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(NASA-CR-151830) REMOTE MANIPULATOR SYSTEM FLEXIBILITY ANALYSIS PROGRAM: MISSION PLANNING, MISSION ANALYSIS, AND SOFTWARE FORMULATION (McDonnell-Douglas Astronautics Co.) 50 p HC A03/MF A01 CSCL 22A G3/16

N79-10116

Unclas 35938

MCDONNELL DOUGLAS TECHNICAL SERVICES CO. HOUSTON ASTRONAUTICS DIVISION

TRANSMITTAL MEMO

CR151830

TM NO.	1.4-7-650
DATE	10 August 1978
CONTRACT NO.	NAS 9-15550
TASK ORDER NO.	D0937
TASK ASSIGNMENT NO.	

SUBJECT: TRANSMITTAL OF WORKING PAPER 1.4-7-245

ENCL: Remote Manipulator System Flexibility Analysis Program

To: NASA/LYNDON B. JOHNSON SPACE CENTER
2101 NASA ROAD I
HOUSTON, TEXAS 77058
ATTN:

Use Brief Informal Language

REMARKS:

Two programs 'A' and 'B' have been developed on the Hewlett Packard-9825.

Program 'A' calculates the flexibility coefficients for the Remote Manipulator System (RMS) modeled as five beams with each beam composed of an arbitrary number of segments.

Program 'B' calculates the end-effector arm flexibility and the joint flexibility terms for the torque motor model for any arbitrary arm configuration.

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SPACE SHUTTLE ENGINEERING AND OPERATIONS SUPPORT

WORKING PAPER NO. 1.4-7-245

REMOTE MANIPULATOR SYSTEM FLEXIBILITY ANALYSIS PROGRAM

MISSION PLANNING, MISSION ANALYSIS, AND SOFTWARE FORMULATION

---

10 AUGUST 1978

This Working Paper is Submitted to NASA Under Task Order

No. D0937, Contract NAS 9-15550.

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1.0 SUMMARY

The 'Payload Deployment and Retrieval System Simulation' (PDRSS) program, which is currently in use for simulating the Remote Manipulator System (RMS) does not have the capability to update the flexibility coefficients as the arm design changes. Any standard structural program can be used to calculate these coefficients. However, most of these structural programs require large computers and the associated turn-around time and expense make them unattractive for calculating just the flexibility coefficients. A program (called 'Program A' in this report) has been developed on the Hewlett Packard-9825 to calculate these flexibility coefficients.

Another program (called 'Program B') has been developed to cut down the number of PDRSS runs required to simulate various RMS maneuvers. This program calculates the end-effector flexibility and the joint flexibility terms for the torque motor model of each joint for any arbitrary arm configuration. Instead of making expensive PDRSS runs for all the required arm operations, program 'B' can be used to select those which involve stiffest arm configurations. Figure 2 shows the use of programs 'A' and 'B' in conjunction with the PDRSS program. Flexibility coefficients calculated by 'A' are required as input to 'B'.

Section 2.0 describes the purpose of this task in greater detail. Equations used in 'A' and 'B' are presented in Section 3.0.

Appendix A discusses the mathematical basis for the equations used in 'A' and 'B'.

Appendix B familiarizes the user with the procedure required to access the programs 'A' and 'B', the input parameters required and the output obtained.

Appendix C presents the results which are obtained through the use of programs 'A' and 'B' for two example problems and compares them with the PDRSS results. Appendix D contains the listing of programs 'A' and 'B'.

2.0 INTRODUCTION

The 'Payload Deployment and Retrieval System Simulation' (PDRSS) program, which is currently in use for simulating the RMS, uses a five beam model to represent the arm. In this model, flexibility coefficients for each of the five beams are required to simulate the arm motion and the loads generated during any commanded arm operation. At present, the flexibility coefficients built-into the PDRSS program, have been calculated by assuming beam one is between the attach point and the shoulder pitch joint and is composed of three different segments. Similarly, beam two is between the shoulder pitch joint and the elbow pitch joint and is composed of three segments; beam three is between the elbow pitch joint and the wrist pitch joint and consists of three segments; beam four is between the wrist pitch joint and yaw joint and consists of two segments; beam five is between the wrist yaw joint and the end effector and consists of three segments. The above model is shown in Figure 1.

The PDRSS program does not have the capability to update the flexibility coefficients as the arm design changes. Any standard structural program can be used to calculate these flexibility coefficients but most of these programs use large computers and the turn-around time and expense involved make them unattractive. Therefore, it was recognized that a program is needed, which could be run on a desk-type computer, to calculate the flexibility coefficients for the arm modeled by any given number of beams each consisting of an arbitrary number of segments. This led to the first objective of this task, i.e., to develop a program on the Hewlett-Packard 9825 to calculate the flexibility coefficients. The second objective of this task was to develop the capability to determine the arm flexibility in any given configuration, i.e., deflection at the end-effector due to unit loads at the end effector for any arbitrary arm configuration. This information would be useful in making a judgement regarding the arm stiffness and thereby anticipating its natural frequency and loads in a given configuration.

For example, to assure that design loads are not exceeded during arm operation, instead of making expensive PDRSS runs for all the required arm operations we can select those which involve arm configurations with higher stiffnesses. To this end another H. P. program has been developed which determines end-effector arm flexibility, when given the individual beam flexibility coefficients and the arm configuration (characterized by specifying six joint angles). This program has been further extended to determine the flexibility coefficients for the torque motor model at each joint. When studying simplified simulation techinques which could be used instead of PDRSS, flexibility for the torque motor model can be calculated with the above program. The use of programs 'A' and 'B' in conjunction with the PDRSS program to simulate the arm is shown in Figure 2.

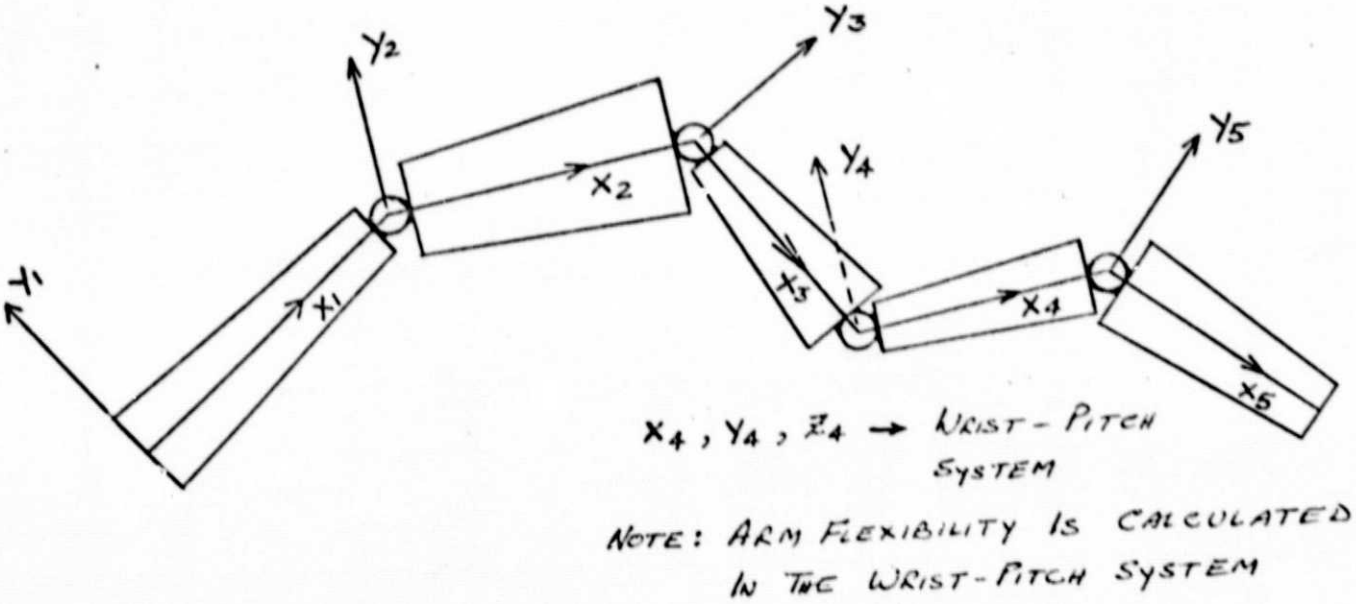

Figure 1

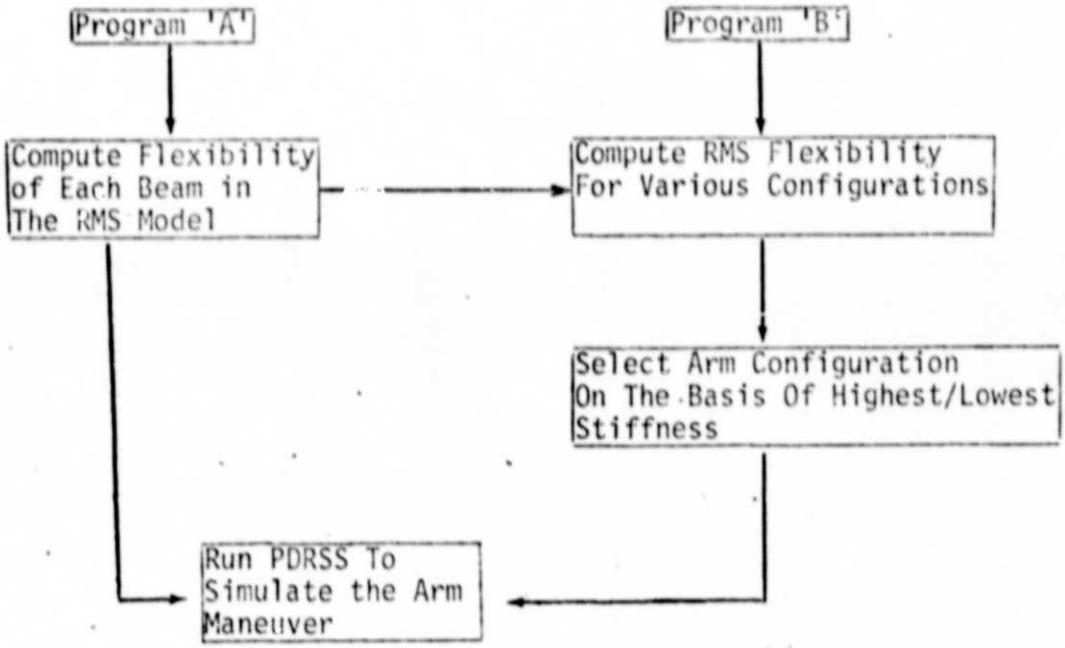
Program 'A'
Program 'B'
Compute Flexibility of Each Beam in The RMS Model
Compute RMS Flexibility For Various Configurations
Select Arm Configuration On The Basis Of Highest/Lowest Stiffness
Run PDRSS To Simulate the Arm Maneuver

Figure 2

3.0 ANALYSIS

Equations used in programs 'A' and 'B' are presented below. Program 'A' uses the standard load/deflection relationship for a beam to calculate the flexibility coefficients. Three rotational and two displacement degrees of freedom are assumed for each beam. In the RMS simulation it is assumed that the displacement along the beam axis is negligible; therefore, the flexibility coefficient in the axial direction is taken to be zero. Program 'B' uses the flexibility coefficients of each beam to calculate the combined arm flexibility at the end-effector. Program 'B' takes into account the effect of arm configuration by using transformation matrices to transform all the flexibility coefficients into the wrist-pitch system. Through the use of the Jacobian matrix, program 'B' uses the end-effector flexibility coefficients to calculate the flexibility at each of the six RMS joints. It further calculates, the joint flexibility terms for the torque motor model at each joint. Appendix A presents the derivation which leads to the equations used in program 'A' and 'B'.

