INTERPOLATION AND EXTRAPOLATION OF CREEP RUPTURE DATA
BY THE MINIMUM COMMITMENT METHOD
PART III - ANALYSIS OF MULTIHEATS

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ABSTRACT

The Minimum Commitment Method was applied to two sets of data for which multiple heat information was available. For one alloy, a 304 stainless steel studied in Japan, data on nine well characterized heats were used, while for a proprietary low alloy carbon steel studied in the United Kingdom data were available on seven heats - in many cases to very long rupture times. For this preliminary study no instability factors were used.

It was discovered that heat-to-heat variations could be accounted for by introducing heat identifiers in the form $A + B \log \sigma$ where $\sigma$ is the stress and the constants $A$ and $B$ depend only on the heat. With these identifiers all the data could be collapsed onto a single master curve, even though there was considerable scatter among heats. Using these identifiers together with the average behavior of all heats made possible the determination of an accurate constitutive equation for each individual heat.

Two basic approaches are discussed for applying the results of the analysis. If it is assumed that the component of interest is to be constructed from the same generic material, but no information will be available on the specific piece from which the construction will be made, then the design stress is based on a statistical analysis of the average behavior and the standard deviation about this average. For the two materials studied excellent results were obtained in establishing lower bounds for potential use in setting design stresses. The second approach relates to the possibility that some information will be available on the particular lot used in construction of the component in question. In this case a minimum amount of data, perhaps only...
3 or 4 strategically selected data points, might suffice to establish the identifier for the special heat of interest, making possible its complete characterization from the equations for average heat behavior. Good results were obtained for one illustration investigated, although further study is needed to generalize the approach.

INTRODUCTION

While in the past the use of time-temperature parameters has been applied primarily to single heat data, as presented in Parts I and II, a major application has arisen in relation to multiple heats. Large technological programs such as nuclear power reactors have focused attention on the large scatter that can result in the high temperature properties of materials arising from compositional and processing variables which in most cases are within acceptable limits of specifications. If each lot or heat could be completely evaluated by conducting all the necessary tests there would be no problem in regarding each as a separate material, and in analyzing it according to the concepts described in Parts I and II which use mainly single heat data. However, the results would apply only to material of the specific composition and thermo-mechanical processing of the prototype evaluated. The probable behavior of a future heat would not be readily estimable, nor would a large data base on one heat be directly useful in characterizing heats with a smaller data base. It is thus important to extend the concepts of single heat analysis to answer such questions as:

1. Given information on samples from various lots of material which possibly differ slightly in composition and processing, how can we combine the information for better prediction of the behavior of each individual lot?

2. How can we estimate the properties of another lot not used in the initial study with a minimum of data on the new lot?

3. How can we set design limits on the material as a class which is statistically compatible with the large scatter frequently found in multiheats?

The above questions have already received considerable attention by other investigators, and some of the results of their studies are reported at this Symposium (e.g. Refs. 1 and 2). It is a relatively new application, however, for the MEGA procedure (Part 1), but one for which its features are advantageous: since the approach already allows for redundancies of data even in a single heat, further redundancies involved in multiple heats can readily be accommodated; use is made of a stress function (master curve) that is inherently stable, and will not display undesirable wiggles or curvature introduced in other functional forms as a result of data scatter; and most importantly, a single functional form is used for all materials - only the constants change from one material to another. Thus it is appropriate to expand the method to
considerations of multiheats. Experience in the treatment of multiheats is, as yet, limited. However, the attempts that have been made have been relatively successful, and it is clear that the promise of the method should be further explored. In this report we shall outline the basis for the procedures adopted and the results that have been obtained on two classes of materials: a 304 stainless steel studied in Japan, and designated NRIM (National Research Institute for Metals); and a proprietary low alloy steel used in the United Kingdom. The data on each of these multiheats are so extensive that the individual lots could, in fact, be analyzed separately according to MCM procedures. Such analyses have actually been made, but will not be presented here. Instead we shall show only the results for the collective heats, first to demonstrate the procedure as we have developed it to date, and second to illustrate results that can be obtained.

BASIS FOR METHOD

To illustrate the procedure we shall refer to the NRIM data which are reported in detail in Ref. 3. Data on nine heats are available, designated A to F and L, M, and N. The last three heats differ considerably in properties from the first six, perhaps because of differences in structural form and processing variables. In our early Committee deliberations it had been decided to omit heats L, M, and N because they were so uncharacteristic in overall behavior. For this reason treatment of this multiheat group in References 1 and 2 do not include heats L, M, and N. However, for our own study, we later decided that the inclusion of these heats would provide an increased challenge to our method and would therefore be especially valuable in developing the treatment for cases involving large data scatter. Thus, the analysis as we present it below includes all nine heats.

