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MILLIMETER WAVE PROPAGATION MODELING OF INHOMOGENEOUS RAIN MEDIA FOR SATELLITE COMMUNICATIONS SYSTEMS
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on

A DEPOLARIZATION AND ATTENUATION
EXPERIMENT USING THE CTS AND COMSTAR
SATELLITES

Millimeter Wave Propagation Modeling Of
Inhomogeneous Rain Media For Satellite
Communications Systems

Text by

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June, 1978
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CHAPTER I

INTRODUCTION

The ever-increasing demand for additional communications capacity has led system design engineers to increase operating frequencies higher into the millimeter wave frequency band. Also, to effectively double communications capacity for a given satellite communications system, frequency reuse systems employing orthogonal polarizations have been suggested [1]. However, for operating frequencies above 10 GHz rain attenuation and depolarization become more severe because of the increased signal scattering and absorption by raindrops. Before future communications systems can be designed for maximum reliability and economy, a clearer understanding of these weather induced phenomena is necessary. Presently an experimental data base is being collected to describe the effects of rain on terrestrial and satellite communications links. Measured signal attenuation, orthogonal channel isolation, and phase shift data are being correlated with weather data to aid the design engineer in the prediction of rain effects on millimeter wave propagation. Since this collection of data is relatively expensive, especially for satellite communications links, theoretical models have been developed to predict weather effects on communications link performance. This report investigates the ability of a theoretical model to account for the inhomogeneous rain conditions present on a satellite communications link and thus accurately predict the effects of weather on system performance.

Theoretical modeling of the scattering properties of an ensemble
of particles was considered by Gustav Mie as early as 1908 [2]. The present development of theoretical modeling as related to a rain-filled space and the restrictions associated with the ensemble of raindrops will be discussed in Chapter II. In Chapter III a new deterministic theoretical model is presented with an associated rain propagation prediction program that can model an inhomogeneous rain and accurately predict the effects of precipitation on a satellite communications link for a variety of frequencies, elevation angles and locations. Finally in Chapter IV, the most recent multiple frequency data from various depolarization experiments are presented as verification of the new model.

The new model presented in Chapter III, although more general than previous models, is theoretically equivalent to each of the classical models presented in Chapter II under the assumptions of the respective model. By taking a slightly different approach in the theoretical derivation of the new model, the scattering properties of an arbitrary inhomogeneous ensemble of raindrops and ice crystals can be determined and the validity of other theoretical models can be verified. Unique aspects of the new model are: 1) attenuation and depolarization are computed directly in terms of the scattered fields from an ensemble of particles containing an arbitrary mixture of particle type (rain or ice), particle shape, particle size, particle density and particle orientation, 2) an inhomogeneous mixture of particles and the presence of a melting layer along the propagation path is accounted for directly in determining the scattering fields exiting the rain medium, 3) a
frequency independent synthetic storm algorithm models the nonuniform
rain rates present on a satellite communications link, 4) antenna-wave
interaction is considered, and 5) signal attenuation, channel isolation
and phase as a function of ground rain rate can be determined for a
variety of site locations from a knowledge of the operation frequency,
system antenna polarization parameters, and the system elevation angle.
CHAPTER II
REVIEW OF CURRENT THEORETICAL PROPAGATION MODELS

Although Gustav Mie in 1908 [2] was responsible for the preliminary foundation for theoretical work relating to the attenuation and depolarization of electromagnetic waves propagating through rain, it was not until 1960 that Oguchi [3] first considered the depolarization of an incident field by an oblate spheroidal raindrop. Oguchi obtained the first-order change in the scattered field from a single oblate raindrop by expanding the scattered field directly in a series of spherical vector wave functions involving powers of the raindrop eccentricity. The single oblate raindrop scattering coefficients provided the foundation for all theoretical rain depolarization models that followed. The scattering coefficients of Oguchi were later verified using different theoretical methods by Morrison and Cross of Bell Labs in 1974 [4] and Uzunoglu, Evans, and Holt of the University of Essex, England in 1977 [5].

Using the theory of van de Hulst [6], Oguchi extended the single drop solution to an ensemble of identical, equioriented oblate spheroidal raindrops. By decomposing the incident field along the orthogonal axes defined by the principal planes of the canted oblate raindrop, Oguchi determined an effective complex refractive index of the rain medium. The application of the work of Oguchi to the problem of depolarization as related to a communications system was first made by Thomas of Bell Labs in 1971 [7]. He showed that a difference in attenuation for waves polarized along the major and minor axes of an
oblate raindrop would lead to depolarization of any wave not polarized parallel to one of the principal axes. However, Thomas neglected the effects of differential phase on the depolarization of an electric field. The assumptions of the theoretical model of Thomas are given below:

1) Uniform rainfall over the entire path,
2) Laws and Parsons [8] raindrop-size distribution,
3) Single scattering (which is to say that each raindrop is illuminated by an incident plane wave and wave interaction between drops is negligible),
4) 100% oblate spheroidal equioriented raindrops at an effective canting angle θ.

The theoretical models of Watson (1973) [9] and Chu (1974) [10] also employ the assumptions of Thomas; however, these models consider the effects of differential attenuation and differential phase. The work of Watson and Chu, referred to as the classical propagation constant formulation, predicted rather well the rain effects experienced on a terrestrial communications link with nearly uniform rain conditions.

Wiley [11] in 1973 checked the validity of the classical propagation constant model. He derived the same results by directly summing the scattered fields located within the first Fresnel zone from an arbitrary equioriented ensemble of raindrops. (A drop size distribution was not included.) Wiley also illustrated that theoretical models could represent an ensemble of raindrops that contained up to 60%
spherical raindrops and accurately predict measured depolarization data. This difference in opinion of the percent of oblate raindrops is complicated by the positions of meteorologists Jones [12] and Pruppacher and Pitter [13]. Using experimental vertical wind tunnel conditions and formulations relating surface tension to particle shape, Pruppacher and Pitter hypothesize that, except for only the largest drops, raindrops are oblate spheroidal particles. Based on real rain measurements, Jones indicates that a real rain is composed of spherical, oblate spheroidal, prolate spheroidal and irregular shaped raindrops for all size classes greater than 0.9 mm spherical radius. Depending on the assumption of the percent of oblate raindrops present in an ensemble of raindrops, channel isolation can vary by a significant amount for a given value of attenuation as indicated in Fig. 2-1. (This figure was obtained from the results of Chapter III.) Since the work of Jones indicates the presence of shapes other than oblate and the effect of different raindrop shapes has a significant impact on the prediction of channel isolation, a theoretical propagation model should include a particle shape distribution.

Although still assuming 100% oblate spheroidal raindrops, Uzunoglu, Evans, and Holt [5] and Nowland, Olsen, and Shkarofsky [14] in 1977 indicated that a distribution of canting angles should be included in a theoretical model. Since the classical propagation constant formulation is restricted to equioriented raindrops, a stochastic averaging process was implemented. By weighting the elements of the scattering matrix as seen in Eq. (2-1) with a Gaussian probability distribution
Figure 2-1. Isolation versus attenuation for different percentages of oblate raindrops. (f = 11 GHz, no canting angle distribution, circular polarization, L = 10 km).
function, a canting angle distribution can be modeled:

\[
\begin{bmatrix}
\hat{E}_x \\
\hat{E}_y
\end{bmatrix} = \begin{bmatrix}
\sum_i w D_{11}(\theta_i) & \sum_i w D_{12}(\theta_i) \\
\sum_i w D_{21}(\theta_i) & \sum_i w D_{22}(\theta_i)
\end{bmatrix} \begin{bmatrix}
\hat{E}_x \\
\hat{E}_y
\end{bmatrix}
\]

where

\[
D_{11} = d_h \cos^2 \theta + d_v \sin^2 \theta \\
D_{22} = d_h \sin^2 \theta + d_v \cos^2 \theta \\
D_{12} = D_{21} = (d_v - d_h) \cos \theta \sin \theta
\]  \hspace{1cm} (2-1)

and the symbols \(w, d_v, d_h, \theta, f_{v,h}, N, \) and \(L\) are defined as

\[w = \text{normalized Gaussian weighting function}\]
\[d_{v,h} = e^{-j\lambda L \sum N(a) f_{v,h}(\bar{a})}\]
\[\theta = \text{drop canting angle}\]
\[f_{v,h} = \text{principal plane single drop scattering coefficients}\]
\[N(a) = \text{number of raindrops per unit volume in the } \bar{a} \text{ to } \bar{a} + \text{equivolumetric drop radius interval.}\]
\[L = \text{rain extent length}\]

Because a stochastic averaging process was used, the above formulation ignores multiple scattering between arbitrarily oriented raindrops. However, the assumption was made that multiple scattering is negligible. This assumption can be verified by the more general model presented in the following chapter because it accounts for the arbitrarily oriented particles directly.
For satellite communications links the assumption of uniform rainfall over the entire rain extent is not adequate in describing the effects of precipitation on system performance. As peak rain rate increases, rain cells of higher rain rates decrease in size. Based on experimental attenuation measurements, researchers [15], [16], [17] have implemented an effective path length ($L_e$) that models the nonuniform rain rates present on a satellite communications link. However, it will be shown in Sec. 3.6 that this method is frequency dependent and overly restrictive. Therefore, a frequency independent synthetic storm algorithm will be presented.

The formulations of Uzunoglu, Evans and Holt [5] and Nowland, Olsen, and Shkarofsky [14] represent the current status in theoretical propagation models. However, these formulations do not consider the effects of an ensemble of arbitrary particles and an inhomogeneous rain medium. The theoretical model presented in the following chapter accounts for the scattering properties of an arbitrary inhomogeneous ensemble of particles without a significant increase in model complexity.
CHAPTER III
DEVELOPMENT OF THE THEORETICAL MODEL

A theoretical rain propagation model should represent accurately the physical nature of a rain medium and allow a wide range of rain parameters; thus, the effect of precipitation on communications system performance can be determined accurately. Parameters such as canting angle, particle size, particle shape and particle type should be described with deterministic distributions based on existing physical knowledge of their behavior. The model also should be flexible so that these parameters can be adjusted to best describe the true physical nature of the rain medium. In this chapter a model is presented that provides this generalized format and henceforth will be referred to as the scattering model.

3.1 Electromagnetic Scattering by an Oblate Spheroidal Particle

To develop a solid foundation for the derivation of the scattering model, the scattering properties of a single oblate raindrop will be considered first. The oblate raindrop is of extreme importance in any model that predicts the attenuation, depolarization, and phase shift of an electric field. The oblate particle through its differential attenuation and differential phase properties provides the key mechanism in the depolarization process. Oguchi and Hosoya [18], Morrison and Cross [4], and Uzunoglu, Evans, and Holt [5] have published single drop scattering coefficients as a function of drop size for the principal planes of the oblate drop. The drop size is represented by an
equivolumetric drop radius, and as the radius increases the drop becomes more oblate. The available scattering coefficients agree rather well and provide the basis for any theoretical model.

An oblate spheroidal raindrop canted at an angle \( \theta \) with respect to an arbitrary \( x-y \) coordinate system is illustrated in Fig. 3.1-1. If the \( x-y \) coordinate system is aligned with true horizontal and vertical, \( \theta \) is the drop canting angle. Following the notation of Oguchi, the \( h-v \) axis in Fig. 3.1-1 represents the orientation of the oblate raindrop where the \( h \) and \( v \) axis are aligned with the drop major and minor axes, respectively. Another important parameter needed in the calculation of the single drop scattering coefficients is the elevation angle. The elevation angle \( \beta \) is the angle between a plane parallel to the local horizontal and the direction of propagation represented by the vector \( \vec{K} \). For a terrestrial communications link, this angle would be zero. A satellite link would have an elevation angle ranging from 0 to 90 degrees. Knowing the equivolumetric drop radius as defined by Uzunoglu, Evans and Holt and given in Fig. 3.1-1, the elevation angle, and the frequency of propagation, the vertical and horizontal scattering coefficients \( f_v \) and \( f_h \) can be determined.

Any incident field can always be decomposed into orthogonal linear components along the principal axes of the oblate spheroidal raindrop. Thus, single drop scattering coefficients are given only for the principal axes of the drop. Single drop scattering coefficients are defined in several ways in the literature; the definition of Oguchi will be used as follows:
Figure 3.1-1. Arbitrary oblate spheroidal raindrop canted in an arbitrary x-y coordinate system.
f_v: ratio of the forward scattered electric field component along the drop minor axis to the incident electric field component along the drop minor axis.

f_h: ratio of the forward scattered electric field component along the drop major axis to the incident electric field component along the drop major axis.

The incident electric field component along the drop major axis (h) experiences more attenuation and phase shift than the component along the minor axis (v). This effect causes the depolarization of incident fields that are not linearly polarized along one of the principal axes.

The hv decomposition is overly restrictive. Wiley [11] has shown that a generalized scattering coefficient can be defined as a function of θ and the single drop scattering coefficients. The generalized scattering coefficients allow the incident field to have any arbitrary angle with respect to the drop. These coefficients are easily derived (see Appendix 6.1.1) and are given below in terms of the arbitrary x-y coordinate system:

\[
\begin{align*}
    f_{xx} &= f_v \sin^2 \theta + f_h \cos^2 \theta \\
    f_{xy} &= (f_v - f_h) \sin \theta \cos \theta \\
    f_{yx} &= f_{xy} \\
    f_{yy} &= f_v \cos^2 \theta + f_h \sin^2 \theta.
\end{align*}
\]
The symbol $f_{pq}$ will be used to represent the general scattering coefficient.

Now the forward scattering effects of an oblate spheroidal raindrop can be examined. The incident field on the drop can be written in the general form

$$\hat{E}^i = \hat{E}_x x + \hat{E}_y y$$

(3.1-2)

for all polarization states where $\hat{E}_x$ and $\hat{E}_y$ are phasor quantities. The forward scattered electric field at point P in Fig. 3.1-1 is expressed by the relation

$$\hat{E}^s(P) = \{E_{xx}^s(P) + E_{yx}^s(P)\}x + \{E_{xy}^s(P) + E_{yy}^s(P)\}y$$

(3.1-3)

where

$$E_{pq}^s(P) = E_p^i(P) f_{pq} \frac{-jk_o r}{r}$$

(3.1-4)

= the scattered electric field component with polarization q as a result of an incident field with polarization p, evaluated at point P

and,

$$f_{pq} = \text{the ratio of the forward scattered electric field with polarization q to the incident electric field with polarization p evaluated for a particular drop orientation, drop size, and frequency.}$$

The $\frac{-jk_o r}{r}$ factor of Eq. (3.1-4) describes the spherical wave behavior.
of the scattered field between the drop and point P. Although the oblate drop does have back and side scattering properties, these are of no consequence in the prediction of rain effects on a point to point millimeter wave communications link. However, back-scattering coefficients are available in the literature [18] and can be used in place of the forward scattering coefficients in Eqs. (3.1-1) and (3.1-4) to obtain the back-scattered electric field.

The forward scattering effects of an oblate raindrop can be described by Eq. (3.1-3) in terms of the drop orientation, drop size, and frequency. The generalized scattering coefficients of Wiley [11] also can be extended to any particle having two axes of symmetry. With this fact, the scattering effects of a slab of arbitrary particles can be investigated.

3.2 Electromagnetic Scattering by a Single Homogeneous Slab of Arbitrary Particles

To predict accurately the effects of precipitation on a given communications link, the propagation path may be subdivided into small intervals of thickness Δz which are commonly called rain slabs. To describe the scattering properties of a slab, and hence the scattering properties of the entire propagation path, a physical understanding of the rain parameters is necessary. Jones [12] has indicated that rain consists of particles with a variety of shapes. An average rain consists of spherical, oblate spheroidal, prolate spheroidal, and irregular shaped raindrops. Furthermore, it is not uncommon to find
that a heavy concentration of ice crystals at higher altitudes affects the performance of a satellite link. The particle shape, particle orientation, and particle density influence electromagnetic waves propagating through a rain medium. All particle shapes except for spherical contribute to the depolarization of the incident field. For this reason the electromagnetic scattering from a single homogeneous slab of arbitrary particles is extremely important to the evaluation of rain effects on millimeter wave propagation.

In 1957 van de Hulst [6] addressed the problem of light scattering by a slab of particles. Although the scattering here is at high radio frequencies, van de Hulst's optical work will serve as a foundation for the development of the scattering model. Consider a homogeneous slab of particles ΔL meters thick and infinite in extent in the transverse plane as shown in Fig. 3.2-1. Since single drop scattering coefficients are not readily available for irregular and prolate spheroidal raindrops, the assumption will be made that all scattering effects of the slab are produced by spherical raindrops, oblate spheroidal raindrops, and ice crystals. However, if coefficients do become available for the irregular and prolate spheroidal raindrops, their inclusion in the theoretical development of the model would further refine the physical representation of the rain medium. With the exception of the work of Wiley [11], most researchers have assumed 100% oblate raindrops which is not physically accurate. Also, any model with the 100% oblate assumption inherently predicts more depolarization for a given rain rate and attenuation because non-depolarizing spherical scatterers are
Figure 3.2-1. Homogeneous rain slab of arbitrary particles.
The incident field on the slab of particles is assumed to be a uniform plane wave propagating in a direction perpendicular to the slab. Also, the magnitude of the incident field is the same for all particles within the slab. The incident field, regardless of polarization, can be represented by its x and y components as in the previous section. The rain slab is assumed to be large-scale homogeneous with a mixture of particles having varying shapes, sizes, and orientations within the volume of the slab. The distribution functions describing these parameters will be discussed in greater detail in Sec. 3.6. The purpose of this section is to determine the total electric field exiting the propagation medium.

The total electric field at point P is the sum of the incident field and the forward scattered field both evaluated at point P.

\[ \mathbf{E}_T(P) = \mathbf{E}_i(P) + \mathbf{E}_s(P) \quad (3.2-1) \]

The total forward scattered field at point P is the sum of the scattered fields from all the particles within the slab. The total scattered electric field with polarization q resulting from an incident field with polarization p evaluated at point P is expressed by

\[ \mathbf{E}_s^{pq}(P) = \mathbf{E}_p(P) \sum_{m}^{\text{All Particles}} f_{pq_m}(\overline{a}, \theta) \frac{e^{-jk_0(r_m - z_m)}}{r_m}. \quad (3.2-2) \]

The factor \( f_{pq_m}(\overline{a}, \theta) \) is the generalized complex forward scattering
coefficient for the \( m \)th particle as found in Eq. (3.1.1) and represents the change in amplitude, phase, and polarization that the incident field experiences as a result of the \( m \)th particle. Note that the scattering coefficient is a function of particle size \((\bar{a})\) and particle orientation \((\bar{\theta})\) and must remain within the summation. The factor \( e^{-j\kappa_0 (r_m - z_m)/r_m} \) represents the spherical nature of the scattered waves and the effect of the location of the \( m \)th particle on the phase of the scattered field at point \( P \).

With a few assumptions Eq. (3.2-2) can be simplified. Consider the geometry for the location of the \( m \)th particle as shown in Fig. 3.2-1. The relative phase difference corresponding to the path difference \((r_m - z_m)\) is all that need be considered in determining the phase of the scattered waves arriving at point \( P \). If the assumption is made that \( z_m \gg \rho_m \), then \( r_m \approx z_m \) and:

\[
r_m^2 = \rho_m^2 + z_m^2
\]

\[
\rho_m^2 = r_m^2 - z_m^2 = (r_m - z_m)(r_m + z_m)
\]

\[
(r_m - z_m) = \rho_m^2 / (r_m + z_m) \approx \rho_m^2 / 2 z_m \quad (z_m \gg \rho_m).
\]

The assumption \( z_m \gg \rho_m \) is reasonable because the scattered fields at point \( P \) are influenced coherently only by the particles located within the first few Fresnel zones. For a satellite link, the radius of the first Fresnel zone corresponding to a rain extent less than 15 km is less than 20 meters. Using Eq. (3.2-3) in Eq. (3.2-2) yields
\[ \hat{E}_{pq}^S(P) = \hat{E}_p^i(P) \sum_{m=1}^{\text{SPH}} f_{pq_m}^{\text{SPH}}(\bar{a}, \bar{b}) g_m + \hat{E}_p^i(P) \sum_{m=1}^{\text{OBL}} f_{pq_m}^{\text{OBL}}(\bar{a}, \bar{b}) g_m + \hat{E}_p^i(P) \sum_{m=1}^{\text{ICE}} f_{pq_m}^{\text{ICE}}(\bar{a}, \bar{b}) g_m \]

where

\[ g_m = \frac{e^{-jk_0 \rho_m^2/2 z_m}}{z_m} \]

The superscripts SPH, OBL, and ICE denote the scattering coefficient for the respective particle type.

Within each class of particle shapes there exists a distribution
of particle size and for the oblate raindrop and ice crystal classes there is also a distribution of particle orientation. Let $N_{i,i,k}^{\text{SPH}}$, $N_{i,k}^{\text{OBL}}$, and $N_{i,k}^{\text{ICE}}$ represent the number of spherical, oblate, and ice particles within a rain slab that are in the $i^{th}$ size class and $k^{th}$ orientation class, respectively. The scattered field can now be expressed as a series of summations:

$$
\tilde{E}_p^{\gamma}(P) = \sum_{m=1}^{N_{i,1}^{\text{SPH}}} f_{pq}^{\text{SPH}}(\alpha, \beta) g_m + \sum_{m=1}^{N_{i,1}^{\text{OBL}}} f_{pq}^{\text{OBL}}(\alpha, \beta) g_m + \sum_{m=1}^{N_{i,1}^{\text{ICE}}} f_{pq}^{\text{ICE}}(\alpha, \beta) g_m + \ldots
$$

$$(3.2-6)$$
Note that the summation over the spherical drops is independent of canting angle. Although Eq. (3.2-6) is somewhat cumbersome, it illustrates the physical significance of the particular class of particles involved in the summation process. The assumption has been made that the canting angle of a given particle is independent of the equilvalometric particle radius. Brussaard [19] has shown that this is not strictly true. As the radius and the terminal velocity of the falling particle increases, wind gradients have more effect on the orientation of the particle. However, the assumption of independence is reasonably accurate and it also leads to a relatively simple model.

Since the scattering coefficients in Eq. (3.2-6) are constant within their respective summations, they can be taken out of the summation signs leaving the summation \( \sum_{m=1}^{N_{i,k}} g_m \) in every term. This summation is easily evaluated (see Appendix 6.1.2) and the result is a constant given below:

\[
\sum_{m=1}^{N_{i,k}} g_m = -j \lambda \Delta \xi n_{i,k} \tag{3.2-7}
\]

The symbol \( n_{i,k} \) is the number of particles per unit volume in the \( i \)th size class and the \( k \)th orientation class. Equation (3.2-6) now becomes:

\[
\tilde{S}_p(P) = -j \lambda \Delta \xi \tilde{S}_p^i(P) \left\{ n_{1} f_{pq}(\bar{a}_1) + \cdots + n_{n} f_{pq}(\bar{a}_n) \right\}
\]
Recognizing that the individual bracketed terms are double sums over the discretized particle size and particle orientation distributions, Eq. (3.2-8) can be further simplified. Let \( N_v(a_i, \theta_k) \) be the total number of particles per unit volume with particle size \( a_i \) and particle orientation \( \theta_k \). Then Eq. (3.2-8) can be rewritten as follows:

\[
\begin{align*}
\tilde{\mathcal{E}}_{pq}(p) &= -j\lambda\Delta z \sum_{i=1}^{M_i} \sum_{k=1}^{M_k} N_v(a_i, \theta_k) F_{pq}(a_i, \theta_k) \quad (3.2-9)
\end{align*}
\]

where,

\[
F_{pq}(a_i, \theta_k) = \left( P_{ES} f_{pq}^{SPH}(a_i) + P_{OBL} f_{pq}^{OBL}(a_i, \theta_k) + P_{ICE} f_{pq}^{ICE}(a_i, \theta_k) \right)
\]

and, the symbols \( M_i \) and \( M_k \) represent the number of discrete intervals in the respective distribution functions.

