AN ULTRASONIC FLOWMETER FOR MEASURING DYNAMIC LIQUID FLOW

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OCTOBER 1978
A dynamic ultrasonic liquid flowmeter was tested in water and LN₂ to determine its precision and repeatability with steady-state flows, and its response to oscillatory flow. The flowmeter (diameter 7.62 cm, length 20.32 cm and range, 4.6 m/sec) was developed by the French Space Agency, Office Nationale d'Etudes et Recherches Aerospatiales (ONERA), for investigating the Pogo phenomenon during launch of liquid propelled vehicles. The flowmeter was mounted in a novel oscillating pipe system which was sinusoidally translated to generate dynamic-velocity differentials between the flowmeter and the pipe fixed flows. The dynamic tests were designed to evaluate the flowmeter's potential for measuring flows in ground studies of Shuttle engine-coupled Pogo under pulsed flow conditions. Test results include those obtained with steady flow calibrations using water and LN₂, and the dynamic flow tests with water where in the presence of steady-state flows of 0.61, 1.22, and 1.82 m/sec small flow oscillations were generated by the oscillating pipe system, and measured. The oscillation amplitudes ranged from 0.5 to 10% of the steady-state flow rate and occurred at fixed frequencies of 7, 19, and 30 Hz. Dynamic flowmeter output signals were compared with corresponding dynamic velocity readings obtained from an accelerometer mounted on the flowmeter. A description and operating principle of the dynamic calibration system is given as well as a discussion of the instrumentation and the cross spectrum signal processing techniques used to retrieve the small dynamic flowmeter signals under extremely poor SNR conditions. Error appeared to increase at the worst SNR conditions (lower amplitude, lower frequency, and lower steady-state flows). However agreement between the flowmeter output and the accelerometer output was generally excellent, attesting to the validity of applying the oscillating-pipe calibration technique in the evaluation of dynamic flowmeters.

INTRODUCTION

Liquid-propellant rocket vehicles have experienced Pogo oscillations resulting from a closed-loop interaction between a longitudinal structural vibration and the liquid propulsion system. The more common form of Pogo instability, and that which could appear in the launch vehicle of the Space Shuttle, is called engine-coupled Pogo and stems from the interactions between the feed lines (including the turbopump) and the engine. Here, when the vehicle (external tank) vibrates longitudinally, the pump and the propellant in the flexible tank are forced into oscillatory motions which in turn produce oscillatory flow in the feedline and the pump's discharge line. The flow oscillations cause oscillations in engine thrust and in pump inlet pressure, which then act as regenerative forcing functions on the tank structure as the loop is closed. The Pogo vibration begins spontaneously, intensifies and then dies away after a period of 10 to 40 seconds and at
frequencies ranging from 5 to 60 Hz. At its maximum amplitudes, as high as 17 g's are felt at the input to payloads, often causing hazardous environments for humans and equipment.

During the Pogo vibrations, the propulsion system is known to exhibit resonance at frequencies determined by turbopump cavitation and suction-line wave propagation characteristics. The greatest difficulty in evaluating these engine-coupled instabilities is the determination of numerical values for the hydraulic compliance (volume change per unit pressure change) due to the cavitation and dynamic turbopump gain (partial derivative of discharge pressure with respect to inlet pressure).

Previous tests to determine the required values for the propulsion parameters of other vehicles have yielded questionable results primarily from the lack of propellant oscillatory flow measurements which have been unobtainable with available flowmeters. Consequently, in order to study more precisely the propellant fluid dynamics which may contribute to Space Shuttle Pogo oscillations, new ultrasonic flowmetering techniques have recently been explored here and abroad to meet the difficult flow measurement requirements.

The prime requisites of a flowmeter which is able to measure the Shuttle's Pogo-induced dynamic flows are that it:

1. Work in cryogenic liquid propellants (LO₂, LH₂) and LN₂ and H₂O for testing purposes,

2. Be essentially nonintrusive (no mechanical obstructions),

3. Have flat response to small oscillatory flow amplitudes at frequencies from 2 to 50 Hz in the presence of a large steady-state flow rate, and

4. Resolve dynamic flows of +1.0% of the steady-state flow rate with an uncertainty of 5% of reading or better.

Since 1971, only two American ultrasonic flowmeters were known to have the capability to meet the first two requirements in that they measured steady-state flows of cryogens for limited periods of time but could not perform in the dynamic mode required for the Shuttle Pogo application. In 1973 an ultrasonic dynamic flowmeter was developed by an American manufacturer (ref. 1) for the Shuttle application and was able to track oscillatory liquid flows occurring at frequencies up to 100 Hz for brief periods of time, but because of recurring instabilities in the signal processing electronics and crystal transducer bonding problems due to severe thermal cycling, it was unable to meet the Shuttle requirements without further costly development.

In France, Pogo specialists and propulsion engineers of the French Space Agency, Office Nationale D'Etudes et Recherches Aerospatiales (ONERA), recognized the need and pressed for the development of a reliable flowmeter.
which could yield dynamic flow data for use in predicting Pogo instabilities which arose from earlier launches of their Diamont B vehicle. As a result, a practical dynamic ultrasonic flowmeter was developed for working in non-cryogenic fluids like hydrazine and nitrogen tetroxide (later adapted to use with cryogens) as reported in reference 2. A 10-centimeter-diameter, 20 liters per second prototype was procured for testing its suitability for application on the Shuttle Pogo problem.

This paper describes the flowmeter's operating principle along with the tests and experiments performed in its evaluation, including the development and application of a novel dynamic calibration system used to assess the flowmeter's dynamic characteristics. Test results include those obtained at Langley in water and at NBS, Boulder, with liquid nitrogen at steady-state flows, and those obtained at Langley in water with simulated Shuttle oscillatory flows over the frequency range 7 to 30 Hz and amplitudes of from ±0.5 to ±10% of the steady-state flows. A description and operating principle of the dynamic calibration system is given as well as a discussion of the instrumentation and the cross spectrum signal processing techniques used to retrieve the small dynamic flowmeter signals under extremely poor SNR conditions.

SYMBOLS

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>A</td>
<td>cross sectional area of pipe, m²</td>
</tr>
<tr>
<td>A'</td>
<td>location of upstream acoustic transducer</td>
</tr>
<tr>
<td>B'</td>
<td>location of downstream acoustic transducer</td>
</tr>
<tr>
<td>C</td>
<td>speed of sound, m/sec</td>
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<tr>
<td>D</td>
<td>diameter of pipe, m</td>
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<td>F( )</td>
<td>Fourier transformation</td>
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<td>FM</td>
<td>frequency modulation</td>
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<td>G</td>
<td>power spectral density</td>
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<tr>
<td>H</td>
<td>frequency response function</td>
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<tr>
<td>H₂O</td>
<td>water</td>
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<tr>
<td>I</td>
<td>integral defined by $I = \int_{a}^{b} Vdl$, m²/sec</td>
</tr>
<tr>
<td>K</td>
<td>accelerometer amplifier gain</td>
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</table>
flowmeter amplifier gain

constant relating \( V_1 \) to \( \Delta t \), volts/sec

constant relating \( V_2 \) to \((t_1 + t_2)^2\), volts/sec^2

constant relating \( V_3 \) to \( \frac{t_2 - t_1}{(t_1 + t_2)^2} \)

liquid hydrogen

liquid nitrogen

liquid oxygen

accelerometer sensitivity, m/sec^2/volt

flowmeter sensitivity, m/sec/volt

division constant used in flowmeter \( N = 2^p \)

power spectral density

volumetric flow, liter/sec

volumetric flow based on \( \bar{V} \), liter/sec

volumetric flow based on \( \bar{V}' \), liter/sec

Fourier transformation of a time function, i.e., \( S_x(f) = F(x) \)