The first step was to treat all the data collectively as if they were from a single heat. For this analysis we used the universalized value $A = -0.05$ without temperature correction. The effect of using $A = -0.05$ is, of course, to make the results slightly more conservative than would be obtained for $A = 0$. Use of $A = 0$ would remove an objection in the interpretation of the statistics of the results (See Part I); however, illustration for $A = -0.05$ provides a more generalized treatment. Later we shall illustrate the use of $A = 0$ which provides estimates of the coefficients in which only the squared deviations of log $t$ have been minimized and which thus could be used in setting confidence limits.

1 Actually, a small part of the analysis presented does include a minor temperature correction used in the early development of the method, but which has since been abandoned. For $A = -0.05$ the effect of this correction is so small, however, that it should not substantially affect the results. We did not deem it worthwhile to redo the manual analysis to be described in order to remove this very small effect.
Fig. 1 shows the analysis for the Focal Point Convergence Method when all the data are treated as from a single heat. The curves of Fig. 1(a) represent the "average" behavior at each temperature as produced by the analysis. The scatter of the data is substantial at all temperatures. Figure 1(b) shows the master curve with individual heats represented by separate symbols, and the fitted curve through them. In order to see more clearly the systematized variations according to heat, the heats are separately represented in Fig. 2. The master curve in each case is the same as in Fig. 1(c), but only one heat is plotted in each subfigure. The points deviate from the curve in a relatively smooth, non-random manner.

Our first attempt in interpreting Fig. 2 was to determine whether an identifier for each heat could be some easily measured quantity such as hot tensile strength. But it appeared that such an approach was inadequate. Heat A, for example, had a higher tensile strength than Heat B over the entire temperature range involved in the analysis. However, the reflection of this higher strength in regard to creep rupture behavior was apparent only at the lower temperatures, whereas at the higher temperatures the creep rupture properties became similar despite the differences in strength. In fact Heat A had lower creep rupture properties than Heat B at the highest temperatures evaluated. This can be seen in Fig. 2 relating the master curve to the data for the two heats. Points for Heat A lie above the curve at the high stresses, while Heat B points lie below the curve (which is the same in both figures). Similar observations were made for other heat combinations. Thus it became clear that an alternate approach, rather than just normalizing relative to hot tensile strength, would be desirable.

A second step was, therefore, taken based on the observation in Fig. 2 that the systematic deviation among heats depends largely on stress. Whether the test is at lower temperature and longer time, or at higher temperature and shorter time, does not seem to matter much; the deviation depends mostly on the stress. Each heat has its own pattern, or "fingerprint", relative to the average behavior of the multihot treated collectively. Since the points in Fig. 2 deviate systematically from the curves according to stress, a plot was made as shown in Fig. 3 of the displacement between the curve and the data points as a function of stress. While considerable scatter occurs, reasonably good least-squares straight lines can be fitted through the data for each heat. In this case it follows that, a power-law relation exists between the actual stress at a given datum point and the stress on the master curve at the same value of G. These relations vary from heat to heat, and can be regarded as the "fingerprints" identifying the individual heats. Thus each heat can be identified by two numbers - the coefficient and
exponent for a modified stress. For example, as seen in Fig. 3, the equation for the modified stress for Heat A is

$$\sigma_m = 1.372 \sigma_i^{0.863}$$  \hspace{1cm} (1)

where $\sigma_i$ = initial, (prior to modification) stress for a given point of Heat A.

$\sigma_m$ = Modified stress for Lot A to bring it into coincidence with the master curve.

For example, if the actual stress is 16 ksi, we must regard it as 15 ksi for Heat A, 17.63 ksi for Heat B, etc. In this way we can then use the same $P$ function of Fig. 1(b), and master curve of Fig. 1(c) for all heats. Figure 4 shows the "master curve" for the entire set when modified stresses are used for each heat. The points now fall much closer to the curve, and are more evenly distributed around it.

Figure 5 shows the isothermals for the individual heats using the modified stress value. In each case the isothermals were obtained using the $P$-function of Fig. 1(b) and the master curve of Fig. 1(c). The curves show the representation of each heat using its characteristic stress modifiers of Fig. 3. Good agreement is seen between the data and their representation by the stress modification approach. To put the results of this analysis in the same perspective as the analyses for the single heat materials discussed in Part I, Figs. 6 and 7 are also included. Figure 6 shows the complete computer analysis for the multiheat when the stresses are unadjusted, while Fig. 7 shows the corresponding analysis for the adjusted stresses. The improvement in individual heat representation by this approach can be seen by comparing Fig. 5 with Fig. 1(a). In Fig. 1(a) all heats are represented by the same isothermals as shown. Obviously there is considerable scatter and misrepresentation of some heats. But in Fig. 5 each heat is individually and properly represented.

