REVERSE CURRENT IN SOLAR FLARES

by

Joshua W. Knight III

National Aeronautics and Space Administration
Grant NGL 05-020-272

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INSTITUTE FOR PLASMA RESEARCH
STANFORD UNIVERSITY, STANFORD, CALIFORNIA
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*Also Department of Applied Physics
ABSTRACT

An idealized steady state model of a stream of energetic electrons neutralized by a reverse current in the pre-flare solar plasma is developed. These calculations indicate that, in some cases, a significant fraction of the beam energy may be dissipated by the reverse current. Joule heating by the reverse current is a more effective mechanism for heating the plasma than collisional losses from the energetic electrons because the Ohmic losses are caused by thermal electrons in the reverse current which have much shorter mean free paths than the energetic electrons.

Analysis of the steady state model indicates that it can not adequately describe the interaction of the beam with the solar plasma because the atmosphere is rapidly heated. If the time scale for this heating is short enough, the density of the atmosphere can be taken constant in time. The charge separation required to drive the reverse current is expected to respond to changes on a time scale very short compared to the time for the ambient plasma temperature to change significantly, so it is a reasonable approximation to use the steady state results for the electric field. With these simplifications, the heating due to reverse currents is calculated for two injected energetic electron fluxes. For the smaller injected flux, the temperature of the coronal plasma is raised by about a factor of two. The larger flux causes the reverse current drift velocity to exceed the critical velocity for the onset of ion-cyclotron turbulence, producing anomalous resistivity and an order of magnitude increase in the temperature. The heating is so
rapid that the lack of ionization equilibrium may produce a soft x-ray and EUV pulse from the corona.
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1. INTRODUCTION

This dissertation examines the consequences of reverse currents that may be expected to develop in the solar atmosphere in response to the imposition of a directed stream of energetic (non-thermal) electrons. The phenomena which indicate the presence of streams of electrons manifest themselves primarily in the "flash phase" of solar flares (Svestka 1975). Not all flares exhibit a "flash phase" (Svestka 1975, Sweet 1969, Sturrock and Coppi 1966) and the existence of directed streams of non-thermal electrons is not universally accepted (Svestka 1975, Brown 1974, Brown and Melrose 1977). A short historical review is presented (cf. 1.1) as an attempt to place the phenomena in perspective. Observations that indicate the presence of energetic electrons in the solar atmosphere are reviewed and the introduction concludes with a short summary of our present theoretical understanding of the flare process.

In Chapter 2 the objections to unneutralized electron beams and previous work on reverse currents are summarized and a steady state model of a stream of energetic electrons neutralized by a reverse current is developed. In Chapter 3 the model is modified to include time dependence for a restricted case. The results of Chapters 2 and 3 are summarized in Chapter 4 and possible extensions of the present work are suggested. Details of the numerical calculations of Chapters 2 and 3 are discussed in the appendices.

1.1 Historical Overview

The sun is the closest star to the earth and the only star which we can presently observe in great detail. Aside from the intrinsic interest of solar phenomena, we can hope that by understanding solar phenomena we
will gain insight into what is likely to happen on other stars like the sun. The sun is a normal G type main sequence star, but by virtue of its position it is the brightest object in the sky. The importance of the sun to life on earth cannot be overstated. In the introduction to his book, The Sun, C. A. Young (1902a) emphasizes this point.

"It is true from the highest point of view the sun is only one of a multitude - a single star among millions - thousands of which, most likely, exceed him in brightness, magnitude and power. He is only a private in the host of heaven. But he alone, among the countless myriads, is near enough to affect terrestrial affairs in any sensible degree; and his influence upon them is such that it is hard to find the word to name it; it is more than mere control and dominance. He does not, like the moon, simply modify and determine certain more or less important activities upon the surface of the earth, but he is almost absolutely, in a material sense, the prime mover of the whole. To him we can trace directly nearly all the energy involved in all phenomena, mechanical, chemical or vital. Cut off his rays for even a single month, and the earth would die; all life upon its surface would cease."

The great preponderance of the energy flux from the sun is, to the best of our knowledge, very nearly constant (Smith and Gottlieb 1974). It is only in those portions of the electromagnetic spectrum where the solar output is small (radio XUV, X-ray), in individual spectral lines (e.g. H, Ca H and K), and in particle emission (the solar wind, energetic electrons and nuclei), that the sun's output varies significantly due to solar activity.

The most obvious manifestations of solar activity are sunspots. Sunspots have been observed telescopically since 1611, shortly after the invention of the telescope, and with the unaided eye on infrequent occasions since ancient times (Bray and Loughhead 1964). It is not clear which of four men, Galileo Galilei, Johann Goldsmid, Thomas Harriot or Christopher Scheiner, actually made the first telescopical observation of
sunspots (Bray and Loughhead 1964). That another manifestation of solar activity, faculae, were observed at about the same time is demonstrated by the title of Christopher Scheiner's (1630) book, *Rosa Ursina Sive Sol ex Admirando Facularum and Macularum Phearum Pheonomeno Varius* (see Eddy et al. 1977, Meadows 1970). In the first half of the 19th century Schwabe (1844) announced the possible existence of the sunspot cycle with a period of about 10 years ("von ungenfahr 10 Jahren"). Wolf (1852) later deduced a more accurate period of 11.1111 ± 0.038 years or "de sorte que neuf periodes equivalent justement a un siecle". Wolf (1852) also deduced from earlier records the years of sunspot minima back to 1700, but the earlier portion of this historical reconstruction has been questioned recently (Eddy 1976).

The first recorded observation of a solar flare occurred on September 1st, 1859. A relatively rare "white light flare", visible against the photosphere, was simultaneously observed by Carrington (1859) and Hodgson (1859). In 1868 Janssen (1869) and Lockyer (1869) independently discovered that prominences could be seen outside eclipse with a spectro-scope with a wide entrance slit. Thereafter various observers, especially Secchi (1877) made extensive visual observations of the forms of the chromosphere and prominences using this technique. Flares in individual lines were observed quite often from the 1870's onward (see Young 1871, 1902a, b, c for early examples). The first photographs of flares were obtained by Hale (1892) with a spectroheliograph of his own invention (Hale 1891). Deslandres (1893) independently developed a similar instrument, and the basic principle of the spectroheliograph was known to Janssen (1869) who actually constructed an instrument similar to the
spectrohelioscope (Millochau and Stefanik, 1906) for observing prominences but abandoned it in favor of a widened spectroscope slit. The basic principle was independently discovered by Braun, and Lohse attempted the construction of a spectroheliograph (Hale 1906). The matter of who actually used a "spectroheliograph" first was the subject of some debate between Deslandres and Hale (Hale 1906, Deslandres, 1905) but this distinction is generally given to Hale. In 1908 Hale (1908) made the first observation of magnetic fields on the sun, and realized soon thereafter that magnetic fields, sunspots and flares were intimately connected (Hale 1929). Because the spectroheliograph took a relatively long time to form an image of the whole sun, the systematic investigation of flares did not begin until the spectrohelioscope, constructed by Hale in 1926 (Hale 1929), was fully developed (Smith and Smith 1963). The development of the polarizing monochromatic filter (Lyot filter) by Ohman in 1938 (Ohman 1938), independently of Lyot's original proposal (Loyt 1933, Evans 1949), allowed photographs of the entire solar disk in one spectral line to be made rapidly. This type of filter is still widely used in flare patrol telescopes and solar observatories.

Jansky (1933) made the first observation of radio emission from an extra-terrestrial source. It was not until 1942 that Hey (1946) discovered meter wavelength radiation from the sun. At about the same time Southworth (1945) discovered centimeter wavelength radiation from the sun. Reber (1944) made the first published report of radio emission from the sun; the earlier work was not published due to its association with the war effort. Appleton (1945) published evidence for radio emission from the sun in the 7-30 meter wavelength band. Appleton's
results were based on amateur radio operators' reports (dating from 1936) of "hiss" heard only during the daylight hours and frequently before sudden fade outs. Appleton and Hey (1946) noted that some radio bursts were associated with flares. Covington (1948) first reported microwave bursts from the sun near the maximum of solar cycle 18.

Burnight (1949) reported the first observation of X-ray emission from the sun. Burnight's observation was made using photographic film with aluminum and beryllium filters flown in a captured V2 rocket. Peterson and Winkler (1958) made the first observation of a flare associated impulsive X-ray burst using a balloon borne proportional counter.

1.2 Review of Observations

The presence of energetic electrons in the solar atmosphere is inferred from impulsive hard X-ray bursts, impulsive microwave bursts and observations of energetic electrons by satellites in earth orbit. Impulsive microwave bursts are rapid enhancements of radio flux at frequencies greater than $\sim 1$ GHz. These impulsive enhancements occur simultaneously with impulsive X-ray and EUV bursts and often show very similar time structure, even down to the fine details of the time profiles (Peterson and Winkler 1959, Kundu 1961, Anderson and Winkler 1962, Kane and Donnelly 1971, deFeiter 1971, Svestka 1970). The impulsive microwave bursts are generally attributed to gyro-synchrotron radiation from electrons with energies greater than $\sim 100$ keV (Holt and Ramaty 1969, Svestka 1970). The gradual post-burst increases can be interpreted as thermal bremsstrahlung from the flare-associated soft X-ray plasma and are usually accompanied by radio emission at lower frequencies (Svestka 1970). The apparent discrepancy between the number of electrons required
to produce the impulsive X-ray bursts and the number of electrons required
to produce the impulsive microwave emission (Peterson and Winkler 1959)
can be resolved if the details of the microwave emission in the solar
atmosphere are considered (Holt and Ramaty 1969, Takakura 1972). The
generation and propagation of microwaves in the solar atmosphere during
a solar flare are complicated processes involving the magnetic field
configuration, ambient plasma density and temperature and density and
energy spectrum of the non-thermal electrons (Holt and Ramaty 1969,
Kruger 1972, Takakura 1972, Svestka 1975). Therefore, it is difficult to
unambiguously infer the number and energy spectrum of the non-thermal
electrons from the observed microwave emission.

In some flares, non-thermal electrons escape into the interplanetary medium and are observed by satellites in earth orbit (Svestka 1975,
Lin 1974). Since the electrons apparently propagate primarily along
magnetic field lines in the interplanetary medium, electrons are observed
primarily from flares in the western half of the visible hemisphere of
the sun or from flares behind the west limb of the sun (Svestka 1975,
Lin 1974). Lin (1974) concludes that there are two distinct types of
non-relativistic electron bursts ($E < 500$ keV) observed at 1 AU, "pure
electron events", that is those not accompanied by energetic ($> 10$ mev)
protons, and "mixed events" during which both energetic electrons and
protons are observed. The energy spectra of the "pure electron" events
can be well fitted between 5 keV and 100 keV by a power-law in energy,
$$\frac{dN}{dE} \propto E^{-\gamma},$$
with $\gamma \sim 2.5-5.5$ but exhibit a rapid steepening at energies above 100 keV (Lin 1974). On the other hand the typical spectra of
energetic electrons for "mixed events" extend smoothly in a power-law
out to and beyond 10 mev and tend to be somewhat harder ($\gamma \sim 2.5-4.5$) (Lin 1974). When impulsive X-ray bursts are associated with electrons observed at 1 AU, $10^2-10^3$ more electrons are required to produce the impulsive hard X-ray bursts than escape to the interplanetary medium (Lin and Hudson 1972, Lin 1974).

It is now generally believed that the mechanism for the production of impulsive hard X-ray bursts is bremsstrahlung from electrons scattering on protons and heavier ions (Kane 1974, Svestka 1971, Brown 1974). Smaller impulsive events generally consist of one or a few spikes with comparable e-folding rise and fall times of $\sim 10$ s (Kane 1975, Kane and Anderson 1970, Crannell et al. 1975). Larger events, with total durations of minutes or tens of minutes, usually have a complex spiky time structure (Svestka 1971, Hoyng et al. 1972). Frost and Dennis (1972) and Frost (1974) have also reported an apparently distinct non-impulsive non-thermal hard X-ray component in some larger events, after the impulsive phase of the flare and possibly associated with a second phase of particle acceleration. In this work, we restrict our attention to the impulsive hard X-ray bursts, and assume that both the later "second phase" hard X-rays and the "gradual components" in the low energy channels ($< 100$ keV) of some instruments are distinct phenomena.

The spectral information on impulsive hard X-ray bursts is limited, but most events can be reasonably fitted to a decreasing power-law in photon energy between $10^{-1}$ keV and $100$ keV (Kane 1974, Brown 1971, Hoyng 1974). The power-law index is typically between 2.1 and 3 (Kane 1974, Svestka 1971) although some bursts have very soft spectra and power-law indices as large as 8 have been reported (Peterson et al. 1974).
Most events show a softening of the spectrum at higher energies (Kane 1974, Svestka 1975). This bend or "knee" in the power-law spectrum usually occurs between 60 and 100 keV (Brown 1975, Svestka 1975) but in some events can occur as high as 500 keV (Brown 1975). Since the high energy cut-off of many instruments is below 500 keV [e.g. OSO-7 (Peterson et al. 1974), OSO-5 (Frost et al. 1970) or OGO-5 (Kane and Anderson 1970)] such a break in the spectrum may be present in many events for which no break is reported. It is obvious that the power-law must flatten at low energies, otherwise the total X-ray flux would diverge. However, the determination of the low energy cut-off is difficult because the X-ray emission at low energies (< 10 keV) is dominated by the gradual quasi-thermal component in most events (Brown 1975, Svestka 1975).

Although the interpretation of the X-ray spectrum as bremsstrahlung from a non-thermal (i.e. non-Maxwellian in energy) distribution of electrons is widely accepted, some workers advocate a thermal interpretation for many impulsive X-ray bursts (for example Chubb 1970, Elcan 1976, Crannell et al. 1977) and some events seem to fit an exponential rather than a single power-law spectrum (Elcan 1976, Crannell et al. 1977). However, the spectral data are poor, particularly at higher energies (primarily due to counting statistics), and it is not clear that an exponential spectrum is to be preferred over two power-laws or some other form for the spectra, Brown (1974) has demonstrated that any observed hard X-ray spectrum can be produced by a thermal plasma with a suitable temperature distribution in the source. Brown (1975) has also pointed out that the emitted X-ray spectrum is rather
insensitive to the source electron energy spectrum and concludes that a power-law electron spectrum is not strongly mandated by the presently available data.

There are theoretical objections to multi-thermal models of impulsive hard X-ray bursts (Kahler 1979) and models that produce power-law spectra by superposing different exponential spectra seem somewhat contrived to this author despite assertions to the contrary by some workers (e.g. Brown 1979). In more recent "thermal" models of impulsive hard X-ray bursts (e.g. Smith and Lilliequist 1980), the electron distribution is not expected to be Maxwellian. Neither the theoretical objections (Kahler, 1979) or the limited observational support for thermal electron distributions (Elsen 1970, Crannell et al. 1977) are relevant to this type of model.

There is some support, from observations of impulsive EUV bursts, for the view that the impulsive hard X-ray bursts are produced by non-thermal, energetic electrons streaming from the corona to the chromosphere. Impulsive EUV bursts have been observed directly by satellites (for example Kelley and Rense 1979, Hall 1971) and indirectly from the ionospheric effects they produce (Donnelly 1974). These bursts show close time coincidence with the impulsive X-ray and microwave bursts and the time profiles closely resemble the X-ray and microwave bursts (Noyes 1977, Donnelly 1977, Kane and Donnelly 1977, Kane 1977). The energy radiated in the 10-10.30 Å band is \( \geq 10^{34} \) times the energy radiated in the associated impulsive hard X-ray burst (Donnelly 1977, Kane and Donnelly 1977). This ratio of energy radiated in the EUV and hard X-ray bands corresponds qualitatively with the expected ratio of
Coulomb collisional losses to bremsstrahlung emission from a thick-target hard X-ray source (Koch and Motz 1959, Petrosian 1973, Donnelly 1974, Brown 1975). There are indications that the EUV radiation originates low in the chromosphere. The density of the EUV emitting region can be estimated to be \( \approx 10^{12} \text{ cm}^{-3} \) (Donnelly 1974, Kane and Donnelly 1971), corresponding to the solar chromosphere, and the EUV bursts exhibit (statistical) limb darkening which would be expected if the radiation originates in the chromosphere (Kane and Donnelly 1971).

Although the observations presently available do not exclude other interpretations, the preponderance of evidence seems to favor bremsstrahlung from a non-thermal distribution of energetic electrons as the source of impulsive hard X-ray bursts (Svestka 1973).

If the emergent X-ray spectrum were known exactly, the spectrum of the energetic electrons that produce the radiation, averaged over the source, could in principle, be recovered (Brown 1975). The two extreme approximations that are usually considered are "thick-target" and "thin-target" (Brown 1974, Svestka 1977, Hudson 1974). In the thin-target approximation the electrons lose a negligible amount of energy in the hard X-ray source (Brown 1975, Svestka 1977, Hudson 1974). In this approximation, the mean electron source spectrum \( [i.e. \text{the instantaneous average of the electron energy spectrum over the emitting volume weighted by the background density, see Brown (1975)]} \) is just the spectrum of accelerated electrons. In the thick-target approximation, this is not the case.

In the thick-target approximation, the electrons lose all their energy (primarily by Coulomb collisions) in the source region. Since
the mean free path of the more energetic electrons is longer, the energy spectrum of the electrons averaged over the emitting volume is harder. If we assume the density is uniform in the source, that the electrons are all streaming downward and that the accelerated electron energy spectrum is a fairly steep power-law, then the approximate difference between the inferred mean electron source spectrum in the thin and thick-target approximations can be estimated. In this case, since the mean free path of an electron against Coulomb collisions is approximately proportional to \( E^2 \), the effective source volume for electrons of energy \( E \) in the accelerated spectrum is also approximately proportional to \( E^2 \). Since the injected spectrum is very steep (by assumption), once an electron has lost an appreciable fraction of its energy, it no longer contributes significantly to the emergent X-ray flux. Therefore, to produce the same power-law index of emergent X-rays the index of the injected electron beam must be \( \sim 2 \) greater (a softer injected spectrum) in the thick-target case than in the thin-target approximation. The preceding simple analysis neglects beaming effects in the case the energetic electron velocity distribution is anisotropic (Petrosian 1973, Brown 1972) and the exact behavior of the Coulomb cross section. However, the conclusion is found to be qualitatively correct in thick-target models of impulsive hard X-ray bursts for X-ray energies below \( \sim 100 \) keV even when a more detailed analysis is performed (Brown 1975, Petrosian 1973, Hudson 1972, Brown 1971). The more detailed calculations indicate that, depending on the assumptions and model characteristics, thick-target models require injected electron power-law indices \( \sim 1.5-2 \) greater than thin-target models for the same emergent X-ray spectra.
Some early models of impulsive X-ray bursts considered impulsive injection of the energetic electrons and the subsequent decay of the impulsively injected electrons in the source region (e.g. Takakura and Kii 1966). In its simplest form this model does not agree with observations since it predicts a systematic hardening of the burst spectra during the decay of the hard X-ray burst (Brown 1975, Petrosian 1973) for the same reason that the source averaged energetic electron spectrum is harder in the thick-target models, i.e. the low energy electrons lose their energy more rapidly than the high energy electrons. Brown (1972) has introduced a modification of the usual coronal impulsive hard X-ray source that removes this particular objection, but it requires the assumption of an average source density that is energy dependent \( \langle n \rangle \propto E^{\alpha}, \alpha > 3/2 \). Brown (1972) motivates this assumption by invoking an energy dependent pitch angle distribution for the accelerated electrons, but the simplicity of the original "coronal trap" model is lost. This type of model can explain the observation of impulsive X-ray bursts from "behind-the-limb" flares since portions of the X-ray source are high in the corona. However, since behind-the-limb flares also produce high energy X-rays, this model requires densities \( \geq 10^{11} \text{ cm}^{-3} \) high \( (\geq 10^{9} \text{ cm}) \) in the solar atmosphere. If this were the case, impulsive X-ray bursts from behind-the-limb flares could be explained by thick-target models as well. Since the product of the instantaneous number of energetic electrons in the source and background density determines the emergent X-ray flux, Brown's (1972) model requires a much larger number of energetic \( (> 20 \text{ keV}) \) electrons than equivalent thick-target models. Additionally, since most of the energy resides in the low energy electrons
which encounter only low densities ($\sim 10^9$), these electrons cannot be invoked to account for the impulsive EUV bursts which are emitted from regions where the density is $\gtrsim 10^{12}$ cm$^{-3}$ (Donnelly 1974) and which are observed simultaneously with impulsive X-ray bursts (this is also true of more recent "thermal" models, e.g. Smith and Lilliequist 1978).

Aside from some difficulty in accounting for impulsive X-ray bursts from behind-the-limb flares, thick-target models for the hard X-ray bursts are at least not excluded by present observations. Since they have the advantage of also providing the energy required for the impulsive EUV bursts (Donnelly 1974), it seems reasonable to accept the thick target approximation for the production of the hard X-ray bursts. In this case, since the time for the electrons to lose all their energy is short compared to the time scale of the impulsive X-ray burst (Brown 1975, Petrosian 1973), variations in the X-ray flux and spectrum are attributed to changes in the (unspecified, c.f. l.3) acceleration process.

In the thick-target model the energy flux of the electron stream required to produce a specified X-ray flux at 1 AU depends on (a) the anisotropy of the electron velocity distribution, (b) the power-law index of the X-ray flux and (c) the lowest energy to which the power-law in energy is assumed to extend for the energetic electrons (Brown 1975, Petrosian 1973). Neglecting possible beaming of the bremsstrahlung radiation (Petrosian 1973, Brown 1972) and backscatter from atmosphere (Langer and Petrosian 1977), we can obtain an order of magnitude estimate for the flux of non-thermal electrons at the sun for an observed flux of X-rays at 1 AU. If the flux of X-rays at some energy $E_0$ at earth is $f$ (photons cm$^{-2}$ s$^{-1}$ keV$^{-1}$), then the total X-ray photon flux
Since the observed power-law spectra are typically fairly steep (Brown 1975, Svestka 1975), most of the X-rays at $E_0$ are produced by electrons with only slightly higher energies. The total efficiency (the ratio of bremsstrahlung losses to Coulomb collisional losses) is approximately $10^{-6}E$ for a thick-target hydrogen plasma (Koch and Motz 1959, Petrosian 1973). Therefore, the total non-thermal electron flux in the source above $E_0$ must be $\approx 10^{33.45} E_0^{-1}$ keV$^{-1}$.

### 1.3 Review of Flare Theories

In the preceding sections we have discussed some of the observed properties of solar flares as they relate to the inferred presence of non-thermal electrons in the solar atmosphere during a flare. We have not dealt with most of the diverse phenomena associated with solar flares. Svestka (1975) lists thirty-seven "basic properties of flares". When all the subtopics are counted, Svestka's list contains more than eighty observational aspects of flares. With such a large number of properties to be considered, it is not surprising that a wide variety of flare theories and models have been proposed. Since reviews of flare theories exist in the literature (Svestka 1975, Sweet 1969), the selection of theoretical ideas discussed here is only representative and not exhaustive. This discussion of flare theories is included only to show how the production of non-thermal electron streams fits in the present theoretical picture of solar flares and therefore no particular model will be treated in detail.

It is now widely believed that the energy released in solar flares is stored in the magnetic fields in the upper solar atmosphere (Rust 1977, Svestka 1975, Sweet 1969). The energy which is available for
release is the excess energy of the non-potential magnetic field configuration above the energy in the potential (current-free) field (Rust 1977, Svestka 1975, Sweet 1969). Because the magnetic field energy density is generally believed to be greater than the thermal energy density of the plasma in the upper solar atmosphere, the non-potential field configurations must be nearly force-free (Gold and Hoyle 1960, Sturrock 1974). Although many non-potential field configurations have been proposed, these configurations can be divided into two broad categories depending on the distribution of currents in the solar atmosphere (Svestka 1975, Sturrock 1974). One possibility is a force-free configuration in the form of twisted flux tubes (Gold and Hoyle 1960, Alfvén and Carlqvist 1967, Spicer 1977) or sheared field lines (Tanaka and Nakagawa 1973). In this case the currents are distributed over a large volume in the atmosphere. The other possibility is that the field is largely current-free with the current concentrated in current sheets (Sweet 1958, Syrovatsky 1966, Sturrock 1968, Priest and Heyvaerts 1974). A large number of flare models have been developed under the assumption that current sheets develop in the solar atmosphere as a result of motions in the photosphere or the emergence of new flux (Svestka 1975, Sweet 1969). Barnes and Sturrock (1972) have studied the development of non-potential force-free fields due to photospheric motions and found that the stored energy in the force-free configuration can exceed that of a configuration with a current sheet. They concluded that one possible sequence of events that would produce a current sheet in the solar atmosphere was the conversion of a more energetic force-free configuration to a configuration with a sheet. Priest and Heyvaerts (1974) examined
the production of a current sheet when new flux emerges into a pre-existing magnetic field configuration.

The earliest electromagnetic models of flares invoked the production of non-thermal electrons and realized the importance of electric fields at "neutral points" in the magnetic field (Giovanelli 1946, 1947, 1948, Hoyle 1948). Dungey (1958) pointed out that, when reconnection of magnetic field lines occurs, a DC electric field will be developed in the reconnection region which could lead to acceleration of charged particles. In "current interruption" models (Alfven and Carlqvist 1967) electrons are accelerated by the DC electric field that develops when the "inductive circuit" is opened. In models in which reconnection occurs in a current sheet (Sturrock 1968, Friedman and Hamberger 1969, Coppi and Friedland 1971), some acceleration by a DC electric field at the neutral point may occur, but the bulk of the acceleration is usually attributed to stochastic acceleration of electrons by high frequency electric fields that develop during the reconnection process due to plasma instabilities (Sturrock 1974, Smith 1974). It has proved difficult to develop a self consistent theoretical model of the rapid acceleration of the number of electrons required to produce the observed X-ray flux (Smith 1977a,b, Brown and Melrose 1977). At present, the mechanism by which electrons are accelerated in the impulsive phase of solar flares is not well understood theoretically (Svestka 1973).

However, simple considerations indicate that if the energy stored in the magnetic field is released in the low density corona, particles can be expected to acquire energies of 10-100 keV (Sturrock 1974). Furthermore, the ingredients of many possible acceleration mechanisms (DC electric
fields, plasma turbulence) are natural by-products of most processes which release the energy stored in the magnetic field. Therefore, since there is observational evidence for the acceleration of electrons in the impulsive phase of flares, we will assume that this acceleration does occur even though the exact mechanism has yet to be elucidated.
2. STEADY STATE MODEL OF BEAM AND REVERSE CURRENT

2.1 Objections to Unneutralized Beams

The simplest thick-target model for the production of impulsive X-ray bursts is that considered by Petrosian (1973). In this model a beam of energetic electrons is assumed to propagate from the corona to the chromosphere. All the electrons are assumed to have their velocities in the same direction until they lose all their energy, approximating a source in which the energetic electrons stream down a nearly vertical magnetic field line with small pitch angles into an atmosphere with a small density scale height (Petrosian 1973). Several authors (Brown 1976, Brown and Melrose 1977, Colgate et al. 1977, Hoyng 1977, Hoyng et al. 1976) have pointed out difficulties if this electron stream is not neutralized by a reverse current.

Brown (1976) pointed out that the number of electrons required to stream from the corona to the denser portions of the solar atmosphere during some impulsive hard X-ray bursts was quite large. Indeed in some events as large as $10^{39}$ (Hoyng et al. 1976), or all the electrons in the solar atmosphere above the level where the electron density is $\sim 10^{33}$ cm$^{-3}$ (Brown 1976). Another objection to the existence of an unneutralized beam is that the magnetic energy that would be stored in this beam is many orders of magnitude larger than the total flare energy (Colgate et al. 1977). If $N$ is the total number of electrons streaming downward over the duration $\tau$ (s) of the impulsive phase, the magnitude (emu) of the current may be estimated from

$$I \approx ec^{-1} NT$$  \hspace{1cm} (2.1)
If the transverse and longitudinal dimensions of the stream are of order \( R \) (cm), an estimate of the strength \( B \) (gauss) of the magnetic field produced by the stream is given by

\[
B \approx \frac{2I}{R};
\]

(2.2)

and the total energy \( U \) (ergs) of this magnetic field may be estimated from

\[
U \approx \frac{1}{8\pi} R^3 B^2 \approx \frac{1}{2\pi} I^2 R,
\]

(2.3)

which becomes

\[
U \approx \frac{1}{2\pi} e^2 c^{-2} N^2 \tau^{-2} \approx 10^{-40} \cdot 10^{-4} \cdot 10^{-2}.
\]

(2.4)

Kane and Anderson (1970) estimate the total energy involved in a typical small flare to be \( \sim 10^{29} \) ergs, the time scale to be \( \sim 10^2 \) s, and the characteristic length scale to be \( \sim 10^{8.5} \) cm and infer from the X-ray data that the total number of energetic electrons is \( \sim 10^{35} \). For these values the above formulae lead to estimates of \( I \approx 10^{13.2}, B \approx 10^5 \), and \( U \approx 10^{34} \). For a large event the total flare energy could be \( \sim 10^{32} \) ergs, the length scale \( \sim 10^{9.5} \), the characteristic time \( \sim 10^3 \) and the total number of energetic electrons \( \sim 10^{39} \) (Hoyng et al. 1976). In this case \( I \approx 10^{16.2}, B \approx 10^7 \) and \( U \approx 10^{41} \). Clearly a model which involves an unneutralized beam leads to unacceptably high values of the magnetic field and magnetic energy associated with the beam.

