SOLVING MAGNETOSTATIC FIELD PROBLEMS WITH NASTRAN

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SUMMARY

Determining the three-dimensional magnetostatic field in current-induced situations has usually involved vector potentials, which can lead to excessive computational times. A recent paper shows how such magnetic fields may be determined using scalar potentials. The present paper shows how the heat transfer capability of NASTRAN Level 17 has been modified to take advantage of the new method.

INTRODUCTION

All classical electromagnetic phenomena are governed by the four Maxwell equations:

\[ \nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \]  
\[ \nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0 \]  
\[ \nabla \cdot \mathbf{D} = \rho \]  
\[ \nabla \cdot \mathbf{B} = 0 \]

where the vector quantities are defined as follows:

- \( \mathbf{H} \) Magnetic field strength or intensity
- \( \mathbf{B} \) Magnetic induction or flux density
- \( \mathbf{J} \) Current density
- \( \mathbf{E} \) Electric field strength
- \( \mathbf{D} \) Electric displacement

and the scalar quantities are defined as follows:

- \( \rho \) Charge density
- \( t \) Time

There is also a constitutive relation between \( \mathbf{B} \) and \( \mathbf{H} \), given by

\[ \mathbf{B} = \mu \mathbf{H} \]
where $\mu$ is the magnetic permeability.

Electromagnetic problems are often solved by introducing and solving for the magnetic vector potential $A$, where

$$B = \nabla \times A$$  \hspace{1cm} (6)

Spreeuw and Reefman (ref. 1) used this method with NASTRAN in solving for harmonically oscillating electromagnetic fields in the presence of conductors carrying alternating currents. However, in order to use the existing structural and heat transfer capabilities in NASTRAN, simplifying assumptions had to be made. In particular, the magnetic vector potential $A$ and the source current densities $J$ were allowed to have components in only one direction, and those components were invariant in that direction. These assumptions effectively reduce the problem to one of solving for a scalar potential, which can be handled by NASTRAN's heat transfer analyses.

In the same paper, Spreeuw and Reefman also considered a problem in which $A$ was not unidirectional and were able to use NASTRAN's structural analysis capability only because the governing equations uncoupled for the components of $A$.

Another problem with this formulation is the requirement that

$$\nabla \cdot A = -\varepsilon \mu \frac{\partial \phi}{\partial t}$$  \hspace{1cm} (7)

where $\varepsilon$ is the electric permittivity.

Spreeuw and Reefman used a separate post-processor to handle this condition. Frye and Kasper (ref. 2), in solving magnetostatic problems using the vector potential, use a Lagrange multiplier method (similar to multi-point constraints) to satisfy constraint (7). They also point out that special boundary conditions are required at boundaries where the permeability $\mu$ changes.

In reference 3, Zienkiewicz, Lyness, and Owen have developed a method for solving general, three-dimensional magnetostatic problems using scalar potentials, so that standard heat transfer analyses may be used and constraint equation (7) is not required. They also indicate that special boundary conditions, such as those mentioned in reference 2, are not needed.

The present paper shows how this new method has been implemented in NASTRAN Level 17.

BASIC EQUATIONS AND ASSUMPTIONS

The problem to be solved is the determination of the magnetostatic field due to a body placed in an existing magnetic field produced, for example, by direct current-carrying loops. The materials are assumed to be linear, but may be anisotropic. The governing equations are:

$$\nabla \times H = J$$  \hspace{1cm} (8)
Zienkiewicz separates $H$ into two parts,

$$H = H_c + H_m$$

$H_c$ is the field in a homogeneous region due to current $J$, satisfies

$$\nabla \times H_c = J$$

and is computed using the Biot-Savart law. $H_m$ is the unknown magnetic field strength and satisfies

$$\nabla \times H_m = 0$$

so that

$$H_m = \nabla \phi$$

and

$$\nabla \cdot \mu \nabla \phi + \nabla \cdot \mu H_c = 0$$

where $\phi$ is the scalar potential. Zienkiewicz then uses standard variational principles, with equations (9) and (10), to arrive at the standard finite element form

$$K\phi = F$$

where $K$ is the "stiffness" matrix,

$F$ is the "load" vector, and

$$k_{ij} = \int_V (\nabla N_i)^T \mu \nabla N_j \, dV$$

$$f_i = \int_V (\nabla N_i)^T \mu H_c \, dV$$

where

$N_i$ is the finite element shape function for the $i^{th}$ grid point, and

$V$ is the volume of the finite element.