3.1 PROGRAM 'A'

For a beam consisting of n segments, the load/deflection relationship is given by

$$\begin{Bmatrix} \xi_R{}^n \\ P_R{}^n \end{Bmatrix} = \begin{Bmatrix} L^n & \alpha^n \\ 0 & L^n \end{Bmatrix} \begin{Bmatrix} L^{n-1} & \alpha^{n-1} \\ 0 & L^{n-1} \end{Bmatrix} \cdots \begin{Bmatrix} L^1 & \alpha^1 \\ 0 & L^1 \end{Bmatrix} \begin{Bmatrix} \xi_L{}^1 \\ P_L{}^1 \end{Bmatrix} \qquad (1)$$

where

$$L^n = \begin{Bmatrix} 1 & \ell_n \\ 0 & 1 \end{Bmatrix}, \quad \alpha^n = \begin{Bmatrix} \ell^2{}_n/2EI_2 & \ell^3{}_n/6EI_2 \\ \ell_n/EI_2 & \ell^2{}_n/2EI_2 \end{Bmatrix}$$

$$\xi_R{}^n = \begin{Bmatrix} Y_R{}^n \\ \phi_R{}^n \end{Bmatrix} \qquad P_R{}^n = \begin{Bmatrix} M_R{}^n \\ F_R{}^n \end{Bmatrix} \qquad G = \begin{Bmatrix} 0 & 0 \\ 0 & 0 \end{Bmatrix}$$

Superscript n = Number of Segment

Subscript L = L.H.S. of Segment

Subscript R = R.H.S. of Segment

ℓ_n = Length of nth Segment

E = Young's Modulus, nth Segment

I_2 = Moment of Inertia About the Z-Axis, nth Segment

$Y_R{}^n$ = Displacement at R.H.S. of nth Segment in the Y-Direction.

$\phi_R{}^n$ = Rotation at R.H.S. of nth Segment About the Z-Axis

$M_R{}^n$ = Bending Moment Applied at R.H.S. of nth Segment About the Z-Axis

$F_R{}^n$ = Force Applied at R.H.S. of nth Segment in the Y-Direction

Another Equation, Similar to (1) can be written for bending about the Y-Axis. The only difference would be the moment of inertia about the Y-Axis, I_1, instead of I_2. For torsional flexibility, the torque/rotation relationship is given by

$$\Theta_R{}^n = \sum_{K=1}^{n} \frac{\ell_K}{G_K \; J_K} \tau_R{}^n \tag{2}$$

where θ_R^n = Rotation at R.H.S. of nth Segment About the Beam Axis

τ_R^n = Applied Torque at R.H.S. of nth Segment About the Beam Axis

G_n = Shear Modulus of nth Segment

J_n = Polar Moment of Inertia of nth Segment

In Program 'A' Equations (1) and (2) have been programmed to calculate flexibility coefficients of a beam consisting of arbitrary number of segments n.

3.2 PROGRAM 'B'

For the RMS consisting of five beams, end-effector flexibility (γ_E) in the Wrist-Pitch System is given by

$$\{\gamma_E\} = \sum_{i=1}^{5} \begin{Bmatrix} I & 0 \\ -r_{i5} & I \end{Bmatrix} \begin{Bmatrix} T_{Wi} & 0 \\ 0 & T_{Wi} \end{Bmatrix} \begin{Bmatrix} \gamma_{11}^i & \gamma_{12}^i \\ \gamma_{12}^i & \gamma_{22}^i \end{Bmatrix} \begin{Bmatrix} T_{iW} & 0 \\ 0 & T_{iW} \end{Bmatrix} \begin{Bmatrix} I & r_{i5} \\ 0 & I \end{Bmatrix} \qquad (3)$$

where $-r_{i5} = -\sum_{K=i}^{4} r_K$

and $r_K = \begin{Bmatrix} 0 & -r_K^3 & r_K^2 \\ r_K^3 & 0 & -r_K^1 \\ -r_K^2 & r_K^1 & 0 \end{Bmatrix}$ where r_K^j = Length of (K + 1)th Beam in jth Direction, K = 1, 2, 3, 4

j = 1, 2, 3

Superscripts j = 1, 2, 3 Represent X, Y, Z Axes of the Orbiter Coordinate System, Respectively.

T_{Wi} , i = 1, 2, 3, 4, 5 are the transformation matrices from the beam i system to the W. Pitch system. $Y11^i$, $Y12^i$, $Y22^i$, i = 1, 2, 3, 4, 5 are the matrices of flexibility coefficients of beam i.

The joint flexibility matrix is given by

$$\left\{Y_J\right\} = \left\{J^{-1}\right\} \left\{Y_E\right\} \left\{J^{T-1}\right\} \tag{4}$$

and the flexibility term for the torque motor model at each joint is:

$$Y_i^e = Y_{ii_J} - \left\{Y_{i_J}^{21}\right\}^T \left\{Y_{i_J}^{22}\right\}^{-1} \left\{Y_{i_J}^{21}\right\} \quad i = 1, 2, 3, 4, \tag{5}$$

$\left\{Y_{i_J}^{22}\right\}$ is a 5x5 matrix derived from $\left\{Y_J\right\}$ by deleting the ith row and column. i being the number of the joint. $\left\{Y_{i_J}^{21}\right\}$ is a 5x1 matrix derived from $\left\{Y_J\right\}$ by taking the ith column and deleting the ith element.

Equations (3), (4) and (5) are used in Program 'B' to calculate the end-effector flexibility, joint flexibility and the flexibility terms for the torque motor model.

4.0 CONCLUSIONS AND RECOMMENDATIONS

Two computer programs, 'A' and 'B', on the Hewlett-Packard 9825 have been developed incorporating the computations described in the previous section. The input and output data formats of these programs are described in Appendix B, sample results are in Appendix C and Program Listings are in Appendix D. Conclusions and recommendations related to the use of these programs are summarized below.

Program 'A' is used to calculate flexibility coefficients for the example problem 1 in Appendix C-1. Flexibility coefficients for beams 2 and 3 as calculated by Program 'A' are presented in Appendix C-3.1 along with corresponding flexibility coefficients currently used in the PDRSS. Comparing the two we conclude that they match closely. The difference in 2nd and 3rd decimal places (8 and 9 significant figures) is attributed to different number of significant figures in the two machines.

Program 'B' is used to calculate the end-effector arm flexibility for the example problem 2 in Appendix C-2. For comparison, end-effector arm flexibility calculated by Program 'B' is presented in Appendix C.3-2 along with end-effector flexbility calculated by PDRSS. We again conclude that these flexibility coefficients match almost exactly. Again, the insignificant difference is attributed to the different machines used to compute these coefficients.

We have concluded from the execution of the sample problems that the formulation and implementation of the programs is correct. The programs are available for use. Further information may be obtained by contacting the author, Lalit Kumar, at McDonnell Douglas Technical Services Company, 488-5660, Extension 216.

5.0 REFERENCES

1. Lockheed Electronics Cc: RMS Massless Arm Dynamics Capability in SVDS, JSC-12632, LEC-10633, June 1977.
2. Lockheed Electronics Co: Kinematic Capability in the SVDS, JSC-12606, LEC-10246, March 1977.

APPENDIX A

APPLICABLE EQUATIONS

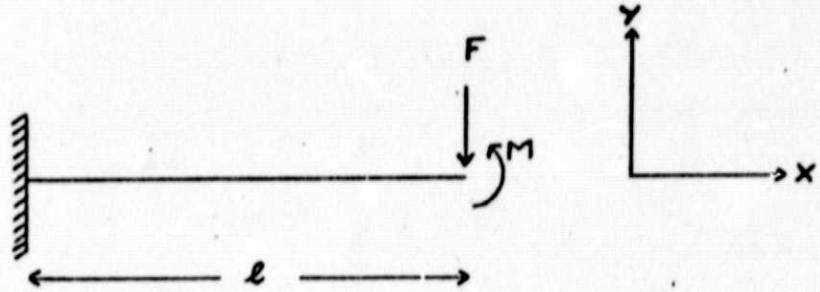

FIGURE A-1

For a cantilever beam of length ℓ shown in Figure A-1, the deflection due to end load F is given by

$$Y^F = \frac{-\ell^3}{3EI_2} F \qquad \text{(A-1)}$$

The deflection due to moment M is :

$$Y^M = \frac{\ell^2}{2EI_2} M \qquad \text{(A-2)}$$

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The total deflection is :

$$Y = Y^F + Y^M = \frac{-\ell^3}{3EI_2} F + \frac{\ell^2}{2EI_2} M \qquad \text{(A-3)}$$

Similarly Total Rotation/Slope is given by:

$$\phi = \frac{-\ell^2}{2EI_2} F + \frac{\ell}{EI_2} M \qquad \text{(A-4)}$$

where E = Young's Modulus

I_2 = Moment of Inertia of the Beam Crossection About the Z-Axis

I_1 = Moment of Inertia of the Beam Crossection About the Y-Axis

For an arbitrary uniform beam segment loaded as shown in Figure A-2

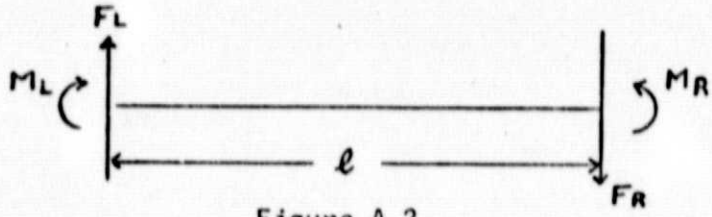

Figure A-2

$$\phi R = \phi L + \frac{\ell}{EI_2} M_R - \frac{\ell^2}{2EI_2} F_R \tag{A-5}$$

$$Y_R = Y_L + \ell\phi_L + \frac{\ell^2}{2EI_2} M_R - \frac{\ell^3}{3EI_2} F_R \tag{A-6}$$

$$M_L = M_R - \ell F_L \tag{A-7}$$

$$F_L = F_R \tag{A-8}$$

Rearranging Equations A-5, --- A-8 and writing in matrix form

$$\begin{Bmatrix} Y_R \\ \phi R \\ M_R \\ F_R \end{Bmatrix} = \begin{Bmatrix} 1 & \ell & \ell^2/2EI_2 & \ell^3/6EI_2 \\ 0 & 1 & \ell/EI_2 & \ell^2/2EI_2 \\ 0 & 0 & 1 & \ell \\ 0 & 0 & 0 & 1 \end{Bmatrix} \begin{Bmatrix} Y_L \\ \phi_L \\ M_L \\ F_L \end{Bmatrix} \tag{A-9}$$

In partitioned form

$$\begin{Bmatrix} \xi_R \\ P_R \end{Bmatrix} = \begin{Bmatrix} L & \alpha \\ 0 & L \end{Bmatrix} \begin{Bmatrix} \xi_L \\ P_L \end{Bmatrix} \tag{A-10}$$

where $L = \begin{Bmatrix} 1 & \ell \\ 0 & 1 \end{Bmatrix}$, $\alpha = \begin{Bmatrix} \ell^2/2EI_2 & \ell^3/6EI_2 \\ \ell/EI_2 & \ell^2/2EI_2 \end{Bmatrix}$, $0 = \begin{Bmatrix} 0 & 0 \\ 0 & 0 \end{Bmatrix}$

