THEORETICAL MODELS OF HELICOPTER ROTOR NOISE

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SUMMARY

This paper examines some of the traditional theoretical models of helicopter noise. For low speed rotors, it is shown that unsteady load models are only partially successful in predicting experimental levels. A new theoretical model is presented which leads to the concept of "unsteady thickness noise." This gives better agreement with test results. For high speed rotors, it is argued that present models are incomplete and that other mechanisms are at work. Some possibilities are briefly discussed.

INTRODUCTION

The considerable theoretical effort expended on rotor noise in recent years has, as yet, had little impact on helicopter design practice. However, the impending era of helicopter noise legislation presents new challenges and opportunities for the researcher since designers will increasingly demand methods for reducing noise levels.

The objectives of this paper are to show that there are several areas of rotor noise where traditional theoretical ideas and mechanisms need revision and to suggest some ways forward. It is not intended to present a balanced state-of-the-art assessment of theoretical models to achieve this end, but rather to discuss a few selected topics which will illustrate the main points. (See ref. 1 for a recent review.) It is important to recognise that each topic has a strong practical motive behind it. The topics arise from a need to predict real helicopter noise in realistic situations. If these areas were more fully understood, it would go a long way towards helping the designer reduce helicopter external noise.

SYMBOLS

Measurements and calculations were made in British Imperial Units. They are presented herein in the International System of Units (SI) with the equivalent values given parenthetically in the British Imperial Units.

\[ a_0 \quad \text{speed of sound} \]
\[ BW \quad \text{analysis band width} \]
c  blade chord
D  in-plane drag force
div  divergence
e  exponential function
f  frequency factor
H  non-dimensional blade thickness distribution
h  physical blade thickness
i,j,β  indices
k  harmonic number
J_n  Bessel function
L  local unsteady lift
l  wave number
M  source Mach number
m  wave number
mB  harmonic number
M_t  tip Mach number
n  surface normal
P  acoustic power spectrum
p  pressure
R  blade radius
r  distance between source and observer
S  true blade surface
SPL  sound-pressure level
T  thrust
t  time
T_{ij}  quadrupole source strength
T_{ref}       reference thrust
U             typical blade speed
u             velocity
V_{c}         convection velocity
V             response velocity
W             power spectrum of input gust
w             gust velocity
x             vector position
x_1,x_2,x_3   axes
x,y,z         axes
\alpha        frequency
\delta        small parameter
\Theta        angle between observation direction and rotor axis
\Lambda       integral scale length
\lambda       negative constant
\rho           density
\Sigma         mean blade planform
\Omega         angular velocity of rotor

A bar over a symbol denotes a non-dimensional value; a bar below a symbol indicates a vector quantity.

UNSTEADY BLADE LOAD MODELS

The problem of main rotor hover noise has been central to rotor noise research over the past decade. The corresponding foundation stone for theoretical models was the realisation that small, oscillatory blade loads can result in a significant noise output far in excess of that resulting from the steady aerodynamic loads. These unsteady loads are assumed to result from disturbed inflow (such as atmospheric turbulence or blade wakes) entering the rotor and are accepted as the physical mechanism responsible for hover noise.
Mathematical models of this mechanism were first confined to the discrete tone generation (refs. 2, 3), but subsequent refinement (refs. 4, 5, 6) has extended it to broad band noise so that a unified treatment of the whole spectrum is now possible.

This concept, that noise arises from blade loads caused by disturbed inflow entering the rotor, has dominated thinking in the subject to the virtual exclusion of other ideas. However, for various reasons, it has not proved possible to convert this idea into a convincing prediction method. The most obvious difficulty is knowing details of the unsteady blade loads. These cannot be measured, and the inference from acoustic results is that they are extremely sensitive to extraneous influences such as wind or atmospheric turbulence, at least for the subjectively important higher frequencies. Thus the theories usually resort to "guestimating" average levels and trends, either of the loads directly (ref. 2) or of an input distortion spectrum which, combined with a simple blade response model, can give the loads indirectly (refs. 5, 6).

