DESIGN OF EXPERIMENT FOR EARTH ROTATION AND BASELINE PARAMETER DETERMINATION FROM VERY LONG BASELINE INTERFEROMETRY

by

Athanasios Dermanis

Prepared for the
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The Ohio State University
Research Foundation
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PREFACE

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1. Introduction

Very Long Baseline Interferometry (VLBI) is one of the new techniques which will probably dominate geodesy and geophysics in the near future. Its main advantage lies in the fact that it brings the accuracy of direction measurements to a level previously possible only for range measurements. This closes the gap between powerful range determination techniques such as laser ranging and the much less accurate determination of directions through photographic tracking of artificial earth satellites.

The technique is geometric in the sense that the observations are independent of the gravity field of the earth. However, the "orbits" of the observed extragalactic radio sources with respect to an earth-fixed system are dominated and perturbed by the rotation of the earth with respect to inertial frame. This allows the determination of polar motion, precession-nutation and length-of-the-day variations, and the technique becomes also "dynamic" in this respect.

The capability of determining the geometry of a network of stations within a short time interval and with a centimeter level accuracy also allows the study of the variation of network geometry with time caused by earth tides and other periodic or secular station drifts.

The obvious importance of VLBI for both geodetic and geophysical applications is only limited by the effect of atmospheric refraction and unwanted noise associated with instrumentation [Shapiro and Knight, 1970, Section 3]. At present the development of the technique is at the stage of establishing its capabilities by direct comparison with classical surveys in relatively short baselines where geophysical effects are negligible, and of making the system more transportable.

In the present work the effect of nonwhite instrumentation noise, atmospheric refraction and earth tides are ignored, and the emphasis is on the possibility of recovering earth rotation and network geometry (baseline) parameters. It is assumed that nonwhite instrumentation noise is either eliminated or separately determined through calibration except for a linear effect which cannot be
distinguished from clock errors included in the adjustment parameters. Errors in the determination of atmospheric refraction are assumed to be either absent or uncorrelated in which case their effect is incorporated in the variance of the considered instrumentation white noise. The effect of earth tides on the variation of station coordinates is assumed to be calculated from separate information and subtracted from the actual observations.

The numerical simulated experiments performed here are set up in an environment where station coordinates vary with respect to inertial space according to a simulated earth rotation model similar to the actual but unknown rotation of the earth.

In Chapter 2 the basic technique of VLBI and its mathematical model are presented. The parametrization of earth rotation chosen is described in Chapter 3, and the resulting model is linearized in Chapter 4. Cartesian station coordinates are not estimable quantities in the analysis of VLBI observations. It is possible to use a model where only estimable quantities are considered (see, for example, [Arnold, 1974]). However, the choice of Cartesian coordinates leads to a simpler model, and the problem of coordinate system definition is resolved in Chapter 5 with the use of inner constraints. A simple analysis of the geometry of the observations in Chapter 6 leads to some useful hints on achieving maximum sensitivity of the observations with respect to the parameters considered. The basic philosophy for the simulation of data and their analysis through standard least squares adjustment techniques is presented in Chapter 7.

The main objective of the present work is the exploration of the capabilities of VLBI for the recovery of earth rotation and baseline parameters. For this purpose, a number of characteristic network designs based on present and candidate station locations is chosen in Chapter 8. The results of the simulations for each design are presented in Chapter 9 together with a summary of the conclusions.
2. **VLBI—Basic Technique and Mathematical Model**

Very Long Baseline Interferometry (VLBI) is a technique for observing time intervals and/or their derivatives at two widely separated antennas between the two instants of reception of the same wavefront emitted from extragalactic radio sources. The fundamental difference with respect to classical radio (short baseline) interferometry is in the fact that the antennas are not directly connected by cables for the comparison of the received signals. Sufficiently stable atomic frequency standards at each site make it possible to record the signals as functions of time. The recorded signals are then crosscorrelated afterwards in a computer. The fact that maximum correlation occurs when the two recordings are shifted in time by an amount equal to the wave travel time between the two instances of reception, makes it possible in principle to determine this time interval as well as its time derivative. Extragalactic radio sources are sufficiently far away for the wavefront to be considered as a plane and thus from the known velocity of the wave (velocity of light) the geometric distance corresponding to the observed time delay can be calculated.

The crosscorrelation procedure is described in detail in a series of articles by Thomas (1972a, 1972b and 1973), and we shall be concerned here only with the geometry of the observations. In Fig. 1, XYZ is a geocentric Cartesian reference frame fixed with respect to the radio sources (assumed to be an inertial frame). A certain wavefront arrives at station i at epoch $t_i$, when the position vector of the station is $X_i(t_i)$, and at the station $j$ at a slightly different epoch $t_j$ when the position vector of this station is $X_j(t_j)$. The true time delay $t_j - t_i$ is the travel time corresponding to the projection $D_{ij}$ of the retarded baseline $X_j(t_j) - X_i(t_i)$ on the direction of the radio source. If e is the unit vector in the radio source direction and $c$ the velocity of light,

$$D_{ij} = c \left( t_j - t_i \right) = [X_j(t_j) - X_i(t_i)] \cdot e$$  \hspace{1cm} (2-1)
Fig. 2.1 The geometry of VLBI observations.
Since \((t_2 - t_1)\) is a very short time interval, the time variation of the position vector \(X_j(t)\) may be assumed to be solely due to the earth's rotation. Therefore

\[
X_j(t_2) = X_j(t_1) + \int_{t_1}^{t_2} \Omega(t) \times X_j(t) \, dt
\]

(2-2)

where \(\Omega(t)\) is the rotational velocity vector of the earth. \(\Omega(t)\) is practically constant over the short time interval \((t_1 - t_2)\), and thus, by setting \(\Omega(t) = \Omega(t_1)\)

\[
X_j(t_2) = X_j(t_1) + (t_2 - t_1) [\Omega(t_1) \times X_j(t_1)]
\]

(2-3)

Setting \(B_{ij}(t) = X_j(t) - X_i(t)\) and \(V_j(t) = \Omega(t) \times X_j(t)\), one obtains

\[
D_{ij} = B_{ij}(t_1) \cdot e - \frac{1}{c} D_{ij} V_j(t_1) \cdot e
\]

(2-4)

If approximate values of \(\Omega(t_1), X_j(t_1)\) and \(e\) are known with sufficient accuracy such that the corresponding uncertainty in the retardation effect

\[
\delta D_{ij} = \frac{1}{c} D_{ij} V_j \cdot e
\]

(2-5)

is negligible, one obtains, by setting \(d_{ij} = D_{ij} + \delta D_{ij}\) and switching to matrix (column vector) notation, for an observation from the \(ij\) baseline to the radio source \(p\) at epoch \(t_k\), the much simpler model

\[
d_{ijpk} = B_{ij}^T(t_k) e_p
\]

(2-6)

Assume that \(d_{ijpk}\) has been corrected for effects giving rise to discrepancies between modeled and true distance, e.g., atmospheric errors, relativistic effects, aberration, except for clock errors. Assuming that the two local oscillators at stations \(i\) and \(j\) differ only by a constant offset \(C_{ij}\) (at some epoch \(t_0\)) and a linear drift \(C_{ij}(t)\), the model becomes

\[
d_{ijpk} = B_{ij}^T(t_k) e_p + c[C_{ij} + C_{ij}(t_k - t_0)]
\]

(2-7)

Since the inner product of two vectors is invariant with respect to the choice of reference frame, the model may be rewritten

\[
d_{ijpk} = b_{ij}^T e_p^*(t_k) + c[C_{ij} + C_{ij}(t_k - t_0)]
\]

(2-8)
with

$$b_{ij} = [x_j - x_i, y_j - y_i, z_j - z_i]$$  \hspace{2cm} (2-9)$$

where $b_{ij}$ and $e_p^*$ are the baseline and the radio source direction unit vector, respectively, with respect to an earth-fixed reference frame $x, y, z$. One also has

$$e_p^* (t_k) = M(t_k) e_p$$  \hspace{2cm} (2-10)$$

where $M(t)$ is the earth rotation matrix of transformation from the inertial to the earth-fixed frame. The model with respect to the earth-fixed reference frame becomes

$$d_{ijk} = b_{ij}^T M(t_k) e_p + c[C_{ij} + C_{ij} (t_k - t_o)]$$  \hspace{2cm} (2-11)$$

Further development of the model depends on the particular parametrization chosen for the rotation of the earth.

3. Earth Rotation Parametrization

The most straightforward way of parametrizing earth rotation is that of expressing the inertial to earth-fixed system transformation matrix $M(t)$ in terms of three rotation angles at every epoch $t$. The most logical choice seems to be that of the Eulerian angles $\varphi, \theta, \psi$, defined as

$$M(t) = R_3 [\varphi(t)] R_2 [\theta(t)] R_3 [\psi(t)]$$  \hspace{2cm} (3-1)$$

where $R_1, R_2, R_3$ denote conventional rotation matrices about the $x, y, z$ axes, respectively. The rotating earth can be viewed as a dynamical system whose state evolves in time according to a second-order differential equation of the form

$$\frac{d^2 E}{dt^2} = f(E, t)$$  \hspace{2cm} (3-2)$$

where $E = [\varphi, \theta, \psi]^T$. If such equations of motion could be exactly known and solved, their solution

$$E(t) = F(E(t_0), \dot{E}(t_0), t) \quad (\dot{E} = dE/dt)$$  \hspace{2cm} (3-3)$$
would provide a parametrization of the transformation matrix

\[ M(t) = M(\varphi_0, \theta_0, \psi_0, \dot{\varphi}_0, \dot{\theta}_0, \dot{\psi}_0, t_0, t) \]  \hspace{1cm} (3-4)

in terms of the six initial values (integration constants):

\begin{align*}
\varphi_0 &= \varphi(t_0) & \theta_0 &= \theta(t_0) & \psi_0 &= \psi(t_0) \\
\dot{\varphi}_0 &= \frac{d\varphi}{dt} \bigg|_{t_0} & \dot{\theta}_0 &= \frac{d\theta}{dt} \bigg|_{t_0} & \dot{\psi}_0 &= \frac{d\psi}{dt} \bigg|_{t_0}
\end{align*}

However, such an approach is not practical because of uncertainties surrounding the current knowledge of the earth's rotation. The alternative is the representation of the functions \( \varphi(t), \theta(t), \psi(t) \) in terms of a finite number of parameters to be estimated from the observations. Such representations as polynomials, trigonometric series, etc. can sufficiently approximate a wide class of functions, provided that appropriate number of terms is included. The efficiency of the approximation can be judged only when the function to be approximated is known over some time interval, while in our case an unknown function is to be approximated. The unknown approximation error (representation error, modeling error) may well be significant compared to the observational errors, thus affecting the validity of the final parameter error estimates based upon real data analysis.

Among various finite parameter representations, the simplest perhaps is that of representing a continuous bounded function over an interval by a step function, which has a constant value over each subinterval, into which the original time interval has been partitioned.

To formally represent a step function, we introduce the characteristic or indicator function \( \chi_A \) of a set \( A \), defined as follows:

\[ \chi_A(x) = \begin{cases} 
1 & \text{if } x \in A \\
0 & \text{if } x \notin A 
\end{cases} \]  \hspace{1cm} (3-5)

Let \([a, b]\) be the original interval and

\[ a = t_0 < t_1 < t_2 < \ldots < t_{a-1} < t_a = b \]
If \( \chi_i \) denotes the characteristic function of the set \([t_{i-1}, t_i]\) (\(i = 1, 2, \ldots, n-1\)) and \( \chi_n \) that of \([t_{n-1}, t_n]\), a step function with respect to the above partition of \([a, b]\) can be written in the form

\[
y(t) = \sum_{i=1}^{n} \alpha_i \chi_i(t)
\]

A continuous bounded function \( f(t) \) can be approximated arbitrarily well by a step function \( y(t) \) with an appropriate choice and increase in the number of subintervals.

In the case of this investigation, the functions to be approximated are the Eulerian angles \( \phi(t), \theta(t), \psi(t) \), and the effectiveness of step function approximation may be increased if the known parts of \( \phi, \theta, \psi \) are filtered out. Let \( \phi^0(t), \theta^0(t), \psi^0(t) \) be the "reference" Eulerian angles as computable from classical astronomy. Then the earth rotation transformation matrix may be represented as

\[
M(t) = R_3[\phi^0 + \delta \phi] \ R_1[\theta^0 + \delta \theta] \ R_3[\psi^0 + \delta \psi]
\]

where \( \delta \phi(t), \delta \theta(t) \) and \( \delta \psi(t) \) are step functions. An alternative parametrization may be of the form

\[
M(t) = M^0(t) \ \delta M(t) = R_3(\phi^0) \ R_1(\theta^0) \ R_3(\psi^0) \ R_3(\delta \phi) \ R_3(\delta \theta) \ R_3(\delta \psi)
\]

Traditionally, earth rotation representations are separated into three parts with the help of the instantaneous rotation vector \( \omega(t) \) related to the Eulerian angles through Euler's geometric equations

\[
\omega_x = \sin \theta \sin \phi \dot{\psi} + \cos \phi \dot{\theta} \\
\omega_y = \sin \theta \cos \phi \dot{\psi} + \sin \phi \dot{\theta} \\
\omega_z = \cos \theta \dot{\psi} + \dot{\phi}
\]

where \( \omega_x, \omega_y, \omega_z \) are the rotation vector components with respect to the earth-fixed system. The variation in the direction of the vector \( \omega \) with respect to the inertial system constitutes precession and nutation, while polar motion is the corresponding variation with respect to the earth-fixed system. The third part of earth rotation is the variation in the length of the vector \( \omega \), i.e., the variation of the rotational velocity (length of the day) \( \Omega = \|\omega\| \).
To obtain a parametrization of earth rotation where precession-nutation, polar motion and rotational velocity are separated, we introduce an intermediate moving geocentric Cartesian reference frame $x', y', z'$. The $z'$-axis direction coincides at any epoch with the direction of the instantaneous rotation axis, while the $x'$-axis direction is arbitrary. The transformation from the $X, Y, Z$ (inertial) to the $x', y', z'$ frame may be expressed in terms of three rotations:

$$R_3 (\Theta_3) R_2 (\Xi) R_1 (H)$$

and the transformation from $x', y', z'$ to the $x, y, z$ (earth-fixed) frame can be similarly expressed as

$$R_1 (-\eta) R_3 (-\xi) R_3 (\Theta)$$

Setting $\Theta = \Theta_1 + \Theta_2$, the total transformation from the inertial to the earth-fixed frame becomes

$$M = R_1 (-\eta) R_3 (-\xi) R_3 (\Theta) R_2 (\Xi) R_1 (H)$$

(3-10)

where $\eta, \xi, \Theta, \Xi, H$ are all functions of time. (Note that the $\Theta_1$ and $\Theta_2$ components of $\Theta$ cannot be separated, and this justifies the arbitrariness of the $x'$-axis direction.) In order to introduce the instantaneous rotational velocity $\Omega$ in the model, noting that the $R_3 (\Theta)$ rotation is about the instantaneous rotation axis (diurnal rotation), one may set

$$\Theta(t) = \Theta_0 + \int_{t_0}^{t} \Omega(\tau) \, d\tau \quad \Theta_0 = \Theta(t_0)$$

(3-11)

The three angles $\Xi, H, \Theta_0$ represent the traditional precession-nutation, and the angles $\eta$ and $\xi$ the polar motion. The angle $\Theta = \Theta_0 - \Theta_0$ is similar to GAST (Greenwich Apparent Sidereal Time). The functions $\eta, \xi, \Omega, \Xi, H$ can now be approximated by step functions. To improve the approximation, we shall include in the transformation $M$ the traditional precession-nutation based on current knowledge, as follows:

$$M - R_1 (-\eta) R_3 (-\xi) R_3 (\Theta) R_2 (\Xi) R_1 (H) R^C$$

(3-12)

where the known part of precession and nutation may be computed from [Mueller, 1969, Ch. 4].
\[ R^0 = R_z(-\epsilon - \Delta \epsilon) \ R_z(-\Delta \psi) \ R_x(\epsilon) \ R_z(-\Delta \zeta) \ R_z(\theta) \ R_z(-\zeta_0) \]  

(3.13)

In the above expression, \( \epsilon, \theta, \zeta_0 \) are the Newcomb components of precession, and \( \Delta \psi \) and \( \Delta \epsilon \) the nutations in longitude and obliquity by Woolard. The transformation \( R^0 \) defines the traditional "true" frame of reference, an intermediate, noninertial, moving reference frame \( X', Y', Z' \) of known orientation, such that the \( Z' \)-axis is at any epoch near the instantaneous rotation axis. The angles \( \Xi, \Theta, \Theta_0 \) are thus small corrections to the reference precession-nutation parameters used in (3.13), and as such they may be considered as the transformation parameters between the "traditional (Newcomb/Woolard) true" frame and the "actual true" frame.

There are two alternatives in the step function representation of the angle \( \Theta \). One may represent \( \Theta \) as a step function with constant value \( \Theta_i \) over the \( i \)th time interval (step), in which case

\[ \Theta(t) = \Theta_0 + \sum_{k=1}^{t-1} \Omega_k (t_k - t_{k-1}) + \Omega_1 (t - t_{t-1}) \]  

(3.14)

\[ t_{t-1} < t < t_t \]

A second choice is to approximate \( \Theta \) over each step by a function of the form

\[ \Theta_i(t) = \Theta_{oi} + \Omega_i (t - t_{t-1}) \]  

(3.15)

This is a weaker representation from the adjustment point of view because of the larger number of parameters \( (\Theta_{oi}, i = 1, 2, \ldots, N \) vs. \( \Theta_0 \)), but it leads to estimates of variances of the estimates of the step function constants \( \eta_i, \xi_i, \Theta_{oi}, \Omega_i, \Xi_i, \Theta_0 \), which are uniform (equal) over all steps \( (t_{t-1}, t_t) \). On the contrary, the first representation leads to variances which are smaller for steps in the middle and larger for steps at the ends of the original time span of the available observations.

Over each step the model for the VLBI observations becomes

\[ d_{ijk} = b_{ij}^* R_z(-\eta) \ R_o(-\xi) \ R_z([\Theta_0 + \Omega (t_k - t_0)] R_z(\Xi) \ R_h(H) R^0 e_p + \]  

\[ + c [C_{oi} + C_{ij} (t_k - t_0)] \]  

(3.16)

where \( \eta, \xi, \Theta_0, \Omega, \Xi, H \) are constants over the considered step (the subscripts identifying the steps have been dropped for simplicity), \( t_k \) is the epoch of observation, and \( t_0 \) is the beginning of the step containing \( t_k \).
4. Linearization of Observation Equations

For the time span of available observations considered in this study (maximum of one month), the effect of precession-nutation on the experimental design (relative radio sources - stations configuration) is negligible. One can therefore drop the known transformation matrix $R^0$ from the model without any significant change on the simulation results. This simplifies somewhat the equations following, but it must be understood that appropriate modifications must be made when analyzing real data by including the effect of $R^0$.

