Calibration Analysis for a Multi-Channel Infrared Scanning Radiometer

Harvey Walden, Edward J. Hurley, and C. Laurence Korb

DECEMBER 1977

National Aeronautics and Space Administration
Goddard Space Flight Center
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ABSTRACT

A procedure for calibrating an infrared scanning spectroradiometer by a computerized parametric error analysis technique is developed and described. The uncertainties in the radiometric measurements of scene radiance and (for the case of a blackbody scene) temperature due to possible uncertainties in the calibration target temperature, calibration target emissivity, and instrument temperature are calculated for a range of uncertainty levels in the parameters, as well as for a gamut of scene temperatures corresponding to a given spectral channel. This technique is applicable to the radiometric calibration of any infrared radiometer; in this paper, it has been applied specifically to the Cloud-Top Scanning (C.T.S.) Radiometer, a three-channel instrument designed for aircraft-borne cloud radiance measurements in the 6.75 and 11.5 μm thermal emission spectral regions.
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INTRODUCTION

The characteristics of cloud formations, viz., water content, temperature and altitude, as well as the temperature of the land and water surface beneath the clouds, are used to predict the development of major weather patterns, in particular the occurrence and movement of severe storm centers. The determination of these characteristics by the measurement of cloud and surface thermal emission is the objective of many spacecraft- and aircraft-borne scanning spectroradiometers.

Fundamentally, a radiometer measures radiant flux within its instantaneous field of view and spectral interval. If a uniform scene fills the radiometer instantaneous field of view and radiates as a blackbody within the spectral interval, then the scene temperature can be determined from the radiance measurement, assuming the intervening path is transparent to the radiation of interest. Radiometers are generally calibrated in terms of radiance by using one or more known blackbody (or, more accurately, graybody) calibration targets. The radiometer can then measure unknown targets in terms of equivalent blackbody radiance.

The purpose of this paper is to provide and describe a procedure for calibrating a spectroradiometer by performing a computerized parametric error analysis. Errors in measured target radiance due to uncertainties in particular
instrument parameters are calculated using a range of scene temperatures for given infrared radiation spectral intervals. This procedure should be applicable to the radiometric calibration of any infrared radiometer, although initially it has been applied to the Cloud-Top Scanning (C.T.S.) Radiometer, a three-channel radiometer designed for cloud radiance measurements. The C.T.S. instrument is designed for operation on a high altitude aircraft, and it measures reflected solar radiation at 0.65 μm and thermal emission in the 6.75 and 11.5 μm spectral regions.

In the 11.5 μm atmospheric window region, clouds, water and land surface features resemble blackbodies reasonably well. Also, in this wavelength interval, reflected solar radiation is generally negligible as compared to terrestrial emission. Thus, even during daylight hours, the measured radiance temperature is close to the true target temperature. If target emissivity and atmospheric transmission are taken into account, the actual target temperature can be determined. Thus, a spectroradiometer is often utilized for its temperature discrimination capabilities.

This analysis calculates the uncertainties in radiometric measurements of scene radiance and (for the case of a blackbody scene) temperature due to uncertainties in calibration target temperature, calibration target emissivity, and instrument temperature. Errors due to such sources as signal to noise ratio and registration error are not included in this study.
ANALYSIS

The spectral projected radiance $N_\lambda$ (measured in ergs/cm$^3$-sec-sterad, or, more commonly, in watts/cm$^2$-$\mu$m-sterad) of a Lambertian graybody with radiant emissivity $\varepsilon$ at absolute temperature $T$ (measured in K) is given by

$$N_\lambda(T, \varepsilon) = \varepsilon N_\lambda(T), \quad (1)$$

where

$$N_\lambda(T) \, d\lambda = \frac{2hc^2 \lambda^3}{\lambda^5 (e^{hc/\lambda kT} - 1)} \quad (2)$$

is the Planck radiation function for the wavelength interval $\lambda$ to $\lambda + d\lambda$ (measured in cm), with the constants,

$$c = 2.997925 \times 10^{10} \text{ cm/sec (speed of light)},$$

$$h = 6.626196 \times 10^{-27} \text{ erg-sec (Planck's constant)},$$

$$k = 1.380622 \times 10^{-16} \text{ erg/K (Boltzmann's constant)}.$$  

Thus, the projected radiance $N$ (measured in ergs/cm$^2$-sec-sterad, or watts/cm$^2$-sterad) in the wavelength interval $\lambda_L$ to $\lambda_U$ is

$$N(T, \varepsilon) = \int_{\lambda_L}^{\lambda_U} g(\lambda)N_\lambda(T, \varepsilon) \, d\lambda = \varepsilon \int_{\lambda_L}^{\lambda_U} g(\lambda)N_\lambda(T) \, d\lambda, \quad (3)$$

where $g(\lambda)$ is the instrument response function. It is assumed that the emissivity $\varepsilon$ is independent of wavelength and temperature for the spectral interval $\lambda_L$ to $\lambda_U$. In a typical radiometer, the instrument response function is close to rectangular, and, for the sake of generality, in this work, $g(\lambda)$ is assumed to be unity in the spectral interval $\lambda_L$ to $\lambda_U$ and zero elsewhere.

