Closed-Form Equations for the Lift, Drag, and Pitching-Moment Coefficients of Airfoil Sections in Subsonic Flow

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CLOSED-FORM EQUATIONS FOR THE LIFT, DRAG, AND PITCHING-MOMENT COEFFICIENTS OF AIRFOIL SECTIONS IN SUBSONIC FLOW

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SUMMARY

Closed-form equations for the lift, drag, and pitching-moment coefficients of two-dimensional airfoil sections in steady subsonic flow are obtained from published theoretical and experimental results. A turbulent boundary layer is assumed to exist on the airfoil surfaces. The effects of section angle of attack, Mach number, Reynolds number, and the specific airfoil type are considered. The equations are applicable through an angle-of-attack range of -180° to +180°; however, above about +20°, the section characteristics are assumed to be functions only of angle of attack. A computer program is presented which evaluates the equations for a range of Mach numbers and angles of attack. Calculated results for the NACA 23012 airfoil section are compared with experimental data.

NOMENCLATURE

c airfoil chord, m
c_d drag coefficient
c_F friction drag coefficient
c_f mean skin friction coefficient
c_l lift coefficient
c_{l_{max}} maximum lift coefficient (first lift-curve peak)
c_m pitching-moment coefficient about quarter-chord
c_{m_0} pitching-moment coefficient at \alpha = 0

\text{c_s pressure (form) drag coefficient}
\text{x correlation parameter for airfoil drag due to lift}
L. airfoil section perimeter, m
M. Mach number

M_{DD}. drag divergence Mach number
M_{MD}. moment divergence Mach number
M_{L}. Mach number at which trend of increasing lift-curve slope with increasing M reverses; also, highest Mach number for which L is invariant with M

R_{N}. Reynolds number
R_{N}. Reynolds number based on free-stream velocity
S_{A}. mean value of airfoil pressure coefficient

t. airfoil section maximum thickness, m

\alpha. airfoil section angle of attack, deg

\alpha_{c}. cutoff \alpha, above which the slope of \frac{c_{l}}{M} with M is assumed to be constant, deg

\alpha_{D}. \alpha corresponding to highest value of M_{DD}, deg

\alpha_{P}. \alpha corresponding to highest value of M_{MD}, deg

\varphi. reference angle about which \frac{dc_{l}}{dM} vs \alpha curve is assumed to be a mirror image, deg

\psi. airfoil section stall angle, approximated as \frac{c_{l_{\text{mix}}}}{dc_{l}/d_{\alpha}} \alpha_{s}, deg

\alpha_{0}. angle of attack for zero lift, deg
\gamma. ratio of specific heats

INTRODUCTION

Piloted simulation of helicopters or other rotorcraft in real time is paced by the speed of solution of the rotor dynamic equations of motion. One method of increasing computation speed is the use of a hybrid (analog plus digital) computer, with all rotor calculations being accomplished in the analog computer. Unfortunately, this may preclude the normal practice of providing rotor blade airfoil section characteristics in tabular form, since the time required for the very large number of table lookups may negate the expected speed advantage. Therefore, it was determined to derive closed-form equations for the lift, drag, and pitching moment of airfoil sections which
would exhibit, 'insofar as possible and practical, the effects of angle of
attack, Mach number, Reynolds number, and the specific section type. Pub-
lished airfoil data were used to establish the form of equations which both
reflect general trends and accommodate specific section type characteristics
where possible. The coefficients of the equations must be obtained from
experimental data for the particular airfoil of interest, but the data
requirements are not as extensive as for construction of airfoil data tables
typically used in helicopter rotor analysis programs.

The equations presented herein are equally applicable to fixed-wing prob-
lems, and should be useful in any case where it is either impractical or
impossible to use an airfoil section data table. Conversely, if a particular
data table is desired but not available, the equations provide a means of
genrating the table (with angle of attack and Mach number as parameters) from
limited input data.

ASSUMPTIONS AND LIMITATIONS

The equations presented herein were derived from experimental airfoil
section data obtained under conditions of two-dimensional, steady-state flow.
The airfoil surface condition is assumed to be representative of in-service
conditions; that is, smooth, but with a turbulent boundary layer over essen-
tially the entire airfoil. Considering the combined effects of manufacturing
defects, service wear, environmental deposits, high Reynolds number and free-
stream turbulence, it is reasonable, if slightly conservative, to assume a
fully turbulent boundary layer for all full-scale rotors and wings for which
no special laminar flow apparatus is provided (ref. 1). The assumption of a
turbulent boundary layer establishes the form of the incompressible drag
coefficient (appendix B).

The subject equations are applicable only for flight conditions in which
the free-stream Mach number is less than one. The Reynolds number, based on
section chord, is assumed to be greater than \(1 \times 10^6\). Certain experimental data
for the particular airfoil type under consideration must be available. Two
examples of such data are: (1) moment coefficient at low Mach number and zero
angle of attack; and (2) at least two values of drag divergence Mach number as
a function of section angle of attack. All such requirements are presented in
a subsequent section.

Equations are provided in this report for section lift, drag, and pitch-
ing moment coefficient through an angle of attack range of \(\pm 180^\circ\). However,
above about \(\pm 20^\circ\), these coefficients are functions only of angle of attack,
and are independent of Mach number, Reynolds number, and the airfoil type.
This simplification is a consequence of the very small amount of experimental
data available at high angle of attack.

It is assumed that the equations presented in this report will be applied
only to airfoils whose characteristics follow the same general trends as the
experimental data used to derive the equations (see the appendixes). For
example, the drag divergence Mach number should be at least approximately linear with section angle of attack.

**AIRFOIL SECTION FORCE AND MOMENT EQUATIONS**

Equations for the lift, drag, and pitching moment of airfoil sections through 180° of attack, derived in appendices A, B, and C, respectively, are presented below. The equations are principally empirical in nature, and in the low angle of attack range require as input experimental data for the particular airfoil type. These requirements are discussed in a subsequent section. Assumptions and limitations are given in the preceding section. To facilitate reference to the appropriate section of the appendices, equation numbers from the appendices are retained here.

**Lift Coefficient**

Small angle of attack (\(\alpha \leq \alpha_{\text{stall}}\))

\[
\frac{dc_l}{d\alpha} = \frac{dc_l}{d\alpha_{\text{stall}}} (\alpha - \alpha_0) , \quad |\alpha| \leq |\alpha_{\text{stall}}|
\]

(A1)

\[
\left( \frac{dc_l}{d\alpha} \right) = \left( \frac{dc_l}{d\alpha_{\text{stall}}} \right)_{\text{inc}} \left\{ \mu + \frac{\mu}{1 + t/c} \left[ \mu (\mu - 1) + 0.6(\mu^2 - 1) \right] \right\}
\]

(A2)

where

\[
\mu = \frac{1}{\sqrt{1 - \frac{\alpha}{\alpha_{\text{stall}}}}}
\]

If \(M > M_1\), use

\[
\frac{dc_l}{d\alpha} = \left( \frac{dc_l}{d\alpha_{\text{stall}}} \right)_{\text{inc}} \left\{ \mu + \frac{\mu}{1 + t/c} \left[ \mu (\mu - 1) + 0.6(\mu^2 - 1) \right] \right\} - (0.45)(M - M_1)
\]

(A3)

in lieu of equation (A2), but set \(\frac{dc_l}{d\alpha} = 0.05\) as a lower limit.

\[
\alpha_0 = (\alpha_0)_1 , \quad 0 \leq M \leq M_1
\]

(A4)

\[
\alpha_0 = (\alpha_0)_1 - \frac{(\alpha_0)_1 - (\alpha_0)}{M_1 - M_1} (M - M_1) , \quad M > M_1
\]

(A5)

\[
c_{l_{\text{max}}} = C_1 + C_2M + C_3M^2 + C_4M^3 + C_5M^4 + (C_6 + C_7M^{C_8}) \sin(C_9 + C_{10}M)
\]

(A6)
For negative angle of attack, the form of \( c_{\text{f,max}} \) is assumed to be given by equation (A6). The sign of \( c_{\text{f,max}} \) will be negative, and for cambered sections the constant term \( C_{\text{l}} \) will have a different magnitude also.