Similarly for two consecutive segments

$$\begin{Bmatrix} \xi_R{}^1 \\ P_R{}^1 \end{Bmatrix} = \begin{Bmatrix} L^1 & \alpha^1 \\ 0 & L^1 \end{Bmatrix} \begin{Bmatrix} \xi_L{}^1 \\ P_L{}^1 \end{Bmatrix} \tag{A-11}$$

and
$$\begin{Bmatrix} \xi_R{}^2 \\ P_R{}^2 \end{Bmatrix} = \begin{Bmatrix} L^2 & \alpha^2 \\ 0 & L^2 \end{Bmatrix} \begin{Bmatrix} \xi_L{}^2 \\ P_L{}^2 \end{Bmatrix} \tag{A-12}$$

but
$$\begin{Bmatrix} \xi_R{}^1 \\ P_R{}^1 \end{Bmatrix} = \begin{Bmatrix} \xi_L{}^2 \\ P_L{}^2 \end{Bmatrix} \tag{A-13}$$

Therefore by substitution

$$\begin{Bmatrix} \xi_R{}^2 \\ P_R{}^2 \end{Bmatrix} = \begin{Bmatrix} L^2 & \alpha^2 \\ 0 & L^2 \end{Bmatrix} \begin{Bmatrix} L^1 & \alpha^1 \\ 0 & L^1 \end{Bmatrix} \begin{Bmatrix} \xi_L{}^1 \\ P_L{}^1 \end{Bmatrix} \tag{A-14}$$

For n consecutive segments

$$\begin{Bmatrix} \xi_R{}^n \\ P_R{}^n \end{Bmatrix} = \begin{Bmatrix} L^n & \alpha^n \\ 0 & L^n \end{Bmatrix} \begin{Bmatrix} L^{n-1} & \alpha^{n-1} \\ 0 & L^{n-1} \end{Bmatrix} \begin{Bmatrix} L^1 & \alpha^1 \\ 0 & L^1 \end{Bmatrix} \begin{Bmatrix} \xi_L{}^1 \\ P_L{}^1 \end{Bmatrix} \tag{A-15}$$

Superscript Refers to the Segment Number

Subscript L → L.H.S. of Beam

Subscript R → R.H.S. of Beam

By Definition

$$\underset{6x1}{\{\delta\}} = \underset{6x6}{\{A\}} \quad \underset{6x1}{\{F\}} \tag{A-16}$$

Where δ is the deflection due to load F and A is a 6x6 matrix of flexibility coefficients. Writing A-16 in expanded form

$$\begin{Bmatrix} \delta_{O_x} \\ \delta_{O_y} \\ \delta_{O_z} \\ \delta_x \\ \delta_y \\ \delta_z \end{Bmatrix} = \begin{Bmatrix} A_{11} & A_{12} & A_{13} & A_{14} & A_{15} & A_{16} \\ A_{21} & A_{22} & A_{23} & A_{24} & A_{25} & A_{26} \\ A_{31} & A_{32} & A_{33} & A_{34} & A_{35} & A_{36} \\ A_{41} & A_{42} & A_{43} & A_{44} & A_{45} & A_{46} \\ A_{51} & A_{52} & A_{53} & A_{54} & A_{55} & A_{56} \\ A_{61} & A_{62} & A_{63} & A_{64} & A_{65} & A_{66} \end{Bmatrix} \begin{Bmatrix} M_x \\ M_y \\ M_z \\ F_x \\ F_y \\ F_z \end{Bmatrix} \tag{A-17}$$

In our model we assume zero flexibility along the beam longitudinal axis, say, X-axis. $A_{4i} = 0$, $i = 1, 2, 3, 4, 5, 6$

Since, by definition A is symmetric, $A_{i4} = A_{4i} = 0$, $i = 1, 2, 3, 4, 5, 6$

Also, for a beam $A_{12} = A_{13} = A_{14} = A_{15} = A_{16} = 0$

$$A_{23} = A_{24} = A_{25} = 0$$

$$A_{34} = A_{36} = 0$$

$$A_{56} = 0$$

And due to symmetry $A_{21} = A_{31} = A_{41} = A_{51} = A_{61} = A_{32} = A_{42} = A_{52}$

$$= A_{43} = A_{63} = A_{65} = 0$$

Therefore the flexibility matrix A reduces to

$$\begin{Bmatrix} A_{11} & 0 & 0 & 0 & 0 & 0 \\ 0 & A_{22} & 0 & 0 & 0 & A_{26} \\ 0 & 0 & A_{33} & 0 & A_{35} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & A_{35} & 0 & A_{55} & 0 \\ 0 & A_{26} & 0 & 0 & 0 & A_{66} \end{Bmatrix} \tag{A-18}$$

i.e., seven non-zero flexibility coefficients need to be determined. For a single beam segment, using equations A-5 and A-6

$$A_{22} = \frac{\ell}{EI_1}, \quad A_{33} = \frac{\ell}{EI_2}, \quad A_{55} = \frac{\ell^3}{3EI_2}$$

$$A_{66} = \frac{\ell^3}{3EI_1}, \quad A_{26} = \frac{\ell^2}{2EI_1}, \quad A_{35} = -\frac{\ell^2}{2EI_2} \tag{A-19}$$

$$A_{11} = \text{Torsional Flexibility} = \frac{\ell}{GJ}$$

For a beam consisting of more than one segment, A_{22}, A_{33}, A_{55}, A_{66}, A_{35}, A_{26} are calculated using equation A-15 and rewriting it in the form of equations A-5, A-6, A-7, A-8

$$\text{And} \quad A_{11} = \frac{\ell_1}{G_1J_1} + \frac{\ell_2}{G_2J_2} + \cdots\cdots \frac{\ell_n}{G_nJ_n} \tag{A-20}$$

The above equations have been programmed to calculate the flexibility coefficients of beams given the lengths, Young's modulus, moment of inertias about the two bending axes, shear modulus (G) and polar moment of inertia (J) of each segment.

END-EFFECTOR ARM FLEXIBILITY

The flexibility matrix, A for a beam can be written in the following form

$$A = \begin{Bmatrix} Y_{11} & Y_{12} \\ Y_{12} & Y_{22} \end{Bmatrix} \tag{A-21}$$

where $Y_{11} = \begin{Bmatrix} A_{11} & 0 & 0 \\ 0 & A_{22} & 0 \\ 0 & 0 & A_{33} \end{Bmatrix}$, $Y_{12} = \begin{Bmatrix} 0 & 0 & 0 \\ 0 & 0 & A_{26} \\ 0 & A_{35} & 0 \end{Bmatrix}$

And $Y_{22} = \begin{Bmatrix} 0 & 0 & 0 \\ 0 & A_{55} & 0 \\ 0 & 0 & A_{66} \end{Bmatrix}$

From Reference 1, Page 14, the End-Effector Flexibility $\{Y_E\}$ is given by

$$\{Y_E\} = \sum_{i=1}^{5} \begin{Bmatrix} I & 0 \\ -r_{i5} & I \end{Bmatrix} \begin{Bmatrix} T_{Wi} & 0 \\ 0 & T_{Wi} \end{Bmatrix} \begin{Bmatrix} Y_{11}{}^{i} & Y_{12}{}^{i} \\ Y_{12}{}^{iT} & Y_{22}{}^{i} \end{Bmatrix} \begin{Bmatrix} T_{iW} & 0 \\ 0 & T_{iW} \end{Bmatrix} \begin{Bmatrix} I & r_{i5} \\ 0 & I \end{Bmatrix} \tag{A-22}$$

where $-r_{i5} = -\sum_{K=i}^{4} r_K$

$$r_K = \begin{Bmatrix} 0 & -r_K{}^3 & r_K{}^2 \\ r_K{}^3 & 0 & -r_K{}^1 \\ -r_K{}^2 & r_K{}^1 & 0 \end{Bmatrix}$$

$r_K{}^j$ = length of K+1 beam in jth direction Superscripts 1, 2, 3 Represent X, Y, Z Axes of Orbiter Coordinate System, Respectively.

T_{Bi} , $i = 1, 2, 3, 4, 5$

is the transformation matrix from beam i system to W. Pitch system

$\gamma_{11}{}^{i}$, $\gamma_{12}{}^{i}$, $\gamma_{22}{}^{i}$ $\qquad i = 1, 2, 3, 4, 5$

Are the matrices of flexibility coefficients of beam i.

Equation A-22 has been programmed to determine the End-Effector flexibility given the length of each beam, orientation of the arm (characterized by 6 joint angles) and the seven flexibility coefficients for each beam.

The joint flexibility matrix $\{\gamma_J\}$ is derived as follows:

The End-Effector flexibility relationship can be expressed as

$$\{\delta_e\} = \{\gamma_e\}\{F_e\} \tag{A-23}$$

Where $\{\delta_e\}$ is a 6x1 matrix containing three rotations and three deflections, $\{F_e\}$ is a 6x1 force matrix containing three moments and three forces; $\{\gamma_e\}$ is a 6x6 flexibility matrix.

End-Effector deflections can be related uniquely to joint rotations using the rate law (valid only for 6 joint system), Reference 2.

$$\{J\}\{\delta\Theta\} = \{\delta_e\} \tag{A-24}$$

$\{J\}$ = 6x6 Jacobian matrix for the given arm (6 Joints) configuration.

$\{\delta\Theta\}$ = 6x1 matrix of rotations at each of the 6 Joints

Similarly End-Effector loads can be related to joint torques

$$\{\tau_F\} = \{J\}^T \{F_e\}, \ \{\tau_F\} = \text{6x1 Matrix of Joint Torques} \tag{A-25}$$

Or $\{J\}^{T-1} \{\tau_F\} = \{F_e\}$ (A-26)

Substituting for F_e and δ_e in Equation A-23, we get

$$\{J\}\ \{\delta\Theta\} = \{Y_e\}\{J\}^{T\ -1}\ \{\tau_F\}$$

$$\text{Or}\quad \{\delta\Theta\} = \{J\}^{-1}\ \{Y_e\}\ \{J\}^{T-1}\ \{\tau_F\} \tag{A-27}$$

$$\text{From A-27}\quad \{Y_J\} = \{J^{-1}\}\ \{Y_E\}\{J\}^{T-1} \tag{A-28}$$

Equation A-28 has been used to calculate the joint flexibility.

Flexibility terms for the torque motor model at each joint are calculated from $\{Y_J\}$ matrix as follows:

$$Y_i^e = Y_{iiJ} - \left\{Y_{iJ}^{21}\right\}^T \left\{Y_{iJ}^{22}\right\}^{-1} \left\{Y_{iJ}^{21}\right\} \quad i = 1, 2, 3, 4, 5, 6 \tag{A-29}$$

Where $\{Y_{iJ}^{22}\}$ is the 5x5 matrix derived from $\{Y_J\}$ by deleting the ith row and column, i being the number of joint. $\{Y_{iJ}^{21}\}$ is the 5x1 matrix derived from $\{Y_J\}$ by taking the ith column and deleting the ith element.

APPENDIX B

PROGRAM USAGE

The following programs may be obtained from L. Kumar, 488-5660, Ext. 217.

