Airplane Stability Calculations
With a Card Programmable Pocket Calculator

Windsor L. Sherman

AUGUST 1978
ERRATA

NASA Technical Memorandum 78678

AIRPLANE STABILITY CALCULATIONS WITH A CARD PROGRAMMABLE POCKET CALCULATOR

Windsor L. Sherman

August 1978

Please make the following corrections:

Page 15: Sentence after equation (11) should read as follows:

Equations (9) and (10) were programmed for the calculator and the program is given in appendix B.

Page 16: Equation (16) should read as follows:

\[ \text{Re}(y) = S + T - \frac{b_2}{3} \]

Page 24, Last sentence: Change step 49 to step 45.

Page 25: Step 100 should read as follows:

\[ \text{STO} \times 9 \quad (g \sigma_T/2U_{ss}) \sin 2\gamma_{ss} \]

Page 26, Step 105: Change - to +

Step 141: Change RCL8 to RCLB

Page 29: Delete the last sentence.

Page 49: In column headed "Output," change the values of \( a_3, \ a_2, \ a_1, \ a_0, \) and \( a_{12} \) to

\[ a_3 = 1.3980958 \]
\[ a_2 = 1.1093007 \]
\[ a_1 = -0.0098076 \]
\[ a_0 = -0.0211448 \]
\[ a_{12} = 0.0373094 \]

ISSUED NOVEMBER 1978
ERRATA

NASA Technical Memorandum 78737

DEVELOPMENT OF A NONLINEAR SWITCHING FUNCTION AND ITS APPLICATION TO STATIC LIFT CHARACTERISTICS OF STRAIGHT WINGS

Donald E. Hewes
September 1978

Page 5: Equation (3) should read

\[ x_{10} = x_e \left( \frac{\ln e}{\ln 10} \right)^{1/2} = x_e \left( \frac{1}{\ln 10} \right)^{1/2} \]
Airplane Stability Calculations
With a Card Programmable
Pocket Calculator

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SUMMARY

Programs are presented for calculating airplane stability characteristics with a card programmable pocket calculator. These calculations include eigenvalues of the characteristic equations of lateral and longitudinal motion as well as stability parameters such as the time to damp to one-half amplitude or the damping ratio. The effects of wind shear are included. Background information and the equations programmed are given. The programs are written for the International System of Units, the dimensional form for the stability derivatives, and stability axes. In addition to the programs for stability calculations, an unusual and short program is included for the Euler transformation of coordinates used in airplane motions. The programs have been written for a Hewlett Packard HP-67 calculator. However, the use of this calculator does not constitute an endorsement of the product by the National Aeronautics and Space Administration.

INTRODUCTION

Over the past several years, the programmable pocket calculator has developed into a highly sophisticated device that has almost computer characteristics. Because of its sophistication, the newer models are capable of being programmed to make very complicated calculations. Since different logics are used in programmable calculators and since the available keyboard instructions vary with models of different manufacturers, it is necessary to identify the make and model of the calculator for which a program is written. The airplane stability programs presented in this paper were written for a Hewlett Packard HP-67 card programmable calculator; however, its use and identification in this report does not constitute an endorsement of the product by the National Aeronautics and Space Administration.

Programs are given for the calculation of the coefficients of the airplane lateral and longitudinal characteristic equations, the eigenvalues, and the stability parameters such as the time to damp to one-half amplitude or the damping ratio. In addition, a unique coordinate transformation program is given for transformations between inertial axes and airplane body axes. This program requires very few program steps and may be useful as part of a larger program. The equations on which the programs are based are given so that the programs can be readily adapted to other calculators that have sufficient program capacity.

The programs presented herein evolved during the study of wind shear and its effect on airplane stability and control. These programs proved useful in making stability calculations in this study and should be of use in other investigations.
SYMBOLS

A aspect ratio

$a_0, a_1, \ldots, a_5$ coefficients of characteristic equations

$a_{12}, a_{13}, a_{14}, \ldots$ elements of longitudinal stability determinant

b wing span

$b_2, b_1, b_0$ coefficients of resolvent cubic

$C_D$ drag coefficient, $\frac{D}{\rho S u_{ss}^2/2}$

$C_{D,0}$ drag coefficient for $C_L = 0$

$C_{D\alpha} = \frac{\partial C_D}{\partial \alpha}$

$C_L$ lift coefficient, $\frac{L}{\rho S u_{ss}^2/2}$

$C_{L,0}$ lift coefficient at zero angle of attack

$C_{L\alpha} = \frac{\partial C_L}{\partial \alpha}$

$C_{L\alpha} = \frac{\partial C_L}{\partial \alpha}$

$C_{L\theta} = \frac{\partial C_L}{\partial \theta}$

$C_l$ rolling-moment coefficient, $\frac{M_X}{\rho S b u_{ss}^2/2}$

$C_{l_p} = \frac{\partial C_l}{\partial p}$

$C_{l_r} = \frac{\partial C_l}{\partial r}$
\[ c_{1\beta} = \frac{\partial c_l}{\partial \beta} \]

\[ c_{1\phi} = \frac{\partial c_l}{\partial \phi} \]

\[ c_m = \text{pitching-moment coefficient, } \frac{M_Y}{\rho S c U_{\infty}^2/2} \]

\[ c_{m,\alpha} = \text{total pitching-moment coefficient at zero angle of attack} \]

\[ c_{m\alpha} = \frac{\partial c_m}{\partial \alpha} \]

\[ c_{m\phi} = \frac{\partial c_m}{\partial \phi} \]

\[ c_n = \text{yawing-moment coefficient, } \frac{M_z}{\rho S b U_{\infty}^2/2} \]

\[ c_{n\rho} = \frac{\partial c_n}{\partial \rho} \]

\[ c_{n_r} = \frac{\partial c_n}{\partial r} \]

\[ c_{n\beta} = \frac{\partial c_n}{\partial \beta} \]

\[ c_{n\phi} = \frac{\partial c_n}{\partial \phi} \]
\begin{align*}
C_{n\phi} & = \frac{\partial C_n}{\partial \phi} \\
C_T & \text{ thrust coefficient} \\
C_{Tu} & = \frac{\partial C_T}{\partial u} \\
C_Y & \text{ side-force coefficient, } \frac{F_Y}{\rho s u^2/2} \\
C_{Yp} & = \frac{\partial C_Y}{\partial p} \\
C_{Yr} & = \frac{\partial C_Y}{\partial r} \\
C_{Y\beta} & = \frac{\partial C_Y}{\partial \beta} \\
C_{Y\phi} & = \frac{\partial C_Y}{\partial \phi} \\
\{C_{11}, C_{21}, C_{30}, b_{11}, b_{12}, b_{13}, \ldots \} & \text{ terms in lateral stability determinant} \\
\bar{c} & \text{ mean aerodynamic chord} \\
D & \text{ drag} \\
F_T & \text{ thrust} \\
F_{T, tr} & \text{ trim thrust} \\
F_{Tu} & = \frac{\partial F_T}{\partial u} \\
F_X, F_Y, F_Z & \text{ forces along } X, Y, \text{ and } Z \text{ stability axis} \\
F_{X\delta_e} & = \frac{\partial F_X}{\partial \delta_e}
\end{align*}
\[ F_{Y\delta_a} = \frac{\partial F_Y}{\partial \delta_a} \]
\[ F_{Y\delta_r} = \frac{\partial F_Y}{\partial \delta_r} \]
\[ F_{Z\delta_e} = \frac{\partial F_Z}{\partial \delta_e} \]

\( g \) acceleration of gravity

\( I_X, I_Y, I_Z \) moments of inertia, stability axes

\( I_{XZ} \) product of inertia, stability axes

\( \text{Im}(\cdot) \) imaginary part of complex root

\( k_X, k_Y, k_Z, k_{XZ} \) radii of gyration, stability axes

\( L \) lift

\( M_X, M_Y, M_Z \) moments about X, Y, and Z stability axes

\[ M_{X\delta_a} = \frac{\partial M_X}{\partial \delta_a} \]
\[ M_{X\delta_r} = \frac{\partial M_X}{\partial \delta_r} \]
\[ M_{Y\delta_e} = \frac{\partial M_Y}{\partial \delta_e} \]
\[ M_{Z\delta_a} = \frac{\partial M_Z}{\partial \delta_a} \]
\[ M_{Z\delta_r} = \frac{\partial M_Z}{\partial \delta_r} \]

\( m \) mass

\( N_D \) number of cycles to double amplitude

\( N_{1/2} \) number of cycles to damp to one-half amplitude
p  rolling velocity
R  radius
Re( ) real part of complex root
Re(y) real root of resolvent cubic
r  yawing velocity
S  wing area
t  period
tD  time to double amplitude
t1/2 time to damp to one-half amplitude
USS  steady-state velocity
Uw  wind velocity
upr  perturbation velocity
uw'  wind shear gradient
ww'  updraft-downdraft gradient
X,Y,Z  stability axes
Xb,Yb,Zb  airplane body axes
Xe,Ye,Ze  Earth-fixed axes
Xsp,Ysp,Zsp  space axes
x,y,z  general variables
xb,yb,zb  body axis coordinates
xsp,ysp,zsp  space axis coordinates
αpr  perturbation angle of attack
αtr  trim angle of attack
{α1,α2,α3,α4}  real roots
β  sideslip angle
γpr  perturbation flight-path angle
\( \gamma_{ss} \) steady-state flight-path angle
\( \Delta \) logarithmic decrement
\( \delta_a \) aileron deflection
\( \delta_e \) elevator deflection
\( \delta_r \) rudder deflection
\( \varepsilon_1, \varepsilon_2 \) angles
\( \zeta \) damping ratio
\( \theta_{tr} \) trim pitch angle
\( \rho \) atmospheric density
\( \sigma_T = \sigma_u + \sigma_w \)
\( \sigma_u = \frac{U_{ss}w'}{g} \)
\( \sigma_w = \frac{U_{ss}w'}{g} \)
\( \psi, \theta, \phi \) airplane yaw (heading), pitch, and roll angles, respectively
\( \omega_n \) undamped circular frequency

Dot over a symbol indicates differentiation with respect to time.

**EQUATIONS PROGRAMMED AND PROGRAM DESCRIPTIONS**

Six programs are presented in this paper. The first three calculate the elements of the lateral and longitudinal stability determinants and the coefficients of the characteristic equations. In addition, program 3 extracts a real root of a fifth-order polynomial when required. Programs 4 and 5 complete the root extraction process and calculate the stability parameters. Program 6 implements the Euler angle transformation by using the polar-rectangular keys found on calculators.

Programs 1, 2, and 3 are written for the International System of Units, stability axes (fig. 1), and the dimensional form of the stability derivatives. The equations programmed are the linearized form of the equations of motion derived in appendix A of reference 1; thus, the effects of wind shear are included.
In deriving these equations, head winds and updrafts were taken as negative. Thus, a positive $u_w'$ will change a head wind into a tail wind, and a positive $w_w'$ will change an updraft into a downdraft. The signs of $u_w'$ and $w_w'$ set the signs of $\sigma_u$ and $\sigma_w$; $u_w'$ is a gradient with altitude and $w_w'$ is a gradient along the flight path.

In writing the programs, the following conventions were used for the labels:

1. Capital letters (A to E) are program labels
2. Lower-case letters (a to e) are subroutine labels
3. Numbers (0 to 9) are used for all other labels

Table I summarizes the programs presented in this paper. The key entries given in appendixes A to F are the standard HP-67 key entries given in the owner's manual. Check cases for all programs are given in appendix G.

