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RESEARCH OF MICROWAVE SCATTERING PROPERTIES OF SNOW FIELDS

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ABSTRACT

A review is presented of the results obtained in the research program of microwave scattering properties of snow fields. The program is a cooperative effort between a group at the University of California, Berkeley (D. J. Angelakos, F. D. Clapp, K. K. Mei, and S. Coen) and NASA-Ames (W. I. Linlor). A goodly portion of the research was undertaken at the Central Sierra Snow Lab and with the active cooperation of the U.S. Forest Service (J. L. Smith).

Experimental results are presented showing backscatter dependence on a) frequency (5.8-8.0 GHz), b) angle of incidence (0-60 degrees), c) snow wetness (time of day), and d) frequency modulation (0-500 MHz). Theoretical studies are being made of the inverse scattering problem yielding some preliminary results concerning the determination of the dielectric constant of the snow layer.

The experimental results lead to the following conclusions: a) snow layering affects backscatter, b) layer response is significant up to 45 degrees of incidence, c) wetness modifies snow layer effects, d) frequency modulation "masks" the layer response, and e) for the proper choice of probing frequency and for nominal snow depths, it appears to be possible to measure the effective dielectric constant and the corresponding water content of a snow pack.

I. INTRODUCTION

The mapping of ice and snow and soil moisture is an important project because such maps provide essential information for water resources management and agricultural or forestry yields. Mapping by microwave remote sensing has recently been identified as an important technology in addition to the optical and infrared techniques. There are two branches of microwave remote sensing techniques currently being investigated for airborne mappings, namely, the radiometry and radar remote sensing. The radiometry is a passive sensing technique which measures the emissivity of the target subject to illumination by natural sources. The radar remote sensing is an active sensing technique which measures the echoes of the target subject to illumination by an airborne microwave source. There are advantages and disadvantages in both approaches, which have been thoroughly discussed in reference [1]. To date, the microwave radiometry appears to have more promise for successful applications than radar remote sensing. That, however, does not necessarily mean that radar methods are basically inferior in remote sensing. Most probably, that situation just reflects the amount of research effort devoted to these techniques. The art of radiometry had been well developed by the radio astronomers before it was brought to use in microwave remote sensing; there have been well-documented theories relating brightness temperature to the atmospheric composition and temperature profiles. On the other hand, radar remote sensing is a rather young technology. So far, research effort on radar remote sensing is mostly concentrated on instrumentation; it lacks the corresponding solid theoretical foundation which supports radiometry.

II. ERL's UNIQUE EXPERIENCE AND QUALIFICATIONS IN MICROWAVE REMOTE SENSING OF SNOW

The Electronics Research Laboratory of the University of California at Berkeley has been supported by NASA under contract NASA-Goddard Grant NSG-5093 to study active microwave remote sensing. The total support for the research is $61,000 for two years. With the limited funds and time, the ERL team, in cooperation with Dr. William I. Linlor of the NASA-Ames Research Center, was able to obtain rather significant results, both theoretically and experimentally, which are basic to radar remote sensing. A report of these results is contained in the paper by Angelakos, which is attached in the appendix of this proposal. These experiments were performed in the frequency band 5.8-12 GHz. An interesting discovery is the strong dependence of the backscattering of the snow pack on time of day. The phenomenon was later confirmed by several other groups (Curie et al [2], Ulaby et al [3]). Some typical backscattering results are shown in Fig. 1. It is noticed that considerable variations were recorded at 15 min. intervals. Due to the time dependence of radar backscattering of the snow pack, empirical correlation between measurements and snow properties is difficult to establish. That is one of the reasons we feel that the optimum frequency band for snow pack remote sensing is yet to be determined. We propose for future research to investigate radar backscattering starting at lower frequencies such as 100 MHz and proceeding upwards in frequency.

On the theoretical front, the ERL team has formulated the integral equation describing the electromagnetic scattering from layered media in a form in which only the amplitude of the backscattering is needed to predict the dielectric constants of the layers. The results of a

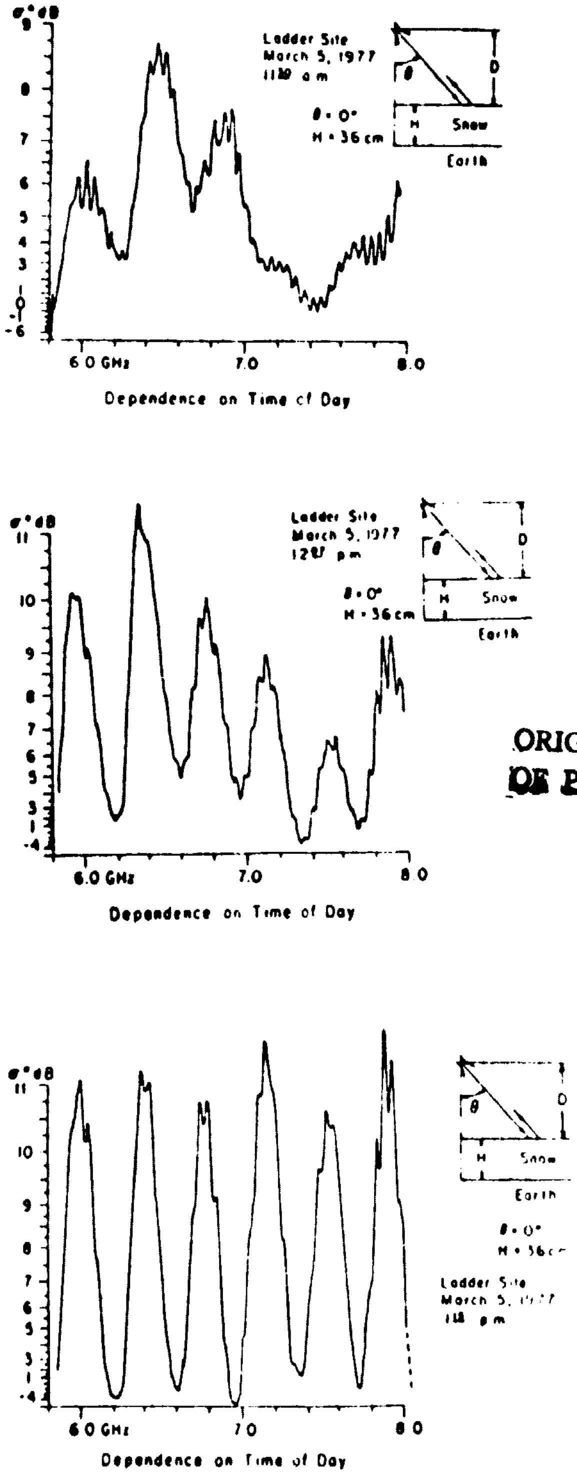

Fig. 1. Backscattering of a snow pack as a function of frequency at 11:30 A.M., 12:09 A.M. and 1:15 P.M.

typical inverse scattering are shown in Fig. 2. It is noted that the inverse algorithm we have used predicts the layered media quite well even with only three (3) measurements of the backscattered amplitude. The details of our formulation and algorithm will be presented in the following sections.