Problems associated with the propagation of high current beams of charged particles not neutralized by a reverse current have been considered in other contexts. Alfven (1939) examined the limitations on the propagation of electrostatically neutralized high current beams of relativistic...
charged particles, motivated by an apparent sidereal day variation in the cosmic ray flux (Alfven 1938, Compton and Getting 1935), which later proved to be spurious (Dorman 1974). Consider a cylindrically symmetric, mono-energetic, uniform beam of charged particles moving through a background of opposite charge so distributed that the charge density (esu) is everywhere zero. If the beam is infinite in extent along the symmetry axis and has a radius of $R$, then the magnetic field as a function of distance from the axis for $r \leq R$ is

$$B(r) = \frac{2I(r)}{r} = 2\pi j r,$$  \hspace{1cm} (2.5)$$

where $I(r)$ is the current inside $r$ and $j$ is the current density (assumed uniform). The gyro radius of a charged particle in a magnetic field is

$$r_g = \frac{pc}{qB} \hspace{1cm} (2.6)$$

where $p$ is the particle momentum and $q$ is the particle charge.

Consider a test particle of the same charge and mass as the beam particles moving in the magnetic field of the beam. Suppose the test particle is initially at the outer edge of the beam ($r=R$) and has the same momentum as the beam particles. We denote by $R_A$ the beam radius for which the gyroradius of this particle in the average field it sees in its trajectory is equal to the beam radius. For a beam of this radius ($R_A$), the particle will cross the axis of symmetry with its momentum perpendicular to that of the beam particles. If the radius of the beam is increased, the particle will cross the symmetry axis with the component of its momentum opposite in sign from that of the beam particles and its average
velocity over the trajectory will also be negative. Clearly increasing the beam radius beyond \( R_A \) will not increase the current. If we estimate the average magnetic field as \( \sim 1/2 \) the field at the edge of the beam, we find that there is a maximum current which can be carried by a beam which satisfies our original assumptions:

\[
\bar{B} \approx \frac{I_A}{R_A} = \frac{I_A}{v} = \frac{I_A}{\frac{pc}{qB}}.
\]  

(2.7)

Therefore

\[
I_A \approx \frac{pc}{q} = \frac{mc^2\gamma}{qB}.
\]  

(2.8)

Here \( \beta = v/c \) and \( \gamma = (1 - \beta^2)^{1/2} \). \( I_A \) is called the "Alfven current limit" or the "Alfven-Lawson current limit" and for electrons we find

\[
I_A \approx 1700 \beta \gamma,
\]  

(2.9)

in agreement with Alfven's (1939) more rigorous derivation. This restriction is much more stringent than the objections to the stored magnetic energy. For an electron energy of 100 keV, the currents estimated for the hypothetical small and large events are \( \sim 10^{10} I_A \) and \( \sim 10^{13} I_A \) respectively. The value of the current limit derived by Alfven depends on all the original assumptions being satisfied. Arbitrarily large currents can in principle be propagated by relaxing the assumption of exact electrostatic neutralization (Lawson 1957, 1958, 1959), the assumption that the beam is mono-energetic (Bennett 1934), the assumption that the current density (particle flux) is uniform (Hammer and Rostoker 1970) or adding a very strong magnetic field along the symmetry axis.
(Hammer and Rostoker 1970). However, none of these mechanisms seem particularly likely to be applicable in solar flare impulsive hard X-ray bursts, although some are relevant to particular laboratory experiments. The simplest resolution to these objections is the existence of a reverse current (cf. 2.2).

2.2 Previous Work on Reverse Currents

It is well known that a plasma tends to preserve charge neutrality. A process which tends to give an excess positive or negative charge in some region will lead to electric fields which act upon the plasma. Movement of electrons in response to this electric field will then restore charge neutrality. One expects that analogous process will also tend to maintain current neutrality. If an electron beam is suddenly introduced into a plasma, a sudden change occurs in the magnetic field structure which will develop induced electric fields opposing the primary current.

Although interest in beams of relativistic electrons is not recent (see for example Bennett 1934, Alfven 1939), theoretical and experimental work on high current relativistic electron beams was stimulated by the development of devices capable of producing relativistic electron beams with currents on the order of or greater than the Alfven-Lawson current limit (See for example Graybill and Nablo 1966, Roberts and Bennett 1968, Yonas and Spence 1969). Roberts and Bennett (1968) injected a beam of 3.5 mev electrons ($\beta=0.992$, $\gamma=7.85$) with a beam current of 3000 emu ($I \simeq 0.23I_A$) into a linear pinch with $n_e \simeq 10^{18.5} \text{cm}^{-3}$. They found that the beam current was nearly completely neutralized by a reverse current in the ambient plasma and that the change in the total current (measured)
was a very small fraction of the beam current. Similar results have been obtained with other experimental apparatus (Prono et al. 1975, Ekdahl et al. 1974, Goldenbaum et al. 1974, Klok et al. 1974, Miller and Kuswa 1973, Levine et al. 1971) when the ambient plasma density was sufficiently high.

Several theoretical models of energetic electron beams neutralized or partially neutralized by reverse currents in the ambient plasma have been developed (for example Cox and Bennett 1970, Hammer and Rostoker 1970, Lee and Sudan 1971, Lovelace and Sudan 1971, Chu and Rostoker 1973). Since these theoretical treatments are primarily concerned with the high current energetic electron beams that are typically produced in laboratory studies and not in the electron beams thought to be responsible for impulsive hard X-ray bursts, some of the results are not relevant to the solar flare case (cf. 2.3). The models cited treat cylindrically symmetric mono-energetic beams of the type considered by Alfven (cf. 2.1) with the possible addition of a uniform magnetic field along the symmetry axis. When the beam current is small compared to \( I_A \), then the induced reverse current flows primarily outside the beam cylinder \( (r > R) \) while for \( I \gg I_A \) the reverse current is confined to \( r \leq R \) and the current neutralization is local in the sense that the ambient electrons drift with the velocity

\[
V_d = -\frac{n_b}{n_e} V_b , \tag{2.10}
\]

where \( V_d \) is the reverse current drift velocity, \( V_b \) is the velocity of the beam electrons and \( n_b \) and \( n_e \) are the beam and plasma electron number densities (Cox and Bennett 1970). Depending upon the sharpness
of the leading edge of the beam, large amplitude coherent plasma oscillations may be generated by the passage of the beam head (Hammer and Rostoker 1970, Cox and Bennett 1970, Lee and Sudan 1971, Chu and Rostoker 1973). The amplitude of these plasma oscillations is $\sim (\omega_p T)^{-1}$, where $\omega_p$ is the plasma frequency $\left(\omega_p = \left(4\pi n_e e^2/m_e\right)^{1/2}\right)$ and $T$ is the rise time of the beam (Lee and Sudan 1971). These oscillations decay with a scale length of $V_b \tau_{ee}$, where $\tau_{ee}$ is a phenomenological momentum relaxation time for the plasma electrons. If the lateral dimension ($R$) of the beam is large compared to the electromagnetic skin depth $(\lambda_E = c/\omega_p)$, after the plasma oscillations decay the net current will be $\sim \lambda_E/R$ times the beam current. The current of the beam will be neutralized for a length of $\sim V_b \tau_{ee} (R/\lambda_E)^2$. The theoretical models for mono-energetic beams are not appropriate for the streams of energetic electrons that are responsible for impulsive X-ray bursts. We argue in Section 2.3 that the beams in solar flares will be current neutralized in steady state.

2.3 Steady State Model

We now examine a simple model for an impulsive X-ray burst. We consider a vertical flux tube extending from the corona to the chromosphere and assume that electrons are accelerated at the top of the flux tube by the development of stochastic electric fields (Sturrock 1966, Hall and Sturrock 1967, Newman 1973) or by some other mechanism (cf. Section 1.3). The injection of these electrons down the field toward the chromosphere then leads to the development of a reverse current both by the mechanisms considered for mono-energetic beams in laboratory plasmas (Cox and Bennett 1970, Hammer and Rostoker 1970, Chu and Rostoker 1973)
and due to an electrostatic field due to charge imbalances. The strong tendency of a plasma to remain charge neutral implies that, if a current is generated in the plasma that would systematically violate $\partial \rho / \partial t = 0$ on time scales much greater than a plasma period (i.e. a non-MHD current), then this current will generate a neutralizing secondary reverse current.

In contrast to the mono-energetic beams typical of laboratory experiments, the streams of energetic electrons that produce impulsive X-ray bursts probably have smooth distributions in energy. This is inferred from observations (cf. 1.2) and theoretical considerations indicate it is likely that the number of electrons does not increase with energy (Brown and Melrose 1977, Smith 1975). We consider below an energetic electron stream with a distribution of this type, that has electrons of all energies present. The low energy electrons are constantly merging with the background plasma and can build up charge imbalances. In the case of a mono-energetic beam considered by other workers (for example Cox and Bennett 1970, Chu and Rostoker 1973), charge imbalance would only build up at the ends of the plasma device since the energetic electrons do not interact with the plasma significantly except through the reverse current. Charge built up at the ends of an experimental plasma column would either be conducted away by external return paths or be shielded from the bulk of the plasma within a few Debye lengths of the ends and not drive reverse currents in most of the volume of the plasma column.

Lovelace and Sudan (1971) pointed out that the microscopic process involved in heating the plasma with reverse currents are equivalent to heating with currents induced by external fields. However, the reverse currents avoid the skin effect limitations of currents induced by
external fields. Similarly, since charge can be supplied by the beam in the case of solar flares, charge imbalances can build up within the plasma and drive reverse currents. Although these charge imbalances arise throughout the plasma, we can estimate the time ($\tau_c$) required to accumulate sufficient charge separation from the time required to accumulate enough charge per unit area on a parallel plate capacitor to produce an electric field sufficient to drive the required reverse current. This required charge separation is related to the current density by

$$j_\eta = 4\pi E = 4\pi \Sigma = 4\pi j_{\text{unn}} c \tau_c,$$  \hspace{1cm} (2.11)

where $\Sigma$ is a surface charge density, $\eta$ is the resistivity, and $j_{\text{unn}}$ is the unneutralized portion of the beam current density. Then the time to accumulate the required charge is

$$\tau_c = \frac{j}{j_{\text{unn}}} \frac{\eta}{4\pi c}.$$  \hspace{1cm} (2.12)

The ratio of unneutralized current density to the beam current density is $\lambda E/R$ (cf. 2.2) so that

$$\tau_c = \frac{\eta \lambda^2}{4\pi p^2}$$  \hspace{1cm} (2.13)

$$\tau_c \approx 10^{-9.16} \left(10^6/T\right) \left(n/10^9\right)^{1/2} \left(R/10^9\right).$$  \hspace{1cm} (2.14)

This assumes that the resistivity is the usual Spitzer value. If the resistivity is "anomalous" the effective collision frequency can be of order the electron plasma frequency ($\omega_p$). Actually this is an upper limit, for the Buneman instability the effective collision frequency is
\[ \tau_c \approx 10^{-1.78} \left( \frac{R}{10^9} \right) . \]  

We see that the time to accumulate charge imbalances sufficient to drive a neutralizing reverse current is short compared to time scales of interest.

If the resistivity is written

\[ \eta = \frac{m e^2}{n e^2 \tau_{ee}} , \quad (2.17) \]

then \( \tau_c \) becomes

\[ \tau_c = \frac{R}{c} \left( \omega_p \tau_{ee} \right)^{-1} . \quad (2.18) \]

and the ratio of the charge accumulation time to the time the current remains neutralized \( (\tau_n) \) by the mechanisms considered for a mono-energetic beam (cf. 2.2) becomes

\[ \frac{\tau_c}{\tau_n} = \frac{\frac{R}{c} \left( \omega_p \tau_{ee} \right)^{-1}}{\frac{R}{c} \frac{R}{c} \left( \omega_p \tau_{ee} \right)^{-1}} = \left( \frac{\omega_p}{\lambda_{ee}} \right)^{-2} \left( \frac{\lambda_{ee}}{R} \right) , \quad (2.19) \]

\[ \frac{\tau_c}{\tau_n} = 10^{-7.77} \left( \omega_p \tau_{ee} \right)^{-2} \left( \frac{10^9}{R} \right) \left( \frac{10^9}{n} \right)^{1/2} . \quad (2.20) \]
so that the charge accumulation time is much shorter than the time the current would remain neutralized if no charge imbalance arose in the plasma. For time scales and length scales of interest in solar impulsive X-ray bursts, the reverse currents will be caused primarily by charge separation (Hoyng and Melrose 1977). Also, since beams of interest for solar impulsive X-ray bursts are not expected to have sharp fronts, the plasma oscillations excited by passage of the "beam head" will be of extremely small amplitude and consequently of no great significance (Melrose 1974). Therefore, we are justified in considering a steady state in which the beam current is exactly balanced by a reverse current in the background plasma. For the present (cf. Chapter 3), we assume that the background plasma can be adequately described by a Maxwellian velocity distribution and use transport coefficients based on this assumption (Spitzer 1962).

We are interested in the case in which the primary electron stream is composed of high-energy electrons with consequently long mean free paths in the tenuous solar corona. However, we shall find that the electric field that develops to drive the reverse current also decelerates the electron stream (cf. Lovelace and Sudan 1971). But when the electron energy becomes comparable with the thermal energy, the mean free path will be sufficiently short that the primary electrons will merge with the background plasma. As a simple representation of this process, we ignore collisions in discussing the primary beam but we assume that an electron of the primary beam is absorbed into the background plasma when it is decelerated to zero energy. This approximation is justified, if the temperature of the ambient plasma is sufficiently low.
If, as a further simplification, we consider a flux tube of uniform cross section, we may use the following simple one-dimensional form of the Vlasov equation:

\[
\frac{\partial f}{\partial s} + \frac{e}{m} \frac{\partial f}{\partial \phi} = 0 ,
\]

(2.21)

where \( s \) measures length along the tube, \( v \) is velocity (along the tube), \( f(s,v) \) is the velocity distribution function of the primary electron stream, and \( \phi \) is the electrostatic potential.

At the top of the flux tube (\( s=0 \)), the primary electron stream is moving with positive velocity and electrons that are decelerated to zero velocity are assumed to be removed from the beam. Hence we may without ambiguity, express \( f \) in terms of \( \Psi \), which is defined by

\[
\Psi = \frac{mv^2}{2e} .
\]

(2.22)

The initial distribution function may therefore be expressed as

\[
f(0,v) = F(\Psi) .
\]

(2.23)

With this initial condition, we find that the solution of the Vlasov equation (2.21) is

\[
f(s,v) = F(\Psi-\phi) .
\]

(2.24)

The current density \( j_s \) in the primary electron stream is given by

\[
j_s = -\frac{e}{c} \int_0^\infty f(s,v)v \, dv ,
\]

(2.25)

which may be expressed as
\[ j_s = -\frac{e^2}{mc} \int_0^s F(y-\phi)dy \quad (2.26) \]

Since \( \phi \) will prove to be negative in the region of interest, it is convenient to write

\[ \Theta = -\phi \quad (2.27) \]

so that Equation (2.26) may be reexpressed as

\[ j_s = -\frac{e^2}{mc} \int_\Theta^s F(x)dx \quad (2.28) \]

We have seen that the beam current will be nearly completely neutralized by currents in the background plasma, so we may write

\[ j_p + j_s = 0 \quad (2.29) \]

where \( j_p \) is the secondary current induced in the background plasma. We here assume that the density and temperature are such that \( j_p \) may be represented by Ohm's law,

\[ j_p = \eta^{-1}E = \eta^{-1} \frac{d\Theta}{ds} \quad (2.30) \]

It is convenient to introduce a new independent variable \( \xi \) to replace \( s \) by the relationship

\[ d\xi = \eta ds \quad (2.31) \]

Then, on substituting Equations (2.28) and (2.30) into Equation (2.29) and differentiating with respect to \( \xi \), we obtain
It is convenient to solve this equation for $g$ in terms of $\Theta$,

$$g = X(\Theta), \quad (2.33)$$

rather than vice versa. Equation (2.32) becomes

$$\frac{mc}{e^2} \left( \frac{dX}{d\Theta} \right)^2 \frac{d^2X}{d\Theta^2} = F(\Theta), \quad (2.34)$$

which may be integrated once to give

$$\frac{mc}{e^2} \left[ \left( \frac{d\Theta}{d\xi} \right)_{\xi=0} - \left( \frac{dX}{d\Theta} \right)_{\|} \right] = \int_0^\Theta F(\Theta')d\Theta', \quad (2.35)$$

if we assume that $\Theta = 0 (\phi=0)$ and $X = 0 (g=0)$ at $s = 0$. We find from Equations (2.28), (2.29) and (2.30) that

$$\frac{e^2}{mc} \left( \frac{d\Theta}{d\xi} \right)_{\xi=0} = \int_0^\infty F(\Theta')d\Theta'. \quad (2.36)$$

Hence Equation (2.35) becomes

$$\frac{dX}{d\Theta} = \frac{mc}{e^2} \left[ \int_\Theta^\infty F(\Theta')d\Theta' \right]^{-1} \quad (2.37)$$

It is now convenient to introduce a specific form for $F(\Psi)$:

$$F(\Psi) = K(\Psi_0 + \Psi)^{-\gamma}. \quad (2.38)$$
This is a power-law distribution at high energy which flattens at low energy, with the "knee" characterized by \( \Psi_0 \).

We introduce the symbol \( H(\Psi, s) \) for the flux of electrons \( \text{cm}^{-2} \text{s}^{-1} \) of energy exceeding \( \Psi \) at the position \( s \):

\[
H(\Psi, s) = \frac{e}{m} \int_{\Psi}^{\infty} F(\Psi' + \Theta(s)) d\Psi' .
\]  

(2.39)

If the initial flux is written as \( H_0(\Psi) \), we find that

\[
H_0(\Psi) = \frac{eK}{(\gamma-1)m} (\Psi_0 + \Psi)^{-\gamma+1} ,
\]

(2.40)

so that the total particle flux is given by

\[
H_0(0) = \frac{eK}{(\gamma-1)m} \Psi_0^{-\gamma+1} .
\]

(2.41)

With the form of Equation (2.38) for \( F(\Psi) \), Equation (2.37) integrates to give

\[
x(\Theta) = \frac{\gamma-1}{\gamma} \frac{mc}{eK} \left[ (\Psi_0 + \Theta)^{\gamma} - \Psi_0^{\gamma} \right] .
\]

(2.42)

We easily obtain from Equation (2.42) an expression for the (negative) electric potential \( \Theta \) in terms of the resistivity weighted distance measure \( \xi \):

\[
\Theta(\xi) = \left( \Psi_0^\gamma + \frac{\gamma}{\gamma-1} \frac{e^2K}{mc} \xi \right)^{1/\gamma} - \Psi_0 .
\]

(2.43)

Hence from Equation (2.39), we find that
\[ H(\psi, s) = \frac{eK}{(\gamma-1)mc} \left[ \psi + \left( \frac{\psi}{\psi_0} + \frac{\gamma}{\gamma-1} \frac{e^2K}{mc} \right)^{1/\gamma} \right]^{-(\gamma-1)}. \] (2.44)

On noting that the electric current carried by the stream is related to \( H(\psi, s) \) by

\[ j_s(s) = -\frac{e}{c} H(0, s), \] (2.45)

we see that

\[ j_s(s) = -\frac{e^2K}{(\gamma-1)mc} \left( \frac{\psi}{\psi_0} + \frac{\gamma}{\gamma-1} \frac{e^2K}{mc} \right)^{-(\gamma-1)/\gamma}. \] (2.46)

In order to specify the current, particle flux, and electric field as functions of \( s \), we must adopt a specific form for \( \eta(s) \). A convenient approximation to the density and temperature structure of the solar atmosphere, which is expressible in analytic form, is provided by the constant heat flux model. If we now assume that \( s \) measures distance vertically downward from the corona, and that \( n = n_0 \) and \( T = T_0 \) at \( s = 0 \), this model (Adams and Sturrock 1975) yields the following expressions:

\[ T(s) = (T_0^{7/2} - bF s)^{2/7}, \] (2.47)

\[ n(s) = n_0 \left[ T_0 / T(s) \right] \exp \left\{ -\left[ (T_0^{7/2} - bF s)^{5/7} - T_0^{5/2} \right] \right\}, \] (2.48)

where \( a \approx 10^{-1.21} \), \( b \approx 10^6.58 \), and \( F \) (ergs cm\(^{-2}\) s\(^{-1}\)) is the downward heat flux.

The resistivity, in modified Gaussian units, may be derived from the expression given by Spitzer (1962):
\[ \eta = gT^{-3/2} \quad (2.49) \]

where \( g \approx 10^{3.64} \). Hence we find from Equation (2.31) that \( \xi \) is related to \( s \) by

\[ \xi = \frac{7}{4} \frac{g}{bF} \left[ \frac{T^2}{T_0} - \left( \frac{T^{7/2}}{T_0} - bF \right)^{4/7} \right] \quad (2.50) \]

Our model is then completely specified by the choice of the coronal temperature, the coronal density, the coronal heat flux, \( \gamma \), the energy corresponding to \( \psi_0 \), and the injected energetic electron flux. For the coronal parameters, we adopt values typical of the corona above an active region (Noyes 1971):

\[ T \approx 3 \times 10^6 \, \text{K} , \]
\[ n \approx 10^9 \, \text{cm}^{-3} , \]
\[ F \approx 5 \times 10^6 \, \text{erg cm}^{-2} \, \text{s}^{-1} . \]

We choose \( \psi_0 \) to correspond to 25 keV; and we choose \( \gamma = 2.5 \). The fraction of the beam energy deposited and the total energy deposited by Joule heating between \( T = 3 \times 10^6 \, \text{K} \) and \( T = 3 \times 10^4 \, \text{K} \) as a function of the energetic electron flux are displayed in Figure 2.1. For a flare area of \( 10^{19.5} \, \text{cm}^2 \), the energetic electron flux inferred from a large impulsive X-ray burst corresponds to \( \sim 10^{17} \, \text{cm}^{-2} \, \text{s}^{-1} \) (Hoyng et al. 1976). Figure 2.2 illustrates the energy deposition rate due to Joule heating as a function of temperature of the atmosphere for this injected energetic electron flux. The ordinate of Figure 2.2 is the time required to raise the ambient plasma temperature by \( 10^7 \, \text{K} \), if the plasma were heated at the steady state rate. As we will see (cf. Chapter 3), the heating rate
Figure 2.1 The fraction of beam energy deposited (solid curve) and the total energy deposited (broken curve) by Joule heating as a function of the energetic particle number flux.
Figure 2.2 Joule heating rate as a function of ambient plasma temperature for an energetic particle number flux of $10^{17} \text{ cm}^{-2} \text{ s}^{-1}$. The heating rate is displayed as the time $\tau_H$ required to heat the plasma by $10^7 \text{K}$.
decreases as the temperature of the plasma increases, so the ordinate of Figure 2.2 is only representative of the heating rate immediately after the beam is turned on.

We can check the assumption that Coulomb collisions are not important for the energetic electrons in the beam. Since the reverse current losses are proportional to the resistivity for a constant current density, these losses are proportional to $T^{-3/2}$. The Coulomb collisional losses for energetic electrons of the same kinetic energy are proportional to $n$. Therefore, we expect the ratio of reverse current losses to collisional losses to be proportional to $(nT^{3/2})^{-1}$. Reverse current losses will be more important than Coulomb collisional losses for an energetic electron in the beam if

$$\alpha = 10^{-3/2} (V/V_t)^2 (V_d/V_t) > 1,$$

(2.51)

where $V$ is the velocity of the energetic electron, $V_t$ is the electron thermal velocity, and $V_d$ (cf. Equation 2.10) is the reverse current drift velocity (Hoyng et al. 1978). As we expected, for the same kinetic energy and current density, $\alpha$ is proportional to $(nT^{3/2})^{-1}$ since $V_t \propto T^{1/2}$ and $V_d \propto n^{-1}$. If we define the injected energetic electron flux as the flux of electrons with kinetic energies greater than $e\psi_0$, then from Equation (2.40) we find

$$H_E = \frac{eK}{(\gamma-1)me^2} \psi_0^{-\gamma-1}.  \quad \quad (2.52)$$

Since the reverse current drift velocity is related to the current density by
\[ V_d = \frac{c}{en} j_p, \quad (2.53) \]

we may write the drift velocity in terms of the injected energy flux as

\[ V_d = \frac{2^{\gamma-1}}{n} H_E \left( 1 + \frac{\gamma_0^{\gamma-1} e}{c r_0} H_E \xi \right)^{-(\gamma-1)/\gamma}. \quad (2.54) \]

For the adopted values \( \gamma = 2.5 \) and \( e_0 = 25 \) keV, we find that the ratio of reverse current losses to Coulomb collisional losses (\( \alpha \)) for a beam electron with kinetic energy 25 keV in the adopted constant heat flux model atmosphere is

\[ \alpha = 10^{2.82} \frac{H_E}{nT^{3/2}} \left[ 1 + 10^{-23.56} \frac{H_E}{F} T_0^2 \right]^{-(\gamma-1)/\gamma}. \quad (2.55) \]

In Figure 2.3, the energy at which \( \alpha = 1 \) is plotted as a function of temperature for several values of \( H_E \). We see that for any energetic electron flux we have considered, the energy at which Coulomb collisions are as important as the reverse current losses for the energetic electrons is reasonably low, indicating that our assumption that Coulomb collisions may be neglected is an adequate approximation.
Figure 2.3 The kinetic energy for which the reverse current losses are equal to Coulomb collisional losses for a beam electron as a function of temperature in the constant heat flux atmosphere, for five different energetic particle number fluxes. From top to bottom the curves correspond to energetic particle number fluxes (cm$^{-2}$s$^{-1}$) of $10^{16.5}$, $10^{17.0}$, $10^{17.5}$, $10^{18.0}$ and $10^{18.5}$. 
3. REVERSE CURRENT HEATING

3.1 Generalization of Steady Model to Time Dependent Case

As we have indicated, for the energetic electron fluxes required to account for the observed X-ray flux by thick-target bremsstrahlung, the ambient plasma is rapidly heated by the reverse current. The rate at which the background plasma is heated by the reverse current depends on the beam current density and the ambient plasma density and temperature. If the ratio of \( V_d \) to the electron thermal velocity \( V_{t,e} = \left( \frac{2kT_e}{m_e} \right)^{1/2} \) is large enough, the background plasma may be unstable to the growth of electrostatic plasma turbulence which can dramatically enhance the plasma resistivity and therefore, the reverse current heating rate. For example, the reverse current will be unstable against the excitation of ion-acoustic or electrostatic ion-cyclotron turbulence unless for \( T_e \) and \( T_i \) the electron and ion temperatures, respectively, (Kindel and Kennel 1971)

\[
\frac{V_d}{V_{t,e}} \leq \begin{cases} 
2.5 & \text{for } T_e \approx 0.1 T_i \text{ (ion-acoustic turbulence)} \\
0.9 & \text{for } T_e \approx 0.3 T_i \text{ (ion-cyclotron turbulence)} \\
0.3 & \text{for } T_e \approx T_i \text{ (ion-cyclotron turbulence)} \\
0.1 & \text{for } T_e \approx 3 T_i \text{ (ion-cyclotron turbulence)} \\
0.05 & \text{for } T_e \approx 10 T_i \text{ (ion-acoustic turbulence)} 
\end{cases}
\]

Reverse current heating is a self quenching process. If the reverse current is stable against the growth of electrostatic turbulence, then, as the plasma is heated by the reverse current, the resistivity decreases and the reverse current losses are reduced. If the reverse current is unstable to the growth of electrostatic turbulence, the plasma will be
heated until the instability criterion is no longer satisfied. The heating of the plasma will also cause a pressure imbalance. The time $\tau (s)$ for the plasma to respond to changes of pressure by bulk motions can be estimated from

$$\tau \approx \frac{L}{V_{t,i}} ,$$

(3.1)

where $L$ is a characteristic length and $V_{t,i}$ is the ion thermal velocity. Even for a temperature as large as $10^7$ K, this time is long ($10^2$ s) compared with the heating time for a length scale of $10^{9.7}$ cm, so that the plasma density will not change appreciably during the heating. Since we expect reverse currents to be established locally on time scales on the order of a plasma period ($\leq 10^{-9}$ s) which is much shorter than the time scale for heating of the plasma ($\geq 10^{-2}$ s), it should be a reasonable approximation to use the results of Chapter 2 for the instantaneous velocity distribution of the energetic electrons as a function of distance from the injection point.

We have calculated the heating due to reverse currents for two injected energetic electron fluxes ($H_{E_j}$). The heating rate was taken to be just that which results from the Ohmic losses suffered by the reverse current and is given by

$$\frac{\partial T}{\partial t} = \frac{c}{3n_e k} \eta J^2 p .$$

(3.2)

The electron and ion temperatures were assumed to be equal. We shall say more about this assumption later. At each time step the current at each spatial grid point was calculated using Equation (2.46). The time step was regulated so that the largest change in temperature at any grid
point was 1% in one time step. Since we have not found an analytic solution to the time dependent problem considered here, the constant heat flux model of the atmosphere was abandoned in favor of a more accurate numerical model which is discussed in Appendix B. The spatial grid spacing was chosen so that for the initial temperature profile \(t=0\) the temperature change between spatial grid points was less than 1%. The atmosphere was assumed static; that is, the number density \(n\) was held constant in time. The details of the numerical methods used are discussed in Appendix A.

We have used the same \(\gamma\) and \(\gamma_0\) as in Chapter 2. The results for an injected energetic electron flux of \(1.414 \times 10^{17}\) are displayed in Figure 3.1 while similar curves for an injected energetic electron flux of \(5.656 \times 10^{17}\) are displayed in Figure 3.2. Figure 3.3 depicts the density structure of the model atmosphere. The abscissa, \(I\), of the figures is integrated number density from the injection point, defined by

\[
I(s) = \int_0^s n(s') ds',
\]

(3.3)

where \(n\) is the total number density (sum of neutral hydrogen and proton density). Because we have used a numerical model rather than the simple analytic constant heat flux model, we were not free to choose the density at the injection point (see Appendix B). The initial density in the adopted model is approximately twice the density in the constant heat flux model used in Chapter 2. Since the reverse current heating rate is proportional to \(j^2_p\) and inversely proportional to density, the
Figure 3.1 Temperature \( T \) as a function of integrated number density \( I \) from the injection point, for an energetic electron number flux of \( 1.414 \times 10^{17} \text{ cm}^{-2} \text{ s}^{-1} \).