The formulation (16) for $k_{ij}$ is exactly that of the standard heat transfer conductivity matrix with magnetic permeability $\mu$ playing the role of thermal conductivity. The formulation (17) for $f_i$, however, is not a standard heat transfer quantity and must be computed either in a separate program and input to NASTRAN or in a new NASTRAN capability. Also, note that $f_i$ is element-dependent, as evidenced by the shape function gradient in equation (17).
Equation (15) is solved for $\phi$ subject to standard natural or forced boundary conditions. $H_m$ can then be computed from equation (14), and the final results can be obtained using equations (11) and (9).

**NASTRAN IMPLEMENTATION**

To solve magnetostatic problems with NASTRAN Level 17 using the methods of the previous section, we select rigid format 1, HEAT approach. However, we have modified the program to

1) compute the $H_c$ field due to circular, direct current-carrying loops;
2) accept a specified $H_c$ field;
3) compute $f_1$ (equation (17)) for the axisymmetric solid ring elements TRAPRG and TRIARG;
4) perform the addition specified in equation (11), where $H_m$ is a transformation of the "temperature" gradients; and
5) output $B$ (equation (9)) for subsequent NASTRAN plotting.

The implementation thus far has been limited to solid axisymmetric problems using TRAPRG and TRIARG finite elements and is running on the DTNSRDC CDC 6000 computers.

**NEW BULK DATA CARDS**

Two new bulk data cards have been introduced into the program for computing $H_c$ fields. They are CEMLOOP, for computing the $H_c$ fields due to circular current loops, and SPCEFLD, for specifying $H_c$ at selected grid points. (See figures 1 and 2 for detailed descriptions of these cards.)

**MODIFIED NASTRAN ROUTINES**

Nineteen existing NASTRAN routines have been modified to accommodate the new capability. The routines and the nature of the modifications are as follows:

<table>
<thead>
<tr>
<th>Routine</th>
<th>Reason for Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>IFP,IFS4P,IFX1BD-IFX7BD</td>
<td>Recognize and check new bulk data cards CEMLOOP and SPCEFLD.</td>
</tr>
<tr>
<td>LD21</td>
<td>Restart with CEMLOOP and SPCEFLD in Static Heat Transfer Analysis.</td>
</tr>
<tr>
<td>GP3A,GP3BD</td>
<td>Recognize CEMLOOP and SPCEFLD as &quot;heat transfer&quot; load specifications and place on HSLT (Heat Static Load Table).</td>
</tr>
<tr>
<td>Routine</td>
<td>Reason for Change</td>
</tr>
<tr>
<td>-------------</td>
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</tr>
<tr>
<td>SSGSLT</td>
<td>Retrieve CEMLOOP and SPCFLD specifications from NWSLT. CEMLOOP cards are copied directly to data block NWSLT for later processing. All SPCFLD specifications are combined into one vector giving total $H_c$ at all grid points. This vector is placed on NWSLT.</td>
</tr>
<tr>
<td>EXTERN</td>
<td>Call new routine EANDM, which retrieves CEMLOOP and SPCFLD specifications from NWSLT and computes $f$ as given in equation (17).</td>
</tr>
<tr>
<td>XBSHD, XMPLBD</td>
<td>Specify table updates so that NASTRAN will recognize new functional module EMFLD.</td>
</tr>
<tr>
<td>XSEMLA</td>
<td>Call new functional module EMFLD.</td>
</tr>
<tr>
<td>OFP1A</td>
<td>Specify new headings to output new data block HOEHL giving magnetic field strength and induction.</td>
</tr>
<tr>
<td>OFPZZZZ</td>
<td>Call for new headings from OFP1A when HOEHL is recognized.</td>
</tr>
</tbody>
</table>

