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Microwave Brightness of Polar Firn As Measured By Nimbus 5 and 6 ESMR

A. T. C. Chang, B. J. Choudhury, and P. Gloersen

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Abstract

The microwave emission from a half-space medium characterized by coordinate dependent scattering and absorbing centers has been calculated by numerically solving the radiative transfer equation by the method of invariant imbedding. A Mie scattering phase function and surface polarization have been included in the calculation. Also included are the physical temperature profile and the temperature variation of the index of refraction for ice. Using published values of grain size and temperature profile data of polar firn, the brightness temperature has been calculated for the 1.55 cm and 0.8 cm wavelengths. For selected regions in Greenland and Antarctica, the results of our calculations are in reasonable agreement with the observed Nimbus-5 and Nimbus-6 ESMR data.
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Introduction

Brightness temperatures obtained from the Electrically Scanning Microwave Radiometers on board the Nimbus-5 and Nimbus-6 spacecrafts (ESMR-5, operating at a wavelength of 1.55 cm and ESMR-6, at 0.8 cm, respectively) over Greenland and Antarctica (Gloersen and others, 1974, and this paper) have shown a lack of correlation with the physical surface temperature. Model studies of microwave emission from a half-space with scattering and absorbing centers were carried out by England (1974) and by Chang and others (1976). They calculated the brightness temperature for a model snow field consisting of randomly spaced ice spheres of different radii and dielectric properties. The scattering and extinction cross-sections were calculated using the Mie-scattering theory and the brightness temperature values were then obtained by numerically solving the radiative transfer equation. Results of the calculation showed that the emerging microwave radiation originates deep within the medium. These results not only provided an explanation for the lack of correlation of the observed brightness temperatures and the physical surface temperatures but also
opened up the possibility of remotely sensing such parameters as the snow accumulation rate and snow temperature profile.

In the model calculations performed by Chang and others (1976) and by England (1974 and 1975), the scattering and absorption properties have been assumed to be independent of the snow depth. This is in contrast to the actual situation where the crystal size does vary with the snow depth (Gow, 1969 and 1971). Also in the former work the physical temperature of snow has been taken to be independent of depth and in the latter calculation a linear temperature variation has been used. The accuracy of the numerical results obtained by England (1975) is difficult to assess because of the convergence problem associated with his method of solution (England, 1974). By using approximate, integrable analytic expressions for the radiative transfer equation, Zwally (1977) has obtained microwave brightness temperatures for polar ice sheet, taking into account the grain size and temperature profile variations in season and location. However, in order to obtain agreement between theory and experiment, he found it necessary to lower the calculated value of scattering coefficient by an order of magnitude. Here we have succeeded in providing an explanation for the radio brightness temperatures observed over Greenland and Antarctica with the ESMR-5 and ESMR-6 without the need for such an adjustment to the scattering coefficient. The expression for the variation of snow grain size with depth and location used in the calculation are those compiled by Zwally (1977) by fitting the crystal size data measured by Gow (1969, and 1971) at different snow depths. The radiative transfer equation (equation (1)), has been solved numerically using these snow parameters by the method of invariant imbedding (Chandrasekhar (1960), Redheffer (1962), Bellman and others (1963), Preisendrager (1965) and Grant and Hunt (1969)).
NUMERICAL SOLUTION FOR THE RADIATIVE TRANSFER EQUATION

The radiative transfer equation for an axially symmetric inhomogeneous medium in which all interactions are linear can be written in the form of an integro-differential equation (Grant and Hunt, 1969)

\[ \mu \frac{dI(x, \mu)}{dx} = -\sigma(x) \ I(x, \mu) + \sigma(x) \left\{ [1 - \omega(x)] \ B(x) \right\} \]

\[ + \frac{1}{2} \omega(x) \int_{-1}^{1} p(x, \mu, \mu') I(x, \mu') \ d\mu' \]  \hspace{1cm} (1)

where the radiation intensity \( I(x, \mu) \) is at depth \( x \) traveling in the direction making an angle whose cosine is \( \mu \) with the normal toward the direction of increasing \( x \) (Figure 1).