B.1 CALCULATION OF FLEXIBILITY COEFFICIENTS - PROGRAM A

To access the program and run it, the following procedure is required:

Step 1 Load the tape on HP-9825, labeled side of the tape toward the back of HP-9825.

Step 2 Turn on the calculator and the printer.

Step 3 Access the appropriate track number by typing trk ^ 1 and pressing the EXECUTE key. ^ indicates a blank space.

Step 4 Load the appropriate file by typing ldf ^ 3 and pressing the EXECUTE key.

Step 5 Run the program by pressing the RUN key.

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Input Parameters

Once the program is running several self-explanatory messages are displayed. The following explains the displays and inputs required sequentially.

HP-9825 Display	Display Duration	Response Required From User
(1) Beam One, First Cross Axis	2 Secs	No Response Required, Info only
(2) No. of segments	—	Input the number of segments in beam 1 and press "Continue" key
(3) Notation used for segment K	2 Secs	No resp. req., Info only
(4) L [K] = Length, Inches	2 Secs	No resp. req., Info only
(5) E [K] = Youngs Mod., Psi	2 Secs	No resp. req., Info only
(6) G [K] = Shear Mod., Psi	2 Secs	No resp. req., Info only
(7) I [K] = Moment of Iner., (IN) ↑ 4	2 Secs	No resp. req., Info only
(8) J [K] = Polar Momt. of Iner., (IN) ↑ 4	2 Secs	No resp. req., Info only
(9) L [1]		Input the length of Seg. 1, Beam 1 and press "Continue" key
(10) E [1]		Input Young's Mod. of Seg. 1, Beam 1 and press "Continue" key
(11) G [1]		Input shear Mod. of Seg. 1, Beam 1 and press "Continue" key
(12) I [1]		Input moment of inertia of Seg. 1, Beam 1 About Cross Axis, 1, Press "Continue" key
(13) J [1]		Input polar moment of inertia of Seg. 1 Beam 1, Press "Continue" key

The last five variables will be displayed repeatedly until L[K], E[K], G[K], I[K], J[K] for the Kth segment, Beam 1 have been input. K = No. of segments in Beam 1 which was entered in Line 1.

HP-9825 Display	Display Duration	Response Required From User
(14) Moment of Iner. -2nd Cross Axis	2 Secs	No resp. req., Info. only
(15) I [1]		Input Momt. of Iner. Seg. 1, Beam 1 about cross axis 2, press "Continue" key
(16) I [2]		
(17) I [K]		Input Momt. of Iner. Seg. K, Beam 1 about cross axis 2, press "Continue" key
(18) Beam two, first cross axis	2 Secs	No resp. req., Info. only

Display will start again from Line (2). Input parameters for Beam 2 from Line (2) to (17).

(19) Beam three, first cross axis	2 Secs	No Resp. req. Info. only

Again it will go back to Line (2). Input Beam 3 parameters from Line (2) to (17).

(20) Beam four, first cross axis	2 Secs	No Resp. req. Info. only

Start again from Line (2). Input Beam 4 parameters from Line (2) to (17).

(21) Beam Five, First Cross Axis	2 Secs	No Resp. req. Info. only

Start again from line (2). Input Beam 5 parameters from Line (2) to (17).

(22) End

NOTE: Currently the program has been set-up for five beams but it can easily be extended to any number of beams by simply changing the beam counter variable YO

Program Output

Referring to Eqn. A-15, for any beam consisting of several segments we can write

$$\begin{Bmatrix} Y_R \\ \phi_R \\ M_R \\ F_R \end{Bmatrix} = \begin{Bmatrix} L & \alpha \\ & \\ & \\ 0 & L \end{Bmatrix} \begin{Bmatrix} Y_L \\ \phi_L \\ M_L \\ F_L \end{Bmatrix}$$

where L is a 2x2 matrix given by $\{L^K\}\ \{L^{K-1}\}\ldots\{L^1\}$

K = No. of segments, $\{L^K\} = \begin{Bmatrix} 1 & \ell_K \\ 0 & 1 \end{Bmatrix}$

α is a 2x2 matrix given by

$$\alpha = \{L^K\}\ \{L^{K-1}\}\ldots\{L^2\}\{\alpha^1\} + \{L^K\}\ \{L^{K-1}\}\ldots\{\alpha^2\}\{L^1\} + \ldots\{\alpha^K\}\{L^{K-1}\}\ldots\{L^1\}$$

The program outputs matrix L and α in each cross-axis for beam 1 to beam 5, followed by flexibility coefficients Gamma [1]... Gamma [7].

Interpretation of Gamma i , i = 1, 2, 3, 4, 5, 6, 7

Numerical values of Gamma i , i=1, 2, 3, 4, 5, 6, 7 as calculated by the program are correct but the signs for some of the Gammas might be reversed due to loading direction chosen in Equations A-3 and A-4. The following signs are associated with each of the Gammas.

Gamma i , i = 1, 2, 3 are the rotations in radians about axes x, y, z or x, z, y (depending upon whether first cross axis is the Y-Axis or Z-Axis) for unit loads in Ft-Lb applied about axes x, y, z or x, z, y respectively. These three Gammas are always positive.

If first cross axis is the Y-Axis

Gamma 5 = Disp. in Z-Dir. For Unit Force in Z-Dir. (FT/LB) Always Positive.
Gamma 4 = Disp. in Y-Dir. For Unit Force in Y-Dir.(FT/LB) Always Positive.
Gamma 6 = Rot. About Y-Axis For Unit Force in Z-Dir. (Rad/LB.) Always Negative.
Gamma 7 = Rot. About Z-Axis For Unit Force in Y-Dir. (Rad/LB.) Always Positive.

If first cross axis is the Z-Axis

Gamma 5 = Disp. in Y-Dir. For Unit Force in Y-Dir. (FT/LB) Always Positive.
Gamma 4 = Disp. in Z-Dir. For Unit Force In Z-Dir. (FT/LB.) Always Positive.
Gamma 6 = Rot. About Z-Axis For Unit Force In Y-Dir. (Rad/LB.) Always Positive.
Gamma 7 = Rot. About Y-Axis For Unit Force In Z-Dir. (Rad/LB.) Always Negative.

B.2 CALCULATION OF END EFFECTOR FLEXIBILITY - PROGRAM B

To access the program and run it same procedure as on page 11 is to be followed. In step 4 instead of typing ldf^3 type ldf^2 and press the EXECUTE key.

Input Parameters

H.P.-9825 Display	Disp. Duration Secs	Response Required From User
(1) Input angles and lengths	2	No Resp. Req. info. only
(2) Y0	-	Enter Outb'd roll angle, press "Continue"
(3) Y1	-	Enter shoulder yaw angle, press "Continue"
(4) Y2	-	Enter shoulder pitch angle, press "Continue"
(5) Y3	-	Enter elbow pitch angle, press "Continue"
(6) Y4	-	Enter wrist pitch angle, press "Continue
(7) Y5	-	Enter wrist yaw angle, press "Continue'
(8) Y6	-	Enter hand roll angle, press "Continue"
(9) Y7	-	Enter Attach pt. to shoulder pitch length in ft. and press "Continue"
(10) Y8	-	Enter Sh. pitch to El. pitch length in ft. and press "Continue"
(11) Y9	-	Enter El. pitch to wrist pitch length in ft. and press "Continue"
(12) Y10	-	Enter wrist pt. to wrist yaw length in ft. and press "continue"
(13) Y11	-	Enter wrist yaw to E.E. tip length in ft. and press "Continue"
(14) Print Transf.-Beam 1 to O.b Sys?	2	No Resp. Req. info only
(15) 1 = Yes 0 = No	2	No Resp. Req. info only
(16) Y22	-	Enter 1 or 0 as desired, press "Continue"

H.P.-9825 Display	Disp. Duration Secs	Response Required From User
(17) Print Transf-Beam 2 to Orb Sys?	2	No Resp. Req. info. only
(18) 1 = Yes 0 = No	2	No Resp. Req. info. only
(19) Y22	-	Enter 1 or 0 as desired, press "Continue"
(20) Print Transf-Beam 3 to Orb Sys?	2	No Resp. Req. info. only
(21) 1 = Yes 0 = No	2	No Resp. Req. info. only
(22) Y22	-	Enter 1 or 0, press "Continue"
(23) Print Transf-Beam 4 to Orb Sys?	2	No Resp. Req. info. only
(24) 1 = Yes 0 = No	2	No Resp. Req. info. only
(25) Y22	-	Enter 1 or 0, press "Continue"
(26) Print Transf-Beam 5 to Orb Sys?	2	No Resp. Req. info. only
(27) 1 = Yes 0 = No	2	No Resp. Req. info. only
(28) Y22	-	Enter 1 or 0, press "Continue"
(29) Use Built-in flex coefficients?	2	No Resp. Req. info. only
(30) 1 = Yes 0 = No	2	No Resp. Req. info. only
(31) Y22	-	Enter 1 or 0, Press "Continue"

If the answer is no, i.e., 0 Entry in (31), Display will start from (32), otherwise from (40).

(32) Input flex. coefficients-Gammas	2	No Resp. Req. info. only
(33) Z [1,1]	-	Enter Gamma 1 for Beam 1, press "Continue"
(34) Z [2,2]	-	Enter Gamma 2 for Beam 1, press "Continue"

H.P.-9825 Display	Disp. Duration Secs	Response Required From User
(35) Z [3,3]	-	Enter Gamma 3 for Beam 1, press "Continue"
(36) Z [5,5]	-	Enter Gamma 4 for Beam 1, press "Continue"
(37) Z [6,6]	-	Enter Gamma 5 for Beam 1, press "Continue"
(38) Z [2,6]	-	Enter Gamma 6 for Beam 1, press "Continue"
(39) Z [3,5]	-	Enter Gamma 7 for Beam 1, press "Continue"

Display will repeat from (32) to (39) until flexibility coefficients for all the five beams have been entered

(40) Print End-Effector arm flex.?	2	No Resp. Req. info. only
(41) 1 = Yes 0 = No	2	No Resp. Req. info. only
(42) Y22	-	Enter 1 or 0, press "Continue"
(43) Print the Jacobian matrix?	2	No Resp. Req. info. only
(44) 1 = Yes 0 = No	2	No Resp. Req. info. only
(45) Y22	-	Enter 1 or 0, press "Continue"
(46) Print joint flex. matrix?	2	No Resp. Req. info. only
(47) 1 = Yes 0 = No	2	No Resp. Req. info. only
(48) Y22	-	Enter 1 or 0, press "Continue"

End of the program

PROGRAM OUTPUT DESCRIPTION

Output obtained depends upon the user's choice. The following data (except the components of beam lengths in the wrist-pitch system and the joint flexibility terms for the torque motor model which is always printed) is optional and can be printed or not printed.

(i) Transformation matrices - from beam coordinate systems to orbiter system.

(ii) End-Effector arm flexibility in W-pitch system i.e., if $\{\delta e\}$ is a 6x1 deflection matrix at the E.E. due to $\{P_e\}$ loads at the E.E. then $\{\delta e\} = \{Y\}\{P_e\}$. And $\{Y\}$ is the 6x6 E.E. flexibility matrix.

(iii) Jacobian 6x6 matrix for the given arm configuration

(iv) Joint flexibility matrix. For a six joint system, six rotations (one rotation at each joint) can be related to torques at each joint thru joint flexibility matrix.

$\{\delta\theta\} = \{Y_J\}\{\tau_F\}$, $\{Y_J\}$ = Jt. Flex. Matrix.