signal-to-noise ratio

analysis time, sec

flow velocity magnitude, m/sec

flow velocity vector, m/sec

average radial flow velocity, m/sec

average cross sectional area flow velocity, m/sec

voltage proportional to uncompensated flow, volts

voltage proportional to sound speed compensation factor, volts

voltage proportional to compensated flow, volts
\( V_x \) voltage proportional to acceleration signal, volts
\( V_u \) voltage proportional to oscillatory flow signal, volts
\( W \) gravimetric flow, kg/sec
\( e \) error in the estimate of the mean square value, \( \% \)
\( f \) frequency, Hz
\( \lambda \) acoustic path length from A' to B', m
\( n \) noise signal, m/sec\(^2\)
\( p \) integer exponent which determines \( N \), where \( N = 2^p \)
\( r \) radial distance from pipe centerline, m
\( \text{rms} \) root mean square
\( \text{rpm} \) revolutions per minute
\( t \) time variable, sec
\( t_1 \) downstream transit time, sec
\( t_2 \) upstream transit time, sec
\( \Delta t \) \( t_2 - t_1 \), sec
\( u(r) \) local velocity, m/sec
\( u \) oscillatory flow, m/sec
\( \dot{u} \) \( \dot{u} = \frac{du}{dt} = x \), flow acceleration, m/sec\(^2\)
\( u' \) rms value of oscillatory flow, m/sec
\( u_p \) peak value of oscillatory flow, m/sec
\( x \) \( \dot{x} \), m/sec\(^2\)
\( x' \) rms value of acceleration, m/sec\(^2\)
\( x_p \) peak value of acceleration, m/sec\(^2\)
\( y \) oscillatory flow + noise, m/sec
\( \dot{z} \) \( \dot{z} = \frac{dz}{dt} = u \), flow velocity, m/sec
\( z_p \) peak displacement, m
DISCUSSION AND RESULTS

Description of Flowmeter

Ultrasonic flowmeters.- Ultrasonic flowmeters make use of the interaction of sound waves with the moving fluid to measure fluid velocity. These flowmeters (also called sonic or acoustic), because of their nonintrusive, no-moving-parts construction, offer high potential for measuring rapidly varying flow velocities which occur in launch vehicle propellant lines under Pogo conditions.

Three basic techniques are the bases for measuring fluid velocity with ultrasonic flowmeters:

1. The beam deflection technique where the deflection of an acoustic beam propagated radially across the moving liquid is measured.

2. The Doppler shift technique where acoustic waves are propagated axially along the flow path and the frequency shift of return signals from scatters in the fluid is measured.

3. The transit time difference technique where acoustic waves are propagated in opposite direction (upstream and downstream) diagonally across the flow and the difference in transit times between the waves is measured.

The transit time difference technique appears to be the most effective for practical flow measurements. In this technique, the three most frequently used mechanisms for measuring the time difference are (1) determining the transit time difference (Δt) directly, (2) determining the phase difference (Δφ), and (3) obtaining a frequency difference (Δf). In (1), acoustic pulses are transmitted simultaneously upstream and downstream and the
difference in transit times is measured directly. In (2), continuous waves are transmitted in both directions and the difference in phase shift is measured. In (3), pulse trains are transmitted upstream and downstream whose repetition frequencies depend on their repetitive transit times, with the difference in repetitive frequencies being measured.

**ONERA flowmeter**.- The French design uses the transit time difference method for sensing fluid velocity. Digital circuit techniques are used to precisely measure the sound propagation times upstream and downstream across the flow and to generate trains of constant energy, square pulses for integrations which give rise to an analog output voltage proportional to axial fluid velocity. Although the difference in transit times is actually measured, the corresponding changes in phase occurring as a consequence of the transit times difference are actually used to generate the flow proportional output. Further, the flowmeter is truly dynamic in that it is capable of tracking flow velocities oscillating at frequencies up to 300 Hz.

The flowmeter system (fig. 1) comprises three parts: The sensing unit consisting of the cylindrical meter body on which two piezoelectric crystal transducers are mounted, a signal processing unit consisting of the digital electronic circuits, including filters and sound speed compensation circuits, and a power supply-readout unit which, in addition to a panel output meter and zero setting potentiometer, also includes outlets for recordable output signals.

**Working principles**.- Figure 2 shows a simple sketch of the flow sensing unit of the ultrasonic flowmeter under discussion. Transducers $A'$ and $B'$ (lead zirconate-titanate (PZT)) are capable of simultaneously transmitting and receiving sound pulses (both upstream and downstream) along path $A'B'$ which is inclined to the flowmeter axis and the flow at an angle $\theta$. When excited by an ac voltage, the transducers vibrate at some natural frequency (5 MHz in this case) and generate sound wave bursts at that frequency. The burst repetition rate is 2 kHz. The waves propagate across the fluid to strike the opposite transducer which is forced to oscillate at the impinging frequency and generate a time delayed version of the original signal which is used to measure flow. With fluid flow established in the pipe, the sound wave transit time from $A'$ to $B'$ (downstream with the flow) is given by

$$ t_1 = \frac{l}{C + V \cos \theta} \quad (1) $$

and that from $B'$ to $A'$ (upstream against the flow) is given by

$$ t_2 = \frac{l}{C - V \cos \theta} \quad (2) $$
where \( l \) is the path length, \( C \) the fluid sound speed, and \( V \) the fluid velocity. The difference in transit times, \( \Delta t \), is the useful information for determining fluid velocity and is given by

\[
\Delta t = t_2 - t_1 = \frac{l}{C-V \cos \theta} - \frac{l}{C+V \cos \theta} = \frac{2lV \cos \theta}{C^2 - V^2 \cos^2 \theta} \tag{3}
\]

Because \( V \) is generally much smaller (on the order of 0.01) than \( C \), the term \( V^2 \cos^2 \theta \) in the denominator can be neglected without changing the fractional value of (3), which becomes

\[
\Delta t \approx \frac{2l}{C^2} V \cos \theta \tag{4}
\]

However, because \( \Delta t \) is also inversely proportional to the square of the fluid sound speed which can change appreciably with fluid temperature and pressure, it is necessary for accurate measurements, that \( C^2 \) be eliminated from equation (3). In the ONERA flowmeter, this is done electronically by dividing the difference in transit times, \( t_2 - t_1 \), by the squared value of the sum of the transit times \( (t_2 + t_1)^2 \):

\[
t_2 + t_1 = \frac{l}{C-V \cos \theta} + \frac{l}{C+V \cos \theta} = \frac{2lC}{C^2 - V^2 \cos^2 \theta} \approx \frac{2l}{C} \tag{5}
\]

Dividing equation (4) by the squared value of equation (5), we have

\[
\frac{t_2 - t_1}{(t_2 - t_1)^2} = \frac{V \cos \theta}{2l} \text{ or } V = \frac{2l(t_2 - t_1)}{\cos \theta(t_2 + t_1)^2} \tag{6}
\]

The measurement of fluid velocity, \( V \), is then made independent of the sound speed \( C \).

To gain a sense of the quantities and dimensions involved in the above equations, let us review an intended application to a shuttle Pogo propellant measurement. Here, liquid oxygen flow rates up to \( 6 \) m/sec (20 ft/sec) are to be measured in a \( 30 \)-cm-diameter (12-in.-diameter) pipe upstream of the low pressure oxidizer pump. With the sound speed of liquid oxygen at \( 925 \) m/sec (3034 ft/sec), the ratio \( \frac{V}{C} = 0.0065 \). This justifies the elimination of the term \( V^2 \cos^2 \theta \) from equation (3),
since, with $\theta = 45^\circ$, the term's value is $(6^2)(0.707)^2 = 18$ compared to the value, $C^2 = (925)^2 = 8.55 \times 10^5$. Further, the path length, $\ell$, is $1.414 \times 30 \text{ cm} = 42.42 \text{ cm}$ (17 in.), and the transit time at no-flow conditions, $\frac{\ell}{C} = \frac{42.42 \text{ cm}}{92,500 \text{ cm/sec}} = 4.58 \times 10^{-4} \text{ seconds}$.

In the measurement of flow rate, the essential quantity desired from the ultrasonic flowmeter is the average axial velocity over the flow area. The quantity which is measured directly, $\Delta t$, however, yields only the average velocity along the path of the ultrasonic beam, $A'B'$, of figure 2. Other than the sound speed, $C$, and fluid velocity, $V$, the measurement of $\Delta t$ is also dependent upon the natural flow irregularities (turbulence) and the shape of the velocity profile, which are functions of the Reynolds number.