Although the scattered field expression has been simplified to a great extent, it is convenient to define an effective scattering coef-
ficient for the entire slab, \( S_{pq} \), such that

\[
E_{pq}^s(P) = S_{pq} E_{p}^i(P).
\]  
(3.2-11)

From Eq. (3.2-9), the effective scattering coefficient \( S_{pq} \) is expressed by the relation

\[
S_{pq} = -j\lambda \Delta \varepsilon \sum_{i=1}^{M_q} \sum_{k=1}^{M_k} N_V(\vec{a}_i, \vec{e}_k) F_{pq}(\vec{a}_i, \vec{e}_k)
\]  
(3.2-12)

= the ratio of the forward scattered electric field with polarization \( q \) to the incident electric field with polarization \( p \) representing the scattering properties of a thin rain slab with arbitrary particle shapes, sizes, and orientations.

Using Eq. (3.1-1) in Eq. (3.2-12), the effective scattering coefficients relative to the arbitrary \( x-y \) axes can be formulated as

\[
S_{xx} = -j\lambda \Delta \varepsilon \sum_{i=1}^{M_q} \sum_{k=1}^{M_k} N_V(\vec{a}_i, \vec{e}_k)(P_{eS} f^{SPH} + \sin^2 \theta_k(P_{eO} f^{OBL} + P_{eI} f^{ICE}))
\]  
(3.2-13)

\[
+ \cos^2 \theta_k(P_{eO} f^{OBL} + P_{eI} f^{ICE}) + \sin \theta_k \cos \theta_k
\]  

\[
S_{xy} = S_{yx} = -j\lambda \Delta \varepsilon \sum_{i=1}^{M_q} \sum_{k=1}^{M_k} N_V(\vec{a}_i, \vec{e}_k)(P_{eO}(f_v - f_h)^{OBL}) + \sin \theta_k \cos \theta_k
\]  

\[
S_{yy} = -j\lambda \Delta \varepsilon \sum_{i=1}^{M_q} \sum_{k=1}^{M_k} N_V(\vec{a}_i, \vec{e}_k)(P_{eS} f^{SPH} + \cos^2 \theta_k(P_{eO} f^{OBL} + P_{eI} f^{ICE}))
\]  

\[
+ \sin \theta_k \cos \theta_k
\]  

(3.2-13)
The cross polarization scattering coefficients $S_{xy}$ and $S_{yx}$ represent the scattering of energy from one polarization component into its orthogonal component. This effect is directly related to the differential phase and differential attenuation ($f_v - f_h$) properties of the oblate raindrop and the ice crystal. The spherical raindrop on the other hand does not contribute to the depolarization process and when $P_{e0}$ and $P_{e1}$ are zero, $S_{xy} = S_{yx} = 0$, and the rain slab does not depolarize the incident field.

Using Eqs. (3.2-1) and (3.2-11), the total field at point $P$ now can be expressed in terms of the effective scattering coefficients as follows:

\[
\begin{align*}
\dot{E}_x(P) &= \dot{E}_x^i(P) + \dot{E}_x^i(P) S_{xx} + \dot{E}_y^i(P) S_{yx} \\
\dot{E}_y(P) &= \dot{E}_y^i(P) S_{xy} + \dot{E}_y^i(P) + \dot{E}_y^i(P) S_{yy}.
\end{align*}
\]  

(3.2-14)

In matrix form, Eq. (3.2-14) becomes

\[
\begin{bmatrix}
\dot{E}_x(P) \\
\dot{E}_y(P)
\end{bmatrix} = \begin{bmatrix}
(1 + S_{xx}) & S_{yx} \\
S_{xy} & (1 + S_{yy})
\end{bmatrix} \begin{bmatrix}
\dot{E}_x^i(P) \\
\dot{E}_y^i(P)
\end{bmatrix}.
\]  

(3.2-15)

The scattering properties of a single homogeneous thin rain slab of arbitrary particles now have been presented. Also an effective scattering coefficient has been formulated that includes the effects of arbitrary particles, a particle size distribution, and a particle orientation distribution. So now with an understanding of the scattering properties of a thin rain slab, the effect of a homogeneous
rain on a communications link can be investigated.

3.3 Electromagnetic Scattering by a Homogeneous Rain

The purpose of the scattering model is to predict the physical changes that an incident electromagnetic wave with a given polarization undergoes when all or part of the propagation path is filled with rain. Now that the scattering properties of this path are understood on a microscopic level, the entire rain extent can be modeled by extending the work in the previous two sections. In this section the assumption will be made that the rain is homogeneous in the direction of propagation. However, this restriction will be relaxed in the next section when a model for a piecewise homogeneous rain is developed.

Consider a propagation path with a homogeneous rain of extent \( \lambda \) meters in the direction of propagation. In the previous section the total electric field was determined at point \( P \) for a single thin rain slab. Since the rain volume is homogeneous in the direction of propagation, the rain extent \( \lambda \) can be modeled by a series of \( M_s \) thin rain slabs. If the incremental length \( \Delta \lambda \) of the thin rain slab is assumed to be one meter (\( \Delta \lambda = 1 \text{ m} \)), \( M_s \) is a large number equal to the homogeneous rain extent length \( \lambda \).

As a result of the assumption of a homogeneous rain in the direction of propagation, the \( M_s \) slabs have identical scattering properties as described by Eq. (3.2-15). The effect of \( M_s \) rain slabs on the incident field can be represented by the multiplication of the \( M_s \) matrices as follows:
The scattering matrix $[S]^M$ in Eq. (3.3-1) represents the changes that the incident field experiences as it propagates through a homogeneous rain medium of thickness $\lambda$. However, raising a matrix to the $M^{th}$ power where $M$ can be a large number (from 1000 to 10000 depending on the length $x$) is numerically awkward. This problem can be solved by the use of the Cayley-Hamilton theorem [20]. The Cayley-Hamilton theorem allows the $2 \times 2$ matrix of Eq. (3.3-1) to be expressed in terms of the identity matrix and two unique constants as follows:

$$[S]^M = \alpha_0[I] + \alpha_1[S]$$  \hspace{1cm} (3.3-2)

where,

$$\alpha_0 = (\lambda_1 \lambda_2 - \lambda_1 \lambda_2^2)/(\lambda_2 - \lambda_1)$$  \hspace{1cm} (3.3-3)

$$\alpha_1 = (\lambda_2 - \lambda_1)/(\lambda_2 - \lambda_1)$$

$$\lambda_{1,2} = (1 + \frac{S_{xx} + S_{yy}}{2}) \pm \sqrt{(1 + \frac{(S_{xx} + S_{yy})}{2})^2 - S_{xy}^2 - (1 + S_{xx})(1 + S_{yy})}.$$  

Thus, the electric field exiting the rain medium now can be represented by...
\[
\begin{bmatrix}
\dot{E}_x \\
\dot{E}_y
\end{bmatrix} = \begin{bmatrix}
\alpha_0 + \alpha_1(1 + S_{xx}) & \alpha_1 S_{yx} \\
\alpha_1 S_{xy} & \alpha_0 + \alpha_1(1 + S_{yy})
\end{bmatrix}
\begin{bmatrix}
\dot{E}_x \\
\dot{E}_y
\end{bmatrix} \tag{3.3-4}
\]

In this section the scattering properties of a homogeneous rain have been modeled to predict the effect of this rain on an arbitrary incident field. Equation (3.3-1) provides a physical insight into the scattering properties of the medium while Eq. (3.3-4) provides an efficient method for computing the electric field exiting a homogeneous rain.

3.4 **Electromagnetic Scattering by a Piecewise Homogeneous Rain**

The development of the scattering model as presented in Sec. 3.2 considered a homogeneous mixture of particles with varying sizes, shapes, and orientations in a thin slab of rain. In the previous section a homogeneous rain was modeled as a series of identical thin rain slabs. In order to permit more accurate modeling of a real rain, the homogeneous rain restriction is removed in this section.

By discretizing the rain extent on a large scale basis as shown in Fig. 3.4-1, a piecewise homogeneous rain can be modeled with \(M\) discrete rain cells. Each rain cell can have arbitrary length and arbitrary rain conditions. The rain cells also can have different distributions of particle shape, particle size, and particle orientation. The scattering properties of each rain cell can be represented by a scattering matrix \([S_i]\) where the effect of the \(i^{th}\) rain cell on
Figure 3.4-1. Rain cell division on a satellite communications link.
the electric field is determined by Eq. (3.3-4). The resulting fields after passing through the piecewise homogeneous rain then can be expressed by the multiplication of the rain cell matrices as follows:

\[
\begin{bmatrix}
E_x' \\
E_y'
\end{bmatrix}
= 
\begin{bmatrix}
S_M \\
S_{M-1} \\
\vdots \\
S_2 \\
S_1
\end{bmatrix}
\begin{bmatrix}
E_x \\
E_y
\end{bmatrix}
\]  
(3.4-1)

Equation (3.4-1) is a very powerful tool in predicting the effects of an inhomogeneous rain on a millimeter wave communications link. However, data describing the true physical nature of a given rain extent are not readily available for most locations. To use this equation to its fullest extent, Sec. 3.6 presents a synthetic storm algorithm that can predict statistically rain effects for a variety of locations. Another use of this large scale discrete model is to allow only the first cell to contain ice particles. This is a physically meaningful situation because ice crystals usually exist only at the higher elevations of a storm. Physical situations of this nature are handled rather well by the above formulation and the generalized format of the scattering model has been achieved.

3.5 A Discussion of Rain Reciprocity

Now that a general model has been developed to predict the effects of an inhomogeneous rain on millimeter waves, the effects of nonreciprocity should be considered. Nonreciprocity as related to rain and millimeter wave propagation has received very little attention in the
literature; however, under certain rain conditions it may have important consequences on millimeter wave communications links. To describe the effects of an inhomogeneous rain on millimeter waves, the medium is modeled by the product of $M$ homogeneous rain cell matrices. This section will show that an inhomogeneous rain may be nonreciprocal and then the order of matrix multiplication in Eq. (3.4-1) is important.

The reciprocity theorem [21] for electromagnetic fields states that the response of a medium to a source (transmitter) is unchanged when source and measurer (receiver) are interchanged. Consider two arbitrary homogeneous rain cells that are placed adjacent to one another to form an inhomogeneous rain cell as shown in Fig. 3.5-1. The measured electric field after passing through the piecewise homogeneous rain medium can be expressed in matrix form as follows:

\[
\begin{bmatrix}
E_x \\
E_y
\end{bmatrix} = [S_2] [S_1] \begin{bmatrix}
E_x^i \\
E_y^i
\end{bmatrix} = \begin{bmatrix} A_2 & B_2 \\ B_2 & C_2 \end{bmatrix} \begin{bmatrix} A_1 & B_1 \\ B_1 & C_1 \end{bmatrix} \begin{bmatrix}
E_x^i \\
E_y^i
\end{bmatrix}
\]

\[
= \begin{bmatrix} A_2 & B_2 \\ B_2 & (A_2 + \epsilon_2) \end{bmatrix} \begin{bmatrix} A_1 & B_1 \\ B_1 & (A_1 + \epsilon_1) \end{bmatrix} \begin{bmatrix}
E_x^i \\
E_y^i
\end{bmatrix}.
\]

The symbols $A_i$, $B_i$, $C_i$, and $\epsilon_i$ represent the entries for the $i^{th}$ rain cell matrix as defined by Eq. (3.3-1). For reciprocity as defined above to be satisfied, the matrices $[S_1]$ and $[S_2]$ must be commutative. Using matrix algebra it can be shown that the two matrices are commuta-
Figure 3.5-1. An inhomogeneous rain, two cells of dissimilar canting angles.
tive and the medium reciprocal if and only if

\[ \Delta = \varepsilon_1 B_2 - \varepsilon_2 B_1 = 0. \]  

(3.5-2)

So that a more direct understanding of the nonreciprocal phenomenon can be achieved, a simplifying assumption will be made. Assume that all the particles within the respective rain cells are equioriented oblate spheroidal raindrops with equal drop size (the distributions are represented by unit impulse functions). With this assumption, the matrix formulation of Eq. (3.3-1) is equivalent to the classical propagation constant model formulation [22]. The matrix entries \( A_i, B_i, \) and \( C_i \) as defined by Uzunoglu, Evans, and Holt [5] are given below:

\[
A_i = d_{h_i} \cos^2 \theta_i + d_{v_i} \sin^2 \theta_i
\]

\[
B_i = \gamma_i \sin \theta_i \cos \theta_i
\]

(3.5-3)

\[
C_i = d_{h_i} \sin^2 \theta_i + d_{v_i} \cos^2 \theta_i
\]

where

\[
\gamma_i = d_{v_i} - d_{h_i}
\]

\[
d_{v_i} = e^{-j\lambda \chi_i} N_i f_{v_i}
\]

(3.5-4)

\[
d_{h_i} = e^{-j\lambda \chi_i} N_i f_{h_i}
\]

Using Eq. (3.5-3) and the definition of \( \varepsilon_i \) as defined by Eq. (3.5-1),
\[ \varepsilon_i = C_i - A_i = (d_y - d_x) \left( \cos^2 \theta_i - \sin^2 \theta_i \right) \]
\[ = Y_i \cos^2 \theta_i . \]

Now, Eq. (3.5-2) can be rewritten as
\[ \Delta = \frac{Y_2 Y_1}{2} \sin 2(\theta_2 - \theta_1) \]
(3.5-6)

where \( \theta_2 \) and \( \theta_1 \) represent the canting angles of the raindrops in the appropriate cells. The inhomogeneous rain medium of Fig. 3.5-1 is reciprocal only when \( \Delta = 0 \) or when
\[ (\theta_2 - \theta_1) = \frac{n\pi}{2}, \quad n = 0, 1, 2, 3 \ldots \]
(3.5-7)

Note that the conditions under which reciprocity fails are independent of the length and the rain rate of the two cells, but only depend on the difference of the respective canting angles.

By definition the classical propagation constant model requires that all drops in a given thin, finite slab be equioriented. As a result, Eq. (3.5-7) indicates that the propagation constant model cannot deterministically model an arbitrary canting angle distribution without mathematically violating reciprocity. For this reason the scattering model accounts for the canting angle distribution within the finite slab with the use of the generalized single particle scattering coefficients discussed in Sec. 3.1.

Now the impact of nonreciprocity on communications link performance will be investigated. Although the assumptions of the classical propagation constant model lead to an exaggerated physical interpreta-
tion of the nonreciprocal phenomena, simple equations which explain the
trends of nonreciprocity can be formulated. As a simple example consider the piecewise homogeneous medium of Fig. 3.5-1 with the incident field being vertically polarized \( E_x^i = 0 \). The exiting fields are found from Eqs. (3.5-1) and (3.5-3) with Case 1 having the incident field \( E_y^i \) incident on Cell 1 and Case 2 having \( E_y^i \) incident on Cell 2. The fields for the two cases are given below:

**Case 1**

\[
\begin{align*}
\hat{E}_x^{1,2} &= (A_1 B_2 + B_1 A_2 + \epsilon_1 B_2) E_y^i \\
\hat{E}_y^{1,2} &= (B_1 B_2 + (A_1 + \epsilon_1)(A_2 + \epsilon_2)) E_y^i
\end{align*}
\]

**Case 2**

\[
\begin{align*}
\hat{E}_x^{2,1} &= (A_1 B_2 + B_1 A_2 + \epsilon_2 B_1) E_y^i \\
\hat{E}_y^{2,1} &= (B_1 B_2 + (A_1 + \epsilon_1)(A_2 + \epsilon_2)) E_y^i
\end{align*}
\]

Clearly, if \( \epsilon_1 B_2 \) equals \( \epsilon_2 B_1 \) (\( \Delta = 0 \)), the fields of Case 1 equal the fields of Case 2 and the medium is reciprocal.

Define \( \Delta A \) to be the difference in attenuation of the incident field for the two cases. The general expression for \( \Delta A \) with any incident polarization is

\[
\Delta A = 20 \log_{10} \left| \frac{\hat{E}_x^{1,2} \cdot E_y^{i*}}{\hat{E}_x^{2,1} \cdot E_y^{i*}} \right| \text{dB} \quad (3.5-9)
\]

For a vertically polarized incident field, \( \Delta A \) is
\[ \Delta A = 20 \log_{10} \left| \frac{E_y(1,2)}{E_y(2,1)} \right| = 0 \quad (3.5-10) \]

Independent of the rain conditions in either cell, attenuation is reciprocal. One can show, using Eq. (3.5-9), that attenuation is reciprocal for any arbitrary linear or circularly polarized incident field. Although it has not been proven for the general case of elliptical polarization, the same result is expected. Since attenuation should depend only on the volume of water present along the propagation path, the above conclusion seems reasonable.

Now define \( \Delta I \) to be the difference in channel isolation on a dual polarized communications system for the two cases of Fig. 3.5-1. The general expression for any arbitrary incident polarization is

\[ \Delta I = 20 \log_{10} \left| \frac{\hat{E}(1,2) \cdot \hat{E}^*}{\hat{E}(2,1) \cdot \hat{E}^*} \right| \quad (3.5-11) \]

where \( \hat{E}^* \) represents the polarization state orthogonal to the incident field. For a vertically polarized incident field \( \hat{E}^* = \hat{E}_x \cdot \hat{x} \) and Eq. (3.5-11) reduces to

\[ \Delta I = 20 \log_{10} \left| \frac{\hat{E}_x(1,2)}{\hat{E}_x(2,1)} \right| \quad (3.5-12) \]

Using Eq. (3.5-8) in Eq. (3.5-12) yields

\[ \Delta I = 20 \log_{10} \left| 1 + \frac{\Delta}{B_1A_2 + A_1B_2 + \varepsilon_2B_1} \right| \quad (3.5-13) \]

Examination of Eq. (3.5-13) yields only two conditions under which reciprocity holds for isolation:
1) \( \Delta = 0 \)

2) \[
\frac{\Delta}{\{B_1A_2 + A_1B_2 + e_2B_1\}} = -2
\]

The first case is when the entire medium is reciprocal. The second is a special case and will be investigated shortly.

In general isolation is nonreciprocal. This is to be expected because it is the difference in the respective canting angles that makes the rain medium nonreciprocal. The magnitude of nonreciprocity depends on the difference of canting angles, input polarization, and other path rain conditions. The interdependence of the rain parameters (rain rate, rain extent, etc.) and their effect on the magnitude of \( \Delta I \) will be presented through two special cases using the classical propagation constant model formulation. From these examples it will be evident that the magnitude of \( \Delta I \) increases with frequency for a given input polarization and path rain conditions.

For the first special case allow the medium of Fig. 3.5-2 to have arbitrary rain rates in either cell. Allow the cells to have any length \( \xi_1 \) and \( \xi_2 \). The canting angle \( e_2 \) is also arbitrary; however, let \( e_1 \) equal zero. Under these conditions and using Eqs. (3.5-3) and (3.5-4) in Eq. (3.5-13) yield

\[
\Delta I = 20 \log_{10} \left| e \left( \lambda \xi_1 N_1 \right) \cdot \text{Im}\{f_{v_1} - f_{h_1}\} \right| \text{dB}
\]  

(3.5-14)

where

\[
\theta_1 = 0
\]
Figure 3.5-2. An example of nonreciprocity.

Figure 3.5-2a. Forward propagation.

Figure 3.5-2b. Reverse propagation.
\[ \theta_2 \neq n\pi/2, \quad n = 0, 1, 2, 3 \ldots \]

\[ RR_2 \neq 0 \]

\[ \epsilon_2 \neq 0 . \]

As long as \( \theta_2 \) is not zero or ninety degrees (or a multiple thereof), the magnitude of nonreciprocity is independent of the rain conditions in the second cell. At first this result may seem surprising; however, it is explained easily with a physical example. Figure 3.5-2a illustrates Case 1. The incident field is not depolarized by Cell 1 and the drops of Cell 2 are oriented in such a manner that the field exiting Cell 2 is depolarized or rotated by \( \phi \) degrees. Case 2 is illustrated in Fig. 3.5-2b. In this case the incident field is depolarized or rotated by \( \phi \) degrees. The field incident upon Cell 1 is now oriented such that it will be depolarized. The depolarization by Cell 2 is the same in both cases since the input polarization is the same. However, the depolarizing properties of cell one are different for the two cases. Thus, the difference in isolation for the two cases depends only on the rain conditions of Cell 1. Equation (3.5-14) is plotted in Fig. 3.5-3 for three different frequencies as a function of rain rate.

The propagation constant model formulation predicts serious non-reciprocal crosstalk effects on a given communications link using frequency reuse techniques. Although it provides simple equations to predict the effects of various rain parameters of rain reciprocity,
Figure 3.5-3. Theoretical predictions of the classical propagation constant model of the difference in isolation experienced on an uplink and a downlink of a millimeter wave satellite communications link.
the magnitude of the predicted results are exaggerated. The scattering model represents the rain in a more physically significant manner. With the inclusion of the spherical scatterer and the orientation and drop size distributions, the magnitude of $\Delta I$ is significantly decreased. Even for an operating frequency of 30 GHz, the scattering model indicates that $\Delta I$ is less than 0.5 dB. (The propagation constant model can also represent a drop size distribution.)

The second special case will illustrate the effect that the difference in the respective canting angles $(\theta_2 - \theta_1)$ has on the magnitude of $\Delta I$. For this example, the rain rates and path lengths of both rain cells are equal, but the canting angles are allowed to be arbitrary. Under these assumptions $\gamma_1$ equals $\gamma_2$ and Eq. (3.5-13) reduces to

$$\Delta I = 20 \log_{10} \left| \frac{1 + g \tan \theta_1 \cot \theta_2}{g + \tan \theta_1 \cot \theta_2} \right|$$  \hspace{1cm} (3.5-15)

where

$$g = -j\alpha_{pl}(f_v - f_h)$$

for both cells. Note for the special case of $\theta_2 = \pi - \theta_1$, $\Delta I$ equals zero. For any arbitrary linearly polarized incident field there is a certain special case of canting angle orientation that makes the medium reciprocal for isolation. This effect for vertical polarization is illustrated in Fig. 3.5-4. For a fixed value of $\theta_1$, the isolation for each case and $\Delta I$ are plotted as a function of $\theta_2$. Note that whenever $(\theta_2 - \theta_1) = n\pi/2$ where $n = 0,1,2 \ldots$, $\Delta I$ is zero. Again, the magnitude
Figure 3.5-4. An example of the effect of canting angle on isolation nonreciprocity. (Classical propagation constant model, f = 30 GHz)
of $\Delta I$ is exaggerated. However, the peaks in $\Delta I$ for the special values of $\theta_2$ suggest that certain physical conditions may exist that could affect the operation of an adaptive isolation correction system on a given satellite communications link.

The purpose of this section was to show that nonreciprocal conditions may exist on an uplink and a downlink of a given satellite communications system. The interdependence of the parameters attributing to the nonreciprocal phenomena have been investigated with the use of simple examples. In general the scattering model does not seem to predict any significant nonreciprocal effects for typical rain situations in the 10 to 30 GHz frequency band. Thus, the prediction of isolation nonreciprocity is exaggerated by models that do not include a canting angle or particle shape distribution. However, there still may be certain physical conditions that could have important consequences on millimeter wave communications links. Further investigation into the problem of rain reciprocity is necessary.