**Table I.- Summary of Programs**

<table>
<thead>
<tr>
<th>Program</th>
<th>Description</th>
<th>Key entries given in</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Calculates the elements of longitudinal stability determinant and normalized coefficients for characteristic equation</td>
<td>Appendix A</td>
</tr>
<tr>
<td>2</td>
<td>Calculates the elements of lateral stability determinant and starts calculating coefficients of the characteristic equation</td>
<td>Appendix B</td>
</tr>
<tr>
<td>3</td>
<td>Label A completes calculating coefficients of characteristic equations of lateral motion; label B calculates a real root of a fifth-order polynomial and reduces the fifth-order polynomial to a fourth-order one; $t_{1/2}$ or $t_D$ for the real root determined; label B can be used as a stand-alone program</td>
<td>Appendix C</td>
</tr>
<tr>
<td>4</td>
<td>Uses Ferrari's method to calculate the roots of a fourth-order polynomial and can be used as a stand-alone program; will also determine roots of cubic, quadratic, and first-order equations</td>
<td>Appendix D</td>
</tr>
<tr>
<td>5</td>
<td>Calculates stability parameters such as $t_{1/2}$, $t_D$, and $N_{1/2}$</td>
<td>Appendix E</td>
</tr>
<tr>
<td>6</td>
<td>Uses the polar-rectangular transformations of the calculator to implement the Euler transformation between space and body axes or body and space axes; this method saves about 57 program steps when compared with the more usual methods of programming</td>
<td>Appendix F</td>
</tr>
</tbody>
</table>
Programs 1 and 2 give solutions from an equilibrium flight condition. There are six parameters, $U_{ss}$, $Y_{ss}$, $\alpha_{tr}$, $P_{T_tr}$, $\sigma_T$, and $\sigma_w$, that must be adjusted correctly to obtain the equilibrium flight condition. There are two equations to accomplish this adjustment. Programs 1 and 2 were set up in the following manner. The parameters $U_{ss}$, $Y_{ss}$, $\sigma_T$, and $\sigma_w$ are specified by the user. The program calculates $\alpha_{tr}$, assuming that $P_{T_tr}$ is 0. For the flight condition $U_{ss} = 77.12$ m/sec, $Y_{ss} = -0.05236$ rad, $\sigma_T = 2.0$, and $\sigma_w = 0.0$, the error introduced in $\alpha_{tr}$ by this method is 0.000081 rad, which is considered acceptable. If it is desired to monitor the calculated value of $\alpha_{tr}$, insert a pause after step 45 of program 1.

**Program 1**

The linearized equation of longitudinal motion is in symbolic form

\[
\begin{bmatrix}
\frac{d}{dt} + a_{12} & a_{21} & a_{31} \\
a_{13} & \frac{d}{dt} + a_{22} & a_{32} + a_{33} \\
a_{14} & \left(\frac{d}{dt}\right)^2 + a_{25} \frac{d}{dt} + a_{26} & \left(\frac{d}{dt}\right)^2 + a_{35} \frac{d}{dt} + a_{33}
\end{bmatrix}
\begin{bmatrix}
u \\ \alpha_{tr} \\ \gamma_{pr}
\end{bmatrix} = \begin{bmatrix}
F_{X_{\delta_e}} \\
F_{Z_{\delta_e}} \\
M_{Y_{\delta_e}}
\end{bmatrix}
\]

(1)

The characteristic equation for longitudinal stability is obtained from the determinant of the $3 \times 3$ matrix and has the form

\[
a_4\left(\frac{d}{dt}\right)^4 + a_3\left(\frac{d}{dt}\right)^3 + a_2\left(\frac{d}{dt}\right)^2 + a_1 \frac{d}{dt} + a_0 = 0
\]

(2)

where $a_0$ to $a_4$ are given by

\[
a_4 = a_{22} - a_{32}
\]

\[
a_3 = a_{12}(a_{22} - a_{32}) + (a_{23} - a_{33} - a_{25}a_{32} + a_{22}a_{35})
\]

\[
a_2 = (a_{23}a_{35} - a_{25}a_{33} - a_{26}a_{32}) + a_{12}(a_{23} - a_{33} - a_{25}a_{32} + a_{22}a_{35})
\]

+ $a_{13}(a_{31} - a_{21})$

(3a, 3b, 3c)
\[ a_1 = a_{12}(a_{23}a_{35} - a_{25}a_{33} - a_{26}a_{32}) + a_{31}(a_{13}a_{25} - a_{14}a_{22}) \]
\[ - a_{21}(a_{13}a_{35} - a_{14}a_{32}) - a_{26}a_{33} \]  
\[ (3d) \]
\[ a_0 = a_{33}(a_{21}a_{14} - a_{26}a_{12}) + a_{31}(a_{13}a_{26} - a_{14}a_{23}) \]  
\[ (3e) \]

and \( a_{12}, a_{13}, \mathrm{etc.}, \) are given by

\[ a_{12} = \left(-\frac{g\sigma_T}{2U_{ss}}\sin2\gamma_{ss} + \left(C_{D,0} + \frac{C_{L}^2}{\pi\Lambda}\right)k_1 - \frac{F_{Tu}}{m}\right) \]

\[ a_{13} = C_Lk_1 - \frac{g}{U_{ss}}(\sigma_T \sin^2 \gamma_{ss} - \sigma_w) \]

\[ a_{14} = \left(-C_{m,0} + C_{m,\alpha tr}\right)k_2 \]

\[ a_{21} = C_{D,\alpha}k_3 \]

\[ a_{22} = \left(C_{L, \alpha} + C_{L, \delta}\right)k_3 \]

\[ a_{23} = C_{L, \alpha}k_3 \]

\[ a_{25} = \left(-C_{m, \delta} + C_{m, \alpha}\right)k_4 \]

\[ a_{26} = -C_{m, \alpha}k_4 \]

\[ a_{31} = g(\cos\gamma_{ss} - \sigma_T \cos2\gamma_{ss}) \]

\[ a_{32} = -U_{ss} + C_{L, \delta}k_3 \]

\[ a_{33} = g(\sin\gamma_{ss} - \sigma_T \sin2\gamma_{ss}) \]

\[ a_{35} = -C_{m, \delta}k_4 \]  
\[ (4) \]
where \( k_1 = \frac{\rho S U_{SS}}{m} \), \( k_2 = \frac{\rho S C U_{SS}}{I_Y} \), \( k_3 = \frac{\rho S U_{SS}^2}{2m} \), and \( k_4 = \frac{\rho S C U_{SS}^2}{2I_Y} \). In addition to the foregoing equations, the following equations are needed to calculate the values of \( C_L \), \( C_D \), and \( \alpha_{tr} \) at trim:

\[
C_L = \frac{2mg}{\rho S U_{SS}^2} (\sigma_T \sin^2 \gamma_{SS} - \sigma_w + \cos \gamma_{SS}) \tag{5a}
\]

\[
C_D = C_{D, o} + \frac{C_L^2}{\pi a} \tag{5b}
\]

\[
\alpha_{tr} = \frac{C_L - C_{L, o}}{C_{L, a}} \tag{5c}
\]

Because large changes in forward speed are encountered in wind shear, the effects of the \( u \) stability derivatives not normally accounted for are included in this program. This was done in the following manner:

\[
D_u = \frac{\partial D}{\partial u} = \left( C_{D, o} + \frac{C_L^2}{\pi a} \right) k_1 \tag{used in eq. 4}
\]

\[
L_u = \frac{\partial L}{\partial u} = C_L k_1 \tag{used in eq. 4}
\]

\[
M_{Y_u} = \frac{\partial M_Y}{\partial u} = \left( C_{m, o} + C_{m_a} \alpha_{tr} \right) k_2 \tag{used in eq. 4}
\]

Equations (3), (4), and (5) were programmed to calculate the coefficients of the characteristic equation, which is equation (2). The key codes for program 1 are given in appendix A.

The program destroys the original input data but preserves the coefficients of the determinant in the secondary registers. The principal output is the 'normalized coefficients of the characteristic equation which are stored in \( R_0, R_1, R_2, \) and \( R_3 \).
The linearized equations of lateral motion with the effects of wind shear included are, in symbolic form,

\[
\begin{bmatrix}
C_{11} \frac{d}{dt} + b_{13} & C_{21} \frac{d}{dt} + b_{22} & C_{30} \frac{d}{dt} + b_{31} \\
\left(\frac{d}{dt}\right)^2 + b_{14} \frac{d}{dt} + b_{15} & b_{23} \left(\frac{d}{dt}\right)^2 + b_{32} \frac{d}{dt} + b_{33} \\
b_{43} \left(\frac{d}{dt}\right)^2 + b_{16} \frac{d}{dt} + b_{17} & \left(\frac{d}{dt}\right)^2 + b_{24} \frac{d}{dt} + b_{25} & b_{34} \frac{d}{dt} + b_{35}
\end{bmatrix}
\begin{bmatrix}
\phi \\
\psi \\
\beta
\end{bmatrix}
= 
\begin{bmatrix}
F_{Y\delta_a} & F_{Y\delta_r} \\
M_{X\delta_a} & M_{X\delta_r} \\
M_{Z\delta_a} & M_{Z\delta_r}
\end{bmatrix}
\begin{bmatrix}
\delta_a \\
\delta_r
\end{bmatrix}
\]  

(6)

and are based on equations (A5), (A8), and (A9) of reference 1. In linearizing these equations, it was assumed that no wind gradient existed in the \( Y_e \) derivative in Earth axes. If the wind gradients are zero (i.e., no wind shear), these equations reduce to the standard form of the linearized equations of lateral motion that are given in many standard works, such as reference 2. The equations are valid in the interval \(-0.17453 \leq \gamma_{ss} \leq 0.17453\).

The characteristic equation is obtained from the \( 3 \times 3 \) matrix on the left-hand side of equation (6) and has the form

\[
a_5 \left(\frac{d}{dt}\right)^5 + a_4 \left(\frac{d}{dt}\right)^4 + a_3 \left(\frac{d}{dt}\right)^3 + a_2 \left(\frac{d}{dt}\right)^2 + a_1 \frac{d}{dt} + a_0 = 0
\]  

(7)

for \( \sigma_T \neq 0 \).