Large angle of attack—For positive angles of attack,

\[
c_{\text{f}} = 0.813 + \frac{c_{\text{f,max}}}{22 - \alpha}, \quad \text{stall} < \alpha < 22^\circ, \quad \text{(A6)}
\]

where

\[
c_{\text{f,max}} = \frac{\text{stall}}{\frac{\partial c_{\text{f}}}{\partial x} + \alpha_0}
\]

\[
c_{\text{f}} = 1.1 - 1.78[0.01745(a - 0.7853)^2], \quad 22^\circ < a < 90^\circ \quad \text{(A7)}
\]

\[
c_{\text{f}} = -1.1 + 1.78[0.01745(a) - 2.356]^2, \quad 90^\circ < a < 160^\circ \quad \text{(A8)}
\]

\[
c_{\text{f}} = -0.763, \quad 160^\circ < a < 172.5^\circ \quad \text{(A9)}
\]

\[
c_{\text{f}} = -5.82[a - 0.01745(a)], \quad 172.5^\circ < a < 180^\circ \quad \text{(A10)}
\]

For negative angles of attack:

\[
c_{\text{f}} = -0.813 + \frac{c_{\text{f,max}}}{22 + \alpha}, \quad -22^\circ < \alpha < \text{neg stall} \quad \text{(A11)}
\]

where

\[
c_{\text{f,max}} = \frac{\left(\frac{c_{\text{f,max}}}{\alpha_0}\right)}{\frac{\partial c_{\text{f}}}{\partial x} + \alpha_0}
\]

\[
c_{\text{f}} = -1.1 + 1.78[0.01745|a| - 0.7853]^2, \quad -90^\circ < a \leq -22^\circ \quad \text{(A12)}
\]

\[
c_{\text{f}} = 1.1 - 1.78[0.01745|a| - 2.356]^2, \quad -160^\circ < a \leq -90^\circ \quad \text{(A13)}
\]

\[
c_{\text{f}} = 0.763, \quad -172.5^\circ < a \leq -160^\circ \quad \text{(A14)}
\]

\[
c_{\text{f}} = 5.82[a - 0.01745|a|], \quad -180^\circ \leq a \leq -172.5^\circ \quad \text{(A15)}
\]
Drag Coefficient

Small angle of attack ($|\alpha| \leq \alpha_{\text{stall}}$):

\[
\frac{M_{\infty} - M_{\text{DD}}}{C_\infty} = A + k_1 \alpha, \quad \alpha = \alpha_{p_{\text{DD}}}
\]  \hspace{1cm} (B6)

\[
\frac{M_{\infty} - M_{\text{DD}}}{C_\infty} = C + k_2 \alpha, \quad \alpha = \alpha_{p_{\text{DD}}}
\]  \hspace{1cm} (B7)

\[
M_{\text{DD}} \geq 0.3 \text{ for any } \alpha
\]

For Mach number less than the drag divergence Mach number,

\[
C_d = (c_d)_{M_{\infty}}, \quad M = M_{\text{DD}}, \quad \alpha = \alpha_{\text{stall}}
\]  \hspace{1cm} (B1)

\[
(c_d)_{M_{\infty}} = (c_f) \left[ \frac{S_A}{c} \left( 1 + \frac{c_s}{c_f} \right) + R(0.017445, 0.1) \right]
\]  \hspace{1cm} (B3)

\[
c_f = \left( \frac{2.555}{\log R_{n_{\text{eff}}}} \right)^{1/8}
\]  \hspace{1cm} (B4)

\[
R_{n_{\text{eff}}} = R_n \left( 1 - \frac{1}{2} \frac{1}{c} \right) S_A
\]  \hspace{1cm} (B5)

For Mach number greater than the drag divergence Mach number:

\[
C_d = (c_d)_{M_{\infty}} + \frac{dc_d}{dM} (M - M_{\text{DD}}), \quad M : M_{\text{DD}}, \quad \alpha \geq \alpha_{\text{stall}}
\]  \hspace{1cm} (B2)

\[
\frac{dc_d}{dM} = A + B \alpha + C \alpha^2 + D \alpha^3, \quad \alpha_R \leq \alpha \leq \alpha_c
\]  \hspace{1cm} (B8)

\[
\frac{dc_d}{dM} \bigg|_{\alpha = \alpha_c}
\]  \hspace{1cm} (B9)

For angles of attack less (more negative) than the reference angle $\alpha_R$, substitute

\[
\alpha = \alpha + 2 \alpha_R, \quad \alpha < \alpha_R
\]  \hspace{1cm} (B10)

in equation (B8). Also the negative angle at which $dc_d/dM$ becomes constant is

\[
\alpha_c' = -\alpha_c + 2\alpha_R
\]  \hspace{1cm} (B11)
Large angle of attack:

\[ c_d = 0.219 - (c_{d,\text{stall}}) (15 - |\alpha|), \quad |\alpha| = 15^\circ \] (B13)

where

\[ c_{\text{max}} = \frac{c_{d,\text{stall}}}{\text{dc}/\text{da} + a_0} \] (B14)

\[ c_{d,\text{stall}} = 2.18(|\sin \alpha|)^{1.7}, \quad 15^\circ \leq |\alpha| \leq 180^\circ \] (B12)

For negative angle of attack, \( c_{\text{max}} \) in equation (B14) is replaced by \( (c_{\text{max}})^{-1} \).

Pitching Moment Coefficient

Small angle of attack \((|\alpha| \leq 20^\circ)\):

\[ M_{MD} = A + B \alpha, \quad \alpha = M_{MD} \] (C2)

\[ M_{MD} = C + D \alpha, \quad \alpha = M_{MD} \] (C3)

\[ M_{MD} \geq 0.3 \quad \text{for any} \alpha \]

\[ c_m = c_m + \frac{d c_m}{d \alpha} (\alpha), \quad M = M_{MD}, \quad |\alpha| = |\alpha_{\text{stall}}| \] (C1)

\[ c_m = -0.077 + \frac{M_{\text{stall}}}{20 - M_{\text{stall}}} (20 - \alpha), \quad 20^\circ \leq |\alpha| < |\alpha_{\text{stall}}| \] (C4)

\[ c_m = (c_m)_{MD} - \frac{(c_m)_{MD} + 0.077}{0.95 - (M_{MD})} (M - M_{MD}), \quad M > M_{MD} \] (C5)