3.6 Model Implementation

Now that the scattering model has been presented formally and a word of caution has been noted about reciprocity, the practical implementation of the scattering model will be presented. The final goal of the scattering model is to predict signal attenuation, isolation, and phase shift for a variety of frequencies and site locations as a function of measured rain rate data. Before this goal can be achieved, a detailed analysis of particle shape, particle size, particle density,
and particle orientation as related to ground rain rate must be formulated. These topics will be discussed in this section. Also, the relationship of the single particle scattering coefficients to elevation angle $\beta$ will be presented. The effect of non-ideal antennas on the predicted results will be investigated. Finally, this section presents a Rain Propagation Prediction program (RPP) that includes a synthetic storm algorithm that can predict the effects of rain on millimeter wave communications links for a range of frequencies, elevation angles, and locations.

It has been shown that spherical raindrops, oblate spheroidal raindrops, and ice crystals may be used to model an arbitrary rain path. In the practical application of the tools developed in Sec. 3.4, the assumption will be made that a given cell consists entirely of ice crystals or entirely of raindrops. The first cell will usually be modeled by ice crystals and the remaining cells will be modeled using an effective percentage of oblate spheroidal and spherical raindrops. The distributions associated with the rain cell and ice cell will be presented separately; however, the assumption is made that both particle density distributions are a function of ground rain rate. It should be noted that the scattering model can include a mixture of rain and ice in a given rain cell, but presently data are not available to describe accurately this physical distribution of particles. The scattering model also will allow varying distributions along the propagation path. However, based on existing physical data, each rain cell is assumed to have the same drop shape distribution, drop size distri-
3.6.1 Raindrop Shape Distribution

First consider the distribution of drop shapes. Jones [12] has found that the following distribution of shapes occur in an average rain for all radii greater than 0.9 mm:

- Spherical: 32%
- Oblate Spheroidal: 28%
- Prolate Spheroidal: 18.5%
- Irregular: 21.5%

Since the scattering model is limited by the availability of single particle scattering coefficients, the above distribution will be modeled by an effective percentage of oblate and spherical raindrops. About one-half of the prolate drops would be expected to be aligned with their elliptical cross section facing the incident field and therefore produce depolarization. On a satellite link, the elevation angles are such that a fraction of the remaining prolate drops would also depolarize the incident field. The prolate drop class may be modeled by assuming the class is made up of 70% oblate drops and 30% spherical drops. A large fraction of the irregular drop class would also depolarize the incident field. If the irregular drop class is modeled by 90% oblate drops and 10% spherical drops, the total effective percentage of oblate drops is approximately 60%. Based on
the above arguments and good correlation with measured data (see Sec. 4.2) a rain cell is assumed to have 60% oblate spheroidal raindrops and 40% spherical raindrops. This distribution is assumed not to be a function of rain rate.

3.6.2 Raindrop Size Distribution

For a given rain rate there will be a distribution of drop sizes. There are several drop size distributions published in the literature [8], [23], [24], [25]. A modification of the Laws and Parsons distribution will be used. The Laws and Parsons distribution as published is somewhat inconvenient in that it has no explicit functional form. However, real rains tend to deviate from any of the published distributions and an approximate functional representation of the Laws and Parsons distribution can be developed with little additional effect on the results. A simple approximation is that of a triangle. In Fig. 3.6-1, the Laws and Parsons distribution for a few rain rates is shown together with the triangular approximation where

\[
\begin{align*}
    s(a) = \begin{cases} 
    \frac{\bar{a}}{a_m^2} & a \leq a_m \\
    \frac{2}{a_m} \left(1 - \frac{\bar{a}}{2a_m}\right) & a_m \leq \bar{a} \leq 2a_m \\
    0 & \text{elsewhere}
    \end{cases} \\
\end{align*}
\] (3.6-1)

\[
\frac{a}{a_m^2} \frac{2}{a_m} \left(1 - \frac{\bar{a}}{2a_m}\right) \frac{2}{a_m} \left(1 - \frac{\bar{a}}{2a_m}\right) \frac{a}{a_m^2} \frac{2}{a_m} \left(1 - \frac{\bar{a}}{2a_m}\right)
\]

= fraction of drops in radius interval \( \bar{a} \) to \( \bar{a} + d\bar{a} \).

The symbol \( a_m \) represents the modal or most frequently occurring drop
Figure 3.6-1. Triangular approximation to the Laws and Parsons size distribution.
radius and $\bar{a}$ is the equivolumetric drop radius. Both radii are in millimeters. The modal drop radius is a function of rain rate and is expressed by the relation \cite{11}

$$a_m = 0.5 + 0.45 \log_{10} RR$$

(3.6-2)

where RR is rain rate in mm/hr. The distribution of drop sizes can be represented by the relation

$$n(\bar{a}) = N_V s(\bar{a})$$

(3.6-3)

The symbol $N_V$ represents the particle density or the total number of drops per cubic meter of space. It can be shown (see Appendix 6.1.3) that $N_V$ is a function of the modal drop size and thus is a function of rain rate,

$$N_V = 5.833 RR a_m^{-7/2}$$

(3.6-4)

The calculation of the effective scattering coefficients (Eq. 3.2-13) include summations over the single drop scattering coefficients weighted by the number of drops in the appropriate finite drop size interval. Single drop scattering coefficients are available typically at 0.25 mm drop radius increments up to 3.25 mm or 3.50 mm, depending on the frequency. By using curve fitting routines, the scattering coefficients can be determined at any point over the interval 0.25 to 3.50 mm. This permits the selection of any convenient drop size
interval. In general define

\[ M_i = \text{number of drop size intervals over the range of drop sizes encountered} \]

\[ \widetilde{a}_i = \text{the equivolumetric drop radius value in mm at the midpoint of the } i^{th} \text{ drop size interval} \]

\[ \Delta \tilde{a} = \text{drop radius interval in mm} \]

Choose \( M_i = 28 \) intervals. This provides a very fine division and the corresponding drop radius interval is \( \Delta \tilde{a} = 0.125 \text{ mm} \). The midpoints of these intervals are

\[ \overline{a}_i = 0.0625 + 0.125(1 - 1) \quad 1 < i \leq 28 \]  

(3.6-5)

The number of drops per unit volume of space in the \( i^{th} \) drop size interval is

\[ N_V(\overline{a}_i) = \int_{\overline{a}_i - \frac{\Delta \tilde{a}}{2}}^{\overline{a}_i + \frac{\Delta \tilde{a}}{2}} n(a) \, da \]  

(3.6-6)

Using Eqs. (3.6-1) and (3.6-3) in Eq. (3.6-6) yield

\[ N_V(\overline{a}_i) = \begin{cases} 
N_V \frac{\Delta \tilde{a}}{a_m} a_i & 0.25 \leq \overline{a} \leq a_m \\
N_V \frac{2\Delta \tilde{a}}{a_m} (1 - \frac{a_m}{2} a_i) & a_m \leq \overline{a} \leq 2a_m \\
0 & \overline{a} > 2a_m 
\end{cases} \]  

(3.6-7)

A rough estimate of usable rain rates for this drop size distri-
bution can be formulated. For $a_m$ less than 0.4 mm the lack of small drop size contributions would become significant. From Eq. (3.6-2) this modal drop radius corresponds to a rain rate of 0.60 mm/hr. For high rain rates the large drop contributions become more important. Drop radii out to $2a_m$ are included in the triangular distribution. Since scattering coefficients are not complete beyond $a = 3.25$ mm, $a_m = 3.25/2 = 1.625$ mm. From Eq. (3.6-2) this corresponds to a rain rate of 316 mm/hr. Thus, the above drop size distribution is reliable for rain rates between 1 mm/hr and 300 mm/hr.

3.6.3 Raindrop Orientation Distribution

The drop orientation distribution is very important to the prediction of cross-polarization. Although Brussaard [19] has shown that wind gradients affect the orientation of falling raindrops, little is known about the nature of the distribution itself. The work of Saunders [26] indicates that the distribution is Gaussian; however, researchers disagree on the range of the mean and the standard deviation. Most have assumed an effective drop orientation and modeled the distribution with a unit impulse function. Although the unit impulse assumption leads to simple formulations, it does not adequately describe the true physical situation. Uzunoglu, Evans, and Holt [5], using the distribution of Saunders, employ a stochastic method of averaging the effects of a canting angle distribution; however, this is not internal to the model and does not consider the effects of arbitrary orientation in the rain cell. Further experimental investiga-
tion into the problem of canting angle distributions is needed; however, until this is accomplished some sort of assumption must be formulated. Experimental measurements of cross-polarization at three frequencies over approximately the same propagation path (see Sec. 4.2) indicate that there is good correlation between theory and measurement with a Gaussian distribution whose mean ranges from \(-10^\circ\) to \(10^\circ\) and with a standard deviation of \(12^\circ\).

The drop orientation distribution within a finite slab can be discretized in the same manner as the drop size distribution. Define

\[ M_k = \text{number of drop orientation intervals over the canting angles encountered} \]

\[ \bar{\theta}_k = \text{the canting angle in radians at the midpoint of the } k^{th} \text{ orientation interval} \]

\[ \Delta \bar{\theta} = \text{canting angle interval in radians} \]

\[ \bar{\theta}_\mu = \text{mean canting angle in radians} \]

\[ \sigma = \text{standard deviation in radians} . \]

Saunders indicates that canting angles ranging from \(-100^\circ\) to \(100^\circ\) can be found in an average rain. Over this range choose \(M_k\) to be \(20\) intervals. Thus, \(\Delta \bar{\theta}\) is \(\pi/18\) or 10 degrees. The midpoints of these intervals are given by the following equation:

\[ \bar{\theta}_k = -1.6581 + 0.1745 k \text{ radians } \quad (1 \leq k \leq 20) . \quad (3.6-8) \]
The number of drops per unit volume of space in the $k^{th}$ orientation interval is

$$N_V(\bar{\theta}_k) = \frac{N_V}{\sqrt{2\pi} \sigma} \int_{\theta_1}^{\theta_2} e^{-\frac{(\theta - \bar{\theta}_k)^2}{2\sigma^2}} d\theta$$

where

$$\theta_1 = (\bar{\theta}_k - \frac{\Delta\theta}{2})$$

$$\theta_2 = (\bar{\theta}_k + \frac{\Delta\theta}{2})$$

To evaluate the effective scattering coefficients of Eq. (3.2-13), the joint distribution of drop size and drop orientation must be known. Since the assumption has been made that the drop orientation is independent of drop size, the joint distribution is simply the product of the two distributions.

$$N_V(\bar{a}_i, \bar{\theta}_k) = N_V(\bar{a}_i) N_V(\bar{\theta}_k).$$

3.6.4 Single Particle Scattering Coefficients and Elevation Angle

It was indicated in Sec. 3.1 that the single particle scattering coefficients are a function of particle size, frequency and elevation angle. Coefficients are available for many frequencies and sizes; however, the selection of coefficients over the spectrum of elevation angles is limited. Uzunoglu, Evans, and Holt [5] have shown that a simple trigonometric relation can be used to evaluate the scattering coefficients for any arbitrary elevation angle. The relation is given below:
\[ f_v(\beta) = f_{\text{SPH}} \sin^2 \beta + f_v(0^\circ) \cos^2 \beta \]

\[ f_h(\beta) = f_{\text{SPH}} \sin^2 \beta + f_h(0^\circ) \cos^2 \beta \]

\[ f_v(\beta) - f_h(\beta) = (f_v(0^\circ) - f_h(0^\circ)) \cos^2 \beta \]  

(3.6-11)

where \( f_{v,h}(0^\circ) \) is the scattering coefficient evaluated for an elevation angle of zero degrees. This approximation is accurate to within 1\% for elevation angles ranging from 0\(^\circ\) to 90\(^\circ\) and provides a simple formulation to evaluate scattering coefficients for a range of elevation angles.

3.6.5 Ice Crystal Distribution

Ice effects on satellite communications links have drawn considerable attention since experimenters recently have observed cross-polarization without a significant fade in the co-polarized signal \([27, 28, 29]\). This so-called "anomalous" depolarization is attributed to a collection of ice crystals at high altitudes. Although the effect is not as severe, ice crystals also affect the isolation of a given communications system when followed by a series of rain cells.

Haworth, McEwan, and Watson \([30]\) have shown that the scattering coefficients for ice particles can be derived using Rayleigh scattering for frequencies up to 30 GHz. The derivation depends on the assumption that the particle eccentricity is very close to one. This physically models the ice particles as ice plates (oblate spheroidal) and ice needles (prolate spheroidal) as shown in Fig. 3.6-2 (see Appendix
Figure 3.6-2. Ice particle shapes.
6.1.4). One disadvantage of this formulation is that it does not lend itself to a distribution of particle orientation. The assumption is made that all particles are equioriented at an effective canting angle. It has been observed by radar that lightning [31] and wind shear [32] align the particles in a given orientation. Since the ice cell is relatively thin, the assumption of an effective canting angle is not necessarily restrictive. The Haworth - McEwan - Watson formulation does include a particle size distribution.

Using the Haworth - McEwan - Watson formulation in Eq. (3.2-13) along with the elevation angle formulation previously mentioned, the effective scattering coefficients of a thin finite slab of ice particles are given below:

\[
S_{xx} = -j \frac{V}{2\lambda} \Delta \xi \left( \sin^2 \theta (A^\text{PRO}_v + A^\text{OBL'}_v) + \cos^2 \theta (A^\text{PRO}_h + A^\text{OBL'}_h) \right)
\]

\[
S_{xy} = S_{yx} = -j \frac{V}{2\lambda} \Delta \xi \left( (A^\text{PRO}_v + A^\text{OBL'}_v) - (A^\text{PRO}_h + A^\text{OBL'}_h) \right) \sin \theta \cos \theta
\]

\[
S_{yy} = -j \frac{V}{2\lambda} \Delta \xi \left( \cos^2 \theta (A^\text{PRO}_v + A^\text{OBL'}_v) + \sin^2 \theta (A^\text{PRO}_h + A^\text{OBL'}_h) \right)
\]

(3.6-12)

where

\[
A^\text{OBL'}_v = \sin^2 \beta A^\text{SPH}_v + \cos^2 \beta A^\text{OBL}_v
\]

(3.6-13)

\[
A^\text{OBL'}_h = \sin^2 \beta A^\text{SPH}_h + \cos^2 \beta A^\text{OBL}_h
\]

and \(A^\text{PRO}_v, A^\text{OBL}_v, A^\text{PRO}_h, A^\text{OBL}_h\) and \(A^\text{SPH}\) are complex constants independent of frequency and are given in Table 3.6-1 (see Appendix 6.1-4). Note that the scattering coefficients describing the prolate particle are independent of the elevation angle \(\beta\). The symbol \(V\) represents the
Table 3.6-1. Ice scattering coefficients.

\[
\begin{align*}
A^\text{PRO}_h &= \{6.812211 - j \ 0.026842\} \\
A^\text{PRO}_v &= \{3.268519 - j \ 0.006179\} \\
A^\text{OBL}_h &= \{6.812211 - j \ 0.026842\} \\
A^\text{OBL}_v &= \{2.150047 - j \ 0.002674\} \\
A^\text{SPH} &= \{3.95417 - j \ 0.009044\}
\end{align*}
\]
total volume of ice in a cubic meter of air. In Eq. (3.6-12) the assumption has been made that the ice slab has 50% ice needles and 50% ice plates.

Haworth, McEwan, and Watson [30] use a value of $V = 10^{-6}$ in their predictions of anomalous depolarization. However, when a ice cell is followed by a series of rain cells, the volume of ice should have some dependence on ground rainfall. The hypothesis is made that the volume of ice per cubic meter of air in the freezing layer of a storm is equal to the volume of rain per cubic meter of air near the ground. The total volume of ice in a cubic meter of air is then a function of rain rate and is expressed by the relation

$$V = (24.43 \times 10^{-9}) \frac{RR}{\sqrt{A_m}}$$  \hspace{1cm} (3.6-14)

One should note that there still can be ice particles present without ground rainfall thus producing anomalous depolarization.

3.6.6 Antenna Effects

The development of the scattering model thus far has not included the interaction of the wave exiting the rain medium with the receive antenna. Even in clear weather conditions, a polarization mismatch is observed with non-ideal antennas. This polarization mismatch is modeled easily and for low rain rates has significant effects in the prediction of attenuation, isolation, and phase shift.

The co-polarized (main) and the cross-polarized (orthogonal) antenna polarization states can be described by a complex vector repre-
sentation. The symbol \( \hat{e}_a \) represents the respective antenna state and is defined below:

\[
\hat{e}_a = \hat{e}_x x + \hat{e}_y y \tag{3.6-15}
\]

where for the co-polarized state

\[
\hat{e}_x = \cos \gamma_{\text{co}} e^{j \delta_{\text{co}}} \\
\hat{e}_y = \sin \gamma_{\text{co}} e^{j \delta_{\text{co}}}
\tag{3.6-16}
\]

and for the cross-polarized state

\[
\hat{e}_x = \cos \gamma_{\text{cross}} e^{j \delta_{\text{cross}}} \\
\hat{e}_y = \sin \gamma_{\text{cross}} e^{j \delta_{\text{cross}}}
\]

The symbols \( \gamma \) and \( \delta \) define the polarization ellipses of the respective antenna states as shown in Fig. 3.6-3.

The wave-antenna interaction can be modeled by the use of a phasor voltage. Define a phasor voltage \( V(w, a) \) which describes the interaction of a wave polarization state \( w \) and an antenna polarization state as

\[
V(w, a) = \hat{E}_w \cdot \hat{e}_a^* \tag{3.6-17}
\]

The wave state exiting the rain medium is found from Eq. (3.4-1) and the antenna states from Eq. (3.6-15).

To evaluate attenuation, isolation, and phase referenced to the antenna ports, the clear weather wave state must be known. The attenu-
\[ \delta = \tan^{-1} \left( \frac{\tan 2\varepsilon}{\sin 2\tau} \right) \]

\[ \gamma = \frac{1}{2} \cos^{-1} \{\cos 2\varepsilon \cos 2\tau\} \]

\[ \varepsilon = \cot^{-1} \{\text{axial ratio}\} \]

Figure 3.6-3. Polarization ellipse.
The attenuation of the co-polarized component is

\[
A = 20 \log_{10} \left| \frac{V(w', a_{\text{co}})}{V(w, a_{\text{co}})} \right| = 20 \log_{10} \left| \frac{E_w - e_{a_{\text{co}}}^*}{E_w' - e_{a_{\text{co}}}^*} \right| \text{ dB} \quad (3.6-18)
\]

where \( E_w \) is the clear weather wave and \( E_w' \) is the wave exiting the rain medium. The isolation between the two receive channels is

\[
I = 20 \log_{10} \left| \frac{V(w', a_{\text{co}})}{V(w', a_{\text{cross}})} \right| = 20 \log_{10} \left| \frac{E_w' - e_{a_{\text{cross}}}^*}{E_w' - e_{a_{\text{co}}}^*} \right| \text{ dB} \quad (3.6-19)
\]

Also of interest is the phase difference between the phasor voltages of the two receive polarization states co and cross. Define

\[
\phi = \text{Cross phase} - \text{Co Phase} = \angle \{ E_w' - e_{\text{cross}}^* \} - \angle \{ E_w' - e_{\text{co}}^* \} \text{ radians} \quad (3.6-20)
\]

The terms attenuation, isolation, and phase appear frequently in the remaining sections of this text and unless otherwise specified, the definitions of Eqs. (3.6-18), (3.6-19), and (3.6-20) will be assumed.

3.6.7 A Synthetic Storm Algorithm

Rain is not usually uniform over the extent of a storm and for increasing rain rates rain cells of higher rain rate tend to decrease in size. However, most researchers assume uniform rains and account for this phenomenon with an effective rain extent referred to in the literature as effective path length. As rain rate increases, effective path length decreases. The effective path length formulation is based
on experimental measurements and theoretical predictions for uniform rain conditions. The intention is to formulate an effective path length based on statistical data that can be used for a variety of locations to predict rain effects on millimeter wave propagation. There is one major disadvantage to this approach: effective path lengths are frequency dependent. Thus, experimental data is required for all frequencies of interest and this is overly restrictive. The frequency dependence of effective path length is investigated in this section and a frequency independent synthetic storm algorithm that models nonuniform rain rates is presented.

The actual or measured attenuation on a given propagation path with physical rain extent \( L \) can be expressed as

\[
A_m = \int_0^L A_1(RR(t), f) \, dt
\]  
(3.6-21)

where \( A_1(RR(t), f) \) is attenuation per meter and is a function of rain rate and frequency. Note that rain rate is also a function of length \( t \). Effective path length in km is defined as

\[
L_{e}(RR_{AVG}, f) = \frac{A_m}{A_{AVG}^{AVG}(RR_{AVG}, f)}
\]  
(3.6-22)

where \( A_{AVG}^{AVG}(RR_{AVG}, f) \) is a theoretically predicted attenuation per kilometer assuming a uniform average rain rate. The average rain rate is based on rain rate measurements taken over a long period of time. The measured attenuation is also indirectly a function of average rain rate. Measured attenuation and measured rain rate data are compared on an equal probability of occurrence basis over a long time base (see
Sec. 4.1). This removes the instantaneous time dependence of the measurements. Note that if rain rate is not a function of length $\xi$, $A_m = A_{\text{AVG}}(\text{RR}_{\text{AVG}}, f) L$, and the effective path length would equal the physical rain extent $L$.

To investigate the frequency dependence of effective path length, consider the ratio of two effective path lengths for two frequencies $f_1$ and $f_2$:

$$
\frac{L_e(\text{RR}_{\text{AVG}}, f_1)}{L_e(\text{RR}_{\text{AVG}}, f_2)} = \frac{\int_0^L A_{\text{AVG}}(\text{RR}_{\text{AVG}}, f_2) d\xi}{\int_0^L A_{\text{AVG}}(\text{RR}_{\text{AVG}}, f_1) d\xi} = R_p R_m.
$$

(3.6-22)

Ratio $R_m$ is the ratio of measured attenuation for the two frequencies. Ratio $R_p$ is a theoretical prediction of the ratio of attenuation assuming uniform rain conditions. For effective path length to be independent of frequency, $R_p$ must equal $R_m$ inverse. The effective path lengths for two frequencies (19.04 GHz and 28.56 GHz) are plotted as a function of statistical (average) rain rate in Fig. 3.6-4. Effective path length data for these curves were calculated from three months (July, August and September, 1977) measured attenuation and rain rate data. The data were collected at the VPI&SU earth station using the COMSTAR D2 satellite. It is obvious from Fig. 3.6-4 that effective path length is frequency dependent. Other researchers have also observed this frequency dependence [33]. This indicates that for higher rain rates a theoretical model should not assume uniform rain conditions, but model the nonuniform rain with a synthetic storm.
Figure 3.6-4. Effective path lengths for the VPI&SU COMSTAR 19 and 28 systems based on measured data collected during the months of July, August, and September, 1977.
algorithm.

The scattering model, because of its ability to represent a piecewise homogeneous rain, can accommodate easily a synthetic storm algorithm. The storm algorithm will model rain rate as a function of position along the path and storm extent as a function of elevation angle. Based on the data of Fig. 3.6-4, two physical assumptions will be made:

1) the rain is uniform for low rain rates
2) as rain rate increases, the rain becomes nonuniform.

The first step in the development of a synthetic storm algorithm is to obtain a functional relationship between effective storm extent and elevation angle. Define effective storm height as

\[ H_e = L_e (10 \text{ mm/hr}) \sin \beta \]

(3.6-23)

where \( L_e \) is the effective path length calculated for a rain rate of 10 mm/hr, and \( \beta \) is the elevation angle. Since for low rain rates the rain is nearly uniform, effective path length at 10 mm/hr is frequency independent \( (R_p = R_m^{-1}) \) and equal to the physical rain extent \( L \). As a result, Eq. (3.6-23) can be rewritten as

\[ H_e = L \sin \beta \text{ km} \]

(3.6-24)

or the physical rain extent is

\[ L = \frac{H_e}{\sin \beta} \text{ km} \quad (\beta \neq 0) \]

(3.6-25)
Note that effective storm height approaches zero as elevation angle approaches zero. The effective storm height formulation provides a limit to the storm extent both in height and ground extent. Using the data of various experimenters [17], [34], [35] at various locations, elevation angles, and frequencies, the functional relationship between elevation angle and effective storm extent can be obtained. Valid statistical attenuation data are very limited over the spectrum of frequencies and elevation angles; however, enough data are available to provide a first order approximation for an effective storm model. The available data at a rain rate of 10 mm/hr are used to calculate effective storm heights and these are tabulated in Table 3.6-2. Using the data of Table 3.6-2, an effective storm model is presented in Fig. 3.6-5. From this figure given an elevation angle, the physical rain extent \( L \) can be calculated where \( L \) is not a function of rain rate.