The angles $\xi, \eta, \Xi, H$ are very small since both the z and Z axes (actually the noninertial $Z'$-axis on account of ignoring $R^0$) are or can be chosen to be near the rotation axis. The usual approximations, $\cos \alpha = 1, \sin \alpha = \alpha, \alpha^2 = 0$, for a small angle $\alpha$ may be used, leading to

$$R_1(-\eta) R_2(-\xi) = \begin{pmatrix} 1 & 0 & \xi \\ 0 & 1 & -\eta \\ -\xi & \eta & 1 \end{pmatrix}$$

and

$$R_2(\Xi) R_1(H) = \begin{pmatrix} 1 & 0 & -\Xi \\ 0 & 1 & H \\ \Xi & -H & 1 \end{pmatrix}$$

The model now becomes

$$d_{1jk} = (r_1 - r_i)^T S'(\xi, \eta) R_3(\Theta_0 + \Omega \tau_k) S(\Xi, H) e_p + c [C_{01} + C_{11} \tau_k] \quad (4-1)$$

where $\tau_k = t_k - t_0$; $e_p = [\cos \delta_p \cos \alpha_p, \cos \delta_p \sin \alpha_p, \sin \delta_p]^T$, $\alpha_p$ and $\delta_p$ are the right-ascension and declination of the $p$th radio source with respect to the $X, Y, Z$ inertial frame; $r_1 = [x_1, y_1, z_1]^T$, $x_i, y_i, z_i$ are the coordinates of the $i$th station with respect to the $x, y, z$ earth-fixed frame; and

$$S(a, b) = \begin{pmatrix} 1 & 0 & -a \\ 0 & 1 & b \\ a & -b & 1 \end{pmatrix} \quad (4-2)$$
The available observations \( d_{ij} \) form a vector of observations \( \mathbf{L}_b \), corresponding to a vector of observables \( \mathbf{L}_a = \mathbf{F}(\mathbf{X}_a) \), where \( \mathbf{X}_a \) is a vector of unknown parameters. The unknowns are the station coordinates \( x_i, y_i, z_i, i = 1, 2, \ldots, N_s \), the earth rotation parameters \( \xi, \eta, \Theta_0, \Omega, \Xi, H \) (one set of six per each step), radio source coordinates \( \alpha_p, \delta_p, p = 1, 2, \ldots, N_{rs} \), and clock synchronization parameters \( C_{0ij}, C_{1ij}, i,j = 1, 2, \ldots, N_s \).

The \( d_{ij} \) components of the vector \( \mathbf{L}_a \) are linear with respect to most of the parameters with the exception of \( \Theta_0, \Omega, \alpha_p, \) and \( \delta_p \). A completely linearized model can be obtained by means of approximate values \( \mathbf{X}^0 \) of the parameters \( \mathbf{X}_a \). Expansion in Taylor series and neglect of second- and higher-order terms leads to

\[
\mathbf{L}_a = \mathbf{F}(\mathbf{X}^0) + \frac{\partial \mathbf{F}(\mathbf{X}_a)}{\partial \mathbf{X}_a} \mathbf{X}^0 (\mathbf{X}_a - \mathbf{X}^0)
\]

The actual observations \( \mathbf{L}_b \) differ from the observables \( \mathbf{L}_a \) because of the presence of unknown observations errors \( \mathbf{V} \), according to \( \mathbf{L}_a = \mathbf{L}_b + \mathbf{V} \). Setting \( \mathbf{A} = (\partial \mathbf{F}/\partial \mathbf{X}_a)|_{\mathbf{X}_a = \mathbf{X}^0} \), \( \mathbf{L}^0 = \mathbf{F}(\mathbf{X}^0), \mathbf{X} = \mathbf{X}_a - \mathbf{X}^0, \) and \( \mathbf{L} = \mathbf{L}^0 - \mathbf{L}_b \), the linearized observation equations are obtained:

\[
\mathbf{V} = \mathbf{A} \mathbf{X} + \mathbf{L}
\]

\( \mathbf{A} \) and \( \mathbf{L} \) are known, while \( \mathbf{X} \) and \( \mathbf{V} \) are unknown. The vector \( \mathbf{V} \) is assumed to be an outcome of a random vector with \( \mathbb{E}[\mathbf{V}] = 0 \) and known covariance matrix \( \mathbb{E}[\mathbf{V} \mathbf{V}^T] = \mathbf{S} \).

Application of standard least squares techniques leads to a solution \( \mathbf{X} \), minimizing the quadratic form \( \mathbf{V}^T \mathbf{P} \mathbf{V} \) where \( \mathbf{P} = \mathbf{S}^{-1} \), which satisfies the "normal equations,"

\[
(A^T \mathbf{P} \mathbf{A}) \mathbf{X} = -A^T \mathbf{P} \mathbf{L} \quad \text{or} \quad \mathbf{NX} = \mathbf{U}
\]

To compute the elements of the design matrix \( \mathbf{A} \), the partial derivatives of \( d_{ij} \) with respect to the parameters are required. Introducing the notation, \( \psi_k = \Theta_0 + \Omega \tau_k, \) \( x_{kp} = \psi_k - \alpha_p, \) \( x_{ij} = x_j - x_i, \) \( y_{ij} = y_j - y_i, \) \( z_{ij} = z_j - z_i, \) and dropping subscripts where confusion is not likely to arise, the partial derivatives are as follows:
\[
\frac{\partial d}{\partial x_j} = - \frac{\partial d}{\partial x_1} = \cos \delta \cos \chi + \sin \delta (- \Xi \cos \psi + H \sin \psi + \xi)
\]

\[
\frac{\partial d}{\partial y_j} = - \frac{\partial d}{\partial y_1} = - \cos \delta \sin \chi + \sin \delta (\Xi \sin \psi + H \cos \psi - \eta)
\]

\[
\frac{\partial d}{\partial z_j} = - \frac{\partial d}{\partial z_1} = \sin \delta - \cos \delta (\zeta \cos \chi + \eta \sin \chi - \Xi \cos \alpha + H \sin \alpha)
\]

\[
\frac{\partial d}{\partial \xi} = x_{1j} [\sin \delta + \cos \delta (\Xi \cos \alpha - H \sin \alpha - \xi \cos \chi)] + \\
+ y_{1j} [\eta \cos \delta \cos \chi] + \\
+ z_{1j} [- \cos \delta \cos \chi - \sin \delta (- \Xi \cos \psi + H \sin \psi + \xi)]
\]

\[
\frac{\partial d}{\partial \eta} = y_{1j} [- \sin \delta + \cos \delta (\xi \cos \chi + \eta \sin \chi - \Xi \cos \alpha + H \sin \alpha)] + \\
+ z_{1j} [- \cos \delta \sin \chi + \sin \delta (\Xi \sin \psi + H \cos \psi - \eta)]
\]

\[
\frac{\partial d}{\partial \Theta_0} = x_{1j} [- \cos \delta \sin \chi + \sin \delta (\Xi \sin \psi + H \cos \psi)] + \\
+ y_{1j} [- \cos \delta \cos \chi + \sin \delta (\Xi \cos \psi - H \sin \psi)] + \\
+ z_{1j} [\cos \delta (\xi \sin \chi - \eta \cos \chi)]
\]

\[
\frac{\partial d}{\partial \tau} = \tau_k \frac{\partial d}{\partial \Theta_0}, \quad \frac{\partial d}{\partial C_{1j}} = c, \quad \frac{\partial d}{\partial C_{1j}} = c \tau_k
\]

\[
\frac{\partial d}{\partial \Xi} = x_{1j} \left\{ - \cos \psi \cos \delta + \cos \delta [\xi \cos \alpha + \cos \psi (- \Xi \cos \alpha + H \sin \alpha)] \right\} + \\
+ y_{1j} \left\{ \sin \psi \sin \delta + \cos \delta [- \eta \cos \alpha + \sin \psi (\Xi \cos \alpha - H \sin \alpha)] \right\} + \\
+ z_{1j} [\cos \delta \cos \alpha + \sin \delta (\xi \cos \psi + \eta \sin \psi - \Xi)]
\]

\[
\frac{\partial d}{\partial \eta} = x_{1j} [\sin \delta \sin \psi + \cos \delta \sin \alpha (\Xi \cos \psi - H \sin \psi - \xi)] + \\
+ y_{1j} [\sin \delta \cos \psi + \cos \delta \sin \alpha (- \Xi \sin \psi - H \cos \psi + \eta)] + \\
+ z_{1j} [- \cos \delta \sin \alpha + \sin \delta (- \xi \sin \psi + \eta \cos \psi - H)]
\]

\[
\frac{\partial d}{\partial \alpha} = x_{1j} [\cos \delta \sin \chi] + y_{1j} [\cos \delta \cos \chi] + \\
+ z_{1j} \cos \delta (- \xi \sin \chi + \eta \cos \chi - \Xi \sin \alpha - H \cos \alpha)
\]
\[
\frac{\partial d}{\partial \delta} = x_{ij} \left[ -\sin \delta \cos \chi + \cos \delta \left( -\Xi \cos \psi + \Pi \sin \psi + \xi \right) \right] + \\
+ y_{ij} \left[ \sin \delta \sin \chi + \cos \delta \left( \Xi \sin \psi + \Pi \cos \psi - \eta \right) \right] + \\
+ z_{ij} \left[ \cos \delta + \sin \delta \left( \xi \cos \chi + \eta \sin \chi - \Xi \cos \alpha + \Pi \sin \alpha \right) \right] \\
\text{(4-6)}
\]

A second type of possible observations is the time derivative of \(d_{ijk}\)

\[
f_{1jkp} = \frac{\partial d_{ijpk}}{\partial t} = \Omega (r_1 - r_1)' S'(\xi, \eta) P_3 R_3(\Theta_0 + \Omega \tau_k) S(\Xi, \Pi) e_p + c C_{ij} \\
\text{(4-7)}
\]

where

\[
P_3 = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \text{ (4-8)}
\]

The related partial derivatives of \(f_{ijkp}\) with respect to the parameters are as follows:

\[
\frac{1}{\Omega} \frac{\partial f}{\partial \xi} = x_{ij} \left[ \xi \cos \delta \sin \chi \right] - y_{ij} \left[ \eta \cos \delta \sin \chi \right] \\
+ z_{ij} \left[ \cos \delta \sin \chi - \sin \delta \left( \Xi \sin \psi + \Pi \cos \psi \right) \right] \\
\text{(4-9)}
\]

\[
\frac{1}{\Omega} \frac{\partial f}{\partial \eta} = y_{ij} \left[ \cos \delta \left( -\xi \sin \chi + \eta \cos \chi \right) \right] \\
+ z_{ij} \left[ -\cos \delta \cos \chi + \sin \delta \left( \Xi \cos \psi - \Pi \sin \psi \right) \right] \\
\text{(4-10)}
\]

\[
\frac{1}{\Omega} \frac{\partial f}{\partial \Xi} = x_{ij} \left[ \sin \delta \cos \psi - \cos \delta \sin \psi \left( -\Xi \cos \alpha + \Pi \sin \alpha \right) \right] \\
+ y_{ij} \left[ \sin \delta \cos \psi - \cos \delta \cos \psi \left( -\Xi \cos \alpha + \Pi \sin \alpha \right) \right] \\
+ z_{ij} \left[ \sin \delta \left( -\xi \sin \psi + \eta \cos \psi \right) \right] \\
\text{(4-11)}
\]

\[
\frac{1}{\Omega} \frac{\partial f}{\partial \Pi} = x_{ij} \left[ \cos \psi \sin \delta - \cos \delta \sin \alpha \left( \Xi \sin \psi + \Pi \cos \psi \right) \right] \\
+ y_{ij} \left[ -\sin \psi \sin \delta + \cos \delta \sin \alpha \left( -\Xi \cos \psi + \Pi \sin \psi \right) \right] \\
+ z_{ij} \left[ \sin \delta \left( -\xi \cos \psi - \eta \sin \psi \right) \right] \\
\text{(4-12)}
\]
\[ \frac{1}{\Omega} \frac{\partial f}{\partial \phi_0} = x_{11} \left( -\cos \delta \cos \chi + \sin \delta \left( \Xi \cos \psi - H \sin \psi \right) \right) \]
\[ + y_{11} \left( \cos \delta \sin \chi - \sin \delta \left( -\Xi \sin \psi - H \cos \psi \right) \right) \]
\[ + z_{11} \left( \cos \delta \left( \xi \cos \chi + \eta \sin \chi \right) \right) \]
\[ \frac{1}{\Omega} \frac{\partial f}{\partial x_1} = - \frac{1}{\Omega} \frac{\partial f}{\partial x_1} = - \cos \delta \sin \chi + \sin \delta \left( \Xi \sin \psi + H \cos \psi \right) \]
\[ \frac{1}{\Omega} \frac{\partial f}{\partial y_1} = - \frac{1}{\Omega} \frac{\partial f}{\partial y_1} = - \cos \delta \cos \chi + \sin \delta \left( \Xi \cos \psi - H \sin \psi \right) \]
\[ \frac{1}{\Omega} \frac{\partial f}{\partial z_1} = - \frac{1}{\Omega} \frac{\partial f}{\partial z_1} = \cos \delta \left( \xi \sin \chi - \eta \cos \chi \right) \]
\[ \frac{1}{\Omega} \frac{\partial f}{\partial \alpha} = x_{11} \left[ \cos \delta \cos \chi \right] + y_{11} \left[ -\cos \delta \sin \chi \right] \]
\[ + z_{11} \left[ \cos \delta \left( \xi \cos \chi + \eta \sin \chi \right) \right] \]
\[ \frac{1}{\Omega} \frac{\partial f}{\partial \delta} = x_{11} \left[ \sin \delta \sin \chi + \cos \delta \left( \Xi \sin \psi + H \cos \psi \right) \right] \]
\[ + y_{11} \left[ \sin \delta \cos \chi + \cos \delta \left( \Xi \cos \psi - H \sin \psi \right) \right] \]
\[ + z_{11} \left[ \sin \delta \left( -\xi \sin \chi + \eta \cos \chi \right) \right] \]
\[ \frac{\partial f}{\partial \Omega} = \tau_k \frac{\partial f}{\partial \phi_0}, \quad \frac{\partial f}{\partial C_{11}} = 0, \quad \frac{\partial f}{\partial C_{11}} = c \]

\[ (4-9) \]
5. Coordinate System Definition and Inner Constraints

The combination of all observations leads to the linearized observation equations

\[ nV_i = nA_u X_1 + nL_1 \]  

(5-1)

Minimization of \( V^T P V \) leads to a solution satisfying the normal equations [Uotila, 1967]

\[ N X = U \]  

(5-2)

where

\[ N = A^T P A \quad \text{and} \quad U = -A^T P L. \]

Because of the lack of coordinate system definition, \( N \) is singular with rank \( (N) = r < u \), where \( u \) is the number of parameters and \( s = u - r \) is the rank deficiency of \( N \).

Originally \( s = 9 \), corresponding to the six degrees of freedom in the definition of the earth-fixed coordinate system (three for origin position and three for orientation) plus the three degrees of freedom in the inertial system definition (orientation only).

Since a radio source catalogue of a certain accuracy is assumed to be available, radio source coordinates are treated as observations (weighted unknowns) rather than as unknown parameters. The inertial frame is therefore defined through the catalogued radio source coordinates and the remaining rank deficiency of \( N \) becomes \( s = 6 \). A unique solution to the normal equations may be obtained if in addition a set of minimal linear constraints are imposed on the parameters, of the form

\[ C^T X = 0 \]  

(5-3)

where \( C \) is a \( u \times s \) matrix and rank \( (C) = s \).

Among the various possible solutions to the normal equations, the unique one given by \( X = N^+ U \), where \( N^+ \) is the pseudoinverse of \( N \), has the following properties [Blaha, 1971]: \( X^T X = \text{min.} \) and trace \( N^+ = \text{min.} \). In view of the interpretation of \( N^+ \) as the variance-covariance matrix of the parameters, the second property makes the solution optimal in the sense that smaller variances of the parameters provide a better representation of parameter related estimable quantities in terms of the
nonestimable parameters. To avoid the use of pseudoinverse computation algorithm, one may resort to a particular set of minimal constraints called inner constraints [Blaha, 1971],

\[ E^T X = 0 \]  

(5-4)

which leads to the same solution for \( X \) as the one obtained with the use of the pseudoinverse \( N^+ \). It can be shown that such a set of inner constraints can be obtained with the help of a \( u \times s \) matrix \( E \) (rank (\( E \)) = \( s \)) satisfying

\[ NE = 0 \]  

(5-5)

An algebraic type of proof can be found in [Pope, 1971] which settles the truth of the matter but throws little light on the interrelation between inner constraints and pseudoinverse. A different type of proof is given in Appendix A of this work, based on the geometry of the operator represented by the matrix \( N \).

In view of the fact that \( AE = 0 \) \( \Rightarrow NE = A^T PAE = 0 \), a set of inner constraints has been analytically constructed using six independent solutions to the set of equations

\[ A^j y = 0 \quad j = 1, 2, \ldots, u \]  

(5-6)

where \( A^j \) denotes the \( j \)th row of \( A \). We assume that the approximate values of \( \eta, \xi, \Xi, H \) are zero in the computation of partials of observations with respect to the unknowns, thus obtaining simpler analytical expressions for the elements of \( A \).

Letting the order of the unknowns in \( X \) to be the following

\[ X^j = [d\eta, d\xi, d\Theta_0, d\Omega, dx_1, dy_1, dz_1, \ldots, dx_N, dy_N, dz_N] \]

one may set

\[ y^j = [w_1, w_2, w_3, w_4, \alpha_1, \beta_1, \gamma_1, \ldots, \alpha_u, \beta_u, \gamma_u] \]

For a row of \( A \) corresponding to a \( d_4kp \) observation, one has

\[ w_1 \frac{\partial d}{\partial \eta} + w_2 \frac{\partial d}{\partial \xi} + w_3 \frac{\partial d}{\partial \Theta_0} + w_4 \frac{\partial d}{\partial \Omega} + \alpha_1 \frac{\partial d}{\partial x_1} + \beta_1 \frac{\partial d}{\partial y_1} + \gamma_1 \frac{\partial d}{\partial z_1} + \]

\[ + \alpha_j \frac{\partial d}{\partial x_j} + \beta_j \frac{\partial d}{\partial y_j} + \gamma_j \frac{\partial d}{\partial z_j} = 0 \]  

(5-7)
With the analytical expressions of the partials and after a considerable algebraic
effort, one arrives at six independent solutions \( y_i, i = 1, 2, \ldots, 6 \), and the inner
constraint matrix \( E \) is formulated by setting

\[
E = \begin{bmatrix} y_1 & y_2 & \ldots & y_6 \end{bmatrix}
\]

The final result is

\[
\begin{bmatrix}
\frac{d\eta}{d\eta} & \frac{d\eta}{d\Omega} & \frac{d\Omega}{dx_1} & \frac{dy_1}{dx_1} & \frac{dz_1}{dx_1} & \ldots & \frac{dx_N}{dx_1} & \frac{dy_N}{dx_1} & \frac{dz_N}{dx_1} \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & \ldots & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\
1 & 0 & 0 & 0 & 0 & -z_1^o & y_1^o & 0 & -z_N^o & y_N^o & 0 \\
0 & 1 & 0 & 0 & z_1^o & 0 & -x_1^o & z_N^o & 0 & -x_N^o & 0 \\
0 & 0 & -1 & 0 & -y_1^o & x_1^o & 0 & -y_N^o & x_N^o & 0 & 0
\end{bmatrix}
\]

(5-8)

where \( x_i^o, y_i^o, z_i^o \) are the approximate values of station coordinates used in the
computation of the design matrix \( A \). More explicitly there are two sets of constraints.
The first one

\[
\sum_i \begin{bmatrix} \frac{dx_1}{dy_i} \\ \frac{dy_1}{dz_i} \end{bmatrix} = 0
\]

(5-9)

"defines" the origin of the earth-fixed system, while the second one

\[
\begin{bmatrix}
-\frac{d\eta}{dx_1} \\
-\frac{d\xi}{dy_1} \\
\frac{d\Omega}{dz_1}
\end{bmatrix} = \sum_i \begin{bmatrix} 0 & -z_i^o & y_i^o \\ z_i^o & 0 & -x_i^o \\ -y_i^o & x_i^o & 0 \end{bmatrix} \begin{bmatrix} \frac{dx_1}{dy_1} \\ \frac{dy_1}{dz_1} \end{bmatrix}
\]

(5-10)

"defines" its orientation.