When \( \sigma_T = 0 \), the \( a_0 \) term in equation (7) becomes 0. Equation (7) now has one zero root and four finite roots and is solved as a quartic. Program 2 tests equation (7) and informs the user if a fourth- or fifth-degree polynomial is present. The coefficients \( a_0 \) to \( a_5 \) are given by

\[
a_5 = C_{30} (1 - b_{43} b_{42})
\]  

(9a)

\[
a_4 = C_{11} (b_{42} b_{34} - b_{32}) - C_{21} (b_{34} - b_{43} b_{32}) + b_{31} (1 - b_{43} b_{42})
\]  

(9b)

+ \[ C_{30} (b_{24} + b_{14} - b_{43} b_{23} - b_{16} b_{42}) \]
The \( C \) and \( b \) terms that are used to generate the coefficients of the characteristic equation are given by

\[
b_{11} = 0.0 \quad (10a)
\]

\[
b_{12} = -C_{y_p} k_5 \quad (10b)
\]

\[
b_{13} = -\frac{g\sigma_w}{u_{S_s}^2} - \frac{g}{u_{S_s}} \cos \gamma_{S_s} \quad (10c)
\]

\[
b_{14} = -C_{i_p} k_6 \quad (10d)
\]

\[
b_{15} = u_w \left( -C_{i_p} k_6 \right) = C_{i_p} k_6 \quad (10e)
\]

\[
b_{16} = -C_{n_p} k_7 \quad (10f)
\]

\[
b_{17} = u_w \left( -C_{n_p} k_7 \right) = C_{n_p} k_7 \quad (10g)
\]

\[
b_{21} = -C_{y_r} k_5 \quad (10h)
\]
\[
b_{22} = \frac{g(\sigma_u + \sigma_w)}{2u_{ss}^2} \sin 2\gamma_{ss} \tag{10i}
\]

\[
b_{23} = -C_{l_k}k_6 \tag{10j}
\]

\[
b_{24} = -C_{n_k}k_7 \tag{10k}
\]

\[
b_{30} = -C_{\theta_k}k_5 \tag{10l}
\]

\[
b_{31} = -C_{\psi_k}k_5 \tag{10m}
\]

\[
b_{32} = -C_{l_{\beta_k}}k_6 \tag{10n}
\]

\[
b_{33} = -C_{l_{\beta_k}}k_6 \tag{10o}
\]

\[
b_{34} = -C_{n_{\beta_k}}k_7 \tag{10p}
\]

\[
b_{35} = -C_{n_{\beta_k}}k_7 \tag{10q}
\]

\[
b_{42} = -\frac{I_{xz}}{I_x} \tag{10r}
\]

\[
b_{43} = -\frac{I_{xz}}{I_z} \tag{10s}
\]

\[
C_{11} = b_{11} + b_{12} \tag{10t}
\]

\[
C_{21} = 1 + b_{21} \tag{10u}
\]

\[
C_{30} = 1 + b_{30} \tag{10v}
\]

where 
\[
k_5 = \frac{\rho u_{ss}}{2m}, \quad k_6 = \frac{\rho Su_{ss}^2}{2I_x}, \quad \text{and} \quad k_7 = \frac{\rho Su_{ss}^2}{2I_z}.
\]

The trim angle of attack
was calculated from

\[ \alpha_T = \left[ \frac{2mg}{\rho S U_{SS}^2} \left( \sigma_T \sin^2 \gamma_{SS} - \sigma_w + \cos \gamma_{SS} \right) \right] - C_{L,0} \left( C_{L,\alpha} \right)^{-1} \]  \tag{11} \]

Equations (9), (10), and (11) were programmed for the calculator and the program is given in appendix B. The stability derivatives \( C_{l,\phi}, C_{n,\phi}, \text{and } C_{\gamma,\phi} \) have been included in this program. The derivatives \( C_{l,\phi} \) and \( C_{n,\phi} \) are always calculated when wind shears are included. This program calculates all \( b \) and \( C \) coefficients in the determinant and starts calculating the coefficients of the characteristic equation.

Program 3

Program 3 completes the calculation of the coefficients of the characteristic equation and tests the contents of register 4 to determine if the equation is a quartic or a quintic. If it is a quartic, the number 4 is displayed and the calculator stops. If it is a quintic, the number 5 is displayed and the calculator stops. Label B of this program calculates a real root of the quintic equation by using the secant method. The initial guess for the root is obtained by dividing the coefficient \( a_4 \) by 5; this operation is done in the program.

Even with an estimate of the root, however, two estimated points are required to start the secant method. These points are obtained by either adding or subtracting 0.08 from \( a_4/5 \). The number 0.08 has proved satisfactory for several different fifth-order polynomials. However, the secant method is sensitive to this number and changes may be necessary. Subsequent estimates of the root were calculated from

\[ x_{i+1} = x_i - \frac{x_i - x_{i-1}}{f(x_i) - f(x_{i-1})} \]  \tag{12} \]

where \( x_i \) is always the present value and \( f(x_i) \) is the value of the function being used for \( x = x_i \). Synthetic division is used to determine when a root had been found. The fifth-order polynomial is then reduced to a quartic for processing by program 4. During the iteration process, the calculator pauses to display the value of the characteristic polynomial so that convergence can be monitored. When the display shows zero, the root has been found. When the root has been found, the calculator will stop and display 8. The root is in the \( Y \) stack register and the time to damp to one-half amplitude or time to double amplitude is in the \( Z \) register. A negative number in the \( Z \) register means that the value given is the time to double amplitude. The number of iterations required to extract the root is in the \( T \) stack register.
The test used for the determination of a root is that the polynomial must be zero to the number of digits in the calculator display; thus, the test for a root assures its accuracy.

Program 4

A quartic equation is the highest order polynomial for which an explicit analytical solution for the root exists. Ferrari's method (refs. 3 and 4) and appendix H was used to obtain the roots of the quartic from the characteristic equation. The general form of the quartic equation is

\[ a_4x^4 + a_3x^3 + a_2x^2 + a_1x + a_0 = 0 \]  (13)

The first step in applying Ferrari's method is to normalize equation (13) so that \( a_4 = 1 \). The determination of a real root of the following resolvent cubic is the next step:

\[ y^3 + b_2y^2 + b_1y + b_0 = 0 \]  (14)

The coefficients of equation (14) are given by

\[
\begin{align*}
    b_2 &= -a_2 \\
    b_1 &= a_1a_3 - 4a_0 \\
    b_0 &= a_0\left(4a_2 - a_3^2\right) - a_1^2
\end{align*}
\]  (15)

and the root \( \text{Re}(y) \) is obtained by

\[
\text{Re}(y) = S + T = -\frac{b_2}{3} \quad (f > 0)
\]  (16)

or

\[
\text{Re}(y) = 2(R^2 + f)^{1/3} \cos \left[ \frac{1}{3} \left( \tan^{-1} \frac{\sqrt{\bar{f}}}{R} \right) \right] - \frac{b}{2} \quad (f \leq 0)
\]  (17)

where

\[
\begin{align*}
    Q &= (3b_1 - b_2^2)/9 \\
    R &= (9b_2b_1 - 27b_0 - 2b_2^3)/54 \\
    f &= R^2 + Q^3
\end{align*}
\]  (18a,b,c)
\[ S = (R + \sqrt{f})^{1/3} \]  
\[ T = (R - \sqrt{f})^{1/3} \]

The root \( \text{Re}(y) \) is any root of the resolvent cubic, equation (14); this program is written to calculate the largest real root of equation (14). Once \( \text{Re}(y) \) is known, the roots of the quartic are obtained by solving the following two quadratic equations:

\[ \begin{align*}
  z^2 + (A + C)z + (B + D) &= 0 \\
  z^2 + (A - C)z + (B - D) &= 0
\end{align*} \]  
(19)

where

\[ A = \frac{a_3}{2} \]

\[ B = \frac{\text{Re}(y)}{2} \]

\[ D = \sqrt{b^2 - a_0} \]

\[ C = \left( AB - \frac{a_1}{2} \right) / D \quad (D \neq 0) \]

\[ C = \sqrt{A^2 - a_2 + \text{Re}(y)} \quad (D = 0) \]

Equations (15) to (20) and a quadratic solution routine were programmed to obtain the roots of a quartic equation. The key codes for program 4 are given in appendix D.

Because \( f \) and \( D \) are tested to determine program direction, special programming is required both to insure that nonsignificant digits do not influence the test and to protect against the small difference of large numbers. The expressions for \( f \) and \( D \) were written as

\[ f = R^2 \left( 1 + \frac{Q^3}{R^2} \right) \]
\[
D = \sqrt{B^2 \left( 1 - \frac{a_0}{B^2} \right)}
\]

for programming. In each case, the quantity in the parenthesis was rounded to the calculator display and then tested. Special routines were added to protect against \( R \) and \( B \) being equal to 0. The introduction of rounding will produce some error if a significant number is truncated. As the rounding is controlled by the number of decimal digits in the calculator display, there is flexibility in the amount of rounding introduced. Experience with a set of 20 test equations indicates that a display of 7 digits is satisfactory for most cases.

The roots of the quartic are stored in registers \( R_1, R_2, S_1, \) and \( S_2 \). The root indicator (\(-1.0\) for complex roots and \(0.0\) for real roots) is stored in registers \( R_0 \) and \( S_0 \). If the roots are complex, the real part is stored in register 1 and the imaginary part in register 2.

This program is a general program for the roots of a quartic equation and may be used as a stand-alone program if the coefficients of the quartic are stored in the following locations:

- \( a_3 \) in register \( R_0 \)
- \( a_2 \) in register \( R_1 \)
- \( a_1 \) in register \( R_2 \)
- \( a_0 \) in register \( R_3 \)

In addition, this program may be used to solve for the roots of lower order equations. For the cubic where the equation has the form

\[ a_3 x^3 + a_2 x^2 + a_1 x + a_0 = 0, \quad a_3 = 1.0 \]

the equation is multiplied by \( x \) so that it is converted to a quartic with a zero root and the coefficients are stored as follows:

- \( a_2 \) in register \( R_0 \)
- \( a_1 \) in register \( R_1 \)
- \( a_0 \) in register \( R_2 \)
- \( 0.0 \) in register \( R_3 \)

Quadratic and first-order equations may be solved in a similar manner by multiplying through by \( x^2 \) or \( x^3 \), respectively.
Program 5

Program 5 calculates the stability parameters (ref. 5, p. 61), such as the time to damp to one-half amplitude or the damping ratio. The equations programmed are given as follows:

Time to damp to one-half amplitude \( t_{1/2} \) or time to double amplitude \( t_D \):

\[
t_{1/2} \text{ or } t_D = -\frac{0.693}{\text{Re}(\ )}
\]

(21)

Period:

\[
t = \frac{2\pi}{\text{Im}(\ )} \quad \rho = \frac{2\pi}{\omega}
\]

(22)

Number of cycles to damp to one-half amplitude \( N_{1/2} \) or time to double amplitude \( N_D \):

\[
N_{1/2} \text{ or } N_D = -0.110 \frac{\text{Im}(\ )}{\text{Re}(\ )}
\]

(23)

Logarithmic decrement:

\[
\Delta = \frac{0.693}{N_{1/2} \text{ or } N_D}
\]

(24)

Undamped circular frequency:

\[
\omega_n = [(\text{Re}(\ ))^2 + (\text{Im}(\ ))^2]^{1/2}
\]

(25)

Damping ratio:

\[
\zeta = \frac{\text{Re}(\ )}{\omega_n}
\]

(26)

If \( \Delta \), \( t \), or \( N \) is negative, unstable conditions are indicated. For instance, if \( \Delta \) is negative, the time calculated is for doubling the amplitude.

The key entries for this program are given in appendix E and the storage at the end of this program contains all the calculated information concerning airplane stability. For real roots, only the time to damp to one-half amplitude or the time to double amplitude is calculated. This program may be used as a stand-alone program.
Program 6

Program 6 uses the polar-rectangular keys of the calculator to implement the Euler transformation used in rigid-body rotation. The transformation programmed is the $\Psi, \Theta, \Phi$ transformation that is frequently used in aeronautics (fig. 1). The use of the polar-rectangular keys permits a short program for this type of transformation.

