d_{ij} \) components of the vector \( L_a \) are linear with respect to most of the parameters with the exception of \( \Theta_0, \Omega, \alpha_p, \) and \( \delta_p \). A completely linearized model can be obtained by means of approximate values \( X^0 \) of the parameters \( X_a \). Expansion in Taylor series and neglect of second- and higher-order terms leads to

\[
L_a = F(X^0) + \frac{\partial F(X_a)}{\partial X_a} \bigg|_{X^0} (X_a - X^0) \quad (4-3)
\]

The actual observations \( L_b \) differ from the observables \( L_a \) because of the presence of unknown observations errors \( V \), according to \( L_a = L_b + V \). Setting \( A = \frac{\partial F}{\partial X_a} \big|_{X^0}, L^0 = F(X^0), \ X = X_a - X^0, \ \text{and} \ L = L^0 - L_b \), the linearized observation equations are obtained:

\[
V = AX + L \quad (4-4)
\]

\( A \) and \( L \) are known, while \( X \) and \( V \) are unknown. The vector \( V \) is assumed to be an outcome of a random vector with \( E[V] = 0 \) and known covariance matrix \( E[VV^T] = S \).

Application of standard least squares techniques leads to a solution \( X \), minimizing the quadratic form \( V^TPV \) where \( P = S^3 \), which satisfies the "normal equations,"

\[
(A^TPA)X = -A^TPL \quad \text{or} \quad NX = U \quad (4-5)
\]

To compute the elements of the design matrix \( A \), the partial derivatives of \( d_{ij} \) with respect to the parameters are required. Introducing the notation, \( \psi_k = \Theta_0 + \Omega \tau_k, \ X_{kp} = \psi_k - \alpha_p, \ x_{ij} = x_j - x_i, \ y_{ij} = y_j - y_i, \ z_{ij} = z_j - z_i \), and dropping subscripts where confusion is not likely to arise, the partial derivatives are as follows:
\[ \frac{\partial d}{\partial x_j} = -\frac{\partial d}{\partial x_1} = \cos \delta \cos \chi + \sin \delta (-\Xi \cos \psi + H \sin \psi + \xi) \]

\[ \frac{\partial d}{\partial y_j} = -\frac{\partial d}{\partial y_1} = -\cos \delta \sin \chi + \sin \delta (\Xi \sin \psi + H \cos \psi - \eta) \]

\[ \frac{\partial d}{\partial z_j} = -\frac{\partial d}{\partial z_1} = \sin \delta - \cos \delta (\xi \cos \chi + \eta \sin \chi - \Xi \cos \alpha + H \sin \alpha) \]

\[ \frac{\partial d}{\partial \xi} = x_{11} [\sin \delta + \cos \delta (\Xi \cos \alpha - H \sin \alpha - \xi \cos \chi)] + \\
+ y_{11} [\eta \cos \delta \cos \chi] + \\
+ z_{11} [-\cos \delta \cos \chi - \sin \delta (-\Xi \cos \psi + H \sin \psi + \xi)] \]

\[ \frac{\partial d}{\partial \eta} = y_{11} [-\sin \delta + \cos \delta (\xi \cos \chi + \eta \sin \chi - \Xi \cos \alpha + H \sin \alpha)] + \\
+ z_{11} [-\cos \delta \sin \chi + \sin \delta (\Xi \sin \psi + H \cos \psi - \eta)] \]

\[ \frac{\partial d}{\partial \Theta_0} = x_{11} [-\cos \delta \sin \chi + \sin \delta (\Xi \sin \psi + H \cos \psi)] + \\
+ y_{11} [-\cos \delta \cos \chi + \sin \delta (\Xi \cos \psi - H \sin \psi)] + \\
+ z_{11} [\cos \delta (\xi \sin \chi - \eta \cos \chi)] \]

\[ \frac{\partial d}{\partial \Omega} = \tau_k \frac{\partial d}{\partial \Theta_0}, \quad \frac{\partial d}{\partial C_{11}} = c, \quad \frac{\partial d}{\partial C_{11}} = c \tau_k \]

\[ \frac{\partial d}{\partial \Xi} = x_{11} \left\{ -\cos \psi \cos \delta + \cos \delta [\xi \cos \alpha + \cos \psi (-\Xi \cos \alpha + H \sin \alpha)] \right\} + \\
+ y_{11} \left\{ \sin \psi \sin \delta + \cos \delta [-\eta \cos \alpha + \sin \psi (\Xi \cos \alpha - H \sin \alpha)] \right\} + \\
+ z_{11} [\cos \delta \cos \alpha + \sin \delta (\xi \cos \psi + \eta \sin \psi - \Xi)] \]

\[ \frac{\partial d}{\partial \Pi} = x_{11} [\sin \delta \sin \psi + \cos \delta \sin \alpha \cdot (\Xi \cos \psi - H \sin \psi - \xi)] + \\
+ y_{11} [\sin \delta \cos \psi + \cos \delta \sin \alpha \cdot (-\Xi \sin \psi - H \cos \psi + \eta)] + \\
+ z_{11} [-\cos \delta \sin \alpha + \sin \delta (-\xi \sin \psi + \eta \cos \psi - H)] \]

\[ \frac{\partial d}{\partial \alpha} = x_{11} [\cos \delta \sin \chi] + y_{11} [\cos \delta \cos \chi] + \\
+ z_{11} \cos \delta (-\xi \sin \chi + \eta \cos \chi - \Xi \sin \alpha - H \cos \alpha) \]
\[ \frac{\partial d}{\partial \delta} = x_{1j} \left[ -\sin \delta \cos \chi + \cos \delta (-\Xi \cos \psi + H \sin \psi + \xi) \right] + \\
+ y_{1j} \left[ \sin \delta \sin \chi + \cos \delta (\Xi \sin \psi + H \cos \psi - \eta) \right] + \\
+ z_{1j} \left[ \cos \delta + \sin \delta (\xi \cos \chi + \eta \sin \chi - \Xi \cos \alpha + H \sin \alpha) \right] \]  

(4-6)

A second type of possible observations is the time derivative of \( d \):

\[ f_{1kp} = \frac{\partial d_{1jk}}{\partial t} = \Omega (r_1 - r_1)' S'(\xi, \eta) P_3 R_3(\Theta_0 + \Omega \tau_k) S(\Xi, H) e_p + c C_{1j} \]  

where

\[ P_3 = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \]  

(4-7)

The related partial derivatives of \( f_{1kp} \) with respect to the parameters are as follows:

\[ \frac{1}{\Omega} \frac{\partial f}{\partial \xi} = x_{1j} [\xi \cos \delta \sin \chi] - y_{1j} [\eta \cos \delta \sin \chi] - z_{1j} [\cos \delta \sin \chi - \sin \delta (\Xi \sin \psi + H \cos \psi)] \]

\[ \frac{1}{\Omega} \frac{\partial f}{\partial \eta} = y_{1j} [\cos \delta (-\xi \sin \chi + \eta \cos \chi)] + z_{1j} [-\cos \delta \cos \chi + \sin \delta (\Xi \cos \psi - H \sin \psi)] \]

\[ \frac{1}{\Omega} \frac{\partial f}{\partial \Xi} = x_{1j} [\sin \delta \sin \psi - \cos \delta \sin \psi (-\Xi \cos \alpha + H \sin \alpha)] + y_{1j} [\sin \delta \cos \psi - \cos \delta \cos \psi (-\Xi \cos \alpha + H \sin \alpha)] + z_{1j} [\sin \delta (-\xi \sin \psi + \eta \cos \psi)] \]

\[ \frac{1}{\Omega} \frac{\partial f}{\partial H} = x_{1j} [\cos \psi \sin \delta - \cos \delta \sin \alpha (\Xi \sin \psi + H \cos \psi)] + y_{1j} [\sin \psi \sin \delta + \cos \delta \sin \alpha (-\Xi \cos \psi + H \sin \psi)] + z_{1j} [\sin \delta (-\xi \cos \psi - \eta \sin \psi)] \]
\[
\frac{1}{\Omega} \frac{\partial f}{\partial \Theta_o} = x_{1j} \left[ - \cos \delta \cos \chi + \sin \delta \left( \Xi \cos \psi - H \sin \psi \right) \right] \\
+ y_{1j} \left[ \cos \delta \sin \chi - \sin \delta \left( -\Xi \sin \psi - H \cos \psi \right) \right] \\
+ z_{1j} \left[ \cos \delta \left( \xi \cos \chi + \eta \sin \chi \right) \right]
\]

\[
\frac{1}{\Omega} \frac{\partial f}{\partial x_j} = - \frac{1}{\Omega} \frac{\partial f}{\partial x_i} = - \cos \delta \sin \chi + \sin \delta \left( \Xi \sin \psi + H \cos \psi \right)
\]

\[
\frac{1}{\Omega} \frac{\partial f}{\partial y_j} = - \frac{1}{\Omega} \frac{\partial f}{\partial y_i} = - \cos \delta \cos \chi + \sin \delta \left( \Xi \cos \psi - H \sin \psi \right)
\]

\[
\frac{1}{\Omega} \frac{\partial f}{\partial z_j} = - \frac{1}{\Omega} \frac{\partial f}{\partial z_i} = \cos \delta \left( \xi \sin \chi - \eta \cos \chi \right)
\]

\[
\frac{1}{\Omega} \frac{\partial f}{\partial \alpha} = x_{1j} \left[ \cos \delta \cos \chi \right] + y_{1j} \left[ - \cos \delta \sin \chi \right] \\
+ z_{1j} \left[ \cos \delta \left( -\xi \cos \chi - \eta \sin \chi \right) \right]
\]

\[
\frac{1}{\Omega} \frac{\partial f}{\partial \delta} = x_{1j} \left[ \sin \delta \sin \chi + \cos \delta \left( \Xi \sin \psi + H \cos \psi \right) \right] \\
+ y_{1j} \left[ \sin \delta \cos \chi + \cos \delta \left( \Xi \cos \psi - H \sin \psi \right) \right] \\
+ z_{1j} \left[ \sin \delta \left( -\xi \sin \chi + \eta \cos \chi \right) \right]
\]

\[
\frac{\partial f}{\partial \Omega} = \tau_x \frac{\partial f}{\partial \Omega_o}, \quad \frac{\partial f}{\partial \Theta_o} = 0, \quad \frac{\partial f}{\partial C_{ij}} = 0
\]

\[
(4-9)
\]
5. **Coordinate System Definition and Inner Constraints**

The combination of all observations leads to the linearized observation equations

\[ nV_1 = nA_u X_1 + nL_1 \]  

Minimization of \( V^T PV \) leads to a solution satisfying the normal equations [Uotila, 1967]

\[ NX = U \]  

where

\[ N = A^T P A \]  

\[ U = -A^T P L. \]

Because of the lack of coordinate system definition, \( N \) is singular with rank

\( (N) = r < u \), where \( u \) is the number of parameters and \( s = u - r \) is the rank deficiency of \( N \).

Originally \( s = 9 \), corresponding to the six degrees of freedom in the definition of the earth-fixed coordinate system (three for origin position and three for orientation) plus the three degrees of freedom in the inertial system definition (orientation only).

Since a radio source catalogue of a certain accuracy is assumed to be available, radio source coordinates are treated as observations (weighted unknowns) rather than as unknown parameters. The inertial frame is therefore defined through the catalogued radio source coordinates and the remaining rank deficiency of \( N \) becomes \( s = 6 \). A unique solution to the normal equations may be obtained if in addition a set of minimal linear constraints are imposed on the parameters, of the form

\[ C^T X = 0 \]  

where \( C \) is a \( u \times s \) matrix and rank \( (C) = s \).

Among the various possible solutions to the normal equations, the unique one given by \( X = N^+ U \), where \( N^+ \) is the pseudoinverse of \( N \), has the following properties [Blaha, 1971]: \( X^T X = \text{min.} \) and \( \text{trace } N^+ = \text{min.} \). In view of the interpretation of \( N^+ \) as the variance-covariance matrix of the parameters, the second property makes the solution optimal in the sense that smaller variances of the parameters provide a better representation of parameter related estimable quantities in terms of the
nonestimable parameters. To avoid the use of pseudoinverse computation algorithm, one may resort to a particular set of minimal constraints called inner constraints [Blaha, 1971],

\[ E^T X = 0 \]  

which leads to the same solution for \( X \) as the one obtained with the use of the pseudoinverse \( N^+ \). It can be shown that such a set of inner constraints can be obtained with the help of a \( u \times s \) matrix \( E \) (rank \( (E) = s \)) satisfying

\[ NE = 0 \]  

An algebraic type of proof can be found in [Pope, 1971] which settles the truth of the matter but throws little light on the interrelation between inner constraints and pseudoinverse. A different type of proof is given in Appendix A of this work, based on the geometry of the operator represented by the matrix \( N \).

In view of the fact that \( AE = 0 \implies NE = A^TPAE = 0 \), a set of inner constraints has been analytically constructed using six independent solutions to the set of equations

\[ A_j^T y = 0 \quad j = 1, 2, \ldots, u \]  

where \( A_j^T \) denotes the \( j \)th row of \( A \). We assume that the approximate values of \( \eta, \xi, \Xi, \) and \( H \) are zero in the computation of partials of observations with respect to the unknowns, thus obtaining simpler analytical expressions for the elements of \( A \).

Letting the order of the unknowns in \( X \) to be the following

\[ X^T = [d\eta, d\xi, d\Theta_0, d\Omega, dx_1, dy_1, dz_1, \ldots, dx_N, dy_N, dz_N] \]

one may set

\[ y^T = [w_1, w_2, w_3, w_4, \alpha_1, \beta_1, \gamma_1, \ldots, \alpha_N, \beta_N, \gamma_N] \]

For a row of \( A \) corresponding to a \( \mathcal{A}_{1k}p \) observation, one has

\[ \frac{\partial d}{\partial \eta} + w_2 \frac{d}{d\xi} + w_3 \frac{d}{d\Theta_0} + w_4 \frac{d}{d\Omega} + \alpha_1 \frac{d}{d\xi} + \beta_1 \frac{d}{dy_1} + \gamma_1 \frac{d}{dz_1} + \]

\[ + \alpha_j \frac{d}{dx_j} + \beta_j \frac{d}{dy_j} + \gamma_j \frac{d}{dz_j} = 0 \]  

(5-7)
With the analytical expressions of the partials and after a considerable algebraic effort, one arrives at six independent solutions $y_i$, $i = 1, 2, \ldots, 6$, and the inner constraint matrix $E$ is formulated by setting

$$E = [y_1 \ y_2 \ \ldots \ y_6]$$

The final result is

$$
\frac{d\eta}{d\eta} \ \frac{d\eta}{d\omega} \ \frac{d\theta}{d\omega} \ \frac{dx_1}{dy_1} \ \frac{dy_1}{dz_1} \ \ldots \ \frac{dx_N}{dy_N} \ \frac{dy_N}{dz_N}
$$

$$
\begin{bmatrix}
0 & 0 & 0 & 0 & 1 & 0 & 0 & \ldots & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & \ldots & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & \ldots & 0 & 0 & 1 \\
1 & 0 & 0 & 0 & 0 & -z_1^0 & y_1^0 & \ldots & 0 & -z_N^0 & y_N^0 \\
0 & 1 & 0 & 0 & z_1^0 & 0 & -x_1^0 & \ldots & z_N^0 & 0 & -x_N^0 \\
0 & 0 & 1 & 0 & -y_1^0 & x_1^0 & 0 & \ldots & -y_N^0 & x_N^0 & 0
\end{bmatrix}
$$

(5-8)

where $x_i^0$, $y_i^0$, $z_i^0$ are the approximate values of station coordinates used in the computation of the design matrix $A$. More explicitly, there are two sets of constraints. The first one

$$\sum_i \begin{pmatrix}
\frac{dx_1}{dy_1} \\
\frac{dy_1}{dz_1}
\end{pmatrix} = 0 \quad (5-9)$$

"defines" the origin of the earth-fixed system, while the second one

$$
\begin{pmatrix}
-d\eta \\
-d\zeta \\
-d\theta
\end{pmatrix} = \sum_i \begin{pmatrix}
z_i^0 & 0 & -z_i^0 \\
0 & -x_i^0 & y_i^0 \\
-y_i^0 & x_i^0 & 0
\end{pmatrix}
\begin{pmatrix}
\frac{dx_1}{dy_1} \\
\frac{dy_1}{dz_1}
\end{pmatrix} \quad (5-10)
$$

"defines" its orientation.