While the type of analysis discussed above could be used as a direct framework for further development, two difficulties are readily apparent. The first is that the determination of the master curve was made before any modifiers were applied to stress; thus the modifiers were developed after-the-fact using a non-optimized master curve. It would have been more proper to introduce the modifiers immediately in terms of unknown coefficients and exponents for each heat, and solve the set of equations simultaneously for optimized values of all the constants involved in the analysis. In this way a more optimum baseline master curve would emerge. However, as will be shown in the next section this procedure would have rendered the equations non-linear, complicating their solution and the statistical interpretation of the results. A second difficulty relates to the fact that the
approach described above involves a certain amount of manual analysis. It will be noted in Fig. 2, for example, that the increment in log stress involved in the analysis is the vertical distance between a given point and the curve at the same value of G. Now the stress value at the datum point is known because it is an experimental value. However, the value stress on the curve for a given value of G must be determined by solving the transcendental equation expressing the master curve on each side of the spline. While this could be done by any one of a number of approaches for the approximate solution of transcendental equations (e.g. Newton-Raphson), it is apparent that the complexity would be greatly increased if this path for extension were chosen.

In the following section we describe, instead, the approach adopted which is based in general on the observations of the behavior shown in Figs. 1 to 5, but overcomes the objections discussed.

AN APPROACH SUITABLE FOR TOTALLY COMPUTERIZED ANALYSIS

In order to examine how the foregoing procedure would be implemented by introducing the corrections of the type shown in Eq. (1) before the regression is made, rather than after the "average" master curve has already been determined, we actually proceed to do so.

Let

\[ \sigma_{m,a} = C_a \sigma^d_a \]
\[ \sigma_{m,b} = C_b \sigma^d_b \]
\[ \vdots \]
\[ \sigma_{m,n} = C_n \sigma^d_n \]

where \( C_a, C_b \ldots C_n \) are the coefficients of the modifiers, and \( d_a, d_b \ldots d_n \) are the exponents for each of the heats A, B, N. The idealized way to simulate what was done in the partially manual analysis of Figures 1 to 5 is to introduce Eqs. (2) into the basic MCM equation (Eq(2) of Part I). For simplicity of analysis let us continue the discussion by assuming that the instability parameter A=0. The basic equations are taken from Eqs (2), (10) and (11) of Part I.

\[ \log t + P = A_1 + B_1 \log \sigma + C_1 \sigma^{\alpha_1} \] for stresses at or below the spline point

and

\[ \log t + P = A_2 + B_2 \log \sigma + C_2 \sigma^{\alpha_2} \] for stresses above the spline point

where in the present context the stress values are the "modified" stresses of Eqs (2). Consider, for example what happens if we substitute into Eqs (3) the relation shown in Eqs (2) for Heat A. We are interested in using
the "initial" stresses since these are the experimental values we know. Thus the relation in terms of initial stresses (below the spline point) becomes

\[ \log t + P = A_1 + B_1 \log \sigma_{m,A} + C_1 \sigma_{m,A}^{\alpha_1} \]

\[ = A_1 + B_1 \left( \log \sigma_{a} + \sigma_{i} \right) + C_1 \left( \sigma_{a} \sigma_{i}^{\alpha_1} \right) \]

\[ = A_1 + B_1 \log \sigma_a + B_1 \sigma_{i} \log \sigma_i + C_1 \sigma_i^{\alpha_1} \sigma_{i1}^{\alpha_1} \]

If we were to write similar equations for each of the other heats we would get different values of the constant, the coefficient of \( \log \sigma_i \) and the coefficient and exponent of \( \sigma_i \). In essence, then, each heat would have its own "master curve" with a different function value, slope, and second derivative at the spline point (although there would be a complex interrelation among all the quantities involved in the system as a whole). From the standpoint of regression analysis, establishing the optimized constants for the problem would become complex.

However, if we examine the problem more closely we realize that perhaps we could accomplish approximately the same final result if we consider that the modification affects mainly the constant and coefficient of \( \log \sigma_i \) in the "master curve", and that the power term of stress could be left unaltered. In other words we seek to determine improved results by allowing just the value of \( G \) (the master curve) at the spline point, as well as its slope, to vary from heat to heat, but not the second derivative. This leads us to be able to write the basic relations (below the spline point, for example), as

\[ \log t + P + R_3 + R_4 \log \sigma_i = A_1 + B_1 \log \sigma_i + C_1 \sigma_i^{\alpha_1} \]