The radiance measured by a radiometer when a calibration target fills its field of view will consist of target emission and instrument case emission
reflected by the target into the signal beam. Therefore, the calibration radiance $N_{cal}$ is given as the sum of two terms:

$$N_{cal}(T_{cal}, T_{instr}, \varepsilon) = N(T_{cal}, \varepsilon) + (1 - \varepsilon) N(T_{instr}, \varepsilon_{instr})$$

$$= \varepsilon \int_{\lambda_L}^{\lambda_U} N_{\lambda}(T_{cal}) \, d\lambda$$

$$+ (1 - \varepsilon) \int_{\lambda_L}^{\lambda_U} N_{\lambda}(T_{instr}) \, d\lambda. \quad (4)$$

In equation (4), $\varepsilon$ is the calibration target emissivity (and, of course, $1 - \varepsilon$ is the reflectivity of the calibration target), and $T_{cal}, T_{instr}$ are the absolute temperatures of the calibration target and instrument case, respectively. It is assumed in equation (4) and for the purposes of this calculation that the emissivity $\varepsilon_{instr}$ of the instrument case is unity. In the above form, it is evident that calibration of the radiometer through a parametric error analysis must consider the uncertainties in calibration target radiance $N_{cal}$ due to uncertainties in target temperature $T_{cal}$, target emissivity $\varepsilon$, and instrument temperature $T_{instr}$. The radiation spectral interval $(\lambda_L, \lambda_U)$ must, of course, also be specified.

Although the Planck function (2) cannot be analytically integrated by closed methods, numerical integration techniques may be applied to solve equation (4) iteratively. For instance, if the composite Simpson's rule [Ref. 1, pp. 78-79] is used, then functional values are required at $2n + 1$ spectral points, $\lambda_i = \lambda_L + iH$, $i = 0, 1, \ldots, 2n$, where $H$ is the subinterval length, given as $H = (\lambda_U - \lambda_L)/2n$. Then $n$ repeated applications of Simpson's rule lead to the approximation:
\[
\int_{\lambda_L}^{\lambda_U} N_{\lambda}(T) \, d\lambda \approx \frac{H}{3} \left[ N_{\lambda_0} + 4N_{\lambda_1} + 2N_{\lambda_2} + 4N_{\lambda_3} + 2N_{\lambda_4} + \ldots + 2N_{\lambda_{2n-2}} + 4N_{\lambda_{2n-1}} + N_{\lambda_{2n}} \right],
\] 

where

\[
N_{\lambda_i} = N_{\lambda_i}(T) = \frac{2hc^2}{\lambda_i^5 (e^{hc/k\lambda_i T} - 1)}, \quad i = 0, 1, \ldots, 2n.
\]

It is assumed in this study that the radiometer contains two graybody calibration targets, at known distinct temperatures, \( T_C \) and \( T_H \), and with equal emissivities \( \varepsilon < 1 \). In order to reduce systematic calibration errors, the known "cold" \( (T_C) \) and "hot" \( (T_H) \) calibration target temperatures should be separated as much as possible within the range of anticipated scene temperatures. It is further assumed that the radiometer detector output voltage is directly proportional to the collected radiance. Thus, it is possible to obtain a linear radiometric calibration curve using the two calibration targets at known temperatures.

To establish a nominal calibration curve, the dynamic ranges of the voltage scale and the scene radiances must be known or estimated. It is assumed here that the voltage output is \( 2V_{\text{max}} \) full range for a given radiation spectral interval \((\lambda_L, \lambda_U)\), and that this range has a center point of zero. Thus, the voltage output \( V \) satisfies \(-V_{\text{max}} \leq V \leq V_{\text{max}}\). Furthermore, the radiometric range is specified by upper and lower limits on the anticipated scene temperatures: \( T_{\text{max}} \) and \( T_{\text{min}} \), respectively. To convert temperature dynamic range to limits on the anticipated projected radiance, equation (3) is invoked, with \( \varepsilon \) assumed equal to unity:
Given the two fixed points, \((-V_{\text{max}}, N_{\text{min}})\) and \((V_{\text{max}}, N_{\text{max}})\), the voltage scale can be calibrated linearly with respect to projected radiance, so that

\[
\frac{V + V_{\text{max}}}{N - N_{\text{min}}} = \frac{2V_{\text{max}}}{N_{\text{max}} - N_{\text{min}}},
\]

or

\[
V = \left(\frac{2V_{\text{max}}}{N_{\text{max}} - N_{\text{min}}}\right)(N - N_{\text{min}}) - V_{\text{max}},
\]

where \(V\) is a voltage in the range \(|V| \leq V_{\text{max}}\) corresponding to an arbitrary radiance measurement \(N\) such that \(N_{\text{min}} \leq N \leq N_{\text{max}}\). If the radiance measurement falls outside this range, then the linear calibration relation (8) still applies, but the corresponding voltage may be off-scale, i.e., \(|V| > V_{\text{max}}\).