In the discussion above, the problem of epoch and time scale definition for the set of station clocks has been ignored. The parameters $C_{ij}$ and $C_{ij}'$ refer only to relative offsets and drifts between the clocks at stations $i$ and $j$. 18
Usually epoch and time scale are provided by the readings of one of the clocks (master clock), and they are transformed to other station clocks through the parameters $C_{opt}$, $C_m$ where the master clock is at the $m^{th}$ station.

Another way to provide epoch and time scale is the use of inner constraints defining a fictitious master clock. We may shift to a new set of parameters by setting

$$
C_{opt} = C_{opt} - C_{opt}
$$

$$
C_{ij} = C_{pl} - C_{pl}
$$

The new parameters are offsets $C_{opt}$ and drifts $C_{pl}$ with respect to the fictitious master clock, and the additional inner constraints are

$$
\sum_i dC_{opt} = 0 \quad \text{and} \quad \sum_i dC_{pl} = 0
$$

6. Remarks Related to Design of Experiment Concepts

Since the usual objective of a geodetic experiment is the estimation of certain parameters, an experimental design may be considered optimal when the errors in parameter estimates are minimized in some certain sense. Within the linear least squares model, the a posteriori variance-covariance matrix of parameter estimates is given in general by

$$
S_i = \sigma_0^2 (A^T P A)^+ = \sigma_0^2 Q^+
$$

where $Q^+$ denotes the pseudoinverse of a square matrix $Q$, $P$ is the weight matrix and $\sigma_0^2$ the a posteriori estimate of the variance of unit weight. The "errors" in parameter estimates therefore depend on the observational accuracy and on the design matrix $A$. The design matrix $A = A(X^0)$ itself depends on the approximate values of parameters $X^0$. The determination of optimal $X^0$ values minimizing some risk function (e.g., trace $S_i$) is usually referred to as the "configuration problem," or the "first-order optimal design" [Grafarend, 1974]. On the assumption that the risk function chosen is insensitive to small changes of $X^0$, the approximate values of parameters may be identified with the true ones. In a similar way the small
angles $\eta, \xi, \Xi, H$ in our model may be considered to be zero. Of the remaining parameters $\Theta_0$ depends on the choice of reference frames, $\Omega$ is approximated by the fixed average rotational velocity of the earth, while in the case of transcontinental baselines the choice of radio source coordinates $\alpha_p, \delta_p$ is strongly restricted by observability conditions. Therefore, the first-order optimal design is practically limited to the determination of the optimal configuration of the network of observing stations.

The order of magnitude of elements in a row of the design matrix $A$ determines the sensitivity of the corresponding observation with respect to each of the parameters. Next a short geometric interpretation of these partials is given to provide possible "hints" towards the determination of the optimal design.

The partial derivatives with respect to $\eta, \xi, \Theta_0, \Omega, \Xi, H$ (earth rotation parameters) and $\alpha_p, \delta_p$ are proportional to the baseline length. Indeed, they all contain linear terms in $x_{ij} = r_{ij} \overline{x}_{ij}, y_{ij} = r_{ij} \overline{y}_{ij}$ and $z_{ij} = r_{ij} \overline{z}_{ij}$, where $r_{ij}$ is the length of the $ij$ baseline and $\overline{x}_{ij}, \overline{y}_{ij}, \overline{z}_{ij}$ are the components of the unit vector $r_{ij}^0$ in the direction of the baseline. It is therefore possible to define "sensitivity per unit of baseline length" by means of

$$\frac{1}{r_{ij}} \frac{\partial d}{\partial \beta} = (r_{ij}^0)^T S_{\beta}$$

(6–2)

i.e., that the sensitivity per unit of baseline length is the projection of some vector $S_{\beta}$ (called the parameter $\beta$ sensitivity vector) on the direction of the baseline.

It can be easily verified that the parameter sensitivity vectors are given by.

$$S_\eta = - R_1 (\pi/2) P_{yz} \ R_3 (X) \ e_6$$
$$S_\xi = - R_2 (\pi/2) P_{xz} \ R_3 (X) \ e_5$$
$$S_{\Theta_0} = \tau_x^3 \ S_{\Omega} = P_{xy} \ R_3 (X + \pi/2) \ e_6$$
$$S_{\Xi} = R_3 (\psi) \ R_2 (\pi/2) \ P_{xz} \ R_3 (-\alpha) \ e_5$$
$$S_{H} = R_3 (\psi) \ R_1 (\pi/2) \ P_{yz} \ R_3 (-\alpha) \ e_5$$
$$S_{\alpha} = - R_3 (X + \pi/2) \ P_{xy} \ e_5$$
$$S_{\delta} = R_3 (X) \ R_2 (\pi/2) \ e_5$$

(6–3)
where

\[ e_5 = [\cos \delta, 0, \sin \delta]^T, \quad e_p = R_3 (-\omega) e_5, \]  

and

\[
\begin{align*}
P_{xy} &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, &
P_{yz} &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, &
P_{xz} &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}
\end{align*}
\]

are projection matrices on the xy, yz and xz planes, respectively. The position of the sensitivity vectors varies with time, and their loci, all contained within the unit sphere, are depicted in Fig. 6.1. Maximum sensitivity occurs when the baseline is parallel to the corresponding parameter sensitivity vector, while an observation is insensitive to a parameter when the baseline and parameter sensitivity vector are perpendicular. With this in mind and with the help of Fig. 6.1, Table 6.1 is constructed, summarizing the sensitivity to various parameters for extreme cases of baseline and radio source directions.

### Table 6.1

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Baseline Parallel to Equator ((z_{ij} = 0))</th>
<th>Baseline Parallel to Rotation Axis ((x_{ij} = y_{ij} = 0))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\xi, \eta)</td>
<td>no</td>
<td>yes</td>
</tr>
<tr>
<td>(\Xi, H)</td>
<td>no</td>
<td>yes</td>
</tr>
<tr>
<td>(\Theta_\alpha, \Omega, \alpha)</td>
<td>yes</td>
<td>no</td>
</tr>
<tr>
<td>(\delta)</td>
<td>no</td>
<td>yes</td>
</tr>
</tbody>
</table>
Note that observations in the fourth column are impossible due to observability conditions (a baseline parallel to the rotation axis must be located near the equator and observe equatorial radio sources only).

The sensitivity with respect to $\Omega$ as well as to $C_{ij}$ is proportional to the time span $\tau$ of the available observations while that to $C_{ij}$ and $C_{ij}'$ is independent of the baseline - radio source configuration.

Sensitivity with respect to station coordinates is independent of baseline length. If $e_x$, $e_y$, $e_z$ denote unit vectors in the directions of the x, y, z axes, respectively, the relevant partial derivatives under the same approximations are

$$\frac{\partial d}{\partial x} = e_x S_c, \quad \frac{\partial d}{\partial y} = e_y S_c, \quad \frac{\partial d}{\partial z} = e_z S_c$$

where $S_c = R_3(-\chi)e_6$. The sensitivity of observations with respect to station coordinates is determined by the projection of the coordinate sensitivity vector $S_c$ on the coordinate axes. The locus of $S_c$ is depicted in Fig. 6.1. It can readily be seen that observations to equatorial radio sources are insensitive to the $z$ station coordinates, while observations to polar radio sources are insensitive to $x$ and $y$ station coordinates.

All the above remarks are of a rather general nature and simply provide "hints" towards the design of an optimal station configuration for the VLBI experiment, which also depends on which of the parameters in particular is to be determined.
Fig. 6.1 Geometric loci of parameter sensitivity vectors.
Fig. 6.1 (cont'd)
7. Simulation and Adjustment Philosophy

The objective of this simulation is the determination of standard deviations of parameter estimates relevant to VLBI observations for a number of characteristic network designs. In addition to this design (relative configuration of station - radio sources and arrangement of observations in time), the a posteriori standard deviations of parameters also depend on a number of other factors. These include the accuracy of observations, their total time span, and the modeling of earth rotation and relative station motions in the adjustment. Since step function models are used here, the time interval over which earth rotation parameters are considered to be constant ("earth step") and the time interval over which station coordinates are considered constants ("station step") become also variables. It has also been assumed that a radio source coordinate catalogue is available, and its accuracy is one additional variable affecting final results. Summarizing, the problem is to examine the variation of standard deviations of the following parameters:

- i. station coordinates
- ii. earth rotation parameters

with respect to the following independent variables:

- a. observational noise
- b. time span of observations
- c. earth step
- d. station step
- e. accuracy of radio source catalogue
- f. station - radio source configuration, arrangement of observations in time.

With respect to the last item above, three different variables appear in the same group. This is justified in view of the fact that once a transcontinental network design has been decided upon, observability conditions impose strong limitations on both the choice of radio sources to be observed and the arrangement of observations in time.
The fact that observations are not instantaneous, but averages over some time interval (integration time of 7-10 minutes), imposes also limitations on the maximum density of observations in time.

The choice of earth and station step in real data analysis depends on the frequency content of the corresponding time functions. In view of the aliasing effect [Koopmans, 1974], the smaller the step size, the higher the spectral resolution of the corresponding functions. On the other hand, larger step size leads to a smaller number of parameters to be estimated and a "stronger" adjustment with more degrees of freedom. The choice of optimal step size in the analysis of real data constitutes a very interesting problem worthy of a separate study.

In the present simulation study, step size has been considered as a variable, while the corresponding functions generated in the simulation of observations are continuous rather than step functions. However, precautions have been taken so that the total variation $V_{f,I}$ of the function $f(t)$ in question over each step time interval $I$ is insignificant compared to observational noise. In fact,

$$V_{f,I} = \max_{t \in I} [f(t)] - \min_{t \in I} [f(t)] \leq \sigma_b/N \quad i = 1, 2, \ldots, n$$

where $\sigma_b$ is the standard deviation of the observations, and $N$ is a large-integer (specifically here $N = 10$).

In view of a 7-10-minute integration time, and allowing for antenna motion, observations (time delays and their time derivatives) are uniformly spaced at 15-minute intervals over the total time span considered. Their corresponding standard deviations $\sigma_b$ and $\sigma_b'$ are taken to be of a constant ratio $\sigma_b/\sigma_b' = 1/2$, where $\sigma_b$ is in nsec and $\sigma_b'$ in nsec/hr. This value is typical, but the ratio may be somewhat different depending on antenna and observed signal characteristics. The choice of a fixed ratio reduces the number of considered parameters by one with a corresponding significant reduction in computational effort.

Once a certain observational pattern has been decided upon, a simulation program (VLBI SIMULATOR) produces perfect observations using the simplified model (Eq. 4-1). This program requires as input station and radio source true coordinates and the pattern of the observations (baseline - radio source - epoch of
observation). The station coordinates are taken to be constants, while the values of earth rotation parameters at the epoch of observation are provided by a subroutine (SUBROUTINE EARTH). $\Theta_0$ is fixed to a realistic value for the beginning of the observations $t_0$. The effect of traditional precession-nutation is absent from the simulation as well as from the adjustment model for computational simplicity. The inertial frame thus is taken to be the true equatorial system of the epoch $t_0$ so that its $Z$-axis is near the rotational axis. For the same reason, the earth-fixed frame is taken to be the CIO/BIH zero-meridian terrestrial system.

The perfect observations are processed through a program which adds "Observational errors" drawn from a Gaussian population with zero mean and desired variance.

For the adjustment of the observations, two separate programs are used. The first one (PROG01) processes observations of total time span equal to both earth and station step (one day). An option is available for processing any desired number of different radio source catalogue accuracies in the same run.

The second program (PROG02) processes observations of time span equal to the station step only, which in turn is taken to be an integer multiple of the earth step. It is, however, possible to process observations using unequal earth steps within the total span of available observations. Within each earth step the linearized observations equations are of the form

$$V_i = \ddot{A}_i \ddot{X} + \dot{A}_i \dot{X} + L_i$$

where $\dot{X}$ refers to parameters common to all earth steps (station and radio source coordinates) and $\ddot{X}_i$ refers to parameters particular to each earth step (earth rotation parameters).

The total set of such individual observation equations is combined in a unified solution through a "second-order partitioned linear regression" scheme, extended to incorporate inner constraints. The detailed algorithm is presented in Appendix B. The related programs and subroutines are in Appendix C.
8. **Arrangement of Experiments**

To obtain more realistic results, station locations considered here are not completely arbitrary, but limited to present and prospective locations of VLBI stations. Such candidate station locations are presented in Table 8.1. In the same table the stations actually used in this study are indicated by asterisks. Two criteria have been considered in the selection. First, stations close to plate boundaries, i.e., in geophysically active areas, have been excluded to avoid possible comparatively large station drifts deviating from those considered in our simulations. Secondly, from groups of stations close to each other, only one station has been included on the assumption that final results would be similar for a nearby station. It is recognized that these criteria probably exclude the most active stations currently in operation, but this should not influence the general conclusions sought.

An experiment consists of a group of stations simultaneously participating in the observations. In Table 8.3 the station groups for the experiments considered in this study are presented. After a certain station group has been selected, the arrangement of observations is one observations every 15 minutes, which interval and choice of the radio sources are limited by observability conditions. The observability condition naturally imposed is that the observed radio source must be above the horizon at both ends of the observing baseline. To avoid large atmospheric refraction errors, the zenith distance of the radio source is limited, in general, to be less than $z_{\text{max}} = 45^\circ$, with some exceptional cases where $z_{\text{max}} = 60^\circ$.

Such observability conditions limit the observable part of the sky to within the intersection of two cones with the local station verticals as axes and $z_{\text{max}}$ as the vertex angle. For transcontinental baselines, the region of observability is small and the choice is between observing a certain quasar or a neighboring one without significant effect on the experimental design.