**NEW ROUTINES**

Three new routines have been developed. Subroutine EANDM reads into open core CEMLOOP and SPCFLD data from data block NWSLT, reads information for an element from the Element Summary Table (EST), and calls an element-dependent routine to compute the load as given in equation (17).

Subroutine EMRING is called by EANDM and computes the load due to CEMLOOP and SPCFLD specifications for solid axisymmetric trapezoidal (TRAPRG) and triangular (TRIARG) rings. We assume that $H_c$ is constant over an element. Therefore, for TRIARG elements, $H_c$ is computed at the centroid due to all CEMLOOP's using an elliptic integral formulation. If SPCFLD's are given, $H_c$ is computed to be the average of the $H_c$'s at the three vertices. Each TRAPRG element is divided into four overlapping triangles, and each triangle is treated as a TRIARG element. Once $H_c$ has been computed for a triangle, equation (17) is used to compute the load at each grid point forming the triangle. Subroutine EMRING also outputs to Fortran file 11 certain element information, including $H_c$, for later processing by functional module EMFLD. This is a temporary method for passing information from EMRING to EMFLD. The normal method, of course, is to use data blocks. However, subroutine EMRING resides in functional module SSGL, and adding a new output data block to that existing module would require a change to every rigid format in the program. These changes will be made at a later time.

Subroutine EMFLD, which is also a new functional module, computes the magnetic field strength and induction, according to equations (11) and (9), for each finite element in the model. EMFLD retrieves from data block HOEFL the
"temperature" gradient for an element. Since the gradient $N_m$ was computed in an element coordinate system, EMFLD transforms it into basic coordinates. Then Fortran file 11 is searched to match the element identification number, and, on a match, $N_c$ for the element is retrieved, added to the transformed $N_m$, and put out to data block HOEHI. Also computed and output to HOEHI is the magnetic induction $B$. HOEHI is later output using normal Output File Processor (OFF) execution. EMFLD also computes and outputs other information for plotting purposes, as explained in the next section.

**PLOTTING MAGNETOSTATIC RESULTS**

Normal NASTRAN plot processing allows for deformed plots based on grid point displacements or contour plots of stresses. In the present analysis, however, the "displacements" (the scalar potentials) are of little use by themselves. The "stresses" (H or B fields coming from EMFLD) are more useful, but what we would really like to see for "nice" plots are the lines of induction. The lines of induction indicate the direction of the magnetic induction $B$, and the number of lines per unit area indicates the magnitude of $B$. While we do not presently plot these lines of induction, we do plot the actual induction, magnitude and direction, for each element. Therefore, functional module EMFLD outputs other quantities as follows. For each element, two coincident grid points are created at the centroid of the element, and the corresponding GRID cards are punched. Also punched is a PLOTEL card connecting the two grid points. (The length of the PLOTEL element is zero.) Then a "displacement" vector is created by assigning zero values to each of the original grid points in the model and assigning the magnetic induction value for an element to each of the two coincident grid points created for the element. (The "displacement" vector uses six degrees of freedom per grid point since $B$ is a vector, not a scalar.) This vector is packed in EMFLD and output in DMAP using module OUTPUT. On a subsequent NASTRAN run, the new GRID and PLOTEL cards are merged with the original data, the "displacement" vector is retrieved using DMAP module INPUT as data block UGV, and a deformed plot is requested with the VECTOR R option. This NASTRAN run is performed with rigid format 1, DISP approach, and ALTERS are used to execute only those modules required for deformed plots. The plots show the original structure as an underlay, and a vector is drawn at each element centroid indicating the magnitude and direction of $B$ in that element.