For the two calibration targets with temperatures \(T_C\) and \(T_H\) and emissivity \(\epsilon\), corresponding nominal projected "cold" and "hot" radiances are, respectively,

\[
N_C = \epsilon \int_{\lambda_L}^{\lambda_U} N_\lambda(T_C) \, d\lambda + (1 - \epsilon) \int_{\lambda_L}^{\lambda_U} N_\lambda(T_{\text{instr}}) \, d\lambda,
\]

\[
N_H = \int_{\lambda_L}^{\lambda_U} N_\lambda(T_H) \, d\lambda.
\]
by equation (4). The corresponding "cold" and "hot" voltages are found by substitution into equation (8), as follows:

\[ V_C = \left( \frac{2 V_{\text{max}}}{N_{\text{max}} - N_{\text{min}}} \right) (N_C - N_{\text{min}}) - V_{\text{max}}, \]  
\[ V_H = \left( \frac{2 V_{\text{max}}}{N_{\text{max}} - N_{\text{min}}} \right) (N_H - N_{\text{min}}) - V_{\text{max}}. \]  

Subtraction of equation (10a) from equation (10b) yields

\[ \frac{V_H - V_C}{N_H - N_C} = \frac{2 V_{\text{max}}}{N_{\text{max}} - N_{\text{min}}}, \]

so that, using the two fixed points, \((N_C, V_C)\) and \((N_H, V_H)\), the nominal linear calibration relation may be written

\[ V = \left( \frac{V_H - V_C}{N_H - N_C} \right) (N - N_C) + V_C. \]  

This is equivalent to equation (8).
Each of the three parameters consisting of the graybody calibration target temperatures, the calibration target emissivities, and the instrument temperature was varied separately in order to isolate the effects of uncertainties in each parameter on the calibration accuracy. The uncertainties in the "cold" and "hot" calibration target temperatures and emissivities were assumed to be equal, in this study. If slight changes are introduced in the values of any one of the three parameters, then the nominal calibration curve is shifted somewhat. Assume that $N_C^-$ and $N_C^+$ are projected "cold" radiances, ordered such that $N_C^- < N_C < N_C^+$, corresponding to equal in magnitude but opposite in sign changes about the nominal value in any one of the three parameters, $T_C$, $\varepsilon$, or $T_{\text{instr}}$, where the other two parameters are maintained at the nominal values. Analogously, let $N_H^-$ and $N_H^+$ represent projected "hot" radiances, ordered such that $N_H^- < N_H < N_H^+$, corresponding to like changes about the nominal value in any one of $T_H$, $\varepsilon$; or $T_{\text{instr}}$, with the remaining two parameters at nominal values. For example, for parametric changes $\Delta T$ in the graybody calibration target temperatures, $N_C^- = N_{\text{cal}}(T_C - \Delta T, T_{\text{instr}}, \varepsilon)$, $N_C^+ = N_{\text{cal}}(T_C + \Delta T, T_{\text{instr}}, \varepsilon)$, $N_H^- = N_{\text{cal}}(T_H - \Delta T, T_{\text{instr}}, \varepsilon)$, and $N_H^+ = N_{\text{cal}}(T_H + \Delta T, T_{\text{instr}}, \varepsilon)$, using the functional notation of equation (4).

The projected radiances, $N_C^-$, $N_C^+$, $N_H^-$, and $N_H^+$, are used in this study to construct upper and lower calibration envelopes, as illustrated in Figure 1. The envelopes bound all possible non-nominal calibration curves which are determined by any two points, $(N_1, V_C)$ and $(N_2, V_H)$, such that $N_C^- \leq N_1 \leq N_C^+$ and $N_H^- \leq N_2 \leq N_H^+$. 
Figure 1. The nominal calibration curve and the upper and lower calibration envelopes are determined by six fixed points.
The upper and lower calibration envelopes are each formed by three joined linear segments, such that the central segments are parallel (or nearly so) to the nominal calibration curve, while the outer segments diverge. Specifically, with reference to Figure 1, the upper calibration envelope is formed of the linear segments defined by \( P_1 \) and \( P_6 \) (for \( V \leq V_C \) and \( N \leq N_C^- \)), by \( P_1 \) and \( P_4 \) (for \( V_C \leq V \leq V_H \) and \( N_C^- \leq N \leq N_H^- \)), and by \( P_3 \) and \( P_4 \) (for \( V \geq V_H \) and \( N \geq N_H^- \)). The lower calibration envelope is formed of the linear segments defined by \( P_3 \) and \( P_4 \) (for \( V \leq V_C \) and \( N \leq N_C^- \)), by \( P_3 \) and \( P_6 \) (for \( V_C \leq V \leq V_H \) and \( N_C^- \leq N \leq N_H^+ \)), and by \( P_1 \) and \( P_6 \) (for \( V \geq V_H \) and \( N \geq N_H^+ \)). The voltage and radiance values contained within the envelopes correspond to all possible values within the accuracy of the calibration, and hence extreme values on the envelopes correspond to the uncertainties in radiometric calibration resulting from the uncertainties in a given parameter, viz., \( T_{cal} \), \( \varepsilon \), or \( T_{instr} \). In this study, a distinct pair of calibration envelopes was constructed for each variation (i.e., assumed uncertainty) of the single parameters, \( T_{cal} \), \( \varepsilon \), and \( T_{instr} \).