The temperature is displayed before the beam is injected \( (t = 0 \text{ s}) \) and for two times after the beam is injected \( (t = 1 \text{ s} \) and \( t = 4 \text{ s}) \).
Figure 3.2 Temperature \( T \) as a function of integrated number density \( I \) from the injection point, for an energetic electron number flux of \( 5.656 \times 10^{17} \text{ cm}^{-2} \text{ s}^{-1} \). The temperature is displayed before the beam is injected \( (t = 0.25 \text{ s and } t = 1 \text{ s}) \).
Figure 3.3 Number density of neutral and ionized hydrogen \( n \) as a function of integrated number density \( I \) from the injection point. The model serves only to represent the gross overall structure of the solar atmosphere above an active region (see Appendix B).
lower energetic electron flux corresponds roughly to the initial heating rate shown in Figure 2.2.

Figure 3.1 shows the temperature as a function of $I$ for two times, 1s and 4s after the injection of the beam. The energetic electron flux used in the calculation of the results displayed in Figure 3.2 is four times that used for Figure 3.1. Figure 3.2 displays the temperature after .25s and 1s corresponding to the same total energy input as the curves for 1s and 4s in Figure 3.1. Thermal conductivity was neglected in these calculations, but computer runs with thermal conductivity included indicated that thermal conductivity did not have significant effects for the short time scales ($\leq 4s$) involved in here (see Appendix A).

3.2 Anomalous Resistivity and Reverse Current Heating Rate

The electrical resistivity used in the calculations depended on the reverse current drift velocity as indicated below:

$$\eta = \begin{cases} 
\eta_s & V_d \leq 13 V_{t,i} \\
\eta_s + \eta_A & V_d \geq 13 V_{t,i}
\end{cases}, \quad (3.4)$$

where $V_{t,i} = (2KT_i/m_i)^{1/2}$ for $T_i$ the ion temperature, $m_i$ is the proton mass, $\eta_s$ is the resistivity due to Coulomb collisions derived by Spitzer (1962), and $\eta_A$ is an anomalous resistivity due to the presence of electrostatic ion-cyclotron turbulence calculated by Ionson (1976). For the purposes of calculating the value of the anomalous resistivity we have adopted $B=100$ gauss, a reasonable value for the pre-flare corona. Since we are considering a flux tube of constant cross section, the field is the same for all values of $s$, the distance
from the injection point. For the smaller energetic electron flux, the reverse current drift velocity did not exceed the critical velocity for the onset of electrostatic ion-cyclotron turbulence. In this case the temperature of the tenuous coronal plasma was raised by a factor of $\approx 2$, but most of the beam energy was deposited in the dense portion of the model atmosphere.

The larger energetic electron flux, however, caused the reverse current drift velocity to exceed the critical velocity for the onset of electrostatic ion-cyclotron turbulence in the low density portion of the atmosphere, resulting in an anomalous resistivity and an order of magnitude increase in the temperature in these regions in a relatively short time. Since Coulomb collisions were neglected in this calculation, the heating of the denser portion of the atmosphere is not calculated accurately after the first few tenths of a second (also see Appendix B). If collisions were taken into account for the primary electrons in the beam, the heating of the denser regions below the corona would be more localized and higher temperatures would be reached. However, these results indicate that an energetic electron beam may significantly heat the low density coronal plasma much more rapidly than would be calculated from considering only the effects of Coulomb collisions on the beam electrons.

The time for electron and ion temperatures to equilibrate by Coulomb collisions assuming only one species rather than both species are heated as we have assumed may be estimated (Spitzer 1962) from

$$ t_{ei} \sim 12.6 \ n^{-1} \left( T_e + \frac{m_e}{m_i} T_i \right)^{3/2} \ s \ . \quad (3.5) $$
For the temperatures, densities, and the time scales considered here, Coulomb collisions alone will not establish equal electron and ion temperatures. We have taken the electron and ion temperatures to be equal for computational convenience; however, and must, therefore, address the question of whether one species is preferentially heated.

For the case depicted in Figure 3.1, for which the resistivity is just classical Spitzer resistivity, only the electrons are heated at first. According to Equation (3.5), the ions are not likely to be heated significantly in turn by energy exchange with the electrons. The heat capacity of the plasma is therefore reduced by a factor of 2, and the times given in Figure 3.1 should simply be reduced by a factor of 2.

The situation in which plasma turbulence develops, as for the case depicted in Figure 3.2, is considerably more complicated. As we have indicated, the critical drift velocity for the onset of electrostatic ion acoustic or ion-cyclotron turbulence depends on the ratio of the electron and ion temperatures. Just what happens when this drift velocity is exceeded is not well understood, however.

The anomalous resistivity which we have assumed to result from the presence of electrostatic ion-cyclotron turbulence was calculated by Ionson (1976) under the assumption that the turbulence saturates by ion resonance broadening (Dum and Dupree 1970) and that the electron and ion temperatures were equal. Palmadesso et al. (1974), on the other hand, made the first of these assumptions and calculated heating rates of electrons and ions. They found that the ions are heated much more rapidly than the electrons, and Papadopoulos (1977) has subsequently concluded that the instability turns off when the ion heating has
proceeded to the point at which the instability criterion is no longer satisfied. If only the ions are heated, the situation will differ from that depicted in Figure 3.2. The temperature plotted should be interpreted as the ion temperature (note that this affects the calculation of the expected excitation and ionization rates) and the times given reduced by a factor of 2 for the same reason those in Figure 3.1 should be reduced if only the electrons are heated.

It has also been suggested that the ion-cyclotron turbulence saturates, not by ion resonance broadening, but by the formation of a plateau on the electron velocity distribution, instead, in which case no significant anomalous resistivity results (Papadopoulos 1977). If this happens, then as in the case without plasma turbulence, only the electrons are heated at first, at a rate given approximately by classical resistivity. In this case, however, larger electron beam current densities must have been involved to begin with in order for the reverse current drift velocity to have exceeded the critical velocity for the onset of ion-cyclotron turbulence. Since \( j \) is larger than in the case without turbulence, the classical heating rate is higher for this case. If the electrons are heated sufficiently in this manner, the critical drift velocity for the onset of ion-acoustic turbulence will be exceeded. In that case, the electrons will be heated until the criterion for instability is no longer satisfied, or until

\[
V_d \sim C_s \sim \left( \frac{kT_e}{m_i} \right)^{1/2},
\]  

(3.6)

where \( C_s \) is the ion sound speed.
This latter scenario for rapid electron heating would apply, for instance, to an electron beam strength equal to that assumed in Figure 3.2. More precisely, assuming the ion-cyclotron turbulence does saturate by electron plateau formation, a beam of this strength would result first in electron heating given approximately by the results in Figure 3.1 with a time scale reduced by a factor of about 32. After roughly .5 s, ion-acoustic turbulence would develop, resulting in rapid heating of the electrons to a final temperature which may be estimated from

\[ T_e \approx \frac{m_i v^2}{k} \approx 10^{10} \text{ K} \quad (3.7) \]

In short, the exact behavior of the ratio of the electron and ion temperatures is not well understood and cannot be determined without a much more detailed analysis than is appropriate for the present work. We have assumed that the electron and ion temperatures are about equal as a useful and reasonable approximation with which to estimate the magnitude of the reverse current heating. As discussed above, however, temperature enhancements much larger than those depicted in Figures 3.1 and 3.2 are possible.
4. CONCLUSIONS AND SUGGESTIONS FOR FURTHER RESEARCH

We have examined a simple model for the production of impulsive hard X-ray bursts during solar flares. The model involves a beam of energetic electrons propagating from the corona to the chromosphere. We have found that if this beam is to exist, the current carried by the beam electrons must be neutralized by a reverse current in the background plasma. The requirement that the reverse current exist has two consequences that have not been previously recognized in the context of this type of simple model of impulsive hard X-ray bursts. The reverse current heats the ambient plasma and the electric field that is developed to drive the reverse current decelerates the primary electrons. Joule heating by the reverse current is a more effective mechanism for heating the tenuous coronal plasma than Coulomb collisional losses from the energetic electrons, because the ohmic losses are caused by thermal electrons in the reverse current which have much shorter mean free paths than do the energetic electrons.

We have found that the time scale for heating the ambient plasma by reverse currents can be comparable with the time scales characteristic of impulsive X-ray bursts (Hoyng et al. 1976). It is possible that thermal bremsstrahlung from the rapidly heated plasma can account for a significant portion of the observed impulsive X-ray flux. Hence this mechanism can offer an explanation of the fact that some flares first produce high-energy X-ray emission near the top of a loop rather than at the footpoints of the loop (Brueckner 1976). Another important consequence of this process is that, if thermal emission can account for a substantial fraction of the impulsive flux up to $\sim 50$ keV, then the
number of electrons required to produce the nonthermal X-ray flux is greatly reduced (Brown 1975).

The time scale for heating can also be short compared to the ionization times of the plasma ions and may therefore produce non-equilibrium line-emission strength enhancements of lines present in the plasma spectrum just prior to the rapid heating (Shapiro and Knight 1978). These non-equilibrium effects are likely to be observable only if plasma turbulence develops causing a large enhancement in the plasma resistivity (Shapiro and Knight 1978).

We have made several simplifying assumptions in order to facilitate the calculations presented in Chapters 2 and 3. In a more realistic model, some or perhaps all of these restrictions could be relaxed. We now briefly discuss how the relaxation of some of these assumptions is likely to change the conclusions we have drawn and suggest possible extensions of the work we have presented in Chapter 3. We have assumed that all the electrons in the beam are moving in the same direction, or equivalently that they have zero pitch angle. The reverse current arises to balance the flux of electrons in a given direction due to any anisotropy in the energetic electron velocity distribution. If the energetic electron velocity distribution is nearly isotropic, no significant reverse current will arise (see for example Smith and Lilliequist 1978). Even if the distribution is strongly anisotropic, but the electrons streaming down from the corona to the chromosphere have non-zero pitch angles, the Coulomb collisional losses will be enhanced relative to reverse current losses since the collisional losses are proportional to the total path length of the energetic electrons in the
atmosphere, while the reverse current losses are proportional to the average component of the energetic electron velocity along the field. Since the emergent X-ray spectrum is relatively insensitive to the angular distribution of the energetic electron velocities (Langer and Petrosian 1977), it is extremely difficult to infer the reverse current drift velocity from measurements of the X-ray flux. A more detailed discussion of this and other difficulties in inferring the reverse current drift velocity from X-ray observations can be found elsewhere (Hoyng et al. 1978).

We have neglected the effects of Coulomb collisions on the primary electrons. As Figure 2.3 demonstrates, this is an adequate approximation immediately after the flux of energetic electrons is initiated; however, Coulomb collisions become relatively more important as the plasma is heated since the reverse current losses are reduced. Until a significant increase in the density of the coronal plasma is effected by the evaporation of material from the chromosphere, Coulomb collisions are unlikely to be important in the upper portions of the atmosphere. In the lower lying dense regions, Coulomb collisions will rapidly dominate over reverse current losses, and, as we have indicated, affect the heating of this portion of the atmosphere. One extension of the work presented in Chapter 3 that should provide additional insight into the behavior of energetic electrons in the solar atmosphere during flares would be to perform a calculation similar to that we have presented, but include the effects of Coulomb collisions and a distribution of pitch angles for the energetic electrons.
We have neglected the dynamics of the background plasma. As we have indicated, the rapid heating of the plasma can cause a large pressure imbalance. For the results presented in Figures 3.1 and 3.2, the pressure is a factor of $\sim 20$ higher in the high density portion of the atmosphere than in the low density regions indicating that evaporation of high density material would occur if the dynamics of the ambient plasma were accounted for. This would not have a large effect on the calculations presented in Chapter 3 because the time scales considered are so short. However, on longer time scales mass motions in the atmosphere could have important effects. Previous work with fluid dynamic models of solar flares (for example see Kostyuk and Pikel'ner 1975, Kostyuk 1975, Craig and McClymont 1976) has not included reverse current losses. The development of a numerical fluid dynamic model of the solar atmosphere heated by a beam of energetic electrons, including reverse current losses could provide valuable information about the formation of the quasi-thermal soft X-ray plasma that is produced during solar flares.

We have not calculated either the radiation from the heated plasma or the bremsstrahlung from the energetic electrons. Since almost all the information we now have and are likely to accumulate in the foreseeable future about solar flares comes from the observation of the emitted radiation, it would be useful to calculate the emitted radiation from any realistic model to ascertain to what degree it resembles the solar atmosphere during a flare.

More realistic models than those we have considered that include the effects of Coulomb collisions, the dynamics of the background plasma,
a reasonable magnetic field configuration, radiation and thermal conduction are necessary to account for the complicated phenomena that are observed in solar flares. However, our study of the reverse current and the heating it can cause indicates that reverse currents can play an important role, at least in the initial heating of the solar plasma during a flare.
Appendix A

NUMERICAL AND COMPUTATIONAL METHODS

As we have indicated in Chapter 3, the results for the time dependent case are calculated by using the steady state results for the current as a function of distance from the injection point and calculating the change in temperature from a suitably discretized form of equation (3.2). In reality, the calculation is done for the more general case of partially ionized hydrogen. Since the reverse current heating calculation is only accurate in the tenuous high temperature portion of the atmosphere, this generalization did not have a substantial effect on the results of the calculation. However, the manner in which the partial ionization is included could in principle be accurate in any astrophysical plasma that is sufficiently tenuous that the gas is optically thin to its own radiation, photo-excitation and ionization are unimportant and collisional de-excitation can be ignored. The ionization state of the plasma is then a function of temperature only provided non-equilibrium effects can be ignored. The only elements in astrophysical plasmas that are sufficiently abundant for their ionization potential to affect the heat capacity of the gas are hydrogen and helium. Only hydrogen is included in the present calculation, but since the effects of partial ionization on the heat capacity are included via a pretabulated interpolation table (discussed below) the effects of helium could be included with only minor modification. The modified version of equation (3.2) actually solved numerically is
where \( T_E \) is defined by

\[
T_E = (1 + \chi)T + \chi \frac{2E_{\text{ION}}}{3k},
\]

where \( \chi(T) \) is the fraction of the hydrogen nuclei that are ionized and \( E_{\text{ION}} \) is the ionization potential of hydrogen. That is, the thermal energy content of the plasma per cubic centimeter is

\[
E_{\text{TH}} = \frac{3}{2} nk T_E.
\]

The temperature is obtained from \( T \) via the interpolation tables mentioned above, and it is obvious that the inclusion of helium only involves calculating a different interpolation table. In fact we have included only hydrogen and used the expression given by Moore and Fung (1972) for \( \chi(T) \):

\[
\chi(T) = \left(1 + 10^{-5.69} e^{\beta \left[.4288 + \frac{1}{2} \ln \beta + .4698 \beta^{-1/3}\right]}\right)^{-1},
\]

where \( \beta = 15800/t \). Then \( T \) is implicitly defined as a function of \( T_E \) by Equations (A.3) and (A.4).

Spitzer gives the resistivity of a hydrogen plasma as:

\[
\eta_s = 10^{2.34} T^{-3/2} \ln \Lambda,
\]

where \( \Lambda \) is defined by
so that we may write \( \eta_s \) as

\[
\eta_s = \begin{cases}
10^{3.31} T^{-3/2} \left[ \frac{3}{2} \ln T - \frac{1}{2} \ln X - \frac{1}{2} \ln n \right] & T \leq 4.2 \times 10^5 k \\
10^{3.54} T^{-3/2} \left[ \ln T - \frac{1}{2} \ln X - \frac{1}{2} \ln n \right] & T \geq 4.2 \times 10^5 k
\end{cases}
\]

Therefore, \( \eta_s \) can be written as a sum of a function of \( T \) only and a function of \( T \) only times \( \ln(n) \):

\[
\eta_s = TL(T) + TM(T) \ln(n) ,
\]

where

\[
TL(T) = \begin{cases}
10^{3.31} T^{-3/2} \left[ \frac{3}{2} \ln T - \frac{1}{2} \ln X \right] & T \leq 4.2 \times 10^5 k \\
10^{3.54} T^{-3/2} \left[ \ln T - \frac{1}{2} \ln X \right] & T \geq 4.2 \times 10^5 k
\end{cases}
\]

and

\[
TM(T) = \begin{cases}
10^{3.01} T^{-3/2} & T \leq 4.2 \times 10^5 k \\
10^{3.24} T^{-3/2} & T \geq 4.2 \times 10^5 k
\end{cases}
\]

The calculation of the current as a function of distance depends only on the resistivity weighted distance measure \( \xi \). In Chapter 2 we were able to write an analytic expression for \( \xi \) as a function of \( s \),
but in the present case the resistivity varies with time. The value of $\xi$ at the $i$th grid point is approximated by

$$\xi_i^j = \xi_{i-1}^j + (\eta_{i-1}^j + \eta_i^j) \left( s_i - s_{i-1} \right)/2$$  \hspace{1cm} (A.11)

where superscripts refer to time steps and subscripts refer to spatial grid points, and $\xi_{i-1}^j = 0$. So long as the reverse current drift velocity is less than the critical velocity for the onset of ion cyclotron turbulence, the calculation of $\xi_i^j$ in this manner is straightforward. However, when the background plasma is unstable to the growth of ion cyclotron turbulence, the situation is somewhat more complicated. In this case the value of $\eta_i^j$ depends on the current, and a transcendental equation must be solved to find $\eta_i^j$ from Equations (2.46), (2.49) and the result for anomalous resistivity due to ion cyclotron turbulence (Ionson 1976)

$$\eta_a = 0.06 \left( c \frac{\Omega_i}{\omega_{pe}} \omega_{pi} \right) \left( 1 - \frac{13V_{t,i}}{V_d} \right) , \hspace{1cm} (A.12)$$

where $\Omega_i = (eB/m_i c)$ and $\omega_{pi} = (4\pi n_i e/\alpha_i)^{1/2}$, we find that $J_i^j$ is defined implicitly by

$$J_i^j = \frac{e K}{(\gamma - 1) mc} \left\{ \psi_0 + \frac{\gamma e^2 K}{(\gamma - 1) mc} \left[ \xi_i^j + (\eta_{i-1}^j + \eta_i^j) \left( s_i - s_{i-1} \right)/2 \right. \right.$$

$$\left. + 0.06 \left( \frac{m_i e}{m_i} \right)^{1/2} \frac{B}{4\pi e} \frac{(s_i - s_{i-1})}{2n_i \alpha_i^j} \left( 1 - \frac{V_i^j}{V_d^j} \right) \right\} \left[ (\gamma - 1)/\gamma \right] / \left( \gamma - 1 \right) , \hspace{1cm} (A.13)$$
where $v_{d_i}^j = c_{d_i}^j / e \mu n x_{d_i}^j$ and $v_{c_i}^j = 1 / \beta v_{t_i}^j$. If we define $G(J_i^j)$ by

$$G(J_i^j) = J_i^j + \frac{e V^2}{2} \left( \frac{\gamma}{\gamma - 1} \right) \left[ \frac{\gamma}{\gamma - 1} + (\eta_s^j + \eta_s^j) (s_i - s_{i-1}) / 2 \right]$$

$$+ \frac{.06 (m_e / m_i)^{1/2}}{n^2} \left( \frac{s_i - s_{i-1}}{2n \chi_i^j} \right) \left( 1 - \frac{c_{c_i}^j e n_i \chi_i^j}{c J_i^j} \right) \right]$$

$$\left( \frac{\gamma}{\gamma - 1} \right) \right]$$

then when $G(J_i^j) = 0$, $J_i^j$ satisfies Equation (A.13). When the current calculated neglecting anomalous resistivity corresponds to a drift velocity that is greater than $v_{c_i}^j$, we take an initial estimate for $J_i^j$, $J_i^j$:

$$J_i^j = \frac{v_{c_i}^j}{e n_i \chi_i^j}$$

and refine this estimate by application of Newton's method to Equation (A.14). Examination of Equation (A.13) shows that Newton's method will always converge for this initial estimate and the convergence is usually reasonably rapid, i.e. usually 6 or fewer iterations are required.

The functions needed for the calculation $[T_L, T_m, \chi$ and the implicitly defined $T(T_E)]$ are evaluated by cubic interpolation on pretabulated tables. The method used is dependent on the architecture of the IBM 360-370 series computers and the internal representation of double precision floating point numbers used on these machines. The use of pretabulated functions is considerably faster than calls to the FORTRAN library routines that would otherwise be necessary. This is particularly true in the case of the implicitly defined function $T(T_E)$ which would
have to be solved iteratively at each spatial grid point for each time step. The internal representation of double precision floating numbers on IBM 360-370 computers is presented diagramatically below.

```
+-------------------+-------------------+
| sign bit          | hexadecimal exponent (excess 64) |
+-------------------+-------------------+
|                   | 14 digit hexadecimal fraction |
|                   | 017863             |
+-------------------+-------------------+
```

FIG. A.1. Internal representation of double precision floating point numbers on IBM 360 and 370 series computers.

In the interpolation procedure, the first 16 bits (bits 0-15) are extracted, an offset subtracted and the result treated as a double word displacement from the base address of an interpolation table. The remaining 48 bits (16-63) are used to form a floating point fractional displacement (frac) from the largest value of the temperature for which the function is tabulated which is smaller than the value of the temperature for which the value of the function is desired. The value of this displacement is such that \( 0 \leq \text{frac} < 1 \); frac is used to calculate weights for the four nearest tabulated values of the function in a cubic polynomial interpolation. Once the weights for the cubic interpolation are calculated only 4 double precision floating point multiplies (\(~ 0.6 \mu s\) each on IBM 370-168 with high speed multiply) and 3 adds (\(~ 0.3 \mu s\) each) are required to produce an interpolated value from a table. Since the weights are to be calculated for \( T_L, T_M, T, X \) they are also used to calculate the critical velocity for the onset of ion-cyclotron turbulence. This would require a call to the FORTRAN library subroutine "DSQRT" which
is sufficiently fast that interpolation would be slower for the calculation of the square root alone. However, since the weights must be calculated for $T_L$, $X$, $T_M$ and $T_E$, interpolation is faster than a call to "DSQRT" because the weights are effectively "free" for this calculation. The semi-logarithmic tabulation scheme allows interpolation from $T=4.0 \times 10^3$ K to $6.7 \times 10^7$ K with a maximum relative displacement from a value of $T$ for which the function is tabulated of $\sim 3\%$ with only 820 table entries. In fact some of the 820 entries are never used due to the nature of the signed magnitude normalized representation of floating point numbers on these machines, but the reason for using this sort of tabulation scheme is that a reasonably large range can be covered with relatively few table entries, and the correct tabulated values can be accessed extremely rapidly.

Listings of two main programs and several subroutines are provided for the sake of completeness. All of the time consuming routines have been hand coded in assembly language. Routines that perform initialization and diagnostic functions as well as the main program are coded in FORTRAN. The first main program produces the tables that are required for the cubic interpolation. The second main program reads in the interpolation tables, model parameters and starting values. The starting values used initially are from the steady state model atmosphere described briefly in Appendix B.

The assembly language programs calculate the current at each spatial grid point ($\text{CURCAL}$), calculate the change in temperature at each point and determine the time step ($\text{TESTP}$) and write out the arrays at the designated intervals ($\text{TOUT}$). In addition the calculation of the current
(CURCAL) requires taking the 
\[-(\gamma-1)/\gamma\] power of a number, which if done with FORTRAN library routines would require taking the natural logarithm and exponentiating. Both the library routines "DLOG" and "DEXP" are slower than "DSQRT" so an assembly language routine was written that calculates the \(3/5\) power of a number (F35), the routine is called by CURCAL. The subroutine DJAG is used primarily for monitoring the performance of the model during program changes and subsequent debugging. In production runs it could be replaced with a subroutine that does nothing (i.e. returns as soon as it is called) without affecting the model calculations; therefore it is not reproduced here. The FORTRAN subprograms initialize the array containing \(T_E\) (EINIT) and read in the starting values (INIT and RDR). The calculation that includes thermal conductivity which is referred to in Chapter 3 is not discussed in detail here. In order to avoid undue restriction on the time step [to satisfy the Courant-Friedrichs-Lewy condition - see Richtmyer and Morton (1967)], the method employed is implicit and requires the inversion of a tridiagonal matrix (dimension 846) and is rather slow. Runs with this program indicated thermal conduction did not change the results substantially so these routines are not reproduced here.
IMPLICIT REAL*8 (A-H,O-Z)

THIS PROGRAM CALCULATES SEMI-LOGARITHMIC TABLES NEEDED FOR
CALCULATION OF REVERSE CURRENT HEATING WITH RESISTIVITY THAT
IS A FUNCTION OF THE DRIFT VELOCITY.

THE TABLES ARE WRITTEN OUT TO LOGICAL UNIT 9

REAL*8 TL(820),TM(820),VITH(820),CHIN(820),CINV(564).
REAL*8 DTO/Z4310000000000000/,TBO/Z4410000000000000/,.
.KAY/1.38054D-16/,EC/4.80298D-10/,MH/1.6753089D-24/.
.HBAR/1.05450D-27/,MU,EION,ONE3/Z0555555555555555/.
.C/2.997925010/,CSIG,LSIG,K32,CKAP,LNS1,LNS2,DCON,BETA,PSI

INTEGER*4 LIM(4) 

CALC TABLES: ELECTRICAL CONDUCTIVITY (TL,TM), CRITICAL
VELOCITY (VITH*13.) AND INVERSE IONIZATION FRACTION (CHIN).

DT=DTO
TO=TBO
DO 20 II=1,4
K=(256*(II-1))+1
L=K+LIM(II)
T=TO-DT
DO 10 J=K,L
BETA=1.5805/T
PSI=4.5DO/(BETA*DEXP(BETA)*(.42880+.5DO*DLOG(BETA)
+.469800+BETA**ONE3))
VITH(J)=DQSRT(VITH*T)
CHIN(J)=CHIN*(1.00+PSI)/PSI
CHI=PSI/(1.00+PSI)
IF(T.GT.4.2D5)GOTO 5
20 T=T+DT
5 TM(J)=CSIG/(T*DQSRT(T))
TL(J)=TM(J)*(LNS1+1.5DO*DLOG(T)-.5DO*DLOG(CHI))
GOTO 10
10 T=T+DT
20 DT=DT*16.00

CALCULATE TABLES FOR CHI INVERSE (((1+CHI)*T+CHI*TION AS FUNC OF T)

TNEW=TBO-DT
DT=DTO
TO=TBO
DO 40 II=1,3
K=(256*(II-1))+1
L = K + LIM(II + 1)
T = TO - DT
DO 30 J = K, L
   TCH = 4.0 - 16*TNEW
   TOLD = TNEW
   BETA = 1.585/TOLD
   EBETA = DEXP(BETA)
   B13 = BETA**ONE3
   TEMP1 = 0.428800 + 0.500*DLG(BETA) + 0.469800*B13
   PSI = 4.505/(BETA*EBETA*TEMP1)
   C = PSI/(1.00 + PSI)
   DC = DCON*C*C*BETA*BETA*EBETA*((1.00 + BETA)**
       TEMP1 + (1.500 - 156600*B13))
   TNEW = TOLD - ((1.00 + C)*TOLD + C**TION - T)/(1.00 + C +
       (TOLD + TION)*DC)
   IF(DABS(TNEW - TOLD) .GT. TCH) GOTO 25
   CINV(J) = TNEW
30   T = T + DT
40   TO = TO*16.0
C
   WRITE OUT TABLES
   WRITE(9, 9001) TL
   WRITE(9, 9001) TM
   WRITE(9, 9001) VITH
   WRITE(9, 9001) CHIN
   WRITE(9, 9001) CINV
   STOP
END
REVERSE CURRENT HEATING MAIN ROUTINE

IMPLICIT REAL*8 (A-H,O-Z)

C
C THIS PROGRAM CALCULATES REVERSE CURRENT HEATING OF A MODEL
C ATMOSPHERE READ IN AS UP TO 1024 VALUES OF TEMPERATURE (T) NUMBER
C DENSITY (N) AND DISTANCE (S) FROM THE INJECTION POINT (TOP OF
C MODEL. THE PROGRAM DOES NOT HAVE TO START AT TIME 0 (INJECTION
C TIME) AS THE CURRENT TIME, TIME STEP AND ITERATIONS TO THIS POINT
C ARE READ IN ALSO. THE PROGRAM READS IN THE MAXIMUM NUMBER OF
C ITERATIONS TO BE PERFORMED (NITER), THE ENERGETIC ELECTRON NUMBER
C FLUX (EFLUX), PSIO WHICH CORRESPONDS TO AN ENERGY CHARACTERISTIC
C OF A LOW ENERGY KNEE IN THE ENERGETIC ELECTRON DISTRIBUTION
C (SEE KNIGHT AND STURROCK 1977), FRAC, THE MAXIMUM PERCENTAGE
C CHANGE IN THE THERMAL ENERGY CONTENT OF THE PLASMA PER HYDROGEN
C NUCLEUS ALLOWED AT ANY GRID POINT IN ONE TIME STEP, TIMMAX
C THE MAXIMUM TIME FOR THIS RUN (REAL TIME NOT COMPUTER TIME) AND
C DTOUT, THE INTERVAL AT WHICH THE ARRAYS CONTAINING THE TEMPERATURE
C AND CURRENT DENSITY AS WELL AS THE CURRENT TIME AND TIME STEP.

CALLED SUBROUTINES:

INIT - READS IN STARTING VALUES OF DENSITY, TEMPERATURE AND
DISTANCE AS WELL AS TIME, TIME STEP AND NUMBER OR PREVIOUS
ITERATIONS.

EINIT - CALCULATES INITIAL TE DEFINED AS (1+CHI)T+2*EION/3*K FOR
EACH SPATIAL GRID POINT. ENERGY INPUT INCREASES TE
AND T IS CALCULATED FROM CHINV.