An additional difficulty arises when the observer lies in or near the rotor plane, which is often the case of practical interest. Here the noise field is dominated by the in-plane "drag" forces; but, the origins and characteristics of these unsteady drag forces are obscure, especially for inviscid flow at high reduced frequencies. Consequently, the acoustic studies have of necessity avoided this issue either by taking the drag forces to be zero or by simply relating them to the unsteady lifting forces.

Some of these difficulties are illustrated in figures 1-4 which show some comparisons between theory and fullscale test results for rotor discrete tones. The experiments were conducted on a whirl tower several years ago, and the test series and equipment are fully reported in references 7 and 8 (although some of the results used here do not appear in those references). The experiments have been compared with the theoretical predictions of reference 2, modified to permit alternative loading laws to be used. (These loading laws assume that the harmonics of the unsteady blade loading decay as \( n^\lambda \), where \( n \) is the harmonic number and \( \lambda \) is an empirically chosen negative constant.)

Figure 1 illustrates both the variability of discrete tone test data and the wide range of prediction levels. The two test spectra were taken under nominally identical conditions, but in different seasons. The theoretical results are based on reference 2 for the different loading laws indicated. The main points to note are the possible scatter on test results - up to 10 dB - and the even wider range of theoretical results which can be obtained by adjusting the loading law. It should be remembered that for main rotors it is the higher rotational harmonics (\( n > 40 \)) which are subjectively important, and these are the least accurately predicted.

Figures 2, 3, and 4 illustrate several points. The experimental data have been collapsed on a \( T^2 \) basis (to a reference thrust of 4 kN (1000 lb)), and this yields a good reduction of the data. This result is commonly found but is puzzling nonetheless; since, apart from general scale ideas, there is no direct reason to expect the unsteady blade loads to vary linearly with total rotor thrust.
Comparisons between the predictions of equations (13) and (14) and the experimental results are shown in Figures 3, 4, 6, and 7. The length scale \( \Lambda \) has been taken equal to the blade chord, and the mean square intensity \( w^2 \) equal to \( 10^{-5} \frac{U_{tip}^2}{(T/T_{ref})^2} \). Again, a \( T^2 \) dependence has been enforced upon the gust intensity, but there is no direct reason why this should be true. These levels of disturbance appear realistic; they vary from 1 m/s to 4 m/s for the test series quoted.

Figures 3 and 4 show the comparison of discrete tone levels for the in- and near-plane microphones. The agreement is very good, both in spectral shape and level, for the important higher rotational harmonics. The present theory clearly performs better than the previous model near the rotor plane.

Figures 6 and 7 show the corresponding broad band levels in a stylised form; and again, reasonable agreement is found near the rotor plane. Exactly the same input spectrum has been used for both the discrete and broad band noise; the difference is a matter of coherence scales. The input is insufficiently coherent at the higher frequencies to generate discrete tones. This transition from discrete tone to broad band noise is a natural feature of the model.

The main objective of this section has now been achieved. It has been shown that an alternative formulation of the rotor-gust interaction problem appears possible; this could lead to a better understanding of experimental results. The theory has the right trends for spectral shape, directionality, and speed dependence. Obviously some empiricism has been incorporated, but no more than in the loads model; and, this is outweighed by the more direct representation of the acoustic field. This is clearly a new avenue for research in the classic area of rotor-inflow distortion noise.

HIGH SPEED FLIGHT NOISE

Another area where the traditional unsteady loads model has not found favour is in rotor noise generation at high advance ratios. In this flight regime, a periodic impulsive noise signature is produced which increases significantly with flight speed. Containment of this increase is essential if future high speed helicopters are to meet certification levels. The cause of this noise has been considered in several theoretical papers (refs. 10, 11, 12), and these conclude that it is dominated by thickness noise. This is the noise caused by the direct volume displacement of the air by the blades as distinct from the forces they exert. The experimental work of reference 13 is usually cited to support this proposition; it shows negative pressure acoustic pulses of precisely the form predicted by the theory. The pulses vary in level very much as expected.