The transformation scheme is illustrated through the use of a two-dimensional transformation. The coordinates of a point $p(x,y)$ in the $xy$ axis system are given in the $x'y'$ axis system, which is rotated through the angle $\epsilon_1$ with respect to the $xy$ axis system by

$$\begin{align*}
x' &= x \cos \epsilon_1 + y \sin \epsilon_1 \\
y' &= -x \sin \epsilon_1 + y \cos \epsilon_1
\end{align*}$$

The polar coordinates of $p(x,y)$ are $R_*, \epsilon_2$ in the $xy$ axis system, where $R_* = (x^2 + y^2)^{1/2}$ and $\epsilon_2 = \tan^{-1} \frac{y}{x}$, and are $R_*, (\epsilon_2 - \epsilon_1)$ in the $x'y'$ axis system. The $x'y'$ axis system coordinates are now given by

$$\begin{align*}
x' &= R_* \cos (\epsilon_2 - \epsilon_1) \\
y' &= R_* \sin (\epsilon_2 - \epsilon_1)
\end{align*}$$

If equation (28) is expanded ($x$ is substituted for $R_* \cos \epsilon_2$ and $y$ is substituted for $R_* \sin \epsilon_2$), equation (27) results and shows that the same transformation is taking place. This result leads to a program for a two-dimensional transformation. It is assumed that $y$ is stored in the Y stack register, $x$ is stored in the X stack register, and $\epsilon_1$ is stored in register Rn. The program is as follows:

1. $\rightarrow p$
2. $x+ y$
3. RCL n
4. __
5. $x+ y$
6. $\rightarrow R$

This program gives $x'$ and $y'$ in 6 steps instead of the usual 18 steps. This two-dimensional program is completely general. If this two-dimensional transformation program is used in conjunction with a bookkeeping program, three-dimensional transformations may be made. In reference 6 (pp. 272 to 275), a method is given that simplifies the bookkeeping problem. A program for one of the three-dimensional Euler transformations used in aeronautics is given in

20
appendix F. This program is for transformations between two right-hand axes systems (fig. 1) in which the Z-axis is positive downwards. The first rotation is through the angle $\psi$ about the $Z_{sp}$-axis; the second is through the angle $\theta$ about the $Y_{sp}$-axis; and the third is through the angle $\phi$ about the $X_{sp}$-axis. The angles $\psi$, $\theta$, and $\phi$ are the airplane heading, pitch, and roll angles, respectively. The program presented in appendix F is a specialized program because, in three-dimensional transformations, the order in which the rotation angles are taken and the axes about which the rotations take place vary from one transformation to another. Similar programs may be written for other three-dimensional transformations by changing the bookkeeping part of program 6. Subroutines B and C would not be changed.

The advantages of using the polar-rectangular keys in program 6 for three-dimensional transformations are not apparent unless program 6 is compared with a program that uses the traditional approach of calculating the direction cosines and then using them to make the transformation. By using direction cosines, a reasonably efficient program for the $\psi, \theta, \phi$ transformation discussed in this section takes 124 program steps and 20 storage registers, compared with 67 steps and 10 storage registers for the polar-rectangular method of this paper. The impact is even more apparent if both the polar-rectangular (P+R) and the direction-cosine (D-C) methods are considered as subprograms to a main program. Take the following example:

A vector has been computed and its components are stored in three consecutive registers. The angles $\psi$, $\theta$, and $\phi$ have also been calculated and are stored in consecutive registers. It is desired to transform the calculated vector components to a new coordinate system rotated from the original by the angles $\psi$, $\theta$, and $\phi$.

Table II summarizes the manner in which the two programs would merge with the main program. Storage for the original vector components and the angles is not counted.

<table>
<thead>
<tr>
<th>Programming considerations</th>
<th>Space to body</th>
<th>Body to space</th>
<th>Two way</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>P+R method</td>
<td>P+R method</td>
<td>P+R method</td>
</tr>
<tr>
<td>Program steps in transformation</td>
<td>26</td>
<td>83</td>
<td>27</td>
</tr>
<tr>
<td>Registers used</td>
<td>----</td>
<td>13</td>
<td>----</td>
</tr>
<tr>
<td>I register</td>
<td>Used</td>
<td>Used</td>
<td>Used</td>
</tr>
<tr>
<td>Storage for new computation</td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>Total program steps used$^a$</td>
<td>26</td>
<td>83</td>
<td>27</td>
</tr>
<tr>
<td>Total registers used$^a$</td>
<td>3</td>
<td>16</td>
<td>3</td>
</tr>
<tr>
<td>Steps available for main program</td>
<td>198</td>
<td>141</td>
<td>197</td>
</tr>
<tr>
<td>Registers available for main program</td>
<td>22</td>
<td>9</td>
<td>22</td>
</tr>
</tbody>
</table>

$^a$I register is not counted.
Analysis of the data presented in table II shows the economics of using the P*R method in programs. In addition, a calculator with only 49 program steps can be programmed one way using the P*R method, and a calculator with 98 steps can handle the two-way P*R transformation. For the D-C method, the smallest programmable calculator that can handle a one-way transformation is one with 98 steps. The two-way transformation will not fit on a 98-step calculator.

USE OF PROGRAMS 1 TO 6

After a program has been keyed into the calculator, the program switch should be set to run. Set the display and trig modes, switch back to program, and record program. The display and trig mode status are now recorded on the magnetic card and the calculator will be set to the indicated status conditions whenever the program is read in. The display and trig mode status are given for each program in the appendixes.

Appendix G contains the check case for the programs in appendixes A to F. To make longitudinal stability calculations, use the following procedure:

(1) Enter program 1 (appendix A)

(2) Enter data as shown on storage map
   Push A
   At stop, coefficients of characteristics equation have been calculated

(3) Enter program 4 (appendix D)
   Push A
   At stop, roots of characteristic have been determined

(4) Enter program 5 (appendix E)
   Push A
   At stop, complete set of longitudinal stability data is stored as indicated on storage map

To make lateral stability calculations, use the following procedure:

(1) Enter program 2 (appendix B)

(2) Enter data as shown on storage map
   Push A

(3) At stop, enter program 3 (appendix C)
   Push A
   If 4 is displayed at stop, go to step 4
   If 5 is displayed, push B

   When 8 is displayed, the real root, the time to damp to one-half amplitude or the time to double amplitude, and the number of iterations are stored in the stacks
(4) Enter program 4 (appendix D)
Push A
At stop, roots of quartic have been calculated

(5) Enter program 5 (appendix E)
Push A
At stop, a complete set of lateral stability data has been calculated. Data relating to quartic is stored in calculator

Programs 4 and 5 may be used as stand-alone programs.

Program 6 may be used in several different ways. To transform from space axes (X_sp, Y_sp, Z_sp) to airplane axes (X_b, Y_b, Z_b), use the following procedure:

(1) Enter z_sp, y_sp, x_sp in stack in order given
Push A

(2) Enter φ, θ, ψ in stack in order given
Push B
Push C to make the transformation
At stop, airplane axis coordinates x_b, y_b, z_b are stored in registers R6, R7, and R8, respectively

To transform from airplane axes (X_b, Y_b, Z_b) to space axes (X_sp, Y_sp, Z_sp), use the following procedure:

(1) Enter z_b, y_b, x_b in stack in order given
Push A

(2) Enter φ, θ, ψ in stack in order given
Push B
Push D to make the transformation
At stop, space coordinates X_sp, Y_sp, Z_sp are stored in registers R6, R7, and R8, respectively

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Hampton, VA 23665
April 28, 1978
APPENDIX A

PROGRAM 1 - LONGITUDINAL AIRPLANE STABILITY

Program 1 uses the basic physical and aerodynamic data of an airplane to calculate the coefficients of the characteristic equation of longitudinal motion. This program calculates normalized coefficients for the characteristic equation. For running this program, the calculator should be set to calculate in radians and to display seven decimal digits. A pause inserted after step 49 causes the calculated value of $\alpha_{tr}$ to be displayed.
APPENDIX A

001 LBL A
010 SIN
020 RCL5
030 RCL6
040 RCL1
050 RCLC

Calculate elements
of determinant

\[ \sigma_T \]

\[ -c/k_Y^2 \]

\[ \sin^2 \gamma_{SS} - \sigma_w \]

\[ g(\sigma_T \sin^2 \gamma_{SS} - \sigma_w + \cos \gamma_{SS}) \]

\[ \rho SU_{SS}/m \]

\[ \rho SU_{SS}/2m \]

\[ C_L \]

\[ C_{m_x} \] stored in R6

\[ C_L \] stored in S5

\[ \alpha_{tr} \]

\[ g(\sigma_T \sin^2 \gamma_{SS}/2U_{SS}) \]

\[ \pi \]

\[ \frac{C_{D,0} + C_L^2/\pi A}{C_{L\phi} + C_{L\psi}} \]

\[ C_{m_{\phi}} + C_{m_{\psi}} \]

\[ a_{14} \]

\[ a_{21} \]

\[ a_{22} \]

\[ a_{26} \]

\[ a_{25} \]

\[ a_{35} \]

\[ p+S \]

\[ RCL9 \]

\[ RCL7 \]

\[ 2 \]

\[ \cos \]

\[ RCL8 \]

\[ RCL9 \]

\[ RCL7 \]

\[ \cos \]

\[ RCL8 \]

\[ RCL9 \]

\[ RCL7 \]

\[ 2 \]

\[ \times \]

\[ \times \]

\[ \times \]

\[ \times \]

\[ \times \]

\[ \times \]

\[ \times \]

\[ \times \]
APPENDIX A

\[ g(\sigma_T \sin^2 \gamma_{SS} - \sigma_w)/U_{SS} \]

\[ \text{RCL0} \times \]
\[ \text{RCL9} \]
\[ \text{RCL3} - \]
\[ \text{RCL6} \]
\[ \text{P} + \text{S} \]
\[ \text{STO-2} \quad \text{a32} \]
\[ \text{R}^+ \]
\[ \text{STO-4} \quad \text{a12} \]
\[ \text{R}^+ \]
\[ \text{STO-5} \quad \text{a13} \]
\[ 1 \]
\[ 0 \]
\[ \text{STO}\text{I} \]
\[ \text{RCL3} \]
\[ \text{RCL2} - \]
\[ \text{STOC} \quad \text{a4} \]
\[ 170 \]
\[ \text{RCL4} \]
\[ \times \]
\[ \text{RCL1} \]
\[ \text{RCLB} - \]
\[ \text{RCL2} \]
\[ \text{RCL7} \times \]
\[ - \]
\[ \text{RCL3} \]
\[ 180 \]
\[ \text{RCL8} \]
\[ \times \]
\[ + \]
\[ \text{STOD} \]
\[ + \]
\[ \text{STO(i) a3} \]
\[ \text{ISZ} \]
\[ \text{STO}\text{I} \]
\[ \text{RCL1} \]
\[ \text{RCL8} \times \]
\[ 190 \]
\[ \text{RCL7} \]
\[ \text{RCL8} \times \]
\[ - \]
\[ \text{RCL6} \]
\[ \text{RCL2} \times \]
\[ - \]
\[ \text{STOE} \]
\[ \text{RCLD} \]
\[ \text{200} \]
\[ \text{RCL4} \]
\[ \times \]
\[ + \]
APPENDIX A