NOUT - WRITES OUT DENSITY AND DISTANCE ARRAYS AS WELL AS INPUT
PARAMETERS (TO FORTRAN LOGICAL UNIT 9)

CURINT - INITIALIZATION FOR CURCAL (SEE CURCAL)

TESTPI - INITIALIZATION FOR TESTP (SEE TESTP)

CURCAL - CALCULATES CURRENT AS A FUNCTION OF DISTANCE USING
STEADY STATE RESULTS OF KNIGHT AND STURROCK AND A
RESISTIVITY THAT DEPENDS ON THE REVERSE CURRENT DRIFT
VELOCITY.

TESTP - UPDATES TEMPERATURE AND ADJUSTS TIME INCREMENT SO THAT
MAXIMUM CHANGE IN TE AT ONE GRID POINT IS FRAC*TE
AT THE GRID POINT

DIAG - OUTPUTS A SMALL SUBSET OF THE CALCULATED CURRENT DENSITY
AT INTERVALS DETERMINED BY VALUES IT READS FROM LOGICAL
UNIT 5 - CAN BE RECOMPILED WITHOUT RECOMPILING THE REST
OF THE PROGRAMS AS IT DOES NOT AFFECT CALCULATIONS.

TOUT - WRITES OUT TEMPERATURE AND CURRENT ARRAYS AND CURRENT
TIME, TIME STEP AND ITERATIONS TO LOGICAL UNIT 9

CTOUT - CLOSES LOGICAL UNIT 9 (I.E. END FILE 9)

DECLARE VARIABLES:

REAL*8 KAY,ME,C,GAM,NORM,SFST,EFLUX,PSIO,DTOUT,
.EL,EFACT,NEWDT
REAL*8 T(1024),N(1024),J(1024),S(1024),OSIG(1024),LN(1024),
.TEV(1024),UP(1024),SD(1024),
.TLC(820),TH(820),CNV(820),VITH(820),CINV(564)
INTEGER*4 IND(2,1024)
5001 FORMAT(5F7.0)
5002 FORMAT(I7)
INITIALIZE CONSTANTS: BOLTZMANN'S CONSTANT, ELECTRON REST MASS, ELECTRON CHARGE (ESU), # ERG/KEV, SPEED OF LIGHT

\[
\begin{align*}
\text{KAY} &= 1.380540 \times 10^{-16} \\
\text{ME} &= 9.1091 \times 10^{-27} \\
\text{EL} &= 4.802980 \times 10^{-10} \\
\text{EFAC} &= 1.602100 \times 10^{-9} \\
\text{C} &= 2.997925 \times 10^8 \\
\end{align*}
\]

READ IN MODEL PARAMETERS

READ(5,5002)NITER
READ(5,5001)EFLUX,PSIO,FRAC,TIMMAX,DTOUT
READ(8,9001)TL
READ(8,9001)TM
READ(8,9001)VITH
READ(8,9001)CNV
READ(8,9001)CINV

UNITS OF INPUT PARAMETERS ARE:

EFLUX = (1.017 (CM**2 SEC)**-1, PSIO IN KEV, FRAC IN PERCENT, TIMMAX IN SEC (MAXIMUM TIME), DTOUT IN SEC (OUTPUT INTERVALS)

SCALE INPUT VARIABLES

EFLUX=EFLUX*1.017
PSIO=(PSIO*EFAC)/EL
FRAC=FRAC*0.0100

CALCULATE CONSTANTS FOR CALCULATION OF CURRENT

\[
\begin{align*}
\text{TEMP1} &= \text{PSIO} + \text{PSIO} \\
\text{GAM} &= 2.500 \\
\text{GAMM1} &= 1.500 \\
\text{TEMP1} &= \text{TEMP1} \times \text{DSQRT(TEMP1)} \\
\text{TEMP2} &= \text{PSIO} \times \text{DSQRT(PSIO)} \\
\text{NORM} &= (\text{EFLUX} \times \text{GAMM1} \times \text{ME} \times \text{TEMP1}) / \text{EL} \\
\text{TEMP1} &= -(\text{EL} \times \text{EL} \times \text{NORM}) / (\text{GAMM1} \times \text{ME} \times \text{C}) \\
\text{J(1)} &= \text{TEMP1} / \text{TEMP2} \\
\text{TEMP2} &= \text{TEMP2} \times \text{PSIO} \\
\text{TEMP3} &= -\text{GAM} \times \text{TEMP1} \\
\end{align*}
\]

INITIALIZE TIME AND ITERATIONS

NEWDT=0.0
IITER=0
TIM=0.0

INITIALIZE TEMPERATURE AND DENSITY

NTAB=1024
CALL INIT(T,N,S,DELT,TIM,NIT,NTAB)
OUTTIME=DTOUT+TIM
NITER=NIT+NITER
IITER=NIT
T(NTAB+1)=T(NTAB)
S(NTAB+1)=S(NTAB)+(S(NTAB)-S(NTAB-1))
CALL EINITCT(T,TE,NTAB)

OUTPUT INPUT PARAMETERS AND INITIAL DENSITY AND DISTANCES

CALL NOUT(EFLUX,PSIO,FRAC,TIMMAX,NTAB,N,S)

NEED 1/N IN LOOP SO WE CHANGE N TO 1/N AND CALCULATE DIFFERENCES OF DISTANCES USED IN TIME STEP.
START TIME STEPPING LOOP

CONTINUE

CALCULATE CURRENT AND RESISTIVITY AT EACH GRID POINT
CALL CURCAL(OSIG,J,T,N,LN,NTAB)

CALCULATE ONE TIME STEP WORTH OF HEATING, UPDATE TEMPERATURE AND ADJUST TIME STEP ACCORDING TO FRAC
CALL TESTPCT(T,TE,J,OSIG,N,TUP,DELT,FRAC,NTAB,CINV)

WRITE OUT SOME STUFF TO MAKE SURE THINGS ARE WORKING RIGHT
CALL DIAGCS(T,TE,J,OSIG,N,TUP,DELT,TIM,NTAB,1ITER)

STEP TIME

IITER=IITER+1
IF(IITER.GT.NITER)GOTO 40
TIM=TIM+DELT
IF(TIM.LT.OUTIME)GOTO 1
OUTIME=OUTIME+DTOUT

OUTPUT CURRENT VALUES OF TIM, TEMP AND J
CALL TOUT(TIM,T,J,DELT,IITER,NTAB)
IF(TIM.LT.TIMMAX1)GOTO 1
40 CALL CTOUT
WRITE(*,*)IITER,DELT
STOP
END
1001 FORMAT(10A8)
READ(10,1001)NTAB,NIT,DELT,TIM
CALL RDR(T,N,S,NTAB)
RETURN
END

SUBROUTINE RDR(T,N,S,NTAB)
REAL*8 T(NTAB),S(NTAB),N(NTAB)
1001 FORMAT(10A8)
READ(10,1001)N
READ(10,1001)S
READ(10,1001)T
RETURN
END
ROUGHLY EQUIVALENT TO THE FORTRAN CODE BELOW, EXCEPT
THE FUNCTIONS PASSED IN THE ARGUMENT LIST OF THE
FORTRAN ENTRY POINT CURINT (TL, TM, VTH, CVN) ARE IMPLEMENTED
IN LINE IN THE ASSEMBLY LANGUAGE VERSION AND A CALL TO
THE ASSEMBLY LANGUAGE VERSION SHOULD PASS THE ADDRESSES
OF SEMI-LOGARITHMIC TABLES (TL(820), TM(820), VTH(820), CVN(820))
VIA THE ENTRY POINT CURINT RATHER THAN FUNCTION NAMES.
CURCAL DOES THE EQUIVALENT OF A FORTRAN RETURN 1 WHEN A
VALUE OF T IS OUTSIDE THE TABULATED RANGE. THERE IS NO
OBVIOUS WAY TO MAKE THIS APPARENT IN THE FORTRAN VERSION.
The arguments passed to CURCAL are ignored and
obtained from local storage where CURINT put them.
NOTE THAT THIS MEANS CURINT MUST BE CALLED BEFORE THE
FIRST TIME CURCAL IS CALLED OR A REAL MESS WILL RESULT.

SUBROUTINE CURINT (TEMP1, TEMP2, TEMP3, GAM, TL, TM, VTH, CVN,
. N, LN, SD, OSIG, J, T, NTAB)

DECLARE VARIABLES
IMPLICIT REAL*8 (A-H, O-Z)
REAL*8 N(NTAB), LN(NTAB), SD(NTAB), J(NTAB), OSIG(NTAB),
. TL, TM, VTH, CVN,
REAL*8 TEMP1, TEMP2, TEMP3, CONAN, ESU, C, CONAN1, B, CHT,
. VC, VTH, TSI, JA, NCON, NCON1, ME, MI, PI, JCON, JC1
DATA C/2.997925D10/, ESU/4.80298D-10/, ME/9.109D-28/,*
. MI/1.67252D-24/, PI/4.341324F6A8885A30/, ERR/1.0-6/

INITIALIZE CONSTANTS FOR CURCAL
INTEGER*4 COUNT/30/
B=1.02
CONAN=6.6D-2*DSQRTCME/MI)*B/(4.0D*PI*ESU)
TCON=((GAM-1.0D)/GAM)
CONAN1=-TCON*TEMP3*CONAN*ESU/C
TCON=TCON*TEMP1
CE=-C/ESU
RETURN
ENTRY CURCAL(OSIG, J, T, N, LN, NTAB, *)

CALCULATE CURRENT AND RESISTIVITY (OSIG) FOR FIRST POINT

OSIG(1)=TL(T(1))+TM(T(1))*LN(1)
CHT=CNVC(T(1))
VTH=VTH(T(1))
VD=J(1)*N(1)*CHT*CE
IF(VD.LT.VTH) GOTO 10

THE NEXT STATEMENT CALCULATES THE ANOMALOUS PART OF THE
RESISTIVITY IF THE DRIFT VELOCITY IS GREATER THAN THE
CRITICAL VELOCITY FOR THE ONSET OF TURBULENCE

OSIG(1)=OSIG(1)+CONAN*N(1)*CHT*(1.0D-VTH/VD)

INITIALIZE TSI TO ZERO

10 TSI=0.0D
DO 20 I=2, NTAB
OSIG(I)=TL(T(I))+TM(T(I))*LN(I)
TSI=TSI+(OSIG(I-1)+OSIG(I))*SD(I)
JCON=(TEMP2+TEMP3*TSI)
THIS SECTION CALCULATES ANOMALOUS PART OF RESISTIVITY USING
NEWTON'S METHOD FOR INTERIOR GRID POINTS - SKIP IF
DRIFT VELOCITY IS LESS THAN CRITICAL VELOCITY

JA=(12.DO/13.DO)*VITH/NCON
JCON=JCON+CJA*TEMP3
CJA=CJA*JA
NCON1=NCON*CONAN1
J(I)=JA*(1.DO+(J(I)-JA)/(NCON1*SD(I)*F35C1*J(I)+JA))
DO 15 K=1,COUNT
CJAT=CJA/J(I)
TC1=TCON-CJAT
JC1=JCON-CJAT
JT=(J(I)*(TC1+TEMP1*JC1)/(J(I)*JC1+F35(JC1)+TC1))
IF(DABS(JT-J(I))/JT.LE.ERR)GOTO 16
15 J(I)=JT
16 TSIG=NCONO*CONAN*(1.DO-JA/JeI)
TSI=TSI+TSIG*SD(I)
OSIG(I)=OSIG(I)+TSIG
CONTINUE
RETURN
END

USING *,15
B,CFIRST
DC X'06'
DC CL'TCURCAL'
ENTRY CURINT
USING *,15
CURINT B,IFIRST
DC X'06'
DC CL'TCURINT'
AREA DS 18F
REG1 DC AL4(ARGA) R1 (ADDR ARG LIST)
REG2 DC FLO' R2 BASE T
REG3 DC FLO' R3 (BASE OSIG - 8 - BASE T)
REG4 DC FLO' R4 (BASE J - BASE T)
REG5 DC FLO' R5 (BASE N - BASE T)
REG6 DC FLO' R6 (BASE LN - BASE T)
REG7 DC FLO' R7 (BASE SD - BASE T)
EIGHT DC F'8' R8 (INCREMENT - 8)
REG9 DC F'0' R9 (BASE T + 8*(NTAB-1) COMPARAND)
TLA DC F'0' R10 (BASE TL TABLE CHANGES IN LOOP)
TMA DC F'0' R11 (BASE TM TABLE CHANGES IN LOOP)
VTHA DC F'0' BASE VTH TABLE
CNVA DC F'0' BASE CNV TABLE
COUNT DC F'30' MAX # OF NEWTON'S METHOD ITERATIONS
ARGA DC X'80' ARGUMENT LIST (ONE LONG)
AL3(F35A)
DC AL3(F35A) ADDR ARGUMENT
X'00000330'
X'00004410'
FORCE DOUBLE WORD ALIGNMENT
WM1 DC D'0.'
WP1 DC D'0.'
WM3 DC D'0.'
WP3 DC D'0.'
FLOAT DC X'4000000000000000'
CONAN DC D'0.'
CONAN1 DC D'0.1'
TEMP1 DC D'0.1'
TEMP2 DC D'0.1'
TEMP3 DC D'0.1'
GAM DC D'0.1'
NCON DC D'0.1'
NCONO DC D'0.1'
TCON DC D'0.1'
JCON DC D'0.1'
CJA DC D'C1'
TC1 DC D'C1'
JA DC D'0.1'
TSI DC D'0.1'
ERR DC D'0.1.E-6'
C DC D'2.997925E10'
ESU DC D'4.80296E-10'
ME DC D'9.1091E-28'
MI DC D'1.67252E-24'
B DC D'0.1.E2'
PI DC X'413243F6A8885A30'
CE DC D'0.1'
F35C1 DC D'0.1'
F35C2 DC D'0.1'
F35A DC D'0.1'
IFIRST STM 14,12,12(13) SAVE CALLING ROUTINE'S GPR'S
LR 2,13 R2 <= ADDR OLD SAVE AREA
LA 13,AREA R13 <= ADDR NEW SAVE AREA
DROP 15 R15 NO LONGER BASE REG
USING AREA,13 R13 NEW BASE REG
ST 2,4(13) LINK SAVE AREAS
ST 13,8(2)
LM 2,9,0(11) R2-R9 <= ADDR'S 1ST 8 ARG'S
LD 0,0,5(5) F0 <= GAM
ST 6,TLA TLA <= BASE TL TABLE
LDR 2,0 F2 <= GAM
ST 7,TMA TMA <= BASE TM TABLE
LD 4,0,4(4) F4 <= TEMP3
ST 8,VTHA VTHA <= BASE VTH TABLE
STD 4,TEMP3 TEMP3 (LOCAL) <= TEMP3
ST 9,CNVA CNVA <= BASE CNV TABLE
LD 6,0,3(3) F6 <= TEMP2
LM 4,10,32(1) R4-R10 ADDR'S REST OF ARG'S
LD 4,5(2) F4 <= TEMP1
STD 6,TEMP2 TEMP2 (LOCAL) <= TEMP2
SLA 10,3 R10 <= NTAB*8
STD 4,TEMP1 TEMP1 (LOCAL) <= TEMP1
S 10,EIGHT R10 <= 8*(NTAB-1)
SD 2,=D'1.1'
SR 4,9 R4 <= BASE N - BASE T
DDR 2,0 F2 <= (GAM-1.0D)/GAM
SR 5,9 R5 <= BASE LN - BASE T
STD 2,0AM GAM <= (GAM-1.0D)/GAM
SR 6,9 R6 <= BASE SD - BASE T
LD 0,ME F0 <= ME
L 1,REG1 R1 <= ADDR ARG LIST
DD 0,MI F0 <= ME/MI
L 15,=V(DSQRT) R15 <= ENTRY ADDR DSQRT
STD 0,F35A F35A <= ME/MI
BALR 14,15 F0 <= DSQRT(ME/MI)
SR 8,9 R8 <= BASE J - BASE T
MD 0,B F0 <= DSQRT(ME/MI)*B
SR 7,9 R7 <= BASE OSIG - BASE T
MD 0,=D'6.E-2' F0 <= 6.0-2*DSQRT(ME/MI)*B
AR 10,9 R10 <= BASE T + 8*(NTAB-1)
LD 2,=D'4.1'
MD 2,PI F2 <= 4.00*PI
S 7,EIGHT R7 <= BASE OSIG - 8 - BASE T
MD 2,ESU F2 <= 4.00*PI*ESU

ERROR TOLERANCE FOR NEWTON'S METHOD
<table>
<thead>
<tr>
<th>Instruction</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>ST 9, REG2</td>
<td>REG2 &lt;= BASE T</td>
</tr>
<tr>
<td>DDR 0, 2</td>
<td>F2 &lt;= 6.0 - 2<em>DSQRT(ME/MI)<em>B/(4.00</em>PI</em>ESU)</td>
</tr>
<tr>
<td>ST 7, REG3</td>
<td>REG3 &lt;= BASE OSIG - 8 - BASE T</td>
</tr>
<tr>
<td>STD 0, CONAN</td>
<td>CONAN &lt;= 6.0 - 2<em>DSQRT(ME/MI)<em>B/(4.00</em>PI</em>ESU)</td>
</tr>
<tr>
<td>ST 8, REG4</td>
<td>REG4 &lt;= BASE J - BASE T</td>
</tr>
<tr>
<td>LD 4, GAM</td>
<td>F4 &lt;= ((GAM-1.00)/GAM)</td>
</tr>
<tr>
<td>MD 0, TEMP3</td>
<td>F0 &lt;= TEMP3*CONAN</td>
</tr>
<tr>
<td>ST 4, REG5</td>
<td>REG5 &lt;= BASE N - BASE T</td>
</tr>
<tr>
<td>MDR 0, 4</td>
<td>F0 &lt;= ((GAM-1.00)/GAM)<em>TEMP3</em>CONAN</td>
</tr>
<tr>
<td>ST 5, REG6</td>
<td>REG6 &lt;= BASE LN - BASE T</td>
</tr>
<tr>
<td>LD 2, C</td>
<td>F2 &lt;= C</td>
</tr>
<tr>
<td>ST 6, REG7</td>
<td>REG7 &lt;= BASE SD - BASE T</td>
</tr>
<tr>
<td>LCDR 2, 2</td>
<td>F2 &lt;= -C</td>
</tr>
<tr>
<td>DDR 0, 2</td>
<td>F0 &lt;= -((GAM-1.00)/GAM)<em>TEMP3</em>CONAN/C</td>
</tr>
<tr>
<td>ST 10, REG9</td>
<td>REG9 &lt;= BASE T + 8*(NTAB-1) COMPARAND</td>
</tr>
<tr>
<td>MD 4, TEMP1</td>
<td>F4 &lt;= TEMP1*((GAM-1.00)/GAM) = TCON</td>
</tr>
<tr>
<td>DD 2, ESU</td>
<td>F2 &lt;= -C/ESU</td>
</tr>
<tr>
<td>STD 4, TCON</td>
<td>TCON &lt;= TEMP1*((GAM-1.00)/GAM)</td>
</tr>
<tr>
<td>L 10, 4(13)</td>
<td>R10 &lt;= ADDR OLD SAVE AREA</td>
</tr>
<tr>
<td>MD 0, ESU</td>
<td>F0 &lt;= -((GAM-1.00)/GAM)<em>TEMP3</em>CONAN*ESU/C</td>
</tr>
<tr>
<td>STD 2, CE</td>
<td>CE &lt;= -C/ESU</td>
</tr>
<tr>
<td>LM 14,10,12(10)</td>
<td>GPR'S RESTORED</td>
</tr>
<tr>
<td>STD 0, CONAN</td>
<td>CONAN &lt;= ((GAM-1.00)/GAM)<em>TEMP3</em>CONAN</td>
</tr>
<tr>
<td>L 13,4(13)</td>
<td>R13 &lt;= ADDR OLD SAVE AREA</td>
</tr>
<tr>
<td>SR 15, 15</td>
<td>R15 &lt;= 0 (RETURN CODE)</td>
</tr>
<tr>
<td>MVI 12(13), X'FF'</td>
<td>INDICATE CONTROL RETURNED</td>
</tr>
<tr>
<td>BR 14</td>
<td>RETURN</td>
</tr>
<tr>
<td>DROP 13</td>
<td>USING CURCAL, 15</td>
</tr>
<tr>
<td>STM 14,12,12(13)</td>
<td>SAVE CALLING ROUTINE'S GPR'S</td>
</tr>
<tr>
<td>LR 2, 13</td>
<td>R2 &lt;= ADDR OLD SAVE AREA</td>
</tr>
<tr>
<td>LA 13, AREA</td>
<td>R13 &lt;= ADDR NEW SAVE AREA</td>
</tr>
<tr>
<td>DROP 15</td>
<td>R15 NO LONGER BASE REG</td>
</tr>
<tr>
<td>USING AREA, 13</td>
<td>R13 NEW BASE REG</td>
</tr>
<tr>
<td>ST 2,4(13)</td>
<td>LINK SAVE AREAS</td>
</tr>
<tr>
<td>ST 13,8(2)</td>
<td></td>
</tr>
<tr>
<td>LM 1,11, REG1</td>
<td>SET UP GPR'S</td>
</tr>
<tr>
<td>LH 12,0(2)</td>
<td>R12 &lt;= HIGH ORDER 2 BYTES OF T(1)</td>
</tr>
<tr>
<td>MVC FLOAT+1(6),2(2)</td>
<td>FLOAT &lt;= FRACTIONAL DISPLACEMENT</td>
</tr>
<tr>
<td>S 12, TDISP</td>
<td>REDUCE R12 BY TDISP. NOW # DOUBLE WORDS</td>
</tr>
<tr>
<td>* FROM BASE OF INTERPOLATION TABLES</td>
<td></td>
</tr>
<tr>
<td>LD 4, FLOAT</td>
<td>F4 &lt;= FRACTION 0 LE FRAC LT 1</td>
</tr>
<tr>
<td>BM BADT</td>
<td>IF RESULT NEGATIVE - OUT OF RANGE</td>
</tr>
<tr>
<td>GOTO BADT</td>
<td></td>
</tr>
<tr>
<td>C 12, TBND</td>
<td>IF R12 GREATER THAN TBND</td>
</tr>
<tr>
<td>BH BADT</td>
<td>GOTO BADT</td>
</tr>
<tr>
<td>SD 4, =D'.5'</td>
<td>R12 &lt;= R2*X8 NOW BYTE DISPLACEMENT</td>
</tr>
<tr>
<td>SLA 12, 3</td>
<td>FROM BASE OF INTERPOLATION TABLES</td>
</tr>
<tr>
<td>* NOW COMPUTE WEIGHTS FOR CUBIC INTERPOLATION OF</td>
<td></td>
</tr>
<tr>
<td>* FUNCTIONS OF T</td>
<td></td>
</tr>
<tr>
<td>*</td>
<td></td>
</tr>
<tr>
<td>LDR 2, 4</td>
<td>F2 &lt;= X</td>
</tr>
<tr>
<td>MDR 4, 4</td>
<td>F4 &lt;= X**2 - X2</td>
</tr>
<tr>
<td>HDR 4, 4</td>
<td>F4 &lt;= X2/2</td>
</tr>
<tr>
<td>SD 4, =D'.125'</td>
<td>F4 &lt;= X2/2 - 9/8</td>
</tr>
<tr>
<td>LDR 6, 4</td>
<td>F6 &lt;= X2/2 - 9/8</td>
</tr>
<tr>
<td>HDR 4, 4</td>
<td>F4 &lt;= X2/4 - 9/16</td>
</tr>
<tr>
<td>MDR 6, 2</td>
<td>F6 &lt;= X3/2 - 9/8</td>
</tr>
<tr>
<td>LCDR 0, 4</td>
<td>F0 &lt;= -X2/4 + 9/16</td>
</tr>
<tr>
<td>ADR 0, 6</td>
<td>F0 &lt;= X3/2 - X2/4 - 9X/8 + 9/16</td>
</tr>
<tr>
<td>STD 0, WM1</td>
<td>WM1 &lt;= WEIGHT FOR TABLE ENTRY CORRESPONDING TO CLOSEST SMALLER VALUE OF T.</td>
</tr>
<tr>
<td>LCDR 0, 4</td>
<td>F0 &lt;= -X2/4 + 9/16</td>
</tr>
<tr>
<td>SDR 0, 6</td>
<td>F0 &lt;= -X3/2 + 9X/8</td>
</tr>
<tr>
<td>STD 0, WP1</td>
<td>WP1 &lt;= WEIGHT FOR TABLE ENTRY CORRESPONDING TO CLOSEST LARGER VALUE OF T.</td>
</tr>
<tr>
<td>ADR 6, 2</td>
<td>F6 &lt;= X3/2 - X8</td>
</tr>
</tbody>
</table>
AD 4,0 1.5 F4 \leq \frac{x^2}{4} - 1/16
MD 6,0 0.5 555555555555555555 F6 \leq \frac{x^3}{6} - x/24
LDR 0,4 F0 \leq \frac{x^2}{4} + 1/16
SDR 0,6 F0 \leq -\frac{x^3}{6} + \frac{x^2}{4} + x/24 - 1/16
ADR 6,4 F6 \leq \frac{x^3}{6} + \frac{x^2}{4} - x/24 - 1/16
STD 0,WM3 WM3 \leq \text{weight for smallest value of T}
STD 6,WP3 WP3 \leq \text{weight for largest value of T}

* NOW CALCULATE INTERPOLATED VALUES OF TL AND TM
* (HAVE WEIGHTS FOR TABLES ENTRIES 1 & 4 IN FPR'S 0 & 6)

LDR 4,0 F4 \leq \text{weight 1}
LDR 2,6 F2 \leq \text{weight 4}
MD 0,0(10,12) F0 \leq \text{weight 1} \times \text{TL 1}
MD 2,24(10,12) F2 \leq \text{weight 4} \times \text{TL 4}
MD 4,0(11,12) F4 \leq \text{weight 1} \times \text{TM 1}
MD 6,24(11,12) F6 \leq \text{weight 4} \times \text{TM 4}
ADR 0,2 F0 \leq \text{W1*TL1 + W4*TL4}
ADR 4,6 F4 \leq \text{W1*TM1 + W4*TM4}
LD 2,WM1 F2 \leq \text{weight 2}
LDR 2,6 F6 \leq \text{weight 2}
MD 2,8(10,12) F2 \leq \text{W2*TL2}
MD 6,8(11,12) F6 \leq \text{W2*TM2}
ADR 0,2 F0 \leq \text{INTERPOLATED VALUE OF TL}
ADR 4,6 F4 \leq \text{INTERPOLATED VALUE OF TM}
MD 4,06,2 F4 \leq \text{TM*LN(1)}
ADR 4,0 F4 \leq \text{TL + TM*LN(1)}
L 10,CTNA R10 \leq \text{base addr CHINV table}
STD 4,83,2 SIG(1) \leq \text{TL + TM*LN(1)}
L 11,VTIHA R11 \leq \text{base VTH table}
LD 0,WM3 F0 \leq \text{weight for smallest value of T}
LD 6,WP3 F6 \leq \text{weight for largest value of T}

* NOW CALCULATE INTERPOLATED VALUES OF CH AND VT
* (HAVE WEIGHTS FOR TABLES ENTRIES 1 & 4 IN FPR'S 0 & 6)

LDR 4,0 F4 \leq \text{weight 1}
LDR 2,6 F2 \leq \text{weight 4}
MD 0,0(10,12) F0 \leq \text{weight 1} \times \text{CH 1}
MD 2,24(10,12) F2 \leq \text{weight 4} \times \text{CH 4}
MD 4,0(11,12) F4 \leq \text{weight 1} \times \text{VT 1}
MD 6,24(11,12) F6 \leq \text{weight 4} \times \text{VT 4}
ADR 0,2 F0 \leq \text{W1*CH1 + W4*CH4}
ADR 4,6 F4 \leq \text{W1*VT1 + W4*VT4}
LD 2,WM1 F2 \leq \text{weight 2}
LDR 6,2 F6 \leq \text{weight 2}
MD 2,8(10,12) F2 \leq \text{W2*CH2}
MD 6,8(11,12) F6 \leq \text{W2*VT2}
ADR 0,2 F0 \leq \text{W1*CH1 + W4*CH4 + W2*CH2}
ADR 4,6 F4 \leq \text{W1*VT1 + W4*VT4 + W2*VT2}
LD 2,WP1 F2 \leq \text{weight 3}
LDR 6,2 F6 \leq \text{weight 3}
MD 2,16(10,12) F2 \leq \text{W3*CH3}
MD 6,16(11,12) F6 \leq \text{W3*VT3}
ADR 0,2 F0 \leq \text{INTERPOLATED VALUE OF CH}
ADR 4,6 F4 \leq \text{INTERPOLATED VALUE OF VT}
LDR 2,0 F2 \leq \text{CHT}
SDFR 6,6 F6 \leq \text{0.00}
MD 2,CE F2 \leq \text{CHT*CE}
L 10,TLA R10 \leq \text{base TL table}
MD 2,015,2 F2 \leq \text{N(1)*CHT*CE}
L 11,TMA R11 \leq \text{base TM table}
STD 6,TSI TSI \leq \text{0.00}
MO    CDR    BL    LO    MDR    MO    LO    OOR    SDR    MDR    AD    STD
ARNO  AR     LOOP    LH    MDC    S    12,0(2)    F2 (= J(1)*CHT*CE = VD
ARNO  ARND   GOTO ARND
LD    6, CONAN    F6 <= CONAN
MDR   6,0    F6 <= CONAN*CHT
MD    6,0(5,2)    F6 <= CONAN*N(1)*CHT
LD    0, =0'1'.1    F0 <= 1.00
DDR   4,2    F4 <= VITH/VD
SDR   0,4    F0 <= 1.00-VITH/VD
MDR   6,0    F6 <= CONAN*N(1)*CHT*(1.00-VITH/VD)
AD    6,8(3,2) F6 <= OSIG(1)+CONAN*N(1)*CHT*(1.00-VITH/VD)
STD   6,8(3,2) OSIG(1)<=OSIG(1)+CONAN*N(1)*CHT*(1.00-VITH/VD)
ARND  AR    2,8    R2 <= ADDR T(2)
LOOP    LH    12,0(2)    R12 <= HIGH ORDER 2 BYTES OF T(1)
MVC    FLOAT+1(6),2(2) FLOAT <= FRACTIONAL DISPLACEMENT
S    12,TDISP    REDUCE R12 BY TDISP. NOW # DOUBLE WORDS FROM BASE OF INTERPOLATION TABLES
LD    4,FLOAT    F4 <= FRACTION 0 LE FRAC LT 1
BM    BADT    IF RESULT NEGATIVE - OUT OF RANGE
C    12,TBND    IF R12 GREATER THAN TBND
BH    BADT    GOTO BADT
SD    4, =0'.5' F4 <= FRAC -.5 LE FRAC LT .5
SLA    12,3    R12 <= R2*8 NOW BYTE DISPLACEMENT FROM BASE OF INTERPOLATION TABLES