**SAMPLE PROBLEMS**

At the time that this paper was being prepared, the only axisymmetric problems run with NASTRAN for which analytical results were readily available were problems with uniform permeability. The comparison between the NASTRAN and analytical results was very good. Although problems with nonuniform permeability have been run with NASTRAN and "reasonable-looking" results have been obtained, analytical results, required for verification, are still forthcoming.
REFERENCES


Input Data Card CEMLOOP  Circular Current Loop

Description: Defines a circular current loop in magnetic field problems.

Format and Example:

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<table>
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</tr>
</thead>
<tbody>
<tr>
<td>CEMLOOP</td>
<td>SID</td>
<td>J</td>
<td>AXI</td>
<td>X1</td>
<td>Y1</td>
<td>Z1</td>
<td>X2</td>
<td>Y2</td>
</tr>
<tr>
<td>CEMLOOP</td>
<td>3</td>
<td>2.5</td>
<td>1</td>
<td>5.2</td>
<td>0</td>
<td>2.25</td>
<td></td>
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</tbody>
</table>

Field Contents

SID  Load set identification number (integer > 0).
J    Current through loop (units of positive charge/sec)(real ≥ 0).
AXI  = 0 nonaxisymmetric problem (not yet implemented)
     = 1 axisymmetric problem; TRAPR1 and TRIARG elements are implied (integer).
X1,Y1,Z1 Coordinates of two points through which the loop passes
             (given in coordinate system CID)(real).
X2,Y2,Z2
XC,YC,ZC Coordinates of center of loop (given in coordinate system CID) (real).
CID  Coordinate system identification number (integer ≥ 0).

Remarks:

1. Load sets must be selected in the Case Control Deck (LOAD=SID) to be used by NASTRAN.

2. If AXI=1, Y1 must be 0. or blank, and all data fields after Z1 must be 0. or blank. (Continuation card need not be present.)

Figure 1. Bulk Data Description of CEMLOOP
Input Data Card **SPCFLD** Specified Magnetic Field

**Description:** Specifies magnetic field at selected grid points.

**Format and Example:**

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
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<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>SPCFLD</td>
<td>SID</td>
<td>HCX</td>
<td>HCY</td>
<td>HCZ</td>
<td>G1</td>
<td>G2</td>
<td>G3</td>
<td>G4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SPCFLD</td>
<td>18</td>
<td>12.25</td>
<td>0.</td>
<td>62.</td>
<td>8</td>
<td>17</td>
<td>103</td>
<td>1</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

or

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</tr>
</thead>
<tbody>
<tr>
<td>SPCFLD</td>
<td>SID</td>
<td>HCX</td>
<td>HCY</td>
<td>HCZ</td>
<td>G1</td>
<td>G2</td>
<td>G3</td>
<td>G4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SPCFLD</td>
<td>18</td>
<td>12.25</td>
<td>0.</td>
<td>62.</td>
<td>9</td>
<td>THRU</td>
<td>27</td>
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or

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<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>SPCFLD</td>
<td>SID</td>
<td>HCX</td>
<td>HCY</td>
<td>HCZ</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SPCFLD</td>
<td>18</td>
<td>12.25</td>
<td>0.</td>
<td>62.</td>
<td>-1</td>
<td></td>
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</tbody>
</table>

**Field**

**Contents**

- **SID**: Load set identification number (integer > 0).
- **HCX, HCY, HCZ**: Components of specified $H_c$ field (real).
- **Gi, GID1**: Grid point identification numbers (integer > 0).
- **-1**: Implies the $H_c$ field applies at all grid points.

**Remarks:**

1. Load sets must be selected in the Case Control Deck (LOAD=SID) to be used by NASTRAN.

2. All grid points referenced by GID1 THRU GID2 must exist.

**Figure 2. Bulk Data Description of SPCFLD**