III. GENERAL BACKGROUND

(a) Measurements and Instrumentation

Measurements of microwave backscattering by snow pack have been made by many investigators. However, with no exception, all previous efforts were channeled towards finding the electrical properties of the snow, assuming the snow pack is one uniform layer. The objective of ERL's snow measurements has been to understand the field environment and to develop instrumentation for untimately collecting data for an inverse scattering theory which predicts the profile of the snow property. Toward that end, it was believed that the repeatability of the measurements should be of the greatest importance. It was to our great surprise that the backscattering of snow pack is not repeatable, even at an interval as short as fifteen minutes. The experiments were made at the Central Sierra Snow Laboratory at Tahoe, performed by a team of the ERL, in cooperation with Dr. William Linlor of NASA-Ames and the U.S. Forest Service (J. L. Smith). The instruments involve horn antennas mounted on towers as shown in the picture in Fig. 3. The results obtained at 15 min. intervals at 8.2 GHz and 10 GHz are shown in Figs. 4 and 5. It is observed that the variation of the backscattering of the same snow pack is so unpredictable that the simplistic approach of predicting snow electrical property by comparing its radar

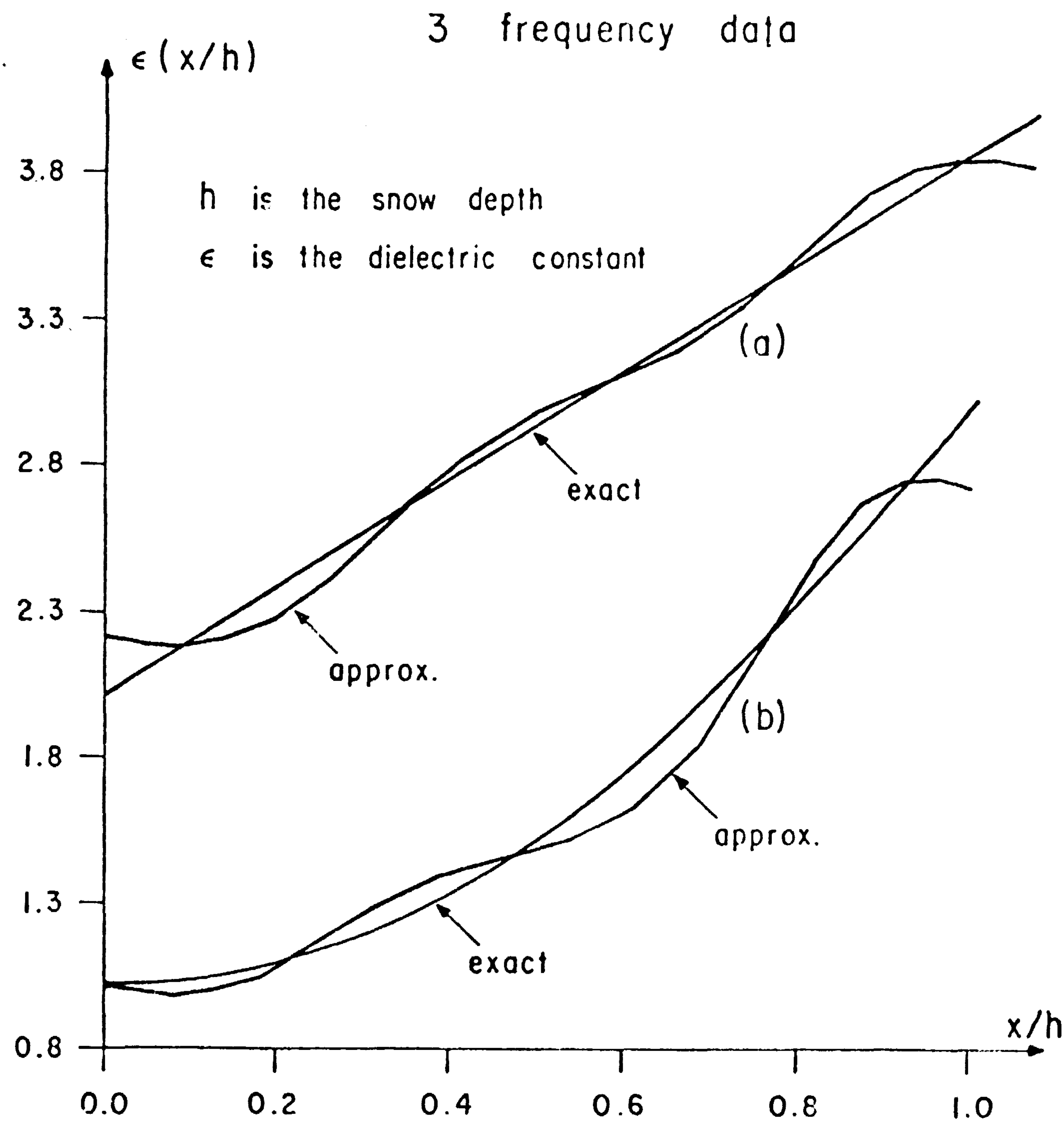

Fig. 2. Approx. vs Exact Snow Profile

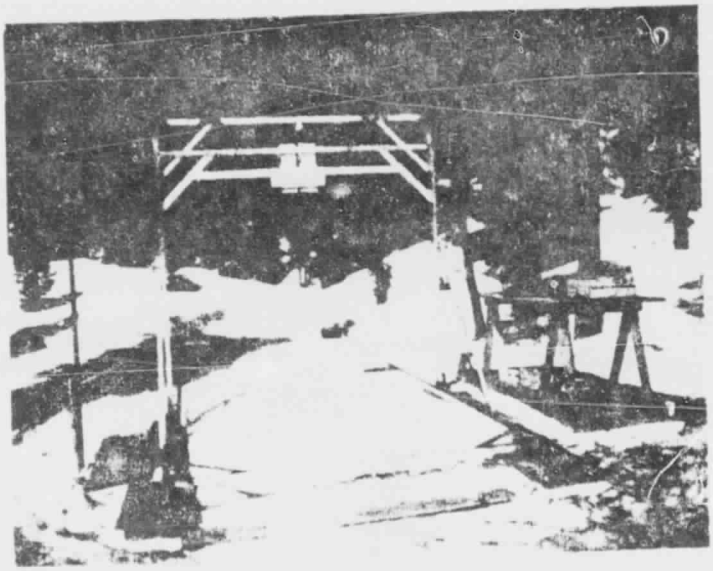

Fig. 3. Snow measurement set-up

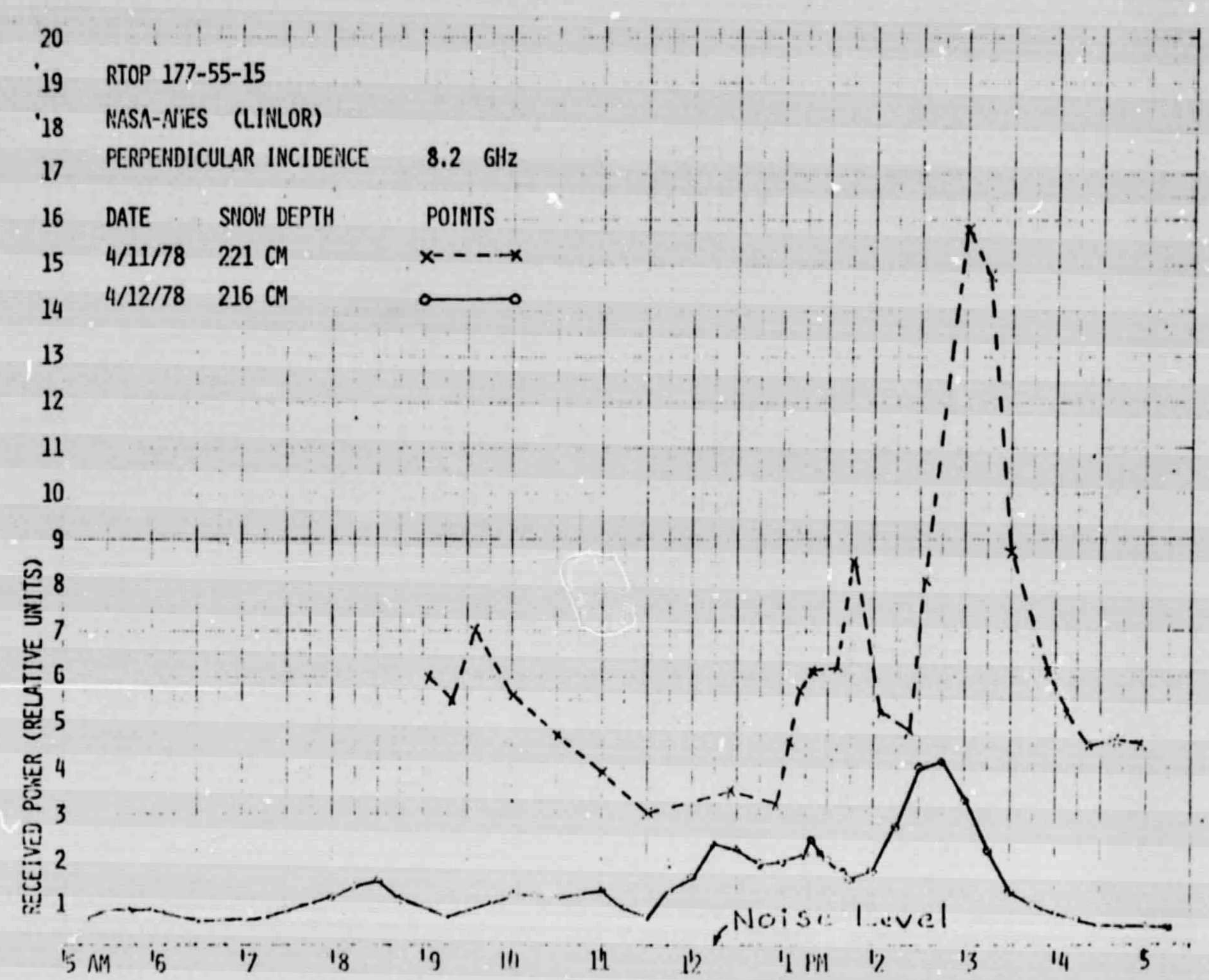