* NOW COMPUTE WEIGHTS FOR CUBIC INTERPOLATION OF FUNCTIONS OF T
*
LDR    2,4    F2 <= X
MDR   4,4    F4 <= X**2 = X2
HDr   4,4    F4 <= X2/2
SD    4, =0'1.125' F4 <= X2/2 - 9/8
LDR   6,4    F6 <= X2/2 - 9/8
HDr   4,4    F4 <= X2/4 - 9/16
MDR   6,2    F6 <= X3/2 - 9X/8
LDr   0,4    F0 <= -X2/4 + 9/16
ARD   0,6    F0 <= -X3/2 - X2/4 - 9X/8 + 9/16
STD   0,WM1    WM1 <= WEIGHT FOR TABLE ENTRY CORRESPONDING TO CLOSEST SMALLER VALUE OF T.
LDr   0,4    F0 <= -X2/4 + 9/16
SDR   0,6    F0 <= -X3/2 - X2/4 + 9X/8 + 9/16
STD   0,WP1    WP1 <= WEIGHT FOR TABLE ENTRY CORRESPONDING TO CLOSEST LARGER VALUE OF T.
ARD   6,2    F6 <= X3/2 - X/8
AD    4, =0'.5' F4 <= X2/4 - 1/16
MD    6, =X'4055555555555555' F5 <= X3/6 - X/24
LDr   0,4    F0 <= -X2/4 + 1/16
SDR   0,6    F0 <= -X3/6 + X2/4 + X/24 - 1/16
ARD   6,4    F6 <= X3/6 + X2/4 - X/24 - 1/16
STD   0,WM3    WM3 <= WEIGHT FOR SMALLEST VALUE OF T
STD   6,WP3    WP3 <= WEIGHT FOR LARGEST VALUE OF T

* NOW CALCULATE INTERPOLATED VALUES OF TL AND TM
* (HAVE WEIGHTS FOR TABLES ENTRIES 1 & 4 IN FPR'S 0 & 6)
*
LDR    4,0    F4 <= WEIGHT 1
LDR    2,6    F2 <= WEIGHT 4
MD    0,0(10,12)    F0 <= WEIGHT 1*TL 1
MD    2,24(10,12)    F2 <= WEIGHT 4*TL 4
MD    4,0(11,12)    F4 <= WEIGHT 1*TM 1
MD    6,24(11,12)    F6 <= WEIGHT 4*TM 4
ADR   0,2    F0 <= W1*TL1 + W4*TL4
ADR   4,6    F4 <= W1*TM1 + W4*TM4
LD    2,WM1    F2 <= WEIGHT 2
LDR    6,2    F6 <= WEIGHT 2
MD    2,0(10,12)    F2 <= W2*TL2
MD    6,8(11,12)    F6 <= W2*TM2
ADR   0,2    F0 <= W1*TL1 + W4*TL4 + W2*TL2
**ORIGINAL PAGE IS**

**OF POOR QUALITY**

ADR 4,6  
LD 2,UP1  
LDR 6,2  
MD 2,16(10,12)  
MD 6,16(11,12)  
ADR 0,2  
ADR 4,6  
ADR 4,0(6,2)  
ADR 0,4  
L 10,CNVA  
STD 0,8(3,2)  
L 11,VTHA  
AD 0,0(3,2)  
MD 0,0(7,2)  
AD 0,TSA  
STD 0,TSA  
MD 0,TMP3  
AD 0,TMP2  
STD 0,JCON  
L 6,WP3  

```
F4 <= W1*TM1 + W4*TM4 + W2*TM2
F2 <= WEIGHT 3
F6 <= WEIGHT 3
F2 <= W3*TL3
F6 <= W3*TM3
F0 <= INTERPOLATED VALUE OF TL
F4 <= INTERPOLATED VALUE OF TM
F4 <= TM*LN(I)
F0 <= TL + TM*LN(I)
R10 <= BASE ADDR CHINV TABLE
OSIG(I) <= TL + TM*LN(I)
R11 <= BASE VTH TABLE
F0 <= OSIG(I)+OSIG(I-1)*SO(I)
F0 <= (OSIG(I)+OSIG(I-1))*SO(I)
F0 <= TSI + (OSIG(I)+OSIG(I-1))*SO(I)
F0 <= TSI*TEMP3
F0 <= TEMP2 + TEMP3*TSI
JCON <= TEMP2+TEMP3*TSI
F6 <= 1.00/(TEMP2+TEMP3*TSI)
F4 <= 1.00/(TEMP2+TEMP3*TSI)
F35 <= 1.00/(TEMP2+TEMP3*TSI)
```

*NOW CALCULATE INTERPOLATED VALUES OF CH AND VT*
*(HAVE WEIGHTS FOR TABLES ENTRIES 1 & 4 IN FPR'S 0 & 6)*

LDR 4,0  
LDR 2,6  
MD 0,0(10,12)  
MD 2,24(10,12)  
MD 4,0(11,12)  
MD 6,24(11,12)  
ADR 0,2  
ADR 4,6  
LDR 2,UP1  
LDR 6,2  
MD 2,8(10,12)  
MD 6,8(11,12)  
ADR 0,2  
ADR 4,6  
L 2,UP1  
LDR 5,2  
MD 2,16(10,12)  
MD 6,16(11,12)  
ADR 3,2  
ADR 4,6  
MD 0,0(5,2)  
LD 6,0  
STD 0,NONO  
MD 6,F0(7,2)  
MD 0,CE  
MD 6,CONAN  
STD 0,NONO  
STD 6,CJA  
LD 2,TEMP1  
MD 2,F35C2  
L 2,TEMP  
STD 2,0(4,2)  
MOR 2,0  
L 8,TLA  
CDR 2,4  
BL ARND1  

```
F4 <= WEIGHT 1
F2 <= WEIGHT 4
F0 <= WEIGHT 1 * CH 1
F2 <= WEIGHT 4 * CH 4
F4 <= WEIGHT 1 * VT 1
F6 <= WEIGHT 4 * VT 4
F0 <= W1*CH1 + W4*CH4
F4 <= W1*VT1 + W4*VT4
F6 <= W2*CH2
F6 <= W2*VT2
F0 <= W1*CH1 + W4*CH4 + W2*CH2
F4 <= W1*VT1 + W4*VT4 + W2*VT2
F0 <= INTERPOLATED VALUE OF CH
F4 <= INTERPOLATED VALUE OF VT
F0 <= N(I)*CH
F6 <= N(I)*CH = NONO
NCONO <= NCI*CHT
F6 <= NONO*SD(I)
F0 <= N(I)*CHT*CE
F6 <= CONAN*NONO*SD(I)
F6 <= CONAN*CHT*CE
CJA <= CONAN*NONO*SD(I)
F2 <= TEMP1
F2 <= TEMP*F35C2 = J
R11 <= BASE TM TABLE
J(I) <= TEMP*F35C2
F2 <= JT*NCON = VD
R10 <= BASE TL TABLE
IF(VD,LT.VITH)
GOTO ARND1
```

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THE FOLLOWING CALCULATES THE CURRENT AND RESISTIVITY IN THE
CASE THE DRIFT VELOCITY EXCEEDS 13 TIMES THE ION THERMAL
VELOCITY. IN THIS CASE AT LEAST PART OF THE SLAB
IS CHARACTERIZED BY ANOMALOUS RESISTIVITY.

LD 2,CJA  
LD 2,TEMP3  
LD 6,4  
STD 2,CJA  
AD 2,JCON  
STD 2,JCON  
DR 6,0  
LD 2,CJA  
MD 0,CONAN1  
STD 6,JA  
STD 2,CJA  
LD 2,0(4,2)  
MD 0,0(7,2)  
MD 0,F35C1  
MDR 0,6  
ADR 6,0  
SDR 2,6  
ADR 2,0  
AD 2,-D'1. '

NEWT

LD 6,CJA  
DDR 6,2  
L 15,=V(F35)  
LD 0,JCON  
STD 0,F35A  
STD 6,TC1  
BALR 14,15  
LD 4,TC1  
LD 6,F35A  
MD 0,6  
LD 2,0(4,2)  
MD 0,2  
MD 6,TEMPl  
ADR 6,4  
ADR 2,0  
DOR 6,0  
MOR 2,6  
LDR 4,2  
LPDR 0,2  
SD 4,0(4,2)  
MD 0,0ERR  
LPDR 4,4  
CBR 0,4  
STD 2,0(4,2)  
BCT 11,NEWT  
OUT  

LD 4,JA  
LD 4,2  
L 11,TMA  
LD 2,0(4,2)  
S2,4  
MD 2,NCON0  
MD 2,CONAN  
LDR 4,2  

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AD 2,8(3,2) F2 <= OSIG(I) + TSI
MD 4.0(7,2) F4 <= TSI*SD(I)
AD 4,TST F4 <= TSI + TSI*SD(I)
STD 2,8(3,2) OSIG(I) <= OSIG(I) ÷ TSI
STD 4,TST TSI <= TSI + TSI*SD(I)

ARND1 BXLE 2,8,LOOP I <= I+1 AND GOTO LOOP IF NOT DONE
L 13,4(13) R13 <= ADDR OLD SAVE AREA
LM 14,12,12(13) GPR'S RESTORED
SR 15,15 R15 <= 0 (RETURN CODE)
MVI 12(13),X'FF' INDICATE CONTROL RETURNED
BR 14 RETURN

BADT L 13,4(13) R13 <= ADDR OLD SAVE AREA
LM 14,12,12(13) GPR'S RESTORED
LA 15,4 R15 <= 4 (RETURN CODE)
MVI 12(13),X'FF' INDICATE CONTROL RETURNED
BR 14 RETURN

'ORIGINAL PAGE IS
DE POOR QUALITY
REAL FUNCTION F3S8(X)
F3S8 RETURNS THE 3./5. POWER OF THE ARGUMENT IF
THE ARGUMENT IS POSITIVE AND THE NEGATIVE OF THE 3./5.
POWER OF THE ABSOLUTE VALUE OF THE ARGUMENT IF THE ARGUMENT
IS NEGATIVE. THE COMMENTS REFER TO THE POSITIVE CASE.
THE ALGORITHM IS:
CUBE X AND CALL THE RESULT Y, THEN WRITE Y AS
Y = (16**(5*N)) * (16**M) * (Z)
WHERE M IS BETWEEN -4 AND +4 AND Z IS BETWEEN 1/16 AND 1.
THEN Z**(1/5) IS APPROXIMATED BY A MINI-MAX LINEAR FIT
FROM TWO TABLES WITH A MAXIMUM RELATIVE ERROR IN THE
APPROXIMATION OF 5.1E-4. THEN THE INITIAL ESTIMATE OF
T**1/5 = (16**(M/5)) * (Z**(1/5))
IS REFINED BY TWO APPLICATIONS OF NEWTON'S METHOD.
X**3/5 IS THEN CALCULATED FROM (16**N) * (T**1/5).

USING *,15 TELL ASSEMBLER NEXT INST ADDR IN R15
B FIRST BRANCH AROUND NAME AND SAVE AREA
DC X'03' LENGTH OF NAME
DC CL3'F35' NAME
AREA
DS 18F SAVE AREA
FIRST
STM 14,12,12(13) SAVE CALLING ROUTINE'S GPR'S
L 1,0(1) R1 <= ADDR ARGUMENT (X)
LD 4,0(1) F4 <= X
LR 9,13 R9 <= ADDR OLD SAVE AREA
STD 4,ARG ARG <= X
LA 13,AREA R13 <= ADDR NEW SAVE AREA
DROP 15 R15 NO LONGER BASE REG
USING AREA,13 R13 NEW BASE REG
LDR 9,4(13) CHECK SIGN OF X
ST 9,4(13) LINK SAVE AREAS
BNM NONNEG IF SIGN POSITIVE - NO FIXES OR FLAGS
NI ARG,X'7F' TURN OFF SIGN BIT OF ARG <= |X|
O 9,=X'80000000' SIGN BIT R9 ON - FLAG
NONNEG
ST 13,8(9) LINK SAVE AREAS
SR 3,3 R3 <= 0
IC 3,ARG R3 <= EXCESS 64 EXPONENTET OF ARG
MVI ARG,X'40' ARG <= FRACTION OF ARG
LD 4,ARG F4 <= FRACTION OF ARG
L 4,=F'64' R4 <= HEX 40
SR 3,4 R3 <= EXPONENT OF ARG
MDR 4,4 F4 <= FRACTION OF ARG **2
M 2,=F'3' R3 <= FRACTION OF ARG**3
LA 6,TAB1-16 R6 <= ADDR TABLE 1 - 16
MD 4,ARG F4 <= FRACTION OF ARG **3
SR 5,5 R5 <= 0
LA 7,TAB2-16 R7 <= ADDR TABLE 2 - 16
STD 4,ARG ARG <= FRACTION ARG **3
IC 5,ARG R5 <= EXCESS 64 EXPONENT OF
* FRACTION OF ARG **3
AR 3,5 R3 <= EXCESS 64 EXPONENT OF |X|**3
SR 2,2 R2 <= 0
SR 3,4 R3 <= EXPONENT OF |X|**3
LA 8,TAB3+16 R8 <= ADDR TABLE 3 + 16
BNM NOEXTD IF R3 > 0 NO SIGN EXTEND
L 2,=F'1' SIGN EXTEND FOR DIVIDE
NOEXTD
D 2,=F'5' R2 <= EXPONENT OF T
R3 <= N (SEE COMMENTS)
BEGIN TWO APPLICATIONS OF NEWTON'S METHOD
WITH SOME GPR FIX-UPS INTERLEAVED.

MDR 2,2 F2 <= EST1 ** 2
STC 3,MUL MUL (= SIGN(X) * 16**N)
MDR 2,1 F2 <= EST 1 ** 4
L 1,24(9) R1 RESTORED
LDR 6,2 F6 <= EST 1 ** 4
MDR 2,0 F2 <= EST 1 ** 5
L 2,28(9) R2 RESTORED
SDR 2,4 F2 <= EST 1 ** 5 - T
DER 2,6 F2 <= (EST 1 ** 5 - T)/EST 1 ** 4
L 7,32(9) R3 RESTORED
ME 2,0,ME5 F2 <= (EST 1 ** 5 - T)/5*EST 1 ** 4
L 4,36(9) R4 RESTORED
SDR 0,2 F0 <= EST 2
LDR 2,0 F2 <= EST 2
MDR 2,2 F2 <= EST 2 ** 2
L 5,40(9) R5 RESTORED
MDR 2,2 F2 <= EST 2 ** 4
L 4,44(9) R6 RESTORED
LDR 6,2 F6 <= EST 2 ** 4
MDR 2,0 F2 <= EST 2 ** 5
L 7,48(9) R7 RESTORED
SDR 2,4 F2 <= EST 2 ** 5 - T
L 8,52(9) R8 RESTORED
DDR 2,6 F2 <= (EST 2 ** 5 - T)/EST 2 ** 4
SR 15,15 R15 <= 0 (RETURN CODE)
MD 2,0,ME5 F2 <= (EST 2 ** 5 - T)/5*EST 2 ** 4
MVI 12(9),X'FF' INDICATE CONTROL RETURNED
SDR 0,2 F0 <= EST 3 (FINAL ESTIMATE OF T**1/5)
L 9,56(9) R9 RESTORED
MD 0,MUL F0 <= ESTIMATE OF [X]**3/5
L 13,4(13) R13 RESTORED
BR 14 RETURN
CNOP 4,8 FORCE CORRECT ALIGNMENT
ORIGINAL PAGE IS
OF POOR QUALITY

TAB3
DC X'3F27CF79'
DC X'3F25DDFD'
DC X'3F2420D4'
DC X'3F2286B5'
DC X'3F2120C2'
DC X'3F1FD408'
DC X'3F1EA350'
DC X'3F1DB817'
DC X'3F1C8870'
DC X'3F1B980F'
DC X'3F1ABA4B'
DC X'3F19EA65'
DC X'3F192921'
DC X'3F1873A6'
DC X'3F17C948'
DC X'3F172902'
DC X'3F1691EC'
DC X'3F160338'
DC X'3F157C30'
DC X'3F14FD40'
DC X'3F1482ED'
DC X'3F140F75'
DC X'3F13A198'
DC X'3F1338E1'
DC X'3F12D4F4'
DC X'3F12757A'
DC X'3F121A27'
DC X'3F11C2B5'
DC X'3F116EE4'
DC X'3F111178'
DC X'3F100130'
DC X'3F108701'
DC X'3F103F97'
DC X'3EFFAD4E'
DC X'3EFBD941'
DC X'3EF78B10'
DC X'3EF380AD'
DC X'3EEFF820'
DC X'3EEC5FC9'
DC X'3EE8E5DA'
DC X'3EE588D6'
DC X'3EE2474D'
DC X'3EDF1FE8'
DC X'3EDC1167'
DC X'3ED91A9C'
DC X'3ED63A6E'
DC X'3ED36FD6'
DC X'3ED0690C'
DC X'3ECE1796'
DC X'401DBB8C'
DC X'40306UC0'
DC X'405472D1'
DC X'409308C1'
DC X'410000D0'
DC X'4119DB8C'
DC X'413080C0'
DC X'415472D1'
DC X'4193088C'
END
**TESTP**

CSECT

* ROUGHLY EQUIVALENT TO THE FORTRAN CODE BELOW EXCEPT THE
  FUNCTION CHINV PASSED IN THE ARGUMENT LIST
  IS IMPLEMENTED IN LINE IN THE ASSEMBLY LANGUAGE VERSION.
  A CALL TO THE ASSEMBLY LANGUAGE VERSION SHOULD
  PASS A SEMI-LOGARITHMIC INTERPOLATION TABLE (CHINV(884))
  RATHER THAN A FUNCTION NAME. ALSO THE PARAMETER ADDRESSES
  ARE OBTAINED FROM LOCAL STORAGE IN THE CALL TO TESTP
  NOT FROM THE PARAMETER LIST. THE PARAMETER ADDRESSES
  ARE INITIALIZED BY THE ENTRY POINT TESTPI IN THE ASSEMBLY
  LANGUAGE VERSION.

  NOTE THAT THIS MEANS TESTPI MUST BE CALLED BEFORE
  THE FIRST CALL TO TESTP OR UNPREDICTABLE ABENDS
  WILL RESULT.

  TESTP(T,TE,J,OSIG,N,TUP,DELT,FRAC,NTAB,CHINV)
  IMPLICIT REAL*8 (A-H.O-Z)
  REAL*8 T(NTAB),TE(NTAB),J(NTAB),OSIG(NTAB),N(NTAB),TUP(NTAB)
  REAL*8 0/2.997925D10/FRAC,FRACI,DELT,DELTS
  ENTRY TESTPI(T,TE,J,OSIG,N,TUP,DELT,FRAC,NTAB,CHINV)
  FRACI=2.0D0/FRAC
  RETURN
  10 DELTS=DELT*FRACI
  DO 20 I=1,NTAB
  TUP(I)=0*J(I)+J(I)*OSIG(I)*N(I)
  IF(DELTS*TUP(I).LE.TE(I))GOTO 20
  DELTS=TE(I)/TUP(I)
  20 CONTINUE
  DELT=FRAC*DELTS
  DO 20 I=1,NTAB
  TE(I)=TE(I)+TUP(I)*DELT
  20 T(I)=CHINV(TE(I))
  RETURN
  END

  USING *,15
  B TFIRST
  DC X'05'
  DC CL5'TESTP'
  ENTRY TESTPI
  USING *,15

  TESTPI
  B IFIRST
  BRANCH AROUND NAME, SAVE AREA ETC.
  DC X'06'
  DC CL7'TESTPI'
  AREA
  DS 10F
  TDISP DC X'000004410'
  TBND DC X'00000230'
  REGS DS 10F
  CHOP 0,8
  REGISTER STORAGE
  FORCE DOUBLE WORD ALIGNMENT
  C DC D'2.997925E10'
  TION DC X'4519AEF8FF9FC62'
  FRAC DC D'0.'
  FRACI DC D'0.'
  DELT DC D'0.'
  WM1 DC D'0.'
  HP1 DC D'0.'
  FLOAT DC X'4000000000000000'
  * FIRST ENTRY POINT
  *
  I FIRST STM 14,12,12(13) SAVE CALLING ROUTINE'S GPR'S
  LR 2,13 R2 <= ADDR OLD SAVE AREA
LA 13, AREA
DROP 15
USING AREA, 13
ST 2, 4(13)
ST 13, S(2)
LM 3, 12, 0(1)
LD 0, 0(10)
L 11, 0(11)
LD 2, =0'2', 1
SLA 11, 3
STD 0, FRAC
SR 3, 4
ST 5, 4
SR 6, 4
STD 2, FRACI
SR 7, 4
STD 8, 4
SR 10, 8
SR 11, 10
AR 11, 4
STM 3, 12, REGS
LM 13, 4(13)
SR 15
MVI 12(13), X'FF'
BR 14
DROD 13
TFIRST
STM 14, 12, 12(13)
LA 2, 13
LA 13, AREA
DROP 15
USING AREA, 13
ST 2, 4(13)
ST 13, S(2)
LM 3, 12, REGS
LD 6, 0(9)
MD 6, FRACI
LD 0, 0(5, 4)
MDR 0
MD 0, 0
MD 0.0(6, 4)
MD 0.0(7, 4)
STD 0.0(8, 4)
MDR 0.6
CD 0.0(4)
BNIH ARND
LD 6, 0(4)
DD 6.0(8, 4)
BXLE 4, 10, LOOP1
MD 6, FRAC
L 4, REGS+4
STD 6, 0(9)
LOOP2
LD 0, 0(8, 4)
MD 0, 0(9)
MD 0.0(4)
AP 0.0(4)
STD 0.0(4)
LM 2.0(4)
RVC 2.0(16), 2.0(4)
LD 4, FLOAT
BM LOWT
S 2, TDISP
LD 4, FLOAT
S 2, TBNND
BM HITE
SD 4, =0'.5'
SLA 2, 3
FROM BASE OF INTERPOLATION TABLE.

NOW COMPUTE WEIGHTS FOR CUBIC INTERPOLATION OF FUNCTIONS OF T

LDR  2,4    F2 <= X
MDR  4,4    F4 <= X**2  = X2
HDR  4,4    F4 <= X2/2
SD   4,'D 1.125' F4 <= X2/2 - 9/8
LDR  6,4    F6 <= X2/2 - 9/8
HDR  4,4    F4 <= X2/4 - 9/16
MDR  6,2    F6 <= X3/2 - 9X/8
LCDR 0,4    F0 <= -X2/4 + 9/16
ADR  0,6    F0 <= X3/2 - X2/4 - 9X/8 + 9/16
STD  0,WM1   WM1 <= WEIGHT FOR TABLE ENTRY CORRESPONDING TO CLOSEST SMALLER VALUE OF T.

LCDR 0,4    F0 <= -X2/4 + 9/16
SDR  0,6    F0 <= -X3/2 - X2/4 + 9X/8 + 9/16
STD  0,WP1   WP1 <= WEIGHT FOR TABLE ENTRY CORRESPONDING TO CLOSEST LARGER VALUE OF T.

ADR  6,2    F6 <= X3/2 - X/8
AD   4,'D 1.5' F4 <= X2/4 - 1/16
MD   6,'X'40555555555555555 F6 <= X3/6 - X/24
LDR  0,4    F0 <= X2/4 - 1/16
SDR  0,6    F0 <= X3/6 + X2/4 - 24/2 - 1/16
ADR  6,4    F6 <= X3/6 + X/24 - 1/16
MD   0,0(12,2) F0 <= WEIGHT1 * CHNV1
MD   6,24(12,2) F6 <= WEIGHT4 * CHNV4
LD   2,WM1   F2 <= WEIGHT2
ADR  0,6    F0 <= W1*CHNV1+W4*CHNV4
LD   4,WP1   F4 <= WEIGHT3
MD   2,8(12,2) F2 <= W2*CHNV2
MD   4,16(12,2) F4 <= W3*CHNV3
ADR  0,2    F0 <= W1*CHNV1+W2*CHNV2+W4*CHNV4
ADR  0,4    F0 <= INTERPOLATED VALUE OF CHINV(TE(I))
STD  0,0(3,4) T(I) <= CHINV(TE(I))
BXLE 4,10,LOOP2 I<=I+1 & GOTO LOOP2 IF NOT DONE

B ARND1
LOWT

STD  0,0(3,4) T(I) <= TE(I) (FOR T < 4096 K)
BXLE 4,10,LOOP2 I<=I+1 AND GOTO LOOP2 IF NOT DONE
B ARND1

HITEST

SD   0,TION F0 <= TE(T) - TION
HDR   0,0    F0 <= (TE(T) - TION)/2.DO
STD   0,0(3,4) T(I) <= (TE(T) - TION)/2.DO
BXLE 4,10,LOOP2 I<=I+1 AND GOTO LOOP2 IF NOT DONE

ARND1
LM  13,4(13) R13 <= ADDR OLD SAVE AREA
LM  14,12,12(13) GPR'S RESTORED
SR   15,15     R15 <= 0 (RETURN CODE)
MVI  12(13),X'FF' INDICATE CONTROL RETURNED
BR  14
END
TOUT

ROUNGLY EQUIVALENT TO THE THREE FORTRAN SUBROUTINES BELOW.
NOUT OPENS LOGICAL UNIT 9 (FT09F001) AND DOES THE OUTPUT
OF THE SUBROUTINE NOUT. THE PARAMETER NTAB IS PASSED
BY ENTRY POINT NOUT AND THE NTAB IN THE CALLING SEQUENCE
TO TOUT IS IGNORED BY THE ASSEMBLY LANGUAGE VERSION OF
THES ROUTINES. FORTRAN CLOSES DATA SETS IT KNOWS ABOUT
 BUT FORTRAN WON'T KNOW ABOUT THIS DATA SET SO CTOUT MUST
BE CALLED BEFORE THE STOP STATEMENT IN THE MAIN ROUTINE.
THE FORTRAN CTOUT DOES THE SAME THING - I.E. IT CAUSES A
CLOSE TO BE ISSUED FOR THE DATA SET REFERENCED BY THE
DDNAME FT09F001. USES QSAM UNDER OS/VS2.