In the discussion above, the problem of epoch and time scale definition for the
set of station clocks has been ignored. The parameters \( C_{ij} \) and \( C_{ij} \) refer only to
relative offsets and drifts between the clocks at stations \( i \) and \( j \).
Usually epoch and time scale are provided by the readings of one of the clocks (master clock), and they are transformed to other station clocks through the parameters $C_{oj}, C_{n}$, where the master clock is at the $m$th station.

Another way to provide epoch and time scale is the use of inner constraints defining a fictitious master clock. We may shift to a new set of parameters by setting

$$C_{otl} = C_{oij} - C_{o11}$$

$$C_{oij} = C_{opi} - C_{p1}$$

(5-11)

The new parameters are offsets $C_{o11}$ and drifts $C_{p1}$ with respect to the fictitious master clock, and the additional inner constraints are

$$\sum_i dC_{o11} = 0 \quad \text{and} \quad \sum_i dC_{p1} = 0$$

(5-12)

6. Remarks Related to Design of Experiment Concepts

Since the usual objective of a geodetic experiment is the estimation of certain parameters, an experimental design may be considered optimal when the errors in parameter estimates are minimized in some certain sense. Within the linear least squares model, the a posteriori variance-covariance matrix of parameter estimates is given in general by

$$S_i = \sigma_0^2 (A^T P A)^+ = \sigma_0^2 Q^+$$

(6-1)

where $Q^+$ denotes the pseudoinverse of a square matrix $Q$, $P$ is the weight matrix and $\sigma_0^2$ the a posteriori estimate of the variance of unit weight. The "errors" in parameter estimates therefore depend on the observational accuracy and on the design matrix $A$. The design matrix $A = A(X^0)$ itself depends on the approximate values of parameters $X^0$. The determination of optimal $X^0$ values minimizing some risk function (e.g., trace $S_i$) is usually referred to as the "configuration problem," or the "first-order optimal design" [Grafarend, 1974]. On the assumption that the risk function chosen is insensitive to small changes of $X^0$, the approximate values of parameters may be identified with the true ones. In a similar way the small
angles \eta, \xi, \Omega, H in our model may be considered to be zero. Of the remaining parameters \Theta_0 depends on the choice of reference frames, \Omega is approximated by the fixed average rotational velocity of the earth, while in the case of transcontinental baselines the choice of radio source coordinates \alpha_p, \delta_p is strongly restricted by observability conditions. Therefore, the first-order optimal design is practically limited to the determination of the optimal configuration of the network of observing stations.

The order of magnitude of elements in a row of the design matrix A determines the sensitivity of the corresponding observation with respect to each of the parameters. Next a short geometric interpretation of these partials is given to provide possible "hints" towards the determination of the optimal design.

The partial derivatives with respect to \eta, \xi, \Theta_0, \Omega, \Xi, H (earth rotation parameters) and \alpha_p, \delta_p are proportional to the baseline length. Indeed, they all contain linear terms in \xi_{ij} = r_{ij} \alpha_{ij}, y_{ij} = r_{ij} \gamma_{ij} and z_{ij} = r_{ij} \delta_{ij}, where r_{ij} is the length of the ij baseline and \alpha_{ij}, \gamma_{ij}, \delta_{ij} are the components of the unit vector r_{ij} in the direction of the baseline. It is therefore possible to define "sensitivity per unit of baseline length" by means of \[ \frac{1}{r_{ij}} \frac{\partial d}{\partial \beta} \] where \beta stands for any of the earth rotation or radio source coordinates parameters. It can be shown that

\[ \frac{1}{r_{ij}} \frac{\partial d}{\partial \beta} = (r_{ij}^o) \cdot S_{\beta} \quad (6-2) \]

i.e., that the sensitivity per unit of baseline length is the projection of some vector \( S_{\beta} \) (called the parameter \beta sensitivity vector) on the direction of the baseline.

It can be easily verified that the parameter sensitivity vectors are given by.

\[ S_{\eta} = - R_1 (\pi/2) \ P_{yz} \ R_3 (X) \ e_{5} \]
\[ S_{\xi} = - R_0 (\pi/2) \ P_{xz} \ R_3 (X) \ e_{5} \]
\[ S_{\Theta_0} = \tau^3 R_3 S_{\Omega} = P_{xy} \ R_3 (X + \pi/2) \ e_{5} \]
\[ S_{\Xi} = R_3 (\psi) \ R_2 (\pi/2) \ P_{xz} \ R_3 (- \alpha) \ e_{5} \]
\[ S_{H} = R_3 (\psi) \ R_1 (\pi/2) \ P_{yz} \ R_3 (- \alpha) \ e_{5} \]
\[ S_{\alpha} = - R_3 (X + \pi/2) \ P_{xy} \ e_{5} \]
\[ S_{\delta} = R_3 (X) \ R_2 (\pi/2) \ e_{5} \quad (6-3) \]
where

\[ e_\delta = [\cos \delta, 0, \sin \delta]^T, \quad e_p = R_3(-\alpha) e_\delta \]

and

\[
P_{xy} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad P_{yz} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad P_{xz} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}
\]

are projection matrices on the xy, yz and xz planes, respectively. The position of the sensitivity vectors varies with time, and their loci, all contained within the unit sphere, are depicted in Fig. 6.1. Maximum sensitivity occurs when the baseline is parallel to the corresponding parameter sensitivity vector, while an observation is insensitive to a parameter when the baseline and parameter sensitivity vector are perpendicular. With this in mind and with the help of Fig. 6.1, Table 6.1 is constructed, summarizing the sensitivity to various parameters for extreme cases of baseline and radio source directions.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Equatorial Radio Sources ((\delta = 0^\circ))</th>
<th>Equatorial Radio Sources ((\delta = 90^\circ))</th>
<th>Polar Radio Sources ((\delta = 0^\circ))</th>
<th>Polar Radio Sources ((\delta = 90^\circ))</th>
</tr>
</thead>
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<tr>
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<td>no</td>
<td>yes</td>
<td>yes</td>
<td>no</td>
</tr>
<tr>
<td>(\Xi, \Omega)</td>
<td>no</td>
<td>yes</td>
<td>yes</td>
<td>no</td>
</tr>
<tr>
<td>(\Theta, \Omega, \alpha)</td>
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<td>no</td>
<td>no</td>
<td>no</td>
</tr>
<tr>
<td>(\delta)</td>
<td>no</td>
<td>yes</td>
<td>yes</td>
<td>no</td>
</tr>
</tbody>
</table>

Table 6.1

Sensitivity of Baseline - Radio Source Extreme Configurations with Respect to Various Parameters
Note that observations in the fourth column are impossible due to observability conditions (a baseline parallel to the rotation axis must be located near the equator and observe equatorial radio sources only).

The sensitivity with respect to $\Omega$ as well as to $C_{ij}$ is proportional to the time span $T_k$ of the available observations while that to $C_{ij}$ and $C_{kj}$ is independent of the baseline - radio source configuration.

Sensitivity with respect to station coordinates is independent of baseline length. If $e_x$, $e_y$, $e_z$ denote unit vectors in the directions of the x, y, z axes, respectively, the relevant partial derivatives under the same approximations are

$$\frac{\partial d}{\partial x_j} = e_x^T S_o, \quad \frac{\partial d}{\partial y_j} = e_y^T S_o, \quad \frac{\partial d}{\partial z_j} = e_z^T S_o$$

where $S_o = R_3(-\chi) e_6$. The sensitivity of observations with respect to station coordinates is determined by the projection of the coordinate sensitivity vector $S_o$ on the coordinate axes. The locus of $S_o$ is depicted in Fig. 6.1. It can readily be seen that observations to equatorial radio sources are insensitive to the z station coordinates, while observations to polar radio sources are insensitive to x and y station coordinates.

All the above remarks are of a rather general nature and simply provide "hints" towards the design of an optimal station configuration for the VLBI experiment, which also depends on which of the parameters in particular is to be determined.
Fig. 6.1 Geometric loci of parameter sensitivity vectors.
Fig. 6.1 (cont'd)
7. Simulation and Adjustment Philosophy

The objective of this simulation is the determination of standard deviations of parameter estimates relevant to VLBI observations for a number of characteristic network designs. In addition to this design (relative configuration of station - radio sources and arrangement of observations in time), the a posteriori standard deviations of parameters also depend on a number of other factors. These include the accuracy of observations, their total time span, and the modeling of earth rotation and relative station motions in the adjustment. Since step function models are used here, the time interval over which earth rotation parameters are considered to be constant ("earth step") and the time interval over which station coordinates are considered constants ("station step") become also variables. It has also been assumed that a radio source coordinate catalogue is available, and its accuracy is one additional variable affecting final results. Summarizing, the problem is to examine the variation of standard deviations of the following parameters:

i. station coordinates

ii. earth rotation parameters

with respect to the following independent variables:

a. observational noise
b. time span of observations
c. earth step
d. station step
e. accuracy of radio source catalogue
f. station - radio source configuration, arrangement of observations in time.

With respect to the last item above, three different variables appear in the same group. This is justified in view of the fact that once a transcontinental network design has been decided upon, observability conditions impose strong limitations on both the choice of radio sources to be observed and the arrangement of observations in time.
The fact that observations are not instantaneous, but averages over some time interval (integration time of 7-10 minutes), imposes also limitations on the maximum density of observations in time.

The choice of earth and station step in real data analysis depends on the frequency content of the corresponding time functions. In view of the aliasing effect [Koopmans, 1974], the smaller the step size, the higher the spectral resolution of the corresponding functions. On the other hand, larger step size leads to a smaller number of parameters to be estimated and a "stronger" adjustment with more degrees of freedom. The choice of optimal step size in the analysis of real data constitutes a very interesting problem worthy of a separate study.

In the present simulation study, step size has been considered as a variable, while the corresponding functions generated in the simulation of observations are continuous rather than step functions. However, precautions have been taken so that the total variation \( V_{f,I} \) of the function \( f(t) \) in question over each step time interval \( I \) is insignificant compared to observational noise. In fact,

\[
V_{f,I} = \max_{t \in I} [f(t)] - \min_{t \in I} [f(t)] \leq \sigma_b / N \quad i = 1, 2, \ldots, n
\]

where \( \sigma_b \) is the standard deviation of the observations, and \( N \) is a large-integer (specifically here \( N = 10 \)).

In view of a 7-10-minute integration time, and allowing for antenna motion, observations (time delays and their time derivatives) are uniformly spaced at 15-minute intervals over the total time span considered. Their corresponding standard deviations \( \sigma_b \) and \( \sigma_b' \) are taken to be of a constant ratio \( \sigma_b / \sigma_b' = 1/2 \), where \( \sigma_b \) is in nsec and \( \sigma_b' \) in nsec/hr. This value is typical, but the ratio may be somewhat different depending on antenna and observed signal characteristics. The choice of a fixed ratio reduces the number of considered parameters by one with a corresponding significant reduction in computational effort.

Once a certain observational pattern has been decided upon, a simulation program (VLBI SIMULATOR) produces perfect observations using the simplified model (Eq. 4-1). This program requires as input station and radio source true coordinates and the pattern of the observations (baseline - radio source - epoch of
observation). The station coordinates are taken to be constants, while the values of earth rotation parameters at the epoch of observation are provided by a subroutine (SUBROUTINE EARTH). $\Theta_0$ is fixed to a realistic value for the beginning of the observations $t_0$. The effect of traditional precession-nutation is absent from the simulation as well as from the adjustment model for computational simplicity. The inertial frame thus is taken to be the true equatorial system of the epoch $t_0$ so that its $Z$-axis is near the rotational axis. For the same reason, the earth-fixed frame is taken to be the CIO/BIH zero-meridian terrestrial system.

The perfect observations are processed through a program which adds "Observational errors" drawn from a Gaussian population with zero mean and desired variance.

For the adjustment of the observations, two separate programs are used. The first one (PROG01) processes observations of total time span equal to both earth and station step (one day). An option is available for processing any desired number of different radio source catalogue accuracies in the same run.

The second program (PROG02) processes observations of time span equal to the station step only, which in turn is taken to be an integer multiple of the earth step. It is, however, possible to process observations using unequal earth steps within the total span of available observations. Within each earth step the linearized observations equations are of the form

$$V_i = \dot{A}_t X + \ddot{A}_t \ddot{X}_i + L_i$$

where $X$ refers to parameters common to all earth steps (station and radio source coordinates) and $\ddot{X}_i$ refers to parameters particular to each earth step (earth rotation parameters).

The total set of such individual observation equations is combined in a unified solution through a "second-order partitioned linear regression" scheme, extended to incorporate inner constraints. The detailed algorithm is presented in Appendix B. The related programs and subroutines are in Appendix C.
8. **Arrangement of Experiments**

To obtain more realistic results, station locations considered here are not completely arbitrary, but limited to present and prospective locations of VLBI stations. Such candidate station locations are presented in Table 8.1. In the same table the stations actually used in this study are indicated by asterisks. Two criteria have been considered in the selection. First, stations close to plate boundaries, i.e., in geophysically active areas, have been excluded to avoid possible comparatively large station drifts deviating from those considered in our simulations. Secondly, from groups of stations close to each other, only one station has been included on the assumption that final results would be similar for a nearby station. It is recognized that these criteria probably exclude the most active stations currently in operation, but this should not influence the general conclusions sought.

An experiment consists of a group of stations simultaneously participating in the observations. In Table 8.3 the station groups for the experiments considered in this study are presented. After a certain station group has been selected, the arrangement of observations is one observations every 15 minutes, which interval and choice of the radio sources are limited by observability conditions. The observability condition naturally imposed is that the observed radio source must be above the horizon at both ends of the observing baseline. To avoid large atmospheric refraction errors, the zenith distance of the radio source is limited, in general, to be less than $z_{\text{max}} = 45^\circ$, with some exceptional cases where $z_{\text{max}} = 60^\circ$.

Such observability conditions limit the observable part of the sky to within the intersection of two cones with the local station verticals as axes and $z_{\text{max}}$ as the vertex angle. For transcontinental baselines, the region of observability is small and the choice is between observing a certain quasar or a neighboring one without significant effect on the experimental design.

In setting out a certain sequence of observations, the choice between possible alternatives has been guided by the hints obtained in Chapter 6, with emphasis on sensitivity with respect to earth motion and station coordinate parameters.
Table 8.1
Present and Prospective VLBI Station Locations and Their Approximate Coordinates

<table>
<thead>
<tr>
<th>No.</th>
<th>Location</th>
<th>Longitude</th>
<th>Latitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Canberra, Australia</td>
<td>149</td>
<td>-35</td>
</tr>
<tr>
<td>2</td>
<td>Kauai, Hawaii</td>
<td>200</td>
<td>22</td>
</tr>
<tr>
<td>3</td>
<td>Fairbanks, Alaska</td>
<td>212</td>
<td>65</td>
</tr>
<tr>
<td></td>
<td>Goldstone, California</td>
<td>243</td>
<td>35</td>
</tr>
<tr>
<td></td>
<td>Algonquin, Canada</td>
<td>277</td>
<td>51</td>
</tr>
<tr>
<td></td>
<td>Greenbank, Virginia</td>
<td>282</td>
<td>38</td>
</tr>
<tr>
<td>4</td>
<td>Haystack, Massachusetts*</td>
<td>288</td>
<td>41</td>
</tr>
<tr>
<td></td>
<td>Santiago, Chile</td>
<td>289</td>
<td>-34</td>
</tr>
<tr>
<td></td>
<td>Arecibo, Puerto Rico</td>
<td>294</td>
<td>18</td>
</tr>
<tr>
<td>5</td>
<td>Sao Paulo, Brazil</td>
<td>313</td>
<td>-24</td>
</tr>
<tr>
<td>6</td>
<td>Madrid, Spain</td>
<td>356</td>
<td>40</td>
</tr>
<tr>
<td></td>
<td>Bonn, Germany</td>
<td>7</td>
<td>51</td>
</tr>
<tr>
<td>7</td>
<td>Onsala, Sweden</td>
<td>17</td>
<td>59</td>
</tr>
<tr>
<td>8</td>
<td>Johannesburg, So. Africa*</td>
<td>28</td>
<td>-26</td>
</tr>
<tr>
<td></td>
<td>Crimea, USSR</td>
<td>34</td>
<td>45</td>
</tr>
</tbody>
</table>

* indicates stations considered in this study
Table 8.2

Fictitious Stations Used in Experiments 6 and 9

<table>
<thead>
<tr>
<th>No.</th>
<th>Location</th>
<th>Longitude</th>
<th>Latitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>equatorial</td>
<td>0°</td>
<td>0°</td>
</tr>
<tr>
<td>10</td>
<td>&quot;</td>
<td>60</td>
<td>0</td>
</tr>
<tr>
<td>11</td>
<td>&quot;</td>
<td>120</td>
<td>0</td>
</tr>
<tr>
<td>12</td>
<td>&quot;</td>
<td>180</td>
<td>0</td>
</tr>
<tr>
<td>13</td>
<td>&quot;</td>
<td>240</td>
<td>0</td>
</tr>
<tr>
<td>14</td>
<td>&quot;</td>
<td>300</td>
<td>0</td>
</tr>
<tr>
<td>15</td>
<td>meridian</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>16</td>
<td>&quot;</td>
<td>0</td>
<td>60</td>
</tr>
<tr>
<td>17</td>
<td>&quot;</td>
<td>180</td>
<td>60</td>
</tr>
<tr>
<td>18</td>
<td>&quot;</td>
<td>180</td>
<td>0</td>
</tr>
<tr>
<td>19</td>
<td>&quot;</td>
<td>180</td>
<td>-60</td>
</tr>
<tr>
<td>20</td>
<td>&quot;</td>
<td>0</td>
<td>-60</td>
</tr>
</tbody>
</table>
Table 8.3
Experiments and Participating Baselines
(Station Nos. Refer to Numbering in Table 8.1 or 8.2)

<table>
<thead>
<tr>
<th>Experiment No.</th>
<th>Participating Baselines</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2-3, 2-4, 3-4</td>
</tr>
<tr>
<td>2</td>
<td>3-4, 3-7, 4-7</td>
</tr>
<tr>
<td>3</td>
<td>4-5, 4-6, 5-6</td>
</tr>
<tr>
<td>4</td>
<td>5-6, 5-8, 6-8</td>
</tr>
<tr>
<td>5</td>
<td>1-2, 2-3, 2-4, 3-4, 3-7, 4-5, 4-6, 4-7, 5-6, 5-8, 6-7, 6-8</td>
</tr>
<tr>
<td>6</td>
<td>9-10, 10-11, 11-12, 13-14, 14-9</td>
</tr>
<tr>
<td>7</td>
<td>2-4, 4-5</td>
</tr>
<tr>
<td>8</td>
<td>2-4, 4-6</td>
</tr>
<tr>
<td>9</td>
<td>15-16, 16-17, 17-18, 18-19, 19-20, 20-15</td>
</tr>
<tr>
<td>10</td>
<td>3-4, 4-7</td>
</tr>
</tbody>
</table>
An effort has also been made to observe radio sources of a variety of declinations to avoid the critical configuration appearing when, for example, all radio sources are of the same declination. From a larger set of available radio sources, only a rather uniformly distributed set has been considered. The criterion for inclusion has been the use of the radio source in previous experiments so that its appropriateness for VLBI observations is assured.