In practice the flow irregularities across the path $A'B'$ occur over distances which are large with respect to the wavelength of the propagated signal. Consequently the path $A'B'$ can be divided into infinitesimal lengths, $d\ell$, which, in the interaction of the fluid velocity with the sound speed of signal propagation, produces the infinitesimal time delay,

$$dt = \frac{2 V \cos \theta}{C^2} d\ell$$  \hspace{1cm} (7)

The total time delay of a wave propagated from $A'$ to $B'$ and that from $B'$ to $A'$, using vectorial notation, can be written as the sum of the infinitesimals

$$t_2 - t_1 = \Delta t = 2 \int_{A'B'} \frac{V \cos \theta}{C^2} d\ell = \frac{2}{C^2} \int_{A'B'} \mathbf{v} \cdot d\mathbf{\ell}$$  \hspace{1cm} (8)

It is assumed that the velocity profiles are the same at $A'$ and $B'$. Then the delay time $\Delta t$ may be considered a function of the integral, $I$, of the velocity profile along a chord which is normal to the flow and $a@$ which is a projection of path $A'B'$ on the pipe centerline such that

$$\Delta t = \frac{2}{C^2} I$$  \hspace{1cm} (9)

where

$$I = \int_{a@} V d\ell$$  \hspace{1cm} (10)
The quantity which can be measured, then, is the flow rate which is an integral of the velocity profile on one chord and not the velocity profile on the area cross section. In this respect the following analysis is useful in understanding the flow measuring capability of the ultrasonic flowmeter.

Flow rates are expressed as $Q = \bar{V}'A$ for volumetric flow (liters/sec) and $W = \rho \bar{V}'A$ for gravimetric flow (kg/sec) where $\bar{V}'$ in each case is the average cross section velocity. Assuming a symmetrical velocity profile, the equation for determining the average flow velocity over the circular flow area is

$$\bar{V}' = \frac{1}{\pi D^2} \int_0^{D/2} \int_0^{2\pi} u(r, \xi) (dr)(r d\xi)$$  \hspace{1cm} (11)

$$= \frac{1}{\pi D^2} \int_0^{2\pi} d\xi \int_0^{D/2} ru(r) dr = \frac{4}{D^2} \int_0^{D/2} ru(r) dr$$  \hspace{1cm} (12)

where $r$ is some distance from the center of the circular pipe area, $u(r)$ the local velocity at $r$, and $D$ the flow area diameter. The average velocity over the radius $D/2$ can be written

$$\bar{V} = \frac{1}{D/2} \int_0^{D/2} u(r) dr$$  \hspace{1cm} (13)

The ratio of these velocities is

$$K = \frac{\bar{V}}{\bar{V}'} = \frac{\frac{1}{D/2} \int_0^{D/2} u(r) dr}{\frac{4}{D^2} \int_0^{D/2} ru(r) dr} = \frac{D/2}{4} \frac{\int_0^{D/2} u(r) dr}{\int_0^{D/2} ru(r) dr}$$  \hspace{1cm} (14)
In more practical terms

\[ K = \frac{\bar{V}_A}{\bar{V}'_A} = \frac{Q_r}{Q_a} \]  

(15)

Obviously, \( K \) is a function of velocity profile, which is in turn, a function of Reynolds number. For fully developed flow having an ideally vertical profile, \( K = 1.00 \); for the known parabolic profile of laminar flow (\( Re < 2000 \)), \( K = 1.33 \), and for representative pipe flow (\( Re > 10^6 \)) where the profile is essentially vertical, \( K \approx 1.05 \). These values for \( K \) have been derived empirically by various flowmetering engineers with good agreement (refs. 3, 4, and 5) and can be applied to determine the area velocity when only the indicated flowmeter velocity is known. In practice, however, when using an ultrasonic flowmeter which measures the average velocity on one chord, as does the ONERA flowmeter, the use of \( K \) can be eliminated by a direct time-volume or time-weight calibration over the meter's designed range. The calibration is actually in terms of \( Q_a \) vs. meter output (with a given pipe geometry) so that knowledge of profile and value of \( K \) are not essential. Because \( Q_a = \bar{V}'A \) (where \( A \) is constant) and the flowmeter is applied with essentially the same pipe geometry at Reynolds numbers similar to those of the calibration, \( \bar{V}' \) is easily obtained by \( \bar{V}' = \frac{Q_a}{A} \). Steady-state calibrations were performed on the ONERA flowmeter (fig. 6) with both ambient temperature water and liquid nitrogen over the volumetric flow range, 0-19 liters/sec (0-5 gallons/sec) which, by \( \bar{V}' = \frac{Q_a}{A} \), yields the velocity range 0-4.16 meters/second (0-13.64 ft/sec).

In evaluating the ONERA meter's potential for measuring the dynamic propellant flows in Shuttle Pogo studies, \( \bar{V}' \) was the quantity of interest. As delineated later in this paper, the nature of the dynamic-flow tests required that the flowmeter-measured dynamic velocities be compared with the oscillating pipe dynamic velocities which were derived from an accelerometer and considered to be a standard.

Signal processing techniques.- As stated previously, the ONERA flowmeter is comprised of four sub-subsystems whose functions are integrated to generate usable flow-related electrical signals. The subsystems include the flowmeter body (spool piece), the piezoelectric transducers, the signal conditioning electronics, and the power supply readout electronics. The transducers, which are mounted in the wall of the spool piece, are used to transmit and receive ultrasonic waves across the fluid to be measured. The signal conditioning electronics amplify, shape, count, integrate, and filter the received signals for proper display and recording. The power supply readout electronics provides (1) the appropriate electrical power to operate the conditioning electronics, (2) zero shift and zero adjustment circuits, (3) a panel meter for reading out flow, and (4) two recorder outlet
receptacles delivering analog signals proportional to flow velocity, \( t_2 - t_1 \), uncompensated for sound-speed changes, and the other \( t_2 - \frac{t_1}{t_2 + t_1} \), which is compensated for sound-speed changes.

The phase shift, \( \Delta \phi \), which occurs between the received signals as a result of the transit time difference, \( \Delta t = t_2 - t_1 \), is actually used to derive the flow proportional signals in the ONERA flowmeter. The phase shift between the received signal transmitted upstream against the flow and that transmitted downstream with the flow is given by

\[
\Delta \phi = 2\pi f \Delta t
\]

where \( \Delta t = t_2 - t_1 \approx \frac{2\theta}{c^2} V \cos \theta \)

and \( f \) is the frequency of the transmitted signal.

Combining the equations for \( \Delta \phi \) and \( \Delta t \) yields the equation

\[
\Delta \phi = \frac{4\pi f \theta}{c^2} V \cos \theta
\]

from which a flowmeter for any reasonable steady state flow range can be designed. The right hand side, it will be noticed, includes the path length, \( \ell \), the angle \( \theta \) which determines \( \ell \), the fluid sound speed, \( C \), the frequency of the propagated signal, \( f \), and the fluid velocity, \( V \).

A block diagram of the signal processing electronics is shown in figure 3 and examples of the processed signals are shown in figures 4 and 5. A 2 kHz oscillator is used to control the entire operation, including the zero resetting of the frequency counters. A 5 MHz oscillator is actually used to generate the ac voltage which is impressed on each PZT transducer. This oscillator is driven by the 2 kHz oscillator such as to apply through impedance matching amplifiers, 5 MHz bursts of approximately 50 microseconds duration to the transducers at a recurrence rate of approximately 2000 times a second. Essentially each wave train comprises about 256 cycles of 5 MHz signals and after processing yields a flow measurement which is updated 2000 times a second, permitting dynamic flow measurements up to 300 Hz.

The wave trains are transformed and launched as acoustic waves by the transducers across the fluid to the opposite transducers where they are received, retransformed to electrical signals, amplified and sent to shaping circuits. The duration of the wave trains is less than the time required for sound to travel the distance between transducers. The transducers transmit one to the other simultaneously and a programmer is used to switch them alternately to the receive and transmit mode. The wave trains arrive simultaneously with zero fluid flow during the pipe crossing, but pick up
velocity moving downstream with the fluid flowing and lose velocity moving upstream against the flow, resulting in a delay time, $\Delta t$.