Equations (5) state, in essence, that we will have a single master curve which below the spline point is \( G_1 = A_1 + B_1 \log \sigma_i + C_1 \sigma_i^{\alpha_1} \), and above the spline point is \( G_2 = A_2 + B_2 \log \sigma_i + C_2 \sigma_i^{\alpha_2} \). This single master curve will apply to all heats, and will use real (unmodified) stress. On the left side of the equations we add the "fingerprint" or heat identifier terms \( R_3 + R_4 \) which is different for different heats. In a sense this replaces the time-temperature parameter by a time-temperature-stress parameter where time is represented by \( \log t \), temperature by \( P \) and stress by \( (R_3 + R_4 \log \sigma) \). The advantage of representation by Eq (5) over Eq (4) is, however, that Eq (5) contains all unknowns as linear coefficients, which readily lend themselves to conventional regression analysis.
One further point should be made in connection with the stress modifiers. They are not all independent of each other. Since the "master curve" represents their "average" behavior, the "average" of the modifying coefficients must also be zero. Then $R_3^{A} + R_3^{B} + \cdots + R_3^{N} = 0$ and $R_4^{A} + R_4^{B} + \cdots + R_4^{N} = 0$. Thus, in making the analysis we choose one heat (any one) and set its coefficients equal to the negative sums of the other coefficients. For example, if we chose Heat A as the reference, the system of equations to be solved would be

$$\log t + P + (-R_3^{B} - R_3^{C} - \cdots - R_3^{N}) + (-R_4^{B} - R_4^{C} - \cdots - R_4^{N}) \log \sigma = A_1 + B_1 \log \sigma + C_1 \sigma^{a_1}$$

A corresponding set of equations apply at stresses above the spline point. These equations, together with the experimental data, permit the determination of all the constants by linear regression. The number of constants involved depend on how the temperature function $P$ is expressed, and how many heats are involved. If $P$ is expressed as a single continuous function, it involves 2 unknowns (see Part I); if it is expressed as a station function there will be one less unknown than the number of isothermals to be characterized (since $P=0$ at the central isothermal). The master curve requires four constants (function, slope, and two second derivatives at the spline point), and each heat above the first (any heat can be regarded as the first) adds two constants to the system.

When the equations are solved what we have is the "average" behavior (obtained by omitting the $R_3$ and $R_4$ terms, since on average they are zero), together with the "master curve", which applies to all heats. This average behavior, however, now has an optimized master curve, which includes consideration of systematic heat-to-heat variations, whereas the master curve obtained in Fig. 3 ignores the systematic heat-to-heat effects and obtains an average by lumping all data together. In a sense, therefore, the procedure just described introduces the heat-to-heat effect before the fact (of the analysis of average behavior), while the first approach illustrated studies the heat to heat variations after the fact. Whether an important difference in final results develops according to the procedure adopted probably depends on the type of data available. For the materials studied in this report the effect was minor, as will be discussed.

Once the analysis has been completed and all constants determined, there are two choices in the application of the results. The first is to use the
average behavior, and the statistics of variation about the mean, to set design stresses for applications wherein it is assumed that no information at all is available on the properties of the particular heat which is to be used in any given application. The other is to assume that some information will be available on the particular heat of interest, and that the overall behavior of the multiheat can be used, in essence, as a lever to amplify the usefulness of the scant information on the particular heat. Thus the particular heat of interest can be fully characterized from the collective information on all available heats. Both these potential uses will be discussed later in this report.

As a matter of interest it should also be pointed out that a similar analysis has been conducted to determine if the heat-to-heat variation could equally well be characterized by linear variations in the P terms (temperature characterizations) instead of stress variations. The study was not comprehensive, but the limited results obtained indicated that the results obtainable by this approach were not as effective as alterations in the stress terms described.

APPLICATION OF THE TOTALLY COMPUTERIZED PROCEDURE

NRIM Steels

Equations (6) provide a relation for each available datum point. Since 133 data points are available, and the system contains 22 constants (2 P values, 4 spline point parameters, and 16 heat characterization constants) the solution is obtained by conventional least squares procedures. For this case we have taken $A=0$ in order to be able to obtain a fit which exactly minimizes the sums of squares of the log time deviations. As already noted, choosing $A=-.05$ would produce a slightly more conservative evaluation.

In presenting the results of the analysis, two views could be taken, as already noted. These relate to whether we wish to assume that some creep rupture data are available for a specific lot to be used in a design, or whether we wish to assume that nothing is known about the specific lot to be used and that we must therefore design to minimum expectations determined through statistical analysis. We shall first present the results, and discuss their potential application in the next section.