To model the nonuniform rain rate along the physical rain extent, assume that the maximum rain rate occurs in the rain cell closest to the receive antenna or cell \( M \) of Fig. 3.4-1. This is a valid assumption over long periods of time since it has been shown that statistical rain rate is independent of the measurement location [35]. The rain rates of all other rain cells will be a function of the \( M^{th} \) cell. This functional relationship is the storm algorithm.

To obtain the functional relationship, another comparison to measured attenuation data is necessary. However, for this comparison only data for a single frequency and a single location are needed. Figure 3.6-6 is a plot of measured attenuation data at 28.56 GHz as a
Table 3.6-2. Effective storm heights obtained from measured attenuation and rain rate statistics from various locations.

<table>
<thead>
<tr>
<th>Data Point</th>
<th>$\beta$</th>
<th>$f$</th>
<th>$H_e$</th>
<th>Location</th>
</tr>
</thead>
<tbody>
<tr>
<td>1)</td>
<td>54°</td>
<td>30 GHz</td>
<td>5.4 km</td>
<td>University of Texas</td>
</tr>
<tr>
<td>2)</td>
<td>44°</td>
<td>28 GHz</td>
<td>5.7 km</td>
<td>VPI&amp;SU</td>
</tr>
<tr>
<td>3)</td>
<td>44°</td>
<td>19 GHz</td>
<td>5.6 km</td>
<td>VPI&amp;SU</td>
</tr>
<tr>
<td>4)</td>
<td>23°</td>
<td>28 GHz</td>
<td>4.4 km</td>
<td>Comsat Labs</td>
</tr>
<tr>
<td>5)</td>
<td>23°</td>
<td>19 GHz</td>
<td>4.4 km</td>
<td>Comsat Labs</td>
</tr>
</tbody>
</table>
Figure 3.6-5. Effective storm model based on data from Table 3.6-2. (Rain extent versus elevation angle.)
Figure 3.6-6. Measured 28 GHz attenuation compared to the theoretical prediction of the scattering model using the synthetic storm algorithm. (28.56 GHz)
function of statistical rain rate. The data were taken over a three
month period (July, August and September, 1977) at the VPI&SU earth
station. To remove the instantaneous time dependence, the curve is
plotted using equal probability techniques. For an elevation angle of
44°, using Fig. 3.6-2 along with Eq. (3.6-25) the rain extent L is
8.2 km.

Assume that the rain extent can be modeled by ten rain cells as
shown in Fig. 3.6-7. The rain rate of cells nine and ten equals the
statistical rain rate of Fig. 3.6-6. Rain cells one through eight are
equal and are related to cell ten by the following relation:

\[ RR_i = \left( \frac{x_i}{10} \right) RR_{10} \text{ mm/hr} \quad (3.6-26) \]

This relation models the nonuniform rain rate over the rain extent.
The value of \( x_i \) can be obtained by trial and error until the attenua-
tion corresponding to a rain rate of 40 mm/hr is predicted by the
scattering model. The value of \( x_{9,10} \) by definition is zero. The
value of \( x_{1-8} \) is found to be

\[ x_{1-8} = -0.66 \quad (3.6-27) \]

Although the value of \( x_{1-8} \) was obtained from one set of attenuation
data, it models the nonuniformity of a rain medium as rain rate
increases and is completely general for a variety of frequencies,
elevation angles, and locations.

The theoretical prediction for 28 GHz attenuation over the entire
spectrum of rain rate is shown in Fig. 3.6-6. Other comparisons to
Figure 3.6-7. Synthetic storm algorithm, rain rate versus physical rain extent.
measured data using the above formulation are presented in Chapter IV. Given an elevation angle, the synthetic storm algorithm models non-uniform rain rates and effective storm extent and reliably predicts rain effects on communications links regardless of site location.

3.6.8 Rain Propagation Prediction Program

The Rain Propagation Prediction program (RPP) uses the scattering model and the synthetic storm algorithm of the previous section to predict attenuation, isolation, and phase as a function of rain rate. The input format allows changes in system parameters so that the effects of rain on millimeter wave propagation as a function of frequency, location and elevation angle can be predicted. The program is written in FORTRAN and its operational format is explained in Appendix 6.2. The program listing is found in Appendix 6.2.3. The RPP program is computationally efficient and it accurately predicts measured experimental data.
CHAPTER IV
EXPERIMENTAL VERIFICATION

In this chapter the theoretical predictions of the scattering model are compared to measured data obtained from three different site locations: (1) VPI&SU, Blacksburg, Virginia; (2) Comsat Labs, Clarksburg, Maryland; and (3) University of Texas, Austin, Texas. The data obtained from these sites cover the existing frequency range of interest for millimeter wave communications links (11 to 30 GHz) and provide a good comparison of theoretical predictions to measured data for a variety of system parameters. Before the data comparison is made a system description of the VPI&SU experiment is presented with an emphasis on general data reduction techniques. Then measured attenuation, isolation, and phase data for three frequencies (11.7 GHz, 19.04 GHz, and 28.56 GHz) collected by the VPI&SU experiment are compared with theory. The scattering model predictions are then compared to attenuation data measured by Comsat Labs and the University of Texas to demonstrate the flexibility of the synthetic storm algorithm. The chapter concludes with an investigation of frequency scaling techniques and the frequency dependence of attenuation and isolation.

4.1 The VPI&SU Experiment

4.1.1 General System Description

The Virginia Polytechnic Institute and State University (VPI&SU) millimeter wave propagation experiment is sponsored by NASA with
important additive support from the Defense Communications Agency and
the U.S. Army Research Office. The experiment monitors continuous
transmissions from the Communications Technology Satellite (CTS) at
11.7 GHz and the COMSTAR D2 satellite at 19.04 GHz and 28.56 GHz. The
VPI&SU experiment is designed to measure attenuation, isolation, and
phase shift and correlate these quantities with rain rate and other
weather data. The experiment has been described in considerable detail
in previous publications [36], [37], [17] and will be reviewed here
only briefly with an emphasis on data reduction techniques.

A block diagram of the VPI&SU system is presented in Fig. 4.1-1.
The experiment is controlled by a PDP-11/10 mini computer which monitors
the experiment through a digital controller and analog to digital
converter interface. The three different frequency systems are moni-
tored simultaneously and the PDP 11/10 stores receiver data along with
weather and time reference data on a storage disk for later transfer to
an IBM/370 for data reduction. The CTS (11.7 GHz) satellite transmits
a right hand circularly polarized wave and the CTS ground antenna
receives both the co-polarized and cross-polarized signals. The
COMSTAR receive antennas are both dual-linearly polarized. The
COMSTAR satellite transmits vertical polarization at 28.56 GHz; however,
the COMSTAR 19.04 GHz beacon switches between vertical and horizontal
polarization at a 1 KHz rate. This feature was incorporated into the
propagation experiment to provide information on the differential
attenuation and differential phase properties of the rain medium. As
a result of this switching polarization, there are four
Figure 4.1-1. VPI&SU system block diagram.
receive channels (Vertical Co, Vertical Cross, Horizontal Co, and Horizontal Cross) for the 19 GHz system. The 19 GHz IF signal processor has a deswitching control unit that provides a switching signal to monitor the various polarization states. The 28 GHz system is also a dual-polarized system and monitors the co-polarized and cross-polarized signals. From these eight channels, attenuation, isolation, and phase data are recorded for the different frequencies and polarizations. The dynamic range of each system and other system parameters are tabulated in Table 4.1-1. The polarization parameters describing the polarization ellipses of the incoming waves and antenna states are tabulated in Table 4.1-2.

4.1.2 Preliminary Data Processing

As indicated in the previous section, on-line data acquisition is controlled by the PDP 11/10 mini computer. Each data channel has an identifying number that is used throughout the data reduction process. The PDP 11/10 also monitors various status indicators that indicate the validity of the data and whether it should be stored on the PDP 11/10 storage disk. Data are stored only when the status indicators indicate valid data and when the specific data input has changed by a predetermined amount. Data are stored with a time flag and other identifiers for later off-line processing.

During periods of low experimental activity, accumulated experimental data can be transferred to the main IBM 370 computer for subsequent data reduction. Before each data transfer all data are
Table 4.1-1. VPI&SU system parameters.

<table>
<thead>
<tr>
<th>System Parameter</th>
<th>CTS  (11.7 GHz)</th>
<th>COMSTAR (19.04 GHz)</th>
<th>COMSTAR (28.56 GHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Antenna</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Reflector size:</td>
<td>12'</td>
<td>4'</td>
<td>4'</td>
</tr>
<tr>
<td>Gain:</td>
<td>50.9 dBi</td>
<td>44.7 dBi</td>
<td>44.7 dBi</td>
</tr>
<tr>
<td>Polarization:</td>
<td>RHCP, LHCP</td>
<td>LV, LH</td>
<td>LV, LH</td>
</tr>
<tr>
<td>Look angle</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Azimuth:</td>
<td>240°</td>
<td>213°</td>
<td>213°</td>
</tr>
<tr>
<td>Elevation:</td>
<td>33°</td>
<td>44°</td>
<td>44°</td>
</tr>
<tr>
<td><strong>RF Front Ends</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Co (Gain/Noise Temp):</td>
<td>41.5 dB/962° K</td>
<td>34.8 dB/1540° K</td>
<td>37.5 dB/1716° K</td>
</tr>
<tr>
<td>Cross (Gain/Noise Temp):</td>
<td>79 dB/184° K</td>
<td>32.8 dB/1450° K</td>
<td>34.5 dB/1716° K</td>
</tr>
<tr>
<td><strong>IF Signal Processors</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dynamic Range (Co/Cross)</td>
<td>39 dB/39 dB</td>
<td>30 dB/30 dB</td>
<td>39 dB/39 dB</td>
</tr>
<tr>
<td>Clear Weather Input Power</td>
<td>-78 dBm/-75 dBm</td>
<td>-90 dBm/-125 dBm</td>
<td>-86 dBm/-121 dBm</td>
</tr>
<tr>
<td>Clear Weather Isolation:</td>
<td>37 dB</td>
<td>35 dB</td>
<td>35 dB</td>
</tr>
</tbody>
</table>
Table 4.1-2. VPI&SU polarization parameters.

<table>
<thead>
<tr>
<th>Polarization Parameters</th>
<th>CTS (11.7 GHz)</th>
<th>COMSTAR (19.04 GHz)</th>
<th>COMSTAR (19.04 GHz)</th>
<th>COMSTAR (28.56 GHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Co-Polarized Wave</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ε:</td>
<td>-44.34°</td>
<td>0.82°</td>
<td>0.82°</td>
<td>1.02°</td>
</tr>
<tr>
<td>τ:</td>
<td>0°</td>
<td>52.5°</td>
<td>142.5°</td>
<td>52.5°</td>
</tr>
<tr>
<td>Co-Polarized Antenna</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ε:</td>
<td>-43.68</td>
<td>0°</td>
<td>0°</td>
<td>0°</td>
</tr>
<tr>
<td>τ:</td>
<td>0°</td>
<td>52.5°</td>
<td>142.5°</td>
<td>52.5°</td>
</tr>
<tr>
<td>Cross-Polarized Antenna</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ε:</td>
<td>43.68</td>
<td>0°</td>
<td>0°</td>
<td>0°</td>
</tr>
<tr>
<td>τ:</td>
<td>90°</td>
<td>142.5°</td>
<td>52.5°</td>
<td>142.5°</td>
</tr>
</tbody>
</table>

Note: See Fig. 3.6-3 for definition of ε and τ.
checked for time sequence and any required calibration is performed at this time. The data are transferred from the PDP-11/10 storage disk to magnetic tape and then read by the IBM 370 onto a system data base tape. The data base tape is a complete time history record of the VPIL4SU experiment. A data exception tape is stored also in the IBM 370. This tape records time markers of any system failure that cannot be determined by the status indicators such as a complete power failure and any other special cases in which erroneous data may have been stored. For a particular data reduction time interval, these two tapes are merged to form a process file. This process file can be accessed through an IBM time sharing system that allows efficient data reduction over the specified time interval. An inter-active graphics system is also available to aid the data reduction process.

4.1.3 Data Reduction

The data reduction software [38] is capable of processing data in a wide range of data reduction formats. A few of these formats will be illustrated in this section. Figures 4.1-2a through 4.1-2e illustrate rain rate and signal behavior during a storm observed on August 9, 1977. These figures are time history plots and illustrate the instantaneous time dependence of signal level and rain rate. Note that the rain began to fall after significant fading had occurred at all three frequencies. The storm was severe enough (125 mm/hr) to cause the 19 GHz and 28 GHz systems to lose phase lock. Note that the peak fades occurred approximately at the same time and the severity of the fades
Figure 4.1-2a. Ground rainfall rate on August 9, 1977. (The rain gauge is located beside the receiving antennas.)

Figure 4.1-2b. CTS 11.7 GHz co-polarized signal behavior on August 9, 1977. (Measured fade 15 dB)
Figure 4.1-2c. 19 GHz vertical co-polarized signal behavior on August 9, 1977.
(Measured fade 32 dB)

Figure 4.1-2d. COMSTAR 28 GHz co-polarized signal behavior on August 9, 1977.
(Measured fade 32 dB)
Figure 4.1-2e. 11.7 GHz cross-polarized signal behavior on August 9, 1977.

Figure 4.1-2. VPI&SU system time history plots.
increased with frequency.

Figures 4.1-3a and 4.1-3b are scatter plots which provide an insight into the instantaneous interdependence of attenuation and isolation at different frequencies. Figure 4.1-3a is a plot of isolation versus attenuation at 11.7 GHz for the storm of August 9, 1977. Figure 4.1-3b is a comparison of attenuation of the 19 GHz vertical and the 28 GHz COMSTAR channels during the storm of August 9, 1977. The attenuation ratio for these two frequencies is approximately 2 to 1. Attenuation ratios will be discussed in more detail in Sec. 4.4.

The data reduction process used in Figures 4.1-2 and 4.1-3 did not include any statistical techniques. However, for the prediction of rain effects on millimeter wave signals the removal of the instantaneous time dependence from measured data is necessary. This is accomplished by using the VPI&SU reduction software coupled with a statistical analysis system called SAS [39]. System engineers designing communications links operating in the millimeter wave band are interested in the percent of time that attenuation, isolation, and rain rate will equal or exceed a given value. Such "exceedence" plots are presented in Fig. 4.1-4 and 4.1-5. Figure 4.1-4 is a plot of the attenuation statistics for July, August, and September, 1977. Figure 4.1-5 is a plot of the rain rate statistics over the same time base. The theoretical prediction of the Rice-Holmberg rain rate model [40] is also plotted in Fig. 4.1-5 and agrees rather well with the measured data. The Rice-Holmberg model will be discussed in more detail in Sec. 4.2.
Figure 4.1-3a. 11.7 GHz isolation versus attenuation for the storm of August 9, 1977.

Figure 4.1-3b. A comparison of attenuation on the 19 GHz vertical and the 28 GHz COMSTAR channels during the storm of August 9, 1977.
Figure 4.1-4. Measured attenuation statistics for July, August, and September, 1977.
Figure 4.1-5. Rain rate statistics for July, August, and September, 1977.
In Fig. 3.6-6 a plot of attenuation versus rain rate was presented using equal probability techniques. This plot was obtained by comparing attenuation data and rain rate data of Figs. 4.1-4 and 4.1-5 for a given percent of time. This process removes the instantaneous time dependence as seen in Fig. 4.1-2 and represents the data on an equal probability of occurrence basis. Figure 3.6-6 is duplicated in Fig. 4.1-6 for comparison to Figs. 4.1-4 and 4.1-5.

4.2 Comparison of Theory with Measured Data from the VPI&SU Experiment

This section demonstrates the ability of the scattering model to predict the effects of rain on a dual-polarized satellite communications link. Theoretical predictions of the scattering model will be compared to measured attenuation, isolation, and phase data collected by the VPI&SU earth station during the months of July, August, and September, 1977. Experimental results are presented using graphs which also contain theoretical predictions of the rain propagation prediction program (RPP). These graphs make the comparison between theory and experiment evident at a glance. The frequency range of the VPI&SU data and the polarization diversity of the VPI&SU experiment provide an excellent opportunity to test the scattering model with a variety of system parameters.

Before theoretical predictions can be formulated for the three systems of the VPI&SU earth station, the input parameters for the RPP program must be determined. The only input parameters needed are the operating frequency, the elevation angle, the effective physical rain
Figure 4.1-6. Measured 28 GHz attenuation versus rain rate. (Plotted using equal probability of occurrence techniques.)
extent, and the system polarization parameters. The polarization parameters and the elevation angles for the three systems are given in Tables 4.1-2 and 4.1-1, respectively. Knowing the system elevation angle, the physical rain extent \( L \) can be determined using Fig. 3.6-5 and Eq. (3.6-25). The effective rain extents for the CTS and COMSTAR systems are 10 km and 8.2 km, respectively. The RPP program is currently limited to the discrete frequency values of 11.0, 14.0, 20.0, and 30.0 GHz. In the following figures the frequencies given are those of the measurement frequency and the particular frequency used in the associated theoretical prediction is the closest available value.

4.2.1 Attenuation

Engineers designing satellite communications links are faced with the problem of determining rain fade margins to provide a reliable communications link during periods of inclement weather. As the operating frequency increases, rain fading becomes more severe. As a result, fade margins have to increase or ground station site diversity must be employed. Either of these solutions is expensive. Data are being collected to help determine the best solution to rain fade problems; however, for satellite links this collection of data is also expensive. For this reason, the development of a reliable theoretical model is necessary. The purpose of this section is to demonstrate the effectiveness of the scattering model in predicting the attenuation experienced on a satellite communications link for a given frequency and ground rain rate.
The relationship between attenuation and ground rain rate as measured by one tipping bucket rain gauge near the VPI&SU antennas is illustrated in Fig. 4.2-1. Measured data are presented using equal probability data reduction techniques. The theoretical predictions are those of the RPP program where the nonuniform rain rates present on a satellite link have been modeled using the synthetic storm algorithm presented in Sec. 3.6. As can be seen, the agreement between theory and experiment is very good over the entire rain rate spectrum for all three frequencies. The severe fading at the higher frequencies (19.04 and 28.56 GHz) indicate that site diversity may be the only solution for communications links operating at these frequencies and requiring a high degree of reliability.

Simple equations relating attenuation to rain rate can be written in the form

\[ A = a \cdot R^b \, \text{dB} \quad (4.2-1) \]

where \( A \) is attenuation in dB and \( R \) is rain rate in mm/hr. Using power curve regression techniques on both the measured and theoretical data presented in Fig. 4.2-1, the constants \( a \) and \( b \) of Eq. (4.2-1) can be determined and are given in Table 4.2-1. These values include the effects of the synthetic storm algorithm and are valid for rain rates up to 60 mm/hr.

Ground station site selection is aided by the knowledge of the percent of time that a given value of attenuation will be exceeded during a particular time interval. Thus, the percent of time that the
Figure 4.2-1. Attenuation versus rain rate for July, August, and September, 1977.
(11.7, 19.04, 28.56 GHz)
Table 4.2-1. Power curve fits to attenuation versus rain rate data presented in Fig. (4.2-1).
(RR \leq 60 \text{ mm/hr})

<table>
<thead>
<tr>
<th>Frequency</th>
<th>Theory (RPP)</th>
<th>Measured (VPI2SU)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>a</td>
<td>b</td>
</tr>
<tr>
<td>11 GHz</td>
<td>0.9646</td>
<td>0.6158</td>
</tr>
<tr>
<td>19 GHz</td>
<td>2.5878</td>
<td>0.5598</td>
</tr>
<tr>
<td>28 GHz</td>
<td>6.4779</td>
<td>0.4569</td>
</tr>
</tbody>
</table>
link performance is degraded beyond acceptable system specifications can be determined. Using the scattering model coupled with the Rice-Holmberg rain rate model [40], attenuation statistics can be predicted that agree rather well with the measured attenuation statistics of the VPI&SU earth station. The Rice-Holmberg rain rate model is an empirical fit to a very large data base of measured rain rate data. The model predicts the percent of time that a given rain rate was exceeded based on the total rain accumulation over the time period in question. The Rice-Holmberg equation is given below:

\[ \%T(RR) = A_{RH} (0.038 e^{-0.03RR} + 0.2(1 - \beta_{RH})e^{-0.258RR} + 1.86 e^{-1.63RR}) \]

(4.2-2)

where \( A_{RH} \) is the total rain accumulation in \( \text{mm times 100 divided by the number of hours in the accumulation period} \), \( \beta_{RH} \) is the ratio of accumulation of "thunder storm rain" to the total accumulation of rain, and \( RR \) is rain rate in \( \text{mm/hr} \). The total accumulation of rain measured by the VPI&SU earth station was 153 mm for the months of July, August, and September, 1977 (2208 hours). As a result,

\[ A_{RH} = 6.93 \text{ \( \text{mm/hr} \)} \]

(4.2-3)

Since the time period in question is the thunder storm season,

\[ \beta_{RH} = 0.6 \]

(4.2-4)

The Rice-Holmberg prediction is compared to measured rain rate statis-
tics in Fig. 4.2-2. The United States Weather Service observer (located within 8 km of the VPI&SU rain gauge) measured an accumulation of 418 mm for the same time period. This indicates that over short statistical time periods (less than a year), the local terrain may bias the observed statistics, particularly in mountainous regions. However, over long statistical time periods the design engineer should be able to use rain accumulation data from the USWS and predict rain rate and attenuation statistics for a given location [41].

Attenuation statistics for the months of July, August, and September, 1977 can be predicted using the theoretical power curve regressions in Table 4.2-1 and the Rice-Holmberg rain rate equation. Given a value of attenuation, the associated value of rain rate can be determined using the inverse of Eq. (4.2-1) given below:

\[ RR = e^{b \ln\left(\frac{A}{a}\right)} \text{ mm/hr} \]  

(4.2-5)

Using Eq. (4.2-5) in Eq. (4.2-2), the percent of time that a given value of attenuation is exceeded can be determined. Figure 4.2-3 compares the theoretical attenuation exceedence plots to those measured at the VPI&SU earth station. As can be seen for rain rates above 10 mm/hr (.1%), experiment and theory agree rather well for all three frequencies.

4.2.2 Isolation

As the channel capacity of single-polarized satellite communications systems become saturated, dual-polarized communications systems
Figure 4.2-2. Measured rain rate statistics for July, August, and September, 1977 compared to the Rice and Holmberg rain rate model.
Figure 4.2-3. Measured attenuation statistics for July, August, and September, 1977 compared to the theoretical predictions using the Rice and Humberg rain rate model and the scattering model.
have been suggested [1] to increase channel capacity without significantly increasing the system cost. However, for operating frequencies above 10 GHz the isolation between the dual-polarized channels is affected significantly by rain. In this section theoretical predictions of isolation as related to attenuation are compared to measured data taken by the VPI&SU earth station during the month of August, 1977. Theoretical predictions of isolation statistics for the months of July, August, and September, 1977 are also presented along with the effects of ice on the isolation of a dual-polarized communications link.