In the shaping circuits, the received signals are formed into trains of constant amplitude pulses operating above the noise levels, a condition insured by voltage comparators. This is required to facilitate the frequency counting, pulse forming and integration which yield the flow-proportional output signals. After signal comparing and shaping, the two received pulse trains are divided by a number $N = 2P$ by way of binary counters (fig. 5). The number $N$ depends on the desired flow range as determined by the geometric parameters of the meter body (pipe diameter, distance between transducers, angle of inclination) included with other parameters in the design equation (17). Practical circuit considerations dictate that $N$ range from 8 to 128 in the ONERA flowmeter. For the tested 7.62 cm prototype, $N = 16$ and the total flow range is 6 meters per sec (20 ft/sec), the total phase shift, $\Delta \phi$, is approximately $8\pi$.

After a second division, this time by 2, a time shift equal to a quarter of the final period is formed between the two pulse trains so that at zero flow the signals appear, after division and phase shifting, as pulses which are 90 degrees out of phase. Then a system of double input gates are used to generate for integration a series of square, constant energy, constant width pulses from both the downstream-launched ($t_1$) and the upstream-launched ($t_2$) signals (fig. 5). The logic of the gate electronics is to produce these integrative pulses with durations which are a function of the degree of overlapping present between the $t_2$ related pulses and the $t_1$ related pulses. In the zero flow case the overlapping is symmetrical (equal portions of the $t_2$ pulses overlap the $t_1$ pulses), producing integrative pulses of equal duration which, after integration and subtraction in a differential amplifier, results in zero flowmeter output. During flow, however, the overlapping is not symmetrical and the series of pulses, although equal in number, vary inversely in duration when the original $\frac{1}{2}$ shift (zero flow) is modified by the flow (pulse duration $t_1$ - pulse duration $t_2 = 2 \Delta t$). These pulses are integrated and also applied to the inputs of a differential amplifier where they are subtracted. The difference output is passed through a low-pass filter and becomes an analog voltage ($V_1$) proportional to the propagation delay $\Delta t$, so that

$$V_1 = K_1 \Delta t = K_1 (t_2 - t_1)$$

(18)

for a flow signal which is uncompensated for sound speed changes.

For a compensated signal, two signals are developed representing the wave travel times $t_1$ and $t_2$, respectively, in the downstream flow direction and upstream flow directions. After passing through a low pass filter, an analog summation of $t_1$ and $t_2$ is carried out in the squarer amplifier, yielding the signal.
\[ V_2 = K_2 (t_1 + t_2)^2 \] \hfill (19)

After division by this signal in the divide amplifier, an analog of the flow signal compensated for sound speed variations appears as

\[ V_3 = K_3 \frac{t_2 - t_1}{(t_1 + t_2)^2} \] \hfill (20)

This output signal was used throughout the tests performed on the ONERA flowmeter under evaluation.

The ONERA flowmeter also provides for premeasurement zero adjustment and a dc component to be added or subtracted from the output signal in order to compensate for steady-state flow when setting up to measure the dynamic flow component.

**Dynamic Flow Test**

Several steady-state calibrations performed both in France and the United States in water and LN$_2$ (figure 6 is typical) had established the ONERA flowmeter's reliability, linearity, and repeatability when the flow rate was changed slowly in discrete increments. Furthermore, preliminary dynamic quantitative tests demonstrated the flowmeter's ability to respond to oscillating flow rates up to 100 Hz. This is not surprising when realizing the electronics is capable of updating the instantaneous flow measurement 2000 times per second on a continuous basis. The dynamic tests, however, were made using various flow-modulating devices (translating piston, modulating-valve) where characterizing the magnitude and profile of the perturbations was exceedingly difficult and mostly unsuccessful. Only the frequency of oscillation could be determined reliably. The question arises, then, given a potentially useful dynamic flowmeter, how does one verify its performance in the dynamic mode? How does one calibrate it dynamically? What reference stimulus of known quality can be used to graph against the output of the flowmeter in order to establish its accuracy in measuring rapidly changing fluid velocities? Ideally the dynamic flowmeter should have the same sensitivity to the oscillating component of fluid velocity as it does to steady-state fluid velocity.

The problem was twofold: How to (1) generate (and apply to the flowmeter) well defined and measurable fluid velocity oscillations at frequencies from 2 to 40 Hz (Shuttle requirements) in the presence of a large steady-state flow rate (fig. 7) and (2) retrieve the small flowmeter response signals obscured by hydraulic noise which in many cases resulted in poor signal-to-noise ratios. This condition is analogous to generating...
a precisely known ac ripple riding atop a relatively large dc bias, and sensing the weak ac signals while suppressing the dc bias and rejecting any spurious ac signals not related to the desired one.

**Oscillating-pipe calibration system.** - To solve these problems a dynamic calibration technique was developed and tested whereby the fluid (water) flows through the flowmeter at constant velocities while the flowmeter is forced to oscillate relative to the fluid. The basic water flow loop is shown in figure 8 and the essential elements of the dynamic test system in figures 9 and 10. The flow loop consisted of a covered steel tank 60.96 cm by 91.44 by 213.36 cm long (2- by 3- by 7-foot long), treated internally to inhibit rust. A 1730-rpm centrifugal pump was used to force up to 12.6 liters/sec (200 gpm) of water through approximately 854 cm (28 ft) of 7.62-cm diameter (3 inch) plastic pipe whose wall thickness was 0.645 cm (0.25 in.). The piping was fastened to the tops of seven stanchions (distributed along the loop) which were anchored to the concrete floor with lag bolts. At maximum flow rate, the pump maintained an absolute head of 1.72 x 10\(^5\) Pa (25 psia) and at 3.2 liters/sec (50 gpm) (the lowest steady-state flow rate used in the dynamic tests), an absolute head of about 2.41 x 10\(^5\) Pa (35 psia). A turbine flowmeter was used to check the steady-state (dc) calibration of the ONERA flowmeter and a throttling valve was located at the tank exit, downstream of the pump which was attached to the tank, and at the tank inlet downstream of the oscillating-pipe test section. A 91.44 cm (36 in.) straightening section to eliminate swirl was installed immediately upstream of the 111 cm (44 in.) metering run which consisted of the test flowmeter installed in the oscillating-pipe section which translated axially with respect to the fixed pipe.

The flowmeter was supported in line with the fixed flow-loop pipe by means of wide flat phosphor-bronze springs (fig. 9) which were 15.24 cm square (6 in.) by 0.08 cm thick (1/32 in.) and permitted axial motion only. The springs were hung on a support structure made of angled dural and fastened to the loop's support stanchions nearest the test section so as to maximize the stability of the complete oscillating section. A tunable spring-mass system consisting of a pair of parallel rectangular bars whose dimensions were 0.635 cm by 5.08 cm by 182.88 cm (1/4 in. by 2 in. by 72 in.) were bolted at right angle to the flowmeter and movable 6.12 kg (13.5 lb) masses were clamped to the bars. The sinusoidal input perturbation frequency relative to the fluid was the resonant symmetric bending frequency of the tunable mass-spring system. This provided low frequency oscillations when the masses were bolted to the ends of the bars and higher calibration frequencies when the masses were moved inward where the higher symmetric bending modes were excited. The sinusoidal exciting force was provided by an electrodynamic shaker attached to one of the bars at a vibration antinode. In this fashion, the dynamic component of the fluid velocity relative to the flowmeter is determined from the velocity derived from the output of an accelerometer mounted on the flowmeter.

The critically essential elements of this oscillating-pipe concept are the flexible joints (fig. 10) required at each end where the moving pipe attaches to the fixed pipe. The joints must permit translation of the
oscillating-pipe (flowmeter) at displacements up to 0.38 cm (0.15 in.) while still maintaining a seal at absolute fluid pressures to \(2.41 \times 10^5\) Pa (35 psia), and without distorting the sinusoidal nature of the forcing function or creating pumping and excessive friction.

Coulomb friction would not only increase the driving force required of the shaker but would distort the accelerometer output signal rendering it impossible to obtain an accurate measurement of the perturbation velocity used as the reference for gaging the performance of the flowmeter. The design illustrated in figure 10 circumvents all these difficulties by the use of Bellofram rolling diaphragm seals which are made of fiber-impregnated rubber. They are long-stroke, deep-convolution, flexible diaphragms which roll off oscillating-pipe sidewalls onto the fixed-pipe sidewalls with a continuous, smooth, essentially frictionless motion. An auxiliary axial guide (such as the wide, thin support springs described above) must be used with seals since they provide no radial support.