For negative angles of attack:

\[ c_m = 0.077 - \frac{(c_m)_{\alpha}}{20 + \frac{\alpha}{\alpha_{\text{stall}}}} (20 + \alpha), \quad 20^\circ \leq |\alpha| < |\alpha_{\text{stall}}| \] (C6)

\[ c_m = (c_m)_{MD} - \frac{(c_m)_{MD} - 0.077}{0.95 - (M_{MD})} (M - M_{MD}), \quad M > M_{MD} \] (C7)

where \( \alpha_{\text{stall}} \) is negative.
Large angle of attack:

\[ c_m = -0.00802 \left( \alpha - 20 \right) - 0.077 \], \hspace{1cm} 20^\circ \leq \alpha \leq 67^\circ \hspace{1cm} (C3) \\
\[ c_m = -0.619 \left[ \sin(0.0260\alpha - 1.26) \right]^{0.387} \], \hspace{1cm} 67^\circ \leq \alpha \leq 162^\circ \hspace{1cm} (C4) \\
\[ c_m = -0.00838 (\alpha - 162) - 0.320 \], \hspace{1cm} 162^\circ \leq \alpha \leq 170^\circ \hspace{1cm} (C5) \\
\[ c_m = 0.0387 (\alpha - 170) - 0.387 \], \hspace{1cm} 170^\circ \leq \alpha \leq 180^\circ \hspace{1cm} (C6)

For negative angles of attack:

\[ c_m = 0.00802 (|\alpha| - 20) + 0.077 \], \hspace{1cm} -67^\circ \leq \alpha \leq -20^\circ \hspace{1cm} (C7) \\
\[ c_m = 0.619 \left[ \sin(0.0260|\alpha| - 1.26) \right]^{0.387} \], \hspace{1cm} -162^\circ \leq \alpha \leq -67^\circ \hspace{1cm} (C8) \\
\[ c_m = 0.00838 (|\alpha| - 162) + 0.320 \], \hspace{1cm} -170^\circ \leq \alpha \leq -162^\circ \hspace{1cm} (C9) \\
\[ c_m = -0.0387 (|\alpha| - 170) + 0.387 \], \hspace{1cm} -180^\circ \leq \alpha \leq -170^\circ \hspace{1cm} (C10)

INPUT DATA REQUIREMENTS

The input data that are necessary for evaluation of the lift, drag, and pitching moment equations can be obtained from airfoil section test results. The specific input variables, and the equations in which they are used, are listed in Table 1. (It should be noted that \( S_A, L/c, c_w/c_p, \) and \( K, \) which are used in the drag equations (B3) and (B5), can be obtained from Figs. 7-10, appendix B, respectively, as functions of section thickness \( t/c. \))

Minimum experimental data (for a particular airfoil) necessary to evaluate the inputs listed in Table 1 are:

- \( c_L \) vs \( \alpha \) for several \( M \) from zero to \( M_n \)
- \( c_d \) vs \( M \) for several \( \alpha \)
- \( c_m \) vs \( M \) for several \( \alpha \)

The effort required for data preparation will be reduced if the following curves are also available:

- \( c_L \max \) vs \( M \)
- \( M_{DD} \) vs \( \alpha \)
- \( M_{MD} \) vs \( \alpha \)
- \( c_m \) vs \( \alpha \), for a low \( M \)
The $c_\alpha$ vs $\alpha$ curves should extend to at least the stall angle. No test data past stall are required because the airfoil section characteristics at high angles of attack are assumed to be independent of section type. The appendices provide a guide to the use of the above data in the preparation of input.

EXAMPLE CASE

The equations presented above were used to calculate lift, drag, and pitching moment coefficients for the NACA 23012 airfoil section. Input data were obtained from wind tunnel test results for the 23012 given in references 2-4, and are listed in table 2. The calculations were done by a digital computer program written to evaluate the equations over a range of Mach number and angle of attack. A FORTRAN listing of the program is presented in appendix D. Section coefficients were calculated through an angle of attack range of $-180^\circ$ to $+180^\circ$, and through a Mach number range of 0.0 to 0.9. From these results, $c_\alpha$, $c_d$, and $c_m$ at $M = 0.1$, for $\alpha = -20^\circ$ to $+180^\circ$, are plotted in figure 1.

Calculated and measured (ref. 2) section aerodynamic characteristics at low angle of attack for three Mach numbers, are compared in figure 2. Overall good agreement was realized, although some details of the 23012 behavior were not reproduced, since the equations are intended to be sufficiently general to represent most airfoil sections. No effort was made to tailor the inputs to improve the correlation. Experimental data for the 23012 at high angle of attack were not available. Figure 3 presents measured and calculated section coefficients as a function of Mach number. In this case, the experimental data are from reference 5. Again, generally good agreement was found.

CONCLUDING REMARKS

The airfoil section equations presented in this report were developed principally by fitting curves to a relatively limited set of experimental data. Some accuracy was sacrificed to obtain general applicability to a range of airfoil types. The utility of such equations can be judged only in the context of their intended use. If high accuracy is not required, these equations provide the ability to calculate airfoil section aerodynamic coefficients at any angle of attack, over a wide range of subsonic Mach numbers.

The principal application of the equations will be in fixed or rotary wing computations for which it is not practical to use graphical or tabulated section data, or for which a data table must be generated from limited input. The equations also provide a method for calculating section lift, drag, and pitching moment at very high angles of attack (i.e., $-180^\circ \leq \alpha \leq 180^\circ$). Experimental data of this type are available for very few airfoils. Another important application is in the prediction of section characteristics at high subsonic Mach numbers in cases where such data are incomplete.
APPENDIX A

DERIVATION OF LIFT COEFFICIENT EQUATIONS

SMALL ANGLE OF ATTACK

In the section angle of attack range \( 0 \leq \alpha \leq 22^\circ \), effects of compressibility, Reynolds number, and the specific airfoil section of interest are considered. For angles of attack below the stall, the section lift curve is assumed to be linear:

\[
c_l = \frac{d c_l}{d \alpha} (\alpha - \alpha_c), \quad \alpha < \alpha_{stall}
\]

Lift-Curve Slope

Mach number and thickness effects—Kaplan's rule (ref. 3) is used to represent the effects of compressibility and airfoil section thickness on lift curve slope:

\[
\left( \frac{d c_l}{d \alpha} \right)_{comp} = \left( \frac{d c_l}{d \alpha} \right)_{inc} \left\{ \gamma + \frac{t/c}{1 + t/c} \left[ \frac{1}{4} (\alpha - 1) + \frac{1}{4} (\alpha - 1) \left( \beta - 1 \right) \right] \right\}
\]

where

\[
\gamma = \frac{1}{\sqrt{1 - \beta}}
\]

\( \frac{t}{c} \) = section thickness to chord ratio

\( \gamma = 1.4 \) for air

so that

\[
\left( \frac{d c_l}{d \alpha} \right)_{comp} = \left( \frac{d c_l}{d \alpha} \right)_{inc} \left\{ \gamma + \frac{t/c}{1 + t/c} \left[ \frac{1}{4} (\alpha - 1) + 0.6(\alpha - 1) \right] \right\}
\]

The utility of this relation was investigated by comparing calculated and measured (ref. 2) lift-curve slopes for several airfoils over a range of Mach numbers, as shown in table 3. Lift-curve slope at \( M \) above 0.3 was obtained from the measured value at \( M = 0.3 \) and the ratio

\[
\frac{(d c_l/d \alpha)_{M>0.3}}{(d c_l/d \alpha)_{M=0.3}}
\]
calculated with equation (A2). The difference between the calculated and the measured lift-curve slope was 10° or less except for one value at the highest airfoil.