However, it is the contention of this section that thickness noise is not the sole cause of high speed flight noise since there are significant anomalies between test data and theory. Although the data of reference 13 tally with many aspects of the theory, the absolute levels do not. The experimental levels are roughly double those predicted theoretically. This is illustrated in figure 93.
However, in practice these surface sources are moved to the mean blade planform \( \Sigma \) (which lies in the rotor disc plane, fig. 5); the upper and lower surface sources are combined. The resultant source strength can be identified with minus local unsteady lift \( L \) and in-plane drag force \( D \). Hence equation (1) becomes

\[
4 \pi p(x,t) = \frac{\partial}{\partial x_3} \int \left[ \frac{L}{r(1-M_r^2)} \right] d\Sigma + \frac{\partial}{\partial x_\beta} \int \left[ \frac{D_\beta}{r(1-M_r^2)} \right] d\Sigma
\]

where \( x_3 \) is the direction normal to \( \Sigma \) and \( \beta = 1,2 \) are the two in-plane directions. This equation forms the mathematical foundation of the unsteady loads model.

Unfortunately, this last step is questionable. Acoustic sources cannot be moved around with impunity (without incurring additional multipoles), and, if the true integral equation status of the solution is to be retained, what boundary condition applies? Is it to be enforced at the true blade surface \( S \) or at the mean planform?

More important, it is very difficult to obtain the correct distribution for the high frequency drag component, even if viscous effects are omitted. This component depends directly on the full pressure distribution over the blade surface, and finite thickness and three dimensional effects must be retained in calculating that pressure distribution. Consequently, simple aerodynamic response theory will not yield it.

These difficulties can be overcome if the sources are placed on the mean planform at the outset of the acoustic analysis rather than afterwards. Source strengths are sought whose associated fields satisfy the equations of motion and the correct boundary conditions. This, of course, is just the classic method of singularities, but its application in the present context appears novel. The key to the analysis is the proper treatment of the boundary condition.

For simplicity, consider a thin symmetrical blade at zero incidence passing through a region of disturbed flow, figure 5. (Camber and incidence effects can easily be included in the analysis but make little difference to the final results). Let \( x, y, z \) be a stationary coordinate system, with \( x, y \) lying in the rotor plane and \( z \) normal to it. Let the upper blade surface be given by the equation \( z = \delta \hat{H}(x,y,t) \), where \( \delta \) is a small parameter. If the gust velocity is denoted by \( w \) and the corresponding response velocity by \( v \), then the unsteady boundary condition on \( S \) is \((v + w).\hat{n} = 0\), where \( \hat{n} \) is the surface normal. Hence at each instant

\[
(w_z + v_z) = \delta \left\{ (w_x + v_x) \frac{\partial \hat{H}}{\partial x} + (w_y + v_y) \frac{\partial \hat{H}}{\partial y} \right\}, \text{ on } S
\]

Since this is to be satisfied at \( z = \delta \hat{H}(x,y,t) \), expand each side of equation (3) in a series about \( z = 0 \),

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The first part of this expression, being symmetric about \( z = 0 \), can be satisfied by another dipole distribution similar to equation (9). This exhibits no novel features since it represents a small second order modification to the lift distribution arising from the displacement of the boundary condition. It will not be considered further. However, the second half of equation (10), being anti-symmetric about \( z = 0 \), requires a monopole type of source distribution to satisfy it. The appropriate form is

\[
(11)
\]

This equation is now multiplied by \( \partial^2 \) and returned to dimensional form; \( 2\partial H \) is written as \( h \), the physical blade thickness. Finally, since the source strength contains space derivations, these may be taken outside the integral. Hence equation (11) becomes

\[
4\pi p(x,t) = 2\rho \frac{\partial}{\partial t} \left( \frac{\partial}{\partial x} \left( \frac{\partial H}{\partial x} \right) + \frac{\partial}{\partial y} \left( \frac{\partial H}{\partial y} \right) \right) \frac{d\Sigma}{r(1-M_r)} \tag{12}
\]

where \( \beta \) is again confined to the two in-plane directions. This expression is taken as the definition of "unsteady thickness noise." It is the direct analogue of the conventional thickness noise equation (ref. 10) with the blade speed \( V \) here replaced by the unsteady gust velocity \( \overline{w} \).