From figure 10, as the pipe attached to the flowmeter is moved axially, the Bellofram convolution acts as a piston displacing fluid as it moves. This pumping action, if not compensated, would result in an incremental flow velocity of unknown magnitude through the flowmeter, yielding an erroneous calibration. If, however, the shoulder is one-half the width of the Bellofram convolution, the volume of the void immediately to the right of the convolution will remain virtually constant throughout the travel thus eliminating the pumping of fluid in and out of the void and through the flowmeter.

The system just described permitted distortion-free sinusoidal translation of the flowmeter relative to the established steady-state flow rates, at discrete frequencies of 7, 19, and 30 Hz and at oscillating velocities having amplitudes from 0.5 to 15% of the steady-state fluid velocity.

**Preliminary tests.** — Because of the difficulties, the uncertainties and high potential for misinterpretation of results attending the unprecedented testing of a newly developed dynamic flowmeter with the oscillating pipe, the test procedures and instrumentation were kept as simple as possible. Several preliminary tests were performed on the flowmeter and oscillating test section to ensure the validity of the actual dynamic evaluation tests where only the accelerometer and the flowmeter were ultimately recorded in order to plot the oscillatory flow velocities.

The first of these tests was designed to check the flowmeter's sensitivity to accelerations in both the axial and radial directions. Here the flowmeter was filled with water and capped at both ends. This permitted the flowmeter to operate with the transmission of sound signals through the water path but the meter would have zero output because of the absence of flow insured by the end caps. The flowmeter was then mounted rigidly in the 7.62 cm (3 in.) pipe flow loop (in the absence of any pipe line fluid with the exception of that trapped in the meter) and excited by the electrodynamic shaker at several frequencies from 7 to 100 Hz with forces up to ±50 g. No measurable signal was evident in the flowmeter's output. The results instilled
confidence in the investigation that the flowmeter would respond only to relative flow velocity differences between the oscillating flowmeter and the steady flow, when forced to translate by the shaker-spring bars system.

In a second test, in order to determine how well the flow loop was anchored to the floor, 50 g accelerometers were mounted on the fixed pipe directly upstream and downstream of the oscillating-pipe section near the flexible seals. The accelerometers were recorded while the oscillating pipe was driven at frequencies of 7, 19, and 30 Hz at steady flows to > 1.82 m/sec (6 ft/sec). Recorded acceleration amplitudes were negligible at all frequencies in both the axial and radial directions. It was concluded that fixed-pipe movements were so small that little or no pipe acceleration effects would distort the dynamic flow measurements.

A third test was performed after the flowmeter was installed in the oscillating-pipe system and the complete flow loop was readied for the dynamic investigation. For this test, two flush-diaphragm pressure transducers were mounted in the rigid piping about 25.4 cm (10 inches) upstream and downstream from the moving-pipe section and close-coupled to the fluid. The transducer circuit sensitivities were set high in order to detect pressure oscillations which would strongly indicate the presence of fluid pumping in the oscillating-pipe section. The dynamic pressures were recorded at fluid line static gage pressures of 0 and 1.38 x 10^2 Pa (psig) and the largest amplitude recorded was about 133 Pa (1 mm Hg), indicating the absence of significant fluid pumping, a condition observed with an early system of inadequate design. In the earlier system where pumping occurred, significant additional unknown quantities of fluid were forced through the flowmeter during each stroke, resulting in the flowmeter readings being consistently higher than those of the accelerometer.

A prime requisite for the effective evaluation of a dynamic flowmeter is to establish a valid method for producing and subjecting the meter to repeatable, measurable flow velocity oscillations in the presence of relatively large steady flows. To achieve this one would normally think of moving oscillating flows through a stationary flowmeter. With the oscillatory-pipe system, however, the reverse is true - the flow remains fixed at zero or some level of steady flow rate while the flowmeter is oscillated axially to simulate flow velocity variations between the flowmeter and the fixed flow rate. The peak velocities resulting from the axially translating flowmeter were computed from measured peak accelerations sensed by the meter-mounted servoed accelerometer. Because the action of the oscillating pipe constitutes simple harmonic motion, the equation for computing the peak velocity is derived as follows. For simple harmonic motion, displacement is given by:

\[ z = z_p \sin \omega t \]  \hspace{1cm} (21)
Velocity is the derivative of displacement with respect to time

\[ u = \dot{z} = z_p \omega \cos \omega t \quad (22) \]

Similarly, acceleration is the derivative of velocity with respect to time

\[ x = \ddot{u} = -z_p \omega^2 \sin \omega t \quad (23) \]

By inspection, the following can be written

- \( z_p \) = peak displacement
- \( z_p \omega \) = peak velocity
- \( z_p \omega^2 \) = peak acceleration

Thus, if peak acceleration is known, peak velocity can be found by

\[ u_p = \frac{z_p}{\omega} \quad (24) \]

If \( x \) is given in \( g \) units, it must be converted to \( m/sec^2 \)

\[ 1 \ g = 9.81 \ m/sec^2 \]

\[ u_p = \frac{(x_p)(9.81)}{2\pi f} = \frac{4.9 x_p}{\pi f} \quad (25) \]

where \( x_p \) is the calibrated accelerometer output and \( f \) the frequency of oscillation. These accelerometer-derived flow velocities were used as the reference velocities against which all flowmeter-measured velocities were graphed, and are presented later in the section on test results.

**Dynamic tests.**—A Shuttle requirement for a Pogo dynamic flowmeter was that it be able to track dynamic flow amplitudes from 0 to 10% of the steady-state flow present. Attention, however, would be focused on amplitudes 2% or less of steady flow. A specific requirement stated that at the dynamic flow
A V = 0.01, flowmeter accuracy be 5% of reading or less. Therefore, measurements were made during the tests of dynamic flow amplitudes ranging from 0.5 to 10% of the three different steady-state flows of approximately 0.61, 1.22, and 1.82 m/sec, respectively (2, 4, and 6 ft/sec). Oscillating-pipe tests were also made at zero steady-state flows. Without flow, the amplitudes were taken at an assumed steady-state flow of 0.61 m/sec (2 ft/sec).

The dynamic tests were run by oscillating the flowmeter at frequencies of 7, 19, and 30 Hz. Earlier Shuttle Pogo specialists had theoretically determined that vehicle longitudinal instabilities would occur at frequencies from 2 to 40 Hz. Consequently, the task was to choose at least three frequencies within the Pogo spectrum at which the tunable spring-mass system which drove the flowmeter could be excited to resonate in symmetric bending. These system resonant frequencies were found experimentally to be approximately 7, 19, and 30 Hz. At these frequencies, the accelerometer output signals appeared on the monitoring oscilloscope to be free of distortion and nearly pure, indicating excellent sinusoidal forcing of the oscillating-pipe section by the shaker.

Following is the step-by-step procedure used in this investigation of the ONERA flowmeter using the oscillating-pipe method:

1. The ONERA flowmeter's steady-state calibration factor was verified at three flow rates against a calibrated turbine flowmeter which served as the in-line standard for this purpose. The turbine had a linearity uncertainty of ±0.5% of reading and a repeatability of ±0.2%. Both the ONERA flowmeter and the turbine had been previously calibrated on Langley's liquid time-weight flow calibration standard having an uncertainty of ±0.15%, traceable to the National Bureau of Standards. The in-line check calibration was performed before each series of runs at a given frequency. The turbine flowmeter's output frequency was continuously monitored with an electronic counter during the entire test. Both flowmeters were recalibrated on the Langley stand after test completion.

2. The accelerometer was also carefully checked with a ±1 g calibration before each series of runs. The instrument's sensitivity factor and frequency response function had been determined on a shaker table prior to the flowmeter tests.

3. Calibrate signals at 7, 19, and 30 Hz from a signal generator were inserted into the flowmeter and accelerometer channels and recorded on tape before each series of runs.

4. Accelerometer peak amplitudes corresponding to nominal peak dynamic velocities in terms of percent of the steady-state flow were used to drive the oscillating pipe at flow modulations of 0.5 to 10% for each of three steady-state flow rates at the chosen frequencies.
5. With the necessary calibrations on tape, the steady-state flow rates verified, and the the oscillating-pipe section in motion, data was recorded at the selected steady-state velocities dwelling at each modulation point for 2 minutes at constant frequency.