Since a radio source catalogue of a certain accuracy is assumed to be available, no effort has been made to optimize radio source coordinate recovery. As a result, this study is of no "astrometric" value, and no significant improvement of radio source coordinate accuracy can be found in the results of the adjustment performed.

One sidereal day can be taken as the "unit" for an observational pattern since a sequence of observations performed over one day can be repeated over the next days. For observations of a total time span of a number of days, the same daily pattern is repeated in the simulation. Various experimental designs are compared to each other on the basis of one day of observations where earth rotation and baseline parameters are considered as constants over the whole day.

The choice of experiments performed has been primarily directed towards the possibility of recovering earth rotation parameters from a minimum number of observing stations. The minimum number is the three stations necessary for the definition of an earth-fixed system through minimal or inner constraints. Experiments 1, 2, 3 and 4 are cases of such minimal "triangle networks" where all the sides of the triangle are observing baselines.

A weaker design (Experiments 7, 8, 10) is a three station configuration where only two of the triangle sides are observing baselines. In particular, Experiment 10 is a counterpart of Experiment 2, such that the effect of removing one baseline (other design parameters remaining the same) can be studied. The opposite to such minimal three-station designs is provided in Experiment 5 where all eight stations participate in the observations.
In addition to these "realistic" station locations, two "fictitious" designs are also considered, their common characteristic being the closure of a network of stations on a great circle around the earth. Experiment 6 is such a network of uniformly spaced stations along the equator, while Experiment 9 is of a similar design along a meridian. The coordinates of stations involved in these experiments are listed in Table 8.2.

Fig. 9.1 depicts the geographic locations of the networks in the above-mentioned experiments.
Fig. 9.1  Geographic locations of stations and baselines in Experiments 1 through 10.
9. Results and Conclusions

The results of the simulations are presented in three parts. The first part (Figs. 9.2 - 9.3) deals with the comparison of the ten designs described in the previous chapter, on the basis of one day of observations. Fig. 9.2 depicts the recovery (standard deviations) of earth rotation parameters \( (\xi, \eta, \Xi, \Theta, \Omega) \), while Fig. 9-3 shows the recovery of network geometry in terms of baseline lengths and angles (estimable quantities).

Standard deviations \( \sigma \) for each parameter are given as a function of the standard deviation \( \sigma_0 \) of radio source coordinates and for various values of observational precision \( \sigma_b \). Values used in the simulation are \( \sigma_b, \sigma_0 = 1, 5, 10, 20, 50 \) cm. To achieve uniformity, \( \sigma_0 \) is expressed as arc length on a reference sphere with a radius of 6371 km \((1^\circ \approx 30 \text{ m})\).

The second part (Figs. 9.4 - 9.5) is a study of the improvement of the standard deviations of the parameters with the total time interval \( T \) of available observations. Station coordinates are assumed to be constant over \( T \) while earth rotation parameters are constant only over the subintervals of duration \( \Delta t \) (earth step). The ratio \( \rho = \sigma / \sigma_b \) is given for each parameter as a function of \( T \) for various values of \( \Delta t \). In the simulations, the following values have been used: \( T = 1, 2, 3, 4, 5, 10, 30 \) days; \( \Delta t = 6, 12, 24 \) hours. Experiment 2 has been chosen for this study. Radio source coordinate standard deviations are assumed to be \( \sigma_0 = \sigma_b \) for all cases. However, the variation of the results with \( \sigma_0 \) is insignificant for \( T \geq 5 \) days.

The third part (Figs. 9.6 and 9.7) is a study of the variation of \( \rho = \sigma / \sigma_b \) with \( \Delta t \) (earth step) based on one day of observations \((T = 1 \text{ day})\) for some of the other designs (Experiments 1, 3, 4, 6, 9, 10) is shown.

Since the results of the simulations are fully presented here, one may draw his own conclusions. However, the following general remarks are made in an attempt to summarize the results:

(1) Earth Rotation Parameters from One-Day Observations

For one day of observations and the recovery of earth rotation parameters, the following are noted:

(a) Three-baseline "triangle" designs (Experiments 1, 2, 3, 4 in Fig. 9.2) are capable of recovering polar motion \( (\xi, \eta) \) with a precision of the same order of magnitude as that of the observations, even for low radio source accuracies \( (\sigma_0 = 50 \text{ cm}) \).
For two-baseline designs (Experiments 7, 8, 10 in Fig. 9.2), polar motion recovery is of the same quality as for the "triangle" station configurations. However, the removal of one baseline in Experiment 10 results in an increase of polar motion standard deviations by a factor of 1.5 compared to Experiment 2. The more complex design of Experiment 5 offers no dramatic improvement in polar motion recovery. An essential improvement can be seen in the equatorial design of Experiment 6 and even more in Experiment 9 which is the "best," but admittedly not very practical design for polar motion recovery.

(b) The standard deviations of the (precession-nutation) parameters $\Xi$ and $H$ are practically the same for all two- or three-baseline designs (Experiments 1, 2, 3, 4, 7, 8, 10). In particular, the removal of one baseline in Experiment 10 with respect to Experiment 2 has no effect on the recovery of $\Xi$ and $H$. The best recovery of $\Xi$, $H$ can be seen in Experiments 5 and 9. Although the station configuration in Experiment 6 is of the same shape as in Experiment 9, the change in their position (equatorial vs. meridian) has a great effect on $\Xi$, $H$ standard deviations. Experiment 6 has the weakest recovery, while Experiment 9 provides the strongest one.

(c) The standard deviation of $\Theta_o$ varies with the station configurations. It is highest for Experiments 4, 8, 7; medium for Experiments 10, 3, 5; and smallest for Experiments 1, 2, 9, and especially 6 (best).

(d) Recovery of $\Omega$ varies strongly even for designs of the same type. It is generally weak for two-baseline designs (Experiments 7, 8, 10) and somewhat better for three-baseline designs with the exception of the extremely weak recovery in Experiment 4. Many-station designs (Experiments 5, 6, 9) improve the recovery especially in the equatorial configuration of Experiment 6.

(e) $\Omega$ and $\Theta_o$ are the only parameters favored by the artificial equatorial design of Experiment 6 vs. the meridian one of Experiment 9. Recovery of the two components $\xi$ and $\eta$ of polar motion is not equal but depends on the position of the stations in each experiment with the exception of Experiment 6. On the contrary, both the $\Xi$ and $H$ parameters of precession-nutation are recovered with practically the same precision for every design.
The recovery of earth rotation parameters varies with the accuracy \( (\sigma_Q) \) of radio source coordinates. However, this dependence is much stronger for \( \Xi, H \); milder for the rest of the parameters; and even disappearing for \( \Omega, \xi, \eta \) in the strongest designs of Experiments 6 and 9.

(f) The results of the idealized designs of Experiments 6 and 9 can be used to verify some of the results of Chapter 6. Recovery of \( \xi, \eta, \Xi, H \) is better for Experiment 9 than 6, while recovery of \( \Theta_0, \Omega \) is better for Experiment 6. This is explained by the fact that Experiment 9 involves baselines parallel to the equator (16-17, 19-20) observing polar radio sources, a design sensitive to \( \xi, \eta, \Xi, H \). Experiment 6 involves equatorial baselines observing equatorial radio sources, the design most sensitive to \( \Theta_0, \Omega \) (see Table 6.1).

(g) The recovery of baselines varies with station configuration. However, larger number of baselines does not in general improve the individual recovery as in the case of earth rotation parameters. For example, recovery in the three-baseline design of Experiment 2 is much better than in any of the many-station designs (Experiments 5, 6, 9). The relative recovery of angles in the same experiment is inversely proportional to the recovery of the opposite baselines. Dependence on radio source accuracy varies and disappears in the case of the equatorial configuration of Experiment 6, especially in comparison to the meridian configuration of Experiment 9.

(2) **Earth Rotation and Baseline Parameters As a Function of the Length of Observations and Earth Step**

For the variation of the recovery of earth rotation and baseline parameters with respect to the total time interval of observations \( T \), it is noted that the standard deviations \( \sigma \) of the parameters are directly proportional to the standard deviations \( \sigma_b \) of the observations. (Note that \( \sigma_Q = \sigma_b \).) For this reason only the ratio \( \rho = \sigma/\sigma_b \) is depicted in Figs. 9.4 and 9.5.

The dependence of \( \rho \) on the earth step \( \Delta t \) is very strong for earth rotation parameters, especially for \( \Omega \) (see Fig. 9.4), while not significant for baseline
parameters (see Fig. 9.5). In general, the recovery improves as $\Delta t$ increases since the corresponding number of unknown parameters in the adjustment decreases. For earth rotation parameters (Fig. 9.4), standard deviation decreases strongly as $T$ increases from 1 to 5 days. An additional increase of $T$ from 5 to 10 days offers a small improvement, while no significant additional improvement is gained from the extension of $T$ from 10 to 30 days. The same is true for baseline parameters (lengths and angles, see Fig. 9.5) with the exception of an additional small improvement with the increase of $T$ from 10 to 30 days.

(3) **Earth Rotation and Baseline Parameters As a Function of Earth Step**

As mentioned, the recovery of earth rotation and network geometry parameters improves with larger earth step $\Delta t$ (Figs. 9.6 and 9.7). The only exceptions are with respect to the $\xi, \eta$ parameters in Experiments 1, 2, 4, 10 and with respect to $\Theta_o$ in Experiment 4 where $\Delta t = 24$ hours results in worse recovery than $\Delta t = 12$ hours or even $\Delta t = 6$ hours in some cases. This "inversion" is correlated to the weak recovery of polar motion in Experiments 1, 2, 4, 10 (see Fig. 9.2) and to the weak recovery of $\Theta_o$ in Experiment 4. On the contrary, for Experiments 6 and 9, where $\xi, \eta$ and $\Theta_o$ recovery is better, variation of $\rho$ with respect to $\Delta t$ is "normal." Such variation is negligible for $\Theta_o$ in Experiment 6 and also for network geometry parameters in Experiments 6 and 9. Naturally an increase in the length of the earth step is a disadvantage when short periodic variations in the earth rotation parameters are sought.
Fig. 9.2  Recovery of earth rotation parameters, Experiments 1 - 10.

Total time span of observations: \( T = 1 \text{ day} \)

Earth step: \( \Delta t = 1 \text{ day} \)

\( \sigma = \) a posteriori standard deviation of earth rotation parameters

\( \sigma_\text{q} = \) a priori standard deviation of radio source coordinates

\( \sigma_\text{b} = \) standard deviation of observational errors

Note: Angular quantities when expressed in length units are arc lengths on a sphere of radius \( R = 6371000 \text{ m} \) (\( 1'' \approx 30 \text{ m} \)).
EXPERIMENT 1

Fig. 9.2 Recovery of earth rotation parameters.
EXPERIMENT 2

Fig. 9.2 (cont'd)
Fig. 9.2 (cont'd)
EXPERIMENT 4

Fig. 9.2 (cont'd)
EXPERIMENT 5

Fig. 9.2 (cont'd)
EXPERIMENT 7

Fig. 9.2 (cont'd)
EXPERIMENT 8

Fig. 9.2 (cont'd)
Fig. 9.2 (cont'd)
EXPERIMENT 10

Fig. 9.2 (cont'd)
Fig. 9.3  Recovery of baseline lengths and angles, Experiments 1 - 10

Total time span of observations: $T = 1$ day

Earth step: $\Delta t = 1$ day

For explanation of other symbols, see Fig. 9.2, p. 40.
EXPERIMENT 1

Fig. 9.3  Recovery of baseline lengths and angles.
EXPERIMENT 2

Fig. 9.3 (cont'd)
EXPERIMENT 3

Fig. 9.3 (cont'd)
EXPERIMENT 4

Fig. 9.3 (cont'd)
EXPERIMENT 5

Fig. 9.3 (cont'd)
BASELINES

w = WORST
b = BEST

ANGLES

EXPERIMENT 6

Fig. 9.3 (cont'd)
EXPERIMENT 7

Fig. 9.3 (cont'd)
EXPERIMENT 9

Fig. 9.3 (cont'd)
EXPERIMENT 10

Fig. 9.3 (cont'd)
Fig. 9.4 Variation of earth rotation parameter recovery with stepsize and the total time interval of observations (Experiment 2).
Fig. 9.5 Variation of baseline length and angle recovery with stepsize and the total time interval of observations (Experiment 2).
Fig. 9.6 Variation of earth rotation parameter recovery with earth step.
Fig. 9.6 (cont'd)
Fig. 9.7  Variation of baseline length and angle recovery with earth step.
Fig. 9.7 (cont'd)
References


Uotila, U.A. (1967). "Introduction to Adjustment Computations with Matrices." Lecture Notes, Department of Geodetic Science, The Ohio State University, Columbus.
APPENDIX A

Inner Constraints and Their Relation to the "Geometry" of the Pseudoinverse of an Operator

Suppose that a system of linear equations
\[ N x = y \]
\[ u \times u \quad u \times 1 \quad u \times 1 \]
is given (\( N, y \) known; \( x \) unknown) which is consistent (i.e., it has at least one solution), and furthermore it has more than one solution. The objective is to show that there exists a set of linear constraints
\[ E^T x = 0 \]
called inner constraints, such that the solution to the simultaneous system of equations \( N x = y \) and \( E^T x = 0 \) is unique and identical to the solution \( x_0 = N^+ y \), where \( N^+ \) is the pseudoinverse of the matrix \( N \).

The matrix \( N \) is a representation of an operator \( N: X \rightarrow Y \), where \( X \) and \( Y \) are both \( u \)-dimensional inner product spaces with elements \( u \times 1 \) vectors and the usual Euclidean inner product
\[ \langle f, g \rangle = g^T f \quad f, g \in X \quad \text{or} \quad f, g \in Y \]
Two subspaces can be introduced with respect to the operator \( N \), the range \( R(N) = R \) of \( N \) defined as
\[ R = \{ y; \ y \in Y \ \text{and} \ y = Nx \ \text{for some} \ x \in X \}, \quad R \subseteq Y \]
and the kernel or null space \( K(N) = K \subseteq X \) of \( N \),
\[ K = \{ x; \ x \in X \ \text{and} \ Nx = 0 \} \]
The consistency of the linear equations \( N x = y \) refers to the fact that \( y \in R \), while the non-uniqueness of solution to the fact that \( K \cap \{ 0 \}^\circ \) is non-empty, i.e., \( K \) has other elements in addition to \( 0 \). If \( x' \) is a solution to \( Nx = y \), then \( x = x' + \bar{x} \) is also a solution where \( \bar{x} \) is any element of \( K \). We can therefore identify the solution

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space $S_y \subset X$, defined as
\[ S_y = \{ x; x \in X \text{ and } N x = y, y \in R \text{ fixed} \} \]
with a linear variety
\[ S_y = \{ x; x = x' + \bar{x}, \bar{x} \in K, x' \text{ fixed with } N x' = y \} \]
If $K^\perp$ is the orthogonal complement of $K$ with respect to $X$, the projection $x_0 = \mathcal{P}_{K^\perp}(x)$ of any element $x \in S_y$ on $K^\perp$ is unique, and furthermore [Dermanis, 1977, §3.5]
\[ \| x_0 \| = \min_{x \in S_y} \| x \| \]
We can now define the pseudoinverse $N^+$ of the operator $N$ as an operator
\[ N^+: R \to X \text{ where} \]
\[ N^+ y = x_0 = \mathcal{P}_{K^\perp}(x) \quad \text{for any } y \in R \text{ and } x \in S_y \]
$N^+$ is therefore the ordinary inverse of the restriction of the operator $N$ to $K^\perp$ (see [Desoer and Whalen, 1963, p. 444] for a more rigorous definition). The domain of definition of $N^+$ can be extended to the whole space $Y$ ($N^+: Y \to X$) by setting
\[ N^+(y) = N^+(y') \]
where $y' = \mathcal{P}_R(y)$ is the projection of $y \in Y$ on $R$.

The range of $N^+$ is $K^\perp$, while its kernel (null) space is $R^\perp$. For $y \in R$ we have $x_0 = N^+ y$ and $x_0$ can be alternatively defined as the unique element in the intersection $S_y \cap K^\perp$. This means that $x_0$ is uniquely defined by its two properties:
\[ x_0 \in S_y \quad \text{and} \quad x_0 \in K^\perp \]
The first property is expressed by the original equations $N x = y$, while a similar expression must be established for the second property. Let $r = \text{rank } (N) < u$ and $s = u - r$ be the rank deficiency of $N$. The dimension of the linear subspace $K \subset X$ is then $s$, and suppose that $\{ e_i \}, i = 1, 2, \ldots, s$ is a basis in $K$. Since $x_0 \in K^\perp$, we have $x_0 \perp e_i$ for $i = 1, 2, \ldots, s$, i.e.,
\[ <x_0, e_i> = e_i^T x_0 = 0 \quad \text{for } i = 1, 2, \ldots, s \]
Fig. A.1  The geometry of the pseudoinverse operator.
Setting

\[ E = [e_1, e_2, \ldots, e_s] \]
\[ s \times u \]

the above equations can be written in the compact matrix form

\[ E' x_0 = 0 \]

Therefore, \( x_0 \) is uniquely determined as the vector satisfying both equations

\[ N x = y \quad \text{and} \quad E' x = 0 \]

where the columns of \( E \) constitute a basis in \( K \), the kernel (null space) of \( N \). The constraints \( E' x = 0 \) are referred to as the "inner" constraints.

The problem has now been reduced to that of finding a basis in \( K \), i.e., of finding a set of \( s \) linearly independent \( s \times 1 \) vectors \( \{e_i\} \) satisfying

\[ N e_i = 0 \quad \text{for } i = 1, 2, \ldots, s \]

The equations above can be written again in matrix form

\[ NE = 0 \quad \text{where} \quad E = [e_1, e_2, \ldots, e_s] \]

We therefore only need to find a \( s \times s \) matrix \( E \) with rank \( (E) = s = u - \text{rank } (N) \)

(to secure linear independence of its columns \( e_i \) satisfying \( NE = 0 \). Summarizing, we have the following theorem

Given the system of linear equations

\[ N x = y \quad \text{(N, y known; x unknown)} \]
\[ u \times 1 \quad u \times 1 \]

where \( \text{rank } (N) = r < u, s = u - r, \) and a \( u \times s \) matrix \( E \) with rank \( (E) = s \) satisfying \( NE = 0 \), the vector \( x \) satisfying simultaneously \( N x = y \) and \( E' x = 0 \) is unique and identical to \( x = N^+ y \), where \( N^+ \) is the pseudoinverse of \( N \).