RCLA
×
+
→ S
STO3

210 RCLC
STO: 0
STO: 1
STO: 2
STO: 3
RTN
R/S
APPENDIX A

Storage Map for Program 1

<table>
<thead>
<tr>
<th>(i) Address</th>
<th>Register</th>
<th>Input storage</th>
<th>Output storage</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>R₀</td>
<td>kₚ</td>
<td>a₂₃</td>
</tr>
<tr>
<td>1</td>
<td>R₁</td>
<td>m</td>
<td>a₂</td>
</tr>
<tr>
<td>2</td>
<td>R₂</td>
<td>ρ</td>
<td>a₁</td>
</tr>
<tr>
<td>3</td>
<td>R₃</td>
<td>Cₜᵤ</td>
<td>a₀</td>
</tr>
<tr>
<td>4</td>
<td>R₄</td>
<td>Cₘ₀</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>R₅</td>
<td>g</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>R₆</td>
<td>Uₚₛ</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>R₇</td>
<td>γₚₛ</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>R₈</td>
<td>σᵤ</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>R₉</td>
<td>σₚₖ</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>S₀</td>
<td>Cₖ₀</td>
<td>a₂₁</td>
</tr>
<tr>
<td>11</td>
<td>S₁</td>
<td>Cₖ₀</td>
<td>a₂₃</td>
</tr>
<tr>
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</tr>
<tr>
<td>13</td>
<td>S₃</td>
<td>Cₖ₀</td>
<td>a₂₂</td>
</tr>
<tr>
<td>14</td>
<td>S₄</td>
<td>Cₖ₀,o</td>
<td>a₁₂</td>
</tr>
<tr>
<td>15</td>
<td>S₅</td>
<td>0.0</td>
<td>a₁₃</td>
</tr>
<tr>
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<td>S₆</td>
<td>0.0</td>
<td>a₂₆</td>
</tr>
<tr>
<td>17</td>
<td>S₇</td>
<td>Cₘ₀</td>
<td>a₂₅</td>
</tr>
<tr>
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<td>Cₘ₀</td>
<td>a₃₅</td>
</tr>
<tr>
<td>19</td>
<td>S₉</td>
<td>Cₘ₀</td>
<td>a₄</td>
</tr>
<tr>
<td>20</td>
<td>Rₐ</td>
<td>ñ</td>
<td>a₃₁</td>
</tr>
<tr>
<td>21</td>
<td>R₉</td>
<td>S</td>
<td>a₃₃</td>
</tr>
<tr>
<td>22</td>
<td>R₉</td>
<td>A</td>
<td></td>
</tr>
<tr>
<td>23</td>
<td>R₉</td>
<td>Cₖ₉,₀</td>
<td></td>
</tr>
<tr>
<td>24</td>
<td>R₉</td>
<td>0.0</td>
<td></td>
</tr>
<tr>
<td>25</td>
<td>I</td>
<td>20</td>
<td></td>
</tr>
</tbody>
</table>

These are the normalized coefficients of the quartic; thus, a₄ = 1.00.
Program 2 uses the basic physical and aerodynamic data of an airplane to generate the coefficients of the lateral stability determinant. After completing the calculation of these coefficients, the program starts but does not finish calculating the coefficients of the characteristic equation for lateral motion. For running this program, the calculator should be set to calculate in radians and to display seven decimal digits. A pause inserted after step 44 causes the calculated value of $\alpha_{tr}$ to be displayed.
APPENDIX B

001 LBLA
RCL1
RCL2
RCL3
×
×
2
÷
RCL4
RCL5
RCL6
STO×0

010 ÷ ρSuSS/2m
CHS
STO3
RCL1
STO÷0
STO×4
RCL2
STOE
RCL5
RCL4
STO×4
ρSbuSS/2m
RCL9
RCL1
STO÷0
STO×4
STO÷(i)
ρSbuSS/2I

020 x^2
RCL5
STO5
STO÷4
ρSbuSS/2I
RCL9
X^2
STO9
STO÷(i)
ρSbuSS/2I
RCL8
X<0
SF2
F?2
CHS
CHS
ENT
ENT
RCL9
÷
STO9
X+Y
RCL5
÷
STO8
IXZ/I
RCL0
RCL7
RCL1
÷
×
STO2
RCLC
COS

RCL0
×
- g
USS
÷
- g
USS
cos γSS

060 ÷
+ 2
÷
RCLC

070 R^+
STO6
RCL3
GSBa
b12, C21
DSZ
GSBa
b21
1
STO+(i)
X+Y
DSZ
GSBa
b30
X+Y
STO+(i)
C30
X+Y
GSBa
RCL4
GSBa
b32
GSBa
b23
RCL(i)
STO6

080 GSBa
b14
GSBa
b33
RCLE
GSBa
b34
GSBa
b16
GSBa
b35
GSBa
b24
STO7

30
APPENDIX B

100 RCL0
STO×6
STO×7
2
3
STOI
RCL8
Calculate coefficient of characteristic equation
P×S
RCL3
×

110 RCL7
- b_{42}b_{34} - b_{32}

150 RCL7
×

160 RCL2
RCL7
×

Secondary called

b_{34} - a_{32}a_{34}

170 RCL4
P×S
RCL9
×

Primary called

b_{35} + b_{34}b_{41}

Secondary called

-b_{16}b_{32} - b_{33}b_{43}

180 RCL3
×

Complete calculations and store terms

End of program

Subroutines for calculation of coefficients
## APPENDIX B

### Storage Map for Program 2

<table>
<thead>
<tr>
<th>Address</th>
<th>Register</th>
<th>Initial Storage</th>
<th>End of Program</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>R0</td>
<td>g</td>
<td>(a)</td>
</tr>
<tr>
<td>1</td>
<td>R1</td>
<td>Uss</td>
<td>(a)</td>
</tr>
<tr>
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<td>R2</td>
<td>( \rho )</td>
<td>(a)</td>
</tr>
<tr>
<td>3</td>
<td>R3</td>
<td>S</td>
<td>(a)</td>
</tr>
<tr>
<td>4</td>
<td>R4</td>
<td>b</td>
<td>(a)</td>
</tr>
<tr>
<td>5</td>
<td>R5</td>
<td>( k_X )</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>R6</td>
<td>( \sigma_u )</td>
<td>b15</td>
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<td>7</td>
<td>R7</td>
<td>( \sigma_w )</td>
<td>b17</td>
</tr>
<tr>
<td>8</td>
<td>R8</td>
<td>( b_{k_X Z} )</td>
<td>b42</td>
</tr>
<tr>
<td>9</td>
<td>R9</td>
<td>( k_Z )</td>
<td>b43</td>
</tr>
<tr>
<td>10</td>
<td>S0</td>
<td>( c_{n_r} )</td>
<td>b24</td>
</tr>
<tr>
<td>11</td>
<td>S1</td>
<td>( c_{n_\beta} )</td>
<td>b35</td>
</tr>
<tr>
<td>12</td>
<td>S2</td>
<td>( c_{n_p} )</td>
<td>b16</td>
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<tr>
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<td>S3</td>
<td>( c_{n_\beta} )</td>
<td>b34</td>
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<tr>
<td>14</td>
<td>S4</td>
<td>( c_{l_\beta} )</td>
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<td>15</td>
<td>S5</td>
<td>( c_{l_p} )</td>
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<td>16</td>
<td>S6</td>
<td>( c_{l_r} )</td>
<td>b23</td>
</tr>
<tr>
<td>17</td>
<td>S7</td>
<td>( c_{l_\beta} )</td>
<td>b32</td>
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<tr>
<td>18</td>
<td>S8</td>
<td>( c_{y_\beta} )</td>
<td>b31</td>
</tr>
<tr>
<td>19</td>
<td>S9</td>
<td>( c_{y_\beta} )</td>
<td>C30</td>
</tr>
<tr>
<td>20</td>
<td>RA</td>
<td>m</td>
<td>b22</td>
</tr>
<tr>
<td>21</td>
<td>RB</td>
<td>( c_{y_r} )</td>
<td>C21</td>
</tr>
<tr>
<td>22</td>
<td>RC</td>
<td>( \gamma_{SS} )</td>
<td>b13</td>
</tr>
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<td>RD</td>
<td>( c_{y_p} )</td>
<td>C11</td>
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</tr>
<tr>
<td>25</td>
<td>I</td>
<td>24</td>
<td></td>
</tr>
</tbody>
</table>

\( a \) Registers R0 to R4 contain the partially calculated coefficients of the characteristic equation.

\( b \) If \( k_{XZ} \) is imaginary, enter \( k_{XZ} \) as a negative number.
Program 3 completes the calculation of the coefficients of the characteristic equation of lateral motion that was started in program 2. The program then determines if the characteristic equation is a quartic or a quintic. If it is a quartic, a 4 is displayed and the program stops. Program 4 is then used to obtain the roots of the quartic. If the characteristic equation is a quintic, a 5 is displayed and the program continues on to extract the real root of the quintic and then calculates the time to damp to one-half amplitude or the time to double amplitude. This program uses the storage that existed at the end of program 2. This program calculates normalized coefficients for the quartic and the quintic. For running this program, the calculator should be set to calculate in radians and to display seven decimal digits.
APPENDIX C

001 LBLA
RCL7
RCL6
P+S
RCL3
X+Y
RCL7
×
010 -
RCL2
RCL4
×
- 
RCL5
RCL1
× 
+ 
RCL4
- 
X>Y
RCL7
× 
020 RCL1
P+S
Primary called
RCL6
× 
X+Y
RCL7
× 
- 
GSBa
030 STO4
CHS
R+ 
STO-3
R+ 
GSBa
STO-3
R+ 
STO-2
R+ 
RCL8
RCL9
040 P+S
Secondary called
RCL6
× 
CHS
X+Y
RCL2
× 
- 
RCL5
+ 
b_{15}b_{34} - b_{32}b_{17}
050 RCL0
P+S 
RCL6
× 
X>Y
RCL7
× 
-  
b_{24}b_{15} - b_{23}b_{17} 
060 b_{35}b_{15} - b_{33}b_{17}
070 STO0
1
1 - b_{42}b_{43}
080 Complete and store calculations

Determine if equation is a quartic or a quintic

GOT01
APPENDIX C

GSBd
4 Indicates quartic RCL8
RTN Stop for quartic RCL9
LBL1 Stop for quintic STO8
GSBd X+Y
5 Indicates quintic STO6
110 Stop for quintic GOTO0
LBLB Calculate real root
RCL0 of quintic
STOA This section
RCL1 positions data
STOB Output routine
RCL2
STOC
RCL3
STOD
RCL4
STOE
120 Initialization for
ST07 secant method
STO7 FIX
ST08 RCLA
5 RTN
÷ ST6 RCL6
. End of program
0 RCL6
180 \+ RCL(i)
8 \× RCL6
÷ RCL6
· Chs
RCL6
3
ST06
ST06
ST05
ST08
ENT
ENT
ENT
 ENT
RTN
SUBROUTINE FOR
SUBROUTINE FOR
SUBROUTINE FOR
X+Y
DSZ
ISZ

Calculating
characteristic
Polynomial evaluation
and tests for
and tests for
and tests for

characteristic
equation
equation
equation

coefficient of
coefficient of
coefficient of

Evaluates polynomial
Evaluates polynomial
Evaluates polynomial

solution
solution
solution

Displays value of
Displays value of
Displays value of

0
1
1

GOT0
GOT0
GOT0

Calculates and stores
Calculates and stores
Calculates and stores

ST02
ST02
ST02

new value of X
new value of X
new value of X

ST3
ST3
ST3

STO4
STO4
STO4

RTN
RTN
RTN

GSBc
GSBc
GSBc

GSBc
GSBc
GSBc

GSBc
GSBc
GSBc

LBlb
LBlb
LBlb

LBlb
LBlb
LBlb

LBlb
LBlb
LBlb

35
APPENDIX C

RCL6
×
RCL(i)
+

210  ISZ
RTN

LBLd    Normalization subroutine
RCLE    
STO¥0
STO¥1
STO¥2
STO¥3
STO¥4
RTN

220  R/S
APPENDIX C

Storage Map for Program 3

<table>
<thead>
<tr>
<th>Address</th>
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<th>Initial storage</th>
<th>End of program</th>
</tr>
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<td></td>
<td>a3</td>
</tr>
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<td>1</td>
<td>R1</td>
<td></td>
<td>a2</td>
</tr>
<tr>
<td>2</td>
<td>R2</td>
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<td>a1</td>
</tr>
<tr>
<td>3</td>
<td>R3</td>
<td></td>
<td>a0</td>
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<tr>
<td>4</td>
<td>R4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>R5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>R6</td>
<td>b15</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>R7</td>
<td>b17</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>R8</td>
<td>b42</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>R9</td>
<td>b43</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>S0</td>
<td>b24</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>S1</td>
<td>b35</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>S2</td>
<td>b16</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>S3</td>
<td>b34</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>S4</td>
<td>b33</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>S5</td>
<td>b14</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>S6</td>
<td>b23</td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>S7</td>
<td>b32</td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>S8</td>
<td>b31</td>
<td></td>
</tr>
<tr>
<td>19</td>
<td>S9</td>
<td>C30</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>R_A</td>
<td>b22</td>
<td>a4</td>
</tr>
<tr>
<td>21</td>
<td>R_B</td>
<td>C21</td>
<td>a3</td>
</tr>
<tr>
<td>22</td>
<td>R_C</td>
<td>b13</td>
<td>a2</td>
</tr>
<tr>
<td>23</td>
<td>R_D</td>
<td>C11</td>
<td>a1</td>
</tr>
<tr>
<td>24</td>
<td>R_E</td>
<td>in use</td>
<td>a0</td>
</tr>
<tr>
<td>25</td>
<td>I</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The initial storage is the same as that at end of program 2. The partially calculated coefficients of the characteristic equation are stored in R0 to R4.