In setting out a certain sequence of observations, the choice between possible alternatives has been guided by the hints obtained in Chapter 6, with emphasis on sensitivity with respect to earth motion and station coordinate parameters.
Table 8.1
Present and Prospective VLBI Station Locations and Their Approximate Coordinates

<table>
<thead>
<tr>
<th>No.</th>
<th>Location</th>
<th>Longitude</th>
<th>Latitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Canberra, Australia</td>
<td>149</td>
<td>-35</td>
</tr>
<tr>
<td>2</td>
<td>Kauai, Hawaii</td>
<td>200</td>
<td>22</td>
</tr>
<tr>
<td>3</td>
<td>Fairbanks, Alaska</td>
<td>212</td>
<td>65</td>
</tr>
<tr>
<td></td>
<td>Goldstone, California</td>
<td>243</td>
<td>35</td>
</tr>
<tr>
<td></td>
<td>Algonquin, Canada</td>
<td>277</td>
<td>51</td>
</tr>
<tr>
<td></td>
<td>Greenbank, Virginia</td>
<td>282</td>
<td>38</td>
</tr>
<tr>
<td>4</td>
<td>Haystack, Massachusetts</td>
<td>288</td>
<td>41</td>
</tr>
<tr>
<td></td>
<td>Santiago, Chile</td>
<td>289</td>
<td>-34</td>
</tr>
<tr>
<td></td>
<td>Arecibo, Puerto Rico</td>
<td>294</td>
<td>18</td>
</tr>
<tr>
<td>5</td>
<td>Sao Paulo, Brazil</td>
<td>313</td>
<td>-24</td>
</tr>
<tr>
<td>6</td>
<td>Madrid, Spain</td>
<td>356</td>
<td>40</td>
</tr>
<tr>
<td></td>
<td>Bonn, Germany</td>
<td>7</td>
<td>51</td>
</tr>
<tr>
<td>7</td>
<td>Onsala, Sweden</td>
<td>17</td>
<td>59</td>
</tr>
<tr>
<td>8</td>
<td>Johannesburg, So. Africa</td>
<td>28</td>
<td>-26</td>
</tr>
<tr>
<td></td>
<td>Crimea, USSR</td>
<td>34</td>
<td>45</td>
</tr>
</tbody>
</table>

* indicates stations considered in this study
Table 8.2
Fictitious Stations Used in Experiments 6 and 9

<table>
<thead>
<tr>
<th>No.</th>
<th>Location</th>
<th>Longitude</th>
<th>Latitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>equatorial</td>
<td>0°</td>
<td>0°</td>
</tr>
<tr>
<td>10</td>
<td>&quot;</td>
<td>60</td>
<td>0</td>
</tr>
<tr>
<td>11</td>
<td>&quot;</td>
<td>120</td>
<td>0</td>
</tr>
<tr>
<td>12</td>
<td>&quot;</td>
<td>180</td>
<td>0</td>
</tr>
<tr>
<td>13</td>
<td>&quot;</td>
<td>240</td>
<td>0</td>
</tr>
<tr>
<td>14</td>
<td>&quot;</td>
<td>300</td>
<td>0</td>
</tr>
<tr>
<td>15</td>
<td>meridian</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>16</td>
<td>&quot;</td>
<td>0</td>
<td>60</td>
</tr>
<tr>
<td>17</td>
<td>&quot;</td>
<td>180</td>
<td>60</td>
</tr>
<tr>
<td>18</td>
<td>&quot;</td>
<td>180</td>
<td>0</td>
</tr>
<tr>
<td>19</td>
<td>&quot;</td>
<td>180</td>
<td>-60</td>
</tr>
<tr>
<td>20</td>
<td>&quot;</td>
<td>0</td>
<td>-60</td>
</tr>
</tbody>
</table>
Table 8.3
Experiments and Participating Baselines
(Station Nos. Refer to Numbering in Table 8.1 or 8.2)

<table>
<thead>
<tr>
<th>Experiment No.</th>
<th>Participating Baselines</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2-3, 2-4, 3-4</td>
</tr>
<tr>
<td>2</td>
<td>3-4, 3-7, 4-7</td>
</tr>
<tr>
<td>3</td>
<td>4-5, 4-6, 5-6</td>
</tr>
<tr>
<td>4</td>
<td>5-6, 5-8, 6-8</td>
</tr>
<tr>
<td>5</td>
<td>1-2, 2-3, 2-4, 3-4, 3-7, 4-5, 4-6, 4-7, 5-6, 5-8, 6-7, 6-8</td>
</tr>
<tr>
<td>6</td>
<td>9-10, 10-11, 11-12, 13-14, 14-9</td>
</tr>
<tr>
<td>7</td>
<td>2-4, 4-5</td>
</tr>
<tr>
<td>8</td>
<td>2-4, 4-6</td>
</tr>
<tr>
<td>9</td>
<td>15-16, 16-17, 17-18, 18-19, 19-20, 20-15</td>
</tr>
<tr>
<td>10</td>
<td>3-4, 4-7</td>
</tr>
</tbody>
</table>
An effort has also been made to observe radio sources of a variety of declinations to avoid the critical configuration appearing when, for example, all radio sources are of the same declination. From a larger set of available radio sources, only a rather uniformly distributed set has been considered. The criterion for inclusion has been the use of the radio source in previous experiments so that its appropriateness for VLBI observations is assured.

Since a radio source catalogue of a certain accuracy is assumed to be available, no effort has been made to optimize radio source coordinate recovery. As a result, this study is of no "astrometric" value, and no significant improvement of radio source coordinate accuracy can be found in the results of the adjustment performed.

One sidereal day can be taken as the "unit" for an observational pattern since a sequence of observations performed over one day can be repeated over the next days. For observations of a total time span of a number of days, the same daily pattern is repeated in the simulation. Various experimental designs are compared to each other on the basis of one day of observations where earth rotation and baseline parameters are considered as constants over the whole day.

The choice of experiments performed has been primarily directed towards the possibility of recovering earth rotation parameters from a minimum number of observing stations. The minimum number is the three stations necessary for the definition of an earth-fixed system through minimal or inner constraints. Experiments 1, 2, 3 and 4 are cases of such minimal "triangle networks" where all the sides of the triangle are observing baselines.

A weaker design (Experiments 7, 8, 10) is a three-station configuration where only two of the triangle sides are observing baselines. In particular, Experiment 10 is a counterpart of Experiment 2, such that the effect of removing one baseline (other design parameters remaining the same) can be studied. The opposite to such minimal three-station designs is provided in Experiment 5 where all eight stations participate in the observations.
In addition to these "realistic" station locations, two "fictitious" designs are also considered, their common characteristic being the closure of a network of stations on a great circle around the earth. Experiment 6 is such a network of uniformly spaced stations along the equator, while Experiment 9 is of a similar design along a meridian. The coordinates of stations involved in these experiments are listed in Table 8.2.

Fig. 9.1 depicts the geographic locations of the networks in the above-mentioned experiments.
Fig. 9.1 Geographic locations of stations and baselines in Experiments 1 through 10.
9. Results and Conclusions

The results of the simulations are presented in three parts. The first part (Figs. 9.2 - 9.3) deals with the comparison of the ten designs described in the previous chapter, on the basis of one day of observations. Fig. 9.2 depicts the recovery (standard deviations) of earth rotation parameters ($\xi, \eta, \Xi, \Theta_o, \Omega$), while Fig. 9-3 shows the recovery of network geometry in terms of baseline lengths and angles (estimable quantities).

Standard deviations $\sigma$ for each parameter are given as a function of the standard deviation $\sigma_Q$ of radio source coordinates and for various values of observational precision $\sigma_b$. Values used in the simulation are $\sigma_b: \sigma_Q = 1, 5, 10, 20, 50$ cm. To achieve uniformity, $\sigma_Q$ is expressed as arc length on a reference sphere with a radius of $6371$ km ($1^\prime = 30$ m).

The second part (Figs. 9.4 - 9.5) is a study of the improvement of the standard deviations of the parameters with the total time interval $T$ of available observations. Station coordinates are assumed to be constant over $T$ while earth rotation parameters are constant only over the subintervals of duration $\Delta t$ (earth step). The ratio $\rho = \sigma/\sigma_b$ is given for each parameter as a function of $T$ for various values of $\Delta t$. In the simulations, the following values have been used: $T = 1, 2, 3, 4, 5, 10, 30$ days; $\Delta t = 6, 12, 24$ hours. Experiment 2 has been chosen for this study. Radio source coordinate standard deviations are assumed to be $\sigma_Q = \sigma_b$ for all cases. However, the variation of the results with $\sigma_Q$ is insignificant for $T \geq 5$ days.

The third part (Figs. 9.6 and 9.7) is a study of the variation of $\rho = \sigma/\sigma_b$ with $\Delta t$ (earth step) based on one day of observations ($T = 1$ day) for some of the other designs (Experiments 1, 3, 4, 6, 9, 10) is shown.

Since the results of the simulations are fully presented here, one may draw his own conclusions. However, the following general remarks are made in an attempt to summarize the results:

(1) Earth Rotation Parameters from One-Day Observations

For one day of observations and the recovery of earth rotation parameters, the following are noted:

(a) Three-baseline "triangle" designs (Experiments 1, 2, 3, 4 in Fig. 9.2) are capable of recovering polar motion ($\xi, \eta$) with a precision of the same order of magnitude as that of the observations, even for low radio source accuracies ($\sigma_Q = 50$ cm).
For two-baseline designs (Experiments 7, 8, 10 in Fig. 9.2), polar motion recovery is of the same quality as for the "triangle" station configurations. However, the removal of one baseline in Experiment 10 results in an increase of polar motion standard deviations by a factor of 1.5 compared to Experiment 2. The more complex design of Experiment 5 offers no dramatic improvement in polar motion recovery. An essential improvement can be seen in the equatorial design of Experiment 6 and even more in Experiment 9 which is the "best," but admittedly not very practical design for polar motion recovery.

(b) The standard deviations of the (precession-nutation) parameters ξ and η are practically the same for all two- or three-baseline designs (Experiments 1, 2, 3, 4, 7, 8, 10). In particular, the removal of one baseline in Experiment 10 with respect to Experiment 2 has no effect on the recovery of ξ and η. The best recovery of ξ, η can be seen in Experiments 5 and 9. Although the station configuration in Experiment 6 is of the same shape as in Experiment 9, the change in their position (equatorial vs. meridian) has a great effect on ξ, η standard deviations. Experiment 6 has the weakest recovery, while Experiment 9 provides the strongest one.

(c) The standard deviation of θ₀ varies with the station configurations. It is highest for Experiments 4, 8, 7; medium for Experiments 10, 3, 5; and smallest for Experiments 1, 2, 9, and especially 6 (best).

(d) Recovery of Ω varies strongly even for designs of the same type. It is generally weak for two-baseline designs (Experiments 7, 8, 10) and somewhat better for three-baseline designs with the exception of the extremely weak recovery in Experiment 4. Many-station designs (Experiments 5, 6, 9) improve the recovery especially in the equatorial configuration of Experiment 6.

(e) Ω and θ₀ are the only parameters favored by the artificial equatorial design of Experiment 6 vs. the meridian one of Experiment 9. Recovery of the two components ξ and η of polar motion is not equal but depends on the position of the stations in each experiment with the exception of Experiment 6. On the contrary, both the ξ and η parameters of precession-nutation are recovered with practically the same precision for every design.
The recovery of earth rotation parameters varies with the accuracy \( \sigma_0 \) of radio source coordinates. However, this dependence is much stronger for \( \Xi, \Omega \); milder for the rest of the parameters; and even disappearing for \( \Omega, \xi, \eta \) in the strongest designs of Experiments 6 and 9.

(f) The results of the idealized designs of Experiments 6 and 9 can be used to verify some of the results of Chapter 6. Recovery of \( \xi, \eta, \Xi, \Omega \) is better for Experiment 9 than 6, while recovery of \( \Theta_0, \Omega \) is better for Experiment 6. This is explained by the fact that Experiment 9 involves baselines parallel to the equator (16-17, 19-20) observing polar radio sources, a design sensitive to \( \xi, \eta, \Xi, \Omega \). Experiment 6 involves equatorial baselines observing equatorial radio sources, the design most sensitive to \( \Theta_0, \Omega \) (see Table 6.1).

(g) The recovery of baselines varies with station configuration. However, larger number of baselines does not in general improve the individual recovery as in the case of earth rotation parameters. For example, recovery in the three-baseline design of Experiment 2 is much better than in any of the many-station designs (Experiments 5, 6, 9). The relative recovery of angles in the same experiment is inversely proportional to the recovery of the opposite baselines. Dependence on radio source accuracy varies and disappears in the case of the equatorial configuration of Experiment 6, especially in comparison to the meridian configuration of Experiment 9.

(2) **Earth Rotation and Baseline Parameters As a Function of the Length of Observations and Earth Step**

For the variation of the recovery of earth rotation and baseline parameters with respect to the total time interval of observations \( T \), it is noted that the standard deviations \( \sigma \) of the parameters are directly proportional to the standard deviations \( \sigma_0 \) of the observations. (Note that \( \sigma_0 = \sigma_0 \).) For this reason only the ratio \( \rho = \sigma/\sigma_0 \) is depicted in Figs. 9.4 and 9.5.

The dependence of \( \rho \) on the earth step \( \Delta t \) is very strong for earth rotation parameters, especially for \( \Omega \) (see Fig. 9.4), while not significant for baseline
parameters (see Fig. 9.5). In general, the recovery improves as $\Delta t$ increases since the corresponding number of unknown parameters in the adjustment decreases. For earth rotation parameters (Fig. 9.4), standard deviation decreases strongly as $T$ increases from 1 to 5 days. An additional increase of $T$ from 5 to 10 days offers a small improvement, while no significant additional improvement is gained from the extension of $T$ from 10 to 30 days. The same is true for baseline parameters (lengths and angles, see Fig. 9.5) with the exception of an additional small improvement with the increase of $T$ from 10 to 30 days.

(3) Earth Rotation and Baseline Parameters As a Function of Earth Step

As mentioned, the recovery of earth rotation and network geometry parameters improves with larger earth step $\Delta t$ (Figs. 9.6 and 9.7). The only exceptions are with respect to the $\xi, \eta$ parameters in Experiments 1, 2, 4, 10 and with respect to $\Theta_0$ in Experiment 4 where $\Delta t = 24$ hours results in worse recovery than $\Delta t = 12$ hours or even $\Delta t = 6$ hours in some cases. This "inversion" is correlated to the weak recovery of polar motion in Experiments 1, 2, 4, 10 (see Fig. 9.2) and to the weak recovery of $\Theta_0$ in Experiment 4. On the contrary, for Experiments 6 and 9, where $\xi, \eta$ and $\Theta_0$ recovery is better, variation of $\rho$ with respect to $\Delta t$ is "normal." Such variation is negligible for $\Theta_0$ in Experiment 6 and also for network geometry parameters in Experiments 6 and 9. Naturally an increase in the length of the earth step is a disadvantage when short periodic variations in the earth rotation parameters are sought.
Fig. 9.2  Recovery of earth rotation parameters, Experiments 1 - 10.

Total time span of observations:  \( T = 1 \) day

Earth step: \( \Delta t = 1 \) day

\( \sigma = \) a posteriori standard deviation of earth rotation parameters

\( \sigma_q = \) a priori standard deviation of radio source coordinates

\( \sigma_b = \) standard deviation of observational errors

Note: Angular quantities when expressed in length units are arc lengths on a sphere of radius \( R = 6371000 \) m (\( 1'' \approx 30 \) m).
EXPERIMENT 1

Fig. 9.2 Recovery of earth rotation parameters.
EXPERIMENT 2

Fig. 9.2 (cont'd)
EXPERIMENT 3

Fig. 9.2 (cont'd)
Fig. 9.2 (cont'd)
Fig. 9.2 (cont’d)
EXPERIMENT 6

Fig. 9.2 (cont'd)
EXPERIMENT 7

Fig. 9.2 (cont'd)
EXPERIMENT 8

Fig. 9.2 (cont'd)
EXPERIMENT 9

Fig. 9.2 (cont'd)
EXPERIMENT 10

Fig. 9.2 (cont'd)
Fig. 9.3  Recovery of baseline lengths and angles, Experiments 1 - 10

Total time span of observations:  $T = 1$ day
Earth step:  $\Delta t = 1$ day

For explanation of other symbols, see Fig. 9.2, p. 40.
Fig. 9.3 Recovery of baseline lengths and angles.

EXPERIMENT 1

51
EXPERIMENT 2

Fig. 9.3 (cont'd)
EXPERIMENT 3

Fig. 9.3 (cont'd)
EXPERIMENT 4

Fig. 9.3 (cont'd)
EXPERIMENT 5

Fig. 9.3 (cont'd)
Fig. 9.3 (cont'd)
EXPERIMENT 7

Fig. 9.3 (cont’d)
Fig. 9.3 (cont'd)
EXPERIMENT 9

Fig. 9.3 (cont'd)
EXPERIMENT 10

Fig. 9.3 (cont'd)

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Fig. 9.4 Variation of earth rotation parameter recovery with stepsize and the total time interval of observations (Experiment 2).
Fig. 9.5  Variation of baseline length and angle recovery with stepsize and the total time interval of observations (Experiment 2).
Fig. 9.6 Variation of earth rotation parameter recovery with earth step.

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Fig. 9.6 (cont'd)
Fig. 9. 7 Variation of baseline length and angle recovery with earth step.
Fig. 9.7 (cont'd)
References


Uotila, U.A. (1967). "Introduction to Adjustment Computations with Matrices." Lecture Notes, Department of Geodetic Science, The Ohio State University, Columbus.
APPENDIX A

Inner Constraints and Their Relation to the "Geometry" of the Pseudoinverse of an Operator

Suppose that a system of linear equations

\[
\begin{align*}
N \times u &= y \\
u \times u &= u \times 1
\end{align*}
\]

is given (\(N, y\) known; \(x\) unknown) which is consistent (i.e., it has at least one solution), and furthermore it has more than one solution. The objective is to show that there exists a set of linear constraints

\[E^\top x = 0\]

called inner constraints, such that the solution to the simultaneous system of equations \(N x = y\) and \(E^\top x = 0\) is unique and identical to the solution \(x_0 = N^+ y\), where \(N^+\) is the pseudoinverse of the matrix \(N\).