For a given pair of calibration envelopes, the uncertainties in the observed scene radiance and in the observed scene temperature may be determined for a range of scene temperatures \( T \) such that \( T_{min} \leq T \leq T_{max} \). For each scene temperature \( T \), a corresponding scene projected radiance \( N \) is calculated by equation (3), assuming the scene consists of a blackbody target (\( \varepsilon = 1 \)). The radiance \( N \) is converted to a corresponding voltage \( V \) through the nominal calibration curve (11). Furthermore, the upper and lower calibration envelopes
determine a corresponding "high voltage" $V^+$ and "low voltage" $V^-$, respectively, given by

$$
V^+ = \begin{cases} 
\left( \frac{V_H - V_C}{N_H^+ - N_C^-} \right) (N - N_C^-) + V_C, & \text{if } N \leq N_C^- , \\
\left( \frac{V_H - V_C}{N_H^- - N_C^-} \right) (N - N_C^-) + V_C, & \text{if } N_C^- \leq N \leq N_H^- , \\
\left( \frac{V_H - V_C}{N_H^- - N_C^+} \right) (N - N_C^+) + V_C, & \text{if } N \geq N_H^- ,
\end{cases}
$$

and

$$
V^- = \begin{cases} 
\left( \frac{V_H - V_C}{N_H^- - N_C^+} \right) (N - N_C^+) + V_C, & \text{if } N \leq N_C^+ , \\
\left( \frac{V_H - V_C}{N_H^+ - N_C^+} \right) (N - N_C^+) + V_C, & \text{if } N_C^+ \leq N \leq N_H^+ , \\
\left( \frac{V_H - V_C}{N_H^+ - N_C^-} \right) (N - N_C^-) + V_C, & \text{if } N \geq N_H^+ .
\end{cases}
$$

The above high and low voltages are then converted back to corresponding high and low projected radiances, $N^+$ and $N^-$, respectively, through use of the nominal calibration curve (11), as follows:

$$
N^+ = \left( \frac{N_H - N_C}{V_H - V_C} \right) (V^+ - V_C) + N_C, \quad (13a)
$$

$$
N^- = \left( \frac{N_H - N_C}{V_H - V_C} \right) (V^- - V_C) + N_C. \quad (13b)
$$
The high and low projected radiances are the uncertainties in the scene projected radiance $N$ due to uncertainties in any one of the three parameters, $T_{cal}$, $\varepsilon$, or $T_{instr}$, represented by the calibration envelopes about the nominal calibration curve (refer to Figure 1). Relative changes in the nominal projected radiance $N$ are given by

$$\frac{\Delta N^+}{N} = \frac{N^+ - N}{N},$$  \hspace{1cm} (14a)\\

$$\frac{\Delta N^-}{N} = \frac{N^- - N}{N}. \hspace{1cm} (14b)$$

The conversion of $N^+$ and $N^-$ to corresponding blackbody temperatures involves inverting the integrals,

$$N^+ = \int_{\lambda_L}^{\lambda_U} N_\lambda(T^+) d\lambda, \hspace{1cm} (15a)$$

$$N^- = \int_{\lambda_L}^{\lambda_U} N_\lambda(T^-) d\lambda. \hspace{1cm} (15b)$$

and solving for the high and low temperatures, $T^+$ and $T^-$, respectively. Since the integral equations (15) are intractable with respect to analytical inversion by closed methods, an iterative computational procedure has been applied. One such method, which has the advantage of not requiring the evaluation of derivatives, is the method of false position [Ref. 1, p. 178; Ref. 2, pp. 13ff]. If $T^+_i$ is the $i$-th approximation to the desired temperature $T^+$, then the following recurrence relation provides an improved approximation:
\[ T^+_{i+1} = \frac{T(N^+_1 - N^+_i) - T^+_i(N - N^+_i)}{N^+_1 - N}, \quad i = 1, 2, 3, \ldots, \quad (16) \]

where \( T \) is the known scene temperature, \( N \) is the corresponding scene projected radiance calculated by equation (3), and \( N^+_1 = N(T^+_1, \varepsilon = 1) \), using the functional notation of equation (3). Similarly, if \( T^-_i \) is the \( i \)-th approximation to the desired temperature \( T^- \), then the method of false position provides the following recurrence relation for the improved approximation:

\[ T^-_{i+1} = \frac{T(N^-_1 - N^-_i) - T^-_i(N - N^-_i)}{N^-_1 - N}, \quad i = 1, 2, 3, \ldots, \quad (17) \]

where \( N^-_1 = N(T^-_1, \varepsilon = 1) \), in the notation of equation (3). Convergence for these iterative procedures occurs when, for a sufficient number of iterations,

\[ |T^+_{i+1} - T^+_i| < \varepsilon^+ \quad (18a) \]

and

\[ |T^-_{i+1} - T^-_i| < \varepsilon^-, \quad (18b) \]

where \( 0 < \varepsilon^+, \varepsilon^- < 1 \) are suitable pre-selected convergence criteria. The absolute changes in the nominal scene temperature \( T \), corresponding to the observed scene radiance \( N \), are, of course, \( \Delta T^+ = T^+ - T \) and \( \Delta T^- = T^- - T \).