Equations (9) and (12) are the central result of this section. They imply that the noise field can be represented by two distributions spread over \( \Sigma \). Equation (9) is an unsteady lift dipole distribution and is equivalent to its counterpart in equation (2). However, equation (12) represents a hybrid source whose strength is related to the product of the blade thickness and gust velocity clearly differing from the drag dipole term of equation (2). This does not mean that equation (2) is incorrect. In the present simplified situation the drag dipoles must possess considerably more structure than hitherto suspected, and consequently the acoustic field can be expressed in an alternative simpler form, equation (12). It is emphasised that equation (1) is an exact result, whereas equation (2) is an approximation to it; equations (9) and (12) are an alternative approximate solution to the same problem. However, since equation (12) relates the field directly to known quantities, many of the difficulties associated with equation (2) are avoided.

The main feature of equation (12), however, is that essentially it represents an in-plane quadrupole field (because of the double derivative outside the integral); and, this is of major acoustic significance. A simple order of magnitude comparison shows that this unsteady thickness noise can be significantly higher than the
Next, the theory of reference 2 has been used incorporating a total loading law of $\lambda = -2$ (not $\lambda = -2.5$) for comparison purposes. It is seen that a fair agreement with the data is obtained, at least for the lower speed, near axis combinations. From an empirical viewpoint, this agreement is encouraging and could form the basis for prediction in this regime. It should be remembered that the use of a simple loading law is an empirical convenience; the precise value of $\lambda$ is not determined by the theory.

However, it is equally clear that nearer the rotor plane (where the drag forces are important), the theory significantly underestimates the levels of the higher harmonics. Furthermore, it is not capable of giving the observed humped spectral shape, no matter what value of $\lambda$ is chosen. Thus, although the theory can be adjusted to fit in some directions, it fails elsewhere and does not provide a self-consistent description of the full discrete tone noise field.

The purpose of the above discussion is to demonstrate one area where present theoretical models are unsatisfactory. The model can be manipulated to match part of the test data set but does not match all of it; the model fails completely to explain some important features. Thus, although the idea that noise results from unsteady blade loads is an important physical notion, present models based on that idea do not appear to tell the whole story. This provides the motivation for a re-examination of the theory to see if any important aspects have been overlooked.

UNSTEADY THICKNESS NOISE

The unsteady blade loads model arises from one particular solution of the governing wave equation. However, there are many other possible solutions to this equation, and the art of aeroacoustics is choosing the form of solution most appropriate to the task in hand. In this section it is shown that a slightly different formulation of the problem, which leads to some different conclusions concerning certain aspects of the noise field, appears possible.

Consider a rotor running through a region of disturbed inflow. The mathematical basis of the unsteady loads model lies in the FW-H equation (ref. 9). If viscosity and quadrupoles are neglected and only the additional sound induced by the inflow distortion is considered, it follows from that equation that the sound field is given by the integral

$$4\pi p(x,t) = -\frac{3}{2\pi} \int \left[ \frac{p_{nj}}{r(1-M^2)} \right] dS$$

(1)