Fig. 4. Snow backscatterings as a function of time and day at 8 GHz.

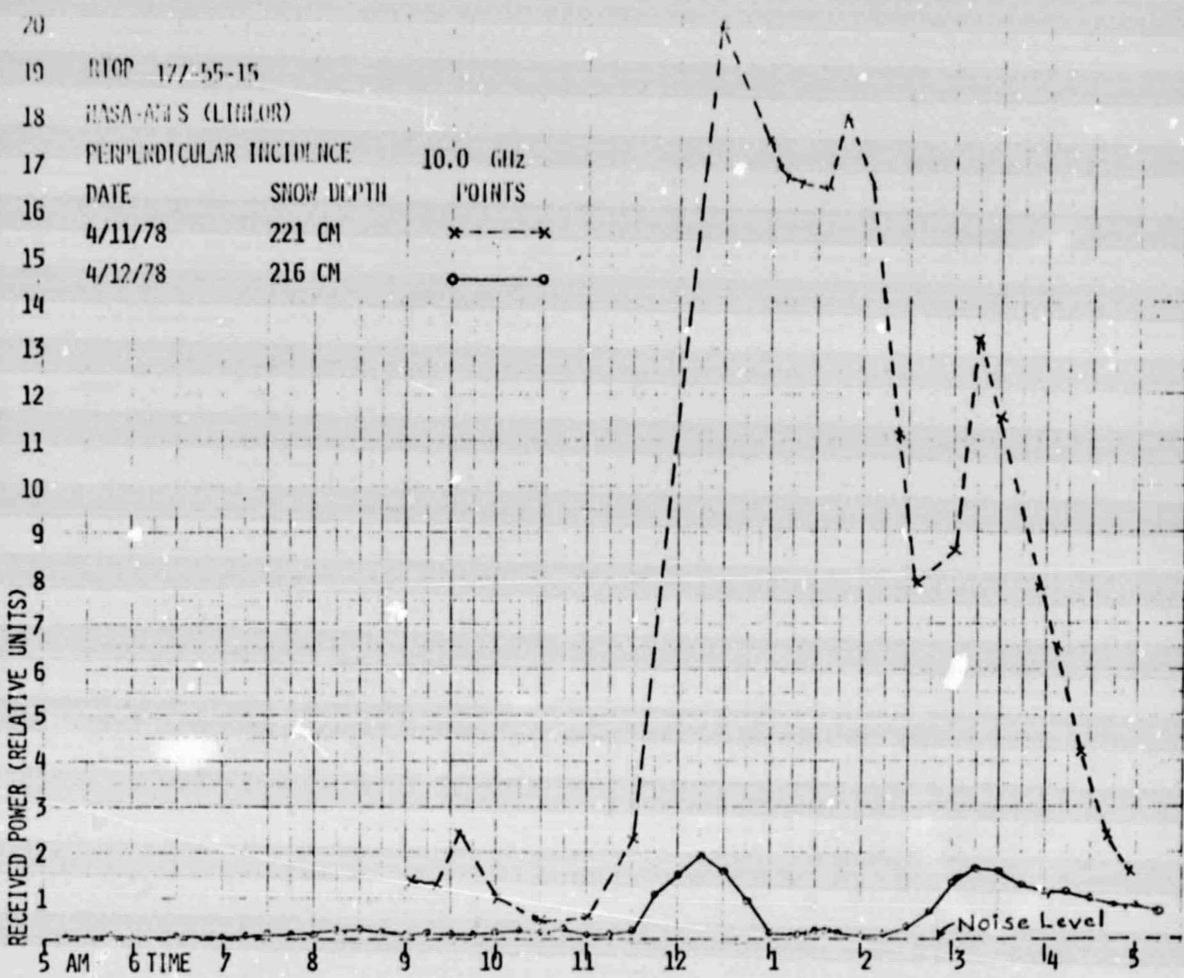

Fig. 5. Snow backscatterings as a function of time and day at 10.2 GHz.

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OF POOR QUALITY

backscatterings with a set of known signatures is hopelessly an inadequate model.

With the above understanding of the dynamic property of snow pack, we feel that two things should be done in order to affirm the feasibility of radar remote sensing of snow packs, namely, first, further measurements be done at much lower frequencies, such as 100 MHz, or even as low as 25 MHz, and secondly, concrete evidence of the applicability of the inverse scattering technique be established.

The purpose of lowering the frequency of observation is two-fold: to determine whether the time dependence of the backscatterings from snow is a surface effect, and to condition the instrumentation for collecting data towards the use of an inverse scattering algorithm. If the time variation of a snow backscattering was a surface effect, lowering the frequency should reduce the variation greatly. Furthermore, for successful prediction of the snow properties by inverse scattering, the signal must penetrate the entire snow pack. It would be impossible to predict the snow properties for regions which the field cannot reach. In many field situations, the snow could be over 20 feet deep, hence it would be unlikely for a 1 GHz wave (λ=30 cm) to penetrate a wet or packed snow layer of that thickness.