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*
DC X'04' LENGTH OF NAME
DC CL5'NOUT' NAME
USING *,15
ENTRY CTOUT
CTOUT B CFIRST BRANCH AROUND NAME, VARIABLES AND AREA
DC X'05' LENGTH OF NAME
DC CL5'CTOUT' DROP 15 USING AREA,13
AREA DS 18F SAVE AREA
BUF DS 20F OUTPUT BUFFER (ONE CARD - WE USE QSAM)
BLNCRD DC 200L4' A BLANK CARD - I'M LAZY
TEN DC F'10' TO BE EXECUTED BY AN EX
LMOV3 MVC BUF(1),O(3) TO BE EXECUTED BY AN EX
LMOV4 MVC BUF(1),O(4)
SREG DS 5F DROP 13 USING CTOUT,15

* CTOUT *

* CFIRST *

STM 14,12,12(13) SAVE CALLING ROUTINE'S GPR'S
LR 2,13 R2 <= ADDR OLD SAVE AREA
LA 13,AREA R13 <= ADDR NEW SAVE AREA
DROP 15 R15 NO LONGER BASE REG
USING AREA,13 R13 NEW BASE REG
ST 2,4(13) LINK SAVE AREAS
ST 13,8(2)

* RETURN SEQUENCE *

L 13,4(13) R13 <= ADDR OLD SAVE AREA
LM 14,2,12(13) GPR'S RESTORED
SR 15,15 R15 <=0, RETURN CODE
MVI 12(13),X'FF' INDICATE CONTROL RETURNED
BR 14 RETURN

* NOUT(EFLUX,PSIO,FRAC,TIMMAX,NTAB,N,S) *

* OFIRST *

USING NOUT,15

STM 14,12,12(13) SAVE CALLING ROUTINE'S GPR'S
LR 2,13 R2 <= ADDR OLD SAVE AREA
LA 13,AREA R13 <= ADDR NEW SAVE AREA
DROP 15 R15 NO LONGER BASE REG
USING AREA,13 R13 NEW BASE REG
ST 2,4(13) LINK SAVE AREAS
ST 13,8(2)
LM 2,8,0(1) R2-R8 <= ADDR'S ARGS
OPEN (LU9DCB,(OUTPUT))
MVC BUF(80),BLNCRD FILL BUFFER WITH BLANKS
MVC BUF(8),O(2) FIRST 8 BYTES OF BUFFER <= EFLUX
MVC BUF+8(8),O(3) 2ND 8 BYTES OF BUFFER <= PSIO
MVC BUF+16(8),O(4) 3RD 8 BYTES OF BUFFER <= FRAC
MVC BUF+24(8),O(5) 4TH I BYTES OF BUFFER <= TIMMAX
MVC BUF+32(4),O(6) 2ND HALF OF 5TH & 8BYTES OF

* BUFFER <= NTAB

PUT LU9DCB,BUF WRITE OUT 1ST RECORD
LR 3,7 R3 <= BASE ADDR N
LR 4,8 R4 <= BASE ADDR S
L 7,0(6) R7 <= NTAB
SR 6,6 R6 <= 0
D 6,TEN R6 <= REMAINDER OF NTAB/10

* LA 5,6 R7 <= INTEGER PART OF NTAB/10
M 6,TEN R7 <= (NTAB/10)*10 INTEGER MODE

SLL 5,3 F5 <= REMAINDER NTAB/10 * 8
SLL 7,3 R7 <= (NTAB/10)*10 * 8
# BYTES IN # CARDS - 1
LA 6,80 R6 <= 80 (INCREMENT FOR LOOP)
LR 2,3 R2 <= BASE ADDR N
LA 4.0(4) HIGH BYTE OF BASE ADDR S ZEROED
SR 7,6 R7 <= (NTAB/10 - 1) * 80
STM 5,7,SREG SAVE INCREMENTS AND OFFSET FOR COMPARAND
STM 6.7, SREG+12 TWO COPIES OF INCREMENTS AND OFFSET
AR 7,3 R7 <= BASE ADDR N + ((NTAB/10)-1)*80
LOOP1
MVC BUF(80),0(3) PUT NEXT 80 BYTES IN BUFFER AND WRITE THEM OUT

PUT LU9DCB,BUF
BXLE 3,6,LOOP1 KEEP GOING TILL WE'RE DONE
LTR 5,5 CHECK IF NO MORE THINGS TO WRITE
BZ ARND1 IF NOT DON'T WRITE OUT ANOTHER CARD BECAUSE FORTRAN WOULDN'T

MVC BUF(80),BLNCRD FILL BUFFER WITH BLANKS
EX 5,LMOV3 MOVE IN LAST FEW (<10) VALUES AND WRITE OUT LAST CARD FOR N

ARND1
SR 7,2 R7 <= (NTAB/10 - 1) * 80
AR 7,4 R7 <= BASE S + ((NTAB/10)-1)*80
LOOP2
MVC BUF(80),0(4) PUT NEXT 80 BYTES IN BUFFER AND WRITE THEM OUT

PUT LU9DCB,BUF
BXLE 4,6,LOOP2 KEEP GOING TILL WE'RE DONE
LTR 5,5 CHECK IF NO MORE THINGS TO WRITE
BZ ARND2 IF NOT DON'T WRITE OUT ANOTHER CARD BECAUSE FORTRAN WOULDN'T

MVC BUF(80),BLNCRD FILL BUFFER WITH BLANKS
EX 5,LMOV4 MOVE IN LAST FEW (<10) VALUES AND WRITE OUT LAST CARD FOR S

PUT LU9DCB,BUF
RETURN SEQUENCE

ARND2
L 13,4(13) R13 <= ADDR OLD SAVE AREA
LM 14,8,12(13) GPR'S RESTORED
SR 15,15 R15 <= 0, RETURN CODE
SWI 12(13),X'FF' INDICATE CONTROL RETURNED
BR 14 RETURN

TOUT(TIM,T,J,DELT,IITER,NTAB) WE IGNORE LAST PARAMETER

USING TOUT,15
STM 14,12,12(13) SAVE CALLING ROUTINE'S GPR'S
LR 2,13 R2 <= ADDR OLD SAVE AREA
LA 13,AREA R13 <= ADDR NEW SAVE AREA
DROP 15 R15 NO LONGER BASE REG
USING AREA,13 R13 NEW BASE REG
ST 2,4(13) LINK SAVE AREAS
ST 13,8(2)
LM 2,6,0(1) R2-R6 <= ADDR'S ARG'S WE USE
MVC BUF(80),BLNCRD FILL BUFFER WITH BLANKS
MVC BUF(80),0(2) FIRST 8 BYTES OF BUFFER <= TIM
MVC BUF+8(8),0(5) 2ND 8 BYTES <= DELT
MVC BUF+20(4),0(6) 2ND HALF 3 8 BYTES <= IITER
PUT LU9DCB,BUF

LM 5,9,SREG R5-R9 <= NUMBER OF VALUES ON LAST CARD FOR T AND J AND INCREMENTS AND COMPARANDS
AR 7,3 R7 <= COMPARAND FOR LOOP3
AR 9,4 R9 <= COMPARAND FOR LOOP4
LOOP3
MVC BUF(80),0(3) PUT NEXT 80 BYTES IN BUFFER AND WRITE THEM OUT

PUT LU9DCB,BUF
BXLE 3,6,LOOP3 KEEP GOING TILL WE'RE DONE
LTR 5,5 CHECK IF NO MORE THINGS TO WRITE
BZ LOOP4 IF NOT DON'T WRITE OUT ANOTHER CARD
BECAUSE FORTRAN WOULDN'T
MVC BUF(80),BLNCRD  FILL BUFFER WITH BLANKS
* EX 5,LMOV3 MOVE IN LAST FEW (<10) VALUES
* PUT LU9DCB,BUF AND WRITE OUT LAST CARD FOR T
LOOP4 MVC BUF(80),O(4) PUT NEXT 80 BYTES IN BUFFER
* PUT LU9DCB,BUF AND WRITE THEM OUT
PUT LU9DCB,BUF BXLE 4,8,LOOP4 KEEP GOING TILL WE'RE DONE
MVC BUF(80),BLNCRD FILL BUFFER WITH BLANKS
LTR 5,5 CHECK IF NO MORE THINGS TO WRITE
* BZ ARND3 IF NOT DON'T WRITE OUT ANOTHER CARD
* EX 5,LMOV4 MOVE IN LAST FEW (<10) VALUES
* PUT LU9DCB,BUF AND WRITE OUT LAST CARD FOR J
* RETURN SEQUENCE
* ARND3 L 13,4(13) R13 <= ADDR OLD SAVE AREA
LM 14,9,12(13) GPR'S RESTORED
SR 15,15 R15 <= 0, RETURN CODE
MVI 12(13),X'FF' INDICATE CONTROL RETURNED
BR 14 RETURN
LU9DCB DCB DEVD=DA,MACRF=PM,DSORG=PS,RECFM=FB,LRECL=80,DDNAME=FT09F001
END
Appendix B

STEADY STATE MODEL OF THE SOLAR ATMOSPHERE

We have constructed a steady state numerical model of the solar atmosphere. The model was developed to investigate the effects of upward velocities and diverging magnetic field patterns on the temperature and density structure of the solar atmosphere; however, for this work the model is used only to provide reasonable temperature and density profiles for the estimation of the effect of reverse current heating on the atmosphere. The computer program calculates the run of temperature and density in an individual flux tube.

The equations governing the behavior of an inviscid compressible fluid in the presence of gravity are

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{u}) = 0 , \tag{B.1}
\]

\[
\frac{\partial (\rho \vec{u})}{\partial t} + \nabla \cdot (\rho \vec{u} \vec{u}) = - \nabla P + \rho g , \tag{B.2}
\]

\[
\frac{\partial (\rho \epsilon)}{\partial t} + \nabla \cdot (\rho \epsilon \vec{u}) = - \nabla \cdot \vec{q} - \rho \nabla \cdot \vec{u} - \mathcal{L} + S , \tag{B.3}
\]

where \( \epsilon \) is the total internal energy per unit mass, \( \vec{q} \) is the heat flux, \( g \) is the gravitational acceleration, \( \mathcal{L} \) is the energy lost via radiation, and \( S \) is the sum of all other non-thermal energy sources or sinks. For flow along a magnetic flux tube, considering variation only along the field lines and assuming the radius of curvature of the field lines to be large compared to the dimensions of the flux tube, we see that the equations become one-dimensional. If we add the definition of the heat flux and an equation of state to Equations (B.1)-(B.3), we may
write a complete set of equations for the steady state ($\partial/\partial t = 0$) case

$$u_s \frac{dp}{ds} + \rho \frac{du_s}{ds} + \frac{u_s \rho}{A} \frac{dA}{ds} = 0 \quad ,$$  

(B.4)

$$u_s \rho \frac{du_s}{ds} = -\frac{dp}{ds} - \rho g_s \quad ,$$  

(B.5)

$$\rho u_s \frac{de}{ds} = -\frac{dq_s}{ds} - \rho \frac{du_s}{ds} - \left(\frac{pu_s + q_s}{A}\right) \frac{dA}{ds} = \rho + S \quad ,$$  

(B.6)

$$u_s = K \frac{dT}{ds} \quad ,$$  

(B.7)

$$p = \frac{1}{\mu} \rho KT \quad ,$$  

(B.8)

where $A$ is the area of the flux tube, $u$ is the mean particle mass, $K$ is the heat conductivity, $k$ is Boltzmann's constant, $T$ is the fluid temperature, $s$ measures distance along the flux tube and the subscript $s$ denotes the component of a vector along the flux tube. We have neglected transport of energy and momentum across field lines in writing equations (B.4)-(B.7). We wish to apply Equations (B.4)-(B.8) to the solar atmosphere. For this case we shall assume the plasma to be pure hydrogen except for computing the radiative losses. To account for radiative losses, we have assumed that it is reasonable to treat the solar atmosphere as optically thin (we discuss this assumption later).

We have adopted the radiative loss function calculated by Raymond et al. (1976) as modified by Raymond (1979) to include radiative losses from Ar and neutral hydrogen excitation, but excluding radiative losses due to forbidden lines for temperatures below $T = 10^5$ K. We have used the values of $u$ and $K$ derived by Moore and Fung (1979) for a pure hydrogen plasma. With these and the equation of state, we may eliminate the
pressure. We choose as our dependent variables $q_s$, $T$, $u_s$ and $n$, the number density of hydrogen nuclei and rewrite (B.6)-(B.7) in a form more convenient for numerical solution:

$$\frac{dT}{ds} = -\frac{q_s}{K}$$

(B.9)

$$\frac{dn}{ds} = \left( \frac{m_H}{(1+\chi)KT} - u_s^2 \right) \left\{ \frac{u_s^2 n}{A} \frac{dA}{ds} - \frac{dT}{ds} \left[ \frac{(1+\chi)nk}{m_H} + \frac{nkt}{m_H} \frac{dX}{dT} \right] - n g_s \right\},$$

(B.10)

$$\frac{dq_s}{ds} = -\ell + S - \frac{3}{2} \frac{k}{s} nk \frac{dT}{ds} \left[ (1+\chi) + (T-T_i) \frac{dX}{dT} \right] + (1+\chi)kT q_s \frac{dn}{ds} - q_s \frac{1}{A} \frac{dA}{ds},$$

(B.11)

$$n u_s A = \text{constant}$$

(B.12)

where $m_H$ is the mass of a hydrogen atom, $\chi$ is the fraction of hydrogen nuclei that are ionized, and $T_i$ is the hydrogen ionization energy expressed as a temperature, $\sim 1.07 \times 10^4$ K. Equation (B.12) is the integral of Equation (B.4), we need only solve three first order ordinary differential equations to calculate the run of temperature and density in a flux tube.

We have written an assembly-language subroutine, to execute on IBM 360 or 370 series computers, to evaluate the quantities $\frac{dT}{ds}$, $\frac{dn}{ds}$ and $\frac{dq_s}{ds}$, given $A$, $\frac{dA}{ds}$, $g_s$, $S$, $n$, $T$ and $q_s$. This subroutine may be used with a standard library ordinary differential solver, or as we have done with one coded specially for this problem. The quantities $K$, $\chi$, $\frac{dX}{dt}$, $\ell$, $A$, $g_s$, $\frac{dA}{ds}$, and $S$ are tabulated as a function of $T$. 


and \( s \), and the values for a particular \( T \) or \( s \) are computed by a cubic interpolation scheme similar to the one described in Appendix A.

The subroutine we have written to solve the coupled set of ordinary differential equations (B.9)-(B.12) uses an Adams-Bashforth-Moulton fourth order linear multistep integration scheme (see Isaacson and Keller 1966) with a fourth order Runge-Kutta scheme (with a smaller step size) to "start up" the linear multistep method and provide intermediate values when halving the step size. The routine returns the values of \( T \), \( u \), \( q \) and \( n \) at intervals from the starting point specified by the calling program and reduces the step size or increases it according to the requested accuracy. The pretabulated quantities are read in by the main program which also reads in starting values, calls the differential equation solver and writes out the results of the integration.

The downward heat flux in the corona above an active region is \( \sim 5 \times 10^6 \) (Noyes 1971). Since the thermal conductivity of the solar plasma is a strong function of temperature \( (\propto T^{5/2}) \), this heat flux must be largely radiated away above the low chromosphere. We have the choice of starting with our initial values where the heat flux is large (in the corona) and calculating the solutions to a region where the heat flux is small (the chromosphere), or proceeding in the reverse direction from the region where the heat flux is small. It is well known that the latter choice is preferable numerically (Acton 1970, Isaacson and Keller 1966). This is basically because the numerical calculation proceeding from the region of large heat flux to the region of small heat flux is not a "well posed" problem (Isaacson and Keller 1966) since a small relative change in the initial value of the heat flux can cause a large

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relative change in the final value. We therefore shall choose our starting point near the temperature minimum.

There are three major difficulties with starting the calculation below $3 \times 10^4$ K. The first is that the atmosphere becomes optically thick and therefore the radiative losses cannot be calculated simply. Second, the radiative loss function calculated by Raymond is not tabulated below $10^4$ K. Third, the approximation that the atmosphere is purely hydrogen breaks down as the fraction of ionized hydrogen becomes very small because the electron density (which appears in the expression for the radiative losses) is grossly underestimated by (A.4), since the major contribution to the electron density is from trace elements with low ionization potentials (e.g. Na). However, for the purposes of this work, we only need a model that represents the overall structure of the atmosphere reasonably well. This is particularly true since (cf. Chapter 3) the calculation of the heating of the cool dense portions of the atmosphere by the reverse current is not accurate after the first few tenths of a second due to the neglect of Coulomb collisions. We do not attempt a solution of the radiative transfer problem. We use a power law extrapolation of Raymond's (1976) radiative loss coefficient. We also use Equation (A.4) to find the electron density. The fact that the atmosphere is not optically thin is compensated for by the underestimate of the electron density. We have extrapolated Raymond's (1976) radiative loss function with a power law above and below the tabulated range $(T=10^4 - T=10^8$ K). For the high temperatures above $T=10^8$ K, this should be a reasonable approximation since the losses for these temperatures are almost completely due to thermal bremsstrahlung and therefore should vary
as \( \sim T^{1/2} \); however, these temperatures are not of importance in the present calculation. The power law extrapolation below \( 10^4 \) K is purely ad hoc, but the range over which extrapolated values are used is small (\( \sim a \) factor of 2) and the calculation of radiative losses for these temperatures is at best approximate in any event. The resulting temperature and density profiles resemble the solar atmosphere in overall structure. Since the atmosphere varies from active region to active region, this should provide an adequate representation for the purposes of the calculations of Chapter 3.

To produce the model used (see Chapter 3), we integrate up from near the temperature minimum \((T=4200 \text{K}, n=1.1025 \times 10^{15})\). The heat flux and velocity are taken to be zero at this point. No non-thermal energy input was included in the calculation. The resulting temperature, density and heat flux at the top of the model (corresponding to the injection point for the beam in Chapter 3) were \( T=3 \times 10^5 \), \( n=1 \times 10^9 \text{ cm}^{-3} \) and \( F=0.30 \times 10^5 \text{ erg cm}^{-2} \text{ s}^{-1} \), in reasonable agreement with the values given by Noyes (1971).

Listings of two main programs and several subroutines are provided for the sake of completeness. The first main program and associated subroutines produce the tables that are required for the cubic interpolation. The second main program reads in starting values for the solution of the coupled set of differential equations and writes out the results both as tables suitable for people to look at and (if desired) for machines to read. The subroutine ABMINT is the differential equation solver described above. The present version is in FORTRAN and is certainly adequate for the purpose of this work. An adaptation of the
present main program to solve a boundary value problem rather than an initial value problem would (absent the wealth of Croesus) require this routine to be hand coded. The assembly language subroutine DIVF calculates the quantities needed by ABMINT to integrate the differential equations.
This program calculates semi-logarithmic interpolation tables for a steady state model of the plasma in a magnetic flux tube under the influence of gravity. Four functions of temperature and four functions of S (distance along the flux tube from the Sun's surface) are tabulated. The functions of temperature (T) are:

- The inverse of the thermal conductivity (ONEK)
- The radiative loss coefficient (LUM)
- The ionization fraction (CHI)
- The derivative of the ionization fraction (DCHI)

The functions of S are:

- The force of gravity along the tube (G)
- The area of the tube (A)
- The logarithmic derivative of the area (DADS)
- The non-thermal energy input (SOR)

The tabulation range in temperature is 4.096E3 - 6.71E7 (K).
The tabulation range in S is 1.677E7 - 2.75E11 (CM).

The tables are written out to FORTRAN logical unit 9 and the program reads in 641 values of temperature and radiative loss coefficient (Raymond, private communication) used to tabulate the radiative loss coefficient.

The program also reads in several parameters that characterize the flux tube and the non-thermal energy input:

- FRAC: The area of the flux tube is ((D+S)/D)**2 WHERE D IS FRAC TIMES A SOLAR RADIUS.
- AMP: The integral of the non-thermal energy deposited in a flux tube of constant area is AMP (ERG PER CM**2 PER SEC).
- SCALE: The form of the non-thermal energy input is (COS((S*PI)/(2*SCALE)))*2 FOR S LESS THAN SCALE AND ZERO FOR S GREATER THAN SCALE. SCALE IS INPUT IN SOLAR RADII (INPUT OF 1. MEANS SCALE IS ABOUT 7.E10 CM).

Initialize tables:

DO 1 I=1,820
   G(I)=0.00
   DADS(I)=0.00
   SOR(I)=0.00
   A(I)=1.00
   ONEK(I)=0.00
   CHI(I)=0.00
   DCHI(I)=0.00
   LUM(I)=0.00
1

Read in parameters
READ(5,5001)FRAC
READ(5,5001)AMP
READ(5,5001)SCALE

CALCULATE FUNCTIONS OF TEMPERATURE:

DO 20 I=1,3
   T=TST-DT
   K=(I-1)*256+1
   DO 10 J=1,243
      CHI(K)=FCHI(T)
      DCHI(K)=FDCHI(T)
      ONEK(K)=FKAP(T)
      LUM(K)=FPLUM(T)
   T=T+DT
   K=K+1
20
   TST=16.00*TST
   DT=16.00*DT

DO 30 K=769,820
   CHI(K)=FCHI(T)
   DCHI(K)=FDCHI(T)
   ONEK(K)=FKAP(T)
   LUM(K)=FPLUM(T)
   T=T+DT
30

CALCULATE FUNCTIONS OF S:

DO 50 I=1,3
   S=SST-DS
   K=(I-1)*256+1
   DO 40 J=1,243
      G(K)=FG(S)
      A(K)=FA(S)
      DADS(K)=FDADS(S)
      SOR(K)=FSOR(S)
      S=S+DS
   K=K+1
50
   SST=16.00*SST
   DS=16.00*DS

DO 60 K=769,820
   G(K)=FG(S)
   A(K)=FA(S)
   DADS(K)=FDADS(S)
   SOR(K)=FSOR(S)
   S=S+DS
60

WRITE OUT TABLES:

WRITE(9,9001)G
WRITE(9,9001)DADS
WRITE(9,9001)SOR
WRITE(9,9001)A
WRITE(9,9001)ONEK
WRITE(9,9001)LUM
WRITE(9,9001)CHI
WRITE(9,9001)DCHI
STOP
END

REAL FUNCTION FCHI*T8(T)

THIS FUNCTION CALCULATES THE IONIZATION FRACTION AS A FUNCTION
OF THE TEMPERATURE (T). THE IONIZATION FRACTION (FCHI) IS
DEFINED AS NE/(NH+NP) WHERE NE IS THE NUMBER DENSITY OF
ELECTRONS, AND NH AND NP ARE THE NUMBER DENSITIES OF HYDROGEN
ATOMS AND PROTONS RESPECTIVELY. SEE MOORE AND FUNG, SOLAR
PHYSICS 23 (1972), 78-102 FOR FORMULAE.
DATA ONE3/ZC0555555555555555/
COMMON BETAM,EBETA,B13,TEMP1,TEMP2,TCHI,D
BETA=1.58D5/T
EBETA=DEXPCBETA(BETA)
B13=BETA**ONE3
TEMP1=0.4288D0+0.5DO*DLOGC1EBETA+.4698DO*B13
TEMP2=2.220D0**BETA*TEMP1*EBETA
TCHI=1.0D0/(1.0D0+TEMP2)
FCHI=TCHI
RETURN
END
REAL FUNCTION FDCHI*8(T)
C THIS FUNCTION CALCULATES THE DERMYVATIVE OF THE IONIZATION FRACTION
(D DHI / DT) AS A FUNCTION OF TEMPERATURE (T). SEE FUNCTION FCHI.
IMPLICIT REAL*8 (A-H,O-Z)
COMMON BETAM,EBETA,B13,TEMP1,TEMP2,TCHI,D
FDCHI=1.406D-11*TCHI*TCHI*BETA*BETA*EBETA*C(1.DO+BETA)*TEMPI
+F(.5DO-.1566DO*B13))
RETURN
END
REAL FUNCTION FKAP*8(T)
C THIS FUNCTION CALCULATES THE INVERSE OF THE TOTAL THERMAL
CONDUCTIVITY AS A FUNCTION OF TEMPERATURE. THE DEPENDENCE OF
THE CONDUCTIVITY ON THE "COULOMB LOGARITHM" IS APPROXIMATED IN A
MANNER SIMILAR TO MOORE AND FUNG, SOLAR PHYSICS 23 (1972), 78-102.
IMPLICIT REAL*8 (A-H,O-Z)
COMMON BETAM,EBETA,B13,TEMP1,TEMP2,TCHI,D
REAL*8 PO/0.00/,CL/0.00/,CK1/0.00/,RKAY/1.38062D-16/, 
.MPR0T/1.6735D-24/,ESU/4.80325D-10/,PI/3.141592653589793/ 
IF(PO.NE.0.00)GOTO 10
PO=1.0541.0100*(2.000*KAY)
CL=(3.000*KAY**2)/(DLOG(SQRT(RKAY)))
CK1=(9.000*KAY**DSQRT(RKAY))/(4.000*DSQRT(MPR0T))
10 CLAM=(T**T*CL)*DSQRT(((1.00+TCHI)/(2.00*TCHI)))
T12=DSQRT(T)
IF(T.GT.4.2D5)CLAM=CLAM*6.480741D2/T12 
TEMPK=(CK1*T)/((9.120-14.7950-11/(TEMPC*T12)) 
FKAP=1.00/(TEMPK+1.890-5**T**T12)/DLOG(CLAM))
RETURN
END
REAL FUNCTION FLUM*8(T)
C THIS FUNCTION CALCULATES THE RADIATIVE LOSS COEFFICIENT
SUCH THAT THE RADIATIVE LOSSES FROM AN OPTICALLY THIN
PLASMA OF SOLAR ABUNDANCES ARE FLUM*NE**2 WHERE NE IS
THE ELECTRON NUMBER DENSITY. THE CALCULATION OF THE
RADIATIVE LOSS COEFFICIENT IS RAYMOND'S (PRIVATE COMM.)
IMPROVEMENT OF THE CALCULATIONS OF RAYMOND, COX AND SMITH
AP. J. 204 (1976), 290-292.
IMPLICIT REAL*8 (A-H,O-Z)
REAL*8 T,L,TDC5411,LD(641),LOGT,ERR,FINT(10),XDIF(10),WRK(10)
REAL*4 RTD(641),RLD(641)
LOGICAL SORT,FALSE.,EXTRAP./.FALSE./,FIRST./.TRUE./
equivalence (TD,RTD),LD(641),ERR)(10),XDIFF(10),WRK(10)
8001 FORMAT(20A4)
110 LOGT=DLOG10(T)
IF(LOGT.LT.TD1))GOTO 200
IF(LOGT.GT.TD(NRADPT)))GOTO 300
ERR=.00 
CALL AITKEN(L,LOGT,10,ERR,TD,LD,NRADPT,SORT,EXTRAP,FINT,XDIFF,WRK,
,610,.4D00,.4D00)
10 FLUM=10.00**L
RETURN
CALCULATING LEAST SQUARE FITS FOR POWER LAW EXTENSION OF CALCULATED RADIATIVE LOSS COEFFICIENT BEYOND TABULATED RANGE. ONLY DO ON FIRST CALL.

```fortran
READ(8,8001)RTD
READ(8,8001)RLD
DO 15 I=1,NRADPT
TD(I)=DBLE(RTD(I))
LD(I)=DBLE(RLD(I))
A1=0.00
B1=0.00
TEMP1=0.00
TEMP2=0.00
DO 20 I=2,10
A1=A1+LD(I)
B1=B1+TD(I)
TEMP1=TEMP1+TD(I)*TD(I)
TEMP2=TEMP2+TD(I)~LD(I)
20 B1=(TEMP2-TD(I)*A1-LD(I)*(B1-9.00*TD(I)))/
     .(TEMP1-2.00*TD(I)*B1+9.00*TD(I)*TD(I))
A1=LD(I)-B1*TD(I)
A2=0.00
B2=0.00
TEMP1=0.00
TEMP2=0.00
DO 30 I=636,640
A2=A2+LD(I)
B2=B2+TD(I)
TEMP1=TEMP1+TD(I)*TD(I)
TEMP2=TEMP2+TD(I)~LD(I)
30 B2=(TEMP2-TD(I)*A2-LD(641)*(B2-5.00*TD(641)))/
     .(TEMP1-2.00*TD(641)*B2+5.00*TD(641))
FIRST=.FALSE.
GOTO 110
200 FLUM=1.00***(A1+B1*LOGT)
RETURN
300 FLUM=1.00***(A2+B2*LOGT)
RETURN
400 WRITE(6,6001)
6001 FORMAT(1H,'OOPS - WE SHOULD NOT BE HERE')
STOP
END
```

THIS FUNCTION CALCULATES THE FORCE OF GRAVITY ALONG THE FLUX TUBE AS A FUNCTION OF S, THE DISTANCE ABOVE THE SOLAR SURFACE.

```fortran
IMPLICIT REAL*8 (A-H,O-Z)
REAL*8 RSUN/6.9599010/,G/6.67D-8/,MSUN/1.989D33/
LOGICAL NOT1ST/.FALSE./
IF(NOT1ST)GOTO 10
GM=MSUN*G
NOT1ST=.TRUE.
10 R=(RSUN+S)
FG=GM/(R**2)
RETURN
END
```


```fortran
IMPLICIT REAL*8 (A-H,O-Z)
COMMON BETA,EBETA,B13,TEMP1,TEMPC,TCHI,D
```
REAL*8 RSUN/6.9599010/,AO/1.0D0/
LOGICAL NOT1ST/.FALSE./
COMMON /PARAM/ FRAC,AMP,SCALE
IF(NOT1ST)GOTO 10
D=FRAC*RSUN
R=S+D
AR=AO/AR
FA=AR*(R**2)
NOT1ST=.TRUE.
RETURN
10 R=D+S
FA=AR*(R**2)
RETURN
END REAL

FUNCTION FDADS*8CS)
C THIS FUNCTION CALCULATES THE LOGARITHMIC DERIVATIVE
C SURFACE OF THE SUN.
C
IMPLICIT REAL*8 (A-H,O-Z)
COMMON BETA,EBETA,B13,TEMP1,TEMPC,TCHI,D
FDADS=2.DO/(D+S)
RETURN
END

REAL FUNCTION FSOR*8(S)
C THIS FUNCTION CALCULATES THE (AD HOC) NON-THERMAL
C ENERGY INPUT INTO THE SOLAR PLASMA AS A FUNCTION OF S,
C THE DISTANCE ABOVE THE SUN'S SURFACE.
C
IMPLICIT REAL*8 (A-H,O-Z)
REAL*8 RSUN/6.9599010/,RSUN/6.9599010/,PIBY2/2.0D0/
REAL*8 RSUN/6.9599010/, PI BY2/2.0D0/
LOGICAL NOT1ST/.FALSE./
COMMON /PARAM/ FRAC,AMP,SCALE
IF(NOT1ST)GOTO 10
SCALE=SCALE*RSUN
ARG=1.DO/SCALE
AMP=AMP*ARG
AMP=AMP+AMP
ARG=ARG*PIBY2
S0=S
NOT1ST=.TRUE.
10 SR=S-S0
IF(SR.GT.SCALE)GOTO 20
C=0.DO
FSOR=AMP*C*C
RETURN
20 FSOR=0.DO
RETURN
END

SUBROUTINE AITKEN(F,X,M,ERR,XTAB,FTAB,N,SORT,EXTRAP,FINT,
.XDIF,WRK,X*,X*)
SUBROUTINE AITKEN INTERPOLATES TO FIND THE VALUE OF THE FUNCTION
(F) AT THE POINT X. IF THE ROUTINE DOES NOT ACHIEVE THE DESIRED
RELATIVE ERROR (ERR) USING M POINTS OR IF ROUND OFF ERROR APPEARS
TO BE PRESENT, THE ROUTINE RETURNS THE CURRENT ERROR ESTIMATE IN
ERR, RETURNING TO THE MAIN PROGRAM AT THE FIRST STATEMENT NUMBER
IN THE ARGUMENT LIST. THE ROUTINE REQUIRES THE TABULATED VALUES
IN FTAB TO BE IN ORDER OF INCREASING VALUE OF X (IN XTAB). IF
SORT IS TRUE ON ENTRY, BOTH TABLES ARE SORTED (SEE NOTE). IF THE
VALUE OF X IS OUTSIDE THE RANGE OF THE TABLES SUPPLIED, THE
ROUTINE RETURNS TO THE SECOND STATEMENT NUMBER IN THE ARGUMENT
LIST - UNLESS EXTRAP IS TRUE. IF THE ROUTINE DISCOVERS TWO
IDENTICAL VALUES OF X IN XTAB, THE ROUTINE RETURNS TO THE THIRD
STATEMENT NUMBER IN THE ARGUMENT LIST.
IF M IS GREATER THAN N OR LESS THAN 2, IT IS SET TO 10.
IF ERR IS LESS THAN 16**-5, IT IS SET TO 16**-5.