In the previous section measured data were presented using equal probability data reduction techniques. However, multiple frequency isolation statistics are not presently available because of data reduction difficulties. Since hard rains often cause the COMSTAR 19 and 28 systems to lose phase lock, uncertainties have developed in how best to handle these periods when computing isolation statistics. This problem was overcome in the evaluation of attenuation statistics by frequency scaling from lower frequencies where the receiver remained phase locked. However, as of this writing no standard format has been decided upon to scale isolation. Frequency scaling is discussed in more detail in Sec. 4.4. Measured isolation data presented in this section were reduced using instantaneous data reduction techniques. For a given value of attenuation, all of the corresponding values of isolation are averaged and a standard deviation computed. For a given value of attenuation, the scatter in the associated isolation values is significant and can be explained by examining the physical scatter-
The scatter in measured isolation data can be accounted for by three physical properties of the rain medium: 1) the oscillation of the mean canting angle with wind gusts [42], 2) the presence or the absence of a freezing layer, and 3) the movement of inhomogeneous rain cells along the propagation path. The assumption is made that wind gusts can cause the mean canting angle to range between $-10^\circ$ and $10^\circ$. However, for a prevailing wind direction, the mean canting angle is predominately negative or positive [42]. Attenuation is relatively insensitive to changing canting angle; however, isolation can vary up to 3 or 4 dB depending on the canting angle and rain intensity. The presence of an ice layer during a rain storm has no significant effect on signal attenuation; however, channel isolation degrades with an increase in ice particle concentration. The effects of canting angle oscillation and the presence of an ice layer on the isolation of a dual-polarized communications link can be predicted by the scattering model. Figures 4.2-4, 4.2-5, and 4.2-6 compare the predictions of the RPP program to measured isolation and attenuation data during the month of August, 1977. In each of the fore-mentioned figures there are four theoretical curves a, b, c, and d with the following mean canting angle and ice content:

\[
\begin{align*}
  a: & \quad \bar{\theta}_\mu = 10^\circ, \quad \text{no ice} \\
  b: & \quad \bar{\theta}_\mu = -10^\circ, \quad \text{no ice} \\
  c: & \quad \bar{\theta}_\mu = 10^\circ, \quad \text{ice layer present} \\
  d: & \quad \bar{\theta}_\mu = -10^\circ, \quad \text{ice layer present} 
\end{align*}
\]
Figure 4.2-4. Isolation versus attenuation for the VPI&SU CTS system for August, 1977. (11.7 GHz)
Figure 4.2-5. Isolation versus attenuation for the VPI&SU COMSTAR 19 system for August, 1977. (19.04 GHz)
Figure 4.2-6. Isolation versus attenuation for the VPI & SU COMSTAR 28 system for August, 1977. (28.56 GHz)
The ice layer is assumed to be 1 km thick and since the ice layer does not affect signal attenuation significantly, the effective physical rain extent $L$ remains the same. Theory and experiment agree rather well for all three frequencies. However, close examination of the three figures indicates that the scatter in isolation increases with frequency. This is not completely accounted for by the oscillation of the canting angles or the presence of an ice layer. Because the data presented in the three figures are well within the system dynamic range, the scatter should not be a result of receiver error. One possible explanation is the movement of inhomogeneous rain cells along the propagation path. As indicated in Sec. 3.5, isolation may be non-reciprocal under inhomogeneous rain conditions and the magnitude of this nonreciprocity increases with frequency. So for a given value of attenuation (attenuation is reciprocal), there could be a range of varying isolation values depending on the inhomogeneous rain conditions.

Mean isolation can be calculated from attenuation through equations of the form

$$I_m = U + V \ln (A) \text{ dB}$$

(4.2-6)

where $I_m$ is the mean isolation and $A$ is attenuation in dB. If reliable values of $U$ and $V$ can be determined for arbitrary earth station locations, then Eq. (4.2-6) could be used to calculate isolation statistics from existing attenuation statistics. This would enable communications engineers to predict the performance that would be available if an existing single-polarized link were replaced by a
dual-polarized link. The theoretical prediction assuming no ice and a canting angle of -10° appears to best describe mean isolation as a function of attenuation for all three frequencies. With this assumption and using logarithmic curve regression techniques, the constants \( U \) and \( V \) of Eq. (4.2-6) can be determined for the mean isolation data presented in Figs. 4.2-4, 4.2-5, and 4.2-6 and are given in Table 4.2-2. The logarithmic curve fits for this case for all three frequencies (11.0, 20.0, and 30.0 GHz) are presented in Fig. 4.2-7 to illustrate the frequency dependence of isolation and attenuation. For a given value of attenuation, channel isolation improves with frequency. It should be noted that the 11 GHz system is circularly polarized and as a result isolation is slightly worse than if the system were linearly polarized.

Isolation statistics can also be predicted without a prior knowledge of attenuation statistics. Using the scattering model coupled with the Rice-Holmberg equation, isolation statistics can be predicted for arbitrary site locations based on total rain accumulation. Mean isolation can be calculated from ground rain rate through equations of the form

\[
I_m = T + W \ln (RR) \text{ dB} \quad (4.2-7)
\]

where \( I_m \) is the mean isolation in dB and RR is rain rate in mm/hr. The constants \( T \) and \( W \) again can be determined using logarithmic curve fitting techniques and are given in Table 4.2-3. For the theoretical assumptions of Fig. 4.2-7 (no ice, \( \bar{\theta}_u = -10^\circ \)), given a value of isolation the associated value of rain rate can be determined using the
Table 4.2-2. Logarithmic curve fits to mean isolation versus attenuation for August, 1977. (For theory, $A < 40$ dB; for measured CTS and COMSTAR 19, $A < 15$ dB; and for measured COMSTAR 28, $A < 30$ dB)

<table>
<thead>
<tr>
<th>Frequency</th>
<th>Theory $\theta_{\mu} - 10^\circ$ No Ice</th>
<th>Measured (VPI&amp;SU)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Theory</td>
<td>RPP</td>
</tr>
<tr>
<td>11 GHz</td>
<td>42.2507</td>
<td>-9.4005</td>
</tr>
<tr>
<td>19 GHz</td>
<td>40.1418</td>
<td>-7.6020</td>
</tr>
<tr>
<td>28 GHz</td>
<td>42.6301</td>
<td>-7.0897</td>
</tr>
</tbody>
</table>

* $r^2$ is low because of the scatter in the data.
Figure 4.2-7. Logarithmic curve fits of theoretical isolation versus attenuation. (August, 1977)
Table 4.2-3. Logarithmic curve fits to theoretical isolation versus rain rate data. (Rain rate < 60 mm/hr, $\bar{\theta}_\mu = -10^\circ$, no ice)

<table>
<thead>
<tr>
<th>Frequency</th>
<th>T</th>
<th>W</th>
<th>$r^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>11 GHz</td>
<td>42.7801</td>
<td>-5.8524</td>
<td>1.00</td>
</tr>
<tr>
<td>20 GHz</td>
<td>32.7645</td>
<td>-4.2008</td>
<td>0.99</td>
</tr>
<tr>
<td>30 GHz</td>
<td>29.0694</td>
<td>-3.1824</td>
<td>0.98</td>
</tr>
</tbody>
</table>
inverse of Eq. (4.2-7) given below:

\[ RR = \exp\left(\frac{I_m - T}{W}\right) \text{ mm/hr} \quad (4.2-8) \]

Using Eq. (4.2-8) in Eq. (4.2-2), the percent of time that isolation is less than a given value can be determined. Theoretical isolation statistics are presented in Fig. 4.2-8 without validation by experiment for all three frequencies. Isolation statistics are affected by parameters other than rain such as tracking error and antenna polarization sensitivity. Figure 4.2-8 only accounts for rainfall degradation.

A decrease in channel isolation also can be observed without an associated rain event. This phenomenon is due to ice crystals which can depolarize incident fields without significantly affecting the copolarized signal. Assuming that a 1 km freezing layer can exist without ground rainfall, Fig. 4.2-9 illustrates the effect this freezing layer would have on the VPI&SU earth station for various particle canting angles. Although these effects have not been observed by the VPI&SU facility, ice depolarization has been observed in other climatic zones [29]. The theoretical data in Fig. 4.2-9 is presented to illustrate the ice depolarization phenomenon and to demonstrate the flexibility of the RPP program. Note that the isolation of the circularly polarized 11 GHz system is relatively constant over the ice crystal canting angle range.

4.2.3 Phase

To improve the channel isolation of a dual-polarized communica-
Figure 4.2-8. Theoretical prediction of the isolation statistics for the VPI\&SU system. (July, August, and September, 1977)
50% Oblate, 50% Prolate Particles
1 KM of Ice, No Rain
Volume of Ice/M³ = 10⁻⁶

Figure 4.2-9. Theoretical predictions of the scattering model of ice depolarization in the absence of rain for different particle orientations.
tions link during precipitation events, static or adaptive cancellation systems are being considered. These systems adaptively change the polarization state of the receive antenna to improve system isolation. To design adaptive cancellation systems, a prior knowledge of the relative phase of the dual-polarized channels is often helpful. For convenience the term "phase" represented by the symbol \( \phi \), is defined as the cross-polarized channel phase minus the co-polarized channel phase. By controlling the relative phase \( \phi \) between the two channels and the differential channel attenuation, system isolation can be improved. This section illustrates the sensitivity of phase changes to changes in antenna polarization parameters and the ability of the scattering model to predict these changes as system isolation degrades with an increase in ground rain rate.

Theoretical phase data versus isolation data are compared to measured data in Figs. 4.2-10 and 4.2-12. The theoretical predictions assume that an ice layer is not present along the propagation path and that the mean canting angle \( \overline{\theta} \) ranges between -10° and 10° as a result of wind gusts. Since adaptive cancellation systems correct for relatively fast changes in phase, measured data are presented on an instantaneous time basis for a particular rain storm. This eliminates slow phase variations over longer periods of time and more accurately describes the requirements of an isolation enhancement system.

Figure 4.2-10 represents a typical response of the CTS (11.7 GHz) system during the storm of August 9, 1977. Note that the scatter in
Figure 4.2-10. CTS isolation versus phase for the storm of August 9, 1977. (CTS is circularly polarized)
isolation and phase supports the assumption of a 20° range in canting angle. The boxed-in area of Fig. 4.2-10 represents the system behavior before the actual rain event. This enhancement of isolation before the onset of a storm is seen often and is attributed to the special characteristics of the system polarization parameters. The circular polarization of the CTS system also accounts for the relatively large phase scatter for a given value of isolation. For circular polarization, Overstreet [43] has shown that phase scatter is usually twice that of the mean canting angle range and this is verified in Fig. 4.2-10.

Phase changes during precipitation events for a dual circularly polarized communications system are very sensitive to changes in antenna polarization parameters. If the satellite and ground station antenna polarization parameters are changed slightly from those found in Table 4.1-2 (the new epsilon's and tau's are given in Table 4.2-4), the phase retardation of Fig. 4.2-10 can change direction and advance as seen in Fig. 4.2-11. Note that for high values of isolation or for low rain rates, the phase has changed by a significant amount (61°) relative to Fig. 4.2-10; however, for higher rain rates the change is relatively minor (7°). For high rain rates, changes in the polarization parameters of the spacecraft antenna are masked because of the large cross-polarized component generated by the rain. Changes in the ground station antenna are then minor for small polarization parameter deviations. However, for low rain rates the interaction of the spacecraft and ground station antennas becomes significant. The co-polarr-
Table 4.2-4. Epsilon's and Tau's used in Fig. 4.2-11.
(Clear weather isolation is 38 dB)

<table>
<thead>
<tr>
<th>Polarization Parameters</th>
<th>CTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varepsilon_w$</td>
<td>-43.35</td>
</tr>
<tr>
<td>$\tau_w$</td>
<td>0°</td>
</tr>
<tr>
<td>$\varepsilon_{co}$</td>
<td>-44.01</td>
</tr>
<tr>
<td>$\tau_{co}$</td>
<td>0°</td>
</tr>
<tr>
<td>$\varepsilon_{cross}$</td>
<td>-44.11</td>
</tr>
<tr>
<td>$\tau_{cross}$</td>
<td>90°</td>
</tr>
</tbody>
</table>
Figure 4.2-11. Theoretical prediction of isolation versus phase for the polarization parameters of Table 4.2-4.
ized phase is relatively insensitive to changes in the antenna polarization parameters; however, for low rain rates the cross-polarized phase is very sensitive to polarization parameter deviations. During low rain rate conditions, the cross-polarized component generated by the satellite is approximately equivalent to the cross-polarized component generated by the rain. Thus, the phase of the resultant wave is very sensitive to the cross-polarized phase of the wave transmitted by the satellite. Because phase \( \phi \) is insensitive to antenna polarization changes during high rain rate conditions, phase for a circularly polarized system can advance or retard depending on the antenna polarization parameters and the associated effect on the cross-polarized phase for low rain rates.

Although the same arguments apply for a dual linearly polarized communications system as for the circularly polarized antenna system, the uniqueness of the polarization parameters for a linear system reduce the dependence of phase on polarization parameter deviations. Also, the associated phase scatter of a linearly polarized system is significantly reduced as seen in Fig. 4.2-12. This figure represents a typical response of the COMSTAR 28 (28.56 GHz) system during a storm on August 30, 1977. From the data presented in this section, a linearly polarized system is recommended over a circularly polarized system when an adaptive polarization system is being considered [43].

4.3 Comparison of Theory to Measured Data from Other Experiments

In the previous section the theoretical predictions of the
Figure 4.2-12. COMSTAR 28 isolation versus phase for the storm of August 30, 1977. (COMSTAR 28 is linearly polarized)
scattering model were compared to measured data from the VPI&SU earth station for a variety of system parameters. Excellent correlation between measurement and theory was seen. However, the true test of a theoretical propagation model is the ability to predict the effects of rain on dual-polarized communications links for different frequencies, elevation angles, and locations. Using the first order approximation of the synthetic storm model (Fig. 3.6-5) and the respective system parameters of the Comsat Labs COMSTAR experiment and the University of Texas ATS-6 experiment, theoretical attenuation predictions of the scattering model are presented in this section that agree rather well with published measured data from those experiments.

Since attenuation is relatively insensitive to changes in polarization parameters and the exact polarization parameters for the experiments of Comsat Labs and the University of Texas are not known by the authors, the polarization parameters of the VPI&SU COMSTAR system will be assumed in the theoretical predictions of this section. Comsat Labs monitors continuous transmissions from the COMSTAR D1 satellite at an elevation angle of 23°. As a result, using Fig. 3.6-5 and Eq. (3.6-25) the effective rain extent is 11.2 km for the 19.04 GHz and 28.56 GHz COMSTAR D1 beacons. Putting this data into the RPP program yields the theoretical attenuation predictions in Fig. 4.3-1 for the 19 GHz and 28 GHz systems of Comsat Labs. The associated measured data were taken from published rain rate and attenuation statistics for July, 1976 through January, 1977 [34]. As can be seen, the agreement between theory and measurement is very good. Since no
Figure 4.3-1. Attenuation versus rain rate, theoretical predictions of the scattering model compared to measured data of Comsat Labs. (19.04, 28.56 GHz)
isolation data are available, no theoretical predictions are presented. The University of Texas monitored transmissions from the ATS-6 satellite on a time-available basis from July, 1974, to May, 1975. Measured attenuation data from the NASA Technical Note [35] along with the theoretical predictions of the scattering model are presented in Fig. 4.3-2. The ATS-6 propagation frequency was 30 GHz and the elevation angle from the University of Texas experiment site was 54°. As a result, the effective rain extent was 6.7 km. Again there is good agreement between theory and measurement.

Figures 4.3-1 and 4.3-2 demonstrate the flexibility of the synthetic storm algorithm of Sec. 3.6 and the scattering model in predicting rain effects on millimeter wave communications links. However, before the scattering model can be used to predict the effect of rain on any arbitrary earth station, more data must be collected to improve the synthetic storm model of Fig. 3.6-5. An effective storm height of 6 km and a ground rain extent of 10 km seems reasonable; however, these values may change for different climatic zones. Before the full potential of the synthetic storm algorithm can be achieved, more data are needed.

4.4 Frequency Scaling

4.4.1 Attenuation

It is often very useful to scale measured attenuation data at one frequency in order to estimate system performance at another frequency. The frequency dependence of attenuation and the relation-
Figure 4.3-2. Attenuation versus rain rate, theoretical predictions of the scattering model compared to measured data of the University of Texas. (30 GHz)
ship to ground rain rate is predicted easily by the scattering model. In this section theoretical predictions of the scattering model are compared to measured data and to the recent frequency scaling algorithm of Hodge [44].

Figures 4.4-1, 4.4-2, and 4.4-3 illustrate the frequency dependence of attenuation measured at the VPI&SU earth station and the corresponding predictions of the scattering model. Measured and theoretical data are plotted using equal probability of occurrence techniques. Figure 4.4-4 presents the measured attenuation data of Comsat Labs along with the associated prediction of the scattering model. Although the rain extent and the elevation angles are different for the two sites, the ratio of the 28 GHz attenuation to the 19 GHz attenuation is the same.

For a given value of attenuation at a particular frequency, the corresponding scaled value of attenuation at a second frequency can be determined using equations of the form

\[ A_2 = u A_1^v \text{ dB} \]  

(4.4-1)

where \( A_1 \) represents the attenuation of the \( i^{\text{th}} \) frequency in dB. Using power regression techniques the constants \( u \) and \( v \) in Eq. (4.4-1) can be determined and are given in Table 4.4-1 for both the theoretical and measured data presented in Figs. 4.4-1, 4.4-2, 4.4-3, and 4.4-4.

Another way of describing the frequency dependence of attenuation is through equations of the form
Figure 4.4-1. 28 GHz attenuation versus 19 GHz attenuation (VPI&SU).
Figure 4.4-2. 19 GHz attenuation versus 11 GHz attenuation (VPI&SU).
Figure 4.4-3. 28 GHz attenuation versus 11 GHz attenuation (VPI&SU).
Figure 4.4-4. 28 GHz attenuation versus 19 GHz attenuation (Comsat Labs).

X-X MEASURED DATA (COMSAT LABS)
--- THEORY (RPP)
Table 4.4-1. Power curve fits to attenuation versus attenuation data presented in Figs. 4.4-1, 4.4-2, 4.4-3, and 4.4-4. \((A_i < 40 \text{ dB})\) See Eq. (4.4-1).

<table>
<thead>
<tr>
<th>Frequency Ratio</th>
<th>VPI&amp;SU Measured</th>
<th>VPI&amp;SU Theory</th>
<th>Comsat Labs Measured</th>
<th>Comsat Labs Theory</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(u)  (v)  (r^2)</td>
<td>(u)  (v)  (r^2)</td>
<td>(u)  (v)  (r^2)</td>
<td>(u)  (v)  (r^2)</td>
</tr>
<tr>
<td>28/19</td>
<td>2.4902  0.9106  0.99</td>
<td>2.9919  0.8148  1.00</td>
<td>1.9377  0.9927  1.00</td>
<td>3.0778  0.8148  1.00</td>
</tr>
<tr>
<td>28/11</td>
<td>2.9722  0.9552  0.98</td>
<td>6.6514  0.7421  1.00</td>
<td>-----  -----  -----</td>
<td>-----  -----  -----</td>
</tr>
<tr>
<td>19/11</td>
<td>0.8019  1.2501  0.97</td>
<td>2.6708  0.9099  1.00</td>
<td>-----  -----  -----</td>
<td>-----  -----  -----</td>
</tr>
</tbody>
</table>
where $RR$ is rain rate in mm/hr and $a_i$ and $b_i$ are found in Table 4.2-1 for the VPI&SU experimental system. It is obvious from Eq. (4.4-2) that the frequency dependence of attenuation is also dependent on ground rainfall.

Hodge [44] also has considered the dependence of attenuation scaling on rain rate. Assuming that rainfall along the propagation path is a Gaussian function of position on the path, the ratio of attenuations at two different frequencies is expressed by the relation

$$\frac{A_2}{A_1} = \frac{a_2}{a_1} \frac{RR}{b_2 - b_1} \xi$$

(4.4-2)

where $a_i$ and $\xi_i$ are constants associated with power curve fits to attenuation per kilometre (specific attenuation) as a function of a uniform rain for the $i$th frequency. These constants are given in Table 4.4-2. The scattering model prediction, the theory of Hodge, and a curve fit to measured data for the ratio of 28 GHz attenuation to 19 GHz attenuation are illustrated in Fig. 4.4-5. The theory of the scattering model and the theory of Hodge both employ a nonuniform distribution of rain rate along the propagation path. As a result both models agree rather well with measured data. However, unlike the Hodge formulation, the synthetic storm algorithm is internal to the scattering model and can be used also to scale isolation (the Hodge formulation can predict only the ratio of two attenuations).
Table 4.4-2. Power curve fits to theoretical attenuation per kilometer versus rain rate assuming uniform rain conditions.

<table>
<thead>
<tr>
<th>Frequency</th>
<th>$\alpha$</th>
<th>$\xi$</th>
<th>$r^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>0.0217</td>
<td>1.2001</td>
<td>0.98</td>
</tr>
<tr>
<td>19</td>
<td>0.089</td>
<td>1.094</td>
<td>0.99</td>
</tr>
<tr>
<td>28</td>
<td>0.276</td>
<td>0.903</td>
<td>0.99</td>
</tr>
</tbody>
</table>
Figure 4.4-5. Theoretical attenuation scaling, comparison of the Hodge and the scattering model formulations to measured data.
4.4.2 Isolation

Frequency scaling of isolation is more complicated than scaling attenuation. Isolation is very sensitive to changes in rain conditions and as a result significant scatter in isolation occurs. Isolation measurements are also very sensitive to antenna polarization parameters and tracking errors. As a consequence of the large variation in isolation for a single frequency, the scaling of isolation presents a difficult problem to communications engineers, researchers and experimenters.

For a particular set of system parameters, the scattering model can predict the frequency dependence of mean isolation. Figure 4.4-6 illustrates the frequency dependence of mean isolation for the system parameters of the VPI&SU earth station. The data presented in Fig. 4.4-6 can be represented with equations of the form

\[ I_{m_2} = t I_{m_1}^w \text{ dB} \] (4.4-4)

where \( I_{m_1} \) is the mean isolation in dB for the \( i^{th} \) frequency. The symbols \( t \) and \( w \) are constants containing the effects of the particular system parameters (canting angle, elevation angle, rain extent, etc.) on mean isolation. These values are tabulated in Table 4.4-3 for the VPI&SU experimental system. Although the data in Fig. 4.4-6 are presented without experimental verification, an insight into isolation scaling has been obtained.
Figure 4.4-6. Theoretical mean isolation versus mean isolation for 11, 20, and 30 GHz.
Table 4.4-3. Power curve fits to isolation versus isolation data presented in Fig. 4.4-6 (15 ≤ I_m ≤ 40)

<table>
<thead>
<tr>
<th>Frequency Ratio</th>
<th>t</th>
<th>w</th>
<th>r²</th>
</tr>
</thead>
<tbody>
<tr>
<td>30/20</td>
<td>1.7238</td>
<td>0.8107</td>
<td>1.00</td>
</tr>
<tr>
<td>30/11</td>
<td>1.6371</td>
<td>0.7707</td>
<td>1.00</td>
</tr>
<tr>
<td>20/11</td>
<td>0.9642</td>
<td>0.9416</td>
<td>1.00</td>
</tr>
</tbody>
</table>
CHAPTER V
SUMMARY AND CONCLUSIONS

The effects of rain on millimeter wave propagation will influence the future design of satellite communications systems. Therefore, it is necessary to predict the impact of precipitation on satellite communications system performance prior to the specification of system parameters. To this end a theoretical model has been developed to predict the degradation in system performance due to precipitation. A new deterministic model has been presented that models an inhomogeneous rain by discretizing the rain into several piecewise homogeneous rain cells. The scattering properties of each homogeneous rain cell are described by effective scattering coefficients. These coefficients describe the changes in polarization, attenuation and phase shift that an incident field experiences as it propagates through an ensemble of particles with a distribution of particle type (rain or ice), particle shape, particle size, particle density, and particle orientation. A frequency independent synthetic storm algorithm also was developed to model the effects of nonuniform rain rates present on satellite communications links. As a result, the scattering model can predict accurately the effects of rain on a millimeter wave communications link for a variety of frequencies, elevation angles and locations.