For the airfoils considered in table 1, and other airfoils presented in reference 2, the trend of increasing lift-curve slope with Mach number reverses above approximately M = 0.8. However, there is no discernible pattern to the rate of decline. References 5 and 6 present "synthesized" airfoil section data (for the NACA 0012 and 0015, respectively) which do exhibit a smooth decline of lift-curve slope above the reverse. This smoothness is due to the manner in which the airfoil characteristics were derived: an iteration between successive assumption of airfoil data values and comparison of calculated with measured rotor performance. The average rate of decline of lift-curve slope for the two sets of section data is

\[ \frac{dc_1}{da} = -0.45 \text{ per deg} \]

Let M at which the trend of the lift-curve slope reverses be M* Then for M > M*, the lift-curve slope will be

\[ \frac{dc_1}{da} = \left( \frac{dc_1}{da} \right)_{\text{comp}} - (0.45)(M - M*) \text{ per deg} \]

\[ \frac{dc_1}{da} = \left( \frac{dc_1}{da} \right)_{\text{inc}} \left\{ - \frac{t/c}{t/c} + \left( \frac{t/c}{t/c} \right) \right\} - (0.45)(M - M*) \]

It is necessary to set a lower limit for equation (A3). Again, the data available (e.g., ref. 2) are not sufficiently regular to provide a trend. Therefore, the lift-curve slope for the highest Mach number presented was measured for several airfoils in reference 2. The average value is 0.05. Thus the lower limit of equation (A3) is assumed to be

\[ \frac{dc_1}{da} \geq 0.05 \text{ per deg} \]

Reynolds number effects- Airfoil section lift-curve slope is a weak function of \( R_m \) up to \( 1 \times 10^7 \) or \( 2 \times 10^7 \), and is essentially independent of \( R_m \) for higher values (refs. 2 and 3). Therefore, for most uses, lift-curve slope can be assumed to be constant with \( R_m \). The value of incompressible lift-curve slope used in equation (A3) should be selected with consideration for the likely \( R_m \) range to be encountered. This value is also a function of the specific airfoil section considered.

Angle of Zero Lift

Mach number effects- The variation of airfoil section angle of zero lift, \( \alpha_0 \), is small with \( M \) and may be neglected, until \( M \) reaches a high subsonic
value. Then, for most airfoils, \( a_0 \) decreases in magnitude with further increases in \( M \). Table 4 presents measured \( a_0 \) values for \( M > M_1 \), where \( M_1 \) is the highest \( M \) in the experimental data for which \( a_0 \) remains at a constant value. (The Mach number \( M_1 \) is also the Mach number at which the trend of increasing lift-curve slope with increasing \( M \) reverses.) The data in Table 4 provide no consistent trend. For six of the sections, there is a strong decrease in the magnitude of \( a_0 \) with \( M \), for \( M > M_1 \). However, for three of the sections, the decline of \( a_0 \) is slight, while for two other sections, the decline of \( a_0 \) is reversed as \( M \) continues to increase. Since a general expression for \( a_0 \) was desired, it was assumed that the variation of section angle of zero lift with \( M \) can be adequately represented by a straight line above \( M_1 \):

\[
a_0 = \begin{cases} 
(a_0)_1, & 0 \leq M \leq M_1 \\
(a_0)_1 - \frac{(a_0)_2 - (a_0)_1}{M_1 - M_2} (M_1 - M), & M > M_1 
\end{cases}
\]  

(Mach number \( M_1 \) is some convenient \( M > M_1 \) for which \( a_0 \) can be determined from the experimental section data. Note that for symmetrical airfoil sections, \( a_0 = 0 \) even for high Mach numbers. Thus, for symmetrical sections, \( a_0 = (a_0)_1 = (a_0)_2 = 0 \).

Reynolds number effects- The angle of zero lift (low Mach number) of an airfoil section is determined by the camber. The extensive section data presented in reference 3 indicate that \( a_0 \) is not significantly affected by Reynolds number.

Maximum Lift Coefficient

Mach number effects- In the angle of attack range below the first lift peak, the variation of airfoil section maximum lift coefficient, \( c_{\text{max}}^l \), with \( M \) may be categorized as: (1) throughout the range of interest, \( c_{\text{max}}^l \) decreases with \( M \), or (2) in part of the range, \( c_{\text{max}}^l \) increases with \( M \) (but decreases otherwise). Experimental data for both types are shown by the solid curves in figure 4, and both can be fitted by polynomials of the form

\[
c_{\text{max}}^l = C_1 + C_2 M + C_3 M^2 + C_4 M^3 + C_5 M^4 + (C_6 + C_7 M^6) \sin(C_9 + C_{10} M) \tag{A6}
\]

Fits for two specific airfoil sections are shown by the dashed lines in figure 4. For the V23010 \(-1.58 \) (type 1 above), only the first three terms of equation (A6) were required. However, for the VR-7 (type 2 above), all ten coefficients are nonzero. In both cases, the functions were obtained by use of a least-squares curve-fit program.

Reynolds number effects- Figure 5, from reference 2, presents the combined effects of \( M \) and \( \frac{V}{M} \) for two airfoil sections. It can be seen that the
effect of increasing \( R_N \) is to shift the complete \( c_{l_{\text{max}}} \) vs \( M \) curve upwards, with the curves tending to collapse together above about \( R_N = 1 \times 10^5 \) in one case, and above about \( R_N = 6 \times 10^5 \) for the other. Therefore, the constant \( C_l \) in equation (A6) above will be determined (from low \( M \) experimental data for the particular section) as a function of the expected \( R_N \) range.

Negative angle of attack—Experimental data for \( c_{l_{\text{max}}} \) at negative angle of attack, as a function of \( M \), are not available for most airfoil sections. It is assumed that the variation of \( c_{l_{\text{max}}} \) with \( M \) at negative \( \alpha \) has the same form as for positive \( \alpha \). The sign of \( c_{l_{\text{max}}} \) is negative, of course, and for cambered sections the constant term \( C_l \) in equation (A6) will have a different magnitude as well as a different sign. The \( C_l \) term is obtained from the experimental lift curve at low \( M \). If no data are available for negative \( c_{l_{\text{max}}} \) even for low \( M \), it can be estimated as

\[
\left( c_{l_{\text{max}}}_{\text{exp}} \right)_{\alpha = -\alpha} = \left( c_{l_{\text{max}}}_{\text{exp}} \right)_{\alpha = \alpha} - 2(M - 1)\frac{d}{dM}
\]

That is, for a cambered section, negative \( c_{l_{\text{max}}} \) is lower in magnitude than the positive \( c_{l_{\text{max}}} \) by an amount double the lift increment due to the camber.

The accuracy of this estimate was evaluated by comparing it with measured values for several airfoil sections, as presented in Table 3. The error is less than 10% in six of the seven cases studied.

LARGE ANGLE OF ATTACK

Very few experimental section lift coefficient data for angle of attack more than a few degrees above the stall angle are available. Section lift coefficients through 180° for the NACA 0012 and the NACA 64A012 are presented in Figure 6. A curve fit of these data in four segments is given by

\[
c_l = 1.1 - 1.78(0.01745(a) - 0.7854)^2, \quad 22^\circ \leq \alpha \leq 90^\circ \tag{A8}
\]

\[
c_l = -1.1 + 1.78(0.01745(a) - 2.356)^2, \quad 90^\circ \leq \alpha \leq 160^\circ \tag{A9}
\]

\[
c_l = -0.768, \quad 160^\circ \leq \alpha \leq 172.5^\circ \tag{A10}
\]

\[
c_l = -5.82(a - 0.01745(a)) \quad 172.5^\circ \leq \alpha \leq 180^\circ \tag{A11}
\]

Equations (A8) through (A11) are taken from reference 7, except that (A8) is begun at \( \alpha = 22^\circ \) rather than 16° since this yields better agreement with the experimental data in Figure 6.