The end storage is the same for display signals 4 and 8; the normalized coefficients of the quartic are in registers R0 to R3. The real root of the quintic is in the Y register and the time to damp to one-half amplitude or the time to double amplitude is in the Z register when 8 is displayed. Pressing R+ moves the real root of the quintic to the X register; pressing R+ again moves the time to damp to one-half amplitude or the time to double amplitude to the X register. The number of iterations required to obtain the root is in the stack T register and may be obtained by pressing R+. 


APPENDIX D

PROGRAM 4 - ROOTS OF A QUARTIC EQUATION

Program 4 applies Ferrari's method for the roots of a quartic equation to the output of either program 1 or program 3 to determine the remaining eigenvalues of the characteristic equation of longitudinal or lateral motion. Normalized coefficients must be used for this program. For running this program, the calculator should be set to calculate in radians and to display seven decimal digits.
APPENDIX D

001  LBLA  Calculate coefficients of resolvent cubic
  4  STO6
  RCL1  ÷
  STOC
  STO×6
  CHS
  STO4  b₂ in R₄  060  LBL1  f
  RCL0
  STO8  ABS
  STOB
  STO5  Calculate largest real root of resolvent cubic
   x²
  RCL2  √
  STOD  F?2
  STO×5  GOTO2
   x
  RCL3  RCL7
  STO6  +P
  3
  STO×6  GSBB
   x
  4
  STO-5  GOTO3
   b₁ in R₅
  x+y
  STO-6  GOTO4
   b₀ in R₆
  3
  STO±4
   Calculate Q, R, Q³, R², and f
  STO±5  STO-7
  RCL5  +
  RCL4  GSBB
   x
  X²
  -
  X+y
  STO×5
   Q
  RCL4
   Q³
  X²
  RCL5
  x
  RCL6
  2
  ÷
  RCL4
  3
  Y²
  -
  R
  STO7
  X²
  RCL2
  STOA
  X≠0
  GOTO0
  GSBa

010
020
030
040
050
39
APPENDIX D

CHS
RCL8
X²
X≠0
GOTO4
110
X+Y
GSBa
GOTO5
LBL4
÷
GSBa
RCL8
X²
×
170
LBL5

120
√
D
STO+7
B + D
STO-8
B - D
F?2
GOTO6
RCL0
X²
RCL1
-
180
X<0
LBL8

130
√
C, D = 0
GOTO7
LBL6
÷
LBL7
STO+0
A + C
STO-6
A - C
+ 
RCL7
Solve for roots
190
RCL0
of quartic
RCL4
X<0
GOTO

140
GSBc
RCL8
RCL6
P+S
GSBc
P+S
RTN
LBLa
Subroutine used in
+ 
calculation of f
200
RND
and D
GOTO1

150
Pause
Displays quantity
X>0
tested
SF2
ENT†
RTN
LBLb
Subroutine for cube
X<0
root of positive or
SF2
negative number
-
R†
APPENDIX D

STO1
R↓

STO2
R↓

RTN
APPENDIX D

Storage Map for Program 4

<table>
<thead>
<tr>
<th>(i) Address</th>
<th>Register</th>
<th>Initial storage(^a)</th>
<th>End of program(^b)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>R(_0)</td>
<td>a(_3)</td>
<td>Root-type indicator</td>
</tr>
<tr>
<td>1</td>
<td>R(_1)</td>
<td>a(_2)</td>
<td>Re( ) or (\alpha(_1))</td>
</tr>
<tr>
<td>2</td>
<td>R(_2)</td>
<td>a(_1)</td>
<td>Im( ) or (\alpha(_2))</td>
</tr>
<tr>
<td>3</td>
<td>R(_3)</td>
<td>a(_0)</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>R(_4)</td>
<td></td>
<td></td>
</tr>
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<td>R(_5)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>R(_6)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>R(_7)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>R(_8)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>R(_9)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>S(_0)</td>
<td></td>
<td>Root-type indicator</td>
</tr>
<tr>
<td>11</td>
<td>S(_1)</td>
<td></td>
<td>Re(_1)() or (\alpha(_3))</td>
</tr>
<tr>
<td>12</td>
<td>S(_2)</td>
<td></td>
<td>Im(_1)() or (\alpha(_4))</td>
</tr>
<tr>
<td>13</td>
<td>S(_3)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>S(_4)</td>
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</tr>
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<td>15</td>
<td>S(_5)</td>
<td></td>
<td></td>
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<td>16</td>
<td>S(_6)</td>
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<td>18</td>
<td>S(_8)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>19</td>
<td>S(_9)</td>
<td></td>
<td></td>
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<td>a(_3)</td>
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<td>R(_B)</td>
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<td>R(_C)</td>
<td>a(_1)</td>
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<td>23</td>
<td>R(_D)</td>
<td>a(_0)</td>
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<tr>
<td>24</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>25</td>
<td>I</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\(^a\) Initial storage is provided by output of program 1 or program 3.

\(^b\) The root-type indicator is 0 for real roots and -1 for complex roots. The real part of the complex root is stored in R\(_1\) or S\(_1\) and the imaginary part in R\(_2\) or S\(_2\).
APPENDIX E

PROGRAM 5 - STABILITY PARAMETERS

Program 5 utilizes the eigenvalues computed by program 4 to calculate stability parameters, such as the time to damp to one-half amplitude or the damping ratio. For running this program, the calculator should be set to calculate in radians and to display seven decimal digits.
APPENDIX E

001 LBLA clears registers
010 STO0 clears and protects
020 P+S roots and root
stores constants
030 CHS and initializes I
040 X≠0 register
050 F?2

060 RCL1
Calculates stability
parameters for
complex roots
Protecs against
zero real part of
complex root

t₁/₂ or t₉

070 X+Y
N₁/₂ or N₉

080 RCL2
Switch for second
set of roots and
program stop
t

090 GOTOB
RCL1
P+S

100 RCLB
Switch for complex
roots

110 STO5

120 CHS

130 STO4

140 ABS

150 RCL2

160 ABS

170 X=0

180 GOTO1

190 GOTO2

200 RCL1

210 ABS

220 +

230 X=0

240 GOTO1

250 RCL2

260 X≠0

270 SF2

280 RCL1

290 ABS

300 RCL2

310 ABS

320 +

330 X=0

340 GOTO1

350 RCL2

360 X≠0

370 SF2

380 RCL1

390 ABS

400 RCL2

410 ABS

420 +

430 X=0

440 GOTO1

450 RCL2

460 X≠0

470 SF2

480 RCL1

490 ABS

500 RCL2

510 ABS

520 +

530 X=0

540 GOTO1

550 RCL2

560 X≠0

570 SF2

580 RCL1

590 ABS

600 RCL2

610 ABS

620 +

630 X=0

640 GOTO1

650 RCL2

660 X≠0

670 SF2

680 RCL1

690 ABS

700 RCL2

710 ABS

720 +

730 X=0

740 GOTO1

750 RCL2

760 X≠0

770 SF2

780 RCL1

790 ABS

800 RCL2

810 ABS

820 +

830 X=0

840 GOTO1

850 RCL2

860 X≠0

870 SF2

880 RCL1

890 ABS

900 RCL2

910 ABS

920 +

930 X=0

940 GOTO1
APPENDIX E

Storage Map for Program 5

<table>
<thead>
<tr>
<th>(i)Address</th>
<th>Register</th>
<th>Initial storage$^a$</th>
<th>End of program</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$R_0$</td>
<td>Root-type indicator</td>
<td>Root-type indicator</td>
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<tr>
<td>1</td>
<td>$R_1$</td>
<td>$\text{Re}(\cdot)$ or $\alpha_1$</td>
<td>$\text{Re}(\cdot)$ or $\alpha_1$</td>
</tr>
<tr>
<td>2</td>
<td>$R_2$</td>
<td>$\text{Im}(\cdot)$ or $\alpha_2$</td>
<td>$\text{Im}(\cdot)$ or $\alpha_2$</td>
</tr>
<tr>
<td>3</td>
<td>$R_3$</td>
<td></td>
<td>$t_{1/2}$ or $t_D$</td>
</tr>
<tr>
<td>4</td>
<td>$R_4$</td>
<td></td>
<td>$\Delta$</td>
</tr>
<tr>
<td>5</td>
<td>$R_5$</td>
<td></td>
<td>$N_{1/2}$ or $N_D$</td>
</tr>
<tr>
<td>6</td>
<td>$R_6$</td>
<td></td>
<td>$t$</td>
</tr>
<tr>
<td>7</td>
<td>$R_7$</td>
<td></td>
<td>$\omega_n$</td>
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<td>$R_8$</td>
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<td>$R_9$</td>
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<td></td>
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<td>$S_0$</td>
<td>Root-type indicator</td>
<td>Root-type indicator</td>
</tr>
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<td>11</td>
<td>$S_1$</td>
<td>$\text{Re}_1(\cdot)$ or $\alpha_3$</td>
<td>$\text{Re}_1(\cdot)$ or $\alpha_3$</td>
</tr>
<tr>
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<td>$\text{Im}_1(\cdot)$ or $\alpha_4$</td>
<td>$\text{Im}_1(\cdot)$ or $\alpha_4$</td>
</tr>
<tr>
<td>13</td>
<td>$S_3$</td>
<td></td>
<td>$t_{1/2}$ or $t_D$</td>
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<td>$S_4$</td>
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<td>$\Delta$</td>
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<td>15</td>
<td>$S_5$</td>
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<td>$N_{1/2}$ or $N_D$</td>
</tr>
<tr>
<td>16</td>
<td>$S_6$</td>
<td></td>
<td>$t$</td>
</tr>
<tr>
<td>17</td>
<td>$S_7$</td>
<td></td>
<td>$\omega_n$</td>
</tr>
<tr>
<td>18</td>
<td>$S_8$</td>
<td></td>
<td>$\zeta$</td>
</tr>
<tr>
<td>19</td>
<td>$S_9$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>$R_A$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>21</td>
<td>$R_B$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>22</td>
<td>$R_C$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>23</td>
<td>$R_D$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>24</td>
<td>$R_E$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>25</td>
<td>$I$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$^a$The initial storage is the same as the storage at the end of program 4.