It should be emphasized here that the time variation of radar backscattering is not a deterrent to the inverse scattering approach, because the backscattering dynamics are most probably due to the effect of changing snow profile, i.e., the backscattering data are fairly sensitive to the snow profile constituency. Since the end result of the inverse scattering technique is to find the snow profile, the sensitivity of the measurements indeed offers prospects for better resolution. We shall discuss the background for inverse scattering in the following section.

(b) Theory of Inverse Scattering

(1) Introduction

Inverse scattering problems in science and engineering concern the determination of the properties of some inaccessible region from observations away from the region. The observations are often taken as the response of the inaccessible region to some source, which may be natural or artificial. It may be located within or on the boundary of the inaccessible region.

The inverse scattering theory falls into two distinct parts. One deals with the ideal case in which the observations are assumed to be known exactly and as dense as required, whereas the other treats the practical problems that are created by incomplete and imprecise data.

To apply these ideas, there must be a valid mathematical model that describes the physical behavior of the inaccessible region. The model is a plane stratified half-space whose properties are characterized by an inhomogeneous, linear and isotropic dielectric.

The equation which describes the model is a one-dimensional wave equation,

$$\left[\frac{d^2}{dx^2} + k^2\,\varepsilon(x)\right] E(k,x) = 0 \qquad -\infty < x < \infty \tag{1}$$

where $\varepsilon(x)$ is the dielectric profile and $E(k,x)$ is the electric field. We have assumed the wave to be polarized parallel to the half-space, hence the polarization direction of the electric field is omitted.

The conventional interest in eq. (1) is to solve the electric field everywhere, given the incident field and the dielectric profile

$\varepsilon(x)$, which, in the remote sensing terminology is known as the forward problem. There are many different ways the forward problem can be solved economically and accurately. In the ensuing discussion, we shall assume that all forward solutions of (1) are available for any given $\varepsilon(x)$.

The inverse problem of (1) can be posed as the following:

Given the incident field $E^i(k,x)$ and the reflected field $E_r(k,d)$ at a fixed position d, find the dielectric profile of $\varepsilon(x)$.

The inverse problem where $E_r(k,d)$ is known for all k, $(0 < k < \infty)$ can be solved by Gelfand and Levitan theory [4], which also proves the uniqueness of the inverse scattering problem. However, Gelfand and Levitan theory is limited to real $\varepsilon(x)$.

If the medium is lossy, i.e., complex $\varepsilon(x)$, and the information on the reflected field is incomplete, i.e., $E_r(k,d)$ is known at a finite number of frequencies, the Backus and Gilbert's iterative technique can be used [6]. We shall discuss the formulation and application of the Backus and Gilbert method in the following paragraph.

(2) Iterative Procedure

Consider two different wave equations of the type of eq. (1),

$$\left[\frac{d^2}{dx^2} + k^2 \varepsilon_n(x)\right] E_n(k,x) = 0 \qquad n = 1,2 \tag{2}$$

The scattering geometries are shown in Fig. 6. Multiplying the equation for n = 1 by E_2, and that for n = 2 by E_1, subtracting the resulting equations and integrating over the snow region, we get

$$\delta\rho(k) = \frac{k}{2j} \int_0^h \delta\varepsilon(x)\, E_1(k,x)\, E_2(k,x)\, dx \tag{3}$$

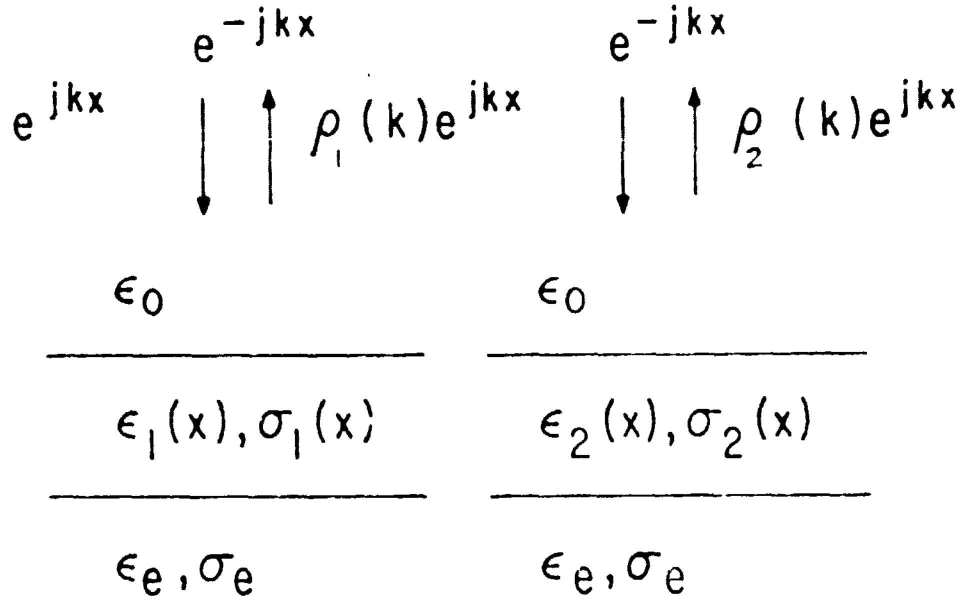

Fig. 6. Relevance of backscattering from layered media.

where $\delta\rho = \rho_2 - \rho_1$ and $\delta\varepsilon = \varepsilon_2 - \varepsilon_1$.

Let $\varepsilon_1(x)$ be a given profile, then by solving the direct problem numerically, $E_1(k,x)$ and $\rho_1(k)$ are known functions. If $\rho_2(k)$ were given from measurements, then eq. (3) is a nonlinear equation for $\varepsilon_2(x)$. One way of solving the nonlinear equation is by developing an iterative scheme, based on Newton's like method in functional space. The linearized equation to be solved at each step of the iteration scheme is given by

$$\delta\rho(k) = \frac{k}{2j} \int_0^h \delta\varepsilon(x)\, E_1^2\,(k,x)\, dx \tag{4}$$

where $\frac{k}{2j} E_1^2\,(k,x)$ is the Frechet derivative.

Normally, eq. (4) requires measurements of $\rho_2(k,d)$ for both magnitude and phase. Phase measurement is, in general, difficult to implement. To circumvent the requirement for phase measurement, we have reformulated eq. (4) by a transformation to,

$$g_i = \int_0^h f(x)\, L_i(x)\, dx \qquad i = 1,2,\ldots,N \tag{5}$$

where $g_i = \log \frac{|\rho_2(k_i)|}{|\rho_1(k_i)|}$, $f(x) = \varepsilon_2(x) - \varepsilon_1(x)$

and $L_i(x) = \mathrm{Im}\left[\frac{k_i}{2\rho_1(k_i)} E_1^2\,(k_i,x)\right]$, where the wave number k is discretized and the medium is assumed to be lossless. N represents the total number of measurements. Note eq. (5) requires only the magnitudes of backscattered signal.

(3) Resolving Power Theory

Suppose that after finite iterations a profile has been determined which fits the data. The question is then, how good is the resultant profile? This question can be answered by Backus and Gilbert theory [6], as follows.