ARGUMENTS (OTHER THAN STATEMENT NUMBERS):

F  INTERPOLATED VALUE OF FUNCTION AT X (REAL - OUTPUT)
X  VALUE OF INDEPENDENT VARIABLE (REAL - INPUT)
M  LARGEST NUMBER OF DATA POINTS TO BE USED (INTEGER - INPUT)
ERR REQUESTED RELATIVE ERROR (REAL - INPUT)
XTAB TABLE OF X VALUES AT WHICH F(X) IS TABULATED (REAL ARRAY - INPUT)
FTAB TABLE OF F(X) AT THE CORRESPONDING POINTS IN XTAB (REAL ARRAY - INPUT)
N  THE LENGTH OF TABLES XTAB AND FTAB (INTEGER - INPUT)
SORT DETERMINES WHETHER OR NOT THE INTERNAL SORTING ROUTINE IS TO BE USED (LOGICAL - INPUT/OUTPUT)
EXTRAP DETERMINES WHETHER OR NOT EXTRAPOLATION OUTSIDE THE RANGE OF THE TABLES IS ALLOWED (LOGICAL - INPUT)
FINT ARRAY OF SUCCESSIVE INTERPOLANTS - WORKING ARRAY (REAL ARRAY DIMENSION > OR = M)
XDIF ARRAY OF DIFFERENCES BETWEEN THE POINTS AT WHICH F(X) IS TABULATED AND X - WORKING ARRAY (REAL ARRAY DIMENSION > OR = M)
WRK WORKING ARRAY FOR CURRENT LEVEL OF INTERPOLATION (REAL ARRAY DIMENSION > OR = M)

INTERNAL VARIABLES:

TEMP TEMPORARY STORAGE LOCATION FOR INTERMEDIATE RESULTS
FDIFF1 PREVIOUS ABSOLUTE RELATIVE DIFFERENCE BETWEEN INTERPOLANTS - COMPARED WITH FDIFF2 TO CHECK FOR CONVERGENCE (ROUND-OFF ERROR INDICATOR)
FDIFF2 PRESENT ABSOLUTE RELATIVE DIFFERENCE BETWEEN INTERPOLANTS - USED TO CHECK FOR CONVERGENCE AT CURRENT LEVEL (ALSO SEE FDIFF1 ABOVE)
DIFFMAX LARGEST REPRESENTABLE FLOATING POINT NUMBER (IBM 360)
IUP USED AS POINTER IN SORT AND INTERPOLATION
IMID USED AS POINTER IN SORT
IDN USED AS POINTER IN SORT AND INTERPOLATION
XUPDIF DIFFERENCE BETWEEN X AND CLOSEST UNUSED LARGER VALUE IN XTAB
XDNDIF DIFFERENCE BETWEEN X AND CLOSEST UNUSED SMALLER VALUE IN XTAB
LEVEL CURRENT LEVEL OF AITKEN TRIANGULAR SCHEME
ISTEP COUNTER FOR INTERMEDIATE INTERPOLANT LOOP
DONE LOGICAL FLAG TO INDICATE CURRENT LEVEL OF SHELL SORT IS COMPLETE
IDISP CURRENT EXCHANGE INTERVAL IN SHELL SORT
ILAST N MINUS IDISP - UPPER LIMIT FOR SORT DO LOOP
I COUNTER IN SORT DO LOOP

REMARKS:


NOTE:

SORT METHOD USED IS SHELL SORT - THIS METHOD MAY BE VERY INEFFICIENT WHEN XTAB IS PARTIALLY SORTED.

DECLARE VARIABLES

REAL F,X,ERR,XTAB(1),FTAB(1),FINT(1),XDIF(1),WRK(1),EPS,XUPDIF,
.XNDIF,FDIff,FDIF2,DFIMAX,TEMP
INTEGER M,N,ISTEP,ILAST,LEVEL,IDISP,IUP,IDN,IMID,I
LOGICAL SORT,EXTRAP,DONE

INITIALIZE VARIABLES

DATA EPS/Z3C100000/,DIFMAX/Z7FFFFFFF/

CHECK TO SEE IF M > N OR IF M < 2, IF SO SET M TO 10 (THIS CARD MUST BE REMOVED FOR WATFIV EXECUTION AND THE WORKING ARRAYS DIMENSIONED TO M)
IF(M.LT.2.OR.M.GT.N)M=10

CHECK TO SEE IF ERR < 16**-5 IF SO SET IT TO 16**-5
I**(ERR.LT.EPS)ERR=EPS

CHECK TO SEE IF TABLES ARE TO BE SORTED - IF NOT GO AROUND SORT SECTION.
IF(.NOT.SORT)GOTO 200

**** SORTING SECTION BEGIN

IDISP=N
101 IDISP=(IDISP+1)/2
ILAST=N-IDISP
102 DONE=.TRUE.
  DO 103 I=1,ILAST
  IF(XTAB(I).LT.XTAB(I+IDISP))GOTO 103
  IF(XTAB(I).EQ.XTAB(I+IDISP))RETURN 3
  TEMP=XTAB(I)
  XTAB(I)=XTAB(I+IDISP)
  XTAB(I+IDISP)=TEMP
  TEMP=FTAB(I)
  FTAB(I)=FTAB(I+IDISP)
  FTAB(I+IDISP)=TEMP
  DONE=.FALSE.
103 CONTINUE
IF(.NOT.DONE) GOTO 102
IF(IDISP.GT.1) GOTO 101

*** SORTING SECTION END ***

200 CONTINUE

CHECK TO SEE IF X IS WITHIN RANGE OF TABLE — IF NOT AND IF EXTRAP IS FALSE RETURN TO SECOND STATEMENT IN ARGUMENT LIST

IF(X.GE.XTAB(1)) GOTO 201
X IS BELOW LOWEST X VALUE IN XTAB — EXIT UNLESS EXTRAP IS TRUE
IF(.NOT.EXTRAP) RETURN 2
EXTRAP IS TRUE — SET UP POINTERS AND GO TO AITKEN INTERPOLATION SECTION

IUP=2
IDN=1
GOTO 400

201 CONTINUE

CHECK TO SEE IF X IS LARGER THAN LARGEST X VALUE IN XTAB — IF NOT BRANCH TO SEARCH SECTION

IF(X.LE.XTAB(N)) GOTO 300
X IS ABOVE HIGHEST X VALUE IN XTAB — EXIT UNLESS EXTRAP IS TRUE
IF(.NOT.EXTRAP) RETURN 2
EXTRAP IS TRUE — SET UP POINTERS AND GO TO AITKEN INTERPOLATION SECTION

IUP=N
IDN=N-1
GOTO 400

SEARCH SECTION — FIND XTAB VALUES THAT BRACKET X — USE BISECTION

300 CONTINUE

SET UP POINTERS FOR BISECTION

IUP=N
IMID=N/2
IDN=1

CHECK TO SEE WHICH SIDE OF EXTAB(IMID) X IS ON AND UPDATE IUP, IMID AND IDN — WHEN NEW IMID EQUALS IDN WE ARE DONE

301 IF(X.GT.XTAB(IMID)) GOTO 302
X LE XTAB(IMID) SO IUP<=IMID & IMID<=(IUP+IDN)/2
IUP=IMID
IMID=(IUP+IDN)/2

IF IMID > IDN WE AREN'T DONE YET — GO BACK AND CHECK AGAIN OTHERWISE GO TO AITKEN INTERPOLATION SECTION

302 CONTINUE

IF(IMID.GT.IDN) GOTO 301
GOTO 400

GOTO 400

X > XTAB(IMID) SO IDN<=IMID & IMID<=(IUP+IDN)/2

104
IDN=IMID
IMID=(IUP+IDN)/2

IF IMID > IDN WE AREN'T DONE YET - GO BACK AND CHECK AGAIN
OTHERWISE ENTER AITKEN INTERPOLATION SECTION

IF(IMID.GT.IDN)GOTO 301

END OF SEARCH SECTION

AITKEN INTERPOLATION SECTION

CONTINUE

IUP AND IDN POINT TO FIRST TWO FUNCTION VALUES USED IN
INTERPOLATION - INITIALIZE VARIABLES

FDIFF2=DIFMAX
XNDXDIFF=XTAB(IUP)-X
XUPDIFF=XTAB(IDN)-X

START AITKEN INTERPOLATION

DO 401 LEVEL=1,M

DECIDE WHICH OF THE TWO TABLE VALUES POINTED TO BY IUP AND IDN
IS TO BE USED NEXT - THE ONE WITH XTAB CLOSER TO X

IF(ABS(XUPDIFF).GT.ABS(XNDXDIFF))GOTO 402

WE WILL USE IUP - PUT INFORMATION IN WORKING ARRAYS

WRK(1)=FTAB(IUP)
XDIF(LEVEL)=XUPDIFF

CHECK TO SEE IF WE JUST USED THE LARGEST VALUE OF X IN XTAB
IF SO GO TO 403 AND DO FIX UP - IF NOT UPDATE IUP AND XUPDIFF

IF(IUP.GE.N)GOTO 403
IUP=IUP+1
XUPDIFF=XTAB(IUP)-X

BRANCH AROUND CODE TO INTERPOLATION LOOP FOR THIS LEVEL
GOTO 404

FIX UP FOR USE OF LARGEST X IS TO SET XUPDIFF TO LARGEST
REPRESENTABLE FLOATING POINT NUMBER

403 XUPDIFF=DIFMAX

BRANCH AROUND CODE TO INTERPOLATION LOOP FOR THIS LEVEL
GOTO 404

WE WILL USE IDN - PUT INFORMATION IN WORKING ARRAYS

402 WRK(1)=FTAB(IDN)
XDIF(LEVEL)=XNDXDIFF

CHECK TO SEE IF WE USED THE SMALLEST VALUE OF X IN X IN XTAB
IF SO GO TO 405 AND DO FIX UP - IF NOT UPDATE IDN AND XNDXDIFF

IF(IDN.EQ.1)GOTO 405
IDN=IDN-1
XNDXDIFF=XTAB(IDN)-X

BRANCH AROUND CODE TO INTERPOLATION LOOP FOR THIS LEVEL
GOTO 404

FIX UP FOR USE OF SMALLEST X IS TO SET XDNDIF TO LARGEST
REPRESENTABLE FLOATING POINT NUMBER

405 XDNDIF=DIFMAX

SKIP INTERPOLATION CALCULATION IF LEVEL IS 1

404 IF(LEVEL.LE.1)GOTO 406

AITKEN INTERPOLATION LOOP

DO 407 ISTEP=2,LEVEL
    TEMP=XDIF(LEVEL)-XDIF(ISTEP-1)
    CHECK TO SEE IF WE ARE GOING TO DIVIDE BY 0 IF SO RETURN
    TO THIRD STATEMENT NUMBER IN ARGUMENT LIST
    IF(TEMP.EQ.0.)RETURN 3

CALCULATE INTERMEDIATE INTERPOLANTS

407 WRK(ISTEP)=($INT(ISTEP-1)*XDIF(LEVEL) -
                      WRK(ISTEP-1)*XDIF(ISTEP-1))/TEMP

ENTER INTERPOLANT IN FINT

406 $INT(LEVEL)=WRK(LEVEL)

SKIP CHECK FOR CONVERGENCE FOR LEVEL LESS THAN 4

401 IF(LEVEL.LT.4)GOTO 401

CHECK FOR CONVERGENCE AT THIS LEVEL - IF SO BRANCH OUT

    FDIFF2=3.*$ABS($INT(LEVEL)-$INT(LEVEL-1))/
            ($INT(LEVEL)+$INT(LEVEL-1))
    IF(FDIFF2.LT.ERR)GOTO 408

SKIP ROUND OFF ERROR CHECK FOR LEVEL LESS THAN 6

408 IF(LEVEL.LT.6)GOTO 408

IF INTERPOLANTS ARE NOT CONVERGING - EXIT

    IF(FDIFF2.GT.FDIFF1)GOTO 501

UPDATE FDFF1 AND CONTINUE

401 FDFF1=FDFF2

IF INTERPOLATED TO LEVEL=M WITHOUT CONVERGENCE - EXIT

GOTO 501

SET F EQUAL TO $INT(LEVEL) AND RETURN

408 F=$INT(LEVEL)
RETURN

TERMINATIONS DUE TO LACK OF CONVERGENCE OR ROUND OFF ERROR

501 LEVEL=LEVEL-1
ERR=FDFF1
F=$INT(LEVEL)
RETURN 1
END
STeady State ATMosphere model main routine

IMPLICIT REAL*8 (A-H,O-Z)

This program calculates the run of temperature, density, heat flux and velocity in an individual flux tube. The program reads in parameters that control the number of sets of tables read in (NTAB), and the independent variable that controls the frequency of tabulation (ITEST). For each set of tables the program reads in the number of different initial conditions for which the integration is to be performed (NRUN) and a variable that controls whether or not the results of the integration are only printed out or both printed out and written out in a format suitable for rereading by another program (NOUT). If NOUT is less than 1, then the results are only printed. If NOUT is greater than or equal to 1, then the results of the integration are both printed out and written out to logical unit 10.

The program reads in 4 tabulated functions of distance and 4 tabulated functions of temperature:

Functions of S:

  G  THE FORCE OF GRAVITY ALONG THE TUBE
  DA THE LOGARITHMIC DERIVATIVE OF THE AREA OF THE TUBE
       WITH RESPECT TO DISTANCE ALONG THE TUBE (1/A DA/DS)
  SO  A PHENOMENOLOGICAL NON-THERMAL HEAT SOURCE
  A  THE AREA OF THE FLUX TUBE

Functions of T:

  OK INVERSE OF THE THERMAL CONDUCTIVITY
  LU THE RADIATIVE LOSS FUNCTION
  CH THE FRACTION OF HYDROGEN NUCLEI THAT ARE IONIZED
  DC DERIVATIVE OF THE FRACITIONAL IONIZATION (CH)

For each run with a set of tables, the program reads in SO, the starting distance, DSO the initial step size, PRCT, the multiplicative factor by which the independent variable selected by ITEST is allowed to change between tabulation points, the initial temperature T0, initial density NO, initial heat flux Q0, initial velocity U0, TSTOP, the temperature at which the integration will stop, SSTOP, the distance at which the integration will stop, EPS, the maximum relative error in an independent variable allowed per DSO, and MSTOP, the Mach number at which the integration will stop. If the initial heat flux read in is greater than 10 to the 50th (a very unphysical value) the initial heat flux is determined by the condition that the net energy flux is zero at the starting point.

The current version interpolates the results of the integration to print out values of distance, temperature, density, heat flux, velocity, pressure and a quantity which can be inferred from EUV observations (P**2 KAPPA/Q where KAPPA is the thermal conductivity) at values of the temperature initialized in the array TMPOUT. In addition the initial and final points of the integration are printed out. If the results are to be written to logical unit 10 all the tabulated results are printed as calculated by ABMINT.
DECLARE AND INITIALIZE ARRAYS AND VARIABLES

REAL*8 EPS, TSTOP, SSTOP, AO, U0, NO, TO, Q0, SO, DS0, DS, PRCT,
  YPAS(4), KAY*Z339F2DB6000000007,
  MSTOP
REAL*8 GC(819), OA(819), SO(819), OK(819), LU(819), CH(819),
  GC(819), YTAB(5, 2048), TAB(8), GRAV, DADS, SOR, AR, OKAP, LAM, CHI, DCHI
INTEGER*4 NRUM, NTAB, IRUN, ITAB, I, J
EQUIVALENCE (TAB(1), GRAV), (TAB(2), DADS), (TAB(3), SOR), (TAB(4), AR),
  (TAB(5), OKAP), (TAB(6), LAM), (TAB(7), CHI), (TAB(8), DCHI)
EXTERNAL OLVINT
REAL*8 SPRIN, SBOT/2.4741438000000000/, TMPDOUT(-40)/
.103, 1.503, 2.03, 3.03, 4.03, 5.03, 6.03, 7.03, 8.03, 9.03,
  .104, 1.504, 2.04, 3.04, 4.04, 5.04, 6.04, 7.04, 8.04, 9.04,
  .105, 1.505, 2.05, 3.05, 4.05, 5.05, 6.05, 7.05, 8.05, 9.05,
  .106, 1.506, 2.06, 3.06, 4.06, 5.06, 6.06, 7.06, 8.06, 9.06/
5001 FORMAT(215)
5002 FORMAT(3216)
5003 FORMAT(-1013, 6)
6001 FORMAT(1H, 'S(CM)', TI9, 'TEMP', T34, 'N', T49, 'Q(CGS)',
  .T64, 'UC(GS)', T79, 'P(CGS)', T94, 'LUM', /
6002 FORMAT(7('PD15.6'))
6004 FORMAT('HMT(1H, T4, 'S(CM)', TI9, 'TEMP', T34, 'N', T49, 'Q(CGS)',
  .T64, 'UC(GS)', T79, 'P(CGS)', T94, 'LUM', /
9004 FORMAT(10A8)

CALL DIVINT - PASS BASE ADDRESSES OF INTERPOLATION TABLES
TO DIVF

CALL DIVINT(6, OA, SO, A, OK, LU, CH, DC)

READ IN NUMBER OF SETS OF TABLES AND INDEX OF INDEPENDENT
VARIABLE THAT CONTROLS TABULATION FREQUENCY

READ(5, 5001) NTAB, ITEST
PRCT=1.0500

READ IN INTERPOLATION TABLES

DO 999 ITAB=1, NTAB
  READ(5, 5001) IRUN, NOUT
  READ(N, 90004) G
  READ(N, 90004) IA
  READ(N, 90004) S0
  READ(N, 90004) A
  READ(N, 90004) OK
  READ(N, 90004) LU
  READ(N, 90004) CH
  READ(N, 90004) DC

READ IN INITIAL CONDITIONS FOR RUNS

DO 99 IRUN=1, NRUN
  READ(8, 5002) SO, DS0, PRCT
  READ(8, 5003) TO, NO, Q0, U0
  READ(8, 5003) TSTOP, SSTOP, EPS, MSTOP

Initialize AO and QO IF NECESSARY

YPASS(1)=TO
YPASS(2)=NO
CALL DIVF(SO, YPAS, TAB)
AO=AR*NO*U0
I=1
IF(Q0.LE.1.50) 50 TO 5
Q0=-.500*U0*NO*(U0*U0*1.67352D-24+5.0*KAY*TO*(1.+CHI))
5 YPAS(3)=Q0
YPASS(4)=U0

108
S=SO
DS=DSo
NMAX=2048

CALL ABMINT TO INTEGRATE EQUATIONS
CALL ABMINT(S,YPASS,DIVF,DS,EPS,TSTOP,SSTOP,MSTOP,YTAB(1,I1),
.AO,ITEST,PACT,NMAX)
I1=I1+NMAX-1
20 CONTINUE

PRINT OUT RESULTS

WE LOOK FOR VALUES OF TEMPERATURE THAT BRACKET VALUES
OF TEMPERATURE IN TMPOUT AND INTERPOLATE. WE ALSO
DO OUR OWN PAGINATION.

WRITE(6,6004)
ITEMP=0
ILINE=1
I=1
SPRIN=YTAB(5,1)
CALL DIVF(SPRIN,YTAB(1,1),TAB)
P=YTAB(2,1)*(.DO+CHI)*YTAB(1,1)
RADOUT=1.D050
IF(YTAB(3,1).EQ.0.DO)GOTO 30
RADOUT=-(P*P)/(OKAP*YTAB(3,1))
30 P=KAY*P
SPRIN=SPRIN-SBOTT
WRITE(6,6002)SPRIN,YTAB(1,1),YTAB(2,1),YTAB(3,1),
.YTAB(4,1),P,RADOUT

FIND NEXT OUTPUT TEMPERATURE

105 ITEMP=ITEMP+1
IF(ITEMP.GT.40)GOTO 130
IF(YTAB(1,1).GT.TMPOUT(ITEMP))GOTO 105

FIND PRINT TEMPERATURE AND PRINT

110 IF(YTAB(1,1+1).GE.TMPOUT(ITEMP))GOTO 115
I=I+1
IF(I.GE.I1)GOTO 130
GOTO 110
115 FRAC=(TMPOUT(ITEMP)-YTAB(1,1))/(YTAB(1,1+1)-YTAB(1,1))
YPASS(1)=TMPOUT(ITEMP)
YPASS(2)=YTAB(2,1+1)+FRAC*YPASS(2,2+1)-YTAB(2,1)
YPASS(3)=YTAB(3,1+1)+FRAC*YPASS(3,3+1)-YTAB(3,1)
YPASS(4)=YTAB(4,1+1)+FRAC*YPASS(4,4+1)-YTAB(4,1)
SPRIN=YTAB(5,1)+FRAC*YPASS(5,5+1)-YTAB(5,1)
CALL DIVF(YTAB(5,1),YTAB(1,1),TAB)
P1=YTAB(2,1)*(.DO+CHI)*YTAB(1,1)
CALL DIVF(YTAB(5,1+1),YTAB(1,1+1),TAB)
P2=YTAB(2,1+1)*(.DO+CHI)*YTAB(1,1+1)
P=P1+FRAC*(P2-P1)
RADOUT=1.D050
IF(YPASS(2).EQ.0.DO)GOTO 125
RADOUT=-(P*P)/(OKAP*YPASS(3))
125 P=KAY*P
SPRIN=SPRIN-SBOTT
WRITE(6,6002)SPRIN,TMPOUT(ITEMP),YPASS(2),YPASS(3),
YPASS(4),P,RADOUT
I=I+1
IF(I.GE.I1)GOTO 130
ILINE=ILINE+1
IF(I1.LT.58)GOTO 105
ILINE=0
WRITE(6,6004)
GOTO 105
130 IF(I.GT.1) GOTO 99
SPRIN=YTAB(5,I)
CALL DIVF(SPRIN,YTAB(1,I),TAB)
P=YTAB(2,I)*(1.0D+CHI)*YTAB(1,I)
RADOUT=1.0D0
IF(YTAB(3,I).EQ.0.DO) GOTO 135
RADOUT=-(P/P)/(OKAP*YTAB(3,I))
135 P=KAY*P
SPRIN=SPRIN-SBOTT
WRITE(6,6002)SPRIN,YTAB(1,I),YTAB(2,I),YTAB(3,I),
.YTAB(4,I),P,RADOUT
99 CONTINUE
WRITE OUT TABULATED RESULTS OF INTEGRATION IF REQUESTED
999 CONTINUE
IF(NOUT.GE.1) CALL WRTR(YTAB,NMAX)
STOP
END
SUBROUTINE WRTR(Y,N)
IMPLICIT REAL*8 (A-H,O-Z)
REAL*8 Y(5,N)
1001 FORMAT(IOA8)
1002 FORMAT(A)
WRITE(10,1002)N
WRITE(10,1001)Y
RETURN
END
SUBROUTINE ABMINT(S,YINT,F,DS,EPS,TSTOP,SSTOP,MSTOP,YTAB,AO,
.ITS,PRCT,NMAX)

ABMINT SOLVES A SET OF THREE COUPLED ORDINARY DIFFERENTIAL
EQUATIONS PLUS A CONSERVATION RELATION THAT DESCRIBE THE
(STEADY STATE) BEHAVIOR OF A COMPRESSIBLE FLUID IN A FLUX TUBE.
THE ROUTINE TAKE THE FOLLOWING INPUT PARAMETERS:

S THE INITIAL DISTANCE (ARBITRARY)

YINT THE INITIAL VALUES OF Y(1)-Y(4), THE INDEPENDENT
VARIABLES (TEMPERATURE, DENSITY, HEAT FLUX AND
VELOCITY)

F THE NAME OF THE SUBROUTINE THAT CALCULATES THE
DERIVATIVES OF THE INDEPENDENT VARIABLE AND THE VELOCITY
(MUST BE DECLARED IN AN EXTERNAL STATEMENT IN THE
CALLING ROUTINE)

DS THE INITIAL STEP SIZE

EPS THE DESIRED ACCURACY (RELATIVE) FOR A DISTANCE DS

TSTOP THE MAXIMUM (OR MINIMUM) TEMPERATURE TO WHICH THE
ROUTINE WILL INTEGRATE

SSTOP THE MAXIMUM (MINIMUM) DISTANCE TO WHICH THE ROUTINE
WILL INTEGRATE

MSTOP THE MAXIMUM MACH NUMBER TO WHICH THE ROUTINE WILL INTEGRATE

YTAB AN ARRAY IN WHICH THE RESULTS OF THE INTEGRATION ARE
RETURNED TO THE CALLING PROGRAM - SHOULD BE DIMENSIONED AT
LEAST 5*NMAX. VARIABLES STORED IN THE FOLLOWING ORDER:
TEMPERATURE, DENSITY, HEAT FLUX, VELOCITY AND DISTANCE

AO THE AREA AT THE STARTING POINT TIMES THE VELOCITY AT THE STARTING POINT
(A CONSERVED QUANTITY)

ITS INDEX OF THE VARIABLE THAT CONTROLS THE FREQUENCY
AT WHICH RESULTS ARE PUT IN YTAB

PRCT THE MULTIPLICATIVE FACTOR BY WHICH THE ITS ELEMENT OF Y
IS ALLOWED TO CHANGE BETWEEN THE TABULATION OF THE RESULTS

NMAX THE MAXIMUM NUMBER OF TABULATION POINTS

THE ROUTINE USES SEVERAL LOCAL WORKING ARRAYS

RUNGE-KUTTA:

Y(4), Y1(4), F0(4), F1(4), F2(4) USED TO STORE INTERMEDIATE
VALUES OF THE INDEPENDENT VARIABLE AND THEIR DERIVATIVES

ADAMS-BASHFORTH-MOULTON PREDICTOR CORRECTOR:

YW(32), FW(32) USED TO STORE LAST 8 STEPS OF INTEGRATION.
THE PRESENT INTEGRATION USES 4 PREVIOUS VALUES TO ESTIMATE
THE NEXT VALUE SO DOUBLING THE STEP SIZE CAN BE DONE IF
AT LEAST 4 INTEGRATION STEPS HAVE OCCURRED SINCE THE LAST
DOUBLING OF THE STEP SIZE

YP(4), YC(4) USED TO STORE THE PREDICTED AND CORRECTED
VALUES OF THE INDEPENDENT VARIABLES

ADAMS-BASHFORTH-MOULTON PREDICTOR-CORRECTOR INTEGRATION SCHEME. START UP IS ACCOMPLISHED BY BACKWARD INTEGRATION WITH A RUNGE-KUTTA SCHEME AND MISSING VALUES NEEDED WHEN HALVING THE STEP SIZE ARE PROVIDED USING THE SAME RUNGE-KUTTA SCHEME.