It is assumed that airfoil section lift coefficient at large angles of attack is an odd, symmetric function about \( \alpha = 0 \) (even for cambered sections). Thus \( c_l \) for large negative angles is given by the above equations, except that the sign of \( c_l \) is reversed.
\[
c_{l} = -1.1 + 1.78[0.01745|a| - 0.7853], \quad -90^\circ < a < -22^\circ \quad (A12)
\]
\[
c_{l} = 1.1 - 1.78[0.01745|a| - 2.356], \quad -160^\circ < a < -90^\circ \quad (A13)
\]
\[
c_{l} = 0.763, \quad -172.5^\circ < a < -160^\circ \quad (A14)
\]
\[
c_{l} = 5.82[1 - 0.01745|a|], \quad -180^\circ < a < -172.5^\circ \quad (A15)
\]

Equations (A8) through (A15) are assumed to hold for all airfoil sections, regardless of camber, thickness, \( M \), or \( R_0 \).

Due to the meager amount of test data for \( c_{l} \) above stall, the section lift coefficient is assumed to be a straight line between \( c_{l} \text{max} \) and \( \alpha = 22^\circ \). Then, from equation (A8),

\[
c_{l} = \frac{c_{l} \text{max} - 0.811}{22 - a} (22 - a), \quad a_{\text{stall}} < a < 22^\circ, \quad (A16)
\]

where it is assumed that \( c_{l} \text{max} \) occurs at

\[
a_{\text{stall}} = \frac{c_{l} \text{max}}{dc_{l}/da + a_0} \quad (A17)
\]

If the particular airfoil section exhibits gradual stall characteristics, equation (A17) does not accurately predict the angle at which \( c_{l} \text{max} \) occurs. However, equation (A17) may still be used in equation (A16), because the purpose of the latter is to approximate the lift curve in the region between the (assumed) linear lift-curve region and the high angle of attack region (\( \alpha = 22^\circ \)). For negative angles of attack,

\[
c_{l} = -0.813 + \frac{c_{l} \text{max}}{22 + a_{\text{neg stall}}} (22 + a), \quad -22^\circ < a < a_{\text{neg stall}} \quad (A18)
\]

where

\[
a_{\text{neg stall}} = \left( \frac{c_{l} \text{max}}{dc_{l}/da} \right) + a_0 \quad (A19)
\]
APPENDIX B

DERIVATION OF DRAG COEFFICIENT EQUATIONS

SMALL ANGLE OF ATTACK

For small section angle of attack less than the stall angle, section drag coefficient is essentially constant with Mach number below the drag divergence Mach number $M_{pp}$. Above $M_{pp}$, the drag rises very steeply. Thus it is assumed that

$$c_d = (c_d)_{M=0} + \frac{\partial c_d}{\partial M} (M - M_{pp}), \quad M > M_{pp}, \quad \alpha < \alpha_{stall}$$  \hspace{1cm} (B1)

$$c_d = (c_d)_{M=0} + \frac{\partial c_d}{\partial M} (M - M_{pp}), \quad M > M_{pp}, \quad \alpha > \alpha_{stall}$$  \hspace{1cm} (B2)

where $(c_d)_{M=0}$, $\frac{\partial c_d}{\partial M}$, and $M_{pp}$ are functions of section angle of attack.

Drag Coefficient at Low $M$

Section drag coefficient for smooth airtails at low Mach number, with fully turbulent boundary layer (the important case for most practical applications), may be estimated (ref. 1) by

$$c_d = \left[ \frac{1}{4} \left( 1 + \frac{0.4}{c_f} \right) \right]^{0.8} (1 - \frac{0.4}{c_f})$$  \hspace{1cm} (B3)

Equation (B1) was derived for symmetrical airtail sections; however, as shown by experimental data presented in reference 1, camber has a small effect on the minimum drag coefficient. Further, an inspection of the section data in reference 1 shows that minimum $c_d$ occurs at approximately $\alpha = 0$, for both symmetrical and cambered sections. Therefore, as discussed in reference 1, the $c_d$ vs $\alpha$ relation obtained for a symmetrical airtail may be applied with good accuracy to an airtail with the same thickness distribution but a cambered mean line. The factors $S_A$, $L/c$, $c_\delta/c_f$, and $k$ are constants for a specific airtail section, and are obtained from graphs provided in reference 1. Those graphs are reproduced here as figures 7 through 10. The turbulent skin friction drag coefficient is represented by

$$c_f = (0.4)^{0.5} \left( \log \frac{k}{R_{N_{eff}}} \right)^{0.5}$$  \hspace{1cm} (B4)

reference 10. The effective $R_N$ (ref. 1) is given by

$$R_{N_{eff}} = R_N \left( \frac{1}{c} \right) S_A$$  \hspace{1cm} (B5)
where \( R_e_p \) is based on free-stream velocity and airfoil chord, except where it is specifically taken as \( 6 \times 10^6 \) (third term of eq. (34)).

**Drag Divergence Mach Number**

The drag divergence Mach number of an airfoil section is defined as the Mach number for which \( (dC_D/dM) = 0 \), as airspeed is increased at constant section angle of attack. Measured drag divergence Mach numbers for several airfoil sections are presented in figure 11. The \( M_{dp} \) data points in figure 11 have been fitted with straight lines, or combinations of two straight lines in the cambered airfoil cases. Correlation is very good for the NACA 6-digit, 5-digit, 5-series, and Hertmann airfoils, throughout the \( \alpha \) range for which data were available. However, the data for the NACA 64A-series airfoils have a shift to different straight-line segments above about \( 5^\circ \) angle of attack. Also, the NACA 64A-series data appear to have an abrupt fluctuation at about \( \alpha = 10^\circ \). It was not established whether a general trend of 6-series airfoils is represented by these data (also note that the NACA 62-215 data are well represented by a straight line). Since a general expression was desired, it was assumed that \( M_{dp} \) can be represented by equations of the form

\[
M_{dp} = A + B\alpha, \quad \alpha \leq \alpha_{dp} \quad (3a)
\]

\[
M_{dp} = C + D\alpha, \quad \alpha > \alpha_{dp} \quad (3b)
\]

where \( \alpha_{dp} \) is the peak of the \( M_{dp} \) data. For symmetrical sections, \( M_{dp} \) is zero, \( C = A \), and \( D = -B \). Study of experimental data in references 4 and 4 indicates that \( M_{dp} \) is never less than 0.1, regardless of angle of attack.