After the iteration of (5) converges, which means $g_i = 0$, for $i = 1,2,\ldots,N$, we have

$$\alpha_i = \int_0^h \varepsilon_1(x)\, L_i(x)\, dx = \int_0^h \varepsilon_2(x)\, L_i(x)\, dx \qquad i = 1,2,\ldots,N. \tag{6}$$

where α_i and $L_i(x)$ are known. Multiply the equations by $a_i(x)$ and sum over i to obtain

$$\langle \varepsilon_2 \rangle_z = \int_0^h \varepsilon_2(x)\, A(z,x)\, dx \tag{7}$$

where

$$\langle \varepsilon_2 \rangle_z = \sum_{i=1}^{N} \alpha_i\, a_i(z) \tag{8}$$

$$A(z,x) = \sum_{i=1}^{N} a_i(z)\, L_i(x) \tag{9}$$

If the functions $a_i(z)$ are so selected that $A(z,x)$ resembles the Dirac Delta function $\delta(z-x)$, then $\langle \varepsilon_2 \rangle_z$ may be viewed as a local average of ε_2 at z. Further, the local average $\langle \varepsilon_2 \rangle_z$ is a common property of all the solutions that satisfy the data. The functions $a_i(z)$ are determined by

$$\min. \int_0^h (z-x)^2\, A^2(z,x)\,dx \tag{10}$$

$$\text{s.t.} \int_0^h A(z,x)\,dx = 1$$

Upon introducing a Lagrange multiplier, the minimization problem is reduced to a set of N+1 linear algebraic equations. Once the coefficients $a_i(z)$ are determined, the local average $\langle \varepsilon_2 \rangle_z$ is given by eq. (8) and the averaging kernel is computed by eq. (9).

Another such "Deltaness" criteria is the Dirichlet Kernel.

$$\begin{aligned} &\text{min.} \int_0^h [\delta(z-x) - A(z,x)]^2 \, dx \\ &\text{s.t.} \int_0^h A(z,x)dx = 1 \end{aligned} \tag{11}$$

Fig. 7 shows the averaging and the Dirichlet kernel. It is clear that there is a trade-off between resolution and side lobes.

In case the data is erroneous, one minimizes the spread of the kernel and the error in the solution. This leads to a trade-off between resolution and certainty.

(4) Results with Artificial Data

Fig. 8 presents results for a typical snow profile; the results shown in Fig. 2 have been obtained with data at 3 frequencies only, whereas the results shown in Fig. 8 have been obtained with data at 7 frequencies. The approximate results are in good agreement with the exact results for smooth profiles, as is evident from Fig. 2. The poor resolution near the edges of the ice layer (Fig. 8) may be improved by using more data. It will be noted that the computation time was about 7 seconds on a CDC 7600 computer for the profile reconstruction shown in Fig. 8. (computation cost ~ 2.5 $).

(5) Best Solution and Error Estimates

The linear Fredholm eq. (5) does not have a unique solution. Further, the data g may have some noise. Miller and Viano [5] have shown how to obtain solutions to eq. (5) with prescribed bounds and

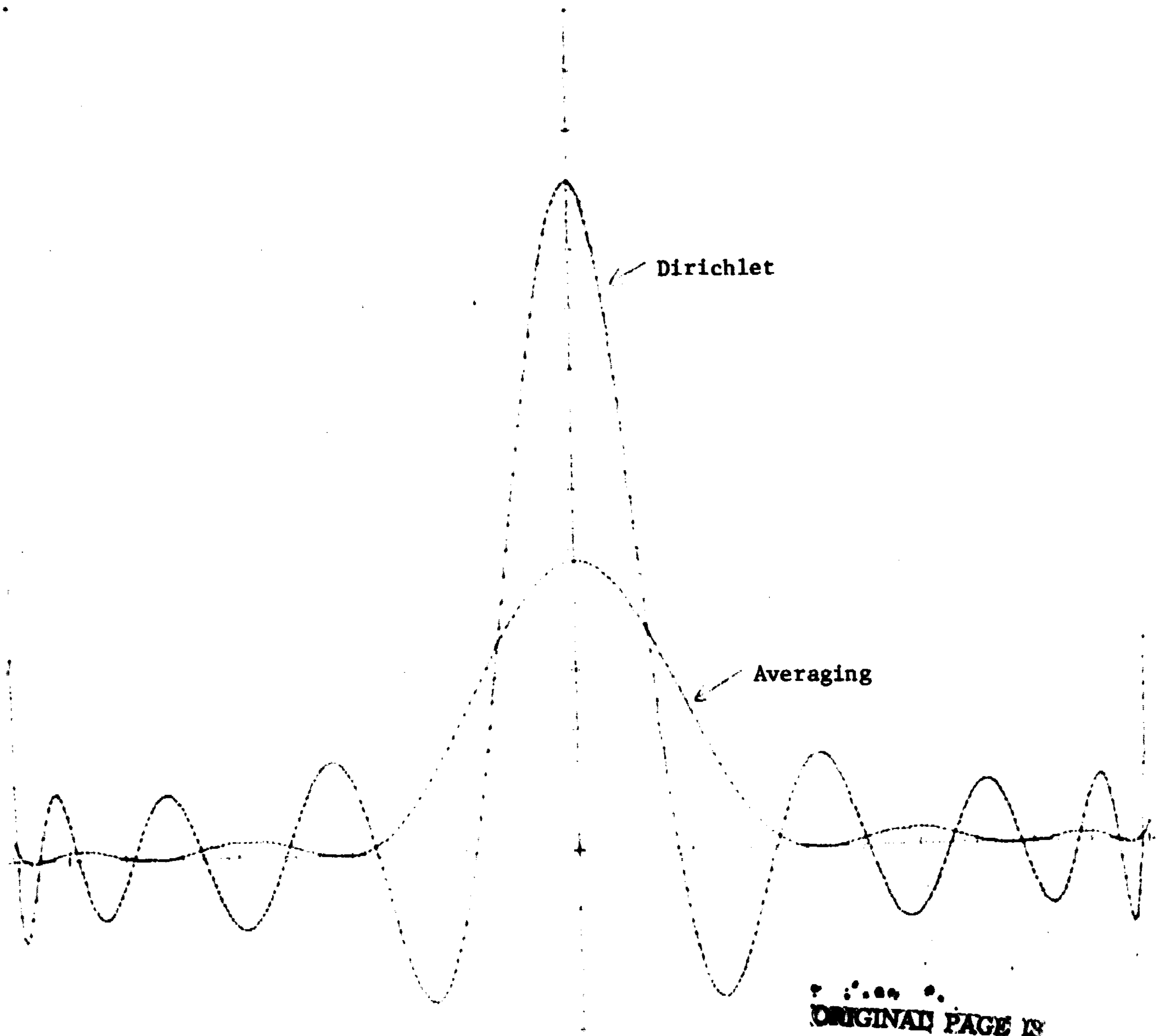

Fig. 7. Dirichlet and averaging kernels.