After a detailed derivation of the scattering model, the predictions of the associated Rain Propagation Prediction program were compared to available measured data. The comparison of theory to experimental data covered a wide range of system parameters and
various site locations. Excellent agreement was obtained.

Although the scattering model has predicted accurately the observed rain effects on satellite link performance at various ground terminals, more experimental data are needed to further refine the synthetic storm algorithm. Data describing the distribution of canting angles and particle shape along the entire rain extent also would lead to further refinements of the physical modeling of the rain conditions present on a satellite link. After these refinements and further verification of the model with a more extensive experimental data base, the scattering model will provide the system design engineer with a low cost, reliable model to predict the effects of weather on arbitrary satellite communications systems.
6.1 Derivations

6.1.1 Derivation of the Generalized Single Particle Scattering Coefficients

In the development of the scattering model, a generalized single particle scattering coefficient was used to allow particles within a thin rain slab to have any arbitrary orientation. Thus, a canting angle distribution within the rain slab is possible. The derivation of the generalized single particle scattering coefficients is straightforward and based on simple geometric principles [11].

Consider Fig. 6.1-1. An incident field $E_x^i$ aligned with the arbitrary x-axis is incident on an arbitrary oblate particle canted at an angle $\theta$ with respect to the x-axis. $E_x^i$ is decomposed into components $E_v^i$ and $E_h^i$ along the principal axes (v, h) of the oblate particle; respectively,

$$
E_v^i = E_x^i \sin \theta \\
E_h^i = E_x^i \cos \theta
$$

(6.1-1)

$E_v^i$ and $E_h^i$ are scattered by the oblate particle producing $E_v^s$ and $E_h^s$,

$$
E_v^s = f_v E_v^i = f_v E_x^i \sin \theta \\
E_h^s = f_h E_h^i = f_h E_x^i \cos \theta
$$

(6.1-2)
Figure 6.1-1. Geometry used in the derivation of the generalized single particle scattering coefficients.
where $f_v$ and $f_h$ are the complex scattering coefficients of the particle minor and major axis, respectively ($f_v$ and $f_h$ are defined in Sec. 3.1). $E_v^S$ and $E_h^S$ have components $E_{vx}^S$ and $E_{hx}^S$ along the x-axis and components $E_{vy}^S$ and $E_{hy}^S$ along the y-axis as a result of the rotation of the incident field by the oblate particle and are given by

$$E_{vx}^S = E_v^S \sin \theta = f_v E_x^i \sin^2 \theta$$
$$E_{hx}^S = E_h^S \cos \theta = f_h E_x^i \cos^2 \theta$$
$$E_{vy}^S = E_v^S \cos \theta = f_v E_x^i \sin \theta \cos \theta$$
$$E_{hy}^S = E_h^S \sin \theta = f_h E_x^i \sin \theta \cos \theta$$

(6.1-3)

The total scattered field component along the x-axis is

$$E_x^S = E_{vx}^S + E_{hx}^S = E_x^i \left( f_v \sin^2 \theta + f_h \cos^2 \theta \right)$$

(6.1-4)

The total scattered field component along the y-axis is

$$E_y^S = E_{vy}^S - E_{hy}^S = E_x^i (f_v - f_h) \sin \theta \cos \theta$$

(6.1-5)

The scattering coefficient $f_{xx}$ is defined as the ratio of $E_x^S$ to $E_x^i$,

$$f_{xx} = \frac{E_x^S}{E_x^i} = f_v \sin^2 \theta + f_h \cos^2 \theta$$

The scattering coefficient $f_{xy}$ is defined as the ratio of $E_y^S$ to $E_x^i$. 

$$f_{xy} = \frac{E_y^S}{E_x^i}$$
The coefficients $f_{yx}$ and $f_{yy}$ are obtained by replacing $\theta$ in Eqs. (6.1-5) and (6.1-6) with $\theta_c$ where

$$\theta_c = \theta - 90^\circ$$

and then

$$f_{yx} = f_{xy} = (f_v - f_h) \sin \theta \cos \theta$$

and

$$f_{yy} = f_v \cos^2 \theta + f_h \sin^2 \theta$$

Equations (6.1-5), (6.1-6), (6.1-7) and (6.1-8) define the generalized single particle scattering coefficients as given in Eq. (3.1-1).

### 6.1.2 Evaluation of $\Sigma_{N_{i,k}} g_m$

Before the effective scattering coefficients of Eq. (3.2-13) can be determined, the evaluation of the summation $\Sigma_{N_{i,k}} g_m$ is necessary. The summation extends over all the particles within the finite rain slab of Fig. 3.2-1 in the $i^{th}$ size interval and the $k^{th}$ orientation interval. If the particles in the $i^{th}$ and $k^{th}$ intervals are numerous, the summation can be expressed as an integral,

$$\Sigma_{N_{i,k}} g_m = n_{i,k} \int_{V_0} \int_{m} \int_{d_1} g_m dV = n_{i,k} \int_{0}^{\Delta \infty} \int_{0}^{2\pi} \int_{0}^{\rho_{d_0 d_0 d_0}} g_m \rho d\rho d\rho d\rho dz$$

(6.1-9)

where $n_{i,k}$ is the number of particles per unit volume in the $i^{th}$ size interval.
interval and the kth orientation interval. Performing the integral yields

\[ N_{i,k} = \int_{j}^{\infty} \frac{e^{-\rho^2/2z}}{\rho} d\rho d\phi dz \]

\[ = 2\pi n_{i,k} \int_{0}^{\infty} \frac{1}{z} \{ -j \frac{z}{k_0} \} dz \]

\[ = -j \frac{2\pi \Delta \lambda}{k_0} n_{i,k} \]

\[ = -j \lambda \Delta \lambda n_{i,k} \tag{6.1-10} \]

where the following indefinite integral \[45\] was used:

\[ \int_{0}^{\infty} x e^{-r^2} x^2 dz = \frac{1}{2r^2} \]

Equation (6.1-10) is the result given in Eq. (3.2-7).

6.1.3 Derivation of \( N_v \)

\( N_v \) is the total number of raindrops per unit volume within a thin rain slab and is a function of rain rate and the distribution of drop size. Since drops of different size fall with different terminal velocities, they will contribute differently to the total measured ground rain rate. To account for this difference consider a time interval \( \Delta t \) over which all drops striking the ground will be summed to obtain the total rain accumulation. The drops in the drop size inter-
val $a$ to $a + da$ have a terminal velocity of $v_T(a)$ given below:

$$v_T(a) = 4.6 \cdot 2a \text{ m/sec} \quad (6.1-11)$$

where $a$ is in mm [46]. The height above the ground corresponding to the height of the last drops in the time interval $\Delta t$ to strike the ground is

$$h(a) = v_T(a) \Delta t \text{ m}$$

$$= 6.5054 \sqrt{a} \Delta t \text{ m} \quad (6.1-12)$$

where $a$ is in mm, $v_T$ is in m/sec and $h$ is in m. The volume of water accumulated on top of the ground per unit area of ground surface in the time interval $\Delta t$ due to drops in the size interval $a$ to $a + da$ is then

$$\text{Vol/m}^2(a) = h(a) \frac{4\pi}{3} (a)^3 n(a) da \quad (6.1-13)$$

where $n(a)$ is the number of drops per cubic meter of space in the size class $a$ to $a + da$. This volume of water equals the rain rate for the drops with drop size $a$ to $a + da$, $RR(a) da$, times the time interval. Using Eq. (6.1-12) in Eq. (6.1-13) and eliminating $\Delta t$ yields

$$RR(a) da = v_T(a) \frac{4\pi}{3} (a)^3 n(a) da \quad (6.1-14)$$

The total rain rate is found by summing over all drop size contributions,

$$RR = \int RR(a) \, da$$

$$= \int_0^\infty v_T(a) \frac{4\pi}{3} (a)^3 n(a) \, da \quad (6.1-15)$$
Using the modified Laws and Parsons drop size distribution discussed in Sec. 3.6 gives

\[
RR = \frac{4\pi}{3} (6.5054) N_v \int_0^\infty \frac{s(a)}{a^{7/2}} \, da
\]

\[
= \frac{4\pi}{3} (6.5054) N_v \{4(\frac{2^{9/2}}{2} - 1)(\frac{1}{9} - \frac{1}{11})a_m^{7/2}\}
\]

\[
= 47.624 N_v a_m^{7/2} \text{ m/s} \quad (6.1-16)
\]

where \(a_m\) is the modal drop radius in mm. Rain rate should be in units of mm/hr, but the above expression arose from terminal velocity in m/s, drop volume in mm\(^3\) and number density in per m\(^3\) of space. Thus, to convert Eq. (6.1-16) into units of mm/hr,

\[
RR = 47.624 N_v a_m^{7/2} \left(\frac{10^3 \text{ mm}}{m}\right) \left(\frac{3600 \text{ s}}{\text{hr}}\right) \left(\frac{10^{-3} \text{ m}}{\text{mm}}\right)^3
\]

\[
= 0.17144 N_v a_m^{7/2} \text{ mm/hr} \quad (6.1-17)
\]

To find the total number of raindrops per unit volume, the inverse of Eq. (3.6-17) yields

\[
N_v = 5.8327 (a_m)^{-7/2} \quad (6.1-18)
\]

which is the result given in Eq. (3.6-4).

6.1.4 Derivation of Single Ice Particle Scattering Coefficients

In the derivation of the single ice particle scattering coefficients, the theory of Rayleigh scattering can be used for frequencies up to 30 GHz [30]. Assume that an incident electric field is propaga-
ting in the $z$ direction and is incident on an ice particle in the $hv$
plane as shown in Fig. 6.1-2. The ice particle can be spherical,
oblative spheroidal, or prolate spheroidal in shape. A dipole moment is
induced in each of the principal planes of the particle. The assump-
tion is made that the size of the particle is sufficiently small to
ignore higher multipole moments. Each of the components of the inci-
dent field along the principal planes ($h, v$) are considered to act
separately in the excitation of the dipole moment in the respective
component direction.

Battan [47] defines the dipole moments in terms of particle shape
as follows:

$$m_h = 4\pi \varepsilon_0 [g, g'] E_h$$  \hspace{1cm} (6.1-19)

$$m_v = 4\pi \varepsilon_0 [g, g'] E_v$$

where $g$ is a complex number describing the effects of particle
geometry on the dipole moment associated with the particle's axis of
revolution and $g'$ is associated with the orthogonal axis. The defini-
tions of $g$ and $g'$ are given below:

$$g = \frac{\varepsilon_{RO} V_p (\varepsilon_R^2 - 1)}{4\pi + (\varepsilon_R^2 - 1)}$$  \hspace{1cm} (6.1-20)

$$g' = \frac{\varepsilon_{RO} V_p (\varepsilon_R^2 - 1)}{4\pi + (\varepsilon_R^2 - 1)(2\pi - P/2)}$$  \hspace{1cm} (6.1-21)

where for a prolate spheroid,
Figure 6.1-2. Geometry used in the derivation of the single ice particle scattering coefficients.
\[ P_{\text{PRO}} = 4\pi \frac{(1 - \frac{e^2}{e^2})}{\ln\left(\frac{1 + e}{1 - e}\right) - 1} \]  \hspace{1cm} (6.1-22)

for an oblate spheroid,

\[ P_{\text{OBL}} = \frac{4\pi}{\frac{e^2}{e^2}} (1 - \sqrt{\frac{1 - \frac{e^2}{e^2}}{\sin^{-1} e}}) \]  \hspace{1cm} (6.1-23)

and for a spherical particle,

\[ P = \frac{4\pi}{3} \quad . \]  \hspace{1cm} (6.1-24)

The various symbols appearing above are defined below:

- \( \varepsilon_{\text{RO}} \): complex refractive index of air \( (\varepsilon_{\text{RO}} = 1) \)
- \( \varepsilon_R \): complex refractive index of ice \([47]\)
  \[ (\varepsilon_R = 1.78 - j 0.0024 @ 0^\circ C) \]
- \( V_p \): volume of the ice particle
- \( e \): eccentricity \( (e = 1 - \frac{b^2}{a^2}) \)

If the eccentricity is close to one, i.e., a long narrow prolate spheroid or a flat platelike oblate spheroid Eq. (6.1-22) and Eq. (6.1-23) reduce to

\[ P_{\text{PRO}} = 0 \]  \hspace{1cm} (6.1-25)

\[ P_{\text{OBL}} = \frac{4\pi}{3} \quad . \]

Using Eq. (6.1-24) and Eq. (6.1-25) in Eq. (6.1-20) and Eq. (6.1-21)

* Haworth, McEwan, and Watson \([30]\) evidently have a misprint in their publication.
and the results in Eq. (6.1-19) yield for the various particle shapes:

**Prolate**

\[
\begin{align*}
\bar{m}_h &= \frac{1}{3} \bar{a}^3 \left( \varepsilon_R^2 - 1 \right) 4\pi \varepsilon_0 \bar{E}_h \\
\bar{m}_v &= \frac{2}{3} \bar{a}^3 \left( \frac{\varepsilon_R^2 - 1}{\varepsilon_R^2 + 1} \right) 4\pi \varepsilon_0 \bar{E}_v
\end{align*}
\]

(6.1-26)

**Oblate**

\[
\begin{align*}
\bar{m}_h &= \frac{1}{3} \bar{a}^3 \left( \varepsilon_R^2 - 1 \right) 4\pi \varepsilon_0 \bar{E}_h \\
\bar{m}_v &= \frac{1}{3} \bar{a}^3 (1 - 1/\varepsilon_R^2) 4\pi \varepsilon_0 \bar{E}_v
\end{align*}
\]

(6.1-27)

**Spherical**

\[
\bar{m}_{h,v} = \bar{a}^3 \left( \frac{\varepsilon_R^2 - 1}{\varepsilon_R^2 + 2} \right) 4\pi \varepsilon_0 \bar{E}_{h,v}
\]

(6.1-28)

where

\(\bar{a}\) is the equivolumetric particle radius in meters.

Now consider the radiation properties of an electric dipole. From Stratton [48] the radiated electric field is

\[
\bar{E}(z) = \frac{k_0^2}{3\pi \varepsilon_0} \left[ \hat{z} \times (\hat{m} \times \hat{z}) \right] \frac{-3k_0 z}{2z^3}
\]

(6.1-29)
Comparing Eq. (6.1-30) to the definition of the scattered field as defined by the scattering model, the single particle scattering coefficients can be determined as seen below:

\[
\begin{align*}
\tilde{E}_{h,v}^s(z) &= \frac{k_0^2}{4\pi \varepsilon_0} m_{h,v} \frac{e^{-jk_0 z}}{z} = \tilde{E}_{h,v}^\text{RAD}(z) \\
&= \frac{k_0^2}{4\pi \varepsilon_0} m_{h,v} \frac{e^{-jk_0 z}}{z} .
\end{align*}
\]

So,

\[
f_{h,v} = \frac{k_0^2}{4\pi \varepsilon_0} m_{h,v} 1/\tilde{E}_{h,v}^s .
\] (6.1-31)

Using Eqs. (6.1-26), (6.1-27), and (6.1-28) in (6.1-31) yield the single particle scattering coefficients for each particle shape and are given below:

**Prolate**

\[
\begin{align*}
f_{h}^\text{PRO} &= \frac{(4\pi \frac{a^3}{3\lambda^2})}{3\lambda^2} (6.812211 - j 0.0268421) \\
f_{v}^\text{PRO} &= \frac{(4\pi \frac{a^3}{3\lambda^2})}{3\lambda^2} (3.268519 - j 0.006179)
\end{align*}
\] (6.1-32)

**Oblate**

\[
\begin{align*}
f_{h}^\text{OBL} &= \frac{(4\pi \frac{a^3}{3\lambda^2})}{3\lambda^2} (6.812211 - j 0.0268421)
\end{align*}
\]
Equations (6.1-32), (6.1-33), and (6.1-34) define the single ice particle scattering coefficients. By summing over all the particle radii within an ice particle concentration, a general scattering coefficient can be obtained. Define

$$F_{h,v} = \sum_{\bar{a}} n(\bar{a}) f_{h,v}(\bar{a})$$  \quad (6.1-35)$$

where $n(\bar{a})$ is the number of particles with radius $\bar{a}$. Since $f_{h,v}$ is directly proportional to the volume of the particular particle size class and inversely proportional to $\lambda^2$,

$$F_{h,v} = \frac{V}{\lambda^2} A_{h,v}$$  \quad (6.1-36)$$

where $V$ is the total volume of ice in a cubic meter of air and $A_{h,v}$ is a complex constant independent of frequency given in Table 3.6-1. This result along with the general scattering coefficient formulation, Eq. (3.1-1), and the elevation angle formulation presented in 3.6 yield Eq. (3.6-12).

6.2 The Rain Propagation Prediction Program (RPP)
6.2.1 General Description

The Rain Propagation Prediction program uses the scattering model and the synthetic storm algorithm to predict attenuation, isolation, and phase as a function of rain rate for a given set of system and rain parameters. A block diagram of the RPP program is found in Fig. 6.2-1. The single input format of Fig. 6.2-2 facilitates changing the system and rain parameters. Although the lack of data has caused certain restrictions to have been placed on the particle distributions along the rain extent, the scattering model and thus the RPP program can model an arbitrary inhomogeneous rain given the appropriate input parameters. The input parameters will be discussed in more detail in Sec. 6.2-2.

After reading the input parameters and calculating the clear weather isolation and phase, the RPP program prints an information header at the beginning of the output section as seen in Fig. 6.2-3. The rain conditions for each cell are then computed and the scattering matrices as defined by Eq. (3.3-1) are determined. After computing the wave exiting the rain medium and the wave-antenna interaction, the RPP program outputs rain rate, attenuation, isolation and phase data as seen in Fig. 6.2-4. Since curve fits to theoretical data are often useful, the RPP program outputs curve fit data for various combinations of rain rate, attenuation, and isolation data as seen in Fig. 6.2-5.
Figure 6.2-1. Block diagram of the rain propagation prediction program (RPP).
Figure 6.2-2. Sample input data to the rain propagation prediction program (RPP).
*** RAIN PROPAGATION PREDICTION PROGRAM (RPP) ***

THE DOWNLIMI PROPAGATION FREQUENCY IS 11.0 GHz
THE LOOK ANGLE, DELTA, IS 33.0 DEGREES

*** PROGRAM PARAMETERS ***

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<tr>
<th>TYPE</th>
<th>START</th>
<th>STOP</th>
<th>INCREMENT</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) RAIN EXTNT</td>
<td>10000</td>
<td>10000</td>
<td>10000</td>
</tr>
<tr>
<td>2) CANT ANGL</td>
<td>10</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>3) RAIN RATE</td>
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<td>60</td>
<td>3</td>
</tr>
</tbody>
</table>

Binned Coefficients

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<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
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<td>0.0</td>
<td>-0.7</td>
<td>-0.7</td>
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<tr>
<td>CANT ANGL</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>FRAC DILT</td>
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<td>0.6</td>
<td>0.6</td>
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<td>0.6</td>
<td>0.6</td>
<td>0.6</td>
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<td>12.0</td>
<td>12.0</td>
<td>12.0</td>
<td>12.0</td>
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</tr>
</tbody>
</table>

*** CLEAR WEATHER INFORMATION ***

THE POLARIZATION PARAMETER EPSILON FOR THE INCIDENT WAVE IS -43.35 DEGREES
THE POLARIZATION PARAMETER TAU FOR THE INCIDENT WAVE IS 0.0 DEGREES
THE POLARIZATION PARAMETER EPSILON FOR THE CROSS ANTENNA POLARIZATION STATE IS -44.01 DEGREES
THE POLARIZATION PARAMETER TAU FOR THE CROSS ANTENNA POLARIZATION STATE IS 0.0 DEGREES
THE POLARIZATION PARAMETER EPSILON FOR THE CROSS ANTENNA POLARIZATION STATE IS -44.11 DEGREES
THE POLARIZATION PARAMETER TAU FOR THE CROSS ANTENNA POLARIZATION STATE IS 0.00 DEGREES
THE CLEAR WEATHER ISOLATION REFERENCED TO THE ANTENNA PORTS IS 37.55 DB
THE CLEAR WEATHER PHASE DIFFERENCE IS -0.01 DEGREES

Figure 6.2-3. Information header section for the output of the rain propagation prediction program corresponding to the input data of Fig. 6.2-2.
The rain extent is 10000 meters.

The mean angle between the Y-axis and the drop minor axis (θ) is 10 degrees.

*************** SCATTERING MODEL ***************

<table>
<thead>
<tr>
<th>Rain Rate (mm/hr)</th>
<th>Attenuation (dB)</th>
<th>Isolation (dB)</th>
<th>Phase (deg)</th>
</tr>
</thead>
<tbody>
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<td>3.00</td>
<td>1.95</td>
<td>31.83</td>
<td>299.96</td>
</tr>
<tr>
<td>6.00</td>
<td>2.90</td>
<td>29.53</td>
<td>290.61</td>
</tr>
<tr>
<td>9.00</td>
<td>3.73</td>
<td>27.80</td>
<td>285.81</td>
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<tr>
<td>12.00</td>
<td>4.39</td>
<td>26.63</td>
<td>283.19</td>
</tr>
<tr>
<td>15.00</td>
<td>5.03</td>
<td>25.61</td>
<td>281.28</td>
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<tr>
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<td>12.20</td>
<td>17.96</td>
<td>271.96</td>
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</tbody>
</table>

Figure 6.2-4. Rain propagation prediction program output, attenuation, isolation and phase as a function of rain rate corresponding to the input data of Fig. 6.2-2.
<table>
<thead>
<tr>
<th>CURVE TYPE</th>
<th>ASSOCIATED EQUATION</th>
<th>A</th>
<th>B</th>
<th>R##?</th>
</tr>
</thead>
<tbody>
<tr>
<td>ATTEN VS RR</td>
<td>ATTEN = A*RR+B</td>
<td>0.7525</td>
<td>0.6197</td>
<td>0.9994</td>
</tr>
<tr>
<td>ISOL VS RR</td>
<td>ISOL = A*LN(RR)</td>
<td>38.4404</td>
<td>-4.9078</td>
<td>0.9908</td>
</tr>
<tr>
<td>ISOL VS ATTEN</td>
<td>ISOL = A*LN(ATFFEN)</td>
<td>38.0867</td>
<td>-7.9350</td>
<td>0.9933</td>
</tr>
</tbody>
</table>

Figure 6.2-5. Rain propagation prediction program output, curve fits to the data in Fig. 6.2-4.
6.2.2 RPP Users Guide

The RPP program was designed to provide an efficient tool to predict the effects of precipitation on satellite communications system performance. The RPP program is written in the FORTRAN programming language, and the program listing is found in Sec. 5.2.3. This section explains the steps in the operational procedure of the RPP program. Each step corresponds to a line of input data in Fig. 6.2-2.

Step 1

Input the operating frequency in GHz in an F5.1 format.*

Step 2

Input the system elevation angle in degrees in an F5.1 format.

Step 3

Input the effective physical rain extent (L) do loop parameters in meters, LSTRT, LSTOP, and LINC. The input format is I5. The do loop parameters allow finer control over the rain extent parameter. To implement the synthetic storm algorithm consult Fig. 3.6-5.