6. The data tape for each series of runs was processed on a Fourier analyzer which employed Fourier transform methods for yielding the rms values of the dynamic acceleration and dynamic flow rates and their phase relationships. Power spectral density plots and a coherence function, \( \gamma^2 \) (which expresses the degree of linear causality between the forcing function and measured outputs as a function of frequency) were also obtained with this analyzer.

**Instrumentation.**—Figure 11 shows a block diagram of the instrumentation used to perform the dynamic flow tests in the presence of an established steady-state flow rate. Once the oscillating-pipe section was found to be free of pumping action, the fixed pipe free of movement and the flowmeter insensitive to accelerations, the instrumentation was reduced to two channels: The accelerometer and the flowmeter with their conditioning equipment. Since the reference dynamic peak velocities for these tests would be derived from the accelerometer readings, extreme care was used in its selection. A \( \pm 50 \text{ g} \) force balance, closed-loop transducer was chosen in which an acceleration force applied to the seismic mass is counterbalanced by an electromagnetic force from a current generated in the servo loop, energizing a movable coil in the permanent magnetic field. The generated current when passed through a suitable range resistor, provides a dc voltage proportional to acceleration. The accelerometer has a natural frequency of \( \approx 680 \text{ Hz} \) and a flat frequency response to nearly 500 Hz. Hysteresis was 0.0005 g/g while zero and sensitivity shifts due to temperature were 0.03 g/100°F and 0.01%/g, respectively. Its sensitivity to transverse accelerations was 0.002 g/g, and because of very minute movements within the seismic mass-servo mechanisms, the accelerometer's resolution was limited only by the associated electronics.

The accelerometer and flowmeter signals were fed through signal conditioners and dc amplifiers to a seven-track FM tape recorder in which the data was recorded at a tape speed of 7½ inches per second.

Because of the nature of the tests, it was important to monitor the character and amplitude of the input forcing function represented by the accelerometer output signal. This was done for each oscillation-pipe modulation point at all three frequencies. Accordingly, an oscilloscope with appropriate switching circuits was used to monitor the input signals into the recorder as well as the output signals before each run was recorded. The data was recorded over a period of 2 minutes at each modulation setting for all three frequencies and reduced on a Fourier analyzer. The rms values of flow rate measurement and accelerations, as well as power spectral densities were determined employing digital filtering techniques and fast Fourier transform methods described in the following section of this paper.

The dynamic tests made with zero steady-state flows, because of the reasonably noise-free signals, were, for the large part, recorded with an
oscillograph and the data reduced manually. Some zero-flow data was also recorded on tape and reduced on the Fourier analyzer. These points agreed with the results obtained with the earlier oscillographic data.

The phase relationship between the accelerometer and flowmeter output signals were also measured because of the importance to Shuttle Pogo tests of knowing the phase lag through the flowmeter system. The prime function of the dynamic flowmeters in Pogo tests of the Shuttle propulsion system would be to help determine the transfer functions through the turbopumps and any Pogo suppressor. Here, the relative amplitudes as well as the phases between inlet and outlet dynamic flows are important for an accurate characterization of the fluid components' transfer functions which can be used to design effective suppressors of Pogo instabilities. The phase lag between accelerometer and flowmeter was determined using the Fourier analyzer over the frequency range from 7 to 87 Hz. Additional phase measurements were made with a phase meter. Although considerable scatter was present in the phase data (fig. 12) because of the hydraulic noise in the flow loop, good agreement was obtained among the measurements and the data clustered around a line whose slope was 0.55°/Hz. The 0.55°/Hz phase lag is due primarily to the low pass filter in the flowmeter electronics, and is in addition to the natural 90° phase lag between the accelerometer and the flowmeter, as seen in figure 12.

Data Analysis

Due to the natural hydraulic turbulence in the flow, the flowmeter signal always contained some noise for every test run. The problems presented by this noise seemed to depend on both the level and frequency of the calibration signal. Whenever a signal is obscured by the presence of noise, signal processing techniques, such as correlation, filtering, averaging, or various combinations of these methods are required to retrieve the flow-related signal. There are a variety of instruments available which enable one to employ these techniques. Recently, the advancement of minicomputer technology and the Fast Fourier Transform (FFT) algorithm has led to an instrument called a Fourier analyzer with which one can perform various time series analysis functions. One of these analyzers was used in the frequency response calibration of the flowmeter. In addition to the techniques mentioned for retrieving signals buried in noise, the Fourier analyzer can provide other valuable functions such as the frequency response and coherence.

Frequency response and coherence functions.—The frequency response function is an extremely important attribute of the system. If the system under test is excited by a broadband random noise, the system's response over a band of input frequencies can be determined. The graph of this response function will reveal regions in the frequency domain where problems can occur. The frequency response function for the oscillating-pipe calibration system is shown in figure 13 where magnitude and phase are plotted against frequency. Also shown is a plot of the coherence function. This function is a measure of what portion of the output is due directly to the input. When the coherence function is less than unity, sources (usually in the form of noise) other than the input are contributing to and distorting the output. It is readily
apparent from the plot in figure 13 that problems would most likely occur if tests in this flow system were attempted when oscillating the pipe test section in the region of 26 Hz. The frequency response function indicates that there is a nonlinear condition in that region, and that is confirmed by the coherence function. Nominal frequencies of 7, 19, and 30 Hz were used to calibrate the flowmeter. The frequency response and coherence functions for the system show that these figures were reasonable values to use for the tests.

Analysis technique.- Appendix A shows the equations for determining the frequency response function \( H \), cross power spectrum \( G_{yx} \) and coherence function \( y^2 \). They are all derived by the FFT analyzer after obtaining the Fourier transform of the electrical signals corresponding to the physical input and output variables of the system. The input to this system was the acceleration of the pipe and the output was the apparent dynamic velocity of the fluid in the pipe as measured by the flowmeter. There are two noise contributions in these calculations. The term \( G_{nx} \) is the average cross power spectrum between the unwanted noise and the desired input. The other term \( G_{nn} \) is the average power spectrum of the unwanted noise alone. The noise is that part of the flowmeter signal due to flow turbulence. When these terms are negligible, \( y^2 \) approaches unity. If enough data samples are used in the average of \( G_{nx} \), and \( x \) does not correlate with \( n \), then \( G_{nx} \) becomes negligible. If the bandwidth of the effective filters in the Fourier transformations are narrow enough, the value of \( G_{nn} \) will be small. As usual in nature, there is a tradeoff. Given the constraint of a set period of time (or a given amount of data), optimizing one condition penalizes the other. For example, in order to get more spectrum samples from a given amount of data, the filter bandwidths must be widened. This causes more noise \( G_{nn} \) in the band occupied by the signal. As is the case in all statistical operations, having more samples produces more valid results. A further discussion of accuracy appears later in this paper. If the coherence function, \( y^2 \), is in excess of 0.8, the noise terms can be considered negligible in the measured cross power functions \( G_{yx} \). Appendix B shows the equations used in determining the flow rate values from the accelerometer and flowmeter signal.

Cross spectrum vs. cross correlation.- The original goal of these tests was to provide a dynamic calibration of a nonintrusive flowmeter. However, during the tests, a problem surfaced which caused a great deal of concern to the eventual users of the flowmeter. The problem was that in order to obtain reasonable calibration results a great deal of data (long time sequences) were required for low dynamic flow rates at low frequencies. The reason for this was that the spectrum of the flow turbulence was concentrated in the low frequencies. The spectra of the flowmeter shown in figures 14 and 15 clearly illustrate the problem. At 30 Hz the signal is separated from the noise while at 7 Hz the noise problem is severe. The requirement of long record lengths would cause problems in the Pogo tests of the Shuttle's propulsion system, since only 10 seconds of dwell time at each frequency had been scheduled.

It was suggested that better results could be realized if cross correlation techniques were used to reduce the data instead of the FFT
procedures. There are two good reasons for not using correlations. First, the best commercially available correlators have less dynamic range than the Fourier analyzer used in the calibration tests. The best correlators have 8-bit digitizers while the Fourier analyzer utilizes 12-bit units. With 12-bit resolution, digitizing errors are negligible, but they can be significant if resolution is only 8 bits. The second reason is that the quality measure given by the coherence function is not available in correlators.