**Slope of \( C_D \) Curve Above \( M_{dp} \)**

The slope of the \( C_D \) curve above \( M_{dp} \) was measured from experimental data for several airfoil sections, and the results are presented in figure 12. The curves plotted in the figure were obtained from least-squares curve fits of the experimental data. The 4-digit series airfoils are easily represented by curves of the form

\[
\frac{dC_D}{dM} = A + B\alpha + C\alpha^2
\]

However, the trend of \( (dC_D/dM) \) is more complex for the other two sections studied. Curve fits of the form

\[
\frac{dC_D}{dM} = A + B\alpha + C\alpha^2 + D\alpha^3
\]

were required to duplicate the rise and then leveling-off of the experimental data. Note that the quadratic equations will turn upwards, and the cubic
equations downward, at high $a$. Therefore, it is necessary to use a cutoff, $a_c$, above which $(dc_d/dm)$ is assumed to be constant:

$$
\frac{dc}{dm} = A + B \cdot a + Ca^2 + Da^4, \quad a \geq a_c
$$

$$
\frac{dc}{dm} \left( \frac{dc}{dm} \right)_{a=a_c}, \quad a > a_c
$$

Experimental data for $(dc_d/dm)$ through a significant range of negative angle of attack are not available. Therefore, it is assumed that the $dc_d/dm$ curve is a mirror image about a reference angle, $\alpha_R$. For example, for the NACA 2412 section shown in figure 12, the negative data point is chosen as $\alpha_R$, and so $\alpha_R = -1^\circ$. For symmetrical sections, $\alpha_R = 0$. Thus for angles less (more negative) than $\alpha_R$, substitute

$$
a' = |a| + 2\alpha_R, \quad a < \alpha_R
$$

for $a$ in equation (88). Also, the negative angle at which $(dc_d/dm)$ becomes constant is

$$
a'_c = -a_c + 2\alpha_R
$$

LARGE ANGLE OF ATTACK

Experimental section drag coefficients through $180^\circ$ are presented for two NACA airfoil sections in figure 13. These data are companion to the lift coefficient data in figure 6. The curve fit of the data shown in figure 13 is given by

$$
c_d = 2.18 \left( \sin a \right)^{1.1}, \quad 15^\circ \leq |a| \leq 180^\circ
$$

The lower limit of $15^\circ$ for equation (B12) is arbitrary, but reflects the fact that the effects of section type and of Mach number are significant only for small $a$. Equation (B12) was obtained from reference 1, but the constant factor has been increased slightly to provide better correlation with the experimental data in figure 13. Equation (B12) is assumed to be applicable to all airfoil sections, regardless of section thickness, Reynolds number or Mach number.

If section angle of attack is greater than $a_{\text{stall}}$, but less than $15^\circ$, the drag coefficient is assumed to be a straight line between $a_{\text{stall}}$ and $15^\circ$:

$$
c_d = 0.219 - \frac{0.219 - (c_d)_{a_{\text{stall}}}}{15 - |a_{\text{stall}}|} \left( 15 - |a| \right), \quad a_{\text{stall}} < |a| < 15^\circ
$$
where $\alpha_{\text{stall}}$ is assumed to be the angle for $c_{t_{\text{max}}}$:

$$\alpha_{\text{stall}} = \frac{c_{t_{\text{max}}}}{dc_{i}/d_{t}} + \alpha_{0}$$  \hspace{1cm} (B14)

For negative angle of attack, $c_{t_{\text{max}}}$ in equation (B14) is replaced by $(c_{t_{\text{max}}})_{-\alpha}$.\n
APPENDIX C

DERIVATION OF MOMENT COEFFICIENT EQUATIONS

SMALL ANGLE OF ATTACK

The variation of airfoil section pitching moment (about the quarter-chord) with Mach number is small below the moment divergence Mach number, and may be neglected in that region. Also the moment is essentially independent of Reynolds number. For most airfoils, the slope of the moment coefficient curve is approximately constant (often zero) with angle of attack until the stall angle is reached, whereupon \( c_m \) breaks sharply. Therefore, it is assumed that

\[
c_m = c_{m_0} + \frac{d c_m}{d \alpha} (\alpha), \quad M \leq M_{MD}, \quad |\alpha| \leq \alpha_{stall}
\]  

(C1)

where \( c_{m_0} \) is \( c_m \) at \( \alpha = 0 \). At positive stall, \( c_m \) breaks in the negative (nose-down) \( c_m \) direction. At negative stall, the reverse is true.

Moment Divergence Mach Number

An inspection of experimental airfoil section pitching moment coefficient plotted against Mach number (e.g., ref. 4) shows that, at a fixed angle of attack, \( c_m \) is essentially constant with \( M \) until a certain \( M \) is reached, whereupon \( c_m \) diverges rapidly with further increases in \( M \). A pitching moment divergence Mach number is defined such that \( M_{MD} \) is the Mach number for which \( |dc_m/dM| = 0.5 \) as airspeed is increased at constant section angle of attack. Using this definition, \( M_{MD} \) as a function of angle of attack was obtained from experimental data for several airfoil sections, and is plotted in figure 14. Although there is some scatter, the data are reasonably well fitted by straight lines, or combinations of straight lines. Therefore, \( M_{MD} \) will be represented by equations of the form

\[
M_{MD} = A + B\alpha, \quad \alpha \geq \alpha_{P_{MD}}
\]  

(C2)

\[
M_{MD} = C + D\alpha, \quad \alpha < \alpha_{P_{MD}}
\]  

(C3)

where \( \alpha_{P_{MD}} \) is the peak of the data. For symmetrical sections \( \alpha_{P_{MD}} = 0 \), \( C = A \), and \( D = -B \) Experimental data in references 3 and 4 indicate that \( M_{MD} \) is always greater than about 0.3, for any angle of attack.
Above Stall or $M_{MD}$

There are very few data available for pitching moment above stall or $M_{MD}$, and no general trends are discernible. It is assumed that $c_m$ varies linearly with $a$ between $\alpha_{stall}$ and 20° (assumed to be the start of the large angle region). A linear relation of $c_m$ with $M$ is also assumed between $M_{MD}$ and $M = 0.95$. At $M = 0.95$ it is assumed that $c_m$ has reached the same large-angle region value of -0.077 as at $a = 20°$. The selection of $M = 0.95$ for this relation is arbitrary. Thus,

$$c_m = \frac{0.077}{(20 - \alpha_{stall})} + 0.077$$

$$c_m = (c_m)_{MD} \frac{(c_m)_{stall} + 0.077}{0.95 - M_{MD}}$$

$$(M - M_{MD}), \quad M > M_{MD}$$

Note that the condition $\alpha > \alpha_{stall}, \quad M > M_{MD}$ may occur, in which case equation (C4) is used to evaluate $(c_m)_{MD}$ in equation (C5). For negative angles of attack, these equations become

$$c_m = \frac{0.077 - (c_m)_{stall}}{20 + \alpha_{stall}} (20 + a), \quad 20° < |a| < |\alpha_{stall}|$$

and

$$c_m = (c_m)_{MD} \frac{(c_m)_{stall} - 0.077}{0.95 - M_{MD}}$$

$$(M - M_{MD}), \quad M > M_{MD}$$

where $\alpha_{stall}$ is negative.

LARGE ANGLE OF ATTACK

Experimental section pitching moment coefficients through '180°' are presented in figure 15 for the NACA 63A012 and NACA 0012 airfoils. A curve through the data was determined in four sections, as shown in figure 15. The curve is given by

$$c_m = -0.00802 (a - 20) - 0.077, \quad 20° \leq a \leq 67°$$

$$c_m = -0.619 [\sin (C.0260a - 1.26)]^{5.394}, \quad 67° < a \leq 162°$$

$$c_m = -0.00838 (a - 162) - 0.320, \quad 162° < a \leq 170°$$

$$c_m = 0.0387 (a - 170) - 0.387, \quad 170° < a \leq 180°$$

The lower limit of 20° for these large-angle equations is arbitrary.
Section pitching moment at large angle of attack is assumed to be an odd, symmetric function about \( \alpha = 0 \), even for cambered sections. Thus \( c_m \) for large negative angles is given by the above equations, except that the sign of \( c_m \) is reversed (\( c_m \) is positive)

\[
\begin{align*}
    c_m &= 0.00802(|\alpha| - 20) + 0.027, \quad -67^\circ \leq \alpha \leq -20^\circ \quad (C12) \\
    c_m &= 0.619\sin(0.0260|\alpha| - 1.26)^{0.198}, \quad -162^\circ \leq \alpha \leq -67^\circ \quad (C13) \\
    c_m &= 0.00838(|\alpha| - 162) + 0.320, \quad -170^\circ \leq \alpha \leq -162^\circ \quad (C14) \\
    c_m &= -0.0387(|\alpha| - 170) + 0.387, \quad -180^\circ \leq \alpha \leq -170^\circ \quad (C15)
\end{align*}
\]
APPENDIX D