* The scattering coefficients \( f_v \) and \( f_h \) are limited to 11.0, 14.0, 20.0, and 30.0 GHz. To increase the frequency selection, curve regressions for published \( f_v \) and \( f_h \) coefficients at other frequencies are easily obtained and the RPP program is easily modified.
Step 4

Input the mean canting angle ($\bar{\theta}_\mu$) and loop parameters ANSTR, ANSTOP, and ANINC in degrees ($|\bar{\theta}_\mu| < 45^\circ$). The format is I5.

Step 5

Input the rain rate do loop parameters RRSTRT, RRSTOP, and RRINC in mm/hr (RR < 250 mm/hr). The format is I5.

Step 6

Input the clear weather wave polarization state parameters $\varepsilon_W$ and $\tau_W$ in degrees. The format is F7.2.

Step 7

Input the co and cross antenna polarization state parameters $\varepsilon_{co}$, $\tau_{co}$, $\varepsilon_{cross}$ and $\tau_{cross}$ in degrees. The format is F7.2.

Step 8

Input the $x_1$ coefficients of the synthetic storm algorithm for each rain cell. To model a uniform rain path $x_{1 \rightarrow 10} = 0$. Note that cell one is closest to the receive antenna. The format is F4.1.

Step 9

If a distribution of mean canting angles is desired along...
the propagation path, input the multiplicative factors for each rain cell \( \bar{\theta}_i = B_i \bar{\theta}_1 \). If a uniform distribution along the rain path is desired \( B_{1-10} = 1.0 \). The format is F4.1.

**Step 10**

Input the fraction of oblate raindrops \( C_i \) in each rain cell. The format is F4.1.

**Step 11**

Input the standard deviation of the canting angle distribution of \( \sigma_i \) of each rain cell in degrees. The format is F4.1.

**Step 12**

Input a "0" to indicate a rain cell or input a "1" to indicate an ice cell for each cell along the propagation path. For an ice cell, steps 9, 10 and 11 are overridden and the assumptions of Sec. 3.6.5 apply. The format is I3.

**Step 13**

Input a "0" for a downlink. Input a "1" for an uplink. The format is I5.

By controlling steps 1 through 13, a wide range of system and rain parameters can be modeled and their effects on the performance
of an arbitrary satellite communications link can be determined.
*************** RAIN PROPAGATION PREDICTION PROGRAM (RPP) ***************

**MAIN PROPAGATION PREDICTION PROGRAM (RPP)**

This program outputs theoretical rain propagation data (attenuation, isolation, and phase) for a variety of frequencies, elevation angles, and locations as a function of ground rain rate.

The input format allows the rain path to be subdivided into ten segments which provide arbitrary variation of the propagation medium parameters.

**Propagation Model:** Scattering Model

**Programmer:** Russell R. Persinger

**Consulting Advisors:** Dr. Warren L. Stutzman and Robert E. Castle

This program was developed in partial fulfillment of the requirements for the degree of Master of Science in Electrical Engineering; however, its development was supported by NASA with important additive support from the Defense Communications Agency and the US Army Research Office.

********************************************
DIMENSION ICE(10), SIGMA(10)
DIMENSION BIN(10)
DIMENSION A(10), B(10), C(10)
DIMENSION THETA(15), RRATE(15), RRATE(300), PE(15)
DIMENSION PHASE(300), ISOL(300), ATTEN(300)
DIMENSION PX(15), PY(15), PYY(15)
INTEGER LSTR, LSTOP, INC
INTEGER ANSTR, ANSTOP, ANINC
INTEGER RRSTR, RRSTOP, RRINC
INTEGER BIN, DIREC
REAL ISOL
REAL ISOL, LOKANG
COMPLEX MINUSJ
COMPLEX*16 CONSNT
COMPLEX*16 EXO, EYO, EXC, EYC, EXX, EYX
COMPLEX*16 FV, FH, FSPH, DIFF
COMPLEX*16 P1, P2, P3, P4
COMPLEX*16 EXN, EYN
COMPLEX*16 PXX, PYX, PYX, PYY
COMPLEX*16 QX, FYX, FYY
COMPLEX*16 SXX, SX, SYX, SYX, SYY
COMPLEX*16 LAMDA1, LAMDA2, ALPHAO, ALPHAI

C
COMMON/BLOC/MINUSJ, CONV
C
C
C*****************************************************************

C
C
C* FIRST THE PROGRAM READS ALL NECESSARY VARIABLE VALUES INTO
C* MEMORY AND WRITES THIS INFORMATION OUT AS A HEADER AT THE
C* BEGINNING OF THE OUTPUT SECTION.
C* ALSO INCLUDED IN THIS SECTION ARE THE CLEAR WEATHER CONDITIONS
AND AS A RESULT, UNIT VECTORS DESCRIBING THE SYSTEM POLARIZATIONS ARE COMPUTED.

FREQ IS FREQUENCY IN GHZ (11.0, 14.0, 20.0, 30.0 ONLY)

LOKANG IS ELEVATION ANGLE IN DEGREES (0...90)

LSTRT, LSTOP, AND LINC ARE THE RAIN EXTENT PARAMETERS IN METERS

ANSTRT, ANSTOP, AND ANINC ARE CANTING ANGLE PARAMETERS IN DEGREES

RRSTRT, RRSTOP, AND RRINC ARE RAIN RATE PARAMETERS IN MM/HR

ALL EPS'S AND TAU's ARE IN DEGREES

BIN ONE IS LOCATED AT THE GROUND ANTENNA

A(1,...10), B(1,...10), AND C(1,...10) ARE MULTIPLICATIVE FACTORS ASSOCIATED WITH THE RAIN EXTENT BINS

A(I) CONTROLS THE ITH BIN RAIN RATE

THE VALUES OF A(I) ARE SET IN ACCORDANCE WITH THE STORM ALGORITHM

B(I) CONTROLS THE ITH BIN CANTING ANGLE

C(I) CONTROLS THE ITH BIN PERCENT OBLATENESS

THE ABSOLUTE VALUE OF THETA(I) IS ALWAYS LESS THAN 45 DEGREES

RRATE(I) IS ALWAYS LESS THAN 250 MM/HR

SIGMA(I) IS THE STANDARD DEVIATION OF THE CANTING ANGLE

DISTRIBUTION FOR THE ITH BIN. (EQUAL TO ZERO OR 10<SIGMA<50)

IF ICE(I) IS EQUAL TO 1, THE ITH BIN IS FILLED WITH ICE PARTICLES

IF ICE(I) IS EQUAL TO 0, THE ITH BIN IS FILLED WITH RAIN

DIREC EQUALS ONE FOR UPLINK

DIREC EQUALS ZERO FOR DOWNLINK

READ(5,100) FREQ
READ(5,100) LOKANG
READ(5,101) LSTRT, LSTOP, LINC
READ(5,101) ANSTRT, ANSTOP, ANINC
READ(5,101) RRSTRT, RRSTOP, RRINC
READ(15,102) EPSW, TAUW
READ(5,103) EPSX, EPSY, TAUW
READ(5,104) (A(I), I=1, 10)
READ(5,104) (B(I), I=1, 10)
READ(5,104) (C(I), I=1, 10)
READ(5,104) (SIGM(I), I=1, 10)
READ(5,105) (ICE(I), I=1, 10)
READ(5,101) DIREC

C
100 FORMAT(F4.1)
101 FORMAT(I5, I5, I5)
102 FORMAT(F7.2, F7.2)
103 FORMAT(4(F7.2))
104 FORMAT(10(F4.1))
105 FORMAT(10I3)
C
WRITE(6,200)
WRITE(6,221)
IF(DIREC.EQ.1) WRITE(6,2031) FREQ
IF(DIREC.EQ.0) WRITE(6,2030) FREQ
WRITE(6,201) LOKANG
WRITE(6,221)
WRITE(6,202)
WRITE(6,204)
WRITE(6,205)
WRITE(6,206) LSTRT, LSTOP, LINC
WRITE(6,207) ANSTRT, ANSTOP, ANINC
WRITE(6,208) RRSTRT, RRSTOP, RRINC
WRITE(6,209)
WRITE(6,210)
WRITE(6,211) (A(I),I=1,10)  
WRITE(6,212) (B(I),I=1,10)  
WRITE(6,213) (C(I),I=1,10)  
WRITE(6,2131) (SIGMA(I),I=1,10)   
WRITE(6,2132) (ICE(I),I=1,10)   
WRITE(6,214)   
WRITE(6,215) EPSW  
WRITE(6,216) TAUW  
WRITE(6,217) EPSC  
WRITE(6,218) TAUC  
WRITE(6,219) EPSX  
WRITE(6,220) TAUX

C

200 FORMAT(1HI,3X,'***** RAIN PROPAGATION PREDICTION PROGRAM (RPP) *****')   
201 FORMAT(/,3X,'THE ELEVATION ANGLE, BETA, IS ',F4.1,2X,'DEGREES')  
202 FORMAT(/,3X,'***** PROGRAM PARAMETERS *****')  
203 FORMAT(/,3X,'THE UPLINK PROPAGATION FREQUENCY IS ',2X,F4.1,2X,'GHZ')  
204 FORMAT(/,3X,' THE DOWNLINK PROPAGATION FREQUENCY IS ',2X,F4.1,2X,'GHZ')  
205 FORMAT(/,3X,'DO LOOP PARAMETERS')  
206 FORMAT(/,3X,'1) RAIN EXTNT ',I6,5X,I6,4X,I6)  
207 FORMAT(/,3X,'2) CANT ANGLE ',I6,5X,I6,4X,I6)  
208 FORMAT(/,3X,'3) RAIN RATE ',I6,5X,I6,4X,I6)  
209 FORMAT(/,3X,'4) BUNNED COEFFICIENTS')  
210 FORMAT(/,15X,'1',5X,'2',5X,'3',5X,'4',5X,'5',5X,'6',5X,'7',5X,'8',5X,'9',5X,'10')  
211 FORMAT(/,3X,'RAIN RATE',2X,10(F4.1,2X))  
212 FORMAT(/,3X,'CANT ANGL',2X,10(F4.1,2X))  
213 FORMAT(/,3X,'FRAC OBLT',2X,10(F4.1,2X))
2131 FORMAT(//,3X,'SIGMA ', '2X,10(F4.1,2X)'), RPP01610
2132 FORMAT(//,3X,'ICE ', '2X,10(I4,2X)'), RPP01620
214 FORMAT(//,3X, "**** CLEAR WEATHER INFORMATION ****") RPP01630
215 FORMAT(//,3X, "THE POLARIZATION PARAMETER EPSILON FOR THE INCIDENT WAVE IS", '2X,F7.2,2X,' 'DEGREES'), RPP01640
216 FORMAT(//,3X, "THE POLARIZATION PARAMETER TAU FOR THE INCIDENT WAVE IS", '2X,F7.2,2X,' 'DEGREES'), RPP01650
217 FORMAT(//,3X, "THE POLARIZATION PARAMETER EPSILON FOR THE CROSS ANNEAR POLARIZATION STATE IS", '2X,F7.2,2X,' 'DEGREES'), RPP01660
218 FORMAT(//,3X, "THE POLARIZATION PARAMETER TAU FOR THE CROSS ANNEAR POLARIZATION STATE IS", '2X,F7.2,2X,' 'DEGREES'), RPP01670
219 FORMAT(//,3X, "THE POLARIZATION PARAMETER EPSILON FOR THE CIRCULAR POLARIZATION STATE IS", '2X,F7.2,2X,' 'DEGREES'), RPP01680
220 FORMAT(//,3X, "THE POLARIZATION PARAMETER TAU FOR THE CIRCULAR POLARIZATION STATE IS", '2X,F7.2,2X,' 'DEGREES'), RPP01690
221 FORMAT(//,3X, "**** SCATTERING MODEL *******"), RPP01700
222 FORMAT(//,9X,'THE RAIN EXTENT IS ', '15,' ' METERS'), RPP01710
223 FORMAT(//,3X, "THE MEAN ANGLE BETWEEN THE Y-AXIS AND THE DROP MINOR AXIS (THETA) IS", '15,' ' DEGREES'), RPP01720
224 FORMAT(//,25X,"******* SCATTERING MODEL *******"), RPP01730
225 FORMAT(//,3X, "RAIN RATE (MM/H), 5X, 'ATTEN (DB)', 9X, 'ISOL (DB)', 8X), RPP01740
227 FORMAT(//,' CURVE FITS TO ABOVE DATA'), RPP01760
229 FORMAT('//, ' ATTEN VS RR', '8X, 'POWER', '12X, 'ATTEN = A*RR**B', '4X, 3(FRPP01780
230 FORMAT('//, ' ISOL VS RR', '10X, 'LOG', '12X, 'ISOL = A+B*LN(9R)', '3X, 3(FRPP0190
18.4,4X)), RPP01910
230 FORMAT('//, ' ISOL VS RR', '10X, 'LOG', '12X, 'ISOL = A+B*LN(9R)', '3X, 3(FRPP01920
18.4,4X)), RPP01920
231 FORMAT(//, ' ISOL VS ATTN', 'LOG', 10X, 'ISOL = A+B*LN(ATTN)', \RPP01930
1X,3(F8.4,4X)) \RPP01940
C \RPP01950
C \RPP01960
PI=3.14159265 \RPP01970
CONV=PI/180. \RPP01980
LOKANG=LOKANG*CONV \RPP01990
MINUSJ=10.*-1.) \RPP02000
C \RPP02010
CALL COMPNT(EPSW,TAUW,EXO,EYO) \RPP02020
CALL COMPNT(EPSC,TAUC,EXC,EYC) \RPP02030
CALL COMPNT(EPSX,TAUX,EXX,EYX) \RPP02040
C \RPP02050
EYC=DCONJG(EYC) \RPP02060
EYX=DCONJG(EYX) \RPP02070
C \RPP02080
CALL CLEWEA(EXO,EYO,EXC,EYC,EXX,EYX) \RPP02090
C \RPP02100
C****************************************************************************** \RPP02110
C****************************************************************************** \RPP02120
C* THE REST OF THE PROGRAM USES THE SCATTERING MODEL \RPP02130
C* TO COMPUTE ATTENUATION, ISOLATION, AND PHASE AS A FUNCTION \RPP02140
C* OF RAIN RATE AND OUTPUTS THEM IN AN ORGANIZED FORM. \RPP02150
C****************************************************************************** \RPP02160
C****************************************************************************** \RPP02170
C IF(ANSTRT LT 0) FLAG=1.0 \RPP02180
IF(ANSTRT LT 0) ANSTOP=ANSTOP+360 \RPP02190
IF(ANSTRT LT 0) ANSTRT=ANSTRT+360 \RPP02200
C \RPP02210
ANSTRT=ANSTRT + ANINC \RPP02220
ANSTOP=ANSTOP + ANINC \RPP02230
ORIGINAL PAGE QUALITY
C DO 1 II=LSTRT,LSTOP,LINC
DO 1 JJ=ANSTRT,ANSTOP,ANINC
C RAINL=II
JJJ=JJ-ANINC
JJJJ=JJJ
IF(FLAG.EQ.0.0) JJJJ=JJJ-360
C WRITE(6,222) II
WRITE(6,223) JJJJ
WRITE(6,224)
WRITE(6,225)
C III=0
THETA(1)=JJJ*CONV
IF(THETA(1).GT.PI) THETA(1)=THETA(1)-2.*PI
C DO 2 KK=RRSTRT,RRSTOP,RRINC
C SET UP THE BINNED PARAMETERS
C RRATEI=KK
C III IS THE NUMBER OF RAIN RATE ENTRIES... LESS THAN 300
C III=III + 1
RRATEI(III)=KK
C DO 3 !=1,10
K=I+1
IF(K.EQ.11) GO TO 9
RRATE(K)=RRATE(1)/10.0**A(K)*RRATE(1)
THETA(K)=B(K)*THETA(1)
9 PE(I)=C(I)
C IF(RRATE(K).LT.1.00) RRATE(K)=1.0
IF(RRATE(K).GT.250.0) RRATE(K)=250.0
IF(THETA(K).GT.0.7854) THETA(K)=0.7854
IF(THETA(K).LT.-0.7854) THETA(K)=-0.7854
3 CONTINUE
C SCATTERING MODEL
C DELTAL=1.0
CONSNT=MINUSJ*DELTAL*0.3/FREQ
NN=RAINL/(DELTAL*10.0)
C COMPUTE THE SYSTEM MATRICES FOR EACH DISCRETE BIN
C USING THE CAYLEY-HAMILTON THEOREM.
C I1=0
BIN(I1)=1
C DO 4 I=1,10
C CHECK FOR UNIQUE BINS, IF PREVIOUSLY COMPUTED DO NOT RECOMPUTE
C IF(I.EQ.1) GO TO 13
C DO 8 K1=1,I1
C K2=BIN(K1)
C

IF(THETA(I).EQ.THE(A(K2)) .AND. (PE(I).EQ.PE(K2)) .AND. (RRATE(I) .EQ.RRATE(K2)) .AND. (ICE(I).EQ.ICE(K2)) .AND. (SIGMA(I).EQ.SIGMA(K2)))
11 GO TO 7

C

8 CONTINUE
13 CONTINUE

C

AMODE=0.5 + 0.45*ALOG10(RRATE(I))

C

IF(ICE(I).NE.1) GO TO 14
CALL ICECOF(FREQ,AMODE,FXX,FYY,FXY,LOKANG,RRATE,I,THETA)
GO TO 15

C

14 CALL CAPF(PE,AMODE,FXX,FYY,FXY,FREQ,LOKANG,RRATE,I,THETA,SIGMA)
15 CONTINUE

C

SXX=1. + CONSNT*FXX
SYY=1. + CONSNT*FYY
SXY=CONSNT*FXY
SYX=SXY

C

LAMDA1=(SXX+SYY)/2. + COSQRT(((SXX+SYY)**2)/4.+SXY**2-SXX*SYY)
LAMDA2=(SXX+SYY)/2. - COSQRT(((SXX+SYY)**2)/4.+SXY**2-SXX*SYY)

C

ALPHAO=(LAMDA2*(LAMDA1**NN) - LAMDA1*(LAMDA2**NN))/(LAMDA2-LAMDA1)
ALPHA1=(LAMDA2**NN - LAMDA1**NN)/(LAMDA2-LAMDA1)

C

PXX(I)=ALPHAO + ALPHA1*SXX
PXY(I)=ALPHA1*SXY
PYX(I)=PXY(I)
PYY(I)=ALPHAO + ALPHA1*SYY
C 
11=II+1 
BIN(I1)=1 
GO TO 4 
7 CONTINUE 
PXX(I)=PXX(K2) 
PYX(I)=PYX(K2) 
PYX(I)=PYX(K2) 
PYY(I)=PYY(K2) 
C 
4 CONTINUE 
C 
DO 5 I=1,9 
C 
IF(DIREC.EQ.1) GO TO 11 
C 
J=II-I 
K=J-1 
GO TO 12 
11 CONTINUE 
K=II+1 
J=I 
12 CONTINUE 
P1=PXX(K)*PXX(J) + PYX(K)*PYX(J) 
P2=PXX(K)*PYX(J) + PYX(K)*PYY(J) 
P3=PYX(K)*PXX(J) + PYY(K)*PYY(J) 
P4=PYX(K)*PYX(J) + PYX(K)*PYY(J) 
PXX(K)=P1 
PYX(K)=P2
PXY(K)=P3
PYK(K)=P4

5 CONTINUE

C COMPUTE THE RESULTING X AND Y COMPONENTS EXITING THE RAIN MEDIUM

EXN=P1*EXO + P2*EYO
EYN=P3*EXO + P4*EYO

C COMPUTE THE ANTENNA OUTPUTS DUE TO RAIN AND POLARIZATION MISMATCH

CALL OUTANT(EXN,EYN,EXC,EYC,EXX,EYX,III,ISOL1,ATTEN1,PHASE1)

2 CONTINUE

C NOW OUTPUT THE DATA IN AN ORGANIZED FORM

DO 6 I=1,III
   WRITE(6,226) RRATE1(I),ATTEN1(I),ISOL1(I),PHASE1(I)
6 CONTINUE

C PERFORM LINEAR REGRESSIONS ON OUTPUT DATA

WRITE(6,227)
WRITE(6,228)
CALL CURVE(RRATE1,ATTEN1,III,AA,BB,R,2)
WRITE(6,229) AA,BB,R
CALL CURVE(RRATE1,ISOL1,III,AA,BB,R,3)
WRITE(6,230) AA,BB,R
CALL CURVE(ATTEN1,ISOL1,III,AA,BB,R,3)
WRITE(6,231) AA,BB,R
CONTINUE

WRITE(6,1000) STOP
SUBROUTINE COMPNT(EPS,TAU,EX,EY)

THIS SUBROUTINE RETURNS THE X AND Y COMPONENTS GIVEN AN EPSILON
AND TAU (IN DEGREES) DESCRIBING AN ARBITRARY POLARIZATION STATE

COMPLEX*16 EX,EY
COMPLEX MINUSJ
COMMON/BLOC1/MINUSJ,CONV

EPSR=EPS*CONV
TAUR=TAU*CONV

IF(ABS(EPSR).EQ.(45.*CONV)) GO TO 1
IF(EPSR.EQ.0.) GO TO 2
IF(TAUR.EQ.0.) GO TO 3
IF(TAUR.EQ.(90.*CONV)) GO TO 4

T1=TAN(2.*EPSR)
T2=SIN(2.*TAUR)
DELTR=ATAN2(T1,T2)
GAMR=0.5*ARCOS(COS(2*EPSR)*COS(2*TAUR))
GO TO 100

DELTR=2.*EPSR
GAMR=45.*CONV
GO TO 100
2 DELTR=0.
   GAMR=TAUR
   GO TO 100
3 DELTR=SIGN(1.*EPSR)*90.*CONV
   GAMR=ABS(EPSR)
   GO TO 100
4 DELTR=SIGN(1.*EPSR)*90.*CONV
   GAMR=90.*CONV-ABS(EPSR)
100 CONTINUE
C
   EX=COS(GAMR)
   EY=SIN(GAMR)*CEXP(-MINUSJ*DELTR)
C
   RETURN
C
END
SUBROUTINE CLEWEA(EXO, EYO, EXC, EYC, EXX, EYX)
C
THIS SUBROUTINE OUTPUTS THE CLEAR WEATHER ISOLATION AND PHASE
C
REAL ISOCL
COMPLEX MINUSJ
COMPLEX*16 EXO, EYO, EXC, EYC, EXX, EYX
COMPLEX*16 VWAC, VWAX
REAL*8 VWACM, VWAXM
REAL*8 REAL, AIMAG
C
COMMON/BLOC1/ MINUSJ, CONV
COMMON/BLOC2/VWACM
C
VWAC=EXO*EXC + EYO*EYC
VWAX=EXO*EXX + EYO*EYX
VWACR=REAL(VWAC)
VWACI = AIMAG(VWAC)
VWACM = CDABS(VWAC)
IF((VWACI.EQ.0).AND.(VWACR.EQ.0)) GO TO 1
VWACP = ATAN2(VWACI, VWACR)
VWACPH = VWACP/CONV
GO TO 2
1 VWACPH = 0.
2 CONTINUE
VWAXR = REAL(VWAX)
VWAXI = AIMAG(VWAX)
VWAXM = CDABS(VWAX)
IF((VWAXI.EQ.0).AND.(VWAXR.EQ.0)) GO TO 3
VWAXP = ATAN2(VWAXI, VWAXR)
VWAXPH = VWAXP/CONV
GO TO 4
3 VWAXPH = 0.
4 CONTINUE
C PHASCW = VWAXPH - VWACPH
C IF(VWAXM.EQ.0.) GO TO 5
C ISOCL = 20.*DLOG10(VWACM/VWAXM)
WRITE(6,8) ISOCL
8 FORMAT(/,3X,'THE CLEAR WEATHER ISOLATION REFERENCED TO THE ANTENNA PORTS IS $j', 19X,F6.2,2X,'#D8)
C GO TO 7
5 WRITE(6,6)
6 FORMAT(/,3X,'THE CLEAR WEATHER ISOLATION IS INFINITY')
7 CONTINUE
WRITE(6,9) PHASCW
9 FORMAT(/,3X,'THE CLEAR WEATHER PHASE DIFFERENCE',46X,F7.2, 
12X,'DEGREES')

RETURN
END

SUBROUTINE ICECOF(FREQ,AMODE,FXX,FYY,FXY,LOKANG,RRATE,I,THETA)

THIS SUBROUTINE RETURNS TO THE CALLING PROGRAM THE EFFECTIVE 
SCATTERING COEFFICIENTS FOR A SLAB OF ICE PARTICLES. THE SLAB 
HAS 50% PROLATE PARTICLES AND 50% OBLATE PARTICLES. THE 
SCATTERING COEFFICIENTS ARE A FUNCTION OF PARTICLE SIZE AND 
ORIENTATION. THE PARTICLES ALL HAVE THE SAME CANTING ANGLE 
THETA. THE COEFFICIENTS ARE DERIVED FROM RAYLEIGH SCATTERING 
TECHNIQUES.