Accuracy vs. analysis time. Since long record lengths would present problems in the Shuttle Pogo tests, it was requested that further investigations be made on the low frequency Langley calibration data. Some indication of the accuracy for various analysis times (record lengths) was desired. It was emphasized at that time and should be again that the amount of analysis time will be dependent on the amount and frequency content of the unwanted noise. This will be completely dependent on the fluid system in which the flowmeter is used. Thus, the answers obtained in the Langley tests would not necessarily apply directly to other systems. Some tests were run, however, and table I and figure 16 illustrate the results.

During discussion with ONERA personnel, they reported using an analysis technique employing a special "windowing" technique to process dynamic flowmeter data before performing spectral analysis. The technique supposedly yielded high accuracies when dealing with noise-laden data. It is suspected that this technique is similar to one which has recently been added to Langley's FFT system. This new capability, called "overlap processing," permits the taking of many more averages than previously possible with the same length of data. The last entry in table I shows that a 15% improvement was realized using the overlap technique.

The statistical error in the estimate of the mean square magnitude when measuring sinusoids in the presence of noise is given by (ref. 6)

\[ e = \left( \frac{G_{nn}}{\frac{V_u^2}{2} T} \right)^{\frac{1}{2}} \]  

(26)

where

- \( e \) = error in the estimate of the mean square value
- \( G_{nn} \) = PSD of the noise at frequencies near the sinusoid
- \( \frac{V_u^2}{2} \) = mean square value of the sinusoidal flow signal
- \( T \) = analysis time

From figure 15 and the data it can be seen that \( V_u^2 \) and \( G_{nn} \) are about the same magnitude. Using the relation for \( e \) the theoretical errors in the rms values, as derived from cross spectral density measurements, are
also plotted in figure 16. The graph shows good agreement between the measurement errors experienced in the tests and the theoretical errors considering the limited amount of data used in the error analysis.

Test Results

Zero steady-state flow.- The experimental dynamic flow data appear in figures 17 through 20. The first tests were performed with zero steady-state flow, considered the simplest from the standpoint of data reduction because of a minimum of hydraulic noise which is characteristic of flow. Although not realistic with respect to the flowmeter's intended Shuttle application, these tests served to build confidence that the oscillating-pipe system was a valid concept for verifying the performance of a fast-response flowmeter. For these zero-flow tests, the flowmeter was oscillated at 0.5, 1, 2, 5, 6, 10, and 15% of an assumed steady flow of 0.61 m/sec and at frequencies of 7, 19, and 30 Hz. Only data at \( \leq 5\% \) and less were plotted for this paper. The flowmeter-measured dynamic velocities were then plotted against the accelerometer-measured velocities in figure 17. The flowmeter readings were reduced to engineering terms using its steady-state calibration factor, and the accelerometer readings according to the method previously described. The test points are seen to cluster closely along a 45° line of perfect agreement. As can be expected, the point scatter becomes more pronounced at a dynamic flow rate of 1% (0.0061 m/sec). Nevertheless, the average error represented by the six data points clustered around the 1% flow modulation point is about 5%; the average error of those data below the 0.5% modulation point is about 8%. These are not unreasonable errors in view of the fact that these zero-flow data were in most part recorded on an oscillograph whose trace deflections were read visually without benefit of the signal processing methods used for all the dynamic data taken under steady-state flow conditions.

Non-zero steady-state flow.- The more important results with respect to the real Shuttle test applications are displayed in the next three figures. Each figure contains the dynamic data taken while dwelling at a specific frequency of oscillation of the ONERA flowmeter at the three selected steady-state flow rates. Again, each graph presents the flowmeter readings in m/sec along the vertical axis plotted against the accelerometer-derived velocity readings along the horizontal.

Figure 18 shows the data taken at the three levels of steady flow, 1.82, 1.22, and 0.61 m/sec while dwelling at the oscillating-pipe frequency of 7 Hz. With the exception of 2 points (0.16% of the 1.82 m/sec steady flow and 4.0% of the 0.61 m/sec steady flow) the flowmeter readings agree quite well with the accelerometer readings over the dynamic velocity range of interest. Considering the dynamic nature of the tests in the presence of flow turbulence and the retrieval of relatively small signals under poor signal-to-noise ratios, particularly at the frequency of 7 Hz, the results appear to be remarkable.

At the higher dwell frequencies of 19 and 30 Hz, noise had less influence on the dynamic flow signals as the aforementioned PSD's showed. Consequently, the data was better behaved as exhibited by figures 19 and 20. In both
graphs, the point scatter reflects errors of <5.0% in the critical range of dynamic flows from 0.5% to 2% of the steady-state flow. Because of the improved SNR, signal processing was achieved with less data sampling time.

The results, it is felt, strongly indicate that the ONERA flowmeter is capable of measuring liquid dynamic flow amplitudes on the order of 0.5 to 10% of the steady flow with uncertainties of <5% of reading. For the smaller amplitudes (0.5 to 1.0% of steady-state flow) where the SNR was extremely poor, up to 100 seconds of signal processing time on the Fourier analyzer was required to retrieve the usable signals with 5% uncertainty. Both the uncertainty of measurement and the signal processing time were reduced sharply with improved SNR data. The results also show that the dynamic flow values can be reduced in a valid fashion using the flowmeter's steady-state flow calibration factor. Furthermore the excellent agreement shown between the flowmeter velocity readings and those derived from the accelerometer readings provides a high degree of confidence in the use of the oscillating pipe system as a valid dynamic flowmeter calibrator.

CONCLUDING REMARKS

A dynamic ultrasonic liquid flowmeter was tested in water and LN$_2$ to determine its precision and repeatability with steady-state flows, and its response to oscillatory flow. The flowmeter (diameter 7.62 cm, length 20.32 cm and range, 4.6 m/sec) was developed by the French Space Agency, Office Nationale d'Etudes et Recherches Aerospatiales (ONERA) for investigating the Pogo phenomenon during launch of liquid-propelled vehicles. After steady-state calibrations (with water at Langley and LN$_2$ at NBS, Boulder, Colorado) the flowmeter was mounted at Langley in an oscillating-pipe system which was specifically developed to generate dynamic-velocity differentials between the flowmeter and the pipe fixed-flows. The dynamic tests were designed to evaluate the flowmeter's potential for measuring flows in ground studies of Shuttle engine-coupled Pogo under pulsed flow conditions.

Test results include those obtained over the flowmeter's designed range (0-6 m/sec) with steady flow calibrations using water and LN$_2$, and the dynamic flow tests with water where in the presence of steady-state flows of 0, 0.61, 1.22, and 1.82 m/sec, small flow oscillations generated by the oscillating pipe system were measured. The oscillations were sustained by a dynamic-shaker, tunable flexbar system at fixed frequencies of 7, 19, and 30 Hz. The flowmeter was oscillated at nominal values of 0.5, 1, 2, 5, 7, 10, and 15% of the steady-state flow rate at each frequency and the flowmeter-measured velocities were plotted against the accelerometer-measured velocities. The flowmeter readings were reduced to engineering terms using its steady-state calibration factor and the dynamic velocity readings were computed from the measured acceleration values.

The test results show the following:

1. That the ONERA flowmeter has the potential for measuring the dynamic flows present during Shuttle Pogo ground tests involving pulsed flows.
2. Flowmeter steady-state calibration exhibits 1% nonlinearity and better than 99% repeatability over the range 0 to 14 meters/sec.

3. Flowmeter has demonstrated its ability to measure dynamic flows whose amplitudes ranged from 0.5% to 2% of the steady-state flows with an uncertainty of 5% of reading; at the higher amplitudes, the uncertainty approaches less than 2%.

4. Results showed that it is valid to use the flowmeter's steady-state calibration factor to reduce the dynamic flow rates.

5. The excellent agreement between the dynamic velocities as represented by the flowmeter's output and those derived from the accelerometer show that the oscillating-pipe system affords a valid technique for evaluating fast-response flowmeters.

6. The fast Fourier transform technique is a valid and powerful method for retrieving small dynamic flow-related signals in extreme SNR conditions.