References


The problem in question is the construction of an algorithm for the least squares adjustment of a group of linearized observation equations of the form

\[ V_i = \tilde{A}_i \tilde{X}_i + \hat{A}_i \hat{X} + L_i \quad i = 1, 2, \ldots, m \]  \hspace{1cm} (A 1)

with \( E \{ V_i \} = 0 \) and \( E \{ V_i V_i^T \} = P_i \). \( \hat{X} \) refers to parameters common to all sets of observations, while \( \tilde{X}_i \) refers to those appearing only in the \( i \)th set of observations. Each set of observation equations gives rise to a contribution to the normal equations of the form

\[
\begin{bmatrix}
\tilde{N}_i & \tilde{N}_i \\
\tilde{N}_i^T & \tilde{N}_i
\end{bmatrix}
\begin{bmatrix}
\tilde{X}_i \\
\tilde{X}_i
\end{bmatrix}
+
\begin{bmatrix}
\tilde{U}_i \\
\tilde{U}_i
\end{bmatrix}
= 0 \hspace{1cm} (A 2)
\]

where

\[
\tilde{N}_i = \tilde{A}_i^T P_i \tilde{A}_i, \quad \tilde{U}_i = \hat{A}_i^T P_i L_i \\
\tilde{N}_i = \tilde{A}_i^T P_i \tilde{A}_i, \quad \tilde{U}_i = \hat{A}_i^T P_i L_i
\]

and

\[ \tilde{N}_i = \tilde{A}_i^T P_i A_4 \]

The combination of such individual sets of normal equations leads to the final set of normal equations incorporating all observations

\[
\begin{bmatrix}
\tilde{N} & \tilde{N} \\
\tilde{N}^T & \tilde{N}
\end{bmatrix}
\begin{bmatrix}
\tilde{X} \\
\tilde{X}
\end{bmatrix}
+
\begin{bmatrix}
\tilde{U} \\
\tilde{U}
\end{bmatrix}
= 0 \hspace{1cm} (A 3)
\]

where

\[
\tilde{N} = \begin{bmatrix}
\tilde{N}_1 & 0 & 0 \\
0 & \tilde{N}_2 & 0 \\
0 & \tilde{N}_3 & \ddots
\end{bmatrix}, \quad \tilde{N} = \begin{bmatrix}
\tilde{N}_1 \\
\tilde{N}_2 \\
\vdots
\end{bmatrix}, \quad \tilde{U} = \begin{bmatrix}
\tilde{U}_1 \\
\tilde{U}_2 \\
\vdots
\end{bmatrix}, \quad \tilde{X} = \begin{bmatrix}
\tilde{X}_1 \\
\tilde{X}_2 \\
\vdots
\end{bmatrix}
\]

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\[
\dot{N} = \sum_i \dot{N}_i, \quad \dot{U} = \sum_i \dot{U}_i
\]

In case \( N = \begin{bmatrix} \ddot{N} & \bar{N} \\ \bar{N}^\prime & \tilde{N} \end{bmatrix} \) is nonsingular (\( \det N \neq 0 \)), the normal equations above can be solved through a First-Order Partitioned Linear Regression scheme as explained in [Brown and Trotter, 1969; and Uotila, 1973]. We give here an extension of this scheme for the case when \( N \) is singular and a unique solution is ascertained only after the introduction of a set of inner constraints

\[
\ddot{E}' \ddot{X} + \ddot{E}' \ddot{X} = 0
\]

with

\[
\ddot{E}' = [\ddot{E}_1 \ddot{E}_2 \ldots \ddot{E}_n] = [\ddot{E}_0 \ddot{E}_0' \ldots \ddot{E}_0']
\]

The normal equations can now be augmented as follows [Uotila, 1967]:

\[
\begin{bmatrix}
\ddot{N} & \ddot{E} \\
\ddot{N}^\prime & \ddot{E}
\end{bmatrix}
\begin{bmatrix}
\ddot{X} \\
\ddot{X}
\end{bmatrix}
+ \begin{bmatrix}
\ddot{U} \\
\ddot{U}
\end{bmatrix}
= 0
\]

(A 5)

The new augmented coefficient matrix is nonsingular and inversion with the introduction of some new notation leads to

\[
\begin{bmatrix}
\ddot{N} & \ddot{E} \\
\ddot{N}^\prime & \ddot{E}
\end{bmatrix}^{-1} = \begin{bmatrix}
\dddot{M} & \ddot{M} \\
\ddot{M}^\prime & \dddot{M}
\end{bmatrix}^{-1} = \begin{bmatrix}
\dddot{Q} & \dddot{Q} \\
\ddot{Q}^\prime & \dddot{Q}
\end{bmatrix} = \begin{bmatrix}
\ddot{G} & \ddot{G} & \dddot{F} \\
\ddot{G} & \ddot{G} & \dddot{F}
\end{bmatrix}
\]

(A 6)

With the help of this inverse, the solution of the normal equations becomes

\[
\ddot{X} = \dddot{G} \ddot{U} + \dddot{G} \ddot{U}
\]

(A 7)

\[
\dddot{X} = \ddot{G} \ddot{U} + \ddot{G} \ddot{U}
\]

If \( \ddot{G}_1, \dddot{G}_{11} \) denote submatrices of \( \ddot{G} \) and \( \dddot{G} \), respectively, with dimensions equal to those of \( \ddot{N}_1 \) and \( \dddot{N}_{11} \), respectively, we have

\[
\dddot{X} = \sum_j \dddot{G}_j \ddot{U}_j + \dddot{G} \ddot{U}
\]

(A 8)

\[
\dddot{X}_1 = \sum_j \dddot{G}_{1j} \ddot{U}_j + \dddot{G}_1 \ddot{U}
\]
The inversion of Eq. (A 6) leads to

\[ \dot{Q} = [\ddot{M} - \dot{M} \dddot{M} \dot{M}^{-1} \dot{M}] \]
\[ \ddot{Q} = -\dddot{M} \dot{M} \dot{Q} \]
\[ \dddot{Q} = \dot{M}^{-1} + \dot{M}^2 \ddot{M} \dot{Q} \dot{M}^T \dot{M}^3 \]

Replacing \( \ddot{M}, \dot{M}, \dddot{M} \) from (A 6) in terms of submatrices of \( \dddot{N}, \dot{N}, \dddot{N}, \) and introducing the following notation

\[ H_i = \dddot{N}_i^3 \dddot{N}_i \]
\[ R_i = \dddot{N}_i^T H_i \]
\[ S_i = \dddot{N}_i - R_i \]
\[ S = \sum_i S_i \]
\[ H = \sum_i H_i \]
\[ K = \sum_i \dddot{N}_i^3 \]
\[ \dddot{L}_i = H_i^T \dddot{U}_i \]
\[ L = \sum_i \dddot{L}_i \]
\[ T_i = \dddot{N}_i^3 \dddot{U}_i \]
\[ T = \sum_i T_i \]

we finally obtain after some algebraic manipulations

\[ \dot{Q}^3 = \begin{bmatrix} S & (\dot{E} - H^T \dddot{E}_0) \\ (E^T - \dddot{E}_0 H) & (-\dddot{E}_0 K \dddot{E}_0) \end{bmatrix} \]

\[ \dot{X} = \dot{G} L + F \dddot{E}_0^T T - \dot{G} \dddot{U} \]

\[ \dddot{X}_i = -T_i - \dddot{N}_i^3 \left[ \dddot{E}_0 \tilde{F}^T \left(-\dddot{U} + L \right) + (\dddot{E}_0 F \dddot{E}_0) T \right] - H_i \dot{X} \]

\[ \dddot{G}_{ij} = \delta_{ij} \dddot{N}_i^3 + H_i \dot{G} H_j^T + H_i (F \dddot{E}_0) \dddot{N}_j^3 + \]
\[ + \dddot{N}_i^3 (\dddot{E}_0 \tilde{F}_i^T) \dddot{H} + \dddot{N}_i^3 (\dddot{E}_0 F \dddot{E}_0) \dddot{N}_j^3 \]

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The algorithm can now be summarized in the following steps:

1. Compute $\tilde{A}_1, \hat{A}_1, L_1; \tilde{E}, \tilde{E}_0$
2. Compute $\tilde{N}_1, \hat{N}_1, \tilde{N}_1, \tilde{U}_1, \hat{U}_1$
3. Compute $H_1, R_1, S_1, T_1, L_1$
4. Compute $H, S, \hat{N}, \hat{U}, K, L, T$
5. Obtain $\hat{G}, \hat{F}, F$ by inversion

$$
\begin{bmatrix}
    S & E - H^T \tilde{E}_0 \\
    (E - H^T \tilde{E}_0)^T & -\tilde{E}_0^T K \tilde{E}_0
\end{bmatrix}
\begin{bmatrix}
    \hat{G} \\
    \hat{F}
\end{bmatrix}
= 
\begin{bmatrix}
    \dot{G} \\
    \dot{F}
\end{bmatrix}
$$

6. Compute $\dot{X}$ from (A11), $\tilde{X}_1$ from (A12), and $\tilde{G}_{1j}$ from (A13)

References


APPENDIX C

Computer Programs

The simulation of observations and their adjustment are carried out with the help of three programs, VLBI SIMULATOR, PROG$1$, and PROG$2$, as explained in Chapter 7. These programs are presented here in detail together with their supporting subroutines.

1. Name of Program: VLBI SIMULATOR

   **Function:** Simulates distance and distance rate VLBI observations corresponding to time delays and their derivatives, according to a previously decided observational pattern.

   **Input:** Number of participating stations and radio sources, radio source and station coordinates, initial and final epochs of observations, time interval between successive observations, identifying numbers of stations and radio source participating in each observation.

   **Output:** Identification numbers for participating stations and radio source, observed (perfect) distance and distance rate, epoch of observation.

Subroutines required:

- **GRESID:** Calculates Greenwich sidereal time for a given epoch
- **JULIA:** Converts Universal Time to Julian date
- **EARTH:** Provides simulated earth rotation parameters for a given epoch
- **REDPI:** Converts an angle in radians to the \((-\pi, \pi)\) interval.

The following is a listing of VLBI SIMULATOR program and supporting subroutines.
C****** CONVERT RA AND DEC TO RADIANS

RA=RA(I)*PI/180.0D
DEC=DEC(I)*PI/180.0D
C=COS(DEC)
SD=SIN(DEC)
CA=COS(RA)
SA=SIN(RA)

F1(I)=CP*CA
EP(I)=CD*SA

LD=DCOS(DECR)
SD=SIN(PECR)
CA=nCOD(RAQ)
SA=DSIN(RAR)

F1(I)=CP*CA
EP(I)=CD*SA

CONTINUE

C****** READ STATION COORDINATES
C****** PHI(LATITUDE) AND LONG ARE IN DEGREES, R IS IN METERS

DO 6 I=1,IN

RAD(15,701) IDUM,IL1,IL2,LONG(I),PHI(I),R(I)
701 FORMAT(1X,I2,1X,A4,A2,3FI5.1)
WRITE(6,701) I,IL1,IL2,LONG(I),PHI(I),R(I)
6025 CONTINUE

C****** CONVERT PHI AND LONG TO RADIANS

PHIR=PHIC(1)*PI/180.0D
LONGR=LONG(I)*PI/180.0D
C=DCOS(PHI)
S=DSIN(PHI)

CALL GRESIN(IYAR,IMO,IDAY,IHOUR,STNO,THO)

CONTINUE

C****** READ INITIAL EPOCH IN UT

READ(5,201) IYEAR,IMO,IDAY,IHOUR,IMIN,SEC
WRITE(6,201) IYEAR,IMO,IDAY,IHOUR,IMIN,SEC
201 FORMAT(1X,14,A15,F15.1)

C****** FIND 'REF' WITCH SIDERIAL TIME AT EPOCH TO
CALL GRESIN(IYAR,IMO,IDAY,IHOUR,IMIN,SEC,THO)

C****** READ FINAL EPOCH TF AND TIME INTERVAL IN HOURS

READ(5,201) JYEAR,IMO,JDAY,JHOUR,JMIN,SEC
WRITE(6,201) JYEAR,IMO,JDAY,JHOUR,JMIN,SEC
201 FORMAT(1X,14,A15,F15.1)

IF(JYEAR.NF.IYAR) GO TO 88
IF(IMO.NF.IMOI) GO TO 88

C****** NOTICtTHAT IF TO, TF APE CLOSE BUT IN DIFFERENT MONTHS SECOND
C****** DATE TF MUST BE EXPRESSED AS A DAY OF PREVIOUS MONTH, E.G. AUG 33

TF=DFLAT(JAY-IAY)*24.0D+DFLAT(JMOM-IHOUR)+DFLAT(JMIN-IMIN)
**C*** TRANSFORM QUASAR UNIT VECTOR TO EARTH FIXED SYSTEM AT EPOCH T

**8**

**C** 8 J=1,IN

**D** 0 DO 8 J=1,IN

**C** 0058 ZNT=ARCOS(X(J)*Q1+Y(J)*Q2+Z(J)*Q3)

**D** 0057 ARGU=X(J)*Q1+Y(J)*Q2+Z(J)*Q3

**C** 0056 IF(ARGU.LT.-1.DO) ARGU=-1.DO

**D** 0055 IF(ARGU.GT.1.DO) ARGU=1.DO

**C** 0054 ZNT=ARCOS(ARGU)

**D** 0053 ZNT=DABS(ZNT)*180.DO/PI

**C** 0052 ZNTMAX=60.DO

**D** 0051 IF(ZNT.GT.ZNTMAX) ZNT=10.DO**10

**C** 0050 INDEX(I,J)=ZNT

**D** 0049 CONTINUE

**C** 0048 WRITE(6,207) T

**D** 0047 207 FORMAT(*10X,EPOCH = *F15.5//20X,*1*2X,2X,*2*,2X,*3*,2X,*4*)

**C** 0046 **2X,*5*,2X,*6*,2X,*7*,2X,*8*,2X,*9*,1X,*10*,1X,*11*,1X,*12*,

**D** 0045 71X,*13*,1X,*14*,1X,*15*,1X,*16*,1X,*17*,1X,*18*,1X,*19*,1X,*20*,

**C** 0044 71X,*21*,1X,*22*,1X,*23*,1X,*24*,1X,*25*,1X,*26*,1X,*27*,1X,*28*,

**D** 0043 71X,*29*,1X,*30*/)

**C** 0042 DO 9 I=1,IM

**D** 0041 WRITE(6,105) I,RA(I),DEC(I),(INDEX(I,J),J=1,IN)

**C** 0040 105 FORMAT(IX,I2,2F7.0,1X,30(IX,I2))

**D** 0039 9 CONTINUE

**C** 0038 IM=IN-1

**D** 0037 DO 555 I=1,IM

**C** 0036 DO 555 J=1,INM

**D** 0035 IF(INDEX(I,J).GT.1000) GO TO 555

**C** 0034 JP=J+1

**D** 0033 DO 555 K=JP,IN
IF(INDEX(I,K).GT.1000) GO TO 555
WRITE(7,705) J,K,I,T
FORMAT(315,F15.2)
CONTINUE

T=T+DT
GO TO 33

WRITE(6,500)
FORMAT(1/)///10X,'WRONG INITIAL AND FINAL DATE'/
?10X,'PROGRAM INTERRUPTED THROUGH STATEMENT 88')
STOP
END
C****** RA AND DEC ARE IN DEGREES
DO 5 I=1,IN
READ(5,321) IDM,IL2,IL2,IL3,RA(I),DEC(I)
WRITE(6,321) I,IL2,IL3,RA(I),DEC(I)
FORMAT(I,3A4,F17.7,F20.9)
5 CONTINUE
C****** CONVERT RA AND DEC TO RADIANS
RA=RA(I)*PI/180.DO
DEC=DEC(I)*PI/180.DO
CD=DCOS(DEC)
SD=DSIN(DEC)
SA=DCOS(RA)
CA=DSIN(RA)
E1(I)=CD*CA
E2(I)=CD*SA
E3(I)=SD
5 CONTINUE
C****** READ STATION COORDINATES
C****** LATITUDE) AND LONG ARE IN DEGREES, R IS IN METERS
DO 6 I=1,IN
READ(5,701) IDM,IL2,IL2,IL3,PHI(I),R(I)
WRITE(6,701) I,IL2,IL3,PHI(I),R(I)
701 FORMAT(I,3A4,A2,3F15.1)
C****** CONVERT PHI AND LONG TO RADIANS
PHIR=PHI(I)*PI/180.DO
LONGR=LONG(I)*PI/180.DO
CF=DCOS(PHIR)
SF=DSIN(PHIR)
CL=DCOS(LONGR)
SL=DSIN(LONGR)
X(I)=CF*CL
Y(I)=CF*SL
Z(I)=SF
6 CONTINUE
C****** READ CORRECTION TO GREENWITCH SIDERIAL TIME DDTH IN ARC SEC
READ(5,101) DDTH
WRITE(6,101) DDTH
101 FORMAT(3F15.5)
C****** CONVERT TO RADIANS
DDTH=DDTH*PI/(180.DO*3600.DO)
C
C****** READ INITIAL EPOCH IN UT
READ(5,201) IYEAR,IM0,IDAY,IHOUR,IMIN,SEC
WRITE(6,201) IYEAR,IM0,IDAY,IHOUR,IMIN,SEC
201 FORMAT(5S1,F15.5)
C
C****** FIND GREENWITCH SIDERIAL TIME AT EPOCH TO
CALL GRESID(IYEAR,IM0,IDAY,IHOUR,IMIN,SEC,ThO)
COPY(1)=ThO
THO=THO+IPI
Y(1)=COPY(1)
WRITE(6,136) ThO
136 FORMAT(///1OXY'APPR. THO= tF25.9//)
WRITE(7,101) ThO
VLBI NETWORK, OBSERVATION SIMULATOR
CONTINUOUS VERSION : EARTH MOTION PARAMETERS

THE MODEL USED IS:

\( \mathbf{D}(i,j,p,k) = (X_j - X_i) \times (X_p, H_p) \times T \)

\( \mathbf{T} \) = \( R_3(\Omega \times T + \Theta_0) \times S(XX, PH) \times Q \)

\( S(A,B) = \begin{pmatrix} 1 & 0 & A \\ 0 & 1 & -B \\ -A & B & 1 \end{pmatrix} \)

\( Q \) = \( \begin{pmatrix} \cos(\text{DECP}) \times \cos(\text{RAP}) \\ \sin(\text{DECP}) \times \sin(\text{RAP}) \times \sin(\text{DECP}) \end{pmatrix} \)

\( X_j, X_i \) : POSITION VECTORS OF J AND I
\( \text{DECP}, \text{RAP} \) : DECLINATION AND RIGHT ASCENSION
\( \Omega, X_p, H_p \) : POLAR MOTION SMALL ANGLES
\( \text{XX}, \text{PH} \) : PRECESSION NUTATION SMALL ANGLES

\( \mathbf{D}(i,j,p,k) \) : OBSERVED DISTANCE INVOLVING IJ AFTER INITIAL REFERENCE EPOCH TO THETA : GREENWICH SIDERIAL TIME AT EPOCH TO
THETA : UT1 ANGLE