Figure 8 shows the overall analysis, corresponding to Figure 6, but wherein $A=0$. This figure is constructed from the "average heat", obtained by using the P values (Fig. 8 b), and the G curve (Fig. 8 c). For this figure no heat identifiers are included, since we seek average heat behavior, without presumed foreknowledge of the particular heat to be used. Again, of course, there is much scatter between the mean isothermals and the experimental data points.
If the heat identifiers are included in the correlation the results are as shown in Fig. 9, which is the analogue of Fig. 5. In Fig. 9 the G curve of Fig. 8(c) has been corrected for each heat by subtracting \((R_3 + R_4 \log \sigma)\), the values of which are listed in each subfigure for the particular heat involved. The agreement between the correlating isothermal lines drawn in each figure and the experimental data points is now greatly improved, actually slightly better than was obtained in Fig. 5. In addition, manual analysis has been by-passed in the generation of Fig. 9.

Proprietary U.K. Low Alloy Carbon Steels

A similar analysis for the UK Steels is shown in Figs. 10 and 11. For this low alloy steel extensive data were available for seven heats (Ref. 4). As in the case of NRIM Steel each heat was so extensively characterized that an individual MCM analysis could be made. However, for the present purpose the entire data base was treated collectively to illustrate application to multi-heats. Figs. 10 and 11 will be discussed in the next section.

DISCUSSION

As already noted, two alternate philosophies could be adopted with regard to using the type of analysis outlined. In the first we assume that all the data available to characterize the multiheat are available in advance, as in the cases of the NRIM 304 and the U.K. Steels already illustrated. However, it is also assumed that during the design stage nothing is known about the particular heat that will be used in the future construction of the component. Therefore, the design must be based only on a statistical analysis of the available information. The second approach also assumes that together with the available data base on a variety of heats of the material of interest, there is also available at least a small sampling of data on the particular heat of special interest (or one having closely controlled similarity). Thus the large data base provides the generalized behavior, but the small sampling of data identifies the particular niche wherein the actual lot to be used in the construction fits within the total spectrum of behavior possibilities. We consider in the following a possible procedure for implementing each of these approaches.

Analysis of NRIM Steels

Approach Based on Overall Statistics

Assuming that we are limited to current information only, and nothing will be known during the design stage about the particular heat to be used in the construction, we rely on the variance statistics of the data base. Consider first the NRIM stainless steel. A starting point is the standard deviation of the data used in the analysis, which is noted in Fig. 8(d) to be 0.31 log cycles of time. A decision must then be made as to what multiple of this standard deviation to use in choosing a lower bound. This is a
major decision, to be made by a Committee of concerned and experienced analysts. Here we shall assume that the 1.65 multiple proposed by Smith (Ref. 5) is a reasonable lower limit which 95% of the data can be expected to exceed. Thus we start by moving the master curve $1.65 \times 0.31 = 0.51 \log$ cycles to the left, the equation of the curve can be determined by simply subtracting 0.51 from the constant of each half of the spline function. From the equations shown in Fig. 9, therefore, the relations become

$$\log t = 0.0102(T-1292) - 69.497 \left(\frac{1}{T+460} - \frac{1}{1752}\right)$$

$$= 8.48 - 5.257 \log \sigma - 0.07890 \text{ for } \sigma < 10.23 \text{ ksi}$$

$$= 14.774 - 10.735 \log \sigma - 16.071/\sigma \text{ for } \sigma > 10.23 \text{ ksi} \quad (7)$$

The results are shown in Fig. 12. The dotted curves represent the median behavior, obtained by using the equations of Fig. 9, letting the Heat term = 0. The continuous curves show the corrected values according to Eqs. (7). It can be seen that the continuous curves do, indeed, represent a lower bound for about 90 to 95% of the data points. However, the curves represent somewhat over-conservative bounds for the 1292 F and 1382 F isothermals, and a slightly under-conservative lower bound for 1112 F isothermal. The reason for this result lies in the fact that the deviations between predicted and experimental results are not uniformly distributed over the stress range covered by the isothermals, being greater at high stresses where the master curve is flat, and smaller at low stresses where the master curve is steep.

Analysis based on statistics of individual isothermals

An alternate approach was therefore also attempted, wherein the displacement of each isothermal was in accordance with the standard deviation of the data relating to only that isothermal, rather than using the single displacement for the entire master curve. The results are shown by the continuous lines of Fig. 13. For example, at 1112 F, the standard deviation of the data is 0.432; thus the appropriate displacement is $1.65 \times 0.432 = .713 \log$ cycles. The result of the displacement is shown in Fig. 13 by the continuous line associated with the 1112 F isothermal. On the other hand, for the 1382 F isothermal, the standard deviation of the data is only 0.126. Therefore the displacement is $1.65 \times 0.126 = 0.208 \log$ t cycles, as shown by the continuous line associated with the 1382 F isothermal. Similarly, the displacements for the 1202 F and 1292 F isothermals are 0.622 and 0.317 log t cycles, as also shown in Fig. 13. It can be seen that the continuous curves associated with each of the isothermals is a better estimate of the lower bound of the data than is the curve which is obtained by a constant displacement for the entire master curve as shown in Fig. 12. Very likely the approach using individual isothermal displacement, according to the standard deviation of available data for that isothermal will find preference in the eventual use of this type of approach. However, it
is not the purpose of this discussion to indicate how best to use the results of the analysis; rather it is to provide options for future deliberations.