The matrix \(N\) is a representation of an operator \(N: X \rightarrow Y\), where \(X\) and \(Y\) are both \(u\)-dimensional inner product spaces with elements \(u \times 1\) vectors and the usual Euclidean inner product

\[< f, g > = g^\top f \quad f, g \in X \text{ or } f, g \in Y\]

Two subspaces can be introduced with respect to the operator \(N\), the range \(R(N) = R\) of \(N\) defined as

\[R = \{y; y \in Y \text{ and } y = Nx \text{ for some } x \in X\}, \quad R \subseteq Y\]

and the kernel or null space \(K(N) = K \subseteq X\) of \(N\),

\[K = \{x; x \in X \text{ and } Nx = 0\}\]

The consistency of the linear equations \(N x = y\) refers to the fact that \(y \in R\), while the non-uniqueness of solution to the fact that \(K \cap \{0\}^c\) is non-empty, i.e., \(K\) has other elements in addition to 0. If \(x'\) is a solution to \(Nx = y\), then \(x = x' + \bar{x}\) is also a solution where \(\bar{x}\) is any element of \(K\). We can therefore identify the solution

68
space $S_y \subset X$, defined as

$$S_y = \{ x; x \in X \text{ and } Nx = y, y \in R \text{ fixed} \}$$

with a linear variety

$$S_y = \{ x; x = x' + \bar{x}, \bar{x} \in K, x' \text{ fixed with } Nx' = y \}$$

If $K^\perp$ is the orthogonal complement of $K$ with respect to $X$, the projection $x_0 = \mathcal{P}_{K^\perp}(x)$ of any element $x \in S_y$ on $K^\perp$ is unique, and furthermore [Dermanis, 1977, §3.5]

$$\| x_0 \| = \min_{x \in S_y} \| x \|$$

We can now define the pseudoinverse $N^+$ of the operator $N$ as an operator $N^+ : R \rightarrow X$ where

$$N^+ y = x_0 = \mathcal{P}_{K^\perp}(x) \text{ for any } y \in R \text{ and } x \in S_y$$

$N^+$ is therefore the ordinary inverse of the restriction of the operator $N$ to $K^\perp$ (see [Desoer and Whalen, 1963, p. 444] for a more rigorous definition). The domain of definition of $N^+$ can be extended to the whole space $Y$ ($N^+ : Y \rightarrow X$) by setting

$$N^+(y) = N^+(y')$$

where $y' = \mathcal{P}_R(y)$ is the projection of $y \in Y$ on $R$.

The range of $N^+$ is $K^\perp$, while its kernel (null) space is $R^\perp$. For $y \in R$ we have $x_0 = N^+ y$ and $x_0$ can be alternatively defined as the unique element in the intersection $S_y \cap K^\perp$. This means that $x_0$ is uniquely defined by its two properties:

$$x_0 \in S_y \quad \text{ and } \quad x_0 \in K^\perp$$

The first property is expressed by the original equations $Nx = y$, while a similar expression must be established for the second property. Let $r = \text{rank } (N) < u$ and $s = u - r$ be the rank deficiency of $N$. The dimension of the linear subspace $K \subset X$ is then $s$, and suppose that $\{ e_i \}, i = 1, 2, \ldots, s$ is a basis in $K$. Since $x_0 \in K^\perp$, we have $x_0 \perp e_i$ for $i = 1, 2, \ldots, s, i.e.,$

$$< x_0, e_i > = e_i^\top x_0 = 0 \quad \text{for } i = 1, 2, \ldots, s$$
Fig. A.1  The geometry of the pseudoinverse operator.
Setting

\[ E = [e_1 e_2 \ldots e_s] \]
\[ u \times s \]

the above equations can be written in the compact matrix form

\[ E^T x_0 = 0 \]

Therefore, \( x_0 \) is uniquely determined as the vector satisfying both equations

\[ N x = y \quad \text{and} \quad E^T x = 0 \]

where the columns of \( E \) constitute a basis in \( K \), the kernel (null space) of \( N \). The constraints \( E^T x = 0 \) are referred to as the "inner" constraints.

The problem has now been reduced to that of finding a basis in \( K \), i.e., of finding a set of \( s \) linearly independent \( u \times 1 \) vectors \( \{e_i\} \) satisfying

\[ N e_i = 0 \quad \text{for} \quad i = 1, 2, \ldots, s \]

The equations above can be written again in matrix form

\[ N E = 0 \quad \text{where} \quad E = [e_1 e_2 \ldots e_s] \]

We therefore only need to find a \( u \times s \) matrix \( E \) with rank \( (E) = s = u - \text{rank} \ (N) \)

(to secure linear independence of its columns \( e_i \)) satisfying \( N E = 0 \).—Summarizing, we have the following theorem

Given the system of linear equations

\[ N x = y \quad \text{(N, y known; x unknown)} \]
\[ u \times u \times 1 \quad u \times 1 \]

where \( \text{rank} \ (N) = r < u \), \( s = u - r \), and a \( u \times s \) matrix \( E \) with rank \( (E) = s \) satisfying \( N E = 0 \), the vector \( x \) satisfying simultaneously \( N x = y \) and \( E^T x = 0 \) is unique and identical to \( x = N^+ y \), where \( N^+ \) is the pseudoinverse of \( N \).

References


APPENDIX B

Algorithm for First-Order Partitioned Linear Regression
Including Inner Constraints

The problem in question is the construction of an algorithm for the least squares adjustment of a group of linearized observation equations of the form

\[ V_i = \tilde{A}_i \tilde{X}_i + \hat{A}_i \hat{X} + L_i \quad i = 1, 2, \ldots, m \]  

(A 1)

with \( E \{ V_i \} = 0 \) and \( E \{ V_i V_i^T \} = P_i \). \( \hat{X} \) refers to parameters common to all sets of observations, while \( \tilde{X}_i \) refers to those appearing only in the \( i \)th set of observations. Each set of observation equations gives rise to a contribution to the normal equations of the form

\[
\left[ \begin{array}{cc} \tilde{N}_i & \bar{N}_i \\ \bar{N}_i^T & \bar{N}_i \end{array} \right] \left[ \begin{array}{c} \tilde{X}_i \\ \tilde{X} \end{array} \right] + \left[ \begin{array}{c} \bar{U}_i \\ \bar{U} \end{array} \right] = 0
\]

(A 2)

where

\[
\tilde{N}_i = \tilde{A}_i P_i \hat{A}_i \quad \bar{U}_i = \hat{A}_i^T P_i L_i \\
\bar{N}_i = \bar{A}_i P_i \hat{A}_i \quad \bar{U}_i = \hat{A}_i^T P_i L_i
\]

and

\[
\bar{N}_i = \tilde{A}_i P_i A_i
\]

The combination of such individual sets of normal equations leads to the final set of normal equations incorporating all observations

\[
\left[ \begin{array}{cc} \tilde{N} & \bar{N} \\ \bar{N}^T & \tilde{N} \end{array} \right] \left[ \begin{array}{c} \tilde{X} \\ \tilde{X} \end{array} \right] + \left[ \begin{array}{c} \bar{U} \\ \bar{U} \end{array} \right] = 0
\]

(A 3)

where

\[
\tilde{N} = \begin{bmatrix} \tilde{N}_1 & 0 & 0 \\ 0 & \tilde{N}_2 & 0 \\ 0 & \cdots & \tilde{N}_n \end{bmatrix}, \quad \bar{N} = \begin{bmatrix} \bar{N}_1 \\ \bar{N}_2 \\ \cdots \\ \bar{N}_n \end{bmatrix}, \quad \tilde{U} = \begin{bmatrix} \tilde{U}_1 \\ \tilde{U}_2 \\ \cdots \\ \tilde{U}_n \end{bmatrix}, \quad \tilde{X} = \begin{bmatrix} \tilde{X}_1 \\ \tilde{X}_2 \\ \cdots \\ \tilde{X}_n \end{bmatrix}
\]

72
\[ \dot{\mathbf{N}} = \sum \dot{N}_i, \quad \dot{\mathbf{U}} = \sum \dot{U}_i \]

In case \( \mathbf{N} = \begin{bmatrix} \dot{N} \\ \dot{N'} \end{bmatrix} \) is nonsingular (\( \det \mathbf{N} \neq 0 \)), the normal equations above can be solved through a First-Order Partitioned Linear Regression scheme as explained in [Brown and Trotter, 1969; and Uotila, 1973]. We give here an extension of this scheme for the case when \( \mathbf{N} \) is singular and a unique solution is ascertained only after the introduction of a set of inner constraints

\[ \ddot{E}' X + \dot{E}' X = 0 \quad (A.4) \]

with
\[ \ddot{E}' = [\ddot{E}_1 \ddot{E}_2 \ldots \ddot{E}_n] = [\ddot{E}_0 \ddot{E}_0 \ldots \ddot{E}_0] \]

The normal equations can now be augmented as follows [Uotila, 1967]:

\[ \begin{bmatrix} \ddot{N} & \ddot{N} & \ddot{E} \\ \ddot{N}' & \ddot{N} & \ddot{E} \\ \ddot{E}' & \ddot{E}' & 0 \end{bmatrix} \begin{bmatrix} \dddot{X} \\ \dddot{X} \\ \mathbf{K}_l \end{bmatrix} + \begin{bmatrix} \dddot{U} \\ \dddot{U} \\ 0 \end{bmatrix} = 0 \quad (A.5) \]

The new augmented coefficient matrix is nonsingular and inversion with the introduction of some new notation leads to

\[ \begin{bmatrix} \dddot{N} & \dddot{N} & \dddot{E} \\ \dddot{N}' & \dddot{N} & \dddot{E} \\ \dddot{E}' & \dddot{E}' & 0 \end{bmatrix}^{-1} = \begin{bmatrix} \dddot{M} & \dddot{M} \\ \dddot{M}' & \dddot{M} \end{bmatrix}^{-1} = \begin{bmatrix} \dddot{Q} & \dddot{Q} \\ \dddot{Q}' & \dddot{Q} \end{bmatrix} = \begin{bmatrix} \dddot{G} & \dddot{G} & \dddot{F} \\ \dddot{G}' & \dddot{G}' & \dddot{F} \end{bmatrix} \quad (A.6) \]

With the help of this inverse, the solution of the normal equations becomes

\[ \dddot{X} = \dddot{G} \dddot{U} + \dddot{G} \dddot{U} \quad (A.7) \]
\[ \dddot{X} = \dddot{G}' \dddot{U} + \dddot{G} \dddot{U} \]

If \( \dddot{G}_1, \dddot{G}_{1j} \) denote submatrices of \( \dddot{G} \) and \( \dddot{G} \), respectively, with dimensions equal to those of \( \dddot{N}_i \) and \( \dddot{N}_{ij} \), respectively, we have

\[ \dddot{X} = \sum \dddot{G}^I_j \dddot{U}_j + \dddot{G} \dddot{U} \quad (A.8) \]
\[ \dddot{X}_1 = \sum \dddot{G}_{1j} \dddot{U}_j + \dddot{G}_1 \dddot{U} \]
The inversion of Eq. (A 6) leads to

\[ \dot{Q} = [\ddot{M} - \tilde{M}^T \tilde{M}^\dagger \tilde{M}]^\dagger \]

\[ \ddot{Q} = -\tilde{M}^\dagger \tilde{M} \dot{Q} \]

\[ \ddot{\dot{Q}} = \tilde{M}^\dagger + \tilde{M}^3 \tilde{M} \dot{Q} \tilde{M}^T \tilde{M} \]

Replacing \( \tilde{M}, \dot{\tilde{M}}, \ddot{\tilde{M}} \) from (A 6) in terms of submatrices of \( \tilde{N}, \dot{\tilde{N}}, \ddot{\tilde{N}} \), and introducing the following notation

\[ H_i = \tilde{N}_i \tilde{N}_i \]

\[ R_i = \tilde{N}_i H_i \]

\[ S_i = \tilde{N}_i - R_i \]

\[ S = \Sigma_i S_i \]

\[ H = \Sigma_i H_i \]

\[ K = \Sigma_i \tilde{N}_i \]

\[ L_i = H_i \tilde{U}_i \]

\[ L = \Sigma_i L_i \]

\[ T_i = \tilde{N}_i \tilde{U}_i \]

\[ T = \Sigma_i T_i \]

we finally obtain after some algebraic manipulations

\[ \dot{\dot{Q}} = \begin{bmatrix} S & (\dot{E} - H^T \tilde{E}_o) \\ (E^T - \tilde{E}_o^T H) & (-\dot{\dot{E}}_o^T K \tilde{E}_o) \end{bmatrix} \] (A 10)

\[ \dot{X} = \dot{G} L + \dot{F} \tilde{E}_o^T T - \dot{G} \tilde{U} \] (A 11)

\[ \ddot{X}_i = -T_i - \tilde{N}_i \left[ \tilde{E}_o \tilde{F}^T \left( -\tilde{U} + L \right) + (\tilde{E}_o F \tilde{E}_o) T \right] - H_i \dot{X} \] (A 12)

\[ \tilde{G}_{13} = \delta_{ij} \tilde{N}_i \tilde{N}_j H_j \]

\[ \tilde{G}_{13} = \tilde{N}_i (\tilde{E}_o \tilde{F}^T) H_j \]

\[ \tilde{G}_{13} = \tilde{N}_i (\tilde{E}_o \tilde{F}^T) \tilde{N}_j \] (A 13)
The algorithm can now be summarized in the following steps:

1. Compute $\tilde{A}_1, \tilde{A}_4, L_4; \tilde{E}, \tilde{E}_0$
2. Compute $\tilde{N}_4, \tilde{N}_4, \bar{N}_1, \bar{U}_1$
3. Compute $H_4, R_1, S_1, T_1, L_1$
4. Compute $H, S, \tilde{N}, \bar{U}, K, L, T$
5. Obtain $\dot{G}, \dot{F}, F$ by inversion

\[
\begin{bmatrix}
S & E - H^T \tilde{E}_0 \\
(E - H^T \tilde{E}_0)^T & -\tilde{E}_0 K \tilde{E}_0
\end{bmatrix}^{-1} =
\begin{bmatrix}
\dot{G} & \dot{F} \\
\dot{F}^T & F
\end{bmatrix}
\]

6. Compute $\dot{X}$ from (A11), $\bar{X}_1$ from (A12), and $\ddot{G}_{ij}$ from (A13)

References


APPENDIX C

Computer Programs

The simulation of observations and their adjustment are carried out with the help of three programs, VLBI SIMULATOR, PROG$1$, and PROG$2$ as explained in Chapter 7. These programs are presented here in detail together with their supporting subroutines.

1. Name of Program: VLBI SIMULATOR

   **Function:** Simulates distance and distance rate VLBI observations corresponding to time delays and their derivatives, according to a previously decided observational pattern.

   **Input:** Number of participating stations and radio sources, radio source and station coordinates, initial and final epochs of observations, time interval between successive observations, identifying numbers of stations and radio source participating in each observation.

   **Output:** Identification numbers for participating stations and radio source, observed (perfect) distance and distance rate, epoch of observation.

Subroutines required:

   - GRESID: Calculates Greenwich sidereal time for a given epoch
   - JULIA: Converts Universal Time to Julian date
   - EARTH: Provides simulated earth rotation parameters for a given epoch
   - REDPI: Converts an angle in radians to the $(-\pi, \pi)$ interval.