COMPUTER PROGRAM FOR EVALUATION OF AIRFOIL

SECTION AERODYNAMIC CHARACTERISTICS

A digital computer program was written to allow rapid evaluation of the airfoil section force and moment coefficient equations presented in this report. The program calculates section lift, drag, and pitching moment coefficients for angles of attack from -180° to +180°, and for a range of Mach numbers (M < 1.0) which may be selected by the user. Input parameters for the specific airfoil type under study are required. Sample input for the NACA 23012 airfoil section is presented in Table 1. Calculated output is in the form of lift coefficient, drag coefficient, and pitching moment coefficient tables. In each table, the calculated coefficient is printed as a function of section angle of attack and of Mach number. Calculated aerodynamic coefficients for the NACA 23012 are plotted in Figures 2 and 3. These data were calculated by the program as a result of the input data shown in Table 1. A FORTRAN listing of the computer program is presented in this appendix.
ORIGINAL PAGE IS OF POOR QUALITY

**THIS PROGRAM CALCULATES AIRFUEL SECTION LIFT, DRAG, AND PITCHING MOMENT COEFFICIENT TABLES AS A FUNCTION OF MACH NUMBER AND ANGLE OF ATTACK.**

EAL INPUTS

RESULT (SCU7 AFDSUR)

366 FORMAT (4,4)

ORIGINAL PAGE IS OF POOR QUALITY
7 IPCLC= TVL  * (CCPLAX; EXC3)

**CONSTANTS**

CC54100C  PIC=0.145
CC62100C  XREF=0.12KL=0.12L(550)
CC63100C  LSF=U1.45/ (ALLU10HNEF) ** 2..58
CC64100C  XREF=0.12U=0.12L(550)
CC65100C  CSF=U1.45/ (ALLU10HNEF) ** 2..58

SET UP ANGLE OF ATTACK

**MDM**

CC56100C  TAU=
CC57100C  ALPHA=10.1
CC67100C  MAX=0.0145*ALPHA
CC68100C  P=0.01

*** MINIMUM RANGE OF ATTACK RANK ***

CC69100C  IF (ALPHA.LT.6CC) UC TL 500
CC70100C  LIF=CCOEFFICIENT
CC71100C  CC59100C  IF (LFAU=LTA2.950) CLFA=1.11.76* (ALPHA=.70231) ** 2
CC89100C  IF (LFA=1.00) CLFA=UC
CC73100C  IF (ALPHA=LTA16.000) CLFA=-1.11.76* (ALPHA=.0250) ** 2
CC76100C  IF (ALPHA=LTA16.000 AND .LT.172.50) CLFA=UC
CC77100C  IF (ALPHA=LTA16.000) CLFA=1.00 (PIE-ALPHA)
CC78100C  IF (ALPHA=LTA16.000) CLFA=UC
CC79100C  IF (LFAU=LTA2.950) CLFA=1.11.76* (ALPHA=.70231) ** 2
CC80100C  IF (LFAU=LTA2.950) CLFA=UC
CC81100C  IF (ALPHA=LTA16.000) CLFA=1.11.76* (ALPHA=.0250) ** 2
CC82100C  IF (ALPHA=LTA16.000) CLFA=UC
CC83100C  IF (ALPHA=LTA16.000) CLFA=UC
CC84100C  IF (ALPHA=LTA16.000) CLFA=UC
CC85100C  IF (ALPHA=LTA16.000) CLFA=UC
CC86100C  IF (ALPHA=LTA16.000) CLFA=UC
CC87100C  IF (ALPHA=LTA16.000) CLFA=UC
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CC95100C  IF (ALPHA=LTA16.000) CLFA=UC
CC96100C  IF (ALPHA=LTA16.000) CLFA=UC
CC97100C  IF (ALPHA=LTA16.000) CLFA=UC
CC98100C  IF (ALPHA=LTA16.000) CLFA=UC
CC99100C  IF (ALPHA=LTA16.000) CLFA=UC
*** LCM NULL CF ATTACK RANGE ***

LAG CLEFFICIENT AT ZENITAL MACH NUMBER

LAG DIVERSION PATH NUMBER

ULLP: LF LL CURVE ABOVE LAG DIVERSION

PRINT DIVERSION PATH NUMBER

LIFT CURVE SLOPE
ANGLE OF ZERO LIFT

\[ \text{ANGLE OF ZERO LIFT} \]

MAXIMUM LIFT COEFFICIENT

\[ \text{MAXIMUM LIFT COEFFICIENT} \]

ANGLE OF ATTACK FOR MAX CL

\[ \text{ANGLE OF ATTACK FOR MAX CL} \]

LIFT COEFFICIENT

\[ \text{LIFT COEFFICIENT} \]

ANGLE DEVIATION COEFFICIENT

\[ \text{ANGLE DEVIATION COEFFICIENT} \]

LWC COEFFICIENT

\[ \text{LWC COEFFICIENT} \]

LWC COEFFICIENT

\[ \text{LWC COEFFICIENT} \]

LWC COEFFICIENT
INC 1:

**INC 2: Increment Angle of Attack**

**INC 3:**

~

\[
\text{IF (ALFA < -25 Deg OR ALFA > 25 Deg) ALFA = ALFA + 4 Deg}
\]

**INC 4: Increment Angle of Attack**

~

\[
\text{IF (ALFA > -25 Deg AND ALFA < 10 Deg) ALFA = ALFA + 1 Deg}
\]

**INC 5: Increment Angle of Attack**

~

\[
\text{IF (ALFA < 10 Deg AND ALFA > 25 Deg) ALFA = ALFA + 1 Deg}
\]

**INC 6: Increment Angle of Attack**

~

\[
\text{IF (ALFA > -180 Deg) GC = 1300}
\]

**INC 7: Print Cut Results**

**INC 8:**

\[
\text{WRITE ('E (4,9)25)}
\]

**INC 9:**

\[
\text{FORMAT ('I (4,30), 'LIFT COEFFICIENT TABLE')}
\]

**INC 10:**

\[
\text{AMACH = SPACH}
\]

**INC 11:**

\[
\text{CC = 1330 N = 1, NMP}
\]

**INC 12:**

\[
\text{PLACH(N) = AMACH}
\]

**INC 13:**

\[
\text{AMACH = AMACH + 0.1}
\]

**INC 14:**

\[
\text{WRITE ('E (4,9)30) (PLACH(N), N = 1, NMP)}
\]

**INC 15:**

\[
\text{WRITE ('E (4,9)40) (AMACH, 'ALPHA')}
\]

**INC 16:**

\[
\text{CC = 1320 J = 1, M}
\]

**INC 17:**

\[
\text{WRITE ('E (4,9)50) PALFA(J), (CC(J, M), N = 1, NMP)}
\]

**INC 18:**

\[
\text{WRITE ('E (4,9)60) (CC(J, M), N = 1, NMP)}
\]

**INC 19:**

\[
\text{WRITE ('E (4,9)70) (AMACH, 'ALPHA')}
\]