The functions \( \sigma(x) \), \( B(x) \), and \( p(x, \mu, \mu') \) are prescribed functions of their arguments. They are referred to as the extinction per unit length, the single scattering albedo, the source, and the phase function, respectively. For a nonuniform medium these functions are generally piecewise continuous functions of depth subject to the conditions

\[ B(x) \geq 0, \sigma(x) \geq 0, 0 \leq \omega(x) \leq 1, \ p(x, \mu, \mu') \geq 0. \] \hspace{1cm} (2)

In the present work, the following normalization for the phase function will be used

\[ \frac{1}{2} \int_{-1}^{1} p(x, \mu, \mu') \ d\mu' = 1 \] \hspace{1cm} (3)

for all values of \( x \).
Instead of working with depth $x$, one generally works with a dimensionless depth variable called optical depth $\tau$, defined in differential form as

$$d\tau = \sigma(x) \, dx.$$  \hfill (4)

In terms of optical depth, equation (1) reduces to

$$\mu \frac{dI(\tau, \mu)}{d\tau} = -I(\tau, \mu) + [1 - \omega(x)] B(x)$$

$$+ \frac{1}{2} \omega(\tau) \int_{-1}^{1} p(\tau, \mu, \mu') \, d\mu \, I(\tau, \mu').$$  \hfill (5)

This equation of radiative transfer was solved numerically by the invariant imbedding technique (Grant and Hunt, 1969). Then by using the proper scattering phase function and the boundary conditions, the brightness temperature emerging from the snow field can be calculated.

**BRIGHTNESS TEMPERATURE OF THE POLAR FIRN**

The calculation of the brightness temperature of the polar firn requires the specification of a single scattering albedo $\omega(\tau)$, physical temperature within the medium $B(\tau)$, the scattering phase matrix $p(\tau, \mu, \mu')$, and the reflection coefficient at the medium boundary. In the single particle scattering approach, the albedo can be calculated from the knowledge of the scattering and the extinction coefficients of the particle. These coefficients depend upon the radius of the particle and its refractive index. For ice particles, with which we are concerned, the specification of size is sufficient to determine the scattering coefficient. The extinction coefficient of the particle on the other hand depends on the imaginary part of the index of refraction which is less sensitive to the crystal size. Calculations of Chang and others (1976) show that for ice at 273 K the extinction
coefficient is about an order of magnitude higher than for ice at 253 K. Although this sensitivity can be used to distinguish temperature profiles, it also introduces some uncertainty in quantitative comparison with remotely measured brightness temperature. It is also to be noted that the imaginary part of the index of refraction changes considerably with the presence of impurities (Hoekstra and Cappillino, 1971). For continental shelf ice the effect of impurities may be negligible.

Zwally (1977) has performed a regression study of the crystal size data for different snow depth obtained by Gow (1969 and 1971) at various locations in the Greenland and Antarctica. In Table 1 we showed the crystal size profiles, for which we have calculated the brightness temperature.

<table>
<thead>
<tr>
<th>Location</th>
<th>( T_m ) (K)</th>
<th>( r ) (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Byrd, Antarctica</td>
<td>245</td>
<td>((0.026 + 0.00166z)^{1/3})</td>
</tr>
<tr>
<td>79°59'S 120°01'W</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Camp Century, Greenland</td>
<td>249</td>
<td>((0.028 + 0.0111z)^{1/3})</td>
</tr>
<tr>
<td>77°11'N 61°10'W</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Inge Lehmann, Greenland</td>
<td>243</td>
<td>((0.028 + 0.0202z)^{1/3})</td>
</tr>
<tr>
<td>77°57'N 39°11'W</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Mean annual surface temperature

\( z \) is snow depth in meters.
We have mentioned that absorption per unit length is quite insensitive to the crystal size and depends largely on the imaginary part of the refractive index \( n'' \). Although the choice of \( n'' \) is not completely arbitrary, its exact value is difficult to obtain. In this study we have developed an interpolation/extrapolation algorithm to determine the index of refraction of ice from experimental data (Figure 7, Evans, 1965). Table 2 showed the experimental data for \( n'' \) and the interpolated and extrapolated values for 1.55 cm and 0.8 cm wavelengths. The interpolated values for 3.2 cm differs from the measured values reported by Cumming (1952). These differences probably caused by structure of different ice types tested.