Here the notation is standard; but, it is emphasized that $p_{nj}$ is the local unsteady force per unit area exerted on the air; $S$ is the true blade surface. Thus the field is represented by a surface distribution of dipoles, but strictly speaking their strengths are unknown and have to be determined from the zero normal velocity condition. Equation (1) is a well posed integral equation.
\[(w_z + v_z)|_{z=0} + \delta H \frac{d}{dz}(w_z + v_z)|_{z=0} + \cdots = \delta A|_{z=0} + \delta^2 H \frac{dA}{dz}|_{z=0} + \cdots \quad (4)\]

where \(\delta A\) temporarily denotes the right side of equation (3). In practice, the gust velocity is small compared with the blade speed; typically \(w/U \sim \delta\). The response velocity \(v\) is also of this order. Thus write \(w\) as \(\delta w\) and expand \(v\) in a series

\[v = \delta v(1) + \delta^2 v(2) + \cdots \quad (5)\]

This expansion is inserted in equation (4) and like powers of \(\delta\) equated. Hence

\[\text{order } (\delta : v_z(1) + \bar{w}_z = 0) \quad \text{on } z = \pm 0 \quad (6)\]

\[\text{order } \delta^2 : v_z(2) + H \frac{\partial}{\partial z}(\bar{w}_z + v_z(1)) = \left\{ (\bar{w}_x + v_z(1)) \frac{\partial H}{\partial x} + (\bar{w}_y + v_z(1)) \frac{\partial H}{\partial y} \right\} \quad \text{on } z = \pm 0 \quad (7)\]

If the input gust is assumed to be incompressible, \(\text{div } \bar{w} = 0\), and equation (7) can be re-written

\[v_z(2) = \frac{\partial}{\partial x} (\bar{w}_x) + \frac{\partial}{\partial y} (\bar{w}_y) + v_z(1) \frac{\partial H}{\partial x} + v_z(1) \frac{\partial H}{\partial y} - H \frac{\partial v_z(1)}{\partial z} \quad \text{on } z = \pm 0 \quad (8)\]

This analysis applies to the upper blade surface; similar results apply to the lower surface.

The acoustic problem is solved by finding appropriate source distributions to satisfy these boundary conditions. It is assumed that linearised acoustic equations hold. The first order boundary condition demands that \(v_z(1)\) equals \(-w_z\) on \(z = \pm 0\), and this can be accomplished formally by a dipole distribution similar to the first part of (2),

\[4\pi p(1)(x,t) = \frac{\partial}{\partial x} \left[ \int \frac{L}{r(1 - M^2)} \right] \quad (9)\]

Of course, the appropriate lift distribution \(L\) is still to be found in terms of \(\bar{w}_x\), but equation (9) is now a well posed integral equation and a formal solution to the first order problem.

Now consider the second order problem. Considerations of symmetry about \(z = 0\) show that the boundary condition equation (8) can be written
drag noise for the acoustically important higher frequencies. A drag dipole typically has a local source strength of order \( O \cdot 10 U_w \) (\( U = \) typical blade speed), whereas the quadrupole source is of order \( O \cdot 1c w_p (c = \) blade chord). However, this has to be multiplied by an extra acoustic frequency factor \( f \) because of the additional time derivative outside the integral. Hence, the ratio of the thickness noise to drag noise is typically \( fc/U \), and this exceeds unity if the frequency exceeds about the 20th harmonic of the rotational frequency. For main rotors, this is the whole of the subjectively important range. Essentially the increased acoustic efficiency of the quadrupoles rapidly overcomes their weaker source strength. Thus the in-plane noise is considerably larger than unwitting use of the drag formula would suggest.

In order to illustrate the application of equation (12), some comparisons have been made with the test data cited earlier. To achieve this, the analysis of equation (12) has been taken considerably further, although only the outline is presented here. The gust velocity \( w_g(x, y, t) \) is expressed in terms of its frequency and spatial wave number spectrum. Then, by following the ideas of references 5 and 6, it is possible to obtain the following expression for the acoustic power spectrum \( P(x, f) \):

\[
P(x, f) = \left\{ \frac{2 f R \sin \theta}{4 \pi a_o r} \right\}^{5/2} \left( \sum_{n=\infty}^{\infty} W(1, m, \alpha_n) F_n^2 \left[ \left( 1 + f \sin \theta \right)^2 + m^2 \right]^{1/2} \right) d\alpha d m (13)
\]

(See Appendix.) Here \( W(1, m, \alpha) \) is the power spectrum of the input gust, as a function of frequency \( \alpha \) and spatial wave numbers \( l, m \). No point source approximation is made in this analysis; the full blade source distribution is retained.