IMPLICIT REAL*8(A-H,L-Z)
DIMENSION X(4), Y(4), Z(4), E1(18), E2(18), E3(18), RA(18), DEC(18)
2, PH1(4), LONG(4), R(4), COPY(3)
GO TO 100
100 FORMAT(215)
0049 THO=COPY(1)+DDTH

C
C ****** READ FINAL EPOCH TF AND TIME INTERVAL IN HOURS

0050 READ(5,201) JYEAR, JMO, JDAY, JHOUR, JMIN, SECJ
0051 WRITE(6,201) JYEAR, JMO, JDAY, JHOUR, JMIN, SECJ
0052 READ(5,101) DT
0053 WRITE(6,101) DT

C
C IF(JYEAR.NE.IYEAR) GO TO 86
0054 IF(JMO.NE.IMO) GO TO 86

C ****** NOTICE THAT IF TF, TF ARE CLOSE BUT IN DIFFERENT MONTHS SECOND
C ****** DATE TF MUST BE EXPRESSED AS A DAY OF PREVIOUS MONTH, E.G. AUG 33

0056 TF=DFLOAT(JDAY-IDAY)*24.00+DFLOAT(JHOUR-IHOUR)*60+DFLOAT(JMIN-IMIN)

C
C TO=0.00
0057 T=0.00
0058 END=0.00
0059 IF(T.GT.TF) GO TO 40

C
C ******* I=O IS A CODE TO INDICATE END OF OBSERVATIONS FOR THIS EPOCH

0060 READ(5,107) I,J,K
0061 IF(I.EQ.0) GO TO 99

CALL EARTH(T,XP,HP,OMG,THETA,XX,HHD)

C
C MODIFY EARTH PARAMETERS AND WRITE / PUNCH

0063 XP=XP*REART
0064 HP=HP*REART
0065 XX=XX*REART
0066 HHD=HHD*REART
0067 OMG=OMG*(12.00/PI)
0068 THETA=THETA
0069 THETA=THETA*(180.00/PI)
0070 TH1=THETA
0071 TH2=(THETA-DFLOAT(ITH1))*60.00
0072 ITH2=ITH2
0073 ITH3=(ITH2-DFLOAT(ITH2))*60.00
0074 WRITE(6,801) T,XP,HP,XX,HHD,ITH1,ITH2,ITH3,OMG0
0075 801 FORMAT(9X,F7.2,2X,F10.4,2X,F10.4,2X,F8.4,F12.9)

C
C COPY(1)= E1(K) -XX*E3(K)
0077 COPY(2)= + E2(K)*HH*E3(K)
0078 COPY(3)=XX*E1(K)-HH*E2(K)+ E3(K)

C
C 'ANGLE=THETA+THO
0079 CA=DCOS(ANGLE)
0080 SA=DSIN(ANGLE)

C
C Q1= CA*COPY(1)+SA*COPY(2)
0082 Q2=SA*COPY(1)-CA*COPY(2)
0084 Q3=COPY(3)

C
C QF1=-SA*COPY(1)+CA*COPY(2)
0085 QF2=CA*COPY(1)-SA*COPY(2)
0087 QF3=XP*QF1+HP*QF2
COPY(1) = Q1 + XP*Q3
COPY(2) = Q2 - HP*Q3
COPY(3) = -XP*Q1 + HP*Q7 + Q3

DS = (X(J)*R(J) - X(I)*R(I)) + COPY(1) + (Y(J)*R(J) - Y(I)*R(I)) + COPY(2) + (Z(J)*R(J) - Z(I)*R(I)) * COPY(3)

FRNG = (X(J)*R(J) - X(I)*R(I)) * QF1 + (Y(J)*R(J) - Y(I)*R(I)) * QF2 + (Z(J)*R(J) - Z(I)*R(I)) * QF3

FRNG = DERIVATIVE OF DELAY DISTANCE DS (METERS/HOUR)

WRITE(6, 107) I, J, K, DS, FRNG, T
WRITE(7, 107) I, J, K, DS, FRNG, T
FORMAT(315, 3F15.5)

T = T + DT
GO TO 33

WRITZ(6, 500)
FORMAT(*1111X, 'Wrong initial and final date'/*10X, 'Program interrupted through statement 88')
STOP
END
**Routine** GRSID(IYEAR, IMD, IDAY, IHOUR, IMIN, SEC, THO)

*** Calculates the Greenwich Sidereal Time (TO) for epoch with date IYEAR IMD IDAY UT.***

**Implicit Real*8(A-H,L-Z)**

`FI=4.10*DATAN(1.0)`

`II=O`  

`I1=O`  

`I2=O`  

`A=O.DO`

`CALL JULIA(IYEAR, IMD, IDAY, I1, I2, A, MJD)`

`T=(MJD-2415020.DO)/36525.DO`  

`T2=T*T`

`THO=99.6904390+3.0000.768900*T+0.000387080*D*T2`

`THO=THO*PI/180.DO`  

`CALL REDPI(THO)`

`GT=DFLOAT(IHOUR)*60.DO+DFLOAT(IMIN)+SEC/60.DO`  

`DT=DTHET=10.DO**3)`  

`THO=THO+DTHET`  

`CALL REDPI(THO)`

END
0001 SUBROUTINE JULIA(YFAR,IM,IDAY,IMMM,S,MJD)

C***************************************************************
C SUBROUTINE TO CONVERT UNIVERSAL TIME TO
C JULIAN DATE
C***************************************************************
C***************************************************************
C***************************************************************
C***************************************************************

0002 IMPLICIT REAL*8 (A-H,L-Z)
0003 DIMENSION INTH(12)
0004 DATA IMONTH /0,31,59,90,120,151,181,212,243,273,304,334/
0005 H=DFLOAT(IMMM)
0006 I=DFLOAT(TMMM)
0007 ICOD=0
0008 INDIS=(YEAR-1877)/4
0009 IF(YEAR.GT.0) INDIS=INDIS-1
0010 ICH=4*(YEAR/4)
0011 IF(YEAR.EQ.1900) ICOD=1
0012 IF(YEAR.EQ.1900) ICOD=0
0013 NJD=2415020+(YEAR-1900)*365.25+INDIS+IMONTH(IM)+ICOD-0.5D0+IDAY
X+H/24.D0+M/1440.D0+S/86400.D0
0014 RETURN
0015 END
SUBROUTINE EARTH(T, XPHP, OMG, THETA, XX, HH)

T : TIME INTERVAL AFTER INITIAL EPOCH (HOURS)
XP, HP : POLAR MOTION COORDINATES (RADIANS)
THETA : UT ANGLE IN RADIANS
OMG : ROTATIONAL VELOCITY OF EARTH (RADIANS PER HOUR)
XX, HH : PRELATION-NUTATION SMALL ANGLES (RADIANS)

IMPLICIT REAL*A-H,L-Z

DIMENSION P(10), A(10), B(10), PS(10), F(10)

DATA P/8760.0, DO, 380.0, DO, 2190.0, DO, 720.0, DO, 360.0, DO,
2160.0, DO, 24.0, DO, 12.0, DO, 1.0, DO, 0.5000/, 
360.0, DO, 0.6500, DO, 0.2800, DO, 0.1200, DO, 0.0600/, 
40.0300, DO, 0.0200, DO, 0.0100, DO, 0.0000, DO, 0.0000/, 
58.0500, DO, 0.7000, DO, 0.2200, DO, 0.1300, DO, 0.0900/, 
60.0100, DO, 0.0040, DO, 0.0010, DO, 0.0000, DO, 0.0000/, 
90.0000, PS/3.1500, DO, 2.8000, DO, 1.3200, DO, 0.1500, DO, 0.0500, DO, 2.0000/, 
86.1400, DO, 7.5000, DO, 4.1200, DO, 0.0600, DO, 1.1300, DO, 2.5000, DO, 6.1700, DO, 4.3500, DO, 0.0500, DO, 5.1800, DO, 3.7500, DO, 1.2400/, 
70.3500, DO, 1.6700/

PI=4.000000.0000+ATAN(1.0000)/2.0000

M1=6.000000.0000
M2=0.000000.0000
PC=29.000000.0000
C=DCOS(PIC/P/365.000000.0000)
S=DSIN(PIC/P/365.000000.0000)

M1=C*M10+S=M20
M2=S*M10+C=M20

CONTINUE

DO 5 I=1, 10

ARG=(PI2*T)/P/365.000000.0000

M1=M1+A(I)*DCOS(ARG+PS(I))
M2=M2+P(I)*DCOS(ARG+ F(I))

CONTINUE

XP=H2/PK/365.000000.0000
HP=M1/PK/365.000000.0000

XX=0.0600+C.0100*DCOS(2.3400+T/24.00)+0.0500*DCOS(0.1000+T/12.00)

HH=0.0400+C.0200*DCOS(3.1100+T/24.00)+0.0050*DCOS(0.0700+T/12.00)

THETA=0.0017+DO*(T/24.00)+25.1000*DSIN(ARG1)-9.2000*DSIN(ARG2)

FACTOR=PI2/664.000000.0000

THETA2=PI2*T/24.00+THETA*FACTOR

OMG=0.001750/24.00*(25.1000*DCOS(ARG1)-18.4000*DCOS(ARG2))

RETURN
SUBROUTINE REDPI(A)

C***********************
C************
C************ - REDUCES AN ANGLE A IN RADIANS TO THE
C************ ( -PI, +PI ) INTERVAL
C************
C***********************

C******************************

0002 IMPLICIT REAL*8(A-H,L-Z)
0003 PI=4.DO*DATAN(1.DO)
0004 P12=PI*2.DO
0005 COPY=A/PI2
0006 IA=COPY
0007 IF(A.GE.0.DO) A=-CFLOAT(IA)*PI2
0008 IF(A.LT.0.DO) A=CFLOAT(IA)*PI2
0009 5 IF(A.GT.-PI.AND.A.LT.PI) GO TO 8
0010 IF(A.LT.-PI.DO) A=-PI2
0011 GO TO 5
0012 8 RETURN
0013 END

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2. Name of Program: PROG01

Function:

Adjustment of a set of observations with total time-span equal to both earth and station step. Earth rotation parameters and station coordinates are treated as constants over the total time span of observations. A number of weighting options for radio source coordinates are available.

Input:

Number of weighting options for radio source coordinates, standard deviations of observations and radio source coordinates, approximate values of parameters, identification numbers of participating stations and radio source, distance and distance rate observations.

Output:

Standard deviations of adjusted parameters.

Subroutines required:

REDPI.

(Following is a listing of PROG01.)
**VLBI ADJUSTMENT PROGRAM**

This version performs an adjustment of one day of observations treating all parameters as constants over the one day interval.

An option is available for applying a number of different weights on radio source coordinates.

Observations: delay distances and their derivatives.

Unknowns: earth motion parameters.

Systems definition:

Quasar fixed: weighted quasar coordinates.

Earth fixed: by means of inner constraints.

Subroutines required: REDPI.

A row of design matrix containing partials of delay observations with respect to parameters.

B row of design matrix containing partials of delay derivative observations with respect to parameters.

C coefficient matrix of inner constraints.

N constant vector of normal equations.

X vector of unknowns (corrections to parameters).

The order of the parameters is:

- $\alpha, \delta$, polar motion angles $\kappa$ and $\eta$.
- $\alpha, \delta$, precession-nutation angles (capital) $\kappa$ and $\eta$.
- $\theta$, theta zero angle (gast at initial epoch).
- $\Omega$, angular velocity of earth rotation.
- $X_{i}(1), Y_{i}(1), Z_{i}(1)$, coordinates of station $i$ $(i = 1, 2, \ldots, N)$.
- $RA_{j}(1), D_{j}(1)$, right ascension and declination of radio source $j$ $(j = 1, 2, \ldots, M)$.
- Clock offset and drift at station $i$ $(i = 1, 2, \ldots, N)$.

**IMPLICIT REAL*8(A-H,L-Z)**

**DIMENSION A(27), N(45,45), X(3), Y(3), Z(3), RA(8), D(8), U(37)**

**CC(8,37), DD(3), RR(37), NKEEP(45,45)**

**DIMENSION A(KK), N(KK), X(N), Y(N), Z(N), RA(N), D(N), U(KK), XX(KK)**

**CC(KK), DD(N), DD(N)**
C 3,B(KK),NKEEP(KK),SDQUA(IO),PRATIO(IO)
C WHERE: N=NO OF STATIONS, M=NO OF QUASARS, K=6+3*N+2*M, KK=K+2*N, KB=KK+8
C
C READ EXPERIMENT IDENTIFICATION NO
0003 READ(5,100) IEXP
0004 WRITE(6,977) IEXP
0005 977 FORMAT(*11///,I1X,*EXPERIMENT NO =",I5/10X
2,"***************,///)
C
C READ NUMBER OF WEIGHTING OPTIONS
0006 READ(5,100) IOPT
0007 WRITE(6,978) IOPT
0008 978 FORMAT(10X,*REQUESTED NO OF WEIGHTING OPTIONS =",I5///)
C
C READ A PRIORI STANDARD DEVIATIONS:
C SDD : OF DELAY DISTANCE IN CM
C SDBF : OF DELAY DISTANCE DERIVATIVE IN M/HOUR
C SDQUA : OF QUASAR COORDINATES IN CM
C
0009 READ(5,101) SDD,SDBF
0010 101 FORMAT(5F15.5)
C
0011 DO 181 I=1,IOPT
0012 READ(5,101) SDQUA(I)
0013 PRATIO(I)=SDBD**2/SDQUA(I)**2
0014 181 CONTINUE
C
C R=6371600.DO
C CLIGHT=3.DO*(10.DO**8)
C PI=4.DO*ATAN(1.DO)
C RICO=R*100.DO
C RHDX=(112.DO/PI)*(10.DO**9)
C RACM=(180.DO/PI)*260000.DO
C
C PF=(SDBD/RICO)/(SDBF/R)
0022 PF=PF**2
0023 WEIGHTS :
C PF IS NOW IN (EARTH RADII/HOUR)**-2, PRATIO IN (EARTH RADII)**-2
C
C READ IN= NO OF STATIONS & IM= NO OF QUASARS
0024 READ(5,100) IN, IM
0025 100 FORMAT(5I5)
0026 WRITE(6,920) IN,IM
0027 920 FORMAT(5X,*NO OF STATIONS =",I5/5X,*NO OF QUASARS =",I5)
0028 K=6+2*IN+2*IM
0029 KK=K+2*IN.
0030 KB=KK+8
C
C INITIALIZE POLAR MOTION (XP,HP) AND PRECESSION-NUTATION (XN,HN)
C PARAMETRS (APPROXIMATE VALUES) TO ZERO
0031 XP=0.DO
0032 HP=0.DO
0033 XN=0.DO
0034 HN=0.DO
0035 DO 51 I=1,IN
C ********************************************
C ** READ APPROXIMATE VALUES OF PARAMETERS THETA ZERO, OMG, **
C ** XI(I), YI(I), ZI(I). (I=1,IN). RA(J), D(J), (J=1,IM) **
C ** ANGLES ARE IN DEGREES, DISTANCES IN METERS AND OMG IN RADIANS PER HOUR **
C ********************************************

READ(5,101) TH
WRITE(6,360) TH
FORMAT(/5X,'THETA ZERO = ',F20.9/) DO = TH*PI/180.DO
GAM = PI/12.DO
WRITE(6,450) FORMAT(5X,'STATION & RADIO SOURCES APPROXIMATE COORDINATES*
?/3X,*STATIONS*/10X,*LONG*
?/3X,*STATIONS*/11X,*LAT'/)
DO 5 I=1,IN
READ(5,701) IDM,ILI,IL2,ALONG,APHI,AR
WRITE(6,701) I,ILI,IL2,ALONG,APHI,AR
FORMAT(1X,I2,1X,A6,A2,3F15.1) APHI = APHI*PI/180.DO
ALONG = ALONG*PI/180.DO
CONV = RT TO CARTESIAN
XII = AR*COS(APHI)*COS(ALONG)
YII = AR*COS(APHI)*SIN(ALONG)
ZII = AR*SIN(APHI)
CONV TO EARTH RADIUS UNITS
X1 = XII/R
Y1 = YII/R
Z1 = ZII/R
CONTINUE

WRITE(6,451) FORMAT(1X,'KADIO SOURCES,10X,RA,17X,DEC//' DO 6 J=1,IM
READ(5,321) IDM,ILI,IL2,IL3,RA(J),D(J)
WRITE(6,321) J,ILI,IL2,IL3,RA(J),D(J)
FORMAT(1X,I3,7X,3A4,F17.7,F20.9) RA(J) = RA(J)*PI/180.DO
D(J) = D(J)*PI/180.DO
CONTINUE

DO 7 J=1,IM
XX(J) = 0.DO
U(J) = 0.DO
CONTINUE

DO 7 I=1,KK
XX(I) = 0.DO
U(I) = 0.DO
CONTINUE
CC(J,I)=O.DO
CONTINUE
DO 16 I=1,K8
DO 16 J=1,K8
N(I,J)=0.DO
NKEEP(I,J)=0.DO
CONTINUE
C FORMULATE CONSTRAINT MATRIX CC
CC(5,1)=1.DO
CC(4*2)=1.DO
CC(4,5)=-1.DO
DO 3 I=1,IN
I81=4+3*T
IC2=IC1+IN
CC(I,IC1)=1.DO
CONTINUE
C READ OBSERVATIONS AND IDENTIFIERS:
C DS= DISTANCE IN METERS OF IJ BASELINE OBSERVATION TO QUASAR IP
C AT TIME TK (IN HOURS) AFTER SOME INITIAL EPOCH TO
C
C I81=4+3*T
IC2=IC1+IN
ICOUNT=0
READ(5,104) IJ,IP,DS,FRNG,TK,ICHEK
IF(ICHEK.EQ.1) GO TO 66
ICOUNT=ICOUNT+1
WRITE(P,100) IJ,IP,DS/FRNG/TK
DX=X(J)-X(I)
DY=Y(J)-Y(I)
DZ=Z(J)-Z(I)
0111 CA=DCOS(RA(I))
0112 SA=DSIN(RA(I))
0113 CD=DCOS(D(I))
0114 SD=DSIN(D(I))
0115 Y1=OMG*TX+TH
0116 Y2=Y1-RA(I)
0117 CK=DCOS(Y1)
0118 SK=DSIN(Y1)
0119 CKF=FCOS(Y2)
0120 SKP=DSIN(Y2)