Of course, it should be pointed out that analytical expressions can easily be generated from Eqs. (7) for each of the possible options chosen. Only the first term (the constant) on the right side of the equations is affected by the choice of displacement. In this respect the manner of representation of the equations is specially advantageous because a complete analytical representation is available, involving a change in only one constant, according to the isothermal involved and according to the choice of applying the pertinent statistics.

Finally, it should be pointed out that the choice of design stresses is a very complex one, and involves considerations beyond the creep-rupture analysis discussed in this report. For example, the design stress is usually required to be below a specified fraction of the ultimate tensile strength, or by relation to the maximum creep that can be tolerated in a specified time, say 100,000 hours. Consideration of these criteria is beyond the scope of this report; we are concerned here only with aspects relating to creep rupture, and how to represent such creep rupture data so as to be of maximum utility in the decisions that must be made.

Analysis of the U.K. Steels

A similar analysis for the U.K. steels is shown in Figs. 14 and 15. The standard deviations involved - both for the overall analysis, and for the specific isothermals - are identified in the figure; hence little further discussion is required. It is clear, however, that Fig. 15, which is constructed according to the statistics of each individual isothermal, provides a remarkably good set of curves for establishing the lower bounds of the scatterband of the data. These curves should find value in setting design stresses for creep rupture applications.

An analysis of the type shown in Figs. 12 to 15 can also be very useful in relation to making decisions regarding the rejection of certain data points. Note, for example in Fig. 14 or 15 the longest time point at 842°F for Heat H. This point is labeled Point V. It clearly lies beyond the expected scatter represented by all other data for all the heats. Although this point was included in this analysis, it is likely that the decision would be made to reject it in a more refined analysis. How to make such decisions on the basis of quantitative considerations still remains, however, to be considered.

Applications

The results of Figs. 9 and 11 clearly demonstrate that it is possible to represent the individual heats by analytical expressions involving the use of heat identifiers which are linear with log stress. Thus if a component is to use material from a specific, known and moderately characterized heat, it should
be possible to take advantage of the low scatter of that specific heat in the choice of design stresses. In a sense, the information on the collective heats serves to establish a better set of general curves (the P function, and the master curve G), than could be obtained from any one set of data alone. The heat identifiers locate the curves, however, in optimal fashion for each individual heat. Thus each heat is better characterized than if the data alone for that heat were known. To this extent the type of analysis shown in Figs. 9 and 11 can be regarded as a useful adjunct to individual heat characterization.

An alternate approach is to take advantage of the generalized analysis to refine the knowledge of the properties of some current or future heat with a minimum of experimental data for that heat. The broad data base serves to establish all the generic constants for the multiheat (the P and G functions), while a very limited amount of data on a specific heat serves to determine the Heat Identifying Constants for that heat. In principle we need only two data points for the special heat to determine the two constants. One test at high stress, and one at a low stress, at temperatures appropriate to cause rupture in a reasonable time period are really, then, the only requirements to be able to "fingerprint" a heat, from which its complete characteristics can be determined using the temperature and stress functions obtained from the broad data base for all available heats. In essence, this method can be regarded as a generalization of the approach whereby the data are normalized relative to high temperature tensile strength. The high stress data corresponds to the tensile strength determination. But the method also requires that at least one datum point be available to establish the creep rupture characteristic at lower stress levels. Of course, if several data points are available, preferably evenly distributed over the stress range of interest, a linear regression can be applied to determine the $R_3$ and $R_4$ constants.

An easy way to illustrate the above approach is to use only a limited amount of information from one heat. The analysis is made using all the data from the other heats, and the truncated data from a special heat used to establish the rupture characteristics of the special heat over the entire range. As an example, suppose we used only four data points from Heat C for the NRIM stainless steel. The results are shown in Fig. 16. Only the four solid data points were used in the analysis together with all information from the other heats. These four data points cover the spectrum of stresses of interest. The results obtained are shown in the figure. The curves are the predicted behaviors for each of the temperatures studied, and they agree well with the actual data shown by the open symbols which were not used in the analysis. Even only two data points were adequate in this case to provide a complete material characterization, but because of data scatter this conclusion will probably not be general. Yet it points to the potential efficiency of a small amount of information on a selected heat in enabling its complete characteriza-
tion.