For real roots, only the time to damp to one-half amplitude or the time to double amplitude is calculated. This quantity is stored in register 3 for the root in register 1 and in register 4 for the root in register 2.
PROGRAM 6 - EULER TRANSFORMATION FOR AERONAUTICS

Program 6 is for the standard Euler transformation that is used in aeronautics between inertial axes and airplane axes. The trigonometric mode and the number of decimal digits in the display are assigned by the user.
### APPENDIX F

<table>
<thead>
<tr>
<th>001</th>
<th>LBLA</th>
<th>STO0</th>
<th>R4-</th>
<th>020</th>
<th>R3-</th>
<th>R4-</th>
<th>RTN</th>
</tr>
</thead>
<tbody>
<tr>
<td>010</td>
<td>LBLB</td>
<td>STO3</td>
<td>R4-</td>
<td>STO4</td>
<td>R*</td>
<td>STO5</td>
<td>RTN</td>
</tr>
<tr>
<td>020</td>
<td>LBLC</td>
<td>3</td>
<td>STOI</td>
<td>RCL1</td>
<td>RCLO</td>
<td>GSBb</td>
<td>RCL2</td>
</tr>
</tbody>
</table>

**Stores $X$, $Y$, $Z$, or $X_b'$, $Y_b'$, $Z_b'$**

| 030 | LBLD | 5   | STOI | RCL2 | RCL1 | GSBc | RCLO | X+Y | GSBc | ST08 | R4- |

**Transforms $X_b$, $Y_b$, $Z_b$ to $X_s$, $Y_s$, $Z_s$**

**Transformation subroutine X_s, Y_s, Z_s to X_s, Y_s, Z_s**

**Transformation subroutine X_s, Y_s, Z_s to X_s, Y_s, Z_s**

**Transformation subroutine X_s, Y_s, Z_s to X_s, Y_s, Z_s**

---

**Transformation subroutine X_s, Y_s, Z_s to X_s, Y_s, Z_s**

**Transformation subroutine X_s, Y_s, Z_s to X_s, Y_s, Z_s**

---

**Transformation subroutine X_s, Y_s, Z_s to X_s, Y_s, Z_s**

**Transformation subroutine X_s, Y_s, Z_s to X_s, Y_s, Z_s**
This appendix gives check cases for each program given in appendixes A to F. Each check case is complete in itself and does not depend on the output of a previous program. For program 3, two check cases are given - one for label A and one for label B. There is no check case given for programs 1, 2, and 3 for $\sigma_u = \sigma_w = 0.0$. All check cases are independent of previous results.
## APPENDIX G

### Check Case for Program 1

<table>
<thead>
<tr>
<th>Register</th>
<th>Input storage</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>R0</td>
<td>(ky = 10.463784)</td>
<td>(a_3 = 1.3924836)</td>
</tr>
<tr>
<td>R1</td>
<td>(m = 90909.1)</td>
<td>(a_2 = 1.1016636)</td>
</tr>
<tr>
<td>R2</td>
<td>(\rho = 1.2929)</td>
<td>(a_1 = -0.0160353)</td>
</tr>
<tr>
<td>R3</td>
<td>(C_{Tu} = -0.000248411)</td>
<td>(a_0 = -0.0210558)</td>
</tr>
<tr>
<td>R4</td>
<td>(C_{m\alpha} = -1.115)</td>
<td>(a_{21} = 5.9757330)</td>
</tr>
<tr>
<td>R5</td>
<td>(g = 9.80665)</td>
<td>(a_{23} = 55.0128915)</td>
</tr>
<tr>
<td>R6</td>
<td>(U_{ss} = 77.12)</td>
<td>(a_{32} = -73.9231523)</td>
</tr>
<tr>
<td>R7</td>
<td>(\Gamma_{ss} = -0.052359878)</td>
<td>(a_{22} = 4.2010871)</td>
</tr>
<tr>
<td>R8</td>
<td>(\sigma_u = 2.0)</td>
<td>(a_{12} = 0.0316972)</td>
</tr>
<tr>
<td>R9</td>
<td>(\sigma_w = 0.0)</td>
<td>(a_{13} = 0.2546699)</td>
</tr>
<tr>
<td>S0</td>
<td>(C_{D\alpha} = 0.529)</td>
<td>(a_{26} = 0.8064467)</td>
</tr>
<tr>
<td>S1</td>
<td>(C_{L\alpha} = 4.87)</td>
<td>(a_{25} = 0.6856605)</td>
</tr>
<tr>
<td>S2</td>
<td>(C_{L\theta} = 0.283)</td>
<td>(a_{35} = 0.5113523)</td>
</tr>
<tr>
<td>S3</td>
<td>(C_{L\alpha} = 0.0889)</td>
<td>(a_{14} = 0.0007159)</td>
</tr>
<tr>
<td>S4</td>
<td>(C_{D,\alpha} = 0.038)</td>
<td>(a_{31} = -9.7126460)</td>
</tr>
<tr>
<td>S5</td>
<td>(0.0)</td>
<td>(a_{33} = 1.5369077)</td>
</tr>
<tr>
<td>S6</td>
<td>(0.0)</td>
<td>(a_{4} = 78.1242394)</td>
</tr>
<tr>
<td>S7</td>
<td>(C_{m\alpha} = -0.241)</td>
<td></td>
</tr>
</tbody>
</table>
## APPENDIX G

Check Case for Program 2

<table>
<thead>
<tr>
<th>Register</th>
<th>Input storage</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>R0</td>
<td>( g = 9.80665 )</td>
<td>0.0</td>
</tr>
<tr>
<td>R1</td>
<td>( U_{ss} = 77.12 )</td>
<td>0.6027688</td>
</tr>
<tr>
<td>R2</td>
<td>( \rho = 1.2929 )</td>
<td>0.3089144</td>
</tr>
<tr>
<td>R3</td>
<td>( S = 267.1 )</td>
<td>0.0064130</td>
</tr>
<tr>
<td>R4</td>
<td>( b = 43.4 )</td>
<td>-11.394069</td>
</tr>
<tr>
<td>R5</td>
<td>( k_X = 6.559296 )</td>
<td></td>
</tr>
<tr>
<td>R6</td>
<td>( \sigma_u = 2.0 )</td>
<td></td>
</tr>
<tr>
<td>R7</td>
<td>( \sigma_w = -0.5 )</td>
<td></td>
</tr>
<tr>
<td>R8</td>
<td>( k_{XZ} = -1.28016 )</td>
<td></td>
</tr>
<tr>
<td>R9</td>
<td>( k_Z = 12.249912 )</td>
<td></td>
</tr>
<tr>
<td>S0</td>
<td>( c_n_r = -0.057 )</td>
<td></td>
</tr>
<tr>
<td>S1</td>
<td>( c_{n\beta} = 0.173 )</td>
<td></td>
</tr>
<tr>
<td>S2</td>
<td>( c_{n\rho} = -0.0182 )</td>
<td></td>
</tr>
<tr>
<td>S3</td>
<td>( c_{n\beta} = 0.0 )</td>
<td></td>
</tr>
<tr>
<td>S4</td>
<td>( c_{l\beta} = -0.21 )</td>
<td></td>
</tr>
<tr>
<td>S5</td>
<td>( c_{l\rho} = -0.111 )</td>
<td></td>
</tr>
<tr>
<td>S6</td>
<td>( c_{l_r} = 0.0614 )</td>
<td></td>
</tr>
<tr>
<td>S7</td>
<td>( c_{l\beta} = 0.0 )</td>
<td></td>
</tr>
<tr>
<td>S8</td>
<td>( c_{Y\beta} = -0.866 )</td>
<td></td>
</tr>
<tr>
<td>S9</td>
<td>( c_{Y\beta} = 0.0 )</td>
<td></td>
</tr>
<tr>
<td>RA</td>
<td>( m = 90909.1 )</td>
<td></td>
</tr>
<tr>
<td>RB</td>
<td>( c_{Y_r} = 0.0881 )</td>
<td></td>
</tr>
<tr>
<td>RC</td>
<td>( Y_{ss} = -0.052359878 )</td>
<td></td>
</tr>
<tr>
<td>RD</td>
<td>( c_{Y\rho} = 0.0539 )</td>
<td></td>
</tr>
<tr>
<td>RE</td>
<td>0.0</td>
<td></td>
</tr>
<tr>
<td>I</td>
<td>24</td>
<td>21</td>
</tr>
</tbody>
</table>

50
**APPENDIX G**

Check Case for Program 3 - Label A

<table>
<thead>
<tr>
<th>Register</th>
<th>Input storage</th>
<th>Output&lt;sup&gt;a&lt;/sup&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>R0</td>
<td>0.0</td>
<td>a&lt;sub&gt;4&lt;/sub&gt; = 1.5838890</td>
</tr>
<tr>
<td>R1</td>
<td>0.6854141</td>
<td>a&lt;sub&gt;3&lt;/sub&gt; = 0.9679675</td>
</tr>
<tr>
<td>R2</td>
<td>0.3031611</td>
<td>a&lt;sub&gt;2&lt;/sub&gt; = 1.1621140</td>
</tr>
<tr>
<td>R3</td>
<td>0.0063915</td>
<td>a&lt;sub&gt;1&lt;/sub&gt; = 0.0095553</td>
</tr>
<tr>
<td>R4</td>
<td>-490.2586228</td>
<td>a&lt;sub&gt;0&lt;/sub&gt; = -0.0001405</td>
</tr>
<tr>
<td>R5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>R6</td>
<td>b&lt;sub&gt;15&lt;/sub&gt; = -0.1779356</td>
<td></td>
</tr>
<tr>
<td>R7</td>
<td>b&lt;sub&gt;17&lt;/sub&gt; = 0.0473607</td>
<td></td>
</tr>
<tr>
<td>R8</td>
<td>b&lt;sub&gt;42&lt;/sub&gt; = 0.0380903</td>
<td></td>
</tr>
<tr>
<td>R9</td>
<td>b&lt;sub&gt;43&lt;/sub&gt; = 0.0109210</td>
<td></td>
</tr>
<tr>
<td>S0</td>
<td>b&lt;sub&gt;24&lt;/sub&gt; = 0.1862234</td>
<td></td>
</tr>
<tr>
<td>S1</td>
<td>b&lt;sub&gt;35&lt;/sub&gt; = -0.5652042</td>
<td></td>
</tr>
<tr>
<td>S2</td>
<td>b&lt;sub&gt;16&lt;/sub&gt; = 0.0594608</td>
<td></td>
</tr>
<tr>
<td>S3</td>
<td>b&lt;sub&gt;34&lt;/sub&gt; = 0.0</td>
<td></td>
</tr>
<tr>
<td>S4</td>
<td>b&lt;sub&gt;33&lt;/sub&gt; = 2.3929305</td>
<td></td>
</tr>
<tr>
<td>S5</td>
<td>b&lt;sub&gt;14&lt;/sub&gt; = 1.2648347</td>
<td></td>
</tr>
<tr>
<td>S6</td>
<td>b&lt;sub&gt;23&lt;/sub&gt; = -0.6996473</td>
<td></td>
</tr>
<tr>
<td>S7</td>
<td>b&lt;sub&gt;32&lt;/sub&gt; = 0.0</td>
<td></td>
</tr>
<tr>
<td>S8</td>
<td>b&lt;sub&gt;31&lt;/sub&gt; = 0.1268488</td>
<td></td>
</tr>
<tr>
<td>S9</td>
<td>c&lt;sub&gt;30&lt;/sub&gt; = 1.0</td>
<td></td>
</tr>
<tr>
<td>RA</td>
<td>b&lt;sub&gt;22&lt;/sub&gt; = -0.0110087</td>
<td></td>
</tr>
<tr>
<td>RB</td>
<td>c&lt;sub&gt;21&lt;/sub&gt; = 0.9870954</td>
<td></td>
</tr>
<tr>
<td>RC</td>
<td>b&lt;sub&gt;13&lt;/sub&gt; = -0.1273814</td>
<td></td>
</tr>
<tr>
<td>RD</td>
<td>c&lt;sub&gt;11&lt;/sub&gt; = -0.0421244</td>
<td></td>
</tr>
<tr>
<td>RE</td>
<td></td>
<td></td>
</tr>
<tr>
<td>I</td>
<td>21</td>
<td></td>
</tr>
</tbody>
</table>