FORMULATE PARTIALS

0121
0122 F1=CD*DX*CK+DP+SD
0123 F1=CD+(-DX*XP*CP+YN*CP)+CD*CD+DP+DX*DP
0124 F2=DX*CD*(YN*CA+YN*SA)+DX*DP*(YN*CK+YN*SK)
0125 F2=DX*CD*(YN*CA+YN*SA)+DX*DP*(YN*CK+YN*SK)
0126 F2=DX*CD*(YN*CA+YN*SA)+DX*DP*(YN*CK+YN*SK)
0127 F3=DX*CD*(YN*CA+YN*SA)+DX*DP*(YN*SK+YN*CK)
0128 F3=DX*CD*(YN*CA+YN*SA)+DX*DP*(YN*SK+YN*CK)
0129 F5=DX*(SKP+DP*CP)+CD
0130 F5=DX*(SKP+DP*CP)+CD
0131 F5=DX*(SKP+DP*CP)+CD
0132 F6=TX*CP
0133 F7=CD*CP
0134 F7=CD*CP
0135 F7=CD*CP
0136 F8=CD*CP
0137 F8=-CD*CP
0138 F8=-CD*CP
0139 F9=SK
0140 F9=-CD*CP
0141 F9=-CD*CP
0142 F10=CD*(YN*SK+YN*CK)
0143 F10=CD*(YN*SK+YN*CK)
0144 F10=CD*(YN*SK+YN*CK)
0145 F11=DX*CP*(YN*CP+YN*CP)
0146 F11=DX*CP*(YN*CP+YN*CP)
0147 F11=DX*CP*(YN*CP+YN*CP)
0148 F12=DP+DD
0149 F12=DP+DD
0150 F12=DP+DD
0151 G1=DX*CP+CP*CP+CP*CP+CP*CP
0152 G1=DX*CP+CP*CP+CP*CP+CP*CP
0153 G1=DX*CP+CP*CP+CP*CP+CP*CP
0154 G1=DX*CP+CP*CP+CP*CP+CP*CP
0155 G1=DX*CP+CP*CP+CP*CP+CP*CP
0156 G1=DX*CP+CP*CP+CP*CP+CP*CP
0157 G1=DX*CP+CP*CP+CP*CP+CP*CP
0158 G1=DX*CP+CP*CP+CP*CP+CP*CP
0159 G1=DX*CP+CP*CP+CP*CP+CP*CP

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0160  G3=G3+DZ*SD*(-XP*SK+HP*CK)
0161  G3=OMG*G3
0162  G4=DX*CK+SD-DY*SK*SD
0163  G4=G4+DZ*SD*(-XP*SK+HP*SK)
0164  G4=G4+DX*CK*SD-DY*SK*SD
0165  G4=OMG*G4
0166  G5=DX*CK+DP*CD*SKP
0167  G5=G5+DZ*DP*CD*SKP
0168  G5=G5+DX*SD+(XP*SK+HP*CK)
0169  G5=OMG*G5
0170  G6=DP*CD*SA*(XN*SK+HN*SK)
0171  G6=G6+DP*CD*SA*(XN*SK+HN*SK)
0172  G6=OMG*G6
0173  G7=-DX*CD*CKP+DY*CD*SKP
0174  G7=GS+DZ*CD*(XP*CKP+HP*SKP)
0175  G7=OMG*G7
0176  G8=-DX*CD*CKP+DY*CD*SKP
0177  G8=OMG*G8
0178  G9=0.DO
0179  G9=0.DO
0180  G9=0.DO
0181  G10=DX*CD*CKP+DY*CD*SKP
0182  G10=OMG*G10
0183  G11=DX*CD*CKP+DY*CD*SKP
0184  G11=OMG*G11
0185  G12=DX*CD*CKP+DY*CD*SKP
0186  G12=OMG*G12
0187  G12=OMG*G12
0188  G12=OMG*G12

A, B ARE ROWS OF THE DESIGN MATRIX CORRESPONDING TO DELAY
AND DELAY DERIVATIVE OBSERVATIONS RESPECTIVELY

INITIATE A AND B TO ZERO

DO 8 II=1, KK
A(II)=0.DO
B(II)=0.DO
8 CONTINUE

SET PARTIALS IN APPROPRIATE LOCATIONS OF A AND B ROWS

A(1)=F1
A(2)=F2
A(3)=F3
A(4)=F4
A(5)=F5
A(6)=F6

B(1)=G1
B(2)=G2
B(3)=G3
B(4)=G4
B(5)=G5
B(6)=G6

I1=I1+4
I2=I1+1

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0207 I3=I1+2
0208 A(I1)=-F7
0209 A(I2)=-F6
0210 A(I3)=-F5

C
0211 B(I1)=C7
0212 B(I2)=C8
0213 B(I3)=C9
0214 IC1=K+1
0215 IC2=IC1+IN
0216 A(IC1)=-F12
0217 A(IC2)=-F13

C
0218 B(IC1)=-G12
0219 B(IC2)=-G13

C
0220 J1=3*J+4
0221 J2=J1+1
0222 J3=J1+2
0223 A(J1)=-F7
0224 A(J2)=-F8
0225 A(J3)=-F9

C
0226 B(J1)=G7
0227 B(J2)=G8
0228 B(J3)=G9

C
0229 JC1=K+J
0230 JC2=JC1+IN
0231 A(JC1)=F12
0232 A(JC2)=F13

C
0233 B(JC1)=G12
0234 B(JC2)=G13

C
0235 IP1=3*IN+2*IP+5
0236 IP2=IP1+1
0237 A(IP1)=F10
0238 A(IP2)=F11

C
0239 B(IP1)=G10
0240 B(IP2)=G11

C FORMULATION OF L VECTOR ENTRY

C
0241 DS0=DX*F7+DY*F6+DZ*F9
0242 DS0=DS0+DG(J)-DG(I)+DG(J)-DG(I)*(TK/24.0)
0243 L=DS0-DS

C
C*********************************************************************************************************
C
C STORE PARTIALS / NON-ZERO ELEMENTS OF A & B ROWS ON DISK & FOR LATER USE IN COMPUTATION OF RESIDUALS AND VTPV
C ADD CONTRIBUTIONS OF CURRENT OBSERVATIONS TO NORMAL EQUATIONS
C
C**********************************************************************************************************
WRITE(8,351) F1,F2,F3,F4,F5,F6,F7,F8,F9,F10,F11,F12,F13,L
0250 CONTINUE
C
0251 FRNGO=FX*G7+DY*G8+DZ*G9
0252 FRNGO=FRNGO+(DD(JJ)-DD(I)))/24.DO
0253 L=FRNGO-FRNG
0254 WRITE(8,351) G1,G2,G3,G4,G5,G6,G7,G8,G9,G10,G11,G12,G13,L
0255 DO 65 II=1,KK
0256 U(IJ)=U(IJ)-L(IJ)*L
0257 CONTINUE
0258 NKEEP(IJ,JJ)=NKEEP(IJ,JJ)+A(IJ)*A(JJ)
0259 DO 65 JJ=IKK
0260 WRITE(B,351) G12,G13,G14,G15,G16,G17,G18,G19,G20,G21,G22,G23,L
0261 CONTINUE

SOLVE NORMAL EQUATIONS, COMPUTE PARAMETER CORRECTIONS

DO 1 I=1,8
DO 1 J=1,KK
IK=I+KK
NKEEP(IK,J)=CC(I,J)
NKEEP(J,IK)=NKEEP(IK,J)
1 CONTINUE
2 IN=7+IN*3
DO 2 I=IN,K
N(I,J)=N(I,J)+PRATIO(IW)
2 CONTINUE
3 CALL DMINV(N,K8,DET,L1,L2)
4 DO 187 I=1,KK
XX(I)=0.DO
187 CONTINUE
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0281 DO 14 I=1,KK
0282 DO 14 J=1,KK
0283 XX(I)=XX(I)+N(I,J)*U(J)
0284 CONTINUE

C
C WRITE XX
C
0285 DO 9 I=1,5
0286 CALL REDPI(XX(I))
0287 CONTINUE

C
C ISTART=7+3*IN

0288 DO 26 I=ISTART,K
0289 CALL REDPI(XX(I))
0290 CONTINUE

C
C READ A AND B (ROWS OF DESIGN MATRIX) FROM DISK 8,
C FORMULATE RESIDUALS AND COMPUTE VARIANCE COVARIANCE MATRIX

0291 REWIND 8

0292 REWIND 8

0293 VTPV=0.000

0294 VABSI=0.000

0295 VABS2=0.000

0296 DO 22 I=1,ICOUNT

0297 READ(8,100) I,J,IP

0298 DO 27 IDUM=1,2

0299 READ(8,351) (F(JJ),JJ=1,13),L

0300 V=0.000

0301 DO 23 JJ=1,6

0302 V=V+F(JJ)*XX(JJ)

0303 CONTINUE

0304 I1=2*I+4

0305 I2=I1+1

0306 I3=I1+2

0307 J1=3*J+4

0308 J2=J1+1

0.09 J3=J1+7

0310 V=V+F(7)*(XX(I1)-XX(I1))+F(8)*(XX(J2)-XX(I2))

0311 IP1=3*IN+2*IP+5

0312 IP2=IP1+1

0313 V=V+F(10)*XX(IP1)+F(11)*XX(IP2)

0314 IC1=K+I

0315 IC2=IC1+IN

0316 JC1=K+J
CJC<JJC+IN
C
V=V*F(12)*(XX(JJC1)-XX(IC1))+F(13)*(XX(JJC2)-XX(IC2))
C
Y=V+L
C
IF(IDUM.EQ.2) GO TO 28
C
VARS1=VARS1+DABS(V)
C
VTPV=VTPV+V**2
C
GO TO 27
C
VABS2=VABS2+DABS(V)
C
VTPV=VTPV+PF*V**2
C
CONTINUE
C
VABS1=VABS1/ICOUNT
VABS2=VABS2/ICOUNT
IIN=3*IIN+6
IIN1=IIN+1
C
ALTERNATIVE FOR S0 COMPUTATION
C
S0=VTPV+DFLOAT(IM2)*(SDBD/RIO0)**2
C
S0=S0/DFLOAT(ICOUNT*2-KK+2*IM)
C
DO 172 I=IIN1,K
C
VTPV=VTPV+PRATIOCIW)*XX(I)**2
C
172 CONTINUE
C
K1=K+1
KIN=K+IN
C
DO 41 J=1,KK
C
N(1, J)=R100*N(1, J)
C
N(2, J)=R100*N(2, J)
C
N(3, J)=R100*N(3, J)
C
N(4, J)=R100*N(4, J)
C
N(5, J)=RASM3*N(5, J)
C
N(6, J)=RASHM3*N(6, J)
C
DO 42 I=7, KIN
C
42 CONTINUE
C
DO 43 I=IIN1, K
C
43 CONTINUE
C
DO 47 I=KL, KIN
C
47 CONTINUE
C
DO 48 I=KIN1+KK
N(I,J)=(N(I,J)/CLIGHT)*1000.DO
CONTINUE
C
DO 44 I=1,KK
N(I,1)=R100*N(I,1)
N(I,2)=R100*N(I,2)
N(I,3)=R100*N(I,3)
N(I,4)=R100*N(I,4)
N(I,5)=RASM3*N(I,5)
N(I,6)=RHRDM9*N(I,6)
C
DO 45 J=7,KIN
N(I,J)=N(I,J)*R100
CONTINUE
C
DO 46 J=11+KIN
N(I,J)=N(I,J)*RASM3
CONTINUE
C
DO 49 J=KI,KIN
N(I,J)=(N(I,J)/CLIGHT)*1000.DO
CONTINUE
C
FORMAT(10X,3F20.4)
FORMAT(10X,2F22.6)
FORMAT(SXF40.9)
GO 86 I=1,KK
N(I,1)=DSQRT(N(I,1))
CONTINUE
C
FORMAT(10X,2F20.4)
FORMAT(10X,2F22.6)
FORMAT(SXF40.9)
GO 86 I=1,KK
N(I,1)=DSQRT(N(I,1))
CONTINUE
C
WRITE(6,696) ICNT2,IDF
FORMAT(1///5X,'NO OF OBSERVATIONS =',I5//5X
2,'DEGREES OF FREEDOM =',I5/)
C
WRITE(6,850) IEXP,SDBD,SDBF,SCOA(IW)
FORMAT(1///15X,'EXPERIMENT NO =',I5///10X
1,'A PRIORI STANDARD DEVIATIONS',//5X
2,'DELAY DISTANCES (CM)',10X,F15.5//5X
3,'DISTANCE DERIVATIVES (M/HOUR)',1X,F15.5//5X
4,'QUASAR COORDINATES (CM)',7X,F15.5/)
C
WRITE(6,356) (N(I,I),I=1,6)
FORMAT(///20X,'STANDARD DEVIATIONS'/20X
1,'***************///5X
2,'POLAR MOTION =',7X,F14.5,3X,'CM',//26X,F14.5,3X,'CM'///5X
5. *PRECESSION-NUTATION = F14.5,3X,*CM*/26X,F14.5,3X,*CM*/5X
6. *THETA ZERO = F14.5,3X,*ARCSEC/1000 */5X
7. *OMG = 16X,F14.5,3X,*REV PER DAY/10**9 */10X
8. *STATION COORDINATES (X,Y,Z) IN CM*/1

0497 WRITE(6,357) (N(I,I),I=I,N)
0598 357 FORMAT(IOX,3F20.5)
0599 WRITE(6,358) FORMAT(/10X, *QUASAR COORDINATES (RA,DEC) IN ARCSEC/1000 */)
0600 WRITE(6,359) (N(I,I),I=I,N)
0601 359 FORMAT(IOX,2F20.5)
C
0603 WRITE(6,361) FORMAT(/10X, *STATION CLOCK OFFSETS IN MSEC */)
0604 361 FORMAT(/10X, *STATION CLOCK DRIFTS IN MSEC PER DAY */)
0605 WRITE(6,362) (N(I,I),I=I,K)
0606 362 FORMAT(IF20.9)
0607 WRITE(6,363)
0608 363 FORMAT(/10X, *STATION CLOCK DRIFTS IN MSEC PER DAY */)
0609 WRITE(6,362) (N(I,I),I=K1,K1)
C
C
0610 CONTINUE
C
0611 STOP
0612 END
3. **Name of Program:** PROG02

**Function:**

Adjustment of observations with a time span equal to the station step. Station coordinates are considered constant over the whole set of observations, while earth rotation parameters are treated as constant but different over a prearranged set of subintervals.

**Input:**

- Standard deviations of observations and radio source coordinates, approximate values of parameters, station and radio source identification numbers, distance and distance rate observations.

**Output:**

Standard deviations of adjusted parameters.

**Subroutines required:**

- MADD, MATPRO, TRAPRO, PROTRA for matrix operations and REDPI.

(Following is a listing of PROG02 and supporting subroutines, except for REDPI already included in VLBI SIMULATOR.)
**VLBI ADJUSTMENT PROGRAM**

This version performs the adjustment of a set of observations treating station coordinates as constants, earth rotation parameters are constant but different over subintervals of the original total time interval of the available observations.

Observations: delay distances and their derivatives

Unknowns: earth motion parameters

Station & quasar coordinates

Clock offsets and drifts

Systems definition:

Quasar fixed: weighted quasar coordinates

Earth fixed: by means of inner constraints

Subroutines required:

REDPI

MADD

MATPRO

TRAPRO

PROTRA

DIMENSION DO(IN),DD(IN),X(IN),Y(IN),Z(IN),RA(IM,IM),ED(O,8)

WHERE: K=5*IN+2*IN, K8=K+6, KB=K+SIN=ND OF STATIONS, TM=NO OF QUASARS

EQUIVALENCE (HTEDO,FB),(UDI,L2K),(GBL,EDO,FETN),(RT,HIXB),

WHERE: K=5*IN+2*IN, K8=K+6, KB=K+6, IN=NO OF STATIONS, TM=NO OF QUASARS

2, (RKEO,EDF,GBH),(EDO,FETN),(RT,HIXB),(L16,TTI),(L26,NSOL)

DIMENSION DO(IN),DD(IN),X(IN),Y(IN),Z(IN),RA(IM,IM),ED(O,8)
READ EXPERIMENT IDENTIFICATION NO
READ(5,100) IEXP
WRITE(6,977) IEXP
FORMAT(*10X,'EXPERIMENT NO =',I5/10X
?,'**************',////)
READ A PRIORI STANDARD DEVIATIONS:
SDBD : OF DELAY DISTANCE IN CM
SCBF : OF DELAY DISTANCE DERIVATIVE IN M/HOUR
SOQUA : OF QUASAR COORDINATES IN CM
READ(5,101) SDBD,SCBF
WRITE(6,240) SDBD,SCBF
FORMAT(10X,'OBSERVATIONAL NOISE'/15X,'DISTANCES
210.5/15X,'DISTANCE RATES '/,F10.5//)
READ(5,101) SOQUA
WRITE(6,240) SOQUA
FORMAT(30X,'STDerrNOISE'/15X,'STDerrRATES'/,
F10.5//)
READ(5,101) SDQUA
RFAD(5,101) SDQUA
PRATIO=SCED**2/SDQUA**2
READ EARTH STEP INTERVAL IN HOURS
READ(5,101) ERSTEP
WRITE(6,243) ERSTEP
FORMAT(/10X,'EARTH STEP INTERVAL (HRS) =',F10.2/
C
R=6371000.00
CLIGHT=3.00*(10.0D0)**8
CLIGHT=CLIGHT/R
PI=4.0D0*ATAN(1.0D0)
R10G=R*100.0D0
KHKM9=(12.0D0/PI)*(10.0D0)**9
RASH3=(180.0D0/PI)*36000000.0D0
PF=(SDOD/RIOC)/(SD5F/R)
PF=PF**2
PF IS NOW IN (EARTH RADII/HOUR)**-2, PRATIO IN (EARTH RADII)**-2
READ IN= NO OF STATIONS & IM= NO OF QUASARS
READ(5,100) IN, IM
C
WRITE(6,920) IN,IM
FORMAT(5X,'NO OF STATIONS =',I5/5X,'NO OF QUASARS =',I5)
K=5*IN+2*IM
K'=K+6
KP=K+8
DO 91 I=1,IN
DO(1)=C.00
DO(1)=C.00
CONTINUE
READ APPROXIMATE VALUES OF PARAMETERS THETA ZERO, OMG,
C ANGLES ARE IN DEGREES, DISTANCES IN METERS AND OMG IN RADIANS PER HOUR
C
C RFAD5,101) TH
0037 WRITE(6,701) TH
0038 FORMAT(/5X,'THETA ZERO = ',F20.9)
0039 THET=TH*PI/180.DO
0040 OMG=PI/12.DO
C
C WRITF(6,450)
0041 450 FORMAT(/8X,'STATION & RADIO SOURCES APPROXIMATE COORDINATES'
C 2*12X,'STATIONS',10X,'LONG'
C 2*12X,'LAT',11X,'R'
C
C DO 5 I=1,IN
C
C RFA5,701) IDM,11,1IL2,ALONG,APHI,AR
C WRITE6,701) I,11,1IL7,ALONG,APHI,AR
C
C 701 FORMAT(I1X,12,1X,A4,A2,3F15.1)
C
C APHI=APHI*PI/180.DO
C ALONG=ALONG*PI/180.DO
C CONVERT TO CARTESIAN
C
C X(I)=AR*COS(APHI)*COS(ALONG)
C Y(I)=AR*COS(APHI)*SIN(ALONG)
C Z(I)=AR*SIN(APHI)
C CONVERT TO EARTH RADII UNITS
C X(I)=X(I)/R
C Y(I)=Y(I)/R
C Z(T)=Z(I)/R
C CONTINUE
C
C WRITF(6,451)
C 451 FORMAT(/8X,'RADIO SOURCE',10X,'RA',17X,'DEC',11X,'/')
C
C DO 6 J=1,IM
C
C RFA5,321) IDM,11,IL2,IL3,RA(J),D(J)
C WRITE6,321) J,11,IL2,IL3,RA(J),D(J)
C
C 321 FORMAT(I1X,13,7X,A4,F17.7,F20.9)
C CONVERT TO RADIAN UNITS
C
C RA(J)=RA(J)*PI/180.DO
C D(J)=D(J)*PI/180.DO
C CONTINUE
C
C FORMULATE CONSTRAINT MATRICES ED, EDO
C
C DO 11 J=1,S
C DO 9 I=1,6
C EDO(I,J)=0.DO
C CONTINUE
C
C 9 CONTINUE
C DO 11 I=1,K
C EDI(I,J)=0.DO
C CONTINUE
C
C 11 CONTINUE
C
C FDO(1,5)=1.DO
C EDO(9,4)=1.DO
C EDC(5,4)=-1.DO
C
C DD 3 I=1,IN
C I=I+5 I=1-2
INITIALIZE NKEFUP TO ZERO