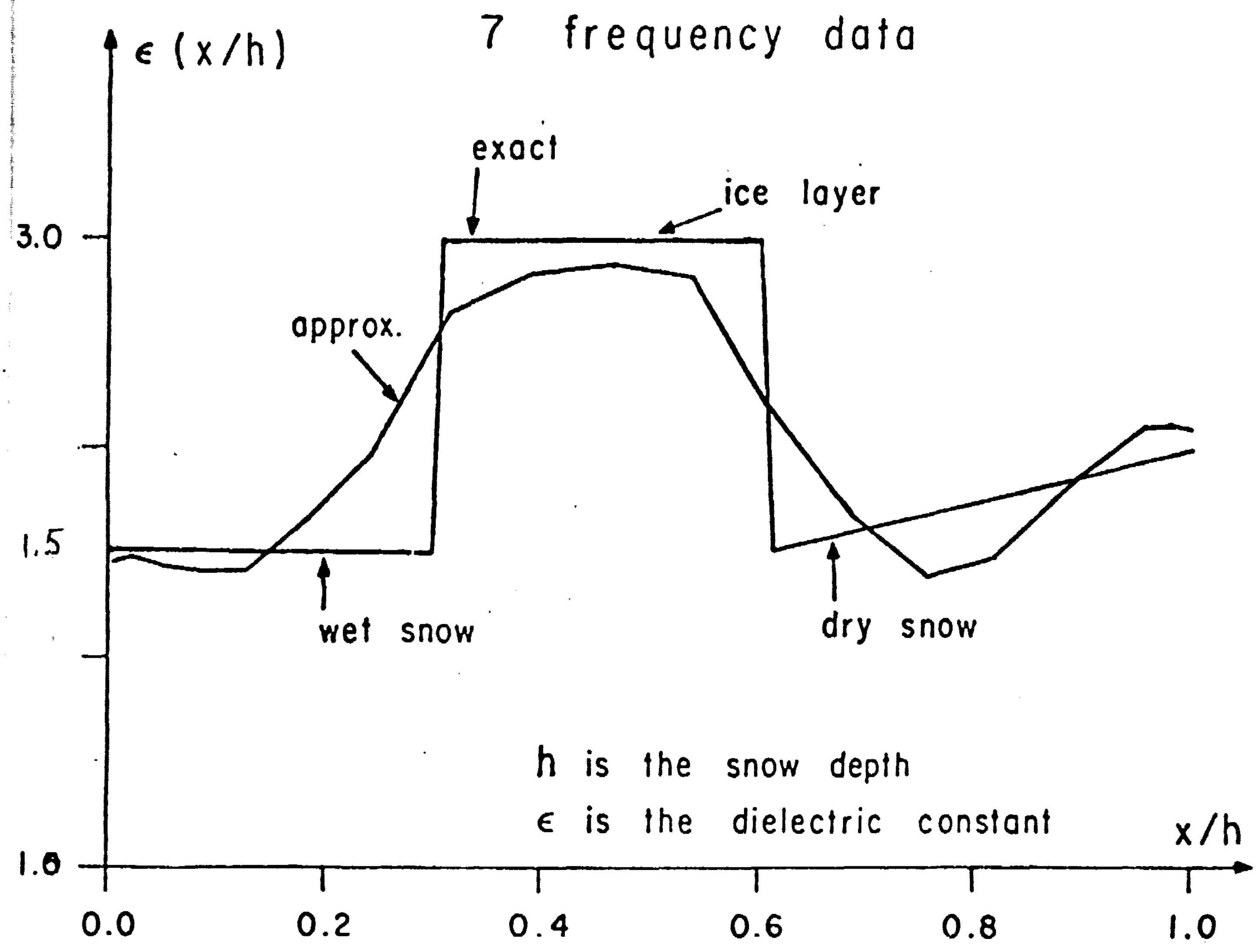

Fig. 8. Inverse scattering prediction of discrete layers.

best possible error estimates. They suggest to find f which satisfies

$$\begin{aligned} \|Lf-g\| &\leq \alpha \\ \|f\| &\leq \beta \end{aligned} \tag{12}$$

where α is a small number and β is a fixed number. Obviously α is related to the noise in the data and β may be obtained from the minimum and maximum values of the dielectric constant of the medium, which are assumed to be known. They further show that f can be obtained by minimizing the functional

$$J(f) = \|Lf-g\|^2 + (\tfrac{\alpha}{\beta})^2 \|f\|^2 \tag{13}$$

which leads to

$$Cf = [L^*L + (\tfrac{\alpha}{\beta})^2 I] f = L^*g \tag{14}$$

Eq. (14) can be inverted since L^*L is symmetric and $(\frac{\alpha}{\beta})^2 > 0$. The solution of (14) is obtained by expanding f in terms of the eigenfunctions of the symmetric operator L^*L. As suggested in ref. [5], the error bound can be extracted from C^{-1} in (14).

IV. FUTURE RESEARCH

Additional research needs to be conducted, both experimental and theoretical. Both are discussed in the following.

(1) Instrumentation and Measurements

(a) Backscattering at Low Frequencies

The type of measurements we have been doing at the snow laboratory at Tahoe should be continued and extended to lower frequencies. Also, the results should be recorded digitally for direct application in inverse scattering.

For operation at lower frequencies down to about 100 MHz the physical arrangement must be changed. Considering the wide percentage bandwidth needed, log periodic antennas could be used. For a range of 100 - 1000 MHz, the dimensions of the log periodic antenna will be on the order of 1-1/2 meters wide and 2-1/4 meters long, which would be too large to be mounted on a frame. It is believed that satisfactory operation can be obtained by suspending the antennas from a cable between two poles or trees with guy stabilizers.

Due to cyclic variation of gain with frequency and the added cable loss at the high frequency end of the 10:1 band, it is recommended that a storage-gain normalizer such as the HP 9750A be used. The response curve of the system as a function of frequency can be generated and stored in this instrument. These data can then be used to automatically normalize the measurements, eliminating the system error.

(b) Measurements in Controlled Environment

Measurements in controlled environment are a necessity in all experimentation. It is particularly true in the snow measurement because at the snow laboratory snow is only available in the winter season, and the instruments must be calibrated and ready when the snow season arrives. A sand pit can be used to simulate layered media, using polyethylene sheets to separate layers of different wetness. The experiments should include the following measurements:

(1) Calibration of instruments to be used in snow measurements.

(2) Gathering data for inverse scattering. This involves layered wet sand.

(3) Studying the effect of rough surfaces.

(c) Ground Truth Collection

(i) Interferometer for dielectric constant and attenuation measurements

One convenient method of obtaining snow truth for comparison with the various measurements has been the use of an X-band (9.3 Hz) interferometer. This set-up can give the dielectric constant and attenuation directly at its frequency of operation. The major problem of operating such equipment at VHF frequencies is the antenna, which illuminates the sample. However, it appears that corner antennas can be designed to fulfill this function in the range 500 - 1000 MHz.

(ii) Attenuation in snow

While attenuation is fairly easily measured in the interferometer by sampling, measurements in situ have not been so simple, due mainly to layering effects. A preliminary test last year made by burying a receiver in the snow and illuminating it vertically from the top appears to be a more satisfactory procedure. For this type of measurement, the receiver antenna and detector are best placed on ground level in a weatherproof enclosure before the snow falls. As the snow builds up during the winter, a transmitting unit can be positioned above as required. Thus no disturbance of the snow need occur.

(2) Theory

(a) Algorithm for Lossy Media Inverse Scattering

The inverse scattering results shown in Fig. 8 are for media of real dielectric constants. Future research will extend the inverse scattering technique to complex media. For this case, eq. (5) will be replaced by

$$g_i = \int_0^h f(x)\ L_i(x)dx + \int_0^h s(x)\ T_i(x)dx \qquad i = 1,2,\ldots,N \qquad (15)$$

where $L_i(x)$ and $T_i(x)$ are known and $f(x)$ and $s(x)$ are sought; here, $s(x) = \sigma_2(x) - \sigma_1(x)$ and $\sigma(x)$ denotes the conductivity profile.

The resolving power of the data in this case may be obtained by constructing averaging kernels which resemble $\delta_{ij}\ \delta(x-\xi)$, where δ_{ij} is the Kronecker delta and $\delta(x-\xi)$ is the Dirac delta function. Such kernels can be constructed by using the criteria suggested by Backus and Gilbert [7].