NUMERICAL RESULTS

The methods previously described for calibration of a radiometer through a parametric error analysis have been applied to the Cloud-Top Scanning (C.T.S.) Radiometer. Significant parameters for the infrared channels of the C.T.S. radiometer [Ref. 3], as pertain to the current study, are displayed in Table 1.
Calculated values for parameters derived from those in Table 1, through equations (7), (9), and (10), are included in Table 2, for each of the two infrared spectral channels. Using the parameter values in Table 2, graphs of the nominal linear calibration curves (11) of voltage output vs. projected radiance were constructed for the two infrared channels and are shown in Figure 2. A voltage of -5 v corresponds to the minimum scene projected radiance $N_{\text{min}}$ at scene temperature $T_{\text{min}}$, and, similarly, +5 v corresponds to the maximum scene projected radiance $N_{\text{max}}$ at temperature $T_{\text{max}}$. The points labeled "240 K" and "280 K" indicate the cold and hot graybody target projected radiances, $N_C$ and $N_H$, at corresponding voltage outputs, $V_C$ and $V_H$, respectively.

Calibration envelopes were constructed, as described previously, for the following variations, or assumed uncertainties, in the nominal values of the graybody calibration target temperatures and emissivities and of the instrument temperature:

$$\Delta T = \Delta T_C = \Delta T_H = \pm 0.1, \pm 0.2, \pm 0.5, \pm 1.0 \text{ K}$$

$$\Delta \epsilon = \pm 0.001, \pm 0.005, \pm 0.01, \pm 0.02$$

$$\Delta T_{\text{instr}} = \pm 1, \pm 2, \pm 5, \pm 10 \text{ K}$$

Corresponding uncertainties in the observed scene radiances and temperatures were calculated for each pair of calibration envelopes for scene temperatures $T'$.
Table 1
Nominal Values of Instrument Parameters

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<thead>
<tr>
<th>Parameter Description and Symbol</th>
<th>Numerical Value and Units</th>
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<tr>
<td>Infrared spectral interval, $\lambda_L$ to $\lambda_U$</td>
<td>Channel 2</td>
</tr>
<tr>
<td>Graybody calibration target temperatures, $T_C$, $T_H$</td>
<td>Channel 3</td>
</tr>
<tr>
<td>Instrument case temperature, $T_{\text{instr}}$</td>
<td>6.6 to 6.9 x 10^{-4} cm</td>
</tr>
<tr>
<td>Graybody calibration target radiant emissivity, $\epsilon$</td>
<td>10.5 to 12.5 x 10^{-4} cm</td>
</tr>
<tr>
<td>Full scale instrument voltage output, $V_{\text{max}}$</td>
<td>240 K, 280 K</td>
</tr>
<tr>
<td>Scene temperature range, $T_{\text{min}}$ to $T_{\text{max}}$</td>
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<tr>
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<td>0.98</td>
</tr>
<tr>
<td></td>
<td>5.0 volts</td>
</tr>
<tr>
<td></td>
<td>Channel 2</td>
</tr>
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<td>Channel 3</td>
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<td>165 to 325 K</td>
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Table 2
Calculated Values of Derived Parameters

<table>
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<th>Numerical Value and Units*</th>
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<td>Scene projected radiance range</td>
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*All projected radiances are in units of ergs/cm² - sec - sterad and all voltages are in volts.
Figure 2. The nominal linear calibration curves of instrument voltage output vs. projected radiance for the two infrared spectral channels.
in the range \( T_{\text{min}} \leq T \leq T_{\text{max}} \) at intervals of 20 K for each of the two infrared spectral channels. The results for all 12 pairs of calibration envelopes (four variations for each of three parameters, as listed above) for each of the two spectral channels are included in Tables A1 through A24 in Appendix A. Each of these tables includes the high and low scene projected radiances, \( N^+ \) and \( N^- \), and the high and low scene temperatures, \( T^+ \) and \( T^- \), as well as the respective relative and absolute changes from nominal values in each scene radiance \( N \) and temperature \( T \). Appendix B contains numerical details on the iterative procedures used in integrating the Planck function and inverting the scene radiance integrals (15) to solve for the scene temperatures.

The results included in the tables in Appendix A are summarized in Figures 3 through 8, in which the absolute changes \( \Delta T^+ \), \( \Delta T^- \) in scene temperature are shown as a function of scene temperature \( T \) for a given set of variations in the nominal values of the instrument parameters. These scene temperature uncertainty curves will be discussed in the following section.