An alternate approach would be to characterize the multiheat without using Heat C at all, and then determine the Heat Identifier for C by regression for the two constants involved. Other possibilities also suggest themselves, for example, involving only an "average" isothermal determination such as shown in Fig. 6, with a subsequent heat by heat fingerprinting using the \( R_3 + R_4 \) identifier. More study is needed, but generally it is clear that the method shows promise for heat characterization from a minimum amount of experimental information on a specific heat.

The method also requires development in other respects. For example, the multiheats used for illustration consists of many highly characterized heats. How the method would be applied to cases wherein numerous but scantily characterized heats were involved requires further study. The choice of the instability parameter A also requires further study. Should we select A=0 as done in the example, in order to feature the unambiguities of the statistics, or should we choose A=-.05 in line with the optimum choice for most steels and superalloys, or should we choose an even higher negative value of A to feature possible instabilities? Furthermore, how well will the method work when long-time extrapolation is required? Here we have used a large data base involving tests at long times, but suppose only short time data were available? Another question of importance is whether the P and G functions determined in conjunction with the heat identifiers are indeed improvements over those determined from a similar analysis without such identifiers. For the materials studied in this program the standard deviations of the data from their "average" representation by the P and G functions were approximately the same whether or not the identifiers were included in the analysis. That is, the "average" heat was equally well represented by an analysis without heat-to-heat identifiers as they were when the identifiers were used (although, of course, the individual heats were far better represented when the identifiers were included). Thus, further work is needed to establish how important it is to include the identifiers in the initial analysis for P and G, or whether P and G can be established without them, applying them only after P and G are known. A considerable saving in complexity can be achieved if the latter approach can be used.

Thus we emphasize that we have only provided a framework for analysis, but that there is room for considerable additional refinement.

CONCLUSIONS AND CONCLUDING REMARKS

In this report we have outlined approaches for treating multiheats on the basis of the Focal Point Convergence Method discussed in Part I. Two philosophies have been discussed: a) as an adjunct to setting design stresses based on analysis of variance of all the available data, but without regard to possible available information on the specific heat to be used in the construction, and
b) as a means of providing a refined characterization of a specific heat using only limited data from that heat in conjunction with a broader data base from other heats of the same material. Using two highly characterized multiheats - a 304 stainless steel studied in Japan, and a low alloy carbon steel studied in England - the method has been shown to produce good results for both of the above possible purposes. The method is quite specific using the same functional form for all materials, and varying only the constants for each multiheat. All the constants are determined by completely computerized procedures and depend relatively little on the judgement of the analyst. Once the basic analysis has been made, the representation of various members in the same system is achieved by adding linear expressions of log stress, changing only two constants in the equations to represent a selected heat. If specific heat characterization is not desired, the approach provides a simple procedure for estimating lower bounds to embrace any desired percentage of the data points within the system. The analytical description of the set, when it is statistically represented in its entirety, is accomplished through the change of a single constant in the constitutive equation expressing the creep rupture behavior. While the method requires further development, it appears that even in its present form it may have promise as a tool for Code-setting bodies seeking to establish design stress criteria for classes of materials wherein large heat-to-heat variations can be expected. Obviously, more work would be useful to apply the approach to other multiheat systems than those studied thus far, and to answer some of the questions raised in this report.

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CONVERSION OF UNITS

To convert units used in this report to the SI units:

1. $C = \frac{5}{9} (F - 32)$
2. $\text{MN/m}^2(\text{MPa}) = 0.14504 \text{ KSI}$
FIG. 1  ANALYSIS OF NRIM 304 STAINLESS STEEL BY MEGA16 PROGRAM TREATING ALL NINE HEATS AS ONE SET.

ORIGINAL PAGE IS OF POOR QUALITY
Fig. 2. Relation Between Data and Average Master Curve on Heat-By-Heat Basis for NRIM 304 Stainless Steel.
Fig. 3. Heat-by-Heat Linearizations of Stress Deviations from the Average Master Curve for NRIM 304 Stainless Steel.
Fig. 4. Master curve for All Heats of NRIM 304 Stainless Steel Based on Heat-By-Heat Stress Modifiers.
Fig. 5  Reconstructed Isothermals for Each Heat of NRIM 304 Stainless Steel Using the Modified Stress Values and Master Curve of Fig. 5-46 or Fig. 5-49.
FIG. 6  COMPLETE COMPUTERIZED SOLUTION FOR NRIM MULTIHEAT, USING $A = -0.05$. STRESSES ARE REAL VALUES, UNADJUSTED.
FIG. 7  COMPLETE COMPUTERIZED SOLUTION FOR NRIM MULTIHEAT, USING $A = -0.05$. ADJUSTED STRESSES UNIFY DATA.
FIG. 8  COMPLETE COMPUTERIZED SOLUTION FOR NRIM MULTIHEAT, USING A = 0.
STRESSES ARE REAL VALUES.
\[ \log t \cdot 0.102 \left( T-1292 \right) = 69.497 \left( \frac{1}{T+460} - \frac{1}{T+1752} \right) + \text{Heat Identifier Term} \]