To use this result, it is necessary to assume a form for the gust spectrum \( W \). Here a "frozen turbulence" model which is convected through the rotor with velocity \( V_c \), with integral length scale \( \Lambda \), is adopted. The form used in the present calculations is

\[
W(1, m, \alpha) = \frac{3 \alpha^2}{4\pi} \frac{\Lambda^3}{V_c} \left\{ 1 + \Lambda^2 \left[ l^2 + m^2 + (\alpha/V_c)^2 \right] \right\}^{-5/2}
\]

(14)

This form has been chosen both for its relative simplicity and its ability to yield good agreement with the experimental results. The justification for this is that the present objective is to show that unsteady thickness noise is possibly a significant noise source, not that it is definitely so. Consequently, it is permissible to show that plausible assumptions concerning spectral levels and shapes can lead to experimentally observed levels. It should be remembered that at present there are no experimental facts on the nature and level of the gusts entering a helicopter rotor; models of laboratory or atmospheric turbulence may not apply. The spectrum in equation (14) is similar to the Dryden spectrum used in references 5 and 6 but decays more rapidly at high frequencies.
8, which shows that although the data collapse reasonably well on the theoretically indicated parameter (ref. 10) there is a 6 dB gap between them. This is probably due to quadrupole sources (see below).

More important, many other aircraft do not show this type of acoustic signature. The literature contains several different examples of helicopter high speed approach signatures, (refs. 10, 14, 15), and it is clear from these that many aircraft exhibit signatures with both negative and positive going pressure spikes of more or less equal magnitude. It is usually argued that the negative spike is thickness noise and the positive one a "compressibility effect." This argument is given credence by the B0105 data (ref. 15), which show that thinner blades remove both the shock waves and the positive acoustic pulse. However, if this is so, why does the UH-1H aircraft (ref. 13) exhibit only an archetypal thickness noise pulse when running furthest into the compressibility regime?

Thus, experimental evidence appears to show that thickness noise is not the complete explanation of this phenomenon. What are the other possibilities? The next most obvious choice is the cyclic blade forces associated with the nominal rotor aerodynamics. The lift force suffers from the disadvantage that its field is zero in the rotor plane where the observed field is apparently a maximum. The cyclic drag, on the other hand, gives insignificant acoustic levels (ref. 16). From a theoretical viewpoint this is not surprising; simple cyclic load models do not contain any of the higher harmonics essential to noise generation by a force mechanism.

The last observation points to another possible explanation for high speed flight noise. It may be that unsteady blade loads associated with rapid and local transient effects, rather than slowly varying cyclic effects, are responsible for the noise. A blade bypassing a vortex is an obvious example, and in a stable flight regime this would be a repetitive event. It would be rich in the higher blade load harmonics and might explain the variability between aircraft types. This idea does not appear to have been sufficiently explored in this flight regime and merits attention. The unsteady thickness formulation described in the previous section may prove useful in this context, since it is capable of predicting the in-plane noise solely from a model of the flow disturbance.

There remains the important question of whether quadrupoles are a possible source of high speed flight noise. There have been several unsuccessful attempts at proving this, but a breakthrough appears to have been made in a very recent paper (ref. 17). Briefly, that paper shows that for a transonic propeller, the $pu^2$ quadrupole makes a significant contribution to the noise only for those blade sections operating above their critical Mach number and below unit Mach number. In this regime, the quadrupole noise is approximately equal to the thickness noise.