THE PARTICLE ECCENTRICITY IS VERY CLOSE TO ONE.

DIMENSION ICE(10),RRATE(15),THETA(15)
REAL LOKANG
COMPLEX*16 FXX,FXY,FYY,APROX,APROY,AOBLX,AOBLY,ASPH

ICE CONCENTRATION INCREASES WITH RAIN RATE.
IF ONLY CONSIDERING ICE IN THE PATH, RAIN RATE IS MEANINGLESS, 
BUT IT CONTROLS PARTICLE CONCENTRATION.
RR=1, VOLICE=E-8...RR=50, VOLICE=E-6

VOLICE=RRATE(I)*(24.434608*1E-9)/SQRT(AMODE)

CON=VOLICE*(FREQ**2)/0.18
CSTH=COS(THETA(I))**2
SNTH=SIN(THETA(I))**2
CSLA=COS(LOKANG)**2
SNLA = SIN(LOKANG)**2

APROX = (6.812211, -0.026842)
APROY = (3.268519, -0.006179)
A0BLX = (6.812211, -0.026842)
AG0LX = (0.150047, -0.002674)
ASPH = (3.95417, -0.009044)

AG0LX = ASPH*CSLA + A0BLX*SNLA
A0BLX = ASPH*CSLA + A0BLX*SNLA

FXX = CON*(SNTH*APROY + A0BLY) + CSTH*(APR0X + A0BLX)
FYY = CON*(CS0H*APROY + A0BLY) + SNTH*(APRX + A0BLX)
FXY = CON*(APROY + A0BLY - APR0X + A0BLX)*SIN(THETA(I))*COS(THETA(I))

RETURN
END

SUBROUTINE CAPF(PE, AMODE, FXX, FYY, FXY, FREQ, LOKANG, RRATE, II, THETA, SIGMA)

THIS SUBROUTINE RETURNS TO THE CALLING PROGRAM THE COMPLEX SCATTERING COEFFICIENTS FXX, FXY, FYY. THESE COEFFICIENTS INCLUDE THE EFFECTS OF A DROP SIZE DISTRIBUTION AND A CANTING ANGLE DISTRIBUTION. (THE ABSOLUTE VALUE OF THE CANTING ANGLE RANGE IS LESS THAN 100 DEGREES)

DIMENSION RRATE(15), THETA(15), PE(15), N(20, 28)
DIMENSION SIGMA(10)
INTEGER P, Q
REAL LOKANG, N
COMPLEX*16 FXX, FYY, FXY
COMPLEX*16 FV, FH, FSPH, DIFF
CALL NOIRRATE,II,N,AMODE,THETA,SIGMA)
FXX=(0.0,0.0)
FYY=(0.0,0.0)
FXY=(0.0,0.0)
DO 1 P=2,28
RADIUS=0.125/2.0 + (P-1)*0.125
CALL COEFIFREQ,RADIUS,FV,FH,FSPH,DIFF,LOKANG)
DO 1 Q=1,20
THETAS=-1.832596 + 0.174533*Q
IF(SIGMA(II).EQ.0.0) THETAS=THETA(II)
SNTH=SIN(THETAS)**2
CSTH=COS(THETAS)**2
FXX=FXX + N(Q,P)*(1.-PE(II))*FSPH+PE(II)*(FV*SNTH+FH*CSTH))
FYY=FYY + N(Q,P)*(1.-PE(II))*FSPH+PE(II)*(FV*CSTH+FH*SNTH))
FXY=FXY + N(Q,P)*PE(II)*DIFF*SNTH*SNTH
1 CONTINUE
RETURN
END
SUBROUTINE NOIRRATE,II,N,AMODE,THETA,SIGMA)
C
C THIS SUBROUTINE RETURNS TO THE CALLING PROGRAM
THE NUMBER OF DROPS IN THE QTH CANTING ANGLE CLASS AND
THE PTH DROP SIZE CLASS.

DIMENSION CANNUM(20), AREA(20), SIGMA(10)
DIMENSION RRATE(15), THETA(15), N(20, 28)
INTEGER P, Q
REAL N, NUMTOT

NUMTOT = RRATE(1) * 5.83333 / (AMODE ** 3.5)

IF (SIGMA(1) .EQ. 0.0) GO TO 2

CALL NUMCAN(NUMTOT, THETA, CANNUM, SIGMA)

GO TO 4

2 CONTINUE
DO 3 J = 1, 20
CANNUM(J) = NUMTOT / 20.0
3 CONTINUE

4 CONTINUE

DELTAA = 0.125

DO 1 Q = 1, 20
DO 1 P = 2, 28
RADIUS = 0.125 / 2.0 + (P - 1) * 0.125

IF (RADIUS .LE. AMODE) N(Q, P) = CANNUM(Q) * DELTAA * RADIUS / AMODE ** 2
IF (RADIUS .GT. AMODE) IN(Q, P) = CANNUM(Q) * 2. * DELTAA * (1. - 5 * RADIUS / AMODE) / AMODE
1 / AMODE
IF (RADIUS .GT. (2. * AMODE)) N(Q, P) = 0.0
CONTINUE
RETURN
END
SUBROUTINE NUMCAN(NUMT,T,THETA,CANNUM,II,SIGMA)

THIS SUBROUTINE RETURNS TO THE CALLING PROGRAM THE
NUMBER OF DROPS IN ALL THE DISCRETE INTERVALS.
THE DISTRIBUTION IS THE NORMAL DISTRIBUTION.
SIGMA CAN EQUAL ZERO OR GREATER THAN 10 DEGREES

DIMENSION AREA(20),CANNUM(20)
DIMENSION SIGMA(10)
DIMENSION THETA(15)
REAL NUMTOT

SIGMAP=SIGMA(II)*0.017453

IF(SIGMAP.LT.0.17453) SIGMAP=0.17453

DO 1 I=1,19
K=I-1
THETAP=-1.57079 + K*0.174533
U=(THETAP-THETA(II))/1.4142136/SIGMAP

IF(U.LT.0.0) AREA(I)=0.5*ERFC(ABS(U))
IF(U.EQ.0.0) AREA(I)=0.5
IF(U.GT.0.0) AREA(I)=0.5*ERF(U) + 0.5

1 CONTINUE
CANNUM(I)=NUMT(I)/AREA(I)
CANNUM(20) = NUMTOT*(1.0 - AREA(19))

IF (CANNUM(20) < 1.E-10) CANNUM(20) = 0.0

DO 2 I = 1, 18
  K = I + 1
  CANNUM(K) = NUMTOT*(AREA(K) - AREA(I))
IF (CANNUM(K) < 1.E-10) CANNUM(K) = 0.0
2 CONTINUE

RETURN
END

SUBROUTINE COEF(FREQ,AMODE,FV,FH,FSPH,DIFF,LOKANG)

! THIS SUBROUTINE RETURNS TO THE CALLING PROGRAM THE SCATTERING
! COEFFICIENTS FOR SPHERICAL AND OBLATE RAIN DROPS. THE COEFFICIENTS ARE A FUNCTION OF FREQUENCY AND DROP SIZE
! AND ELEVATION ANGLE. THE COEFFICIENTS USED ARE THOSE OF UZUNOGLU, EVANS AND HOLT.

COMPLEX CMPLX
DOUBLE PRECISION U1, U2, U3, U4, US
REAL LOKANG

COMPLEX*16 FV, FH, FSPH, DIFF

IF (AMODE < 0.25) AMODE = 0.25
IF ((INT(FREQ) = 11).AND. (AMODE > 3.5)) AMODE = 3.5
IF(INT(FREQ).EQ.14).AND.(AMODE.GT.3.5)) AMODE=3.5
IF((INT(FREQ).EQ.20).AND.(AMODE.GT.3.0)) AMODE=3.0
IF((INT(FREQ).EQ.30).AND.(AMODE.GT.3.0)) AMODE=3.0

C
U1=AMODE
U2=AMODE**2
U3=AMODE**3
U4=AMODE**4
U5=AMODE**5

C
11.0 GHZ COEFFICIENTS
C
IF(INT(FREQ).NE.11) GO TO 1
C
C SPHERICAL DROP COEFFICIENTS
C
IF(AMODE.GT.1.00) GO TO 10
C
EOR=-0.0020548+0.01638947*U1-0.0417568*U2+0.08832213*U3
E01=-0.0025154+0.01928553*U1-0.0456816*U2+0.03655147*U3
GO TO 11
10 CONTINUE
C
EOR=-1.28155706+2.83287718*U1-2.07399678*U2+0.60190887*U3
1-0.00984194*U4-0.01096155*U5
E01=2.60278025-7.52434662*U1+8.14691632*U2-4.12133971*U3
1+0.99089467*U4-0.08731788*U5
C
GO TO 12
11 CONTINUE
OBLATE DROP COEFFICIENTS

EV90R = -0.001322 + 0.01036867*U1 - 0.025372*U2 + 0.07072533*U3
EV90I = -0.0024306 + 0.01861967*U1 - 0.0440732*U2 + 0.03507413*U3
EH90R = -0.0023684 + 0.01878307*U1 - 0.0473704*U2 + 0.09255573*U3
EH90I = -0.0030366 + 0.02320527*U1 - 0.0546296*U2 + 0.04299093*U3
DIFFR = 0.0010464 + 0.0084144*U1 - 0.0219984*U2 + 0.0218304*U3
DIFFI = -0.0060606 + 0.0045856*U1 - 0.0106064*U2 + 0.0079168*U3

GO TO 1000

12 CONTINUE

1 + 0.020309215*U4 - 0.02432204*U5
EV90I = 2.20618555 - 6.19251766*U1 + 6.43873903*U2 - 3.090414*U3
1 + 0.70600211*U4 - 0.0607173*U5
EH90R = 2.20618555 - 6.19251766*U1 + 6.43873903*U2 - 3.090414*U3
1 + 0.70600211*U4 - 0.0607173*U5
EH90I = 3.28654422 - 9.68217951*U1 + 10.71102231*U2 - 5.5491184*U3
1 + 1.36694565*U4 - 0.12512466*U5
DIFFI = 1.08034369 - 3.48962081*U1 + 4.2722407*U2 - 2.45868344*U3
1 + 0.66093861*U4 - 0.06440692*U5

IF(AMODE.GT.2.0) GO TO 120

DIFFR = -0.32599997 + 0.7531995*U1 - 0.57039996*U2 + 0.15039999*U3
GO TO 1000

120 CONTINUE

DIFFR = 11.94200847 - 14.59334299*U1 + 5.80600359*U2 - 0.7346671*U3
C GO TO 1000
C
1 CONTINUE
C
14 GHZ COEFFICIENTS
C
IF(INT(FREQ).NE.14) GO TO 2
C
SPHERICAL DROP COEFFICIENTS
C
IF(AMODE.GT.1.00) GO TO 20
C
EOR=-0.001376+0.012776*U1-0.040137*U2+0.128736*U3
EOI=-0.008796+0.06731467*U1-0.1588*U2+0.12418133*U3
C
GO TO 21
20 CONTINUE
C
EOR=-12.13707993+34.85683676*U1-37.84887378*U2+19.48995359*U3
1-4.69149894*U4+0.42544877*U5
EOI=-8.4550178+22.79996178*U1-23.38444001*U2+11.09553161*U3
1-2.41694538*U4+0.19907664*U5
C
GO TO 22
21 CONTINUE
C
OBLATE DROP COEFFICIENTS
C
EV90R=-0.000248+0.003354567*U1-0.013928*U2+0.10001233*U3
EV90I=-0.006366+0.06455567*U1-0.152256*U2+0.11893333*U3
EH90R=-0.000656+0.007656*U1-0.029656*U2+0.122656*U3
C
EH901 = -0.10366 + 0.079154 * U1 - 0.18596 * U2 + 0.143872 * U3
DIFFR = -C.000408 + 0.00430133 * U1 - 0.015728 * U2 + 0.02263467 * U3
DIFFI = -0.00193 + 0.01459533 * U1 - 0.03704 * U2 + 0.02493817 * U3

C

GO TO 1000

22 CONTINUE

C

EV90R = -4.48493663 + 12.45481829 * U1 - 12.90182271 * U2 + 6.32901913 * U3
1 - 1.42805671 * U4 + 0.11875301 * U5
EV90I = -0.1923376 + 0.3425132 * U1 - 0.196242 * U2 + 6.07468968 * U3

C

IF (AMODE .ST. 2) GO TO 24

C

EH901 = 1.10278458 - 3.74696237 * U1 + 4.48042001 * U2 - 2.22964317 * U3
1 + 0.42010037 * U4
DIFFR = -0.15268404 + 0.39830845 * U1 - 0.35275772 * U2 + 0.11767014 * U3
DIFFI = -1.29960041 + 2.93303524 * U1 - 2.15255009 * U2 + 0.5206384 * U3

C

IF (AMODE .GT. 2.001) GO TO 25

C

EH90R = 2.51000033 - 2.03000022 * U1 + 0.62000004 * U2
1 + 0.85000022 * U4
DIFFR = -15.85152202 + 16.27189723 * U1 - 6.68140666 * U2 + 0.794111153 * U3
DIFFI = 22.32321586 - 44.48681458 * U1 + 30.17926125 * U2 - 8.4692555 * U3

C

GO TO 1000

24 CONTINUE

C

EH901 = 2.51000033 - 2.03000022 * U1 + 0.62000004 * U2
DIFFR = -15.85152202 + 18.27189723 * U1 - 6.68140666 * U2 + 0.794111153 * U3
DIFFI = 22.32321586 - 44.48681458 * U1 + 30.17926125 * U2 - 8.4692555 * U3

C

GO TO 1000

25 CONTINUE
C
E90R=-19.65+20.6567*U1-6.66*U2+0.6933*U3
C
GO TO 1000
2 CONTINUE
C
20 GHZ COEFFICIENTS
C
IF(INT(FREQ).NE.20) GO TO 3
C
SPHERICAL DROP COEFFICIENTS
C
IF(AMODE.GT.1.0) GO TO 30
C
EOR=0.020296-0.145276*U1+0.297656*U2+0.008224*U3
C
ECI=-0.015488+0.12709133*U1-0.334568*U2+0.3058367*U3
C
GO TO 31
30 CONTINUE
C
FOR=3.35557425-8.61818659*U1+7.8157989*U2-2.68217917*U3
C
EUI=1.85636463-3.70750898*U1+2.19599302*U2-0.25148145*U3
C
GO TO 32
31 CONTINUE
C
OBLATE COEFFICIENTS
C
E90R=0.02454800-0.17912133*U1+0.384448*U2-0.07447467*U3
C
E90I=-0.013492+0.11190533*U1-0.299032*U2+0.27853867*U3
EH90R=0.02282-0.16596667*U1+0.34976*U2-0.03061333*U3  
EH90I=-0.015178+0.12623933*U1-0.33848*U2+0.31533867*U3  
DIFFR=0.000072+0.00175467*U1-0.039448*U2+0.0368*U3  
DIFFI=-0.001686+0.014334*U1-0.039448*U2+0.0368*U3

GO TO 1000
32 CONTINUE

EV90R=1.56037246-3.97019757*U1+3.66524972*U2-1.24458239*U3
1+0.14526662*U4
EV90I=4.77376463-13.63610787*U1+14.83397997*U2
1-7.66136891*U3+1.96881573*U4+0.19749381*U5
EH90R=4.63026679-11.97374109*U1+11.0254653*U2
1-4.0050613*U3+0.50244373*U4
EH90I=-1.5390057+6.3417866*U1-9.35328879*U2+6.20987895*U3
1-1.74544867*U4+0.17820925*U5
DIFFR=3.06985428-8.00354339*U1+7.36021547*U2
1-2.76047886*U3+0.3571771*U4
1-3.71425705*U4+0.3757024*U5

GO TO 1000
3 CONTINUE

IF(INT(FREQ).NE.30) GO TO 2000
30 GHZ COEFFICIENTS
SPHERICAL DROP COEFFICIENTS
IF(AMODE.GT.1.0) GO TO 40
EQR=0.028004 - 0.221564*U1 + 0.530744*U2 + 0.022816*U3
E01=-0.007072 + 0.09329866*U1 - 0.390112*U2 + 0.55688533*U3

GO TO 41

40 CONTINUE

EQR=-1.95096901 + 3.4107597*U1 - 0.82790307*U2 - 0.399824*U3
E01=5.77798866 - 16.3876166*U1 + 16.10102407*U2 - 6.06482662*U3

GO TO 42

41 CONTINUE

OBLATE DROP COEFFICIENTS

EV90R=0.03542 - 0.2838733*U1 + 0.7038*U2 - 0.15834666*U3
EV90I=0.013876 - 0.064756*U1 - 0.026904*U2 + 0.290784*U3
EH90R=0.031652 - 0.252652*U1 + 0.614952*U2 - 0.049952*U3
EH90I=-0.000532 + 0.04746533*U1 - 0.299512*U2 + 0.50957867*U3
DIFFR=-0.003768 + 0.03122133*U1 - 0.088848*U2 + 0.10839467*U3
DIFFI=-0.014408 + 0.03122133*U1 - 0.088848*U2 + 0.21879467*U3

GO TO 1000

42 CONTINUE

EV90R=3.93161268 - 10.97208846*U1 + 11.43870068*U2
EV90I=3.12729055 - 9.08249632*U1 + 9.16246329*U2
EH90R=-6.97583584 + 16.79179963*U1 - 13.53607686*U2
EH90I=-4.63811854*U3 - 0.57535653*U4
EH901 = 1.39373246 + 1.72736066 * U1 - 0.14076091 * U2 + 0.05474057 * U3
DIFFR = -10.90744851 + 27.76388807 * U1 - 24.97477753 * U2
1 + 9.42764918 * U3 - 1.26983545 * U4
DIFFI = 5.33984552 - 17.20924077 * U1 + 20.7812541 * U2
1 - 11.64360435 * U3 + 3.07942984 * U4 - 0.30370378 * U5

1000 CONTINUE

C  ALPH = 1.576796 - LOKANG
C    CSLA = 0.001 * COS(ALPH)**2
C    SNLA = 0.001 * SIN(ALPH)**2
C    FVR = CSLA * EOR + SNLA * EV90R
C    FVI = -CSLA * EO1 - SNLA * EV90I
C    FHR = CSLA * EOR + SNLA * EH90R
C    FHI = -CSLA * EO1 - SNLA * EH90I
C    DIFFR = -SNLA * DIFFR
C    DIFF = SNLA * DIFFI
C    EOR = 0.001 * EOR
C    EO1 = -0.001 * EO1
C
C    FV = CMPLX(FVR, FVI)
C    FH = CMPLX(FHR, FHI)
C    FSPH = CMPLX(EOR, EO1)
C    DIFF = CMPLX(DIFFR, DIFFI)
C
C  GO TO 3000
C
C 2000 WRITE(6, 2001)
C 2001 FORMAT(///, 3X, 'FREQUENCY NOT ALLOWED, ONLY 11, 14, 20, 30 GHZ ALLOWED', RPP09250)
C        1)
C
C  STOP

This data is stored in program memory for later output.

REAL ISOL1
COMMON/BLOC1/MINUSJ, CONV
COMMON/BLOC2/VWACH
DIMENSION PHASE1(100), ATTEN1(100), ISOL1(100)
COMPLEX MINUSJ
COMPLEX*16 EXC, EYC, EXX, EYX
COMPLEX*16 VW PAC, VW PAX
COMPLEX*16 EXN, EYN
REAL*8 VW PACR, VW PAXR
REAL*8 REAL, AIMAG
VW PAC = EXN*EXC + EYN*EYC
VW PAX = EXN*EXX + EYN*EYX
VW PACR = REAL(VW PAC)
VW PACI = AIMAG(VW PAC)
VWPACM=CDABS(VWPAC)
VWPACR=ATAN2(VWPACI,VWPACR)
VWPACP=VWPACR/CONV

C
VWPAXR=REAL(VWPAX)
VWPAXI=AIMAG(VWPAX)
VWPAXM=CDABS(VWPAX)
VWPAXR=ATAN2(VWPAXI,VWPAXR)
VWPAXP=VWPAXR/CONV
IF(VWPAXM.LE.0.000001) VWPAXP=0.0

C
PHASE1(I)=VWPAXP-VWPACP
C
IF(PHASE1(I).LT.0.0) PHASE1(I)=PHASE1(I)+360.0
C
ISOL1(I)=20.*DLOG10(VWPACM/VWPAXM)
ATTEN1(I)=20.*DLOG10(VWACH/VWPACM)
C
RETURN
END

SUBROUTINE CURVE(X,Y,III,A,B,R,K)

C
C* THIS SUBROUTINE PERFORMS A LINEAR REGRESSION ON ARRAYS X,Y
C*
C* POWER FITS, LOG FITS, AND EXPONENTIAL FITS ARE DONE BY
C* TAKING THE LOG OF THE INPUT DATA WHERE APPROPRIATE
C*
C* THE CALL STATEMENT FOR THIS SUBROUTINE IS
C*
C* CALL CURVE(X,Y,III,A,B,R,K)
C*
C*
WHERE

X IS A SINGLE DIMENSIONED ARRAY WITH III ENTRIES
Y IS A SINGLE DIMENSIONED ARRAY WITH III ENTRIES
A AND B ARE THE RETURNED COEFFICIENTS
R IS THE RETURNED CORRELATION COEFFICIENT R**2
K IS THE FLAG FOR THE TYPE OF CURVE FIT

K=1  LINEAR FIT  Y=A+B*X
K=2  POWER FIT  Y=A*X**B
K=3  LOG FIT   Y=A+B*LN(X)
K=4  EXP FIT   Y=A*EXP(B*X)

K=1 GO TO 20
K=2 GO TO 30
K=3 GO TO 20

DIMENSION X(300),Y(300)

IF (III.LE.1) GOTO 50

X1=0
X2=0
Y1=0
Y2=0
XY=0
DO 10 I=1,III
XX=X(I)
YY=Y(I)
IF(K.EQ.1) GO TO 20
IF(K.EQ.4) GO TO 30
XX=ALOG(XX)
IF(K.EQ.3) GO TO 20

20  X1=X1+XX
X2=X2+XX*XX
Y1 = Y1 + YY
Y2 = Y2 + YY*YY
XY = XY + XX*YY

CONTINUE
XN = FLOAT(III)
B = (XN*XY - X1*Y1) / (XN*X2 - X1*X1)
A = Y1 / XN - B*X1 / XN
R = (XN*XY - X1*Y1)**2 / (XN*X2 - X1*X1)*(XN*Y2 - Y1*Y1)

IF (K.EQ.1. OR. K.EQ.3) RETURN
A = EXP(A)
RETURN

50 A = 1.0
B = 1.0
R = 0.0
RETURN
END