The single scattering albedo is defined as

\[
\omega = \frac{\gamma_s}{\gamma_a + \gamma_s}
\]  

(6)

Therefore, the depth variation of \( \omega \) depends on the depth variation of \( \gamma_s \) and \( \gamma_a \). Since \( \gamma_a \) varies with the physical temperature (Cumming, 1952) of the snow, an analytic expression for depth and seasonal variation of polar snow temperature has been used (Zwally, 1977).

\[
B(z, t) = 250 - 15 e^{-0.3z} \cos \left[ 0.99(t - 84) - (97 + 20z) \right]
\]  

(7)

The maximum surface temperature occurs at time \( t = 0 \) and \( t = 365 \) and minimum temperature is for time \( t = 365/2 \). The mean temperature of the surface and the asymptotic temperature is 250 K and the peak to peak variation at the surface is 30 K. This mean temperature is quite acceptable for the locations given in Table 1.
Table 2

The Imaginary Part Index of Refraction for Ice at Different Wavelengths and Temperatures

<table>
<thead>
<tr>
<th>Temperature (K)</th>
<th>Wavelength (cm)</th>
<th>Experimental</th>
<th>Interpolated</th>
<th>Extrapolated</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>30</td>
<td>10</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>273</td>
<td>0.00037</td>
<td>0.00042</td>
<td>0.00052</td>
<td>0.00023</td>
</tr>
<tr>
<td>253</td>
<td>0.00015</td>
<td>0.00023</td>
<td>0.00029</td>
<td>0.00019</td>
</tr>
<tr>
<td>233</td>
<td>0.00010</td>
<td>0.00017</td>
<td>0.00018</td>
<td>0.00014</td>
</tr>
<tr>
<td>213</td>
<td>0.00004</td>
<td>0.00012</td>
<td>0.00009</td>
<td>0.00012</td>
</tr>
</tbody>
</table>
The correct value of \( n'' \) is crucial because the absorption coefficient \( (\gamma_a) \) depends on this value. To illustrate the effect of different values of \( n'' \), a single calculation has been performed. The Camp Century, Greenland site was chosen for the comparison. Figure 2 showed the variation of vertically polarized brightness temperature by fixing the \( n'' \) in each calculation, while the other parameters were remained the same as in the other calculations.

Calculated brightness temperature values for several locations and time are given in Table 3. The agreement between the calculated and the observed ESMR-5 values are considered quite good. The r.m.s. deviation between the calculated brightness temperature and the observed brightness temperature for ESMR-5 is 8.4K. The agreement between the calculated brightness temperature and the observed ESMR-6 values is not as good as the ESMR-5, with r.m.s. deviation of 14.5K and 20.4K for ESMR-6 vertical and horizontal polarization respectively. The differences in r.m.s. deviation for the calculated ESMR-5 and ESMR-6 values are attributable at least in part to the following factors: (1) the observed seasonal variations in the brightness temperature larger than those calculated indicate that the microwave emission for 0.8 cm wavelength largely emanates from a thin surface layer and thus the values assumed for \( n'' \) at this wavelength were too low. As a result, the calculated extinction coefficient is too small, resulting in too large a penetration depth, (2) sensitivity to near surface moisture due to solar heating of snow in the presence of below freezing air temperature, and (3) the inherent difficulties in the calibration of ESMR-6 due to the spacecraft thermal conditions.