The following is a listing of VLBI SIMULATOR program and supporting subroutines.
**C**** CONVERT RA AND DEC TO RADIANS**

```fortran
RA = RA(I) * PI/180.0
DEC = DEC(I) * PI/180.0
CO = COS(DEC)
SD = SIN(DEC)
CA = COS(RA)
SA = SIN(RA)

F1(I) = CO * CA
F2(I) = CD * SA
F3(I) = SD
```

**C**** READ STATION COORDINATES**

```fortran
DO 6 I = 1, IN

RRA(I) = RRA(I) * PI/180.0
LON(I) = LON(I) * PI/180.0
CF = COS(PHI(I))
SF = SIN(PHI(I))
CL = COS(LONG(I))
SL = SIN(LONG(I))

X(I) = CF * CL
Y(I) = SF
Z(I) = SF
```

**C**** CONVERt PHI AND LONG TO RADIANS**

```fortran
PHIR = PHI(I) * PI/180.0
LONR = LONG(I) * PI/180.0

IF(JYEAR.NE.IYAR) GO TO 88
IF(JMO.NE.IMO) GO TO 88
```

NOTICE THAT IF TO, TF APE CLOSE BUT IN DIFFERENT MONTHS SECOND

```fortran
DATE TF MUST BE EXPRESSED AS A DAY OF PREVIOUS MONTH, E.G. AUG 33
```

TF = DFLAT(JDAY-IDAY) * 24.0 + DFLAT(JHOUR-IMHOUR) + DFLAT(JMIN-IMIN)
C*** TRANSFORM QUASAR UNIT VECTOR TO EARTH FIXED SYSTEM AT EPOCH T

ANGLF=DEG*(T-TO)+THO
CA=DCOS(ANGLE)
SA=DSIN(ANGLE)
Q1=CA*E1(I)+SA*E2(I)
Q2=-SA*F1(I)+CA*E2(I)
Q3=E3(I)

DO 8 I=1,IM

8 CONTINUE

DO 9 J=1,IN

CONTINUE

INM=IN-1

DO 555 J=1,INM

555 CONTINUE
0084 IF(INDEX(I,K).GT.1000) GO TO 555
0085 WRITE(7,705) J,K,I,T
0086 705 FORMAT(315,F15.2)
0087 555 CONTINUE
C
C
0088 99 T=T+DT
0089 GO TO 33
0090 88 WRITE(6,500)
0091 500 FORMAT(10X,'WRONG INITIAL AND FINAL DATE'
0092 40 STOP
0093 END
C****** RA AND DEC ARE IN DEGREES
DO 5 I=1,IN
READ(5,321) IDM,ILL,IL2,IL3,RA(I),DEC(I)
WRITE(6,321) I,ILL,IL2,IL3,RA(I),DEC(I)
5 CONTINUE
C****** CONVERT RA AND DEC TO RADIANS
RAR=RA(I)*PI/180.DO
DECR=DEC(I)*PI/180.DO
CD=DCOS(DECR)
SD=DSIN(DECR)
CA=DCOS(RAR)
SA=DSIN(RAR)
E1(I)=CD*CA
E2(I)=CD*SA
E3(I)=SD
C****** READ STATION COORDINATES
C****** LATITUDE) AND LONG ARE IN DEGREES, R IS IN METERS
DO 6 I=1,IN
READ(5,701) IDM,ILL,IL2,LONG(I),PHI(I),R(I)
WRITE(6,701) I,ILL,IL2,LONG(I),PHI(I),R(I)
6 CONTINUE
C****** CONVERT PHI AND LONG TO RADIANS
PHIR=PHI(I)*PI/180.DO
LONGR=LONG(I)*PI/180.DO
CF=DCOS(PHIR)
SF=DSIN(PHIR)
CL=DCOS(LONGR)
SL=DSIN(LONGR)
X(I)=CF*CL
Y(I)=CF*SL
Z(I)=SF
C****** READ CORRECTION TO GREENWITCH SIDERIAL TIME DDTH IN ARC SEC
READ(5,101) DDTH
WRITE(6,101) DDTH
101 FORMAT(3F15.5)
C****** CONVERT TO RADIANS
DDTH=DDTH*PI/(180.DO*3600.DO)
C****** READ INITIAL EPOCH IN UT
READ(5,201) IYEAR,IMO,IMDAY,IMHR,IMIN,SEC
WRITE(6,201) IYEAR,IMO,IMDAY,IMHR,IMIN,SEC
201 FORMAT(5S1,F15.5)
C****** FIND GREENWITCH SIDERIAL TIME AT EPOCH TO
CALL GRESID(IYEAR,IMO,IMDAY,IMHR,IMIN,SEC,THO)
COPY(I)=THO
THD=THO*(1P0.DO/PI)
WRITE(6,136) THD
136 FORMAT(///1OXY'APPR. THO= tF25.9//)
WRITE(7,101) THO
C********* VLBI NETWORK, OBSERVATION SIMULATOR
C********* CONTINUOUS VERSION : EARTH MOTION PARAMETERS
C********* PROVIDED BY SUBROUTINE EARTH
C*********
C*********

THE MODEL USED IS:

D(I,J,P,K) = (XJ-XI) *S(XP,HP)* T

*R3(OMG*TK+THO)*S(XX,HH)*QP

S(A,B) = ( 0 1 -B )
( -A 1 0 )

T = (COS(DECP)*COS(RAP), COS(DECP)*SIN(RAP), SIN(DECP)

QP = (COS(DECP)*COS(RAP), COS(DECP)*SIN(RAP), SIN(DECP)

XJ,XI : POSITION VECTORS OF J AND I STATIONS IN EARTH FIXED SYSTEM
DECP,RAP : DECLINATION AND RIGHT ASCENSION OF P QUASAR IN QUASAR FIXED SYSTEM
QP : UNIT VECTOR OF P QUASAR IN QUASAR FIXED SYSTEM
OMG : ROTATIONAL VELOCITY OF EARTH
XP, HP : POLAR MOTION SMALL ANGLES
XX, MH : PRECESSION NUTATION SMALL ANGLES
D(I,J,P,K) : OBSERVED DISTANCE INVOLVING IJ FASELINF AND P QUASAR AT TIME TK
THO : GREENWICH SIDERIAL TIME AT EPOCH TO
THFRA : UT1 ANGLE

0001 IMPLICIT REAL*8(A-H,L-Z)
0002 DIMENSION X(4),Y(4),Z(4),E1(18),E2(18),E3(18),RA(18),DEC(18)
Z,PHI(4),LONG(4),R(4),COPY(3)
C X(IN),Y(IN),Z(IN),E1(IM),E2(IM),E3(IM),INDEX(IM,IN),RA(IM),DEC(IM),
C PHI(IN),LONG(IN),R(IN)
C IN=NO OF STATIONS, IM=NO OF QUASARS, IN,LT,35
C
0003 PI=4.DO*DATAN(1.D0)
0004 REART=6371000.DO
C
0005 READ(5,100) IN,IM
0006 WRITE(6,100) IN,IM
0007 100 FORMAT(2I5)
C
C********* READ QUASAR COORDINATES
THO=COPY(1)+DDTH

C****** READ FINAL EPOCH TF AND TIME INTERVAL IN HOURS
READ(5,201) JYEAR,JMO,JDAY,JHOUR,JMIN,SECl
WRITE(6,201) JYEAR,JMO,JDAY,JHOUR,JMIN,SECl
READ(5,101) DT
WRITE(6,101) DT

IF(JYEAR.NE.IYEAR) GO TO 88
IF(JMO.NE.IMO) GO TO 88
C******* NOTICE THAT IF TF, TF ARE CLOSE BUT IN DIFFERENT MONTHS SECOND DATE TF MUST BE EXPRESSED AS A DAY OF PREVIOUS MONTH, E.G. AUG 33
TF=DFLOAT(JDAY-IDAY)*24.00+DFLOAT(JHOUR-IHOUR)+DFLOAT(JMIN-IMIN)
7/60.00+(SECJ-SEC)/3600.00

TO=O.DO
T=O.DO
33 IF(T.GT.TF) GO TO 40

C******* I=O IS A CODE TO INDICATE END OF OBSERVATIONS FOR THIS EPOCH
READ(5,107) I,J,K
IF(I.EQ.0) GO TO 99
CALL EARTH(TrXP,HP,(OMG,THET,XX,HH)
XPP=XP*REART
HPD=HP RFART
XXD=XX*REART
HHD=HH*RPART
OMGD=OMG*(180.DO/PI)
THtT=THETA
THET=THET*(180.DO/PI)
ITHL=THET
ITH2=(THET-DFLOAT(ITHI3)*60.DO)
ITHZ=THZ
TH3=(TH2-DFLOAT(ITH2))*60.DO
WRITE(6,801) T,XPD,HPD,XXD,HHD,ITHI,ITH2,TH3,DMGD
801 FORMAT(3XF7.2,4FI0.4, X,214,IXFFs.4,FI.9)

COPY(1)= E1(K)-XX*E3(K)
COPY(2)= + E2(K)+HH*E3(K)
COPY(3)=XX*E1(K)-HH*E2(K)+ E3(K)

ANGLE=THETA+THO
CA=DCOS(ANGLE)
SA=DSIN(ANGLE)
Q1= CA*COPY(1)+SA*COPY(2)
Q2=SA*COPY(1)+CA*COPY(2)
Q3=COPY(3)
QF1=SA*COPY(1)+CA*COPY(2)
QF2=CA*COPY(1)-SA*COPY(2)
QF3=XP*QF1+HP*QF2
COPY(1) = Q1 + XP*Q3
COPY(2) = Q2 - HP*Q3
COPY(3) = -XP*Q1 + HP*Q3

DS = (X(J)*R(J) - X(I)*R(I)) * COPY(1) + (Y(J)*R(J) - Y(I)*R(I)) * COPY(2)
    + (Z(J)*R(J) - Z(I)*R(I)) * COPY(3)

FRNG = (X(J)*R(J) - X(I)*R(I)) * QF1 + (Y(J)*R(J) - Y(I)*R(I)) * QF2
      + (Z(J)*R(J) - Z(I)*R(I)) * QF3

FRNG = DERIVATIVE OF DELAY DISTANCE DS (METERS/HOUR)

WRITE(6,107) I,J,K,DS,FRNG,T
WRITE(7,107) I,J,K,DS,FRNG,T
FORMAT(315,3F15.5)

T = T + DT
GO TO 33
STOP
END
**ROUTINE GRESID(IYEAR, IMD, IDAY, IHOUR, IMIN, SEC, THO)**

```
C*******
C***********  ***
C***********  ***
C***********  ***
C***********  ***
C***********  ***
C***********  ***
C***********  ***
C***********  ***
C***********  ***
C***********  ***
C***********  ***
C***********  ***
C***********  ***
C***********  ***
```

```
IMPLICIT REAL*8(A-H,L-Z)
FI=4.1O*DATAN(1.0)
II=0
I2=0
A=0.0
CALL JULIA(IYEAR, IMD, IDAY, II, I2, A, MJD)
T=(MJD-2415020.0)/36525.0
T2=T*T
THC=99.69019P3*T3+0.001689D0*T+0.000038708D0*T2
THO=THO*PI/180.0
CALL REDPI(THO)
TH0=TH0+D*T*DTH0
CALL REDPI(TH0)
END
```
SUBROUTINE JULIA(IY, IM, ID, H, M, JD)

SUBROUTINE TO CONVERT UNIVERSAL TIME TO JULIAN DATE

IMPLICIT REAL*8 (A-H,L-Z)

DIMENSION IMONTH(12)

DATA IMONTH /0,31,59,90,120,151,181,212,243,273,304,334/

H=DFLOAT(H)

DO (V=DFLOAT(T+++M)

INIS=(7*YEAR-187)/4

IF(IYEAR.GT.0) IDIS=IDIS-1

ICH=4*(IYEAR/4)

IF(IYEAR.EQ.1900) ICD=0

MJ0=2415020+(IYEAR-1900)*365.25+IDIS+IMONTH(IM)+ICOD-0.5DO+IDAY

RETURN

END

ORIGINAL PAGE IS OF POOR QUALITY
SUBROUTINE EARTH(T,XP,HP,OMG,THETA,XX,HH)

IMPLICIT REAL*8(A-H,L-Z)
DIMENSION P(10),A(10),B(10),PS(10),F(10)

DATA P/8760.00,380.00,2190.00,720.00,360.00,
2160.00,240.00,120.00,1.00,0.500,
34/1.1600,0.6500,0.2800,0.1200,0.0600,
40/0.0300,0.00200,0.00100,0.00000,0.00000/,
58/0.5500,0.7500,0.2500,0.1300,0.0500/,
60/0.0100,0.00400,0.00010,0.00000,0.00000/,
7P/3.1500,5.2800,1.3200,0.1500,0.0600,0.9900,2.0000,
86/1400,2.7500,4.1200,0.1300,0.0300,0.0100,0.0000/,
9F/1.1300,2.5500,6.1700,4.3500,0.0500,5.1800,3.7500,1.2400,
70/3500,1.6700/,

PI=4.000*ATAN(1.00)
PI2=2.000*PI
REART=6371000.00
M10=6.00
M20=0.00
PC=299.00*24.00
C=DCOS(P12+T/PC)
S=DSIN(P12+T/PC)
M1=C*M10-S*M20
M2=S*M10+C*M20

M=1.00+M10+M20

ARG=(PI2*T)/P(x)
M1=M1+A(1)*DCOS(ARG+PS(1))
M2=M2+P(1)*DCOS(ARG+F(1))

 CONTINUE

XP=-M2/REART
HP=-M1/REART
XX=0.0600+0.0100*DCOS(2.3400+T/24.00)-0.0003*DCOS(0.1000*T/12.00)
HH=0.0400+0.0200*DCOS(3.1100+T/24.00)-0.0005*DCOS(0.0700+T/12.00)

THETA=0.001700*T/24.00+25.1000*DSIN(ARG1)-9.2000*DSIN(ARG2)
THETA=THETA*FACTOR
OMG=0.001700*T/24.00*(25.1000*DCOS(ARG1)-18.4000*DCOS(ARG2));
THETA=PI2/664.000000.00
THETA=PI2/12.00+THETA*FACTOR
THETA=PI2/786.00

RETURN
END
SUBROUTINE REDPI(A)

C*****************************************************************
CREDUCES ANGLE A IN RADIANS TO THE
C( -PI, +PI ) INTERVAL
C*****************************************************************

IMPLICIT REAL*8(A-H,L-Z)
PI=4.DO*DATAN(1.DO)
P12=PI*2.DO
COPY=A/PI2
IA=COPY
IF(A.GE.0.DO) A=A-CFLOAT(IA)*PI2
IF(A.LT.0.DO) A=A+DFLOAT(IA)*PI2
IF(A.GT.-PI.AND.A.LT.PI) GO TO 8
IF(A.GT.0.DO) A=A-P12
GO TO 5
IF(A.LT.0.DO) A=A+P12
GO TO 5
RETURN
END
2. Name of Program: PROG01

Function:

Adjustment of a set of observations with total time-span equal to both earth and station step. Earth rotation parameters and station coordinates are treated as constants over the total time span of observations. A number of weighting options for radio source coordinates are available.

Input:

Number of weighting options for radio source coordinates, standard deviations of observations and radio source coordinates, approximate values of parameters, identification numbers of participating stations and radio source, distance and distance rate observations.

Output:

Standard deviations of adjusted parameters.

Subroutines required:

REDPI.