In order to assess the effects of a change in the nominal values of the instrument parameters, as given in Table 1, a supplementary set of results was generated in which the nominal graybody calibration target radiant emissivity \( \varepsilon \) was assumed to be either 0.96 or 0.99, as contrasted with the nominal value of 0.98 assumed previously. Calibration envelopes were constructed for the set of variations in the nominal instrument temperature parameter \( \Delta T_{\text{inst}} \) only, as given in the final line of (19). The results are displayed in Figures 9 through 12 as
GRAYBODY TEMPERATURE VARIATIONS

\[ \pm 0.1 \text{K} \]
\[ \pm 0.2 \text{K} \]
\[ \pm 0.6 \text{K} \]
\[ \pm 1.0 \text{K} \]

SPECTRAL CHANNEL 2  \(6.6 - 6.9 \mu \text{m}\)

NOMINAL INSTRUMENT PARAMETERS

\[ T_C = 240 \text{K} \]
\[ T_{\text{instr}} = 255 \text{K} \]
\[ T_H = 280 \text{K} \]
\[ e = 0.98 \]

Figure 3. Scene temperature uncertainty curves due to graybody calibration target temperature variations about nominal (spectral channel 2)
Figure 4. Scene temperature uncertainty curves due to graybody calibration target emissivity variations about nominal (spectral channel 2)
Figure 5. Scene temperature uncertainty curves due to instrument temperature variations about nominal (spectral channel 2)
GRAYBODY TEMPERATURE VARIATIONS

- ±0.1 K
- ±0.2 K
- ±0.5 K
- ±1.0 K

SPECTRAL CHANNEL 3  10.5-12.5 μm

NOMINAL INSTRUMENT PARAMETERS

- $T_C = 240$ K
- $T_{\text{instr}} = 235$ K
- $T_H = 280$ K
- $\epsilon = 0.98$

Figure 6. Scene temperature uncertainty curves due to graybody calibration target temperature variations about nominal (spectral channel 3)
GRAYBODY EMISSIVITY VARIATIONS

0 ± 0.001

± 0 ± 0.005

± 0 ± 0.01

V ± 0.02

SPECTRAL CHANNEL 3 10.6–12.5 μm

NOMINAL INSTRUMENT PARAMETERS

Tc = 240K  Tinsr = 255K

TH = 280K  α = 0.98

Figure 7. Scene temperature uncertainty curves due to graybody calibration target emissivity variations about nominal (spectral channel 3)
Figure 8. Scene temperature uncertainty curves due to instrument temperature variations about nominal (spectral channel 3)
Figure 9. Scene temperature uncertainty curves due to instrument temperature variations about nominal (spectral channel 2) for reduced nominal graybody emissivity
Figure 10. Scene temperature uncertainty curves due to instrument temperature variations about nominal (spectral channel 3) for reduced nominal graybody emissivity.
Figure 11. Scene temperature uncertainty curves due to instrument temperature variations about nominal (spectral channel 2) for increased nominal graybody emissivity
Figure 12. Scene temperature uncertainty curves due to instrument temperature variations about nominal (spectral channel 3) for increased nominal graybody emissivity.
scene temperature uncertainty curves for the two spectral channels for each of
the "reduced" and "increased" nominal graybody emissivities. These curves
will also be discussed in the following section.

DISCUSSION

The results of this parametric error analysis are clearly shown in the scene
temperature uncertainty curves (Figures 3 through 8). These curves were de­
termined by using a full range of assumed uncertainty levels (19) in the three
parameters, $T_{cal}$, $\epsilon$, and $T_{instr}$, from extremely small values to values larger
than actually anticipated based upon the specifications and data relating to the
C.T.S. radiometer [Refs. 3, 4]. It is obvious from these uncertainty curves
that the best calibration accuracies are obtained for scene temperatures in the
range between the two graybody calibration target temperatures, viz., 240 K and
280 K. Outside this range, the calibration uncertainties become progressively
more severe, particularly for low scene temperatures.* At a scene temperature
of 165 K, even the lower sets of uncertainty levels in the radiometer parameters
result in calibration temperature errors of several K. For the chosen range of
uncertainty levels (19) in the three parameters, the largest uncertainties in scene
temperature are produced by variations in the graybody calibration target tem­
peratures, $T_C$ and $T_H$, and the smallest uncertainties are produced by variations
in the instrument temperature, $T_{instr}$, with variations in graybody emissivities,
$\epsilon$, producing scene temperature uncertainties of intermediate magnitudes.

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*The omission of certain data points at low scene temperatures from the uncer­
tainty curves of spectral channel 2 results from a type of numerical divergence
experienced, which is more fully explained in Appendix A.
For a given scene temperature and a given set of parameter uncertainty levels, the curves in Figures 3 through 8 (or, for greater precision, the tables in Appendix A) can be used to determine the total root-sum-square (R.S.S.) temperature calibration error of the radiometer infrared spectral channels. (Errors due to sources other than uncertainties in the parameters mentioned above, e.g., registration and signal to noise ratio, are not included in this analysis.) As an example, for a 185 K blackbody scene, the calibration uncertainties in spectral channel 2 are as follows (using mean values of the magnitudes of the positive and negative uncertainties): ±3.92 K due to an uncertainty of ±0.2 K in the calibration target temperatures, ±1.86 K due to an uncertainty of ±0.005 in the calibration target emissivities, and ±0.88 K due to an uncertainty of ±2 K in the instrument temperature. The R.S.S. total calibration error is then ±4.43 K for a 185 K blackbody scene in spectral channel 2. An analogous calculation, using the same parameter uncertainty levels, for spectral channel 3 yields an R.S.S. total calibration error of ±1.79 K for a 185 K blackbody scene.