APPENDIX C

C.1

Example 1

Calculate the flexibility coefficients of an arm composed of five beams with the following number of segments, section and material properties.

Beam 1

Number of segments = 3

	Segment 1	Segment 2	Segment 3
Length (IN)	1.0	16.0	17.0
Youngs Modulus E (PSI)	1.0×10^7	1.0×10^7	1.0×10^7
Shear Modulus G (PSI)	3.84615×10^6	3.84615×10^6	3.84615×10^6
Moment of Inertia, First Cross Axis (IN^4)	1.05	6.8	30.6
Second Cross Axis (IN^4)	2.04	6.8	30.6
Polar Moment of Inertia (IN^4)	10.55	400.00	63.00

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Beam 2

Number of Segments = 3

	Segment 1	Segment 2	Segment 3
Length (IN)	15.0	235.0	15.0
Young's Modulus (PSI)	1.0×10^7	2.22×10^7	1.0×10^7
Shear Modulus (PSI)	3.84615×10^6	5.63×10^6	3.84615×10^6
Moment of Inertia First Cross Axis (IN^4)	27.0	98.5	27.0
Second Cross Axis (IN^4)	27.0	98.5	27.0
Polar Moment of Inertia (IN^4)	55.6	197.0	55.6

Beam 3

Number of Segments = 3

	Segment 1	Segment 2	Segment 3
Length (IN)	15.0	235.0	15.0
Young's Modulus (PSI)	1.0×10^7	2.22×10^7	1.0×10^7
Shear Modulus (PSI)	3.84615×10^6	5.63×10^6	3.84615×10^6
Moment of Inertia First Cross Axis (IN^4)	27.0	37.4	12.0
Moment of Inertia Second Cross Axis (IN^4)	27.0	37.4	2.8
Polar Moment of Inertia (IN^4)	55.6	74.8	124.5

Beam 4

Number of Segments = 2

	Segment 1	Segment 2
Length (IN)	15.0	10.0
Young's Modulus (PSI)	1.0×10^7	1.0×10^7
Shear Modulus (PSI)	3.84615×10^6	3.84615×10^6
Moment of Inertia First Cross Axis (IN^4)	12.0	1.868
Second Cross Axis (IN^4)	2.8	8.0
Polar Moment of Inertia (IN^4)	124.5	83.0

Beam 5

Number of Segments = 3

	Segment 1	Segment 2	Segment 3
Length (IN)	10.0	20.0	15.0
Young's Modulus (PSI)	1.0×10^7	1.0×10^7	1.0×10^7
Shear Modulus(PSI)	3.84615×10^6	3.84615×10^6	3.84615×10^6
Moment of Inertia First Cross Axis (IN^4)	1.868	50.0	1000.0
Moment of Inertia Second Cross Axis (IN^4)	8.0	50.0	1000.0
Polar Moment of Inertia (IN^4)	83.0	100.0	2000.0

Output From Program 'A'

```
ATTACH PT. TO SHOULDER PITCH=BEAM NO. ONE
SHOULDER PITCH TO EL. PITCH=BEAM NO. TWO
ELBOW PITCH TO WRIST PITCH=BEAM NO. THREE
WRIST PITCH TO WRIST YAW=BEAM NO. FOUR
WRIST YAW TO END EFFECTOR=BEAM NO. FIVE

FOR BEAM NO = 1--MATRIX L[2X2] & r[2X2] FOR BENDING AXIS=1
L[1,1] =  1.000000E 00
L[1,2] =  3.400000E 01
L[2,1] =  0.000000E 00
L[2,2] =  1.000000E 00
r[1,1] =  9.545051E-06
r[1,2] =  6.021257E-05
r[2,1] =  3.860878E-07
r[2,2] =  3.581933E-06
FOR BEAM NO = 1--MATRIX L[2X2] & r[2X2] FOR BENDING AXIS=2
L[1,1] =  1.000000E 00
L[1,2] =  3.400000E 01
L[2,1] =  0.000000E 00
L[2,2] =  1.000000E 00
r[1,1] =  7.996732E-06
r[1,2] =  5.944227E-05
r[2,1] =  3.398693E-07
r[2,2] =  3.553324E-06
BEAM NO. = 1----FLEXIBILITY COEFF.
GAMMA[1] =  1.262441E-06 rad/ft-lb
GAMMA[2] =  4.633053E-06 rad/ft-lb
GAMMA[3] =  4.078431E-06 rad/ft-lb
GAMMA[4] = -1.770389E-05 ft/lb
GAMMA[5] = -2.202660E-05 ft/lb
GAMMA[6] = -9.545051E-06 rad/lb
GAMMA[7] = -7.996732E-06 rad/lb

FOR BEAM NO = 2--MATRIX L[2X2] & r[2X2] FOR BENDING AXIS=1
L[1,1] =  1.000000E 00
L[1,2] =  2.650000E 02
L[2,1] =  0.000000E 00
L[2,2] =  1.000000E 00
r[1,1] =  2.396172E-05
r[1,2] =  1.604657E-03
r[2,1] =  2.185790E-07
r[2,2] =  2.896172E-05
FOR BEAM NO = 2--MATRIX L[2X2] & r[2X2] FOR BENDING AXIS=2
L[1,1] =  1.000000E 00
L[1,2] =  2.650000E 02
L[2,1] =  0.000000E 00
L[2,2] =  1.000000E 00
r[1,1] =  2.896172E-05
r[1,2] =  1.604657E-03
r[2,1] =  2.185790E-07
r[2,2] =  2.896172E-05
BEAM NO. = 2----FLEXIBILITY COEFF.
GAMMA[1] =  4.226034E-06 rad/ft-lb
GAMMA[2] =  2.622948E-06 rad/ft-lb
GAMMA[3] =  2.622948E-06 rad/ft-lb
GAMMA[4] = -5.058498E-04 ft/lb
GAMMA[5] = -5.058498E-04 ft/lb
GAMMA[6] = -2.896172E-05 rad/lb
GAMMA[7] = -2.896172E-05 rad/lb
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FOR BEAM NO = 3--MATRIX L[2X2]& r[2X2] FOR BENDING AXIS=1
L[1,1] =  1.000000E 00
L[1,2] =  2.650000E 02
L[2,1] =  0.000000E 00
L[2,2] =  1.000000E 00
r[1,1] =  5.274546E-05
r[1,2] =  4.011322E-03
r[2,1] =  4.635926E-07
r[2,2] =  7.010658E-05
FOR BEAM NO = 3--MATRIX L[2X2]& r[2X2] FOR BENDING AXIS=2
L[1,1] =  1.000000E 00
L[1,2] =  2.650000E 02
L[2,1] =  0.000000E 00
L[2,2] =  1.000000E 00
r[1,1] =  5.582532E-05
r[1,2] =  4.797313E-03
r[2,1] =  8.743069E-07
r[2,2] =  1.758655E-04
BEAM NO. = 3----FLEXIBILITY COEFF.
GAMMA[1] =  7.913995E-06 rad/ft-lb
GAMMA[2] =  5.563111E-06 rad/ft-lb
GAMMA[3] =  1.049168E-05 rad/ft-lb
GAMMA[4] = -8.330442E-04 ft/lb
GAMMA[5] = -8.304772E-04 ft/lb
GAMMA[6] = -5.274546E-05 rad/lb
GAMMA[7] = -5.582582E-05 rad/lb

FOR BEAM NO = 4--MATRIX L[2X2]& r[2X2] FOR BENDING AXIS=1
L[1,1] =  1.000000E 00
L[1,2] =  2.500000E 01
L[2,1] =  0.000000E 00
L[2,2] =  1.000000E 00
r[1,1] =  4.864160E-06
r[1,2] =  6.313459E-05
r[2,1] =  6.603319E-07
r[2,2] =  1.164414E-05
FOR BEAM NO = 4--MATRIX L[2X2]& r[2X2] FOR BENDING AXIS=2
L[1,1] =  1.000000E 00
L[1,2] =  2.500000E 01
L[2,1] =  0.000000E 00
L[2,2] =  1.000000E 00
r[1,1] =  1.000000E-05
r[1,2] =  7.172619E-05
r[2,1] =  6.507143E-07
r[2,2] =  6.517357E-06
BEAM NO. = 4----FLEXIBILITY COEFF.
GAMMA[1] =  7.518030E-07 rad/ft-lb
GAMMA[2] =  7.923983E-06 rad/ft-lb
GAMMA[3] =  7.923571E-06 rad/ft-lb
GAMMA[4] = -1.485615E-05 ft/lb
GAMMA[5] = -4.872450E-06 ft/lb
GAMMA[6] = -4.864160E-06 rad/lb
GAMMA[7] = -1.000000E-05 rad/lb
```

```
FOR BEAM NO = 5--MATRIX L[2X2]& r[2X2] FOR BENDING AXIS=1
L[1,1] =  1.000000E 00
L[1,2] =  4.500000E 01
L[2,1] =  0.000000E 00
L[2,2] =  1.000000E 00
r[1,1] =  2.242453E-05
r[1,2] =  1.216657E-04
r[2,1] =  5.763319E-07
r[2,2] =  3.532910E-06
FOR BEAM NO = 5--MATRIX L[2X2]& r[2X2] FOR BENDING AXIS=2
L[1,1] =  1.000000E 00
L[1,2] =  4.500000E 01
L[2,1] =  0.000000E 00
L[2,2] =  1.000000E 00
r[1,1] =  6.011250E-06
r[1,2] =  4.301375E-05
r[2,1] =  1.665000E-07
r[2,2] =  1.431250E-06
BEAM NO. = 5----FLEXIBILITY COEFF.
GAMMA[1] =  1.023305E-06 rad/ft-lb
GAMMA[2] =  6.921983E-06 rad/ft-lb
GAMMA[3] =  1.993000E-06 rad/ft-lb
GAMMA[4] = -1.895729E-05 ft/lb
GAMMA[5] = -7.395317E-05 ft/lb
GAMMA[6] = -2.242453E-05 rad/lb
GAMMA[7] = -6.011250E-06 rad/lb
```

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C.2 Example 2

Calculate end effector arm flexibility in the wrist-pitch system for a five beam arm defined by the following beam lengths and orientation. Use the built-in flexibility coefficients for each beam.