REFERENCES


APPENDIX A

FFT ANALYSIS

\[ x(t) \rightarrow S_x(f) \rightarrow H(f) \rightarrow y(t) \rightarrow S_y(f) \]

\[ n(t), S_n(f) \]

\[ S = \text{frequency representation} \]
\[ S^* = \text{complex conjugate} \]
\[ H = \text{frequency response function} \]
\[ G = \text{power spectral density} \]
\[ \bar{G} = \text{avg. power spectral density} \]
\[ n = \text{noise signal} \]
\[ x = \text{input signal (accelerometer signal)} \]
\[ y = \text{output signal (flowmeter signal)} \]
\[ F(\ ) = \text{Fourier transform} \]
\[ \gamma^2 = \text{coherence function} \]

1. \[ S_x = F(x) \]
2. \[ S_y = F(y) \]
3. \[ S_n = F(n) \]
4. \[ G_{xx} = S_x S_x^* \]
5. \[ G_{yy} = S_y S_y^* \]
6. \( G_{nn} = S_n S_n \)
7. \( G_{yx} = S_y S_x \)
8. \( G_{nx} = S_n S_x, \ G_{xn} = S_x S_n \)
9. \( H = \frac{S_y}{S_x + S_n} \)
10. \( H = \frac{S_y S_x^*}{S_x S_x^* + S_n S_n^*} \)
11. \( H = \frac{G_{yx}}{G_{xx} + G_{nx}} \)
12. \( \overline{G_{yx}} = H (\overline{G_{xx}} + \overline{G_{nx}}) \)

From (9),
13. \( S_y = H (S_x + S_n) \)
14. \( S_y^* = H^* (S_x^* + S_n^*) \)

From (5),
15. \( \overline{G_{yy}} = |H|^2 (\overline{G_{xx}} + \overline{G_{nx}} + \overline{G_{xn}} + \overline{G_{nn}}) \)
16. \( \gamma^2 = \frac{1}{\overline{G_{xx}} \overline{G_{yy}}} \overline{G_{yx}}^2 \)

Using (12) for \( \overline{G_{yx}} \) and (15) for \( \overline{G_{yy}} \),
17. \( \gamma^2 = \frac{|H|^2 (\overline{G_{xx}} + \overline{G_{nx}})^2}{\overline{G_{xx}} |H|^2 (\overline{G_{xx}} + \overline{G_{nx}} + \overline{G_{xn}} + \overline{G_{nn}})} \)

Dividing numerator and denominator by \( (\overline{G_{xx}})^2 \),

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\[ y^2 = \frac{\left(1 + \frac{G_{nx}}{G_{xx}}\right)^2}{1 + \frac{G_{nx}}{G_{xx}} + \frac{G_{xn}}{G_{xx}} + \frac{G_{nn}}{G_{xx}}} \]

If \( y^2 \neq 1 \), the following is possible

a. \( \overline{G_{nx}} \) and \( \overline{G_{xn}} \) are not negligible due to coherent noise or not enough averages being taken.

b. \( \overline{G_{nn}} \) is not negligible due to FFT filter bandwidth too wide.

c. Errors in the estimates of power spectral terms.
APPENDIX B

CALCULATION OF FLOW RATE FOR PSD's

Accelerometer PSD

\[ \frac{V^2}{2} = \int_{f-\Delta f}^{f+\Delta f} G_{xx} \, df \]

\[ x' = \left( \frac{V^2}{2} \right)^{\frac{1}{2}} \left( M_x \right) \left( \frac{1}{K_x} \right) \]

\( G_{xx} \) = power spectral density of accelerometer signal
\( f \) = frequency of calibration (7, 19, 30 Hz)
\( \Delta f \) = enough of frequency spectrum to pick up energy leakage caused by FFT process
\( M_x \) = accelerometer sensitivity (m/sec^2/volt)
\( K_x \) = accelerometer amplifier gain
\( x' \) = acceleration of pipe (rms) (m/sec^2)

Dynamic flow (rms) = \( \frac{x'}{\omega} \) (m/sec)
\( \omega \) = frequency of calibration (rad/sec)

Flowmeter-accelerometer cross PSD

\[ \frac{V_{yx}^2}{2} = \int_{f-\Delta f}^{f+\Delta f} G_{yx} \, df \]

\[ u' = \left( \frac{V_{yx}^2}{2} \right) \left( \frac{1}{0.707 \, V_x} \right) \left( M_u \right) \left( \frac{1}{K_u} \right) \]

30
$G_{yx}$ = cross power spectral density of accelerometer and flowmeter signals

$0.707 V_x$ = accelerometer voltage (rms) from its PSD

$f & \Delta f$ = same as in accelerometer PSD

$M_u$ = flowmeter sensitivity (m/sec/volt)

$K_u$ = flowmeter amplifier gain

$u'$ = dynamic flow (rms) as calculated from above (m/sec)
FIGURE 3.- BLOCK DIAGRAM OF SIGNAL PROCESSING SYSTEM.
2 kHz

TRANSMIT (CHANNEL t₁, DOWNSTREAM) RECEIVE (t₁)

TRANSMIT (CHANNEL t₂, UPSTREAM) RECEIVE (t₂)

SIGNAL SHAPING (CHANNEL t₁) SIGNAL SHAPING (CHANNEL t₂)

GATES OUT TRANSMIT SIGNAL (EACH CHANNEL)

COUNTING GATE OF RECEIVE SIGNAL (EACH CHANNEL)

COUNTS NO. OF PULSES TO INTEGRATE (EACH CHANNEL) (8 IN THIS CASE)

8 PULSES TO BE INTEGRATED (CHANNEL t₁)

8 PULSES TO BE INTEGRATED (CHANNEL t₂)

CHANNEL t₁ INTEGRATION

CHANNEL t₂ INTEGRATION

DIFFERENCE OF INTEGRALS

FILTERED OUTPUT SIGNAL (ANALOG OF FLUID VELOCITY)

E₀

TIME

FIGURE 4 - FLOWMETER SIGNAL DIAGRAM
FIGURE 5 — SIGNAL DIVISION AND FORMATION OF INTEGRATIVE PULSES (WITH AND WITHOUT FLOW) BY FLOWMETER ELECTRONICS LEADING TO FLOW-PROPORTIONAL ANALOG.
FIGURE 6.- STEADY-STATE FLOW CALIBRATIONS OF ONERA FLOWMETER.
Figure 7.- Graph depicting dynamic flow measurement problem.
FIGURE 8 - CALIBRATION FLOW SYSTEM WITH 7.62 CM DIAMETER PIPE.
Figure 10 - Detail of Flexible Joint in Oscillating Pipe System.

NOTE: Dimensions in centimeters.
FIGURE II - BLOCK DIAGRAM OF TEST INSTRUMENTATION.
Figure 13 - Frequency Response and Coherence Functions

H'(f) [db] vs. Degrees
Figure 14 - PSD plot (30 Hz)

ACCELEROMETER

FLOWMETER, (m/sec)^2

BW .25 Hz
STEADY STATE FLOW 0.61 m/sec
DYNAMIC FLOW 1%
ORIGINAL PAGE IS OF POOR QUALITY
Figure 17 - Dynamic Calibration Curve (Zero Flow)

- Modulating Frequency:
  - 7 Hz
  - 19 Hz
  - 30 Hz
  - Zero Steady State Flow

Flowmeter, m/sec

Accelerometer, m/sec
FREQUENCY 7 Hz

\[ \begin{align*}
\triangle & \text{ VELOCITY 1.82 m/sec} \\
\circ & \text{ VELOCITY 1.22 m/sec} \\
\square & \text{ VELOCITY 0.61 m/sec}
\end{align*} \]

FIGURE 18 - DYNAMIC CALIBRATION CURVE (7 Hz)
FIGURE 20 - DYNAMIC CALIBRATION CURVE (30 Hz)
Tests on a nonintrusive, dynamic flowmeter utilizing ultrasonic wave propagation techniques are described. The flowmeter was developed by the French Space Agency, Office Nationale d'Etudes et Recherches Aerospatiales (ONERA), for investigation of the Pogo phenomenon experienced by liquid propelled vehicles during launch. A novel oscillating pipe system was developed to provide dynamic calibration wherein small sinusoidal signals with amplitudes of 0.5 to 10% of the steady-state flow were added to the steady-state flow by oscillating the flowmeter relative to the fixed pipes in the flow system. Excellent agreement was obtained between the dynamic velocities derived from an accelerometer mounted on the oscillating pipe system and those sensed by the flowmeter at frequencies of 7, 19, and 30 Hz. Also described are the signal processing techniques used to retrieve the small sinusoidal signals which were obscured by the fluid turbulence.