It is possible that the nominal value of the graybody calibration target emissivity could be significantly higher (in accord with a manufacturer's claim, for instance) or lower (due to degradation effects) than the value of 0.98 included in Table 1 and used in generating the curves in Figures 3 through 8. The nominal value of the calibration target emissivity has a significant relative effect on the calibration radiance contribution from the instrument, i.e., the reflection term, as given in equation (4). Therefore, the effects of different nominal values of
the emissivity were studied by recalculating the uncertainty curves due to variations in the instrument temperature parameter only. The "reduced" and "increased" nominal graybody emissivity values of 0.96 and 0.99, respectively, represent reasonable extremes of calibration target emissivities for an aircraft-borne instrument. The results for $e = 0.96$, given in Figures 9 and 10 for the two infrared spectral channels, show temperature errors of approximately double the magnitude, uniformly for all scene temperatures, of the curves in Figures 5 and 8, respectively, for $e = 0.98$. The results for $e = 0.99$, given in Figures 11 and 12 for the two spectral channels, show temperature errors of approximately half the magnitude, uniformly for all scene temperatures, of the curves in Figures 5 and 8, respectively. Thus, the relative magnitudes of the calibration temperature errors due to uncertainties in the instrument temperature are approximately proportional to $(1 - e)$, the factor in the reflection term of equation (4).

The numerical results provided in this paper are based upon instrument parameters characteristic of the particular case of the C.T.S. radiometer; however, the methods described herein are quite general and the computer programs developed (available upon request to the authors) can be used to model the calibration accuracy of any infrared radiometer.
REFERENCES


APPENDIX A: TABULATION OF RADIANCE AND TEMPERATURE VARIATIONS

The radiance and temperature variations included in Tables A1 through A24 comprise the full set of results for 12 pairs of calibration envelopes for each of the two infrared spectral channels. These calibration envelopes correspond to assumed uncertainties in the nominal values of the graybody calibration target temperatures and emissivities and of the instrument case temperature, as listed in the relations (19). For each scene temperature $T$, the corresponding scene projected radiance $N$, calculated by equation (3) and assuming a blackbody target ($\varepsilon = 1$), is given. The center two columns provide the corresponding low and high projected radiances, $N^-$ and $N^+$, given by equations (13), as well as their relative changes from nominal, given by equations (14). The final two columns include the low and high temperatures, $T^-$ and $T^+$, obtained in equations (17) and (16), respectively, as well as the absolute changes from nominal.

In certain tables, data are replaced by dashes, indicating that a type of divergence has occurred for the given parameter values. Notice that the physical limitation that the low projected radiance $N^- > 0$ is not present in the defining equation (13b). When the lower calibration envelope diverges sufficiently from the nominal calibration curve, a large relative change in the scene projected radiance (most likely, at a low scene temperature) can mathematically produce $N^- \leq 0$ by equation (13b). Such a condition, indicated by the dashes in the tables, occurs only when the corresponding relative change in the high projected radiance
exceeds unity (or nearly so, since the symmetry between upper and lower calibration envelopes is only approximate). This condition occurs only for the lower scene temperatures in spectral channel 2, since larger scene radiances are present in spectral channel 3 even at the lowest scene temperatures.

NOTE: In Tables A1 through A24, all projected radiance parameters are given in units of ergs/cm²·sec·sterad, and all temperatures (including absolute changes in temperature) are in units of K. The relative changes in projected radiances are dimensionless.
<table>
<thead>
<tr>
<th>Scene Temperature, $T$</th>
<th>Scene Projected Radiance, $N$</th>
<th>Low Projected Radiance, $N^-$</th>
<th>High Projected Radiance, $N^+$</th>
<th>Low Temperature, $T^-$</th>
<th>High Temperature, $T^+$</th>
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</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Relative Change, $\Delta N^-/N$</td>
<td>Relative Change, $\Delta N^+/N$</td>
<td>Absolute Change, $\Delta T^-$</td>
<td>Absolute Change, $\Delta T^+$</td>
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Table A2
Radiance and Temperature Variations in Spectral Channel 2
due to $\Delta T = \pm 0.2$ K Uncertainties

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<th>Scene Temperature, $T$</th>
<th>Scene Projected Radiance, $N$</th>
<th>Low Projected Radiance, $N^-$</th>
<th>High Projected Radiance, $N^+$</th>
<th>Low Temperature, $T^-$</th>
<th>High Temperature, $T^+$</th>
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</thead>
<tbody>
<tr>
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<td>Relative Change, $\Delta N^-/N$</td>
<td>Relative Change, $\Delta N^+/N$</td>
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<td>Absolute Change, $\Delta T^+$</td>
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Table A3
Radiance and Temperature Variations in Spectral Channel 2
due to $\Delta T = \pm 0.5$ K Uncertainties