Outboard Roll Angle	= 19.2°
Shoulder Yaw Angle	= 0.0°
Shoulder Pitch Angle	= 90.0°
Elbow Pitch Angle	= -30.0°
Wrist Pitch Angle	= 30.0°
Wrist Yaw Angle	= 0.0°
Hand Roll Angle	= 0.0°
Beam 1 Length, FT	= 2.874083
Beam 2 Length, FT	= 20.920833
Beam 3 Length, FT	= 23.1625
Beam 4 Length, FT	= 1.5
Beam 5 Length, FT	= 4.416667

Output From Program B

The following output is obtained by answering "Yes" to all the print options available.

```
OUTBOARD ROLL ANGLE=r0
SHOULDER YAW ANGLE=r1
SHOULDER PITCH ANGLE=r2
ELBOW PITCH ANGLE=r3
WRIST PITCH ANGLE=r4
WRIST YAW ANGLE=r5
HAND ROLL ANGLE=r6
ATTACH PT. TO SHOULDER PITCH LENGTH=r7
SHOULDER PITCH TO ELBOW PITCH LENGTH=r8
ELBOW PITCH TO WRIST PITCH LENGTH=r9
WRIST PITCH TO WRIST YAW LENGTH=r10
WRIST YAW TO END EFFECTOR LENGTH=r11

TRANSFORMATION FROM BEAM ONE SYSTEM TO ORBITER SYSTEM
 0.000000E 00      0.000000E 00      1.000000E 00

-3.288666E-01      9.443764E-01      0.000000E 00

-9.443764E-01     -3.288666E-01      0.000000E 00

TRANSFORMATION FROM BEAM TWO TO ORBITER SYSTEM
 0.000000E 00      0.000000E 00     -1.000000E 00

-3.288666E-01     -9.443764E-01      0.000000E 00

-9.443764E-01      3.288666E-01      0.000000E 00

TRANSFORMATION FROM BEAM THREE SYSTEM TO ORBITER SYSTEM
-5.000000E-01      0.000000E 00     -8.660254E-01

-2.848069E-01     -9.443764E-01      1.644333E-01

-8.178539E-01      3.288666E-01      4.721832E-01

TRANSFORMATION FROM BEAM FOUR SYSTEM TO ORBITER SYSTEM
 0.000000E 00      0.000000E 00     -1.000000E 00

-3.288666E-01     -9.443764E-01      1.000000E-12

-9.443764E-01      3.288666E-01      0.000000E 00

TRANSFORMATION FROM BEAM FIVE SYSTEM TO ORBITER SYSTEM
 0.000000E 00      0.000000E 00     -1.000000E 00

-3.288666E-01     -9.443764E-01      1.000000E-12

-9.443764E-01      3.288666E-01      0.000000E 00
```

CALCULATE COMPONENTS OF LENGTH VECTORS IN THE WRIST-PITCH SYSTEM

COMPONENTS OF BEAM TWO LENGTH IN THE WRIST-PITCH SYSTEM

2.092083E 01	-2.092083E-11	-6.880164E-12

COMPONENTS OF BEAM THREE LENGTH IN THE WRIST-PITCH SYSTEM

2.005931E 01	2.316250E-11	1.158125E 01

COMPONENTS OF BEAM FOUR LENGTH IN THE WRIST-PITCH SYSTEM

1.500000E 00	0.000000E 00	0.000000E 00

COMPONENTS OF BEAM FIVE LENGTH IN THE WRIST-PITCH SYSTEM

4.416667E 00	8.333334E-12	0.000000E 00

END EFFECTOR ARM FLEXIBILITY IN THE WRIST-PITCH SYSTEM

1.453700E-05	2.035253E-17	1.013250E-06	-1.471053E-16	-8.407181E-05	4.121706E-16
2.035253E-17	2.707000E-05	-8.009401E-18	1.054631E-04	-1.105091E-16	-4.583615E-04
1.013250E-06	-8.009401E-18	2.825700E-05	-2.839293E-16	4.699904E-04	2.618868E-16
-1.471053E-16	1.054631E-04	-2.839293E-16	1.[illegible]363E-03	-7.129275E-15	-3.967946E-03
-8.407181E-05	-1.105091E-16	4.699904E-04	-7.129275E-15	1.765481E-02	1.473612E-15
4.121706E-16	-4.583615E-04	2.618868E-16	-3.967946E-03	1.473612E-15	1.545254E-02

JACOBIAN FOR THIS ARM CONF. IN THE WRIST-PITCH SYSTEM

1.000000E 00	-1.000000E-12	2.000000E-12	0.000000E 00	0.000000E 00	1.000000E 00
-1.000000E-12	1.000000E 00	1.000000E 00	1.000000E 00	0.000000E 00	2.000000E-12
-3.288666E-13	-9.443764E-13	-7.835097E-14	0.000000E 00	1.000000E 00	0.000000E 00
-1.158125E-11	1.158125E 01	1.158125E 01	0.000000E 00	-8.833334E-12	0.000000E 00
-1.158125E 01	-3.270699E-11	-2.519774E-11	0.000000E 00	4.416667E 00	0.000000E 00
5.797131E-11	-4.689681E 01	-2.597598E 01	-5.916667E 00	0.000000E 00	0.000000E 00

JOINT FLEXIBILITY MATRIX

1.047853E-04	6.996754E-17	3.631368E-17	1.779747E-16	-2.980581E-05	-9.714012E-05
6.996754E-17	6.195181E-06	-3.027294E-07	-3.702052E-07	-2.973642E-17	-6.156699E-17
3.631368E-17	-3.027294E-07	2.659004E-06	1.227840E-06	2.715993E-18	-4.781779E-17
1.779747E-16	-3.702052E-07	1.227840E-06	1.710600E-05	-6.693703E-17	-1.624015E-16
-2.980581E-05	-2.973642E-17	2.715993E-18	-6.693703E-17	2.825700E-05	3.081906E-05
-9.714012E-05	-6.156699E-17	-4.781779E-17	-1.624015E-16	3.081906E-05	1.040814E-04

JOINT FLEXIBILITIES FOR THE TORQUE MOTOR MODEL

SHOULDER YAW JT.	SHOULDER PITCH JT.	ELBOW PITCH JT.	WRIST PITCH JT.	WRIST YAW JT.	HAND ROLL
1.4067E-05	6.157[illegible]E-[illegible]	2.5585E-06	1.6530E-05	1.9054E-05	1.3515E-05

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C.3 Comparison of Results

C.3.1 Flexibility Coefficients - Program 'A'

Beam Number	Flexibility Coefficients Calculated by Program 'A'	Flexibility Coefficients Used in the PDRSS, Calculated by LEC
	GAMMA (1) = 4.226034×10^{-6}	4.24×10^{-6}
	GAMMA (2) = 2.622948×10^{-6}	2.616×10^{-6}
	GAMMA (3) = 2.622948×10^{-6}	2.628×10^{-6}
2	GAMMA (4) = 5.058498×10^{-4}	5.0585×10^{-4}
	GAMMA (5) = 5.058498×10^{-4}	5.0585×10^{-4}
	GAMMA (6) = -2.896172×10^{-5}	-2.895×10^{-5}
	GAMMA (7) = 2.896172×10^{-5}	2.896×10^{-5}
	GAMMA (1) = 7.913996×10^{-6}	7.908×10^{-6}
	GAMMA (2) = 5.563111×10^{-6}	5.568×10^{-6}
	GAMMA (3) = 1.049168×10^{-5}	1.0438×10^{-5}
3	GAMMA (4) = 8.330442×10^{-4}	8.3304×10^{-4}
	GAMMA (5) = 8.304772×10^{-4}	8.3048×10^{-4}
	GAMMA (6) = 5.274546×10^{-5}	5.274×10^{-5}
	GAMMA (7) = -5.582582×10^{-5}	-5.583×10^{-5}

C.3.2 End-Effector Flexibility, Program 'B'

End-Effector Flexibility Calculated by Program 'B'

1.458700×10^{-5}	0.0	1.013250×10^{-6}	0.0	-8.407181×10^{-5}	0.0
0.0	2.707000×10^{-5}	0.0	1.054631×10^{-4}	0.0	-4.583615×10^{-4}
1.013253×10^{-6}	0.0	2.825700×10^{-5}	0.0	4.699904×10^{-4}	0.0
0.0	1.054631×10^{-4}	0.0	1.106363×10^{-3}	0.0	-3.967946×10^{-3}
-8.407181×10^{-5}	0.0	4.699904×10^{-4}	0.0	1.765481×10^{-2}	0.0
0.0	-4.583615×10^{-4}	0.0	-3.967946×10^{-3}	0.0	1.545254×10^{-2}

End-Effector Flexibility Calculated by PDRSS

1.458700×10^{-5}	0.0	1.013249×10^{-6}	0.0	-8.407181×10^{-5}	0.0
0.0	2.707000×10^{-5}	0.0	1.054631×10^{-5}	0.0	-4.583615×10^{-4}
1.013249×10^{-6}	0.0	2.825700×10^{-5}	0.0	4.699903×10^{-4}	0.0
0.0	1.054631×10^{-5}	0.0	1.106363×10^{-3}	0.0	-3.967945×10^{-3}
-8.407181×10^{-5}	0.0	4.699903×10^{-4}	0.0	1.765480×10^{-2}	0.0
0.0	-4.583615×10^{-4}	0.0	-3.967945×10^{-3}	0.0	1.545253×10^{-2}

APPENDIX D

D.1 Listing of Program A

```
0: flt 6
 : fmt 1,el3.6,z
2: fmt 2,fl.0,c22
3: fmt 3,1/
4: fmt 4,fl.0,c41,fl.0
5: fmt 5,el3.6
6: wrt 6,"ATTACH PT. TO SHOULDER PITCH=BEAM NO. ONE"
7: wrt 6,"SHOULDER PITCH TO EL. PITCH=BEAM NO. TWO"
8: wrt 6,"ELBOW PITCH TO WRIST PITCH=BEAM NO. THREE"
9: wrt 6,"WRIST PITCH TO WRIST YAW=BEAM NO. FOUR"
10: wrt 6,"WRIST YAW TO END EFFECTOR=BEAM NO. FIVE"
11: wrt 6.3
12: dim L[50],E[50],I[50],G[50],J[50]
13: dim A[2,2],N[2,2],B[2,2],C[2,2],D[2,2],F[2,2],H[2,2],M[2,2]
14: dim P[7]
15: dsp "BEAM ONE,FIRST CROSS AXIS";wait 2000
16: 1→r0
17: enp "NO. OF SEGMENTS",S
18: 1→O
19: 1→K
20: dsp "NOTATION USED FOR SEGMENT K";wait 2000
21: dsp "L[K] = LENGTH, INCHES";wait 2000
22: dsp "E[K] = YOUNGS MOD., PSI";wait 2000
23: dsp "G[K] = SHEAR MOD., PSI";wait 2000
24: dsp "I[K]=MOMENT OF INER.,(IN)^4";wait 2000
25: dsp "J[K]=POLAR MOMT. INER.,(IN)^4";wait 2000
26: enp L[K],E[K],G[K],I[K],J[K]
27: if K<S;K+1→K;gto 26
28: 1→A[1,1]→A[2,2]
29: L[1]→A[1,2]
30: 0→A[2,1]
31: 0→N[1,1]→N[1,2]→N[2,1]→N[2,2]
32: 1→K
33: L[K]L[K]/2E[K]I[K]→B[1,1]→B[2,2]
34: L[K]L[K]L[K]/6E[K]I[K]→B[1,2]
35: L[K]/E[K]I[K]→B[2,1]
36: if K<S;K+1→K;gto 38
37: gto 51
38: 1→C[1,1]→C[2,2]
39: L[K]→C[1,2]
40: 0→C[2,1]
41: L[K]L[K]/2E[K]I[K]→D[1,1]→D[2,2]
42: L[K]L[K]L[K]/6E[K]I[K]→D[1,2]
43: L[K]/E[K]I[K]→D[2,1]
44: mat CA→F
45: mat CB→H
46: mat DA→M
47: ara H+1→H
48: ara F→A
49: ara H→B
50: gto 36
51: wrt 6.4,"FOR BEAM NO = ",r0,"--MATRIX L[2X2] & r[2X2] FOR BENDING AXIS=",O
52: wrt 6.5,"L[1,1] = ",A[1,1]
53: wrt 6.5,"L[1,2] = ",A[1,2]
54: wrt 6.5,"L[2,1] = ",A[2,1]
55: wrt 6.5,"L[2,2] = ",A[2,2]
56: wrt 6.5,"r[1,1] = ",B[1,1]
57: wrt 6.5,"r[1,2] = ",B[1,2]
58: wrt 6.5,"r[2,1] = ",B[2,1]
59: wrt 6.5,"r[2,2] = ",B[2,2]
60: if O>1;gto 70
61: 12B[2,1]→P[2]
62: (B[1,2]-B[1,1]A[1,2])/12→P[5]
63: B[2,2]-B[2,1]A[1,2]→P[6]
```