\[ = 8.99 - 5.257 \log \sigma - 0.0789 \sigma \text{ for } \sigma < 10.23 \text{ ksi} \]

\[ = 15.284 - 10.735 \log \sigma - 16.071/\sigma \text{ for } \sigma > 10.23 \text{ ksi} \]

\[ \text{HEAT A} \quad \begin{array}{c} \cdot 37 - 0.587 \log \sigma \end{array} \]

\[ \text{HEAT B} \quad \begin{array}{c} \cdot 462 + 0.725 \log \sigma \end{array} \]

\[ \text{HEAT C} \quad \begin{array}{c} \cdot 583 + 0.820 \log \sigma \end{array} \]

\[ \text{HEAT D} \quad \begin{array}{c} 11.12F \end{array} \]

\[ \text{HEAT E} \quad \begin{array}{c} 12.02F \end{array} \]

\[ \text{HEAT F} \quad \begin{array}{c} \cdot 82F \end{array} \]

\[ \text{HEAT L} \quad \begin{array}{c} 1292F \end{array} \]

\[ \text{HEAT M} \quad \begin{array}{c} 1382F \end{array} \]

\[ \text{HEAT N} \quad \begin{array}{c} 1482F \end{array} \]

Fig. 9. Individual Heat characterization for NRIM Steel obtained from Completely Computerized Analysis. Curves are derived from Equations shown above with Heat Identifiers as shown in each insert.
FIG. 10  
COMPUTERIZED ANALYSIS FOR PROPRIETARY U.K. LOW ALLOY STEEL MULTIHEAT, USING A = 0.

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Figure 11. Individual Heat characterization for U.K. ALLOY obtained from completely computerized Analysis. Curves are Derived from following Equations using Heat Identifiers shown in each insert.

\[
\log t = 0.04681 (T - 932) - 131.122 \left( \frac{1}{T+460} - \frac{1}{T+932} \right) + \text{Heat Identifier Term}
\]

- \( \log t = 8.009 - 1.538 \log c - 0.0491c \) for \( c < 36.31 \) ksi
- \( \log t = 105.892 - 52.408 \log c - \frac{737.38}{c} \) for \( c > 36.31 \) ksi
Fig. 12. Analysis of NRIM 304 S.S. Multiheat In Which the 95% Confidence Lower Bound is Based on the Standard Deviation of All Data.
Subtract 1.65 x .377
\[ = 0.622 \]

1112°F

Subtract 1.65 x .432
\[ = 0.713 \]

1202°F

See Fig. for Symbol Designations according to Heat

Subtract 1.65 x .192
\[ = 0.317 \]

1292°F

--- Median Isothermals
--- Subtract 1.65 x St. Dev. of Individual Isothermal, As Shown

Subtract 1.65 x .126
\[ = 0.208 \]

1382°F

Fig. 13. Summary of NRIM 304 SS Analysis In which the 95% Confidence Lower Bound is Based on the Individual Standard Deviation of Each Separate Isothermal.
Fig. 14. Summary of UK Proprietary Low Allow Steel Analysis, In Which the 95% Confidence Lower Bound is Based on the Standard Deviation of All Data.
Subtract 1.65 x 0.368 = 0.607

Subtract 1.65 x 0.177 = 0.292 logt cycles

Subtract 1.65 x 1.41 = 0.233

Subtract 1.65 x 0.142 = 0.234

--- Median Isothermals

Subtract 1.65 x St. Dev. of Individual Isothermal, As Shown

Fig. 15. Summary of UK Proprietary Low Alloy Steel Analysis in which the 95% Confidence Lower Bound is Based on the Individual Standard Deviation of Each Separate Isothermal.
Points Used in Analysis to characterize Heat C

Data Points Not Used in Analysis. Check points to determine Validity of Approach

Isothermals Predicted from Equations Resulting from Analysis

\[
\log t = 0.0106 (T-1292) - 70.791 \left( \frac{1}{T+460} - \frac{1}{1752} \right) - 0.603 + 0.835 \log \sigma
\]

\[
= 9.058 - 5.415 \log \sigma - 0.0702 \sigma \text{ for } \sigma < 10.23 \text{ ksi}
\]

\[
= 15.717 - 11.021 \log \sigma - 17.555/\sigma \text{ for } \sigma > 10.23 \text{ ksi}
\]

Figure 16. Comparison of Experimental Data with Predictions of Isothermals for Heat C Using only 4 Data Points from this Heat in Conjunction with Data from other Heats.