(b) Error Estimate

The best error estimates of the complex medium inverse scattering can also be achieved using the Miller and Viano (5) method, except now we have to replace (12) by

$$
\begin{aligned}
&\| Lf + Ts - g \| \leq \alpha \\
&\| f \| < \beta \\
&\| s \| \leq \gamma
\end{aligned}
\qquad (16)
$$

(c) Inverse Processing Experimental Data

The ultimate goal should be to process the data collected in the field and compare the predicted snow profile with ground truth.

V. CONCLUSION

The remote sensing of a snow pack is a complex technical problem. So far, neither the passive microwave technique (radiometry) nor the active microwave technique (radar) can claim to be able to solve the problem completely. Our effort has been unique in that its objective is to join experimentation and theory of inverse scattering. Our future goal is to predict the electrical properties of snow pack by remote sensing, which we believe is attainable. However, the useful information of the snow is its water content, (both liquid phase water and water equivalent) which is not just a function of the electrical property.

Temperature, for example, is an important parameter, affecting the relation between the electrical properties of snow and its water content. It is possible that the information gathered by the radiometry method will provide the temperature profile and together we have a good chance to eventually predict the water content of snow. We are confident that the fruit of our future research will bring the remote sensing technology to a new level of sophistication and push the frontier much closer to the final goal.

The purpose of this investigation is to provide radar remote sensing with the basic theoretical and experimental supports which it so badly needs. The objectives of future research can be summarized as follows:

(1) To identify the frequency bands most advantageous for remote sensing of ice, snow and soil moisture.

(2) To establish the theoretical relation between the radar return signals (at selected frequencies) and the dielectric profiles of the target areas.

(3) To design experiments in controlled environment to test the theory, and the statistical properties of the target area.

(4) To design experiments for measurement in field conditions and to develop techniques of gathering reliable ground truth information.

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VII. PARTICIPATING PERSONNEL

The personnel involved with the research of this grant were:

Prof. D. J. Angelakos,	Principal Investigator
Prof. K. K. Mei	Faculty Participant
F. D. Clapp	Associate Research Engineer
S. Coen	Research Assistant
J. Uyemura	Research Assistant

In addition, considerable cooperation was obtained from:

Dr. W. I. Linlor	NASA-Ames
Dr. J. L. Smith	U.S. Forest Service (use of Central Sierra Snow Lab)

VIII. DISSEMINATION OF INFORMATION (Research Results)

A. Coherent Microwave Backscatter of Natural Snowpacks
by W. I. Linlor, D. J. Angelakos, F. D. Clapp and J. L. Smith. Memorandum No. UCB/ERL M77/75, 15 Nov. 1977.

B. Microwave Scattering Properties of Snow Fields
by D. J. Angelakos. Microwave Remote Sensing Symposium, Houston, Texas, Dec. 6-7, 1977 (to appear as a Proceedings).

C. Microwave Remote Sensing Workshop
D. J. Angelakos participated as member of Water Resources Panel, Houston, Texas, Dec. 8-9, 1977.

D. Ph.D. Dissertation - S. Coen will be obtaining his Ph.D. degree late in 1978 shortly after termination of this grant. His dissertation work is based on the theoretical portion of the project.

E. Other Presentations - The Technical Officers have used the results for presentation at their review cycle (July, 1977).

APPENDIX

MICROWAVE SCATTERING PROPERTIES OF SNOW FIELDS

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ABSTRACT

A review is presented of the results obtained in the research program of microwave scattering properties of snow fields. The program is a cooperative effort between a group at the University of California, Berkeley (D. J. Angelakos, F. D. Clapp, K. K. Mei, and S. Coen) and NASA-Ames (W. I. Linlor). A goodly portion of the research was undertaken at the Central Sierra Snow Lab and with the active cooperation of the U.S. Forest Service (J. L. Smith).

Experimental results are presented showing backscatter dependence on a) frequency (5.8–8.0 GHz), b) angle of incidence (0–60 degrees), c) snow wetness (time of day), and d) frequency modulation (0–500 MHz). Theoretical studies are being made of the inverse scattering problem yielding some preliminary results concerning the determination of the dielectric constant of the snow layer.

The experimental results lead to the following conclusions: a) Snow layering affects backscatter, b) Layer response is significant up to 45 degrees of incidence, c) Wetness modifies snow layer effects, and d) Frequency modulation "masks" the layer response.

Introduction

At the Electronics Research Laboratory, University of California, Berkeley, the research program of microwave scattering properties of snow fields along two directions. One is the experimental determination of the microwave properties of snow fields and the other is a theoretical program of mathematical modeling and the inverse scattering problem. The ultimate goal of the project is the coalescence of the two into a useful diagnostic technique for the determination of water content in snow.

Experimental Portion of the Project - D. J. Angelakos, F. D. Clapp, W. I. Linlor and J. L. Smith: The primary objective of the experimental program was to determine the effect of snow conditions on the amount of backscattered electromagnetic energy. In addition the investigation was to seek out any effects on the backscatter due to snow layering and to determine the dependence on frequency and frequency deviation.

In proceeding towards the objective several additional phenomenological effects appeared which added insight to backscatter signal response. What follows is a sampling of results which indicates the partial attainment of our goal.

Equipment: Most of the data were taken for swept-frequency ranging from 5.8 to 8.0 GHz. The sweepers were the "Alfred" and the "Hewlett-Packard" sources. These were connected with coax cable to horns having nominally 20-dB gain. The receiver horns were coupled to crystal detectors and HP SWR meters, whose output was plotted on XY plotters, on a logarithmic scale. For some measurements a converter unit was employed (Wiltron manufacturer) that permitted a uniform dB scale for the Y-axis.

For the 13.5 GHz measurements a fixed-frequency solid state oscillator was used (Varian manufacturer). Time was not available to take measurements at other frequencies, either fixed or swept.

Sites: Two sites were used: one is called the "tower site" which is on the roof of the snow hut, for which only two fixed angles (39° and 45°) could be used; the other is called the "ladder site" where all desired angles could be obtained.

The limited available time — dictated by the snow depth — for measurements and also the limited manpower made a choice necessary between taking data or ground truth (i.e., snow electrical characteristics). Under the circumstances the ground truth had to be limited to:

a. Density profile of the snow
b. Visual inspection of layers in the snow and granularity
c. Dielectric constant measurement of a few samples, near the top and middle of the snowpack
d. Wetness measurement of those samples

Discussion of Experimental Results: The solid line of Fig. 1 shows the backscatter in dB versus frequency in the range of 5.8 to 8.0 GHz, at the fixed angle of 39° from the vertical, and a snow depth of 70 cm. The electric field vector of the horns (20 dB gain each for transmitter and receiver) was parallel to the earth surface.

The various peaks are produced by layering effects in the snow. The peak values are about 25 dB greater than the minima.

To then determine what effect frequency deviation (more specifically, frequency modulation) had on the measurement the signal source was reset to the frequency for each maximum and minimum, but a modulation of ±100 MHz was superimposed for the + points; and a modulation of ±200 MHz was superimposed for the 0 points. Evidently the effect of frequency modulation is merely the averaging of the unmodulated response over the bandwidth represented by the modulation.