(Following is a listing of PROG01.)
**VLBI ADJUSTMENT PROGRAM**

**This version performs an adjustment of one day of observations treating all parameters as constants over the one day interval. An option is available for applying a number of different weights on radio source coordinates.**

**Observations: delay distances and their derivatives.**

**Unknowns: Earth motion parameters, Station & Quasar coordinates, Clock offsets and drifts.**

**Systems definition:**
- **Quasar fixed:** weighted Quasar coordinates
- **Earth fixed:** by means of inner constraints

**Subroutines required:** REDPI

**The order of the parameters is:**
- $X_P, H_P$ Polar motion angles $\kappa$ and $\eta$
- $X_H, H_N$ Precession-nutation angles (capital) $\kappa$ and $\eta$
- $\Theta$ Theta $\theta$ angle ( gast at initial epoch)
- $\Omega$ Angular velocity of Earth rotation
- $X(I), Y(I), Z(I)$ Coordinates of Station $I$ ($I = 1, 2, \ldots, n$)
- $R(A), D(A)$ Right ascension and declination of radio source $J$ ($J = 1, 2, \ldots, m$)
- $C$ Clock offset and drift at Station $I$ ($I = 1, 2, \ldots, n$)

**Subroutines:**
- A: Row of design matrix containing partials of delay observations with respect to parameters
- B: Row of design matrix containing partials of delay derivative observations with respect to parameters
- C: Coefficient matrix of inner constraints
- U: Constant vector of normal equations
- XX: Vector of unknowns (corrections to parameters)

**Definitions:**
- $A$: Row of design matrix containing partials of delay observations with respect to parameters
- $B$: Row of design matrix containing partials of delay derivative observations with respect to parameters
- $C$: Coefficient matrix of inner constraints
- $U$: Constant vector of normal equations
- $XX$: Vector of unknowns (corrections to parameters)

**Variables:**
- $X_P, H_P$: Polar motion angles $\kappa$ and $\eta$
- $X_H, H_N$: Precession-nutation angles (capital) $\kappa$ and $\eta$
- $\Theta$: Theta $\theta$ angle ( gast at initial epoch)
- $\Omega$: Angular velocity of Earth rotation
- $X(I), Y(I), Z(I)$: Coordinates of Station $I$ ($I = 1, 2, \ldots, n$)
- $R(A), D(A)$: Right ascension and declination of radio source $J$ ($J = 1, 2, \ldots, m$)
- $C$: Clock offset and drift at Station $I$ ($I = 1, 2, \ldots, n$)

**Subroutines Required:**
- REDPI

**Variables Defined:**
- $SQUA(T), PRATIO(T)$
- $A(KK), N(KB), X(N), Y(N), Z(N), RA(N), D(N), U(KK), XX(KK)$
- $SQUA(T), PRATIO(T)$
- $A(KK), N(KB), X(N), Y(N), Z(N), RA(N), D(N), U(KK), XX(KK)$
- $A(KK), N(KB), X(N), Y(N), Z(N), RA(N), D(N), U(KK), XX(KK)$
- $A(KK), N(KB), X(N), Y(N), Z(N), RA(N), D(N), U(KK), XX(KK)$
C 3,B(KK),NKEEP(KB,K8),SDQUA(IO),PRATIO(IO)
C WHERE: N=NO OF STATIONS, M=NO OF QUASARS, K=6+3*N+2*M, KK=K+2*N, K8=KK+8
C
C READ EXPERIMENT IDENTIFICATION NO
0003 READ(5,100) IEXP
0004 WRITE(6,977) IEXP
0005 977 FORMAT('**/10X,'EXPERIMENT NO =',I5/10X
               2,'***************************************/')
C
C READ NUMBER OF WEIGHTING OPTIONS
0006 READ(5,100) IOPT
0007 WRITE(6,978) IOPT
0008 978 FORMAT(10X,'REQUESTED NO OF WEIGHTING OPTIONS =',I5///)
C
C READ A PRIORI STANDARD DEVIATIONS:
C SDRD: OF DELAY DISTANCE IN CM
C SDBF: OF DELAY DISTANCE DERIVATIVE IN M/HOUR
C SDQUA: OF QUASAR COORDINATES IN CM
0009 READ(5,101) SDRD,SDBF
0010 101 FORMAT(5F15.5)
0011 DO 181 I=1,IOPT
0012 READ(5,101) SDQUA(I)
0013 PRATIO(I)=SDBF**2/SDQUA(I)**2
0014 181 CONTINUE
C
C R=6371000.D0
C CLIGHT=3.00*(10.00**8)
P=
R10=R*100.00
RHD=122.00/PI*(10.00**9)
RM=180.00/PI*2600000.00
C
C PF=(SDBF/R100)/{SDBF/R}
0023 PF=PF**2
C WEIGHTS:
C PF IS NOW IN (EARTH RADII/HOUR)**-2, PRATIO IN (EARTH RADII)**-2
C
C READ IN= NO OF STATIONS & IM= NO OF QUASARS
0024 READ(5,100) IN, IM
0025 100 FORMAT(5I5)
0026 WRITE(6,920) IN,IM
0027 920 FORMAT(5X,'NO OF STATIONS =',I5/5X,'NO OF QUASARS =',I5)
0028 K=6+2*IN+2*IM
0029 KK=K+2*IM
0030 X8=KK+8
C
C INITIALIZE POLAR MOTION (XP,HP) AND PRECESSION-NUTATION (XN,HN)
C PARAMETFRS (APPROXIMATE VALUES) TO ZERO
0031 XP=0.00
0032 HP=0.00
0033 XN=0.00
0034 HN=0.00
0035 C DO 51 I=1,IN
**FORTRAN IV Release 2.0**

**Main Date** = 76300

**21/55/22**

```
0036  DO(I)=0.DO
0037  DD(I)=0.DO
0038
0039  CONTINUE
0040
C
C******************************************************************************
C READ APPROXIMATE VALUES OF PARAMETERS THETA ZERO, OMG,
C X(I), Y(I), Z(I), (I=I,N), RA(J), D(J), (J=1,IM)
C ANGLES ARE IN DEGREES, DISTANCES IN METERS AND OMG IN RADIANS PER HOUR
C******************************************************************************
C
0041  READ(5,101) TH
0042  WRITE(6,560) TH
0043  FORMAT(//5X,'THETA ZERO = ',F20.9/)
0044  TH=TH*PI/180.DO
0045  OMG=PI/12.DO
0046
C
0047  WRITE(6,450)
0048  FORMAT(6,450)
0049  450  FORMAT(//,10X,'STATION & RADIO SOURCES APPROXIMATE COORDINATES'
0050  ?.//',/1X,'STATION',/1X,'LONG',/1X,'LAT',/1X,'R',//)
0051  DO 5 I=1,N
0052  READ(5,701) IDM,II,II2,ALONG,APHI,AR
0053  WRITE(6,701) I,II,II2,ALONG,APHI,AR
0054  701  FORMAT(IX,12,1X,A4,A2,3F15.1)
0055  APHI=APHI*PI/180.DO
0056  ALONG=ALONG*PI/180.DO
0057  C
0058  DO 5 I=1,N
0059  X(I)=AR*VCOS(APHI)*DCOS(ALONG)
0060  Y(I)=AR*r'COS(APHI)*DSIN(ALONG)
0061  Z(I)=AR*DSIN(APHI)
0062  CONTINUE
0063
C
0064  WRITE(6,451)
0065  451  FORMAT(//11X,'RADIO SOURCE',/1X,'RA',/1X,'DEC',//)
0066  DO 6 J=1,IM
0067  READ(5,321) IDM,II,II2,II3,RA(J),D(J)
0068  WRITE(6,321) J,II,II2,II3,RA(J),D(J)
0069  321  FORMAT(IX,3,7X,3A4,F17.7,F20.9)
0070  C
0071  R(J)=R(J)*PI/180.DO
0072  D(J) = D(J)*PI/180.DO
0073  C
0074  CONTINUE
0075
C
C******************************************************************************
C INITIATE N, U, XX, CC TO ZERO
C******************************************************************************
C
0076  DO 7 J=1,KK
0077  XX(I)=0.DO
0078  U(I)=0.DO
0079  DO 7 J=1,8
```
CC(J,I)=.0.D0  
CONTINUE

DO 16 I=1,K8
DO 16 J=1,K8
N(I,J)=.0.D0
NKEEP(I,J)=.0.D0
CONTINUE

FORMULATE CONSTRAINT MATRIX CC

CC(5,1)=1.D0
CC(4,2)=1.D0
CC(4,5)=-1.D0

DO 3 I=1,IN
I0=I+4*3*T
I1=I+1
IH=I0+1
CC(1,IB1)=1.D0
CC(2,IR2)=1.D0
CC(3,IR3)=1.D0
CC(4,IR2)=-2(I)
CC(4,IR3)=Y(I)
CC(5,IB1)=2(I)
CC(6,IB2)=X(I)
CC(7,IC1)=1.D0
CC(8,IC2)=1.D0
CONTINUE

READ OBSERVATIONS AND IDENTIFIERS:

DS= DISTANCE IN METERS OF IJ BASELINE OBSERVATION TO QUASAR IP
AT TIME TK (IN HOURS) AFTER SOME INITIAL EPOCH TO

ICOUNT=0

READ(5,104) I,J,IP,DS,FRNG,TK,ICHEK
FORMAT([3I5,3F15.5,2I5])

ICHEK IS A CODE INDICATING END OF DATA FOR ICHEK=1
IF(ICHEK.EQ.1) GO TO 66
ICOUNT=ICOUNT+1
WRITE(R,100) I,J,IP

DS=DS/R
FRNG=FRNG/R
DX=X(J)-X(I)
DY=Y(J)-Y(I)
DZ=Z(J)-Z(I)
0111  CA=DCOS(RA(IP))
0112  SA=DSIN(RA(IP))
0113  CD=DCOS(D(IP))
0114  SD=DSIN(D(IP))
0115  Y1=OMG*TX+TH
0116  Y2=Y1-RA(IP)
0117  CK=DCOS(Y1)
0118  SK=DSIN(Y1)
0119  CKF=FCOS(Y2)
0120  SKP=DSIN(Y2)

FORMULATE PARTIALS

0121  F1=-CD*DY*CKP+DX*SD
0122  F1=F1+(-DX*SP+C*Y*HP)*CD*CKP-DZ*XP*SD
0123  F2=F1+DYP*CD*XP*CP*HP*SKP-DZ*MP*SD
0124  F2=F2+DY*CD*XP*CP*HP*SKP-DZ*MP*SD
0125  F3=F3+DY*CD*CP*XP*CP+CP*CP*CP
0126  F3=F3+DY*CD*CP*XP*CP+CP*CP*CP
0127  F4=DX*(CD*CP*XP*CP+CP)*CD
0128  F4=DX*(CD*CP*XP*CP+CP)*CD
0129  F5=DX*CD*SP*CP*CP+CP
0130  F5=DX*CD*SP*CP*CP+CP
0131  F6=TX*TP
0132  F7=CD*CP
0133  F7=CD*CP
0134  F7=CD*CP
0135  F7=CD*CP
0136  F8=-CD*SP
0137  F8=-CD*SP
0138  F8=-CD*SP
0139  F9=SD
0140  F9=SD
0141  F9=SD
0142  F10=CD*NP*CP*CP+CP
0143  F10=CD*NP*CP*CP+CP
0144  F10=CD*NP*CP*CP+CP
0145  F10=CD*NP*CP*CP+CP
0146  F11=CD*NP*CP*CP+CP
0147  F11=CD*NP*CP*CP+CP
0148  F12=1.00
0149  F12=1.00
0150  G1=CD*SP
0151  G1=CD*SP
0152  G1=CD*SP
0153  G1=CD*SP
0154  G1=CD*SP
0155  G1=CD*SP
0156  G1=CD*SP
0157  G1=CD*SP
0158  G3=DX*SP*CP*CP
0159  G3=DX*SP*CP*CP

FORMULATE PARTIALS
FORTRAN IV

G1

RELEASE 2.0

MAIN

DATE = 76300 21/55/22

0160  G3=G3+DZ*SD*(-XP*SK+HP*CK)
0161  G3=OMG*G3
0162  G4=DX*CK+SD-DY*SK*SD
0163  G4=G4+DZ*SD*(-XP*CK-HP*SK)
0164  G4=G4+DX*CK*SA*(-XP*SK-HN*CK)+DY*CD*SA*(-XN*CK+HN*SK)
0165  G4=OMG*G4
0166  G5=DX*CK*CKP+DY*CD*SKP
0167  G5=G5+DZ*CD*(XP*LPK+HP*SKP)
0168  G6=G5+DX*SA*(-XN*SK-HN*CK)-DY*CD*SA*(-XN*CK+HN*SK)
0169  G5=OMG*G5
0170  G6=OMG*G6
0171  G7=CD*SKP
0172  G7=G7+SD*(XN*SK+HN*CK)
0173  G7=OMG*G7
0174  G8=-CD*CKP
0175  G8=G8+SD*(XN*CK-HN*SK)
0176  G8=OMG*G8
0177  G9=0,D0
0178  G9=G9+CD*(XP*SKP-HP*CKP)
0179  G9=OMG*G9
0180  G10=DX*CD*CKP-DY*CD*SKP
0181  G10=G10+DZ*CD*(XP*CKP+HP*SKP)
0182  G10=OMG*G10
0183  G11=DX*CD*CKP+DY*CD*SKP
0184  G11=G11+DZ*CD*(XP*SKP-HP*CKP)
0185  G11=G11+DX*CD*(XN*SK+HN*CK)-DY*CD*(XN*CK-HN*SK)
0186  G11=OMG*G11
0187  G12=0,D0
0188  G13=1,D0/24,D0

C

A, B ARE ROWS OF THE DESIGN MATRIX CORRESPONDING TO DELAY
AND DELAY DERIVATIVE OBSERVATIONS RESPECTIVELY

C

INITIATE A AND B TO ZERO

DO 8 II=1, KK
0190  A(II)=0. D0
0191  B(II)=0. D0
0192  CONTINUE

C

SET PARTIALS IN APPROPRIATE LOCATIONS OF A AND B ROWS

0193  A(1)=F1
0194  A(2)=F2
0195  A(3)=F3
0196  A(4)=F4
0197  A(5)=F5
0198  A(6)=F6

C

B(1)=G1
0200  B(2)=G2
0201  B(3)=G3
0202  B(4)=G4
0203  B(5)=G5
0204  B(6)=G6

C

I1=I1+4
0205  I2=I1+1
FORTRAN IV G1 RELEASE 2.0

0207 I3=I1+2
0208 A(I1)=-F7
0209 A(I2)=-F6
0210 A(I3)=-F5

0211 B(I1)=-G7
0212 B(I2)=-G8
0213 B(I3)=-G9
0214 IC1=K+I
0215 IC2=IC1+IN
0216 A(IC1)=-F12
0217 A(IC2)=-F13

0218 B(IC1)=-G12
0219 B(IC2)=-G13

0220 J1=3*J+4
0221 J2=J1+1
0222 J3=J1+2
0223 A(J1)= F7
0224 A(J2)= F8
0225 A(J3)= F9

0226 B(J1)=G7
0227 B(J2)=G8
0228 B(J3)=G9

0229 JC1=K+J
0230 JC2=JC1+IN
0231 A(JC1)=-F12
0232 A(JC2)=-F13

0233 B(JC1)=G12
0234 B(JC2)=G13

0235 IP1=3*IN+2*IP+5
0236 IP2=IP1+1
0237 A(IP1)=F10
0238 A(IP2)=F11

0239 B(IP1)=G10
0240 B(IP2)=G11

C FORMULATION OF L VECTOR ENTRY

0241 DSG=DX*F7+DY*F8+DZ*F9
0242 DSO=D50+DG(J)-DO(I)+(DG(J)-DO(I))*(TK/24.0)
0243 L=DSO-DS

C STORE PARTIALS / NON-ZERO ELEMENTS OF A & B ROWS ON DISK FOR
C LATER USE IN COMPUTATION OF RESIDUALS AND VTPV
C ADD CONTRIBUTIONS OF CURRENT OBSERVATIONS TO NORMAL EQUATIONS
C
C
WRITE(8,351) F1,F2,F3,F4,F5,F6,F7,F8,F9,F10,F11,F12,F13,L

DO 12 II=1,KK
U(II)=U(II)-A(II)*L
12 CONTINUE

DO 65 JJ=IKK
NKEEP(II,JJ)=NKEEP(II,JJ)+A(II)*A(JJ)
65 CONTINUE

WRITE(8,351) G1,G2,G3,G4,G5,G6,G7,G8,G9,G10,G11,G12,G13,L

DO 65 II=1,KK
U(II)=U(II)-B(II)*L*PF
65 CONTINUE

CALL DMINV(NKR,DET,L1,L2)