SUMMARY AND CONCLUSIONS

A microscopic single particle scattering model has been used along with measured crystal size and temperature variation with depth to provide a quantitative explanation of observed brightness temperatures of the South Polar and North Polar regions. This extends the calculations performed by Chang and others (1976) by taking into account the variations of grain size and temperature
Figure 2. Dependence of 0.8 cm Vertically Polarized Brightness Temperature on $n''$, the Imaginary Part of Index of Refraction.
### Table 3
Comparison of Observed and Calculated Brightness Temperatures of Locations in Table 1

<table>
<thead>
<tr>
<th>Location</th>
<th>t*</th>
<th>Nimbus-5 ESMR</th>
<th></th>
<th>Nimbus-6 ESMR</th>
<th></th>
<th>Nimbus-6 ESMR</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Calculated</td>
<td>Observation</td>
<td>Calculated</td>
<td>Observation</td>
<td>Calculated</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Byrd, Antarctica</td>
<td>0</td>
<td>220</td>
<td>210</td>
<td>210</td>
<td>226</td>
<td>174</td>
</tr>
<tr>
<td></td>
<td>365/4</td>
<td>215</td>
<td>205</td>
<td>204</td>
<td>196</td>
<td>170</td>
</tr>
<tr>
<td></td>
<td>365/2</td>
<td>202</td>
<td>200</td>
<td>192</td>
<td>197</td>
<td>160</td>
</tr>
<tr>
<td>Camp Century, Greenland</td>
<td>0</td>
<td>232</td>
<td>225</td>
<td>214</td>
<td>236</td>
<td>175</td>
</tr>
<tr>
<td></td>
<td>365/4</td>
<td>224</td>
<td>215</td>
<td>207</td>
<td>213</td>
<td>173</td>
</tr>
<tr>
<td></td>
<td>365/2</td>
<td>212</td>
<td>210</td>
<td>195</td>
<td>206</td>
<td>163</td>
</tr>
<tr>
<td>Inge Lehmann, Greenland</td>
<td>0</td>
<td>215</td>
<td>225</td>
<td>210</td>
<td>203</td>
<td>175</td>
</tr>
<tr>
<td></td>
<td>365/4</td>
<td>209</td>
<td>209</td>
<td>204</td>
<td>180</td>
<td>170</td>
</tr>
<tr>
<td></td>
<td>365/2</td>
<td>196</td>
<td>210</td>
<td>192</td>
<td>175</td>
<td>162</td>
</tr>
</tbody>
</table>

t* is the time parameter as shown in equation (7).
with depth. The calculated brightness temperatures for 1.55 cm wavelength are in good agreement with the observations, but one of the calculated brightness temperatures for 0.8 cm wavelength differs by 32 K with the observations. Probable cause of these discrepancies are: (1) uncertainty in the calibration of ESMR-6 instrument and (2) uncertainty in the values of n". In some areas of Greenland, percolation is known to occur which accompanied by large change in n". Since we felt sufficiently accurate values of n" in this region are not available, we did not consider these areas in our comparisons.

Based on the single particle scattering model, the calculated brightness temperatures correspond to the data obtained from the ESMR-5 and ESMR-6 over selected sites in Greenland and Antarctica. Using essentially the same temperature and grain size profile that were used by Zwally (1977), we find that our calculated 1.55 cm wavelength brightness temperatures are significantly higher than his. This discrepancy is probably due to the approximate analytic solution of the radiative transfer equation used in his calculations of the brightness temperature values. In view of these findings, our calculations would provide a more realistic brightness temperature estimate for the polar firn.

Apart from the independent particle scattering model discussed above, there are alternate explanations for the source of microwave scattering and absorption for radio brightness temperatures of Antarctica and continental glaciers which have been studied by Gurvich (1973), Stogryn (1974) and Tsang and Kong (1976). In these studies, the source of microwave scattering is the fluctuation of the dielectric constant of the media (Tatarskii, 1961). A special case of such fluctuations occurring where freeze/thaw cycles have taken place, accompanied by percolation of melt water and formation of subsurface ice lenses, has been identified as important particularly at the longer wavelengths where single particle scattering is of lesser importance (Gudmundsen, 1978). The brightness temperature calculated, based on these models, depends upon the variance and the correlation length of the fluctuation of the dielectric media. Physically this
scattering mechanism is as plausible as scattering by independent ice grains, and it is reasonable to believe that these two scattering mechanisms co-exist in the medium and should be considered together in the quantitative calculation.
REFERENCES


Zwally, H. J., "Microwave Emission and Accumulation Rate of Polar Firn,"

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Polar Firn

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