In the course of taking measurements, whether at normal incidence or otherwise it was observed that the backscattering data depended on the observation time and temperature. Figures 2, 3 and 4 show the backscatter response at normal incidence. The experimental conditions for the three sets of data were identical except for the time taken and accompanying ambient temperature. Variation of backscatter with snow conditions, at 13.5 GHz and incidence angle of 39° is shown in Fig. 5.

The decrease in backscatter shortly before noon was initially observed by us in March 1976 for a snowpack 150 cm deep, and has been confirmed at other frequencies by several groups (Currie et al., 1976, Ulaby et al., 1977). Evidently the increase in surface snow wetness by solar heating is the reason for the backscatter decrease, totaling about 15 dB. The air temperature variation included is useful in correlation the phenomenological effects.

Snow wetness produced during a few hours of sunlight can significantly affect the frequency response. March 5, 1977, when the measurements were taken, was a clear, warm day with a temperature of about 10° C between noon and 16:00, so the snow was essentially saturated with water in the upper surface.

Figure 6 shows typical data demonstrating that variation in backscatter as a function of frequency can be observed even for other than normal incidence (39°). Significant backscatter data has been obtained for incidence angles approaching 45°.

A continuous variation of incidence angle from the vertical is shown in Figure 7, giving the backscatter in relative dB for the fixed frequency of 11.8 GHz (no modulation, with snow depth of 53 cm. The backscatter is down about 20 dB at the angle of 40° from the vertical.

Theoretical Portion of the Project - K. K. Mei and S. Coen: It is suggested that certain useful properties of the snow-pack may be approximately determined by using inverse scattering theory; the inverse problem is that of determining the properties of inaccessible snow pack from the back-scattered electric field due to normally incident plane waves at varies discrete frequencies.

Let the back-scattered frequency response of the snow-pack be taken as the reflection coefficient amplitude $|\rho(\omega_i)|$, $i = 1,2,\ldots,N$. Neglecting for the moment the experimental errors, the question is: how can we learn as much as possible about the characteristics of the snow-pack from a set of numbers $|\rho(\omega_i)|$, $i = 1,2,\ldots,N$? Alternatively what do the numbers $|\rho(\omega_i)|$, $i = 1,2,\ldots,N$ tell us about the dielectric constant and conductivity of the snow-pack? The density and wetness of the snow may be then approximately determined by using Weiner's theory of dielectric mixtures or other available experimental relations.

In what follows, we show that there is enough information in the numbers $|\rho(\omega_i)|$, $i = 1,2,\ldots,N$, to construct local averages of the snow profile.

Here the experimental errors are neglected and a mathematical model that describes the physical behavior of the snow-pack is chosen. The simplest model that we choose, is that of a plane stratified snow-pack, whose properties are characterized by a linear, isotropic and nonhomogeneous dielectric medium, as shown in Fig. 8a. Maxwell's equations now show that, for time harmonic dependence $e^{j\omega t}$, the electric field E, satisfies the one dimensional wave equation

$$\left\{\frac{\partial^2}{\partial x^2} + k^2 q(x,k)\right\} E(x,k) = 0 \tag{1}$$

where $k = \omega(\mu_o \varepsilon_o)^{1/2}$ is the free space wave number and $q(x,k) = \varepsilon(x) - j\,\frac{\sigma(x)}{\omega\varepsilon_o}$ is the profile function. Here $\varepsilon(x)$ is the dielectric constant of the snow and $\sigma(x)$ its conductivity.

Consider two media as shown in Fig. 8b.c. The electric field in the respective medium satisfies

$$\left\{\frac{\partial^2}{\partial x^2} + k^2 q_n(x,k)\right\} E_n(x,k) = 0 \qquad n = 1,2 \tag{2}$$

and the scattered fields are outgoing. Equation (2) can be reformulated into an integral equation form suitable for iterative solutions,

$$\delta\rho(k) = \frac{k}{2j}\int_0^h \delta q(x,k)\; E_1^2(x,k)dx + \alpha(k) \tag{3}$$

where we have used the relation $E_2 = E_1 + \delta E$ and $\delta\rho = \rho_2 - \rho_1$ and $\rho_2(k)$ were given from measurement without error. It may be shown that the term $\alpha(h)$ goes to zero as $\|\delta q\|^2$, where $\|\cdot\|$ is the L^2 norm. Thus, if δq is small enough, this term may be dropped, and we obtain a linear approximation

$$\delta\rho(k) = \frac{k}{2j}\int_0^h \delta q(x,k)\; E_1^2(x,k)\; dx \tag{4}$$

which is a linear Fredholm integral equation, for the determination of δq from the given functions $\delta\rho$ and E_1. Once δq is found, the profile q_2 is given by $q_2 = q_1+\delta q$. However, equation (4) is not exact, so the profile q_2 will not in general, satisfy the equation (2). To overcome this, we solve the equation by iterations, until convergence occurs. In practice few iterations will suffice.

In practice, the amplitude of the reflection coefficient is much to be preferred than its phase, and equation (4) can be rewritten into the form

$$g_i = \int_0^h \delta\varepsilon(x)\ F_i(x)dx \qquad i = 1,2,\ldots,N \tag{5}$$

where $g_i = \log\left|\frac{\rho_2(k_i)}{\rho_1(k_i)}\right|$, $\delta\varepsilon = \varepsilon_2-\varepsilon_1$ and $F_i(x) = \mathrm{Re}\{\frac{k_i}{2j\rho_1(k_i)}\ E_1^2(k_i,x)\}$; here the frequency k is discretized and the medium is assumed lossless. The approach to be discussed is however applicable to lossy medium as well.

We now show how to solve Eq. (5) by the Backus and Gilbert approach. Multiply the equation by $\mathcal{a}_i(\xi)$ and sum over i; this results in

$$\langle\delta\varepsilon\rangle_\xi = \int_0^h \delta\varepsilon(x)\ A(\xi,x)dx \tag{6}$$

where

$$\langle\delta\varepsilon\rangle_\xi = \sum_{i=1}^{N} \mathcal{a}_i(\xi)\ g_i \tag{7}$$

is the local average of $\delta\varepsilon$, and

$$A(\xi,x) = \sum_{i=1}^{N} \mathcal{a}_i(\xi)\ F_i(x) \tag{8}$$

is the averaging kernel.

The coefficients $\mathcal{a}_i(\xi)$, $i = 1,2,\ldots,N$ are determined, such that the averaging kernel resembles the Dirac-Delta function most closely. One way of doing this is by minimizing the quadratic form

$$\int_0^h (\xi-x)^2 A^2(\xi,x)dx \tag{9}$$

subject to

$$\int_0^h A(\xi,x)\,dx = 1 \tag{10}$$

for each ξ; this minimization problem may be reduced to the linear equation upon introducing a Lagrange multiplier.

Once the coefficients $a_i(\xi)$ are determined, the local average $\langle \delta\varepsilon \rangle_\xi$ is computed via equation (7).

In Figs. 9 and 10 we present results for 3 different snow profiles; the results shown in Figs 9a and 9b have been obtained with data at 3 frequencies only, whereas, the results shown in Fig. 10 have been obtained with data at 7 frequencies. The approximate results obtained from our analysis are in good agreement with the exact results for smooth profiles, as is evident from Figs. 9a and 9b. The poor resolution near the edges of the ice layer (Fig. 10) may be improved by using more data. It will be noted that the computation cost is about 5 dollars per profile on CDC 7600.

We are currently investigating the influence of experimental errors on the resulting snow profiles.

We find that there is a tradeoff between resolution and uncertainty, which is in agreement with the Backus and Gilbert theory.

References

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