It remains to be seen to what extent this mechanism applies to helicopter rotors. At present the model takes no account of tip relief effects, cyclic velocity variations, or possible hysteresis effects in the transonic blade flow. All of these details may be important in a helicopter application. Nonetheless, at first sight it does appear that this mechanism applies to helicopter rotors and consequently could offer an explanation for the anomaly between theory and
the data of reference 13 noted earlier. However, it is not immediately apparent whether it could also explain the negative-positive pulse shapes exhibited by most aircraft. This is a promising area which needs to be explored.

It is worthwhile to conclude this discussion of quadrupole effects on a cautionary note. There is a great temptation in performing quadrupole studies to base the source strength on any model which can be readily calculated. However, since the FW-H equation is exact (applying to all fluid flows), only the full solution to the problem will strictly provide the correct quadrupole source strength for use in that equation. This, of course, is never known. Thus the quadrupole integral only becomes useful if some local approximation to $T_{ij}$ can be employed, but as yet the existence and nature of this approximation is not established. It can only be revealed by alternative analysis, involving carefully considered physical arguments and mathematical modelling. The resultant quadrupole source strength may then turn out to be very different from that indicated by the superficial form of $T_{ij}$. The quadrupole integral is not just another source to be added on; it plays a more subtle role, and its relevance to the high speed noise problem remains an open question.

It is clear from this that the high speed noise question is not fully answered. There is sufficient experimental evidence to show that it is not all thickness noise, although this undoubtedly plays a prominent role and provides a reasonable basis for 'first-cut' predictions. However, there still appear to be some unexplored mechanisms which could considerably improve our understanding in this area.

CONCLUSIONS

The purpose of this paper has been to highlight some deficiencies in traditional ideas about rotor noise and to suggest some possible improvements. Section 2 was concerned with the conventional unsteady loads model. It was concluded that although this model could provide a reasonable basis for prediction at the lower speed near axis configurations, it fared badly for the higher frequencies near the rotor plane. This regime is subjectively important and the theory significantly underpredicts the noise in this situation.

In section 3, a new approach to this problem was described. It was shown that the rotor-gust interaction problem could be reformulated, and this led to a description of the in-plane noise in terms of a set of unsteady thickness quadrupole sources. These replace the traditional drag dipoles and overcome many of their practical difficulties. Better agreement with the experimental levels in and near the rotor plane was obtained from this model, for both discrete tone and broad band noise.

In the final section, the problem of high speed flight noise was discussed. Although no new mathematical models were proposed, it was argued that this noise could not be due entirely to blade thickness sources. It was suggested that repetitive transient effects may be important for some aircraft in this flight regime. However, since it now appears proven that quadrupole sources do become
important at transonic blade speeds, several new mechanisms may soon emerge relevant to this problem. This appears to offer a very promising direction for further research.
APPENDIX

In equation (13), \( R \) denotes the rotor tip radius, and \( \theta \) the angle between the observation direction and the rotor axis. If the angular velocity of the rotor is \( \Omega \) rad/s, then \( \alpha_n = \Omega_n \). \( F_n \) and \( h_n \) are defined as follows:

\[
F_n (X) = \frac{1}{X} \int_0^X J_n (x) \, dx
\]

which is the average value of Bessel function, and

\[
h_n = \int_{-c/2}^{+c/2} h(\xi) e^{-i\xi / R} \, d\xi
\]

which is the Fourier component of chordwise distribution of blade thickness, assumed independent of spanwise station. If there are \( B \) blades, \( n \) must be restricted to multiples of \( B \) and value of integral multiplied by \( B \).
REFERENCES


Figure 1.— Variability of rotor discrete tone noise.

Figure 2.— Rotor discrete tone noise, microphone A (75° from rotor plane).
Figure 3.- Rotor discrete tone noise, microphone B (11.5° from rotor plane).

Figure 4.- Rotor discrete tone noise, microphone C (in rotor plane).
Figure 5.- Rotor geometry and coordinate system.

Figure 6.- Rotor broad band noise, microphone B.
Figure 7.- Rotor broad band noise, microphone C.

Figure 8.- UH-1H impulsive noise.