DO 187 I=IKK
XX(I)=0.DO
187 CONTINUE
FORTRAN IV GI
RELEASE 2.0
MAIN
DATE = 76300 21/55/22

DO 14 I=1, KK
DO 14 J=1, KK
XX(I)=XX(I)+N(I,J)*U(J)
CONTINUE

WRITE XX

DO 9 I=1, K
CALL REDPI(XX(I))
CONTINUE

ISTART=7+3*IN
DO 26 I=ISTART, K
CALL REDPI(XX(I))
CONTINUE

READ A AND B (ROWS OF DESIGN MATRIX) FROM DISK 8,
FORMULATE RESIDUALS AND COMPUTE VARIANCE COVARIANCE MATRIX

REWIND 8
VTPV=0.000
VABSI=0.00
VABSZ=0.00
DO 22 II=1, ICOUNT
READ(R, 100) I, J, IP
DO 27 IDUM=1, 2
READ(R, 351) (F(JJ), JJ=1,13), L
V=0.00
DO 23 JJ=1,6
V=V+F(JJ)*XX(JJ)
CONTINUE
I1=2*I+4
I2=I1+1
I3=I1+2
J1=3*J+4
J2=J1+1
J3=J1+7
V=V+F(7)*(XX(J1)-XX(I1))+F(8)*(XX(J2)-XX(I2))
+ F(9)*(XX(J3)-XX(I3))
IP1=3*IN+2*IP+5
IP2=IP1+1
V=V+F(10)*XX(IP1)+F(11)*XX(IP2)
IC1=K+1
IC2=IC1+IN
JCI=K+J
0317        JCI=JCI+IN
0318        V=V+F(12)*(XX(JCI)-XX(IC1))+F(13)*(XX(JCI)-XX(IC2))
              C
0319        V=V+L  
              C
0320        IF(IDUM.EQ.2) GO TO 28
0321        VABS1=VABS1+DBS(V)
0322        VTPV=VTPV+V**2
0323        GO TO 27
              C
0324        VABS2=VABS2+DBS(V)
0325        VTPV=VTPV+P+F*V**2
0326        CONTINUE
              C
0327        CONTINUE
              C
0328        VABS1=VABS1/ICOUNT
0329        VABS2=VABS2/ICOUNT
0330        IIN=3*IN+6
0331        IIN=IIN+1
              C
              C ALTERNATIVE FOR SO COMPUTATION
              C
0332        DO 172 I=IN1,K
0333        VTPV=VTPV+PRATIOIW)*XX(I)**2
0334        CONTINUE
0335        CONTINUE
              C
0336        CONTINUE
0337        DO 24 I=IN1,K
0338        DO 24 J=I,KK
0339        N(I,J)=N(I,J)*SO
0340        CONTINUE
              C
0341        K=K+1
0342        KIN=K*IN
0343        CONTINUE
              C
0344        DO 41 J=IN1,KK
0345        N(1,J)=RIOO*N(1,J)
0346        N(2,J)=RIOO*N(2,J)
0347        N(4,J)=RIOO*N(4,J)
0348        N(5,J)=RASM3*N(5,J)
0349        CONTINUE
              C
0350        DO 42 I=IN1,KK
0351        N(I,J)=N(I,J)*RIO0
0352        CONTINUE
              C
0353        CONTINUE
0354        N(I,J)=N(I,J)*RASM3
0355        CONTINUE
0356        DO 47 I=K1,KIN
0357        N(I,J)=(N(I,J)/CLIGHT)*RIOO*DO
0358        CONTINUE
0359  DO 48 I=KINI,KK
0360       N(I,J)=(N(I,J)/CLIGHT)*1000.DO
0361  CONTINUE
0362  CONTINUE
0363  DO 44 I=1,KK
0364       N(I,1)=R100*N(I,1)
0365       N(I,2)=R100*N(I,2)
0366       N(I,3)=R100*N(I,3)
0367       N(I,4)=R100*N(I,4)
0368       N(I,5)=RASM3*N(I,5)
0369       N(I,6)=RASM3*N(I,6)
0370  CONTINUE
0371  DO 45 J=7,KIN
0372       N(I,J)=N(I,J)*R100
0373  CONTINUE
0374  DO 46 J=KIN1,KIN
0375       N(I,J)=N(I,J)*RASM3
0376  CONTINUE
0377  DO 49 J=KIN2,KIN
0378       N(I,J)=(N(I,J)/CLIGHT)*1000.DO
0379  CONTINUE
0380  DO 52 J=KINI,KK
0381       N(I,J)=(N(I,J)/CLIGHT)*1000.DO
0382  CONTINUE
0383  FORMAT(IOX,3F20.4)
0384  FORMAT(IOX,2F22.6)
0385  FORMAT(SXF40.9)
0386  GO TO 86 I=1,KK
0387  N(I,1)=DSQRT(N(I,1))
0388  CONTINUE
0389  ICNT2=ICOUNT+2
0390  IDF=ICNT2-KK+2*IM
0391  WRITE(6,964) ICNT2,IDF
0392  696  FORMAT('NO OF OBSERVATIONS =',I5,'/5X
0393              DEGREES OF FREEDOM =',I5,'/5X
0394            A PRIORI STANDARD DEVIATIONS',/5X
0395              DELAY DISTANCES (CM)',10X,F15.5/5X
0396              DISTANCE DERIVATIVES (M/HOUR)',10X,F15.5/5X
0397              QUASAK COORDINATES (CM)',10X,F15.5)
0398  C
0399  WRITE(6,850) IEXP,SDBD,SDBF,SCOUA(IW)
0400  850  FORMAT('EXPERIMENT NO =',I5,'/10X
0401              A PRIORI STANDARD DEVIATIONS',/5X
0402              DELAY DISTANCES (CM)',10X,F15.5/5X
0403              DISTANCE DERIVATIVES (M/HOUR)',10X,F15.5/5X
0404              QUASAK COORDINATES (CM)',10X,F15.5)
0405  C
0406  WRITE(6,356) (N(I,I),I=1,6)
0407  356  FORMAT('STANDARD DEVIATIONS',/20X
0408              POLAR MOTION =',7X,F14.5,3X,'CM',/26X,F14.5,3X,'CM'
0409  C
IFGRTRAN IV G1 RELEASE 2.0

MAIN

DATE = 76300 21/55/22

2. *THETA ZER0 = 9X,F14.5,3X,ARCSEC/1000*5X
3. OMG = 16X,F14.5,3X,REV PER DAY*10**9*10X
4. *STATION COORDINATES (X,Y,Z) IN CM*/3

0397 WRITE(6,357) (N(I,I),I=7,11N)
0398 357 FORMAT(10X,3F20.5)
0399 WRITE(6,358)
0400 358 FORMAT(10X,*QUASAR COORDINATES (RA,DEC) IN ARCSEC/1000*)
0401 WRITE(6,359) (N(I,I),I=1NN1,N)
0402 359 FORMAT(10X,2F20.5)

C
C
0403 WRITE(6,361)
0404 361 FORMAT(//10X,*STATION CLOCK OFFSETS IN MSEC*)
0405 WRITE(6,362) (K(I,I),I=K1,KN)
0406 362 FORMAT(2X,F20.9)
0407 WRITE(6,363)
0408 363 FORMAT(//10X,*STATION CLOCK DRIFTS IN MSEC PER DAY*)
0409 WRITE(6,362) (N(I,I),I=K1,KN)

C
C
C
0410 CONTINUE
0411 86 STOP
0412 END
3. Name of Program: PROG02

Function:

Adjustment of observations with a time span equal to the station step.
Station coordinates are considered constant over the whole set of observations, while earth rotation parameters are treated as constant but different over a prearranged set of subintervals.

Input:

Standard deviations of observations and radio source coordinates, approximate values of parameters, station and radio source identification numbers, distance and distance rate observations.

Output:

Standard deviations of adjusted parameters.

Subroutines required:

MADD, MATPRO, TRAPRO, PROTRA for matrix operations and REDPI.

(Following is a listing of PROG02 and supporting subroutines, except for REDPI already included in VLBI SIMULATOR.)
FORTRAN IV G1 RELEASE 2.C

MAIN
DATE = 76300 21/56/32

### VLBI ADJUSTMENT PROGRAM

THIS VERSION PERFORMS THE ADJUSTMENT OF A SET OF OBSERVATIONS TREATING STATION COORDINATES AS CONSTANTS, EARTH ROTATION PARAMETERS ARE CONSTANT BUT DIFFERENT OVER SUBINTERVALS OF THE ORIGINAL TOTAL TIME INTERVAL OF THE AVAILABLE OBSERVATIONS.

OBSERVATIONS: DELAY DISTANCES AND THEIR DERIVATIVES

UNKNOWNs: EARTH MOTION PARAMETERS

STATION & QUASAR COORDINATES CLOCK OFFSETS AND DRIFTS

SYSTEMS DEFINITION:

QUASAR FIXED: WEIGHTED QUASAR COORDINATES

EARTH FIXED: BY MEANS OF INNER CONSTRAINTS

SUBROUTINES REQUIRED:

REDP1
MADD
MATPRO
TRAPRO
PROTRA

DIMENSION DO(3),DD(3),X(3),Z(3),RA(9),D(9),ED(33,8),U(39)
2, NKEEP(33,9),A(9),B(9),NB(6,33),UDI(33),HI(6,33),RI(33,33)
2, OR(41,41),RLI(33),A16(33),RL(33),UD(33),HTED(33,8),QK(41)
4, L(41),GC(33,32),FB(33,8),GBL(33),GFT(33),GU(33),FBTE(33,6)
5, GB(33,6),GFTN(33,6),XBU(6,20),STD(153),XB(33)
6, F(6,8),UDI(6),VI(6),N(6),IK(6),L(6),L2(6),TI(6),K(6),R(6)
7, REDO(6,8),ETKE(8,8),FF(8,8),ETOT(8),FET(8),FBTL(8),FBTU(8)
8, SL(6),DF(6,8),EFET(6,8),GDD(6,8),HGGT(6,8),HPEN(6,8),ENEN(6,8)
9, NFF(6,8),HXB(6),NSOL(6),F(13)

WHERE: K=5*IN+2*IM K=K+6 KB=K+S IN=NO OF STATIONS, IM=NO OF QUASARS
READ EXPERIMENT IDENTIFICATION NO
READ(5,100) IEXP
WRITE(6,977) IEXP
FORMAT(*1///10X,'EXPERRIMENT NO =',I5/10X
7,*************///)

READ A PRIORI STANDARD DEVIATIONS:
SDBD : OF DELAY DISTANCE IN CM
SCBF : OF DELAY DISTANCE DERIVATIVE IN M/HOUR
SOQUA : OF QUASAR COORDINATES IN CM

READ(5,101) SDBD,SCBF
FORMAT(5F15.5)
WRITE(6,240) SDBD,SCBF
FORMAT(*1///10X,'OBSERVATIONAL NOISE',/15X,'DISTANCES
2,=10.5/15X,'DISTANCE RATES *,F10.5/)

READ(5,101) SQUA
PRATIO=SCBD**2/SDQUA**2

READ EARTH STEP INTERVAL IN HOURS
WRITE(6,243) ERSTEP
FORMAT(*1///10X,'EARTH STEP INTERVAL (HRS) =',F10.2/)

R=6371000.00
CLIGHT=3.00*(10.00**8)
CLIGHT=CLIGHT/R
PI=4.00*ATAN(1.00)
R100=R=100.00
KHM9=(12.00/PI)*(10.00**9)
RASHB=(180.00/PI)*36000000.00

PF=(SDOD/R100)/(SDBF/R)
PF=PF**2
PF IS NOW IN (EARTH RADII/HOUR)**-2, PRATIO IN (EARTH RADII)**-2

READ IN= NO OF STATIONS & IM= NO OF QUASARS
READ(5,100) IN, IM
FORMAT(*1///10X,'NO OF STATIONS =',I5/10X,'NO OF QUASARS =',I5)

K=5*IN+2*IM
F*K=K+6
KP=K+8

DO 31 I=1,IN
30 DO(I)=0.00
31 CONTINUE

READ APPROXIMATE VALUES OF PARAMETERS THETA ZERO, OMEGA

CONTINUE
C       ANGLES ARE IN DEGREES, DISTANCES IN METERS AND OMG IN RADIANS PER HOUR
C
C
C
C
C
C
C
C
C
C
C
C
C
C
C
C
C
C
C
C
C
C
C
C
C
C
C
C
C
C
C
C
C
C
      READ(5,101) TH
      WRITE(6,60) TH
      FORMAT(/5X,"THETA ZERO = '*F20.9/")
      THET=TH*PT/180.DO
      OMG=PI/12.DO
      WRITE(6,450)
      FORMAT(/11X,"STATION & RADIO SOURCES APPROXIMATE COORDINATES"
      ?","/X,"STATIONS","/X,"LONG"
      ?,/12X,"LAT","/11X,"R"
      )
      DO 5 I=1,IN
      READ(5,701) IDM,ILL,ILL,ALONG,APHI,AR
      WRITE(6,701) I,ILL,ILL,ALONG,APHI,AR
      FORMAT(1X,I2,1X,APHI*PI/180.DO)
      ALONG=ALONG*PI/180.DO
      CONVERT TO CARTESIAN
      X(I)=AR*DCOS(APHI)*DCOS(ALONG)
      Y(I)=AR*DCOS(APHI)*DSIN(ALONG)
      Z(I)=AR*OSIN(APHI)
      CONVERT TO EARTH RADII UNITS
      X(I)=X(I)/R
      Y(I)=Y(I)/R
      Z(T)=Z(I)/R
      CONTINUE
      WRITE(6,451)
      DO 6 J=1,IM
      READ(5,321) IDM,ILL,ILL,RA(J),D(J)
      WRITE(6,321) J,ILL,ILL,RA(J),D(J)
      FORMAT(1X,J1,1X,3A4,F17.7,F20.9)
      CONVERT TO RADIAN UNITS
      RA(J)=RA(J)*PI/180.DO
      D(J) = D(J)*PI/180.DO
      CONTINUE
      FORMULATE CONSTRAINT MATRICES ED, EDO
      DO 11 J=1,IM
      ED(I,J)=1.DO
      CONTINUE
      DO 9 I=1,K
      ED(I,J)=0.DO
      CONTINUE
      DO 8 I=1,IN
      EDO(I,J)=1.DO
      CONTINUE
      EDO(2,4)=1.DO
      EDC(5,6)=-1.DO
      CONTINUE
      DO 3 I=1,IN
      CONTINUE
      I=I+1
      CONTINUE
FORTRAN IV G1 RELEASE 2.0  MAIN  DATE = 76300  21/56/32

0077  IE2=IB1+1
0076  I83=I82+1
0079  ED(I81,1)=1.0D0
0080  ED(I87+2)=1.0D0
0081  ED(I81,3)=1.0D0
0082  ED(I82,4)=Z(I)
0083  ED(I83,4)=Y(I)
0084  ED(I81,5)=Z(I)
0085  ED(I89,5)=X(I)
0086  ED(I81,6)=Y(I)
0087  ED(I82,6)=X(I)
0088  IC1=3*IN+2*IM+I
0089  IC2=IC1+IN
0090  ED(IC1,7)=1.0D0
0091  ED(IC2,6)=1.0D0
0092  3 CONTINUE
0093  IGCNT=0
0094  IGCNT=0

C INITIATE H, RK, RL, RT, UD, QB TO ZERO
C*************************************************************************
C
0095  DO 71 I=1,K
0096    KL(I)=0.0D0
0097    UD(I)=0.0D0
0098    [O 71 J=1,6
0099         W(J,I)=0.0D0
0100       71 CONTINUE
C
0101  DO 72 I=1,6
0102    RT(I)=0.0D0
0103    SK(I)=0.0D0
0104    R(I,J)=0.0D0
0105       72 CONTINUE
0106  DO 73 I=1,K6
0107    DO 73 J=1,K6
0108         Q(I,J)=0.0D0
0109       73 CONTINUE
C
C INITIATE NKEEP,U TO ZERO
C*************************************************************************
C
0110  55 DO 1A I=1,KK
0111    U(I)=0.0D0
0112    UN I6 J=1, KK
0113    NKEEP(I,J)=0.0D0
0114   1A CONTINUE
C
0115    TH=THET+(ESTEP/12.0D0)*PI*FLOAT(IDCNT)
C*************************************************************************
C
C READ OBSERVATIONS AND IDENTIFIERS:
C
C DS= DISTANCE IN METERS OF IJ BASELINE OBSERVATION TO QUASAR IP
C AT TIME TK [IN HOURS] AFTER SOME INITIAL EPOCH TO
C*************************************************************************
C C
READ(5,104) I,J,IP,DS,FRNG,TK,ICHEK
C
C ICHEK IS A CODE INDICATING END OF DATA FOR ICHEK=1
C
IF(ICHEK.EQ.1) GO TO 66
IF(ICHEK.EQ.2) GO TO 66
ICOUNT=ICOUNT+1
C
C
C
DS=DS/R
FRNG=FRNG/R
DX=X(J)-X(I)
DY=Y(J)-Y(I)
DZ=Z(J)-Z(I)
CA=CDOS(RA(IP))
SA=DSIN(RA(IP))
CD=CDOS(D(IP))
SD=DSIN(D(IP))
Y1=OMG*TK+TH
Y2=Y1-RA(IP)
CK=CDOS(Y1)
SK=DSIN(Y1)
CKP=PCOS(Y2)
SKP=DSIN(Y2)