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```
64: 0+1+0
65: dsp "MOMENT OF INER.-2ND CROSS AXIS";wait 2000
66: 1+K
67: enp I[K]
68: if K<S;K+1+K;gto 67
69: gto 28
70: 12B[2,1]+P[3]
71: (B[1,2]-B[1,1]A[1,2])/12+P[4]
72: B[2,2]-B[2,1]A[1,2]+P[7]
73: 1+K
74: 12L[K]/G[K]J[K]+P[1]
75: if K<S;K+1+K;gto 77
76: gto 78
77: 12L[K]/G[K]J[K]+P[1]+P[1];gto 75
78: wrt 6.2,"BEAM NO. = ",r0,"----FLEXIBILITY COEFF."
79: wrt 6.1,"GAMMA[1] = ",P[1]," rad/ft-lb"
80: wrt 6.1,"GAMMA[2] = ",P[2]," rad/ft-lb"
81: wrt 6.1,"GAMMA[3] = ",P[3]," rad/ft-lb"
82: wrt 6.1,"GAMMA[4] = ",P[4]," ft/lb"
83: wrt 6.1,"GAMMA[5] = ",P[5]," ft/lb"
84: wrt 6.1,"GAMMA[6] = ",P[6]," rad/lb"
85: wrt 6.1,"GAMMA[7] = ",P[7]," rad/lb"
86: wrt 6.3
87: if r0<5;r0+1+r0;gto 89
88: gto 93
89: if r0=2;dsp "BEAM TWO,FIRST CROSS AXIS";wait 2000;gto 17
90: if r0=3;dsp "BEAM THREE,FIRST CROSS AXIS";wait 2000;gto 17
91: if r0=4;dsp "BEAM FOUR,FIRST CROSS AXIS";wait 2000;gto 17
92: if r0=5;dsp "BEAM FIVE,FIRST CROSS AXIS";wait 2000;gto 17
93: end
21141
```

D.2 Listing Of Program B

```
0: flt 9
1: wrt 6,"OUTBOARD ROLL ANGLE=r0"
2: wrt 6,"SHOULDER YAW ANGLE=r1"
3: wrt 6,"SHOULDER PITCH ANGLE=r2"
4: wrt 6,"ELBOW PITCH ANGLE=r3"
5: wrt 6,"WRIST PITCH ANGLE=r4"
6: wrt 6,"WRIST YAW ANGLE=r5"
7: wrt 6,"HAND ROLL ANGLE=r6"
8: wrt 6,"ATTACH PT. TO SHOULDER PITCH LENGTH=r7"
9: wrt 6,"SHOULDER PITCH TO ELBOW PITCH LENGTH=r8"
10: wrt 6,"ELBOW PITCH TO WRIST PITCH LENGTH=r9"
11: wrt 6,"WRIST PITCH TO WRIST YAW LENGTH=r10"
12: wrt 6,"WRIST YAW TO END EFFECTOR LENGTH=r11"
13: fmt 1,1/
14: wrt 6.1
15: dim A[3,3],B[3,3],C[3,3],D[3,3],E[3,3],F[3,3],G[3,3],H[3,1]
16: dim I[3,1],J[3,1],K[3,1],L[3,1],M[3,1],N[6,6],O[6,6],P[6,6]
17: dim Q[6,6],R[6,6],S[6,6],T[6,6],X[6,6],Y[6,6],Z[6,6],U[3,3]
18: dim V[6,36],W[36,6]
19: dsp "INPUT ANGLES AND LENGTHS";wait 2000
20: 0→r12
21: enp rr12
22: if r12<11;1+r12→r12;gto 21
23: -1→U[1,1];0→U[1,2]→U[1,3]→U[2,1]→U[3,1]
24: -cos(r0)→U[2,2];cos(r0)→U[3,3];sin(r0)→U[2,3]→U[3,2]
25: 0→A[1,1]→A[1,2]→A[2,3]→A[3,3]
26: 1→A[1,3]
27: -sin(r0)→A[2,1]→A[3,2]
28: cos(r0)→A[2,2];-cos(r0)→A[3,1]
29: A[1,1]→W[1,1];A[2,1]→W[2,1];A[3,1]→W[3,1]
30: fmt 2,el3.6,5x,el3.6,5x,el3.6,2/
31: fmt 3,el3.6,3x,el3.6,3x,el3.6,3x,el3.6,3x,el3.6,3x,el3.6
32: dsp "Print Transf-Beam 1 To Orb Sys?";wait 2000
33: dsp " 1=YES  0=NO ";wait 2000
34: ent r22
35: if r22=0;gto 39
36: 1→r23
37: wrt 6,"TRANSFORMATION FROM BEAM ONE SYSTEM TO ORBITER SYSTEM"
38: wrt 6.2,A[r23,1],A[r23,2],A[r23,3];jmp (1+r23→r23)>3
39: 1→B[3,3];0→B[1,3]→B[2,3]→B[3,1]→B[3,2]
40: cos(r1)→B[1,1]→B[2,2]
41: sin(r1)→B[1,2];-sin(r1)→B[2,1]
42: 1→C[2,2];0→C[1,2]→C[2,1]→C[2,3]→C[3,2]
43: cos(r2)→C[1,1]→C[3,3];sin(r2)→C[3,1];-sin(r2)→C[1,3]
44: trn B→D
45: trn C→E
46: mat DE→F
47: mat UF→G
48: G[1,2]→W[7,2];G[2,2]→W[8,2];G[3,2]→W[9,2]
49: dsp "Print Transf-Beam 2 To Orb Sys?";wait 2000
50: dsp " 1=YES , 0=NO ";wait 2000
51: ent r22
52: if r22=0;gto 56
53: 1→r23
54: wrt 6,"TRANSFORMATION FROM BEAM TWO TO ORBITER SYSTEM"
55: wrt 6.2,G[r23,1],G[r23,2],G[r23,3];jmp (1+r23→r23)>3
56: cos(r3)→C[1,1]→C[3,3];sin(r3)→C[3,1];-sin(r3)→C[1,3]
57: trn C→E
58: mat GE→D
59: D[1,2]→W[13,3];D[2,2]→W[14,3];D[3,2]→W[15,3]
60: dsp "Print Transf-Beam 3 To Orb Sys?";wait 2000
61: dsp " 1=YES ,0=NO ";wait 2000
62: ent r22
63: if r22=0;gto 67
```

```
64: 1→r23
65: wrt 6,"TRANSFORMATION FROM BEAM THREE SYSTEM TO ORBITER SYSTEM"
66: wrt 6.2,D[r23,1],D[r23,2],D[r23,3];jmp (1+r23+r23)>3
67: cos(r4)→C[1,1]→C[3,3];sin(r4)→C[3,1];-sin(r4)→C[1,3]
68: trn C→F
69: mat DF→E
70: E[1,2]→W[19,4];E[2,2]→W[20,4];E[3,2]→W[21,4]
71: dsp "Print Transf-Beam 4 To Orb Sys?";wait 2000
72: dsp " 1=YES ,0=NO ";wait 2000
73: ent r22
74: if r22=0;gto 78
75: 1→r23
76: wrt 6,"TRANSFORMATION FROM BEAM FOUR SYSTEM TO ORBITER SYSTEM"
77: wrt 6.2,E[r23,1],E[r23,2],E[r23,3];jmp (1+r23+r23)>3
78: cos(r5)→B[1,1]→B[2,2];sin(r5)→B[1,2];-sin(r5)→B[2,1]
79: trn B→F
80: mat EF→B
81: F[1,3]→W[25,5];F[2,3]→W[26,5];F[3,3]→W[27,5]
82: 1→C[1,1];0→C[1,2]→C[1,3]→C[2,1]→C[3,1]
83: cos(r6)→C[2,2]→C[3,3];sin(r6)→C[2,3];-sin(r6)→C[3,2]
84: trn C→F
85: mat BF→C
86: C[1,1]→W[31,6];C[2,1]→W[32,6];C[3,1]→W[33,6]
87: dsp "Print Transf-Beam 5 To Orb Sys?";wait 2000
88: dsp " 1=YES ,0=NO ";wait 2000
89: ent r22
90: if r22=0;gto 94
91: 1→r23
92: wrt 6,"TRANSFORMATION FROM BEAM FIVE SYSTEM TO ORBITER SYSTEM"
93: wrt 6.2,C[r23,1],C[r23,2],C[r23,3];jmp (1+r23+r23)>3
94: inv A→A
95: mat AE→B
96: trn B→A
97: A[1,1]→W[1,1];A[2,1]→W[2,1];A[3,1]→W[3,1]
98: inv G→G
99: mat GE→B
100: trn B→G
101: G[1,2]→W[7,2];G[2,2]→W[8,2];G[3,2]→W[9,2]
102: inv D→D
103: mat DE→B
104: trn B→D
105: D[1,2]→W[13,3];D[2,2]→W[14,3];D[3,2]→W[15,3]
106: 0→W[19,4]→W[21,4];1→W[20,4]
107: inv E→F
108: mat FC→B
109: ara B→C
110: C[1,1]→W[31,6];C[2,1]→W[32,6];C[3,1]→W[33,6]
111: idn E
112: wrt 6,"CALCULATE COMPONENTS OF LENGTH VECTORS IN THE WRIST-PITCH SYSTEM"
113: wrt 6.1
114: r8→H[1,1];0→H[2,1]→H[3,1]
115: mat GH→I
116: wrt 6,"COMPONENTS OF BEAM TWO LENGTH IN THE WRIST-PITCH SYSTEM"
117: wrt 6.2,I[1,1],I[2,1],I[3,1]
118: r9→H[1,1]
119: mat DH→J
120: wrt 6.1
121: wrt 6,"COMPONENTS OF BEAM THREE LENGTH IN THE WRIST-PITCH SYSTEM"
122: wrt 6.2,J[1,1],J[2,1],J[3,1]
123: r10→H[1,1]
124: mat EH→K
125: wrt 6.1
126: wrt 6,"COMPONENTS OF BEAM FOUR LENGTH IN THE WRIST-PITCH SYSTEM"
127: wrt 6.2,K[1,1],K[2,1],K[3,1]
```