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<th>Scene Temperature, $T$</th>
<th>Scene Projected Radiance, N</th>
<th>Low Projected Radiance, $N^-$</th>
<th>High Projected Radiance, $N^+$</th>
<th>Low Temperature, $T^-$</th>
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Table A4
Radiance and Temperature Variations in Spectral Channel 2
due to $\Delta T = \pm 1.0$ K Uncertainties

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<th>Low Projected Radiance, $N^-$</th>
<th>High Projected Radiance, $N^+$</th>
<th>Low Temperature, $T^-$</th>
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Radiance and Temperature Variations in Spectral Channel 2
due to $\Delta \varepsilon = \pm 0.001$ Uncertainties

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due to $\Delta\varepsilon = \pm 0.01$ Uncertainties

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due to $\Delta T_{\text{instr}} = \pm 5$ K Uncertainties

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Radiance and Temperature Variations in Spectral Channel 2
due to $\Delta T_{\text{instr}} = \pm 10$ K Uncertainties

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Note: $N^-$ and $N^+$ represent low and high projected radiance, respectively.
Table A13
Radiance and Temperature Variations in Spectral Channel 3
due to $\Delta T = \pm 0.1$ K Uncertainties

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Table A14
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due to $\Delta T = \pm 0.2$ K Uncertainties

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due to $\Delta T = \pm 0.5$ K Uncertainties

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Table A16
Radiance and Temperature Variations in Spectral Channel 3
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### Table A17
Radiance and Temperature Variations in Spectral Channel 3
due to $\Delta\varepsilon = \pm 0.001$ Uncertainties

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Radiance and Temperature Variations in Spectral Channel 3
due to $\Delta \varepsilon = \pm 0.005$ Uncertainties

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Radiance and Temperature Variations in Spectral Channel 3
due to $\Delta \epsilon = \pm 0.01$ Uncertainties

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Radiance and Temperature Variations in Spectral Channel 3 
due to $\Delta$e = ±0.02 Uncertainties

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Table A21

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due to $\Delta T_{\text{instr}} = \pm 1$ K Uncertainties

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Table A23
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due to $\Delta T_{\text{instr}} = \pm 5$ K Uncertainties

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APPENDIX B: NUMERICAL DATA ON CONVERGENCE OF ITERATIVE PROCEDURES

In the numerical integration of Planck's function, as indicated in equations (5) and (6), the value adopted throughout this study for 2n, equal to the number of subintervals used in the application of the composite Simpson's rule, is 20. This value for n was found to produce adequate precision and leads to a sub-interval length H of $1.5 \times 10^{-6}$ cm for spectral channel 2 and $1.0 \times 10^{-5}$ cm for spectral channel 3.

In the iterative method of false position used to calculate $T^-$ and $T^+$, the nominal temperature T was decreased or increased, respectively, by 10 K (half the interval width of 20 K between successive T values) as an initial ($i = 1$) approximation in equations (17) and (16), for $T^-$ and $T^+$, respectively. These initial approximations were generally sufficient to produce convergence within 3 to 15 iterations using pre-selected convergence criteria $\epsilon^+$, $\epsilon^-$ of $10^{-5}$, as given in equations (18). The few isolated exceptions to this generalization occur at times when either of the absolute changes, $\Delta T^-$ or $\Delta T^+$, in the nominal temperature exceed 10 K. These cases, in which more than 15 iterations were required to converge to the $10^{-5}$ criteria, are as follows:

Table A3 (Channel 2, $\Delta T = \pm0.5$ K)

| $T$ = 165 K | $\Delta T^+$ = 17.41 K | 27 iterations |

Table A4 (Channel 2, $\Delta T = \pm1.0$ K)

| $T$ = 165 K | $\Delta T^+$ = 25.79 K | 81 iterations |
Table A7 (Channel 2, $\Delta \epsilon = \pm 0.01$)

$T = 165$ K  \hspace{1cm} $\Delta T^- = -35.67$ K  \hspace{1cm} 85 iterations

Table A8 (Channel 2, $\Delta \epsilon = \pm 0.02$)

$T = 165$ K  \hspace{1cm} $\Delta T^+ = 14.76$ K  \hspace{1cm} 20 iterations

Table A16 (Channel 3, $\Delta T = \pm 1.0$ K)

$T = 165$ K  \hspace{1cm} $\Delta T^- = -26.29$ K  \hspace{1cm} 19 iterations
A procedure for calibrating an infrared scanning spectroradiometer by a computerized parametric error analysis technique is developed and described. The uncertainties in the radiometric measurements of scene radiance and (for the case of a blackbody scene) temperature due to possible uncertainties in the calibration target temperature, calibration target emissivity, and instrument temperature are calculated for a range of uncertainty levels in the parameters, as well as for a gamut of scene temperatures corresponding to a given spectral channel. This technique is applicable to the radiometric calibration of any infrared radiometer; in this paper, it has been applied specifically to the Cloud-Top Scanning (C.T.S.) Radiometer, a three-channel instrument designed for aircraft-borne cloud radiance measurements in the 6.75 and 11.5 μm thermal emission spectral regions.