C C
FORMULATE PARTIALS

C
F1=-CD*Z*CKP+DX*SD
F2=-CD*DZ*CKP+DY*SD
F3=-DX*SD*CK*DY*SK+DZ*CD*CA
F4=DX*SK*SD+DY*CK*SD-DZ*CD*SA
F5=(DX*SKP+DY*CKP)*CD
F6=TK*F5
F7=CD*CKP
F8=-CD*SKP
F9=SD
F10=CD*(DX*SKP+DY*CKP)
F11=(-DX*CKP+DY*SKP)*SD+DZ*CD
F12=1.00
F13=TK/24.00

C
G1=DZ*CD*SKP
G1=OMG*G1
G2=-DZ*CD*SKP
G2=OMG*G2
G3=(DX*SK+DY*CK)*SD
G3=OMG*G3
G4=DX*CK*SD-DY*SK*SD
G4=OMG*G4
G5=-DX*CD*CKP+DY*CD*SKP
G5=OMG*G5
G6=OMG*G6
G7=-CD*SKP
G8=OMG*G8
G9=0.00
C INITIATE A AND B TO ZERO
C******************************************************************************
C
0171 DD R II=1,KK
0172 A(II)=0.DO
0174 A(II)=0.DO
0175 CONTINUE
C
C SET PARTIALS IN APPROPRIATE LOCATIONS OF A ROW
C******************************************************************************
C
0176 A(1)=F1
0177 A(2)=F2
0178 A(3)=F3
0179 A(4)=F4
0180 A(5)=F5
0181 A(6)=F6
C
0182 B(1)=G1
0183 B(2)=G2
0184 B(3)=G3
0185 B(4)=G4
0186 B(5)=G5
0187 B(6)=G6
C
0188 I1=3*I+4
0189 I2=I1 +1
0190 I3=I1 +2
0191 A(I1)=-F7
0192 A(I2)=-F8
0193 A(I3)=-F9
C
0194 R(I1)=-G7
0195 B(I2)=-G8
0196 B(I3)=-G9
C
0197 IC1=6*3*IM+2*IM+I
0198 IC2=IC1+I
0199 A(IC1)=-F12
0200 A(IC2)=-F13
C
0201 B(IC1)=-G12
0202 B(IC2)=-G13
C
0203 J1=3*J+4
0204 J2=J1 +1
0205 J3=J1 +2
0206 A(J1)=F7
0207 A(J2)=F8
0208 A(J3)=F9
0209  B(J1)=G7
0210  B(J2)=G8
0211  B(J3)=G9
0212  JC1=6+3*IN+2*IM+J
0213  JC2=JC1+IN
0214  A(JC1)=F12
0215  A(JC2)=F13
0216  B(JC1)=G12
0217  B(JC2)=G13
0218  IP1=3*IN+2*IP+5
0219  IP2=IP1+1
0220  A(IP1)=F10
0221  A(IP2)=F11
0222  B(IP1)=G10
0223  B(IP2)=G11
0224  FORMULATION OF L VECTOR ENTRY
0225  DSO=DX*F7+DY*F8+DZ*F9
0226  DS0=DS0+DO(J)-DO(I)*24.DO)
0227  L=DS0-DS
0228  LW1=L*R100
0229  SIORF A, B, L AND OBSERVATION IDENTIFIERS ON DISK 8 FOR LATER
0230  USE IN COMPUTATION OF RESIDUALS AN COVARIANCES
0231  WRITE(8,100) ICHEK,I,J,IP
0232  WRITE(8,351) F1,F2,F3,F4,F5,F6,F7,F8,F9,F10,F11,F12,F13,L
0233  CONTINUE
0234  FRNG0=DX*G7+DY*G8+DZ*G9
0235  FRNG0=FRNG0+(DD(J)-DD(I))/24.DO
0236  L=FRNG0-FRNG
0237  LW2=L*R
0238  WRITE(6,350) LW1,LW2
0239  WRITE(8,351) G1,G2,G3,G4,G5,G6,G7,G8,G9,G10,G11,G12,G13,L'
0240  L=FRKG0-FRNG
0241  CONTINUE
0242  CONTINUE
0243  65  CONTINUE
0244  350  FORMAT(EDO,X,7F19.9)
GO TO 33

WRITE(8,100) ICHEK

DO 20 I=1,6
  UDDII(I)=U(I)
  DO 20 J=1,6
  NDD(I,J)=NKEEP(I,J)
  CONTINUE

DO 21 I=1,6
  Do 21 J=1,K
  J6=J+6
  NB(I,J)=NKEEP(I,J6)
  CONTINUE

DO 22 I=1,K
  16=1+6
  UD(I)=U(16)
  CONTINUE

CALL DMINV(NDD,6,DFTLI6,L26)
CALL MATPRO(NDD,NDI,6,6,K)
CALL TRAPRO(NB,HI,RLI,6,6,K)

DO 23 I=1,6
  DO 23 J=I,K
  J6=J+6
  QB(I,J)=QB(I,J)+NKEEP(I,J6)-RI(I,J)
  CONTINUE

CALL MATPRO(NDI,UD,6,6,1)
CALL TRAPRO(NB,RLI,6,6,1)
CALL MAD(4,HI,6,K)
CALL MAD(4,K,6,6)
CALL MAD(4,RLI,6,1)
CALL MAD(4,HI,6,1)
CALL MADD(UD,UDI,K,1)

STORE UDD,NDI,NDH,TI ON DISK 9
```
0286 WRITF(9,351) III(I)
0287 CONTINUE
0288 IDCNT=IDCNT+1
0289 IF(ICHK.EQ.2) GO TO 67
0290 C
0291 C
0292 CALL TRAPRO(H,EDO+HTEDO+K,6,8)
0293 C
0294 DO 1=1,9
0295 QN(JK,I)=Q(I,JK)
0296 CONTINUE
0297 C
0298 CALL MATPRO(RK,EDO,RKEDO,6,6,8)
0299 C
0300 DO 29 I=1,8
0301 Q(IJK)=QD(IJK)-HTEDO(IJK)
0302 CONTINUE
0303 C
0304 IF(IPF.EQ.1)
0305 IPL=3*IN+2*IK
0306 DO 29 I=IPF,IPL
0307 QB(I,I)=Q(I,I)+PRATIO
0308 CONTINUE
0309 C
0310 CALL DMINV(GB,K8,DET,L1K,L2K)
0311 C
0312 DO 30 I=1,8
0313 G(I,J)=GS(I,J)
0314 CONTINUE
0315 C
0316 DO 31 I=1,8
0317 J=1,8
0318 FB(I,J)=QS(I,J)
0319 CONTINUE
0320 C
0321 DO 32 I=1,8
0322 J=1,8
0323 FF(I,J)=QB(I,J)
0324 CONTINUE
0325 C
0326 CALL MATPRO(GB,RL,GBL,K,K8,1)
0327 C
0328 CALL TRAPRO(EDO,RT,ETOT,8,6,1)
0329 C
```
CALL MATPRO(GB, UD, GU, K, K, 1)

DO 41 I = 1, K
X8(I) = GB(L(I)) + FET(I) - GU(I)
CONTINUE

CALL MATPRO(FE, ECT, FBT, 8, 8, 1)
CALL TRAPRO(FB, RL, FBTL, 8, K, 1)
CALL MANO(FEDT, FBTL, 8, 1)
CALL TRAPRO(FB, UD, BFTU, 8, K, 1)

DO 44 I = 1, 6
FEDT(I) = FEDT(I) - BFTU(I)
CONTINUE

CALL MATPRO(ED0, FEDT, SOL, 6, 8, 1)
CALL PROTRA(FB, ED0, FEDT, K, 8, 6)
CALL MATPRO(ED0, FF, EDF, 6, 8, 8)
CALL PROTRA(EDF, ED0, EDF, 6, 8, 8)

REWIN9 9

DO 60 IDAY = 1, INCNT

DO 35 I = 1, 6
DO 35 J = 1, 6
READ(9, 351) NDD(I, J)
CONTINUE

DO 36 I = 1, 6
DO 36 J = 1, K
READ(9, 351) HIC(I, J)
CONTINUE

DO 37 1 = 1, 6
DO 37 J = 1, 6
GDD(I, J) = GDD(I, J) + HFEN(I, J) + HFEN(I, J)
CONTINUE

CALL PROTRA(GB, HI, GBHT, K, K, 6)
CALL MATPRO(HI, GBHT, HGT, 6, K, 6)
CALL MANO(GD, HGT, 6, 6)
CALL MATPRO(FBEO, NDD, FETN, K, 6, 6)
CALL MATPRO(FET, NDD, HDFN, 6, 6, 6)

DO 39 I = 1, 6
DO 39 J = 1, 6
GDD(I, J) = GDD(I, J) + HFEN(I, J) + HFEN(J, I)
CONTINUE

DO 39 I = 1, 6
DO 39 J = 1, 6
GDD(I, J) = GDD(I, J) + HFEN(I, J) + HFEN(J, I)
CONTINUE

CALL MATPRO(FET, NDD, HDFN, 6, 6, 6)
CALL MATPRO(NDD, HDFN, NDFEN, 6, 6, 6)
CALL MANO(GDD, NEFEN, 6, 6, 6)
```fortran
C
C
C
C074 DD 48 I=1,6
C075 IF(IPF=6-IDAY-6+I)
C076 STD(IPF) = GDD(I, I)
C077 CONTINUE
C
C078 CALL MATPRO(NDD, NSOL, S, 6, 6, I)
C079 CALL MATPRO(H, X, MIX, 6, K, I)
C
C080 DO 49 I=1, 6
C081 XBI(I, IDAY) = -TI(I) - NSOL(I) - MIX(I)
C082 CONTINUE
C
C083 DO 40 I=1, K
C084 IPL = 6*IDCNI + I
C085 STD(IPL) = GB(I, I)
C086 CONTINUE
C
C087 REWIND 8
C088 IDAY = 1
C089 VTPV = 0. DO
C
C090 READ(R, 100) ICHEK, I, J, IP
C091 IF(ICHEK .EQ. 1) GO TO 88
C092 IF(ICHEK .EQ. 2) GO TO 89
C093 IS1 = 3*I-2
C094 IS2 = IS1 + 1
C095 JS1 = 3*J-2
C096 JS2 = JS1 + 1
C097 JS3 = JS2 + 1
C098 IP1 = 2*IP - I
C099 IP2 = IP1 + 1
C100 ID1 = 3*IN + 2*IM + I
C101 ID2 = ID1 + IN
C102 JD1 = JDI + IN
C103 JD2 = JD1 + IN
C104 DO 40 II = 1, 6
C105 V = F(I) + XB(J, IDAY)
C106 CONTINUE
C107 V = F(I) + XB(J, IS1) + F(J) + XB(J, IS2)
C108 DO 40 II = 1, 6
C109 V = V + F(I) + XB(J, IP1) + F(I) - XB(IP2)
C110 V = V + F(I) + XB(J, ID1) + F(I) - XB(ID2)
C111 IF(JJ .EQ. 2) GO TO 43
C
C012 V = V + F(I) + XB(J, JS1) + XB(JS2) - XB(IS2)
C013 V = V + F(I) + XB(J, JS1) + XB(JS2) - XB(IS2)
C014 V = V + F(I) + XB(J, JS1) + XB(JS2) - XB(IS2)
C015 V = V + F(I) + XB(J, JS1) + XB(JS2) - XB(IS2)
```
C WRITE(6,453) VW
VTPV = VTPV + V**2
C GO TO 45
0417
0418
C WRITE(6,454) VW
VTPV = VTPV + PF*V**2
C VW = VW + 1
C WRITE(6,455) VW
C FORMAT(50X,F30.15)
0453
C FORMAT(20X,F30.15)
0454
C CONTINUE
0419
0420
0421
0422
0423
0424
0425
0426
C VTPV = VTPV + PRATIO*XB(I)**2
CONTINUE
0427
0428
0429
C 2S = 2*SICOUNT
C JS = 5*N + 6*SICOUNT
C ISC = ISC - JS2
C IDC = IDC - JS2
C IC = IC - JS2
C CONTINUE
0430
0431
0432
0433
0434
C WRITE(6,242) IDC, ISC2, JS2, ISC3
0435
C IPC = IPC + 1
C CONTINUE
0436
0437
0438
C IF(STD(I).GT.0.000) GO TO 999
0439
0440
0441
C WRITE(6,888) STD(J), J = IPF, IPL
C FORMAT(/IX, I4, 6FI5.5)
0442
C C*********** WRITE STANDARD DEVIATIONS
0443
0444
C WRITE(6,889)
IPF=6*IDCNT+1
IPL=6*IDCNT+3*IN+2*IM
II=IPL-2*IM+1
DO 91 I=II,IPL
CALL REDPI(STD(I))
CONTINUE

DO 75 I=IPF,IPL
STD(I)=STD(I)*R100
CONTINUE

IPL=IPL-2*IM
WRITE(6,222) (STD(I),I=IPF,IPL)
FORMAT(10X,3F10.2)

WRITE(6,223)
FORMAT(///10X,*OUASAR COORDINATES*,//16X,*RA*,8X,*DEC*,/15X
Z,(CM)*,6X,(CM)*//)

IPF=IPL+1
IPL=IPL+2*IM
WRITE(6,224) (STD(I),I=IPF,IPL)
FORMAT(10X,2F10.2)

DO 76 I=I,IN
IC1=IPL+I
IC2=IC1+IN
STC(IC1)=(STD(IC1)/CLIGHT)*1000.D0
STD(IC2)=(STC(IC2)/CLIGHT)*1000.D0
WRITE(6,226) STD(IC1),STD(IC2)
FORMAT(4X,2F16.12)

CONTINUE
STOP
DEBG SUBCHK
END
SUBROUTINE MADD(A,B,L,M)

A = A + B
L*M  L*M  L*M

IMPLICIT REAL*8(A-H,O-Z)
DIMENSION A(L,M),B(L,M)

DO 5 I=1,L
DO 5 J=1,M
A(I,J) = A(I,J) + B(I,J)
CONTINUE
RETURN
END
SUBROUTINE MATPRO(A,B,C,L,M,N)

C = A * B
L*N L*M M*N

IMPLICIT REAL*8(A-H,O-Z)
DIMENSION A(L,M),B(M,N),C(L,N)

DO 5 I=1,L
DO 5 J=1,N
DO 5 K=1,M
C(I,J)=C(I,J)+A(I,K)*B(K,J)
5 CONTINUE
RETURN
END
SUBROUTINE TRAPRO(A,B,C,L,M,N)

C = A * B
L*N  L*M M*N

IMPLICIT REAL*8(A-H,O-Z)
DIMENSION A(M,L),B(M,N),C(L,N)

DO 5 I=1,L
DO 5 J=1,N
C(I,J)=0.0
5 CONTINUE

DO 5 K=1,M
C(I,J)=C(I,J)+A(K,I)*B(K,J)
5 CONTINUE

RETURN
END
SUBROUTINE PROTRA(A*B*C*L*M*N)

C = A * B
L*M = L*M M*N

IMPLICIT RFALL*8(A-H,D-Z)
DIMENSION A(L,M),B(N,M),C(L,N)
DO 5 I=1,L
DO 5 J=1,N
C(I,J)=0.00
DO 5 K=1,M
C(I,J)=C(I,J)+A(I,K)*B(J,K)
5 CONTINUE
RETURN
END