```
128: r11→M[1,1]
129: mat CH→L
130: wrt 6.1
131: wrt 6,"COMPONENTS OF BEAM FIVE LENGTH IN THE WRIST-PITCH SYSTEM"
132: wrt 6.2,L[1,1],L[2,1],L[3,1]
133: 1→r13
134: if r13>1;gto 142
135: ara K+L→M
136: ara J+M→A
137: ara I+A→A
138: M[3,1]→V[4,2]→V[4,8];-M[3,1]→V[5,1]→V[5,7]
139: -M[2,1]→V[4,3]→V[4,9];M[2,1]→V[6,1]→V[6,7]
140: M[1,1]→V[5,3]→V[5,9];-M[1,1]→V[6,2]→V[6,8]
141: gto 161
142: if r13>2;gto 149
143: ara K+L→M
144: ara J+M→A
145: M[3,1]→V[4,14];-M[3,1]→V[5,13]
146: -M[2,1]→V[4,15];M[2,1]→V[6,13]
147: M[1,1]→V[5,15];-M[1,1]→V[6,14]
148: gto 168
149: if r13>3;gto 155
150: ara K+L→M
151: M[3,1]→V[4,20];-M[3,1]→V[5,19]
152: -M[2,1]→V[4,21];M[2,1]→V[6,19]
153: M[1,1]→V[5,21];-M[1,1]→V[6,20]
154: gto 168
155: if r13>4;gto 172
156: ara L→M
157: M[3,1]→V[4,26];-M[3,1]→V[5,25]
158: -M[2,1]→V[4,27];M[2,1]→V[6,25]
159: M[1,1]→V[5,27];-M[1,1]→V[6,26]
160: gto 168
161: 1→N[1,1]→N[2,2]→N[3,3]→N[4,4]→N[5,5]→N[6,6]
162: 0→N[1,2]→N[1,3]→N[1,4]→N[1,5]→N[1,6]
163: 0→N[2,1]→N[2,3]→N[2,4]→N[2,5]→N[2,6]
164: 0→N[3,1]→N[3,2]→N[3,4]→N[3,5]→N[3,6]
165: 0→N[4,1]→N[4,5]→N[4,6]
166: 0→N[5,2]→N[5,4]→N[5,6]
167: 0→N[6,3]→N[6,4]→N[6,5]
168: M[3,1]→N[4,2];-M[3,1]→N[5,1]
169: -M[2,1]→N[4,3];M[2,1]→N[6,1]
170: M[1,1]→N[5,3];-M[1,1]→N[6,2]
171: jmp 2
172: idn N
173: trn N→O
174: 1→r14
175: 1→r15
176: 0→P[r14,3+r15]→P[3+r15,r14]
177: if r15<3;1+r15→r15;gto 176
178: if r14<3;1+r14→r14;gto 175
179: if r13>1;gto 186
180: 1→r14
181: 1→r15
182: A[r14,r15]→P[r14,r15]→P[3+r14,3+r15]
183: if r15<3;1+r15→r15;gto 182
184: if r14<3;1+r14→r14;gto 181
185: gto 213
186: if r13>2;gto 193
187: 1→r14
188: 1→r15
189: G[r14,r15]→P[r14,r15]→P[3+r14,3+r15]
190: if r15<3;1+r15→r15;gto 189
191: if r14<3;1+r14→r14;gto 188
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192: gto 213
193: if r13>3;gto 200
194: 1→r14
195: 1→r15
196: D[r14,r15]→P[r14,r15]→P[3+r14,3+r15]
197: if r15<3;1+r15→r15;gto 196
198: if r14<3;1+r14→r14;gto 195
199: gto 213
200: if r13>4;gto 207
201: 1→r14
202: 1→r15
203: E[r14,r15]→P[r14,r15]→P[3+r14,3+r15]
204: if r15<3;1+r15→r15;gto 203
205: if r14<3;1+r14→r14;gto 202
206: gto 213
207: if r13>5;gto 373
208: 1→r14
209: 1→r15
210: C[r14,r15]→P[r14,r15]→P[3+r14,3+r15]
211: if r15<3;1+r15→r15;gto 210
212: if r14<3;1+r14→r14;gto 209
213: trn P→Q
214: if r13>1;gto 218
215: dsp "Use Built-In Flex Coefficients?";wait 2000
216: dsp "1=YES,0=NO";wait 2000
217: ent r22
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218: if r22=0;gto 244
219: if r13>1;gto 224
220: 1.26e-6→Z[1,1];4.08e-6→Z[2,2];4.632e-6→Z[3,3]
221: 3.3263e-5→Z[5,5];2.3993e-5→Z[6,6];-8.968e-6→Z[2,6];1.12e-5→Z[3,5]
222: Z[2,6]→Z[6,2];Z[3,5]→Z[5,3]
223: gto 248
224: if r13>2;gto 229
225: 4.24e-6→Z[1,1];2.616e-6→Z[2,2];2.628e-6→Z[3,3];5.0585e-4→Z[5,5]
226: 5.0585e-4→Z[6,6];-2.895e-5→Z[2,6];2.895e-5→Z[3,5]
227: Z[2,6]→Z[6,2];Z[3,5]→Z[5,3]
228: gto 253
229: if r13>3;gto 234
230: 7.903e-6→Z[1,1];1.0438e-5→Z[2,2];5.568e-6→Z[3,3];8.3048e-4→Z[5,5]
231: 8.3304e-4→Z[6,6];-5.583e-5→Z[2,6];5.274e-5→Z[3,5]
232: Z[2,6]→Z[6,2];Z[3,5]→Z[5,3]
233: gto 253
234: if r13>4;gto 239
235: 7.44e-7→Z[1,1];7.932e-6→Z[2,2];7.92e-6→Z[3,3];4.87e-6→Z[5,5]
236: 1.4856e-5→Z[6,6];-1e-5→Z[2,6];4.86e-6→Z[3,5]
237: Z[2,6]→Z[6,2];Z[3,5]→Z[5,3]
238: gto 253
239: if r13>5;gto 373
240: 1.02e-6→Z[1,1];2.004e-6→Z[2,2];6.924e-6→Z[3,3];7.388e-5→Z[5,5]
241: 1.89575e-5→Z[6,6];-6.01e-6→Z[2,6];2.242e-5→Z[3,5]
242: Z[2,6]→Z[6,2];Z[3,5]→Z[5,3]
243: gto 253
244: dsp "INPUT FLEX. COEFFICIENTS-GAMMAS";wait 2000
245: enp Z[1,1],Z[2,2],Z[3,3],Z[5,5],Z[6,6],Z[2,6],Z[3,5]
246: Z[2,6]→Z[6,2];Z[3,5]→Z[5,3]
247: if r13>1;gto 253
248: 0→Z[1,2]→Z[1,3]→Z[1,4]→Z[1,5]→Z[1,6]
249: 0→Z[2,1]→Z[2,3]→Z[2,4]→Z[2,5]
250: 0→Z[3,1]→Z[3,2]→Z[3,4]→Z[3,6]
251: 0→Z[4,1]→Z[4,2]→Z[4,3]→Z[4,4]→Z[4,5]→Z[4,6]
252: 0→Z[5,1]→Z[5,2]→Z[5,4]→Z[5,6]→Z[6,1]→Z[6,3]→Z[6,4]→Z[6,5]
253: mat QO→R
254: mat ZR→S
255: mat PS→T
256: if r13>1;gto 259
257: mat NT→X
258: gto 261
259: mat NT→Y
260: ara Y+X→X
261: 1+r13→r13
262: if r13<=5;gto 134
263: dsp "Print End-Effector Arm Flex.?";wait 2000
264: dsp " 1=YES , 0=NO ";wait 2000
265: ent r22
266: if r22=0;gto 271
267: wrt 6,"END EFFECTOR ARM FLEXIBILITY IN THE WRIST-PITCH SYSTEM"
268: 1→r23
269: wrt 6.3,X[r23,1],X[r23,2],X[r23,3],X[r23,4],X[r23,5],X[r23,6]
270: 1+r23→r23;if r23<=6;gto 269
271: 1→r16
272: 1→r17
273: 1→r18
274: if r16=r17;gto 278
275: 0→V[r16,r17]
276: 1+r17→r17
277: 1+r13→r18;gto 274
278: 1→V[r16,r17]
279: 6+r17→r17
280: 1+r13→r18
281: if r18=r17;gto 278
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282: 0→V[r16,r18]
283: if r18<36;gto 280
284: 1+r16→r16
285: if r16<=3;gto 272
286: 4→r16
287: 1→r17
288: 0→V[r16,r17]→V[1+r16,1+r17]→V[2+r16,2+r17]
289: 1→V[r16,3+r17]→V[1+r16,4+r17]→V[2+r16,5+r17]
290: 6+r17→r17
291: if r17<=31;gto 288
292: 4→r16
293: 1→r17
294: 0→V[r16,4+r17]→V[r16,5+r17]
295: 6+r17→r17
296: if r17<=31;gto 294
297: 1+r16→r16
298: if r16>5;gto 304
299: 1→r17
300: 0→V[r16,3+r17]→V[r16,5+r17]
301: 6+r17→r17
302: if r17<=31;gto 300
303: gto 297
304: 1→r17
305: 0→V[r16,3+r17]→V[r16,4+r17]
306: 6+r17→r17
307: if r17<=31;gto 305
308: 0→V[4,32]→V[4,33]→V[5,31]→V[5,33]→V[6,31]→V[6,32]
309: rdm Z[6,6]
310: mat VW→Z
311: dsp "Print The JACOBIAN ?";wait 2000
312: dsp " 1=YES , 0=NO ";wait 2000
313: ent r22
314: if r22=0;gto 320
315: wrt 6.1
316: wrt 6,"JACOBIAN FOR THIS ARM CONF. IN THE WRIST-PITCH SYSTEM"
317: 1→r23
318: wrt 6.3,Z[r23,1],Z[r23,2],Z[r23,3],Z[r23,4],Z[r23,5],Z[r23,6]
319: 1+r23→r23;if r23<=6;gto 318
320: rdm O[6,6],R[6,6],S[6,6],T[5,5],N[5,1],Q[1,5],P[5,1],V[1,1]
321: trn Z→O
322: inv O→O
323: inv Z→Z
324: mat XO→R
325: mat ZR→S
326: dsp "Print Joint Flex. Matrix ?";wait 2000
327: dsp " 1=YES , 0=NO ";wait 2000
328: ent r22
329: if r22=0;gto 335
330: wrt 6.1
331: wrt 6,"JOINT FLEXIBILITY MATRIX"
332: 1→r23
333: wrt 6.3,S[r23,1],S[r23,2],S[r23,3],S[r23,4],S[r23,5],S[r23,6]
334: 1+r23→r23;if r23<=6;gto 333
335: 1→r16
336: 1→r17
337: 1→r1[illegible]
338: 1→r18
339: 1→r20
340: if r16=r17;gto 347
341: if r16=r18;gto 344
342: S[r17,r18]→T[r19,r20]
343: 1+r20→r20
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344: 1+r18+r18
345: if r18<=6;gto 341
346: 1+r19+r19
347: 1+r17+r17
348: if r17<=6;gto 338
349: r16+r18
350: 1+r17
351: 1+r19
352: if r18=r17;gto 355
353: S[r17,r18]+N[r19,1]
354: 1+r19+r19
355: 1+r17+r17
356: if r17<=6;gto 352
357: trn N+Q
358: inv T+T
359: mat TN+P
360: mat QP+V
361: rdm R[6,1]
362: S[r16,r16]-V[1,1]+R[r16,1]
363: 1+r16+r16
364: if r16<=6;gto 336
365: fmt 4,ell.4,1x,ell.4,1x,ell.4,1x,ell.4,1x,ell.4,1x,ell.4
366: wrt 6.1
367: wrt 6,"JOINT FLEXIBILITIES FOR THE TORQUE MOTOR MODEL"
368: wrt 6.1
369: wrt 6," SHOULDER     SHOULDER     ELBOW        WRIST        WRIST        HAN
370: wrt 6," YAW JT.       PITCH JT.    PITCH JT.    PITCH JT.    YAW JT.      ROL
371: wrt 6," ----------   ----------   ----------   ----------   ----------   ---
372: wrt 6.4,R[1,1],R[2,1],R[3,1],R[4,1],R[5,1],R[6,1]
373: end
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