READ OBSERVATIONS AND IDENTIFIERS: OBSERVED TO QUASAR

OF POOR QUALITY

ORIGINAL PAGE IS
READ(5,104) I,J,IP,DS,FRNG,TK,ICHEK
FORMAT(315,3F15.5,2I5)

ICHEK IS A CODE INDICATING END OF DATA FOR ICHEK=1

IF(ICHEK.EQ.1) GO TO 66
IF(ICHEK.EQ.2) GO TO 66
ICOUNT=ICOUNT+1

TK=TK-FRSTEP*DFLOAT(IDCNT)

DS=DS/R
FRNG=FRNG/R
DX=X(J)-X(I)
DY=Y(J)-Y(I)
DZ=Z(J)-Z(I)
CA=DCOS(RA(IP))
SA=DSIN(RA(IP))
CD=DCOS(D(IP))
SD=DSIN(D(IP))
Y1=OMG+TK+TH
Y2=RA(IP)
CK=DCOS(Y1)
SK=DSIN(Y1)
CKP=PCOS(Y2)

FORMULATE PARTIALS

F1=-CD*Z*CKP+DX*SD
F2=-CD*DZ*CKP+DY*SD
F3=-DX*SD*CK+DY*SD*SK+DZ*CD*CA
F4=DX*SK*SD+DY*CK*SD-DZ*CD*SA
F5=(DX*SKP+DY*CKP)*CD
F6=TK*F5
F7=CD*CKP
F8=-CD*SKP
F9=SD
F10=CD*(DX*SKP+DY*CKP)
F11=(-DX*CKP+DY*SKP)*SD+DZ*CD
F12=1.0D0
F13=TK/24.0D0

G1=SZ*CD*SKP
G1=OMG*G1
G2=SZ*CD*CKP
G2=CMG*G2
G3=(DX*SK+DY*CK)*SD
G3=CMG*G3
G4=DX*CK*SD-DY*SK*SD
G4=OMG*G4
G5=-DX*CD*CKP+DY*CD*SKP
G5=QG*G5
G6=TK*G5
G7=-CD*SKP
G7=OMG*G7
G8=-CD*CKP
G8=OMG*G8
G9=0.0D0
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MAIN

DATE = 76300 21/56/32

0166
G10=DX*CD*CKP-DY*CD*SKP
0167
G10=OMG*G10
0164
G11=DX*SD*SKP+DY*SD*CKP
0169
G11=OMG*G11
0170
G12=0.DO
0171
G13=1.DO/24.DO

C
C INITIATE A AND B TO ZERO
*********
C
0172
DO II=1,II=KK
0173
A(II)=0.DO
0174
B(II)=0.DO
0175
CONTINUE

C
C SET PARTIALS IN APPROPRIATE LOCATIONS OF A ROW
*********

0176
A(1)=F1
0177
A(2)=F2
0178
A(3)=F3
0179
A(4)=F4
0180
A(5)=F5
0181
A(6)=F6

0182
B(1)=G1
0183
B(2)=G2
0184
B(3)=G3
0185
B(4)=G4
0186
B(5)=G5
0187
B(6)=G6

0188
I1=3*I+4
0189
I2=I1+1
0190
I3=I1+2
0191
A(I1)=-F7
0192
A(I2)=-F8
0193
A(I3)=-F9

0194
B(I1)=-G7
0195
B(I2)=-G8
0196
B(I3)=-G9

0197
IC1=I+3*IN+2*IM+I
0198
IC2=IC1+IN
0199
A(IC1)=-F12
0200
A(IC2)=-F13

0201
B(IC1)=-G12
0202
B(IC2)=-G13

0203
J1=3*J+4
0204
J2=J1+1
0205
J3=J1+2
0206
A(J1)=F7
0207
A(J2)=F8
0208
A(J3)=F9
**FORTRAN IV G1 RELEASE 2.0**

**MAIN**

**DATE = 76300**  21/56/32

0209  B(J1)=G7
0210  B(J2)=G8
0211  B(J3)=G9

0212  JC1=6+3*IN+2*IN+J
0213  JC2=JC1+IN
0214  A(JC1)=F12
0215  A(JC2)=F13

0216  B(JC1)=G12
0217  B(JC2)=G13

0218  IP1=3*IN+2*IP+5
0219  IP2=IP1+1
0220  A(IP1)=F10
0221  A(IP2)=F11

0222  B(IP1)=G10
0223  B(IP2)=G11

**C**

**FORMULATION OF L VECTOR ENTRY**

0224  DS0=DX*F7+DY*F8+DZ*F9
0225  DS0=DS0+O(J)-O(D(J)-O(D(J)-D(D(I)))*(TK/24.DO)
0226  L=DS0-DS
0227  LW1=L*W100

0228  50 DS0=DS0+O(J)-O(D(J)-O(D(J)]*(TK/24.DO)

0229  351 FORMAT(10X,F40.15)

0230  WRITE(10,100) ICHEK,I,J,IP
0231  WRITE(8,351) F1,F2,F3,F4,F5,F6,F7,F8,F9,F10,F11,F12,F13,L

0232  FORMAT(10X,F40.15)

0233  12 CONTINUE

0234  FRNG0=DX*G7+DY*G8+DZ*G9
0235  FRNG0=FRNG0+(DD(J)-DD(I))/24.DO
0236  L=FRNO0-FRNG

0237  LW2=L*W2

0238  WRITE(6,350) LW1,LW2

0239  WRITE(10,100) ICHEK,I,J,IP
0240  WRITE(8,351) F1,F2,F3,F4,F5,F6,F7,F8,F9,F10,F11,F12,F13,L

0241  65 CONTINUE

0242  NKEEP(I,J)=NKEEP(I,J)+A(I)*A(JJ)

0243  CONTINUE

0244  FORMAT(6OX,2F19.9)
C
C 0245 GO TO 33
C
C***********************************************************************
C
C 0246 WRITE(8,100) ICHEK
C
C 0247 DO 20 I=1,6
C 0248 UDDII(I)=U(I)
C 0249 DO 20 J=1,6
C 0250 NDP(I,J)=NKEEP(I,J)
C 0251 20 CONTINUE
C 0252 DO 21 I=1,6
C 0253 DO 21 J=1,K
C 0254 J6=J+6
C 0255 NB(I,J)=NKEEP(I,J6)
C 0256 21 CONTINUE
C
C 0257 DO 22 I=1,K
C 0258 I6=I+6
C 0259 UD(U(I))=U(I6)
C 0260 22 CONTINUE
C
C 0261 CALL DMINV(NOD,6,DFT,L16,L26)
C 0262 CALL MATPRO(MDD,ND,HI,6,6,K)
C 0263 CALL TRAPRO(HI,RI,K,6,6)
C 0264 DO 23 I=1,K
C 0265 DO 23 J=1,K
C 0266 I6=I+6
C 0267 J6=J+6
C 0268 QB(I+J)=QB(I,J)+NKEEP(I6,J6)-RI(I+J)
C 0269 23 CONTINUE
C
C 0270 CALL MATPRO(NDD,UDDII,TI,6,6,1)
C 0271 CALL TRAPRO(NB,HI,RLI,K,6,1)
C 0272 CALL MADD(HI,6,K)
C 0273 CALL MADD(RK,NDD,6,6)
C 0274 CALL MADD(RK,RLI,6,1)
C 0275 CALL MADD(AR,TI,6,1)
C 0276 CALL MADD(U,UDI,K,1)
C
C***********************************************************************
C
C STORE UDDII,NDD,HI,TI ON DISK 9
C***********************************************************************
C
C 0277 DO 25 I=1,6
C 0278 DO 25 J=1,6
C 0279 WRITE(9,351) NDD(I,J)
C 0280 25 CONTINUE
C
C 0281 DO 26 I=1,6
C 0282 DO 26 J=1,K
C 0283 WRITE(9,351) HI(I,J)
C 0284 26 CONTINUE
C
C 0285 DO 27 I=1,6
0286     WRIF(9,351) II(I)
0287     CONTINUE
0288     IDCNT=IDCNT+1
0289     IF(ICHK.EQ.2) GO TO 67
0290     GO TO 55
0291     CALL TRAPRO(H,EDO,HTEDO,K,6,8)
0292     DO 28 I=1,K
0293         DO 28 J=1,8
0294             JK=J+I
0295             QB(I,JK)=ED(I,J)-HTEDO(I,J)
0296     CONTINUE
0297     CALL MATPRO(RK,EDO,RKEDO,FK,6,8)
0298     CALL TRAPRO(EDO,RKEDO,ETKE,FK,6,8)
0300     DD 29 I=1,8
0301         IK=I+K
0302         DO 29 J=1,8
0303             JK=J+K
0304             QB(IK,JK)=-ETKE(I,J)
0305     CONTINUE
0306     IPF=3*IN+1
0307         IPL=3*IN+2*IM
0308         DO 34 J=IPF,IPL
0309             QB(I,J)=CS(I,J)*PRATIO
0310     CONTINUE
0311     CALL DMINV(QB,K8,DET,L1K,L2K)
0312     DD 36 I=1,K
0313         DO 36 J=1,8
0314             GP(I,J)=CS(I,J)
0315     CONTINUE
0316     DD 31 I=1,K
0317         DO 31 J=1,8
0318             JK=J+K
0319             FB(I,J)=QB(IK,JK)
0320     CONTINUE
0321     DD 32 I=1,8
0322         IK=I+K
0323         DO 32 J=1,8
0324             JK=J+K
0325             FF(I,J)=QB(IK,JK)
0326     CONTINUE
0327     CALL MATPRO(QB,RL,GBL,K,K,1)
0328     CALL TRAPRO(EDO,RTETOT,6,1)
0329     CALL MATPRO(FB,ETOT,FET,K,8,1)
CALL MATPRO(GB,UD,GU,K,K+1)

DO 41 I=1,K
X8(I)=GBL(I)+FET(I)-GU(I)
CONTINUE

CALL MATPRO(F,F,ET,FEDT,8,8,1)
CALL TRAPRO(FB,RL,FETL,8,K+1)
CALL MAND(FEDT,FETL,8,1)
CALL TRAPRO(FB,UD,FETU,8,K+1)

DO 44 I=1,6
FET(I)=FET(I)-FETU(I)
CONTINUE

CALL MATPRO(EDO,FEDT,SOL,6,8,1)
CALL PROTRA(FB,EDO,FBEOT,K,6)
CALL MATPRO(EDO,FF,FEDF,6,8,6)
CALL PROTRA(EDF,EDO,EFET,6,8,6)

REWRIND 9
DO 60 IDAY=1,IDCNT

DO 35 I=1,6
DO 35 J=1,6
READ(*,351) NDD(I,J)
CONTINUE

DO 36 I=1,6
DO 36 J=1,6
READ(*,351) HIC(I,J)
CONTINUE

DO 37 I=1,6
DO 37 J=1,6
GDD(I,J)=GDD(I,J)+HFEN(I,J)+HFEN(J,I)
CONTINUE

CALL PROTRA(GB,HGT,GBHT,K,K,6)
CALL MATPRO(HI,GBHT,HGT,6,K,6)
CALL MAND(GDI,HGT,6,6)
CALL MATPRO(FBEOT,NDD,FETN,K,6,6)
CALL MATPRO(HI,FETN,HFEN,6,K,6)

DO 39 I=1,6
DO 39 J=1,6
GDD(I,J)=GDD(I,J)+HFEN(I,J)+HFEN(J,I)
CONTINUE

CALL MATPRO(FET,NDD,EFEN,6,6,6)
CALL MATPRO(NDD,EFEN,NFEN,6,6,6)
CALL MAND(GDD,NFEN,6,6,6)
DO 48 I=1,6
IPF=6*IDAY-6+I
STD(IPF)=GDD(I+I)
48 CONTINUE

CALL MATPRO(NDD,SOL,NSOL,6+6+I)
CALL MATPRO(HI,XB,HIXB,6,K,1)

DO 40 I=1,6
XBI(I,IDAY)=-TI(I)-NSOL(I)-HIXB(I)
40 CONTINUE

CONTINUE

DO 49 I=1,K
IPL=6*KCNT+I
STD(IPL)=GB(I+I)
49 CONTINUE

REWIND 8

IDAY=1
VTPV=0.DO

READ(8,100) ICHEK,IJ,IP
IF(ICHEK.EQ.1) GO TO 88
IF(ICHEK.EQ.2) GO TO 89
IS1=3*I-2
IS2=IS1+1
JS1=3*J-2
JS2=JS1+1
IPI=3*IN+2*IP-I
IP2=IP1+1
ID1=3*IN+2*IM+I
JD1=3*IN+2*IM+J
ID2=ID1+IN
JD2=JD1+IN
DD=45 JJ=1+2
READ(8,351) (F(II),II=I,13),L

V=L
DO 42 II=1,6
42 CONTINUE

V=V+F(II)*XBI(II,IDAY)

IF(JJ.EQ.2) GO TO 43

V=V+V*(X(II)-X(IS1))+F(8)*(X(II)-X(IS2))
2 +F(9)*(X(IS2)-X(IS3))
V=V+F(10)*X(IP1)+F(11)*X(IP2)
V=V+F(12)*(X(JD1)-X(ID1))+F(13)*(X(JD2)-X(ID2))

C        WRITE(6,453) VW
C416      VTPV = VTPV + V**2
C417      GO TO 45
C418      VTPV = VTPV + PF*V**2
C        WRITE(6,454) VW
C454      FORMAT(50X,F30.15)
C453      FORMAT(20X,F30.15)
C419      CONTINUE
C420      GO TO 77
C421      IDAY = IDAY + 1
C422      GO TO 77
C423      IPF = 2*IN + 1
C424      IPL = 3*IN + 2*IM
C425      DO 46  I = IPF, IPL
C426      VTPV = VTPV + PRATIO*XB(I)**2
C427      CONTINUE
C428      IS2 = 2*ICOUNT
C429      JS2 = 5*IN + 6*IDCNT
C430      IS3 = IS2 - JS2
C431      IDC = (24.DO/ERSTEP + O.IDO)
C432      IDC = IDCNT/IDC
C433      WRITE(6,242) IDC, IS2, JS2, IS3
C435      VTPV = VTPV / DFLOAT(IS3)
C        WRITE(6,498) I, STD(I)
C440      IF (STD(I) .GE. 0. DO) GO TO 999
C441      STD(I) = DSQRT(STD(I)*SO)
C442      CONTINUE
C443      WRITE(6,888)
C444      WRITE(6,889)
C445      DO 85 IDAY = 1, IDCNT
C446      IPL = 6*IDAY
C447      IPF = IPL - 5
C448      DO 86 II = 1, 5
C449      JJ = IPF + II - 1
C450      CALL REDPI(STD(JJ))
C451      STD(JJ) = STD(JJ)**100
C452      CONTINUE
C453      STD(IPL) = STD(IPL)**RHRDM9
C454      WRITE(6,889) IDAY, STD(I), J = IPF, IPL
C455      WRITE(6,888) IDAY, I, IPL
C456      85  CONTINUE
C457      WRITE(6,221)
C459      9X, 'Z'(CM)*, 6X, 'Y'(REV/OAY)*10**9/)
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0450 IPF=6*IDCNT+1
0451 IPI=6*IDCNT+3*IN+2*IM
0461 II=IPI-2*IM+1
0462 DO 91 I=II,IPI
0463 CALL REDPI(STD(I))
0464 91 CONTINUE

C
0465 DO 75 I=IPF,IPI
0466 STD(I)=STD(I)*R100
0467 75 CONTINUE
C
0468 IPL=IPF-IM
0469 WRITE(6,222) (STD(I),I=IPF,IPI)
0470 222 FORMAT(10X,3F10.2)

C
0471 WRITE(6,223)
0472 223 FORMAT( ///10X,"OUASAR COORDINATES",//16X,"RA",8X,"DEC",/15X
Z,*(CM)*,6X,*(CM)="/"
0473 IPF=IPI+1
0474 IPL=IPI+2*IM
0475 WRITE(6,224) (STD(I),I=IPF,IPI)
0476 224 FORMAT(10X,2F10.2)
C
0477 WRITE(6,225)
0478 225 FORMAT( ///10X,"STATION CLOCK OFFSETS AND DRIFTS","/3X
2,*(MSEC)*,6X,*(MSEC/DAY)="/"
0479 DO 76 I=1,IN
0480 IC1=IPI+I
0481 IC2=IC1+IN
0482 STD(IC1)=$STD(IC1)/CLIGHT)*1000.DO
0483 STD(IC2)=$STD(IC2)/CLIGHT)*1000.DO
0484 WRITE(6,226) STD(IC1),STD(IC2)
0485 226 FORMAT(4X,2F16.12)
0486 76 CONTINUE
C
0487 STOP
0488 DEBUG SUBCHK
0489 END
SUBROUTINE MADD(A,B,L,M)

      C
      C     A = A + B
      C     L*M L*M L*M
      C
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION A(L,M),B(L,M)
      DO 5 I=1,L
      DO 5 J=1,M
      A(I,J)=A(I,J)+B(I,J)
      CONTINUE
      RETURN
      END
SUBROUTINE MATPRO(AtB,CLtMN)

C = A * B

IMPLICIT RFAL*8(A-H,O-Z)
DIMENSION A(L,M),B(M,N),C(L,N)

DO 5 I=1,L
  DO 5 J=1,N
    C(I,J)=0.DO

DO 5 K=1,M
  C(I,J)=C(I,J)+A(I,K)*B(K,J)

5 CONTINUE
RETURN
END
SUBROUTINE TRAPRO(A,B,C,L,M,N)

C

C C T

C = A * B

L*N  L*M  M*N

IMPLICIT REAL*8(A-H,O-Z)

DIMENSION A(M,L),B(M,N),C(L,N)

DO 5 I=1,L

DO 5 J=1,N

C(I,J)=0.DO

DO 5 K=1,M

C(I,J)=C(I,J)+A(K,I)*B(K,J)

5 CONTINUE

RETURN

END
SUBROUTINE PROTRA(A,B,C,L,M,N)

C C C T
C C C
C I = A * B
C L*N L*M M*N
C
C IMPLICIT RFAL*8(A-H,D-Z)
DIMENSION A(L,M),B(N,M),C(L,N)
DO 5 I=1,L
DO 5 J=1,N
C(I,J)=0.DO
DO 5 K=1,M
C(I,J)=C(I,J)+A(I,K)*B(J,K)
5 CONTINUE
RETURN
END