**INC 20:**

\[
\text{WRITE ('E (4,9)80) (PLACH(N), N = 1, NMP)}
\]

**INC 21:**

\[
\text{WRITE ('E (4,9)90) (AMACH, 'ALPHA')}
\]

**INC 22:**

\[
\text{WRITE ('E (4,9)100) (PLACH(N), N = 1, NMP)}
\]

**INC 23:**

\[
\text{WRITE ('E (4,9)110) (AMACH, 'ALPHA')}
\]

**INC 24:**

\[
\text{WRITE ('E (4,9)120) (PLACH(N), N = 1, NMP)}
\]

**INC 25:**

\[
\text{WRITE ('E (4,9)130) (AMACH, 'ALPHA')}
\]

**INC 26:**

\[
\text{WRITE ('E (4,9)140) (PLACH(N), N = 1, NMP)}
\]

**INC 27: Stop**

\[
\text{STOP}
\]

**INC 28: End**

\[
\text{EAE}
\]
REFERENCES


### TABLE 1: DATA REQUIREMENTS

<table>
<thead>
<tr>
<th>Input data</th>
<th>Equation numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td>((\text{dc}<em>x/\text{da})</em>\text{inc})</td>
<td>(A2), (A3)</td>
</tr>
<tr>
<td>(t/c)</td>
<td>(A2), (A3), also figs. 7-10</td>
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<tr>
<td>(M_1, M_2)</td>
<td>(A3), (A4), (A5)</td>
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<td>((a_0)_1, (a_0)_2)</td>
<td>(A4), (A5)</td>
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<td>(C_{Y,Y_\text{max}}) coefficients</td>
<td></td>
</tr>
<tr>
<td>(C_1, C_{10}) for +(\alpha) case</td>
<td>(A6)</td>
</tr>
<tr>
<td>(C_1, C_{10}) for -(\alpha) case</td>
<td>(A6)</td>
</tr>
<tr>
<td>(M_{DD}) coefficients A, B, C, D</td>
<td>(B6), (B7)</td>
</tr>
<tr>
<td>(\alpha_{PD})</td>
<td>(B6), (B7)</td>
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<tr>
<td>C2</td>
<td>C2</td>
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<tr>
<td>C2</td>
<td>C2N</td>
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<td>C3</td>
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<td>C5N</td>
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<td>C7</td>
<td>C7N</td>
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<td>C9N</td>
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<td>C10N</td>
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The table below shows the measured and calculated lift-curve slope for several airfoil sections:

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<tr>
<th>Airfoil section</th>
<th>Mach number</th>
<th>Measured $dc_{l}/d\alpha$ (per deg)</th>
<th>Calculated $dc_{l}/d\alpha$ (per deg)</th>
<th>Difference $\gamma$</th>
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<td>.100</td>
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<td>---</td>
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<tr>
<td>VERTOL V23010-1.58</td>
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<td>.129</td>
<td>4.0</td>
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<td>.151</td>
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Reference 2.
Equation (A2).
<table>
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<th>Airfoil section</th>
<th>$M_1$</th>
<th>$(\alpha_{1n})_1$</th>
<th>$M$</th>
<th>$\alpha_n$</th>
<th>Reference</th>
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<td>with T.E. Tab</td>
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<td>0.95</td>
<td>-6</td>
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*$M_1$ is the highest Mach number in the experimental data for which $\alpha_n$ remains at a constant value.
<table>
<thead>
<tr>
<th>Airfoil section</th>
<th>Measured $c_{max}$</th>
<th>Measured $\Lambda$</th>
<th>Calculated $c_{max}$</th>
<th>Error</th>
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</table>

Reference 3.

Equation (A7).
Figure 1. - Calculated lift, drag, and pitching-moment coefficients for the NACA 23012 airfoil section, at $M = 0.1$. 
Figure 2.- Calculated and measured aerodynamic characteristics for NACA 23012 airfoil section.

(a) Section lift coefficient.
Figure 2. Continued.

(b) Section drag coefficient.
Figure 2—Concluded.

(c) Section pitching-moment coefficient.
(a) Section normal force coefficient.

Figure 3c: Calculated and measured aerodynamic characteristics for XA-114-100 airfoil section.
(b) Section drag coefficient.

Figure 3.- Continued.
(c) Section pitching-moment coefficient.

Figure 3.—Concluded.
(a) Airfoil section V23010-1.58.

(b) Airfoil section VR-7.

Figure 4.- Maximum lift coefficient.
Figure 5. - Effect of Mach number and Reynolds number on airfoil section maximum lift coefficient.
Figure 6.- Airfoil section lift coefficient through 180°.
Figure 7.- Airfoil section calculated mean pressure coefficient.
Figure 8. Airfoil section perimeter/chord ratio.
Figure 9.- Airfoil section form drag/friction drag.

FROM REF. 1
\[ (\Delta C_D)_\alpha = K(\alpha)^{2.7} \]

*WITH \( \alpha \) IN RADIANS*

Figure 10.- Correlation parameter for airfoil section drag due to lift.
(a) NACA symmetrical four-digit series airfoil sections.
(b) NACA cambered four-digit series airfoil sections.

Figure 11. - Measured airfoil section drag divergence Mach number.
(c) NACA five-digit series airfoil sections.

(d) NACA 63A symmetrical series airfoil sections.

Figure 11.- Continued.
$M_{DD} = M$ for which $dC_d/dM = 0.1$, at constant $\alpha$

- $\bigcirc$ NACA 64A010, Ref. 12
- $\bigtriangleup$ NACA 64A310, Ref. 12

$M_{DD}$ vs $\alpha$, deg

- $\bigcirc$ NACA 662-215, $\alpha = 0.5$, Ref. 11
- $\bigtriangleup$ WORTMANN FX69H-098, Ref. 4

(e) NACA 64A series airfoil sections.

(f) NACA 65 series and Wortmann airfoil sections.

Figure 11.- Concluded.

51
\[ \frac{dC_d}{dM} = 0.274 + 0.0253\alpha + 0.9273\alpha^2 - 0.00264\alpha^3 \]

\[ 0.466 - 0.0261\alpha + 0.00137\alpha^2 \]

\[ 0.399 - 0.0378\alpha + 0.00245\alpha^2 \]

\[ 0.147 + 0.0127\alpha + 0.0111\alpha^2 - 0.000874\alpha^3 \]

\[ \alpha, \text{ deg} \]

\[ -2 \quad 0 \quad 2 \quad 4 \quad 6 \quad 8 \quad 10 \quad 12 \]

Figure 12.- Slope of \( c_d \) curve above drag divergence Mach number.
<table>
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<tr>
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<th>REFERENCE</th>
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</table>

**Figure 14.** Airfoil section drag coefficient through 180°.
Figure 14. Measured airfoil section moment divergence Mach number.
Figure 14. Airfoil section pitching moment coefficient through 180°.
Closed-form equations for the lift, drag, and pitching-moment coefficients of Aviation Research Center (AVRACOM) research and technology sections in subsonic flow are presented. Published theoretical and experimental results. A turbulent boundary layer is assumed to exist on the aileron surface. Effects of section angle of attack, Mach number, Reynolds number, and specific aircraft geometry considered. The equations are applicable through an angle-of-attack range of -18° to +18°; however, above about 5°, the section characteristics are assumed to be functions only of angle of attack. A computer program is presented which evaluates the equations for a range of Mach numbers and angles of attack. Calculated results for the NASA 2012 airfoil section are compared with experimental data.