<sup>a</sup>They are normalized coefficients for the quintic; thus, a<sub>5</sub> = 1.0.
APPENDIX G

Check Case for Program 3 - Label B

<table>
<thead>
<tr>
<th>Register</th>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>R0</td>
<td>a₄ = 1.583889</td>
<td>a₃ = 1.5915011</td>
</tr>
<tr>
<td>R₁</td>
<td>a₃ = 0.9679675</td>
<td>a₂ = 0.9800821</td>
</tr>
<tr>
<td>R₂</td>
<td>a₂ = 1.1621140</td>
<td>a₁ = 1.1695744</td>
</tr>
<tr>
<td>R₃</td>
<td>a₁ = 0.0095552</td>
<td>a₀ = 0.0184581</td>
</tr>
<tr>
<td>R₄</td>
<td>a₀ = -0.0001405</td>
<td></td>
</tr>
<tr>
<td>R₅</td>
<td></td>
<td></td>
</tr>
<tr>
<td>R₆</td>
<td></td>
<td></td>
</tr>
<tr>
<td>R₇</td>
<td></td>
<td></td>
</tr>
<tr>
<td>R₈</td>
<td></td>
<td></td>
</tr>
<tr>
<td>R₉</td>
<td></td>
<td></td>
</tr>
<tr>
<td>S₀</td>
<td></td>
<td></td>
</tr>
<tr>
<td>S₁</td>
<td></td>
<td></td>
</tr>
<tr>
<td>S₂</td>
<td></td>
<td></td>
</tr>
<tr>
<td>S₃</td>
<td></td>
<td></td>
</tr>
<tr>
<td>S₄</td>
<td></td>
<td></td>
</tr>
<tr>
<td>S₅</td>
<td></td>
<td></td>
</tr>
<tr>
<td>S₆</td>
<td></td>
<td></td>
</tr>
<tr>
<td>S₇</td>
<td></td>
<td></td>
</tr>
<tr>
<td>S₈</td>
<td></td>
<td></td>
</tr>
<tr>
<td>S₉</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rₐ</td>
<td></td>
<td></td>
</tr>
<tr>
<td>R₋</td>
<td></td>
<td></td>
</tr>
<tr>
<td>R₃</td>
<td></td>
<td></td>
</tr>
<tr>
<td>R₄</td>
<td></td>
<td></td>
</tr>
<tr>
<td>R₅</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The stack contains the quintic data as follows:

Stack register T  Number of iterations  12
Stack register Z  t₀  or  t₁/₂  t₀ = 91.0398496  (displayed as negative number)
Stack register Y  Root  0.0076121
Stack register X  8.0  Indicates root has been found

Use the R⁻ to move data into the X register for recording.
APPENDIX G

Check Case for Program 4

Store:
\[ a_3 = 1.4007102 \text{ in } R_0 \]
\[ a_2 = 1.1058038 \text{ in } R_1 \]
\[ a_1 = -0.0158317 \text{ in } R_2 \]
\[ a_0 = -0.0227494 \text{ in } R_3 \]

Results:

| \( R_0 \) | Root indicator | \(-1.00 \) (indicates complex roots) |
| \( R_1 \) | Real part | \(-0.6946683 \) |
| \( R_2 \) | Imaginary part | \( 0.7924165 \) |

| \( S_0 \) | Root indicator | \( 0.00 \) (indicates real roots) |
| \( S_1 \) | First real root | \(-0.1489289 \) |
| \( S_2 \) | Second real root | \( 0.1375553 \) |
## APPENDIX G

**Check Case for Program 5**

<table>
<thead>
<tr>
<th>Register</th>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>R₀</td>
<td>-1.0</td>
<td>-1.0 Root</td>
</tr>
<tr>
<td>R₁</td>
<td>-0.6946683</td>
<td>-0.6946683 indicator</td>
</tr>
<tr>
<td>R₂</td>
<td>0.7924165</td>
<td>0.7924165 and roots</td>
</tr>
<tr>
<td>R₃</td>
<td></td>
<td>t₁/₂ = 0.9975984</td>
</tr>
<tr>
<td>R₄</td>
<td></td>
<td>Δ = 5.5228662</td>
</tr>
<tr>
<td>R₅</td>
<td></td>
<td>N₁/₂ = 0.1254783</td>
</tr>
<tr>
<td>R₆</td>
<td></td>
<td>t = 7.9291450</td>
</tr>
<tr>
<td>R₇</td>
<td></td>
<td>ωₙ = 1.0537969</td>
</tr>
<tr>
<td>R₈</td>
<td></td>
<td>ζ = 0.6592051</td>
</tr>
<tr>
<td>R₉</td>
<td></td>
<td></td>
</tr>
<tr>
<td>S₀</td>
<td>0.0</td>
<td>0.0 Root</td>
</tr>
<tr>
<td>S₁</td>
<td>-0.1489288</td>
<td>-0.1489288 indicator</td>
</tr>
<tr>
<td>S₂</td>
<td>0.1375553</td>
<td>0.1375553 and roots</td>
</tr>
<tr>
<td>S₃</td>
<td></td>
<td>t₁/₂ = 4.6532303  First root</td>
</tr>
<tr>
<td>S₄</td>
<td></td>
<td>t₉ = -5.0379738  Second root</td>
</tr>
<tr>
<td>S₅</td>
<td></td>
<td></td>
</tr>
<tr>
<td>S₆</td>
<td></td>
<td></td>
</tr>
<tr>
<td>S₇</td>
<td></td>
<td></td>
</tr>
<tr>
<td>S₈</td>
<td></td>
<td></td>
</tr>
<tr>
<td>S₉</td>
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<td></td>
</tr>
<tr>
<td>R₆</td>
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<td></td>
</tr>
<tr>
<td>R₇</td>
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<td></td>
</tr>
<tr>
<td>R₈</td>
<td></td>
<td></td>
</tr>
<tr>
<td>R₉</td>
<td></td>
<td></td>
</tr>
<tr>
<td>I</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
APPENDIX G

Check Case for Program 6

Space axes \((X_{sp}, Y_{sp}, Z_{sp})\) to body axes \((X_{b}, Y_{b}, Z_{b})\):

\[ x_{sp} = y_{sp} = z_{sp} = 1.0 \quad \psi = 25^\circ; \quad \theta = 10^\circ; \quad \phi = 30^\circ \]

Results:

\(x_b = 1.1351\) in \(R_6\)
\(y_b = 1.0267\) in \(R_7\)
\(z_b = 0.8109\) in \(R_8\)

Body axes \((X_{b}, Y_{b}, Z_{b})\) to space axes \((X_{sp}, Y_{sp}, Z_{sp})\):

\(x_b = 1.1351\) \(y_b = 1.0267\) \(z_b = 0.8109\)

Results:

\(x_{sp} = 1.0000\) in \(R_6\)
\(y_{sp} = 1.0000\) in \(R_7\)
\(z_{sp} = 1.0000\) in \(R_8\)
APPENDIX H

A DISCUSSION OF FERRARI'S METHOD FOR THE SOLUTION OF A QUARTIC EQUATION

Ferrari (1522-1575), an Italian mathematician, obtained the solution of a quartic by reducing the problem to the solution of two quadratic equations. As the details of obtaining the quadratic equations are not consistent among authors, the details of obtaining the quadratics used for the solution in this paper are presented.

The general quartic equation is
\[ x^4 + a_3x^3 + a_2x^2 + a_1x + a_0 = 0 \]  
(H1)

Rewrite this equation as
\[ x^4 + a_3x^3 = -a_2x^2 - a_1x - a_0 \]  
(H2)

and complete the square

\[ \left(x^2 + \frac{a_3}{2}x\right)^2 = \left(\frac{a_3^2}{4} - a_2\right)x^2 - a_1x - a_0 \]  
(H3)

Now, add \(x^2 + \frac{a_3}{2}x + \frac{y^2}{4}\) to each side of equation (H3), \(y\) being a dummy variable

\[ \left(x^2 + \frac{a_3}{2}x + \frac{y}{2}\right)^2 = \left(\frac{a_3^2}{4} - a_2 + \frac{y}{2}\right)x^2 + \left(\frac{a_3}{2}y - a_1\right)x + \left(\frac{y^2}{4} - a_0\right) \]  
(H4)

The left-hand side of equation (H4) is a perfect square. If the right-hand side is also a perfect square, it can be written as the square of a linear function of \(x\), say \(Cx + D\). Thus, the pair of quadratics that must be solved for the roots of the quartic are

\[ x^2 + \frac{a_3}{2}x + \frac{y}{2} = \pm(Cx + D) \]  
(H5)

The right-hand side of equation (H4) is a perfect square if, and only if, its discriminant is 0.
\( \left( \frac{a_3 y - a_1}{a_2 + y} \right)^2 - \left( \frac{a_3^2}{a_2 + y} \right)^2 = 0 \) \hspace{1cm} (H6)

In this equation \( y \) has not been defined, and if equation (H6) is written as a function of \( y \), it becomes

\[
y^3 - a_2 y^2 + (a_3 a_1 - 4a_0) y + \left[ a_0 (4a_2 - a_3^2) - a_1^2 \right] = 0 \hspace{1cm} (H7)
\]

This equation is called the resolvent cubic and any root \( y_1 \) of equation (H7) insures that equation (H6) is 0.

All that remains is the determination of the coefficients \( C \) and \( D \). The discriminant equation (H6)

\[
\left( \frac{a_3 y}{a_2 + y} \right)^2 = \left( \frac{a_3 y}{a_2 + y} \right)^2 \left( \frac{y^2}{a_2 + y} \right)^2
\]

permits the right-hand side of equation (H4) to be written as

\[
\frac{\left( a_3 y - a_1 \right)^2}{y^2} x^2 + \left( \frac{a_3 y}{a_2 + y} \right) \left( \frac{y^2}{a_2 + y} \right)
\]

which is a perfect square, and the coefficients \( C \) and \( D \) are

\[
C = \left( \frac{a_3 y}{a_2 + y} \right) \sqrt{\frac{y^2}{a_2 + y} - a_0} \hspace{1cm} (H8)
\]

\[
D = \sqrt{\frac{y^2}{a_2 + y} - a_0} \hspace{1cm} (H9)
\]
only if \( D \neq 0 \). The right-hand side of equation (H4) as written is a perfect square because

\[
\frac{a_3 y}{2} - a_1 = 2 \sqrt{\left(\frac{a_3^2}{4} - a_2 + y\right)\left(\frac{y^2}{4} - a_0\right)}
\]

so that

\[
C = \sqrt{\frac{a_3^2}{4} - a_2 + y}
\]  \hspace{1cm} (H10)

and are used in place of equation (H8) if \( D = 0 \).
REFERENCES


Figure 1.- Coordinate systems and Euler angles. Order of rotation for Euler angles is $\Psi$, $\Theta$, and $\Phi$. Moving axes translate with airplane and remain parallel to Earth fixed axes. Positive directions are shown.
Airplane stability programs are presented for calculating airplane stability characteristics with a card programmable pocket calculator. These calculations include eigenvalues of the characteristic equations of lateral and longitudinal motion as well as stability parameters such as the time to damp to one-half amplitude or the damping ratio. The effects of wind shear are included. Background information and the equations programmed are given. The programs are written for the International System of Units, the dimensional form of the stability derivatives, and stability axes. In addition to the programs for stability calculations, an unusual and short program is included for the Euler transformation of coordinates used in airplane motions. The programs have been written for a Hewlett Packard HP-67 calculator. However, the use of this calculator does not constitute an endorsement of the product by the National Aeronautics and Space Administration.