DECLARE VARIABLES

IMPLICIT REAL*8 (A-H,O-Z)
REAL*8 S,EPS,ERR,DS,DT,TSTOP,TDF1,TDF2,H2,H3,H6,H8,H,T,ERR1,
.YP(4),YC(4),YINT(4),YTAB(5,1),F0(4),F1(4),F2(4),Y1(4),Y(4),FP(4),
.ON24/Z3FAAAAAAMNBBBBBBB/EST,FU(32)/32*0.00,DM(32)/32*0.00/,
.MSTOP,MACH,SSTOP,SDIF1,SDIF2,H924,CCC1/Z4161C71C71C71C71C72/,,
.CCC2/Z4168E38E38E38E39/,CCC3/Z4141C71C71C71C71C72/,FRAC
INTEGER*4 I,J,K,IND,IN1,IN2,IN3,IN4,IT,DOUBLE
LOGICAL*4 DONE

START UP USING INTEGRATION BY 4TH ORDER R-K & 1/32 DS

VCH1=PRCT*YINT(I)
VCH2=PRCT*VCH1
FRAC=1.00
MACH=1.6499599*D*MSTOP*MSTOP
MMAX=MMAX
15 DT=.0312500*DS*FRAC
H2=DT*.500
H3=DT*.333
H6=H3*.500
H8=H2*.2500
T=S
IND=4
DO 11 I=1,4
11 YTAB(I,1)=YINT(I)
YTAB(5,1)=S
CALL F(T,YINT,F0,AO,999)
DO 1 I=1,4
1 Y(I)=YINT(I)
YW(I)=YINT(I)
1 FW(I)=F0(I)
DO 2 I=1,3
2 DO 3 J=1,2
3 Y1(K)=F0(K)*H3+Y(K)
DO 5 K=1,3
5 Y1(K)=(F0(K)+F1(K))*H6+Y(K)
DO 8 K=1,3
8 Y1(K)=(F1(K)*3.00+F0(K))*H8+Y(K)
CALL F(T+H3,Y1,F1,AO,999)
DO 7 K=1,3
7 Y1(K)=(F2(K)*4.00-F1(K)*3.00+F0(K))*H2+Y(K)
T=T+DT
CALL F(T,Y1,F1,AO,999)
DO 9 K=1,3
9 Y(K)=(F2(K)*4.00-F1(K)*3.00+F0(K))*H6+Y(K)
CALL F(T,Y,F0,AO,999)
DO 12 J=1,4
12 FW(J)=F0(J)
DO 11 J=IND+4
11 IF(YH(12+ITS).LT.VCH1)GOTO 16
FRAC=FRAC*0.500
GOTO 15
16 SDIF1=S-SSTOP
S=T

ORIGINAL PAGE IS OF POOR QUALITY
I=2

50 DOUBLE=DOUBLE-1
55 CONTINUE
 DO 20 J=1,3
  YP(J)=YW(IN4+J)+H924*(CCC1*FW(IN4+J)-CCC2*FW(IN3+J)
      +CCC3*FW(IN2+J)-FW(IN1+J))
 CALL F(S+DT,YP,FP,AO,&999)
 DO 30 J=30
  YC(J)=YW(IN4+J)+H*(9.*FP(J)+19.*FW(IN4+J)-5.*FW(IN3+J)
      +FU(IN2+J))
 TDIF2=YC(1)-TSTOP
 IF(DSIGN(TDIF2,TDIF1).NE.TDIF2)DONE=.TRUE.
 SDIF2=S-SSTOP
 IF(DSIGN(SDIFF2,SDIFF1).NE.SDIFF2)DONE=.TRUE.
 CALL F(S+DT,YC,FP,AO,&999)
 IF(YC(4)*YC(4).GT.MACH*YC(1))DONE=.TRUE.
 ERST=DMAX1(DABS(YP(2)-YC(2))/(DABS(YP(2))+DABS(YC(2))),ERST)
 IF(DABS(YC(3))+DABS(YP(3)).LT.1.E-30)GOTO 220
 ERST=DMAX1(DABS(YP(3)-YC(3))/(DABS(YP(3))+DABS(YC(3))),ERST)
 IF(DABS(YC(4))+DABS(YP(4)).LT.1.E-30)GOTO 215
 ERST=DMAX1(YYC(4)-YC(4))/(DABS(YP(4))+DABS(YC(4))),ERST)
 IF(YC(ITS).LT.VCH2)GOTO 90
 IF(ERST.GT.ERR)GOTO 100
 IF(ERST.LT.ERR1)GOTO 200
 5=S+S+DT
 IN1=IN2
 IN2=IN3
 IN3=IN4
 IN4=MOO(IN4+4,32)
 DO 40 J=1,4
  YW(IN4+J)=YC(J)
  FW(IN4+J)=FP(J)
 IF(YC(ITS).LT.VCH1)GOTO 50
 DO 60 J=1,4
  YTAB(J,1)=YC(J)
 YTAB(5,1)=S
 I=I+1
 IF(I.GT.NMAX)GOTO 10
 VCH1=VCH2
 VCH2=FRCT*VCH1
 IF(DONE)GOTO 10
 GOTO 50
 90 IACC=1
 GOTO 103
 100 IACC=0
 103 CONTINUE

SECTION THAT HALVES INTEGRATION STEP
 IF(DT.LT.1.D-1)GOTO 205
 IF(DT.GT.DS)GOTO 109
 ERR=ERR*.5D0
 ERR1=ERR1*.5D0
REARRange \( \text{WORKING ARRAYS} \)

\[
\begin{align*}
\text{IT} &= \text{MOD}((\text{IN}4+11, 32)) \\
\text{J} &= \text{IN}4 \\
\text{DO} 101 \quad \text{K} &= 1, 4 \\
\text{FW}(\text{IT}+\text{K}) &= \text{FW}(\text{IN}4+\text{K}) \\
\text{YW}(\text{IT}+\text{K}) &= \text{YW}(\text{IN}4+\text{K}) \\
\text{IN}4 &= \text{IT} \\
\text{DO} 102 \quad \text{K} &= 1, 4 \\
\text{FW}(\text{IT}+\text{K}) &= \text{FW}(\text{IN}3+\text{K}) \\
\text{YW}(\text{IT}+\text{K}) &= \text{YW}(\text{IN}3+\text{K}) \\
\text{FW}(\text{IN}3+\text{K}) &= \text{FW}(\text{IN}2+\text{K}) \\
\text{YW}(\text{IN}3+\text{K}) &= \text{YW}(\text{IN}2+\text{K}) \\
\text{IN}2 &= \text{IT} \\
\text{IT} &= \text{IN}3 \\
\text{IN}1 &= \text{J} \\
\text{IN}3 &= \text{MOD}((\text{IN}2+4, 32)) \\
\end{align*}
\]

GENERate missing information with 4th order \( \text{R-K} \)

\[
\begin{align*}
\text{H2} &= \text{DT}* 2.500 \\
\text{H3} &= \text{DT}/6.00 \\
\text{H6} &= \text{DT}* 5.00 \\
\text{H8} &= \text{H2}* 2.500 \\
\text{T} &= \text{S} - (\text{DT}+\text{DT}) \\
\text{DO} 110 \quad \text{J} &= 1, 4 \\
\text{Y}(\text{J}) &= \text{YW}(\text{IN}2+\text{J}) \\
\text{DO} 110 \quad \text{J} &= 1, 2 \\
\text{DO} 110 \quad \text{J} &= 1, 3 \\
\text{DO} 110 \quad \text{J} &= 1, 4 \\
\end{align*}
\]

\[
\begin{align*}
\text{Y1}(\text{K}) &= \text{FO}(\text{K})* \text{H3} + \text{Y}(\text{K}) \\
\text{CALL F}(T+\text{H3}, \text{Y1}, \text{F1}, \text{A0}, \&999) \\
\text{DO} 140 \quad \text{K} &= 1, 3 \\
\text{Y1}(\text{K}) &= (\text{FO}(\text{K})+\text{F1}(\text{K}))* \text{H6} + \text{Y}(\text{K}) \\
\text{CALL F}(T+\text{H3}, \text{Y1}, \text{F1}, \text{A0}, \&999) \\
\text{DO} 150 \quad \text{K} &= 1, 3 \\
\text{Y1}(\text{K}) &= (\text{F1}(\text{K})*3.00+\text{FO}(\text{K}))* \text{H8} + \text{Y}(\text{K}) \\
\text{CALL F}(T+\text{H2}, \text{Y1}, \text{F2}, \text{A0}, \&999) \\
\text{DO} 120 \quad \text{J} &= 1, 3 \\
\text{Y1}(\text{K}) &= \text{F2}(\text{K})*4.00 + \text{F1}(\text{K})* \text{H2} + \text{Y}(\text{K}) \\
\text{CALL F}(T, \text{Y1}, \text{F1}, \text{A0}, \&999) \\
\text{DO} 110 \quad \text{J} &= 1, 3 \\
\text{Y1}(\text{K}) &= \text{FO}(\text{K})* \text{H6} + \text{Y}(\text{K}) \\
\text{CALL F}(T, \text{Y}, \text{F0}, \text{A0}, \&999) \\
\text{CONTINUE} \\
\text{DO} 170 \quad \text{J} &= 1, 4 \\
\text{FW}(\text{IN}3+\text{J}) &= \text{FO}(\text{J}) \\
\text{YW}(\text{IN}3+\text{J}) &= \text{Y}(\text{J}) \\
\text{DO} 115 \quad \text{J} &= 1, 4 \\
\text{Y}(\text{J}) &= \text{YW}(\text{IT}+\text{J}) \\
\text{DO} 115 \quad \text{J} &= 1, 4 \\
\text{FW}(\text{IN}3+\text{J}) &= \text{FO}(\text{J}) \\
\text{T} &= \text{S} - 4.00*\text{DT} \\
\text{DO} 125 \quad \text{J} &= 1, 2 \\
\text{DO} 135 \quad \text{K} &= 1, 3 \\
\text{Y1}(\text{K}) &= \text{FO}(\text{K})* \text{H3} + \text{Y}(\text{K}) \\
\text{CALL F}(T+\text{H3}, \text{Y1}, \text{F1}, \text{A0}, \&999) \\
\text{DO} 145 \quad \text{K} &= 1, 3 \\
\text{Y1}(\text{K}) &= (\text{FO}(\text{K})+\text{F1}(\text{K}))* \text{H6} + \text{Y}(\text{K}) \\
\text{CALL F}(T+\text{H3}, \text{Y1}, \text{F1}, \text{A0}, \&999) \\
\text{DO} 155 \quad \text{K} &= 1, 3 \\
\text{Y1}(\text{K}) &= (\text{F1}(\text{K})*3.00+\text{FO}(\text{K}))* \text{H8} + \text{Y}(\text{K}) \\
\text{CALL F}(T+\text{H2}, \text{Y1}, \text{F2}, \text{A0}, \&999) \\
\text{DO} 165 \quad \text{K} &= 1, 3 \\
\]
165 \[ Y_1(K) = (F2(K) \times 4.00 - F1(K) \times 3.00 + F0(K)) \times H2 + Y(K) \]

\[ T = T + H2 + H2 \]

CALL F(T, Y1, F1, A0, &999)

DO 185 K = 1, 3

185 \[ Y(K) = (F2(K) \times 4.00 + F1(K) + F0(K)) \times H6 + Y(K) \]

CALL F(T, Y, F0, A0, &999)

125 CONTINUE

DO 175 J = 1, 4

175 \[ F(W(IN1+J)) = F0(J) \]

\[ YW(IN1+J) = Y(J) \]

GOTO 55

RETURN TO ABM P-C INTEGRATION WITH NEW STEP SIZE

185 CONTINUE

SECTION THAT DOUBLES INTEGRATION STEP SIZE

IF (IACC.EQ.1) GOTO 205

IF (DOUBLE.GE.0) GOTO 205

DOUBLE = 4

S = S - DT

DT = DT + DT

H = DT \times N24

H924 = DT \times 3.7500

IF (DT.GT.DS) GOTO 209

ERR = ERR + ERR

ERR1 = ERR1 + ERR1

209 K = MOD(IN4+4,32)

IT = MOD(IN4+12,32)

DO 210 J = 1, 4

210 \[ F(W(IN4+J)) = F(W(IN3+J)) \]

\[ YW(IN4+J) = Y(W(IN3+J)) \]

\[ F(W(IN3+J)) = F(W(IN1+J)) \]

\[ YW(IN3+J) = Y(W(IN1+J)) \]

\[ F(W(IN1+J)) = F(W(K+J)) \]

\[ YW(IN1+J) = Y(W(K+J)) \]

\[ F(W(IN2+J)) = F(W(IT+J)) \]

\[ YW(IN2+J) = Y(W(IT+J)) \]

GOTO 205

10 NMAX = I - 1

RETURN

999 CONTINUE

6001 FORMAT (' FATAL ERROR T OR S OUT OF TABULATED RANGE')

WRITE (6,6001)

S = S

NMAX = I

RETURN

END
DIVF

CSECT

DIVF(S,Y,DY,AO,*) OR DIVF(S,Y,TAB)

REAL*8 S,Y(4),DY(4),AO
REAL*8 S,Y(4),TAB(8)

THE FIRST FORM OF THE CALL CALCULATES THE DERIVATIVES

DT/DS = DY(1)/DS, DN/DS = DY(2)/DS & DQ/DS = DY(3)/DS

AND STORES THEN IN ARRAY DY. THE VALUE OF V=Y(4) IS COMPUTED
FROM THE CONSERVATION LAW NVA = CONSTANT AND STORED IN Y(4).
THE ROUTINE INTERPOLATES THE VALUES OF PARAMETERS NEEDED FOR
THE CALCULATIONS FROM TABULATIONS OF 4 FUNCTIONS OF S ONLY AND
4 FUNCTIONS OF T ONLY. IF S OR T IS OUT OF THE TABULATED
RANGE, THE OFFENDING QUANTITY IS NEGATED AND THE ROUTINE DOES
THE EQUIVALENT OF A FORTRAN RETURN.

THE SECOND FORM OF THE CALL STATEMENT (DISTINGUISHED FROM THE
FIRST BY THE NUMBER OF ARGUMENTS) CALCULATES THE INTERPOLATED
VALUES OF G(S),DA/DS(S),SOURCE(S), AREA(S) AND 1/KAPPA(T),
LAMBDA(T), CHI(T) AND DCHI(DT) AND STORES THEM IN TAB.

THERE IS A SECOND ENTRY POINT (DIVINT) WHICH PICKS UP
AND STORES LOCALLY THE ADDRESSES OF THE TABULATIONS OF
THE FUNCTIONS NEEDED FOR THE CALCULATIONS.

NOTE THAT THIS MEANS THAT MEANINGLESS RESULTS
WILL BE PRODUCED IF DIVINT IS NOT CALLED BEFORE
THE FIRST TIME DIVF IS CALLED. IT IS EVEN POSSIBLE
THAT SOME SORT OF ABEND WILL RESULT.

THE FOLLOWING TWO FORTRAN SUBROUTINES ARE ROUGHLY EQUIVALENT
TO THE TWO CALLS TO DIVF (DIVINT IS NOT REPRODUCED)

SUBROUTINE DIVF(S,Y,DY,AO,*)
IMPLICIT REAL*8 (A-H,O-Z)
DIMENSION Y(4),DY(4)
REAL*8 KOMH/8.299776D7/,TION/1.0464606D5/,KAY/1.380620-16/
DY(1)=-Y(3)*OKAP(T)
Y(4)=AO/Y(2)*A(S))
DY(2)=Y(2)*Y(4)*Y(4)*DADS(S)-DY(1)*KOMH
.((1.0+CHI(T))*Y(2)*Y(2)*DY(2))-DY(3)*KAY
.((1.0+CHI(T))*Y(1)*TIAN)*DCHI(T))+(1.0+CHI(T))*KAY
.*Y(1)*Y(4)*DY(2))+SOR(S)-Y(3)*DADS
RETURN
END

SUBROUTINE DIVF(S,Y,TAB)
IMPLICIT REAL*8 (A-H,O-Z)
DIMENSION Y(4),TAB(8)
TAB(1)=G(S)
TAB(2)=DADS(S)
TAB(3)=SOR(S)
TAB(4)=A(S)
TAB(5)=OKAP(T)
TAB(6)=CHI(T)
TAB(7)=DCHI(T)
TAB(8)=DCHI(T)
RETURN
END

NOTE: IN THE COMMENTS 'R' REFERS TO GENERAL PURPOSE REGISTERS
AND 'F' REFERS TO FLOATING POINT REGISTERS.

*USING *,15 TELL ASSEMBLER NEXT INST ADDR IN R15
B DFIRST BRANCH AROUND NAME AND OTHER ENTRY POINT
DC X'04'
DC CL5'DIVF'

DI VINT(G(DADS,SOR,AREA,OKAP,LUM,CHI,DCHI)
REAL*8 G(563),DADS(563),SOR(563),AREA(563),
REAL*8 OKAP(820),LUM(820),CHI(820),DCHI(820)

ENTRY DIVINT USING *,15 TELL ASSEMBLER NEXT INSTR ADDR IN R15
DIVINT
B TFIRST BRANCH AROUND NAME
DC X'06'
DC CL7'DIVINT'

TFIRST STM 14,12,12(13) SAVE CALLING ROUTINES GPR'S
LM 2,9,0(1) GET BASE ADDR'S OF INTERPOLATION TABLES
STM 2,9,ADDR SAV TABLE BASE ADDR'S
LM 2,9,26(13) RESTORE CALLING ROUTINE'S GPR'S
MVI 12(13),X'FF' INDICATE CONTROL RETURNED
BR 14 RETURN FROM INITIALIZATION

MAIN ROUTINE RESUMES

USING DIVF,15 SAVE CALLING ROUTINES GPR'S
DFIRST STM 14,12,12(13) SAVE CALLING ROUTINES GPR'S
L 2,0(1) R2 <= ADDR S
LM 3,6,ADDR R3-R6 <= ADDR'S OF TABLES FOR S
MVC FLOAT+1(6),2(2) FLOAT <= FRACTIONAL DISTANCE FROM
NEXT SMALLER VALUE OF S TABULATED.

LH 12,0(2) R12 <= HIGH ORDER BYTES OF S
S 12,SDISP REDUCE R12 BY SDISP - # WORDS FROM
BASE OF TABLES
LD 4,FLOAT F4 <= FRACTION 0 LE FRAC LE 1
BM BADS IF RESULT IS NEGATIVE OUT OF RANGE
GOTO BADS
C 12,SBND COMPARE R12 TO SBND - IF GREATER
(b) OUT OF RANGE GOTO BADS
SD 4,=0'.5' F4 <= X = FRAC -.5 -.5 LE X LE .5
SLA 12,3 R12 <= R12*X8 NOW BYTE DISPLACEMENT
FROM BASE OF INTERPOLATION TABLE.

NOW COMPUTE WEIGHTS FOR CUBIC INTERPOLATION OF
FUNCTIONS OF S

LDR 2,4 F2 <= X
MDR 4,4 F4 <= X**2 = X2
HDR 4,4 F4 <= X2/2
SD 4,0'1.125' F4 <= X2/2 - 9/8
LDR 6,4 F6 <= X2/4 - 9/16
HDr 4,4 F4 <= X2/4 - 9/16
MDR 6,2 F6 <= X3/2 - 9x/8
LCRD 0,4 F0 <= -X2/4 + 9/16
ADR 0,6 F0 <= X3/2 - X2/4 - 9X/8 + 9/16
STD 0,WM1 WM1 <= WEIGHT FOR TABLE ENTRY CORRESPOND-
ing to CLOSEST SMALLER VALUE OF S.

LCSR 0,4 F0 <= -X2/4 + 9/16
SDR 0,6 F0 <= -X3/2 - X2/4 + 9x/8 + 9/16
STD 0,WP1 WP1 <= WEIGHT FOR TABLE ENTRY CORRESPOND-
ing to CLOSEST LARGER VALUE OF S.

ADR 6,2 F6 <= X3/2 - X/8
AD 4,=0'.5' F4 <= X2/4 - 1/16
MD 6,=X'4055555555555555' F6 <= X3/6 - X/24
LDR 0,4 F0 <= X2/4 - 1/16
SDR 0,6 F0 <= -X3/6 + X2/4 + X/24 - 1/16
ADR 6,4 F6 <= X3/6 + X2/4 - X/24 - 1/16
STD 0,WM3 WM3 <= WEIGHT FOR TABLE ENTRY CORRESPOND-
ing to 2ND CLOSEST SMALLER VALUE OF S.
STD 6, WP3

WP3 <= WEIGHT FOR TABLE ENTRY CORRESPONDING TO 2ND CLOSEST LARGER VALUE OF S.

* NOW CALCULATE INTERPOLATED VALUES OF GRAVITY AND DA/DS
* (HAVE WEIGHTS FOR TABLES ENTRIES 1 & 4 IN FPR'S 0 & 6)

LDR 4, 0
LDR 2, 6
MD 0, 0(3, 12)
MD 2, 24(3, 12)
MD 4, 0(4, 12)
MD 6, 24(4, 12)
ADR 0, 2
ADR 4, 6
LD 2, WM1
MD 6, 2
MD 2, 8(3, 12)
MD 6, 8(4, 12)
ADR 0, 2
ADR 4, 6
LD 2, WP1
LD 6, 2
MD 2, 16(3, 12)
MD 4, 16(4, 12)
ADR 0, 2
ADR 4, 6
STD 4, DADS
LD 0, WM3
LD 2, WP3

* NOW CALCULATE VALUES OF SOURCE AND AREA
* (HAVE WEIGHTS 1 & 4 IN FPR'S 0 & 2)

LDR 4, 0
LDR 6, 2
MD 0, 0(5, 12)
MD 2, 24(5, 12)
MD 4, 0(6, 12)
MD 6, 24(6, 12)
ADR 0, 2
ADR 4, 6
LD 2, WM1
MD 6, 2
MD 2, 8(5, 12)
MD 6, 8(6, 12)
ADR 0, 2
ADR 4, 6
STD 4, SOR
LD 0, WM3
LD 2, WP3

* CALCULATE INDEX AND FRACTIONAL DISPLACEMENT FOR INTERPOLATION
* ON TEMPERATURE (T) TABLE

L 2, 4(1)
LM 3, 6, KADDR
MVC FLOAT+1(6), 2(2)
LH 12, 0(2)
S 12, TDISP
LD 4, FLOAT

R2 <= BASE ADDR Y ARRAY
R3-R6 <= BASE ADDR'S TABLES FOR INTERPOLATION OF FUNCTIONS OF T
FLOAT <= FRACTIONAL DISTANCE FROM NEXT SMALLER VALUE OF T TABULATED.
R12 <= HIGH ORDER BYTES OF T
R12 <= FRACTION 0 LE FRAC LE 1
BM BADS
IF RESULT IS NEGATIVE OUT OF RANGE
GOTO BADS
C 12,TBND
COMPARE R12 TO TBND - IF GREATER
BH BADS
OUT OF RANGE GOTO BADS
SD 4,=D'1.125'
F4 <= X - FRAC - 5 - .5 LE X LE .5
SLA 12,3
R12 <= R12*8 NOW BYTE DISPLACEMENT
FROM BASE OF INTERPOLATION TABLE.

NOW COMPUTE WEIGHTS FOR CUBIC INTERPOLATION
FUNCTIONS OF T

LDR 2,4
F2 <= X
MDR 4,4
F4 <= X**2 = X2
HDR 4,4
F6 <= X2/2 - 9/8
SD 4,=D'1.125'
F4 <= X2/2 - 9/8
LDR 6,4
F4 <= X2/4 - 9/16
HDR 4,4
F6 <= X3/2 - 9X/8
MCR 6,2
F6 <= X3/2 - 9X/8
LCMR 0,6
F0 <= -X2/4 + 9/16
ADR 0,6
F0 <= X3/2 - X2/4 - 9X/8 + 9/16
STD 0,WM1
WM1 <= WEIGHT FOR TABLE ENTRY CORRESPONDING
TO CLOSEST SMALLER VALUE OF T.

LCMR 0,4
F0 <= -X2/4 + 9/16
SDR 0,6
F0 <= -X3/2 - X2/4 + 9X/8 + 9/16
STD 0,WP1
WP1 <= WEIGHT FOR TABLE ENTRY CORRESPONDING
TO CLOSEST LARGER VALUE OF T.

ADR 6,2
F6 <= X3/2 - X/8
AD 4,=D'1.5'
F4 <= X2/4 - 1/16
MD 6,='40555555555555555
F6 <= X3/6 - X/24
LDR 0,4
F0 <= X2/4 - 1/16
SDR 0,6
F0 <= -X3/6 + X2/4 + X/24 - 1/16
ADR 6,4
F6 <= X3/6 + X2/4 - X/24 - 1/16
STD 0,WM3
WM3 <= WEIGHT FOR TABLE ENTRY CORRESPONDING
TO 2ND CLOSEST SMALLER VALUE OF T.

STD 6,WP3
WP3 <= WEIGHT FOR TABLE ENTRY CORRESPONDING
TO 2ND CLOSEST LARGER VALUE OF T.

NOW CALCULATE INTERPOLATED VALUES OF 1/KAPPA(T) AND L
(HAVE WEIGHTS FOR TABLES ENTRIES 1 & 4 IN FPR'S 0 & 6)

LDR 4,0
F4 <= WEIGHT 1
LDR 2,6
F6 <= WEIGHT 4
MD 0,(3,12)
F0 <= WEIGHT 1*K1
MD 2,24,(3,12)
F2 <= WEIGHT 4*K4
MD 4,0,(4,12)
F4 <= WEIGHT 1*L1
MD 6,24,(4,12)
F6 <= WEIGHT 4*L4
ADR 0,2
F0 <= W1*K1 + W4*K4
ADR 4,6
F4 <= W1*L1 + W4*L4
AD 4,=D'1.5'
F4 <= X2/4 - 1/16
MD 6,='40555555555555555
F6 <= X3/6 - X/24
LDR 0,4
F0 <= X2/4 - 1/16
SDR 0,6
F0 <= -X3/6 + X2/4 + X/24 - 1/16
ADR 6,4
F6 <= X3/6 + X2/4 - X/24 - 1/16
STD 0,WM3
WM3 <= WEIGHT FOR TABLE ENTRY CORRESPONDING
TO 2ND CLOSEST SMALLER VALUE OF T.

STD 6,WP3
WP3 <= WEIGHT FOR TABLE ENTRY CORRESPONDING
TO 2ND CLOSEST LARGER VALUE OF T.

CHECK TO SEE IF GAS FULLY IONIZED (T > 65,536, BYTE DISP > 3BC(HEX)),
IF SO SKIP INTERPOLATION OF CHI AND DCHI/DT, IF NOT CONTINUE
C 12,HBYTE
BC 2,HIGHT
LD 0,WM3
FO <= WEIGHT 1
\[ F_2 = N \times (U^2 \times DA/DS - DT/DS \times \frac{K}{MH} (1+CHI) + DCHI/DT) \]

\[ F_4 = \frac{3}{2} \times F_2 \]

\[ \text{INTERPOLATED VALUE OF CHI} \]

\[ \text{INTERPOLATED VALUE OF DCHI/DT} \]

\[ \text{INTERPOLATED VALUE OF CHI DCHI} \]

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\[ \text{INTERPOLATED VALUE OF CHI DCHI} \]
* F4 <= 3/2*DT/DS*(1+CHI + (T-TI)*DCHI/DT) *

SDR 2.4  F2 <= (1+CHI)*DN/DS*T – F4(*)
MD 2.24(2,0)  F2 <= F2(6) * U
MD 2.24(2,0)  F2 <= K * F2(*)
MDR 0.0  F0 <= N**2
MD 0.CH1  F0 <= N**2 * CHI
MD 0.LUM  F0 <= RADIATIVE LOSSES
SDR 2.0  F2 <= F2(*) - F0(*)
LD 0.16(2,0)  F0 <= Q
MD 0.DADS  F0 <= Q/A*DA/DS
SDR 2.0  F2 <= DQ/DS LESS SOURCE TERM
AD 2.SOR  F2 <= DQ/DS
STD 2.16(3,0)  STORE DQ/DS
LM 14,12,12(13)  GPR'S RETURNED TO ORIGINAL STATE
MVI 12(13),X'FF'  TELL CALLING PROGRAM WE'RE RETURNING
BCR 15,14  RETURN

* END OF SECTION FOR T, 65,536 NEXT SECTION DOES SAME CALCULATIONS *
* FOR FULLY IONIZED CASE *

HIGHT LM 3.4,8(1)  FPR'S 364 <= BASE ADDR'S OF DY & AO
MVC 0KAP+16(16),=X'4110000000000000000000000000000001'

* LTR 3,3  CHECK IF 3 NEGATIVE IF SO IS LAST
BM TABL  PARAMETER - GOTO TABL
MD 0.16(2,0)  F0 <= Q/KAPPA
LCDR 0.0  F0 <= -Q/KAPPA = DT/DS
STD 0.16(3,0)  STORE DT/DS
LD 2.0(4,0)  F2 <= AO = NO*UO*AO
LD 4.0(4,0)  F4 <= AREA
MD 4.8(2,0)  F4 <= AREA*N
DDR 2.4  F2 <= (NO*UO*AO)/(AREA*N) = U
STD 2.24(2,0)  STORE U (VELOCITY)
MDR 2.2  F2 <= U**2
LDR 6.0  F6 <= DT/DS
LD 4.KOMH2  F4 <= 2K/MH
MDR 6.4  F6 <= 2K/MH*DT/DS
MD 4.0(2,0)  F4 <= 2KT/MH
SDR 4.2  F4 <= 2KT/MH - U**2
MD 2.DADS  FS <= U**2/A*DA/DS
SDR 2.6  F2 <= U**2/A*DA/DS - 2K/MH*DT/DS
SD 2.6  F2 <= F2(*) - G
LD 6.8(2,0)  F6 <= N
MDR 2.6  F2 <= F2(*) * N
DDR 2.4  F2 <= DN/DS
STD 2.8(3,0)  STORE DN/DS
MDR 0.6  F0 <= N*DT/DS
LDR 4.0  THIS AND THE NEXT TWO INSTRUCTIONS
ADR 0.0  EFFECTIVELY MULTIPLY FO BY 3.
ADR 0.4  F0 <= 3N*DT/DS
MD 2.0(2,0)  F2 <= DN/DS*T
ADR 2.2  F2 <= 2T*DN/DS
SDR 2.0  F2 <= 2T*DN/DS - 3N*DT/DS
MD 2.24(2,0)  F2 <= K*(2T*DN/DS - 3N*DT/DS)
MDR 6.6  F6 <= N**2
MD 6.LUM  F6 <= RADIATIVE LOSSES
SDR 2.6  F2 <= F2(*) - F6(*)
LD 4.16(2,0)  F4 <= Q
MD 4.DADS  F4 <= Q/A*DA/DS
SDR 2.4  F2 <= DQ/DS LESS SOURCE TERM
AD 2.SOR  F2 <= DQ/DS
STD 2.16(3,0)  STORE DQ/DS
LM 14,12,12(13)  GPR'S RETURNED TO ORIGINAL STATE
MVI 12(13),X'FF'  TELL CALLING PROGRAM WE'RE RETURNING
BR 14  RETURN

* RETURN INTERPOLATED FUNCTION VALUES - NOT DERIVITIVES
* TABL MVC 0(64,3),G PUT INTERPOLATED VALUES IN TAB
LM 14,12,12(13) RESTORE GPR'S
MVI 12(13),X'FF' INDICATE CONTROL RETURNED
BR 14 RETURN
*
* CASE OF S OR T OUTSIDE OF THE TABULATED RANGE
*
BADS LD 0,0(2,0) FO <= OFFENDING QUANTITY (T OR S)
LDR 0,0 FO NOW NEGATIVE
STD 0,0(2,0) STORE OFFENDING QUANTITY - FLAG
LM 14,12,12(13) GPR'S RETURNED TO ORIGINAL STATE
LA 15,4 GPR 14 <= 4 (RETURN 1)
MVI 12(13),X'FF' TELL CALLING PROGRAM WE'RE RETURNING
BR 14
CNOP 4,8
*
* STORAGE FOR ADDR'S, CONSTANTS, AND INTERNAL VARIABLES
*
GADDR DC X'00000000'
DADDR DC X'00000000'
SADDR DC X'00000000'
ADDR DC X'00000000'
KADDR DC X'00000000'
LADDR DC X'00000000'
CADDR DC X'00000000'
BCADDR DC X'00000000'
SUISP DC X'000004710'
TDISP DC X'000004410'
TBND DC X'00000330'
SBND DC X'00000330'
HBYTE DC X'00000778'
WM3 DC X'0000000000000000'
WM1 DC X'0000000000000000'
WP1 DC X'0000000000000000'
WP3 DC X'0000000000000000'
G DC X'0000000000000000'
DADS DC X'0000000000000000'
SOR DC X'0000000000000000'
AREA DC X'0000000000000000'
OKAP DC X'0000000000000000'
LUM DC X'0000000000000000'
CHI DC X'0000000000000000'
DCHI DC X'0000000000000000'
TION DC X'45198C6100000000'
KAY DC X'339F2CB600000000'
KOMH DC X'474EA81800000000'
KOMH2 DC X'479D5A3600000000'
FLOAT DC X'4000000000000000'
END

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